THE IMPACT OF INFLATION ON THE THEORY OF LIFE CONTINGENCIES

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ABSTRACT: Current actuarial notation permits the calculation of annuities given annual benefit amounts that are consistent or that increase annually by a specified dollar amount. This paper, entitled "The Impact of Inflation on the Theory of Life Contingencies," provides a methodology that can be used to calculate the total value of annuities for which benefits are expected to increase annually by a specified percentage rate. The methodology permits the calculation of the value of the annuity on both nominal (undiscounted) and discounted bases.

The Impact of Inflation on the Theory of Life Contingencies

By Cecilia M. LePere

It is often necessary to consider the effect of annual increases in benefit amounts when calculating the value of workers compensation payments expected to be made over a period of time. This situation occurs most frequently when evaluating the cost of workers compensation benefits that will ultimately be paid to persons injured in states that provide for annual increases in indemnity benefits. Other circumstances arise in which it is necessary to approximate the value of future medical benefits that are expected to increase as a result of economic inflation.

The annual increase in indemnity benefits, often called escalation, is commonly calculated as a percentage of the current indemnity benefits an injured worker receives. The amount of the increase is usually determined based on the calculated percentage increase in the average weekly wage of workers in the state of jurisdiction governing the benefits. The increase may be capped at a maximum which is stipulated in the state's workers compensation law.

The annual increase in medical benefits is typically measured by changes in the medical consumer price index. The annual changes in medical benefits are commonly expressed as a percentage increase over the current year's medical benefits.

The intent of this article is to provide a methodology that can be used to calculate the value of the escalated benefits. The formulas are developed in a way that permits the evaluation of the expected costs on both nominal and present value bases.

A life contingency commutation formula is available for situations in which the benefits paid increase by a specified dollar amount for each year the claimant survives. This formula is commonly represented $(Ia)_x$ and denotes the present value of an increasing immediate life annuity payable to a person who is x years of age. The formula assumes an annual benefit of one unit is paid at the end of year one, two units are paid at the end of year two, three units are paid at the end of year three, etc. $(Ia)_x$ has been defined as follows.

$$(Ia)_x = \sum_{t=1}^{\infty} t v^t \; {}_tp_x$$

$$= \sum_{t=0}^{\infty} {}_{t|}a_x$$

$$= \sum_{t=0}^{\infty} \frac{N_{x+t+1}}{D_x}$$

$$= \frac{S_{x+1}}{D_x} \quad \text{where} \quad S_x = \sum_{t=0}^{\infty} N_{x+t} \qquad (1)$$

Similarly, an n-year temporary increasing annuity can be expressed as follows.

$$(Ia)_{x:\overline{n}|} = \sum_{t=o}^{n-1} {}_{t|n-t}a_x$$

$$= \sum_{t=o}^{n-1} \frac{N_{x+t+1} - N_{x+n+1}}{D_x}$$

$$= \frac{S_{x+1} - S_{x+n+1} - nN_{x+n+1}}{D_x} \qquad (2)$$

In the formulas referenced, the payments increase at a constant dollar amount equal to one unit each year. However, as discussed previously, there are situations which may occur in which payments are expected to increase at a constant percentage rate rather than by a constant dollar amount.

The annual percentage rate the payments are expected to increase has been defined as the annual rate of escalation. The calculation of the present value of these payments can be simplified through an adjustment in the life contingency formulas previously referenced. The adjustment is made in the portion of the commutation formula that reflects the annual rate of discount for interest. For example, Assume:

Annual Escalation Percentage Rate = e
Annual Interest Percentage Rate = i
Current Age of Life = x

Present Value of Future Benefits =

$$a_x^e = \sum_{t=1}^{\omega-x-1} v_e^t \; {}_tp_x \qquad where\ v_e^t = \left(\frac{1+e}{1+i}\right)^t$$

$$= \sum_{t=0}^{\omega-x-1} \frac{D_{x+t}^e}{D_x^e}$$

$$= \frac{N_x^e}{D_x^e}$$

It is important to point out that, in the case in which benefits are constant over the duration of the claim payments, the escalation percentage equals zero. Therefore, v_e^t in the above formula becomes v^t (i.e. $\left(\frac{1+e}{1+i}\right)^t$ becomes $\left(\frac{1}{1+i}\right)^t$. As a result, the formula (3) above condenses to the form of an immediate life annuity, a_x.

Utilizing this formula simplifies the calculation of the present value of an annuity that increases annually at a constant percentage. A similar procedure can be used to consider a constant percentage of escalation in deferred and temporary annuities as well as in annuities due and continuous annuities. Therefore, this methodology can be utilized to reflect annual changes in the escalation rate as well as the unique pattern of payments for the annuities.

As can be determined from the formulas shown, the discount factor for interest can offset the impact of escalation to some extent. If the rate of interest equals the escalation rate, the effective rate equals zero percent. This means that any increases in payments that are expected as a result of escalation will be totally offset by the interest expected to be earned on the invested funds. For example, if i=6% and e=6%, the effective rate equals 1.06/1.06. The resulting factor of 1.0 implies there is no effective annual increase in cost. Likewise, an escalation rate of 1.06 and an interest rate of 1.035 imply an effective annual increase of 1.06/1.035 or 1.024. Finally, given an escalation rate of 1.060 and an interest rate of 1.070, an effective annual decrease of 1.060/1.070 or .991 is implied.

Commutation formulas have been derived in Table 1 based on the mortality table shown in Table 2. The mortality table has been computed using the U.S. Life Tables for Total Population and are based on data compiled by the U.S. Department of Commerce from the 1969-71 census of the United States. These tables are utilized to calculate the present value of future payments in the following example.

Example:

Claimant's age: 35 years
Annual benefit: $5,200
Interest rate: 3.5%
Escalation rate: 6.0%
Duration of benefits: Life
Benefits are paid at the end of each year.

$$\$5,200 \times a^e_{35} = \$5,200 \times \frac{N^e_{35}}{D^e_{35}}$$

$$= \$5,200 \times \frac{15,315,238}{217,842}$$

$$= \$365,583$$

$$\text{where; } D^e_{35} = v_e^{35}\ l_{35}$$

$$= \left(\frac{1.060}{1.035}\right)^{35} \times 94,482$$

$$= 217,842$$

$$\text{and; } N^e_{35} = \sum_{t=35}^{110} D^e_t$$

$$= D^e_{35} + D^e_{36} + D^e_{37} + \ldots + D^e_{110}$$

$$= 217,842 + 222,639 + 227,504 + \ldots + 0$$

$$= 15,315,238$$

TABLE 1

Commutation Amounts Considering Escalation*

Age	D^e_x	N^e_x	Age	D^e_x	N^e_x	Age	D^e_x	N^e_x
0	100,000	20,532,860	36	222,639	15,097,396	73	313,587	4,047,766
1	100,365	20,432,860	37	227,504	14,874,757	74	306,248	3,734,179
2	102,661	20,332,495	38	232,430	14,647,253	75	297,724	3,427,931
3	105,051	20,229,834	39	237,410	14,414,823	76	287,990	3,130,207
4	107,514	20,124,783	40	242,440	14,177,413	77	277,072	2,842,217
5	110,047	20,017,269	41	247,514	13,934,973	78	265,047	2,565,145
6	112,649	19,907,222	42	252,632	13,687,459	79	252,029	2,300,098
7	115,316	19,794,573	43	257,774	13,434,827	80	238,139	2,048,069
8	118,050	19,679,257	44	262,934	13,177,053	81	223,418	1,809,930
9	120,856	19,561,207	45	268,091	12,914,119	82	207,946	1,586,512
10	123,732	19,440,351	46	273,238	12,646,028	83	191,900	1,378,566
11	126,682	19,316,619	47	278,359	12,372,790	84	175,515	1,186,666
12	129,703	19,189,937	48	283,447	12,094,431	85	158,994	1,011,151
13	132,789	19,060,234	49	288,481	11,810,984	86	142,383	852,157
14	135,934	18,927,445	50	293,447	11,522,503	87	125,778	709,774
15	139,130	18,791,511	51	298,315	11,229,056	88	109,521	583,996
16	142,374	18,652,381	52	303,065	10,930,741	89	94,037	474,475
17	145,666	18,510,007	53	307,668	10,627,676	90	79,660	380,438
18	149,010	18,364,341	54	312,084	10,320,008	91	66,490	300,778
19	152,415	18,215,331	55	316,289	10,007,924	92	54,553	234,288
20	155,887	18,062,916	56	320,245	9,691,635	93	43,932	179,735
21	159,427	17,907,029	57	323,930	9,371,390	94	34,709	135,803
22	163,038	17,747,602	58	327,303	9,047,460	95	26,897	101,094
23	166,724	17,584,564	59	330,344	8,720,157	96	20,447	74,197
24	170,490	17,417,840	60	333,009	8,389,813	97	15,301	53,750
25	174,345	17,247,350	61	335,272	8,056,804	98	11,273	38,449
26	178,294	17,073,005	62	337,090	7,721,532	99	8,200	27,176
27	182,338	16,894,711	63	338,418	7,384,442	100	5,896	18,976
28	186,478	16,712,373	64	339,198	7,046,024	101	4,178	13,080
29	190,707	16,525,895	65	339,380	6,706,826	102	2,932	8,902
30	195,025	16,335,188	66	338,909	6,367,446	103	2,045	5,970
31	199,423	16,140,163	67	337,728	6,028,537	104	1,400	3,925
32	203,908	15,940,740	68	335,790	5,690,809	105	956	2,525
33	208,475	15,736,832	69	333,062	5,355,019	106	653	1,569
34	213,119	15,528,357	70	329,242	5,021,957	107	437	916
35	217,842	15,315,238	71	325,114	4,692,715	108	290	479
			72	319,835	4,367,601	109	189	189
						110	0	0

* Based on the Life Tables for Total Population compiled from the 1969-71 census.

Annual Rate of Interest 3.5%
Annual Rate of Escalation 6.0%

TABLE 2

Mortality Table*

Age	l_x	Age	l_x	Age	l_x
0	100,000	36	94,285	73	54,913
1	97,998	37	94,073	74	52,363
2	97,876	38	93,843	75	49,705
3	97,792	39	93,593	76	46,946
4	97,724	40	93,322	77	44,101
5	97,668	41	93,028	78	41,192
6	97,619	42	92,712	79	38,245
7	97,573	43	92,368	80	35,285
8	97,531	44	91,995	81	32,323
9	97,494	45	91,587	82	29,375
10	97,460	46	91,144	83	26,469
11	97,430	47	90,662	84	23,638
12	97,401	48	90,142	85	20,908
13	97,367	49	89,579	86	18,282
14	97,322	50	88,972	87	15,769
15	97,261	51	88,315	88	13,407
16	97,181	52	87,605	89	11,240
17	97,083	53	86,838	90	9,297
18	96,970	54	86,007	91	7,577
19	96,846	55	85,110	92	6,070
20	96,716	56	84,142	93	4,773
21	96,580	57	83,103	94	3,682
22	96,438	58	81,988	95	2,786
23	96,292	59	80,798	96	2,068
24	96,145	60	79,529	97	1,511
25	96,000	61	78,181	98	1,087
26	95,859	62	76,751	99	772
27	95,721	63	75,236	100	542
28	95,586	64	73,631	101	375
29	95,448	65	71,933	102	257
30	95,307	66	70,139	103	175
31	95,158	67	68,246	104	117
32	95,003	68	66,254	105	78
33	94,840	69	64,166	106	52
34	94,666	70	61,934	107	34
35	94,482	71	59,715	108	22
		72	57,360	109	14
				110	0

* Based on the Life Tables for Total Population compiled from the 1969-71 census as compiled by the U.S. Department of Commerce.

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