REINSURANCE AND RETENTIONS - A LONDON MARKET ACTUARIES GROUP PAPER, VOLUMES I AND II (GISG CONVENTION, 10/90)

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REINSURANCE AND RETENTIONS

A London Market Actuaries'
Group Paper

VOLUME I

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Section 1

INTRODUCTION AND CONCLUSIONS

Introduction

The working party adopted the following terms of reference:

1. To provide a review of some current practices in the field of reinsurance retentions.

2. To investigate and discuss those aspects of general insurance operations which we believe should influence the reinsurance decision process.

3. To present a synopsis of practical methods that may be used in order to translate the identified objectives of reinsurance into an explicit programme and retention policy.

We have defined the retention of a general insurance operation as all business which is not ceded including coinsured layers of excess of loss reinsurance, and any unplaced parts of the operation's reinsurance programme. We stress that we have used the word retention in its literal sense, namely, an amount retained. We consider that a company which has, for example, reinsured itself £90 million excess of £10 million has decided to retain claims excess of £100 million.

The remainder of the paper is divided into three sections. Section 2 covers some aspects of the current reinsurance market, Section 3 a discussion of the factors that influence the reinsurance programme and retention philosophy, and Section 4 summarises the practical methods for estimating aggregate claim distributions and retentions that we have reviewed. Detailed documentation of the application of these methods is contained in the appendices.

We have attempted to address the problems of retentions separately for all types of general insurer, including proprietary and mutual companies and Lloyd's syndicates.

We have assumed throughout that companies reserve accurately for claims since reserving problems and their effects on reinsurance strategy are, properly, the subject of a separate paper. We have not addressed the question of reinsurance security. In practice there is likely to be a trade-off between the cost and the quality of any reinsurance that is to be purchased.

Conclusions

During the last decade computer technology has leapt forward, but, reinsurance practices do not appear to have kept pace. This revolution enables insurance companies to store previously unimagined amounts of data. It also allows the technicians within those companies to experiment with much more ambitious risk management procedures. Therefore, it is likely that many opportunities exist for organisations who exploit the new technology to gain competitive advantage. This is because, historically, reinsurance practice must have applied unnecessary caution in the face of inadequate data and methodology.

A point of particular importance is that a seller of reinsurance will require a return on capital. The purchaser of the reinsurance must be aware of this fact. This is discussed further in Section 3. We have avoided use of the term "probability of ruin" because of the unhelpful connotations of the word ruin. We think that words such as "the probability of a £10 million reduction in earnings" are of more use and importance.

We can try to summarise this paper in one paragraph. First, we believe a retention should be defined as all business that is written but not ceded. Second, an insurer should review its objectives, and from this base develop a retention strategy. The insurer should view reinsurance as a benefit which will incur a cost. The aim must, therefore, be to use reinsurance as efficiently as possible. The retention strategy should be considered from the top down. The requirements of the entire operation must be determined and from this the implications for internal operating units should follow. Third, the estimation of the aggregate retained claims distribution is essential input into the retention process. This is an area where the actuary in particular can add considerable value. In the paper we present a number of methods which can be helpful in calculating these aggregate claim distributions and determining retentions.

Section 2

SOME ASPECTS OF THE CURRENT REINSURANCE MARKET

2.1 Introduction

This section reviews current practices and some of the rules of thumb for determining retentions.

There are a wide variety of reinsurance products. These range from a straightforward Quota Share treaty for a small proprietary insurance company to a financial reinsurance arrangement for a Lloyd's syndicate. We have not attempted here to cover the market practice across the whole field, but rather have concentrated on those aspects which we believe are important to the market as a whole.

Many insurance companies consider their retentions at three levels, "individual account" level, "company" level and "group" level. The overall retention that results is often built from the bottom up.

2.2 Retentions in Practice

It is worth pointing out that despite the increasing array of mathematical techniques available, decisions regarding retention levels are still based on rules of thumb, and a desire to conform to market norms. This is due, in part, to the impractical data requirements of some theoretical methods, and their often unrealistic assumptions (for example, independence of risks).

Companies may, for commercial reasons, also purchase more (or less) reinsurance than they need, or that various theories might imply. The practical importance of these commercial factors needs to be borne in mind when considering the validity of any methods, or theories, for setting retention levels.

In many instances, the choice of retention level is made by the underwriter of the account under consideration. He will use his skill and judgement, based on his knowledge of the account, to decide the best retention level. The aim, in deciding on this level is more likely to be to balance the relationship between profits and stability, rather than to reduce the risk that capital is exhausted. The probability of ruin is not a concept which underwriters are likely to consider.

A survey of U.S. insurance companies conducted by the Munich Re in 1976, showed that the main factors which were then considered when setting retention levels were, (in order of priority) level of capital, cost of reinsurance and smoothing of earnings fluctuations.

We are not aware of any more up to date surveys, but some previous studies (References 6 and 10) had highlighted the commonly held belief that retention levels should be positively correlated with the size of the company (as measured by premium income or capital/reserves). It is however thought that some composite insurers hold much lower retention levels than their size would indicate, perhaps due to the relatively low cost of reinsurance during a soft market, the risk aversion of the company, or other commercial reasons such as reciprocity. Also, a company which operates a profit-centre approach for each of its categories of business, without any central rationalisation, will probably have lower retentions than one which looks at its retentions on a more global basis.

Retentions considerations should focus on the amount of cover purchased as well as the size of the deductible. This is particularly true for event covers such as catastrophe excess of loss. Several insurance and reinsurance companies have developed their own loss accumulation systems which help them to decide how much catastrophe reinsurance to purchase. These systems can also prove useful in deciding the level of the catastrophe deductibles.

In practice, deciding on the deductible is only part of the process. The structure of the reinsurance programme will affect how much protection is provided. Factors such as the number of reinstatements purchased, inclusion of any drop-down facilities in the contract, vertical versus horizontal cover, and the availability of back-up covers will need to be considered. Underwriters look for continuity of cover: changes are gradual rather than sudden and will generally be in one direction (that is, upwards). There is often reluctance to increase the retention voluntarily.

Other important factors include the risk willingness of the company's management and the capacity (and, therefore, price) of the reinsurance market. Regardless of what retention may be theoretically correct, the market conditions may be such that cover is simply not available. An example of this was the upheaval of the retrocessional market which occurred following the windstorms in Europe in early 1990.

The extent to which companies/syndicates use brokers for advice about retention levels is unclear, but their use to provide alternative quotations for different reinsurance programmes is one way in which a company can obtain help to decide on the best retention. It should be noted, however, that a broker has traditionally earned a living from the placing of reinsurance rather than advising clients to retain risk.

2.3 Rules of Thumb for Setting Retention Levels.

2.3.1 Risk Theory Approach

This approach, which is based on a Normal approximation, assumes that the optimum retention is defined in terms of a per risk excess. Reduction of the probability of ruin to a certain minimum is the target. The theory is developed in Reference 1 and leads to formulae relating the retention, premium loading and free reserves.

These formulae, in turn, lead to a rule of thumb described below, where the maximum retention should not exceed a certain percentage of the free reserves.

Other risk theory approaches involve modelling the aggregate claims distribution. The effect of different forms of reinsurance and different retentions is assessed by analysing the changes in the net retained aggregate claims distribution. The aggregate claims distribution can be modelled by combining the claims severity and claims frequency distributions using a range of possible techniques.

2.3.2 Rules Based on Maximum Percentages

Perhaps the most commonly quoted rules of thumb are those which link the retention level, again a per risk excess, with items such as free reserves and premium income:-

TABLE 1 - RETENTION RULES OF THUMB

As a percentage of:-	Retention
Capital and free reserves..................	1 - 5%
Retained premium income (by class)..........	1 - 10%
Liquid assets.............................	400 - 600%

These rules assume that the aim of the reinsurance programme is to smooth out fluctuations in the net retained account. This is achieved by setting the retention so that a single large claim cannot impact the company by more than, say 5% of its free capital or 10% of premium. By measuring the retention against its liquid assets a company can try to ensure that it has enough cash available to meet a single claim.

Claim in this context means either a single large claim affecting a single risk or an accumulation of relatively small claims arising out of a single event.

These rules of thumb can be expressed differently. The company can determine what percentage of the profits of a class of business they are prepared to lose. This amount combined with estimates of the maximum operating ratio and written premium of the Quota Share treaty will imply a retention.

For excess of loss reinsurance, the retention can be based at the level at which claims become very infrequent or alternatively the level at which the average claim up to that point starts to show significant variation year on year. The basis for this method is that if a claim of a certain severity occurs frequently then claims of that severity are not giving rise to significant variation in results.

For property portfolios, the common practice when designing a Surplus treaty is to compile a table of limits which shows the company's retention for different risk categories.

This could be constructed by firstly deciding on a minimum retention. The retentions for each risk category are then calculated by scaling this minimum in relation to the relative premium loadings for each risk category (Reference 4). In practice, of course, the individual underwriter's experience and judgement will play a major part in determining the retention levels in the table of limits.

Companies do, in practice, vary their retention levels both by risk category within a class, and between classes of business. It is common practice for underwriters to fix their Surplus retention levels so that they are, broadly, inversely proportional to the original premium rates which they charge (In other words, they keep more of the less hazardous risks). It is preferable that retention levels should be based on some assessment of the quality of the risk (for example, as measured by the construction type for Fire insurance) rather than in direct proportion to the actual premium rates.

Section 3

RETURN TO FIRST PRINCIPLES

3.1 Introduction

This section sets out the general considerations relevant to determining reinsurance retentions. Our intention is to return to first principles and consider why companies require reinsurance. We believe that it is from this point that a reinsurance strategy should be built.

The first key point is that the aims of the general insurer in its entirity must be the starting point for a retention policy. As we have seen, in many instances individual units within a general insurer develop their own retention strategy. The retention of the total is the sum of the pieces and may, or may not, be appropriate. In other words retention strategy develop from the bottom up; it should be designed from the top down.

We now consider the major influences in determining the retention of general insurers at the top level. Many of the ideas presented are equally relevant when determining retention strategy for individual business units based upon a global strategy.

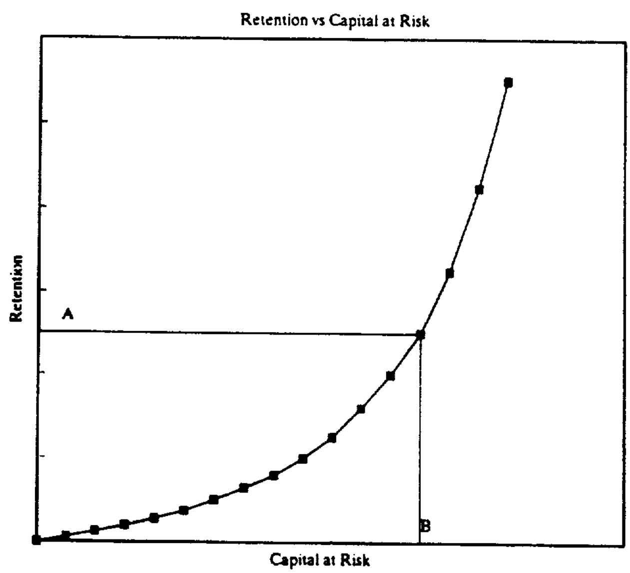
The process of setting a retention level is related to the control of exposure. The control of exposure is the last part of a three stage process.

1. Identify Exposure
2. Quantify Exposure
3. Control Exposure

For an employers' liability insurer, the process of exposure identification should focus both on large claims, and aggregation arising either from industrial disease, or an explosion. A property insurer may have exposure to aggregation from one natural catastrophe in addition to aggregation from adjacent sites and exposure to total loss on one risk. These identified exposures represent potential claims for which insurance may be required.

The second step in the process is the quantification of the severity of potential loss from the identified exposures together with their associated probabilities. Some techniques for achieving this are described in Section 4.

We have adopted a standard presentation of the results of these techniques, which is to show the effect on free reserves of having different retentions. An example of these graphs is shown below:



The graph is to be read as follows: if the retention is set at Level A there is a likelihood of 5% of losing an amount equal to B of free reserves. The actuary can use these graphs to help management quantify a subjective assessment of risk.

The objectives of a company play an important part in determining its retention. Some of these are discussed below for each type of insurer. We then review two general considerations which should effect retentions, namely, the underwriting cycle and the cost of reinsurance.

3.2 Exposure Control

We feel it is important to stress that an insurer's retention should be as much a reflection of its perceived risk aversion as of the underlying distribution of its claims or of conditions in the reinsurance market. Risk aversion depends on the financial condition of the company, and its corporate culture, and is reflected in the reinsurance protection it purchases.

In determining retentions, we need to consider measures by which to quantify unacceptable claim deviation. Possible measures, at a "group level", are the effect on earnings, the effect on shareholders funds, on share price or on Names. We have only presented results in terms of the effect on shareholders' funds.

The insurer must consider its objectives. These objectives may be different for the following three groups:

1. Proprietary insurance companies
2. Mutual insurance companies
3. Lloyd's syndicates.

Proprietary Insurance Companies

For a proprietary company the objective must broadly be to produce a long-term return on capital employed commensurate with the risks involved, and, in the short term, to distribute part of this return as a smoothly increasing dividend.

For a publicly quoted proprietary company, there is also a need to maintain the market share price. This price, to a great extent, is influenced by the return on capital and dividends. Other influences include analysts' comments and market perception of the company.

Some companies form part of conglomerates which have higher quality earnings streams from other activities which may allow the general insurance operation greater variability in results without jeopardising the overall corporate objectives.

Some proprietary companies are set up as captives to write the insurance risks of a larger parent company. In such a case, setting profit objectives is purely an internal or tax accounting process. The objectives of the captive will be aimed at controlling the variability of the results, thus protecting solvency, and developing the captive.

Companies can attempt to control the emergence of profit in the following ways:-

1. Via alterations in reserve surplus.
2. By realising investment gains.
3. Using reinsurance.

At the start of any trading period, the status of the company's reserve surplus and unrealised investment gains must be taken into account. The first two methods of smoothing are cost effective for the company, however, it is only the third that has an elastic supply. The company may determine its retention by examining:-

1. The expected profit in the ensuing period.
2. The variability associated with that expected profit.
3. The desired variability in profit in the ensuing period.
4. The availability of reserve surplus and unrealised investment gains to smooth the difference between the actual and desired variability.

Mutual Insurance Companies

It is likely that the main objective of a mutual is to build up the solvency of the company in order to enable it to write more risk. The control of variability will be pitched at a level that protects solvency rather than annual earnings.

As a result, the mutual is more likely to focus on the maximum amount it wishes to lose in one year. For a large well established mutual the Estimated Maximum Loss from one event may be very small in comparison to the financial resources. In such a case reinsurance is probably not required.

For a small mutual, such as the one of the professional indemnity vehicles that have become commonplace during the last ten years, incurring gross claims in excess of called capital may be a very real possibility. To reinsure very heavily defeats the object of the mutual. The managers might focus on the maximum capital the members wish to have at risk in any year (which may well be much greater than the called capital) at given levels of probability.

The mutual may determine its retention by considering:-

1. The variability associated with the claims costs.
2. The desired capital at risk during the ensuing period.

The retention should be fixed to ensure that items 1. and 2. are consistent. The reserve surplus and unrealised capital gains do not feature directly because revenue account profit is not of overwhelming importance. However, in determining the desired capital at risk, the members will consider the capital already available in the mutual which should include the above items. A small mutual provides an example of where a desired retention profile might be achieved by alteration of the gross portfolio rather than by via reinsurance.

Lloyd's Syndicates

Lloyd's syndicates are different from insurance companies in two ways. First, the shareholders on each underwriting year are separately identified. Second, investment income is only earned on insurance funds which are invested in similar assets for every syndicate until the underwriting year is closed. The investments are generally risk free in nature. Thus, the underwriting result becomes the major source of variation in results between different syndicates and different years of account on the same syndicate. This differs from proprietary companies in two respects, first investment income is of secondary importance and second separate cohorts are considered rather than the change in the overall financial state of the company during the period.

The retention philosophy must focus on controlling the variability of the underwriting result for the individual underwriting year during the three year period prior to closure. It is fair to assume that all underwriters work on the basis that they will close the year in the normal fashion after thirty-six months and set their retention accordingly.

If we suppose that all names require the same variability then a further complication arises from Names participating in varying numbers of "independent" syndicates. Even if all syndicates have identically distributed underwriting results, different Names would experience different variability due to different participations.

Reserve surplus and unrealised capital gains should not have a role in the control of variability at Lloyd's. The syndicate may determine its retention by examining:-

1. The expected result of the underwriting year.
2. The variability associated with that expected result.
3. The desired variability in the underwriting result.

Since Names are generally risk averse, we believe that the retention is primarily aimed at obtaining the desired level of variability. The Lloyd's syndicate can be faced with a unique problem since attaining the desired level of variability could imply purchasing so much reinsurance that the expected profits will be unacceptably low. The underwriter is faced with a dilemma, either reduce the profit or increase the variability.

Variability in Claims Costs

Variability in claims costs are dependent on the amount and nature of the business written. For a major composite insurance group the gross book of business may very nearly conform to that which is desired. For a small company writing LMX business, the gross distribution is likely to be extremely unsuitable and require considerable alteration.

Variability can be reduced by reciprocal reinsurance with another insurer. We define a reciprocal reinsurance as one where the quantum of risk ceded and accepted are equal. The point of this contract is to reduce the variability in the book of business via diversification. Many large insurance operations will already have optimised their diversification via world wide operations and will not add value via reciprocity.

After effecting the reciprocal reinsurance the insurer is left with a redefined book of business. If the characteristics of this business are still incompatible with the objectives then reinsurance can be utilised.

If reinsured and reinsurer both accept that "reinsurance costs money", then long term good relationships with reinsurers can be very valuable. Once this relationship exists and the purpose of reinsurance is established, there should be no barriers to the type of reinsurance cover available provided both parties are satisfied. This, in turn, might allow a simplification of current reinsurance programmes and thus savings on the administration side.

3.3 The Underwriting Cycle

We have not yet discussed the affects of the insurance cycle. An analogy can be drawn between the general insurer and a geared investment trust. Premiums represent borrowed funds. In this analogy a softening market leads to an increase in the cost of borrowing. Usually, there will be no correlated or predictable change in the investment return, and hence, the unit profitability is squeezed. In this situation most types of general insurer will become more variation averse. The expected profit is low, and hence, the acceptable downside is reduced. A priori, the insurer will wish to change the retention to reduce variability.

Under these circumstances the company may cede business at unprofitable rates (for the reinsurer) and in this way improve the short term profitability without loss of business. The cedent should acknowledge that a pay back to the reinsurer will be required in the future. However, this will occur at times of greater unit profitability and so the objective will have been achieved. This is the second way in which the insurance cycle may affect the retention.

This concept is particularly true of the London Market where the rates at the bottom of the cycle can be extremely soft, but each player in the market is supported by equally cheap reinsurance. However, historically there have been reinsurers of London Market companies who have been "fair game" and not received a pay back. The London Market operation of the Insurance Corporation of Ireland is one such company.

The London Market may be considered from a different perspective - as one insurance entity, with each company or syndicate a "department", often the last retrocessionaire for much of the world's market. The reinsurance rates that individual "departments" charge each other are unimportant to the entity as a whole since these merely constitute internal accounting. If we view the market from this perspective, the entity suffers from the cycle when the rates it receives for business ceded into the market are too low. It overcomes the cycle by reducing the profit of each department and by "cannibalising" one or two departments. In other words, the market cedes much of it's loss to these "departments" who never recover. The LMX spiral partly arises out of each "department's" desire not to be one of the "cannibalised".

3.4 The Cost of Reinsurance

Any purchaser of reinsurance needs to bear in mind that the reinsurer is a commercial enterprise and requires a return on capital. The cedent should expect reinsurance premiums to exceed recoveries in the long term and, as such, this represents a cost. The purchase of reinsurance, therefore, reduces profits in the long term. In return the reinsurance provides some stability of claims costs to the cedent.

A principle that we consider should underlie any discussion of an appropriate retention for a company is that the company should avoid purchasing any unnecessary reinsurance.

Section 4

MATHEMATICAL MODELS

4.1 Introduction

At whatever level within a general insurer while investigating retention philosophy, understanding the variability associated with the relevant aggregate claim distribution is essential. In this section we demonstrate some methods that can be used when estimating aggregate claim distributions and investigating retentions. Where possible, we have demonstrated the use of these methods on three case studies. The details of the calculations are given in Appendices 1 to 4.

The three case studies consist of aviation, liability and property risks. Exhibit 1 contains the underlying severity distributions used to derive the aggregate claims distributions on which our analysis is based.

We express the effect of different retention levels as reductions in free reserves together with associated probabilities. Equally, results could be expressed in terms of premium income, earnings or other measures. An increase in retention should not necessarily be seen as increasing the probability that a company will face ruination. It can more usefully be seen as increasing the probability of a specified reduction in free assets or earnings. This increased variability is compensated for by an increase in the expected profitability.

We have used four methods to quantify these effects. The methods used are not intended to be exhaustive, nor, to be necessarily the best methods available. They are methods which have either been used by the members of the working party or which are believed to be commonly used.

We would like to stress that the results of these methods are only as good as the assumptions underlying them which may, in some instances, be very limited. In particular, the assumptions concerning the tail of the probability distribution can be critical when examining retentions.

4.2 Straub's Method of Calculating Retention Levels

This method is based on the theory developed in Erwin Straub's book "Non-life Insurance Mathematics" (Reference 18). Straub develops a mathematical representation of the following intuitively reasonable relationship:-

$$\text{RETENTION} = \frac{\text{CAPITAL} \times \text{RISK WILLINGNESS} \times \text{PROFIT MARGIN}}{\text{UNBALANCEDNESS}}$$

If four of the elements of the equation are known then the fifth is implied. The formula can be used to investigate the relations between capital and retention. A different formula is developed for each of the common types of reinsurance. The method takes the classical risk theory approach and considers an infinite future time period. This is different from the approaches presented in the next three sections which consider a finite future period.

The capital item refers to the free reserves backing the class of business under consideration. Risk willingness is expressed as a function of the tolerated ruin probability (or probability in the examples of Appendix 1). The smaller the tolerated ruin probability, the lower the risk willingness of the company. Unbalancedness is dependent on the type of business written and is determined essentially by the distribution of total aggregate claims.

The relationship follows certain intuitive rules. For example, if we increase the unbalancedness of the portfolio, then ceteris paribus, we would expect the retention to decrease. Alternatively, as the risk willingness of the insurer increases then so should the retention.

In its most general form, Straub's formula relies on very few assumptions about the risk process which is being considered. However, for the purposes of the examples used to demonstrate the method in this paper, we have assumed that:-

1. There are equal loadings used by the insurer and reinsurer. (This makes the mathematics easier!).
2. The claim amount distributions can be approximated by discrete distributions.
3. The claim count distribution is Poisson.
4. Either Quota Share or Risk Excess reinsurance is used.

After fixing the various components of the formula, the method calculates either the Quota Share or the Risk Excess retention. By varying key components such as risk willingness and capital, graphs may be drawn to summarise their inter-relationship.

This method has the advantage that it allows explicitly for all of the important items when setting retentions. The items are linked together in a neat formula.

In addition to calculating a retention level, it is also possible to use the method to calculate a measure of the need for reinsurance. This is clearly an important consideration before deciding what retention to hold. However, given that a particular company needs reinsurance, the method provides little help in deciding what form of reinsurance is the most efficient.

4.3 Heckman and Meyers' Method for the Calculation of Aggregate Loss Distributions (Appendix 2)

The basis of this method is published in a paper entitled "The Calculation of Aggregate Loss Distributions From Claim Severity and Claim Count Distributions" published in 1983 (Reference 11). The method works by convoluting the severity distributions of individual claims. This is achieved by the use of characteristic functions and then inverting the resulting integral by means of numerical integration techniques as described in the paper.

This gives a powerful and practical tool for calculating probability points on the aggregate claim distribution together with excess pure premiums (that is, stop loss risk premiums). Furthermore, the method allows aggregate distributions to be calculated for the combination of a number of lines of business.

Once the method has been set up on a computer, it is quick to use. For example, it is easy to amend the severity distribution to allow for changes in retentions and then recalculate the aggregate claim distributions. By reading off the sizes of aggregate claims at various retentions and probability levels, the effect of various retention strategies can be assessed.

The assumptions underlying the method are:-

Claim Count Distribution

The method can be constructed on a Poisson, Binomial or Negative Binomial claim count distribution. The distribution is, thus, described by two parameters, namely, the expected number of claims and the contagion or contamination parameter. If this second parameter is zero then the Poisson distribution is assumed. If it is positive then we have the Negative Binomial or Polya distribution and if it is negative, then we have the Binomial distribution.

Use of positive contagion is helpful in practice as it makes some allowance for non independence of claims, that is, a higher than expected number of claims in one period can increase the expected number of claims in a future period.

Claim Severity Distribution

The method requires a cumulative probability distribution that is piecewise linear. This results in a great deal of flexibility because any distribution can be represented to any desired degree of accuracy by increasing the number of points in the approximation.

In contrast to the recursive method (Section 4.5), this approach does not require equally spaced intervals. The approach facilitates the use of empirical distributions as exhibited by the underlying data without the need to fit a standard distribution.

The analysis of claim severity is relatively straightforward. In practice, though, it is often helpful to pay special attention to the upper tail of the distribution. In most cases, use of a distribution fitted only to the largest claims can be of value, particularly when coupled with an examination of the underlying claims process and exposures.

Parameter Uncertainty

In practical situations, parameter uncertainty can far outweigh the variation that can occur from randomness within known frequency and severity distributions. The Heckman and Meyers' approach can reflect both sources of variability by introducing a mixing parameter which has an Inverse Gamma distribution and is applied to rescale the claim severity distributions, increasing the level of variability. The effect of this parameter may be removed from the method by setting it to zero.

4.4 A Simulation Method for Retention Determination (Appendix 3)

The essence of the method is to simulate both gross and net aggregate claims distributions in order to assess the effectiveness of different reinsurance programmes. Here a retention is defined as in Section 3 to be everything that is not ceded.

Simulation is very flexible and facilitates the examination of the distribution of claim costs on a per claim, per event or per year basis. Even if the probability distribution of the severity of an individual catastrophe claim is a standard one that can be treated analytically, the distribution of the aggregate annual catastrophe costs to an insurer can be very complex.

Some of the alternative methods used for calculating aggregate claims distributions rely on assumptions such as the independence of individual claims. There are many instances in general insurance where such an assumption is invalid. A strength of the simulation approach is that it does not require this assumption. All this work is based around the use of simple spreadsheet models on a personal computer.

Any random variable with a known density function can be simulated provided that random samples from the uniform distribution over the unit interval (0,1) are available. (U(0,1) random variables) The practitioner can therefore define any empirical distribution for gross claims. Similarly, the effects of most reinsurance programs on the gross claims can be defined parametrically.

The example given in Appendix 3 considers all aspects of a model for UK property catastrophes. The limitations of the analysis are as important as the results themselves. In particular, the use of the standard deviation as a variability measure needs investigation.

The simulation in Appendix 3 depends on claim distribution assumptions. Claims are, of course, the result of random events such as hurricanes. Models can be built for catastrophes where the underlying natural phenomena themselves are simulated, and a separate stage is required to calculate the impact of the event on the insurer. This allows the modeller to use larger and more credible data, such as meteorological records, and thus improve the reliability of the simulations.

A particularly fine example of this, in our opinion, is a methodology for estimating US windstorms claims described in "A Formal Approach to Catastrophe Risk Assessment and Management" by Karen M. Clark (Reference 7) contained in the 1986 Casualty Actuarial Society discussion paper programme.

In this model, windpaths are represented by frequency and severity probability distributions which vary by location. The derivation of these distributions depends on an understanding of the dynamics of hurricanes and the use of historical meteorological data.

Insured properties are classified by location, age and structure. The connection between the windstorm and insured risks made by applying damage and vulnerability factors to the insured values. These factors are based on engineering studies.

Monte Carlo simulation is then used to produce two thousand years of experience. Each simulation results in a hurricane severity at each location (which is zero if the hurricane does not reach the location). The combination of simulated severities and insured values produces simulated claims at each location. Aggregated claims for each simulation gives a distribution of catastrophe claims.

The methodology has certain attractive features. It combines a practical understanding of meteorology, of engineering and of the distribution of insured risks and it has particular value where historical claim experience is limited or where external factors (for example, climatic changes) are considered important. The method does, however, require the insurer to maintain an extensive and detailed exposure database.

4.5 The Recursive Method for the Calculation of Aggregate Claim Distributions (Appendix 4)

The objective of the method is to estimate the aggregate claims generated by an insurance portfolio. The approach is to assume the aggregate claims can be represented as the sum of a number of individual claims where the number of claims is, itself, a random variable. The aggregate claim distribution can be calculated directly from a straightforward recursive formula.

To make the model more tractable, two assumptions are made:-

1. The individual claim severities are identically distributed random variables.
2. The number of claims and the individual severities are independent random variables.

If the mass function assumed for the claim frequency is of the type where successive values are related by a recursive relationship (Reference 1 eq^n 2.9.13) then the formula is easily manipulated. The model is referred to as the Collective Risk Model in risk theory. In the special case where number of claims has a Poisson distribution, claims are said to have a Compound Poisson distribution.

The mass function of the aggregate claims can be found by direct numerical calculation if the severity distribution of individual claims is a discrete equi-distant distribution according to which only the values

$$Z_i = iZ_1 \qquad i = 1, 2, 3 \ldots$$

can occur. In the simplest case, this reduces to a subset of the natural numbers.

The required aggregate claims mass function can then be calculated using the recursive formula (Reference 1). The effects of different per risk retentions are reflected in the distribution selected for the individual claim severities. Repetition of the calculations with different retentions facilities a comparison of the effects of these retentions on the aggregate claims distribution.

Section 5

GLOSSARY OF TERMS

Aggregate claim distribution - The distribution function of total claims during the specified period for example, a year.

Annual aggregate stop loss - A reinsurance cover capping the aggregate claims incurred in a period.

Coefficient of variation - The ratio of the standard deviation of a random variable to its mean.

Convolution - The combination of the density functions of two or more random variables to yield the density function of the combined variable.

Deductible - The amount of risk retained below the attachment point of a reinsurance cover.

Density function - The function representing the probability mass of a continuous random variable.

Distribution function - The function representing the cumulative probability mass of a random variable.

Drop-down cover/Top and drop - Excess of loss reinsurance cover with flexible attachment points and limits.

Financial reinsurance - Reinsurance where the quantum of recovery is known and only the timing of payment is uncertain.

LMX - London Market Excess, that is, reinsurance of a London Market reinsurer.

Mass function - The function representing the probability mass of a discrete random variable.

Per risk excess - Excess of loss reinsurance for individual insured risks.

Polya - An alternative name for the Negative Binomial distribution.

Probability of ruin - The probability that the free reserves of an insurer are exhausted.

Profit centre - An individual unit within an organisation with separate financial objectives.

Reinstatement - The process of replacing an excess of loss reinsurance once a claim has been made.

Unbalancedness - The degree of fluctuation inherent in the profitability of a portfolio of business.

Section 6

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A London Market Actuaries'
Group Paper

VOLUME II

CONTENTS

Appendix 1

EXAMPLE APPLICATIONS OF STRAUB'S METHOD

A1.1 Introduction

Straub's method has been applied to the aviation, liability and property examples mentioned in the Introduction to Section 4. For simplicity we shall only consider the use of either Quota Share or Risk Excess reinsurance. These may not be the most appropriate forms of reinsurance for the class of business in the examples, but they serve to illustrate the use of Straub's method. In each example a discrete distribution was used for claim amounts (Exhibit 1) and a Poisson distribution for claim numbers.

The results are shown in Exhibit 2 pages 1-12. The graphs demonstrate the effect on the retention level of varying the capital at risk and the desired probability of exhausting that capital over an infinite period. The tables show the numeric results of using Straub's method. The graphs are not directly comparable with those of the other methods, which consider finite future time periods.

A summary of the results for a 60% solvency margin (that is, capital at risk of 60% of gross premiums) and probabilities at a one in one thousand level are shown in Table 2 below:-

Table 2 - Results of Straub's Method (Amounts in £000s)

	Aggregate Claims Coefficient of Variation	Quota Share Retention	Risk Excess Retention	Capital* at Risk for no R/I
Aviation	0.79	3%	405	1500%
Property	0.23	46%	75	130%
Liability	0.17	87%	1,875	68%

* Expressed as a percentage of premium.

The following general observations can be made from the results:-

1. The relationship between capital at risk and retention level is linear for a Quota Share, whereas it depends on the claim amount distribution for Risk Excess reinsurance. This is a direct result of the structure of Straub's formula.

2. The Quota Share graphs can be used to determine the point at which no reinsurance is required - that is, the level of capital at the point where the Quota Share retention is 100%. For a probability of one in one thousand this point is shown in the final column of Table 2.

3. For a given probability, the retention increases as the available capital at risk increases.

4. For a given capital at risk, the retention increases for companies which are less risk averse (that is, as the probability increases).

5. The rate of change of retention with respect to capital at risk is lower for a lower probability. In other words, the more risk averse a company is, the less will be the effect on its retention policy of an increase in available capital at risk (due to capital injections etc.)

6. The coefficient of variation (CV) of the aggregate claim amount distribution summarises the variability of this distribution. The above table indicates that the higher the CV, the greater the need for reinsurance and, hence, the lower the retention.

Some brief comments on each example based upon the stated capital and probability assumptions, are as follows:-

A1.2 <u>Aviation Example</u> (Exhibit 2 Pages 1 to 4)

1. There is a very high coefficient of variation, leading to very low retentions.

2. Annual expected gross claims are about £74 million.

3. Across a range of practical levels of capital at risk, the retention level changes very little and is very low.

4. These results indicate the highly volatile nature of this business. In practice, the use of coinsurance or pooled arrangements helps to spread the risk across the market.

A1.3 Liability Example (Exhibit 2 Pages 5 to 8)

1. In this example, annual expected claims are about £10m, with approximately 260 claims per annum.

2. Risk Excess reinsurance is likely to be used here (in conjunction with other forms of reinsurance).

3. The method suggests a retention of about £75,000 which seems reasonable.

4. Such a retention would lead to the reinsurer being involved in 10% of claims.

5. As the capital at risk approaches 100% of premium then there is a rapid increase in the retention and a reduced need for reinsurance.

A1.4 Property Example (Exhibit 2 Pages 9 to 12)

1. This example has the lowest coefficient of variation of the three examples and hence we might expect the retention to be higher. The graphs demonstrate that reinsurance is not needed when the capital at risk is greater than the 70% of premium.

2. The retention is quite high at 87% for a Quota Share and £1.9 million for a Risk Excess (above which there might only be three out of 13,000 claims!).

3. 87% could be considered as an average retention for a Surplus treaty, which is the commonly used form of reinsurance for this class. It is doubtful whether, in practice, an insurer would have a Surplus treaty which ceded such a small percentage of the business.

4. In practice, Catastrophe Excess of Loss would also be used to cover against events such as windstorm.

Appendix 2

EXAMPLE APPLICATIONS OF HECKMAN AND MEYERS' METHOD

A2.1 Introduction

This appendix demonstrates the use of the approach as published by Heckman and Meyers (Reference 11). The Heckman and Meyers (H & M) method was applied to the same three data sets, namely, aviation, liability and property.

A2.2 Outline of Approach

The core of the approach is to use the H & M method to produce an aggregate claim distribution for given input frequency and severity distributions. In order to use this to provide information on varying retention levels, the algorithm must be used a number of times allowing for varying retention and reinsurance costs. The objective is to calculate the capital at risk for a given retention level and probability level. Capital at risk for a given probability level may be defined as follows:-

Capital at Risk = Net aggregate claims at given probability level, less net premium received.

Where:

Net Premium = Gross premium received
less expenses
less cost of reinsurance

The cost of reinsurance will depend on the retention level and market conditions. In this section we (unrealistically) assumed the cost of reinsurance is related to the risk premium with a constant percentage loading, regardless of the retention level. We have also assumed that the expenses are split in proportion to the risk premium independently of the retention level. This may also be unrealistic. In practice, one would aim to use realistic figures based on the current state of the reinsurance market. For all the examples in this paper we have:-

Table 3 - Cost of Reinsurance

		Percentage
Gross Premium	:	100
Risk Premium	:	70
Expenses	:	20
Profit Loading	:	10

For readers more familiar with the λ of Risk Theory, the above represents a λ equal to 1/7, (that is, approximately 14%).

For a particular retention, the first step is to calculate the reinsurance risk premium. The cost of reinsurance is then calculated as that risk premium loaded for profit and expenses. For example, say the net risk premium is 50% of the gross premium, we then have:-

Table 4 - Calculation of Net Risk Premium

	Percentage
Gross Premium	100
Less total expenses	20
Less reinsurance (net of expenses)	57
Net Premium	23

The next step is to adjust the gross claim severity distribution for the effect of the reinsurance retention. The frequency distribution does not require adjustment. The H & M algorithm is then run to produce a table of net aggregate claims at various probability levels. The amount of aggregate claims at the desired probability level is then read off and the net premium subtracted to give the capital at risk for that retention and probability.

The exercise is repeated a number of times to build up a picture of the capital at risk for varying retention levels. These may be represented graphically and interpreted to select an appropriate retention level. Exhibit 3 Page 1 shows an example graph.

For a given retention level, the capital at risk of the various probability levels may be determined from the graph. Alternately, for a given capital at risk the retention consistent with various probability levels may be read from the graph.

For a company as a whole, there are often many lines of business with differing retention levels. The H & M method is specified in their paper to handle multiple lines and so the corresponding capital at risk for an entire company can be easily derived for a given set of retention levels.

This general approach can also be used for other methods of calculating aggregate claims, for example analytical or recursive methods.

A2.3 Assumptions Made in Calculating Aggregate Claims

Claim count distribution : Poisson

This implicitly assumes that the variance of the number of claims is equal to the expected number of claims. A larger variance could have been assumed by use of the negative binomial distribution (that is by using a positive contagion parameter in the H & M algorithm)

Similarly, a smaller variance could have been assumed by use of the binomial distribution (negative contagion parameter).

Claim Severity distribution: Piecewise linear.

The distribution used is based on past claims experience. Past claims were sorted into ascending order and assumed to be equally spaced on the probability scale. The cumulative probability was then calculated and various claim sizes selected to represent the severity distribution. In the case of the liability claims, a log-normal distribution was fitted to the large claims and the actual largest two or three claims were replaced by their fitted values.

Parameter Uncertainty: None

The variation was assumed to come only from that implicit in the claim count and severity distributions. Additional variation could have been incorporated, for example to allow for uncertain future inflation by using a non-zero mixing parameter in the H & M algorithm.

A2.4 Aviation Example (Exhibit 3 Pages 2 to 7)

The frequency and severity distributions used are summarised in the table below. All figures in the example are in thousands. The underlying claim severity distribution is shown in Exhibit 1 Page 1.

Table 5 - Aviation Example Frequency and Severity Distributions

Severity Mean	=	9175
Claim Frequency Distribution	=	Poisson
Mean Claims Per Year	=	8.000

Multiplying the means of the severity and claim count distributions gives expected aggregate claims of £73,398,000. Loading for expenses and profit produces a gross risk premium of £104,854,000. The gross data is initially used unadjusted as input into the H & M algorithm. The output produced from the calculation is contained in Exhibit 3 Page 2.

The column headed 'Entry Ratio' in the table refers to the ratio of claims on the aggregate distribution to the aggregate mean. The column headed 'Excess Pure Premium' refers to the stop loss risk premium. Some diagnostics from the numerical integration process are also included is the output.

From the columns of aggregate claim amounts and probabilities, the aggregate claims at 90%, 99% and 99.9% may be determined by interpolation.

Having calculated aggregate claims from the gross claims, the next step is to adjust the claim severity distribution for a retention level. The mean of the truncated distribution is easily calculated as the distribution remains piecewise linear; this is multiplied by the expected number of claims to obtain the net risk premium. The reinsurance risk premium is calculated as the difference between the gross and net risk premiums. This leads to figures for the capital at risk for the retention level under consideration. Repeating the process for a number of retention levels builds up the complete picture. Exhibit 3 Page 3 below summarises the results for this class of business. These results are plotted in the graphs in Exhibit 3 Page 4 to 7.

Checks for reasonableness

Beard, Pentikäinen and Pesonen (Reference 3) give a formula for a distribution free upper limit for the capital at risk (based on the normal power approximation):

$$U \leq y\sqrt{PM} - \lambda P + \tfrac{1}{6}\,(y^2-1)\,M \qquad (1)$$

Where	U =	capital at risk
	P =	Net Risk Premium
	λ =	Profit loading
	M =	Retention
and	y =	normal variate for a given probability level

A further quick check on the level of aggregate claims may be constructed by assuming that all the claims are equal in size to the retention, and applying a poisson distribution to claim numbers. This gives:

$$\textit{Aggregate Losses} \leq Mw \qquad (2)$$

w is the point where $\sum_{r=0}^{w} e^{-n} \frac{n^r}{r\,!}$ first exceeds the desired probability level. This check is only really helpful at small retention levels. Applying these checks to the results for a probability level of 99%, we have:

Table 6 - Reasonableness Checks on H & M Aviation Results

Retention (£000s)	**100**	**1,000**	**10,000**
H & M Capital at Risk	604	5,210	35,431
Compared with (1) above	608	5,585	41,696
H & M Aggregate claims	1,481	12,102	66,253
Compared with (2) above	1,500	15,000	150,000

This confirms the reasonableness of the results for the 99% probability level.

Interpretation of Results - Aviation

The results as presented show that very large amounts of capital would be needed if aviation were insured on a simple risk excess basis unless the retention were very small. Whilst this may be the case for consideration of the self insured deductible for a fleet operator, the actual aviation LMX market is based around some very complicated programmes involving numerous layers, co-insurance, aggregate deductibles, use of top and drops and so on. However, with some additional work, most of these features can be modelled by repeated application of the H & M method, and hence, the effectiveness of particular reinsurance programmes may be assessed.

A2.5 Liability Example (Exhibit 3 Pages 8 to 13)

Tables and graphs of results similar to the aviation example are set out in the exhibits as follows:-

Underlying claim severity distribution - Exhibit 1 Page 2

H&M aggregate claim distribution - Exhibit 3 Page 8

H&M results table - Exhibit 3 Page 9

Graphs of aggregate claim distribution vs retention - Exhibit 3 page 10

Graphs of capital at risk vs retention - Exhibit 3 Page 11

Graphs of capital at risk vs retention as a percentage of gross written premium - Exhibit 3 Page 12

Graphs of capital at risk vs retention as a percentage of net written premium - Exhibit 3 Page 13

Interpretation of Results - Liability

The tables and graphs indicate that relatively high retentions are possible without putting unreasonable amounts of capital at risk. This arises as a consequence of the high profit loading applied to the risk premium coupled with the assumption that there is no parameter uncertainty. It is interesting to note that the capital at risk at the 90% level becomes negative for a retention of 50,000. This means that at that retention and assumed cost of reinsurance, the premium loading is such that a profit can be expected for 9 out of 10 years.

A2.6 Property Example (Exhibit 3 Pages 14 to 19)

Tables and graphs of results similar to the aviation and liability examples are set out in the exhibits as follow:-

Underlying claim severity distribution - Exhibit 1 Page 3

H&M aggregate claim distribution - Exhibit 3 Page 14

H&M results table - Exhibit 3 Page 15

Graphs of aggregate claim distribution vs retention - Exhibit 3 Page 16

Graphs of capital at risk vs retention - Exhibit 3 Page 17

Graphs of capital at risk vs retention as a percentage of gross written premium - Exhibit 3 Page 18

Graphs of capital at risk vs retention as a percentage of net written premium - Exhibit 3 Page 19

Interpretation of Results - Property

As was the case for the liability example, the tables and graphs indicate that relatively high retentions are possible without putting unreasonable amounts of capital at risk. As before, this arises as a consequence of the high profit loading applied to the risk premium coupled with the assumption that there is no parameter uncertainty. The unrealistic loadings applied to the reinsurance risk premiums also reduce the calculated figures for capital at risk.

In this example the capital at risk at the 90% level remains negative for all retentions shown in the results table, although the gross capital at risk is positive. This means that the premium loading is such that a profit can be expected for 9 out of 10 years for any retention of at least up to £1 million. At the 99.9% probability level, the results show positive capital at risk for retentions above £100,000. In a case like this, solvency aspects may not be as important in the analysis as the maximisation of expected profit subject to the cost and availability of reinsurance.

Appendix 3

AN EXAMPLE APPLICATION OF SIMULATION

A3.1 Introduction

This particular example is of a large insurer writing UK personal and commercial lines. The gross retention is acceptable to the company except for the aggregation exposure to weather events such as flood, windstorm and freeze. We shall consider the effect of weather catastrophes on the company. For this purpose, a catastrophe will be defined as any event giving rise to an insured claim in excess of £100 million to the market at 1990 values.

The results of the simulations lead us to the following conclusions for a hypothetical insurance company with a 10% share of the UK property market.

1. The company could reduce the variability of retained claims at no additional cost by purchasing higher layers of excess of loss reinsurance and retaining a greater coinsured share.

2. The company could raise the lower limit of the reinsurance programme. The outwards reinsurance premiums recouped from this could be used to purchase higher layers of reinsurance and reduce the variability of the claim retention.

3. The company could investigate other forms of reinsurance that will achieve the same level of variability at a reduced cost. One such reinsurance could be an annual aggregate stop loss on claims arising from catastrophe events.

4. The company's annual catastrophe excess of loss reinsurance premium is £22 million. The simulations indicate that the expected claim ratio to the reinsurer in the long term is 40%-60%. On this basis the annual long term cost to the company of smoothing their retentions using excess of loss reinsurance is £8.8 - £13.2 million.

5. If the company management are able to advise on their desired variability then the optimum reinsurance programme can be investigated.

A3.2 Methodology

The simulation divides into four parts:-

1. Determination of the model for the gross market claims distribution.

2. Estimation of the parameters for the gross market claims model.

3. Calculation of the effect of individual events on the company concerned.

4. Analysis of the retention strategy required to achieve the target net claims distribution.

A3.3 Model Identification and Parameter Estimation

It is possible to argue that a catastrophe occurrence is a Poisson process. In other words it satisfies:-

1. The probability of an event occurring in a time period t_1 to t_2 is proportional to $(t_2 - t_1)$.

2. The probability of two or more events occurring at the same time or an infinite number of events in a finite period is zero.

3. The events in two disjoint time periods are independent.

If this is so, then the number of occurrences in a year has a Poisson distribution. Notice that for condition 2 to hold a catastrophe must be defined as all claims arising from one event. Counting two aeroplanes that crashed into each other as two events breaks condition 2. Further, the cyclical nature of weather conditions also undermines condition 1.

We commenced by examining the data concerning past losses above £40 million original cost in order to estimate parameters for the frequency and severity distributions. This is shown in Exhibit 4 Page 1. During the 11.5 years of experience there have been 12 claims in excess of £100 million at current costs or approximately one per year.

We decided to use a Pareto distribution to simulate the severity scaling all claims by £100 million. Thus a simulated value of 1.5 would correspond to a market claim of £150 million. The maximum likelihood estimator of the Pareto parameter based upon experience is 0.84. This gives a very skew distribution which has no mean.

This is probably a result of the fact that the sample of twelve claims includes two very large catastrophes which we expect to occur with much lower frequency than once every six years (unless weather patterns have changed significantly, which should be of more immediate concern to those responsible for gross pricing as well as those responsible for reinsurance pricing!). An adjustment to the severity distribution is required to reflect the finite amount of insured property that is at risk. We chose £10 billion as an upper limit to the severity distribution.

Table 7 shows what we consider to be a reasonable range of parameters to use in the simulations.

Table 7 - Simulation Parameters

Frequency	Severity
0.75	1.25
1.00	1.33
1.25	1.50

The combination of three frequency and three severity parameters gives nine possible distributions for the gross catastrophes. The three severity parameters 1.25, 1.33 and 1.5 indicate events such as the 1987 and 1990 storms as being one in thirty, forty or fifty occurrences respectively. That is one every so many events not years. The frequency of these measured in years will depend upon the number of events assumed per year. A low severity parameter has a high probability of yielding very large claims.

The actual simulation can be performed using the U(0,1) random variable function of the spreadsheet package. The practitioner should consider the randomness of the generator. Simple algorithms for the generation of the U(0,1) can be set up if required.

A3.4 <u>The Company's Claims Distributions and Retention Policy</u>

The estimation of a company's gross claim from that of the market has been assumed to follow a linear relationship with market share measured by premium volume. We believe that this is a reasonable approach due to the very high number of relatively homogeneous small units which compose the exposure of a large company. This assumption may not hold for smaller companies who could have very regionalised exposure. More complex methods can be used. A good example is the method described in Section 4.4 and used by some US insurers to estimate hurricane losses. Exhibit 4 Page 2 shows the mean and standard deviation of the aggregate gross annual cost of claims under the simulation for the company in our example on each of the nine bases.

For each set of parameters, a simulation of perhaps five thousand years' of claims should be performed. The higher the number of simulations, the greater the amount of information available concerning the extremes of the aggregate claims distribution. On the other hand, should events that occur once in ten thousand years have a material influence on the management of the operation?

The next stage is to set up a parameterised programme which calculates the net financial impact to the company for each year of simulated claims. The parameters determining the precise details of the reinsurance programme are required. The premiums paid plus reinstatements payable should be included in the costs of the reinsurance. For some purposes it may be best to use current market premium rates, for others an estimate of the mean long term rate chargeable may be better.

The mean of the resulting net claims distribution can be subtracted from that of the gross distribution to indicate the mean claims recovery. This in turn can be compared to the mean cost of the reinsurance including reinstatement premiums. This should demonstrate the cost of reinsurance to the company over the long term.

The aim of the reinsurance however is to reduce the variability of the retained claims distribution. One problem is to determine how to measure this variability. The standard deviation, 95% confidence limit or 99% confidence limit could be used. Again, a benefit of simulation is that any moment of the distribution can be estimated. The advantage of measures such as the standard deviation is that they look at the shape of the whole distribution. Two identical companies with the same capital and probability of losing that capital could have entirely different claims variability due to different reinsurance. As a result, they will experience very different profits. This demonstrates one problem of the probability of loss concepts: they look at only one point in the claims distribution.

It is worth investigating the effect that the truncation of the claim severity has on the measure of variability selected. Table 8 shows the results for a simulation of 5,000 years with a Poisson parameter of 1.25 and a Pareto parameter of 1.25.

Table 8 - Gross Market Catastrophe Claims

	No Truncation	£10 Billion	£5 Billion
Average Annual Cost	549	448	433
SD of Average Annual Cost	2,771	920	753

Clearly, if conclusions are being drawn on the basis of the value of standard deviations it is important to investigate whether the conclusions are the same whatever the truncation point.

We are now ready commence investigation of the retention of the company. As we have touched on earlier, the retention philosophy must come from a consideration of the objectives of the company and may well incorporate shareholder utility curves. These discussions are outside the scope of this section. Here, we shall demonstrate some of the ways in which we can use this work to improve retention decisions.

Our starting point is to assume that the company in question has a catastrophe reinsurance programme covering claims arising from one event for £170 million excess of £30 million. The cover has been 95% placed at an initial cost of £22 million and has unlimited reinstatements paid 100% for time irrespective of the unelapsed exposure and pro-rata to the size of the recovery.

Exhibit 4 Page 3 shows the mean gross and net claims costs for this company for each combination of simulation parameters. The standard deviations are also shown. As expected the reinsurance programme results in a lower coefficient of variation for the net claims distribution than for the gross. Even under the most severe claim assumptions the expected reinsurance recovery net of reinstatements is £13 million against the original premium of £22 million. Can the reinsurance programme be improved without increasing the cost? We can investigate what happens when the height of the layers purchased is changed, both above £30 million and above £200 million. The cost is kept the same by increasing the amount of coinsurance, after all, who said "Placing 100% of the layer is the most efficient thing to do."!?

The graphs in Exhibit 4 Pages 4 to 6 show that with a fixed lower limit the standard deviation of the net claims reduces as the upper limit is raised! Further, raising the lower limit also reduces the standard deviation as is shown in Exhibit 4 Page 7. Perhaps the result of this is that companies should be encouraged to take higher layers of cover with more coinsurance? This will provide a reduction in the standard deviation of the retained claims at no additional cost.

We have concentrated, thus far, on one type of reinsurance. The variability that we are trying to control is the standard deviation of the retained catastrophe claims in one year. So why are we considering a reinsurance programme focusing on each event? What about an aggregate stop loss contract that caps the aggregate claims from all catastrophe events in the year? In order to perform a full analysis of this, the company would have to obtain quotes for this insurance.

The simulation allows us to investigate the levels of variability that would result from such contracts. These variabilities are shown in Exhibit 4 Page 8 for a stop loss of £100 million xs £50 million. The results look very promising. This is not wholly surprising since this reinsurance protects against frequency as well as severity of catastrophe.

We have not really discussed which of the nine sets of parameters we consider to be the most appropriate. The main reason for this is that our conclusions have been non-parametric. The results have held for all nine combinations. Exhibit 4 Pages 9 and 10 shows a hundred year simulation of catastrophes under each of these nine combinations. We hope that you will agree, based on your experience of UK weather claims, that they cover a reasonable range from the optimistic to the pessimistic.

Finally, a word of caution: we have used the standard deviation as a measure of variability. Exhibit 4 Page 11 compares the actual 95% and 99% confidence limits for the simulated net claims with the same limits estimated using the normal approximation. There are very considerable differences which demonstrate the skewness of these distributions and the care required when interpreting simulation results.

On the same note, examination of simulation results in Exhibit 4 Page 2 shows that the most severe set of claim assumptions, Pareto 1.25 and Poisson 1.25, do not have the highest standard deviation. The Pareto 1.33 and Poisson 1.25 standard deviation is higher. This could either be a genuine result, a random variation in the simulation or an effect of capping the claim severity distribution. If the same sample of U(0,1) variables are used for both sets of simulations then the Pareto 1.25 and Poisson 1.25 has the highest standard deviation. This is shown in Table 9 below:-

Table 9 - Comparison of Simulations (£ millions)

Simulation Parameters	Simulation Mean	Simulation Standard Deviation
* Pareto 1.33 Poisson 1.25	421	940
* Pareto 1.25 Poisson 1.25	448	920
+ Pareto 1.25 Poisson 1.25	469	1,064

* As shown in Exhibit 4 Page 2.

\+ Calculated using the U(0,1) variables from the simulation of Pareto 1.33 and Poisson 1.25 in Exhibit 4 Page 2.

It would appear that the results arose from random variations in the simulation.

APPENDIX 4

EXAMPLE APPLICATIONS OF THE RECURSIVE METHOD

A4.1 Introduction

We have applied the recursive method to the aviation and liability data sets in order to estimate the aggregate claims distributions. The property data set is so large that we would not recommend the use of the recursive method. There are two reasons for this: first, the normal approximation should be reasonably robust when used with such a high number of claims; second, if the number of claims assumed for the future is very high then the computation of the aggregate claims distribution using the recursive formula becomes arduous.

A4.2 Methodology

The data sets are rescaled. The rescaled data points are then rounded to the nearest integer. This results in an approximation for the severity distribution. Essentially, the continuous severity distribution is substituted by a mass function on the first few dozen integers. We input the empirical severity distributions as implied by the data. An alternative approach would be to fit one of the classical distributions to the data before scaling and grouping the severities for use in the recursive formula.

The choice of scaling factor represents a trade-off. If the scaling factor chosen is too small, then the number of mass points for the proxy distribution is large, and the application of the recursive formula becomes more difficult. However, if the scaling factor is too large the recursive formula may be more easily applied, but the proxy distribution may not reflect all the characteristics of the parent distribution from which it is derived.

Fortunately, this process is quite robust in that the accuracy gained at having three hundred mass points rather than forty, say, is outweighed by the added computational complexity when applying the recursive formula. The scaled data sets are shown in Exhibit 5 Pages 1 and 2.

We assumed a Poisson distribution for claim frequency taking the number of claims as assumed in Appendices 2 and 3 as the estimate of the mean of the distribution.

A4.3 Aviation Example

Exhibit 5 Page 3 shows graphs of various classical points on the aggregate claims distribution against the per risk claim retention. These graphs are directly comparable to those produced by the H & M method as shown in Exhibit 3 Page 4.

A4.4 Liability Example

Exhibit 5 Page 4 shows graphs of various classical points on the aggregate claims distribution against the per risk claim retention. These graphs are directly comparable to those produced by the H & M method as shown in Exhibit 3 Page 10.

A4.5 Property Example

For the reasons outlined above, we used the normal approximation on this data set. Exhibit 5 Page 5 shows graphs of various classical points on the aggregate claims distribution against the per risk claim retention. These graphs can be compared to those produced by the H & M method as shown in Exhibit 3 Page 16 in order to assess the reasonableness of normal approximation.

Section 8

EXHIBITS

Exhibit 3 - Exhibits for Appendix 2

Page 11 - Graph of retention vs capital at risk for the liability example.

Page 12 - Graph of retention vs capital at risk as percentages of gross premium for the liability example.

Page 13 - Graph of retention vs capital at risk as percentages of net premium for the liability example.

Page 14 - H & M method output for the property example.

Page 15 - H & M method results summary for the property example.

Page 16 - Graph of retention vs net aggregate claims for the property example.

Page 17 - Graph of retention vs capital at risk for the property example.

Page 18 - Graph of retention vs capital at risk as percentages of gross premium for the property example.

Page 19 - Graph of retention vs capital at risk as percentages of net premium for the property example.

Exhibit 4 - Exhibits for Appendix 3

Page 1 - UK property catastrophe past claims experience.

Page 2 - Simulation results for gross aggregate claims.

Page 3 - Simulation results for gross and net aggregate claims.

Exhibit 5 - Exhibits for Appendix 4

Page 4 - Graphs of retention vs net aggregate claims for the liability example.

Page 5 - Graphs of the normal approximation confidence intervals for the property example.

Reinsurance and Retentions Working Party
Sample Data Distribution Used in Examples
Aviation LMX
Amounts in £000s

Claim Amount	Probability Point
22	2.439%
35	4.878%
235	7.317%
236	9.756%
244	12.195%
280	14.634%
332	17.073%
332	19.512%
338	21.951%
360	24.390%
598	26.829%
666	29.268%
693	31.707%
723	34.146%
750	36.585%
766	39.024%
795	41.463%
997	43.902%
1,006	46.341%
1,035	48.780%
1,080	51.220%
1,615	53.659%
2,507	56.098%
2,635	58.537%
2,635	60.976%
3,622	63.415%
3,832	65.854%
4,042	68.293%
4,551	70.732%
4,868	73.171%
5,800	75.610%
6,247	78.049%
8,865	80.488%
15,714	82.927%
20,160	85.366%
24,670	87.805%
25,587	90.244%
49,912	92.683%
52,211	95.122%
83,445	97.561%

Severity Mean = 9175

Claim Frequency Distribution = Poisson

Mean Claims Per Year = 8.000

Reinsurance and Retentions Working Party
Sample Data Distribution Used in Examples
Liability
Amounts in £s

Claim Amount	Probability Point
46	0.16%
210	2.03%
464	3.91%
604	5.79%
726	7.67%
864	9.55%
1,005	11.42%
1,168	13.30%
1,366	15.18%
1,542	17.06%
1,689	18.94%
1,843	20.81%
1,996	22.69%
2,218	24.57%
2,342	26.45%
2,545	28.33%
2,773	30.20%
3,135	32.08%
3,423	33.96%
3,880	35.84%
4,094	37.72%
4,458	39.59%
4,922	41.47%
5,252	43.35%
5,678	45.23%
6,104	47.10%
6,268	48.98%
6,917	50.86%
7,560	52.74%
8,024	54.62%
8,685	56.49%
9,763	58.37%
10,648	60.25%
11,784	62.13%
13,018	64.01%
14,054	65.88%
15,421	67.76%
17,940	69.64%
19,019	71.52%
21,568	73.40%
23,442	75.27%
25,230	77.15%
27,479	79.03%
30,405	80.91%
34,021	82.79%
37,906	84.66%
44,741	86.54%
58,040	88.42%
71,214	90.30%
91,109	92.18%
136,713	94.05%
199,848	95.93%
282,685	97.81%
291,012	97.97%
302,705	98.12%
384,537	98.28%
386,191	98.44%
388,151	98.59%
390,770	98.75%
508,880	98.90%
818,551	99.06%
835,373	99.22%
854,870	99.37%
975,428	99.53%
1,151,688	99.69%
1,756,234	99.84%
2,275,973	99.90%

Severity Mean = 38,134

Claim Frequency Distribution = Poisson

Mean Claims Per Year = 262.233

Reinsurance and Retentions Working Party
Sample Data Distribution Used in Examples
Property
Amounts in £s

Claim Amount	Probability Point	Claim Amount	Probability Point
0	0.01%	32,390	98.87%
20	6.59%	32,930	98.91%
30	13.18%	34,130	98.95%
50	19.77%	36,630	99.00%
70	26.36%	38,530	99.04%
90	32.95%	39,740	99.09%
120	39.53%	42,920	99.13%
170	46.12%	45,000	99.17%
220	52.71%	46,660	99.22%
270	59.30%	48,480	99.26%
370	65.88%	50,080	99.30%
510	72.47%	51,540	99.35%
760	79.06%	53,840	99.39%
830	80.16%	56,250	99.44%
900	81.25%	60,150	99.48%
980	82.35%	67,290	99.52%
1,070	83.45%	72,370	99.57%
1,200	84.55%	77,400	99.61%
1,330	85.65%	93,070	99.66%
1,500	86.74%	117,940	99.70%
1,710	87.84%	122,760	99.71%
1,990	88.94%	131,250	99.71%
2,290	90.04%	132,210	99.72%
2,700	91.14%	134,780	99.73%
3,140	92.23%	143,440	99.74%
3,750	93.33%	144,740	99.74%
4,590	94.43%	148,820	99.75%
6,130	95.53%	149,540	99.76%
8,540	96.63%	150,000	99.77%
11,090	97.37%	153,600	99.78%
11,250	97.42%	153,940	99.79%
11,420	97.46%	165,000	99.80%
11,530	97.50%	168,750	99.80%
11,720	97.55%	186,880	99.81%
12,070	97.59%	187,500	99.82%
12,440	97.64%	192,450	99.82%
12,620	97.68%	194,580	99.83%
12,850	97.72%	214,990	99.84%
13,080	97.77%	222,240	99.85%
13,360	97.81%	225,000	99.85%
13,820	97.86%	234,250	99.86%
14,210	97.90%	234,800	99.87%
14,490	97.94%	247,220	99.88%
14,780	97.99%	248,000	99.88%
15,000	98.03%	305,650	99.89%
15,480	98.07%	313,440	99.90%
15,750	98.12%	326,630	99.90%
16,160	98.16%	334,860	99.91%
16,490	98.21%	375,000	99.92%
16,960	98.25%	381,080	99.93%
17,670	98.29%	427,500	99.93%
18,730	98.34%	450,000	99.94%
19,650	98.38%	549,600	99.95%
20,270	98.43%	601,230	99.96%
21,170	98.47%	626,700	99.96%
22,030	98.51%	1,117,730	99.97%
22,760	98.56%	1,261,060	99.98%
24,290	98.60%	3,753,050	99.99%
25,110	98.65%	4,305,000	99.99%
26,250	98.69%		
26,580	98.73%		
28,130	98.78%		
30,180	98.82%		

Severity Mean = 2956.1

Claim Frequency Distribution = Poisson

Mean Claims Per Year = 13661

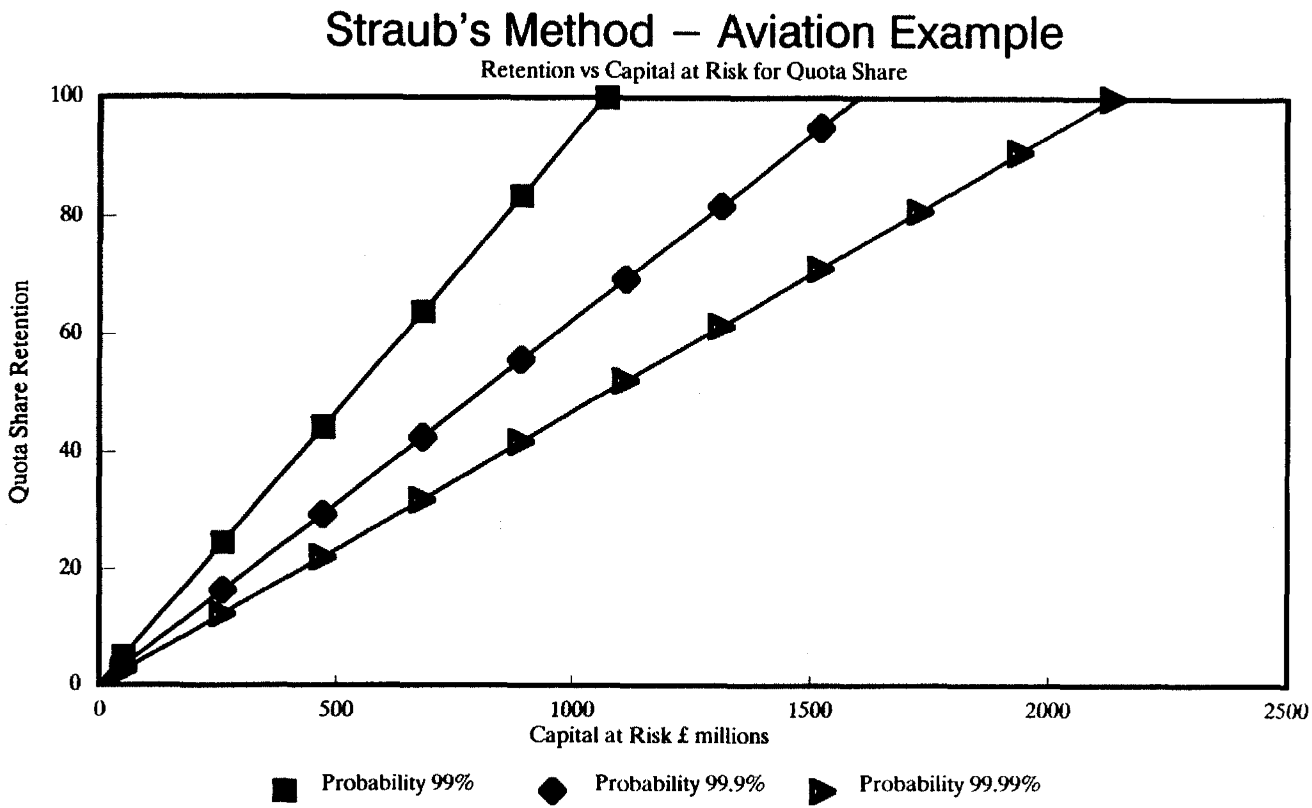


Exhibit 2
Page 1

Straub's Method – Aviation Example

Retention vs Capital at Risk for Risk XL

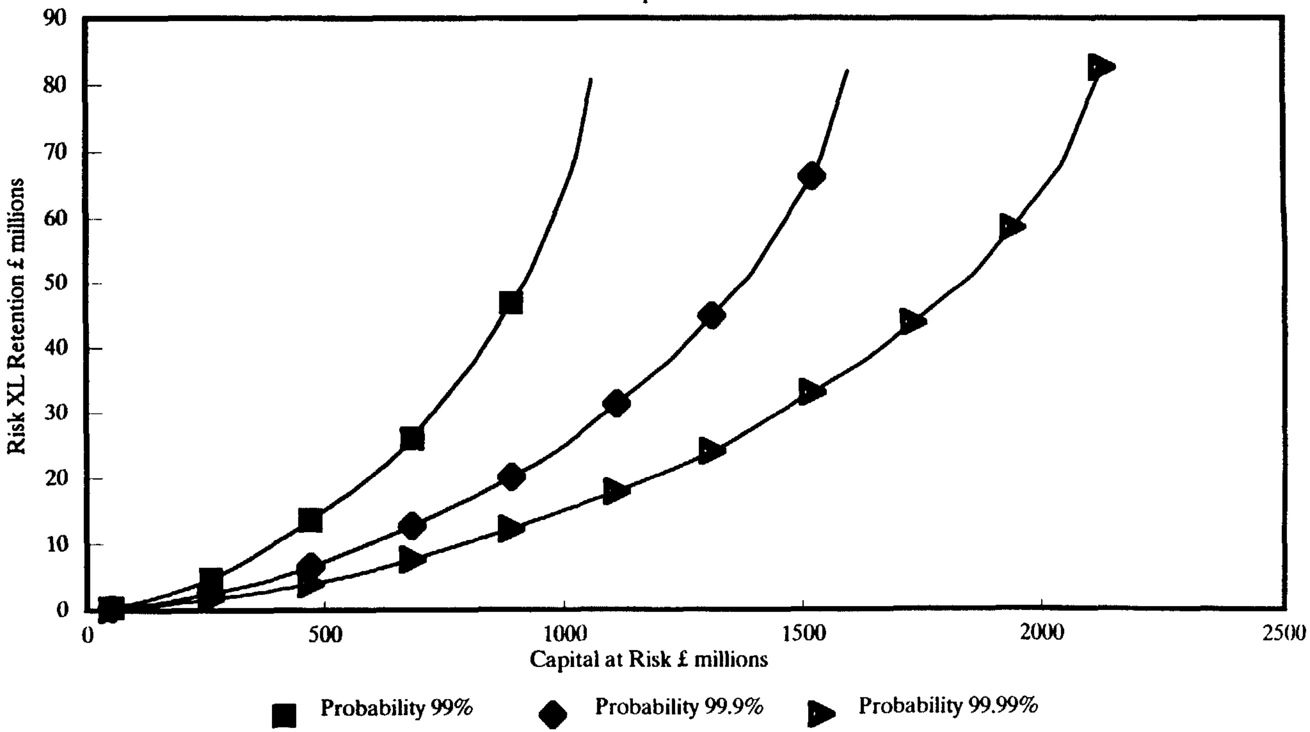


Exhibit 2
Page 2

Straub's Method - Aviation Example

Assumptions

Reference........................ Aviation example
Claim amount dist'n..............User Defined
Claim number dist'n..............Poisson
Total Gross Premiums(M)..........104.9
Capital at risk(M)...............62.9
Total Loading in prems(%)........30
Profit Loading(%)................10
Probability (1 in ...)...........1000
Reinsurance type................. Quota Share

Summary statistics

Claim amount Average............9174719
Claim amount CV.................2.01
Number of claims................8
Aggregate claim average.........73430000
Aggregate claim CV..............0.79

Results

The above assumptions imply Retention = 3%

For different Probabilities:-

Probability (1 in ...)	Retention
1,000	Retention = 3%
100,000	Retention = 2%
1,000,000	Retention = 1%
100,000,000	Retention = 1%
1,000,000,000	Retention = 1%

For different Capital at risk:-

Capital at risk As % prem	Amount(M)	Retention
5%	5.25	Retention = 0%
18%	18.88	Retention = 1%
100%	104.90	Retention = 6%
500%	524.50	Retention = 32%
1000%	1049.00	Retention = 65%

Straub's Method - Aviation Example

Assumptions

Reference........................ Aviation example
Claim amount dist'n..............User Defined
Claim number dist'n..............Poisson
Total Gross Premiums(M)...........104.9
Capital at risk(M)................62.9
Total Loading in prems(%).........30
Profit Loading(%).................10
Probability (1 in ...)...........1000
Reinsurance type.................. Risk XL

Summary statistics

Claim amount Average.............9174719
Claim amount CV..................2.01
Number of claims.................8
Aggregate claim average..........73430000
Aggregate claim CV...............0.79

Results

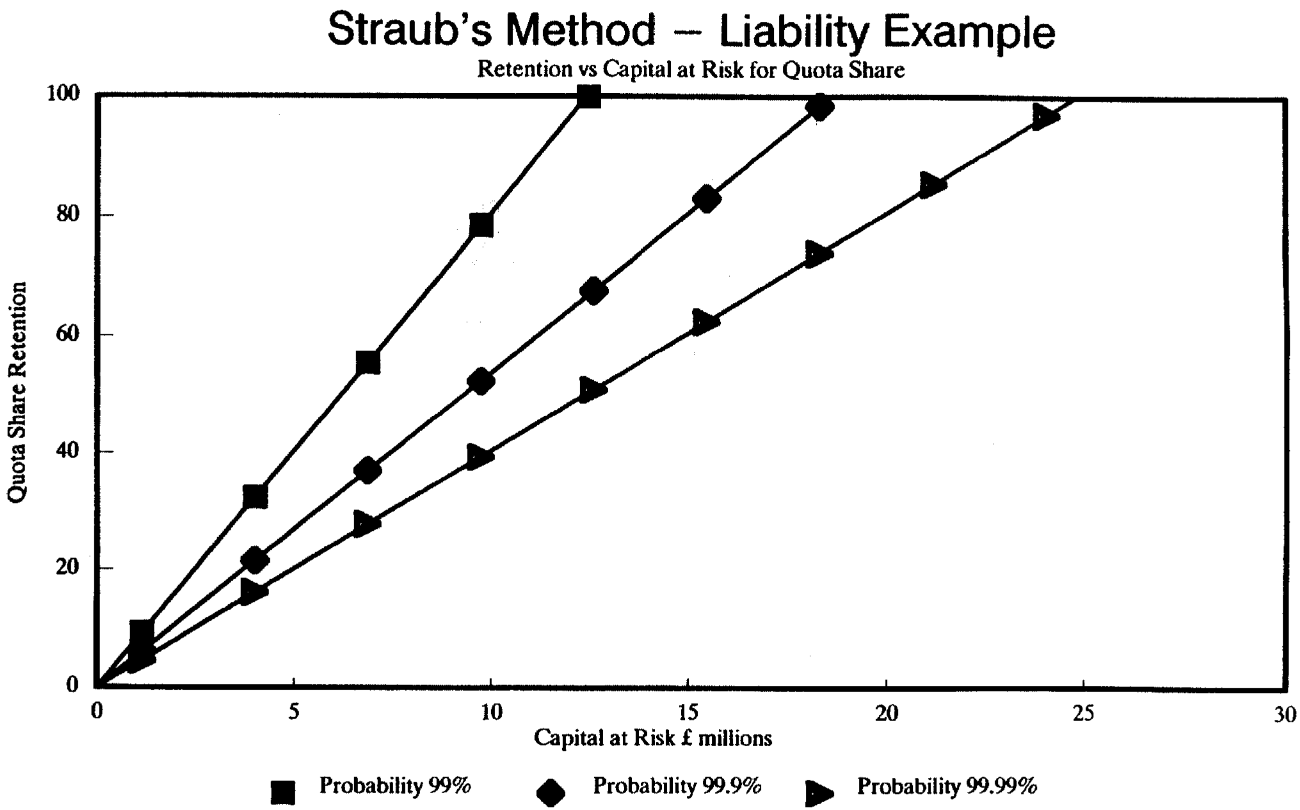
The above assumptions imply Retention = 404796

For different Probabilities:-

Probability (1 in ...)	Retention
1,000	Retention = 404796
100,000	Retention = 228819
1,000,000	Retention = 189917
100,000,000	Retention = 141289
1,000,000,000	Retention = 125335

For different Capital at risk:-

Capital at risk As % prem	Amount(M)	Retention
5%	5.25	Retention = 30594
18%	18.88	Retention = 112772
100%	104.90	Retention = 737865
500%	524.50	Retention = 8119757
1000%	1049.00	Retention = 27613906



Straub's Method – Liability Example
Retention vs Capital at Risk for Quota Share
Quota Share Retention
100
80
60
40
20
0
0
5
10
15
20
25
30
Capital at Risk £ millions
Probability 99%
Probability 99.9%
Probability 99.99%

Straub's Method – Liability Example

Retention vs Capital at Risk for Risk XL

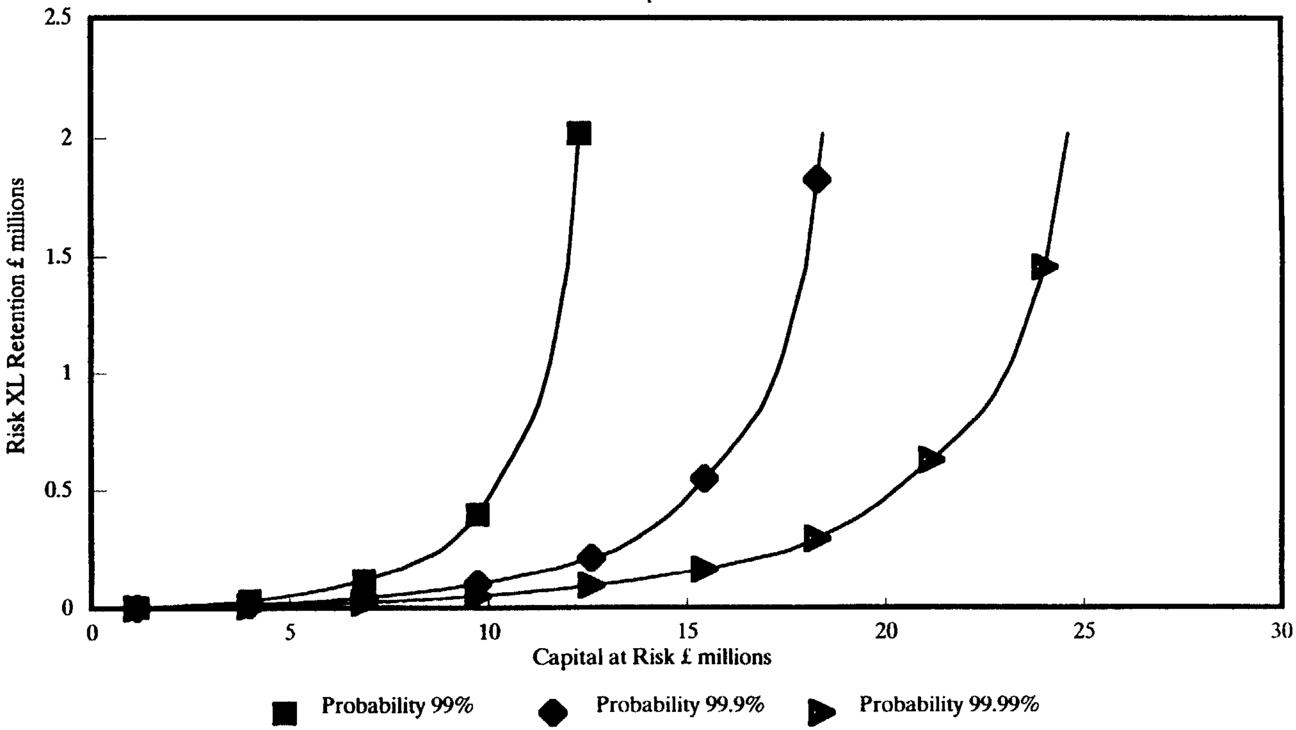


Exhibit 2
Page 6

Straub's Method - Liability Example

Assumptions

Reference......................... Liability example
Claim amount dist'n..............User Defined
Claim number dist'n..............Poisson
Total Gross Premiums(M)...........14.3
Capital at risk(M)................8.6
Total Loading in prems(%).........30
Profit Loading(%).................10
Probability (1 in ...)...........1000
Reinsurance type..................Quota Share

Summary statistics

Claim amount Average.............38133
Claim amount CV..................3.62
Number of claims.................262.5
Aggregate claim average..........10010000
Aggregate claim CV...............0.23

Results

The above assumptions imply Retention = 46%

For different Probabilities:-

Probability (1 in ...)	Retention
1,000	Retention = 46%
100,000	Retention = 27%
1,000,000	Retention = 23%
100,000,000	Retention = 17%
1,000,000,000	Retention = 15%

For different Capital at risk:-

Capital at risk As % prem	Amount(M)	Retention
5%	0.72	Retention = 3%
18%	2.57	Retention = 13%
100%	14.30	Retention = 76%
500%	71.50	No Quota Share reinsurance required!
1000%	143.00	No Quota Share reinsurance required!

Straub's Method - Liability Example

Assumptions

Reference........................ Liability example
Claim amount dist'n...............User Defined
Claim number dist'n...............Poisson
Total Gross Premiums(M)...........14.3
Capital at risk(M)................8.6
Total Loading in prems(%).........30
Profit Loading(%).................10
Probability (1 in ...)...........1000
Reinsurance type.................. Risk XL

Summary statistics

Claim amount Average.............38133
Claim amount CV..................3.62
Number of claims.................262.5
Aggregate claim average..........10010000
Aggregate claim CV...............0.23

Results

The above assumptions imply Retention = 75178

For different Probabilities:-

Probability (1 in ...)	Retention
1,000	Retention = 75178
100,000	Retention = 24110
1,000,000	Retention = 17610
100,000,000	Retention = 11137
1,000,000,000	Retention = 9327

For different Capital at risk:-

Capital at risk As % prem	Amount(M)	Retention
5%	0.72	Retention = 1606
18%	2.57	Retention = 7959
100%	14.30	Retention = 359715
500%	71.50	No Risk XL reinsurance required!
1000%	143.00	No Risk XL reinsurance required!

Straub's Method – Property Example

Retention vs Capital at Risk for Quota Share

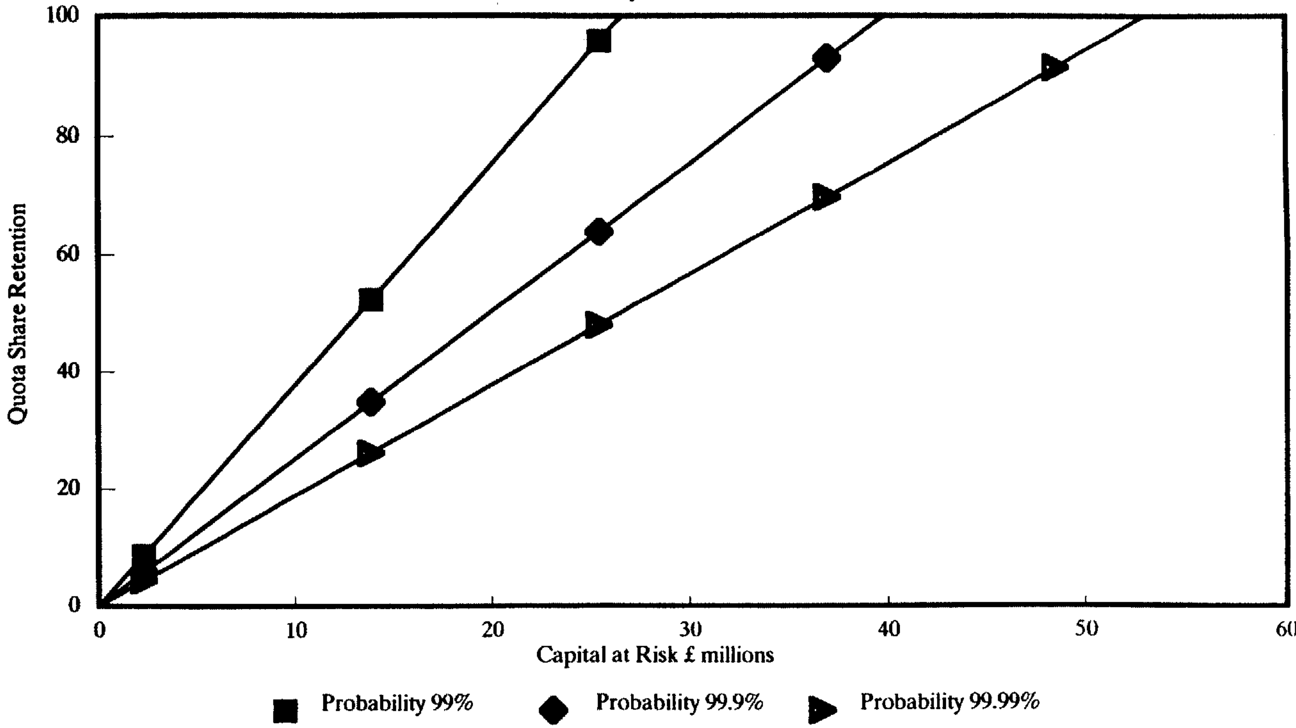


Exhibit 2
Page 9

Straub's Method – Property Example

Retention vs Capital at Risk for Risk XL

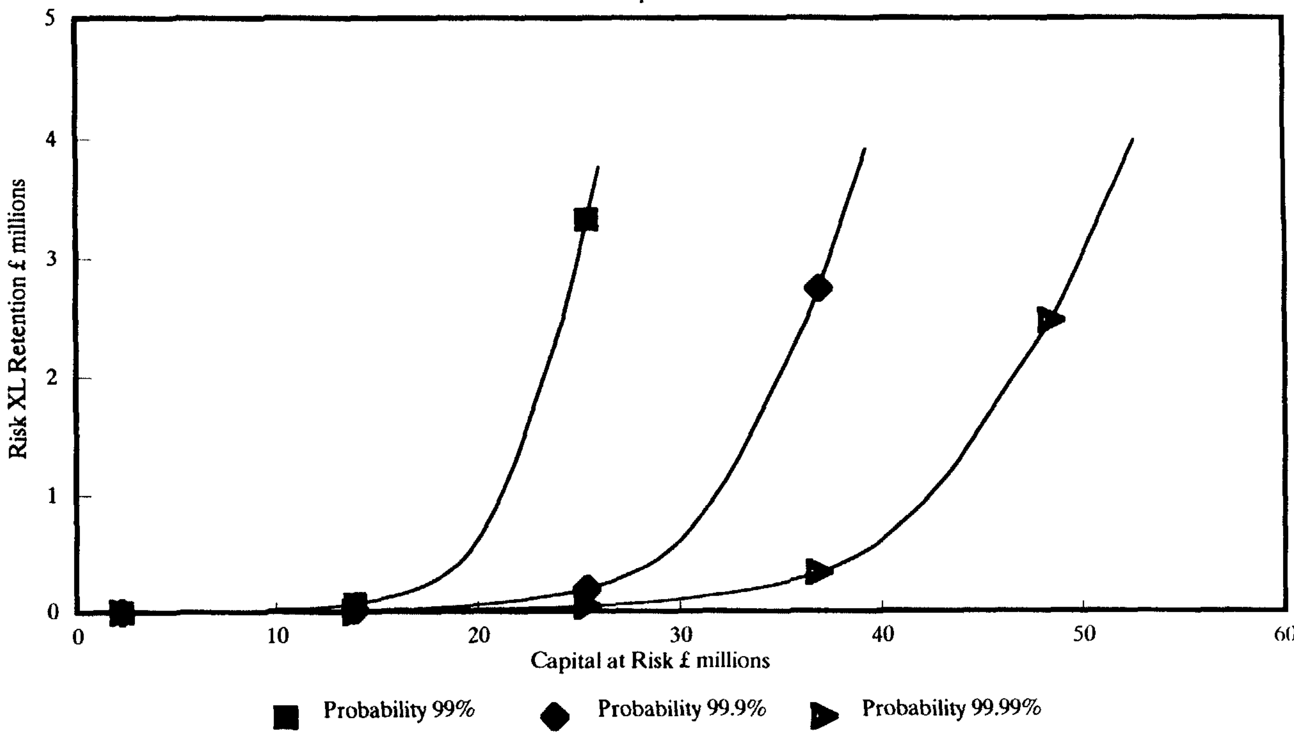


Exhibit 2
Page 10

Straub's Method - Property Example

Assumptions

Reference........................ Property example
Claim amount dist'n..............User Defined
Claim number dist'n..............Poisson
Total Gross Premiums(M)..........57.7
Capital at risk(M)...............34.6
Total Loading in prems(%).........30
Profit Loading(%).................10
Probability (1 in ...)...........1000
Reinsurance type................. Quota Share

Summary statistics

Claim amount Average.............2956
Claim amount CV..................19.7
Number of claims.................13659.44
Aggregate claim average..........40390000
Aggregate claim CV...............0.17

Results

The above assumptions imply Retention = 87%

For different Probabilities:-

Probability (1 in ...)	Retention
1,000	Retention = 87%
100,000	Retention = 52%
1,000,000	Retention = 43%
100,000,000	Retention = 32%
1,000,000,000	Retention = 29%

For different Capital at risk:-

Capital at risk As % prem	Amount(M)	Retention
5%	2.89	Retention = 7%
18%	10.39	Retention = 26%
100%	57.70	No Quota Share reinsurance required!
500%	288.50	No Quota Share reinsurance required!
1000%	577.00	No Quota Share reinsurance required!

Straub's Method - Property Example

Assumptions

Reference........................ Property example
Claim amount dist'n...............User Defined
Claim number dist'n...............Poisson
Total Gross Premiums(M)...........57.7
Capital at risk(M)................34.6
Total Loading in prems(%).........30
Profit Loading(%).................10
Probability (1 in ...)............1000
Reinsurance type.................. Risk XL

Summary statistics

Claim amount Average.............2956
Claim amount CV..................19.7
Number of claims.................13659.44
Aggregate claim average..........40390000
Aggregate claim CV...............0.17

Results

The above assumptions imply Retention = 1874895

For different Probabilities:-

Probability (1 in ...)	Retention
1,000	Retention = 1874895
100,000	Retention = 69274
1,000,000	Retention = 32947
100,000,000	Retention = 12044
1,000,000,000	Retention = 8336

For different Capital at risk:-

Capital at risk As % prem	Amount(M)	Retention
5%	2.89	Retention = 392
18%	10.39	Retention = 6181
100%	57.70	No Risk XL reinsurance required!
500%	288.50	No Risk XL reinsurance required!
1000%	577.00	No Risk XL reinsurance required!

Heckman and Meyers' Method – Example

Retention vs Capital at Risk

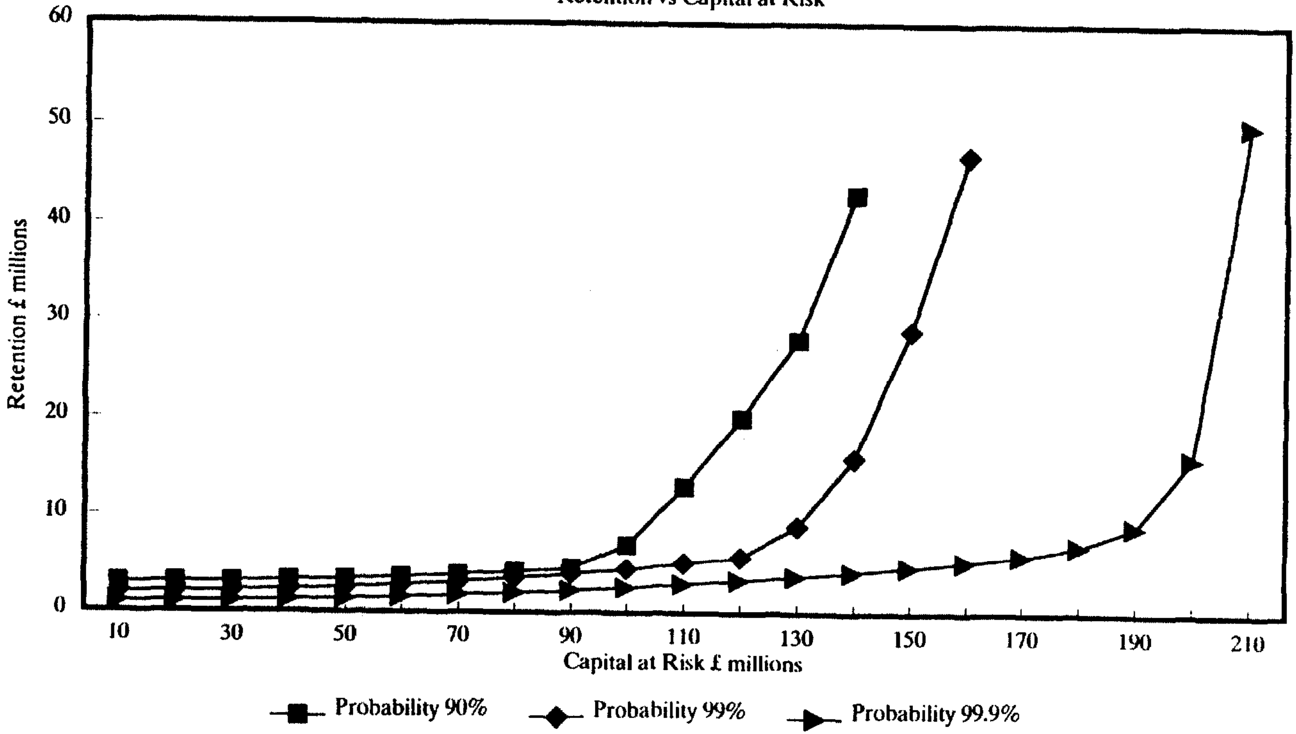


Exhibit 3
Page 1

Heckman and Meyers' Method - Aviation Example
Aggregate Claims Distribution

Aggregate Mean	Claim Severity Distribution	Contagion Parameter	Claim Freq. Mean	Claim Freq. Std Dev
73398		0.0000	8.000	2.828

Mixing Parameter	Aggregate Mean	Aggregate Std Dev	H	Number Of Intervals	Est Trunc Er In EPP Ratio
0.00%	73,398	58,553	0.8350	134	0.000008

Aggregate Claim Amount	Entry Ratio	Cumulative Probability	Excess Pure Premium	Excess Pure Premium Ratio
7,340	0.10000	6.46%	66,252	90.3%
14,680	0.20000	15.42%	59,726	81.4%
22,019	0.30000	21.91%	53,769	73.3%
29,359	0.40000	28.12%	48,257	65.8%
36,699	0.50000	34.49%	43,222	58.9%
44,039	0.60000	39.39%	38,602	52.6%
51,378	0.70000	43.42%	34,303	46.7%
58,718	0.80000	48.26%	30,323	41.3%
66,058	0.90000	52.97%	26,704	36.4%
73,398	1.00000	56.89%	23,399	31.9%
80,738	1.10000	60.62%	20,371	27.8%
88,077	1.20000	64.84%	17,627	24.0%
95,417	1.30000	69.56%	15,223	20.7%
102,757	1.40000	73.40%	13,135	17.9%
110,097	1.50000	76.72%	11,306	15.4%
117,436	1.60000	79.84%	9,715	13.2%
124,776	1.70000	82.40%	8,332	11.4%
132,116	1.80000	84.51%	7,120	9.7%
139,456	1.90000	86.50%	6,055	8.3%
146,796	2.00000	88.38%	5,135	7.0%
161,475	2.20000	91.30%	3,655	5.0%
176,155	2.40000	93.74%	2,561	3.5%
190,834	2.60000	95.56%	1,784	2.4%
205,514	2.80000	96.87%	1,234	1.7%
220,193	3.00000	97.78%	845	1.2%
234,873	3.20000	98.46%	572	0.8%
249,552	3.40000	98.94%	383	0.5%
264,232	3.60000	99.29%	255	0.4%
278,911	3.80000	99.52%	168	0.2%
293,591	4.00000	99.68%	110	0.2%
308,271	4.20000	99.79%	72	0.1%
322,950	4.40000	99.86%	46	0.1%
337,630	4.60000	99.91%	30	0.0%
352,309	4.80000	99.94%	19	0.0%
366,989	5.00000	99.96%	12	0.0%
381,668	5.20000	99.98%	7	0.0%
396,348	5.40000	99.99%	5	0.0%
411,027	5.60000	99.99%	3	0.0%
425,707	5.80000	99.99%	2	0.0%
440,387	6.00000	100.00%	1	0.0%

Heckman and Meyers' Method - Aviation Example

Retention	Net Aggregate Claims 90%	Net Aggregate Claims 99%	Net Aggregate Claims 99.9%	Mean	Reinsurance Risk Premium	Reinsurance Premium Net of Expenses	Premium Net of Reinsurance and Expenses
Infinity	154,940	252,069	333,960	73,398	0	0	83,883
75,000	149,531	240,757	319,190	71,527	1,871	2,138	81,746
50,000	125,913	199,173	260,325	63,186	10,212	11,671	72,213
25,000	85,422	129,077	164,548	46,028	27,370	31,280	52,603
15,000	60,711	88,903	112,555	34,256	39,142	44,733	39,150
10,000	46,002	66,253	82,448	26,969	46,429	53,062	30,822
5,000	30,001	41,930	51,304	18,417	54,981	62,835	21,048
1,000	9,110	12,102	14,472	6,030	67,368	76,992	6,892
500	5,138	6,778	8,071	3,452	69,946	79,938	3,945
100	1,130	1,481	1,755	767	72,631	83,007	877

Retention	Capital at Risk 90%	Capital at Risk 99%	Capital at Risk 99.9%
Infinity	71,057	168,186	250,077
75,000	67,785	159,011	237,444
50,000	53,700	126,960	188,112
25,000	32,819	76,474	111,945
15,000	21,561	49,753	73,405
10,000	15,180	35,431	51,626
5,000	8,953	20,882	30,256
1,000	2,218	5,210	7,580
500	1,193	2,833	4,126
100	253	604	878

Retention	Capital at Risk as a Percentage of Total Gross Premium 90%	99%	99.9%	Retention
Infinity	67.77	160.40	238.50	Infinity
75,000	64.65	151.65	226.45	71.53
50,000	51.21	121.08	179.40	47.69
25,000	31.30	72.93	106.76	23.84
15,000	20.56	47.45	70.01	14.31
10,000	14.48	33.79	49.24	9.54
5,000	8.54	19.92	28.86	4.77
1,000	2.12	4.97	7.23	0.95
500	1.14	2.70	3.93	0.48
100	0.24	0.58	0.84	0.10

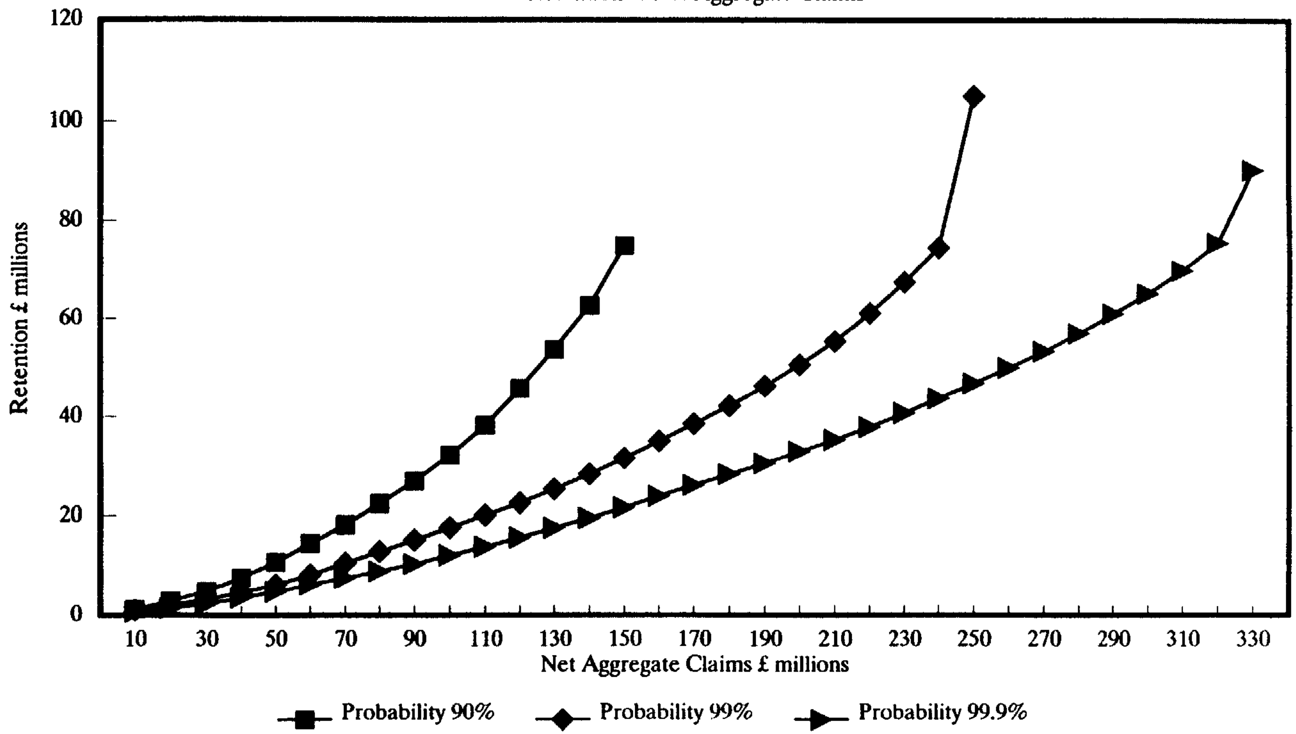
Retention	Capital at Risk as a Percentage of Net Premium 90%	99%	99.9%	Retention
Infinity	67.77	160.40	238.50	Infinity
75,000	66.34	155.62	232.38	73.40
50,000	59.49	140.65	208.40	55.39
25,000	49.91	116.30	170.25	38.02
15,000	44.06	101.67	150.00	30.65
10,000	39.40	91.96	134.00	25.96
5,000	34.03	79.37	115.00	19.00
1,000	25.75	60.49	88.00	11.61
500	24.19	57.44	83.66	10.14
100	23.13	55.16	80.17	9.13

Total Gross Premium = 104,854
Total Loading in Premiums = 30%
Expense Element of Premium = 20%

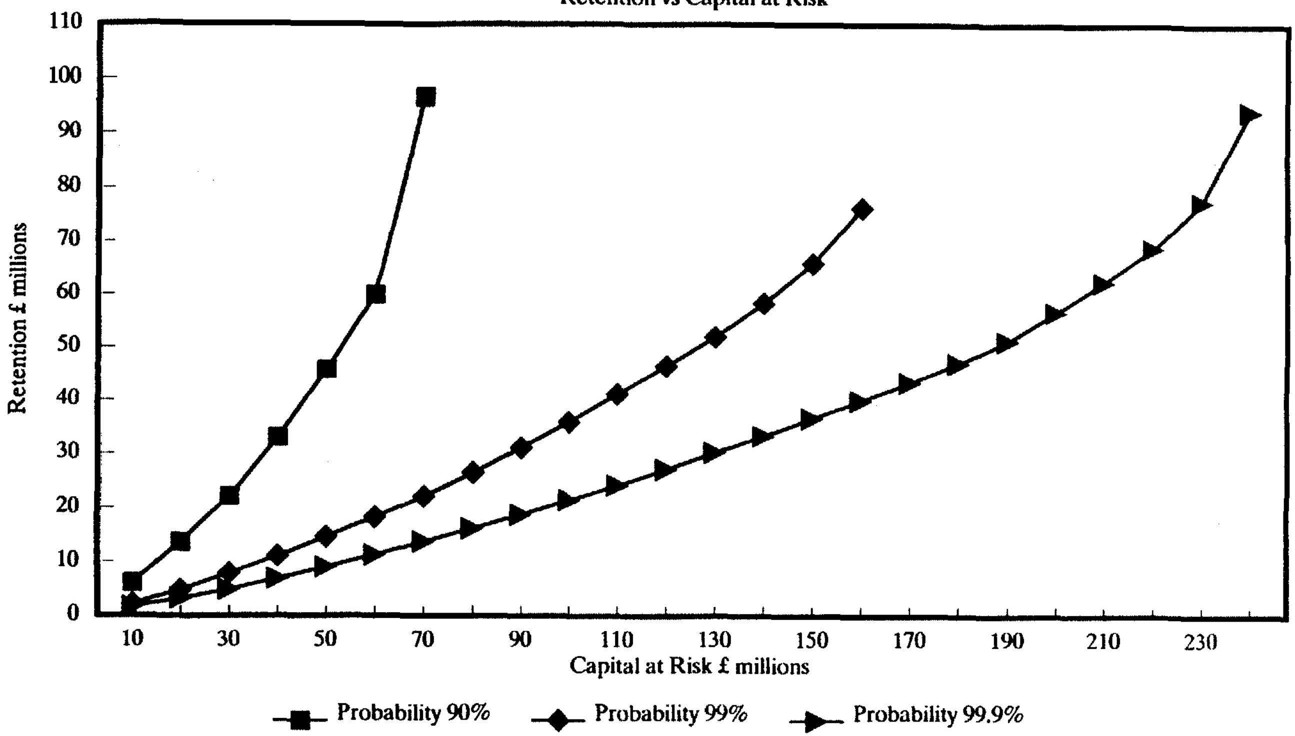
Capital at Risk = Net Aggregate Claims - Premium Net of Reinsurance and Expenses

Heckman and Meyers' Method – Aviation Example

Retention vs Net Aggregate Claims



Heckman and Meyers' Method – Aviation Example



Heckman and Meyers' Method – Aviation Example

Retention vs Capital at Risk (as %s of Gross Premium)

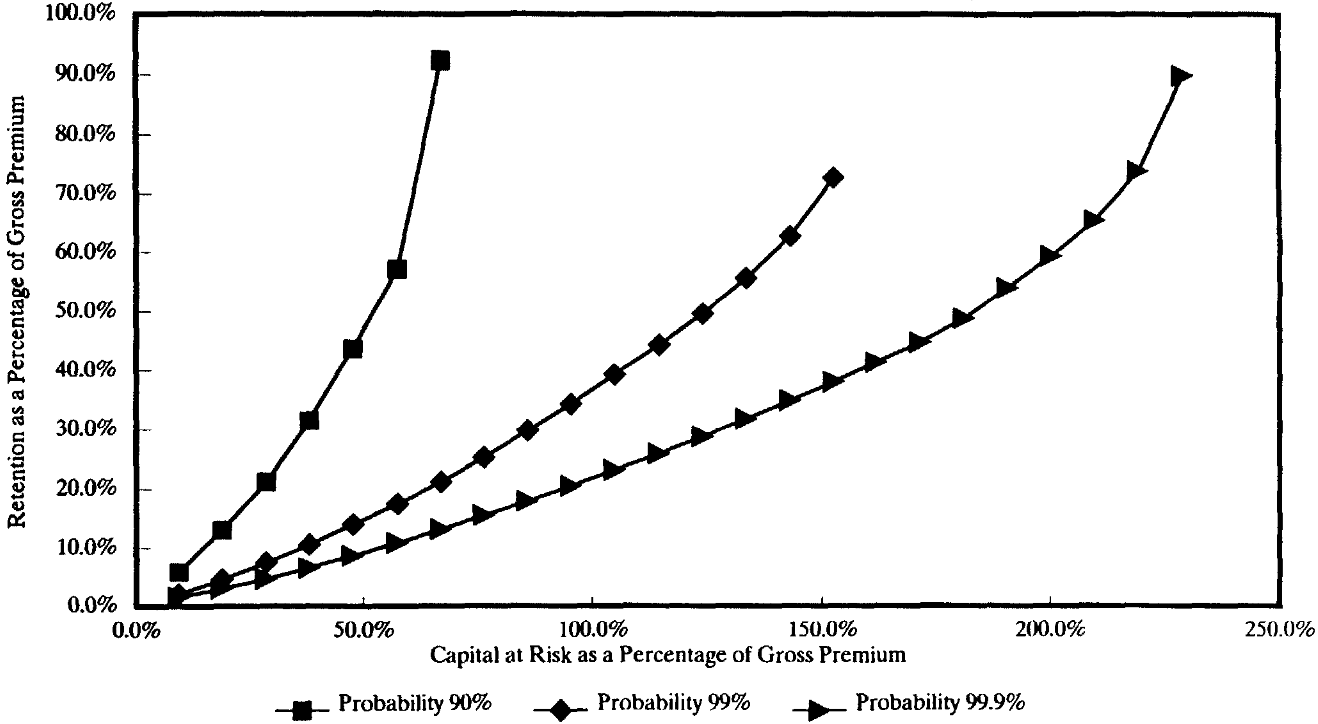
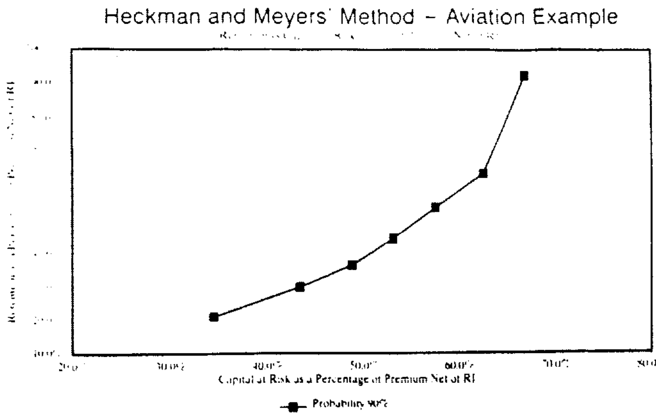
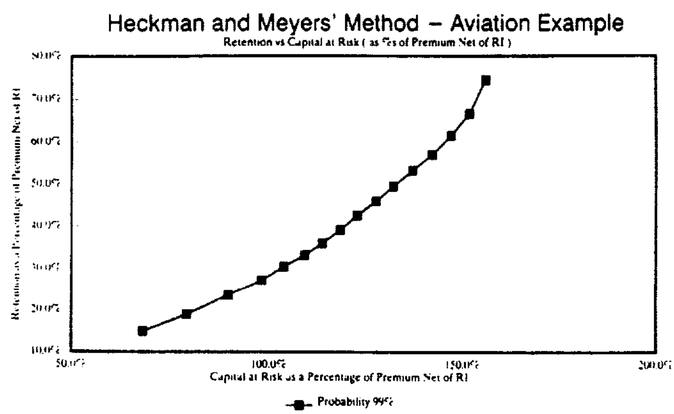
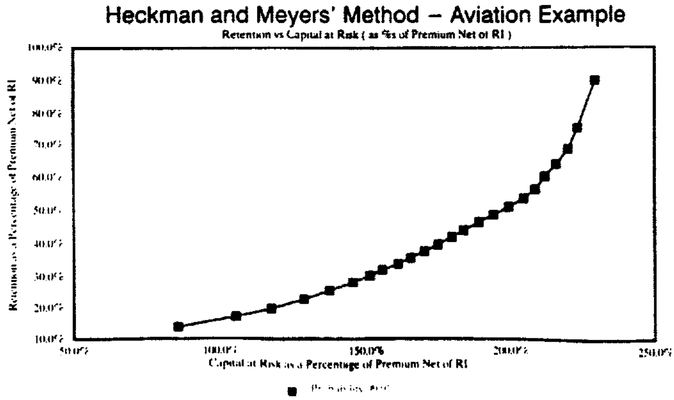


Exhibit 3
Page 6


Heckman and Meyers' Method - Aviation Example
Capital at Risk as a Percentage of Premium Net of RI


Heckman and Meyers' Method - Aviation Example
Retention vs Capital at Risk (as % of Premium Net of RI)
80.0%
70.0%
60.0%
50.0%
40.0%
30.0%
20.0%
10.0%
50.0%
100.0%
150.0%
200.0%
Capital at Risk as a Percentage of Premium Net of RI
Retention as a Percentage of Premium Net of RI
Probability 99%


Heckman and Meyers' Method - Aviation Example
Retention vs Capital at Risk (as % of Premium Net of RI)
100.0%
90.0%
80.0%
70.0%
60.0%
50.0%
40.0%
30.0%
20.0%
10.0%
50.0%
100.0%
150.0%
200.0%
250.0%
Capital at Risk as a Percentage of Premium Net of RI
Retention as a Percentage of Premium Net of RI

Heckman and Meyers' Method - Liability Example

Retention	Net Aggregate Claims 90%	Net Aggregate Claims 99%	Net Aggregate Claims 99.9%	Mean	Reinsurance Risk Premium	Reinsurance Premium Net of Expenses	Premium Net of Reinsurance and Expenses
Infinity	13,126,253	16,438,356	19,000,000	10,000,000	0	0	11,428,571
2,000,000	12,913,981	16,049,921	18,593,955	9,916,776	83,224	95,113	11,333,458
1,500,000	12,478,817	15,287,212	17,455,986	9,697,770	302,230	345,406	11,083,166
1,000,000	11,774,667	14,103,513	15,929,301	9,301,782	698,218	797,963	10,630,608
500,000	9,918,055	11,493,230	12,862,369	8,173,411	1,826,589	2,087,530	9,341,041
250,000	8,381,349	9,599,945	10,490,672	7,099,776	2,900,224	3,314,542	8,114,030
100,000	5,966,927	6,625,811	7,142,855	5,165,027	4,834,973	5,525,683	5,902,888
50,000	4,376,672	4,782,366	5,090,672	3,905,886	6,094,114	6,964,702	4,463,870

Retention	Capital at Risk 90%	Capital at Risk 99%	Capital at Risk 99.9%
Infinity	1,697,682	5,009,785	7,571,429
2,000,000	1,580,523	4,716,463	7,260,497
1,500,000	1,395,651	4,204,046	6,372,820
1,000,000	1,144,059	3,472,905	5,298,693
500,000	577,014	2,152,189	3,521,328
250,000	267,319	1,485,915	2,376,642
100,000	64,039	722,923	1,239,967
50,000	(87,198)	318,496	626,802

Retention	Capital at Risk as a Percentage of Total Gross Premium 90%	99%	99.9%	Retention as a Percentage of Total Gross Premium
Infinity	11.88	35.07	53.00	Infinity
2,000,000	11.06	33.02	50.82	14.00
1,500,000	9.77	29.43	44.61	10.50
1,000,000	8.01	24.31	37.09	7.00
500,000	4.04	15.07	24.65	3.50
250,000	1.87	10.40	16.64	1.75
100,000	0.45	5.06	8.68	0.70
50,000	(0.61)	2.23	4.39	0.35

Retention	Capital at Risk as a Percentage of Net Premium 90%	99%	99.9%	Retention as a Percentage of Net Premium
Infinity	11.88	35.07	53.00	Infinity
2,000,000	11.16	33.29	51.25	14.12
1,500,000	10.07	30.35	46.00	10.83
1,000,000	8.61	26.14	39.87	7.53
500,000	4.94	18.43	30.16	4.28
250,000	2.64	14.65	23.43	2.46
100,000	0.87	9.80	16.80	1.36
50,000	(1.56)	5.71	11.23	0.90

Total Gross Premium = 14,285,714
Total Loading in Premiums = 30%
Expense Element of Premium = 20%

Capital at Risk = Net Aggregate Claims - Premium Net of Reinsurance and Expenses

Heckman and Meyers' Method – Liability Example

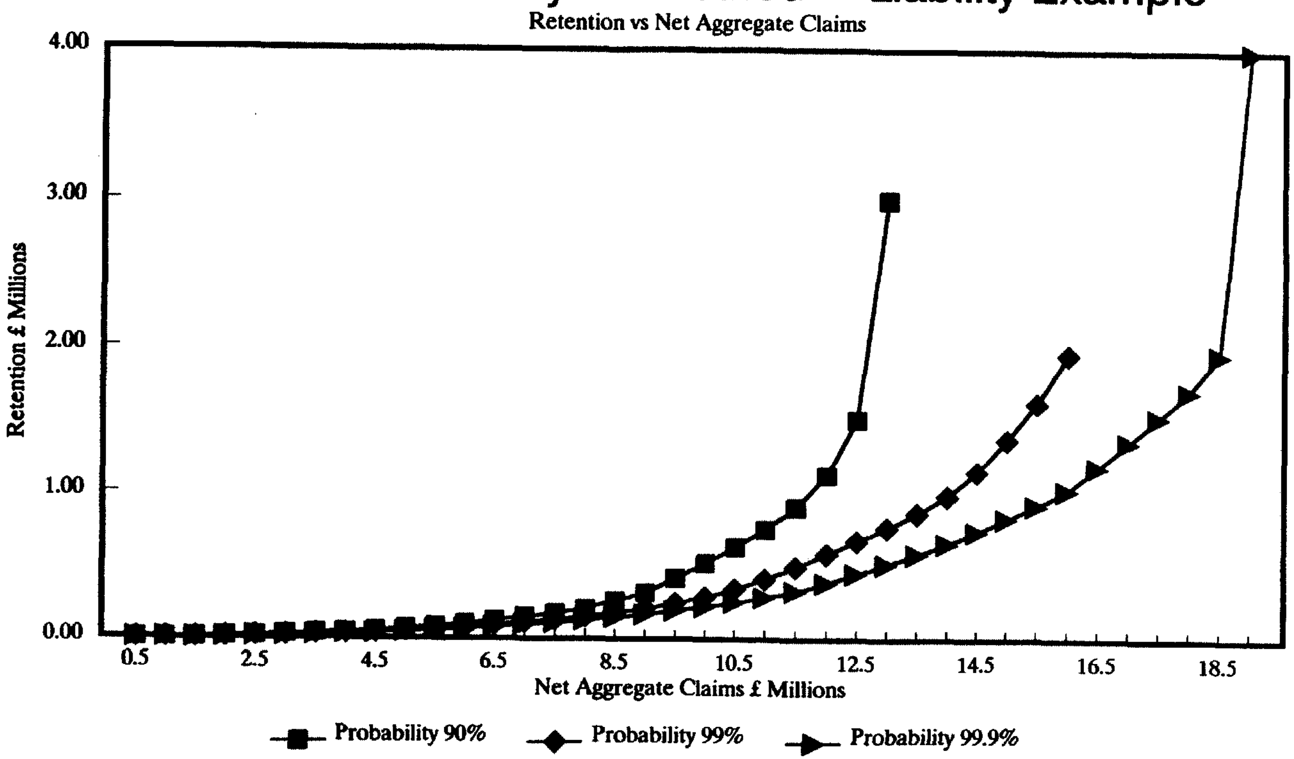


Exhibit 3
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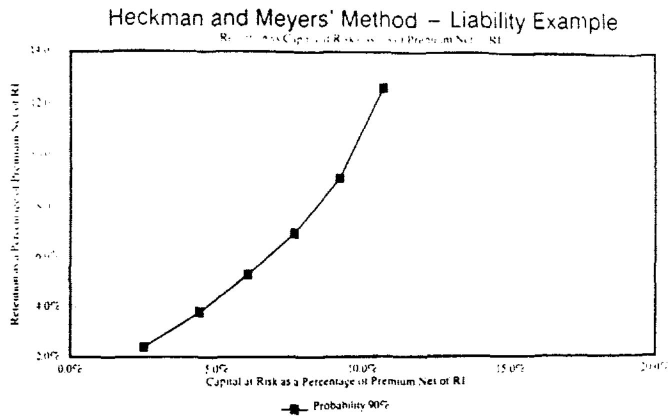
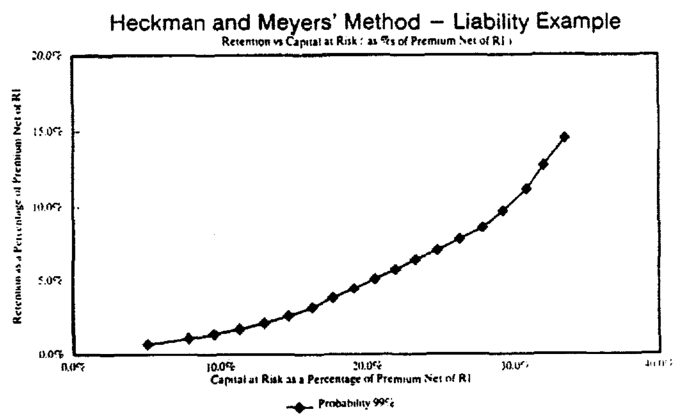
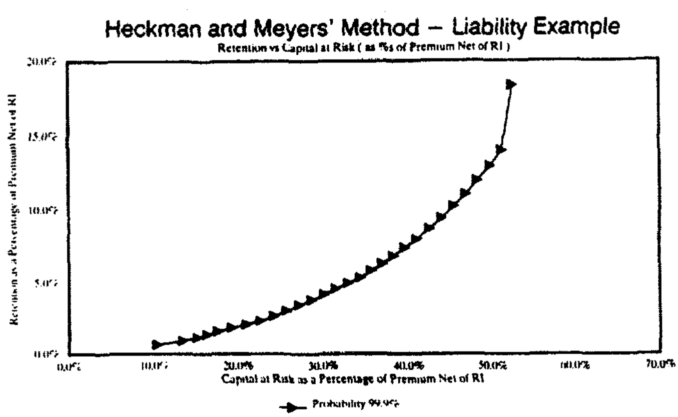

Heckman and Meyers' Method – Liability Example
Retention as a Percentage of Premium Net of RI
0.0%
5.0%
10.0%
15.0%
Capital at Risk as a Percentage of Premium Net of RI
Probability 90%


Heckman and Meyers' Method – Liability Example
Retention vs Capital at Risk (as %s of Premium Net of RI)
20.0%
15.0%
10.0%
5.0%
0.0%
Retention as a Percentage of Premium Net of RI
0.0%
10.0%
20.0%
30.0%
Capital at Risk as a Percentage of Premium Net of RI
Probability 99%


Heckman and Meyers' Method – Liability Example
Retention vs Capital at Risk (as %s of Premium Net of RI)
20.0%
15.0%
10.0%
5.0%
0.0%
Retention as a Percentage of Premium Net of RI
0.0%
10.0%
20.0%
30.0%
40.0%
50.0%
60.0%
70.0%
Capital at Risk as a Percentage of Premium Net of RI
Probability 99.9%

Heckman and Meyers' Method - Property Example
Aggregate Claims Distribution

Aggregate Mean	Claim Severity Distribution	Contagion Parameter	Claim Freq. Mean	Claim Freq. Std Dev	
40,383,860		0.0000	13,661.000	116.960	
Mixing Parameter	**Aggregate Mean**	**Aggregate Std Dev**	**H**	**Number Of Intervals**	**Est Trunc Er In EPP Ratio**
0.00%	40,383,860	6,858,151	0.6100	24	0.000002

Aggregate Claim Amount	Entry Ratio	Cumulative Probability	Excess Pure Premium	Excess Pure Premium Ratio
28,268,702	0.70000	1.83%	12,138,070	30.1%
32,307,088	0.80000	11.01%	8,325,236	20.6%
36,345,474	0.90000	29.98%	5,086,805	12.6%
40,383,860	1.00000	53.49%	2,732,753	6.8%
44,422,246	1.10000	73.94%	1,287,311	3.2%
44,826,085	1.11000	75.62%	1,185,492	2.9%
45,229,923	1.12000	77.24%	1,090,339	2.7%
45,633,762	1.13000	78.78%	1,001,556	2.5%
46,037,600	1.14000	80.25%	918,847	2.3%
46,441,439	1.15000	81.64%	841,917	2.1%
48,460,632	1.20000	87.55%	533,773	1.3%
48,864,471	1.21000	88.53%	485,497	1.2%
49,268,309	1.22000	89.45%	441,057	1.1%
49,672,148	1.23000	90.31%	400,209	1.0%
50,075,986	1.24000	91.11%	362,716	0.9%
50,479,825	1.25000	91.86%	328,350	0.8%
50,883,664	1.26000	92.55%	296,896	0.7%
51,287,502	1.27000	93.20%	268,144	0.7%
51,691,341	1.28000	93.80%	241,899	0.6%
52,095,179	1.29000	94.35%	217,975	0.5%
52,499,018	1.30000	94.86%	196,194	0.5%
52,902,857	1.31000	95.33%	176,392	0.4%
53,306,695	1.32000	95.76%	158,411	0.4%
53,710,534	1.33000	96.16%	142,106	0.4%
54,114,372	1.34000	96.52%	127,340	0.3%
54,518,211	1.35000	96.86%	113,985	0.3%
56,537,404	1.40000	98.14%	64,467	0.2%
60,575,790	1.50000	99.40%	19,121	0.1%
64,614,176	1.60000	99.83%	5,193	0.0%
68,652,562	1.70000	99.96%	1,340	0.0%
70,671,755	1.75000	99.98%	691	0.0%

Heckman and Meyers' Method - Property Example

Retention	Net Aggregate Claims 90%	Net Aggregate Claims 99%	Net Aggregate Claims 99.9%	Mean	Reinsurance Risk Premium	Reinsurance Premium Net of Expenses	Premium Net of Reinsurance and Expenses
Infinity	49,819,650	59,303,860	67,099,340	40,383,860	0	0	46,152,987
1,000,000	36,041,260	39,664,130	42,220,870	32,339,390	8,044,470	9,193,680	36,959,307
500,000	32,466,640	34,852,050	36,703,350	29,779,590	10,604,270	12,119,166	34,033,821
250,000	28,937,770	30,639,720	31,943,310	26,979,140	13,404,720	15,319,680	30,833,307
200,000	27,733,100	29,247,760	30,362,690	25,978,770	14,405,090	16,462,960	29,690,027
150,000	26,160,000	27,462,100	28,426,640	24,647,370	15,736,490	17,984,560	28,168,427
100,000	23,881,340	24,905,960	25,678,690	22,667,190	17,716,670	20,247,623	25,905,364
50,000	20,524,950	21,237,130	21,790,610	19,660,700	20,723,160	23,683,611	22,469,376
10,000	12,782,330	13,056,330	13,277,990	12,439,930	27,943,930	31,935,920	14,217,067
5,000	9,987,180	10,173,110	10,320,780	9,765,680	30,618,180	34,992,206	11,160,781
2,500	7,550,660	7,666,410	7,765,550	7,408,590	32,975,270	37,686,023	8,466,964
1,000	4,935,690	4,998,530	5,047,590	4,859,290	35,524,570	40,599,509	5,553,478
500	3,420,490	3,459,020	3,489,060	3,373,200	37,010,660	42,297,897	3,855,090

Retention	Capital at Risk 90%	Capital at Risk 99%	Capital at Risk 99.9%
Infinity	3,666,663	13,150,873	20,946,353
1,000,000	(918,047)	2,704,823	5,261,563
500,000	(1,567,181)	818,229	2,669,529
250,000	(1,895,537)	(193,587)	1,110,003
200,000	(1,956,927)	(442,267)	672,663
150,000	(2,008,427)	(706,327)	258,213
100,000	(2,024,024)	(999,404)	(226,674)
50,000	(1,944,426)	(1,232,246)	(678,766)
10,000	(1,434,737)	(1,160,737)	(939,077)
5,000	(1,173,601)	(987,671)	(840,001)
2,500	(916,304)	(800,554)	(701,414)
1,000	(617,788)	(554,948)	(505,888)
500	(434,600)	(396,070)	(366,030)

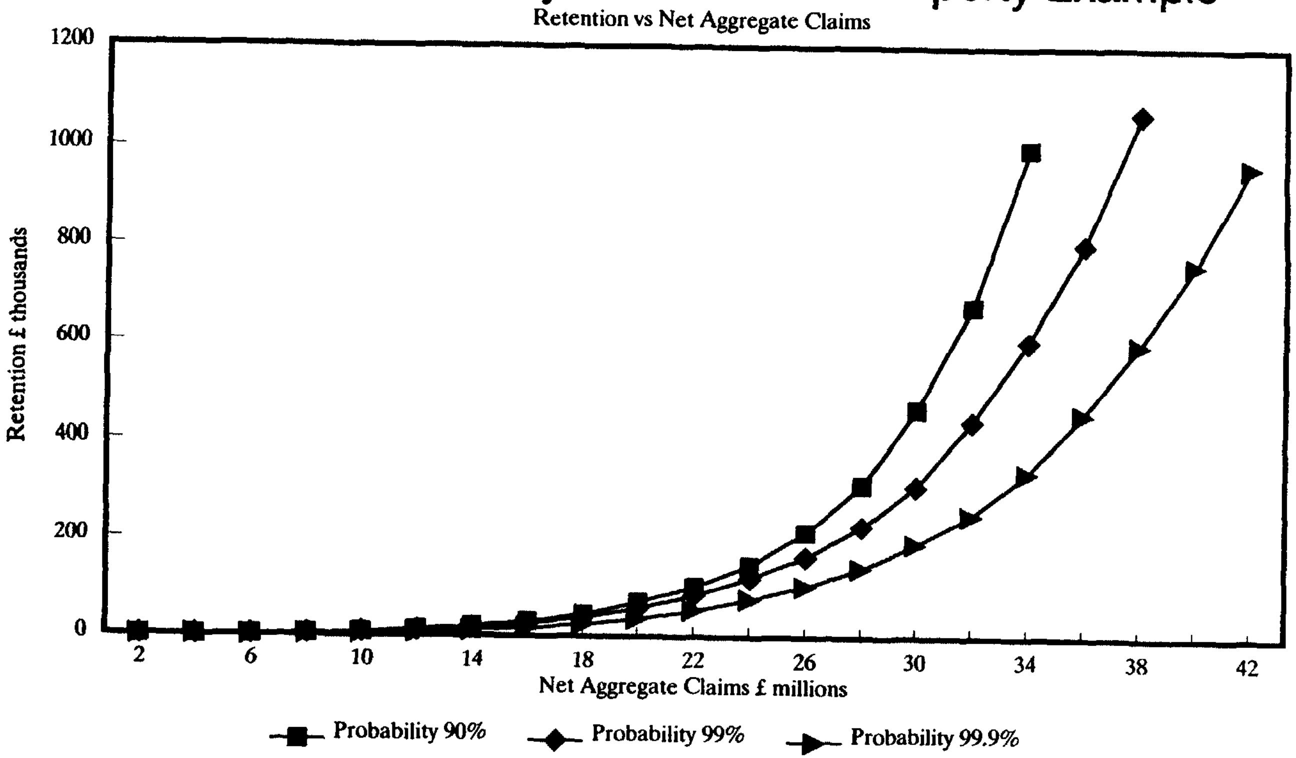
Retention	Capital at Risk as a Percentage of Total Gross Premium 90%	99%	99.9%	Retention
Infinity	6.36	22.80	36.31	Infinity
1,000,000	(1.59)	4.69	9.12	1.73
500,000	(2.72)	1.42	4.63	0.87
250,000	(3.29)	(0.34)	1.92	0.43
200,000	(3.39)	(0.77)	1.17	0.35
150,000	(3.48)	(1.22)	0.45	0.26
100,000	(3.51)	(1.73)	(0.39)	0.17
50,000	(3.37)	(2.14)	(1.18)	0.09
10,000	(2.49)	(2.01)	(1.63)	0.02
5,000	(2.03)	(1.71)	(1.46)	0.01
2,500	(1.59)	(1.39)	(1.22)	0.00
1,000	(1.07)	(0.96)	(0.88)	0.00

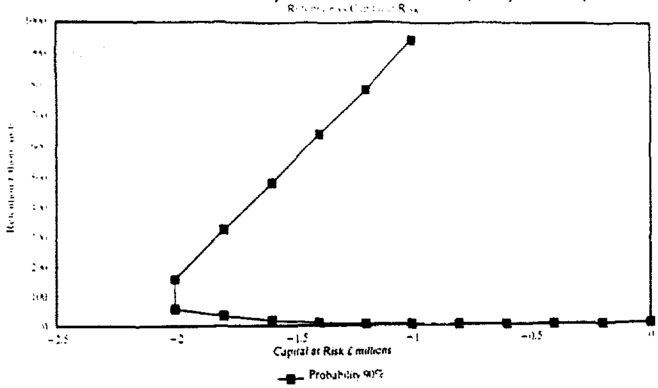
Retention	Capital at Risk as a Percentage of Net Premium 90%	99%	99.9%	Retention
Infinity	7.94	28.49	45.38	Infinity
1,000,000	(2.48)	7.32	14.24	2.71
500,000	(4.60)	2.40	7.84	1.47
250,000	(6.15)	(0.63)	3.60	0.81
200,000	(6.59)	(1.49)	2.27	0.67
150,000	(7.13)	(2.51)	0.92	0.53
100,000	(7.81)	(3.86)	(0.88)	0.39
50,000	(8.65)	(5.48)	(3.02)	0.22
10,000	(10.09)	(8.16)	(6.61)	0.07
5,000	(10.52)	(8.85)	(7.53)	0.04
2,500	(10.82)	(9.46)	(8.28)	0.03
1,000	(11.12)	(9.99)	(9.11)	0.02
500	(11.27)	(10.27)	(9.49)	0.01

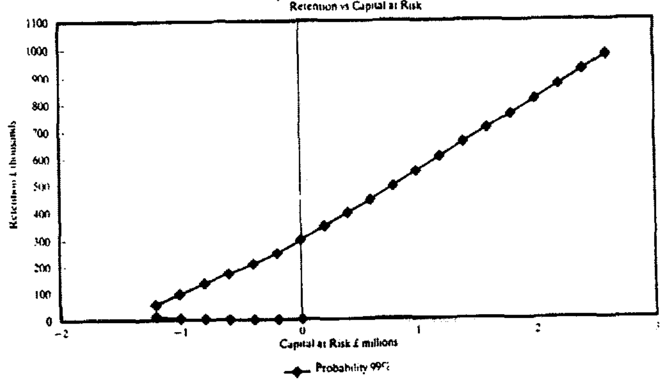
Total Gross Premium = 57,691,234
Total Loading in Premiums = 30%
Expense Element of Premium = 20%

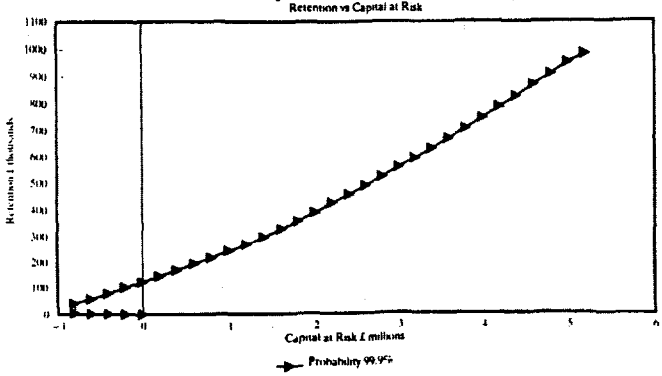
Capital at Risk = Net Aggregate Claims - Premium Net of Reinsurance and Expenses

Heckman and Meyers' Method – Property Example

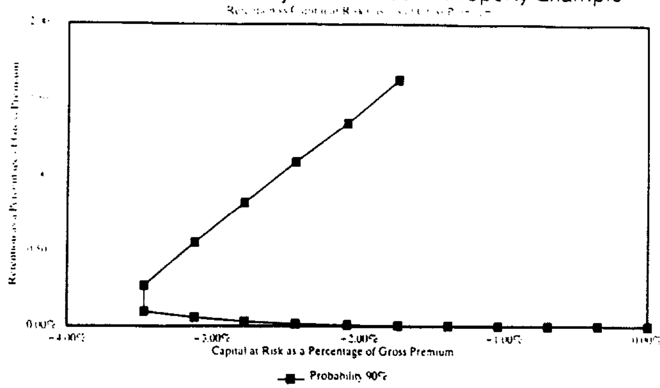



Heckman and Meyers' Method – Property Example
Capital at Risk £ millions
Probability 90%

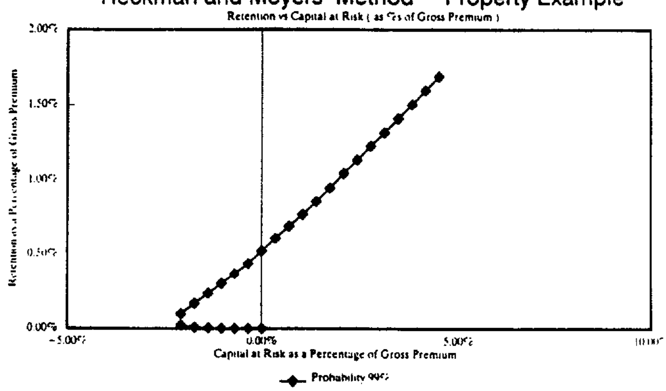

Heckman and Meyers' Method – Property Example
Retention vs Capital at Risk
Retention £ thousands
1100
1000
900
800
700
600
500
400
300
200
100
0
-2
-1
0
1
2
Capital at Risk £ millions
Probability 99%


Heckman and Meyers' Method – Property Example
Retention vs Capital at Risk
Capital at Risk £ millions
Probability 99.9%

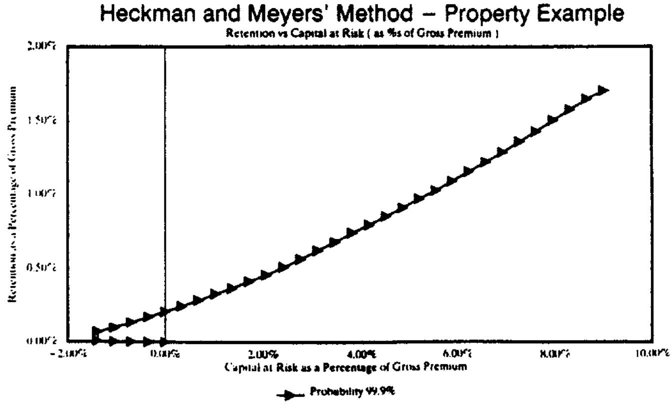
Heckman and Meyers' Method – Property Example

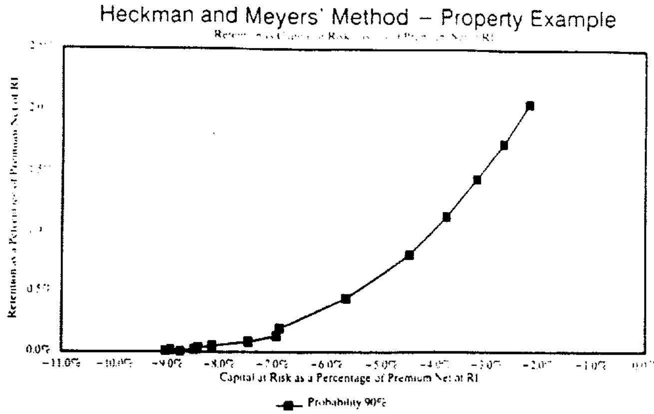


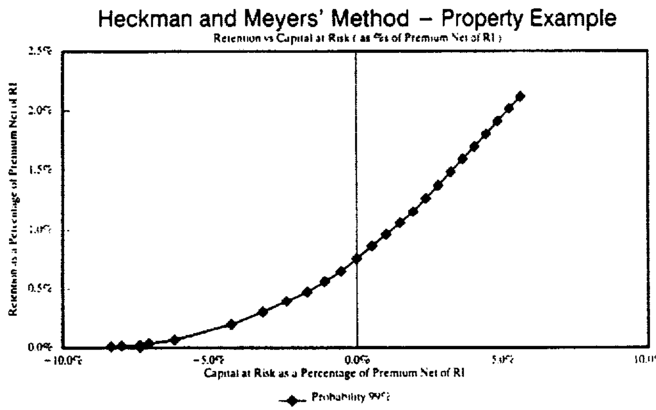
Heckman and Meyers' Method – Property Example

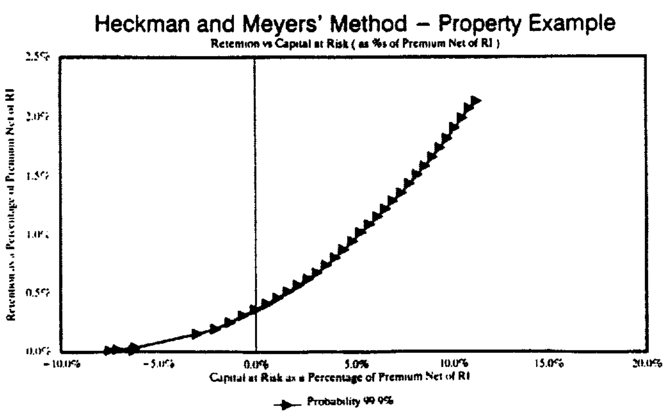


Heckman and Meyers' Method – Property Example




Heckman and Meyers' Method – Property Example
0.0%
0.5%
−11.0%
−9.0%
−8.0%
−7.0%
−6.0%
−5.0%
Capital at Risk as a Percentage of Premium Net of RI
Probability 90%


Heckman and Meyers' Method – Property Example
Retention vs Capital at Risk (as %s of Premium Net of RI)
2.5%
2.0%
1.5%
1.0%
0.5%
0.0%
Retention as a Percentage of Premium Net of RI
−10.0%
−5.0%
0.0%
5.0%
10.0%
Capital at Risk as a Percentage of Premium Net of RI
Probability 99%


Heckman and Meyers' Method – Property Example
Retention vs Capital at Risk (as %s of Premium Net of RI)
2.5%
2.0%
1.5%
1.0%
0.5%
0.0%
Retention as a Percentage of Premium Net of RI
−10.0%
−5.0%
0.0%
5.0%
10.0%
15.0%
20.0%
Capital at Risk as a Percentage of Premium Net of RI
Probability 99.9%

UK Property Catastrophes
Past Claims in excess of £40 million original cost
Amounts in £ millions

Date of Claim	Original Cost	Adjusted to 1990 Values	Claims above £100 million
Jan 79	60	327	327
Dec 81	100	349	349
Dec 81	60	174	174
Jan 82	60	153	153
Jan 82	160	349	349
Jan 84	120	196	196
Jan 85	40	87	0
Jan 85	40	87	0
Feb 85	100	174	174
Dec 85	80	131	131
Mar 86	60	87	0
Jan 87	320	414	414
Oct 87	1,240	1,591	1,591
Jan 90	1,700	1,700	1,700
Feb 90	300	300	300
Average			488

UK Property Catastrophes
Simulation Results for Gross Aggregate Annual Claims Cost
Number of Simulations 5000
Amounts in £ millions

	ESTIMATED COMPANY GROSS COSTS		
	Pareto 1.25	Pareto 1.33	Pareto 1.50
Poisson 0.75			
Average	27	25	21
SD	72	70	51
Poisson 1.00			
Average	37	33	28
SD	83	79	64
Poisson 1.25			
Average	45	42	34
SD	92	94	65

UK Property Catastrophes
Simulation Results for Gross Aggregate Annual Claims Cost
Number of Simulations 5000
Amounts in £ millions

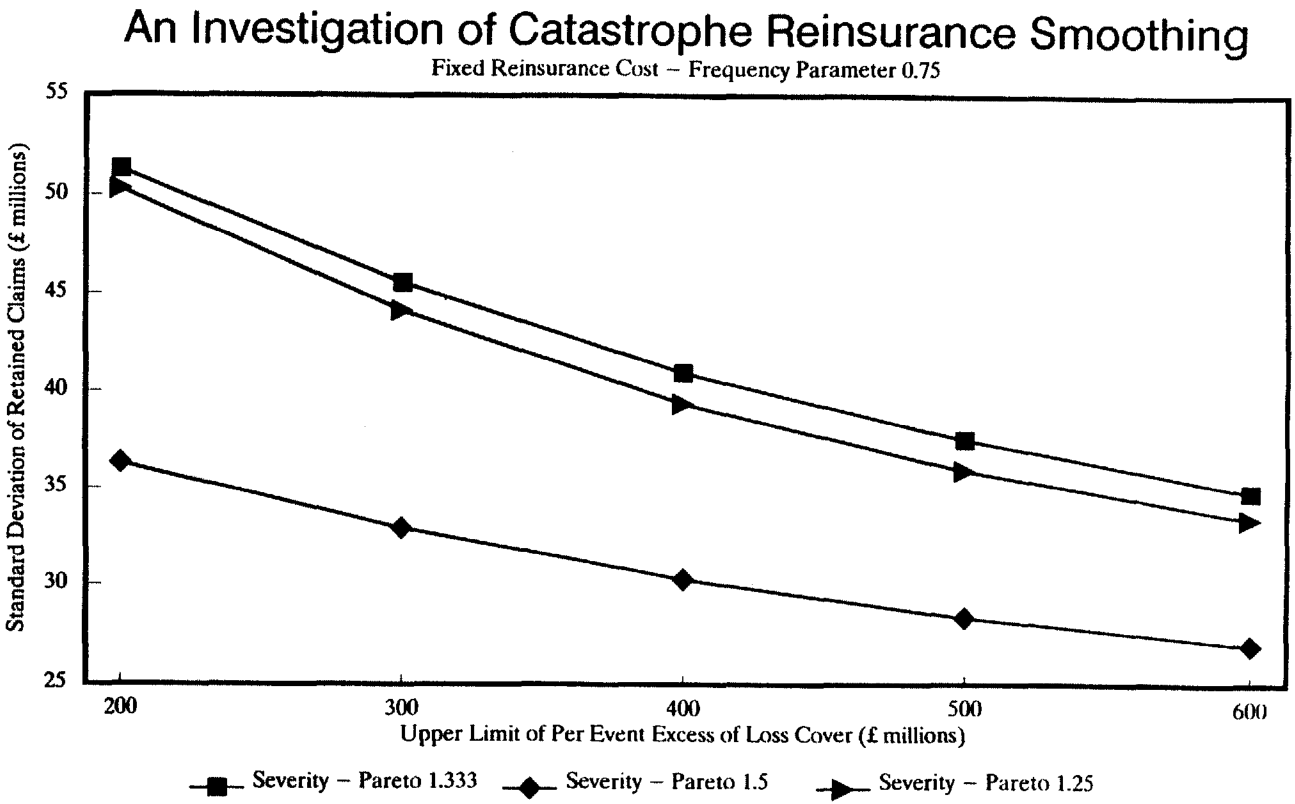
ESTIMATED COMPANY GROSS COSTS

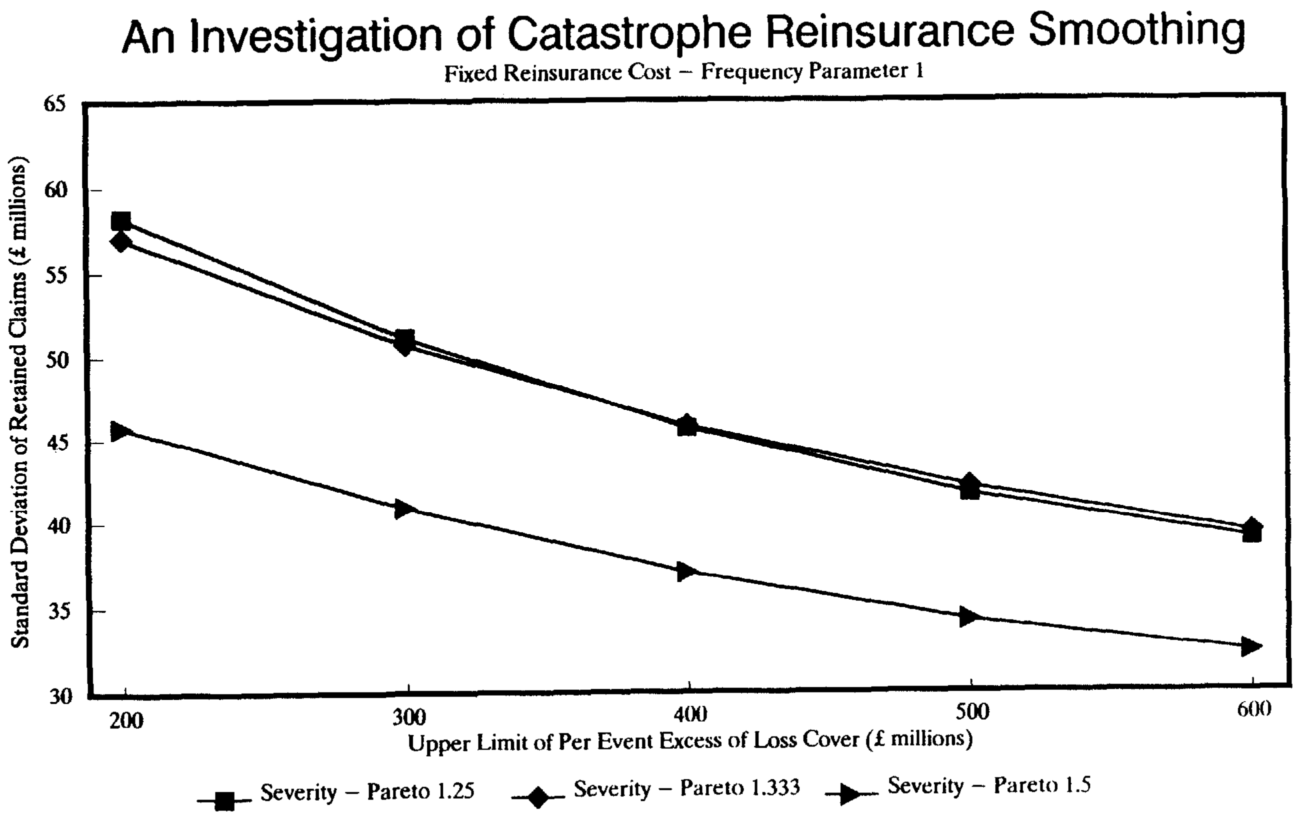
	Pareto 1.25	Pareto 1.33	Pareto 1.50
Poisson 0.75			
Average	27	25	21
SD	72	70	51
Poisson 1.00			
Average	37	33	28
SD	83	79	64
Poisson 1.25			
Average	45	42	34
SD	92	94	65

ESTIMATED COMPANY NET COSTS

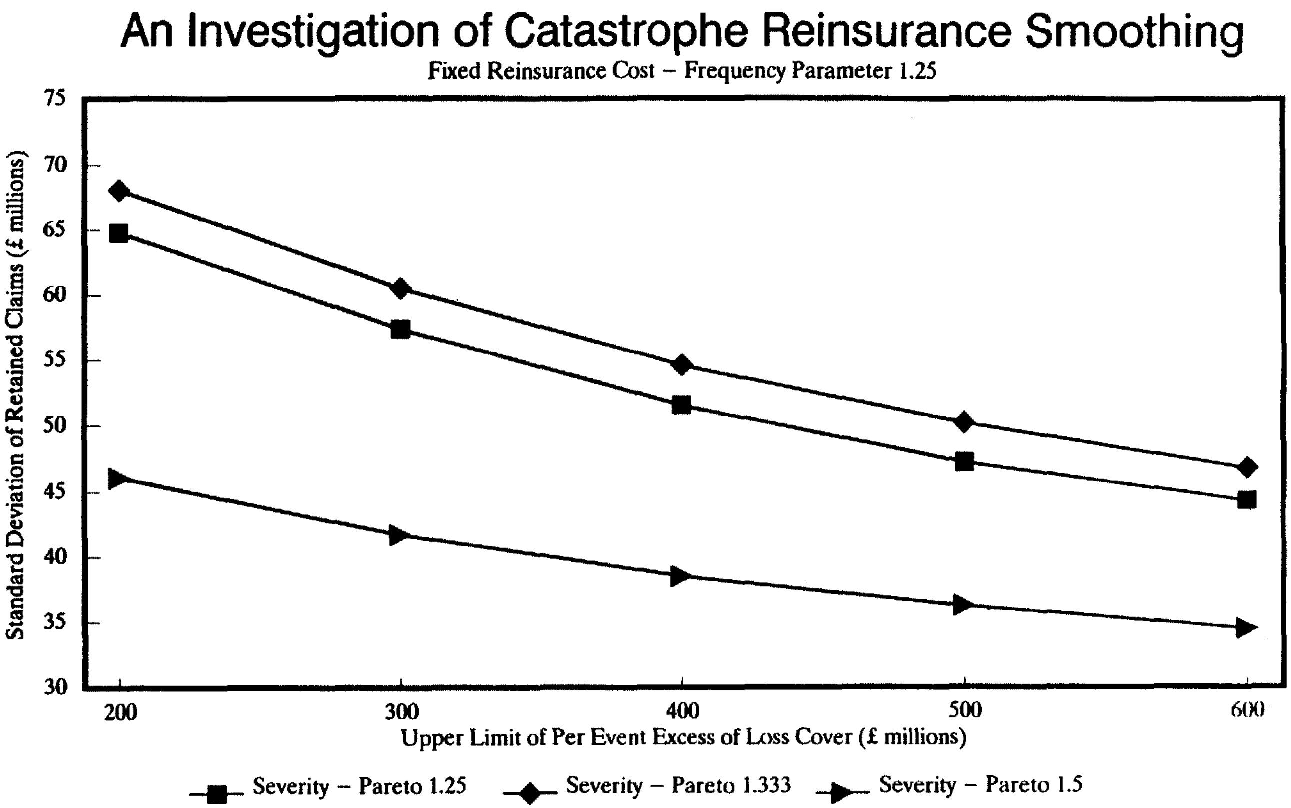
	Pareto 1.25	Pareto 1.33	Pareto 1.50
Poisson 0.75			
Average	19	18	16
SD	50	51	36
Poisson 1.00			
Average	25	24	22
SD	58	57	46
Poisson 1.25			
Average	32	31	26
SD	65	68	46

Note : Reinsurance Per Event Catastrophe £170 XS £30 million


An Investigation of Catastrophe Reinsurance Smoothing
Fixed Reinsurance Cost – Frequency Parameter 0.75
Standard Deviation of Retained Claims (£ millions)
55
50
45
40
35
30
25
200
300
400
500
600
Upper Limit of Per Event Excess of Loss Cover (£ millions)
Severity – Pareto 1.333
Severity – Pareto 1.5
Severity – Pareto 1.25



An Investigation of Catastrophe Reinsurance Smoothing
Fixed Reinsurance Cost – Frequency Parameter 1
Standard Deviation of Retained Claims (£ millions)
65
60
55
50
45
40
35
30
200
300
400
500
600
Upper Limit of Per Event Excess of Loss Cover (£ millions)
Severity – Pareto 1.25
Severity – Pareto 1.333
Severity – Pareto 1.5



An Investigation of Catastrophe Reinsurance Smoothing
Fixed Reinsurance Cost – Frequency Parameter 1.25
Standard Deviation of Retained Claims (£ millions)
75
70
65
60
55
50
45
40
35
30
200
300
400
500
600
Upper Limit of Per Event Excess of Loss Cover (£ millions)
Severity – Pareto 1.25
Severity – Pareto 1.333
Severity – Pareto 1.5

An Investigation of Catastrophe Reinsurance Smoothing

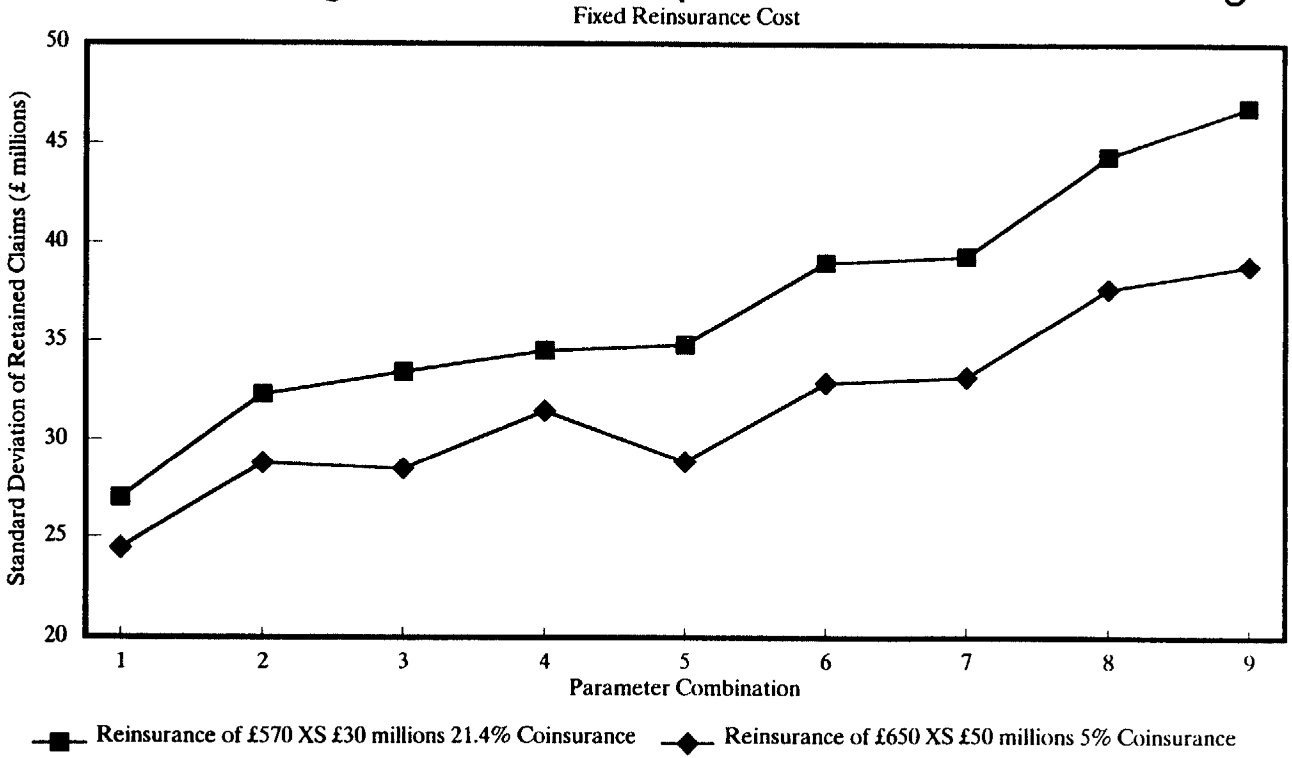
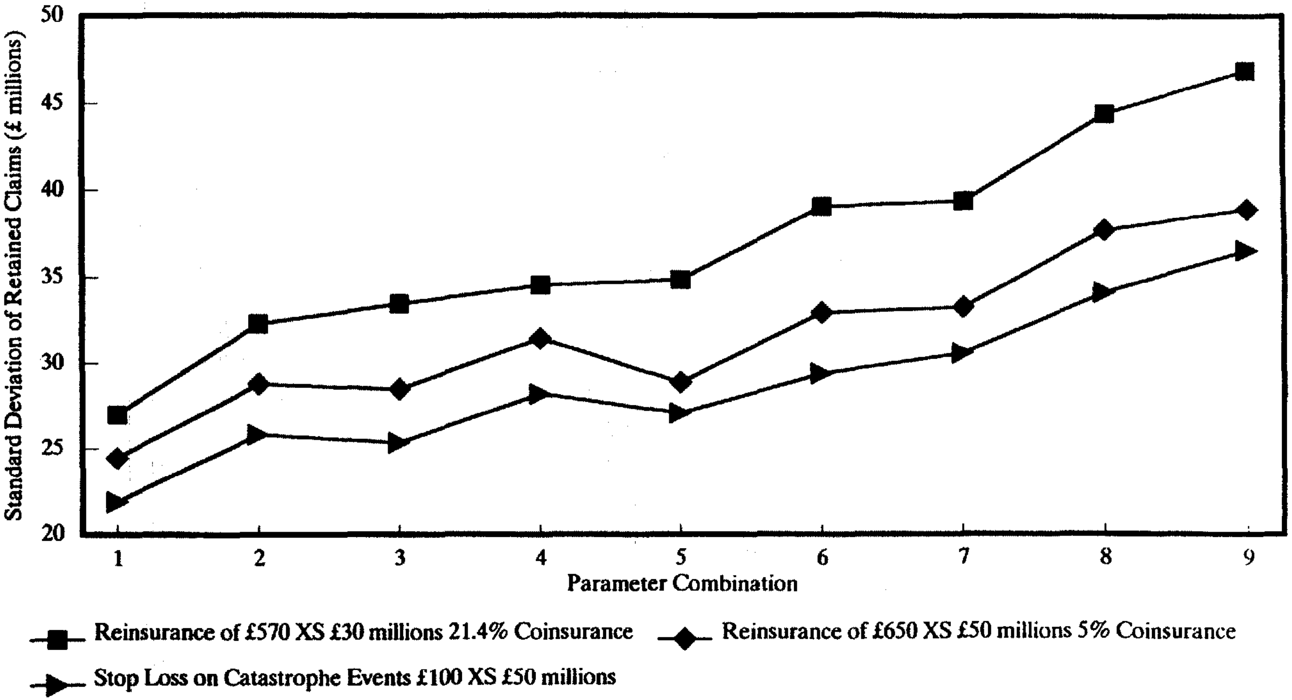


Exhibit 4
Page 7

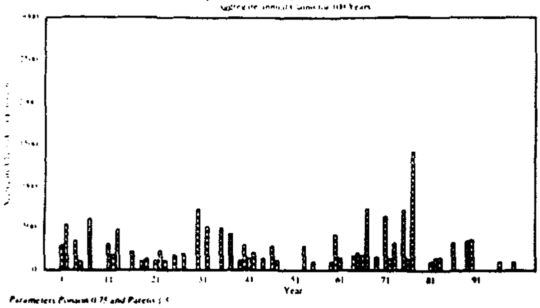
An Investigation of Catastrophe Reinsurance Smoothing

Fixed Reinsurance Cost

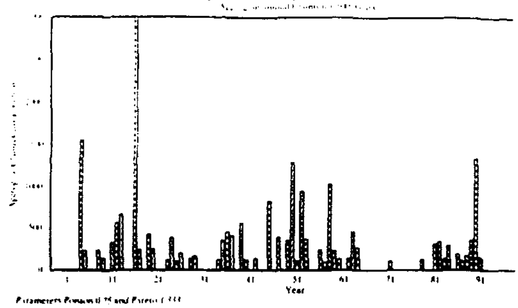


The cost of the stop loss cover is not known

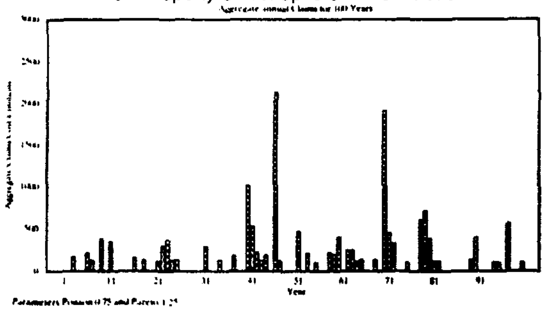
UK Property Catastrophe Claim Simulations



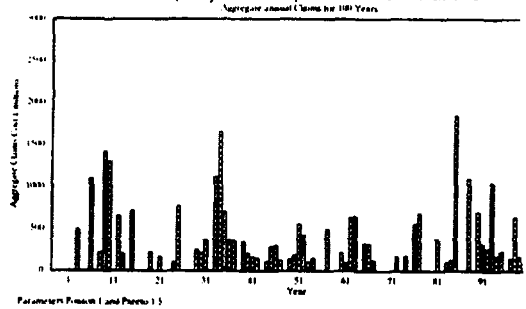
UK Property Catastrophe Claim Simulations



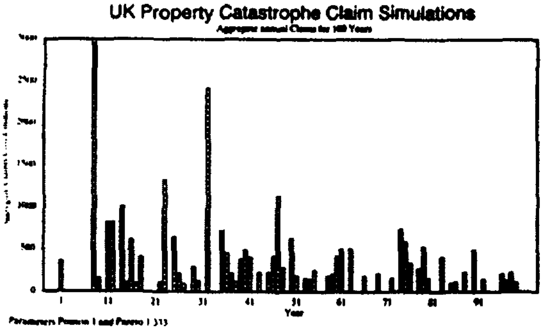
UK Property Catastrophe Claim Simulations



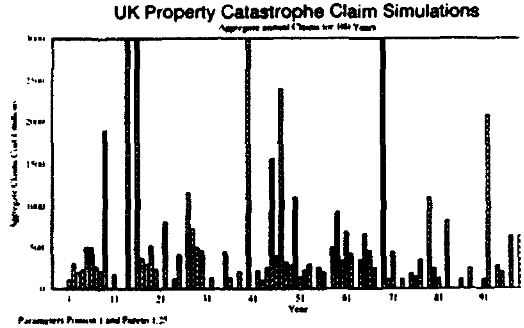
UK Property Catastrophe Claim Simulations

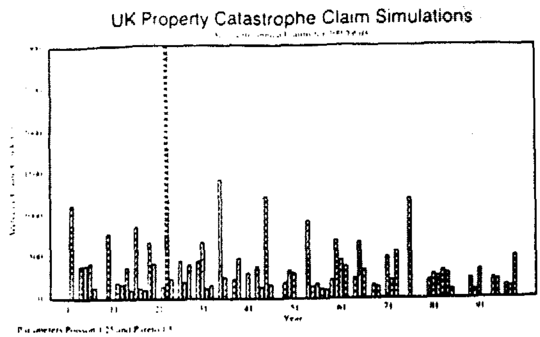


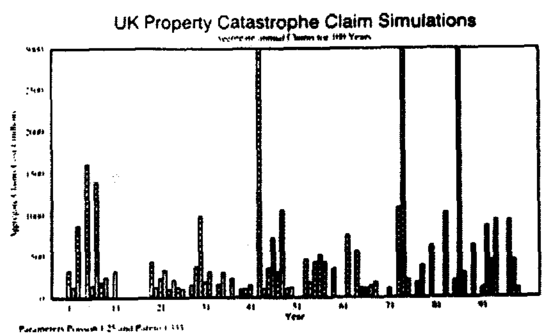
UK Property Catastrophe Claim Simulations

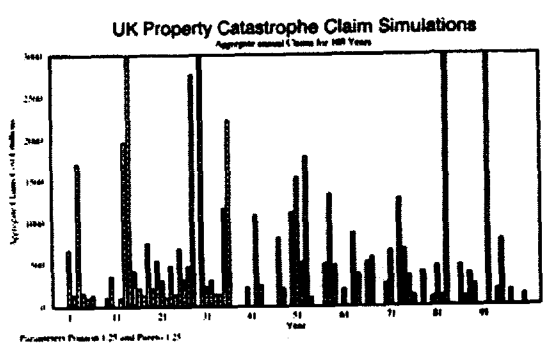


UK Property Catastrophe Claim Simulations




UK Property Catastrophe Claim Simulations


UK Property Catastrophe Claim Simulations


UK Property Catastrophe Claim Simulations
Year

UK Property Catastrophes
Actual versus Central Limit Theorem Confidence Limits
Demonstration on Net of Reinsurance Distributions
Reinsurance - Per Event Catastrophe £570 XS £30 million

	<--------- Actual --------->		<--------- One Tail Central Limit --------->	
	95% Confidence	99% Confidence	95% Confidence	99% Confidence
Poisson 0.75 Pareto 1.5	54.6	85.9	40.1	57.0
Poisson 0.75 Pareto 1.333	58.2	105.8	47.3	67.3
Poisson 0.75 Pareto 1.25	63.0	116.3	46.8	66.4
Poisson 1 Pareto 1.5	66.5	106.3	47.2	67.1
Poisson 1 Pareto 1.333	72.3	124.2	54.5	77.4
Poisson 1 Pareto 1.25	75.7	124.8	54.0	76.7
Poisson 1.25 Pareto 1.5	73.2	114.7	51.6	73.3
Poisson 1.25 Pareto 1.333	85.1	146.6	63.7	90.5
Poisson 1.25 Pareto 1.25	87.6	145.9	61.8	87.7

Recursive Method - Aviation Example
Individual Claim Severity Distribution

No. of Data Points 40
Scaling Factor 1,500

Discretised Distribution of X	Frequency	Relative Frequency f(x)	Cumulative Frequency F(x)	x*f(x)	Col (5) Cumulative	x^2 * f(x)	Col (7) Cumulative
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
0	14	0.350	0.350	0.000	0.000	0.000	0.000
1	8	0.200	0.550	0.200	0.200	0.200	0.200
2	4	0.100	0.650	0.200	0.400	0.400	0.600
3	4	0.100	0.750	0.300	0.700	0.900	1.500
4	2	0.050	0.800	0.200	0.900	0.800	2.300
5	0	0.000	0.800	0.000	0.900	0.000	2.300
6	1	0.025	0.825	0.150	1.050	0.900	3.200
7	0	0.000	0.825	0.000	1.050	0.000	3.200
8	0	0.000	0.825	0.000	1.050	0.000	3.200
9	0	0.000	0.825	0.000	1.050	0.000	3.200
10	1	0.025	0.850	0.250	1.300	2.500	5.700
11	0	0.000	0.850	0.000	1.300	0.000	5.700
12	0	0.000	0.850	0.000	1.300	0.000	5.700
13	1	0.025	0.875	0.325	1.625	4.225	9.925
14	0	0.000	0.875	0.000	1.625	0.000	9.925
15	0	0.000	0.875	0.000	1.625	0.000	9.925
16	1	0.025	0.900	0.400	2.025	6.400	16.325
17	1	0.025	0.925	0.425	2.450	7.225	23.550
18	0	0.000	0.925	0.000	2.450	0.000	23.550
19	0	0.000	0.925	0.000	2.450	0.000	23.550
20	0	0.000	0.925	0.000	2.450	0.000	23.550
21	0	0.000	0.925	0.000	2.450	0.000	23.550
22	0	0.000	0.925	0.000	2.450	0.000	23.550
23	0	0.000	0.925	0.000	2.450	0.000	23.550
24	0	0.000	0.925	0.000	2.450	0.000	23.550
25	0	0.000	0.925	0.000	2.450	0.000	23.550
26	0	0.000	0.925	0.000	2.450	0.000	23.550
27	0	0.000	0.925	0.000	2.450	0.000	23.550
28	0	0.000	0.925	0.000	2.450	0.000	23.550
29	0	0.000	0.925	0.000	2.450	0.000	23.550
30	0	0.000	0.925	0.000	2.450	0.000	23.550
31	0	0.000	0.925	0.000	2.450	0.000	23.550
32	0	0.000	0.925	0.000	2.450	0.000	23.550
33	1	0.025	0.950	0.825	3.275	27.225	50.775
34	0	0.000	0.950	0.000	3.275	0.000	50.775
35	1	0.025	0.975	0.875	4.150	30.625	81.400
36+	1	0.025	1.000	0.900	5.050	32.400	113.800
Total		1.000		5.050		113.800	

Recursive Method - Liability Example
Individual Claim Severity Distribution

No. of Data Points 638
Scaling Factor 10,000

Discretised Distribution of X	Frequency	Relative Frequency f(x)	Cumulative Frequency F(x)	x*f(x)	Col (5) Cumulative	x^2 * f(x)	Col (7) Cumulative
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
0	268	0.4201	0.4201	0.0000	0.0000	0.0000	0.0000
1	161	0.2524	0.6724	0.2524	0.2524	0.2524	0.2524
2	63	0.0987	0.7712	0.1975	0.4498	0.3950	0.6473
3	41	0.0643	0.8354	0.1928	0.6426	0.5784	1.2257
4	20	0.0313	0.8668	0.1254	0.7680	0.5016	1.7273
5	8	0.0125	0.8793	0.0627	0.8307	0.3135	2.0408
6	11	0.0172	0.8966	0.1034	0.9342	0.6207	2.6614
7	7	0.0110	0.9075	0.0768	1.0110	0.5376	3.1991
8	4	0.0063	0.9138	0.0502	1.0611	0.4013	3.6003
9	7	0.0110	0.9248	0.0987	1.1599	0.8887	4.4890
10	4	0.0063	0.9310	0.0627	1.2226	0.6270	5.1160
11	2	0.0031	0.9342	0.0345	1.2571	0.3793	5.4953
12	2	0.0031	0.9373	0.0376	1.2947	0.4514	5.9467
13	2	0.0031	0.9404	0.0408	1.3354	0.5298	6.4765
14	4	0.0063	0.9467	0.0878	1.4232	1.2288	7.7053
15	0	0.0000	0.9467	0.0000	1.4232	0.0000	7.7053
16	3	0.0047	0.9514	0.0752	1.4984	1.2038	8.9091
17	2	0.0031	0.9545	0.0533	1.5517	0.9060	9.8150
18	1	0.0016	0.9561	0.0282	1.5799	0.5078	10.3229
19	0	0.0000	0.9561	0.0000	1.5799	0.0000	10.3229
20	4	0.0063	0.9624	0.1254	1.7053	2.5078	12.8307
21	3	0.0047	0.9671	0.0987	1.8041	2.0737	14.9044
22	0	0.0000	0.9671	0.0000	1.8041	0.0000	14.9044
23	2	0.0031	0.9702	0.0721	1.8762	1.6583	16.5627
24	1	0.0016	0.9718	0.0376	1.9138	0.9028	17.4655
25	2	0.0031	0.9749	0.0784	1.9922	1.9592	19.4248
26	0	0.0000	0.9749	0.0000	1.9922	0.0000	19.4248
27	2	0.0031	0.9781	0.0846	2.0768	2.2853	21.7100
28	1	0.0016	0.9796	0.0439	2.1207	1.2288	22.9389
29	1	0.0016	0.9812	0.0455	2.1661	1.3182	24.2571
30	1	0.0016	0.9828	0.0470	2.2132	1.4107	25.6677
31	0	0.0000	0.9828	0.0000	2.2132	0.0000	25.6677
32	0	0.0000	0.9828	0.0000	2.2132	0.0000	25.6677
33	0	0.0000	0.9828	0.0000	2.2132	0.0000	25.6677
34	0	0.0000	0.9828	0.0000	2.2132	0.0000	25.6677
35	0	0.0000	0.9828	0.0000	2.2132	0.0000	25.6677
36	0	0.0000	0.9828	0.0000	2.2132	0.0000	25.6677
37	0	0.0000	0.9828	0.0000	2.2132	0.0000	25.6677
38	1	0.0016	0.9843	0.0596	2.2727	2.2633	27.9310
39	3	0.0047	0.9890	0.1834	2.4561	7.1520	35.0831
40	0	0.0000	0.9890	0.0000	2.4561	0.0000	35.0831
41+	7	0.0110	1.0000	0.4498	2.9060	18.4436	53.5266
Total		1.0000		2.9060		53.5266	

Recursive Method – Aviation Example

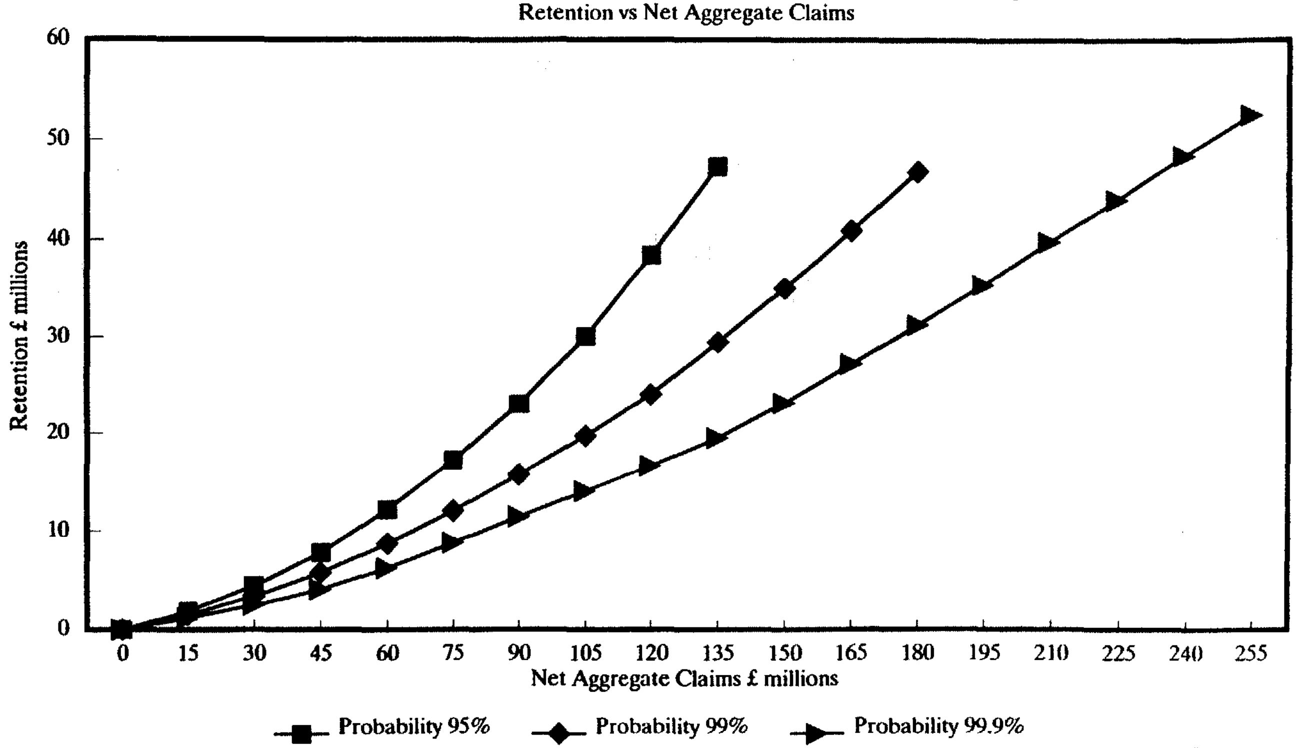
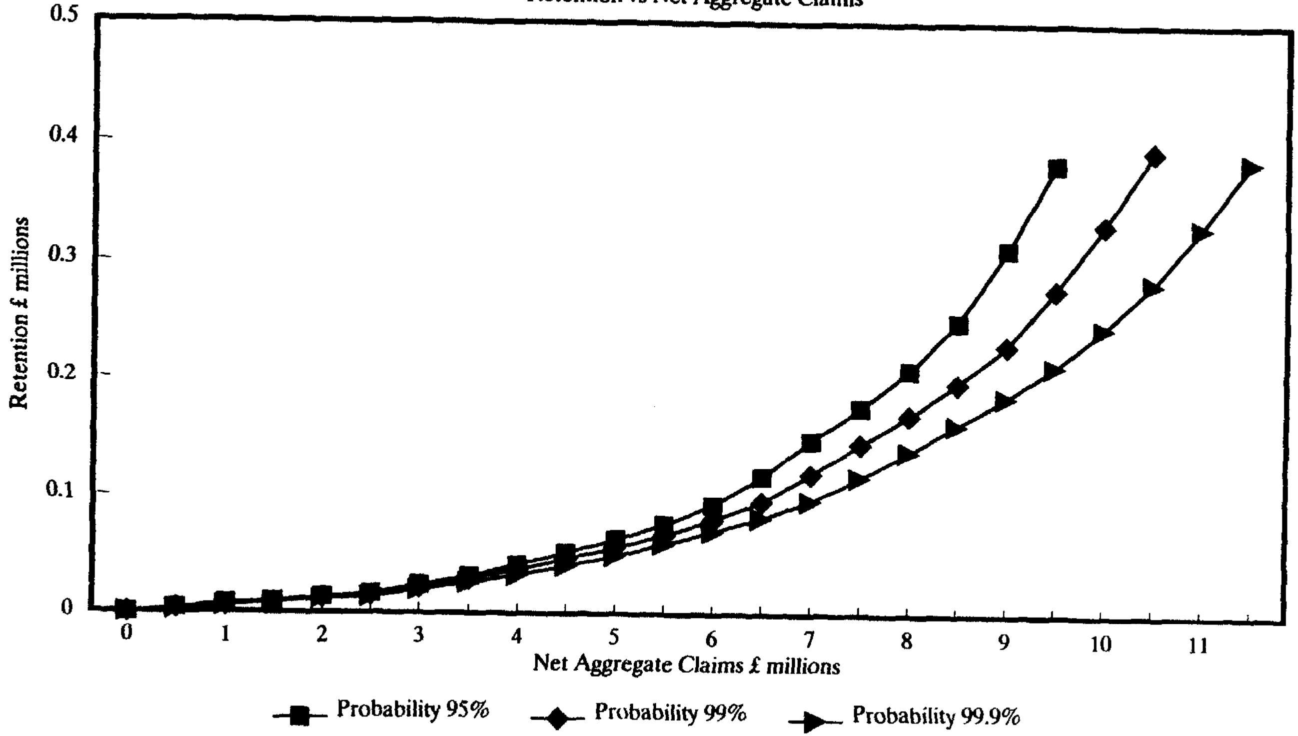


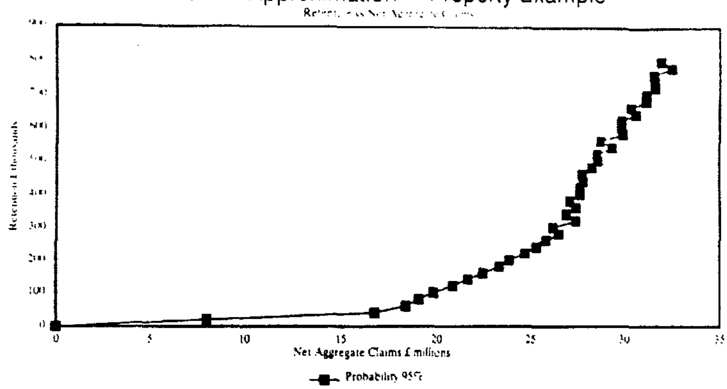
Exhibit 5
Page 3

Recursive Method – Liability Example

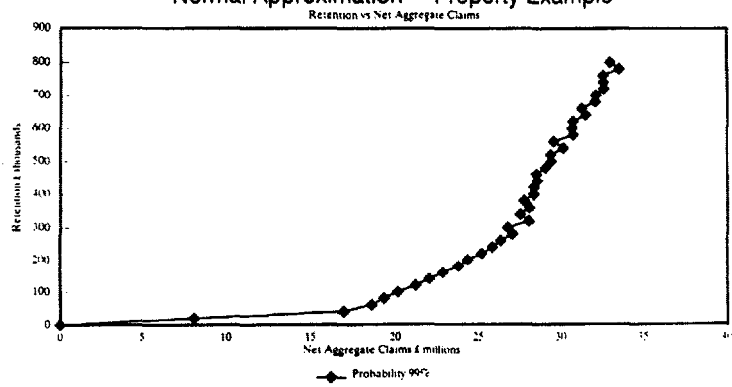
Retention vs Net Aggregate Claims



Normal Approximation – Property Example



Normal Approximation – Property Example



Normal Approximation – Property Example