CARe RESEARCH COMMITTEE – MINUTES FOR MEETING OF AUGUST 23, 1990 – THE REVISIONS TO ISO's INCREASED LIMITS PROCEDURE

CARe Research Committee

CARE RESEARCH COMMITTEE
MINUTES FOR THE MEETING OF AUGUST 23, 1990

Attendee	Company
Bear, Robert	North Star Reinsurance
Cellars, Ralph	North American Reinsurance
Gaydos, Eugene	ISO
Francis, Louise	Tillinghast
Giambo, Robert	Trenwick Reinsurance
Grady, David	Prudential Reinsurance
Handte, Malcolm	Zurich Reinsurance
Hughes, Brian	Skandia America Group
Iafrate, Anthony	General Reinsurance
Krakowski, Israel	CNA Insurance
Licht, Peter	ISO
Mahon, John B.	American Reinsurance
Mashitz, Isaac	North American Reinsurance
Meyers, Glenn	ISO
Moller, Karl	Home Insurance
Newville, Benjamin S.	U.S. International Reinsurance
Norton, Jonathan	Guy Carpenter
Patrik, Gary	North American Reinsurance
Robbins, Ira	CIGNA Insurance
Speigler, David	American Reinsurance
Weissner, Edward	Prudential Reinsurance

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INTRODUCTION:

Gary Patrik introduced the meeting, outlined the topic and presented the agenda (Attachment 1).

SHORT-TERM CHANGES:

ISO distributed a handout entitled Pilot Increased Limits Ratemaking Procedure (Attachment 2). Glenn Meyers outlined the current ISO ILF procedure and noted the more significant changes which will be made.

1. Four Parameter Mixed Pareto Distribution:

Introduction: The intent of ISO is to use a mixed distribution fit to settled claims (paid claims) to estimate the severity distribution underlying the ILF's.

Using a mixed distribution would eliminate the problem of selecting a truncation point T. It has been shown that the selection of T under the current procedure can significantly affect the ILF's, particularly at higher policy limits (Attachment 2 Page 4). With the mixed distribution, the selection of the mixing parameter p is estimated through the maximum likelihood estimation process.

Using settled claims (paid claims) eliminates the current incurred claim development procedure (Attachment 2 Page 3). Mixed distributions tested by ISO fit equally well for settled data as for incurred data.

However for 1991, ISO does not expect to have this procedure in place. Instead ISO intends to use incurred loss data (indemnity occurrences), with the current development procedure, to fit a mixed Pareto distribution (Attachment 2 Page 5) for Commercial Auto, Premises/Operations and Products/Completed Operations.

Discussion: ISO is proposing a mixed Pareto distribution i.e. two different Paretos F(x:b1,q) and F(x:b2, q+2) with the mixing parameter p. If the idea is that small claims have a less severe distribution, then why use two Paretos? Why not use one distribution with a less severe tail? As an example, why not use an

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Exponential and a Pareto? Why are the shape parameters of the Pareto distribution q and q+2?

ISO tried various other pairs of distribution on Products Liability settlement data, e.g. Exponential/Pareto and Pareto/Pareto with shape parameter pairs q,q+1; q,q+3; and q,q+4. ISO's conclusion was that the proposed mixed Pareto distribution resulted in the best fit. ISO noted however that they have not finalized their decision and that testing is still being done. ISO intends to fit the mixed Pareto distribution to all lines of business, not just Products, and test the results before any ILF's will be published using this model. ISO also encourages others to try different models. It was noted that similar type of fitting is being tried at Wharton and that ISO is not aware of any better results.

Did ISO try using distributions with more than two parameters? Yes, but the results were not satisfactory.

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How sensitive is the fit of large claims to the selected fit on the small claims? Because of the large volume of small claims, it is not difficult to get a model to fit well for smaller claims, but how well does the model fit for larger claims? How many claims are there above $1 million?

Because the mixing parameter is estimated from the maximum likelihood estimation, the fit for large claims should not be unduly affected by the fit to small claims.

By graphing the two Pareto distributions and noting the intersection of the curves, an intuitive judgement as to the correct "split" of the distributions can be made.

Because there is not much data in the ISO data base above $1 million, the fit to large claims is somewhat an extrapolation process. It is believed that significant large claim data exists in other lines, such as D&O liability (data outside the ISO data base)

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Professional Liability, and that the model should be tested on such lines.

Under the current ISO ILF procedure, there is a problem with the truncation point drastically changing from review to review. Is it possible that the mixing parameter will drastically change from review to review?

The mixing parameter is expected to be stable from review to review. For each accident year (currently using accident years 1973-1986) at any evaluation age the same shape parameter q (and consequently q+2) will be used to fit the data. The scale parameter b is expected to increase by accident year and will be investigated for trend. The mixing parameter will be required to be the same for each accident year.

Further the number of accident years used to fit the mixed Pareto will be stable. Currently fourteen (14) accident years are used. Subsequent reviews will add additional accident years while dropping a minimal number of the oldest accident years (possible none).

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Consequently the parameter constraints combined with a stable data base should result in stable mixing parameters from review to review.

What type of statistical testing is being done to judge the fitted distribution?

General statistical tests such as Kolmogorov Smirnov or Chi- square tests do not work well on insurance data. ISO uses a set of diagnostic tests including a comparison of limited average severity (LAS) to judge the goodness of fit.

2. Risk Load:

Introduction: Originally ISO used a variance based risk load in the ILF's. This resulted in too large a risk load for higher limits with consequential inconsistencies between limits. ISO changed and is currently using a standard deviation risk load. This has resulted in apparent inconsistencies in risk load between lines of business and/or ILF tables within a line.

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ISO is proposing a Commercial Market Equilibrium Risk Load (CMERL) procedure which incorporates both process risk and parameter risk.

Discussion: Two views emerged concerning CMERL. One view is that although there are problems with the variance and standard deviation based risk loads, it is clear how these risk loads are being calculated and what they measure. It is not clear what CMERL is. The correct risk load needs to be defined and estimated to measure how far CMERL differs from it.

Small insurance companies will use the ILF's blindly, so the best estimate of the correct risk load should be used.

Furthermore, ISO previously tried to build a model of the insurance market. It is a very difficult task and the model did not fit well. Why does ISO think it can build a better model now?

The alternative view is that no one knows what the correct risk load is, but ISO is moving in the right direction. That is, risk load is market driven.

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In spite of this affirmation of CMERL, some concerns with the ISO model are that it does not include the effects of the reinsurance market, the flow of capital in and out of the industry, insurance transaction costs, or investment income.

Conclusion: Even in light of ISO's decision to move away from providing rates to providing loss costs, ISO still intends to provide ILF's with risk load. That eventually will mean CMERL.

ISO also proposes to provide computer software to allow companies to compute ILF's with risk load based on the company's own selected parameters.

3. Composite Rated Risks/U.E.C.F.

Introduction: Composite Rated Risk (CRR) claims cannot be identified by class code, so CRR claims cannot be matched to ILF table, for example Premises/Operations Table 1, 2 or 3. Hence severity distributions for Tables 1, 2 or 3 do not include CRR experience. The Uniform Excess Change Factor (U.E.C.F.) is selected to reflect the effect of CRR claims on the ILF tables, by

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comparing severities fit separately to all Tables with and without CRR claims. The U.E.C.F. is the same for each table within a subline. While this results in ILF's which reflect CRR data, the underlying severity distribution for the tables do not. There was strong argument that the U.E.C.F. method be eliminated and that the final ISO ILF tables should each be based upon an underlying probability distribution for claim severity.

ISO intends to change the procedure it uses on CRR claims to produce severity distributions by table, which reflect CRR claims.

LONG-TERM CHANGES:

1. Pareto Soup Model:

Introduction: ISO gave a handout (Attachment 3) which depicted a Pareto Soup model with 43 parameters. This model is typical of other Pareto Soup models.

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In this example, nine different four parameter mixed Pareto distributions are fit to accident year 1974 paid claims at settlement lags 1 through 9. Trends (S, T1, T2, T3, T4 and T5) are used to adjust the nine mixed Paretos to fit different accident year settlement lag cells.

The parameters for the mixed Pareto distributions, the trends and the mixing parameters are all simultaneously estimated via maximum likelihood techniques.

Discussion: It is difficult to comprehend a model with 43 or more parameters. It is important that the parameters satisfy intuitive opinions on how they should behave. It is especially important that the asymptotic behavior of the patterns be checked as settlement lags increase.

In the example given for AY 1974 it is not intuitively clear why the trend parameter S=0.8865 is less than 1.00 (Attachment 3 Page 2), nor why the mixing parameter P(J) does not decrease to zero as the

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settlement lag increases. For longer settlement lags, small settled claims should have less effect.

ISO is currently investigating a technique to treat the Q(J) parameters as a function of the settlement lag which would require the Q's to decrease with increasing lag. Possibly a similar approach could be used on the P(J) parameters. The intuitive progression of the B(J) parameters is not as easily identified because each B(J) is associated with a different Q(J) parameter.

Once the various Pareto distributions are estimated, how can they be combined into one distribution? Using a settlement distribution W(J), the various mixed Paretos are weighed together by the proportion of occurrences in each settlement period.

Isn't the settlement distribution effected by partial payments? It probably has a minor effect. In fact, the settlement distribution is fit to average per occurrence settlement dates and not actual settlement dates.

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How is the model tested for settlement lags of 30 years? In the example given, the B2(30) parameter trends to 145. Is this reasonable? ISO is developing a set of diagnostic tests, including diagnostics based on incurred loss, to be used in testing the Pareto Soup model particularly for long settlement lags. The reasonableness of these diagnostics will strongly impact the final model selected.

It is expected that a model with a large number of parameters should result in a good model. How much predictive improvement is gained by a model with such a large number of parameters? Can the model be reduced to a simpler format for others to use?

Parsimony is a nice objective, but ISO has a lot of data so even when the data is subdivided into many accident year settlement lag cells there is still sufficient data in each cell to get good fits. The final model can be described as in the example by a matrix of parameters (Attachment 3 Page 2) which can then be used by others.

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The Pareto Soup model doesn't reflect policy limits. Isn't there a correlation between the size of loss and the size of the policy limit? ISO has tested and found that for a fixed settlement lag, the size of the settled losses is independent of the policy limit. That is, it appears that the settlement lag reflects policy limits.

Doesn't ALAE vary by policy limits. In preliminary tests ISO also found the ALAE is independent of policy limit for a fixed settlement lag. Further tests will be done.

For reinsurers, however, settlement lags are hard to get from ceding companies, but policy limit distributions are easier to obtain. Couldn't ISO build a similar model reflecting policy limits instead of settlement lags?

Possibly ISO could relate settlement lags to the more common policy limits. A problem with this might be what policy limit is reported. For example, if an insured has an umbrella policy over its primary policy the settlement of the loss may be affected by the umbrella limit even though only the primary policy limit is reported to ISO.

In the example, data from accident years 1973 through 1986 are used to project accident year 1991. A rhetorical question was asked whether the lag between the end of the data and the projection date could be shortened?

2. Paid Versus Reported Loss Data?

Introduction: ISO has found in examining inconsistencies in reported data that most inconsistencies involve open claims. There is less of a problem with reporting actual paid loss. Furthermore, paid claims lead open claims with respect to major changes in claims settlement practices. For example, stacking of UM/UIM had to result in a settled claim against an insurer before open claim reserves were increased to reflect stacking.

Discussion: For lines of business with long settlement lags, there aren't many large claims, e.g. excess of $1 million, that are likely to settle quickly enough to be

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included in the settled claim experience. This seems to be a high cost to pay for somewhat cleaner data.

While it may be true that paid claims lead open claims in reflecting major changes in claim settlement practices, the impact of the change is delayed if only settled claims are used. The increased reserves on open claims will not enter the data until the claims are settled. Valuable information will not be incorporated as quickly as it should.

By use of diagnostic tests on open claims the ISO results based on settled claims should indicate whether the settled claim data is failing to reflect the open claim reserves correctly. Also the delay in incorporating changes in claim settlement practices will vary by company. ISO data is reported from many different companies all with different claim reserving practices. It is more difficult for ISO to adjust open claim reserving practices for all the different companies reporting to ISO than to reflect such practices for one company.

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ISO has not yet finalized the ILF methodology using settled claim data. The diagnostics tests are still evolving. If the methodology using settled claims fails, the incurred loss methodology is still available.

3. Discounted Increased Limit Factors

Introduction: Discounted limited average severities (LAS) can be calculated by settlement lag for a fixed interest rate (interest rates may vary by settlement lag). Weighing together the LAS, the discounted LAS can be calculated. The discounted LAS can then be used to calculate discounted ILF's.

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Discussion: Many concerns were raised. Will the discounted LAS be used in the risk load calculations? Will variation in interest rates be considered? Will discounted ILF's be used in filings? If ILF's will reflect investment income on loss payments shouldn't they also reflect other expenses such as overhead or commissions?

Many of these concerns have yet to be addressed by ISO. ISO has no intention of filing discounted ILF's. Using discounted LAS to calculate ILF's was noted as a point of information only.

Attachment 1

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AGENDA

ISO INCREASED LIMITS PROCEDURE

Introduction:

9:30 - Overview of short-term and longer term changes

Short-Term Changes:

10:00 - 1. severity model (4-parameter Pareto)

10:40 - 2. risk load

11:40 - 3. composite rated risk data and uniform excess change

12:00 - LUNCH

Longer-Term Changes:

1:00 - 1. Pareto Soup Model (36 or 43 or more parameters)

2:45 - 2. paid versus reported loss data

3:15 - 3. discounted increased limit factors

Closing:

3:45 - Summarization and wrap-up

4:00 - Adjournment

Pilot Increased Limits Ratemaking Procedure

- Developed by ISO staff and Actuarial Research Committee

- Significant new features

   1. Pareto "Soup"

   2. Distribution fit to settled (paid) occurrences

   3. Explicit loss development model

   4. Empirical testing procedures

   5. New risk load formula

      - Derived from economic equilibrium assumptions

      - Explicit recognition of parameter uncertainty

Current Increased Limit Procedure

- Truncated Pareto distribution
- Development of number of occurrences by layer
- Risk load based on standard deviation of loss

Short Term Changes

- Mixed Pareto distribution ???
- Development of number of occurrences by layer
- Competitive Market Equilibrium Risk Load Formula

PRODUCT
ACCIDENT YEAR

INTERVAL ENDPOINT	27-ULT	39-ULT	51-ULT	63-ULT	75-ULT	87-ULT	99-ULT
27866	1.22714	0.94869	0.90311	0.93339	0.95704	0.96580	0.97279
28677	1.23678	0.95290	0.90520	0.93521	0.95855	0.96741	0.97441
29552	1.24651	0.95722	0.90718	0.93687	0.95987	0.96885	0.97581
30433	1.25639	0.96147	0.90909	0.93841	0.96106	0.97009	0.97706
31387	1.26651	0.96576	0.91097	0.93988	0.96213	0.97123	0.97817
32396	1.27692	0.97015	0.91286	0.94129	0.96314	0.97228	[illegible]
33468	1.28768	0.97466	0.91478	0.94268	0.96409	0.97326	[illegible]
34408	1.29885	0.97933	0.91675	0.94406	0.96501	0.97419	0.98099
35823	1.31050	0.98419	0.91882	0.94546	0.96592	0.97509	0.98182
37122	1.32269	0.98929	0.92098	0.94690	0.96683	0.97597	0.98261
38615	1.33549	0.99465	0.92329	0.94839	0.96776	0.97684	0.98339
40812	1.34898	1.00051	0.92574	0.94995	0.96872	0.97772	0.98416
41627	1.36324	1.00632	0.92839	0.95159	0.96973	0.97862	0.98492
43376	1.37838	1.01272	0.93124	0.95335	0.97080	0.97956	0.98570
46276	1.39448	1.01957	0.93433	0.95522	0.97195	0.98053	0.98650
47381	1.41169	1.02691	0.93770	0.95725	0.97318	0.98155	0.98732
49627	1.43015	1.03481	0.94137	0.95940	0.97450	0.98262	0.98818
52130	1.44995	1.04333	0.94537	0.96173	0.97592	0.98376	0.98906
54924	1.47131	1.05253	0.94973	0.96421	0.97741	0.98493	0.98996
58059	1.49433	1.06243	0.95442	0.96680	0.97892	0.98610	0.99085
61547	1.52087	1.07427	0.96051	0.97054	0.98146	0.98804	0.99228
65856	1.55114	1.08807	0.96796	0.97531	0.98497	0.99064	0.99418
70128	1.58471	1.10341	0.97628	0.98058	0.98882	0.99345	0.99617
78485	1.62199	1.12038	0.98547	0.98627	0.99290	0.99632	0.99810
81760	1.66320	1.13494	0.99537	0.99209	0.99690	0.99888	0.99960
89582	1.71000	1.15994	1.00685	0.99847	1.00120	1.00148	1.00118
98718	1.77492	1.19139	1.02547	1.01179	1.01212	1.01032	1.00825
110898	1.84942	1.22682	1.04670	1.02572	1.02320	1.01956	1.01562
126525	1.94234	1.27112	1.07323	1.04328	1.03725	1.03122	1.02490
136320	2.02388	1.30921	1.09849	1.05725	1.04807	1.03974	1.03167
148855	2.09089	1.34026	1.11381	1.06804	1.05715	1.04692	1.03737
165187	2.16845	1.37703	1.13561	1.08265	1.06805	1.05556	1.04421
182027	2.26306	1.42136	1.16196	1.09953	1.08127	1.06604	1.05249
206948	2.38851	1.47623	1.19460	1.11991	1.09764	1.07901	1.06272
241852	2.53208	1.54670	1.23646	1.14616	1.11856	1.09556	1.07574
298190	2.73934	1.64232	1.29508	1.18132	1.14662	1.11769	1.09509
309929	3.05089	1.78438	1.37647	1.23247	1.18740	1.14973	1.11809
419085	3.32196	1.90586	1.44687	1.27488	1.22111	1.17609	1.13865
454006	3.44352	1.95994	1.47806	1.29358	1.23590	1.18761	1.14747
497134	3.58634	2.02310	1.51430	1.31500	1.25295	1.20087	1.15770
551905	3.75779	2.09859	1.55724	1.34028	1.27296	1.21640	1.16967
624327	3.96958	2.19061	1.60947	1.37073	1.29702	1.23502	1.18397
725727	4.24174	2.30789	1.67532	1.40874	1.32695	1.25810	1.20164
880617	4.61237	2.46655	1.76284	1.46856	1.36603	1.28811	1.22481
1155871	5.16734	2.69754	1.88974	1.52942	1.42137	1.33035	1.25683
1815234	6.17024	3.10617	2.10821	1.64814	1.51332	1.39992	1.30883
2909430	7.80630	3.74613	2.45816	1.82001	1.64476	1.49806	1.38173
7.9E+08	13.6127	5.92292	3.51535	2.35115	2.04336	1.78880	1.59239

Policy Limit	Current T=6000	Current T=12000	Pilot 87 Call	Pilot 88 Call
$25,000	$4,077	$4,049	$5,381	$5,177
50,000	5,459	5,381	7,243	7,016
100,000	7,096	6,898	9,407	9,109
300,000	10,244	9,518	12,931	12,606
500,000	11,959	10,785	14,489	14,193
1,000,000	14,573	12,532	16,462	16,254
2,000,000	17,573	14,298	18,265	18,200
5,000,000	22,137	16,651	20,402	20,626

Pareto Distribution:

$$F(x: b,q) = 1 - \left[\frac{b}{x + b}\right]^q$$

Mixed Pareto Distribution:

$$G(x) = \underbrace{(1\text{-}p) \cdot F(x: b_1,q)}_{\text{Long Tail}} + \underbrace{p \cdot F(x: b_2,q+2)}_{\text{Short Tail}}$$

Trending and Developing the Occurrence Severity Distribution

Delay in Settlement

		0	1	2	3	4	...	S	
P	0	o	o	o	o	o	o	o	x →
o	1	o	o	o	o	o	o	x	x →
l	2	o	o	o	o	o	x	x	x →
	3	o	o	o	o	x	x	x	x →
Y	4	o	o	o	x	x	x	x	x →
e	...	o	o	x	x	x	x	x	x →
a	S	o	x	x	x	x	x	x	x →
r		x	x	x	x	x	x	x	x →
		↓	↓	↓	↓	↓	↓	↓	↓

o - observed occurrence severity distribution

x - projected occurrence severity distribution.

Trended Mixed Pareto Distribution

$$G_{y,d}(x) = (1-p_d)\cdot F(x: b_{1d}\cdot t_d^y , q_d) + p_d\cdot F(x: b_{2d}\cdot t_d^y , q_d+2)$$

y = Accident Year
d = Delay in Settlement

Relationship between parameters

1. Trend factors, t_d's, are equal for selected d's.

2. Shape parameters, q_d's and p_d's, are equal for selected d's.

3. Scale parameters, b_{id}'s are equal for selected d's.

$$\text{Likelihood} = \prod_{y=0}^{S} \prod_{d=0}^{y} \prod_{i=1}^{130} \left(G_{y,d}(L_i) - G_{y,d}(L_{i-1})\right)^{n_{y,d,i}}$$

The final claim severity distribution for year S+1:

$$\sum_{d=0}^{\infty} w_d G_{S+1,d}(x)$$

w_d = proportion of occurrences in settlement period d.

w_d's are estimated by maximum likelihood.

We assume w_d's have an exponential tail.

Note

The final occurrence severity distribution is a mixture of Pareto distributions. The proportion of each Pareto is determined by the w_d's and the p_d's. Hence the term:

Pareto Soup

Fitting Diagnostics

Compare Case Reserves with Projected Future Settlements

PRODUCTS CGL TABLE B AY 73 OPEN

POLICY LIMIT	SAMPLE L.A.S.	MODEL L.A.S.	% DIFF
$25,000	7,346	9,669	31.62%
$50,000	10,541	13,972	32.55%
$100,000	15,274	18,893	23.69%
$300,000	21,739	27,208	25.16%
$500,000	25,210	31,135	23.50%
$1,000,000	28,412	36,382	28.05%
$2,000,000	32,264	41,527	28.71%
$5,000,000	38,571	48,118	24.75%
# OF OCCS.	194	313	

PRODUCTS CGL TABLE B AY 74 OPEN

POLICY LIMIT	SAMPLE L.A.S.	MODEL L.A.S.	% DIFF
$25,000	8,237	9,751	18.38%
$50,000	12,824	14,167	10.48%
$100,000	18,059	19,194	6.29%
$300,000	21,511	27,739	28.95%
$500,000	23,289	31,749	36.33%
$1,000,000	25,702	37,164	44.59%
$2,000,000	26,582	42,441	59.66%
$5,000,000	27,358	49,205	79.86%
# OF OCCS.	360	390	

PRODUCTS CGL TABLE B AY 76 OPEN

POLICY LIMIT	SAMPLE L.A.S.	MODEL L.A.S.	% DIFF
$25,000	5,488	9,972	81.70%
$50,000	7,583	14,542	91.77%
$100,000	10,017	19,825	97.91%
$300,000	14,857	28,786	93.76%
$500,000	17,469	33,030	89.08%
$1,000,000	20,267	38,720	91.05%
$2,000,000	22,433	44,303	97.49%
$5,000,000	25,655	51,456	100.57%
# OF OCCS.	544	721	

PRODUCTS CGL TABLE B AY 78 OPEN

POLICY LIMIT	SAMPLE L.A.S.	MODEL L.A.S.	% DIFF
$25,000	8,150	10,180	24.91%
$50,000	11,316	14,945	32.07%
$100,000	14,406	20,446	41.93%
$300,000	18,871	29,880	58.34%
$500,000	20,481	34,331	67.63%
$1,000,000	21,821	40,340	84.87%
$2,000,000	22,115	46,225	109.02%
$5,000,000	22,305	53,786	141.14%
# OF OCCS.	819	1,118	

PRODUCTS CGL TABLE B AY 80 OPEN

POLICY LIMIT	SAMPLE L.A.S.	MODEL L.A.S.	% DIFF
$25,000	7,470	10,321	38.16%
$50,000	10,092	15,107	49.69%
$100,000	12,762	20,615	61.53%
$300,000	16,703	30,050	79.91%
$500,000	18,435	34,495	87.12%
$1,000,000	20,191	40,469	100.43%
$2,000,000	22,158	46,317	109.03%
$5,000,000	25,859	53,797	108.04%
# OF OCCS.	2,122	2,990	

PRODUCTS CGL TABLE B AY 82 OPEN

POLICY LIMIT	SAMPLE L.A.S.	MODEL L.A.S.	% DIFF
$25,000	11,893	10,595	-10.91%
$50,000	16,758	15,423	-7.97%
$100,000	21,490	20,907	-2.71%
$300,000	28,194	29,973	6.31%
$500,000	30,446	34,107	12.02%
$1,000,000	32,448	39,529	21.82%
$2,000,000	33,939	44,696	31.70%
$5,000,000	35,378	51,141	44.55%
# OF OCCS.	2,438	4,370	

Parameter Uncertainty - Severity

$$b_{id} \dashrightarrow B_y \cdot b_{id}$$

y+1973	B_y	
1973	1.000	(by definition)
1974	1.053	
1975	1.016	
1976	0.964	
1977	1.013	
1978	1.013	
1979	0.990	
1980	1.001	
1981	1.014	
1982	1.103	
1983	0.982	
1984	1.060	
1985	0.975	
1986	0.987	

The distribution of B_y is estimated in the maximum likelihood equation.

Parameter Uncertainty - Occurrence Count

Let n = expected claim count for an insurance company

$$n \text{ -----------> } C_y \cdot n$$

$E[C_y] = 1$

$Var[C_y] = c$

Poisson - No Parameter Uncertainty

Negative Binomial - Parameter Uncertainty

$$c = (\text{Coefficient of Variation of Gamma Prior})^2$$

c is estimated by maximum likelihood.

Risk Load

Goals of the Risk Load Formula

The risk load should be sufficient to attract an adequate supply of coverage for all desired policy limits.

The risk load should reflect stable, yet competitive, market conditions. It should not reflect such effects as the underwriting cycle.

The risk load should reflect the risks faced by the insurer in estimating the price of its product. It should recognize parameter uncertainty.

Risk Load

Insurance Market Assumptions

The insurance market is highly competitive. The risk load cannot be influenced by the actions of a single insurer.

Insurers can decide how much insurance to write in each line of business and policy limit.

Insurers will write line/limit combinations in such a way as to maximize the risk load subject to a constraint on the variance of its total insurance portfolio.

The result of all insurers competing for business as described above will result in an equilibrium characterized by the supply of insurance equaling the demand for insurance for each line/limit combination.

Risk Load

Characterization of Equilibrium

Technical note: vectors and matricies will have cells corresponding to each line/limit combination.

Define

m - number of insurance companies

n(k) - vector of expected occurrence counts for the kth company

$\bar{n}$ - average $n(k) = \frac{1}{m} \cdot \sum_{k=1}^{m} n(k)$

U - vector quantifying process risk

V - covariance matrix quantifying parameter risk

L - constant of proportionality

R - vector for risk load per expected occurrence

Then $$R = L \cdot (U + 2 \cdot V \cdot \bar{n})$$

Risk Load

Outline of Derivation of Risk Load Formula

Step 1

For a given risk load vector, R, each insurance company decides how much insurance it will write in each line and policy limit by solving the constrained optimization problem.

> Maximize total risk load subject to the constraint on total variance of its insurance portfolio. This is a standard Lagrange multiplier problem.

This exercise will tell how much insurance will be supplied at each line and policy limit as a function of the risk load vector, R.

Step 2

Do a market survey to determine how much is demanded for each line and policy limit.

Step 3

Select the risk load vector, R, that will cause the total supply equal to the total demand for each line and policy limit.

Risk Load

Sample Calculations

Limit (000)	Severity	Process Risk	Parameter Risk	ILF w/o RL	ILF w RL
25	12032	44	708	1.000	1.000
50	14082	109	965	1.170	1.186
100	16387	257	1252	1.362	1.400
300	20140	859	1723	1.674	1.777
500	21799	1431	1931	1.812	1.968
1000	23901	2763	2194	1.986	2.257
2000	25821	5195	2434	2.146	2.617
5000	28097	11716	2720	2.335	3.327

Risk Load

Risk Reduction by Layering

Common Practice - Calculate the ILF for an excess layer by subtracting the ILF for the lower limit from the ILF for the upper limit.

Sample Calculations

Limit (000)	Severity	Process Risk	Parameter Risk	Total Risk	ILF w RL
1000	23901	2763	2194	4957	2.257
2000	25821	5195	2434	7629	2.617
Diff	1920	2432	240	2762	0.359

Which would an insurer rather sell?

1. A ground up $2,000,000 policy limit, or

2. A ground up $1,000,000 policy limit to one insured, and a $1,000,000 over $1,000,000 policy limit to a second insured.

Concluding Remarks on Risk Load

Our goal is to provide a generic risk load formula which accounts for basic economic conditions.

This risk load formula is, at best, an approximation. It should be judged on its usefulness.

It is up to insurers to make whatever modifications they feel should be made. It is ISO's goal to make common changes easy.

Risk Load

Note that the "subtraction" method implies indifference between the two options.

However, the risk load expression, $R = L \cdot (U + 2 \cdot V \cdot \bar{n})$, implies preference for separate layers.

Sample Calculations

Limit (000)	Severity	Process Risk	Parameter Risk	Total Risk	ILF w RL
1000	23901	2763	2194	4957	2.257
2000	25821	5195	2434	7629	2.617
Diff	1920	2432	240	2762	0.359
RL Eqn	1920	737	240	977	0.227

Note that the subtraction method works for parameter risk but not for process risk.

OSD PRODUCTS TABLE B SEVERITY MODEL

ACCIDENT YEAR	LAG 1	2	3	4	5	6	7	8	9	10	11	12	13	14
1974	Q1(1)	Q1(2)	Q1(3)	Q1(4)	Q1(5)	Q1(6)	Q1(7)	Q1(8)	Q1(9)	Q1(9)	Q1(9)	Q1(9)	Q1(9)	Q1(9)
	Q2(1)	Q2(2)	Q2(3)	Q2(4)	Q2(5)	Q2(6)	Q2(7)	Q2(8)	Q2(9)	Q2(9)	Q2(9)	Q2(9)	Q2(9)	Q2(9)
	B1(1)	B1(2)	B1(3)	B1(4)	B1(5)	B1(6)	B1(7)	B1(8)	B1(9)	B1(9)*S	B1(9)*S**2	B1(9)*S**3	B1(9)*S**4	B1(9)*S**5
	B2(1)	B2(2)	B2(3)	B2(4)	B2(5)	B2(6)	B2(7)	B2(8)	B2(9)	B2(9)*S	B2(9)*S**2	B2(9)*S**3	B2(9)*S**4	B2(9)*S**5
	P(1)	P(2)	P(3)	P(4)	P(5)	P(6)	P(7)	P(8)	P(9)	P(9)	P(9)	P(9)	P(9)	P(9)
1975	Q1(1)	Q1(2)	Q1(3)	Q1(4)	Q1(5)	Q1(6)	Q1(7)	Q1(8)	Q1(9)	Q1(9)	Q1(9)	Q1(9)	Q1(9)	Q1(9)
	Q2(1)	Q2(2)	Q2(3)	Q2(4)	Q2(5)	Q2(6)	Q2(7)	Q2(8)	Q2(9)	Q2(9)	Q2(9)	Q2(9)	Q2(9)	Q2(9)
	B1(1)*T1	B1(2)*T2	B1(3)*T3	B1(4)*T4	B1(5)*T5	B1(6)*T5	B1(7)*T5	B1(8)*T5	B1(9)*T5	B1(9)*T5*S	B1(9)*T5*S**2	B1(9)*T5*S**3	B1(9)*T5*S**4	B1(9)*T5*S**5
	B2(1)*T1	B2(2)*T2	B2(3)*T3	B2(4)*T4	B2(5)*T5	B2(6)*T5	B2(7)*T5	B2(8)*T5	B2(9)*T5	B2(9)*T5*S	B2(9)*T5*S**2	B2(9)*T5*S**3	B2(9)*T5*S**4	B2(9)*T5*S**5
	P(1)	P(2)	P(3)	P(4)	P(5)	P(6)	P(7)	P(8).	P(9)	P(9)	P(9)	P(9)	P(9)	P(9)

ETC.

NOTE: Q2 = Q1 + 2

Attachment 3
Page 1

PARAMETERS FROM THE FULL 14X14 TRIANGLE MODEL WITH SEVERITY TREND
BASED ON EDITED 1988 CALL PRODUCTS CGL TABLE B OSD DATA
FOR ACCIDENT YEARS 1974 TO 1987

LAG(J)	Q1(J)	Q2(J)	B1(J)	B2(J)	P(J)	T(J)	W(J)
1	2.1730	4.1730	2,155	665	0.8513	1.0889	0.4057
2	1.5905	3.5905	2,057	800	0.7520	1.1044	0.2669
3	1.2644	3.2644	5,096	2,047	0.7028	1.1235	0.0753
4	1.2748	3.2748	8,181	3,082	0.6007	1.1185	0.0552
5	1.3772	3.3772	18,460	6,527	0.5540	1.0518	0.0436
6	1.2196	3.2196	12,963	4,893	0.3843	1.0518	0.0284
7	1.3469	3.3469	15,993	4,312	0.3269	1.0518	0.0215
8	0.8381	2.8381	3,635	54	0.0398	1.0518	0.0176
9	0.9456	2.9456	10,491	1,818	0.3386	1.0518	0.0146
10	0.9456	2.9456	9,300	1,612	0.3386	1.0518	0.0121
11	0.9456	2.9456	8,245	1,429	0.3386	1.0518	0.0101
12	0.9456	2.9456	7,309	1,267	0.3386	1.0518	0.0084
13	0.9456	2.9456	6,480	1,123	0.3386	1.0518	0.0069
14	0.9456	2.9456	5,744	996	0.3386	1.0518	0.0058

S = 0.8865 SIGMA = 0.0387

LIMITED AVERAGE SEVERITY ANALYSIS FOR PRODUCTS TABLE B
MODEL INCLUDING TREND ACROSS LAGS FOR EDITED 1988 CALL DATA

PRODUCTS CGL TABLE 2 AY 74 ALL LAGS

Policy Limit	Sample L.A.S.	Model L.A.S.	% Diff
25000	2167	1915	-11.64
50000	2824	2425	-14.13
100000	3523	2954	-16.15
300000	4330	3779	-12.73
500000	4614	4149	-10.10
1000000	4967	4637	-6.63
2000000	5409	5116	-5.42
5000000	5816	5740	-1.30

NUMBER OF OCCURRENCES = 5525

PRODUCTS CGL TABLE 2 AY 75 ALL LAGS

Policy Limit	Sample L.A.S.	Model L.A.S.	% Diff
25000	1810	1771	-2.14
50000	2302	2215	-3.77
100000	2818	2671	-5.22
300000	3568	3368	-5.61
500000	3753	3675	-2.07
1000000	3919	4077	4.03
2000000	4073	4463	9.59
5000000	4261	4961	16.41

NUMBER OF OCCURRENCES = 7181

PRODUCTS CGL TABLE 2 AY 76 ALL LAGS

Policy Limit	Sample L.A.S.	Model L.A.S.	% Diff
25000	1983	2187	10.28
50000	2447	2796	14.25
100000	2957	3431	16.01
300000	3738	4419	18.24
500000	4020	4860	20.89
1000000	4361	5437	24.69
2000000	4651	5997	28.94
5000000	5040	6721	33.36

NUMBER OF OCCURRENCES = 7764

PRODUCTS CGL TABLE 2 AY 77 ALL LAGS

Policy Limit	Sample L.A.S.	Model L.A.S.	% Diff
25000	2301	2297	-0.16
50000	2969	2922	-1.57
100000	3708	3567	-3.81
300000	4728	4547	-3.84
500000	5124	4972	-2.97
1000000	5471	5519	0.88
2000000	5813	6039	3.88
5000000	6264	6699	6.94

NUMBER OF OCCURRENCES = 9637

PRODUCTS CGL TABLE 2 AY 78 ALL LAGS

Policy Limit	Sample L.A.S.	Model L.A.S.	% Diff
25000	2491	2495	0.14
50000	3168	3204	1.13
100000	3946	3939	-0.16
300000	5015	5055	0.80
500000	5363	5537	3.24
1000000	5654	6151	8.79
2000000	5848	6729	15.06
5000000	6082	7453	22.53

NUMBER OF OCCURRENCES = 8680

PRODUCTS CGL TABLE 2 AY 79 ALL LAGS

Policy Limit	Sample L.A.S.	Model L.A.S.	% Diff
25000	2870	2850	-0.71
50000	3678	3660	-0.50
100000	4512	4495	-0.38
300000	5742	5746	0.08
500000	6158	6278	1.95
1000000	6550	6949	6.08
2000000	6836	7570	10.74
5000000	7145	8338	16.70

NUMBER OF OCCURRENCES = 15123

PRODUCTS CGL TABLE 2 AY 80 ALL LAGS

Policy Limit	Sample L.A.S.	Model L.A.S.	% Diff
25000	2797	2671	-4.52
50000	3568	3397	-4.80
100000	4388	4137	-5.71
300000	5702	5217	-8.51
500000	6222	5659	-9.04
1000000	6677	6199	-7.16
2000000	6976	6678	-4.27
5000000	7290	7241	-0.67

NUMBER OF OCCURRENCES = 19612

PRODUCTS CGL TABLE 2 AY 81 ALL LAGS

Policy Limit	Sample L.A.S.	Model L.A.S.	% Diff
25000	2756	2658	-3.52
50000	3492	3364	-3.67
100000	4302	4076	-5.25
300000	5583	5088	-8.86
500000	6066	5486	-9.57
1000000	6596	5949	-9.81
2000000	6938	6333	-8.72
5000000	7200	6738	-6.43

NUMBER OF OCCURRENCES = 20940

LIMITED AVERAGE SEVERITY ANALYSIS FOR PRODUCTS TABLE B
MODEL INCLUDING TREND ACROSS LAGS FOR EDITED 1988 CALL DATA

PRODUCTS CGL TABLE 2 AY 82 ALL LAGS

Policy Limit	Sample L.A.S.	Model L.A.S.	% Diff
25000	2833	2682	-5.30
50000	3588	3380	-5.79
10000C	4378	4089	-6.59
300000	5477	5106	-6.76
500000	5876	5509	-6.25
1000000	6170	5981	-3.07
2000000	6391	6374	-0.27
5000000	6620	6792	2.60

NUMBER OF OCCURRENCES = 20619

PRODUCTS CGL TABLE 2 AY 83 ALL LAGS

Policy Limit	Sample L.A.S.	Model L.A.S.	% Diff
25000	2761	2603	-5.71
50000	3459	3226	-6.71
100000	4247	3851	-9.31
300000	5381	4736	-12.00
500000	5739	5080	-11.48
1000000	6183	5479	-11.37
2000000	6494	5807	-10.58
5000000	6810	6147	-9.74

NUMBER OF OCCURRENCES = 19304

PRODUCTS CGL TABLE 2 AY 84 ALL LAGS

Policy Limit	Sample L.A.S.	Model L.A.S.	% Diff
25000	2526	2445	-3.17
50000	3050	2974	-2.47
100000	3579	3497	-2.32
300000	4360	4237	-2.82
500000	4701	4530	-3.64
1000000	5081	4873	-4.09
2000000	5358	5160	-3.69
5000000	5652	5466	-3.30

NUMBER OF OCCURRENCES = 18696

PRODUCTS CGL TABLE 2 AY 85 ALL LAGS

Policy Limit	Sample L.A.S.	Model L.A.S.	% Diff
25000	2161	2025	-6.28
50000	2550	2347	-7.96
100000	2947	2636	-10.55
300000	3518	3015	-14.31
500000	3766	3157	-16.16
1000000	4034	3320	-17.70
2000000	4248	3454	-18.70
5000000	4462	3594	-19.45

NUMBER OF OCCURRENCES = 14921

PRODUCTS CGL TABLE 2 AY 86 ALL LAGS

Policy Limit	Sample L.A.S.	Model L.A.S.	% Diff
25000	1631	1653	1.33
50000	1775	1815	2.24
100000	1867	1930	3.38
300000	1962	2041	4.04
500000	2007	2073	3.27
1000000	2027	2103	3.71
2000000	2027	2122	4.69
5000000	2027	2138	5.48

NUMBER OF OCCURRENCES = 9955

PRODUCTS CGL TABLE 2 AY 87 ALL LAGS

Policy Limit	Sample L.A.S.	Model L.A.S.	% Diff
25000	1127	1236	9.69
50000	1207	1300	7.73
100000	1265	1335	5.52
300000	1316	1357	3.08
500000	1344	1361	1.24
1000000	1346	1364	1.32
2000000	1347	1365	1.36
5000000	1347	1366	1.38

NUMBER OF OCCURRENCES = 5170

1988 CALL EDITED PRODUCTS TABLE 8 DATA — MODEL INCLUDING TREND ACROSS LAGS — LIMIT = 25,000

MODEL DEVIATIONS FROM SAMPLE

AY/LAG	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1974	4.08%	-4.60%	-0.47%	-2.82%	7.59%	-8.07%	10.54%	-20.93%	-30.48%	-39.00%	-51.81%	-4.36%	23.93%	-25.27%
1975	-5.47%	-7.10%	-3.66%	5.20%	8.87%	6.11%	-5.53%	-23.52%	9.84%	-20.19%	-4.55%	52.37%	-13.36%	
1976	5.86%	1.85%	0.51%	-3.85%	22.29%	0.12%	-0.42%	4.46%	36.50%	-0.11%	125.64%	50.66%		
1977	1.48%	-1.97%	7.46%	4.46%	-11.67%	5.08%	-4.99%	-10.64%	15.63%	38.57%	-11.74%			
1978	2.08%	11.72%	5.40%	-8.66%	5.18%	-11.21%	0.01%	1.16%	-8.54%	2.61%				
1979	-4.22%	5.09%	-3.36%	0.98%	-1.54%	-6.27%	-1.41%	5.46%	-1.88%					
1980	-3.69%	-6.47%	-1.05%	-5.27%	-2.43%	-9.18%	0.45%	-9.82%						
1981	-3.97%	-5.96%	-2.32%	1.84%	-9.06%	-2.15%	-1.80%							
1982	-13.43%	-6.85%	-8.04%	-5.19%	-2.34%	6.11%								
1983	-5.17%	-8.12%	-4.68%	-4.55%	-5.34%									
1984	-4.96%	-3.28%	-0.73%	-4.01%										
1985	-5.03%	-5.94%	-7.79%											
1986	-0.21%	2.50%												
1987	9.69%													

1988 CALL EDITED PRODUCTS TABLE 8 DATA — MODEL INCLUDING TREND ACROSS LAGS — LIMIT = 100,000

MODEL DEVIATIONS FROM SAMPLE

AY/LAG	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1974	-0.20%	-9.26%	-5.03%	-12.63%	21.20%	-12.91%	8.55%	0.77%	-12.51%	-44.28%	-62.31%	18.77%	-4.54%	-22.46%
1975	-19.18%	-7.21%	0.09%	13.13%	20.69%	7.79%	-11.66%	-33.61%	27.75%	-39.09%	-2.72%	9.60%	-17.13%	
1976	6.57%	4.10%	-2.31%	5.36%	34.69%	12.42%	0.29%	7.15%	34.64%	-6.73%	137.58%	57.14%		
1977	3.85%	-1.94%	2.62%	8.04%	-18.83%	9.25%	-10.74%	-21.88%	19.74%	38.97%	-30.13%			
1978	4.89%	13.66%	6.19%	-14.61%	4.45%	-14.71%	8.53%	5.80%	-11.53%	11.21%				
1979	-7.26%	4.64%	-4.74%	3.29%	0.91%	-4.66%	-5.45%	6.94%	2.99%					
1980	-3.68%	-7.75%	-0.65%	-9.85%	2.74%	-13.33%	-2.16%	-13.36%						
1981	-4.90%	-4.87%	-2.65%	0.96%	-10.56%	-5.63%	-9.64%							
1982	-14.71%	-11.80%	-11.17%	-6.46%	-5.93%	8.14%								
1983	-7.24%	-13.34%	-6.21%	-7.57%	-11.34%									
1984	-8.04%	-1.10%	1.22%	-3.42%										
1985	-9.71%	-8.13%	-13.22%											
1986	0.47%	5.35%												
1987	5.52%													

1988 CALL EDITED PRODUCTS TABLE B DATA   MODEL INCLUDING TREND ACROSS LAGS   LIMIT = 500,000

MODEL DEVIATIONS FROM SAMPLE

AY/LAG	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1974	-20.06%	-5.69%	-8.90%	-12.00%	43.81%	-0.40%	15.46%	41.84%	28.43%	-36.09%	-58.62%	-4.85%	-12.46%	10.00%
1975	-18.68%	-2.84%	-15.56%	15.29%	37.53%	1.43%	3.24%	-22.76%	100.33%	-45.26%	20.34%	-3.50%	2.68%	
1976	7.30%	5.39%	-4.14%	17.06%	52.42%	23.60%	13.62%	38.11%	10.30%	-8.32%	121.00%	26.06%		
1977	6.63%	-1.21%	5.09%	11.94%	-25.84%	16.85%	-7.11%	-9.62%	7.03%	48.58%	-32.96%			
1978	5.75%	15.26%	-0.11%	-21.25%	4.61%	-7.01%	12.83%	22.88%	-6.30%	50.66%				
1979	-9.52%	-0.41%	6.49%	10.13%	2.00%	-12.52%	3.81%	10.68%	16.66%					
1980	-4.72%	-9.91%	-3.96%	-16.17%	6.64%	-24.30%	-6.43%	-7.68%						
1981	-4.59%	-9.70%	-5.78%	-2.00%	-15.60%	-11.47%	-14.75%							
1982	-14.09%	-18.07%	-12.17%	-1.05%	-9.78%	11.68%								
1983	-6.99%	-20.27%	-1.16%	-8.08%	-17.86%									
1984	-9.01%	-2.80%	5.08%	-8.51%										
1985	-14.03%	-12.91%	-19.31%											
1986	0.87%	4.73%												
1987	1.24%													

98

1988 CALL EDITED PRODUCTS TABLE B DATA   MODEL INCLUDING TREND ACROSS LAGS   LIMIT = 2,000,000

MODEL DEVIATIONS FROM SAMPLE

AY/LAG	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1974	-20.00%	-4.43%	-6.62%	-7.96%	57.11%	13.45%	28.14%	73.65%	58.41%	-29.04%	-53.89%	30.70%	-54.46%	41.21%
1975	-18.61%	-1.01%	-24.56%	17.15%	56.74%	10.33%	19.82%	12.65%	181.77%	-32.26%	57.76%	4.53%	17.59%	
1976	7.40%	7.49%	5.61%	33.45%	61.36%	37.52%	28.75%	101.85%	0.43%	-4.96%	103.07%	32.90%		
1977	4.74%	-0.30%	10.04%	28.55%	-15.17%	24.30%	-4.12%	9.28%	-8.04%	37.56%	-21.45%			
1978	5.88%	17.69%	5.70%	-22.12%	16.53%	2.85%	24.98%	67.74%	7.48%	99.21%				
1979	-9.40%	-3.15%	17.52%	13.05%	12.08%	-6.90%	13.42%	28.30%	46.85%					
1980	-4.58%	-8.61%	4.12%	-9.27%	12.63%	-25.58%	-2.10%	7.84%						
1981	-4.48%	-13.35%	-9.91%	5.90%	-16.45%	-7.55%	-14.29%							
1982	-13.94%	-19.37%	-11.11%	11.66%	-6.25%	27.04%								
1983	-6.93%	-24.24%	7.13%	-5.16%	-19.90%									
1984	-9.01%	-3.72%	6.32%	-9.00%										
1985	-13.91%	-19.68%	-19.71%											
1986	1.13%	6.83%												
1987	1.36%													

Attachment 3
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AGENDA
ACTUARIAL RESEARCH COMMITTEE
MEETING OF JANUARY 23, 1990

ARC 89-4C OCCURRENCE SETTLEMENT PATTERNS

REFERENCES

ARC 89-4, Agenda & Minutes for Meeting of March 14, 1989
ARC 89-4A, Agenda & Minutes for Meeting of June 28, 1989
ARC 89-12, Agenda & Minutes for Meeting of June 28, 1989
ARC 89-13, Agenda & Minutes for Meeting of June 28, 1989
ARC 89-4B, Agenda & Minutes for Meeting of September 26, 1989

BACKGROUND

The increased limits procedure being developed is based on a model which separates data by year into "time of settlement" periods or lags for which severity distributions, trend parameters, and ultimately fitted trended curves are developed. ARC 89-4 began the analysis of the distribution of occurrences by settlement period on data organized by accident year, rather than by policy year.

At the June 28, 1989 meeting, results of fitting the full triangle with roof function models, that is, exponential models having piecewise linear mixing distributions (see ARC 89-4A) and of fitting individual years with mixed Cauchy models (see ARC 89-12) were presented.

At the September 26, 1989 meeting, results of fitting the full triangle with various mixed distribution models (see ARC 89-4B) were presented. The committee suggested using simpler actuarial techniques or models for fitting the available data and an exponential decay curve for the tail.

SIMPLE MODELS

Two simple models were tested: a three-year average link ratio model and a maximum likelihood estimation (MLE) of lag probabilities model (see ARC 89-4B and ARC 89-13). Staff then focused on testing various ways of splicing an exponential tail derived from the pre-1979 data to the available data for earlier lags.

RESULTS

Attachment I summarizes the results of staff's analysis of occurrence settlement patterns including results of the other attachments to the current item. This attachment exhibits the loss distribution by lag resulting from the occurrence settlement pattern obtained with the currently recommended procedure and the severity model. Attachment II gives the results of fitting an exponential tail to the available data for earlier lags. Attachment III presents the key results using the currently recommended settlement pattern procedure for the revised Products CGL Table 2 data.

The MLE model had a lower chi-squared total over all settled cells than the link ratio model. Analysis suggested a difference between the GLSP-data (pre-1979) and the CSP-data (post-1979). The MLE approach was applied to obtain separate

AGENDA
ACTUARIAL RESEARCH COMMITTEE
MEETING OF JANUARY 23, 1990

ARC 89-4C	**OCCURRENCE SETTLEMENT PATTERNS**
RESULTS (CONTINUED)	fits for the time spans 1973-1978 and 1980-1986. The combined results were the best achieved so far. Exponentials were fit to various tails of the earlier time span. The fit to six lags and beyond did best. But, when only the relativities for lags eight and on from this exponential tail were spliced to the MLE-derived relativities for the first seven lags, the fits were improved. When the exponential was used to project the open cases for 1973 before deriving the MLE lag probabilities, the fits were further improved.
STAFF RECOMMENDATION	THAT the Committee discuss this item and offer guidance for further investigations.
ATTACHMENTS TO AGENDA	I. Occurrence Settlement Patterns. II. Exponential Tail Fit to Settlement Patterns. III. Settlement Patterns and Exponential Tails for Revised Products CGL Table 2 Data.

MODEL BASED ON MLE OF EXPONENTIAL FIT
USING REVISED PRODUCTS CGL TABLE 2 DATA FROM ACCIDENT YEARS 1973-1978, LAGS 6-14

ACTUAL VALUES

YR/LAG	1	2	3	4	5	6	7	8	9	10	11	12	13	14	OPEN	TOT STL 6-LAST	TOTAL 6 & UP
1973	3,017	1,706	431	257	221	128	121	58	36	108	134	144	87	38	194	854	1,048
1974	2,991	2,119	473	303	259	189	138	113	91	86	88	40	42		360	787	1,147
1975	4,463	2,551	506	416	263	247	146	115	100	124	91	160			1,784	983	2,767
1976	4,302	2,619	627	411	402	292	203	156	148	97	124				544	1,020	1,564
1977	4,520	3,061	776	488	420	355	241	199	122	140					456	1,057	1,513
1978	5,151	2,929	761	524	489	487	419	234	239						819	1,379	2,198
MLE WT																	
73-78	0.4302	0.2637	0.0629	0.0422	0.0362	0.0299	0.0223	0.0154	0.0130	0.0125	0.0131	0.0147	0.0093	0.0057	0.0290	0.1357	0.1648
80-86	0.4072	0.2747	0.0749	0.0526	0.0434	0.0279	0.0248								0.0944	0.0527	0.1472

EXPONENTIAL FITTED VALUES

YR/LAG	1	2	3	4	5	6	7	8	9	10	11	12	13	14	OPEN	TOT STL 6-LAST	TOTAL 6 & UP
1973						159	137	118	102	88	76	66	57	49	313	854	1,167
1974						155	134	116	100	87	75	65	56		354	787	1,141
1975						209	180	156	135	116	100	87			552	983	1,535
1976						238	205	177	153	132	114				727	1,020	1,747
1977						277	240	207	179	154					982	1,057	2,039
1978						424	366	316	273						1,736	1,379	3,115
EXPO WT						0.0224	0.0194	0.0167	0.0145	0.0125	0.0108	0.0093	0.0081	0.0070	0.0442	0.1206	0.1648

CHI-SQUARED CONTRIBUTIONS (SIGNED)

YR/LAG	1	2	3	4	5	6	7	8	9	10	11	12	13	14	OPEN
1973						+6	+2	+31	+43	-4	-43	-92	-16	+3	+45
1974						-7	-0	+0	+1	+0	-2	+9	+3		-0
1975						-7	+7	+11	+9	-1	+1	-62			-2,754
1976						-12	+0	+3	+0	+9	-1				+46
1977						-22	-0	+0	+18	+1					+281
1978						-9	-8	+21	+4						+484

CHI-SQUARED CONTRIBUTIONS

YR/LAG	1	2	3	4	5	6	7	8	9	10	11	12	13	14	OPEN	TOT STL 6-LAST	TOTAL 6 & UP
1973						6	2	31	43	4	43	92	16	3	45	240	285
1974						7	0	0	1	0	2	9	3		0	23	24
1975						7.	7	11	9	1	1	62			2,754	96	2,850
1976						12	0	3	0	9	1				46	26	71
1977						22	0	0	18	1					281	42	323
1978						9	8	21	4						484	43	527
TOTAL															3,611	469	4,080

Attachment 4
Page 4

ALL-YEARS CURVE FIT

USING REVISED PRODUC[illegible]GL TABLE 2 DATA FROM ACCIDENT YEARS 1[illegible]-78 & 1980-86

MODEL APPLYING LAG-6 EXPONENTIAL TAIL TO LAG 8 & BEYOND

ACTUAL VALUES

ACCIDENT YEAR	1	2	3	4	5	6	7	8	9	10	11	12	13	14	TOTAL PAID	OUTSTANDING
1973	3,017	1,706	431	257	221	128	121	58	36	108	134	144	87	38	6,486	194
1974	2,991	2,119	473	303	259	189	138	113	91	86	88	40	42		6,932	360
1975	4,463	2,551	506	416	263	247	146	115	100	124	91	160			9,182	1,784
1976	4,302	2,619	627	411	402	292	203	156	148	97	124				9,381	544
1977	4,520	3,061	776	488	420	355	241	199	122	140					10,322	456
1978	5,151	2,929	761	524	489	487	419	234	239						11,233	819
1979	5,794	5,396	1,424	1,264	819	722	481	700							16,600	1,864
1980	8,851	6,360	1,689	1,232	1,060	597	557								20,346	2,122
1981	9,742	6,558	1,712	1,298	888	693									20,891	1,813
1982	9,958	6,144	1,663	1,216	1,080										20,061	2,438
1983	10,774	7,536	1,796	1,316											21,422	3,713
1984	9,324	6,121	2,082												17,527	4,594
1985	8,795	6,028													14,823	4,624
1986	6,388														6,388	5,817

FITTED VALUES

ACCIDENT YEAR	1	2	3	4	5	6	7	8	9	10	11	12	13	14	TOTAL PAID	OUTSTANDING
1973	2,874	1,762	420	282	241	200	149	118	102	88	76	66	57	49	6,486	313
1974	3,095	1,897	453	304	260	215	161	128	110	95	82	71	61		6,932	390
1975	4,136	2,536	605	406	348	287	215	170	147	127	110	95			9,182	604
1976	4,270	2,618	624	419	359	297	222	176	152	131	114				9,381	721
1977	4,756	2,916	695	467	400	330	247	196	169	146					10,322	930
1978	5,250	3,218	768	515	441	365	272	216	187						11,233	1,188
1979	7,318	4,936	1,347	946	781	502	446	325							16,600	2,066
1980	9,149	6,171	1,684	1,183	976	628	557								20,346	2,990
1981	9,658	6,515	1,777	1,248	1,030	662									20,891	3,744
1982	9,578	6,461	1,763	1,238	1,022										20,061	4,370
1983	10,777	7,269	1,983	1,393											21,422	6,067
1984	9,431	6,361	1,735												17,527	6,528
1985	8,852	5,971													14,823	7,756
1986	6,388														6,388	9,906

CHI-SQUARED CONTRIBUTIONS

ACCIDENT YEAR	1	2	3	4	5	6	7	8	9	10	11	12	13	14	OUTSTANDING
1973	-7	+2	-0	+2	+2	+26	+5	+31	+43	-4	-43	-92	-16	+3	+45
1974	+3	-26	-1	+0	+0	+3	+3	+2	+3	+1	-0	+14	+6		+2
1975	-26	-0	+16	-0	+21	+6	+22	+18	+15	+0	+3	-44			-2308
1976	-0	-0	-0	+0	-5	+0	+2	+2	+0	+9	-1				+44
1977	+12	-7	-9	-1	-1	-2	+0	-0	+13	+0					+241
1978	+2	+26	+0	-0	-5	-41	-79	-1	-14						+115
1979	+317	-43	-4	-107	-2	-96	-3	-432							+20
1980	+10	-6	-0	-2	-7	+1									+252
1981	-1	-0	+2	-2	+20	-1									+996
1982	-15	+16	+6	+0	-3										+854
1983	+0	-10	+18	+4											+913
1984	+1	+9	-69												+573
1985	+0	-1													+1265
1986	0														+1688

ALL-YEARS CURVE FIT
USING REVISED PRODUCTS CGL TABLE 2 DATA FROM ACCIDENT YEARS 1973-78 & 1980-86

MODEL APPLYING LAG-6 EXPONENTIAL TAIL TO LAG 8 & BEYOND

CHI-SQUARED CONTRIBUTIONS

ACCIDENT														TOT	STL
YEAR	1	2	3	4	5	6	7	8	9	10	11	12	13	14	CHI SQ	OUTSTANDING
1973	7	2	0	2	2	26	5	31	43	4	43	92	16	3	276	45
1974	3	26	1	0	0	3	3	2	3	1	0	14	6		63	2
1975	26	0	16	0	21	6	22	18	15	0	3	44			171	2,308
1976	0	0	0	0	5	0	2	2	0	9	1				20	44
1977	12	7	9	1	1	2	0	0	13	0					46	241
1978	2	26	0	0	5	41	79	1	14						169	115
1979	317	43	4	107	2	96	3	432							1,005	20
1980	10	6	0	2	7	1	0								26	252
1981	1	0	2	2	20	1									26	996
1982	15	16	6	0	3										40	854
1983	0	10	18	4											32	913
1984	1	9	69												79	573
1985	0	1													1	1,265
1986	0														0	1,688
TOTALS: 73-86															1,954	9,316
W/O 79															950	9,296
W/O 75&79															778	6,988

LOSS DISTRIBUTION BY LAG RESULTING FROM
OCCURRENCE SETTLEMENT PATTERN AND SEVERITY MODELS
FOR PRODUCTS CGL TABLE 2 DATA

LAG J	OCCURRENCE SETTLEMENT DISTRIBUTION W(J)	LIMITED AVG SEVERITY (LIMIT=$500K) LAS(J)	LOSS DISTRIBUTION LD(J)	CUMULATIVE LOSS DISTRIBUTION CLD(J)
1	0.3920	1,981	5.39%	5.39%
2	0.2644	5,070	9.30%	14.68%
3	0.0721	22,814	11.41%	26.09%
4	0.0507	39,491	13.89%	39.98%
5	0.0418	29,828	8.65%	48.62%
6	0.0269	36,917	6.89%	55.51%
7	0.0239	36,297	6.02%	61.53%
8	0.0174	34,507	4.16%	65.69%
9	0.0151	44,970	4.71%	70.40%
10	0.0130	45,250	4.08%	74.48%
11	0.0112	45,532	3.54%	78.02%
12	0.0097	45,815	3.08%	81.10%
13	0.0084	46,100	2.69%	83.79%
14	0.0072	46,386	2.32%	86.10%
15	0.0063	46,675	2.04%	88.14%
16	0.0054	46,965	1.76%	89.90%
17	0.0047	47,256	1.54%	91.44%
18	0.0040	47,549	1.32%	92.76%
19	0.0035	47,844	1.16%	93.92%
20	0.0030	48,139	1.00%	94.92%
21	0.0026	48,438	0.87%	95.80%
22	0.0023	48,736	0.78%	96.58%
23	0.0019	49,037	0.65%	97.22%
24	0.0017	49,340	0.58%	97.80%
25	0.0015	49,643	0.52%	98.32%
26	0.0013	49,948	0.45%	98.77%
27	0.0011	50,255	0.38%	99.15%
28	0.0009	50,563	0.32%	99.47%
29	0.0008	50,874	0.28%	99.75%
30	0.0007	51,186	0.25%	100.00%
	0.9955	14,484	100.00%	

LOSS DISTRIBUTION BY LAG RESULTING FROM
OCCURRENCE SETTLEMENT PATTERN AND SEVERITY MODELS
FOR PRODUCTS CGL TABLE 2 DATA

LAG J	OCCURRENCE SETTLEMENT DISTRIBUTION W(J)	LIMITED AVG SEVERITY (LIMIT=$1M) LAS(J)	LOSS DISTRIBUTION LD(J)	CUMULATIVE LOSS DISTRIBUTION CLD(J)
1	0.3920	1,987	4.75%	4.75%
2	0.2644	5,219	8.42%	13.18%
3	0.0721	25,468	11.21%	24.39%
4	0.0507	45,437	14.06%	38.45%
5	0.0418	33,234	8.48%	46.93%
6	0.0269	41,799	6.86%	53.80%
7	0.0239	40,130	5.86%	59.65%
8	0.0174	39,651	4.21%	63.86%
9	0.0151	53,691	4.95%	68.81%
10	0.0130	54,047	4.29%	73.10%
11	0.0112	54,406	3.72%	76.82%
12	0.0097	54,768	3.24%	80.06%
13	0.0084	55,131	2.83%	82.89%
14	0.0072	55,496	2.44%	85.33%
15	0.0063	55,866	2.15%	87.48%
16	0.0054	56,236	1.85%	89.33%
17	0.0047	56,609	1.62%	90.96%
18	0.0040	56,984	1.39%	92.35%
19	0.0035	57,362	1.23%	93.57%
20	0.0030	57,741	1.06%	94.63%
21	0.0026	58,123	0.92%	95.55%
22	0.0023	58,507	0.82%	96.38%
23	0.0019	58,892	0.68%	97.06%
24	0.0017	59,282	0.62%	97.67%
25	0.0015	59,673	0.55%	98.22%
26	0.0013	60,066	0.48%	98.70%
27	0.0011	60,461	0.41%	99.10%
28	0.0009	60,859	0.33%	99.44%
29	0.0008	61,260	0.30%	99.74%
30	0.0007	61,662	0.26%	100.00%
	0.9955	16,455	100.00%	