DISTRIBUTION OF PENSION BENEFITS ON DIVORCE: SOME UNRESOLVED ACTUARIAL ISSUES

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Abstract

The author was involved in a study of the analytical procedures and assumptions for
the distribution of pension benefits on divorce. The purpose of the study was (1) to review
the positions that the courts have taken; (2) to organize them within an analytical
framework; and (3) to identify and articulate the unresolved issues which impede the court's
ability to render economically unbiased decisions. This presentation presents some of the
findings of that study.
GENERAL STATEMENT OF THE PROBLEM

Given some assignment date, t,\(^1\) let:

\[ \tilde{V}_{ti} = \text{economic value at time } t \text{ of marital property } i \]

\[ \tilde{V}_t = \sum_i \tilde{V}_{ti} \]

\[ a_{ij} = \text{proportion of property } i \text{ assigned to spouse } j \text{ at time } t, \]

where a tilde over a factor indicates a random variable. Then, disregarding the expenses associated with divorce,\(^2\) the problem becomes one of assigning:

\[ \sum_i a_{ij} \tilde{V}_a, \; j=1,2, \]

the total allocation to each spouse, such that

\[ \sum_i a_{ai} \tilde{V}_a - k \cdot \sum_i a_{ai} \tilde{V}_a, \; k \geq 0 \]

and

\[ a_{ai} + a_{a2} = 1, \; \forall t, i. \]

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\(^1\)In this formulation, the date of assignment is taken as given. In practice, since the date of assignment is a principal determinant of property values, it is a critical factor, and, as such, is often a major point in the litigation.

\(^2\)The expenses associated with a divorce include such things as attorney’s fees, expert witness fees, and so on. The formulation is easily extended to incorporate this type of slippage.
THE MODEL BEFORE THE COURTS

Since \( \tilde{V}_n \) is a random variable, an economically unbiased model generally would require that

\[
Pr[| \sum_i a_{n1} \tilde{V}_n - k \cdot \sum_i a_{n2} \tilde{V}_n | > \delta | \leq \alpha
\]

where \( \delta \) is the maximum tolerable deviation from economic unbiasedness and \( \alpha \) is the probability of that occurrence. This fact, notwithstanding, the courts have invariably relied on expected value models which merely require that

\[
\sum_i a_{n1} E(\tilde{V}_n) - k \cdot \sum_i a_{n2} E(\tilde{V}_n)
\]

where \( E(\tilde{V}_n) \) denotes the expected value.
THE PENSION BENEFITS PAYABLE TO THE NONEMPLOYEE SPOUSE

The pension benefits payable to the nonemployee spouse is given by the general formula:

\[
\frac{\text{non-EE Spouse's Portion}}{\text{[Coverture Fraction]}} \times \frac{\text{Value of Pension Benefits}}{\text{[Value of Pension Benefits]}}.
\]

A common example of the formulation of a coverture fraction is the case where separation occurred while the employee spouse was still a plan participant. In this instance, the coverture fraction is given by the ratio

\[
\frac{\text{Date of Separation} - \max(\text{Date of Marriage}, \text{Date of Hire})}{\text{Date of Valuation} - \text{Date of Hire}}
\]

The present value (PV) is:

\[
\tilde{PV}_t = \int_{t-0}^{t} \tilde{B}_{r,s} \int_{t-0}^{t} e^{-\lambda s} ds \, dt
\]

and

\[
\tilde{PV}_t = \left[ \int_{t-0}^{t} e^{-\lambda s} ds \right] \tilde{PV}_t,
\]
ESTIMATING THE RETIREMENT AGE

Before taxes, and assuming a specific set of discount and decrement factors, an economically optimal retirement age is the current age, $x$, plus the $n$ which satisfies:

$$\max_n \left( \int_0^n (ES)_t v^t \pi^{x+t}_t dt + \int_n^\infty B_t v^t \pi^{x+t}_t dt \right),$$

where $(ES)_t$ is the expected salary at time $t$, $v^t$ is the discount factor, $\pi^{x+t}_t$ is the probability that a participant aged $x$ will persist as an active participant to age $x+t$, and $B_t$ is the nominal annual benefit at time $t$. 
The most notable characteristic of the PBGC rates is that the interest rates vary with duration. The general form for the expected discounted annuity purchase rate is:

\[
\prod_{k=1}^{K} [1+i_k]^{-n_k} \cdot r \cdot p_x \cdot APR(r)
\]

where \(i_k\) is the interest rate earned for \(n_k\) years of the deferral period. For PBGC purposes, \(K=3\), and the maximum values for the \(n\)'s are \(n_1=7\), \(n_2=8\), and \(n_3=r-x-n_1-n_2\). The interest assumptions are chosen so that, when used with the mortality assumptions mandated by the regulations, the present values for immediate and deferred annuities are comparable with similar annuity purchase rates found in the industry. The rates contain an allowance for expenses.
THE IMPACT OF TAXES

The impact of taxes on pension benefit is captured in the equations:

\[ a_r^* = \sum_{i=0}^{\infty} v_{i}^* \cdot p_x^* B_r^* \]

where:

\[ v_{i}^* = \prod_{k=1}^{i} \left[ 1 + t_{i_k} \cdot (1-t_{i_k}) \right]^{-1} \]

and

\[ B_r^* = B_r \cdot (1-t_r) \]

Here, \( t \) denotes the taxes paid on the retirement benefit and \( t' \) denotes the taxes paid on the investment income. As indicated, the present value of the tax adjusted annuity at the retirement age \( r \), \( a_r^* \), is derived from a tax-adjusted discount factor, \( v_{i}^* \), and a tax-adjusted periodic retirement payment, \( B_r^* \).
EXPECTED VESTING

The expected vesting (EV) takes the form:

$$
EV(x,j,h) = \int_{\gamma}^{r} V(y,j,h) \gamma_x p_x^{\text{as}} \mu^w_y dy + V(r,j,h) r_x p_x^{\text{as}}
$$

where $z$ is the larger of the initial vesting age or the current age, $x$; $V(x,j,h)$ is the vesting at age $x$ under vesting schedule $j$, given that the participant was hired at age $h$; $\gamma_x p_x^{\text{as}}$ is the probability that a participant aged $x$ will persist as an active participant to age $y$; and the force of withdrawal operating during the interval of age $y$ to $y+dy$ is $\mu^w_y$. Of course, the implementation of the foregoing may be problematic because of the difficulty of procuring the appropriate decrement data.