DISCUSSION OF "THE COMPETITIVE MARKET EQUILIBRIUM RISK LOAD FORMULA FOR INCREASED LIMITS RATEMAKING," BY GLENN MEYERS

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"The Competitive Market Equilibrium Risk

Load Formula for Increased Limits Ratemaking"

Yet another fine paper by Glenn Meyers will appear in PCAS 1991. In it Glenn derives a formula for risk loadings per (expected) occurrence; the risk loading R is proportional to the (partial) derivitative of the Variance with respect to the number of occurrences n.

$$R \propto \frac{\partial \text{ Variance}}{\partial n}$$

This result follows directly from his assumption that each insurer will maximize its collected risk load subject to a constraint on the total variance of its book of business.[1]

Maximize Rn subject to the constraint:

$$\text{Variance} = A^2$$

The solution[2] via the method of Lagrange multipliers is:

$$\frac{\partial (Rn - \lambda \text{Variance} + \lambda A^2)}{\partial n} = 0$$

$$R = \lambda \frac{\partial \text{ Variance}}{\partial n}$$

$$R \propto \frac{\partial \text{ Variance}}{\partial n}$$

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[1] If some other type of constraint were chosen which depended on something other than the total variance (or standard deviation), a different formula for the risk load would follow.

[2] The author shows that one can average the (expected) number of occurrences n over the insurers writing the given line/limit combination. The author refers to this average as $\bar{n}$.

The author breaks the variance into two pieces, process risk and parameter risk. As is usual, the process risk varies with n, while the parameter risk varies with n^2. In the author's matrix notation:

$$\text{Variance} = n^T U + n^T V n$$

The first term quantifies process risk, while the second term quantifies parameter risk. Therefore,

$$\frac{\partial \text{ Variance}}{\partial n} = U + 2Vn$$

Therefore,

$$R \propto U + 2Vn$$

which is the author's equation 5.6.

It should be noted that this differs from the usual variance based risk loadings. First, it considers parameter variance as well as process variance. Second, the parameter variance enters multiplied by n, a measure of size.[3,4] One still has to select the proportionality constant for the risk load. The author suggests looking at average risk loads in the insurance market.

The ideas in this paper are being applied by Glenn in the calculation of I.S.O Increased Limits Factors. This has stirred up some controversy, which was discussed at the March 1992 CAS Seminar on Ratemaking.

In any case, this paper is a very significant step forward in the theory of risk loads.

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[3]These two features are analogous to those found in the computation of credibilities. See for example, the discussion of parameter uncertainty in H.C. Mahler, Discussion of G.G. Meyers, "An Analysis of Experience Rating", PCAS LXXIII, 1987.

[4]The proposed risk loading does not depend on the size of the particular insurer. (It does depend on the average amount of the particular line/limit combination written by all insurers.)