SPLITTING ALLOCATED LOSS ADJUSTMENT EXPENSE

Splitting Allocated Loss Adjustment Expense

by

LeRoy J. Simon*

Facultative casualty reinsurance certificates and working layer casualty excess of loss reinsurance treaties will often provide that the primary company and its reinsurer are to share Allocated Loss Adjustment Expense (ALAE) in proportion to their respective amounts of the indemnity loss. This works well in most cases and can be properly priced by the reinsurer and evaluated by the primary company before entering into the reinsurance contract.

A conflict may occur when a subrogation opportunity arises, however. The reinsurance will usually provide that the apportionment is based on the indemnity loss payments as determined after the subrogation is finalized. Unless the expected value of the primary company's loss plus ALAE after subrogation is less than beforehand, there is no incentive to pursue the matter. However, the reinsurer would be anxious to do so in most cases and here lies an opportunity for some actuarial help to both parties.

A careful analysis of the situation may help each party focus more accurately on the implied probability distributions and thus more accurately evaluate the expected values. It may also show the way for two parties with different probability distributions to come closer together in their agreement on a common distribution. Failing to reach agreement on a course of action, the mediator/actuary might show the parties how to fashion a division of the ALAE that will bring them into agreement on how to proceed.

A simple case for illustrative purposes might be a casualty excess of loss treaty (or a facultative casualty certificate if you prefer) where the primary company's retention (R) is 5 (all figures can be thought of as being in millions of US$). A loss (L), which is covered by reinsurers through one or more layers, has been paid for 25 with an ALAE (A) of 0.2. It is now proposed to spend an additional amount of ALAE (b) to recover an uncertain amount of subrogation (x) which has a probability distribution f(x). Capital letters are used for those values which are known when the analysis is to be made and lower case letters are used for those which must be estimated or are variables in the solution.

Now let us consider the simple probability distribution case (see Table 1 for a convenient summary of the equations involved) where there are only two possible outcomes: (1) to win and get a total recovery with probability p, or (2) to lose and recover nothing with probability 1 - p. Under (1) the cost to the primary company will be (A + b). Under (2) the cost will be R*(1 + (A + b)/L). Hence the expected value of the cost to the primary company is:

$$(A + b)*(p) + (R + R*(A + b)/L)*(1 - p)$$

---

* Presented during the Speaker's Corner of the March 29, 1989 meeting of the Casualty Actuaries of Greater New York.

If this value is less than the cost of not pursuing the subrogation, then the primary company will be interested in pursuing it. That cost is:

$$R*(1 + A/L)$$

A little algebra shows that the no-go situation exists for the primary company when the following decision function, PCDF, is negative:

$$PCDF_1 = R*(p - b/L) - p*(A + b)*(1 - R/L)$$

When the function is positive, it is in the primary company's best economic interest to pursue the subrogation. Remember, however, that we are making the (1) and (2) outcome assumptions above.

TABLE 1

Summary of Equations for the Simple Case

	Primary Company	Reinsurance Company	Third Party	TOTAL
If win (p)	A + b	0	L	L + A + b
If Lose (1-p)	R*(1 + (A + b)/L)	(L - R)*(1 + (A + b)/L)	0	L + A + b
If No Action	R*(1 + A/L)	(L - R)*(1 + A/L)	0	L + A
Decision Function	R*(p - b/L) - p*(A + b)*(1 - R/L)	(1 - R/L)*(p*(L + A + b) - b)	-	p*L - b
Critical Probability	b/(L + (A + b)*(1 - L/R))	b/(L + A + b)	-	-

In our numeric example, suppose b is estimated to be 4 and p is ½. The $PCDF_1$ evaluates to 0.02. Therefore the primary company would have an interest in pursuing the subrogation. The reinsurers (with 20 of indemnity loss at stake and a 50-50 chance of winning) no doubt would also, but we will investigate that below.

If the probability of winning were just slightly different, say p = .40, the primary company decision function would evaluate to -0.144 and it would not be interested in going forward. In fact, under the assumptions made, the critical probability value is .4878 (that is, b/(L + (A + b)*(1 - L/R))); above that the primary company is willing to pursue subrogation; below it, it is unwilling.

Now let's look at things from the reinsurer's standpoint. Under the same (1) and (2) assumptions, the expected value of the cost to the reinsurer under (1) is zero and under

(2) is $(L - R)*(1 + (A + b)/L)*(1 - p)$. If this value is less than the cost of not pursuing the subrogation, then the reinsurer will be interested in pursuing it. That cost is:

$$(L - R)*(1 + A/L)$$

Again, the reinsurer's no-go situation exists when the following decision function, $RCDF_1$, is negative:

$$RCDF_1 = (1 - R/L)*(p*(L + A + b) - b)$$

When the function is positive, it is in the reinsurer's best economic interest to pursue the subrogation. Since $(L + A)$ will ordinarily be quite large, it takes a combination of a large b and a small p to make this function negative; hence the reinsurer's preference will most likely be to pursue subrogation.

In our numeric example, $b = 4$ and $p = ½$, $RCDF_1$ evaluates to 8.48 and thus a go situation for the reinsurer as well.

If the probability of winning were quite low, say $p = .10$, the reinsurer's decision function would evaluate to -0.864 and it would not be interested in going forward. Under the assumptions made, the critical probability value for the reinsurer is .1370 (that is, $b/(L + A + b)$); below that it is unwilling to pursue subrogation; above it, it is willing.

Hence, in this illustrative case, even if the two parties could agree on the estimate of additional ALAE to pursue subrogation (b) and upon the probability of success (p), if b was 4 and p fell between .1370 and .4878, they would reach opposite conclusions.

Certain conclusions are quite clear from the decision functions. In the $PCDF_1$, for example, a very small retention makes it unlikely that the primary company will be interested in pursuing subrogation. In such treaties you should have a split of ALAE which is a better incentive to both parties. The reinsurer who writes an entire layer and considers the $RCDF_1$ will find it compelling in most cases to want to purse subrogation. However, if the reinsurer were to only write a small piece of a layer or the entire amount of a narrow ribbon of a layer, it would be much easier to rationalize the waiving of the subrogation which the primary company was unlikely to wish to pursue anyway.

Reinsurers will also want to consider carefully the situation where the primary company is in liquidation -- the liquidator may have a different viewpoint from that of an ongoing company. He might be quite amenable to, and much more flexible in, negotiating the split of the additional ALAE to pursue the subrogation. Also note how much easier it will be for a reinsurer to negotiate if it has a single, substantial commitment instead of being a small part of various covers spread over an extended set of layers.

As a final exercise, consider the situation where all results between no recovery and full recovery of the indemnity loss are considered to be equally likely (see Table 2 for a convenient summary of the equations involved). In that event the PCDF would again come in two parts: (3) to recover more than the reinsurer's interest (that is, bring the loss down under the retention) in which case the cost to the primary company would be (L - x + A + b) and (4) to recover less than the reinsurer's interest in which case the cost would be R*(1 + (A + b)/(L - x)).

If this value is less than the cost of not pursuing the subrogation then the primary company will not be interesting in pursuing it. That cost continues to be:

$$R*(1 + A/L)$$

We now integrate over the respective ranges of (3) and (4):

$$\int_{L-R}^{L} (L - x + A + b)\, dx \; + \; \int_{0}^{L-R} R*(1 + (A + b)/(L - x))\, dx$$

We then evaluate, divide by the integral of dx over the range 0 to L, and deduct the cost of not pursing subrogation to arrive at the decision function under the new probability function assumption:

$$PCDF_2 = R*(R/2 - b - (A + b)*\ln(L/R))/L$$

where ln is the natural logarithm.

When the function is positive, it is in the primary company's best economic interest to pursue the subrogation.

TABLE 2

Summary of Equations for the Equal Probability Case

	If Recovery is Greater than reinsurer's interest (Range = L-R to L)	If Recovery is Less than reinsurer's interest (Range = 0 to L-R)	If No Action
Primary Company	L - x + A + b	R*(1 + (A + b)/(L - x))	R*(1 + A/L)
Reinsurer	0	(L - x - R)*(1 + (A + b)/(L - x))	(L - R)*(1 + A/L)
Third Party	x	x	0
TOTAL after integration over the range	R*(L + A + b)	(L - R)*(L + A + b)	L + A

If we keep all the values of our previous example but now use the new probability function, $PCDF_2$ evaluates to -1.65; that is, do not pursue.

Now let's again look at this situation from the reinsurer's standpoint. Under (3) its expected value of the cost is zero and under (4) it is (L - x - R)*(1 + (A + b)/(L - x)).

If this value is less than the cost of not pursuing the subrogation then the reinsurer will not be interesting in pursuing it. That cost continues to be:

$$(L - R)*(1 + A/L)$$

We now integrate (4) over the range:

$$\int_0^{L-R} (L - x - R)*(1 + (A + b)/(L - x))\, dx$$

We then evaluate, divide by the integral of dx over the range 0 to L, and deduct the cost of not pursing subrogation to arrive at the decision function under the new probability function assumption:

$$RCDF_2 = (R*((A + b)*\ln L/R) - (L - R)*(b - L/2 - R/2))/L$$

When the function is positive, it is in the reinsurer's best economic interest to pursue the subrogation.

In our numeric example, $RCDF_2$ evaluates to 10.15; that is, pursue. The reinsurer and primary company reach opposite conclusions.

This analysis has focused entirely on the expected value of the decision to be made and has not considered factors such as the working relationship between the primary company and its reinsurer. That bond may be strong enough to override an expected value calculation because of long term -- past or future -- values. Neither have we considered the effect of retrospective or prospective experience rating on treaties. This may again cause a sufficient effect on the primary company's total expected value in the long run that it would reach a different conclusion in some instances. Further sophistication could introduce present value concepts since pursing subrogation can sometimes be a time consuming process.

Note that the unallocated loss expenses of the primary company and of the reinsurer are not considered here at all. ALAE for the Third Party is not mentioned, of course, since it is not relevant to the decision process although it does contribute to the global cost of the entire system.

In closing, it would be of interest to increase the number of discrete probabilities in our first approach and study the situation then. It would also be of interest to have the second approach assume probability curves which were quite optimistic of success or quite pessimistic and have the reinsurer and the primary company choose them in the four possible combinations. Finally it might be worthwhile to study a very different contractual agreement on splitting allocated loss adjustment expense to see if the conflict presented by this proportional method could be avoided.

* * * * *