Chapter 1

PRINCIPLES OF RATEMAKING

by Charles L. McClenahan, FCAS, ASA, MAAA

Introduction

The Concept of Manual Ratemaking

From the earliest days of marine insurance, premium charges have been based upon specific characteristics of the individual risk being priced. Lloyd's of London based early hull rates in part upon the design and protection of each specific ship, and the classification assigned to each vessel was written down in a book for use by the individual underwriters. Eighteenth century dwelling fire insurance rates in the U.S. were based upon roof type and basic construction. While these early rate manuals were meant to provide general guidance to the underwriters in setting the specific rates, rather than the actual rates to be charged, they contained many of the elements associated with present-day property and liability rate manuals including recognition of differing loss costs between classifications, expense loading, and provision for adverse deviation and profit.

One of the most persistent misconceptions associated with property and liability insurance is the level of accuracy which actuaries are believed to achieve in the assessment of individual loss propensity. Over the years, as the doctrine of *caveat emptor* has been eroded and insurance risks have become increasingly complex, rate manuals have evolved to the point that, for many lines of insurance, they provide the exact premium to be charged for providing a specific coverage to a specific risk for a specific period. It is important, however, not to confuse the level of precision inherent in the rate manual with the level of accuracy. The latter will be judged in the cold light of actual loss experience. No matter how refined the classification and rating process may become, manual rates are still *estimates of average costs* based upon a combination of statistical methods and professional judgment.

This chapter will deal with the basic actuarial methods and assumptions underlying the development of manual rates. While a complete treatment of the subject might well fill several books, the key elements will be covered to such an extent that the reader of this chapter will gain an understanding of the basic actuarial concepts and techniques involved in the review and analysis of manual rates for property and liability coverages.

Basic Terminology

While ratemaking is neither pure science nor pure art, both the scientific and artistic elements of the subject demand the use of precise language. Property and casualty insurance is a complicated business which can be best represented and understood in a technical financial context. Many of the misconceptions about property and liability insurance can be directly attributed to either the failure to use precise terminology, or the failure to understand the terminology in precise terms. This section will introduce some definitions of some of the more important terms used by casualty actuaries.

Exposure

The basic rating unit underlying an insurance premium is called an exposure. The unit of exposure will vary based upon the characteristics of the insurance coverage involved. For automobile insurance, one automobile insured for a period of twelve months is a car year. A single policy providing coverage on three automobiles for a six month term would involve 1.5 car years. The most commonly used exposure statistics are **written exposures**, those units of exposures on policies written during the period in question, **earned exposures**, the exposure units actually exposed to loss during the period, and **in-force exposures**, the exposure units exposed to loss at a given point in time. In order to illustrate these three statistics, consider the following four twelve-month, single-car automobile policies:

Effective Date	Written Exposure 1987	Written Exposure 1988	Earned Exposure 1987	Earned Exposure 1988	In-Force Exposure 12/31/87
1/1/87	1.00	0.00	1.00	0.00	0.00
4/1/87	1.00	0.00	0.75	0.25	1.00
7/1/87	1.00	0.00	0.50	0.50	1.00
10/1/87	1.00	0.00	0.25	0.75	1.00
Total	4.00	0.00	2.00	2.00	3.00

Note that the in-force exposure counts a full car year for each twelve-month policy in force at the end of 12/31/87, regardless of the length of the remaining term.

The specific exposure unit used for a given type of insurance depends upon several factors including: reasonableness; ease of determination; responsiveness to change; and historical practice.

Reasonableness - it is obvious that the exposure unit should be a reasonable measure of the exposure to loss. While every exposure unit definition represents some level of compromise of this principle, for example a 1988 Rolls Royce and a 1978 Chevrolet might each represent a car year exposure, the selected measure should directly relate to loss potential to the extent possible.

Ease of Determination - the most reasonable and responsive exposure definition is of no use if it cannot be accurately determined. While the most appropriate exposure for products liability insurance might be the number of products currently in use, this would generally be impossible to determine. If an exposure base is not subject to determination, then an insurer can never be assured of receiving the proper premium for the actual exposure.

Responsiveness to Change - an exposure unit which reflects changes in the exposure to loss is preferable to one which does not. The exposure unit for workers' compensation insurance, which provides benefits which are keyed to average wage levels, is payroll. This is obviously preferable to number of employees, for example, as the payroll will change with the prevailing wage levels.

Historical Practice - where a significant body of historical exposure data is available, any change in the exposure base would render the prior history unusable. Since ratemaking generally depends upon the review of past statistical indications, exposure bases are rarely changed once they have been established.

Claim

A **claim** is a demand for payment by an insured or allegedly injured third party under the terms and conditions of an insurance contract. The individual making the claim is the **claimant**, and there can be multiple claimants within a single claim. Claim statistics are key elements in the ratemaking process. Generally insurers maintain claim data based upon **accident date** - the date of the occurrence which gave rise to the claim, and **report date** - the date the insurer receives notice of the claim. Claim data can then be aggregated based upon these dates. For example, the total of all claims with accident dates during 1988 is the **accident year** 1988 claim count.

Frequency

Because the number of claims is directly related to the number of exposures, actuaries express claim incidence in terms of **frequency** per exposure unit.

$$F_k = \frac{kC}{E} \qquad (1)$$

Where: F_k = frequency per k exposure units
k = scale factor
C = claim count
E = exposure units

For example, if we earned 32,458 car years of exposure during 1988 and we incur 814 claims with 1988 accident dates, then the 1988 accident year claim frequency per 1,000 earned exposures is 25.08 as follows:

$$F_{1000} = \frac{1000(814)}{32,458} = 25.08$$

Where the context is established by either data or previous exposition it might be appropriate to simply refer to this as the *frequency*. In general, however, the need for precision would require that the more specific language *accident year frequency per 1,000 earned car years* be used.

Losses and Loss Adjustment Expenses

Amounts paid or payable to claimants under the terms of insurance policies are referred to as **losses**. **Paid losses** are those losses for a particular period which have actually been paid to claimants. Where there is an expectation that a payment will be made in the future, a claim will have an associated **case reserve** representing the estimated amount of that payment. The sum of all paid losses and case reserves for a specific accident year at a specific point in time is known as the **accident year case-incurred losses.** The term *case-incurred* is used to distinguish this statistic from **ultimate incurred losses** which include losses which have not yet been reported to the insurance company as of the case-incurred evaluation date.

Over time, as more losses are paid and more information becomes available about unpaid claims, accident year case-incurred losses will tend to approach their ultimate value. Generally, because of the reporting of additional claims which were not included in earlier evaluations, accident year case-incurred losses tend to increase over time. In order to keep track of the individual evaluations of case-incurred losses for an accident year, actuaries use the concept of the **accident year age.** The accident year age is generally expressed in months. By convention, the accident year is age 12 months at the end of the last day of the accident year. Therefore, the 1987 accident year evaluated as of 6/30/88 would be referred to as the age 18 evaluation of the 1987 accident year.

Figure 1 represents a graphical interpretation of a typical case-incurred loss development pattern - in this case for automobile liability.

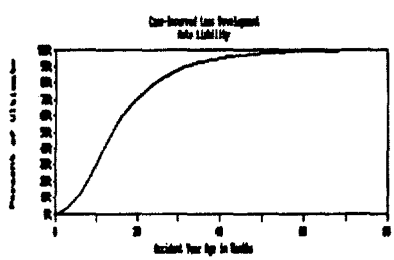


Figure 1

Insurance company expenses associated with the settlement of claims, as opposed to the marketing, investment or general administrative operations, are referred to as **loss adjustment expenses**. Those loss adjustment expenses which can be directly related to a specific claim are called **allocated loss adjustment expenses** and those which cannot are called **unallocated loss adjustment expenses.**

Severity

Average loss per claim is called **severity** by actuaries. Severities can be on a **pure loss** basis, excluding all loss adjustment expenses, or they can include allocated or total loss adjustment expenses. The loss component can be paid, case-incurred or projected ultimate and the claims component can be reported, paid, closed, or projected ultimate. This profusion of available options again requires that the actuary be precise in the references to the components. Note the differences between *accident year case-incurred loss severity per reported claim* and *report year paid loss and allocated severity per closed claim*. However the loss and claim components are defined, the formula for severity is simply:

$$S = \frac{L}{C} \qquad (2)$$

Where: S = severity
L = losses
C = claim count

Pure Premium

Another important statistic is the average loss per unit of exposure or the **pure premium.** The reader will by now appreciate the need for precise component definition either in terminology or through context, so the various options will not be recited. The formula for the pure premium is:

$$P = \frac{L}{E} \tag{3}$$

Where: P = pure premium
L = losses
E = exposure units

Note that the pure premium can also be expressed as:

$$P = \frac{C}{E} \times \frac{L}{C}$$

Where: C = claim count

Or, where frequency is per unit of exposure:

$$P = F_1 \times S \tag{4}$$

In other words, *pure premium equals the product of frequency per unit of exposure and severity.*

Expense, Profit and Contingencies

In order to determine the price for a specific insurance coverage, appropriate provisions must be made for expenses (other than any loss adjustment expenses included in the pure premium) and profit. Because insurance pricing involves the estimation of the cost of future contingencies, the profit provision is generally termed the **profit and contingencies** loading. This term properly reflects the fact that profits, if any, will be based upon actual results and not expectations or projections. While the topic of expenses will be treated in detail in chapter 7, for the purposes of this discussion we will distinguish between **fixed expenses** per unit of exposure, those expenses which do not depend upon premium, and **variable expenses** which vary directly with price. This treatment gives rise to the following formula for the **rate per unit of exposure**:

$$R = \frac{P+F}{1-V-Q} \qquad (5)$$

Where:

- **R** = rate per unit of exposure
- **P** = pure premium
- **F** = fixed expense per exposure
- **V** = variable expense factor
- **Q** = profit and contingencies factor

As an example, assume the following:

Loss and loss adjustment expense pure premium	$75.00
Fixed expense per exposure	$12.50
Variable expense factor	17.50%
Profit and contingencies factor	5.00%

The appropriate rate for this example would be calculated as follows:

$$\text{Rate} = \frac{\$75.00 + \$12.50}{1 - .175 - .050} = \$112.90$$

The individual components of the rate would therefore be as follows:

Pure premium	$75.00
Fixed expenses	12.50
Variable expenses ($112.90 x .175)	19.76
Profit and contingencies ($112.90 x .050)	5.64
Total	$112.90

Premium

Application of the rate(s) to the individual exposures to be covered by an insurance policy produces the **premium** for that policy. If, in the above example, the unit of exposure is a commercial vehicle and we are rating a policy for 15 commercial vehicles, the premium would be calculated as follows:

$112.90 x 15 = $1,693.50

Premium, like exposure, can be either *written*, *earned*, or *in-force*. If the policy in question was written for a twelve month term on 7/1/87 then that policy would have contributed the following amounts as of 12/31/87:

Calendar year 1987 written premium	$1,693.50
Calendar year 1987 earned premium	846.75
12/31/87 premium in-force	$1,693.50

Loss Ratio

Probably the single most widely-used statistic in the analysis of insurance losses is the **loss ratio** or losses divided by premium. Again the need for precision cannot be overemphasized. There is a great difference between a loss ratio based upon paid losses as of accident year age 12 and written premium (termed an *age 12 accident year written-paid pure loss ratio*) and one which is based upon ultimate incurred loss and loss adjustment expenses and earned premium (*ultimate accident year earned-incurred loss and loss adjustment expense ratio*) although either can be properly referred to as a loss ratio.

The Goal of the Manual Ratemaking Process

Broadly stated, the goal of the ratemaking process is to determine rates which will, when applied to the exposures underlying the risks being written, provide sufficient funds to pay expected losses and expenses; maintain an adequate margin for adverse deviation; and produce a reasonable return on (any) funds provided by investors. In addition, manual rates are generally subject to regulatory review and, while detailed discussion of regulatory requirements is beyond the scope of this text, this review is often based upon the regulatory standard that *rates shall not be inadequate, excessive, or unfairly discriminatory between risks of like kind and quality.*

Adequately pricing a line of insurance involves substantial judgment. While actuaries are trained in mathematics and statistics, the actuarial process underlying manual ratemaking also requires substantial understanding of the underwriting, economic, social and political factors which have in the past impacted the insurance results and will impact those results in the future.

If it were sufficient that manual rates be adequate to cover losses and expenses for some past period, then the pricing problem would be basically identical to the reserving problem which is the topic of chapter 3. But rates which were adequate in the past, or even those which are adequate today, may not be adequate when applied to policies providing insurance coverage into the future.

In discussing the goal of the ratemaking process from an actuarial perspective it is important to note that actuarially-determined rates are often subjected to review by others both within and outside the insurance company. Internally, there will generally be a review of the competitiveness of the rate levels in the marketplace. Externally there is the previously mentioned regulatory review, occasionally involving the political acceptability of the proposed rates. While the actuary may be directly involved in both internal and external discussions relating to these reviews, it is the actuary's primary responsibility to recommend rates which can be reasonably expected to be adequate over the period in which they are to be used.

Structure of the Rating Plan

Up to this point the discussion of manual rates has related to the concept of an identified unit of exposure. In practice, manual rates are based upon a number of factors in addition to the basic exposure unit. For example, the elements involved in the rating of a single private passenger automobile insurance policy might include the following:

Age of insured(s)
Sex of insured(s)
Marital status of insured(s)
Prior driving record of insured(s)
Annual mileage driven
Primary use of vehicle(s)
Make and model of vehicle(s)
Age of vehicle(s)
Garaging location of vehicle(s)

The structure of the various elements involved in the manual rating of a specific risk is known as the **rating plan**. Various specific elements are often referred to as **classifications, sub-classifications** or **rating factors**. Rating plans serve to allow the manual rating process to reflect identified differences in loss propensity. To fail to so reflect such known factors gives rise to two separate situations. Where a known positive characteristic is not reflected in the rating plan, the rate applied to risks possessing that positive characteristic will be too high. This would encourage the insuring of these risks to the partial or total exclusion of risks not possessing the positive characteristic, a practice referred to as **skimming the cream**. On the other hand, the failure to reflect a known negative characteristic will result in the application of a rate which is too low. If other companies are reflecting the negative factor in their rating plans, the result will be a tendency towards insuring risks possessing the negative characteristic, a situation known as **adverse selection.**

Risk characteristics underlying a manual rating plan can be broadly identified as those generally impacting frequency and those generally impacting severity. Prior driving record is an example of a factor which has been demonstrated to correlate with frequency. Individuals with recent automobile accidents and traffic violations have, *as a class*, higher frequencies of future claims than do those individuals with no recent accidents or violations. Individuals driving high-powered sports cars have, *as a class*, higher frequencies than those driving family sedans. Annual mileage driven has an obvious impact on frequency.

On the severity side, large vehicles tend to do more damage in collisions than do small vehicles. A Rolls Royce costs more to repair than does a Chevrolet. A late model automobile is more valuable than a ten-year-old "clunker" and will therefore, *on average*, have a higher associated severity.

The above examples deal with private passenger automobile insurance; but other lines have identifiable risk characteristics as well. In commercial fire insurance, restaurants generally have a higher frequency than do clothing stores. The presence or absence of a sprinkler system will impact severity as will the value of the building and contents being insured. Workers' compensation statistics detail higher frequencies for manufacturing employees than for clerical workers. For every type of property and casualty insurance there are identifiable factors which impact upon frequency and severity of losses.

The subject of risk classification will be discussed in detail in chapter 4. In addition the reflection of specific individual risk differences, as opposed to class differences, will be treated in chapter 2. For the purposes of this chapter it is sufficient to be aware of the existence of and need for a rating plan reflecting identifiable risk classification differences.

The Ratemaking Process

In this section we will deal with the basic techniques used by casualty actuaries in the development of manual rates. The reader must bear in mind that this discussion will be general in nature - a complete discussion of the elements involved in a single complex line of insurance might require several hundred pages. Nevertheless, the key elements of manual ratemaking will be addressed to such an extent that a good understanding of the actuarial process of manual ratemaking will result.

Basic Manual Ratemaking Methods

There are two basic approaches to addressing the problem of manual ratemaking; the **pure premium method** and the **loss ratio method**. We will examine the mathematics underlying each method and then develop a relationship between the two.

Pure Premium Method

The pure premium method develops indicated rates based upon formula (5).

$$R = \frac{P+F}{1-V-Q} \tag{5}$$

Where:
- **R** = (indicated) rate per unit of exposure
- **P** = pure premium
- **F** = fixed expense per exposure
- **V** = variable expense factor
- **Q** = profit and contingencies factor

The pure premium used in the formula is based upon experience losses, which are trended projected ultimate losses (or losses and loss adjustment expenses) for the experience period under review, and the exposures earned during the experience period. The methods underlying the trending and projection of the losses will be discussed later in this chapter.

Loss Ratio Method

The loss ratio method develops indicated rate *changes* rather than indicated rates. Indicated rates are determined by application of an adjustment factor, the ratio of the experience loss ratio to a target loss ratio, to the current rates. The experience loss ratio is the ratio of the experience losses to the

on-level earned premium - the earned premium which would have resulted for the experience period had the current rates been in effect for the entire period. In mathematical terms the loss ratio method works as follows:

$$R = AR_0 \qquad (6)$$

Where:

R = indicated rate

R_0 = current rate

A = adjustment factor

= W/T

W = experience loss ratio

T = target loss ratio

Looking first at the target loss ratio:

$$T = \frac{1-V-Q}{1+G} \qquad (7)$$

Where:

V = premium-related expense factor

Q = profit and contingencies factor

G = ratio of non-premium-related expenses to losses

And then the experience loss ratio:

$$W = \frac{L}{ER_0} \tag{8}$$

Where:

L = experience losses

E = experience period earned exposure

R_0 = current rate

Using (7) and (8) we can see:

$$A = \frac{L/(ER_0)}{(1-V-Q)/(1+G)}$$

$$= \frac{L(1+G)}{ER_0(1-V-Q)} \tag{9}$$

and, substituting (9) into (6):

$$R = \frac{L(1+G)}{E(1-V-Q)} \tag{10}$$

Relationship Between Pure Premium and Loss Ratio Methods

It has been emphasized in this chapter that manual rates are *estimates*. Nevertheless, they generally represent *precise* estimates based upon reasonable and consistent assumptions. This being the case, we should be able to demonstrate that the pure premium and loss ratio methods will produce identical rates when applied to identical data and using consistent assumptions. This demonstration is quite simple. It starts with formula (10), the formula for the indicated rate under the loss ratio method:

$$R = \frac{L(1+G)}{E(1-V-Q)} \qquad (10)$$

Now, the loss ratio method uses experience losses while the pure premium method is based upon experience pure premium. The relationship between the two comes from (3):

$$P = \frac{L}{E} \qquad (3)$$

which can be expressed as:

$$L = EP$$

Also, the loss ratio method relates non-premium-related expenses to losses while the pure premium method uses exposures as the base for these expenses. The relationship can be expressed as follows:

$$G = \frac{EF}{L} = \frac{F}{P}.$$

Substituting for L and G in formula (10) produces the following:

$$R = \frac{EP[1+(F/P)]}{E(1\text{-}V\text{-}Q)}$$

Or:

$$R = \frac{P+F}{1\text{-}V\text{-}Q} \qquad (5)$$

Which is the formula for the indicated rate under the pure premium method. The equivalence of the two methods is therefore demonstrated.

Selection of Appropriate Method

Because the two methods can be expected to produce identical results when consistently applied to a common set of data, the question arises as to which approach is the more appropriate for any given situation. Having dealt with the mathematical aspects of the two methods, let us now look at some of the practical differences.

Pure Premium Method	Loss Ratio Method
Based on exposure	Based on premium
Does not require existing rates	Requires existing rates
Does not use on-level premium	Uses on-level premium
Produces indicated rates	Produces indicated rate changes

Noting the above differences, the following guidelines would seem to be reasonable:

Pure premium method requires well-defined, responsive exposures - the pure premium method is based on losses per unit exposure. Where the exposure unit is not available or is not reasonably consistent between risks, as in the case of commercial fire insurance, the pure premium method cannot be used.

Loss ratio method cannot be used for a new line - because the loss ratio method produces indicated rate *changes*, its use requires an established rate and premium history. Where manual rates are required for a new line of business, where there are loss statistics available, the pure premium method must be used.

Pure premium method preferable where on-level premium is difficult to calculate - in some instances, such as commercial lines where individual risk rating adjustments are made to individual policies, it is difficult to determine the on-level earned premium required for the loss ratio method. Where this is the case it is more appropriate to use the pure premium method if possible.

Need for Common Basis

Whichever ratemaking method is selected, the actuary needs to make certain that the experience losses are on a basis consistent with the exposures and premiums being used. This requires that adjustments be made for observed changes in the data. This section will deal with some of the more common sources of change in the underlying data and will discuss methods for dealing with those changes.

Selection of Experience Period

Determination of the loss experience period to be used in the manual ratemaking process involves a combination of statistical and judgmental elements. There is a natural preference for using the most recent incurred loss experience available since it is generally most representative of the current situation, however this experience will also contain a higher proportion of unpaid losses than will more mature periods and is therefore more subject to loss development projection errors. Where the business involved is subject to catastrophe losses, as in the case of windstorm coverage in hurricane-prone areas, the experience period must be representative of the average catastrophe incidence. Finally, the experience period must contain sufficient loss experience that the resulting indications will have statistical significance or **credibility**.

Reinsurance

Ceded reinsurance, which is discussed in depth in chapter 5, serves to reduce an insurer's exposure to large losses, either individual or in the aggregate, in exchange for a reinsurance premium. While there may be instances in which a reinsurance program represents such a significant transfer of risk that separate and distinct provision for the reinsurance premium is appropriate, such cases are beyond the scope of this chapter. In general, manual rates should be based upon direct, that is before reflection of reinsurance, premium and loss data.

Differences in Coverage

Wherever possible, major coverages within a line of insurance should be treated separately. For example, liability experience under homeowners policies should be reviewed separately from the property experience. Auto collision data should be analyzed separately by deductible. Professional liability policies written on a claims-made basis should not be combined with those written on an occurrence basis for ratemaking purposes. Note that unless the mix has been consistent over the entire experience period these separations will require the segregation of premium and exposure data as well as the loss experience.

Treatment of Increased Limits

Liability coverage rate manuals generally provide rates for a basic limit of liability along with **increased limits factors** to be applied to these base rates where higher limits are desired. As will be seen in a later section, these increased limits factors tend to change over time. In addition there will be a general movement toward higher limits as inflation erodes purchasing power. For these reasons premiums and losses used in the manual ratemaking process should be adjusted to a basic limits basis.

On-Level Premium - Adjusting for Prior Rate Changes

Where, as is the general case, the experience period extends over several years there have typically been changes in manual rate levels between the beginning of the experience period and the date as of which the rates are being reviewed. If the actuary is using the loss ratio method in the development of the indicated rate level changes, the earned premium underlying the loss ratio calculations must be on a current rate level basis.

Where the capability exists, the best method for bringing past premiums to an on-level basis is to re-rate each policy using current rates. Doing this manually is generally far too time-consuming to be practical, but where sufficient detail is available in the computer files and if rating software is available, the resulting on-level premiums will be quite accurate. This method is referred to as the **extension of exposures** technique.

When extension of exposures cannot be used, an alternative, called the **parallelogram method**, is available. This method adjusts calendar year earned premiums to current rate levels based upon simple geometric relationships and an underlying assumption that exposure is uniformly distributed over time.

As an example, assume that the experience period in question consists of the three years 1985, 1986 and 1987. Further assume that each policy has a twelve month term. Finally, assume that rate increases have been taken as follows:

+17.8% effective 7/1/82
+12.5% effective 7/1/84
+10.0% effective 7/1/86

Because we are dealing with twelve-month policies, all of the premium earned during the earliest year of the experience period - 1985 - was written at either the 7/1/82 rate level or the 7/1/84 rate level. If we assign the 7/1/82 rate level a relative value of 1.000, then the 7/1/84 rate level has a relative value of 1.125 and the 7/1/86 rate level has a relative value of (1.125)(1.100) = 1.2375.

Figure 2 provides a representation of these data under the parallelogram method. The x-axis represents the date on which a policy is effective, and the y-axis represents the portion of exposure earned.

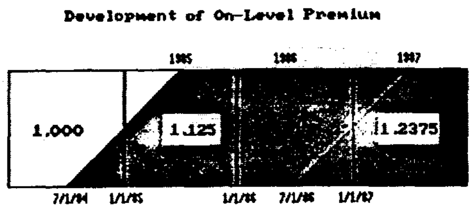


Figure 2

Each calendar year of earned premium can now be viewed as a unit square one year wide and 100% of exposure high. Figure 3 illustrates this treatment of the 1985 year.

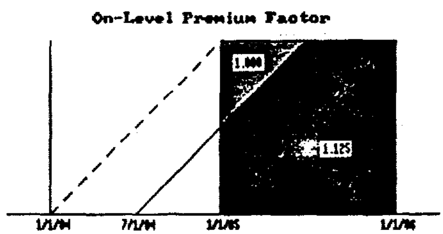


Figure 3

As shown in Figure 4, we can now use simple geometry to determine the portions of 1985 earned exposure written at the 1.000 and 1.125 relative levels.

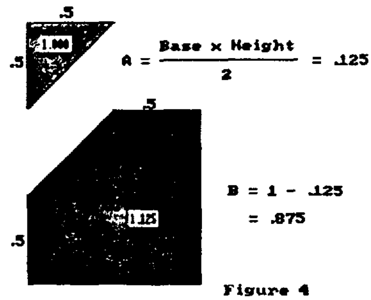


Figure 4

According to the parallelogram model, .125 of the 1985 earned exposure arises from policies written at the 1.000 relative level and .875 of the exposure was written at a relative level of 1.125. The average 1975 relative earned rate level is therefore [(.125)(1.000) + (.875)(1.125)] = 1.1094. Since the current relative average rate level is 1.2375, the 1985 calendar year earned premium must be multiplied by (1.2375/1.1094) = 1.1155 to reflect current rate levels. The 1.1155 is referred to as the 1985 **on-level factor.**

We can repeat this process for the 1986 and 1987 years to generate the following:

Calendar Year	Portion of Earned at Relative Level 1.000	1.125	1.2375	On-Level Factor
1985	.125	.875	0	1.1155
1986	0	.875	.125	1.0864
1987	0	.125	.875	1.0115

These on-level factors are then applied to the calendar year earned premiums to generate approximate on-level earned premiums. For example:

Calendar Year	Calendar Year Earned Premium	On-Level Factor	Approximate On-Level Earned Premium
1985	$1,926,981	1.1155	$2,149,547
1986	$2,299,865	1.0864	$2.498,573
1987	$2,562,996	1.0115	$2,592,470
Total	$6,789,842		$7,240,590

As noted earlier, the parallelogram method is based upon an assumption that exposures are written uniformly over the calendar period. In cases where material changes in exposure level have occurred over the period, or where there is a non-uniform pattern to the written exposures, the parallelogram method may not produce a reasonable approximation of on-level earned premium. While a discussion of adjustments to the simple model underlying the parallelogram method is beyond the scope of this chapter, Miller and Davis[1] have proposed an alternative model which reflects actual exposure patterns.

---

[1]Miller, D.L. and Davis, G.E. "A Refined Model for Premium Adjustment." *Proceedings* of the Casualty Actuarial Society LXIII, 1976. p. 117.

Trended, Projected Ultimate Losses

We are now ready to discuss the methodology underlying the development of the trended, projected ultimate losses. This element represents the most significant part of any ratemaking analysis and requires both statistical expertise and actuarial judgment. Whether the pure premium method or the loss ratio method is being used, the accuracy with which losses are projected will determine the adequacy of the resulting manual rates.

Inclusion of Loss Adjustment Expenses

The actuary must determine whether to make projections on a pure loss basis, or whether to include allocated loss adjustment expenses with losses. Unallocated loss adjustment data are rarely available in sufficient detail for inclusion with losses and allocated loss adjustment expenses, and are generally treated as part of the expense loading - frequently as a ratio to loss and allocated loss expenses.

While the decision whether to include allocated loss expense data with losses is generally made based upon data availability, there is one situation in which it is essential that the allocated loss adjustment expenses be combined with the losses. Some liability policies contain limits of liability which apply to both losses and allocated loss adjustment expenses. Where manual rates are being developed for such policies, allocated loss adjustment expenses should be treated as losses.

Projection to Ultimate - the Loss Development Method

A significant portion of the entirety of casualty actuarial literature produced in this century deals with the methods and techniques for projecting unpaid, and often unreported, losses to their ultimate settlement values. Even a casual treatment of the subject is beyond the scope of this chapter. Nevertheless, the general concepts discussed in this section will be based upon the use of projected ultimate losses and claim counts. A thorough understanding of the issues involved in manual ratemaking requires that the context of the problem be clear. At least one technique for projection to ultimate is needed and we will use the most common - the **loss development method.**

The loss development method is based upon the assumption that claims move from unreported to reported-and-unpaid to paid in a pattern which is sufficiently consistent that past experience can be used to predict future development. Claim counts, or losses, are arrayed by accident year (or report year or on some other basis) and accident year age. The resulting data form a triangle of known values. As an example, consider the following accident year reported claim count development data:

Accident Year	Age 12	Age 24	Age 36	Age 48	Age 60	Age 72
1982	1,804	2,173	2,374	2,416	2,416	2,416
1983	1,935	2,379	2,424	2,552	2,552	
1984	2,103	2,384	2,514	2,646		
1985	2,169	2,580	2,722			
1986	2,346	2,783				
1987	2,337					

Remembering the concept of accident year age it can be seen, for example, that as of 12/31/85 there were 2,424 claims reported for accidents occurring during 1983. By 12/31/86 this number had developed to 2,552. Horizontal movement to the right represents *development*, vertical movement downward represents *change in exposure level*, and positive-sloped diagonals represent *evaluation dates*. The lower diagonal represents the latest available evaluation - in this case 12/31/87.

Accident Year	Age 12	Age 24	Age 36	Age 48	Age 60	Age 72
1982	1,804	2,173	2,374	2,416	2,416	2,416
1983	1,935	2,379	2,424	2,552	2,552	
1984	2,103	2,384	2,514	2,646		
1985	2,169	2,580	2,722			
1986	2,346	2,783				
1987	2,337					

12/31/87 Evaluation

The next step in the process is to reflect the development history arithmetically. This involves the division of each evaluation subsequent to the first by the immediately preceding evaluation. The resulting ratio is called an **age-to-age development factor** or, sometimes, a **link ratio**. For example, the *accident year 1982 12-24 reported claim count development factor* from our example is 2,173/1,804 = 1.2045.

Accident Year	Age 12	Age 24	Age 36	Age 48	Age 60	Age 72
1982	1,804	2,173	2,374	2,416	2,416	2,416
1983	1,935	2,379	2,424	2,552	2,552	
1984	2,103	2,384	2,514	2,646		
1985	2,169	2,580	2,722			
1986	2,346	2,783				
1987	2,337					

2,173/1,804 = 1.2045

Repeating this process, we can produce a second data triangle consisting of age-to-age development factors.

Accident Year	Age 12	Age 24	Age 36	Age 48	Age 60	Age 72
1982	1,804	2,173	2,374	2,416	2,416	2,416
1983	1,935	2,379	2,424	2,552	2,552	
1984	2,103	2,384	2,514	2,646		
1985	2,169	2,580	2,722			
1986	2,346	2,783				
1987	2,337					

Accident Year	12-24	24-36	36-48	48-60	60-72
1982	1.2045	1.0925	1.0177	1.0000	1.0000
1983	1.2295	1.0189	1.0528	1.0000	
1984	1.1336	1.0545	1.0525		
1985	1.1895	1.0550			
1986	1.1863				

Based upon the observed development factors, age-to-age factors are selected and successively multiplied to generate **age-to-ultimate factors.** These age-to-ultimate factors are then applied to the latest diagonal of the development data to yield projected ultimate values.

Accident Year	Accident Year Age	Selected Age-to-Age Factor	Age-to-Ultimate Factor	Reported Claims 12/31/87	Projected Ultimate Claims
1982	72		1.0000	2,416	2,416
1983	60	1.0000	1.0000	2,552	2,552
1984	48	1.0000	1.0000	2,646	2,646
1985	36	1.0450	1.0450	2,722	2,844
1986	24	1.0550	1.1025	2,783	3,068
1987	12	1.1900	1.3120	2,337	3,066

An identical process can be applied to either paid or case-incurred losses. Generally case-incurred values are used, especially where the development period extends over several years. Note that losses tend to take longer to fully develop than do reported claims. This is due to the **settlement lag** - the period between loss reporting and loss payment - which affects losses but not reported claims and represents additional development potential beyond the **reporting lag** - the period between loss occurrence and loss reporting - which affects both claims and losses.

An example of the loss development method applied to case-incurred loss and allocated loss adjustment expense data is contained in the Appendix to this chapter.

In some instances, most notably where premiums are subject to audit adjustments as is often true for workers' compensation insurance, premium data requires projection to ultimate in order that the premium being used in the ratemaking calculations properly reflects the actual exposure level which gave rise to the ultimate losses. One method for handling this situation is to aggregate data on a **policy year**, rather than an accident year, basis. Policy year data is based upon the year in which the policy giving rise to exposures, premiums, claims and losses is effective. Another method involves the projection of written premium to ultimate and the recalculation of earned premium, referred to as **exposure year earned premium**, based upon the projected ultimate written premium. In either case, the projection techniques involved are similar to the loss development method.

Identification of Trends

Once claims and losses have been projected to an ultimate basis it is necessary to adjust the data for any underlying trends which are expected to produce changes in indications between the experience period and the period during which the manual rates will be in effect. For example, if rates are being reviewed as of 12/31/87 based upon 1985 accident year data and the new rates are expected to go into effect on 7/1/88, the projected ultimate losses for the 1985 accident year are representative of loss exposure as of approximately 7/1/85 and the indicated rates must cover loss exposure as of approximately 12/31/88. To the extent that there are identifiable trends in the loss data, the impact of those trends over the 42 months between the midpoint of the experience period and the average exposure date to which the rates will apply.

The most obvious trend affecting the ratemaking data is the trend in severity. Monetary inflation, increases in jury awards, and increases in medical expenses are examples of factors which cause upward trends in loss severities. Frequency is also subject to trend. Court decisions may open new grounds for litigation which would increase liability frequencies. Legal and social pressures might reduce the incidence of driving under the influence of alcohol, thus reducing automobile insurance frequencies.

Some exposure bases also exhibit identifiable trends. Workers' compensation uses payroll as an exposure base and products liability coverage might be based upon dollars of sales. Both of these exposures will reflect some degree of trend. Automobile physical damage rates are based upon the value of the automobiles being insured. As automobile prices increase the physical damage premiums will reflect the change, even though no rate change has been made. When using the loss ratio method for ratemaking it is important that the effect of such trends on premium be properly reflected.

While frequency and severity trends are often analyzed separately, it is sometimes preferable to look at trends in the pure premium, thus combining the impacts of frequency and severity.

Reflection of Trends

Actuaries generally approach the problem of how to reflect observed trends by fitting an appropriate curve to the observed data. The most important word in the preceding sentence is *appropriate*. Consider the following hypothetical projected accident year severity data:

Accident Year	**Projected Severity**
1980	$309
1981	532
1982	763
1983	996
1984	1,225
1985	1,444
1986	1,647
1987	1,828

It so happens that the third-degree polynomial $y = -x^3 + 10x^2 + 200x + 100$ produces a perfect fit to these data where x is defined as the accident year minus 1979. Figure 5 shows the result of this fit graphically.

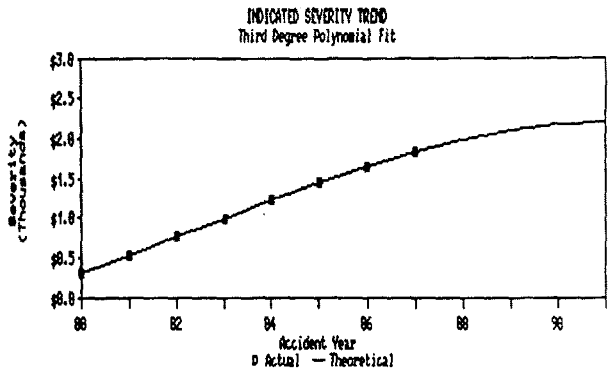


Figure 5

Based upon the strength of the fit one might be tempted to use the third-degree polynomial to project future severity changes. But is a third-degree polynomial really *appropriate* for a severity trend model?

If we extend the x axis out through accident year 1998 we see the following results. Viewed in this manner it is apparent that, regardless of how well it might fit our observations, the third-degree polynomial model is not one which is reasonable for the projection of severity changes.

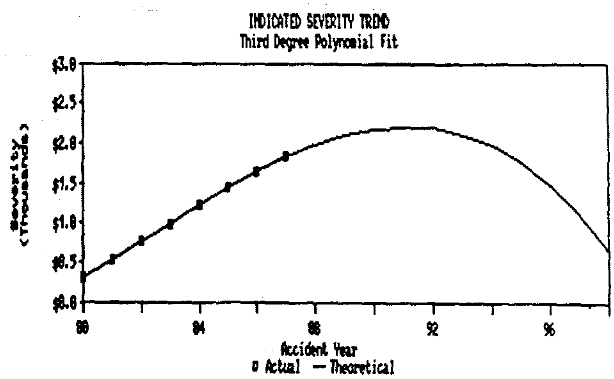


Figure 6

While other appropriate models are available, most of the trending models used by casualty actuaries in ratemaking take one of two forms:

Linear	$y = ax + b$	, or
Exponential	$y = be^{ax}$	

Note that the exponential model can be expressed as:

$$\ln(y) = ax + \ln(b)$$

Or, with the substitutions $y' = \ln(y)$ and $b' = \ln(b)$:

$$y' = ax + b'$$

Since either model can therefore be expressed in terms of a linear function, the standard first-degree least-squares regression method can be applied to the observed data to determine the trend model. Note that the linear model will produce a model in which the projection will increase by a constant *amount* (a) for each unit change in x. The exponential model will produce a constant *rate* of change of $e^a - 1$, with each value being e^a times the prior value. Drawing an analogy to the mathematics of finance, the linear model is analogous to *simple* interest while the exponential model is analogous to *compound* interest.

While either linear or exponential models can be used to reflect increasing trends, where the observed trend is decreasing the use of a linear model will produce negative values at some point in the future. The use of a linear model in such cases is clearly inappropriate since frequency, severity, pure premium and exposure must all be greater than or equal to zero.

Exhibits IV, V, VII and VIII of the Appendix to this chapter provide examples of the application of both linear and exponential trend models using both loss ratio and pure premium methods.

Effects of Limits on Severity Trend

Where the loss experience under review involves the application of limits of liability, it is important that the effects of those limits on severity trend be properly reflected. In order to understand the interaction between limits and severity trend, consider the hypothetical situation in which individual losses can occur for any amount between $1 and $90,000. Assume that insurance coverage against these losses is available at four limits of liability: $10,000 per occurrence; $25,000 per occurrence; $50,000 per occurrence; and $100,000 per occurrence. Note that, since losses can only be as great as $90,000 the $100,000 limit coverage is basically unlimited.

In order to analyze the operation of severity trend on the various limits it will be necessary to look at losses by layer of liability. The following chart illustrates this layering for four different loss amounts.

Loss Amount	Distribution of Loss Amount by Layer First $10,000	$15,000 excess of $10,000	$25,000 excess of $25,000	$50,000 excess of $50,000
$5,000	$5,000			
$20,000	$10,000	$10,000		
$40,000	$10,000	$15,000	$15,000	
$70,000	$10,000	$15,000	$25,000	$20,000
Total	$35,000	$40,000	$40,000	$20,000

The total line represents the distribution of the $135,000 of losses by layer, assuming that one claim of each amount occurred. Consider now the effect of a constant 10% increase in each claim amount.

Loss Amount	Distribution of Loss Amount by Layer First $10,000	$15,000 excess of $10,000	$25,000 excess of $25,000	$50,000 excess of $50,000
$5,500	$5,500			
$22,000	$10,000	$12,000		
$44,000	$10,000	$15,000	$19,000	
$77,000	$10,000	$15,000	$25,000	$27,000
Total	$35,500	$42,000	$44,000	$27,000
Increase	1.43%	5.00%	10.00%	35.00%

While the total losses have increased by 10% from $135,000 to $148,500, the rate of increase is not constant across the layers. This is due to the fact that the larger claims have already saturated the lower layers, thus reducing the impact of severity increases on these layers. Figure 7 provides a graphical representation of this effect by claim size.

For each layer let us define the following:

L = lower bound of layer

U = upper bound of layer

X = unlimited loss size

T = severity increase rate

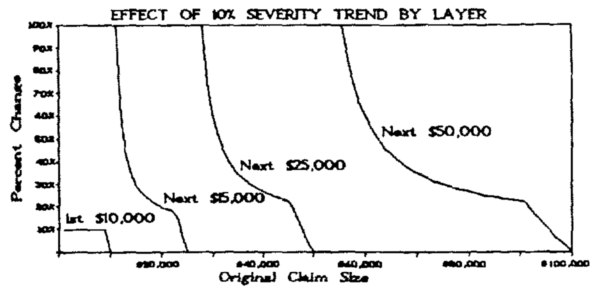


Figure 7

The impact of the severity increase on any given layer can be expressed as follows:

Original <u>Loss Size</u>	**<u>Rate of Increase in Layer</u>**
$X < \frac{L}{(1+T)}$	Undefined
$\frac{L}{(1+T)} < X < \frac{U}{(1+T)}$	$\frac{(1+T)(X) - L}{X-L} - 1 = \frac{TX}{X-L}$
$\frac{U}{(1+T)} < X < U$	$\frac{U-L}{X-L} - 1 = \frac{U-X}{X-L}$
$U < X$	0

The four-loss distribution used in the illustration of the impact of policy limit on severity trend is not realistic for most liability lines. In general we see frequency decreasing as loss size increases. If we assume a loss distribution as shown in Figure 8 then the impact of a 10% severity increase on each limit will be as shown in Figure 9.

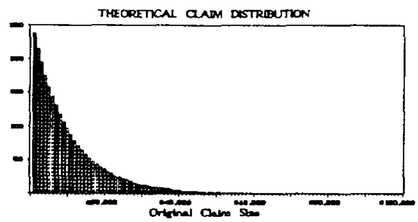


Figure 8

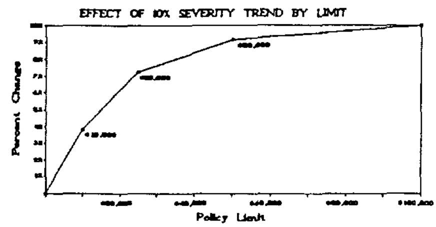


Figure 9

Where severity trend has been analyzed based upon unlimited loss data or loss data including limits higher than the basic level, the resulting indicated severity trend must be adjusted before it is applied to basic limits losses. Because such adjustment will require knowledge of the underlying size-of-loss distribution it is generally preferable to use basic limits data in the severity trend analysis.

Trend Based Upon External Data

Where sufficient loss or claim experience to produce reliable trend indications is not available the actuary should supplement or supplant the available experience with external data. Insurance trade associations, statistical bureaus and the U.S. Government produce insurance and general economic data regularly. While the appropriate source for the data will, of course, depend upon the specific ratemaking situation, Masterson[2] provides a good general reference on the subject. Lommele and Sturgis[3] provide an interesting example of the application of economic data to the problem of forecasting workers' compensation insurance results.

Trend and Loss Development - The "Overlap Fallacy"

It has occasionally been suggested that there is a double-counting of severity trend in the ratemaking process where both loss development factors - which reflect severity changes as development on unpaid claims - and severity trend factors are applied to losses. Cook dealt with this subject in detail, and with elegance, in a 1970 paper[4]. In order to properly understand the relationship between loss development and trend factors assume a situation in which the experience period is the 1986 accident year and indicated rates are expected to be in effect from 7/1/87 through 6/30/88. Now consider a single claim with accident date 7/1/86 and which will settle on 12/31/88. If a similar claim should occur during the effective period of the indicated rates, say on 1/1/88, we would expect an equivalent settlement lag and would project that the 1/1/88 claim would settle on 6/30/90. Figure 10 illustrates the hypothetical situation graphically.

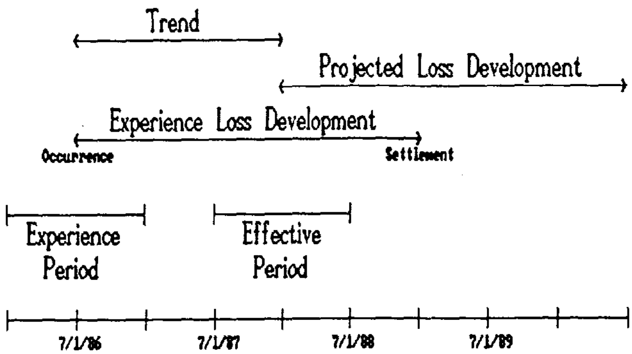
---

[2]Masterson, N.E. "Economic Factors in Liability and Property Insurance Claim Costs, 1935-1967." *Proceedings* of the Casualty Actuarial Society LV, 1968. p. 61.

[3]Lommele, J.A. and Sturgis, R.W. "An Econometric Model of Worker's Compensation." *Proceedings* of the Casualty Actuarial Society LXI, 1974. p. 170.

[4]Cook, C.F. "Trend and Loss Development Factors." *Proceedings* of the Casualty Actuarial Society LVII, 1970. p. 1.

Figure 10



Note that the ratemaking problem, as respects this single hypothetical claim, is to project the ultimate settlement value as of 6/30/90 based upon the single observed claim which occurred on 7/1/86 - a total projection period of 48 months. The loss development factor will, assumedly, reflect the underlying severity trend during the 30 months between occurrence on 1/1/88 and settlement on 6/30/90. The trend factor will reflect the severity trend between the midpoint of the experience period (7/1/86) and the midpoint of the effective period (1/1/88) which accounts for the remaining 18 months of the projection period. Note that while both trend and loss development factors do reflect underlying severity trends there is no overlap between the two, and both are required.

Trended Projected Ultimate Losses

The application of loss development and trending techniques to the underlying loss data produces the trended projected ultimate losses which are the *experience losses* underlying the application of either the pure premium or the loss ratio methods to produce the indicated rates or rate changes.

Expense Loadings

The topic of expenses in ratemaking will be covered in detail in chapter 7, the need for continuity requires at least a limited treatment at this point. For purposes of illustration of the general concepts involved in the reflection of expense loadings in manual rates, assume that the loss ratio method is being used to develop base rate indications for a line of business, and assume further that allocated loss adjustment expenses are being combined with the experience losses. Suppose that for the latest year the line of business produced the following results on a direct basis:

Written Premium	$11,540,000
Earned Premium	$10,832,000
Incurred Loss and Allocated Loss Adjustment Expense	$7,538,000
Incurred Unallocated Loss Adjustment Expenses	$484,000
Commissions	$1,731,000
Taxes, Licenses & Fees	$260,000
Other Acquisition Expenses	$646,000
General Expenses	$737,000
Total Loss and Expense	$11,396,000

Since our losses and expenses exceeded the earned premium $564,000 for the year it is probably reasonable that we review the adequacy of the underlying rates. Since we are using the loss ratio method we need to develop a target loss ratio. Referring back to formula (7):

$$T = \frac{1-V-Q}{1+G} \tag{7}$$

Where:	**T**	=	target loss ratio
	V	=	premium-related expense factor
	Q	=	profit and contingencies factor
	G	=	ratio of non-premium-related expenses to losses

In order to develop the target loss ratio we therefore need factors for premium-related and for non-premium-related expenses and a profit and contingencies factor. Deferring the discussion of profit and contingencies loadings to the next section we will look at the expense factors.

Traditional application of the loss ratio method assumes that only the loss adjustment expenses are non-premium-related. Using this approach we can determine the value for G in formula (7) by dividing the unallocated loss adjustment expenses of $484,000 by the loss and allocated loss expense of $7,538,000. G is therefore (484/7538) = .0642.

The determination of V in formula (7) is then simply the ratio of the other expenses to premiums. But which premiums - written or earned? Since commissions and premium taxes are generally paid based upon direct written premium it would seem appropriate to use written premium in the denominator for these expenses. Other acquisition expenses are expended to produce premium so it might be appropriate to relate those to written premium as well. But the general expenses of the insurance operation involve functions unrelated to the production of premium and which could not be immediately eliminated if the company were to cease writing business. For this reason the general expenses are usually related to earned premium.

Based upon the above, we now calculate V as follows:

Ratio of commissions to written	(1,731/11,540)	.1500
Ratio of taxes, licenses & fees to written	(260/11,540)	.0225
Ratio of other acquisition to written	(646/11,540)	.0560
Ratio of general to earned	(737/10,832)	.0680
Total premium-related expense factor		.2965

If, for the moment, we assume that the profit and contingencies factor is zero, we can apply formula (7) and determine our target loss ratio:

$$T = \frac{1 - .2965 - 0}{1 + .0642} = .6611$$

Profit and Contingencies

While generally among the smallest of the elements in any calculation of indicated manual rates, the profit and contingencies loading represents the *essence* of insurance in that it is designed to reflect the basic elements of risk and rewards associated with the transaction of the insurance business. The risk elements are the *contingencies* portion of the provision while the *profit* portion represents the reward elements.

Sources of Insurance Profit

Highly simplified, the property and casualty insurance operation involves the collection of premium from insureds, the investment of the funds collected, and the payment of expenses and insured losses. If the premiums collected exceed the expenses and losses paid, the insurer makes what is called an **underwriting profit**, if not then there is an **underwriting loss**. In addition, the insurer will generally make an **investment profit** arising out of the investment of funds between premium collection and payment of expenses and losses. In this simplified context, the insurer might be viewed as a leveraged investment operation, with underwriting profits or losses being analogous to (negative or positive) interest expenses on borrowed funds.

Profit Loadings in Manual Rates

Until the mid 1960s insurance rates would typically include a profit and contingencies loading of approximately 5% of premium. While this practice was rooted more in tradition than in financial analysis, it must be understood that the practice existed in an environment in which property insurance represented a much greater portion of the insurance business than it does today, and in which inflation and interest rates were generally low. In that environment investment income tended to be viewed as a *lagniappe* rather than the major source of income it has become. The 5% loading produced sufficient underwriting profits to support the growth of the industry, and it was not generally viewed as being excessive.

The growth of the liability lines, increased inflation and higher interest rates resulted in investment profits which dwarfed the underwriting profits. Not only did this change the way insurance management viewed their financial results and plans, but it also focused regulatory attention on the overall rate of return for insurers rather than the underwriting results. This regulatory involvement generally took the form of adjustments of the traditional 5% profit and contingencies loading downward to reflect investment income on funds supplied by policyholders. In some jurisdictions the allowed profit loadings for certain lines became *negative*.

One of the major problems inherent in the development of a general methodology for the reflection of profit in manual rates is that premium may not be the proper benchmark against which profits should be assessed. Going back to our leveraged investment operation analogy, the specific inclusion of a profit loading based upon premium is the analog to the measurement of profit against borrowed funds - the more you borrow, the more you should earn. If, on the other hand, premiums are viewed in the traditional way, as sales, premium-based profit loadings make more sense.

Unfortunately, the obvious alternative to basing profits on premiums - using return-on-equity as the benchmark - has its own disadvantages. From a regulatory standpoint it both rewards highly leveraged operations and discourages entry to the market, both of which run contrary to regulatory desires. In addition, where rates are made by industry or state rating bureaus, the rates cannot be expected to produce equal return on equity for each company using the rates.

The foregoing discussion provides some of the historical and technical factors entering into the problems associated with profit loadings in manual rates. In practice however, one of two situations generally exists. Either the manual rates will be filed for regulatory approval, in which case the allowable profit provision or methodology will be dictated by the regulators, or the rate levels will reflect the perceived market conditions and will be based upon competitive considerations. In either case the operant decision becomes whether to write business at the resulting rate levels, not what the proper profit loading might be.

Contingencies

The contingencies portion of the profit and contingencies loading represents a provision for adverse deviation or a **risk loading**. Like profit loadings, contingencies provisions are of more theoretical than practical interest. The reader should be aware, however, of the two separate and distinct risk elements inherent in the ratemaking function. These risks are generally termed **parameter risk** and **process risk**. *Parameter risk* is simply the risk associated with the selection of the parameters underlying the applicable model of the process. Selecting the wrong loss development factors, resulting in erroneous experience losses, is an example of parameter risk. *Process risk*, in contrast, is the risk associated with the projection of future contingencies which are inherently variable. Even if we properly evaluate the mean frequency and the mean severity, the actual observed results will generally vary from the underlying means.

From a financial standpoint it is important to understand that the primary protection against adverse deviation is provided by the surplus (equity) of the insurer. If manual rates alone were required to produce sufficient funds to adequately protect the policyholders and claimants from sustaining any economic loss arising out of the policy period in which they were in effect, most property and casualty coverages would be unaffordable. It is more proper to view the contingencies provision as providing sufficient funds to offset the economic costs associated with the net borrowings from the insurer's surplus required to offset the adverse deviations.

One method for determination of an appropriate contingencies provision is the **ruin theory** approach. This method involves the development of a probabilistic model of the insurance operation and then, generally through Monte Carlo simulation, determining the probability of ruin (insolvency) over a fixed period of time. A maximum acceptable probability of ruin is then determined and the rate level assumption underlying the model is adjusted to the minimum rate level producing a ruin probability less than or equal to the acceptable level. The difference between the resultant adjusted rate level assumption and the rate level assumption with no risk margin is then used as the contingencies loading.

Overall Rate Indications

The determination of the overall average indicated rate change will be made on the basis of the experience losses, expense provisions, profit and contingencies loadings and, in the case of the loss ratio method, on-level earned premium. Each of these components has been discussed at such length that their confluence seems almost anticlimactic. However, as shall be seen, the development of the overall rate change indication is generally only the *beginning* of the manual ratemaking process, not the end.

For illustrative purposes, assume that the loss ratio method is being applied to the following data:

(1) Experience loss and allocated - accident years 1985-87	$22,562,119
(2) On-level earned premium - calendar years 1985-87	$31,811,448
(3) Experience loss and allocated ratio [(1)/(2)]	.7092
(4) Target loss and allocated ratio	.6611

The rate change indication follows directly:

(5) Indicated overall rate level change [(3)/(4)] - 1.0	.0728

Credibility Considerations

The concept of *credibility*, the weight to be assigned to an indication relative to one or more alternative indications, is the topic of chapter 6. For the purposes of this chapter it is only necessary to understand that a statistical indication I_1 has an associated credibility z, between 0 and 1, relative to some other indication I_2. The resulting **credibility-weighted indication** $I_{1,2}$ is determined by the formula:

$$I_{1,2} = z(I_1) + (1-z)(I_2)$$

If, for example, the credibility associated with our overall rate level indication of +7.28% is .85, and we have an alternative indication, from some source, of +4.50%, the credibility-weighted indication would be 6.86% as follows:

$$(.85)(.0728) + (.15)(.0450) = .0686$$

In the application of credibility-weighting, the actuary must be careful to use only reasonable alternative indications. For example, the assumption that the complement of the credibility ($1-z$) should be applied to an indication of 0 - that is no change in rates - would be clearly inappropriate where there was a consistent upward trend in pure premium. In this case it would be preferable to use the indicated pure premium trend between the effective date of the current rates and the proposed effective date of the new rates as the alternative indication.

Classification and Territorial Rates

If rate manuals contained a single rate for a given state, the overall rate change indication would be all that was required. But a rate manual will generally contain rates based upon individual classification and sub-classification. In addition, where geographical location of the risk is an important factor, rates may also be shown by rating territory. While classification ratemaking will be discussed in chapter 4, the basics of the process will be illustrated in this section.

Base Rates

In order to facilitate the process of individual rate determination, especially where rates are computer-generated, classification and territorial rates are generally related to some **base rate**. The advantages to this system are apparent when one considers that there may be as many as 200 classifications for as many as 50 territories in a private passenger automobile rate manual for a given state. Determination of 250 classification and territorial relativities is obviously less time-consuming, and more reasonable from a statistical standpoint, than is the determination of 10,000 classification and territorial rates.

Indicated Classification Relativities

The relationship between the rate for a given classification (or territory) to the base rate is the **classification (or territorial) relativity**. The determination of indicated classification relativities is similar to the process used in the overall rate level analysis. If the pure premium method is used, the pure premium for the classification is divided by the pure premium for the base classification to generate the indicated relativity.

If the loss ratio method is used, the on-level earned premium for each classification must be adjusted to the base classification level before the experience loss ratios are calculated. Consider the following three-class situation:

(1) Class	(2) Current Relativity to Class 1	(3) On-Level Earned Premium	(4) Class 1 On-Level Earned (3)/(2)	(5) Experience Loss and Allocated	(6) Loss and Allocated Ratio (5)/(4)	(7) Indicated Relativity to Class 1
1	1.0000	$14,370,968	$14,370,968	$10,718,070	0.7458	1.0000
2	1.4500	$9,438,017	$6,508,977	$6,371,919	0.9789	1.3126
3	1.8000	$8,002,463	$4,445,813	$5,472,130	1.2309	1.6503
Total		$31,811,448	$25,325,758	$22,562,119		

In practice, the resulting indicated relativities are generally credibility-weighted with the existing relativities. This prevents the relativities for smaller classifications against short-term fluctuations in experience.

Correction for Off-Balance

Assume that the existing base rate is $160. If we have determined that we need a 7.28% increase overall, the indicated base rate is (1.0728)($160) = $171.65. The indicated rate changes by classification are therefore:

Class 1: [($171.65)(1.0000)/($160)(1.0000)] - 1 = +.0728

Class 2: [($171.65)(1.3126)/($160)(1.4500)] - 1 = -.0288

Class 3: [($171.65)(1.6503)/($160)(1.8000)] - 1 = -.0164

Applying these indicated classification rate changes to the on-level earned premium we get the following:

Class 1:	\$14,370,968 x 1.0728 =	\$15,417,174
Class 2:	\$9,438,017 x 0.9712 =	\$9,166,202
Class 3:	\$8,002,463 x 0.9836 =	\$7,871,223

The on-level earned premium at these base rates and classification relativities would be (\$15,417,174 + \$9,166,202 + \$7,871,223) = \$32,454,599. This represents only a 2.02% increase over the \$31,811,448 on-level earned premium at the current rate levels. The difference between this and the 7.28% overall indication is the **off-balance**. The off-balance exists because the indicated classification relativities produce an average classification relativity different from the average classification relativity underlying the current rates. In this case, the Class 1 relativity is unchanged while the relativities for the other two classes are decreased.

We correct for this off balance by increasing the indicated base rate by an off-balance factor of (1.0728/1.0202) = 1.0516. The corrected indicated base rate is then (1.0516)(\$171.65) = \$180.51. This will produce the following corrected indicated rate changes by classification:

Class 1:	[(\$180.51)(1.0000)/(\$160)(1.0000)] - 1 =	+.1282
Class 2:	[(\$180.51)(1.3126)/(\$160)(1.4500)] - 1 =	+.0213
Class 3:	[(\$180.51)(1.6503)/(\$160)(1.8000)] - 1 =	+.0344

Applying these corrected indicated classification rate changes to the on-level earned premium we get the following:

Class 1:	\$14,370,968 x 1.1282 =	\$16,213,326
Class 2:	\$9,438,017 x 1.0213 =	\$9,639,047
Class 3:	\$8,002,463 x 1.0344 =	\$8,277,748

The resulting on-level premium aggregates to \$34,130,121 or 7.29% more than the current on-level earned. The corrected base rate of \$180.51, in conjunction with the revised classification relativities, now provides the overall level of rate increase indicated.

The Appendix to this chapter contains a more complex example involving both classification and territorial relativities.

Limitation of Rate Changes

Occasionally, due to regulatory requirements or marketing considerations, it is necessary that individual rate changes be limited to a maximum increase or decrease. In the above example, assume that it has been determined that no classification rate may increase or decrease by more than 10%. Since the Class 1 rate change indicated is 12.82% it needs to be limited to 10.00% or a revised rate of (\$160)(1.1000) = \$176.00.

Reducing the Class 1 rate to \$176.00 has two effects. First, it reduces the indicated on-level earned premium for Class 1 from \$16,213,326 to

$15,808,065, a reduction of $405,261. If we are to make up for this loss by increasing the rates for the remaining classes, we need an increase of [$405,261/($9,639,047 + $8,277,748)] or .0226 in Class 2 and Class 3 rates. The second effect of the limitation arises because Class 1 is the base rate. Since the base rate is being reduced, the class relativities must be increased by a factor of (1.1282/1.1000) = 1.0256 to compensate for the change. The factor necessary to correct for the off-balance due to the limitation is therefore (1.0226)(1.0256) = 1.0488. The resulting class relativities are:

Class 2: (1.3126)(1.0488) = 1.3767

Class 3: (1.6503)(1.0488) = 1.7308

The calculations of the resulting increases by classification and overall increase in on-level premium are left as exercises for the reader.

Claims Made

Certain insurance coverages, most notably professional liability, are offered on what is called a **claims-made** basis. Instead of being insured against losses occurring during the policy period, as is the case for most property and casualty lines and is referred to as the **occurrence basis**, the claims-made policy insures against all losses for which a claim is first asserted during the policy period. When making rates for claims-made coverages, several factors need to be considered.

A Simplified Example

In order to examine the basic aspects of the claims-made coverage let us assume that ultimate losses for actuarial professional liability insurance arise according to the following schedule:

Claims made in year of occurrence	30%
Claims made in first year following occurrence	25%
Claims made in second year following occurrence	20%
Claims made in third year following occurrence	15%
Claims made in fourth year following occurrence	10%

Consider now an actuary who starts a consulting practice on 1/1/88 and takes out a claims-made policy to protect against professional liability losses. Had the coverage been written on an occurrence basis, the first year premium would need to be sufficient to provide for all losses expected to occur during 1988. On a claims-made basis, however, only the 1988 occurrences for which claims are first made during 1988 need to be covered. According to our simple model, this is 30% of the 1988 occurrences. Figure 11 illustrates the growth in exposure to loss over the first five years of claims-made coverage.

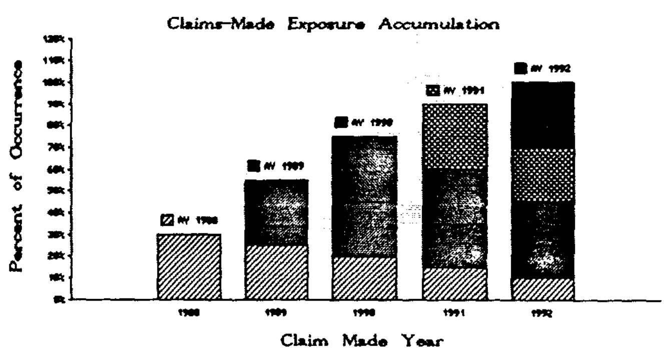


Figure 11

Because our model has a five year reporting period, the fifth and subsequent years will contain the equivalent of 100% of occurrences, although each claims-made year will consist of losses from five accident years.

Step Rates

In order to properly reflect the growth in exposure to loss, claims-made rate manuals contain rates which vary according to the number of years the coverage has been in effect. These are referred to as **step rates.** Referring to our simple model, and conveniently ignoring the effect of fixed expenses, trend, investment income and profit and contingencies loadings, the indicated step rates would be as follows:

First year rate (% of occurrence)	30%
Second year rate (% of occurrence)	55%
Third year rate (% of occurrence)	75%
Fourth year rate (% of occurrence)	90%
Mature rate (% of occurrence)	100%

Reduced Projection Error under Claims-Made

Because claims-made policies cover only those losses reported during the policy period, projections of ultimate losses do not need to consider the **incurred-but-not-reported** (IBNR) losses which arise under occurrence-based coverages. This reduces the potential for projection errors in the ratemaking process.

Reduced Investment Income under Claims-Made

Because premiums for claims-made coverages contain no provision for losses which will be reported subsequent to the policy period, the loss reserves held on account of claims-made policies are less than those under equivalent occurrence policies. As a result, claims-made coverage produces substantially less investment income than does occurrence coverage. This fact will often require recognition in the profit provision underlying the manual rates.

Extended Reporting Endorsement

Returning to our example, suppose that at 12/31/92 our actuary, having made a fortune as a high-priced consultant, decides to retire. While there will be no additional exposure to professional liability claims during retirement, there is the potential for new claims to be reported on 1989 through 1992 occurrences. In order to cover these losses, the actuary must purchase an **extended reporting endorsement** which will cover any claims arising out of occurrences during the claims-made coverage period which are reported subsequent to 12/31/92.

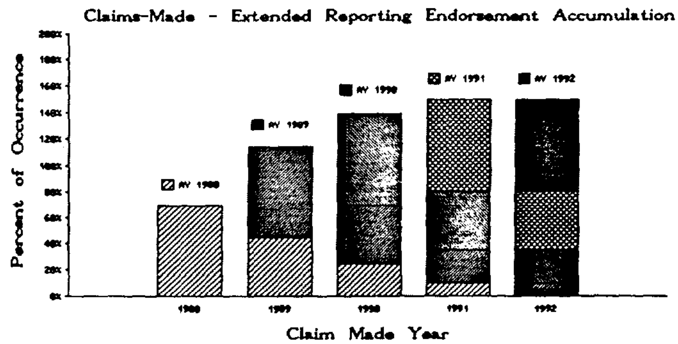


Figure 12

Figure 12 illustrates the growth of the accumulated exposure subject to coverage by the extended reporting endorsement under our simple model.

It is reasonable to assume that every claims-made insured will, at some point, purchase an extended reporting endorsement. Death, disability, retirement or conversion to occurrence-based coverage all produce a need for the extension provided by the endorsement. If we make this assumption, and if we ignore the impact of inflation on limits carried - the policy limits tending to increase over time - then the claims covered under the combination of the successive claims-made policies and the extended reporting endorsement will be the identical claims which would have been covered under successive occurrence-based policies over the same period. Stated differently, the economic value of the total of the pure premiums underlying the combination of the claims-made policies and the extended reporting endorsement must equal the economic value at the same point in time of the pure premiums underlying the equivalent occurrence-based policies.

Extended Reporting Guarantees

Some claims-made policies contain guarantees that extended reporting endorsements will be offered at the end of a continuous claims-made coverage period. In some cases a maximum price, generally stated as a percentage of the mature claims-made rate in effect at the time the extended reporting endorsement is issued, is guaranteed. In a few cases the issuance of the extended reporting endorsement as a result of death or disability (and, occasionally, retirement) is guaranteed at *no additional cost*. When pricing claims-made policies containing the guaranteed offer of extended reporting endorsement endorsements at a maximum price, the actuary needs to examine the need for a specific provision in the claims-made rates for the accrual of any shortfall of the guaranteed maximum price for the endorsement.

Increased Limits

The final topic to be addressed in this section is increased limits ratemaking. While the level of attention to the development of rates for increased limits is generally less than that given the development of basic limits rates, the number of increased limits factors which exceed 2.000 should serve to focus attention on this important element of manual ratemaking. In an earlier discussion we saw how the severity trend in excess layers increases as the lower bound of the layer increases. This effect alone is sufficient to produce a general upward movement in increased limits factors. When combined with the effects of our increased litigiousness as a society, the need for regular review of increased limits rate adequacy should be apparent. In this section we will provide brief descriptions of three methods available for the review of increased limits experience.

Trending Individual Losses

This method involves the application of severity trend to a body of individual loss data. Generally closed claim data are used in order to avoid the problems associated with projecting loss development on individual claims. In order to apply the method, an annual severity trend factor is first determined. This trend factor is then applied to each closed claim for the period from date of closure to the applicable effective period for the indicated increased limits factors. The resulting distribution of trended closed claims is then used to determine the appropriate increased limits factors.

Note that the application of this method requires the use of *unlimited* losses as the projection base. Since insurers are frequently unaware of the unlimited loss amounts associated with closed claims, this method is often based upon special data surveys.

Loss Development by Layer

Another method which can be used to analyze increased limits experience is to look at loss development patterns by layer. This process involves the segregation of case-incurred loss data by policy limit and loss layer and then tracking the observed loss development factors in each layer. Generally the sparsity of data in the upper limits precludes the use of this method.

Fitted Size-of-Loss Distribution

The third method is related to the individual loss trending method. In this method, a theoretical size-of-loss distribution is fitted to existing individual loss data. The resulting distribution can then be used to examine the effects of severity trend on various limits and as a basis for the increased limits factors.

Summary

While this section has covered most of what could be considered the basics of manual ratemaking, every line of insurance will have characteristics requiring specialized treatment. For each method illustrated in this chapter there are situations in which its application would be clearly inappropriate. There is no substitute for informed judgment arising out of a thorough understanding of the characteristics of the insurance coverage being priced. The actuary who becomes a slave to ratemaking methodology rather than a student of the business will, at some point, be led astray.

Appendix

The following appendix contains a complete, though simplified, example of a manual rate analysis of private passenger automobile bodily injury. The data is totally fictitious but is meant to be reasonably representative of actual data which might be observed in practice. The appendix consists of 16 sheets which are meant to provide an example of the exhibits which might accompany a rate filing with a regulatory body. This section will provide a brief description of each of these sheets.

Sheet 1 is meant to represent the existing rate manual, effective 7/1/86, for the coverage under review. The manual contains basic limits rates for each of three classifications within each of three territories, along with a single increased limits factor to adjust the rates for basic limits of $20,000 per person, $40,000 per occurrence (20/40) to limits of $100,000 per person, $300,000 per occurrence (100/300). Territorial and classification rates are keyed to a base rate of $160 for Territory 2, Class 1.

Sheet 2 demonstrates the computation of the on-level earned premium based upon the extension of exposures technique. The experience period is the three years 1985-1987 and the earned exposures, by class and territory, for each of those years are multiplied by the appropriate current rate to yield the on-level earned.

Sheet 3 shows the projection of ultimate loss and allocated loss adjustment expense for accident years 1982-1987 using the case-incurred loss development method.

Sheet 4 contains the projected ultimate claim counts for accident years 1982-1987 based upon the reported count development method.

Sheet 5 details the calculation of the severity trend factor based upon the projected incurred losses and ultimate claims for accident years 1982-1987. The trend factor is based upon a linear least-squares fit.

Sheet 6 addresses the frequency trend factor based upon the earned exposures and projected ultimate claims for accident years 1982-1987 based upon an exponential least-squares fit.

Sheet 7 contains the calculation of the target loss and allocated loss expense ratio. Note that there is no specific provision for profit and contingencies in this example, the assumption being that the investment profits will be sufficient.

Sheet 8 presents the calculation of the indicated statewide rate level change using the loss ratio method.

Sheet 9 contains projections of trended projected ultimate losses and allocated loss expenses by accident year, classification and territory for accident years 1985-1987.

Sheet 10 demonstrates the calculation of indicated classification and territorial pure premiums and pure premium relativities.

Sheet 11 shows the calculation of credibility-weighted classification relativities and the selection of relativities to be used.

Sheet 12 shows the calculation of credibility-weighted territorial relativities and the selection of relativities to be used.

Sheet 13 details the correction for off-balance resulting from the selected classification and territorial relativities.

Sheet 14 shows the development of the revised basic limits rates and the calculation of the resulting statewide rate level change.

Sheet 15 describes the calculation of the revised 100/300 increased limits factor using the individual trended loss approach.

Sheet 16 is the proposed rate manual to be effective 7/1/88.

Appendix
Sheet 1

EXAMPLE AUTO INSURANCE COMPANY

Rate Manual - 7/1/86

Private Passenger Auto Bodily Injury
20/40 Basic Limits

Territory	Class 1 Adult Drivers, No Youthful Operators	Class 2 Family with Youthful Drivers Not Principal Op.	Class 3 Youthful Owners or Principal Operators
1 - Central City	$224	$325	$403
2 - Midway Valley	$160	$232	$288
3 - Remainder of State	$136	$197	$245

Increased Limits

Limit	Factor
100/300	1.300

Appendix
Sheet 2

EXHIBIT I

EXAMPLE AUTO INSURANCE COMPANY

Private Passenger Auto Bodily Injury
Basic Limits

Development of Indicated Statewide Rate Level Change

A. Earned Premium at Current Rate Level

		Class 1	Class 2	Class 3	Total
Earned Exposures:					
1985	Territory 1	7,807	3,877	1,553	13,237
	Territory 2	11,659	4,976	3,930	20,565
	Territory 3	5,760	2,639	3,030	11,429
	Total	25,226	11,492	8,513	45,231
1986	Territory 1	8,539	4,181	1,697	14,417
	Territory 2	12,957	5,442	4,262	22,661
	Territory 3	5,834	2,614	3,057	11,505
	Total	27,330	12,237	9,016	48,583
1987	Territory 1	9,366	4,551	1,870	15,787
	Territory 2	14,284	5,939	4,669	24,892
	Territory 3	5,961	2,591	3,036	11,588
	Total	29,611	13,081	9,575	52,267
Current Rate Level:					
	Territory 1	$224	$325	$403	
	Territory 2	$160	$232	$288	
	Territory 3	$136	$197	$245	
On-Level Earned Premium:					
1985	Territory 1	$1,748,768	$1,260,025	$625,859	$3,634,652
	Territory 2	$1,865,440	$1,154,432	$1,131,840	$4,151,712
	Territory 3	$783,360	$519,883	$742,350	$2,045,593
	Total	$4,397,568	$2,934,340	$2,500,049	$9,831,957
1986	Territory 1	$1,912,736	$1,358,825	$683,891	$3,955,452
	Territory 2	$2,073,120	$1,262,544	$1,227,456	$4,563,120
	Territory 3	$793,424	$514,958	$748,965	$2,057,347
	Total	$4,779,280	$3,136,327	$2,660,312	$10,575,919
1987	Territory 1	$2,097,984	$1,479,075	$753,610	$4,330,669
	Territory 2	$2,285,440	$1,377,848	$1,344,672	$5,007,960
	Territory 3	$810,696	$510,427	$743,820	$2,064,943
	Total	$5,194,120	$3,367,350	$2,842,102	$11,403,572

Appendix
Sheet 3

EXHIBIT II

EXAMPLE AUTO INSURANCE COMPANY

Private Passenger Auto Bodily Injury
Basic Limits

Development of Indicated Statewide Rate Level Change

B. Projected Ultimate Accident Year Loss and Allocated Loss Expense

Acc. Year	Cumulative Basic Limits Case-Incurred Loss and Allocated Loss Expense					
	Age 12	Age 24	Age 36	Age 48	Age 60	Age 72
1982	$2,116,135	$3,128,695	$3,543,445	$3,707,375	$3,854,220	$3,928,805
1983	$2,315,920	$3,527,197	$3,992,805	$4,182,133	$4,338,765	
1984	$2,743,657	$4,051,950	$4,593,472	$4,797,194		
1985	$3,130,262	$4,589,430	$5,230,437			
1986	$3,625,418	$5,380,617				
1987	$3,919,522					

Acc. Year	Incremental Loss Development Factors					
	12-24	24-36	36-48	48-60	60-72	72-Ultimate
1982	1.4785	1.1326	1.0463	1.0396	1.0194	
1983	1.5230	1.1320	1.0474	1.0375		
1984	1.4768	1.1336	1.0444			
1985	1.4661	1.1397				
1986	1.4841					
Selected	1.4800	1.1350	1.0450	1.0385	1.0200	1.0000
Ultimate	1.8595	1.2564	1.1070	1.0593	1.0200	1.0000

Accident Year	Loss & Allocated 12/31/87	Ultimate Factor	Projected Ultimate Loss & Allocated
1982	$3,928,805	1.0000	$3,928,805
1983	$4,338,765	1.0200	$4,425,540
1984	$4,797,194	1.0593	$5,081,668
1985	$5,230,437	1.1070	$5,790,094
1986	$5,380,617	1.2564	$6,760,207
1987	$3,919,522	1.8595	$7,288,351

EXHIBIT III

EXAMPLE AUTO INSURANCE COMPANY

Private Passenger Auto Bodily Injury
Basic Limits

Development of Indicated Statewide Rate Level Change

C. Projected Ultimate Accident Year Claim Counts

Acc. Year	Cumulative Reported Claims					
	Age 12	Age 24	Age 36	Age 48	Age 60	Age 72
1982	1,804	2,173	2,374	2,416	2,416	2,416
1983	1,935	2,379	2,424	2,552	2,552	
1984	2,103	2,384	2,514	2,646		
1985	2,169	2,580	2,722			
1986	2,346	2,783				
1987	2,337					

Acc. Year	Incremental Loss Development Factors					
	12-24	24-36	36-48	48-60	60-72	72-Ultimate
1982	1.2045	1.0925	1.0177	1.0000	1.0000	
1983	1.2295	1.0189	1.0528	1.0000		
1984	1.1336	1.0545	1.0525			
1985	1.1895	1.0550				
1986	1.1863					
Selected	1.1900	1.0550	1.0450	1.0000	1.0000	1.0000
Ultimate	1.3120	1.1025	1.0450	1.0000	1.0000	1.0000

Accident Year	Reported Claims 12/31/87	Ultimate Factor	Projected Ultimate Claims
1982	2,416	1.0000	2,416
1983	2,552	1.0000	2,552
1984	2,646	1.0000	2,646
1985	2,722	1.0450	2,844
1986	2,783	1.1025	3,068
1987	2,337	1.3120	3,066

EXHIBIT IV

EXAMPLE AUTO INSURANCE COMPANY

Private Passenger Auto Bodily Injury
Basic Limits

Development of Indicated Statewide Rate Level Change

D. Development of Severity Trend Factor - Basic Limits

Accident Year	Projected Loss & Allocated (Exhib. II)	Projected Ultimate Claims (Exhib. III)	Projected Ultimate Average Severity	Linear Least-Squares Fit [1]
1982	$3,928,805	2,416	$1,626	$1,605.90
1983	$4,425,540	2,552	$1,734	$1,756.67
1984	$5,081,668	2,646	$1,921	$1,907.44
1985	$5,790,094	2,844	$2,036	$2,058.21
1986	$6,760,207	3,068	$2,203	$2,208.98
1987	$7,288,351	3,066	$2,377	$2,359.75

Annual Severity Trend Factor (1987/1986 Least-Squares) 1.0683

[1] y=mx+b where: x = Accident Year - 1981
m = 150.77
b = 1455.13

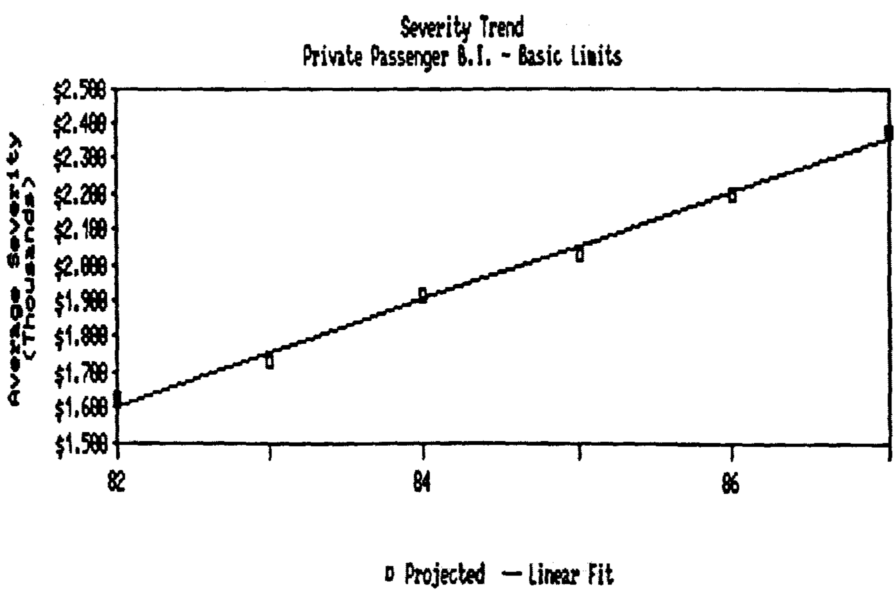


EXHIBIT V

EXAMPLE AUTO INSURANCE COMPANY

Private Passenger Auto Bodily Injury
Basic Limits

Development of Indicated Statewide Rate Level Change

E. Development of Frequency Trend Factor

Accident Year	Projected Ultimate Claims (Exhib. III)	Earned Exposures	Projected Ultimate Frequency	Exponential Least-Squares Fit [2]
1982	2,416	37,846	0.0638	0.0647
1983	2,552	39,771	0.0642	0.0638
1984	2,646	42,135	0.0628	0.0630
1985	2,844	45,231	0.0629	0.0621
1986	3,068	48,583	0.0631	0.0613
1987	3,066	52,267	0.0587	0.0605

Annual Frequency Trend Factor (1987/1986 Least-Squares) 0.9867

[2] y=ae^bx where: x = Accident Year - 1981
a = .065562
b = -.013417

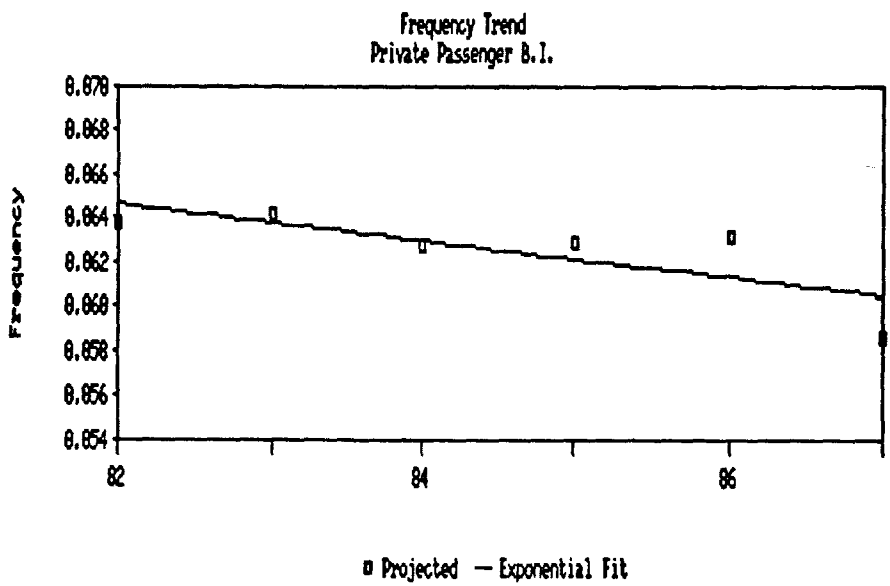


EXHIBIT VI

EXAMPLE AUTO INSURANCE COMPANY

Private Passenger Auto Bodily Injury
Basic Limits

Development of Indicated Statewide Rate Level Change

F. Development of Target Loss & Allocated Loss Expense Ratio

(1)	Commissions as % of Premium	15.00%
(2)	Taxes, Licenses, Fees as % of Premium	2.25%
(3)	Other Acquisition Expense as % of Premium	5.60%
(4)	General Expense as % of Premium	6.80%
(5)	Premium-Based Expense [(1)+(2)+(3)+(4)]	29.65%
(6)	Unallocated Loss Expense as % of Loss & Allocated Loss Expense	6.42%
(7)	Target Loss and Allocated Loss Expense Ratio [1.0 - (5)] / [1.0 + (6)]	66.11%

EXHIBIT VII

EXAMPLE AUTO INSURANCE COMPANY

Private Passenger Auto Bodily Injury
Basic Limits

Development of Indicated Statewide Rate Level Change

G. Development of Statewide Indication

[1] Accident Year	[2] Projected Loss & Allocated (Exhib. II)	[3] Midpoint Experience Period	[4] Years to 12/31/88	Trend Factor to 12/31/88: [5] Severity 1.0683^[4] (Exhib. IV)	Trend Factor to 12/31/88: [6] Frequency .9867^[4] (Exhib. V)
1985	$5,790,094	7/1/85	3.5	1.2602	0.9542
1986	$6,760,207	7/1/86	2.5	1.1796	0.9671
1987	$7,288,351	7/1/87	1.5	1.1042	0.9801

[7] Accident Year	[8] Trended Loss & Allocated [2]x[5]x[6]	[9] On-Level Earned Premium (Exhib. I)	[10] Trended On-Level Loss & Allocated Ratio [8]/[9]	[11] Target Loss & Allocated Ratio (Exhib. VI)	[12] Indicated Statewide Rate Level Change (([11]/[10]) -1.000)
1985	$6,962,489	$9,831,957	70.81%		
1986	$7,711,984	$10,575,919	72.92%		
1987	$7,887,646	$11,403,572	69.17%		
Total	$22,562,119	$31,811,448	70.92%	66.11%	7.28%

EXHIBIT VIII

EXAMPLE AUTO INSURANCE COMPANY

Private Passenger Auto Bodily Injury
Basic Limits

Development of Indicated Rate Level Change by Class and Territory

H. Development of Trended Loss & Allocated by Class and Territory

Terr	Class	Acc. Year	[1] Loss & Allocated 12/31/87	[2] Ultimate Factor (Exhib. II)	[3] Severity Trend to 12/31/88 (Exh. VII)	[4] Frequency Trend to 12/31/88 (Exh. VII)	[5] Trended Projected Loss & Allocated ([1]x[2])x([3]x[4])
1	1	1985	$986,617	1.1070	1.2602	0.9542	$1,313,334
1	1	1986	$982,778	1.2564	1.1796	0.9671	$1,408,606
1	1	1987	$797,650	1.8595	1.1042	0.9801	$1,605,191
1	2	1985	$680,769	1.1070	1.2602	0.9542	$906,205
1	2	1986	$703,406	1.2564	1.1796	0.9671	$1,008,185
1	2	1987	$456,899	1.8595	1.1042	0.9801	$919,464
1	3	1985	$325,397	1.1070	1.2602	0.9542	$433,152
1	3	1986	$343,738	1.2564	1.1796	0.9671	$492,676
1	3	1987	$252,790	1.8595	1.1042	0.9801	$508,715
2	1	1985	$1,062,395	1.1070	1.2602	0.9542	$1,414,206
2	1	1986	$1,170,978	1.2564	1.1796	0.9671	$1,678,351
2	1	1987	$848,551	1.8595	1.1042	0.9801	$1,707,624
2	2	1985	$597,044	1.1070	1.2602	0.9542	$794,754
2	2	1986	$575,004	1.2564	1.1796	0.9671	$824,147
2	2	1987	$449,123	1.8595	1.1042	0.9801	$903,815
2	3	1985	$557,332	1.1070	1.2602	0.9542	$741,892
2	3	1986	$650,645	1.2564	1.1796	0.9671	$932,563
2	3	1987	$469,963	1.8595	1.1042	0.9801	$945,754
3	1	1985	$401,622	1.1070	1.2602	0.9542	$534,619
3	1	1986	$394,358	1.2564	1.1796	0.9671	$565,229
3	1	1987	$243,943	1.8595	1.1042	0.9801	$490,911
3	2	1985	$252,439	1.1070	1.2602	0.9542	$336,034
3	2	1986	$228,313	1.2564	1.1796	0.9671	$327,239
3	2	1987	$174,954	1.8595	1.1042	0.9801	$352,077
3	3	1985	$366,822	1.1070	1.2602	0.9542	$488,295
3	3	1986	$331,397	1.2564	1.1796	0.9671	$474,988
3	3	1987	$225,649	1.8595	1.1042	0.9801	$454,096

EXHIBIT IX

EXAMPLE AUTO INSURANCE COMPANY

Private Passenger Auto Bodily Injury
Basic Limits

Development of Indicated Rate Level Change by Class and Territory

I. Development of Trended Pure Premium by Class and Territory

Terr	Class	Acc. Year	[1] Trended Projected Loss & Allocated (Exh. VIII)	[2] Earned Exposure (Exhib. I)	[3] Trended Pure Premium [1]/[2]	[4] Relativity to Class 1	[5] Relativity to Terr. 2
1	1	1985	$1,313,334	7,807	$168.23	1.0000	1.3869
1	1	1986	$1,408,606	8,539	$164.96	1.0000	1.2735
1	1	1987	$1,605,191	9,366	$171.38	1.0000	1.4336
1	2	1985	$906,205	3,877	$233.74	1.3894	1.4635
1	2	1986	$1,008,185	4,181	$241.13	1.4618	1.5923
1	2	1987	$919,464	4,551	$202.04	1.1788	1.3276
1	3	1985	$433,152	1,553	$278.91	1.6580	1.4775
1	3	1986	$492,676	1,697	$290.32	1.7599	1.3268
1	3	1987	$508,715	1,870	$272.04	1.5873	1.3430
2	1	1985	$1,414,206	11,659	$121.30	1.0000	1.0000
2	1	1986	$1,678,351	12,957	$129.53	1.0000	1.0000
2	1	1987	$1,707,624	14,284	$119.55	1.0000	1.0000
2	2	1985	$794,754	4,976	$159.72	1.3167	1.0000
2	2	1986	$824,147	5,442	$151.44	1.1691	1.0000
2	2	1987	$903,815	5,939	$152.18	1.2730	1.0000
2	3	1985	$741,892	3,930	$188.78	1.5563	1.0000
2	3	1986	$932,563	4,262	$218.81	1.6892	1.0000
2	3	1987	$945,754	4,669	$202.56	1.6944	1.0000
3	1	1985	$534,619	5,760	$92.82	1.0000	0.7652
3	1	1986	$565,229	5,834	$96.89	1.0000	0.7480
3	1	1987	$490,911	5,961	$82.35	1.0000	0.6889
3	2	1985	$336,034	2,639	$127.33	1.3719	0.7972
3	2	1986	$327,239	2,614	$125.19	1.2921	0.8266
3	2	1987	$352,077	2,591	$135.88	1.6500	0.8929
3	3	1985	$488,295	3,030	$161.15	1.7363	0.8537
3	3	1986	$474,988	3,057	$155.38	1.6037	0.7101
3	3	1987	$454,096	3,036	$149.57	1.8162	0.7384

EXHIBIT X

EXAMPLE AUTO INSURANCE COMPANY

Private Passenger Auto Bodily Injury
Basic Limits

Development of Indicated Rate Level Change by Class and Territory

J. Development of Indicated Class Relativity to Class 1

Class	Terr.	Acc. Year	[1] Earned Exposure (Exh. IX)	[2] Relativity to Class 1 (Exh. IX)	[3] Weighted Relativity [1]x[2]
2	1	1985	3,877	1.3894	5,386.70
	1	1986	4,181	1.4618	6,111.79
	1	1987	4,551	1.1788	5,364.72
	2	1985	4,976	1.3167	6,551.90
	2	1986	5,442	1.1691	6,362.24
	2	1987	5,939	1.2730	7,560.35
	3	1985	2,639	1.3719	3,620.44
	3	1986	2,614	1.2921	3,377.55
	3	1987	2,591	1.6500	4,275.15
	Total	Total	36,810	1.3206	48,610.84

Current Class 2 Relativity	1.4500
Credibility = [Exposure/(Exposure+25,000)]	0.5955
Credibility Weighted Indication	1.3729
Selected Relativity	1.3700

Class	Terr.	Acc. Year	[1] Earned Exposure (Exh. IX)	[2] Relativity to Class 1 (Exh. IX)	[3] Weighted Relativity [1]x[2]
3	1	1985	1,553	1.6580	2,574.87
	1	1986	1,697	1.7599	2,986.55
	1	1987	1,870	1.5873	2,968.25
	2	1985	3,930	1.5563	6,116.26
	2	1986	4,262	1.6892	7,199.37
	2	1987	4,669	1.6944	7,911.15
	3	1985	3,030	1.7363	5,260.99
	3	1986	3,057	1.6037	4,902.51
	3	1987	3,036	1.8162	5,513.98
	Total	Total	27,104	1.6763	45,433.94

Current Class 3 Relativity	1.8000
Credibility = [Exposure/(Exposure+25,000)]	0.5202
Credibility Weighted Indication	1.7356
Selected Relativity	1.7400

EXHIBIT XI

EXAMPLE AUTO INSURANCE COMPANY

Private Passenger Auto Bodily Injury
Basic Limits

Development of Indicated Rate Level Change by Class and Territory

K. Development of Indicated Territorial Relativity to Territory 2

Territory	Class	Acc. Year	[1] Earned Exposure (Exh. IX)	[2] Relativity to Terr. 2 (Exh. IX)	[3] Weighted Relativity [1]x[2]
1	1	1985	7,807	1.3869	10,827.53
	1	1986	8,539	1.2735	10,874.42
	1	1987	9,366	1.4336	13,427.10
	2	1985	3,877	1.4635	5,673.99
	2	1986	4,181	1.5923	6,657.41
	2	1987	4,551	1.3276	6,041.91
	3	1985	1,553	1.4775	2,294.56
	3	1986	1,697	1.3268	2,251.58
	3	1987	1,870	1.3430	2,511.41
	Total	Total	43,441	1.3941	60,559.89

Current Territory 1 Relativity	1.4000
Credibility = [Exposure/(Exposure+25,000)]	0.6347
Credibility Weighted Indication	1.3962
Selected Relativity	1.4000

Territory	Class	Acc. Year	[1] Earned Exposure (Exh. IX)	[2] Relativity to Terr. 2 (Exh. IX)	[3] Weighted Relativity [1]x[2]
3	1	1985	5,760	0.7652	4,407.55
	1	1986	5,834	0.7480	4,363.83
	1	1987	5,961	0.6889	4,106.53
	2	1985	2,639	0.7972	2,103.81
	2	1986	2,614	0.8266	2,160.73
	2	1987	2,591	0.8929	2,313.50
	3	1985	3,030	0.8537	2,586.71
	3	1986	3,057	0.7101	2,170.78
	3	1987	3,036	0.7384	2,241.78
	Total	Total	34,522	0.7663	26,455.23

Current Territory 3 Relativity	0.8500
Credibility = [Exposure/(Exposure+25,000)]	0.5800
Credibility Weighted Indication	0.8015
Selected Relativity	0.8000

EXHIBIT XII

EXAMPLE AUTO INSURANCE COMPANY

Private Passenger Auto Bodily Injury
Basic Limits

Development of Indicated Rate Level Change by Class and Territory

L. Adjustment of Base Rate Change for Class and Territory Off-Balance

Terr	Class	Acc. Year	[1] On-Level Earned Premium (Exhib. I)	[2] Current Class Relativity (Exhib. X)	[3] Current Territorial Relativity (Exhib. XI)	[4] Current Relativity to Terr. 2 Class 1 [2]x[3]
1	1	1987	$2,097,984	1.0000	1.4000	1.4000
1	2	1987	$1,479,075	1.4500	1.4000	2.0300
1	3	1987	$753,610	1.8000	1.4000	2.5200
2	1	1987	$2,285,440	1.0000	1.0000	1.0000
2	2	1987	$1,377,848	1.4500	1.0000	1.4500
2	3	1987	$1,344,672	1.8000	1.0000	1.8000
3	1	1987	$810,696	1.0000	0.8500	0.8500
3	2	1987	$510,427	1.4500	0.8500	1.2325
3	3	1987	$743,820	1.8000	0.8500	1.5300
		Total	$11,403,572			

Terr	Class	Acc. Year	[5] Proposed Class Relativity (Exhib. X)	[6] Proposed Territorial Relativity (Exhib. XI)	[7] Proposed Relativity to Terr. 2 Class 1 [5]x[6]	[8] Effect of Relativity Changes ([7]/[4])-1	[9] Premium Effect [1]x[8]
1	1	1987	1.0000	1.4000	1.4000	0.00%	$0
1	2	1987	1.3700	1.4000	1.9180	-5.52%	($81,604)
1	3	1987	1.7400	1.4000	2.4360	-3.33%	($25,120)
2	1	1987	1.0000	1.0000	1.0000	0.00%	$0
2	2	1987	1.3700	1.0000	1.3700	-5.52%	($76,019)
2	3	1987	1.7400	1.0000	1.7400	-3.33%	($44,822)
3	1	1987	1.0000	0.8000	0.8000	-5.88%	($47,688)
3	2	1987	1.3700	0.8000	1.0960	-11.08%	($56,530)
3	3	1987	1.7400	0.8000	1.3920	-9.02%	($67,090)
		Total				-3.50%	($398,873)

Indicated Statewide Rate Change (Exhibit VII)	7.28%
Indicated Base Rate Change (1.0728/.9650)-1	11.17%
Current Class 1 Territory 2 Rate	$160
Indicated Class 1 Territory 2 Rate	$178

EXHIBIT XIII

EXAMPLE AUTO INSURANCE COMPANY

Private Passenger Auto Bodily Injury
Basic Limits

Development of Indicated Rate Level Change by Class and Territory

M. Development of Basic Limits Rates by Class and Territory

[1] Class	[2] Territory	[3] Class Relativity (Exhib. X)	[4] Territorial Relativity (Exhib. XI)	[5] Base Rate (Exh. XII)	[6] Class & Territory Rate [3]x[4]x[5]
1	1	1.0000	1.4000	$178	$249
	2	1.0000	1.0000	$178	$178
	3	1.0000	0.8000	$178	$142
2	1	1.3700	1.4000	$178	$341
	2	1.3700	1.0000	$178	$244
	3	1.3700	0.8000	$178	$195
3	1	1.7400	1.4000	$178	$434
	2	1.7400	1.0000	$178	$310
	3	1.7400	0.8000	$178	$248

[1] Class	[2] Territory	[7] 1987 Earned Exposures (Exhib. I)	[8] New Level Earned Premium [6]x[7]	[9] Current Level 1987 Earned Premium (Exhib. I)	[10] Statewide Rate Level Change ([8]/[9])-1
1	1	9,366	$2,332,134	$2,097,984	
	2	14,284	$2,542,552	$2,285,440	
	3	5,961	$846,462	$810,696	
2	1	4,551	$1,551,891	$1,479,075	
	2	5,939	$1,449,116	$1,377,848	
	3	2,591	$505,245	$510,427	
3	1	1,870	$811,580	$753,610	
	2	4,669	$1,447,390	$1,344,672	
	3	3,036	$752,928	$743,820	
Total	Total	52,267	$12,239,298	$11,403,572	7.33%

EXHIBIT XIV

EXAMPLE AUTO INSURANCE COMPANY

Private Passenger Auto Bodily Injury

Development of Indicated 100/300 Increased Limits Factor

Unlimited Loss Amount	Claim Count	Distribution of Trended Losses [a]		
		Unlimited	20/40	100/300
$1 - $20,000	4,249	$17,706,594	$17,706,594	$17,706,594
$20,001 - $30,000	244	$5,842,632	$5,340,562	$5,842,632
$30,001 - $40,000	150	$5,102,257	$3,884,463	$5,102,257
$40,000 - $50,000	107	$4,819,591	$2,902,869	$4,819,591
$50,001 - $60,000	54	$2,910,399	$1,436,150	$2,910,399
$60,001 - $70,000	25	$1,641,237	$743,278	$1,641,237
$70,001 - $80,000	21	$1,587,230	$611,920	$1,587,230
$80,001 - $90,000	20	$1,660,283	$588,525	$1,660,283
$90,001 - $100,000	13	$1,268,376	$368,077	$1,268,376
$100,001 - $200,000	6	$681,544	$193,968	$660,723
$200,001 - $500,000	16	$4,354,732	$439,906	$2,031,077
	4,905	$47,574,875	$34,216,312	$45,230,399

[1] Indicated 100/300 Factor ($45,230,399/$34,216,312)	1.3219
[2] 100/300 Factor Indicated as of 12/31/85	1.2683
[3] Annual Trend [(1.3219/1.2683)^(1/2)]-1.0000	2.09%
[4] Projected 12/31/88 100/300 Factor {[1]x(1+[3])}	1.3495
[5] Selected 100/300 Factor	1.3500

[a] Based upon unlimited claims closed from 1975 through 1987 trended to 12/31/87 at an annual rate of 8.5%.

Appendix
Sheet 16

EXAMPLE AUTO INSURANCE COMPANY

Proposed Rate Manual - 7/1/88

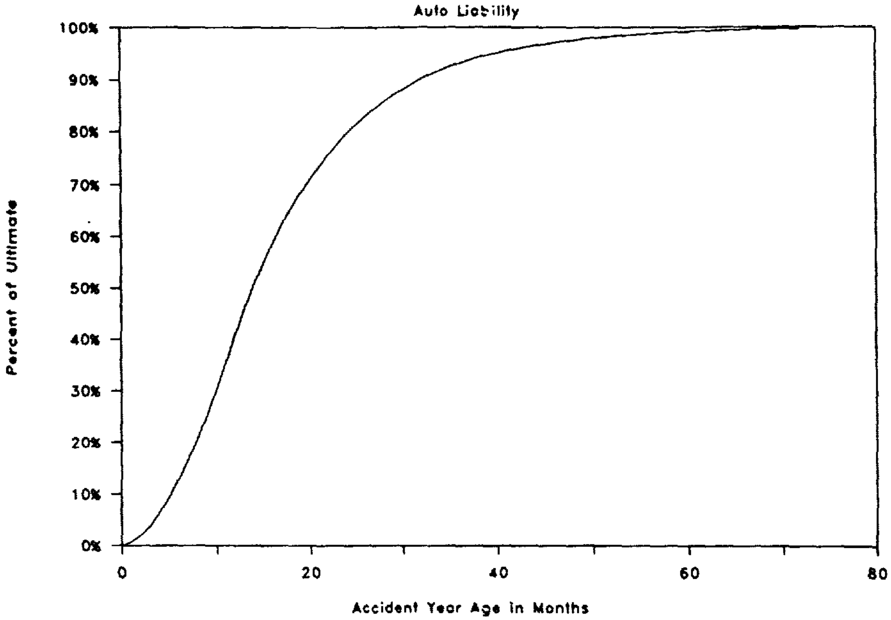
Private Passenger Auto Bodily Injury
20/40 Basic Limits

Territory	Class 1 Adult Drivers, No Youthful Operators	Class 2 Family with Youthful Drivers Not Principal Op.	Class 3 Youthful Owners or Principal Operators
1 - Central City	$249	$341	$434
2 - Midway Valley	$178	$244	$310
3 - Remainder of State	$142	$195	$248

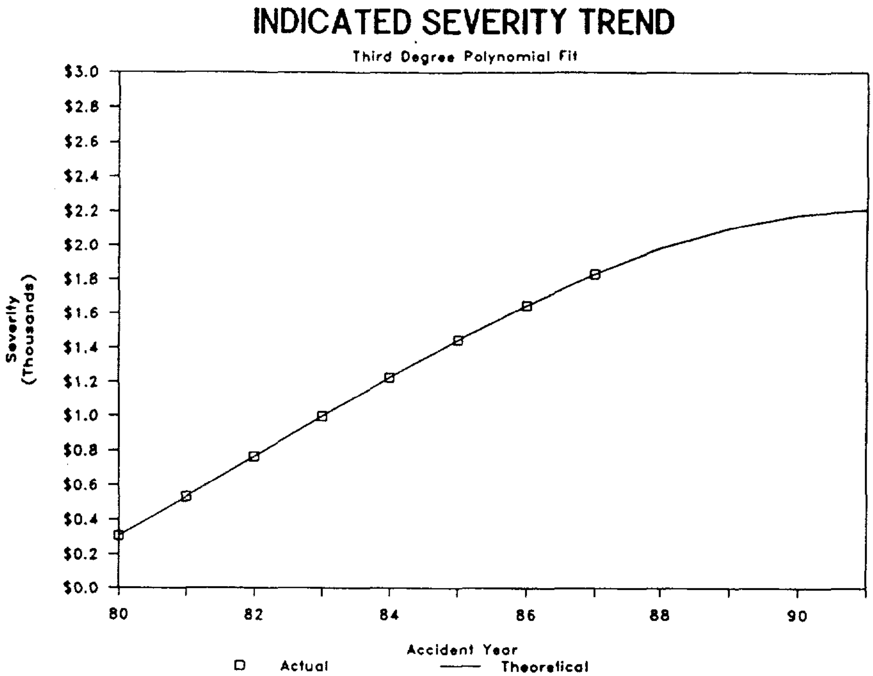
Increased Limits

Limit	Factor
100/300	1.35

Case-Incurred Loss Development



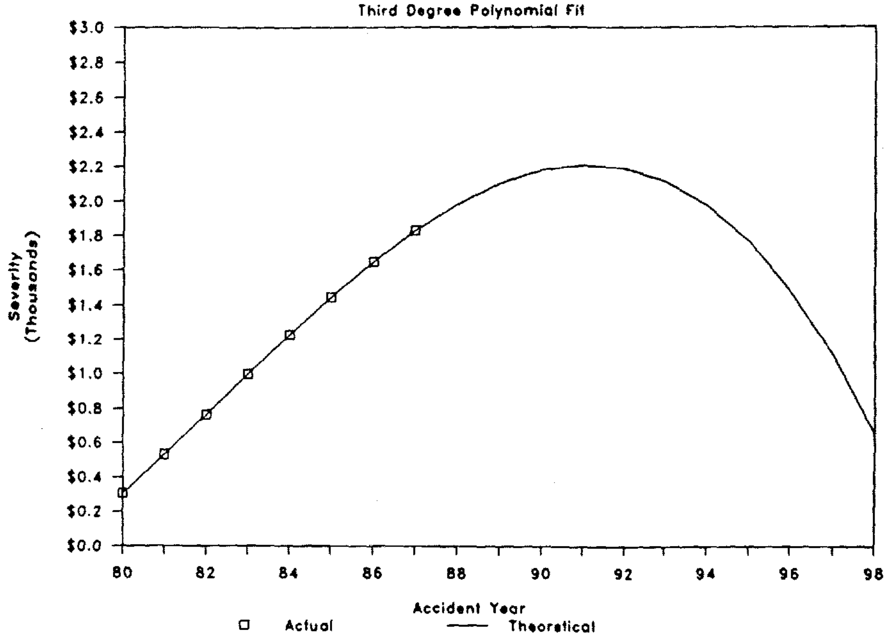
Chapter 1
Figure 1



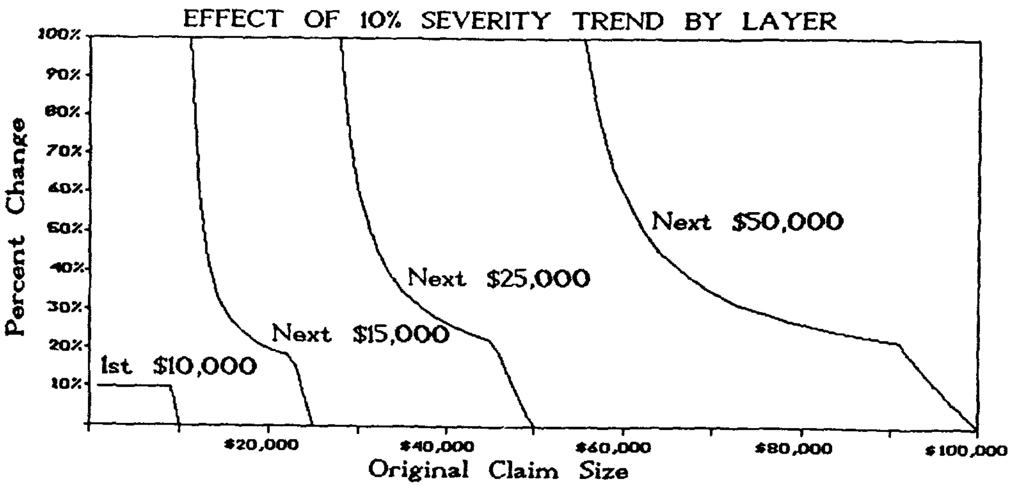
Chapter 1
Figure 5

INDICATED SEVERITY TREND

Third Degree Polynomial Fit

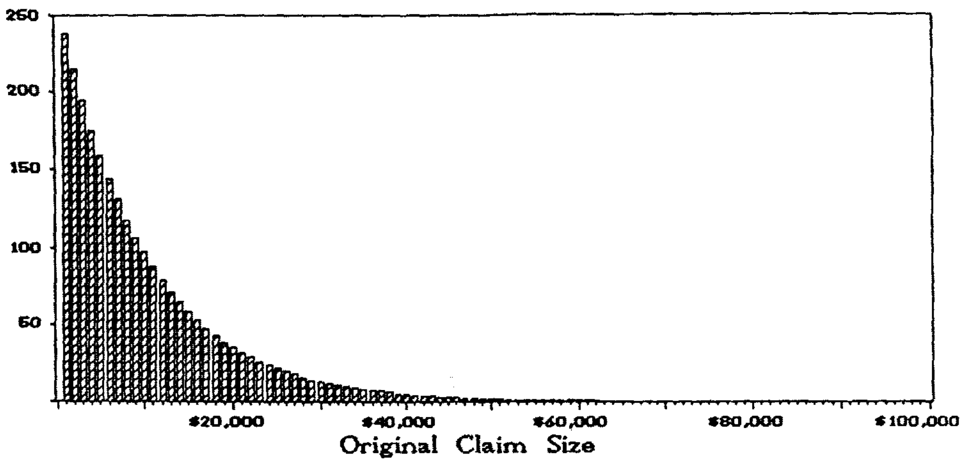


Chapter 1
Figure 6

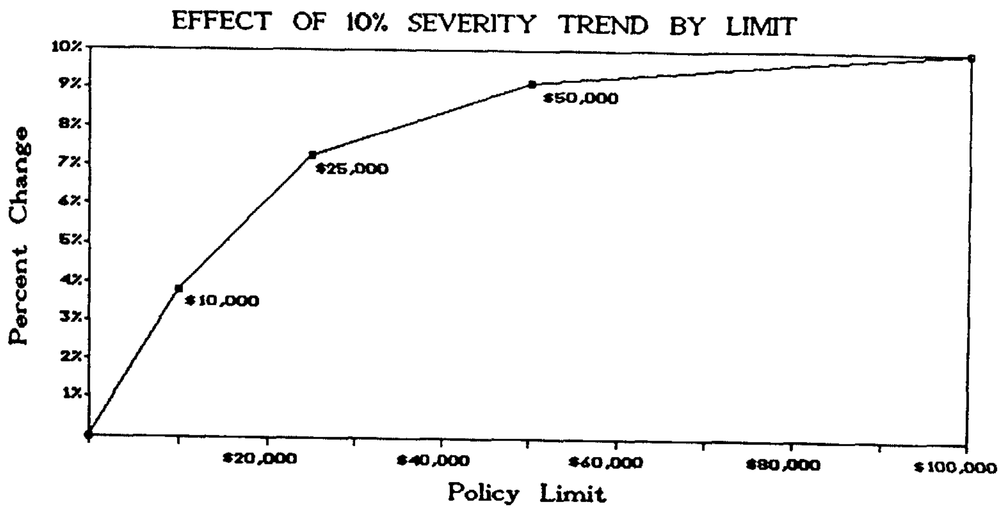


Chapter 1
Figure 7

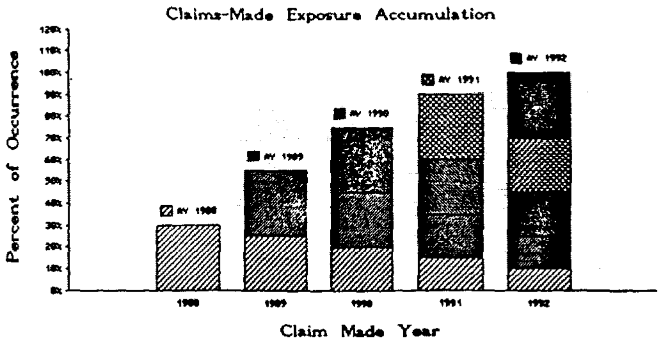
THEORETICAL CLAIM DISTRIBUTION



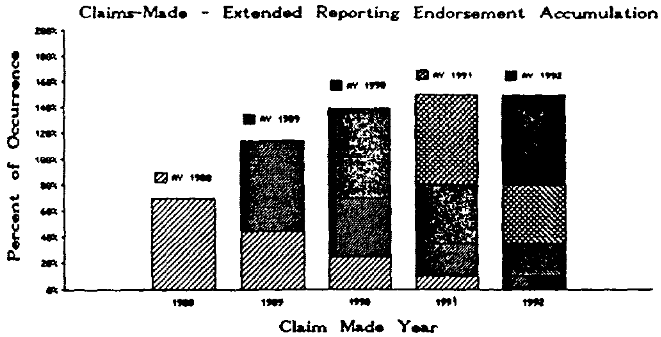
Chapter 1
Figure 8

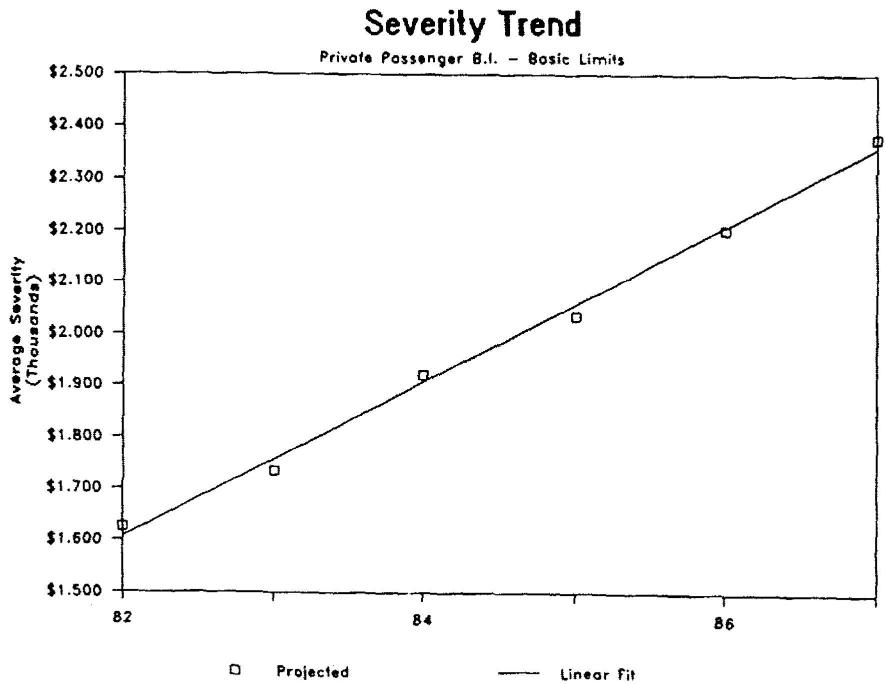


Chapter 1
Figure 9

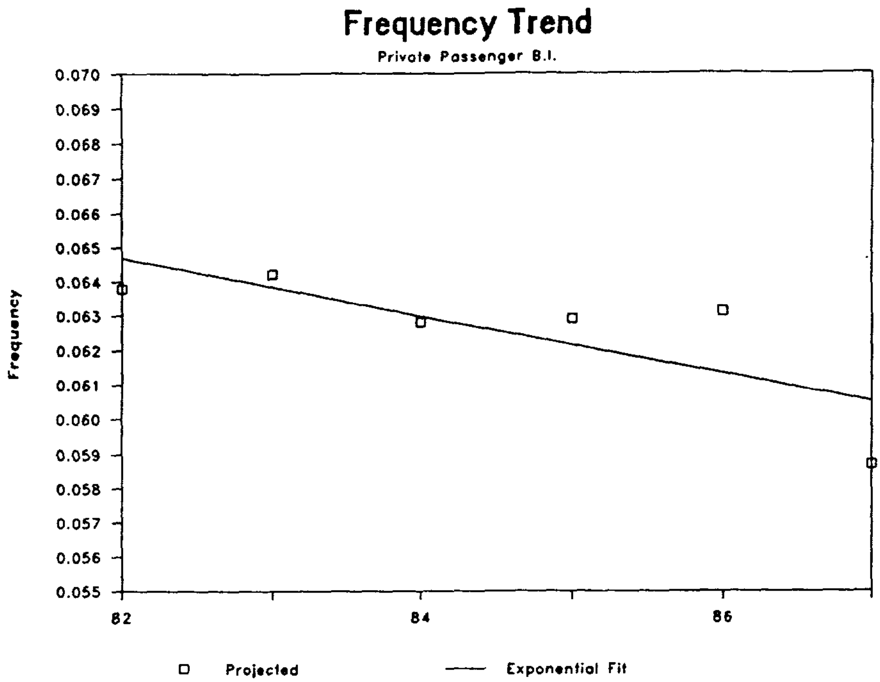


Chapter 1
Figure 11


Claims-Made - Extended Reporting Endorsement Accumulation
Percent of Occurrence
Claim Made Year



Appendix
Sheet 5 Graph



Appendix
Sheet 6 Graph