The Market Value Margin Within The Distribution-Free Chain Ladder Model - A Way To Account For Calendar Year Effects And Aggregating Lines Of Business

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Abstract

Under European and Swiss solvency directives, general insurance companies have to calculate a market value margin (aka risk margin or MVM) for the prediction uncertainty of reserves over each accounting year and until the end of the runoff. The prediction uncertainty is generally split into a process error and an estimation error. In the distribution-free chain ladder framework, [10] derived analytical formulas for the prediction uncertainty over accounting years and showed that they add up to the total runoff uncertainty as given by the Mack error. We suggest a way to modify their methodology in order to account for calendar year uncertainties like a legal reform. Further, we derive the minimum and the maximum market value margin that can result with our modification, which is useful to quantify model uncertainty. Besides, we highlight the simplifications and omissions of the presented ways to infer the MVM. Finally, we discuss aggregating different lines of business. The presented formulas can be calculated in a spreadsheet.

Keywords. market value margin, distribution-free chain ladder model, reserving risk, calendar year effects, SST, Solvency II

1. INTRODUCTION

In Europe and Switzerland, insurance companies are regulated by the Solvency II directive (scheduled to be in full effect on 1 January 2016) and the Swiss Solvency Test (SST, in use since 2006). A comparison of Solvency II with the SST can be found in [4]. These two regulatory frameworks ask insurance companies to back their liabilities based on a one-year distribution of assets and liabilities. In addition, companies have to calculate the market consistent value of technical provisions which is defined as best estimate reserves (defined as the expected present value of future cash flows) plus the market value margin (MVM).

The MVM (aka risk margin) of the general insurance runoff (also called the reserve risk) is the focus of this paper. In this context, the MVM is a margin for the prediction uncertainty of the ultimate claim liabilities. Predictions are usually updated annually when new information is incorporated. These updates have an effect on the result of the insurance company and therefore need to be taken account of in solvency considerations. The prediction uncertainty is generally split into a process error and an estimation error. The process error represents random variations not explained by the model of the reserving actuary. The estimation error represents updates in the estimates of the model's parameters. In [3] and [5] the MVM is defined as the cost of the present value of future solvency capital requirements which will have to be put up during the runoff of the portfolio of assets and liabilities for the in-force book of business one year in the future.

A mathematically consistent calculation of the MVM is a complicated task which usually requires the application of numerical methods. Analytical approximations have been proposed, many of which rely on Bayesian statistics. [11] and [15], for example, describe how to infer the MVM within a Bayesian log-normal model. [13] derived, within a Gamma-Gamma Bayes chain ladder model, three approximations for the MVM whereof two can be computed analytically. The one-year view in the context of a Bayes chain ladder model was discussed by [1].

Bayesian models have the advantage that they include, in a natural way, the estimation error (also referred to as parameter uncertainty). Further, they can be similar to a classical chain ladder in the sense that the expected ultimate claim is given by a product formula involving factors and the latest cumulative payment (or incurred liability). However, Bayesian methods require selecting and calibrating prior distributions and justification of these selections is sometimes difficult. This might be a reason why the distribution-free chain ladder model, discussed in [6], still is one of the most popular reserving methods. Based on the distribution-free chain ladder model, [10] and [12] independently derived formulas which can be used to calculate the MVM. These formulas generalize the one-year solvency view presented in [9].

We take the methodology of [10] as a starting point and discuss a modification in order to account for calendar year effects like a legal reform or inflation. We propose a straight forward correction for the process error. Regarding the estimation error, the reserving actuary might have an idea when relevant information about parameters will become available (maybe the timing of the legal reform is known) and therefore can judge in which years the estimation error will be high. We show how to incorporate this judgment. Further, we provide a result useful to quantify the error of the actuary's judgment, that is to say we derive the minimum and maximum MVM which can possibly result based on different considerations of the estimation error. Finally, we discuss aggregating different lines of business.

The remainder of the paper is organized as follows. Section 2 reviews the classical chain ladder assumption, introduces the MVM and contains a literature review. Section 3 discusses our approach to accounting year effects, derives the minimum and maximum MVM and provides a numerical example. Section 4 treats the issue of aggregating different lines of business. Section 5 concludes.

2. BACKGROUND AND METHODS

2.1 Definitions and Assumptions

This section introduces the notation, revises the classical chain ladder (CL), aka distribution-free chain ladder model, and introduces our assumptions.

We write $C_{i,j}$ for the cumulative payments (or incurred liabilities) for accident years $i \in \{0, \dots, I\}$ and development years $j \in \{0, \dots, I\}$ and suppose that there is a $J \leq I$ such that $C_{i,J}=C_{i,J+1}=\dots=C_{i,I}$ for all i. If we refer to triangle we mean the following set

$$F_I = \{C_{i,j} : 0 \leq i \leq I, 0 \leq j \leq I, i+j \leq I\}$$

ordered as in Table 1 of Section 3.1. We denote the accounting years by $k \in \{0, \dots, I\}$ meaning that I refers to today and $I+k$ to the year k years in the future. We use 'accounting year' and 'calendar year' as synonyms. We define

$$F_{I+k} = \{C_{i,j} : 0 \leq i \leq I, 0 \leq j \leq I, i+j \leq I+k\}.$$

We assume stochastic independence between cumulative claims $C_{i,j}$ of different accident years i and that there exist constants $f_j>0$ and $\sigma_j>0$ and random variables $\varepsilon_{i,j}$ such that

$$C_{i,j} = f_j\, C_{i,j-1} + \sigma_{j-1}\sqrt{C_{i,j-1}}\,\varepsilon_{i,j} \tag{2.1}$$

where $\varepsilon_{i,j}$ are conditionally, given $S_0 = \{C_{i,0} : 0 \leq i \leq I\}$, independent with expectation $E[\varepsilon_{i,j}|S_0]=0$, $E[\varepsilon_{i,j}^2|S_0]=1$ and distribution guaranteeing $C_{i,j}>0$ with probability one. These assumptions imply the assumptions of the distribution-free chain ladder model, see e.g. [16].

We write $\hat{f}_j^{I+k}$ and $\hat{\sigma}_j^{I+k}$ for the estimators of f_j and σ_j given all information up to accounting year $I+k$ and define, for $k \leq j+1$,

$$\hat{f}_j^{I+k} = \frac{\sum_{i=0}^{I+k-j-1} C_{i,j+1}}{\sum_{i=0}^{I+k-j-1} C_{i,j}} \tag{2.2}$$

and $\hat{f}_j^{I+k} = \hat{f}_j^{I+j+1}$ for $k>j+1$ (since we focus on the runoff of past accident years only, the estimators remain unchanged for $k>j+1$). We observe that for $k=0$ the classical chain ladder factors are obtained. We obtain the $\hat{\sigma}_j^{I+k}$s as suggested in [6]. The estimated chain ladder ultimate claim is

$$\hat{C}_{i,J}^{I} = C_{i,I-i} \prod_{j=I-i}^{J} \hat{f}_j^{I} \tag{2.3}$$

and we abbreviate

$$C_J = \sum_{i=0}^{I} C_{i,J} \text{ and } \hat{C}_J^I = \sum_{i=0}^{I} \hat{C}_{i,J}^I.$$

If $C_{i,j}$ are cumulative payments then the liabilities estimated today to remain outstanding in accounting year $I+k$ are, for $k=0,\ldots,J-1$,

$$\hat{R}_k^I = \sum_{i=I-J+1+k}^{I} \left(\hat{C}_{i,J}^I - \hat{C}_{i,I-i+k}^I\right) = \hat{C}_J^I - \sum_{i=0}^{I} \hat{C}_{i,\min(I-i+k,J)}^I. \tag{2.4}$$

Accordingly, $\hat{R}_0^I$ are the chain ladder reserves estimated in the current accounting year (which equals accident year I).

Finally, the claims development result (*CDR*) of accident year i in accounting year $k \in \{0,\ldots,J\}$ is

$$CDR_{i,k+1} = E\left[C_{i,J} \middle| F_{I+k}\right] - E\left[C_{i,J} \middle| F_{I+k+1}\right]. \tag{2.5}$$

with the sigma-algebras defined before. We have $CDR_{i,k+1}=0$ for $i \leq I+k-J$. We write

$$CDR_k = \sum_{i=I-J+k}^{I} CDR_{i,k}. \tag{2.6}$$

The claims development result reflects how the valuation of the ultimate claim changes over a one year period. These changes are due to prediction updates as new information is incorporated. The prediction uncertainty is caused by two risk factors:

1. $\varepsilon_{i,j}$ in (2.1), referred to as the **process error**
2. updates of the chain ladder factors $\hat{f}_j^{I+k}$ in (2.2), referred to as the **estimation error.**

The MVM is the cost of the present value of future capital required to back adverse movements of the CDR caused by these two risk factors - we introduce its formal definition in the next chapter.

2.1.1 A Remark About Implicit Assumptions

We highlight that the CDR as defined in (2.5) does not consider discounting of liabilities. In the standard models of Solvency II and the SST adverse changes in discount factors (adverse meaning that they lead to higher best estimate reserves) are captured by the market risk (which is not the topic of this paper) and, in our understanding, they should also be taken into account in the MVM. Ignoring discounting further implies that the timing of the claims payments has no influence on the best estimate of discounted ultimate liability. However, the CDR as defined above is analogous to how it is defined in existing literature. Indeed, all papers we cite abstract from discounting. We leave it to future research to introduce the missing risk factors like stochastic discount factors, uncertainty in claim payments and potential mismatches in asset-liability cash-flows.

2.2 The MVM of the Runoff

We next introduce the MVM formally. For this purpose, we first define the following quantities:

Definition 1

- deterministic (F_I-measurable) discount factors $D_{I,k}$, $k=0,\ldots,J$ giving the value, in accounting year I, of a unit of money received in year $I+k$
- a risk measure $\rho()$ which quantifies the amount of capital needed to back adverse movements in the CDR (2.5)
- the cost c of capital (6% in Solvency II and the SST)

Assuming, as do [10], that we do not need to put up capital for adverse movements in the MVM itself, the MVM in accounting year I is

$$MVM = c\sum_{k=1}^{J} D_{I,k}\rho(CDR_k). \tag{2.7}$$

We repeat our remark in Section 2.1.1 namely that the MVM as just defined is based on variations of the *nominal* best estimate ultimate liability and therefore neither fluctuations in future discount rates nor the timing of the claims payments play a role. The MVM can be thought of as the present value of dividends required to compensate an investor for providing the risk capital to back the runoff risk.

If we knew the true chain ladder factors f_j then the estimation error would be zero. In this case we would only have to take care of the process error and we could easily calculate a variance (or standard deviation) risk measure for (2.7). We would obtain, for $1\leq k\leq J$,

$$Var(CDR_k|F_I) = \sum_{i=I-J+k}^{I} Var\left(CDR_{i,k}|F_I\right) = \sum_{i=I-J+k}^{I} (E[C_{i,J}|F_I])^2 \frac{\sigma_{I+k-i-1}^2/f_{I+k-i-1}^2}{E[C_{i,I+k-i-1}|F_I]}$$

which can be estimated by

$$\widehat{Var}(CDR_k|F_I) = \sum_{i=I-J+k}^{I} \left(\hat{C}_{i,J}^I\right)^2 \frac{(\hat{\sigma}_{I+k-i-1}^I)^2/\left(\hat{f}_{I+k-i-1}^I\right)^2}{\hat{C}_{i,I+k-i-1}^I}. \tag{2.8}$$

For $k=1$ this corresponds to the estimator of the process error in [8] and [9]. Taking the sum over all k gives the process variance of the total runoff which is one term of the Mack error (the other being the estimation error of the total runoff). Note that it would not matter if instead of $Var(CDR_k|F_I)$ we used $E[Var(CDR_k|F_{I+k-1})|\ F_I]$ for the risk measure, as we show in the next lemma.

Lemma 1. Suppose the true chain ladder factors are known. The classical CL assumptions imply

$$Var(CDR_k|F_I) = E[Var(CDR_k|F_{I+k-1})|F_I], \qquad k = 1, \dots, J.$$

Proof. By independence of the accident years it is sufficient to prove the equality for $CDR_{i,k}$. The total variance formula gives us

$$\begin{aligned} Var\big(CDR_{i,k}\big|F_I\big) &= E\big[Var\big(CDR_{i,k}\big|F_{I+k-1}\big)|F_I\big] + Var\big(E\big[CDR_{i,k}\big|F_{I+k-1}\big]\big|F_I\big) \\ &= E\big[Var\big(CDR_{i,k}\big|F_{I+k-1}\big)|F_I\big] \end{aligned}$$

where the second equality follows because definition (2.5) implies

$$E\big[CDR_{i,k}\big|F_{I+k-1}\big] = 0, \qquad 1 \le k \le J.$$

Q.E.D.

Unfortunately, the true chain ladder factors are unknown and the estimation error needs to be taken into account. We next review how this has been done in existing literature.

2.3 A Brief Literature Review

Instead of the notation of the original papers we use the notation introduced earlier, in particular c and $D_{I,k}$ as given in Definition 1. Indeed, the discount factors $D_{I,k}$ are omitted in the cited literature and we introduced them to be consistent with (2.7) which defines the MVM as the present value of future dividends.

As an example of a paper using Bayesian methods (which allows a natural treatment of the estimation error) and since it introduces notation, we cite [13]. They employed the standard deviation as a risk measure and discussed the following three ways to estimate the MVM.

A. Regulatory Solvency Proxy

$$MVM = c \sum_{k=1}^{J} D_{I,k} \frac{\hat{R}_k^I}{\hat{R}_0^I}\ \phi\ Stdev(CDR_1|F_I)$$

where $\hat{R}_k^I$ are the reserves (estimated to remain in accounting year k) as obtained with the Bayesian methodology of [13], $\phi > 0$ is a loading and the CDR_1 is as defined in (2.6).

B. Split of Total Uncertainty Approach

$$MVM = c \sum_{k=1}^{J} D_{I,k}\, \phi\, Stdev(CDR_k|F_I).$$

The name comes from the property that the total uncertainty about the ultimate $C_{i,J}$ can be split into single one-year uncertainties for different accounting years as follows

$$Var(C_{i,J}) = \sum_{k=1}^{J+i-I-1} Var\big(CDR_{i,k}|F_I\big).$$

C. Expected Stand Alone Measure

$$MVM = c \sum_{k=1}^{J} D_{I,k}\, \phi\, E[Stdev(CDR_k|F_{I+k-1})|F_I].$$

[13] derived analytic formulas for A and B and relied on simulations to solve C. They discussed a fourth approach to calculate the MVM for which instead of $\rho(CDR_K)$ they considered $\rho(CDR_K + MVM_k - MVM_{k-1})$ with MVM_k being the MVM calculated in accounting year $I+k$. This means that a markup for the MVM is included. While this approach is certainly more realistic - dividends can be thought of as a liability too - the computation becomes complicated and they had to rely on simulations. Fortunately, a numerical example in their paper supports B to be a good approximation for their fourth approach. As a side note, we remark that Lemma 1 and Jensen's inequality imply that for the distribution-free chain ladder model with known parameters approach B would yield a larger MVM than approach C.

[10] derived the prediction uncertainties for the CDR in the distribution-free chain ladder model. They computed the mean square errors of prediction (MSEP) for the CDRs as defined by

$$MSEP_{CDR_k|F_{I+k-1}}(0) = E[(CDR_k - 0)^2|F_k], \qquad k = 1, \dots, J$$

with CDR_k as in (2.6), and derived an estimator for the expected value at time I of the MSEP. They then defined the MVM for a variance risk measure given by

$$MVM = c \sum_{k=1}^{J} D_{I,k}\, \phi\, E\big[MSEP_{CDR_k|F_{I+k-1}}(0)|F_I\big] \tag{2.9}$$

and the MVM for a standard deviation motivated risk measure given by

$$MVM = c \sum_{k=1}^{J} D_{I,k}\, \phi\, \sqrt{E\big[MSEP_{CDR_k|F_{I+k-1}}(0)|F_I\big]} \tag{2.10}$$

where $\phi > 0$ is a loading. This is a generalization of [9] who suggested to use $MSEP_{CDR_1|F_I}(0)$ for a one-year view of solvency considerations. [10] further showed that

$$MSEP_{C_{i,J}|F_I}\left(\hat{C}_{i,J}^{I}\right) = \sum_{k=1}^{J+i-I} \phi \, E\left[MSEP_{CDR_{i,k}|F_{I+k-1}}(0)|F_I\right] \tag{2.11}$$

with $\hat{C}_{i,J}^{I}$ defined in (2.3). The left-hand side is the Mack error as introduced in [6]. Hence, the total runoff uncertainty as given by the Mack error splits across accounting years and so their approach is similar to B of [13] stated earlier. Both, the Mack error and $E[MSEP_{CDR_k|F_{I+k-1}}(0)|F_I]$ can be written as a sum of two terms corresponding to the process error variance and the estimation error. Not surprisingly, the process variance in $E[MSEP_{CDR_k|F_{I+k-1}}(0)|F_I]$ equals (2.8).

3. CONSIDERING ACCOUNTING YEAR UNCERTAINTIES

The distribution-free chain ladder is probably the most popular reserving method. There is therefore a good chance that the formulas of [10] for the MVM will become popular, too. Moreover, these formulas can be computed in a spreadsheet, simulations are not required, and they are even implemented in a new package for the statistical software R, see [2]. There are however situations where a modified approach to the MVM is preferable. Suppose, for example, that we are at the dawn of a legal reform which will affect the chain ladder factors. Regarding the process error, a method to filter out accounting year effects (as, for example, described in [14] or chapter 3 of [7]) could be employed and accordingly modified development factors f_j and σ_js (potentially depending on accident years) could be used in (2.8). A correction of this kind would not be enough for the estimation error for the following reason. For any accident year i, the squared estimation error associated with accounting year k is proportional to

$$\frac{1}{\sum_{l=0}^{i-1} C_{l,I-i+k}}$$

(see (1.4) in [10]) which means that claims of all prior accident years reduce the estimation error. We doubt whether this is meaningful when dealing with legal reforms or other uncertain accounting year effects. The following algorithm provides an alternative way.

Algorithm.

1. Compute an error for the entire runoff given current information F_I and taking into account the legal reform (a possible solution could involve simulations assuming appropriate distributions on the parameter space). We denote the resulting quantity by

$$\widehat{MSEP}_{\sum_{i=I-J+1}^{I} C_{i,J}|F_I}\left(\sum_{i=I-J+1}^{I} \hat{C}_{i,J}^{I}\right).$$

2. Compute the total squared estimation error (SEE) according to the difference

$$SEE = \widehat{MSEP}_{\sum_{i=I-J+1}^{I} C_{i,J}|F_I}\left(\sum_{i=I-J+1}^{I} \hat{C}_{i,J}^{I}\right) - \sum_{k=1}^{J} \widehat{Var}(CDR_k|F_I)$$

   using (2.8) for the process error with the mentioned modification for accounting year effects.

3. Split the total estimation error across accounting years according to

$$SEE_k = \varpi_k SEE, \qquad 1 \le k \le J \tag{3.1}$$

   with weights $\varpi_k \ge 0$ and $\sum_{k=1}^{J} \varpi_k = 1$ calibrated in a way to reflect the timing of the legal reform (actuarial judgment may be required).

4. Approximate future 'accounting year' prediction uncertainties by

$$E\left[MSEP_{CDR_k|F_{I+k-1}}(0)|F_I\right] \approx \widehat{MSEP}_k = SEE_k + \widehat{Var}(CDR_k|F_I), 1 \le k \le J.$$

   Use these quantities in (2.9). This is the end of the algorithm.

We remark the following.

a) The total uncertainty still splits over accounting years, i.e.

$$\widehat{MSEP}_1 + \cdots + \widehat{MSEP}_J = \widehat{MSEP}_{\sum_{i=I-J+1}^{I} C_{i,J}|F_I}\left(\sum_{i=I-J+1}^{I} \hat{C}_{i,J}^{I}\right)$$

b) If $C_{i,j}$ are cumulative payments then

$$\varpi_k = \frac{(\hat{R}_{k-1}^{I})^2}{\sum_{l=0}^{J-1} (\hat{R}_l^{I})^2} \tag{3.2}$$

   with $\hat{R}_k^I$ as defined in (2.4), yields a regulatory solvency proxy similar to how it is defined in approach A of [13] (see our literature review). In this case, all coefficients of variation given by $\sqrt{SEE_k}/\hat{R}_{k-1}^{I}$ are equal.

c) Instead of doing step 3, the estimation error could be calculated directly for each accounting year. However, this might require nested simulations which we expect to be computationally more involved than calculating the error for the entire runoff as suggested in step 1.

There is no reason why the regulatory solvency proxy should describe the estimation uncertainty due to reforms. Indeed, suitable weights ϖ might be hard to find and even though the total estimation error is unaffected by these weights, the MVM generally depends on them. The next proposition highlights this dependency for risk measures as defined in (2.9) and (2.10).

Proposition 1. Define

$$MVM_m = c\sum_{k=1}^{J} D_{I,k}\rho_m\left(\widehat{MSEP}(\xi_k)\right), \qquad m \in \{1,2\}$$

with

$$\widehat{MSEP}(\xi_k) = \xi_k + \widehat{Var}(CDR_k|F_I),\ \rho_1(x) = \phi x,\ \rho_2(x) = \phi\sqrt{x}$$

for positive numbers ξ_k, a loading $\phi > 0$ and c and $D_{I,k}$ as given in Definition 1 and $\widehat{Var}(CDR_k|F_I)$ describes the process error given in (2.8) with the mentioned modification for accounting year effects.

The solution to the maximization problem

$$\max_{\xi_k\in\Omega,k=1,\ldots,J} MVM_m, \tag{3.3}$$

with Ω being the set of positive numbers ξ_k satisfying $\sum_{k=1}^{J}\xi_k = SEE$ is as follows.

- Let *m=1*. Then $\xi_{k^*} = SEE$ where k^* is the index of the largest $D_{I,k}$ (or one of the largest if there is more than one maximum $D_{I,k}$), and $\xi_k = 0$ for all other k solves (3.3).
- Let *m=2*. Define, for $k \in \{1,\ldots,J\}$,

$$\xi_k^* = \frac{D_{I,k}^2}{\sum_{j=1}^{J} D_{I,j}^2}\left(SEE + \sum_{j=1}^{J}\widehat{Var}(CDR_j|F_I)\right) - \widehat{Var}(CDR_k|F_I).$$

  If $\xi_k^* \geq 0$ for all $k \in \{1,\ldots,J\}$ then these ξ_k^*s solve (3.3). If $\exists k$ with $\xi_k^* < 0$ then

$$\xi_k^c = \frac{D_{I,k}^2}{\sum_{j\in\mathrm{P}} D_{I,j}^2}\left(SEE + \sum_{j\in\mathrm{P}}\widehat{Var}(CDR_j|F_I)\right) - \widehat{Var}(CDR_k|F_I), \qquad if\ k \in \mathrm{P}$$

  and $\xi_k^c = 0$ if $k \notin \mathrm{P}$, where P is the set of all indices k for which $\xi_k^c > 0$, solves (3.3).

The solution to the minimization problem

$$\min_{\xi_k\in\Omega,k=1,\ldots,J} MVM_m, \tag{3.4}$$

with Ω as in (3.3), is as follows.

- Let *m=1*. Then $\xi_{k^*} = SEE$, where k^* is the index of the smallest $D_{I,k}$ and $\xi_k = 0$ for all other k solves (3.4).

- Let *m=2*. Define k^* to be the index of the smallest

$$\frac{D_{I,k}}{\sqrt{\widehat{Var}(CDR_k|F_I)}}$$

Then $\xi_{k^*} = SEE$ and $\xi_k = 0$ for all other k solves (3.4).

Proof. The proof for *m=1* is obvious. Consider *m=2*. Ignoring the positivity constraints $\xi_k \geq 0$, the Lagrangian of the maximization problem is

$$L = \sum_{k=1}^{J} D_{I,k}\sqrt{\xi_k + \widehat{Var}(CDR_k|F_I)} + \lambda\left(SEE - \sum_{k=1}^{J}\xi_k\right)$$

Thanks to a negative definite Hessian, the first order conditions, given by

$$\frac{D_{I,k}}{2\sqrt{\xi_k + Var(\widehat{CDR}_k|F_I)}} = \lambda \;\forall\; k \in \{1, \dots, J\}, \qquad SEE = \sum_{k=1}^{J}\xi_k,$$

are sufficient for a maximum and therefore the ξ_{k^*}s solve (3.3) if they are all positive. If this is not the case, the Kuhn-Tucker conditions provide the maximum. The solution to (3.4) is obvious.

Q.E.D.

We think that the previous proposition is useful, be it for reporting purposes if the regulator asks about the impact of the selected weights in (3.1) or be it for budget-planning to have an idea how much resources should be spent on calculating the MVM. That is to say the actuary can provide to the company management a range within which the MVM obtained with a more accurate method will fall. The next corollary readily follows from the proposition.

Corollary 1. Consider the maximization problem (3.3) for the standard deviation risk measure (meaning *m=2*). If all discount factors $D_{I,k}$ are equal to 1 then the resulting prediction uncertainties of calendar years with a positive estimation error (where $\xi_k^c > 0$) are identical and smaller than the prediction uncertainty of any other calendar year.

We next provide a numerical example before we proceed with the final topic about aggregation.

3.1 Numerical Example

We borrow an example from [10] and compare their prediction uncertainties to what we obtain based on our Proposition 1 abstracting from discounting, that is to say $D_{I,k}=1$ for all accounting years k. Our intention is to highlight the impact on the MVM of different weights selected in (3.1) and used to split the estimation error across the runoff. We therefore use (2.8) without any modification for accounting year effects which means that the process errors are identical across the different prediction uncertainties.

Table 1 contains the data, the estimated chain ladder factors and the sigmas obtained with the estimator in [6].

Table 1: Cumulative claims payments $C_{i,j}$ and estimated parameters $\hat{f}_j^I$ and $\hat{\sigma}_j^I$.

$i \backslash j$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0	13'109	20'355	21'337	22'043	22'401	22'658	22'997	23'158	23'492	23'664	23'699	23'904	23'960	23'992	23'994	24'001	24'002
1	14'457	22'038	22'627	23'114	23'238	23'312	23'440	23'490	23'964	23'976	24'048	24'111	24'252	24'538	24'540	24'55	
2	16'075	22'672	23'753	24'052	24'206	24'757	24'786	24'807	24'823	24'888	24'986	25'401	25'681	25'705	25'732		
3	15'682	23'464	24'465	25'052	25'529	25'708	25'752	25'770	25'835	26'075	26'082	26'146	26'150	26'167			
4	16'551	23'706	24'627	25'573	26'046	26'115	26'283	26'481	26'701	26'718	26'724	26'728	26'735				
5	15'439	23'796	24'866	25'317	26'139	26'154	26'175	26'205	26'764	26'818	26'836	26'959					
6	14'629	21'645	22'826	23'599	24'992	25'434	25'476	25'549	25'604	25'709	25'723						
7	17'585	26'288	27'623	27'939	28'335	28'638	28'715	28'759	29'525	30'302							
8	17'419	25'941	27'066	27'761	28'043	28'477	28'721	28'878	28'948								
9	16'665	25'370	26'909	27'611	27'729	27'861	29'830	29'844									
10	15'471	23'745	25'117	26'378	26'971	27'396	27'480										
11	15'103	23'393	26'809	27'691	28'061	29'183											
12	14'540	22'642	23'571	24'127	24'210												
13	14'590	22'336	23'440	24'029													
14	13'967	21'515	22'603														
15	12'930	20'111															
16	12'539																
$\hat{f}_j^I$	1.51105	1.05369	1.02681	1.01709	1.01284	1.01098	1.00292	1.01098	1.00698	1.00141	1.00574	1.00386	1.00359	1.00042	1.00035	1.00004	
$\hat{\sigma}_j^I$	29.4993	16.8327	2.6478	5.4444	3.2477	11.4238	0.1882	2.6132	2.0863	0.0542	0.9079	0.5397	0.7202	0.0079	0.0002	0.0000	

Table 2 presents the following quantities: the estimated prediction uncertainties $\sqrt{E[MSEP_{CDR_k|F_{I+k-1}}(0)|F_I]}$ calculated with the formulas in [10] (column 2), the approximate prediction uncertainties $\sqrt{\widehat{MSEP}_k}$ obtained with the solvency proxy (3.2) (column 3), $\sqrt{\widehat{MSEP}_k}$ resulting from solving the optimization problems (3.3) and (3.4) using the standard deviation i.e. $m=2$ (columns 4 and 5) - given that all discount factors are identical the optimization problems would not have unique solutions for the variance risk measure - and the rooted process error variance (2.8) (column 6); in "Total" we find the rooted sums of all squared elements in the respective columns - by construction it is identical for columns 1 to 4 and corresponds to the Mack error of the entire runoff, for column 6 it corresponds to the rooted process error variance of the entire runoff. The squared estimation error of the total runoff is given by $(3233.7)^2$-$(2454.7)^2$=$(2105.0)^2$ and it is this

Table 2: Prediction uncertainties by accounting year over the runoff

k	M.&W. 2015	Solvency Proxy	Maximum	Minimum	$\sqrt{\widehat{Var}(CDR_k\|\mathcal{F}_I)}$
1	1842.9	2077.2	1338.7	2494.6	1338.7
2	1485.1	1419.2	1080.7	1080.7	1080.7
3	1208.3	1118.1	885.2	885.2	885.2
4	1071.1	981.7	834.2	834.2	834.2
5	901.1	831.3	733.2	733.2	733.2
6	785.3	730.7	705.8	669.0	669.0
7	525.2	475.7	705.8	424.1	424.1
8	476.3	438.2	705.8	409.6	409.6
9	366.4	336.6	705.8	320.7	320.7
10	269.3	243.3	705.8	234.0	234.0
11	245.0	229.4	705.8	225.4	225.4
12	180.5	171.3	705.8	170.1	170.1
13	130.1	126.9	705.8	126.6	126.6
14	13.7	13.5	705.8	13.3	13.3
15	2.0	2.0	705.8	1.9	1.9
16	0.3	0.3	705.8	0.3	0.3
Total	3233.7	3233.7	3233.7	3233.7	2454.7

quantity that we split across accounting years according to (3.1) in order to obtain the values in columns 2 to 4. We observe that the prediction uncertainties obtained with [10] are not very different from our solvency proxy. Further, we see that the prediction uncertainties in column "Maximum" are identical for accounting years $k\geq 6$ and equal to the process error $\sqrt{\widehat{Var}(CDR_k|F_I)}$ for $k<6$ which is consistent with Corollary 1. Finally, column "Minimum" shows that the minimum MVM (for $m=2$) is obtained if the entire estimation error is attributed to $k=1$ leaving only the process error for the remaining ks.

In order to quantify the MVM for each approach in Table 2 we assume a cost of capital of $c=6\%$ and a loading of $\phi = 3$ (this calibration corresponds to [13]), that is to say we have

$$MVM = 6\% \sum_{k=1}^{J} 3\sqrt{\widehat{MSEP}_k}$$

The results are in Table 3 where "MVM" shows the monetary values and "Rel. to min." the values relative to the "Minimum."

Table 3: MVM

	M.&W. 2015	Solvency Proxy	Maximum	Minimum
MVM	1'710	1'655	2'274	1'552
Rel to min.	110%	107%	147%	100%

Hence, the error due to wrong weights cannot be larger than 47% of the smallest MVM possible.

4. AGGREGATING LINES OF BUSINESS

Before concluding we discuss aggregation. Suppose that we would like to use correlations to aggregate lines of business in order to obtain the MVM on a company level. Regarding dependencies between lines of business, we need to answer the following questions

- What are the correlations between the process errors?
- What are the correlations between the estimation errors?
- What are the correlations between the estimated claims development results (CDR)?
- What are the correlations between the ultimate liabilities?

These questions cannot be answered independently. For example, if we define correlations for the yearly process errors and for the estimation errors, then the correlations between the CDRs and the correlations between the ultimate liabilities are determined. And it is not difficult to show that the correlations between the ultimate liabilities will be smaller, in absolute value, than the correlations between the CDRs. Or if we define the correlations between the ultimate liabilities, then this will likely imply time-varying correlations for the CDRs. We therefore suggest to reflect well before deciding on a dependency structure and be clear when documenting about it - which can only help in order to fulfill regulatory reporting requirements.

As a side note, we remark that if two individual triangles satisfy the classical chain ladder assumptions then an aggregated triangle obtained by adding up the individual triangles will, in general, no longer satisfy the classical chain ladder assumptions.

5. CONCLUSIONS

We discussed the market value margin (MVM) for a general insurance runoff based on the distribution-free chain ladder model and suggested an easy way to modify the approach of [10] in order to take accounting year effects into consideration. Further, we showed that different splits of the estimation error over the runoff lead to different MVMs even if the estimation error of the total runoff is unchanged. We derived the splits which minimize and maximize the MVM which could be useful to quantify model uncertainty. Finally, we argued that one has to be careful when estimating an aggregated MVM for two lines of business because the correlations between quantities like the process errors, the estimation errors, the claims development results and the ultimate liabilities depend on each other.

We believe that our results are helpful in daily actuarial practice. We leave it to future research to shed light on how the MVM is affected by risk factors like stochastic discount rates and other factors which we mentioned but omitted in our analysis.

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Abbreviations and notations

c, cost of capital
CDR, claims development result
CL, chain ladder
$D_{I,k}$, discount factors
E[ ], expectation operator
MSEP, mean squared error of prediction
MVM, market value margin
Var( ), variance
ϕ, a loading
ρ, a risk measure

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