CREDIBILITY — AN AMERICAN IDEA

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by

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PROLOGUE

Setarcos: What may we say about the basis for human decision and human action?

Student: I assume you mean conscious decision and action as opposed to instinct; all animals possess instinct to some degree. If so, we may say that human decisions (and actions) are based upon a set of beliefs - we may call this knowledge.

Setarcos: Of what does knowledge consist?

Student: Knowledge may consist of a set of suppositions or ideas, many of which exist in the absence of corroborating evidence. Also, knowledge consists of a set of facts accumulated throughout a lifetime.

Setarcos: How does a human being obtain knowledge?

Student: In the instance of ideas, these are obtained from many sources - parents, teachers, friends, and, importantly, from the creativity of the human mind. In the case of facts, these are obtained from personal observation and the reported observations of others.

Setarcos: What if there is a conflict between one's suppositions and the facts?

Student: Of course, we must first dispose of the situations where there are either no facts or no suppositions. Where there are no facts, the supposition prevails; where there is no basis for supposition, facts constitute the only knowledge.

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Setarcos: Very well, but suppose a set of facts contradicts a set of ideas?

Student: The wise person must then weigh the ideas against the evidence and make a judgment. The judgment may be to alter the ideas to fit the facts or, alternatively, to question the facts themselves. Of course, one may compromise by giving partial weight to each.

Setarcos: But what weight do we assign to supposition and what weight do we assign to the evidence?

Student: That's the real question! Perhaps some day there will be a mathematical answer.

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BAYESIAN ANALYSIS

In 1763 Thomas Bayes proposed that 'a priori' probabilities could be assigned to the several hypotheses (suppositions). Next, calculate the probability of each outcome for each hypothesis. After making an observation, or a series of observations, the 'a posteriori' probabilities of the hypotheses could be calculated. Hence, an improved knowledge of the underlying (but unknown) probabilities could be obtained. This approach is generally referred to as Bayesian analysis (sometimes as inverse or, where the judgment is mostly intuitive, subjective probability).

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CREDIBILITY - AN ANSWER TO AN INSURANCE PROBLEM

The Casualty Actuarial Society was formed in 1914 - in part to deal with the problem of making rates for a new (to the United States) line of insurance - Workmens' Compenation. (Many of the charter members of the CAS were also members of the Actuarial Society of America.) Workmens' Compensation had been available in Europe for some time but there was almost no data which might represent American experience, except for Employer's Liability Insurance which Workmens' Compensation Insurance was designed to replace.

As data became available for the new line of insurance the question arose as to how to revise the hypothetical rates to admit this new evidence in ratemaking. A group of casualty actuaries, whose spokesperson was Albert Whitney (12), conceived the idea of assigning weights in the linear expression:

$$\text{New Rate} = \text{Credibility} \times \text{Observed Rate} + (1 - \text{Credibility}) \times \text{Old Rate}$$

The underlying mathematics which determined the value of the term 'Credibility' was implicitly Bayesian but not generalized. Rather, it dealt with a specific 'a priori' distribution and a specific random process. It produced the expession:

$$\text{Credibility} = \frac{\text{Number of Observations (Exposure)}}{\text{Number of Observations (Exposure)} + K}$$

where "K" was a positive constant to be determined from the underlying factors.

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CLASSICAL STATISTICS - THE DARK AGES FOR CREDIBILITY

As will be shown later, the development of "K" required a Bayesian approach. However, in the time of Whitney, and for many years to come, the theory underlying mathematical statistics refused to admit that element of Bayesian analysis which required a subjective approach to the assignment of 'a priori' probabilities. In the Neyman-Pearson school of classical statistics, each hypothesis was something to be either accepted or rejected, but only after an examination of some data which admitted or denied the hypothesis. In Bayesian analysis the original set of hypotheses is not totally rejected, but, rather, the 'a priori' probabilities of the hypotheses may be changed ('a posteriori') to adjust to the new evidence!

Thus, it was to come to pass that casualty actuaries were to "sell their birthright for a mess of pottage". They "copped out" on calculating "K" on the basis required by Whitney's mathematics and settled for an arbitrary assignment of a value chosen by means most convenient. Having yielded to the demands of the then popular approach of statisticians, they capitulated even further by yielding to the demand by insurance buyers and insurance marketing people for 'full' credibility.

The 'Credibility Formula' as set forth above does not admit of 'full' credibility being assigned to the observations - the expression may approach unity as the number of observations (or exposure) increases, but only asymptotically. Insurance buyers with better-than-average experience wanted full recognition in their rates. Since these buyers were, more often than not, the larger customers and also, a fortiori, the preferred

risks, their wishes had to be respected. Once again an arbitrary assignment was made - the point at which exposures were sufficient to admit of 'full' credibility - and, of course, on the basis of convenience.

The actuarial literature during this period - Dark Ages - was almost exclusively in the Proceedings of the Casualty Actuarial Society and dealt with questions of determining standards for 'full' credibility and with interesting, but rather baroque, approaches as to how to make the transition from the 'Whitney' formula to 'full' credibility as the size of the risks being rated increased.

There were, however, notable exceptions. A paper by Ralph Keffer (1929) in the Transactions of the Actuarial Society of America (5)* suggested a Bayesian approach in group life insurance. In 1940 Ove Lundberg (6) in Sweden presented a - now classic - paper, using a Bayesian approach to ratemaking for health insurance. And in 1950 Arthur Bailey (1) rediscovered the original approach of Whitney and his colleagues, using subjective methods in probability theory rather than the classical approach. In so doing Bailey** reaffirmed the underlying strength of the Bayesian roots of Credibility Theory.

Following this "Renaissance" of Bayesian Credibility, successive papers by Dropkin (4), Robert Bailey (2) - son of Arthur Bailey - and

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* Keffer provides an interesting footnote (p. 135) identifying the Poisson Distribution as the "Bortkewitsch Law of Small Numbers". Readers familiar with the history of the use of the Poisson Distribution may readily infer the identity of (Oberst) Bortkewitsch.

** Many persons, including Matt Rodermund and Ben Zehnwirth, regard Arthur Bailey as the "Father of Modern Credibilty Theory".

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Simon (11)* applied specialized Bayesian approaches to the experience rating of private passenger automobile insurance.

It remained for two individuals in the 1960's to reawaken and restore interest in the true nature of the credibility concept, Hans Buhlmann in Switzerland and Allen Mayerson in the United States. The latter individual provided a paper on Bayesian Credibility (7) using an approach which was not generalized, but, rather, used two specific cognate processes, the Beta-Binomial and the Gamma-Poisson.

Meanwhile in Europe, Hans Buhlmann took the more general approach to the problem. In a paper to ASTIN (3) - the non-life section of the International Actuarial Association, Buhlmann demonstrated that the "K" in the much earlier Whitney formula was:

$$K = \frac{\text{Variance of the Processes}}{\text{Variance of the Hypotheses}}$$

These 'Renaissance' protagonists recognized that the straight line produced by the Credibility formula represented a least-squares fit to the 'process means' generated by the differing hypotheses (with weight given to each point on the basis of the 'a priori' probability of each hypothesis).

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* The brief history in my paper omits reference to many works in this field. It is deliberately intended to highlight American actuarial efforts, with reference to foreign contributions where appropriate. A fairly complete, if not up-to-date, historical bibliography may be folund on pp. 6-7 of the Simon paper (11).

CURRENT DEVELOPMENTS

With the increased acceptance of subjective probabilities - see Savage (Yale) (9) and Raiffa and Schlaiffer (Harvard) (8) & (10) - the new/old approach to credibility has flowered - at least theoretically. Many latter-day members of the Casualty Actuarial Society have written on Credibility - Philbrick, Van Slyke and Venter - just to name a few.

Professor William Jewell at the University of California (Berkeley) has been prolific in his research and publication. In Switzerland, Buhlmann and his pupils, Hans Gerber and Erwin Straub, have contributed, and, in Belgium, Fl. DeVylder. In Australia, Gregory Taylor has published frequently on Credibilty.

Credibility may be thought of as a form of 'short-hand' for the more descriptive Bayesian analysis. When Credibility Theory was first developed there were no modern high-speed computers; hence a simplified, two-dimensional linear substitute for the more complex Bayesian analysis was eminently desirable and acceptable.

Modern Credibility Theory has expanded to include non-linear and multi-dimensional approaches. Ultimately, with the advent of high-speed methods of calculation, Credibility, as a form of 'short-hand', may yield to full Bayesian analysis. In the meantime, practical application of theoretical credibility has lagged far behind the state-of-the-art. The Neyman-Pearson training of many mathematical statisticians seems to produce a revulsion for any subjective approach. However, the ideas are there!

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POSTLOGUE

Sometime in the 1920's Sinclair Lewis, the author, was to receive an award as a distinguished alumnus of Yale University. In accepting the award, Lewis took the occasion to assert his known atheism.

"I do not believe there is a God," said Lewis (in substance). "If, in fact, there is one, let him strike me down here and now!" And, of course, nothing happened. However, several days later the noted newspaper columnist, Arthur Brisbane took Lewis to task.

"Lewis, you poor misguided fool," wrote Brisbane (in substance). "You remind me of the ants who lived along the right-of-way of the Atchison, Topeka and Santa Fe Railroad. This colony of ants depended for its existence upon the crumbs thrown from the dining cars of the railroad trains as they passed by.

"It came to pass that the ant colony fell upon hard times because - through chance - no crumbs were thrown out near its particular place along the right-of-way. The situation became desperate and the colony decided to hold a meeting. It was suggested that they all pray to the President of the Atchison, Topeka and Santa Fe Railroad to send more dining cars so that crumbs would be thrown off in their area.

"So they did pray and the following day they waited, but no crumbs were thrown off where they lived. So the ants concluded that there was no such person as the President of Atchison, Topeka and Santa Fe Railroad"

+ + + NUMBER OF OBSERVATIONS + + +

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** Four boys decided to play "hooky" from school, because they knew there was to be a test that morning. About 11 A. M. their consciences got the better of them and they decided to show up at school after all. Upon reaching their classroom, they explained to their teacher that they had been on their way to school in a car, but the car had a flat tire. This made them late because they had to have the flat tire fixed.

"No problem!", said the teacher. "Just come back here during the lunch hour and I'll give you a make-up test." At lunch time, when they reported back to the classroom, the teacher instructed the four boys to take seats in opposite corners of the room.

"Now," said the teacher, "there is only one question on this make-up test. Which tire was flat?"

+ + + VARIANCE OF THE PROCESSES + + +

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*** Television interviewer: "Do you believe in miracles?"

Guest: "Of course."

Television interviewer: "Have you ever seen a miracle?"

Guest: "No."

Television interviewer: "Do you know any one who has actually seen a miracle?"

Guest: "No, but that doesn't prove anything!"

+ + + VARIANCE OF THE HYPOTHESES + + +

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