REVIEW OF "THE MATHEMATICS OF EXCESS OF LOSS COVERAGES AND RETROSPECTIVE RATING – A GRAPHICAL APPROACH", PCAS, 1988

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Introduction

The first exposure that many children have to numbers in school is through the use of a number line, or a picture displaying bundles of sticks. In high school the Pythagorean theorem is often proven by comparing the areas of the triangle to a surrounding rectangle. While teaching calculus, many instructors depict areas and volumes through involved diagrams. But, later courses in advanced mathematics often lead to more obscure illustrations or more often no illustration at all. I would like to thank the author for reminding us of the value of pictures. This review contains a description of how pictures were used to produce a practical solution to an insurance problem.

The Setting

As the allocated loss adjustment expense (ALAE) portion of the premium dollar has become much more significant recently, new variations of defense options have arisen. Defense costs are as major a concern as loss costs in many of the general liability lines for the insurer and insured alike. The traditional policy where defense costs are supplemental to the policy limit is more or less priced by loading the unlimited average defense costs into the basic limits rate. Supplemental defense costs are paid by the insurer in full and do not erode the policy limit. With new policy options whereby defense may be included within the limit, and more frequent use of self insured retentions (SIR's) with liability policies, it becomes necessary to modify the ratemaking techniques associated with defense costs. When defense costs are included in the limit they are usually combined with the loss dollars before the policy limit is applied to determine the insurer's liability.

Ideally one would like to model the joint distribution of loss and defense costs and use this model to estimate any costs of factors associated with different policy defense options. Perhaps the simplest fashion in which one may try to include defense costs is either as a flat percentage of loss or a flat dollar amount per loss. The following solution is somewhat of a middle ground between these two extremes.

The setting under which the following solution arose is as follows. A software package that had already encoded a loss distribution and readily calculated limited expected losses was available. Further, this package allowed one to manipulate the parameters in order to account for inflation. The package had some other features that were useful and would have been rather tedious to program. Time and money were constrained in such a way as to make the calculation of the ideal joint distribution infeasible. It was decided to use the software package and alter it in such a fashion as to hopefully reflect reality with respect to the joint distribution.

This alteration took the form of a combination of a fixed dollar piece of allocated expense and a piece that is a fixed percentage of the individual loss. The fixed percentage piece will be referred to as the variable piece of the defense cost.

ALAE = Fixed plus Variable

Intuitively one may view this combination as fitting the small claims mostly through the fixed piece and the large claims through the variable piece. If only a percentage of loss were used to estimate ALAE, many smaller claims that incur ALAE as a larger percentage of loss would be incorrectly represented. As an extreme example consider a claim that settles for $1. If an ALAE to loss ratio of 40% is used then this would suggest that 40 cents covers the ALAE. It seems more reasonable to assume these smaller claims incur some fixed costs. For the larger claims the variable portion may become the dominant portion of the ALAE estimate. If the fixed piece of the average unlimited ALAE is $16,000 and the variable piece is 8% of loss then a two million dollar claim would incur $16,000 plus $160,000 of ALAE.

The choice of the amounts of the fixed and variable pieces was solved in another expeditious manner. The software package contained an expected unlimited ALAE amount that was judged to be reasonable for use. To apportion this estimate into fixed and variable pieces a simple linear regression was performed on a file of individual closed claim and ALAE amounts. The dependent variable was the ALAE amount and the independent variable was the loss amount.

$$ALAE = a + b * Loss$$

The fixed portion was determined as "a" divided by the average ALAE of the closed claim and ALAE file. The complement of this was the variable portion. For example, if the constant is $14,400 and the average ALAE is $18,000, the fixed portion is 80% and the variable portion is 20%. It is interesting to note that if representing the ALAE as entirely fixed or entirely variable was truly "better", in the least squares sense of the word, than a mix of the two, one of the fitted parameters of the regression would have been close to zero. This was not the case.

Let us return to the software package with the allocated 80/20 split in hand. Assume that the unlimited expected ALAE from the software package is $20,000 and the unlimited expected loss from the software package is $50,000. The fixed part of the ALAE is $16,000. The variable part as a percentage of loss is 8%. Using the inflation adjusting capabilities of the package, the distribution was simply increased 8% to account for the variable piece of the ALAE.

Graph 1 displays the cumulative distribution function (CDF) for loss and the CDF for loss and variable allocated. The latter distribution is the same the loss distribution adjusted for inflation.

Graph 2 incorporates the fixed defense costs. Note the area marked "variable defense" is the average variable defense cost, $4,000. The area marked "fixed defense" is the average fixed defense cost, $16,000. The area marked "loss" is the average loss cost, $50,000. Graph 2 is essentially graph 1 placed atop the fixed costs.

For clarity, names are assigned to three of the four random variables whose distributions are illustrated in graphs 1 and 2. Let X be the random variable of loss size only, the lower function in graph 1. Let Y be the random variable for loss plus variable defense, the upper function in graph 1. Let Z be the random variable for loss plus all defense, both fixed and variable, the upper function in graph 2. Note in the example Y = 1.08X and Z = 1.08X + 16000.

Suppose we want a rate for a policy with a limit of $100,000 per occurrence with defense included in the limit. Using the notation of Hogg and Klugman and ignoring risk loads and ULAE, if the rate for a basic limits policy with a limit of $25,000 with defense costs supplemental to the policy limit is $5, then the rate for the first policy is:

$$\frac{5 * E[Z;100{,}000]}{E[X;25{,}000] + 16{,}000 + 4{,}000}$$

Graph 3 depicts E[Z;P] for some P as the heavily shaded area under the horizontal at P plus the lightly shaded rectangle representing the average fixed expense (fd). Graph 4 depicts E[Y;P-fd]. It is readily apparent that E[Z;P] = E[Y;P-fd] + fd. The software package readily calculates limited expected values for Y, hence for Z. The fact that E[Y;P-fd] = 1.08 * E[X;(P-fd)/1.08] could have been used if the package was not able to model Y so readily.

As one last illustration suppose we want a rate for a policy with a limit of $1 million per occurrence with defense included excess of a SIR of $50,000 per occurrence with defense included. The rate for this policy is:

$$\frac{5 * (E[Z;1{,}050{,}000] - E[Z;50{,}000])}{E[X;25{,}000] + 16{,}000 + 4{,}000}$$

Which is now readily calculable.

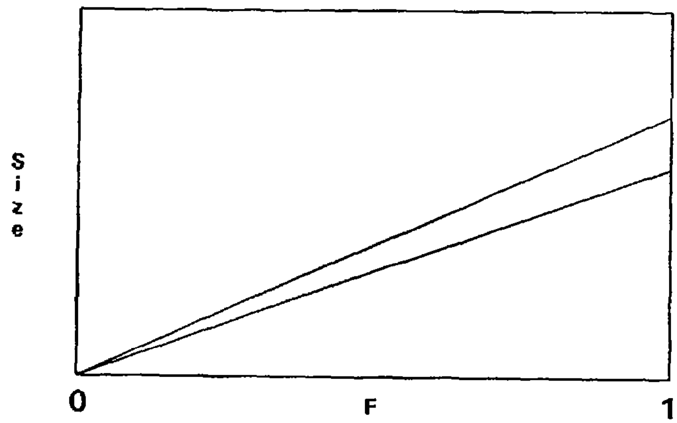
Considerations

There were several considerations that arose while this procedure was being devised. Most fundamental of them all was the nature of the ALAE separation. A line necessarily implies a decreasing percentage of ALAE to loss. The closed claim file used was the subject of several questions concerning maturity and policy limits contained. Finally, the software package had some distributional implications that had to be thought through. These conceptual problems were wrestled with and accounted for where possible and necessary. The determining criteria was reasonableness.

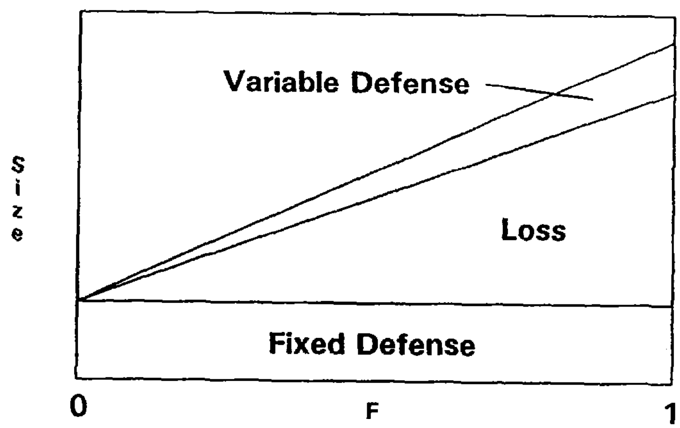
Conclusion

Pictures are a very useful tool that the actuary should keep ready in his or her toolbox. The concepts conveyed through a picture are often so much simpler to grasp than the sometimes tedious algebra that accompanies them. Understanding the concept often makes the algebra that much more palatable. I welcome any tool that aids in my understanding. Once again, I thank the author for reminding me of the usefulness of pictures.

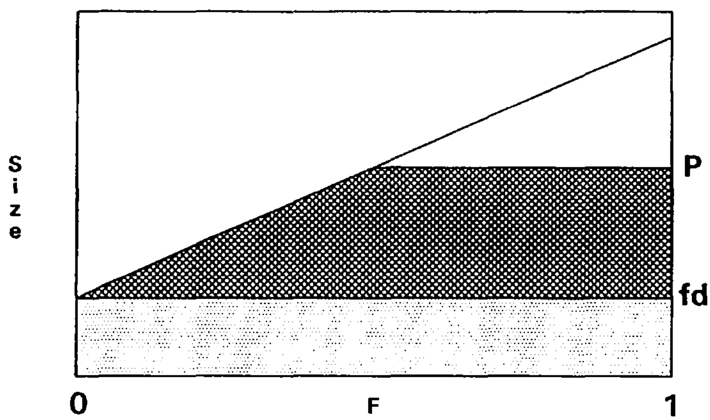
Graph 1
CDF for Loss and Loss + Var ALAE



Graph 2
CDF for L + All ALAE and L + Fixed ALAE



Graph 3
CDF for Loss plus all ALAE



Graph 4
CDF for Loss plus Variable ALAE