INVESTMENT INCOME, UNDERWRITING PROFIT AND CONTINGENCIES; FUTURE DEVELOPMENTS

INVESTMENT INCOME, UNDERWRITING PROFIT & CONTINGENCIES: FUTURE DEVELOPMENTS

By James R. Garven

1. Introduction

In this session's previous presentations, Steve Lehman and Rich Derrig have rather capably set forth the logic underlying the use of financial models in ratemaking as opposed to more traditional "markup" models. Since insurance firms exist in an economic environment within which they must compete for capital with other insurers, financial intermediaries, and even nonfinancial firms, they must therefore concern themselves with delivering competitive rates of return to capital in order to prosper. Furthermore, since the cost of producing insurance services is jointly determined by the firm's investment and underwriting activities, any ratemaking model which ignores the role of investment income will only coincidentally produce a realistic estimate of the actual cost of doing business.

A number of different types of financial ratemaking models have been developed within the last decade. Although these models differ widely in terms of underlying assumptions, parameter specifications, and methods of calculation, they are generally organized around the basic principle that certain targets must be met so as to justify continued or even further allocation of capital to a particular set of insurance activities. The models of Fairley [9], Hill [10], and Hill and Modigliani [11] in particular address this issue by applying the Capital Asset Pricing Model (CAPM) to derive the risk-adjusted rate of return on equity that capital markets require of property-liability insurers.

Unfortunately, CAPM-based ratemaking models suffer from a number of non-trivial problems. First, there are some peculiar difficulties related to parameter estimation.[1] Second, these models do not address the effect of the risk of insolvency on the return to shareholders despite the attention given to this prospect by actuaries and regulators. Third, in spite of the fact that the underwriting and investment activities of

[1]In order to determine appropriate risk premium loadings on policies, the CAPM approach requires that an "underwriting beta" be estimated. However, as Cummins and Harrington [3] have shown, underwriting betas are extremely difficult, if not impossible to calculate with any degree of accuracy.

property–liability firms often result in underutilized tax shields (this is especially true in recent years), these models typically either ignore taxation altogether or implicitly assume that, once realized, tax shields are always fully utilized.

The purpose of my presentation will be to summarize some recent developments in the theory of ratemaking. In particular, I will focus upon the use of option pricing theory as an alternative to the CAPM in calculating the underwriting profit margin. As Neil Doherty and I have shown elsewhere (see Doherty and Garven [8]), the rationale for applying the theory of option pricing to ratemaking is that the values of the claims held by shareholders, policyholders, and the government are contingent upon the amount of investment income earned by the insurance firm. In addition to its intuitive appeal, the option pricing approach also has several practical advantages over the CAPM. Most importantly, it is not plagued by the problems noted to exist for the CAPM. Furthermore, I will show that CAPM–based ratemaking models can be characterized as special cases of option–based models.

With this general framework in mind, let me provide you with a brief summary of the intuition underlying the option pricing approach to ratemaking. I also plan to provide numerical illustrations in which I compare CAPM–based with option–based underwriting profit margin calculations using workers compensation insurance data. Interested readers can refer to the appendix for further details on the mathematical structure of both the CAPM and the option models. Also, references for further reading are included at the end of this paper.

2. The Option Pricing Approach to Ratemaking

2.1. Some General Comments on Options

Before I demonstrate the option pricing approach to ratemaking, some general comments regarding options are clearly in order. First of all, I will define what an option is. An option is a financial contract which endows its holder with the privilege to either buy or sell a particular asset at a given price within a specified period of time. It is not an obligation to buy or sell, but a choice which may be exercised at the option of the holder. Call options derive value from the possibility that the underlying asset can be purchased at some point in time for a price which is less than the market price, thus securing a profit to the holder. Similarly, put options derive value from the possibility that the underlying asset can be sold at some point in time for a price which exceeds its market price.

Next, consider the source of value from holding an option. To keep the analysis as simple as possible, I will consider the case of a European call option. The holder of a European call option is endowed with the right to buy a security at a future date for a price agreed upon now. The future date is known as the expiration date, and the agreed upon price is the exercise, or striking, price. To clarify the example, we will insert values. Suppose the current price of the underlying stock, P_0, is \$95, the exercise price, X, is \$100 and the expiration date is 6 months from now. When the option is purchased, the buyer and seller of the option do not know what the price of the stock will be at expiration. The unknown terminal value of the underlying stock is denoted P_t. If the price at expiration is less than the exercise price of \$100, the holder of the option would allow the option to expire worthless, since it would not be rational to purchase an asset for a price in excess of its market value. But if the price at expiration exceeds the exercise price of \$100, the holder will find it worthwhile to exercise his option and purchase the stock at a price less than the market price. The difference between the terminal market price of the stock and the exercise price represents pure profit to the holder. Thus, the holder is in the enviable position of holding an security that yields nonnegative payoffs at maturity; viz., there is only upside potential. Such a "no lose" position has value, and in a competitive market, the option will trade for this value.

The payoff to the option I just described can be written in the following manner:

$$\text{Payoff on Call Option} = \text{MAX}[P_t - X, 0]. \tag{1}$$

Figure 1 depicts equation (1) graphically.

2.2. Payoffs to Insurance Claimholders

Next, I will show how the limited liability rule as well as the existence of underutilized tax shields cause the payoffs to the claimholders of the insurance firm to resemble options. I start by identifying the principle cash flow to and from the insurance firm. Imagine that the insurance firm is set up at one point in time (e.g., at the beginning of the year, subsequently referred to as t_0) and operated until the end of the period (subsequently referred to as t_1), at which time all liabilities are either discharged or reserved. At t_0, the insurer receives surplus (equity) and premiums and pays its marketing and production expenses. Thus the initial cash flow is

$$Y_0 = S_0 + P_0, \tag{2}$$

Where:

S_0 = the initial surplus;

P_0 = the premiums (net of expenses).

At t_1, allowing for the accumulation of investment income at a rate r_i, the insurer's assets will assume the following value:

$$Y_1 = S_0 + P_0 + (S_0 + kP_0)r_i. \tag{3}$$

The term k is commonly referred to as the funds generating coefficient. This parameter represents the average time delay between premium receipts and claims payments. While this aspect of the model is a somewhat crude correction for the multiperiod nature of claims payments, it is nevertheless a feature common to most financial models, including the CAPM.

2.2.1. The No-Tax Case

Next, consider the manner in which Y_1 would be allocated in the absence of taxes. By issuing insurance policies at t_0, the shareholders are essentially selling the firm's assets to the policyholders in exchange for premium income plus a call option to repurchase these assets at t_1. This call option has an exercise price which is equal to the claims costs (L) which are realized at t_1. Consequently, the terminal payoffs to shareholders and policyholders, S_1 and P_1, can be written

$$S_1 = \text{MAX}[Y_1\text{–}L,0], \text{ and} \tag{4}$$

$$P_1 = Y_1 - \text{MAX}[Y_1\text{–}L,0] = \text{MIN}[L,Y_1]. \tag{5}$$

Should the firm fare poorly (i.e., if $Y_1\text{–}L<0$), then shareholders will rationally choose to not exercise their option to repurchase the firm's assets from the policyholders; consequently, the firm will now be owned by the policyholders. However, should things go well (i.e., if $Y_1\text{–}L>0$), then shareholders will find it worthwhile to exercise their option to repurchase the firm's assets by making good on the policies. These payoffs are depicted in Figure 2.

Before considering the effect of taxes, it is worthwhile to reflect for a moment on the relationship between the option model and a CAPM model without taxes, such as that of Fairley. Fairley's no–tax version of the CAPM ratemaking model is a essentially a special case of the option model described here. The primary difference is due to the CAPM's implicit assumption that either the function $\text{MAX}[Y_1\text{–}L,0]$ is always positive, or that shareholders have unlimited liability. Consequently, under the CAPM model, the terminal payoffs to shareholders and policyholders, S_1 and P_1, are written

$$S_1 = Y_1\text{–}L, \text{ and} \tag{6}$$

$$P_1 = L \tag{7}$$

for all possible values of Y_1 and L.

2.2.2. The Effect of Taxes

The analysis is complicated somewhat by taxes. Tax shields are created whenever the insurer incurs losses from either its underwriting or investment activities. Furthermore, it is common practice for insurers to shelter at least a portion of their investment incomes from taxation by purchasing tax-favored financial assets such as municipal bonds and common stocks. Therefore, depending upon how well or poorly the insurer fares, it is possible for some of these tax shields to be underutilized. Although insurers are able to make use of the tax loss carryback/carryforward provision in the tax code, the net effect of tax shield underutilization is to increase the burden of the corporate tax on the insurer as compared to a tax system which would allow for the complete and contemporaneous realization of tax rebates as well as liabilities.

For the sake of simplicity and in the interest of determining an upper bound for the effect of underutilized tax shields on insurance rates, we will assume that a tax liability is incurred if and only if the terminal asset value of the firm (Y_1) exceeds the terminal value of the firm's tax shields (TS).[2] Consequently, the government can be characterized as holding a fractional position in a call option on Y_1, the exercise price of which is equal to TS. The payoff to this option (T_1) is given in equation (8), and depicted in Figure 3; viz.,

$$T_1 = \tau \mathrm{MAX}[Y_1 - TS, 0], \qquad (8)$$

[2]By using a single period model, I have implicitly assumed away the possibility of the insurer making use of tax loss carrybacks and carryforwards (CB–CF) which could be introduced in a multi–period framework. However, the effect of the CB–CF provision can nevertheless be readily inferred. Since tax shield underutilization effectively increases the burden of the corporate tax on the insurer, this burden will be passed on to policyholders in the guise of higher insurance prices and underwriting profit margins, everything else the same. However, since the effect of the CB–CF provision is to reduce this tax burden, lower insurance prices and underwriting profit margins would be implied than are predicted by the option model presented here. Interested readers are referred to the recent paper by Majd and Myers [13] which numerically simulates the valuation effects of the CB–CF provision in a multi–period option pricing framework.

where τ is the statutory corporate income tax rate.

In Figure 4, I show the payoffs to the policyholders, shareholders, and the government. The effect of taxes is to decrease the payoff received by shareholders whenever Y_1 exceeds TS. Although taxes do not affect policyholders' terminal payoffs, the burden of taxes falls squarely on the policyholders in the guise of higher premiums than would be the case in the absence of a corporate tax.

As Rich Derrig noted in his presentation, there is much concern in ratemaking over the effects of the Tax Reform Act of 1986. The option pricing model presented here is capable of accomodating all of the effects which he addressed (specifically, tax rate changes, discounting reserves, and unearned premium reserve offsets). Furthermore, the alternative minimum tax (AMT) could be incorporated by changing the specification of the payoff to the tax option shown in equation (8) to the payoff given in equation (9):

$$T_1^{AMT} = \mathrm{MAX}[\tau(Y_1\text{–}TS),\mathrm{AMT},0]. \tag{9}$$

The effect of the AMT on the tax option is shown in Figure 5. Since this provision can only increase the corporate tax burden, the AMT is therefore likely to give rise to even higher insurance prices and underwriting profit margins.

It is interesting to note that the after–tax versions of the CAPM (e.g., see Fairley and Hill and Modigliani) are special cases of the after–tax version of the option model. The primary difference is due to the CAPM's implicit assumption that either the function $\mathrm{MAX}[Y_1\text{–}TS,0]$ is always positive, or that the tax system allows for the complete and contemporaneous realization of tax rebates as well as liabilities. Consequently, under the CAPM model, the terminal payoff to the government, T_1, is written

$$T_1 = \tau(Y_1 - TS) \tag{10}$$

for all possible values of Y_1 and TS.

2.3. Using Option Pricing Theory to Value Insurance Payoffs

Now that the terminal payoffs to the insurer's claimholders have been determined, it is a fairly simple matter to value them. By applying the appropriate valuation functions to the payoffs given in equations (4), (5), and (8), the current (t_0) values of the claims held by the policyholders (P_0), the government (T_0) and the shareholders (V_e) can be written

$$P_0 = V(Y_1) - C(Y_1;L), \text{ and} \tag{11}$$

$$T_0 = \tau C[Y_1;TS], \tag{12}$$

$$V_e = C[Y_1;L] - \tau C[Y_1;TS]. \tag{13}$$

In the above equations, the function $V(Y_1)$ represents the t_0 value of the cash flow Y_1, $C[Y_1;L]$ represents the t_0 value of the shareholder's option to repurchase the firm's assets from the policyholders at t_1, and $\tau C[Y_1;TS]$ represents the t_0 value of the government's tax option. It is worthwhile noting that the sum $(P_0 + T_0 + V_e)$ is equal to $V(Y_1)$.

2.4. Determining the Competitive Insurance Price and Underwriting Profit Margin

Given the values for P_0, T_0, and V_e as determined by equations (11)–(13), the ratemaking problem is to price the insurance policies such that the shareholders receive a competitive rate of return on their investment in the insurance firm. Such a return would be made for shareholders if the current value of their future payoff is equal to the value of the capital they invest in the firm; viz.,

$$V_e = C[Y_1(P_0^*);L] - \tau C[Y_1(P_0^*);TS(P_0^*)] = S_0. \tag{14}$$

This is an implicit solution to the competitive insurance price. Among other things, the values of the two call options $C[Y_1;L]$ and $C[Y_1;TS]$ depend upon the premiums charged to policyholders. The premiums affect the value of the underlying asset against which these call options are written, as well as the exercise price of the tax option. Thus the solution requires that a level of premiums P_0^* be chosen such that equation (14) is satisfied.

Before equation (14) can be solved for P_0^*, an explicit pricing model must be implemented. Doherty and Garven provide two such models, both of which are summarized in the appendix. Their first pricing model requires assuming that the insurer's claims costs and investment returns are jointly normally distributed, while their second model requires joint lognormality. Furthermore, each model requires further assumptions regarding the nature of investors' risk preferences. Although neither option model provides a closed form solution for P_0^*, P_0^* can be solved for numerically by implementing appropriately parameterized versions of equation (14). Furthermore, P_0^* may be translated into the underwriting profit margin by the routine solution of equation (15):

$$UPM^* = \frac{P_0^* - E(L)}{P_0^*}, \tag{15}$$

where E(L) is the expected value of the insurer's claims costs.

3. Numerical Illustration

In this section I provide a numerical example which illustrates the points of comparison between option–based and CAPM–based ratemaking models. The option–based models were solved iteratively from equations (A–2) and (A–4), whereas the corresponding CAPM models were solved from equation (A–1) and (A–3).[3] The solutions were derived from a set of parameters presented in Table I which are intended as a crude representation of a typical workers compensation insurance business. Table II and Figures 6–11 show the underwriting profit margins required to deliver a competitive rate of return on equity over different ranges of values for the model parameters. Furthermore, I also show the implied probabilities of insolvency and tax shield underutilization for the option–based models in Table II.

The points of interest include the following. In general, the option–based models provide higher underwriting profit margins than the CAPM. The most useful comparison is between the CAPM results and those produced under the normal option pricing model. Since the distributional assumptions are comparable, the differences in underwriting profit margins are explained by the attention paid in the option pricing model to the probabilities of insolvency and underutilized tax shields. The results of the simulations generally reveal the following set of relationships between UPM^* and the model parameters:[4]

$$UPM^* = f(\underset{+}{S_0},\ \underset{-}{k},\ \underset{+,-}{\sigma_i},\ \underset{+,-}{\sigma_L},\ \underset{-}{r_f},\ \underset{+}{\theta}) \tag{16}$$

[3]Since the results obtained with the lognormal CAPM do not differ materially from the results obtained with the normal CAPM, only the latter model's results are presented here.

[4]See the appendix for definitions of the parameters shown in equation (16).

4. Summary and Discussion of the Relative Merits of the Option Pricing Model

The option–based ratemaking model discussed in this paper is based upon straightforward principles. The insurance firm must discharge a sequence of liabilities to policyholders, the tax authorities and to its shareholders. The values of these respective claims are contingent upon the terminal value of the insurer's assets. Therefore, the option model presented here values the various claims as options written on the insurer's asset portfolio. The competitive price for insurance is derived by choosing the premium such that the present value of the shareholders' claim is equal to the value of their equity (surplus) investment in the firm.

I will conclude my presentation by comparing the features of the option pricing model with CAPM–based models. As my analysis demonstrated, CAPM–based ratemaking models can generally be characterized as special cases of option–based models. Not only are the option–based models more general; they also have several important practical advantages over earlier CAPM models. First, the option model gets around some peculiar difficulties related to parameter estimation. Second, the option model explicitly accounts for the risk of insolvency and will therefore yield an estimate of the probability of ruin which is implicit in the calculation of the competitive insurance price. Third, the option model explicitly models the effects of underutilized tax shields. My numerical calculations reveal that this can have a major impact on the results.

Because of its practical advantages and greater generality, my expectation is that the option pricing approach is likely to do a better job of approximating competitive insurance prices than will the CAPM and other previous, more *ad hoc* models.[5]

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[5]I hope to provide some empirical support for this expectation fairly soon. Steve D'Arcy and I are currently working on a paper which will examine the goodness of fit for several pricing techniques (including target underwriting profit margin, total rate of return, discounted cash flow analysis, the CAPM, and the option pricing model) by comparing predicted model values with actual property–liability insurance industry experience over

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the 60 year period from 1926–1985 (see D'Arcy and Garven [6]). Interested readers are welcome to contact either of us for a copy of this paper.

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Appendix

In this appendix, I present the analytics which underly the normal and lognormal CAPM and option pricing solutions to the competitive underwriting profit margin as derived originally in Doherty and Garven [8]. I start by assuming that 1) the insurer's claims costs and investment returns are jointly normally distributed, and 2) investors' utility functions exhibit constant absolute risk aversion (CARA). Under these assumptions, the CAPM solution can be written

$$UPM^* = -\frac{(1-\theta\tau)}{(1-\tau)}kr_f + (V_e/P_0)\frac{\theta\tau}{(1-\tau)}r_f + \lambda COV(r_u, r_m), \qquad \text{(A-1)}$$

Where:

θ = proportion of investment income that is taxable ($\theta \in [0,1]$);
r_f = rate of return on a riskless asset;
r_u = rate of return on underwriting;
λ = the market price of risk;
= $[E(r_m) - r_f]/\sigma_m^2$.

Hill and Modigliani derive a comparable expression for UPM^*, and a similar relationship is derived by Fairley.

Assuming CARA preferences and jointly normally distributed investment returns and claims costs, the functional form of equation (13) is written

$$V_e = (1+r_f)^{-1}\Big[\hat{E}(X)N[\hat{E}(X)/\sigma_x] - \tau\hat{E}(W)N[\hat{E}(W)/\sigma_w] + \sigma_x n[\hat{E}(X)/\sigma_x] - \tau\sigma_w n[\hat{E}(W)/\sigma_w]\Big] \qquad \text{(A-2)}$$

Where:

$\hat{E}(\cdot)$ = the certainty–equivalent expectation operator;[6]
$\hat{E}(X) = \hat{E}(Y_1) - \hat{E}(L) = S_0 + (S_0+kP_0)r_f + P_0 - \hat{E}(L)$;
$\sigma_x^2 = (S_0+kP_0)^2\sigma_i^2 + \sigma_L^2 - 2(S_0+kP_0)COV(L,r_i)$;
$\hat{E}(W) = \theta(S_0+kP_0)r_f + P_0 - \hat{E}(L)$;
$\sigma_w^2 = (S_0+kP_0)^2\theta^2\sigma_i^2 + \sigma_L^2 - 2(S_0+kP_0)\theta COV(L,r_i)$;
$N[\cdot]$ = the standard normal distribution function;
$n[\cdot]$ = the standard normal density function.

[6]Mathematically, a certainty–equivalent expectation of cash flow is equal to the difference between the expected value of cash flow and an appropriate risk premium as implied by the capital asset pricing model.

As I discuss in the paper, equation (A–2) can be solved iteratively for P_0^*. Once P_0^* is known, UPM^* can be determined by applying equation (15).

Next, I present the lognormal CAPM and option pricing solutions to the competitive underwriting profit margin. By assuming that 1) the insurer's claims costs and investment returns are jointly lognormally distributed, and 2) investors' utility functions exhibit constant relative risk aversion (CRRA), the CAPM solution to the competitive underwriting profit margin is given by equation (A–3):

$$UPM^* = 1 - \left[1+\frac{(1-\theta\tau)}{(1-\tau)}kr_f-(V_e/P_0)\frac{\theta\tau}{(1-\tau)}r_f\right]\exp\{\psi COV[\ln L,\ln R_m]\}, \qquad (A\text{–}3)$$

Where:

$$\begin{aligned} \psi &= \text{the market price of risk} \\ &= \frac{E(\ln R_m) - \ln R_f}{VAR(\ln R_m)} + \frac{1}{2}; \\ R_m &= 1 + r_m; \\ R_f &= 1 + r_f. \end{aligned}$$

Assuming CRRA preferences and jointly lognormally distributed investment returns and claims costs, the functional form of equation (13) is written

$$V_e = V_0^U N(d_1^U) - \tau V_0^T N(d_1^T) - R_f^{-1}P_0\left[N(d_2^U) - \tau N(d_2^T)\right], \qquad (A\text{–}4)$$

Where:

$$\begin{aligned} V_0^U &= \text{the contemporaneous value of the claim U} \\ &= V_0^Y - V_0^L + R_f^{-1}P_0 = S_0 + R_f^{-1}P_0(2+kr_f) - V_0^L; \\ V_0^L &= R_f^{-1}\hat{E}(L) \\ &= R_f^{-1}E(L)\exp\{-\psi COV[\ln L,\ln R_m]\}; \\ d_1^U &= \frac{\ln(V_0^U/P_0) + \ln R_f + \sigma_u^2/2}{\sigma_u}; \\ d_2^U &= d_1^U - \sigma_u; \\ \sigma_u &= \text{the standard deviation of the natural logarithm of U} \\ &= [\sigma_y^2 + \sigma_l^2 - 2COV(\ln Y,\ln L)]^{1/2}; \\ \sigma_y &= \text{the standard deviation of the natural logarithm of } Y_1; \\ \sigma_l &= \text{the standard deviation of the natural logarithm of L}; \end{aligned}$$

V_0^T = the contemporaneous value of the claim T

$= R_f^{-1}[\theta(S_0+kP_0)r_f + 2P_0] - V_0^L;$

$$d_1^T = \frac{\ln\ (V_0^T/P_0) + \ln R_f + \sigma_t^2/2}{\sigma_t};$$

$d_2^T = d_1^T - \sigma_t;$

σ_t = the standard deviation of the natural logarithm of T

$= [\sigma_{\theta\Delta y}^2 + \sigma_1^2 - 2COV(\ln[\theta(Y_1-Y_0)], \ln L)]^{1/2};$

$\sigma_{\theta\Delta y}$ = the standard deviation of the natural logarithm of $\theta(Y_1-Y_0)$.

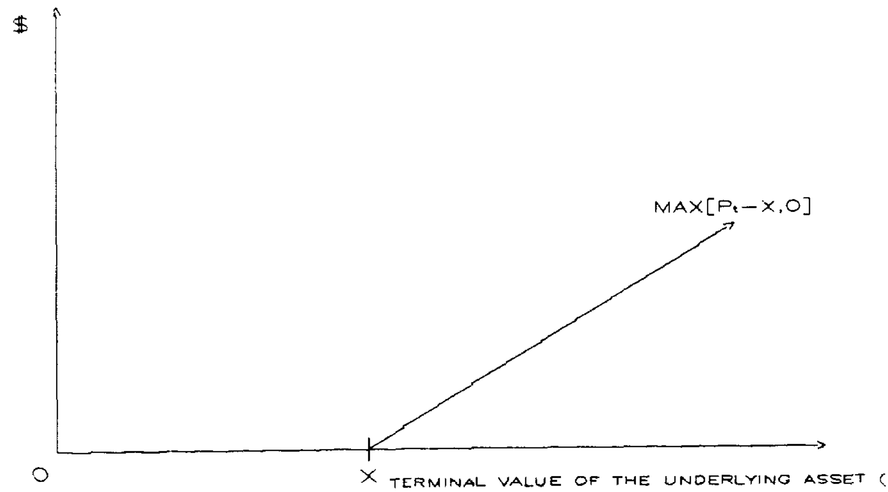

Figure 1: Terminal Payoff on a European Call Option

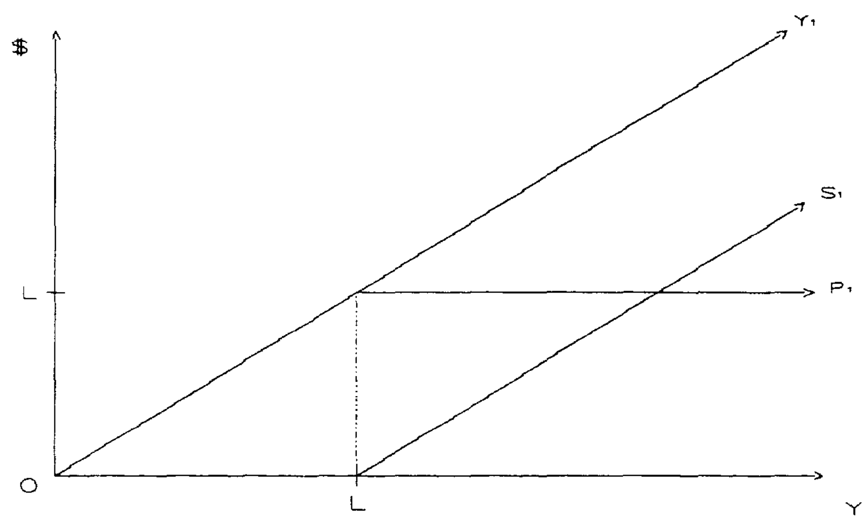

Figure 2: Payoffs to Policyholders (P_1) & Shareholders (S_1) in the Absence of Taxes

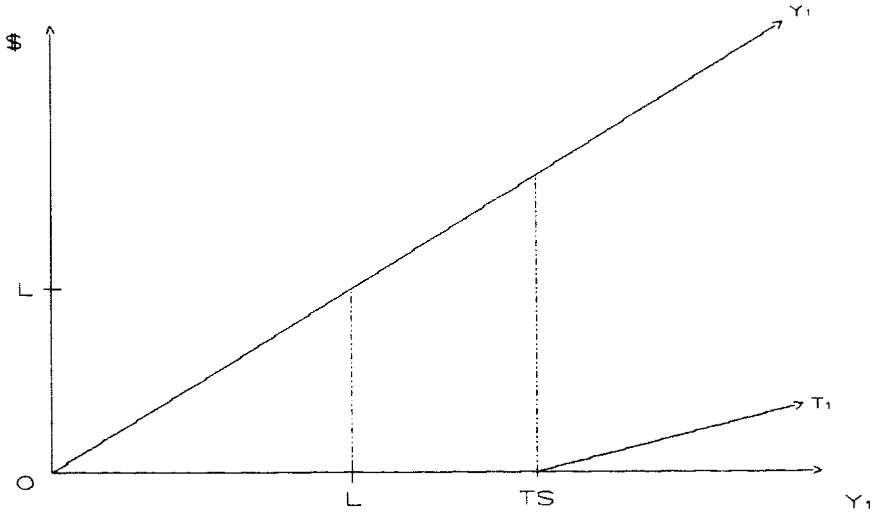

Figure 3: Payoff to the Government (T_1)

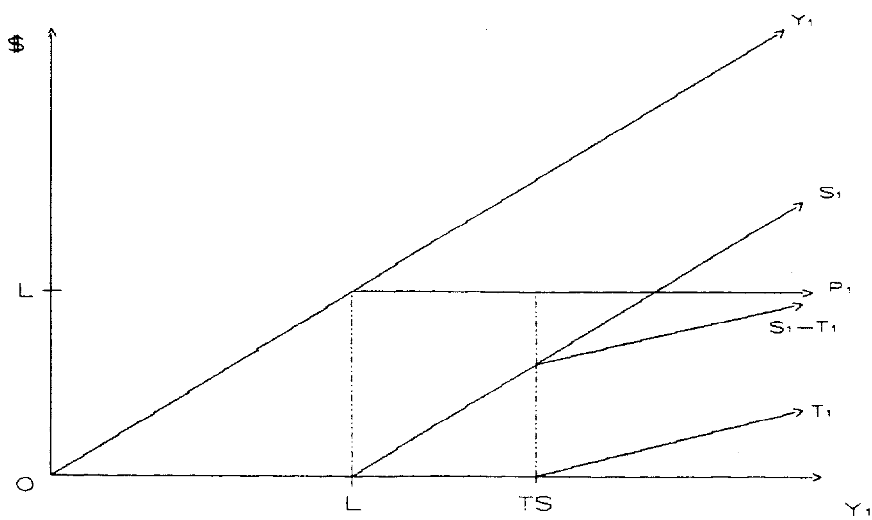

Figure 4: Payoffs to Policyholders (P_1), Shareholders (S_1) and the Government (T_1)

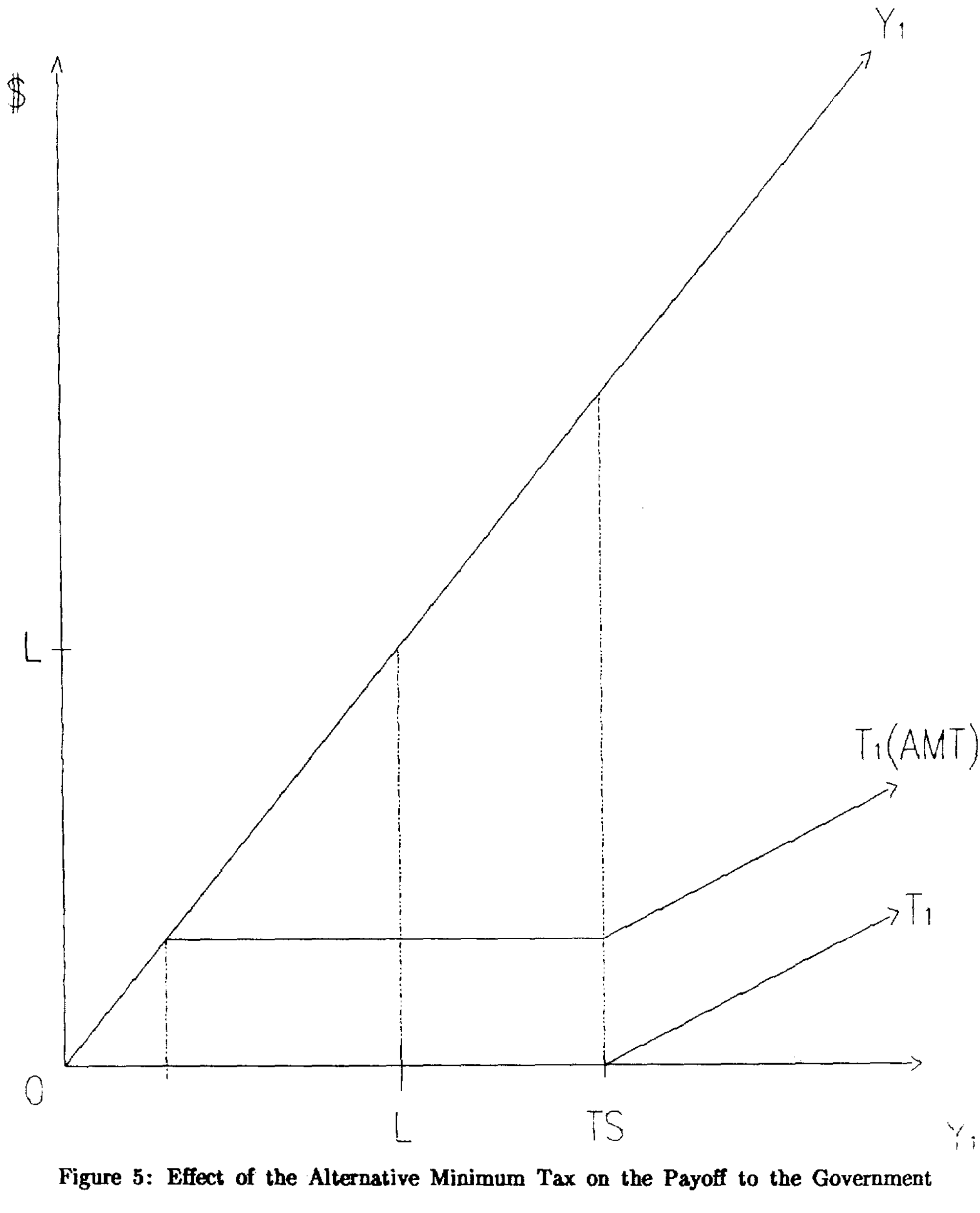

Figure 5: Effect of the Alternative Minimum Tax on the Payoff to the Government

Table I
Model Parameterization: The Base Case

Initial Equity (S_0)	1.00
Funds-Generating Coefficient (k)	2.00
Standard Deviation of Investment Returns (σ_i)	0.0427
Expected Claims Costs (E(L))	1.80
Standard Deviation of Claims Costs (σ_L)	0.142
Correlation Between Investment Returns/Claims Costs (ρ_{iL})	0.114
Riskless Rate of Interest (r_f)	0.07
Statutory Tax Rate (τ)	0.34
Tax Adjustment Parameter (θ)	0.60
Beta of Investment Portfolio (β_i)	0.20
Expected Return on the Market ($E(r_m)$)	0.15
Standard Deviation of Market Return (σ_m)	0.2137

Table II
Effects of Variations in Model Parameters Upon the Equilibrium Rate of Return on Underwriting

Panel A: Effects of Variations in Initial Equity (S_0)

	CAPM	OPM (Normal)			OPM (Lognormal)		
S_0	UPM	UPM	P(default)	P(no tax)	UPM	P(default)	P(no tax)
0.25	−0.1653	−0.1409	0.0469	0.6161	−0.1381	0.0873	0.6430
0.50	−0.1619	−0.1352	0.0013	0.5614	−0.1301	0.0053	0.5884
0.75	−0.1584	−0.1338	0.0000	0.5239	−0.1284	0.0002	0.5575
1.00	−0.1550	−0.1324	0.0000	0.4876	−0.1268	0.0000	0.5287
1.50	−0.1481	−0.1292	0.0000	0.4187	−0.1234	0.0000	0.4741
2.00	−0.1414	−0.1256	0.0000	0.3566	−0.1197	0.0000	0.4241

Panel B: Effects of Variations in the Funds Generating Coefficient (k)

	CAPM	OPM (Normal)			OPM (Lognormal)		
k	UPM	UPM	P(default)	P(no tax)	UPM	P(default)	P(no tax)
0.50	−0.0298	−0.0223	0.0000	0.3449	−0.0167	0.0000	0.3371
1.00	−0.0715	−0.0596	0.0000	0.4005	−0.0538	0.0000	0.4089
2.00	−0.1550	−0.1324	0.0000	0.4875	−0.1268	0.0000	0.5287
3.00	−0.2384	−0.2036	0.0000	0.5453	−0.1988	0.0000	0.6189
4.00	−0.3218	−0.2741	0.0000	0.5836	−0.2704	0.0000	0.6877
5.00	−0.4052	−0.3443	0.0002	0.6098	−0.3416	0.0001	0.7416
6.00	−0.4887	−0.4144	0.0007	0.6287	−0.4127	0.0001	0.7852

Table II (continued)
Effects of Variations in Model Parameters Upon the Equilibrium Rate of Return on Underwriting

Panel C: Effects of Variations in Investment Risk (σ_i)

	CAPM	OPM (Normal)			OPM (Lognormal)		
σ_i	UPM	UPM	P(default)	P(no tax)	UPM	P(default)	P(no tax)
0.00	−0.1550	−0.1311	0.0000	0.4797	−0.1323	0.0000	0.5518
0.20	−0.1550	−0.1389	0.0846	0.5039	−0.1052	0.0606	0.5364
0.40	−0.1550	−0.2949	0.2712	0.5941	−0.1404	0.2530	0.6197
0.60	−0.1550	−0.5582	0.3730	0.6555	−0.2042	0.3868	0.6883

Panel D: Effects of Variations in Underwriting Risk (σ_L)

	CAPM	OPM (Normal)			OPM (Lognormal)		
σ_L	UPM	UPM	P(default)	P(no tax)	UPM	P(default)	P(no tax)
0.25	−0.1550	−0.1261	0.0000	0.4770	−0.1182	0.0011	0.5176
0.50	−0.1550	−0.1107	0.0103	0.4685	−0.1044	0.0495	0.5326
0.75	−0.1550	−0.1057	0.0560	0.4746	−0.1115	0.1469	0.5721
1.00	−0.1550	−0.1177	0.1193	0.4886	−0.1347	0.2399	0.6140
1.50	−0.1550	−0.1950	0.2402	0.5232	−0.1987	0.3778	0.6835
2.00	−0.1550	−0.3555	0.3359	0.5566	−0.2633	0.4680	0.7331

Panel E: Effects of Variations in the Riskless Rate of Interest (r_f)

	CAPM	OPM (Normal)			OPM (Lognormal)		
r_f	UPM	UPM	P(default)	P(no tax)	UPM	P(default)	P(no tax)
0.05	−0.1111	−0.0913	0.0000	0.3370	−0.0854	0.0000	0.4304
0.07	−0.1550	−0.1324	0.0000	0.4875	−0.1268	0.0000	0.5287
0.09	−0.1986	−0.1733	0.0000	0.6316	−0.1680	0.0000	0.6212
0.11	−0.2419	−0.2140	0.0000	0.7528	−0.2091	0.0000	0.7037
0.13	−0.2849	−0.2546	0.0000	0.8442	−0.2500	0.0000	0.7738

Panel F: Effects of Variations in the Tax Parameter Theta (θ)

	CAPM	OPM (Normal)			OPM (Lognormal)		
θ	UPM	UPM	P(default)	P(no tax)	UPM	P(default)	P(no tax)
0.00	−0.1977	−0.1430	0.0000	0.9436	−0.1394	0.0000	0.9602
0.20	−0.1835	−0.1418	0.0000	0.8573	−0.1381	0.0000	0.8585
0.40	−0.1692	−0.1387	0.0000	0.6923	−0.1342	0.0000	0.6975
0.60	−0.1550	−0.1324	0.0000	0.4875	−0.1270	0.0000	0.5286
0.80	−0.1407	−0.1224	0.0000	0.3122	−0.1166	0.0000	0.3869
1.00	−0.1265	−0.1095	0.0000	0.1940	−0.1037	0.0000	0.2807

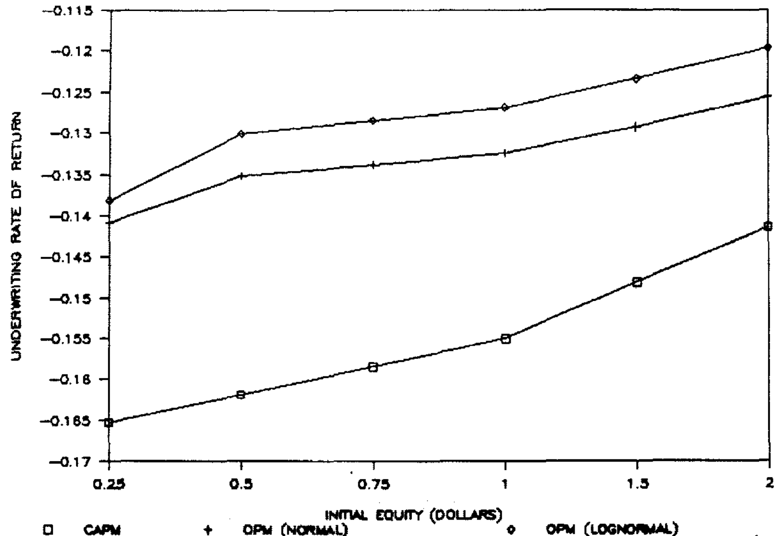

Figure 6: Effect of Variations in Initial Equity (S_0) on UPM*

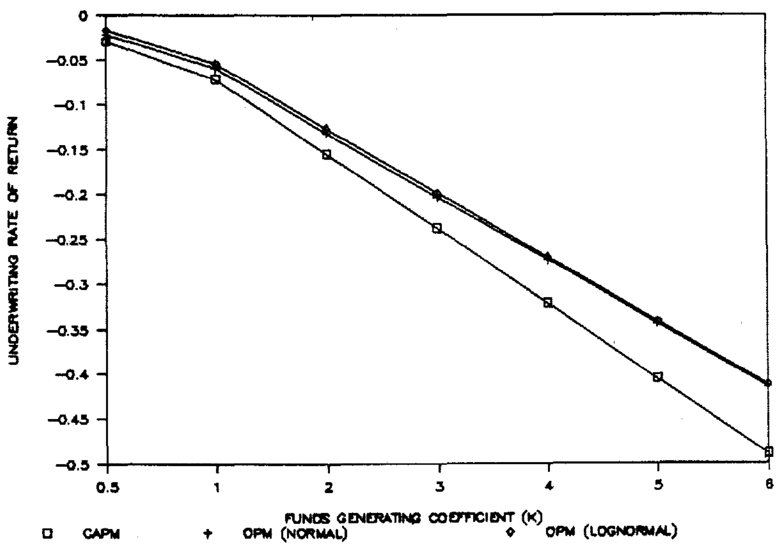

Figure 7: Effect of Variations in the Funds Generating Coefficient (k) on UPM*

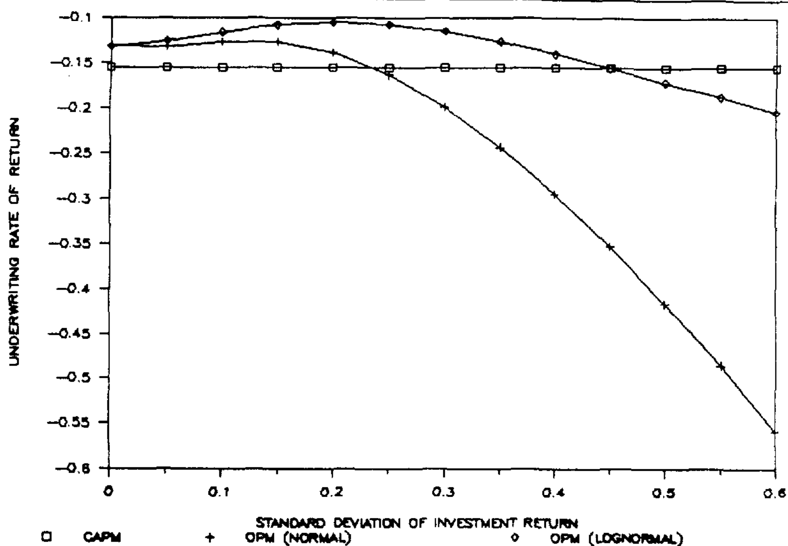

Figure 8: Effect of Variations in Investment Risk (σ_i) on UPM*

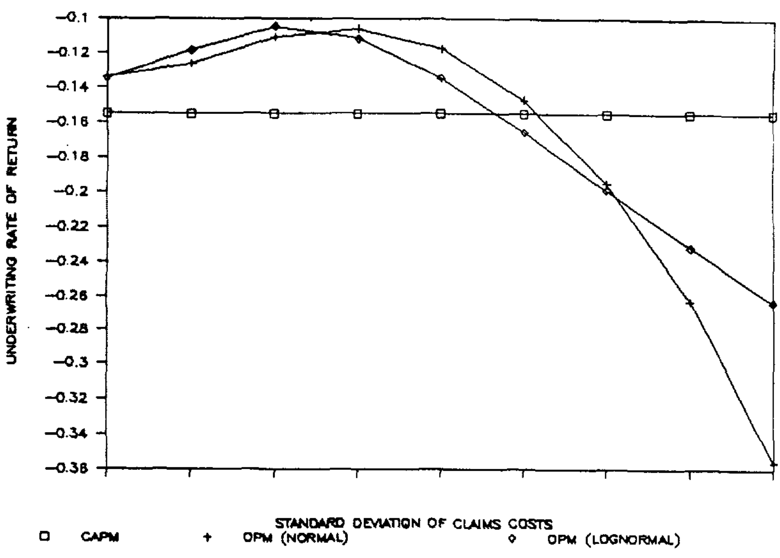

Figure 9: Effect of Variations in Underwriting Risk (σ_L) on UPM*

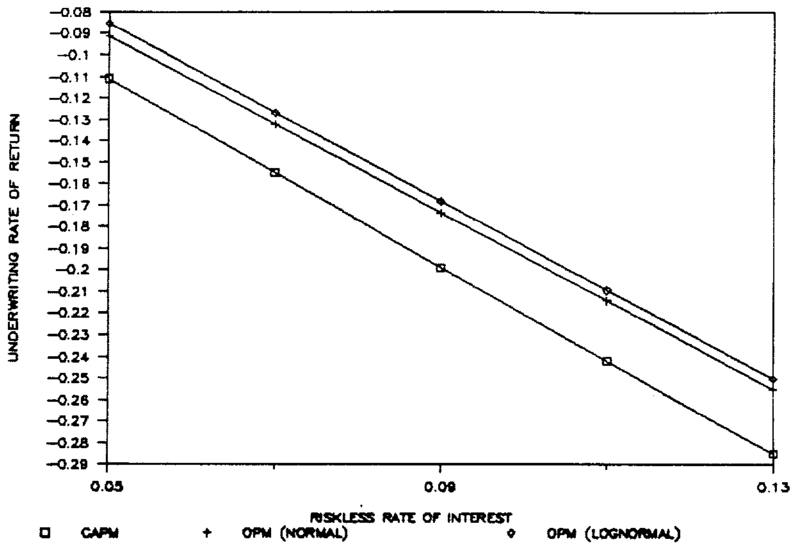

Figure 10: Effect of Variations in the Riskless Rate of Interest (r_f) on UPM*

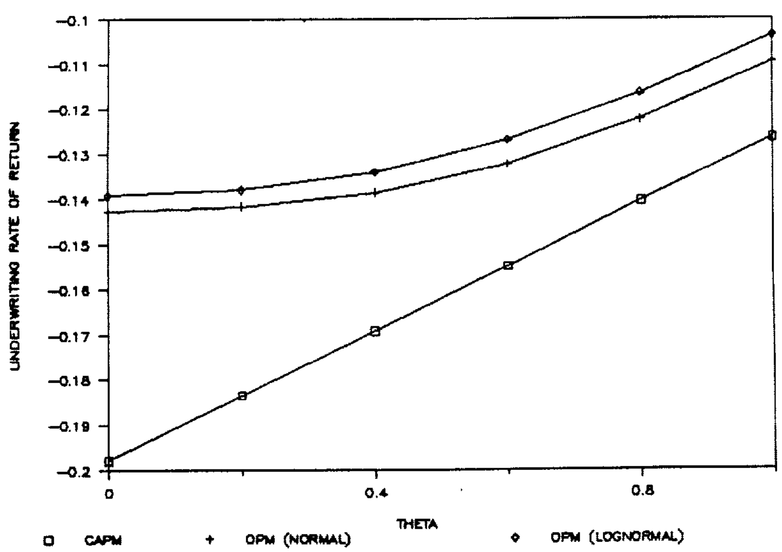

Figure 11: Effect of Variations in the Tax Parameter Theta (θ) on UPM*