REINSURANCE
(TEXTBOOK CHAPTER DRAFT)
5.1 INTRODUCTION

This introduction is only a brief review of basic reinsurance concepts and terminology. The interested reader can further pursue this through the general reinsurance texts listed in the bibliography to this section.

A. What is reinsurance?

Reinsurance is a form of insurance. A reinsurance contract is legally an insurance contract. Under a reinsurance contract, the reinsurer agrees to indemnify the cedent insurer for a specified share of the insurance losses paid by the cedent for a single insurance policy or a designated set of policies. The terminology used is that the reinsurer assumes the liability ceded. The cession, or share of losses to be paid by the reinsurer, may be defined as a percentage share of losses or on some other basis.

The nature and purpose of insurance is to reduce the financial impact upon individuals and corporations from the potential occurrence of specific kinds of contingent events. An insurance company sells many policies which for fixed or bounded (e.g., retro-rated plans) prices guarantee the policyholders that the insurer will indemnify them for part of their financial losses.
arising from these events. This pooling of liabilities allows the insurer's total losses to be more predictable than is the case for each individual insured.

Insurance enables individuals and corporations to perform tasks and manufacture products which might be too risky for one entity. This increases competition and efficiency in a capitalistic marketplace.

The nature and purpose of reinsurance is to reduce the financial impact upon insurance companies of insurance claims; thus further enhancing competition and efficiency in the marketplace. The cession of shares of liability spreads risk further throughout the insurance system. Just as an individual may purchase an insurance policy from a large insurer, a small insurance company may purchase fairly comprehensive reinsurance from a large reinsurer. And a large insurer or reinsurer may spread its assumed insurance risk by purchasing reinsurance coverages from many other reinsurers, both domestically and worldwide. A cession from one reinsurer to another of some part of assumed reinsurance liability is called a retrocession.

Reinsurers write business either directly through their own employed account executives or through reinsurance intermediaries. More than 50% of U.S. reinsurance is estimated to be placed through intermediaries.
A reinsurance contract is a contract of indemnification; the reinsurer agrees to compensate the cedent for a specified share of insurance payments made on certain of the cedent's insured policies. Except for special cases (e.g., cut-through endorsements), the cedent's policyholders are not parties to the contract, and thus have no direct legal recourse against the reinsurer.

The form and wording of reinsurance contracts are not as closely regulated as insurance contracts and there is no rate regulation of reinsurance between private companies. Reinsurance contracts are often manuscript contracts setting forth the unique agreement of the two parties. Because of the many special cases and exceptions in reinsurance, it is extremely difficult to make correct generalizations. Thus whenever analyzing reinsurance experience, one should be careful that when comparing data, the coverages producing the data are reasonably similar. We will be encountering this problem often throughout this chapter.
B. The functions of reinsurance

Reinsurance does not change the basic nature of an insurance coverage; on a long-term basis, it cannot be expected to make bad business good. But it does provide certain direct assistance to the cedent:

1. Capacity

With reinsurance, the cedent can write larger policy limits. By ceding shares of all policies or just of larger policies, the net retained loss exposure per individual policy or in total can be kept in line with the cedent's surplus. Thus smaller insurers can compete with larger insurers, and policies beyond the capacity of any single insurer can be written.

2. Stabilization

Reinsurance can help stabilize the cedent's underwriting and financial results over time and help protect the cedent's surplus against shocks due to especially large and infrequent losses. Reinsurance can be written so that the cedent keeps smaller predictable losses, but shares larger infrequent losses. It can also be written to provide protection against a larger than predicted accumulation of claims either from one catastrophic event or from many. Thus the financial effects of large losses or large accumulations of loss are spread out over many years. This decreases the cedent's probability of financial ruin.
3. Financial results management

Reinsurance can alter the timing of income, enhance statutory and/or GAAP surplus and improve various financial ratios by which insurers are judged. We will see this as we discuss the effects of various covers below.

4. Management advice

Many professional reinsurers have the knowledge and ability to provide an informal consulting service for their cedents regarding underwriting, claims handling, marketing, pricing, reserving, investments, loss prevention and personnel. Enlightened self-interest forces the reinsurer to critically review the cedent's operation and be in a position to offer advice. The reinsurer probably has more expertise in the pricing of high limits policies and in the handling of large and rare claims. Also, through its contact with many similar cedent companies, the reinsurer might be able to provide an overview of general trends.

C. The forms of reinsurance

1. Facultative certificates

A facultative certificate reinsures just one primary policy. Its main function is to provide additional capacity. It is used to cover exposure in excess of or in addition to that covered by the cedent's treaties. A cedent may also use facultative certificates for certain large or especially hazardous policies or exposures to limit their potential impact upon his ongoing
treaty results. The reinsurer underwrites and accepts each certificate individually; it is very similar to primary insurance for large risk underwriting.

Most property certificate coverage is on a proportional basis wherein the reinsurer reimburses a fixed percentage of each claim on the subject policy. Most casualty certificate coverage is on an excess basis wherein the reinsurer reimburses a share (up to some limit) of the part of each claim on the subject policy which lies above some fixed retention (deductible).

2. Facultative automatic programs

A facultative automatic agreement reinsures many policies, usually with the reinsurer taking a proportional share of each policy ceded. It may also be written on an excess basis. It is like a treaty except that either the cedent may not be required to cede or the reinsurer may not be required to assume every single policy of a certain type. It is usually written for new or special programs marketed by the cedent, and the reinsurer is usually very much involved in the primary pricing of the policies. For example, a facultative automatic agreement may cover 90% of the personal umbrella business written by the cedent, and the reinsurer may help the cedent establish underwriting guidelines and rates.
3. Treaty proportional covers

A treaty reinsures a share of a certain part of the loss exposure for a whole set of insurance policies for a certain time period. The set of policies are those of a specified type written during the term of the treaty, but may also include those policies in effect at inception. The subject exposure is usually defined by Annual Statement line of business or some subsets thereof. The treaty may be exposed only to those losses occurring during the term of the treaty on subject policies, or may include coverage for losses occurring after termination upon policies in force at termination (run-off exposure). One benefit of a treaty is that it creates a close working relationship between the parties so that the expertise and services of the reinsurer are available to the cedent.

A quota share treaty reinsures a fixed percentage of each subject policy. Its main functions are capacity and financial results management. Quota share treaties may assume inforce exposure at inception and return unearned premium at termination. This cession of unearned premium reserve creates a financing effect because of the ceding commission thereon. Quota share treaties sometimes attach net of all other reinsurance covers in order to cede an amount of premium necessary to protect the cedent's premium-to-surplus ratio. However, the term quota share is sometimes used improperly when there is a cession of a proportional share of an excess layer.
A surplus share treaty also reinsures a fixed percentage of each subject policy, but the share varies by policy according to the relation between the limit of the policy and the cedent's net retention. Its main function is capacity, but it also provides some stabilization and financing. Surplus share treaties also may assume inforce exposure at inception and return unearned premium at termination. They are used for property coverage and only rarely used for casualty.

4. Treaty excess covers

An excess treaty reinsures, up to a limit, a share of the part of loss in excess of some specified cedent retention (attachment point of the treaty). Its main functions are capacity and stabilization. An excess treaty usually covers exposure earned during its inforce term on a losses-occurring basis with no run-off. The definition of "loss" is important.

For a per-risk excess treaty, a loss is defined to be that loss occurring on one policy for one event. Per-risk excess is used for property exposures to provide protection net of facultative coverage and possibly also net of proportional treaties. It is used for casualty less often than per-occurrence coverage.

For a per-occurrence excess treaty, a loss is defined to be all losses arising from one loss event or occurrence for all subject policies. Per-occurrence excess is used for casualty covers to protect a cedent all the way up from working layers through class layers. A working layer is loosely defined as a layer for which a number of losses are expected each year. A higher exposed

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layer is above the working layer(s), but within some policy limits. A clash layer usually attaches above policy limits and is only exposed by extra-contractual-obligations or excess-of-policy-limit damages (if covered), by catastrophic workers' compensation accidents and by the "clash" of losses from multiple coverages or policies stemming from one loss event. The main purpose of a clash layer is catastrophe protection.

A per-occurrence excess treaty used for property exposure is called a catastrophe cover. It is used to protect the net position of the cedent against the accumulation of claims stemming from a single large natural event. It is usual to stipulate that two or more insureds must be involved before coverage attaches. The cessions are usually 90% to 95% of layers excess of the maximum retention the cedent can absorb or can afford.

For an aggregate excess treaty, a loss is the accumulation of all subject losses during one time period, usually one year. It usually covers the net retention of the cedent, either property or casualty or both. It protects net results, providing very strong stabilization and catastrophe protection (unless catastrophes are excluded from coverage).

5. Nontraditional covers

These are almost always treaties whose main, and sometimes only, purpose is financial. The reinsurer's risk is reduced by various contractual conditions. And the reinsurer's expected margin is reduced to reflect this.
A financial proportional cover usually has a ceding commission which varies within some range inversely to the subject loss ratio. The ceded loss share may also decrease somewhat if the loss ratio exceeds some maximum, or the loss share may be fixed at some percentage less than the premium share. The cover may also have some kind of funding mechanism wherein the aggregate limit of coverage is based upon the fund (net cash position less margin of the reinsurer) plus, of course, some risk layer at least at the beginning of the contract.

A loss portfolio transfer is a cession of some part of the loss liability of the cedent as of some accounting date. It may be a cession of the total liability or, more usually, some aggregate excess layer. It is almost always subject to a limit, and may have sublimits upon payment timing. The retention may be stated in terms of dollars and/or time. A loss portfolio transfer may be a pure risk cover, but usually it is essentially a present value funding of liabilities. It may include profit commissions to be paid to the cedent if the actual loss experience is better than originally anticipated.

A funded aggregate excess cover is, as one might suspect, an aggregate excess cover for which the losses are essentially funded. It is analogous to a funded loss portfolio transfer except that it covers future occurring losses. Besides financing, it may provide strong stabilization.
D. A typical reinsurance program

Of course there is no such thing as a typical reinsurance program. Every company is in a unique situation regarding loss exposure, financial solidity, management culture and future plans. Thus each company needs a unique reinsurance program, a combination of ceded reinsurance covers tailor-made for that company.

Nevertheless, Table 5.1.1 displays a reinsurance program for a medium sized insurance company that we might regard as being typical:
### A "TYPICAL" REINSURANCE PROGRAM

FOR A MEDIUM SIZED COMPANY

<table>
<thead>
<tr>
<th>Lines of Business</th>
<th>Type of Reinsurance</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Fire and Allied Lines</strong></td>
<td>1. Proportional facultative certificates to bring each individual policy's net exposure down to $1,000,000</td>
</tr>
<tr>
<td>HO Section I</td>
<td>2. Surplus share of 4 lines not to exceed $800,000; maximum cedent retention of $200,000</td>
</tr>
<tr>
<td>SMP Section I</td>
<td>3. Per risk excess working layer $100,000 excess of $100,000</td>
</tr>
<tr>
<td></td>
<td>4. Catastrophe covers:</td>
</tr>
<tr>
<td></td>
<td>a) 95% of $3,000,000 excess of $2,000,000</td>
</tr>
<tr>
<td></td>
<td>b) 95% of $5,000,000 excess of $5,000,000</td>
</tr>
<tr>
<td></td>
<td>c) 95% of $5,000,000 excess of $10,000,000</td>
</tr>
<tr>
<td></td>
<td>d) 95% of $5,000,000 excess of $15,000,000</td>
</tr>
<tr>
<td><strong>B. Casualty Lines</strong></td>
<td>1. Facultative certificates for primary per policy coverage excess of $1,000,000</td>
</tr>
<tr>
<td>excluding Medical</td>
<td>2. Working layer excess:</td>
</tr>
<tr>
<td>Malpractice and Umbrella</td>
<td>$700,000 excess of $300,000</td>
</tr>
<tr>
<td></td>
<td>3. Clash layers:</td>
</tr>
<tr>
<td></td>
<td>a) $4,000,000 excess of $1,000,000</td>
</tr>
<tr>
<td></td>
<td>b) $5,000,000 excess of $5,000,000</td>
</tr>
<tr>
<td></td>
<td>c) $10,000,000 excess of $10,000,000</td>
</tr>
<tr>
<td><strong>C. Personal Umbrellas</strong></td>
<td>1. 90% share facultative automatic program</td>
</tr>
</tbody>
</table>
If the company writes Surety, Fidelity, Marine, Medical Malpractice or other special business, other similar reinsurance covers would be purchased. If the company is entering a new market (e.g., a new territory or type of business), it may purchase quota share coverage to lessen the financial impact of the new premium volume (the ceding commissions on unearned premium) and to obtain the reinsurer's assistance. If the company is exiting a market, it may purchase a loss portfolio transfer to cover the run-off of loss payments.

E. The cost of reinsurance to the cedent

1. The reinsurer's margin

The reinsurer charges a margin over and above ceded loss expectation, commissions and brokerage fees (to the intermediary, if any). It is usually stated as a percentage of the reinsurance premium and is theoretically based upon the reinsurer's expenses, the degree of risk transfer and the magnitude of capacity and financial support, but it is practically influenced by competition in the reinsurance market. The actual resulting margin can differ greatly from that anticipated because of the stochasticity of the loss liability and cash flow transferred.
2. Lost investment income

By transferring premium funds (net of ceding commission) to the reinsurer, the cedent naturally loses the use of those funds until returned as loss payments or as profit commissions, and the reinsurer theoretically keeps a margin and the intermediary, if any, keeps a fee. On the surface, this loss may be diminished if the reinsurer agrees to allow the cedent to withhold funds and keep an account of the funds withheld. But of course the reinsurer will charge a higher margin for this. The actual lost investment income depends upon the actual cash flow on the cover; as with (1), this may be highly stochastic.

3. Additional cedent expenses

The cedent incurs various expenses for ceding reinsurance. These include the cost of negotiation, the cost of a financial analysis of the reinsurer, accounting and reporting costs, etc. If an intermediary is involved, the fee covers some of these services to the cedent. In general, facultative is more expensive than treaty because of individual policy negotiation, accounting and loss cessions.

4. Reciprocity

In order to cede reinsurance, the cedent might be required to assume some liability from the reinsurer. If this assumption is unprofitable, the loss should be considered in the cost of the cession. Reciprocity is more prevalent outside the U.S.
F. Balancing costs and benefits

In balancing the costs and benefits of a reinsurance cover or of a whole reinsurance program, the cedent should consider not only the direct loss coverage benefit and functions. A major consideration is the reinsurer's financial solidity: will the reinsurer be around to pay late-settled claims many years from now? Also important may be the reinsurer's services, including underwriting, marketing, investment, claims, loss prevention, actuarial and personnel advice and assistance.

References 5.1:


A. General considerations

In general, reinsurance pricing is more uncertain than primary pricing. Coverage terms can be highly individualized, especially for treaties. These terms determine the coverage period, premium and loss payment timing, commission arrangements, application of limits, etc. It is often difficult and sometimes impossible to get meaningful and credible loss experience pertinent to the cover being evaluated. Often the data are not as it first appears, so one must continually ask questions in order to discover their true nature. Because of these problems of coverage definition and of the meaning of loss and exposure statistics, the degree of risk relative to premium volume is usually much greater for reinsurance business.

Additional risk arises from the low claim frequency/high severity nature of many reinsurance coverages, from the lengthy time delays between the occurrence, reporting and settlement of covered loss events, and also from the leveraged effect of inflation upon excess claims. In general, the lower the expected loss frequency, the higher the variance of results relative to expectation, and thus the higher the risk level. Also, IBNR emergence and case reserve development are severe problems for casualty excess business. Development beyond 10 years can be
large, highly variant and extremely difficult to evaluate. Concomitant is the increased uncertainty for asset/liability matching because of the very long tail and extreme variability of the loss payout timing. Future predictability is decreased by greater uncertainty affecting loss severity inflation above excess cover attachment points. All these elements create a situation where the variance (and higher moments) of the loss process and its estimation are much more important relative to the expected value than is the case for primary coverage. For some reinsurance covers, the higher moments (or at least the underwriter/actuary's beliefs regarding fluctuation potential) determine the price.

There are many ways to price reinsurance covers. For any given situation, there is no one right way. In this section, we will discuss a few reasonable methods. As in most actuarial work, one should try as many reasonable methods as time permits (and reconcile the answers, if possible).

B. Pricing facultative certificates

Since a facultative certificate covers a share of a single insurance policy, the individual insured can be underwritten and priced. The exposure of the individual insured can be evaluated and manual rates and rating factors can be used. However, since most facultative certificates are written on larger or more hazardous exposures, manual rates and rating factors may not exist or must often be modified. Thus individual loss experience and a great deal of underwriting judgment are important.
To the extent that actuaries are involved with facultative certificate business, they can be useful in the following ways:

1. Be sure that the facultative underwriters are provided with and know how to use the most current and accurate manual rates and rating factors, e.g., increased limits factors, loss development factors, trend factors, actuarial opinions on rate adequacy by exposure type and by territory (state), etc.

2. Work with the underwriters to design and maintain good pricing methodologies, perhaps in the form of interactive computer programs.

3. Work with the underwriters to design and maintain good portfolio monitoring systems for meaningful categories of their business, both for relative price level and for the monitoring of loss experience.

4. Work with the underwriters to evaluate and determine which lines of business and which exposure layers to concentrate upon as market/pricing conditions change.

In contemplating any form of facultative coverage, the underwriter first evaluates the exposure to decide if the risk is acceptable, and then evaluates the rate used by the cedent to decide if it is adequate. The underwriter also determines if the ceding commission fairly covers the cedent's expenses, but does not put the cedent into a greatly more advantageous situation than the reinsurer.
Property certificate coverage on a proportional share basis usually needs little further actuarial assistance. However, the actuary should be involved in the corporate discussion and evaluation of catastrophe accumulation potential.

Evaluating and pricing property certificate coverage on an excess basis is more difficult. There exist very little reliable published information on the rating of property excess coverage. Some underwriters use so-called Lloyds Scales, tables of excess loss factors determining the average excess loss as part of the total according to the relationship of excess attachment point to the MPL (maximum possible loss). The MPL, sometimes also called "amount subject", is a very conservative estimate by the individual underwriter of the maximum loss possible on the policy. It includes the maximum full value of contiguous buildings together with contents and also reflects maximal time element (e.g., business interruption) coverage. The actuarial basis for the Lloyds Scales, if any, is lost in the murky remembrance of post-war (World War II) London. I know of no published actuarially sound tables for rating property per-risk excess coverage.

One actuarially sound concept for developing a table of property per risk excess rating factors would be to express the excess loss cost for coverage above an attachment point up to the MPL as a percent of the total loss cost. The curve would depend upon the class of business (its severity potential) and upon the size of the MPL, and also upon the relative size of the PML (probable maximum loss). The PML is a less conservative estimate of the
CLAIM SEVERITY (FICTIONAL EXAMPLE)

CUMULATIVE DISTRIBUTION FUNCTIONS

0
MPL = $10,000,000

0.1
MPL = $100,000

Cumulative Probability

0
Loss as a Fraction of Total

0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
largest loss, assuming for example, that the sprinkler system works, that the contents are normal, etc. The difference between MPL and PML is illustrated by considering an office building: the MPL is the total value; the PML is usually thought to be three to five floors. The MPL and PML affect the shape of the loss cost curve because one expects, for example, very different loss severity distributions for an insured with a $100,000 MPL and PML versus an insured with a $10,000,000 MPL and $5,000,000 PML. This is illustrated by the accompanying graph (5.2.1).

Appropriate risk loadings could be incorporated in the table or could be recommended as additional loading factors.

An appropriate pricing formula for an excess cover is as follows:

\[
RR = \frac{ELCF \times PPLR \times RCF}{(1 - CR - BF) \times (1 - IXL) \times (1 - TER)}
\]

(5.2.2)

where

\begin{align*}
RR & = \text{reinsurance rate} \\
& \quad \text{(as a percent of total premium)} \\
ELCF & = \text{excess loss cost factor} \\
& \quad \text{(from the table; as a percent of total loss cost)} \\
PPLR & = \text{primary company permissible loss ratio} \\
RCF & = \text{rate correction factor} \\
& \quad \text{(for adequacy of primary rate)} \\
CR & = \text{reinsurance ceding commission rate} \\
BF & = \text{reinsurance brokerage fee (if any)} \\
IXL & = \text{reinsurer's internal expense loading} \\
& \quad \text{(as a percent of premium net of CR and BR)} \\
TER & = \text{reinsurer's target economic return} \\
& \quad \text{(as a percent of pure risk premium, net of CR, BF and IXL)}
\end{align*}
The reinsurance rate is applied to the primary total premium to determine the reinsurance premium. If the reinsurer wishes to reflect the investment income to be earned on the contract, then the ELCF would include an appropriate loss discount factor. We will see this later for casualty coverage. To maintain consistent terminology with casualty pricing where investment income is more likely to be reflected, we use the term "target economic return" instead of simply "risk (and profit) loading".

For example, suppose we have the following situation:

(5.2.3) EXAMPLE

Facts:

1. Primary total premium = $100,000.
2. MPL = PML = $10,000,000.
3. Attachment point = $1,000,000.
4. Reinsurance limit = $4,000,000.
5. PPLR = 65%
6. CR = 30%
7. BF = 0% (no broker)
8. IXL = 8%
Suppose that for this class of business and for this layer ($4 million excess of $1 million), we (the reinsurer) want to price to a TER of 10%. Also suppose that we believe that the cedent's rate is inadequate by 5%; thus we believe the total expected loss cost to be $100,000 * .65 * 1.05 = $68,250.

Now assume that we believe that the loss severity for this class of business and this MPL is given by a censored (at MPL) Pareto distribution of the following form:

\[
1 - F(x) = \text{Prob}[X > x] = \begin{cases} 
\frac{b}{b + x} & \text{for } x < 1 \\
0 & \text{for } x = 1
\end{cases}
\]

where the loss size \(X\) is expressed as a percent of MPL.

(Properties of the Pareto are outlined in Appendix A.)

Suppose that the parameters are given by \(b = .1\) and \(q = 2\).

The reader can verify the following facts:

9. \( \text{Prob}[X = 1] = \left(\frac{b}{b + 1}\right)^q = .008 \) (probability of a loss, if it occurs, hitting the MPL)

10. \( E[X;c] = \left(\frac{b}{q - 1}\right) \ast \left(1 - \left(\frac{b}{b + c}\right)^{q-1}\right) \) (expected loss cost up to any censor \(c \leq 1\))

11. \( E[X;1] = .091 \) (as a percent of MPL) (Thus, if a loss occurs, its average size is $910,000)
12. \[ E[X \cdot .1 \mid (.1, .5)] = \frac{q-1}{q-1} \left( \frac{b}{(q - 1)} \right) \left( \frac{b}{b + .1} \right) - \left( \frac{b}{b + .5} \right) \]

\[ = .033 \text{ (as a percent of MPL)} \]

(per-occurrence expected loss cost in the reinsured layer)

13. \[ \text{ELCF} = \frac{(12)/(11)}{.033/.091} = .367 \]

14. \[ \text{RR} = .432 \]

15. Reinsurance gross premium

\[ = .432 \times 100,000 = 43,200 \]

The reader can also verify the following facts:

16. \[ E[N(\text{excess})] = (68,250/910,000) \times \text{Prob}[x > .1] = .01 \]

(expected number of claims excess of \$1,000,000)

17. The reinsurance gross premium for the layer \$5 million excess of \$5 million, with a 15% TER, is \$10,400.

Of course, quite often the pricing situation is much more complicated, with multiple locations and coverages. The underwriter/pricer generally determines a price for each location, each coverage, and adds. Instead of working directly with an estimated underlying loss severity distribution like this Pareto, the ELCF (13) might be obtained from a table such as the Lloyds Scale.

Clearly, a pricing procedure such as this can be easily programmed into an interactive PC package for the underwriters. The package would contain all the appropriate rates and rating factors to be called upon by the user. It would ask most of the pertinent questions of the user and would document the decision.
trail for each submission seriously contemplated by the underwriters.

For facultative certificate property coverage as with any reinsurance business segment, the pricing cycle is very severe. This is mainly due to naive capital flowing into the market because of easy access, but also due to the short-term nature of most peoples' memories. Thus it is very important to monitor the results very closely. Renewal pricing and rate competition in the marketplace should be watched monthly; perhaps summaries derived from the aforementioned pricing system would be appropriate. Quarterly underwriting results in appropriate business segment detail are very important.

Evaluating and pricing facultative certificate casualty covers is even trickier, due mainly to the uncertainty arising from delayed loss reporting and settlement. Because of this increased uncertainty, the actuary's role can be more important in pricing and monitoring the results.

Analogously to property excess, a cover may be exposure rated via manual rates and increased limits factors, together with exposure evaluation and underwriting judgement. The same formula (5.2.2) may be used except that the ELCF will be based upon increased limits loss cost tables and the RCF may be determined both by a judgement regarding the cedent's basic limits rate level and by a judgement regarding the cedent's increased limits factors.
Since most companies use Insurance Services Office (ISO) increased limits factors for casualty pricing (especially for commercial lines), it is very important that the actuaries very closely monitor ISO factors and understand their meaning.

**NOTE:** Most increased limits factors, including those published by ISO, have no provision for allocated loss adjustment expense (ALAE) outside of the basic limit. ALAE is usually covered on an excess basis either 1) proportional to the indemnity loss share of the excess cover vis-a-vis the total or 2) by adding the ALAE to the indemnity loss before applying the attachment point and limit. Thus ELCFs based upon increased limits factors must be adjusted to cover the reinsurer's share of ALAE.

Since policies subject to facultative coverage are larger than usual, experience rating often comes into play. The simplest method of experience rating is to first experience rate the basic limits experience or the experience below the proposed excess attachment point, if it is not too high, to get an experience base rate. This rate may be used together with the reinsurer's ELCF table to determine an excess rate.

For a buffer layer of coverage where the likelihood of loss penetration is significant, a more difficult to estimate but perhaps more relevant excess experience rate may be determined directly from a careful analysis of the large loss experience of the insured. To see this let us consider the following example:
(5.2.5) EXAMPLE

Facts:

1. Estimated 1990 basic limits premium ($25,000 limit) = $100,000
2. Policy limit = $1,000,000, no aggregate
3. Estimated 1990 total limits premium = $260,000
4. PPLR = 75%
5. Attachment point = $250,000
6. Reinsurance limit = $750,000
7. ALAE covered pro rata
8. General liability premises/operations exposure
9. CR = 20%
10. BF = 5%
11. IXL = 10%
12. Pricing for 1990 policy period
13. Have exposure and loss experience for policy years 1984 through 1988, consisting of exposures, basic and total limits premiums, current evaluation of basic limits losses and a detailed history for each known claim larger than $25,000

Suppose that for this class of business and for this layer ($750,000 excess of $250,000), we (the reinsurer) want to price to an undiscounted TER of 10%. Also suppose that the cedent's basic limits premium was determined from a standard experience and schedule rating plan which we believe to be adequate, and the cedent uses the appropriate ISO increased limits factors which
the reinsurer believes to be adequate, but which include the ISO risk loading but no ALAE provision for the layer. Suppose the ISO increased limits factors for this exposure are as follows:

(5.2.6) (Fictional) ISO Increased Limits Factors

<table>
<thead>
<tr>
<th>Limit</th>
<th>Published Factor</th>
<th>Factor without risk load</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100,000</td>
<td>1.80</td>
<td>1.769</td>
</tr>
<tr>
<td>$250,000</td>
<td>2.25</td>
<td>2.154</td>
</tr>
<tr>
<td>$1,000,000</td>
<td>2.60</td>
<td>2.429</td>
</tr>
</tbody>
</table>

Suppose that the cedent is offering us a manual difference excess premium of $35,000 calculated by:

14. Manual difference excess premium

\[
= $100,000 \times (2.60 - 2.25) = $35,000
\]

Suppose that, based upon a study of the relationship of ALAE to claim size for this type of exposure, we believe that an appropriate loading for pro rata ALAE is 10% of indemnity loss cost for this layer. Then if we believe the ISO increased limits factors to be adequate for this exposure, the reinsurance rate as a percentage of the basic limits premium would be calculated from formula (5.2.2) with \(E_{\text{LCF}} = 1.10 \times (2.429 - 2.154) = 0.3025\).

It is left to the reader to check that this exposure rate premium would be:

15. Reinsurance premium 1 = $37,346
Now suppose that we are willing to price to a discounted loss basis. Suppose that we have an estimated expected loss payout pattern for this type of exposure and this layer. And suppose that the corresponding discount factor, using current U.S. Treasury rates (risk-free) timed to the payout pattern and reflecting the implications of the current Tax Act, is .80. Assume that with the reflection of investment income in the pricing, we wish to increase the TER to 20%. Then the reader can check that the new price is:

16. Reinsurance premium 2 = $33,611

In this case, the offered $35,000 premium looks adequate. But what about the large loss experience?

Assume that we believe that the ISO claim severity distribution is reasonably accurate for this insured's large loss exposure and that the ISO distribution can be used to compute probabilities of loss for points above $25,000 and also to compute severity moments. Assume for convenience that this distribution is a Pareto of form (5.2.3) with parameters $b = 50,000$ and $q = 2$ (this is consistent with (5.2.6) with a 30% loading for ALAE in the basic limit rate).

The reader can verify the following facts:

17. $\text{Prob}[X > 250,000 \mid X > 25,000] = 0.0625$

(Use Formula (5.2.4.)
18. $E[25; 975] = 69,643$

(expected claim severity in the excess layer $975,000$
excess $25,000$ where $X25$ notates the excess claim size
for claims strictly greater than $25,000$ - see Appendix
A(10))

19. $E[250; 750] = 214,286$

Suppose that we believe, based upon ISO and other industry
information, that the large loss severity trend from 1984 to 1990
is about 13% per annum and the ground-up frequency trend is 2%
per annum. (For simplicity, assume constant trends.) And
suppose that we believe, based upon the claim severity model,
that the severity and frequency trends translate into a 12.2%
frequency trend excess of $25,000$. (NOTE: This can be seen by
deflating the Pareto parameter $b$ by 13% per annum back for four
years and measuring the exponential effect on the probability
$\text{Prob}[x > 25,000]$. Combine this exponentially smoothed annual
change with the 2% ground-up frequency trend to get the 12.2%
frequency trend excess of $25,000$.) Suppose that for accident
year 1984-1988, there are three claims known as of June 30, 1989
whose indemnity values are greater than $25,000$. We will use the
trended frequency excess of $25,000$ to price the layer $750,000$
excess $250,000$. 

Suppose that we expect that, based upon reinsurance data for this type of business, claims will be reported over time in a pattern defined by a lognormal distributional with mean 3 and coefficient of variation 1.311 (u = 0.6, s=1 in the usual parameterization — see Appendix B), with time measured from the midpoint of the accident year. Note that this means that 5% of the claims are expected to be reported beyond 10 years after the beginning of the accident year. Suppose that for this cedent, for this insured, we expect claim emergence above $25,000 to be no different from the portfolio information. Thus American-style chain ladder development may be used to estimate the ultimate developed claims. Note that the basic limits premiums are adjusted to 1990 rate level:

\[(5.2.7)\quad \text{DATA AS OF 6/30/89 AND DEVELOPMENT}\]

<table>
<thead>
<tr>
<th>Year</th>
<th>Adjusted BL Prem (in $000' s)</th>
<th># Claims &gt;$25,000</th>
<th>Expected Report Lag</th>
<th># Devel. (3)/(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1984</td>
<td>75</td>
<td>1</td>
<td>.844</td>
<td>1.185</td>
</tr>
<tr>
<td>1985</td>
<td>80</td>
<td>0</td>
<td>.784</td>
<td>0</td>
</tr>
<tr>
<td>1986</td>
<td>85</td>
<td>1</td>
<td>.691</td>
<td>1.447</td>
</tr>
<tr>
<td>1987</td>
<td>90</td>
<td>1</td>
<td>.537</td>
<td>1.862</td>
</tr>
<tr>
<td>1988</td>
<td>95</td>
<td>0</td>
<td>.274</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>425</td>
<td>3</td>
<td>na</td>
<td>4.494</td>
</tr>
</tbody>
</table>

We may further adjust the claims for the assumed 12.2% per annum trend excess of $25,000:
### Adjusted Data ($ in 1,000's)

<table>
<thead>
<tr>
<th>Year</th>
<th>Adjusted BL Prem</th>
<th># Devel.</th>
<th>Trend Factor</th>
<th>(3)*(4)</th>
<th>Frequency 100*(5)/(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1984</td>
<td>75</td>
<td>1.185</td>
<td>1.995</td>
<td>2.364</td>
<td>3.152</td>
</tr>
<tr>
<td>1985</td>
<td>80</td>
<td>0</td>
<td>1.778</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1986</td>
<td>85</td>
<td>1.447</td>
<td>1.585</td>
<td>2.293</td>
<td>2.698</td>
</tr>
<tr>
<td>1987</td>
<td>90</td>
<td>1.862</td>
<td>1.412</td>
<td>2.629</td>
<td>2.921</td>
</tr>
<tr>
<td>1988</td>
<td>95</td>
<td>0</td>
<td>1.259</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>425</td>
<td>4.494</td>
<td>na</td>
<td>7.287</td>
<td>1.715</td>
</tr>
<tr>
<td>1984-87</td>
<td>330</td>
<td>4.494</td>
<td>na</td>
<td>7.287</td>
<td>2.208</td>
</tr>
</tbody>
</table>

Most actuaries would discard the 1988 data as being too immature. Thus this experience rating indicates an expected frequency per $100,000 of basic limits premium of 2.208 excess of $25,000, and

\[
2.208 \times \text{Prob} \left[ X > 250,000 \mid X > 25,000 \right] \\
= 2.208 \times 0.0625 = 0.138 \text{ excess of } 250,000.
\]

Since the premium for 1990 is $100,000, the indicated expected number of claims excess of $250,000 is the same 0.138. We can combine this with the expected excess claim size (19) of $214,286 and the 10% ALAE loading to compute undiscounted and discounted excess expected losses (including ALAE) and reinsurance premiums via Formula (5.2.9), which is a slight generalization consistent with (5.2.2):
(5.2.9)

\[
RP = \frac{RLC}{(1-CR-BF) \times (1-IXL) \times (1-TER)}
\]

where \( RP \) = reinsurance premium
\( RLC \) = reinsurance loss cost (expected aggregate loss, either discounted or undiscounted)
\( CR, BF, IXL, TER \) as defined in (5.2.2)

The reader can verify the following facts:

20. Reinsurance expected loss = $29,571

21. Discounted (20) = $23,657 (discount factor = .80)

22. Reinsurance premium 3 = $48,676
   (Use (20) in Formula (5.2.9.))

23. Reinsurance premium 4 = $43,809
   (Use (21) in Formula (5.2.9.))

Now it's time for the underwriter to sharpen his pencil. Is the RP4 = $43,809 significant? That is, how certain is this experience rate? Let us look first at the excess $250,000 frequency indication of 0.138 per $100,000 of basic limits premium. This is based upon a developed loss count of 7.287 excess of $25,000 and upon the ISO loss severity distribution. If the offered reinsurance premium of $35,000 were correct on a discounted loss basis and the loss severity curve were correct for this insured, then the expected frequency excess of $25,000 would be 1.764 (= 2.208 * (35,000/43,809)), and the expected
number of developed claims excess of $25,000 would be 5.822 
\( (= 7.287 \times (35,000/43,809) ) \). Under the assumption that the 
excess claims are distributed Poisson with mean 5.822, the 
probability of seeing a number 7.287 or more is approximately 
27%. Thus our observed 7.287 is not too unlikely.

The underwriter might also consider the average known loss size 
excess of $25,000 to see if it is significantly different, after 
considering development and trend, than the expected excess size 
of $69,643. The underwriter must now ponder these facts, 
together with all his other knowledge of the particular insured's 
exposure and general rate adequacy/inadequacy for this class of 
business in order to make a decision. He may require at least 
$38,500 (adjusting the offered manual difference $35,000 to cover 
ALAE prorata); or he may want $45,000 in light of RP3 and RP4 
(remember the total premium is $260,000); or he may decide not to 
write the cover at all in fear of the large loss exposure 
indicated by the experience.

As with property excess, it is clear that this rating method can 
be programmed into an interactive PC package for underwriters. 
Also, as with property coverage, it is very important to monitor 
relative rate level and results in appropriate business segment 
detail. The actuarial evaluations and opinions regarding future 
case reserve development and IBNR emergence should be very 
important to the underwriters.
C. Pricing facultative automatic programs

These large multi-insured programs are very similar to treaties. One difference however is that the reinsurance premium for a facultative automatic excess cover is usually computed on a policy-by-policy basis using agreed upon excess rates, instead of as a rate times total subject premium. Thus the reinsurance premium may be more responsive to the individual exposures ceded to the reinsurer. The risk of anti-selection against the reinsurer on a non-obligatory contract should be evaluated by the underwriter.
D. Pricing reinsurance treaties in general

Since a treaty covers a share of an indeterminant (at the beginning) set of insurance policies, insureds are rarely individually underwritten and priced by the reinsurer. Instead the reinsurance underwriter/pricer considers the whole set (book of business) of subject policies. To do this, the reinsurer evaluates first the management of the potential cedent. What is their management philosophy and ability? Are they honest, fair-dealing? Do they know what they are doing? Is the company financially solid? What are their business plans? Why do they want, why do they need reinsurance?

Once the reinsurance underwriter is satisfied that this is a company and these are people he would like to deal with on a long-term basis, he can then evaluate their underwriting, primary pricing, marketing and claims handling ability. Since individual insureds are not underwritten by the reinsurer except on an exception basis, he must be satisfied with the cedent's underwriting expertise and pricing for the exposure he may assume. For any treaty, he must understand the cedent's insurance exposures, rate level and limits sold. Many direct-marketing reinsurers will send a team of marketing and underwriting people to perform a pre-quote audit, and will also send claimspeople to review the company's claims handling and reserving practices.

The reinsurer also reviews the structure of the cedent's reinsurance program, that is, how all the reinsurance contracts, facultative and treaty, fit together to provide benefits to the
cedent. Lastly, he evaluates the particular reinsurance treaties and suggested rates if offered, or he creates a program and rates to offer to the cedent company.

Actuaries can provide extremely useful, and often necessary, technical support for treaty business. Besides the list of four items mentioned for the support of facultative certificate business, for treaty (and facultative automatic) business they can also get involved in the technical evaluation and pricing of individual large and/or difficult treaties. Experience rating is much more important for treaties, so the actuarial tools of data analysis and loss modeling can be critical to a reinsurer's ability to write difficult exposures, especially for casualty exposures where long tail loss development and IBNR are critical factors.

E. Pricing proportional treaties

A traditional quota share treaty covers a share of the cedent's net retention after all other reinsurance covers. The cedent's historical experience net of all other reinsurance must be evaluated. If the cedent's other reinsurance covers have been approximately the same for many years, then Schedules 0 and P of the cedent's Annual Statement may be used for this evaluation. If the other covers have changed significantly so that the remaining net exposure to be covered differs from the cedent's past net, then the reinsurer must request historical data which can be recast to the proper net exposure. The underwriter/actuary must be careful that the data includes an adequate provision for reported case reserve development and IBNR.
The reinsurer's evaluation of the cedent's net historical experience should not only consider averages, but should reflect the effects of the underwriting/pricing cycle and of random fluctuations. And this history should be adjusted to the future coverage period by the reinsurer's estimates of relative rate level (including the underwriting cycle).

Proportional treaties often have contingent or sliding scale ceding commissions. In each case, the reinsurer pays the cedent a provisional commission on the reinsurance gross written premium as it is transferred to the reinsurer. At suitable dates (often quarterly), the cumulative experience on the treaty (usually from the beginning if there is a deficit carryforward; or over some period such as three years) is reviewed. If it is profitable, the reinsurer pays the cedent an additional commission; if it is unprofitable, the cedent returns some of the provisional commission to the reinsurer. An example should clarify this.

(5.2.10) EXAMPLE

Facts:

1. 25% quota share on various property lines
2. Cumulative subject written premium = $34,000,000
3. Cumulative subject earned premium = $30,000,000
4. Provisional commission = 35%
5. Commission slides 0.5% for each loss ratio 1%

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6. Minimum commission = 30%

7. Reinsurer provisional expense and profit margin = 10% (at 55% loss ratio)

8. Cumulative subject incurred loss = $19,800,000

The reader can verify the following facts:

9. Subject loss ratio = 66%

10. Indicated cumulative reinsurance commission
    = $11,250,000

11. Cumulative commission adjustment = - 5% (minimum)

12. Reinsurance written premium = $11,250,000

13. Reinsurance earned premium = $10,000,000

14. Return commission (to reinsurer) = $500,000

(5% of earned premium; some part may have already been adjusted at previous evaluation dates)

To properly evaluate the historical results on this treaty, the reinsurer must be sure that appropriate loss development is accounted for, and, if the reinsurer wishes to evaluate the bottomline result, then appropriate investment income must be assigned. Also, the reinsurer must consider the long-term required economic return (RER) for this type of treaty and this type of exposure. For each type of cover, each type of exposure, the RER is some fraction of the reinsurer's TER defined earlier. The
fraction may be less than one if the reinsurer is willing to be satisfied with a long-term return lower than the pricing formula target.

A simplified evaluation formula parallels the pricing formulas (5.2.2) and (5.2.9) we saw earlier:

(5.2.11) EVALUATION FORMULA

\[ \text{AER} - \text{RER} \quad \text{(Evaluation formula)} \]

where

\[ \begin{align*}
\text{RER} &= \text{required economic return (on pure premium)} \\
\text{AER} &= \text{actual economic return (on pure premium)} \\
(1 - \text{CR} - \text{BF})*(1 - \text{IXL})*(1 + \text{UPRF})*\text{REP} - \text{DF} * \text{RIL} &= \frac{-\text{---------}}{(1 - \text{CR} - \text{BF})*(1 - \text{IXL})*\text{REP}} \\
\text{UPRF} &= \text{unearned premium reserve factor} = 0.5*\text{UPRR} * \text{UPIF} \\
\text{UPRR} &= \text{average ratio of unearned premium reserve to earned premium (for each year)} \\
\text{UPIF} &= \text{unearned premium reserve investment return factor} \\
\text{REP} &= \text{reinsurer earned premium} \\
\text{DF} &= \text{loss discount factor} \\
\text{RIL} &= \text{reinsurer incurred loss} \\
\text{CR, BF and IXL as defined in (5.2.2)}
\end{align*} \]
Suppose that with respect to a conservative risk-free interest rate, the cash flow for this type of contract (or for this contract in particular, if the cash flow is known) allows a loss discount factor of .96 on losses and a 7% short-term investment rate on unearned premium reserve balances held by the reinsurer. Suppose that UPRR = 40% for the contract in Example (5.2.11), and the reinsurer needs IXL = 5%.

The reader can verify that, with respect to the minimum 30% commission, the actual economic return on pure premium has been:

15. $AER = 6.12\%$

The reinsurer's required economic return should be based upon the degree of risk transferred and upon the statutory surplus relief arising from the ceding commission on the unearned premium reserve. The surplus effect arises from the fact that the cedent's unearned premium liability decreases by the amount of gross unearned premium ceded, while assets decrease only by the amount of the cash transfer, premium net of provisional commission. Since the subject unearned premium is currently $4,000,000, the reader can verify that the current surplus relief is:

16. Cedent's surplus relief = $350,000.

This is, in effect, a statutory surplus loan; which is why a reinsurer will charge an increment on top of the usual risk margin. Suppose in this case, that the reinsurer wants a 7% return on the surplus loan. To keep this simple, suppose that the unearned premium reserve has been constant over time, so that the surplus
relief has been constant. From the assumption that $UPRR = 40\%$, the reader can verify the following facts:

17. One year reinsurance earned premium = $2,500,000$

18. One year required surplus loan return = $24,500$

19. Surplus loan return stated as part of RER on earned pure premium (with $CR = 35\%$ and $IXL = 5\%$) = $1.59\%$

If the reinsurer needs a minimum $5\%$ risk RER (with respect to pure premium) for this treaty plus a $1.59\%$ surplus loan return for a total RER of $6.59\%$, then the $6.12\%$ ARR might prompt the reinsurer to consider nonrenewal unless the future profitability looks better or the minimum ceding commission can be negotiated downward. The reader can verify the following:

20. If the loss ratio is $65\%$ ($CR = 30\%$), then the reinsurer's $5\%$ margin on gross ceded premium translates into a $7.57\%$ AER.

21. If the loss ratio is $55\%$, then the reinsurer's $10\%$ margin on gross ceded premium translates into a $15.89\%$ AER.

The evaluation of a (true ground-up net retention) quota share on casualty exposure would be similar except that the reinsurer would have to be very careful about loss development. And because of the additional uncertainty arising from loss development, most likely the RER would be higher.
A property surplus share treaty is somewhat more difficult to evaluate. Since the reinsurer does not provide coverage for small insureds and covers larger insureds in proportion to their size above some fixed retention, the reinsurer must be more concerned with the cedent’s pricing of larger insureds. An example should clarify this.

(5.2.12) EXAMPLE

Facts:

1. Four line first surplus not to exceed $800,000
2. Maximum cedent retention = $200,000

Then the following statements are true:

3. Maximum reinsurance limit per risk = $800,000

4. For a policy with limit <= $200,000, the reinsurer receives no premium and pays no losses.

5. For a policy with limit = $500,000, the reinsurer receives 60% of the policy’s premium less ceding commission and brokerage fee and pays 60% of the policy’s losses.

6. For a policy with limit = $1,000,000, the reinsurer receives 80% of the policy’s premium less ceding commission and brokerage fee and pays 80% of the policy’s losses.
7. For a policy with limit = $2,000,000, the reinsurer receives 40% of the policy's premium less ceding commission and brokerage fee and pays 40% of the policy's losses.

It is easy to see that, given this complicated proportional structure depending upon the limit of each policy, the premium and loss accounting for a surplus share treaty is somewhat complex. Despite this, surplus share treaties are popular because they provide more large loss protection than a quota share and are much easier for the reinsurer to evaluate and price (usually only the ceding commission and slide is the subject of negotiations) than excess treaty.

A surplus share treaty is generally riskier than a simple quota share. So the reinsurer will charge a correspondingly higher margin for risk assumption.

F. Pricing working cover excess treaties

A working cover is an excess layer where losses are expected. The reinsurer will consider the cedent's policy limits distribution by line of business and will want to examine the historical gross large loss experience in order to determine the types of losses generated by the exposure and to study the development patterns. As we discussed for facultative certificates, an excess cover is usually riskier than a proportional cover. So the reinsurer will be more mindful of the predictive error and fluctuation potential, and will charge a higher margin for assuming this risk.
If losses are covered per-occurrence, then the reinsurer is exposed by policy limits below the attachment point because of the "clash" of losses on different policies or coverages arising from the same occurrence. If ALAE is added to individual claims in order to determine the reinsurer's excess share, then losses from some policy limits below the attachment point will bleed into the excess layer.

The reinsurance premium is usually specified by a reinsurance rate times subject premium as we saw in formula (5.2.2). However, for liability coverage, it may be on an increased limits premium collected basis. Here the total reinsurance premium is the sum of the individually-calculated reinsurance premiums for each policy, as we saw for the premium offered in example (5.2.5).

In either case, ideally the reinsurance pricing consists of both an exposure rating and an experience rating, and a reconciliation of the two rates. The exposure rating differs from facultative certificate pricing in that the reinsurer deals with broad classes of business instead of individual insureds. The reinsurer considers manual rate relativities to bureau rates and/or to other companies writing the same exposure and evaluates the cedent's experience and schedule rating plans and pricing abilities. The increased limits factors used by the cedent for liability coverages are especially important. The same formulas (5.2.2) and (5.2.9) can be used except that the rate correction factor RCF now adjusts for both basic limits and increased limits (in)adequacy. If the coverage is per occurrence, the reinsurer must load the manual difference rate for the clash exposure. If the coverage includes
ALAE, the reinsurer must adjust the manual increased limits factors to account for this additional exposure.

As with the facultative example we saw earlier, the reinsurer must adjust the historical experience to the estimated level of the proposed coverage period by trend factors for the losses and rate on-level factors for the premiums. Working cover treaties are often large enough so that many of the risk parameters can be determined either directly from the exposure and loss history or by a credibility weighting of the history with more general information.

For working covers, the provisional reinsurance premium is often subject to retrospective rating, with the final premium over certain coverage periods, such as three years, being adjusted according to the actual loss experience. A simple example should clarify this.

(5.2.13) EXAMPLE

Facts:

1. Proposed attachment point = $300,000
2. Proposed reinsurance limit = $700,000
3. Coverage is per-occurrence
4. Coverage is on an accident year basis (losses occurring on or after the effective date up through the termination date)
5. ALAE added to indemnity for each claim

6. All liability and workers compensation exposure for a medium-sized primary company

7. Cedent wants a proposal for a three-year retrospective rated treaty incepting Jan. 1, 1990

8. CR = 0%

9. BF = 0%

10. Estimated 1990 subject premium = $100,000,000

11. Possible reinsurance premium range to $10,000,000

12. An underwriting review has been performed

13. A claims review has been performed

14. Have Annual Statements and Annual Reports for last five years; a more detailed breakdown of premiums, deviations from bureau manual rates, limits profiles, increased limits factors, basic limits premiums, total subject premiums and basic and total subject losses as of June 30, 1989 by subline for last five years plus predictions for 1990; a detailed history for each known claim larger then $25,000 occurring in the last ten years.

15. Have the names of contact people at the ceding company; in particular, an actuary to talk with.
The exposure consists of private passenger and commercial automobile liability, premises/operations general liability with incidental products coverage, homeowners and SMP section II, and workers compensation. The cedent writes limits up to $10,000,000, but purchases facultative coverage for coverage excess of $1,000,000. The cedent also purchases facultative coverage above $300,000 for any difficult exposures on the reinsurer's exclusion list, and a 90% facultative automatic cover for his umbrella programs.

Before getting into the complications arising from a retrospective rating plan, let us first consider how to go about determining a flat (fixed) rate.

**NOTE:** A traditional excess rating methodology prevalent among reinsurers is the "burning cost" method. To compute a burning cost rate, the underwriter divides the sum of known losses in the excess layer occurring over some time period, usually five years, by the cedent's subject premium for the same time period. This ratio is then multiplied by a selected loss development factor, perhaps multiplied by some selected trend factor, loaded by some "free cover" factor and divided by a permissible loss ratio \((1 - CR - BF - IXL)\) to get a rate. Clearly, a problem with this summary approach is that it does not allow one to carefully take into account underlying exposure changes, rate changes, policy limits changes, true excess IBNR emergence and development, true excess claim frequency growth and severity growth and the aggregate excess loss fluctuation potential. I would argue that burning cost rating is not very informative even for property excess covers for which
it was designed. Unfortunately, it has been misapplied to the pricing of casualty covers. For more on this topic, see Ferguson (1978) and Patrik's review.

Let us return to the example. For a full discussion of the pricing of casualty working covers, see Patrik and John (1980). We will only sketch an outline of the procedure and add a few improvements developed since then.

**STEP 1:** The first step is to reconcile with the cedent's audited financial reports as best as possible all the exposure and loss data received from the cedent. This is an ongoing process as we ask for and receive more data.

**STEP 2:** The second step is to segregate the main types of underlying exposure for separate consideration. In this case, we might want to consider the following breakdown:

(5.2.14) **EXPOSURE CATEGORIES**

- Private passenger automobile
- Commercial automobile
- Premises/operations
- Homeowners Section II
- SMP Section II
- Workers compensation
- Umbrella
These categories can or must be further broken down as desirable or feasible. If we can, it is desirable to split the underlying exposure at least according to applicable increased limits table and by policy limit.

**STEP 3:** The next step is to perform an exposure rating. This is best done by estimating the aggregate excess loss cost for 1990 based upon the estimated 1990 exposure and general pricing information. The overall exposure loss cost would be the sum of the exposure loss costs for the individual exposure categories. The exposure loss cost for each category would be determined as in Example (5.2.5), items 15 and 16, leaving out the loading factors CR, BF, IXL and TER.

For example, suppose that the company writes commercial automobile light exposure and the following policy limits distribution is estimated for 1990 for a group of states with the same 10/20 basic limit and the same increased limits tables and, for simplicity, adequate basic rates:

(5.2.15) **COMMERCIAL AUTOMOBILE LIABILITY**

<table>
<thead>
<tr>
<th>(1) Policy Limit</th>
<th>(2) Estimated Total Subject Premium</th>
<th>(3) Cedent's Inc. Limits Factor</th>
<th>(4) Adequate ELCF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100,000</td>
<td>$3,000,000</td>
<td>2.10</td>
<td>NA</td>
</tr>
<tr>
<td>$300,000</td>
<td>2,000,000</td>
<td>2.40</td>
<td>.0346</td>
</tr>
<tr>
<td>$500,000</td>
<td>2,000,000</td>
<td>2.51</td>
<td>.1070</td>
</tr>
<tr>
<td>$750,000</td>
<td>1,000,000</td>
<td>2.60</td>
<td>.1449</td>
</tr>
<tr>
<td>$1,000,000 or more</td>
<td>2,000,000</td>
<td>2.66</td>
<td>.1526</td>
</tr>
<tr>
<td>Total</td>
<td>10,000,000</td>
<td>2.38</td>
<td>NA</td>
</tr>
</tbody>
</table>
The Adequate ELCFs are stated as fractions of 10/20 basic limits losses including all ALAE; but they allocate to the layer $700,000 excess of $300,000 the expected loss plus ALAE per accident arising from the policy limit. The reinsurer has tables of Adequate ELCFs for each type of coverage, attachment point and rate jurisdiction or has an exposure rating computer program to compute them. These are based upon claim severity distributions, as are increased limits factors.

Assuming that the cedent's permissable loss ratio for this business is 65%, the reader can verify:

16. Manual difference pure premium = $234,040
   (From (5.2.15), columns 2 and 3: remember that the subject premium is for coverage up to a $1,000,000 limit)

Suppose we believe that the basic limits loss costs implied by the basic limits premiums and permissable loss ratio are adequate. The reader can then verify:

17. Expected loss based upon the ELCFs = $184,964
   (From (5.2.15), columns 2 and 4)

Note that this is not yet loaded for clash exposure or for risk.

An even better way of estimating an exposure loss cost is to break the estimation down to an estimate of the number of excess claims and an estimate of their sizes. For example, suppose we believe that the indemnity loss distribution is Pareto with $b = 25,000$ and $q = 2$ and that the distribution of the sum of indemnity loss and ALAE per claim is Pareto with $b = 30,000$ and $q = 2$. Then the
reader can verify the entries in Table (5.2.16), where the Table (5.2.15) implied excess loss costs are assumed and we simplistically assume that the effect of adding ALAE to each claim increases the effective policy limit by 20% (along with the parameter change):

(5.2.16) EXCESS EXPECTED LOSS, CLAIM SEVERITY AND COUNT

<table>
<thead>
<tr>
<th>Policy Limit</th>
<th>Effective Policy Limit</th>
<th>Expected Loss Cost</th>
<th>Expected Claim Severity</th>
<th>Expected Claim Count (3)/(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>120</td>
<td>$0</td>
<td>$0</td>
<td>0</td>
</tr>
<tr>
<td>300</td>
<td>360</td>
<td>18,742</td>
<td>50,769</td>
<td>.3692</td>
</tr>
<tr>
<td>500</td>
<td>600</td>
<td>55,418</td>
<td>157,143</td>
<td>.3527</td>
</tr>
<tr>
<td>750</td>
<td>900</td>
<td>36,225</td>
<td>212,903</td>
<td>.1701</td>
</tr>
<tr>
<td>1000</td>
<td>1000</td>
<td>74,579</td>
<td>224,272</td>
<td>.3325</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>184,964</td>
<td>151,053</td>
<td>1.2245</td>
</tr>
</tbody>
</table>

Now we have exposure-determined estimates of excess expected claim count and severity. Why this added complication? The answer is that with a few mild assumptions regarding second (and perhaps third) moments for the claim count and claim amount distributions, we can use a standard risk theoretic model to estimate the distribution of the aggregate excess loss, the loss the reinsurer will cover. This standard model writes the aggregate loss naturally as the sum of the individual claims (events) as follows:
(5.2.17) \[ L = X(1) + X(2) + \ldots + X(N) \]

where \( L = \text{rv (random variable)} \) for aggregate loss

\[ N = \text{rv for number of claims (occurrences, events)} \]

\[ X(i) = \text{rv for the dollar size of the \textit{i}th claim} \]

For us, \( N \) and \( X(i) \) refer to the excess number of claims and the excess amount of the \( i \)th excess claim respectively. The model relates the distributions of \( L, N \) and the \( X(i)'s \). In particular, under reasonable assumptions, the \( k \)th moment of \( L \) is completely determined by the first \( k \) moments of \( N \) and the \( X(i)'s \). The model is outlined in Appendix C. Here we will only discuss its usage.

Underlying Table (5.2.16) is an assumption that the claim severities follow a censored (at policy limits) Pareto distribution. With this model, or with any other reasonable and tested model, we can easily add another column to the table consisting of the individual claim variance for each policy limit. See Patrik (1980) or Hogg and Klugman (1984). Likewise, if we assume that the total ground-up claim count follows either a Poisson or a Negative Binomial distribution, then so do the excess claim counts. So a column of claim count variance for each policy limit can be added.

We can assemble these moments into estimates of the variance of the aggregate loss \( L \) for each policy limit. For this case where the parameters for \( N \) and the \( X(i)'s \) are assumed to be known, the \( L 's \) are independent and their variances can be added to estimate the overall variance for the excess losses arising from this Commercial
Automobile exposure category. Likewise, third and higher moments can be handled. For the more interesting and realistic case where the parameters themselves are uncertain, the L's are no longer independent from the point of view of the observer/actuary, so the second and higher moments do not add. This is covered thoroughly in Patrik and John (1980).

From the first, second and third moments of L, we can get a pretty good idea of its distribution, and riskiness. The really ambitious reader can verify the following:

18. Assuming that the excess claim counts are Poisson, then for the case with known parameters, the standard deviation of the aggregate excess loss arising from the Table (5.2.15) exposure is approximately $170,000.

The estimation of excess exposure for other categories would be similar. For workers compensation, for example, excess loss factor differences would be weighted by estimated subject premium by hazard group, by major state grouping.

We assemble our estimates of the excess claim count, claim severity and aggregate excess loss from each exposure category and perform the appropriate additions to get total expectations and higher moments. The total expectations should be loaded for clash exposure arising from more than one policy or coverage for the same occurrence. If we assume that, based upon the cedent's book of business, the loading should be 5%, I would prefer to increase the claim count (and thus aggregate) expectation by 5%. This has the
effect of increasing the variance of aggregate excess loss by approximately 5% (see Appendix C, item 3).

STEP 4: For a working cover such as this, the next step is an experience rating. Since we have a detailed reserving and payment history for each claim over $25,000 for the past ten years (14), we should work with these data. As with the earlier facultative example, we would use the data for each exposure category to evaluate claim count and claim severity excess of an attachment point lower than the proposed $300,000. For example, if the ground-up claim severity inflation has been less than 13% per annum for the last ten years (and into 1990), then we can evaluate a $100,000 attachment point. The preferred method is to inflate the individual claim values to a common 1990 level and work from there.

Suppose we now have ten-year development triangles for excess of $100,000 inflated claim counts and average sizes for the Commercial Automobile category we were exposure rating. A very delicate issue in the trending is that of policy limits. If a 1980 claim on a policy with limit less than $100,000 inflates to above $100,000, would the policy sold in 1990 be greater than $100,000, so to allow this excess claim? Of course, the counter-argument is that policy limits are not increasing as rapidly as claim severity inflation. If possible, information on the cedent's policy limits distribution over time should be obtained; otherwise, Solomon-like judgment must be displayed by the underwriter/actuary.
The excess claim counts can be developed by the usual methods, or better methods if one can manage (see Section 3). Claim development statistics from the reinsurer's book of similarly exposed treaties can also be used, and credibility methods can balance the answers. Excess claim severity development can be studied in total by accident year, by report year, etc. using various actuarial methods, or individual claims can be analyzed judgementally.

Besides the usual development triangles, a fairly good exhibit for examining excess development, making judgements and explaining the answers to oneself and to underwriters and marketing people is displayed in Exhibit (5.2.18). This type of format allows you to see if your purely technical answers make sense.

Sections 1 and 2 of Exhibit (5.2.18) have no actuarial estimates all except for the adjusted on-level subject premiums. Displayed are the actual closed and known excess claim count and average size growth over the past two years.

The actuarial estimates are concentrated in Section C, columns (5), (6) and (7). The reasonableness of these estimates may be judged in comparison with the facts to date. Try various reasonable estimates on for size. The mean frequency and severity estimates derived from these are displayed in columns (12) and (10) respectively. If we use the estimates which exclude the last two accident years of data, we have a mean frequency of 1.266 per $1,000,000 of subject risk earned premium and a mean severity of $92,348 in the layer $500,000 excess of $100,000. This frequency...
### SECTION A: CLOSED CLAIMS (TRENDED) $400,000 EXCESS $100,000

<table>
<thead>
<tr>
<th>YEAR (1,000's)</th>
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<th>12/31/88</th>
<th>12/31/88</th>
<th>6/30/88</th>
<th>6/30/89</th>
<th>6/30/89</th>
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<td>4</td>
<td>146,114</td>
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<td>3</td>
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<td>7</td>
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<td>0</td>
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### SECTION B: REPORTED CLAIMS (TRENDED) $400,000 EXCESS $100,000

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<th>YEAR (1,000's)</th>
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<th>12/31/88</th>
<th>12/31/88</th>
<th>6/30/88</th>
<th>6/30/89</th>
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<td>$78,391</td>
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<td>70,820</td>
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<td><strong>TOTAL</strong></td>
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<td>95</td>
<td>78,639</td>
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<td><strong>EXCLUDES</strong></td>
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<td>80,419</td>
<td>84</td>
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<td>87</td>
<td>88,267</td>
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<td><strong>LAST 2 YRS</strong></td>
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</tbody>
</table>
**EXHIBIT (5.2.18)**

**COMMERCIAL AUTOMOBILE LIABILITY**

**SECTION C: PREDICTIONS FOR CLAIMS (TREDDED) $400,000 EXCESS $100,000**

<table>
<thead>
<tr>
<th>YEAR (1,000's)</th>
<th>6/30/89</th>
<th>6/30/89 EST. ULT.</th>
<th>6/30/85 (6)*(7) AGGREGATE AVER SIZE (3)/(2) FREQUENCY</th>
<th>80 $1,055,059 $81,158 12.92% 1.5</th>
</tr>
</thead>
<tbody>
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<td>1979</td>
<td>$8,165</td>
<td>0</td>
<td>$100,000 0.00 $100,000 $0</td>
<td>$1,055,059 $81,158 12.92% 1.5</td>
</tr>
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<td>30,000 100,000 4.06 100,000 402,220</td>
<td>691,620 97,518 6.37% 2.7</td>
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</tbody>
</table>

**TOTAL** 96,106 25 79,471 , 100,000 20.12 100,000 2,012,176 11,007,727 90,881 11.45% 1.2 |

**EXCLUDES** 76,156 20 85,847 100,000 9.40 100,000 940,136 8,902,465 92,249 11.69% 1.2 |

**LAST 2 YRS**

**STANDARD DEVIATION**

<table>
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<th>4.29% 2.4</th>
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<td>11.45% 1.2</td>
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<tr>
<td>1988</td>
<td>1.111</td>
<td>4.29% 2.4</td>
</tr>
</tbody>
</table>

372
translates into 8.229 claims excess of $100,000 for $6,500,000 of risk premium ($10,000,000 * .65).

The reader can verify the following exposure-rating values:

19. \( \text{Prob}(X > 300,000 \mid X > 100,000) = 0.1479 \)
    \( \text{Prob}(X + A > 300,000 \mid X + A > 100,000) = 0.1552 \)

20. Expected claim count excess $100,000 = 7.8898 if ALAE is added to each claim, or 5.9277 otherwise.

21. Expected claim severity excess $100,000 and up to $500,000 = $98,113 if ALAE is added to each claim, or $95,238 otherwise.

If the Exhibit 5.2.18 experience includes ALAE added to claims, then the experience claim count estimate of 8.229 is incredibly close to the exposure expectation of 7.8898. Likewise, the experience claim severity estimate of $92,348 is reasonably close to the exposure expectation of $98,113. Patrik (1980), Hogg and Klugman (1984) and Patrik and John (1980) discuss methods for estimating Pareto or other model parameters directly from the excess claims severity data, and the testing of those estimates.

If the various answers differ but cannot be further reconciled, final answers for excess $100,000 and excess $300,000 claim count and severity can be based upon a credibility balancing of the separate estimates. However, all these differences should not be ignored, but should indeed be included in your estimates of parameter (and model) uncertainty, thus giving rise to more realistic measures of variances, etc. and to risk.
As with the exposure rating, these credibility balanced exposure/experience rating estimates are assembled into totals for expectations and variances, and perhaps higher moments. Any other overall adjustments are made, such as for clash exposure.

**STEP 5:** The last step is specifying the cover terms, and explaining and negotiating. Suppose our totals come out too high with respect to (11). Suppose that the estimated 1990 expected aggregate loss is $10,000,000 with estimated standard deviation (including all of our uncertainty) of $5,000,000 ($u = 2.191, s = .4724$). The reader can verify that if the distribution of the aggregate excess loss is assumed to be lognormal (a simplistic assumption), then:

22. $\text{Prob}[L > 10,000,000] = .41$
23. $\text{Prob}[L > 15,000,000] = .14$
24. $\text{Prob}[L > 20,000,000] = .04$

Suppose that, based upon both the excess loss data and upon data for similarly exposed contracts, the reinsurer believes that a loss discount factor of .80 is reasonable and the reinsurer needs $\text{IXL} = 5\%$ for this cover. If the reinsurer believes that, based both upon the above probabilities (22) through (24) and upon general considerations for this type of exposure, he needs an $\text{RER} = 15\%$, then the reader can verify that the reinsurer determines a flat premium of:

25. Reinsurance premium = $9,900,000$ (rounded)
Suppose that the cedent believes that the excess loss potential is significantly less than our estimate. And he wants either a flat rate no higher than 8% of subject premium or a retrospectively-rated treaty with a maximum 10% rate. It's time again for the reinsurer to sharpen his pencil.

The best bet is to recommend that the attachment point be increased to $350,000 or $400,000. Attachment points should naturally increase over time in an inflationary environment. Many cedents have trouble accepting this fact, and the marketing of indexed attachment point contracts in the U.S. market in the late 1970's was an attempt to unobtrusively enforce a status quo balance between the cedent and reinsurer. However, this reasonable idea turned out to be impractical in the U.S. reinsurance market because it complicated the accounting of claims and because of anti-selection against the reinsurers who sold the idea (the best cedents found fixed attachment point excess coverage).

The ambitious reader can verify that for the exposure in Table (5.2.15):

26. Expected claim count excess $350,000 = .9231

28. Expected claim severity excess $350,000 = $142,775

29. Expected aggregate loss excess $350,000 = $131,793
compare these results to Table (5.2.16). For other exposure categories the effects will be similar. Suppose that for a cover of $650,000 excess of $350,000 the reinsurer believes that the expected aggregate loss is $7,000,000 with a standard deviation of $4,000,000.

The reader can verify that analogously to (22) through (24):

30. \( \text{Prob}[L > 10,000,000] = .27 \)

31. \( \text{Prob}[L > 15,000,000] = .04 \)

32. \( \text{Prob}[L > 20,000,000] = .01 \)

In this case the reinsurer might offer a retrospectively-rated treaty of the following form:

33. Provisional rate = 7%

34. Rate = reinsurer's aggregate loss (including an IBNR provision) divided by subject premium + 0.5% (reinsurer's margin).

35. Minimum rate = 5%

36. Maximum rate = 10%

37. Profit and deficit carryforward (into successive coverage periods)
The reader can verify, using Formula (5.2.11):

38. If $L = 7,000,000$, then $AER = 21\%$

39. If $L = 10,000,000$, then $AER = 16\%$

40. If $L = 15,000,000$, then $AER = -26\%$

The astute reader will recognize that Formula (5.2.11) should be modified slightly for retrospective-rated contracts since, if the aggregate loss is large, the additional premium is only transferred to the reinsurer as the losses are reported and is thus not available for investment from the beginning. This could be handled by modifying the discount factor.

As with facultative covers, it is clear that much of the above can and should be programmed into an interactive PC package for the underwriters and actuaries. And it is also extremely important to monitor the results of large treaties and groups of treaties. The monitoring of the pricing experience and the monitoring of the loss development and IBNR experience on the reinsurer's books and the reconciliation of both is important.
G. Pricing higher exposed layer and clash layer excess treaties

Since losses are not "expected" for higher exposed and clash layers, historical loss data is sparse or nonexistent. And yet the layers are exposed, or else the cedent wouldn't want to buy cover. Prices for these covers are usually set by market conditions: what the traffic will bear. The actuarial price is largely determined by the risk loading, and may or may not be comparable to the market price.

Where there is policy limits exposure, an exposure rate may be determined in the same manner as for a working cover. An experience rate may be determined by experience rating the working layers below and using these working cover rates as bases for estimating the higher exposed layer rate. The higher the layer, the more significant becomes the workers compensation exposure, if any, and the clash exposure.

For pure clash layer pricing, the reinsurer should keep experience statistics on all clash covers combined to see how often and to what degree these covers are penetrated, and to see if the historical market-dictated rates have been reasonable overall. The rates for various clash layers should bear reasonable relationships to each other depending upon the underlying exposures and the distance of the attachment points from the policy limits sold. Underwriters sometimes view the market rates with regard to a notion of payback - the premium should cover one loss every m years for some selected m - but this is technically inexplicable.
H. Pricing property catastrophe treaties

The price for a windstorm catastrophe treaty should depend upon the attachment point, upon the cedent's accumulation of property exposure in storm-prone localities and upon the cedent's net position on each policy after all other reinsurance. Historical experience, large losses and exposure, should be adjusted to the level of the contemplated coverage period. Changes in the cedent's non-catastrophe net retentions may have a great effect upon the catastrophe exposure excess of a given attachment point. That is, a reinsurance program can be very tricky: a small change here can have a big effect there.

The actuary can be very useful in estimating the reinsurer's total accumulated exposure to catastrophe shock losses. For each windstorm-prone locality, reinsurance exposure from every property contract should be estimated. For each contract, this would be based upon the cedent's exposed policy limits and the reinsurance coverage. Thus the reinsurer can see where too much catastrophe potential is accumulated and can better structure his own catastrophe retrocessional program. Similarly for earthquake exposure.

Various actuaries are working on catastrophe simulation computer programs which estimate loss distributions based upon an insurer's geographic exposure distribution. It may be possible that such programs can be modified for reinsurance use.
I. Pricing aggregate excess treaties

Aggregate excess treaties are sometimes called stop loss covers. They may be used to protect the cedent's net loss ratio. For example, suppose the cedent's expected loss ratio is 65% of net earned premium. A stop loss cover might cover 50% of all net loss payments in excess of a 75% loss ratio up to a 90% loss ratio. (the loss ratios are with respect to subject premium net of all other reinsurance coverage). The exposure subject to the treaty could be all or part of the cedent's net exposure.

In a sense, this is the ultimate reinsurance cover for protecting the cedent's net position if all else fails. Because of the magnitude of the risk transfer, one can guess that these covers are quite expensive, and often not available unless written on a nontraditional basis, as we shall see.

Another form of aggregate excess treaty provides coverage over an aggregate deductible on an excess layer. This is more interesting; so we will illustrate this alternative. The concepts for pricing a net stop loss are similar.

(5.2.19) EXAMPLE

Facts:

1. The facts are the same as in Example (5.2.13) except that the cedent wants to retain the first $10,000,000 of payments on the layer $700,000 excess of $300,000.
The cedent might want a $10,000,000 deductible to avoid trading dollars with the reinsurer for fairly certain loss payments. Keeping the premium for the deductible, the cedent also keeps the investment income.

We (reinsurer) would have to perform the same analysis we did in Example (5.2.13) except we would now be much more careful in our estimation of the distribution of aggregate excess loss. As we noted before, the best way to build up this distribution is from individual excess claims and amounts in Formula (5.2.17). But now, instead of simply estimating the first two or three moments, we may want to use more exact methods such as simulation or the methods espoused by Phillip Heckman and Glenn Meyers (Heckman-Meyers, 1981) or by Harry Panjer (Panjer, 1984). Gary Venter's method (Venter, 1985) of using continuous models for aggregate loss are also useful when the expected number of excess claims is large, so that the probability of having no claims is close to zero. For most low frequency cases, there is a positive probability, Prob[L = 0] > 0, a cluster point at 0, that must be accounted for. The Panjer method is especially good for the low frequency case, and has the advantage of being fairly easily explainable to non-mathematicians. We would also be very careful to account for our model and parameter uncertainty, so to get a proper spread for the aggregate loss distribution.
Suppose we have done all our homework and now have a rigorous estimate of the distribution of aggregate loss in the layer $700,000 excess of $300,000 together with an analytic representative of our model and parameter uncertainty. Suppose that once again, we believe that this can be described by a lognormal with mean $10,000,000 and standard deviation $5,000,000. For the degree of accuracy we need here, this is now probably not a good model, but we use it here simply for convenience.

Remembering item 22 under Example (5.2.13), we have:

2. \( \text{Prob}[L > 10,000,000] = .41 \)

So even though the deductible is set at the reinsurer's expectation, there is significant probability of loss and thus there should be a substantial expectation in excess of the deductible. For notational convenience, define the reinsurer's loss \( L_{10} \) in terms of the aggregate excess loss \( L \) as follows:

\[
L_{10} = \begin{cases} 
0 & \text{if } L < 10,000,000 \\
L - 10,000,000 & \text{otherwise}
\end{cases}
\]
The reader can verify the following (using Appendix D):

3. \( \text{Prob}[L_{10} > \$5,000,000] = .14 \)  
   (See Example (5.2.131), #23)

4. \( \text{Prob}[L_{10} > \$10,000,000] = .05 \)

5. \( \text{E}[L_{10}] = \$1,867,000 \)

6. Standard deviation of \( L_{10} = \$3,686,500 \)

For an unlimited cover, the risk transfer (if measured by (6)) overwhelms the expectation of $1,867,000, so the reinsurer must charge a substantial margin. The cedent will probably be reluctant to pay so large a premium and the reinsurer may be reluctant to assume so great a risk. To cut down the risk transfer and the price, the reinsurer would want an aggregate limit. Suppose the aggregate limit is $5,000,000. The reader can verify the following:

7. \( \text{E}[L_{10};5] = \$1,251,700 \)  
   (expectation of \( L_{10} \) in the layer $0 to $5,000,000)

8. Standard deviation of \( L_{10};5 = \$1,886,500 \)

The aggregate of $5,000,000 may be acceptable to the cedent if he believes an aggregate excess loss of $15,000,000 is impossible. Remember that our lognormal says that the probability of exceeding $15,000,000 is about 14%.
Along with our evaluation of the incurred aggregate loss distribution, we would more carefully estimate the excess loss payout distribution. Suppose that for each value of \( L_{10;5} \), we estimate a discount factor \( DF(L_{10;5}) \) based upon the expected payout pattern of \( L_{10;5} \), available U.S. Treasury instruments, the effect of taxes and the risk that the payout may be different than expected.

Suppose we need \( IXL = 10\% \) for this limited cover, and \( RER = 10\% \) on an undiscounted basis and \( RER = 30\% \) on a discounted basis, where an average discount factor is \( E[DF(L_{10;5})] = .60 \). Remembering that \( CI = BF = 0 \), the reader can use Formula (5.2.9) to verify the following rounded premiums:

9. Reinsurance premium 1 (undiscounted) = $1,545,000

10. Reinsurance premium 2 (discounted) = $1,200,000

The cedent may object to paying such a large premium for a pure risk cover - where we believe the chance of the aggregate losses reaching the reinsurer is only 41\%, and the chance of a complete $5,000,000 loss is 14\%. So a cover with a long-term funding mechanism and decreased risk transfer (thus lower reinsurance margin) may be more desirable. We will discuss these types of covers in the next section.
J. Pricing nontraditional reinsurance covers

The simplest example of a reinsurance cover which might be classified as nontraditional is a financial quota share. The Example (5.2.10) quota share could be modified in various ways to emphasize the financial aspects of the cover and diminish the risk transfer, thus diminishing the reinsurer's margin. For example:

(5.2.21) EXAMPLE

Facts: Same as Example (5.2.10) except for:

1. Commission slides 1% with each loss ratio 1%
2. Minimum commission = 20%
3. Reinsurance aggregate limit = 90% of reinsurance premium
4. Premium and loss payments quarterly
5. Penalty negative commission if canceled in a deficit position by cedent
6. Reinsurer expense and profit margin = 2% (at a 63% loss ratio)

The cedent still gets surplus relief from the commission on the ceded unearned premium reserve, and still decreases his premium-to-surplus ratio. You can see the drop in the reinsurer's margin from 10% in Example (5.2.10); the 2% here is just for illustration. The reinsurer's margin is meant to cover the surplus loan as before, a small charge for absorbing premium volume and thus indirectly utilizing further surplus (the reinsurer's surplus is thus not
available to support other business), the reinsurer's expenses and a small risk charge. If casualty exposure were covered, the reinsurer would credit the cedent with most of the investment income earned on the contract's cash balance according to some specified formula. As long as the contract is in a profitable position, this would be returned to the cedent as an additional commission upon cancellation or sooner.

Assuming IXL = 2% for this case, the reader can verify the following, with the other facts remaining as in Example (5.2.10):

1. Actual economic return (AER) = 6.32%

2. AER at a 78% loss ratio = 5.89%

3. AER at an 82.81% loss ratio = 0%

4. AER at a 90% loss ratio = -8.80%

When most insurance people think of nontraditional reinsurance, they think of loss portfolio transfers. A cedent may cede all or part of the its liability as of a specified accounting date; this may be for a line of business or territory no longer written, or for other reasons. Most loss portfolio transfers in the U.S. market are written in order to give the cedent the statutory surplus benefit arising from the implicit discounting of loss reserves. That is, the reinsurance premium is essentially the present value of the transferred estimated liability, plus reinsurer's expense, surplus use and risk charges. And the cedent can take reinsurance credit for the liability ceded, thus offsetting all or part of the loss reserve previously set up.
An example may clarify this. Suppose the cedent in Example (5.2.13) has been told by its domiciliary Insurance Department that it should increase loss reserves as of December 31, 1989 for the subject exposure by 20%. With Insurance Department approval, the cedent wishes to purchase a loss portfolio cover for this additional liability. Suppose the cedent would like to minimize the statutory surplus effect as much as possible. Suppose we have the following situation:

(5.2.22) EXAMPLE

Facts: Cedent as in Example (5.2.13), plus:

1. Carried loss reserve 12/31/89 = $150,000,000
2. Required additional reserve = $30,000,000

Based upon a thorough analysis of the cedent's financial reports, historical exposure, historical reinsurance program, net loss development and payout distributions by line and in aggregate, we determine that the additional $30,000,000 could easily be funded by a $15,000,000 payment. To get to this point, besides evaluating the adequacy of the cedent's loss reserves, we would pay careful attention to the historical loss payout patterns and their fluctuations. Has the recent exposure changed in such a way to cause a significant change in future loss payout? Have there been major changes in the cedent's Claim Department or claims processing? A common analytical technique is to study ratios of cumulative loss payout for each accident year divided by ultimate estimates for each category of exposure.
### PRIVATE PASSENGER AUTOMOBILE LIABILITY (FICTIONAL EXAMPLE)

**CUMULATIVE PAID LOSS AS A PERCENTAGE OF ULTIMATE LOSS**

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Est. ULT. Loss</th>
<th>Evaluation Year (End Of):</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1984</td>
<td>$6,000,000</td>
<td></td>
<td>0.50</td>
<td>0.65</td>
<td>0.80</td>
<td>0.95</td>
<td>1.00</td>
</tr>
<tr>
<td>1985</td>
<td>$7,000,000</td>
<td></td>
<td>0.40</td>
<td>0.55</td>
<td>0.70</td>
<td>0.90</td>
<td></td>
</tr>
<tr>
<td>1986</td>
<td>$8,000,000</td>
<td></td>
<td>0.63</td>
<td>0.70</td>
<td>0.75</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1987</td>
<td>$9,000,000</td>
<td></td>
<td>0.35</td>
<td>0.55</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1988</td>
<td>$10,000,000</td>
<td></td>
<td>0.40</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. **Weighted Average**: 0.450, 0.510, 0.748, 0.923, 1.000
2. **3-Year MTQ Average**: 0.461, 0.600, 0.748, 0.923, 1.000
3. **Maximum**: 0.630, 0.700, 0.900, 1.000
4. **Minimum**: 0.350, 0.550, 0.700, 0.900
5. **Trimmed Average**: 0.433, 0.600, 0.750, 0.923, 1.000
6. **Selected Mean**: 0.450, 0.610, 0.748, 0.923, 1.000
7. **Selected Extreme**: 0.630, 0.700, 0.900, 1.000

### SCENARIO I: MEAN PAYOUT

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<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>1984</td>
<td>$0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1985</td>
<td>7.7%</td>
<td></td>
<td>100%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>1986</td>
<td>25.2%</td>
<td></td>
<td>69.5%</td>
<td>30.5%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
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<tr>
<td>1987</td>
<td>39.0%</td>
<td></td>
<td>35.3%</td>
<td>45.0%</td>
<td>19.7%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>1988</td>
<td>55.0%</td>
<td></td>
<td>29.1%</td>
<td>25.0%</td>
<td>31.5%</td>
<td>14.0%</td>
<td>0%</td>
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</tbody>
</table>

### SCENARIO II: EXTREME PAYOUT

<table>
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<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1984</td>
<td>$0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1985</td>
<td>538,462</td>
<td></td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>1986</td>
<td>2,019,048</td>
<td></td>
<td>615,385</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>1987</td>
<td>3,510,068</td>
<td></td>
<td>1,238,571</td>
<td>1,576,121</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>1988</td>
<td>5,502,500</td>
<td></td>
<td>1,502,500</td>
<td>1,375,190</td>
<td>1,754,579</td>
<td>769,231</td>
<td>0%</td>
</tr>
</tbody>
</table>

**TOTAL**: 11,570,009, 4,783,196, 3,570,696, 2,446,866, 769,231, 0
A simplified example is displayed in Table (5.2.23). This table also displays the "mean" dollar payout prediction by calendar year. We would also want to determine maximal extremes based upon extreme values for the ultimate incurred loss by accident year and upon faster than expected payout. The payout predictions from all the covered liabilities would be assembled. If a lower risk, lower margin treaty were contemplated, greater care would be taken with the loss discounting: the reinsurer might consider pricing the payout stream via the use of an immunizing asset portfolio. The bond maturities would be selected to allow adequate margin for the stochastic nature of the payout.

To zero-out the surplus effect on the cedent, we would look for an attachment point where the cash payment for the loss portfolio transfer would approximately match the resulting loss reserve takedown. For example, suppose a reinsurance premium of $35,000,000 is sufficient for a cover of $65,000,000 excess of $115,000,000. This transaction would not change the cedent's beginning surplus (before reserving the additional $30,000,000).

Another example of a nontraditional reinsurance treaty is a funded aggregate excess cover. Let us transform Example (5.2.19) into such a cover.

(5.2.24) EXAMPLE

Facts: Same as Example (5.2.19) except for:

1. Aggregate limit of $5,000,000

2. Cedent desires a low cost funding cover.
Recall that the reinsurer's loss expectation is $E[L_{10}; S] = $1,251,700 with a standard deviation of $1,886,500 and a (dis-
counted) reinsurance premium of $1,200,000 ((5.2.19), #7, #8 and
#10). One possible structure for a funded cover would be that the
reinsurer takes an initial premium of $2,000,000, deducts an
expense and profit margin of only 5%, instead of the 40%
previously, and allocates 90% of the investment income to the "loss
fund". The aggregate limit might be equal to the fund plus
$1,000,000 up to a maximum of $5,000,000, and loss payments might
be only annually, to allow the fund to grow as large as possible.
As with the quota share (5.2.21), there probably would be a penalty
negative commission if the cedent cancelled in a deficit position.

Recalling that $\text{Prob}[L_{10} > 0]$ is estimated to be 0.41 and the
likelihood of a total loss in one year is estimated to be 0.14
((5.2.19), #2 and #3), we would expect the fund to build to
$5,000,000 fairly rapidly, at which time the cedent and reinsurer
could decide to increase the limit. It should be noted that the
aggregate excess attachment point of $10,000,000 should be adjusted
appropriately each coverage year to reflect aggregate loss
inflation.

K. Conclusion of the reinsurance pricing section

We have seen some examples of how standard actuarial methods and
some not-so-standard actuarial methods apply to reinsurance
pricing. We must remember that there is no one right way of
pricing reinsurance. But there are plenty of wrong ways. Common
actuarial methods should be used only to the extent they make
sense. To avoid major blunders, an underwriter/actuary must always understand as well as possible the underlying primary insurance exposure and must always be aware of the differences between the reinsurance cover contemplated and that primary exposure. The differences usually involve much less specificity of information, report and settlement timing delays and often much smaller frequency together with much larger severity, all inducing a distinctly higher risk situation. But with this goes a glorious opportunity for actuaries and other technically sophisticated people to use their theoretical mathematical and stochastic modeling abilities fully.

In the next section, we will see how reinsurance loss reserving differs from primary insurance loss reserving, and we will discuss some simple methods for dealing with these differences.

References 5.2:
Ferguson, Ronald (1978), "Actuarial Note on Loss Rating" and discussion by Gary Patrik, PCAS.
Heckman, Philip E. and Meyers, Glenn G. (1983), "The Calculation of Aggregate Loss Distributions from Claim Severity and Claim Count Distributions" and the discussion by Gary Venter, PCAS, and exhibits omitted from paper in next volume PCAS.


CAST2C.DOC August 22, 1988
5.3 REINSURANCE LOSS RESERVING

A. General considerations

For a reinsurance company, the loss reserve is usually the largest indeterminant number in the statement of the company's financial condition. It is also the most uncertain. To properly estimate a loss reserve, we must study the runoff of the past business of the company. As a result of this process, we should not only be able to estimate a loss reserve as of a certain point in time, but we should also be able to estimate historical loss ratios, loss reporting patterns and loss settlement patterns by year, by line and by type of business in enough detail to know whether or not a particular contract or business segment is unprofitable and when. This information should also be applicable to future pricing and decision-making. The goal is to deliver good management information regarding the company's historical contract portfolio, and also deliver some indications of where the company may be going.

Reinsurance loss reserving has many of the same problems as primary insurance loss reserving, and many of the same methods can be used. But there are also various technical problems which make reinsurance loss reserving somewhat more difficult. I will outline some of these problems and then suggest various techniques for handling them.
B. Reinsurance loss reserving problems

There seem to be seven major technical problems which make reinsurance loss reserving somewhat more difficult than loss reserving for a primary company. These technical problems are:

(5.3.1) Claim report lags to reinsurers are generally longer, especially for casualty excess losses.

The claim report lag, the time from date of accident until first report to the reinsurer, is exacerbated by the longer reporting pipeline - a claim reported to the cedent must first be perceived as being reportable to the reinsurer, then must filter through the cedent's report system to his reinsurance accounting department, then may journey through an intermediary to the reinsurer, then must be booked and finally appear upon the reinsurer's claim system. The report lag is also lengthened by the undervaluation of serious claims by the cedent - for a long time, an ultimately serious claim may be valued below the reinsurance reporting threshold. This is not an indictment of primary company claims staffs, but simply an observation that a claims person, faced with insufficient and possibly conflicting information about a potentially serious claim, may tend to reserve to probable "expectation". While this "expectation" may be sufficient for most claims with a certain probable fact pattern, it is those few which blow up above this average which will ultimately be covered by the reinsurer. Thus these larger claims generally are reported later to the reinsurer than are the smaller claims the cedent carries net.
Also, certain kinds of mass tort claims with really extreme discovery or reporting delays to the cedent, such as asbestosis, are reinsured heavily, so their extreme report lags have a big impact on the reinsurer's experience. Just as we saw these time delays adding greatly to the uncertainty in reinsurance pricing, they also add greatly to the uncertainty in reinsurance loss reserving.

(5.3.2) There is a persistent upward development of most claim reserves.

Economic and social inflation cause this development. It may also be caused by a tendency of claimspeople to reserve at average values, as noted in (5.3.1). Also, there seems to be a tendency to under-reserve allocated loss adjustment expenses. Thus, early on, the available information may indicate that a claim will pierce the reinsurance retention, but not yet indicate the ultimate severity.

(5.3.3) Claims reporting patterns differ greatly by reinsurance line, by type of contract and specific contract terms, by cedent and possibly by intermediary.

The exposure assumed by a reinsurance company can be extremely heterogeneous. This is a problem because most loss reserving methods require the existence of large homogeneous bodies of data. The estimation methods depend upon the working of the so-called law of large numbers; that is, future development en masse will duplicate past development because of the sheer volume of data with similar underlying exposure. Reinsurers do not have this theoretical luxury, since many reinsurance contracts are unique,
and even when there exists larger aggregates of similar exposure, loss frequency may be so low and report lags so long that there is extreme fluctuation in historical loss data. Thus, normal actuarial loss development methods may not work very well.

As we discussed in Section 2, a reinsurer knows much less about the specific exposures being covered than does a primary carrier. Also, the heterogeneity of reinsurance coverages and specific contract terms creates a situation where the actuary never has enough information and finds it difficult to comprehend what is being covered and the true exposure to loss. This is especially true for a reinsurer writing small shares of brokered business.

(5.3.4) Because of the heterogeneity in (5.3.3), it is difficult to use industry statistics.

Every two years, the Reinsurance Association of America (RAA) publishes a summary of casualty reinsurance loss development statistics. These statistics give a very concrete demonstration of the long report and development lags encountered by reinsurers. However, as is noted by the RAA, the heterogeneity of the exposure and reporting differences by company make the statistics of questionable use for particular loss reserving situations.

Likewise, for any two reinsurers, Annual Statement Schedules O and P by-line exposure and loss development are essentially incomparable. Annual Statement lines of business do not provide a good breakdown of reinsurance exposure into reasonably homogeneous exposure categories useful for loss reserving; proper categori-
zation follows the pricing categories we have already seen, and will vary by reinsurance company according to the types of business the company specializes in. This is a problem for two reasons:

1. many people who are not expert in reinsurance insist upon evaluating a reinsurer's loss reserves according to Schedule O and P statistics, and 2. for an actuary examining a reinsurer for the purpose of loss reserving, an appropriate exposure categorization for the particular company may not be as apparent or as easily accomplished as for a primary company.

(5.3.5) The reports the reinsurer receives may be lacking certain information.

Most proportional covers require only summary claims information; often the data are not even split by accident year or by coverage year. Since loss liabilities must be evaluated as of the end of an accident period, calendar year or underwriting year (akin to policy year - all premiums and losses for a contract are assigned to effective or renewal date of the contract) statistics are not sufficient; various interpretations and adjustments must be made. Even when there is individual claims reporting such as on excess-of-loss covers, there often is insufficient information for the reinsurer's claimspeople to properly evaluate each claim without exerting great effort in pursuing information from the cedent.
This is why it is necessary to have a reasonably large, professional reinsurance claims staff even though the cedent's claims staff is handling the claims. Also, reinsurance claims-people are more accustomed to handling large claims with catastrophic injuries, thus being able to advise the cedent's staff and limit the ultimate payments (and advise in the rehabilitation of seriously injured parties).

For loss reserving, it is useful to have an exposure measure against which to compare loss estimates. One possible measure is reinsurance premium by year by Annual Statement line. On most contracts, losses may be coded correctly by Annual Statement line, but very often the reinsurance premium is assigned to line according to a percentage-breakdown estimate made at the beginning of the contract. To the degree that this estimate does not accurately reflect the reinsurer's loss exposure by Annual Statement line, any by-line comparisons of premiums and losses may be distorted. This adds to the problems noted in (5.3.4).

For most treaties, premiums and losses are reported quarterly in arrears; they may not be reported (and paid) until some time in the following quarter. Thus there is an added IBNR exposure for both premiums and losses. The actuary must remember that the latest-year premiums may be incomplete, so they may not be a good measure of latest-year exposure.

(5.3.6) Because of the heterogeneity in coverage and reporting requirements, reinsurers often have data coding and EDP systems problems.
All reinsurers have management information systems problems. The business has grown in size and complexity faster, and expectations regarding the necessary level of data detail have grown faster, than reinsurers' data systems' abilities to handle and produce the reports requested by marketing, underwriting, claims, accounting and actuarial staffs. This problem may be endemic to the insurance business, but it is even more true for reinsurance.

(5.3.7) The size of an adequate loss reserve is greater for a reinsurer.

This is not a purely technical problem; it is more a management problem, and many reinsurance companies have stumbled over it. All the above problems act to increase the size of an adequate loss reserve and also make it more uncertain. It is difficult for the actuary to overcome the disbelief on the part of management and marketing people and convince them to allocate adequate resources for loss liabilities. Eventually, claims emerging on old exposure overwhelms this disbelief, at least for those who listen. A cynic might say that many reinsurance managers change jobs often enough to stay ahead of their IBNR. Start-up operations in particular have this problem - if there is no concrete runoff experience to point to, why believe a doom-saying actuary.

These seven problems imply that uncertainty in measurement and its accompanying financial risk are large factors in reinsurance loss reserving. This has become an even more important item because of the 1986 Tax Act requiring discounting of loss reserves for income tax purposes. This discounting eliminates the implicit margin for
adverse deviation which had been built into insurance loss reserves simply by not discounting. Insurers have lost this implicit risk buffer. Since this buffer now flows into profits and thus is taxed sooner, assets decrease. This clearly increases insurance companies' risk level. The effect on reinsurers is greater.

C. Components of a reinsurer's loss reserve

The general components of a reinsurer's statutory undiscounted loss reserve are as follows:

(5.3.8) COMPONENTS OF A REINSURER'S LOSS RESERVE

1. Case reserves reported by the ceding companies.

These may be individual case reports or may be reported in bulk, depending upon the loss reporting requirements of each individual contract. Most excess-of-loss contracts require individual case reports, while most proportional contracts allow summary loss reporting.

2. Reinsurer additional reserves on individual claims.

The reinsurer's claims department usually reviews individual case reserve reports and specifies additional case reserves (ACR) on individual claims as necessary. This second component of additional case reserves may vary considerably by contract by cedent.

3. Actuarial estimate of future development on (1) and (2).
The future development on known case reserves in total is sometimes known is IBNRR, incurred (and reported) but not enough reserved.

4. Actuarial estimate of pure IBNR.

Most actuaries would prefer that separate estimates be made for (3) and (4) the estimate of pure IBNR, incurred but not reported. However, because of limitations in their data systems, in practice most reinsurers combine the estimates of (3) and (4). Depending upon the reinsurer's mix of business, these together may amount to more than half the total loss reserve.

Unless otherwise noted, the term IBNR henceforth shall stand for the sum of IBNHR and pure IBNR.

5. Risk load.

The last component of a loss reserve should be the risk loading or adverse deviation loading necessary to keep the reserve at a suitably conservative level, so as not to allow uncertain income to flow into profits too quickly. Many loss reserving professionals prefer to build this into the reserve implicitly by employing conservative assumptions and methodology. Many actuaries would prefer to see it estimated and accounted for explicitly; however, this would violate current AICPA standards. Because of the long-tailed nature of much of their exposure and its heterogeneity and the uncertainty of their statistics, this component is theoretically more important for reinsurers.
The above items (1) through (5) refer to undiscounted statutory loss reserves. Not considered is a loss reserve component for the offset arising from future investment income. Even when we must estimate this and record it on our financial statements, most actuaries would prefer to account for it separately from the undiscounted statutory loss reserve. See the chapter on loss reserving for more discussion on this.

D. A general procedure

The steps involved in reinsurance loss reserving methodology are as follows:

(5.3.9) A GENERAL PROCEDURE

1. Partition the reinsurance portfolio into reasonably homogeneous exposure groups.

2. Analyze the historical development patterns. If possible, consider individual case reserve development and the emergence of IBNR claims separately.

3. Estimate the future development. If possible, estimate the bulk reserves for IBNER and pure IBNR separately.

4. Monitor and test the predictions, at least by calendar-quarter.

Let us now proceed to discuss the first step in some detail.
E. Exposure groups

It is obviously important to segregate the contracts and loss exposure into categories of business on the basis of loss development potential. Combining loss data from nonhomogeneous exposures into large aggregates can increase measurement error rather than decrease it.

Reasonably homogeneous exposure categories for reinsurance loss reserving have been discussed in the actuarial literature and follow closely the categories used for pricing.

Table (5.3.10) lists various important variables for partitioning a reinsurance portfolio. All affect the pattern of claim report lags to the reinsurer and the development of individual case amounts. The listing is meant to be in approximate priority order.

**TABLE 5.3.10**

**PARTITIONING THE REINSURANCE PORTFOLIO INTO REASONABLY HOMOGENEOUS EXPOSURE GROUPS**

Important variables:

1. Type of contract (1): Facultative, Treaty

2. Line of business (1): Property, Casualty, Bonding, Ocean Marine, etc.
3. Type of contract (2): Quota share, surplus share, excess per-risk, excess per-occurrence, aggregate excess, catastrophe, loss portfolio transfer, etc.

4. Layer: primary, working, higher excess, clash.

5. Contract Terms: flat-rated, retro-rated, sunset clause, share of loss adjustment expense, claims-made or occurrence coverage etc.

6. Type of Cedent: Small, large, or E&S company.


8. Intermediary

Obviously, a partition by all seven variables would split a portfolio into numerous pieces with too little credibility. However, after partitioning by the first three variables, it may be desirable to recognise the effects of various of the other variables. For example, for Treaty Casualty Excess business, certain reinsurers have found that the type of cedent company (6) is an important indicator of report lag.

Since each reinsurer's portfolio is unique and extremely heterogeneous, in order to determine a suitable partition of exposure for reserving and results analysis, one must depend greatly upon the knowledge and expertise of the people writing and underwriting the exposures, the people examining individual claim reports and the people processing data from the cedents. Their
knowledge, together with elementary data analysis (look at the loss development statistics), point the actuary toward the most important variables.

One possible first-cut partition of assumed reinsurance exposure is shown in Table (5.3.11), remembering that there is no such thing as a "typical" reinsurance company.

TABLE 5.3.11
EXAMPLE OF MAJOR EXPOSURE GROUPS FOR A "TYPICAL" REINSURANCE COMPANY

Treaty Casualty Excess
Treaty Casualty Proportional
Treaty Property Excess
Treaty Property Proportional
Treaty Property Catastrophe
Facultative Casualty
Facultative Property
Surety Excess
Surety Proportional
Fidelity Excess
Fidelity Proportional
Ocean Marine Treaty
Ocean Marine Facultative
Nontraditional Reinsurance
Miscellaneous Special Contracts
Within these major categories, the exposure should be further refined. For example, Treaty Casualty Excess exposure may be further segregated by type of retention (per occurrence excess vs aggregate excess), by type of cedent company (small vs large vs E&S carriers), by layer of coverage (working vs higher clash layers) and by line of business(2) (automobile liability, general liability, workers compensation, medical professional liability). Each of these categories would be expected to have distinctly different lags for claims reported to the reinsurer.

Categories for Treaty Casualty Proportional business would be similar. As we have seen, many contracts classified as proportional are not shares of first dollar primary layers, but rather shares of higher excess layers; thus, whether the exposure is ground-up or excess may be an important variable.

Loss reserving categories for Facultative Casualty would certainly separate out automatic primary programs and automatic umbrella programs; the certificate exposure could be split into buffer versus umbrella, and then further by line of business(2).

Likewise for property and other exposures, the loss reserving categories should correspond closely to the pricing categories.

It is convenient to split the consideration of the historical analysis and estimation ((5.3.9), #2 and #3) according to the length of the claim report lag for different exposure categories.
Methodology for short-tailed exposure categories

As is generally true, the best methodologies to use are those which provide reasonable accuracy for least effort and cost. For short-tailed lines of business, such as most property coverage exposure, losses are reported and settled quickly, so loss liabilities are relatively small and run off very quickly. Thus, elaborate loss development estimation machinery is unnecessary.

Reinsurance categories of business which may usually be considered to be short-tailed (as with anything about reinsurance, be careful of exceptions) are listed in Table (5.3.12).

TABLE 5.3.12

REINSURANCE CATEGORIES WHICH ARE USUALLY SHORT-TAILED

(WITH RESPECT TO CLAIM REPORTING AND DEVELOPMENT)

<table>
<thead>
<tr>
<th>CATEGORY</th>
<th>COMMENTS*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treaty Property Proportional</td>
<td>Beware of recent catastrophes</td>
</tr>
<tr>
<td>Treaty Property Catastrophe</td>
<td>Beware of recent catastrophes</td>
</tr>
<tr>
<td>Treaty Property Excess</td>
<td>Possibly exclude high layers</td>
</tr>
<tr>
<td>Facultative Property</td>
<td>Exclude construction risks</td>
</tr>
<tr>
<td>Fidelity Proportional</td>
<td></td>
</tr>
</tbody>
</table>

* Exclude all international exposure, if possible.
Many reinsurers reserve property business by setting IBNR equal to some percentage of the latest-year earned premium. A rule of thumb in the industry seems to be that 5-6% is a reasonable percentage. Since most property losses are not coded by accident year, the possible improvement in estimation by using more sophisticated methodology is probably not cost effective.

However, it is a good idea to separately consider major storms and other major catastrophes. A recent catastrophe may cause real IBNR liability to far exceed the 5-6% rule loss reserve. Hurricane losses, even on proportional covers, may not be finalized for a few years.

Another simple method used for short-tailed exposure, for new lines of business or for other situations where the reinsurer has little or no loss statistics is to reserve up to a selected loss ratio. For short-tailed exposure, as long as the selected loss ratio bears some reasonable relationship to past years' experience and as long as it is larger than that computed from already-reported losses, this may be a reasonable method.

Another useful method for short-tailed lines of business is to use the standard American Chainladder (CL) Method of age-to-age factors on cumulative aggregate incurred loss triangles. As long as accident year data exists and the report lags are small, this is sufficiently good methodology. An advantage of this method is that it correlates future development with an overall lag pattern and very definitely correlates it with reported losses for each accident year. A major disadvantage, at least for long-tailed
lines, is simply that the IBNR is so heavily correlated with reported losses; so for recent, immature years, the reported, very random nose wags the extremely large tail estimate.

For some proportional treaties, summary loss reporting may assign claims by underwriting year, according to inception or renewal date of the reinsurance treaty instead of by accident year; the reinsurer's claims accounting staff has no choice but to book the claims thusly. So the loss statistics for each false "accident" year may show great development because of future occurring accidents. To get a more accurate loss development picture and estimate IBNR properly, one can assign these "accident" year losses to approximate true accident year by percentage estimates based upon the underwriting year inception date and the general report lag for the type of exposure. Summary claims reported on a calendar (accounting) year basis can likewise be assigned to accident year by percentage estimates, if necessary. For short-tailed lines reserved by a percentage of premium or reserved up to a selected loss ratio, these re-assignments are unnecessary.

G. Methodology for medium-tailed exposure categories

Let us consider any exposure for which claims are almost completely settled within five years and with average aggregate report lag of one to two years to be medium-tailed for this discussion. Reinsurance categories of business one might consider here are listed in Table (5.3.13).
TABLE 5.3.13

REINSURANCE CATEGORIES WHICH ARE USUALLY MEDIUM-TAILED
(WITH RESPECT TO CLAIM REPORTING AND DEVELOPMENT)

<table>
<thead>
<tr>
<th>CATEGORY</th>
<th>COMMENTS*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treaty Property Excess</td>
<td>If it is possible to separate these from working layers</td>
</tr>
<tr>
<td>Higher Layers</td>
<td></td>
</tr>
<tr>
<td>Construction Risks</td>
<td>If it is possible to separate these from other property exposure</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Surety</td>
<td></td>
</tr>
<tr>
<td>Fidelity Excess</td>
<td></td>
</tr>
<tr>
<td>Ocean Marine</td>
<td></td>
</tr>
<tr>
<td>Any International Property</td>
<td></td>
</tr>
<tr>
<td>Exposure</td>
<td></td>
</tr>
</tbody>
</table>

Even if a property claim is known almost immediately, its ultimate value may not. Thus it may take longer to penetrate a higher peril risk excess attachment point. This happens more often if time element coverage is involved. The discovery period for construction risk covers may extend years beyond the contract (loss occurrence) period. So for both these exposures, claim report lags may be significantly longer than normal for property business.

For Surety exposure, it is wise to consider losses gross of salvage and, separately, salvage recoveries. The gross losses are reported fairly quickly, but the salvage recoveries have a long tail. It is instructive to consider for mature years the ratio of salvage to
gross loss; this ratio is fairly stable and may help explain predictions for recent coverage years as long as the underwriters can predict how the salvage ratio may have slowly changed over time.

For medium-tailed exposure, the CL Method using aggregate reported losses, with or without ACRs, will yield reasonably accurate answers. An alternative estimation method is the so-called Bornheuter-Ferguson (BF) Method (Bornheuter, R. and Ferguson, R., 1973) which is discussed in the chapter on loss reserving. This method uses a selected loss ratio for each coverage year and an aggregate dollar report lag pattern specifying the percentage of ultimate aggregate loss expected to be reported as of any evaluation date. An advantage of this method is that it correlates future development with an exposure measure:

$$\text{exposure} = \text{reinsurance premium} \times \text{selected loss ratio}.$$ 

A disadvantage is that the BF IBNR estimate is very dependent upon arbitrarily selected loss ratios; also, the estimate for each accident year does not reflect the particular to-date reported losses for that year, unless the selected loss ratio is chosen with this in mind. The to-date reported loss for a given accident year is strongly correlated with the place of that year in the reinsurance profitability cycle; it would seem to be desirable to use this fact in the IBNR estimate. As noted before, the reinsurance profitability cycles are more extreme than primary insurance cycles. Thus, when using the BF Method, one must select the accident year loss ratios carefully.
An estimation method which overcomes some of the problems with the CL and BF Methods was independently derived by Edward Weissner (written up in Patrik (1978) and Weissner (1981)) and Hans Buehlmann (internal Swiss Re publications). As with the CL and BF Methods, this method, let us call it the Weissner-Buehlmann (WB) Method, uses an aggregate known loss lag pattern which may be estimated via the CL Method. The key innovation is that the ultimate expected loss ratio for all years combined is estimated from the overall known loss experience, instead of being selected arbitrarily. The problem with the WB Method is that it does not tell the user how to adjust the overall loss ratio an appropriate a priori loss ratio by accident year. It is left to the user to adjust each year's premium to reflect the profit cycle on a relative basis. A simple example will explain this.

(5.3.14) EXAMPLE:

For a given exposure category with five years experience, assume that the yearly risk earned premiums (net of reinsurance commissions and brokerage fees) can be adjusted to remove any suspected rate differences by year and the internal expense ratio is constant year to year, so that a single ELR can represent the expected loss ratio for each year. In primary insurance terms, assume that the premiums have been put on-level. Let ELR represent this expected loss ratio to adjusted risk earned premium. Suppose that Table (5.3.15) displays the current experience for this category:
### TABLE 5.3.15

**EXAMPLE**

Data as of 12/31/88 in 1,000's

<table>
<thead>
<tr>
<th>Cal/Acc Year</th>
<th>(1) Risk Earned Premium</th>
<th>(2) Adjusted Earned Premium</th>
<th>(3) Aggregate Reported Loss</th>
<th>(4) Aggregate Reported Loss Lag</th>
</tr>
</thead>
<tbody>
<tr>
<td>1984</td>
<td>$ 6,000</td>
<td>$ 8,000</td>
<td>$ 7,000</td>
<td>95%</td>
</tr>
<tr>
<td>1985</td>
<td>7,000</td>
<td>7,000</td>
<td>5,000</td>
<td>85%</td>
</tr>
<tr>
<td>1986</td>
<td>8,000</td>
<td>6,000</td>
<td>3,000</td>
<td>70%</td>
</tr>
<tr>
<td>1987</td>
<td>9,000</td>
<td>7,000</td>
<td>2,000</td>
<td>50%</td>
</tr>
<tr>
<td>1988</td>
<td>10,000</td>
<td>10,000</td>
<td>4,000</td>
<td>30%</td>
</tr>
<tr>
<td>TOTAL</td>
<td>40,000</td>
<td>38,000</td>
<td>21,000</td>
<td>NA</td>
</tr>
</tbody>
</table>

The IBNR estimate is given by (5.3.16):

\[
(5.3.16) \quad \text{WB IBNR est.} = \sum \text{year i IBNR est.} = \sum (\text{ELR est. x adj. earned premium x (1 - lag)})
\]

\[
= \text{ELR est. x Sum (adj. earned premium x (1 - lag))}
\]

\[
= \text{ELR est. x ((8,000 x .05) + (7,000 x .15) + (6,000 x .3) + (7,000 x .5) + (10,000 x .7))}
\]

\[
= \text{ELR est. x 13,700}
\]
The ELR estimate may be written as in (5.3.17):

\[(5.3.17) \quad \text{WB ELR est.} = \frac{\text{Total reported losses} + \text{Total IBNR est.}}{\text{Total adjusted earned premium}} = \frac{($21,000 + \text{IBNR est.})}{38,000}\]

The trick is putting these two together:

\[(5.3.18) \quad \text{WB ELR est.} \times 13,700 = \text{WB IBNR est.} = (\text{WB ELR est.} \times 38,000) - 21,000\]

or

\[(5.3.19) \quad \text{WB ELR est.} \times (38,000 - 13,700) = 21,000\]

or

\[(5.3.20) \quad \text{WB ELR est.} = \frac{21,000}{24,300} = .864\]

Table (5.3.21) compares IBNR and estimated ultimate loss ratios for CL and WB Methods; the BF and WB Methods cannot be compared, since the BF loss ratios are not estimated by formula.
### TABLE 5.3.21

**COMPARISON OF CHAINLADDER AND WEISSNER-BEUHLMANN METHODS**

<table>
<thead>
<tr>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cal/ Accident Year</td>
<td>Risk Premium</td>
<td>Chainladder Estimates</td>
<td>Weissner-Buehlmann Estimates</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1984</td>
<td>$6,000</td>
<td>$368</td>
<td>123%</td>
<td>$346</td>
<td>122%</td>
</tr>
<tr>
<td>1985</td>
<td>7,000</td>
<td>882</td>
<td>84%</td>
<td>907</td>
<td>84%</td>
</tr>
<tr>
<td>1986</td>
<td>8,000</td>
<td>1,386</td>
<td>54%</td>
<td>1,555</td>
<td>57%</td>
</tr>
<tr>
<td>1987</td>
<td>9,000</td>
<td>2,000</td>
<td>50%</td>
<td>3,024</td>
<td>56%</td>
</tr>
<tr>
<td>1988</td>
<td>10,000</td>
<td>9,333</td>
<td>133%</td>
<td>6,048</td>
<td>100%</td>
</tr>
<tr>
<td>TOTAL</td>
<td>40,000</td>
<td>13,969</td>
<td>87%</td>
<td>11,880</td>
<td>82%</td>
</tr>
</tbody>
</table>

As long as the rate relativity adjustments to yearly risk earned premium are reasonably accurate, the yearly and overall results are more accurate with the WB Method. It is easy to see that the above example would be more vivid if a longer-tailed example were used.

### H. Methodology for long-tailed exposure categories

Just as for pricing, the real problem in loss reserving is long-tailed exposure, especially excess-of-loss casualty reinsurance. Reinsurance categories of business usually considered to be long-tailed are listed in Table (5.3.22).
TABLE 5.3.22

REINSURANCE CATEGORIES WHICH ARE USUALLY LONG-TAILED
(WITH RESPECT TO CLAIM REPORTING AND DEVELOPMENT)

<table>
<thead>
<tr>
<th>CATEGORY</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treaty Casualty Excess</td>
<td>Includes the longest lags</td>
</tr>
<tr>
<td>Treaty Casualty Proportional</td>
<td>Some of this exposure may possibly be medium-tailed</td>
</tr>
<tr>
<td>Facultative Casualty</td>
<td>Some of this exposure may possibly be medium-tailed</td>
</tr>
</tbody>
</table>

The first step is to separate these exposures into finer categories. This is, of course, an iterative process. Depend upon the company's marketing, underwriting, claims and accounting personnel for the first stage categorization. Further refinements will then depend upon your hypothesis testing and upon your investigation of various comments from the marketing and underwriting people as they receive the estimated IBNR by major contract or category based upon the latest categorization. It may be desirable to treat claims-made exposure separately, if possible.

It may be necessary to separate out certain types of losses for special consideration; for example, claims arising from asbestosis, environmental and other mass tort claims. Because of the catastrophic significance of these types of claims (nothing for many years, then suddenly, gigantic totals), they would terribly distort the development statistics if left in the normal loss data. Also, it is unlikely that normal actuarial loss development techniques, if used blindly, would yield reasonable answers for
these types of losses. The question of which claims should be specially treated is difficult and should be discussed thoroughly with the company's claims staff.

For long-tailed exposure, current methodology is usually the CL, BF or WB Methods. However, with the extreme lags encountered here, it may pay to consider the estimation of IBNER separately from the estimation of pure IBNR. For the estimation of pure IBNR, it is appropriate to consider the estimation of IBNR counts and amounts separately. These separate estimates can be input to standard risk theoretic models for aggregate losses so that the loss runoff can be viewed as a stochastic process.

An advantage of using a claim count/claim severity model is that we can contemplate intuitively satisfying models for various lag distributions, such as the time from loss event occurrence until first report and the lag from report until settlement. We can then connect these lags with appropriate models for the dollar reserving and payments on individual claims up through settlement.

Perhaps the best way to describe a simple modeling approach is through use of a simple example.

(5.3.23) EXAMPLE

Facts:

1. We want to estimate gross assumed IBNER and pure IDNR for our Treaty Casualty Excess working cover business assumed from large companies; these working covers would have fairly high attachment points.
2. We have the usual development triangles for reported and paid aggregate dollars and claim counts for the last 25 accident years; the claims are separated by major line: automobile liability, general liability, workers compensation.

3. We have risk earned premiums by year for the last 15 calendar years.

4. We have talked with the marketing people, underwriters and claimspeople to see if there are any special contracts, exposures or types of claims which should be treated separately or particularly large individual claims which should be censored so as not to have an undue random impact on the estimation.

As a result of our discussions(4), we decide to separate out only asbestosis-related claims. Also, we decide to censor (limit) to a value of $5,000,000 each six large reported claims, so to smooth their impact upon the claim severity estimates; the final claim severity estimates will be adjusted to account for potential severity excess of $5,000,000.

We shall pay particular attention to the estimation of the report lag distribution, the time from claim occurrence until first report. By first report we shall mean the month in which the claim first appears in the reinsurer's claims database with a significant (nonprecautionary) dollar value. If, in addition to the summary claim count development triangles, we also have individual claims data with accident date and first report date for each claim, then
for various selected probabilistic models, we can obtain parameters via maximum likelihood estimation as discussed by Weissner (1978 and 1981) and John (1982).

Alternatively, suppose we have only the summary development triangles of reported claims. Also in this case, maximum likelihood estimates of model parameters may be made on these data by treating the increments for each development interval as grouped data, exactly as discussed by Hogg and Klugman (1984) for claims severity. The reported claim counts for each accident year can be considered to be a sample from a truncated model (unknown tail). A slight practical problem here may be negative increments, but for the estimation, the time intervals for the grouping of the data need not necessarily all be one year periods, so the intervals can always be adjusted to avoid negative increments. Or negative increments can be handled separately by estimating claim dropout rates (closed no payment). To simplify this discussion, let us assume that claims closing without payment drop out of the count.

Various statistical and reasonableness tests can then help us decide which model best describes the data and which we believe will best predict future claims arrivals. This model with the fitted parameters can then be used to predict IBNR claim emergence.

Assume now that we have estimated claim report lag distributions for each line. Assume also that we don't trust the breakdown of reinsurance premiums by line, so they cannot serve as a by-line exposure base for IBNR estimation. Without a reasonable by-line exposure, the only achievable by-line IBNR estimates are via the CL
Method, hardly credible for immature accident years. An alternative is to estimate the overall report lag distribution by weighing together the lags for each line. Technically, the weights could vary by accident year. Let us assume in this case that constant weights over all accident years are reasonable.

For simplicity, let us suppose that our combined all-lines report lag is estimated to be a lognormal with mean = 6 years and coefficient of variation = 1 (u = 1.2918, s = 1). Suppose that Table (5.3.24) displays our claims situation:

TABLE 5.3.24
TREATY CASUALTY EXCESS WORKING COVER EXAMPLE - as of 12/31/88

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Reported Claims</th>
<th>Estimated Report Lag</th>
<th>CL IBNR Est. Claims</th>
</tr>
</thead>
<tbody>
<tr>
<td>1964</td>
<td>39</td>
<td>96.89%</td>
<td>1.3</td>
</tr>
<tr>
<td>1965</td>
<td>27</td>
<td>96.58%</td>
<td>1.0</td>
</tr>
<tr>
<td>...</td>
<td>:</td>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td>1984</td>
<td>20</td>
<td>58.41%</td>
<td>14.2</td>
</tr>
<tr>
<td>1985</td>
<td>11</td>
<td>48.45%</td>
<td>11.7</td>
</tr>
<tr>
<td>1986</td>
<td>13</td>
<td>35.36%</td>
<td>23.8</td>
</tr>
<tr>
<td>1987</td>
<td>5</td>
<td>18.77%</td>
<td>21.6</td>
</tr>
<tr>
<td>1988</td>
<td>0</td>
<td>2.36%</td>
<td>0</td>
</tr>
<tr>
<td>TOTAL</td>
<td>473</td>
<td>NA</td>
<td>231.4</td>
</tr>
</tbody>
</table>
Suppose that we can adjust the annual reinsurance premiums for rate relativities and relative excess frequency year by year with some, but not total, confidence. Then we can also estimate IBNR via the WB Method and a reasonable "credibility" method in Table (5.3.25):

TABLE 5.3.25
TREATY CASUALTY EXCESS WORKING COVER EXAMPLE - as of 12/31/88

<table>
<thead>
<tr>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
<td>Adj. Earn Premium (1,000's)</td>
<td>Reported Claims</td>
<td>Report Lag</td>
<td>CL IBNR Claims</td>
<td>WB IBNR Claims</td>
<td>Cred IBNR Claims</td>
<td>Claim Freq. (3+7)/2</td>
</tr>
<tr>
<td>1964</td>
<td>$8,000</td>
<td>39</td>
<td>96.89%</td>
<td>1.25</td>
<td>1.25</td>
<td>1.325</td>
<td>NA</td>
</tr>
<tr>
<td>1965</td>
<td>8,000</td>
<td>27</td>
<td>96.58%</td>
<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
<td>NA</td>
</tr>
<tr>
<td>'74-'88</td>
<td>100,000</td>
<td>166</td>
<td>6.00</td>
<td>205.93</td>
<td>234.02</td>
<td>231.21</td>
<td>3.972</td>
</tr>
<tr>
<td>TOTAL</td>
<td>100,000</td>
<td>473</td>
<td>Average</td>
<td>231.42</td>
<td>259.54</td>
<td>256.66</td>
<td>NA</td>
</tr>
</tbody>
</table>

The reader can verify the following:
5. For those years with adjusted premium, the WB IBNR estimated claims in column 6 are computed with respect to a claim frequency of 4.0 per $1,000,000 (estimated via the WB Method for the most recent 15 years). The earlier years simply use the CL IBNR estimated claims.

6. The "credibility" IBNR estimated claims in column 7 is a weighing of the CL and WB estimated claims using the report lag as a weight for each CL estimate.

Exhibit (5.3.26) is a picture of claim count and claim reporting lags for accident year 1984 from an actuarial point of view (using the total credibility claim count of 33.9 and the expected report lag distribution). Suppose that the commonly used Poisson distribution, with parameter \( n \) say, is a good model for the total claim count \( N \). Then the number of claims reported in the \( i \)th year, \( N(i) \), will also be Poisson with parameter \( n*p(i) \), where \( p(i) \) is the lag probability for the \( i \)th year; that is, \( p(i) \) is the probability that a claim will be reported between \( i-1 \) and \( i \) years after its occurrence. This Poisson assumption allows us to make interconnected probability statements about claim reports from year to year. Under these assumptions, the reader can verify the following:

7. The standard deviation of the accident year 1984 Credibility IBNR is 3.72 claims.

8. \( \text{Prob}[225 < \text{Cred. IBNR total} < 288] = .95 \) (approximate)
Please note that these estimates simplistically assume that we know the true parameter \( n \) for the Poisson. In reality, sample error on the estimate of \( n \) should be considered, perhaps, also simplistically, inducing a Negative Binomial distribution.

In addition to estimating report lag distributions and estimating IBNR claim counts, we must also estimate IBNER and the IBNR claim severities. Various techniques from the chapter on Loss Reserving may be used. Or, if you have the information, an approach similar to that displayed by Table (5.2.18) might be used.

Once the various distributions for counts and amounts are estimated, the aggregate losses reported in the \( i \)th year of run-off can then be modeled via the standard risk theoretic model under suitable assumptions for the claim sizes. We already saw this model in (5.2.17), repeated here for convenience:

\[
(5.3.27) \quad L(t) = X(t,1) + X(t,2) + \ldots + X(t,N)
\]

where \( N = \) number of (paid) claims (or occurrences) and \( X(t,i) = \) amount of the \( i \)th claim at time \( t \)

The \( N, L \) and \( X \)'s here may represent the pure IBNR for a particular accident year. Given appropriate models for \( N \) and \( X \) and suitable assumptions, we can approximate the probability distribution of \( L \). Then we can ask various probability questions as we did in Section 2.

Given models like (5.3.27) for each part of IBNER and IBNR, the various \( L \)'s can be added and we can talk about the joint distribution of the sum.
We should note here that various authors have used very different approaches to estimate the distribution of aggregate IBNER and IBNR. The reader should refer to the bibliography for advanced and/or different methodologies, especially see Taylor (1985), Eegehn, J. van (1981) and various Advanced Techniques sessions in Casualty Loss Reserve Seminar transcripts.

A problem with increasingly sophisticated methodologies is that the answers may become less intuitive and may be much more difficult for the actuary to understand and explain to management and others. Here I recommend the use of an exhibit format like (5.2.17); the few actual estimates (expected settlement average size for reported open and IBNR claims and the expected IBNR count) are cleanly separated from the known numbers, but are juxtaposed for comparison. Various probable future settlement scenarios can be displayed for comparison. These and the monitoring reports to be discussed in the next section are important for management (and actuarial) decision-making.

I. Monitoring and testing predictions

A loss reserve or an IBNR reserve is an hypothesis about future claims settlements for past events. In order to validate your methodology, you must test your predictions against actual future experience. Monitoring and testing quarterly claims runoff against predictions may provide early warning of problems.
For short-tailed and medium-tailed lines, this can be fairly simple. As long as current accident year claims can be reasonably separated from past accident year runoff, the runoff can be compared with the previous year-end reported open and IBNR reserves.

For long-tailed lines, slightly more sophisticated comparisons are necessary. Table (5.3.28) is one possible format:

Table 5.3.28
TREATY CASUALTY EXCESS WORKING COVER EXAMPLE

<table>
<thead>
<tr>
<th>ACC YEAR</th>
<th>REPORTED CLAIMS</th>
<th>CRED IBNR ESTIMATE</th>
<th>PREDICTED EMERGENCE</th>
<th>REPORTED CLAIMS</th>
<th>ACTUAL EMERGENCE</th>
<th>ACTUAL - PREDICTED</th>
<th>REPORT LAG</th>
</tr>
</thead>
<tbody>
<tr>
<td>1964</td>
<td>39</td>
<td>1.25</td>
<td>0.03</td>
<td>39</td>
<td>0</td>
<td>-0.03</td>
<td>96.96%</td>
</tr>
<tr>
<td>1965</td>
<td>27</td>
<td>0.96</td>
<td>0.02</td>
<td>28</td>
<td>1</td>
<td>0.98</td>
<td>96.66%</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>1984</td>
<td>20</td>
<td>13.85</td>
<td>0.70</td>
<td>20</td>
<td>0</td>
<td>-0.70</td>
<td>60.50%</td>
</tr>
<tr>
<td>1985</td>
<td>11</td>
<td>14.17</td>
<td>0.76</td>
<td>13</td>
<td>2</td>
<td>1.24</td>
<td>51.20%</td>
</tr>
<tr>
<td>1986</td>
<td>13</td>
<td>20.10</td>
<td>1.12</td>
<td>12</td>
<td>(1)</td>
<td>-2.12</td>
<td>38.97%</td>
</tr>
<tr>
<td>1987</td>
<td>5</td>
<td>25.17</td>
<td>1.38</td>
<td>9</td>
<td>4</td>
<td>2.62</td>
<td>23.21%</td>
</tr>
<tr>
<td>1988</td>
<td>0</td>
<td>34.32</td>
<td>1.16</td>
<td>2</td>
<td>2</td>
<td>0.82</td>
<td>5.71%</td>
</tr>
<tr>
<td>TOTAL</td>
<td>473</td>
<td>256.66</td>
<td>10.26</td>
<td>487</td>
<td>14</td>
<td>3.74</td>
<td>6.00%</td>
</tr>
</tbody>
</table>

\[ \frac{\text{TOTAL(7)}}{\text{TOTAL(4)}} = 36\% \]
Columns 2 and 3 are from Table (5.3.25). Column 8 is the lognormal adjusted by one quarter. The reader can verify the following:

9. Column 4 is obtained from column 3 using the lags at 12/31/88 and 3/31/89.

10. Assuming that the emergence is Poisson with mean = Total(4), then there is approximately a 12% probability that the actual emergence is 14 or more (use a Normal approximation to the Poisson).

As a result of (10), perhaps we wouldn't alter our opinion of the magnitude of the accuracy of the report lags and the estimated IBNR. However, we may want to pay close attention to the claim emergence over the next quarter. Note the negative emergence for 1986; most likely a claim was settled for less than the excess attachment point.

In addition to monitoring and testing claim count predictions, one should also review claim severities. Besides just report emergence, one can and should monitor and test claim settlements.

J. Conclusion of the reinsurance loss reserving section

We have seen some examples of how standard actuarial methods and some not-so-standard actuarial methods apply to reinsurance loss reserving. We must remember that there is no one right way of estimating reinsurance loss reserves. But there are plenty of wrong ways. Common actuarial methods should be used only to the extent they make sense. To avoid major blunders, the actuary must always understand as well as possible the types of reinsurance
exposure in his company's portfolio. The differences from primary company loss reserving mainly involve much less specificity of information, report and settlement timing delays and often much smaller claim frequency together with much larger severity, all inducing a distinctly higher risk situation. But with this goes a glorious opportunity for actuaries to use their theoretical mathematical and stochastic modeling abilities and data analytical abilities fully.

References 5.3:


Bornheuter, R. L. and Ferguson, R. E. (1972), "The Actuary and IBNR", PCAS.

CAS Committee on Loss Reserves (1988), "Statement of Principles Regarding Property and Casualty Loss and Loss Adjustment Expense Reserves".


APPENDIX A: PARETO DISTRIBUTION

1. Support: $X > 0$

2. Parameters: $b > 0, q > 0$

3. C.d.f.: $F(X) = 1 - \frac{b}{(b + X)^q}$

4. P.d.f.: $f(X) = \frac{qb^q}{(b + X)^{q+1}}$

5. Moments: $E[X] = b^q \frac{(n!)}{(q-1)(q-2)\ldots(q-n)}$

6. Censored c.d.f.: $F(X;c) = \begin{cases} F(X) & \text{if } X < c \\ 1 & \text{otherwise} \end{cases}$

7. Censored moments: If $q$ is not an integer, then

$$E[X;c] = E[X] - q^*\left(\frac{b}{(b+c)}\right)^* \left(\frac{b+c}{q-n}\right) + . . . + \left((-1)^i \frac{(n!)}{(i!)(n-i)!}\right) \left(\frac{b}{q-n+i}\right)$$

8. Censored expectation: $E[X;c] = E[X] \cdot \left(1 - \frac{b}{(b+c)}\right)$
APPENDIX A: PARETO DISTRIBUTION

9. Conditional probability:
\[
\text{Prob}[X > y | X > x] = \frac{(b+x)/(b+y)}
\]

10. Truncated (conditional) distribution:

Definition: \( X_d = X - d \) for \( X > d \)

Then \( X_d \) is Pareto with parameters \( b+d, q \):
\[
F(X_d) = 1 - \left( \frac{(b+d)/((b+d)+X_d)} \right)^q
\]

11. Trended distribution:

Definition: \( Y = t^*X \)

Then \( Y \) is Pareto with parameters \( t^*b, q \):
\[
F(X_d) = 1 - \left( \frac{(t^*b)/((t^*b)+Y)} \right)^q
\]
APPENDIX B: LOGNORMAL DISTRIBUTION

1. Support: \( X > 0 \)
2. Parameters: \(-\infty < u < \infty, \sigma > 0\)
3. C.d.f.: \( F(X) = \Phi((\ln X - u)/\sigma) \)
4. P.d.f.: \( f(X) = (1/X) \Phi((\ln X - u)/\sigma) \)
5. Moments: \( E[X^n] = \exp(\nu + n \sigma^2 / 2) \)
6. Censored moments:

\[
E[X^n; c] = E[X^n] \Phi((\ln(c) - u)/\sigma - n\sigma) + c \cdot (1 - \Phi((\ln(c) - u)/\sigma))
\]

7. Truncated (conditional) distribution: \( (X_d = X - d) \)
   a) \( E[X_d] = E[X] - E[X; d] \)
   b) \( E[(X_d)^2] = (E[X]^2 - E[X; d]^2) - 2d^2 E[X_d] \)
   c) \( \text{Var}[X_d] = E[(X_d)^2] - E[X_d]^2 \)
8. Censored truncated distribution:
   a) \( E[X_d; c-d] = E[X; c] - E[X; d] \)
   b) \( E[(X_d)^2; c-d] = (E[X; c]^2 - E[X; d]^2) - 2d^2 E[X_d; c-d] \)
   c) \( \text{Var}[X_d; c-d] = E[(X_d)^2; c-d] - E[X_d; c-d]^2 \)
APPENDIX C: AGGREGATE LOSS MODEL

1. Aggregate loss \( L = X(1) + X(2) + \ldots + X(N) \)
   where \( N = \text{rv denoting number of claims} \)
   \( X(i) = \text{rv denoting the value of the } i^{th} \text{ claim} \)

Assume that \( N \) and the \( X(i) \)'s are mutually independent and the \( X(i) \)'s are identically distributed with c.d.f. \( F(x) \).
(Note: These are usually reasonable assumptions when the parameters for the distributions of \( N \) and \( X \) are assumed to be known.) Then the following statements are true:

2. C.d.f.: \( F(x) = \prod_{n=1}^{n} \text{Prob}[N=n] \ast F(x) \)
   where \( F \) is the \( n^{th} \) convolution

3. Moments:
   \( E[L] = E[N] \ast E[X] \)
   \( \text{Var}(L) = E[N]^2 \ast E[X]^2 + \left( \text{Var}(N) - E[N] \right) \ast E[X]^2 \)
   \( = E[N]^2 \ast (\text{Var}(X) + \left( \text{Var}(N)/E[N] \right) \ast E[X]^2) \)
   \( E[(L - E[L])^3] = E[N]^3 \ast E[(X - E[X])^3] \)
   \( + E[(N - E[N])^3] \ast E[X]^3 \)
   \( + 3 \ast \text{Var}(N) \ast E[X] \ast \text{Var}(X) \)