AN ACTUARIAL ANALYSIS OF THE NCCI REVISED EXPERIENCE RATING PLAN

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Abstract

Recently the National Council on Compensation Insurance has significantly revised the Experience Rating Plan for Workers' Compensation. The new plan is referred to as the Revised Experience Rating Plan. It is a practical application of credibility theory using parameter uncertainty and risk heterogeneity.

This paper compares the revised plan to the prior experience rating plan, with particular emphasis on a comparison of the credibility formulas used in the two plans.

Examples are shown to illustrate the overall pattern and general conclusions concerning the differences between the prior and revised plans.

The dependence of credibility on size of risk is discussed from a more theoretical point of view in an Appendix.

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AN ACTUARIAL ANALYSIS OF THE NCCI *REVISED EXPERIENCE RATING PLAN*

INTRODUCTION

Recently the National Council on Compensation Insurance has significantly revised the Experience Rating Plan for Workers' Compensation. This followed a detailed actuarial study of the performance of the prior plan and possible alternatives. This study is explained in Venter [1] and Gillam [2].

The new plan that is the result of this study was originally given the acronym SERA (Simplified Experience Rating Adjustment), but it is now referred to as the Revised Experience Rating Plan. This paper compares the revised plan to the prior experience rating plan.

As shown in Exhibit 1, the revised plan shares many of the features of the prior plan. Administratively the plans are the same. Actuarially there have been important changes. The revised plan is a single split plan rather than a multi-split plan. Also, the credibilities that are determined by the W and B values are very different.[1]

The first section reviews the actuarial formulas underlying the two experience rating plans. Readers who do not want to deal with a lot of formulas may wish to go right to the second section.

The second section compares the credibilities under the two experience rating plans. Examples are shown to illustrate the overall pattern and general conclusions concerning the differences between the prior and revised plans.

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[1]The W (Weighting) and B (Ballast) values are defined in formulas 6 and 8, and are used in formula 1.

The Revised Experience Rating Plan is a practical application of Credibility Theory using parameter uncertainty and risk heterogeneity. The dependence of credibility on size of risk is discussed from a more theoretical point of view in an Appendix.

ACTUARIAL FORMULAS UNDERLYING EXPERIENCE RATING

The following formula is used in both the prior plan and the revised plan in order to calculate the experience modification.

$$M = \frac{A_p + B + WA_e + (1-W)E_e}{E_p + B + WE_e + (1-W)E_e} \qquad (1)$$

Where M = Experience Modification

A_p = Actual Primary Losses

A_e = Actual Excess Losses

E_p = Expected Primary Losses

E_e = Expected Excess Losses

B = Ballast Value

W = Weighting Value

Under both plans the W and B values vary with the expected losses and are displayed in a table. However, the formulas used to determine W and B are significantly different under the two plans. An example of W and B values for both plans is shown in Exhibit 5.

In order to compare the plans, it is useful to reframe the formulas in terms of credibilities. Following the development in Snader [3]:

$$\text{Let } Z_p = \frac{E}{E + B} \qquad (2.a)$$

$$Z_e = \frac{E}{E + \frac{B + (1-W)\,E}{W}} = \frac{W\,E}{E + B} = WZ_p \qquad (2.b)$$

This can also be written in terms of the usual Bayesian formula for credibility as:

$$Z_p = \frac{E}{E+K_p} \qquad (3.a)$$

$$Z_e = \frac{E}{E+K_e} \qquad (3.b)$$

with the credibility parameters K_p and K_e depending on the expected losses E, W and B:

$$K_p = B \qquad (4.a)$$

$$K_e = \frac{B + (1-W)\,E}{W} \qquad (4.b)$$

Then the modification formula 1 becomes in terms of the credibilities:

$$M = \frac{(1-Z_p)\,E_p + Z_p\,A_p + (1-Z_e)\,E_e + Z_e\,A_e}{E} \qquad (5)$$

under the prior plan:

$$B = (1-W)\ 20000 \qquad (6.a)$$

$$W = \begin{cases} 0 & E \le 25000 \\ \dfrac{E-25000}{S-25000} & S \ge E \ge 25000 \\ 1 & E \ge S \end{cases} \qquad (6.b)$$

Where S is the self-rating point.

Under the revised plan, the values of the credibility parameters K_p and K_e are given via formula, and then B and W follow from them. The formulas in terms of the state reference point S are:

$$K_p = E\left[\frac{.1E + .01028S}{E + .0028S}\right] \qquad (7.a)$$

K_p is subject to a minimum of 7500. K_p subject to this minimum is labeled B by the NCCI.

$$K_e = E\left[\frac{.75E + .8153S}{E + .0204S}\right] \qquad (7.b)$$

K_e is subject to a minimum of 150,000. K_e subject to this minimum is labeled C by the NCCI.

Formulas 7 can also be stated in terms of g.[2] These formulas are the ones used by the NCCI.[3]

$$K_p = E\left[\frac{.1E + 2500g}{E + 700g}\right] \qquad (7.a')$$

$$K_e = E\left[\frac{.75E + 200,000g}{E + 5100g}\right] \qquad (7.b')$$

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[2]The state specific parameter g is defined by the NCCI as the average claim cost in the state divided by $1000; g is rounded to the nearest .05.

[3]The two sets of formulas only differ due to rounding. The NCCI has rounded 2570 to 2500 and 203,825 to 200,000.

Thus under the revised plan, the credibility parameters have the form

$E_{\frac{\text{Linear}}{\text{Linear}}}$.

As explained in the Appendix, this is the form that is expected when the phenomena of parameter uncertainty and risk heterogeneity are important.[4] The NCCI determined the particular coefficients used in the revised plan by empirical testing.[5]

By solving the set of equations 4 one can express W and B in terms of K_p and K_e. These equations are used to determine W and B from K_p and K_e.[6]

$$B = K_p \qquad (8.a)$$

$$W = \frac{E + K_p}{E + K_e} \qquad (8.b)$$

W is subject to a minimum of .07.

CREDIBILITIES, PRIOR PLAN VS. REVISED PLAN

Under the revised plan the credibilities differ from the prior plan. The credibilities assigned to the primary[7] and excess losses are each significantly

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[4]In Mahler [4] at page 178, the result for a split plan is given as $E_{\frac{\text{Quadratic}}{\text{Quadratic}}}$. However, when the covariance of excess and primary losses is not extremely important, the *no-split plan* result of $E_{\frac{\text{Linear}}{\text{Linear}}}$ is a sufficiently close approximation. Since the observed correlation between the excess and primary losses is usually 95% or more, this is an area for further research.

[5]See Venter [1] and Gillam [2].

[6]The NCCI actually defines B as K_p subject to the minimum. The NCCI defines C as K_e subject to the minimum. Then W=(E+B)/(E+C).

[7]Under the revised plan the definition of primary losses is changed. Thus the D-ratios, which measure the expected portion of the losses that will be
(Footnote Continued)

different, as can be seen in Exhibits 2, 3 and 4:

1. For small risks, Primary Credibilities are larger.
2. For large risks, Primary Credibilities are smaller. The maximum Primary Credibility is 91%, rather than 100% as under the prior plan. This means no more self-rating.
3. For small risks, Excess Credibilities are a little larger. Even very small risks have a small non-zero Excess Credibility, as opposed to zero under the prior plan.
4. For large risks, Excess Credibilities are much smaller. The maximum Excess Credibility is 57%, rather than 100% as under the prior plan.

Thus one important change is that under the revised plan there are no longer self-rated risks. The primary losses are assigned a maximum credibility of 91%, while the excess losses are assigned a maximum credibility of 57%. Thus the maximum credibility assigned to any risk is approximately 70%.[8]

It follows from formulas 3 and 7 that under the revised plan the credibilities as a function of the size of risk are of the form $\frac{\text{linear}}{\text{linear}}$. This can be written as:

$$Z = \frac{E + I}{JE + I + K} \qquad \begin{matrix} 0 \leq I \\ J \leq I \\ 0 \leq K \end{matrix} \qquad (8)$$

(Footnote Continued)
primary, have to be recalculated with the adoption of the revised plan. In one state (Massachusetts) the average D-ratio decreased from about .35 to about .30. The results will vary by state, depending on the size of loss distribution, which depends heavily on the particular state Workers' Compensation Law.

[8]Assuming a D-ratio of D, the maximum credibility is (Dx91%) + ((1-D)x57%). For D=.50 the maximum credibility is 74%. For D=.35 the maximum credibility is 69%. For D=.20 the maximum credibility is 64%.

with one formula for primary credibility and one formula for excess credibility, each with different constants I, J, and K. This is the form of credibility one expects if both parameter uncertainty and risk heterogeneity are important.[9] The more familiar formula for credibility is a special case of formula 8, with I = 0 and J = 1.

In the more familiar formula Z = E/(E+K) the parameter K is a "scale parameter." Changing K changes the overall scale of the credibility curve without changing its shape. As will be discussed below K, and thus the scale of the curve, depends on a state specific inflation sensitive parameter.

In formula 8 used in the revised plan, there are two additional parameters I and J which are "shape parameters." Changing I and/or J changes the shape of the credibility curve. The size of the parameter I relative to the parameter K adjusts the shape of the credibility curve for small risks. The minimum credibility is given by I/(I+K), which is determined by the ratio of I to K.

The parameter J adjusts the shape of the credibility curve for large risks. The maximum credibility is given by 1/J.

Thus the revised plan uses a more general formula for credibility, which is better able to approximate those credibilities that would have performed well in the past and thus are expected to work well in the future.[10] As shown in the Appendix, one could derive even more general formula than formula 8. As a function of the size of risk, the credibilities given by formulas A.11 in the Appendix are of the form quadratic.
quadratic

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[9]See Equation 1.6 in Mahler [4]. What was denoted as K there, is denoted as I+K here. This is a matter of notation rather than substance. The notation used here allows K to have the same underlying source in both formula 8 and the more familiar formula for credibility.

[10]The criterion used by the NCCI to determine which credibilities performed well are discussed in Venter [1] and Gillam [2].

This more general formula for credibility is somewhat better able to approximate those credibilities that would have performed well in the past. The two additional parameters can be adjusted so as to adjust the shape of the credibility curve for medium-size risks. In any given application, one has to decide whether the extra generality introduced by these additional parameters is worth the extra complications also introduced.

The specific formulas for Z_p and Z_e used in the revised plan are:

$$Z_p = \frac{E + .0028S}{1.1E + .01308S} \qquad (9.a)$$

$$Z_e = \frac{E + .0204S}{1.75E + .8357S} \qquad (9.b)$$

where S is the State Reference Point.[11]

These formulas can also be stated in terms of the parameter g:[12]

$$Z_p = \frac{E + 700g}{1.1E + 3270g} \qquad (9.a')$$

$$Z_e = \frac{E + 5100g}{1.75E + 208925g} \qquad (9.b')$$

Thus under the revised plan, the primary and excess credibilities are each given by formula 8, with the following parameters:

	Primary	Excess
I	.0028S = 700g	.0204S = 5100g
J	1.1	1.75
K	.01028S = 2570g	.8153S = 203825g

[11]The State Reference Point is calculated as 250 times the average cost per case in the particular state.

[12]The parameter g is calculated as the average cost per case in the particular state divided by 1000. g is rounded to the nearest .05.

If for example, S=$500,000, and g=2,[13] then the parameters would be:

	Primary	Excess
I	$1,400	$10,200
J	1.1	1.75
K	$5,140	$407,650

Note that the curves for primary and excess credibilities under the revised plan have a significantly different scale from each other due to their vastly different values of the parameter K. As is shown in Exhibit 2, the two curves also have significantly different shapes due to their different values of the parameter J and different ratios of I to K.

The values for the credibilities underlying actual experience ratings may differ slightly from those calculated using formulas 9 due to the rounding process involved in establishing a table of W and B values. Also they will differ for small risks (those with expected losses below about $20,000) because of the minimums imposed on the parameters W, Kp and Ke.[14]

For the smaller risks, there are maximum values imposed on the experience rating modification under the revised plan.

Expected Losses	Maximum Modification
0 to $5,000	1.6
$5,000 to $10,000	1.8
$10,000 to $15,000	2.0

The maximum debit and credit for small risks are compared in Exhibit 6.

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[13]These correspond to an average case of $2,000.

[14]The imposition of minimums on Kp and Ke reduces the credibility assigned to very small risks (those with expected losses below about $6,000). The imposition of a minimum on W increases the credibility assigned to the excess losses of small risks.

POSSIBLE FURTHER RESEARCH

It would be interesting to compare the more general formula A.11 versus formula A.10 using the same types of tests as performed by the NCCI.

Another area for possible research is the number of years of data used in the experience period. Currently three years are given equal weight.[15] One could test whether some other combination of number of years and weights could produce a more accurate result.[16]

SUMMARY

The Revised Experience Rating Plan is based on significantly different credibility formulas than the prior plan. This results in a significantly more responsive plan for small risks and a significantly less responsive plan for large risks.

While the Revised Experience Rating Plan has a firmer theoretical and empirical basis than the prior plan, there remain areas for further actuarial research.

ACKNOWLEDGEMENTS

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[15] Actually since more recent years have more payroll on average due to inflation, the most recent year on average has somewhat more weight.

[16] As pointed out in Mahler [4], the optimal set of years and weights will depend on to what extent the risk parameters of an insured are shifting over time.

Comparison of Workers' Compensation Experience Rating Plans

Prior	Revised
Primary and Excess Losses	Primary and Excess Losses
Multi-split Plan: Primary portion of a loss is determined via formula[1] or from a table.	Single Split Plan: Primary portion of a loss is the first $5000.
Experience Modification depends on a comparison of actual losses to expected losses, taking into account credibilities.	Experience Modification depends on a comparison of actual losses to expected losses, taking into account credibilities.
W and B values are shown in a table, and depend on the expected losses for the risk.	W and B values are shown in a table, and depend on the expected losses for the risk.
The table of W and B values depends on a state specific value, the Self-Rating Point. (SRP)	The table of W and B values depends on a state specific value, the State Reference Point. (SRP)[2]
The per claim accident limitation is 10% of the State's Self-Rating Point.	The per claim accident limitation is 10% of the State Reference Point.
The State Multiple Claim Accident Limitation is twice The State Per Claim Accident Limitation.	The State Multiple Claim Accident Limitation is twice The State Per Claim Accident Limitation.

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[1] $A_p = 10000\ A/(A + 8000)$. For losses less than 2000, the whole loss is considered primary.

[2] The State Reference Point is equal to 250 times the average claim cost in the particular state. The NCCI uses the state specific parameter g which is defined as the average claim cost in the state divided by $1000; g is rounded to the nearest .05. g=SRP/250,000.

NCCI Revised Experience Rating

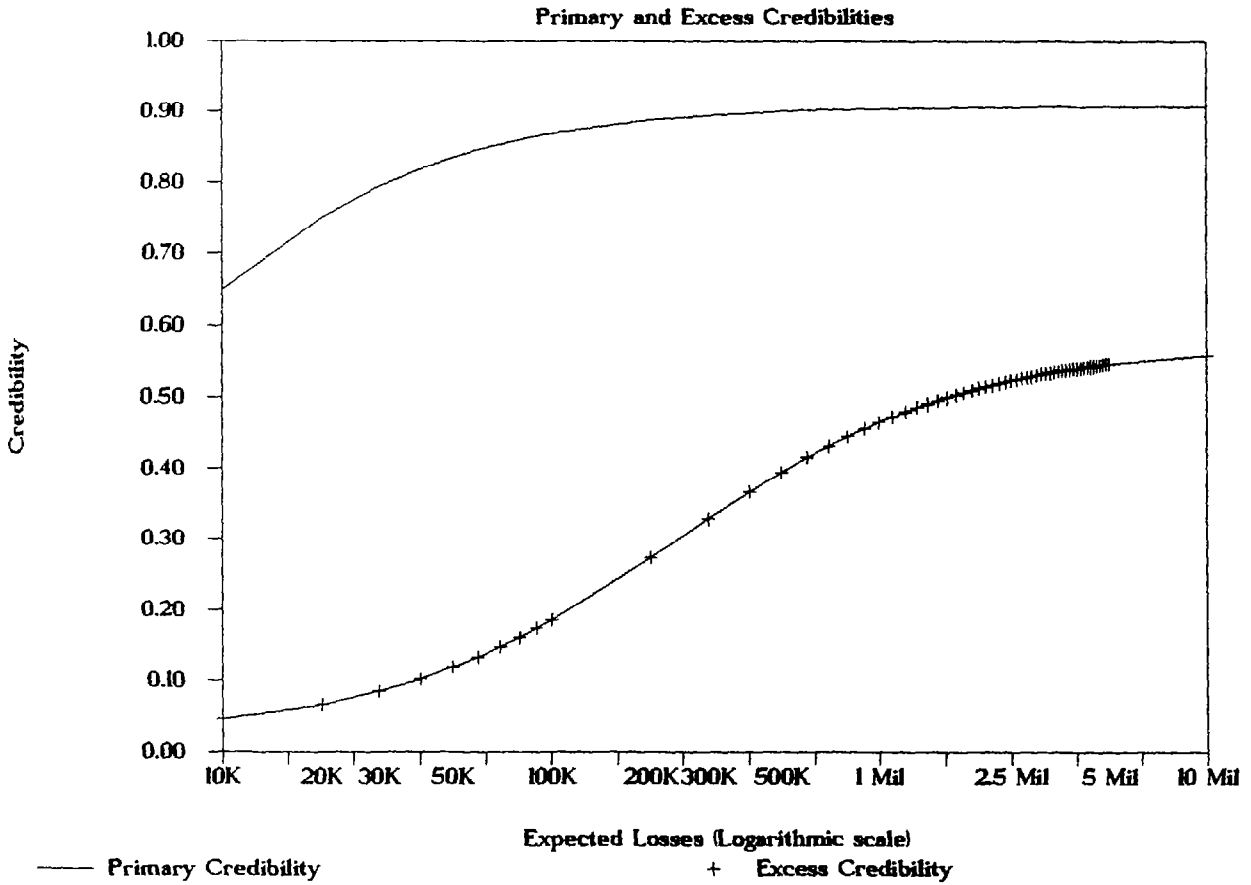

EXHIBIT 3

Workers' Compensation Experience Rating

Credibilities
(Weighted Average of Primary & Excess Credibilities)

Expected Losses ($000)	Prior*	Revised**	Revised Minus Prior***
3****	5%	10%	5%
5	7	14	7
7.5	10	18	8
10	12	20	9
15	15	24	9
20	18	26	9
25	19	28	9
50	27	33	7
75	31	37	6
100	34	39	5
125	36	41	5
150	39	43	4
200	43	46	3
300	51	50	-1
400	58	53	-5
500	66	55	-11
750	83	58	-24
1000	100	59	-41
2000	100	63	-37
3000	100	65	-35
4000	100	65	-35
5000	100	65	-35
7500	100	66	-34
10000	100	66	-34
∞	100	67	-33

* NCCI Experience Rating Plan prior to revision, assuming a Self-Rating Point of $1,000,000 and a D-ratio of .35.

** Revised Experience Rating Plan, assuming a State Reference Point of $500,000 and a D-ratio of .30.

*** Result may differ slightly due to intermediate rounding.

**** Eligibility requirements vary by state. In most states $3,000 in expected losses is currently close to the minimum size ever experience rated.

EXHIBIT 4

Workers' Compensation Experience Rating

Credibilities

Expected Losses ($000)	Primary		Excess	
	Prior*	Revised**	Prior*	Revised**
3***	13%	29%	0%	2%
5	20	40	0	3
7.5	27	50	0	4
10	33	57	0	5
15	43	67	0	6
20	50	73	0	7
25	56	77	0	8
50	72	83	2	12
75	80	86	4	15
100	84	87	7	18
125	87	88	9	21
150	90	88	12	24
200	92	89	17	28
300	95	90	27	33
400	97	90	37	37
500	98	90	48	40
750	99	90	73	44
1000	100	90	100	46
2000	100	91	100	52
3000	100	91	100	54
4000	100	91	100	54
5000	100	91	100	54
7500	100	91	100	55
10000	100	91	100	55
∞	100	91	100	57

* NCCI Experience Rating Plan prior to revision, using Self-Rating Point of $1,000,000 (assumes average serious case of $40,000).

** Revised Experience Rating Plan, using State Reference Point of $500,000 (assumes average case of $2,000).

*** Eligibility requirements vary by state. In most states $3,000 in expected losses is currently close to the minimum size risk ever experience rated.

Workers' Compensation Experience Rating

W and B Values

Expected Losses ($000)	B ($00) Prior*	B ($00) Revised**	W Prior*	W Revised**
3***	200	75	0	.07
5	200	75	0	.08
7.5	200	75	0	.08
10	200	75	0	.08
15	200	75	0	.09
20	200	75	0	.09
25	200	75	0	.10
50	194	99	.03	.14
75	190	124	.05	.18
100	184	149	.08	.21
125	180	174	.10	.24
150	174	200	.13	.27
200	164	250	.18	.31
300	144	350	.28	.37
400	124	450	.38	.41
500	102	550	.49	.44
750	52	800	.74	.49
1000	0	1050	1.00	.51
2000	0	2050	1.00	.57
3000	0	3050	1.00	.59
4000	0	4050	1.00	.60
5000	0	5050	1.00	.60
7500	0	7550	1.00	.61
10000	0	10050	1.00	.61

* NCCI Experience Rating Plan prior to revision using a Self-Rating Point of $1,000,000 (assumes average serious case of $40,000).

** Revised Experience Rating Plan, using State Reference Point of $500,000 (assumes average case of $2,000).

*** Eligibility requirements vary by state. In most states $3000 in expected losses is currently close to the minimum size risk ever experience rated.

EXHIBIT 6

Workers' Compensation Experience Rating

Revised Experience Rating Plan*

Expected Losses ($000)	Maximum Credit** D-ratio=.40	D-ratio=.30	D-ratio=.20	Maximum Debit
3***	13%	10%	7%	60%
4	15	12	9	60%
5	18	14	11	60%
6	20	16	12	80%
7	22	17	13	80%
8	23	18	14	80%
9	24	19	14	80%
10	26	20	15	80%
11	27	21	16	100%
12	28	22	16	100%
13	28	23	17	100%
14	29	23	17	100%
15	30	24	18	100%
16	31	25	19	No Limit

* Revised Experience Rating Plan, using State Reference Point of $500,000 (assumes average case of $2,000).

** The maximum credit depends on the particular D-ratio. The maximum credit is the credibility which is equal to D x primary credibility + (1-D) x excess credibility.

*** Eligibility requirements vary by state. In most states $3,000 in expected losses is currently close to the minimum size risk ever experience rated.

References

[1] G. G. Venter, "Experience-Rating-Equity and Predictive Accuracy," NCCI Digest, Volume II, Issue I, April 1987, p. 27.

[2] W. R. Gillam, "Parametrizing the Workers' Compensation Experience Rating Plan," CAS Discussion Paper Program 1990.

[3] R. H. Snader, "Fundamentals of Individual Risk Rating and Related Topics," Part I of Study Note, Casualty Actuarial Society, 1980.

[4] H. C. Mahler, Discussion of G. G. Meyers "An Analysis of Experience Rating," PCAS LXXIV, 1987, p. 119.

[5] G. G. Meyers, "An Analysis of Experience Rating," PCAS LXXII 1985, p. 278.

Appendix, Dependence of Credibility On Size of Risk

In this appendix, the variation of credibility with size of risk will be discussed. Formulas A.10 are those used in the revised experience rating plan. The theoretical underpinnings of these formulas, as well as the more general formulas A.11, are discussed.

Following the development in Mahler [4] let

a = total variance of the primary losses
b = total variance of the excess losses
c = variance of the hypothetical means of the primary losses
d = variance of the hypothetical means of the excess losses
r = total covariance of the primary and excess losses
s = covariance of hypothetical means of the primary and excess losses

Then the optimum least squares credibilities Z_p and Z_e are derived in Appendix F of Mahler [4] and given in equations 5.3 and 5.4 of that paper as:

$$Z_p = \frac{(c+s)b - (d+s)r}{ab-r^2} \qquad \text{(A.1.a)}$$

$$Z_e = \frac{(d+s)a - (c+s)r}{ab-r^2} \qquad \text{(A.1.b)}$$

Thus both the primary and excess credibilities can be written in terms of variances and covariances.

Thus the dependence of the credibilities on the size of the risk can be derived from the dependence of the various variances and covariances on the size of the risk.

Again following Mahler [4] let

t = a-c = process variance of the primary losses
u = b-d = process variance of the excess losses
v = r-s = process covariance of the primary and excess losses

Then substituting into equations (A.1) one gets:

$$Z_p = \frac{(c+s)(u+d)-(d+s)r}{(t+c)(u+d)-(v+s)^2} \qquad \text{(A.2.a)}$$

$$Z_e = \frac{(d+s)(t+c)-(c+s)r}{(t+c)(u+d)-(v+s)^2} \qquad (A.2.b)$$

The NCCI credibility parameters K_p and K_e are defined so that:

$$Z = \frac{E}{E+K}$$

and therefore

$$K = E(\frac{1}{Z}-1) \qquad (A.3)$$

Substituting into equation A.3 the expressions for Z_p and Z_e given in equations A.2 one obtains:

$$K_p = E\frac{tu+td+vd-su-sv-v^2}{cu+su+cd-s^2-sv-dv} \qquad (A.4.a)$$

$$K_e = E\frac{tu+uc+vc-st-sv-v^2}{dt+st+cd-s^2-sv-cv} \qquad (A.4.b)$$

If the covariances between the primary and excess losses are zero, $v=s=0$,[1] i.e., if there is no useful information about the primary losses contained in the excess losses and vice versa, then these equations are greatly simplified:

$$K_p = E\frac{t}{c} \qquad (A.5.a)$$

$$K_e = E\frac{u}{d} \qquad (A.5.b)$$

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[1]This assumption would yield a good approximation if these covariances are small in magnitude compared to the variances and covariances that enter into the formulas. In fact these covariances are observed to be significantly different from zero. The total covariance of primary and excess losses, $r=s+v$, is generally positive in actual applications. For Workers' Compensation the correlation between primary and excess losses is generally 95% or more.

Each of the two separate pieces, which are assumed to be uncorrelated with each other, has credibility parameter given by the familiar Buhlmann result.

It is formulas A.5 that form the theoretical bases of the credibilities used by the NCCI in the revised experience rating plan, rather than the more complicated but more general formulas A.4.[2]

It is generally assumed that process variances and covariances (so-called "within" variances and covariances) such as t, u and v, increase proportionally with E, the size of risk.

$$t \sim E \qquad \text{(A.6.a)}$$
$$u \sim E \qquad \text{(A.6.b)}$$
$$v \sim E \qquad \text{(A.6.c)}$$

However, as shown in Meyers [5] when the phenomena of parameter uncertainty is important, formulas A.6 do not hold. Instead, t, u, and v increase partially proportionally with E and partially proportionally with E squared.[3] When parameter uncertainty is important:

$$t \sim E \text{ Linear } [E] \qquad \text{(A.7.a)}$$
$$u \sim E \text{ Linear } [E] \qquad \text{(A.7.b)}$$
$$v \sim E \text{ Linear } [E] \qquad \text{(A.7.c)}$$

It is generally assumed that variances and covariances of the hypothetical means (so-called "between" variances and covariances) such as c, d, and s, increase proportionally with the square of E, the size of risk.

$$c \sim E^2 \qquad \text{(A.8.a)}$$
$$d \sim E^2 \qquad \text{(A.8.b)}$$
$$s \sim E^2 \qquad \text{(A.8.c)}$$

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[2]However, both formulas A.5 and A.4 will be treated in the remainder of this appendix. Formulas A.11 follow from formulas A.5, while formulas A.10 follow from formulas A.4.

[3]As discussed in Mahler [4], the portion of the process variance or covariance which is proportional to the square of E represents the variation of the parameters due to the different states of the universe.

However, as shown in Mahler [4] in the presence of risk heterogeneity, formulas A.8 do not hold. Instead, c, d, and s increase partially proportionally with E and partially proportionally with E-squared.[4] When risk heterogeneity is important:

$$c \sim E \text{ Linear } [E] \qquad \text{(A.9.a)}$$
$$d \sim E \text{ Linear } [E] \qquad \text{(A.9.b)}$$
$$s \sim E \text{ Linear } [E] \qquad \text{(A.9.c)}$$

One can substitute the behavior of the variances and covariances with size of risk into the equations for the credibility parameters K. The revised experience rating plan is based on formulas A.5, with parameter uncertainty (formulas A.7) and risk heterogeneity (formulas A.9). Substituting formulas A.7 and A.9 into formulas A.5 gives:

$$K_p \sim \frac{E \text{ Linear } [E]}{\text{Linear } [E]} \qquad \text{(A.10.a)}$$

$$K_e \sim \frac{E \text{ Linear } [E]}{\text{Linear } [E]} \qquad \text{(A.10.b)}$$

This is the form of the credibility parameters used in the revised experience rating plan shown in the equations 7 in the main text.[5] This form of the credibility parameters, leads directly to the form of the credibilities shown in equations 9 in the main text.

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[4]As discussed in Mahler [4], the portion of the variance or covariance of the hypothetical means which is proportional to E represents the variation caused by grouping submits together to form a single risk. For example, several factories might belong to a single insured.

[5]This is the form for the No-Split Plan with parameter uncertainty and risk heterogeneity given at page 178 of Mahler [4].

If instead of the special case equation A.5, one starts with the more general equations A.4, one gets a different form for the credibility parameters. Substituting equations A.7 and A.9 into equations A.4, gives the following general form of the credibility parameters with parameter uncertainty and risk heterogeneity.[6]

$$K_p \sim E\frac{\text{Quadratic [E]}}{\text{Quadratic [E]}} \qquad \text{(A.11.a)}$$

$$K_e \sim E\frac{\text{Quadratic [E]}}{\text{Quadratic [E]}} \qquad \text{(A.11.b)}$$

Formulas A.10 are a special case of formulas A.11.

Therefore, formulas A.11 will always perform at least as well as and usually perform better than formulas A.10 in any empirical tests, including the type of studies conducted by the NCCI in its development of the revised experience rating plan. Practical considerations will determine whether in a particular application the extra generality represented by formulas A.11 is worth the extra complication introduced by the additional parameters contained in formulas A.11.

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[6]This is the form for the Split Plan with parameter uncertainty and risk heterogeneity given at page 178 of Mahler [4].