Interpolation Hacks and their Efficacy

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Abstract
Actuaries are consistently faced with the decision of how to interpolate loss development factors. Methods vary from linear to more theoretical. This paper explores how various methods hold up to actual data and each other by estimating errors in reserve prediction when using paid loss development, incurred loss development and Bornhuetter Ferguson methods. It also lays out a variety of methods for actuaries to use. Lastly, this paper adds an additional process to account for unique situations such as seasonal fluctuations in claims activity. Along with this paper, I have included a practical tool programmed with interpolation formulae and the seasonal method.

Keywords. Interpolation, Development, Quarterly Reserving

1. INTRODUCTION

As Actuaries, we are challenged with producing estimates which are assumed to be accurate given our vast background and industry knowledge. In practice, the documentation of our thought process is a crucial part of third-party assessment of our work product. The hindsight accuracy of the estimate is something we seldom evaluate.

One of the crucial assumptions we make is the selection of development factors and how to interpolate them. Although practices vary widely, it is something that in my experience is not well documented. In fact, in many instances, third party software is relied upon to determine the interpolated amount.

Extrapolated development factors, such as those used for a 9 month old accident year, are more frequently inconsistent and poorly documented. Actuarial practice also varies in determining how to treat the exposure growth portion of a development factor and how to document this treatment.

While differences in judgement regarding a loss development selection and ancillary differences in judgment due to the nuances of a particular interpolation/extrapolation method may not seem material in the context of a reserve review when compared to other more substantial judgments, they do have an impact. More importantly, they impact the quality of our documentation.

Interpolation is heavily relied upon for interim reserves studies and year-end studies where the practicalities of timing only allow for a third quarter in-depth review. Interpolation is also relied upon to form opinions of actual versus expected loss emergence.

Methodology for interpolation varies from sophisticated curve fits, to shortcut methods, to straight linear. This paper will examine various methods (known to the author) and how they compare to actual results produced with sample data that covers various lines of business. This paper will examine the relative degrees of error each method might be expected to produce and the overall effect on reserves estimation.

In addition, the paper will address a special situation where there is specific knowledge of development patterns which are expected to vary on a seasonal basis.

1.1 Research Context

These general concepts are covered by other authors (Flannery, Press, Teukolsky and Vetterling) in "Numerical Recipes" and more recently in Variance Magazine by Joseph Boor in "Interpolation Along a Curve."

Richard Sherman also explored these concepts in "Extrapolating, Smoothing and Interpolating Development Factors."

1.2 Objective

The objective of this paper is to provide options, easy to follow formulae, and tools for the purpose of interpolation, along with context regarding the efficacy of various methods. The hope for this paper is to be a useful reference source for actuaries and students familiarizing themselves with actuarial methods and shortcuts.

1.3 Outline

The remainder of the paper proceeds as follows:

Section 2: Background

Section 3: Interpolation Methods and Formulae

Section 4: Extrapolation Methods and Formulae

Section 5: Handling of Exposure Growth

Section 6: Testing of Methods and Results for Interpolation

Section 7: Testing of Methods and Results for Extrapolation

Section 8: Seasonal Adjustment Method

Section 9: Conclusions

2. BACKGROUND

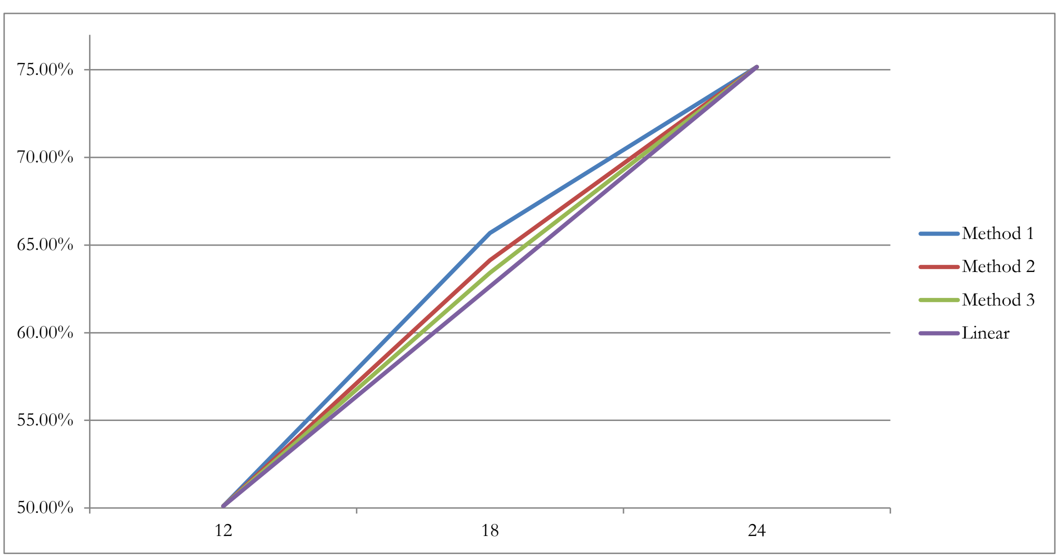
When we think of the concept of interpolation, it is extremely simple – how will losses paid or reported vary over the course of a year (or other specified period)? The easiest concept to grasp is linear interpolation, which assumes development proportional to time over the period. However, sometimes actuaries are more comfortable with the assumption that more will be reported or paid earlier rather than later (and sometimes vice versa). This assumption gives rise to alternative methods, including curve fitting methods.

Since performing a power curve or Weibull regression inside of a spreadsheet can be cumbersome, actuaries have developed many "shortcut" methods and formulae which mimic the effects of a curve regression and consequently mimic the effect that more losses emerge sooner over the interpolation period. We can see this graphically using a simple example. Suppose annual

Ages	12	24	36
	12 - 24	24 - 36	36 - 48
Selected Result	1.500	1.200	1.050
FacToUlt	1.996	1.331	1.109
Percent of Ult	50.11%	75.16%	90.19%

development is as follows:

If we apply various interpolation methods which assume more losses emerge sooner and compare the implied development to that indicated by linear interpolation, we would observe the following:



The corresponding 18 month development factors would be as follows:

Age	18
Method 1	1.522
Method 2	1.559
Method 3	1.577
Linear	1.597

As can be seen above, since linear interpolation assumes steady development, the factor at 18 months is higher than that given by the other methods (which assume accelerated development earlier in the period). These methods are used for illustration and all methods will be given in detail in Section 3.

Although the above demonstrates the general goal of interpolation, in practice, we seldom evaluate the results of one method versus others. The following sections will outline several methods. While the derivations of some of the formulae are quite obvious, some of the shortcut formulae have been passed down from actuary to actuary. It is beyond the scope of this paper to understand the derivation of each method; rather this paper will evaluate the efficacy of each given certain assumptions, which that individual practitioner might make. Since the data evaluated in this paper is far from exhaustive, the link between assumptions and accuracy of the each method is important. The choice of the curve should be driven by what the actuary assumes about the true shape of the curve. This paper will also not explore all possible curve fitting methods, but only some of the more common ones to compare to other shorter methods.

Lastly, based on the same notion that the assumptions about emergence are important, the actuary may use these methods for accident year or policy year methods equally. Some of the observation about early maturities made in the sections to follow would obviously apply for a longer period of time when using policy year data.

3. INTERPOLATION METHODS AND FORMULAE

The following is a list of methods I will explore:

1. Linear
2. Inverse Power Curve on Remaining Development (CDF-1)

3. Weibull
4. Inverse Power Curve on Total Development (CDF)
5. Exponential Curve on Remaining Development (CDF-1)
6. Exponential Curve on Total Development (CDF)
7. Logarithmic Proportions Shortcut (Shortcut 1)
8. Exponential Weighted Shortcut (Shortcut 2)

The formulae included below will contain the following terms. Since in practice most of these formulae will be utilized in Excel, I have used shorthand geared toward excel functions.

LDF_T – Incremental Loss Development Factor at age T

CDF_T – Cumulative Loss Development Factor at age T

T – Development Age in Months

PR_T or PP_T – Percent Reported or Percent Paid or 1/ CDF_T

EXP (Value) – e^{Value}

* - x or multiplication

3.1 Linear Method

The most commonly used method is the Linear Method, which as stated above assumes that the percent paid or reported grows at a constant rate with time. For the purpose of demonstrating the methods, I will assume that we are interpolating between 12 and 24month factors in all of our examples. I will also use the following Paid Development Factors:

$CDF_{12} = 1.996$

$CDF_{24} = 1.331$

$PP_{12} = 50.11\%$

$PP_{24} = 75.16\%$

I will also suppose I am interpolating to 15 months. To derive a linear interpolation estimate, I use the following formula.

$PP_{15} = PP_{12} * (24 – 15)/(24 – 12) + PP_{24} * (15 – 12) / (24 – 12) =$

$CDF_{15} = 1.774$

3.2 Inverse Power Curve Regression on Remaining Development (IVP Decay)

An Inverse Power Curve Regression assumes that development and loss emergence behave in such a way that interim CDFs can be expressed as follows:

$$CDF_T - 1 = a * T^{-b}$$

In more qualitative terms, it is assumed that the remaining development at any point in time varies inversely with time.

Translating this into a convenient linear regression results in the following equation:

$$\ln(CDF_T - 1) = b * \ln(1/T) + \ln(a)$$

In Excel, the function to find the 15 month CDF would be as follows:

EXP(TREND(ln(CDF_{12}-1):ln(CDF_{24}-1),ln(1/12):ln(1/24),ln(1/15)))+1

Using the values stated in the linear example, the resulting value for CDF_{15} = 1.698

Note that I do not try to interpolate between points any wider than the two adjacent development points, as fitting a large curve is more complex and often results in aberrant values. The theoretical considerations for best fit are outside the scope of this paper.

3.3 Weibull Method

The Weibull Method assumes that development and loss emergence behave in such a way that interim CDFs can be expressed as follows:

$$1 - PP_T = EXP(-a \times T^b)$$

Translating this into a convenient linear regression results in the following equation:

$$\ln(-\ln(1-\%PP_T)) = \ln(a) + b \ln(T)$$

In excel, the function to find the 15 month CDF would be as follows:

1/[1 – EXP{-EXP(TREND(ln(-ln(1-1/CDF_{12})):ln(-ln(1-1/CDF_{24})),ln(12):ln(24),ln(15)))}]

Using the values stated in the linear example, the resulting value for CDF_{15} = 1.722

Note the difference between the above curves is merely the form of the equation. The basic principal is the same: remaining development varies inversely with time. This paper does not lay out every possible combination of type of curve and dependent variable, but rather some of the more commonly used ones.

3.4 Inverse Power Curve Regression on Total Development (IVP)

The Inverse Power Curve Regression on Total Development assumes that development and loss emergence behave in such a way that interim CDFs can be expressed as follows:

$$CDF_T = a * T^{-b}$$

In more qualitative terms, it is assumed that the total development at a point in time varies inversely with time.

Translating this into a convenient linear regression results in the following equation:

$$\ln(CDF_T) = b * \ln(1/T) + \ln(a)$$

In excel, the function to find the 15 month CDF would be as follows:

EXP(TREND(ln(CDF_{12}):ln(CDF_{24}),ln(1/12):ln(1/24),ln(1/15)))

Using the values stated in the linear example, the resulting value for CDF_{15} = 1.752

Note that this method will not create errors when the CDF is less than or equal to 1.000. While this is an advantage, in practice, I often have formulae default to linear values (and have done so in the practical tool) when CDFs are less than one as the differences in small factors are less material. When CDFs are large, the method tends to produce much higher values than the regression on remaining development.

3.5 Exponential Curve Regression on Remaining Development (Expo Decay)

Exponential Regression assumes that development and loss emergence behave in such a way that interim CDFs can be expressed as follows:

$$CDF_T - 1 = a * EXP(bT)$$

In more qualitative terms, it is assumed that the remaining development at a point in time varies inversely with time.

Translating this into a convenient linear regression results in the following equation:

$$\ln(CDF_T - 1) = b * T + \ln(a)$$

In excel, the function to find the 15 month CDF would be as follows:

EXP(TREND(ln(CDF_{12}-1):ln(CDF_{24}-1),12:24,15))+1

Using the values stated in the linear example, the resulting value for CDF_{15} = 1.756

This has some properties of the Weibull curve and some properties of the inverse power curve and acts as a variation.

3.6 Exponential Curve Regression on Total Development (Expo)

Exponential Regression assumes that development and loss emergence behave in such a way that interim CDFs can be expressed as follows:

$$CDF_T = a * EXP(bT)$$

In more qualitative terms, it is assumed that the total development at a point in time varies inversely with time.

Translating this into a convenient linear regression results in the following equation:

$$\ln(CDF_T) = b * T + \ln(a)$$

In excel, the function to find the 15 month CDF would be as follows:

$EXP(TREND(ln(CDF_{12}):ln(CDF_{24}),12:24,15))$

Using the values stated in the linear example, the resulting value for $CDF_{15} = 1.803$.

3.7 Logarithmic Proportions Shortcut (Shortcut 1)

This shortcut will produce results which are generally about midway between linear results and curve fitted results.

In excel, the formula to find the 15 month CDF would be as follows:

$CDF_{15} = CDF_{12}$^((ln(CDF_{24})/ln(CDF_{12}))^((15-12)/(24-12)))

Using the values stated in the linear example, the resulting value for $CDF_{15} = 1.740$.

This formula is easier to program into a spreadsheet than regressions and provides a directionally similar result. Regressions require the logarithm to be made in a separate cell first which is cumbersome when dealing with multiple development points.

3.8 Exponential Weighted Shortcut (Shortcut 2)

This shortcut will produce results which are generally higher than Shortcut 1, but lower than linear. The results tend to hover near the exponential regression as well.

In excel, the formula to find the 15 month CDF would be as follows:

$CDF_{15} = 1/ln(EXP(1/CDF_{12})*(24-15)/(24-12) + EXP(1/CDF_{24})*(15-12)/(24-12)))$

Using the values stated in the linear example, the resulting value for $CDF_{15} = 1.755$.

This formula is easier to program into a spreadsheet than regressions and provides a directionally similar result.

3.9 Summary of Values from Various Methods

The following table summarizes the results of the methods extended out to further maturities:

Development Factor Selection						
Ages	12	24	36	48	60	72
	12 - 24	24 - 36	36 - 48	48-60	60-72	72-84
Selected Result	1.500	1.200	1.050	1.025	1.020	1.010
FacToUlt	1.996	1.331	1.109	1.056	1.030	1.010
Percent of Ult	50.11%	75.16%	90.19%	94.70%	97.07%	99.01%
Interim Ages		15	27	39	51	63
Linear		1.774	1.267	1.095	1.049	1.025
IVP Decay		1.698	1.239	1.090	1.047	1.022
Weibull		1.722	1.248	1.092	1.048	1.023
IVP		1.752	1.262	1.094	1.049	1.025
Expo Decay		1.756	1.250	1.092	1.048	1.023
Expo		1.803	1.271	1.095	1.049	1.025
Logarithimic Proportions Shortcut 1		1.740	1.248	1.092	1.048	1.023
Exponential Weighting Shortcut 2		1.755	1.264	1.095	1.049	1.025

It is obvious that our choice of method matters less as accident years mature. In this particular example the Exponential regression of total development actually provides a development factor that is higher than linear. This implies that more losses will emerge in the latter part of the year than would be indicated proportionally with time. Later in this paper we will explore how actual data relates to this assumption.

4. EXTRAPOLATION METHODS AND FORMULAE

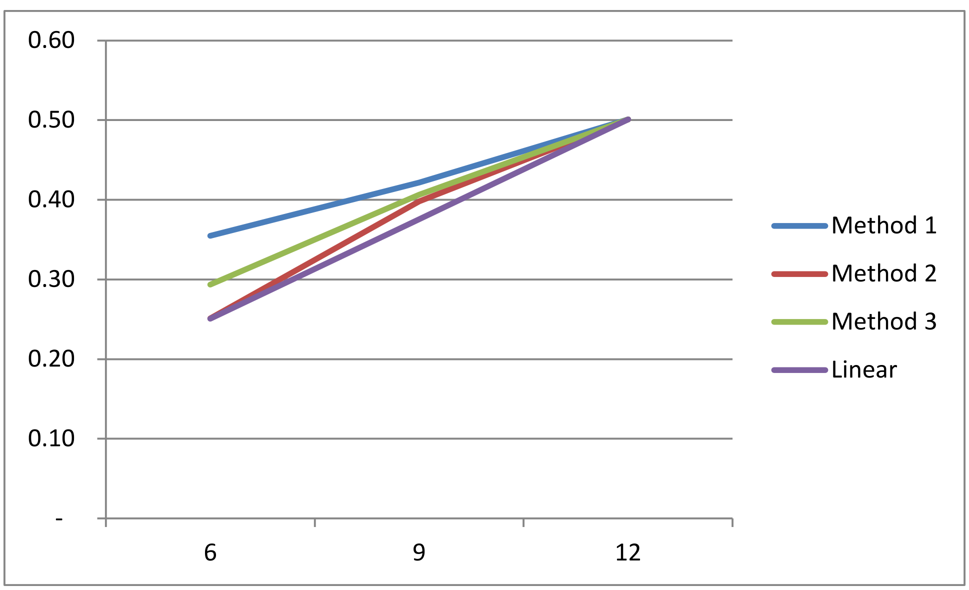
Probably the only concept more elusive than interpolation methods is extrapolation methods. All of the methods are either linear or based on shortcuts and the theoretical bases for these methods are more tenuous than for interpolation methods.

Development prior to 12 months or prior to the first known development point is complicated by a more rapid rate of growth as well as exposure growth. An extrapolation formula needs to consider both of these factors and the actuary should document each piece separately. All of the shortcuts provided mimic the general concept that since more losses are reported or paid closer to the time of the accident, the development will be less than linear within the 12 months. In other words, more than half of losses reported by 12 months on losses occurring within the first six months will be reported as of six months. While this may not always be the case, it is the concept behind these shortcuts.

Section 5 will deal more directly with exposure growth and how it should be handled.

For example, suppose that as of 12 months, we expected reported losses to grow by an additional 40% or that we have a reported development factor of 1.40. Assuming that all premium is earned as of 12 months in time, the development factors pulled from triangles would not include any exposure growth. When evaluating a development factor as of 6 months, we might assume an extrapolation curve that assigns a value of 1.80. The curve would take into account the expected additional development growth, but not necessarily the exposure growth. Assuming even earning of premium throughout the year, the exposure growth factor is 2.00. Therefore the factor to apply to losses at 6 months in order to get a full year of losses would be 1.80 x 2.00 or 3.60. It is beyond the scope of this paper to analyze why specific formulae do not account for exposure growth. In section 7, we will test the adequacy against actual data using our assumption that most actuarial shortcuts do not include an exposure growth component. The following formulae all assume extrapolation without exposure growth.

I will explore several shortcut methods in additional to linear extrapolation. A graph of the percent reported implied by the various formulae might look as follows with most methods assuming the percent reported (or paid) is greater than that implied by the linear method:



Note, as we see in our examples, that when development factors are very high, the reverse is true and shortcut methods produce higher development than linear.

4.1 Linear Method

The easiest method to apply is the Linear Method, which, as stated above, assumes that the percent paid or reported grows evenly with time. For the purpose of demonstrating the methods, I will assume that we are extrapolating from a 12 month factor in all of our examples. I will also use the following Paid Development Factors:

$CDF_{12} = 1.996$

$PP_{12} = 50.11\%$

I will also suppose I am interpolating to 6 months. To derive a linear extrapolation estimate, I use the following formula.

$PP_6 = PP_{12} * (6/12)$

$CDF_6 = 3.992$

4.2 The Plus 12 Method (Method 1)

This method raises the base development factor to a power which increases as the number of months decreases by using subtraction. The formula is as follows:

$CDF_6 = CDF_{12}$^((12+12 – 6)/12) where the first 12 in the exponent represents the age of the base factor and the second 12 is always present.

$CDF_6 = 2.819$.

4.2 The Power Ratio Method (Method 2)

This method raises the base development factor to a power which increases as the number of months decreases by using a ratio. The formula is as follows:

$CDF_6 = CDF_{12}$^(12/6)

$CDF_6 = 3.983$

This method tends to reach uncommonly high values when applied to smaller maturities.

4.3 The Natural Log Method (Method 3)

This method uses the natural log of the remaining development, applies a ratio based on the extrapolation month, and converts it back using Euler's number, e.

CDF_6 =1/(1-EXP(ln(1-1/ CDF_{12})*(6/12)))

$CDF_6 = 3.405$

This method tends to be more stable at lower maturities.

4.4 Summary of Values from Various Methods

The following table summarizes the results of the methods:

Ages	12
	12 - 24
Selected Result	1.500
FacToUlt	1.996
Percent of Ult	50.11%
Interim Age	6
Linear	3.992
Method 1	2.819
Method 2	3.983
Method 3	3.405

5. HANDLING OF EXPOSURE GROWTH

As mentioned earlier, varying practices exist with regards to exposure growth and unfortunately many actuaries are unaware of whether their extrapolation method accounts for it. The use of reserving software has created, to some degree, a "black box" that obscures the derivation of early maturity loss development factors. To be fair, with the use of exposure based methods such as Bornhuetter Ferguson or Cape Cod, most actuaries feel that factors for immature periods are immaterial to an analysis. While this is true, it is preferable to have extrapolation methods explicitly used and documented.

It is difficult to extrapolate a factor for a short accident period. This is further complicated by the existence of exposure growth in exposure based methods. Essentially, there are two ways to look at a short period: it can be viewed as a short period on its own or as a fraction of the full year. From the former viewpoint, we would use factors (utilizing the methods above) which do not include exposure growth. From the latter viewpoint, we would adjust our factors for exposure growth and then scale the final ultimate produced. For the loss development method, it seems arbitrary to make a distinction as the two answers will never be different. However, for the Bornhuetter Ferguson (BF) method, the assumption can make a difference in the final answer (sometimes a large one).

Consider the example and factors from Section 4. Suppose we chose Method 3. Further

	Earned Premium	Paid Losses	CDF	LDM	Proration	Ultimate	Loss Ratio
Partial Year Method	50	10	3.405	34.05	100%	34.05	68.1%
Full Year Method	100	10	6.811	68.11	50%	34.05	68.1%

suppose that earned premium for the full year is 100 and that paid losses as of 6 months are 10. As mentioned above, the assumption of partial year or full year makes no difference to the Loss Development Method (LDM):

Now assume that the Initial Expected Loss Ratio is 60%. The following shows the results of the BF Method under each assumption:

	Earned Premium	Paid Losses	CDF	IELR	BF	Proration	Ultimate	Loss Ratio
Partial Year Method	50	10	3.405	60.0%	31.19	100%	31.19	62.4%
Full Year Method	100	10	6.811	60.0%	61.19	50%	30.60	61.2%

In the second example, the differences in loss ratio are not due to inaccuracies in either method. The difference is driven by the assumed maturity of the year. The BF method assigns more weight to the loss development method based on the maturity of the accident year as measured by the inverse of the loss development factor. Since the partial year method uses a smaller development factor, the loss development method receives more weight. Given the shortened period relative to the full year, one could argue that it is more mature (i.e., that the average accident date is earlier than a full year) and that the partial year method is therefore preferable. In reality, the relative weighting assigned to the ELR and the loss development method is subjective and many actuaries prefer to give less weight to a loss development method based on highly leveraged and extrapolated factors. In any event, awareness is important as the weighting can have material effects on results.

6. TESTING OF METHODS AND RESULTS FOR INTERPOLATION

The sample data was based on actual triangles from two different insurance companies. The lines of business underlying the data include 30 different lines and sublines with a mix of property and casualty. Lines were matched into groups based on their development properties (using 12 month factor and length of tail to group them). I began with quarterly loss triangles for many lines of business on a paid and incurred basis. Accident years within our data span from 2003 to 2014. For some lines the latest evaluation is December 31, 2014 and for others is June 30 or September 30 of

2014. First, I calculated quarterly development factors using several averages including simple, weighted, and simple excluding high and low. Then I created 4 separate annual triangles, incepting at 3, 6, 9 and 12 months respectively. Since only the weighted average ties back between quarterly and annual triangles and since I wanted to isolate the error solely due to interpolation method, I made the selections for these triangles equal to their quarterly triangle equivalent (by type of average).

I applied the interpolation methods to each triangle as described in Section 3. Then I projected the quarterly results by accident year using the most recent data and evaluation. Each result was then compared to the interpolated result based on various annual triangles. I evaluated the interpolated results using interpolations from 3, 6, and 9 months prior to the latest quarterly date. To illustrate, if the latest data was evaluated as of 12 months, I would use factors from my 3 month annual triangle to determine the 9 months prior interpolation error. The 6 month annual triangle interpolated to 12 months gave the 6 month prior error and the 9 month annual triangle gave the three month prior error. The errors were also calculated on a paid and incurred basis and for all three averages.

Error was measured in terms of IBNR for the incurred triangles and total reserves for the paid triangles. The percent error was calculated as a percent of total IBNR or reserve. Therefore percent error for paid losses would equal:

[Ultimate losses derived from Interpolated method – Ultimate losses derived from quarterly triangle factors] / [Ultimate losses derived from quarterly triangle factors – Paid losses at latest evaluation].

6.1 Results for Short Tailed Lines of Business

I grouped lines where the incurred development factor at 12 months was not greater than 2.0. These lines were primarily various types of personal auto business. The average incurred loss development factor at 12 months was roughly 1.09 and typical development dropped off at about 84 months. The average paid loss development factor at 12 months was approximately 1.5.

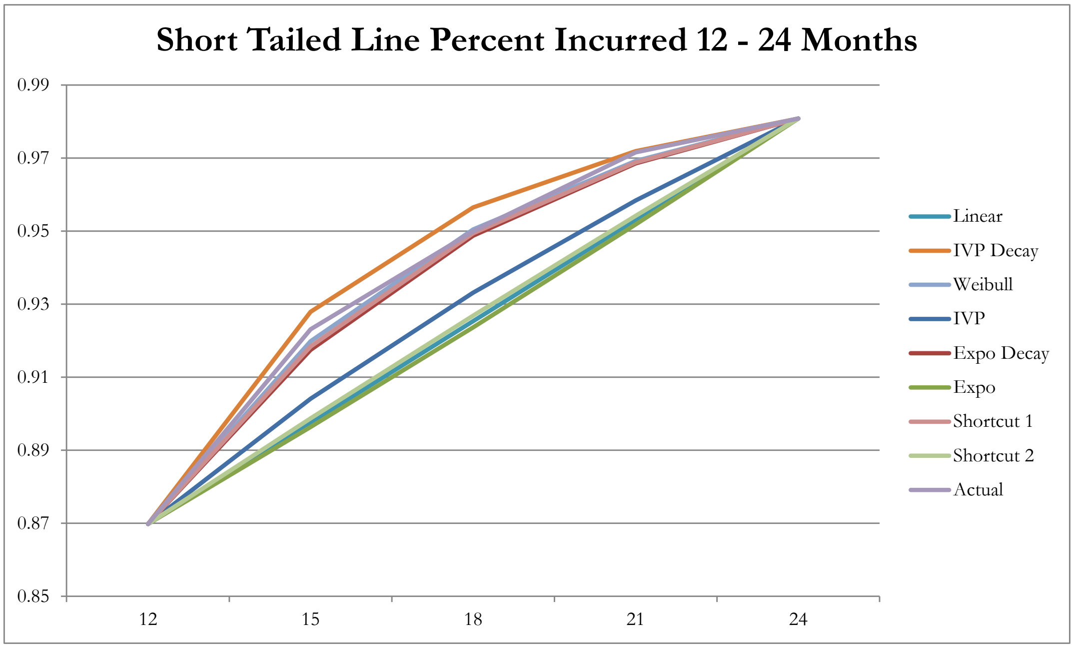
6.1.1 The Factors

Looking at the group as a whole, the following table displays the actual and estimated incurred factors from various methods measured on interpolating from various points during the year (using weighted average factors).

Short Tailed Lines - Incurred Development Factors

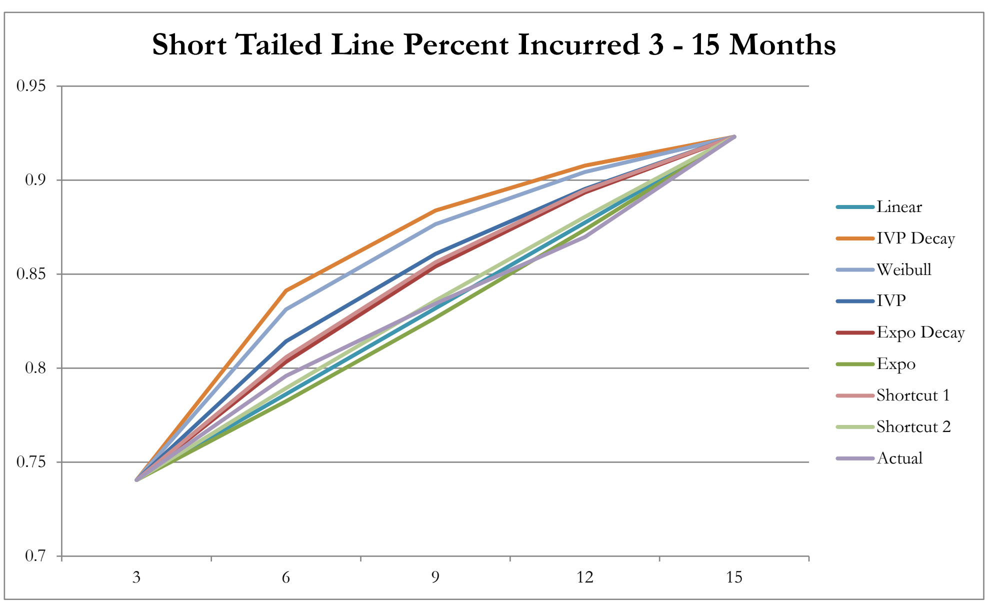
Accident Year	Age	From Quarterly "Actual"	Linear	IVP Decay	Weibull	IVP	Expo Decay	Expo	Shortcut 1	Shortcut 2
			Using factors interpolated from 9 months prior to date							
2006	108	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2007	96	1.000	1.001	1.001	1.001	1.001	1.001	1.001	1.001	1.001
2008	84	1.002	1.001	1.001	1.001	1.001	1.001	1.001	1.001	1.001
2009	72	1.002	1.002	1.002	1.002	1.002	1.002	1.002	1.002	1.002
2010	60	1.006	1.005	1.005	1.005	1.005	1.005	1.005	1.005	1.005
2011	48	1.005	1.004	1.004	1.004	1.004	1.004	1.005	1.005	1.004
2012	36	1.012	1.012	1.011	1.011	1.011	1.011	1.012	1.012	1.012
2013	24	1.029	1.032	1.027	1.028	1.030	1.028	1.032	1.029	1.031
2014	12	1.090	1.096	1.063	1.067	1.079	1.077	1.105	1.076	1.090
			Using factors interpolated from 6 months prior to date							
2006	108	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2007	96	1.000	1.001	1.001	1.001	1.001	1.001	1.001	1.001	1.001
2008	84	1.002	1.001	1.001	1.001	1.001	1.001	1.001	1.001	1.001
2009	72	1.002	1.002	1.002	1.002	1.002	1.002	1.002	1.002	1.002
2010	60	1.006	1.005	1.005	1.005	1.005	1.005	1.005	1.005	1.005
2011	48	1.005	1.005	1.004	1.004	1.004	1.004	1.005	1.005	1.005
2012	36	1.012	1.012	1.012	1.012	1.012	1.012	1.013	1.012	1.012
2013	24	1.029	1.033	1.028	1.029	1.031	1.029	1.033	1.028	1.032
2014	12	1.090	1.097	1.061	1.067	1.083	1.077	1.103	1.077	1.093
			Using factors interpolated from 3 months prior to date							
2006	108	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2007	96	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2008	84	1.002	1.001	1.001	1.001	1.001	1.001	1.001	1.001	1.001
2009	72	1.002	1.002	1.002	1.002	1.002	1.002	1.003	1.002	1.002
2010	60	1.006	1.006	1.006	1.006	1.006	1.006	1.006	1.006	1.006
2011	48	1.005	1.005	1.004	1.005	1.005	1.005	1.005	1.005	1.005
2012	36	1.012	1.012	1.011	1.012	1.012	1.012	1.012	1.012	1.012
2013	24	1.029	1.031	1.029	1.030	1.031	1.030	1.032	1.030	1.031
2014	12	1.090	1.098	1.071	1.078	1.090	1.083	1.100	1.082	1.096

An examination of actual quarterly incurred factors between 12 and 24 months for a typical line in this group reveals the differences between actual factors and our interpolation methods.



Note that the shape of the curve of methods indicates that most methods anticipate more losses emerging in the beginning of the period versus the latter part of the period. Also all methods are higher than linear in terms of percent reported. The actual data agrees with the majority of the methods in the accelerated emergence of losses. This would suggest that linear interpolation overstates estimates for short tailed lines. Early maturities are shown in these graphs because visually it is easier to see the shape. From the table above, one can see that choice of method matters less once factors are less than 1.05.

However looking further back at the period between 3 and 15 months we see the following results:



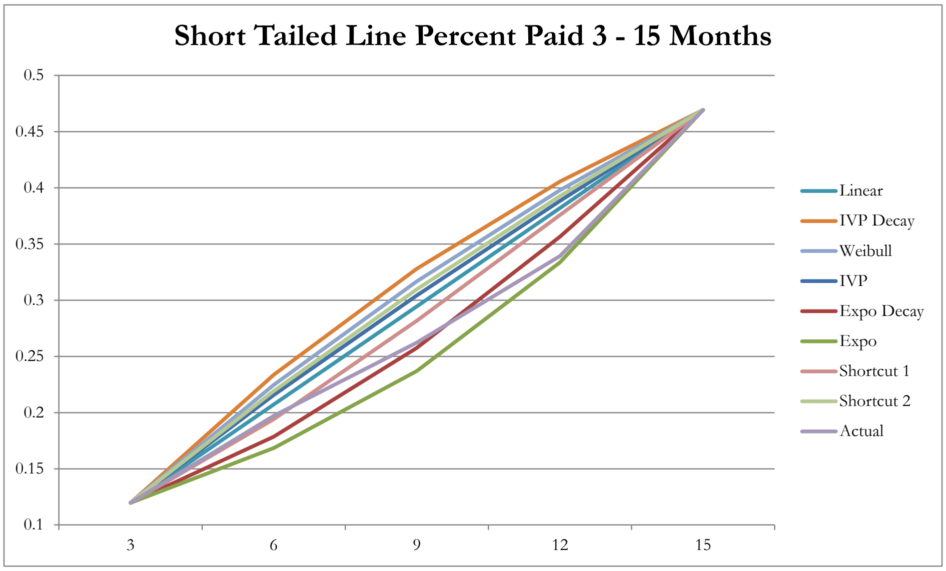
Although the methods are interpolating along the same general curve, the actual results are much nearer to linear. Note to avoid confusion about these factors, these factors are not extrapolated even though they are lower than 12 months. These are interpolated factors from a 3, 15, 27, etc. triangle. They use the same methods as other interpolated factors. They are added here to show how the shapes may differ during this time period.

The table of paid factors (using weighted average factors) is as follows:

Short Tailed Lines - Paid Development Factors

Accident Year	Age	From Quarterly "Actual"	Linear	IVP Decay	Weibull	IVP	Expo Decay	Expo	Shortcut 1	Shortcut 2
				Using factors interpolated from 9 months prior to date						
2006	108	1.002	1.002	1.002	1.002	1.002	1.002	1.002	1.002	1.002
2007	96	1.003	1.003	1.003	1.003	1.003	1.003	1.003	1.003	1.003
2008	84	1.005	1.005	1.005	1.005	1.005	1.005	1.005	1.005	1.005
2009	72	1.009	1.009	1.009	1.009	1.009	1.009	1.009	1.009	1.009
2010	60	1.017	1.018	1.017	1.017	1.018	1.017	1.018	1.017	1.018
2011	48	1.033	1.036	1.033	1.034	1.036	1.034	1.037	1.034	1.036
2012	36	1.074	1.080	1.073	1.075	1.079	1.075	1.082	1.075	1.078
2013	24	1.175	1.193	1.176	1.183	1.195	1.189	1.212	1.183	1.185
2014	12	1.502	1.419	1.372	1.389	1.420	1.485	1.570	1.428	1.396
				Using factors interpolated from 6 months prior to date						
2006	108	1.002	1.002	1.002	1.002	1.002	1.002	1.002	1.002	1.002
2007	96	1.003	1.003	1.003	1.003	1.003	1.003	1.003	1.003	1.003
2008	84	1.005	1.005	1.005	1.005	1.005	1.005	1.005	1.005	1.005
2009	72	1.009	1.009	1.009	1.009	1.009	1.009	1.009	1.009	1.009
2010	60	1.017	1.018	1.017	1.017	1.018	1.017	1.018	1.017	1.018
2011	48	1.033	1.037	1.033	1.034	1.036	1.034	1.037	1.034	1.036
2012	36	1.074	1.081	1.072	1.075	1.080	1.075	1.083	1.074	1.079
2013	24	1.175	1.194	1.174	1.182	1.195	1.188	1.210	1.182	1.187
2014	12	1.502	1.414	1.365	1.390	1.429	1.470	1.550	1.410	1.385
				Using factors interpolated from 3 months prior to date						
2006	108	1.002	1.002	1.002	1.002	1.002	1.002	1.002	1.002	1.002
2007	96	1.003	1.003	1.003	1.003	1.003	1.003	1.004	1.003	1.003
2008	84	1.005	1.005	1.005	1.005	1.005	1.005	1.005	1.005	1.005
2009	72	1.009	1.009	1.009	1.009	1.009	1.009	1.009	1.009	1.009
2010	60	1.017	1.018	1.017	1.017	1.018	1.017	1.018	1.017	1.018
2011	48	1.033	1.036	1.033	1.034	1.036	1.034	1.036	1.034	1.036
2012	36	1.074	1.079	1.073	1.075	1.078	1.075	1.080	1.074	1.078
2013	24	1.175	1.189	1.174	1.180	1.188	1.183	1.196	1.179	1.184
2014	12	1.502	1.447	1.408	1.434	1.467	1.479	1.532	1.442	1.421

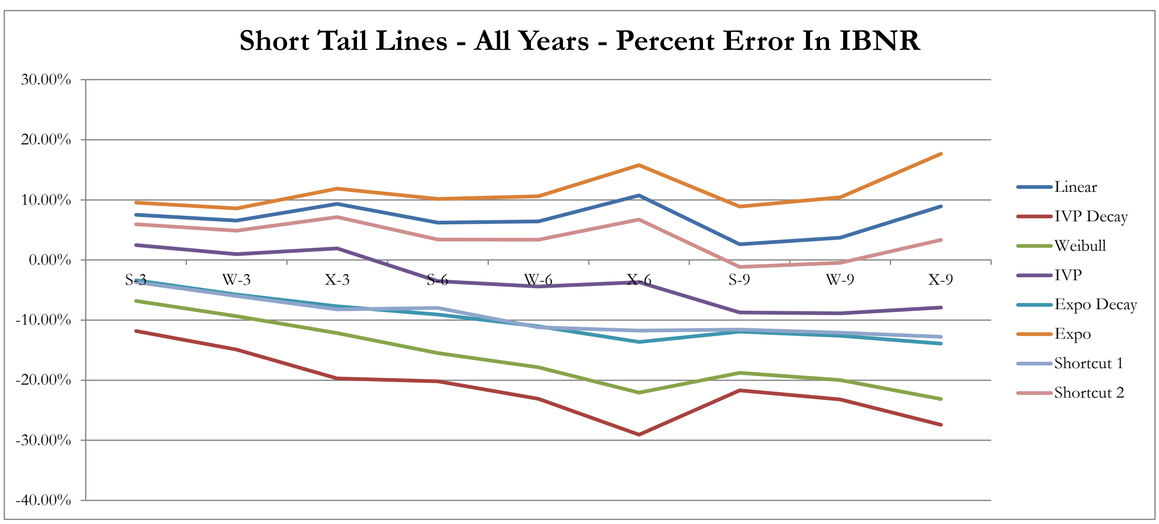
Looking at the paid factors for the 3 – 15 month period for a typical line, the curve reverses itself:



Both the method results and the actual data support the notion that prior to 15 months on a paid basis, less losses are paid earlier in the period and more losses are paid later in the period.

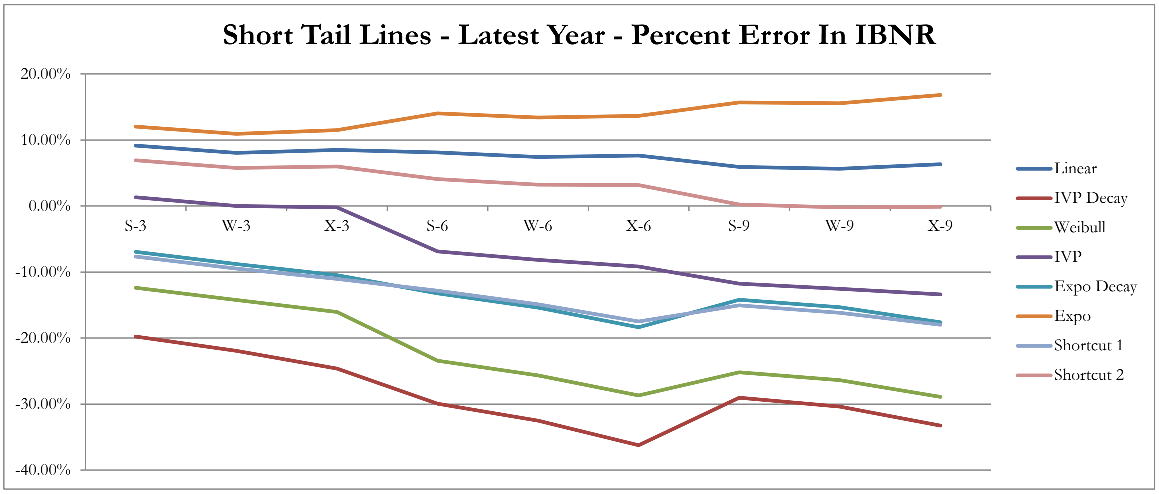
6.1.2 The Errors

The actual errors in each method for all years combined are as follows for incurred factors:

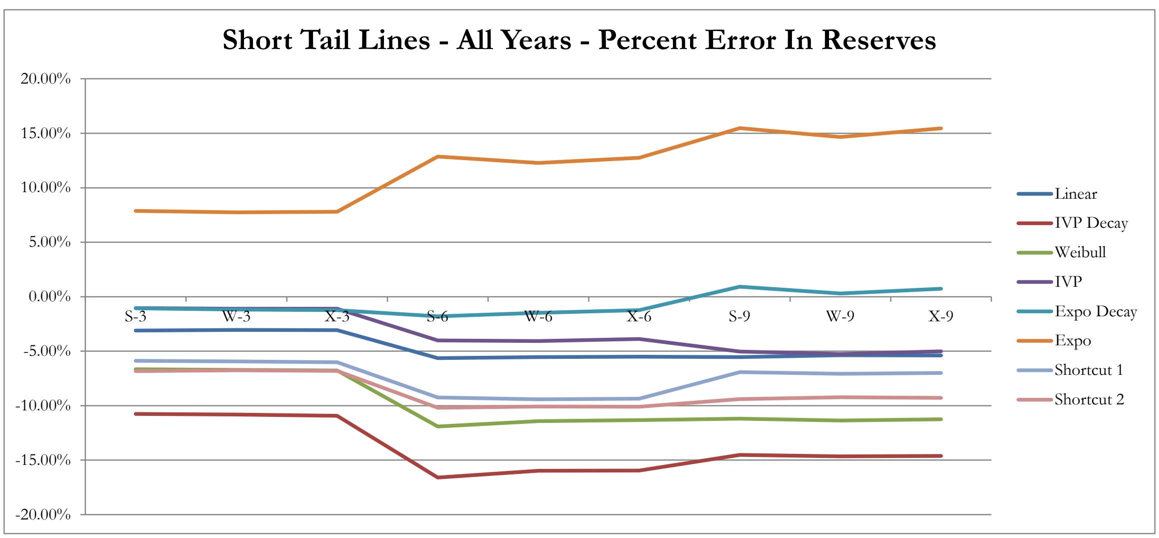


In this graph, S – 3 represents the method performed on simple average factors 3 months prior to evaluation date. W represents weighted factors and X represents simple average excluding high and low factors. Seemingly the most appealing method for short tailed incurred business based on this graph would be Shortcut 2. Many of the curves understate reserves and the linear method overstates reserves.

Looking at the latest year only, the pattern is more exaggerated:



For paid development, the error in all years is illustrated in the following graph. Where the factors are higher, in this case, the exponential method seems to work best:



A complete set of graphs pertaining to short tailed lines is provided in Appendix A.

6.2 Results for Medium Tailed Lines of Business

The medium tailed business had an average 12 month incurred development factor of approximately 7.00 and paid development factor of approximately 25.00. It consists primarily of claims made liability. The pattern becomes negligible after 96 months.

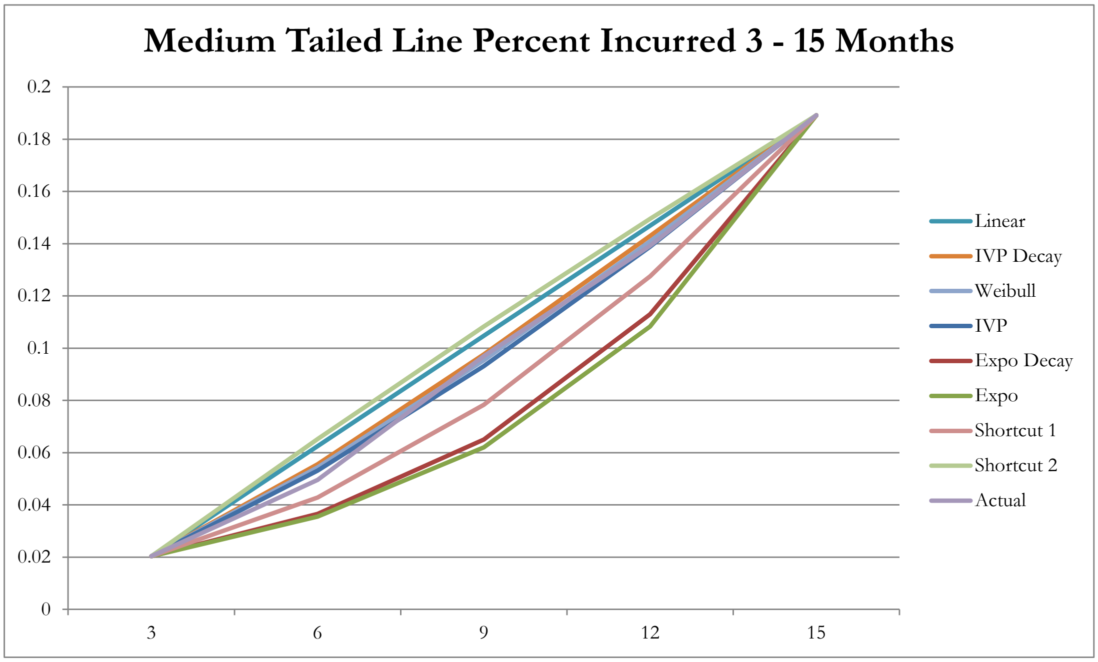
6.2.1 The Factors

The following table displays the results for weighted average factors:

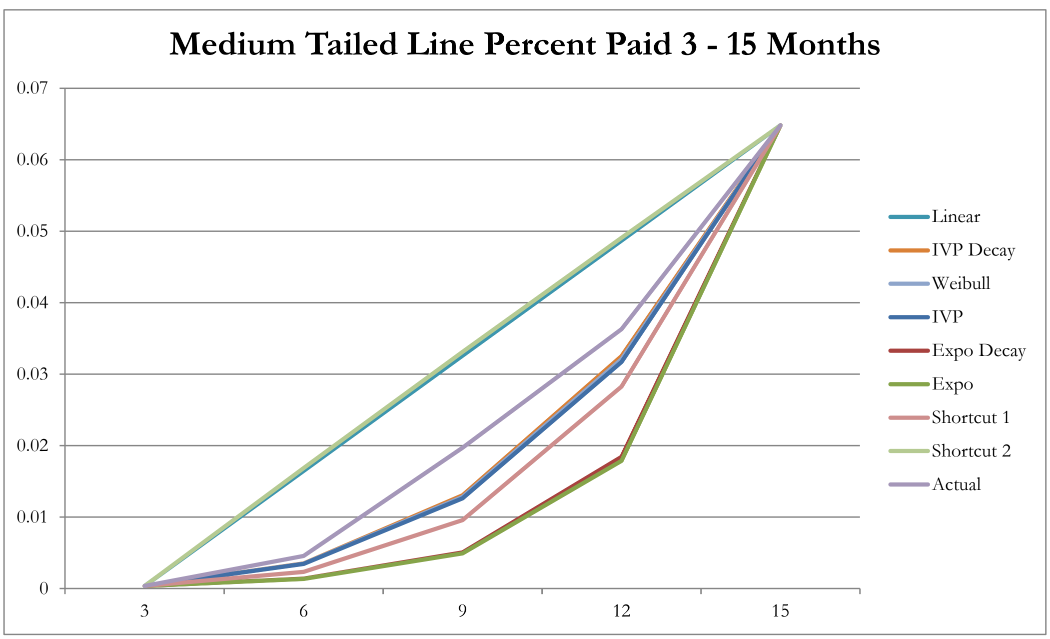
Medium Tailed Lines - Incurred Development Factors

Accident Year	Age	From Quarterly "Actual"	Linear	Regression	Weibull	IVP Whole	Expo	Expo whole	Shortcut 1	Shortcut 2
Using factors interpolated from 9 months prior to date										
2006	102	1.006	1.007	1.006	1.006	1.007	1.006	1.007	1.006	1.007
2007	90	1.016	1.028	1.021	1.022	1.028	1.021	1.028	1.021	1.028
2008	78	1.121	1.099	1.091	1.093	1.099	1.092	1.100	1.092	1.098
2009	66	1.191	1.214	1.209	1.210	1.213	1.211	1.215	1.211	1.213
2010	54	1.335	1.347	1.338	1.341	1.345	1.342	1.349	1.341	1.345
2011	42	1.609	1.582	1.571	1.574	1.579	1.579	1.587	1.576	1.578
2012	30	2.092	2.065	2.035	2.053	2.077	2.076	2.125	2.053	2.043
2013	18	4.014	3.962	3.953	3.997	4.047	4.249	4.375	4.089	3.902
2014	6	40.322	-	-	-	-	-	-	-	-
Using factors interpolated from 6 months prior to date										
2006	102	1.006	1.004	1.004	1.004	1.004	1.004	1.004	1.002	1.004
2007	90	1.016	1.037	1.021	1.025	1.037	1.022	1.038	1.022	1.037
2008	78	1.121	1.110	1.102	1.104	1.110	1.103	1.111	1.103	1.110
2009	66	1.191	1.209	1.202	1.204	1.208	1.204	1.210	1.203	1.208
2010	54	1.335	1.351	1.340	1.344	1.349	1.345	1.354	1.344	1.349
2011	42	1.609	1.581	1.565	1.570	1.577	1.575	1.588	1.572	1.576
2012	30	2.092	2.128	2.087	2.107	2.133	2.134	2.181	2.111	2.104
2013	18	4.014	3.953	3.932	4.000	4.076	4.268	4.426	4.081	3.855
2014	6	40.322	-	-	-	-	-	-	-	-
Using factors interpolated from 3 months prior to date										
2006	102	1.006	1.006	1.006	1.006	1.006	1.006	1.006	1.004	1.006
2007	90	1.016	1.059	1.041	1.047	1.058	1.043	1.059	1.042	1.058
2008	78	1.121	1.123	1.119	1.120	1.122	1.120	1.123	1.119	1.122
2009	66	1.191	1.216	1.210	1.211	1.215	1.212	1.216	1.211	1.215
2010	54	1.335	1.361	1.353	1.355	1.359	1.356	1.362	1.355	1.359
2011	42	1.609	1.584	1.573	1.577	1.582	1.581	1.588	1.578	1.581
2012	30	2.092	2.184	2.152	2.168	2.186	2.186	2.218	2.170	2.167
2013	18	4.014	4.096	4.059	4.109	4.162	4.266	4.360	4.160	4.019
2014	6	40.322	31.981	35.997	36.814	37.648	54.642	56.336	46.673	30.635

Similar to our short tailed paid curve, the incurred curve for 3 – 15 months shows development higher than linear (or percent incurred lower than linear interpolation would suggest):



The actual data falls closer to linear. On a paid basis, the actual data follow the curves but only to a limited degree:



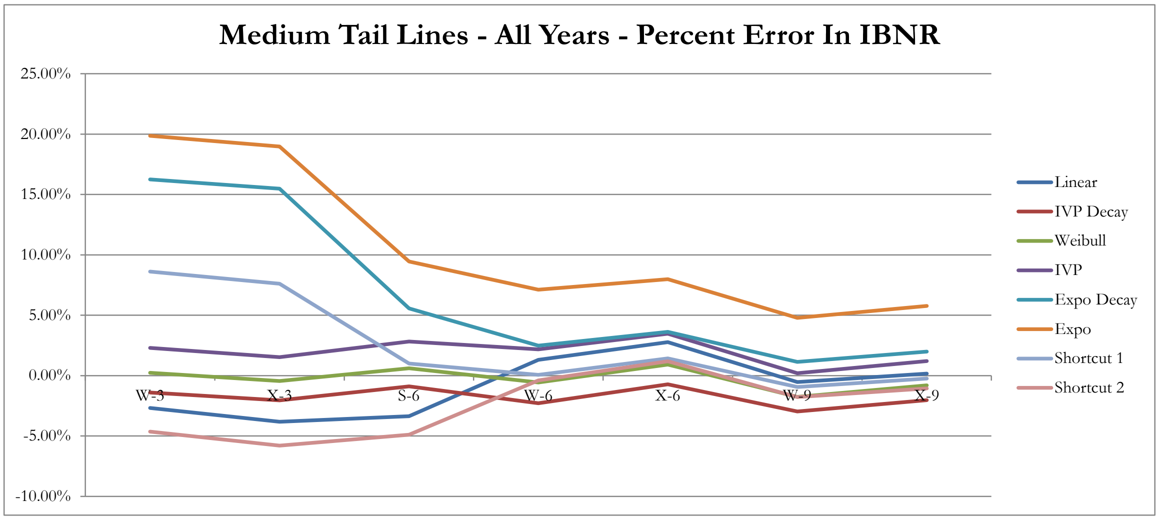
The following is a table of paid weighted average results:

Medium Tailed Lines - Paid Development Factors

Accident Year	Age	From Quarterly "Actual"	Linear	IVP Decay	Weibull	IVP	Expo Decay	Expo	Shortcut 1	Shortcut 2
Using factors interpolated from 9 months prior to date										
2006	102	1.062	1.071	1.069	1.070	1.071	1.070	1.071	1.070	1.071
2007	90	1.104	1.120	1.115	1.117	1.120	1.116	1.121	1.116	1.120
2008	78	1.215	1.240	1.231	1.234	1.240	1.233	1.243	1.232	1.238
2009	66	1.442	1.430	1.427	1.428	1.429	1.429	1.431	1.429	1.430
2010	54	1.618	1.634	1.620	1.626	1.636	1.629	1.646	1.624	1.627
2011	42	2.214	2.246	2.232	2.242	2.254	2.256	2.281	2.242	2.233
2012	30	3.718	3.643	3.669	3.699	3.734	3.785	3.861	3.701	3.603
2013	18	10.286	9.291	10.176	10.280	10.389	11.561	11.854	10.558	9.183
2014	6	438.308	-	-	-	-	-	-	-	-
Using factors interpolated from 6 months prior to date										
2006	102	1.062	1.070	1.065	1.066	1.069	1.065	1.070	1.065	1.070
2007	90	1.104	1.134	1.129	1.130	1.134	1.130	1.135	1.130	1.134
2008	78	1.215	1.242	1.233	1.236	1.241	1.235	1.244	1.234	1.240
2009	66	1.442	1.403	1.394	1.397	1.402	1.398	1.406	1.397	1.401
2010	54	1.618	1.614	1.605	1.608	1.612	1.612	1.619	1.609	1.612
2011	42	2.214	2.069	2.044	2.059	2.077	2.071	2.105	2.056	2.052
2012	30	3.718	3.474	3.493	3.541	3.594	3.632	3.737	3.527	3.408
2013	18	10.286	9.171	10.401	10.565	10.735	11.931	12.310	10.780	8.954
2014	6	438.308	-	-	-	-	-	-	-	-
Using factors interpolated from 3 months prior to date										
2006	102	1.062	1.064	1.062	1.062	1.063	1.062	1.064	1.062	1.063
2007	90	1.104	1.137	1.130	1.132	1.137	1.131	1.138	1.131	1.137
2008	78	1.215	1.206	1.204	1.204	1.205	1.205	1.206	1.205	1.206
2009	66	1.442	1.403	1.388	1.394	1.403	1.393	1.408	1.390	1.399
2010	54	1.618	1.620	1.614	1.616	1.618	1.618	1.623	1.617	1.619
2011	42	2.214	2.168	2.147	2.160	2.176	2.169	2.197	2.156	2.154
2012	30	3.718	3.703	3.703	3.735	3.769	3.795	3.857	3.731	3.653
2013	18	10.286	9.616	10.309	10.422	10.537	11.135	11.345	10.561	9.403
2014	6	438.308	121.292	565.378	572.677	580.003	1,448.769	1,471.631	851.796	118.463

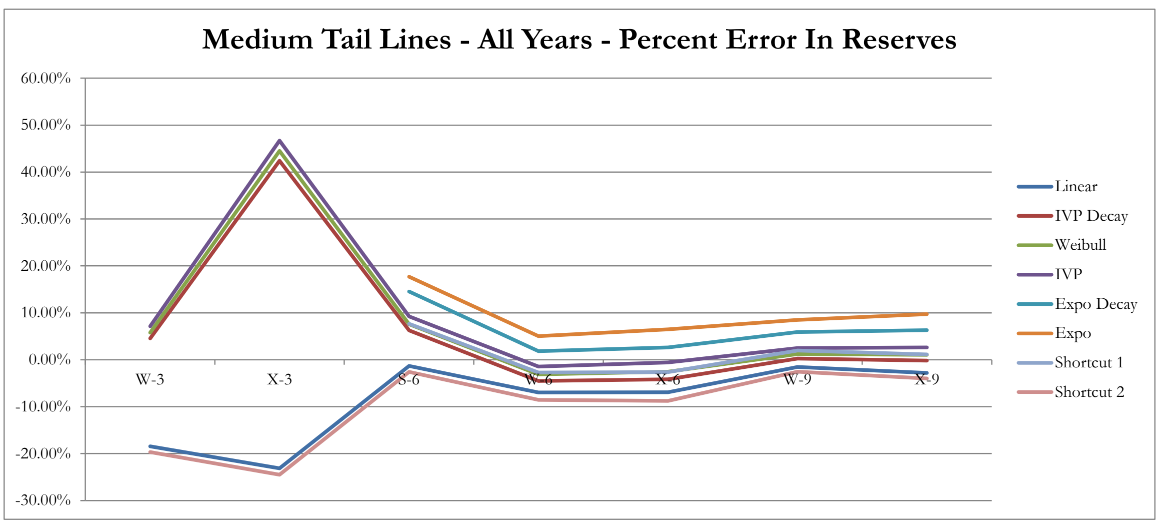
6.2.2 The Errors

The errors for all years on an incurred basis are fairly small except for the exponential curves which overestimate the liability:



Note that the reason the errors are smaller when estimated from 9 months prior is because the latest year is not estimated in the 9 month prior (or 6 month prior) scenario due to data limitations and therefore errors are smaller than the 3 month prior scenario.

Paid data shows a similar effect:



Note that values that were extraordinarily high (greater than 50% error) are shown as blank so as not to distort the graph. These types of large errors only occur with overestimation and in this case with the exponential methods.

Graphs which isolate the latter years are included in Appendix A.

6.3 Results for Long Tailed Lines of Business

The long tailed business had an average 12 month incurred development factor of approximately 15.00. Paid data was unavailable. The pattern has a tail of 1% at 126 months. It is mainly comprised of high layer property lines.

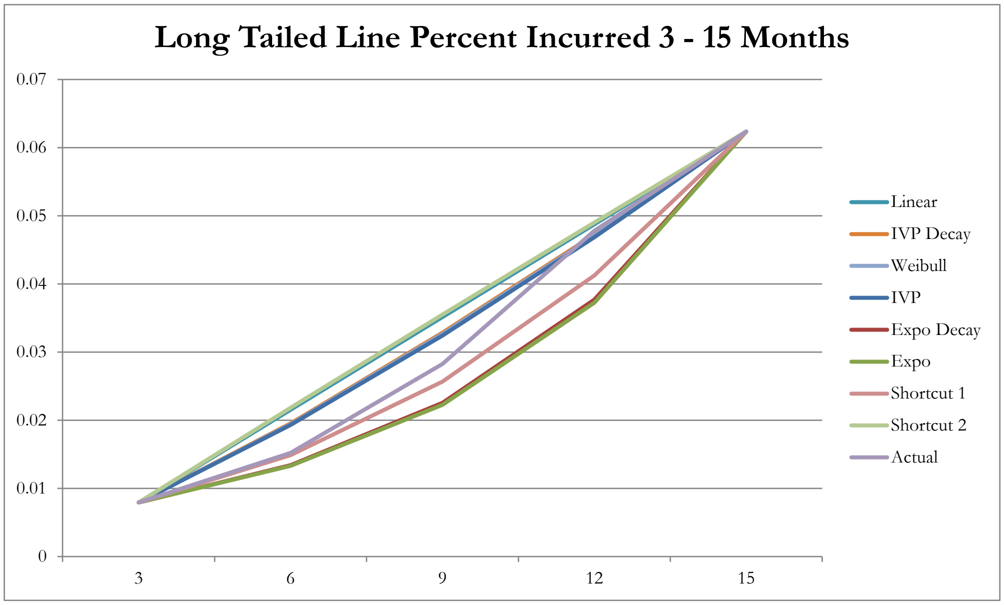
6.3.1 The Factors

Long Tailed Lines - Incurred Development Factors

Accident Year	Age	From Quarterly "Actual"	Linear	IVP Decay	Weibull	IVP	Expo Decay	Expo	Shortcut 1	Shortcut 2
				Using factors interpolated from 9 months prior to date						
2006	105	1.117	1.112	1.109	1.110	1.112	1.110	1.112	1.110	1.112
2007	93	1.203	1.196	1.189	1.191	1.196	1.191	1.198	1.190	1.195
2008	81	1.359	1.346	1.342	1.343	1.345	1.343	1.347	1.343	1.345
2009	69	1.522	1.506	1.500	1.502	1.505	1.503	1.509	1.502	1.504
2010	57	1.773	1.767	1.759	1.762	1.767	1.766	1.775	1.763	1.764
2011	45	2.305	2.276	2.264	2.273	2.285	2.286	2.309	2.273	2.265
2012	33	3.679	3.532	3.531	3.549	3.569	3.605	3.648	3.560	3.507
2013	21	7.146	6.874	6.928	6.960	6.995	7.307	7.391	7.118	6.826
2014	9	32.831	-	-	-	-	-	-	-	-
				Using factors interpolated from 6 months prior to date						
2006	105	1.117	1.110	1.107	1.108	1.110	1.108	1.110	1.108	1.110
2007	93	1.203	1.205	1.194	1.198	1.205	1.196	1.207	1.195	1.203
2008	81	1.359	1.350	1.344	1.346	1.349	1.346	1.352	1.346	1.349
2009	69	1.522	1.526	1.518	1.521	1.526	1.523	1.530	1.521	1.524
2010	57	1.773	1.797	1.786	1.790	1.796	1.795	1.805	1.791	1.792
2011	45	2.305	2.336	2.318	2.330	2.345	2.346	2.374	2.329	2.320
2012	33	3.679	3.615	3.610	3.634	3.662	3.703	3.757	3.647	3.578
2013	21	7.146	6.923	6.996	7.042	7.092	7.430	7.533	7.212	6.846
2014	9	32.831	28.282	33.502	33.905	34.318	53.458	54.753	43.627	27.802
				Using factors interpolated from 3 months prior to date						
2006	105	1.117	1.108	1.105	1.106	1.108	1.105	1.109	1.105	1.108
2007	93	1.203	1.198	1.192	1.194	1.197	1.193	1.199	1.193	1.197
2008	81	1.359	1.334	1.328	1.330	1.334	1.330	1.336	1.329	1.333
2009	69	1.522	1.510	1.504	1.507	1.510	1.508	1.513	1.506	1.509
2010	57	1.773	1.764	1.756	1.759	1.763	1.762	1.768	1.759	1.761
2011	45	2.305	2.290	2.275	2.283	2.293	2.293	2.310	2.283	2.280
2012	33	3.679	3.635	3.630	3.654	3.679	3.701	3.746	3.655	3.599
2013	21	7.146	7.036	7.089	7.126	7.166	7.373	7.443	7.231	6.968
2014	9	32.831	30.369	33.421	33.700	33.981	40.888	41.407	37.789	29.838

The incurred factors based on weighted averages were as follows:

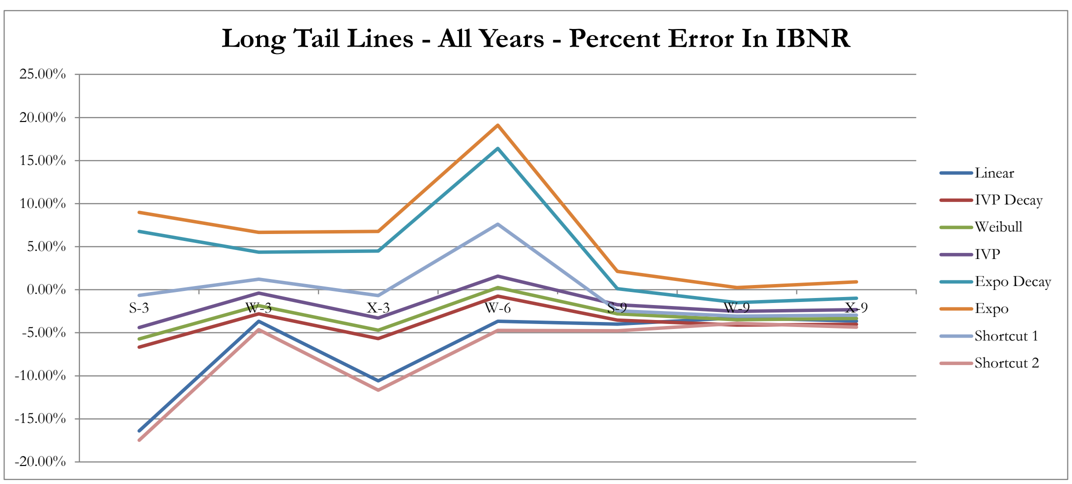
As seen below, the curve of percent reported between 3 and 15 months shows a reverse arc with



actual results falling between this arc and the linear method:

6.3.2 The Errors

The graph of results shows that Shortcut 1 behaves closest to actual data:



6.4 Results for Very Long Tailed Lines of Business

The very long tailed business had an average 12 month incurred development factor greater than 20.00 and an average paid development factor nearing 90.00. The age to age paid factors are around 3% at 120 months. This data set is mainly comprised of casualty lines.

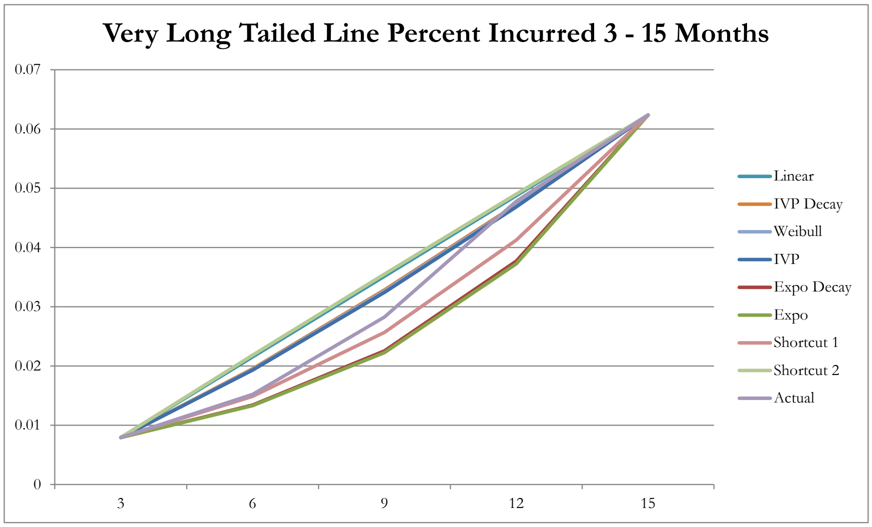
6.4.1 The Factors

The factors for weighted averages on an incurred basis were as follows:

Very Long Tailed Lines - Incurred Development Factors

Accident Year	Age	From Quarterly "Actual"	Linear	IVP Decay	Weibull	IVP	Expo Decay	Expo	Shortcut 1	Shortcut 2
Using factors interpolated from 9 months prior to date										
2006	105	1.064	1.029	1.020	1.022	1.029	1.020	1.030	1.020	1.029
2007	93	1.138	1.125	1.110	1.114	1.125	1.111	1.127	1.110	1.123
2008	81	1.313	1.310	1.305	1.307	1.310	1.307	1.312	1.307	1.309
2009	69	1.479	1.476	1.471	1.473	1.475	1.474	1.478	1.473	1.474
2010	57	1.753	1.714	1.703	1.708	1.715	1.712	1.724	1.707	1.709
2011	45	2.388	2.277	2.264	2.274	2.286	2.286	2.309	2.273	2.265
2012	33	3.884	3.536	3.535	3.553	3.574	3.611	3.654	3.565	3.510
2013	21	7.367	7.130	7.363	7.416	7.473	7.900	8.040	7.559	7.060
2014	9	47.131	-	-	-	-	-	-	-	-
Using factors interpolated from 6 months prior to date										
2006	105	1.064	1.040	1.023	1.027	1.040	1.023	1.041	1.023	1.040
2007	93	1.138	1.134	1.121	1.125	1.134	1.123	1.136	1.122	1.132
2008	81	1.313	1.294	1.281	1.286	1.295	1.284	1.298	1.282	1.291
2009	69	1.479	1.480	1.475	1.476	1.479	1.478	1.482	1.477	1.479
2010	57	1.753	1.749	1.734	1.741	1.750	1.745	1.761	1.739	1.742
2011	45	2.388	2.342	2.323	2.336	2.352	2.352	2.380	2.335	2.326
2012	33	3.884	3.653	3.651	3.677	3.706	3.748	3.805	3.687	3.614
2013	21	7.367	7.713	8.124	8.211	8.302	8.830	9.023	8.359	7.580
2014	9	47.131	37.923	40.563	40.828	41.097	59.057	59.851	51.933	37.528
Using factors interpolated from 3 months prior to date										
2006	105	1.064	1.051	1.051	1.051	1.051	1.051	1.051	1.051	1.050
2007	93	1.138	1.127	1.121	1.123	1.126	1.122	1.127	1.122	1.126
2008	81	1.313	1.276	1.264	1.268	1.276	1.267	1.278	1.265	1.274
2009	69	1.479	1.454	1.448	1.450	1.453	1.451	1.456	1.450	1.452
2010	57	1.753	1.755	1.743	1.749	1.755	1.751	1.763	1.747	1.750
2011	45	2.388	2.332	2.317	2.326	2.336	2.336	2.355	2.325	2.321
2012	33	3.884	3.805	3.809	3.835	3.863	3.887	3.939	3.833	3.764
2013	21	7.367	8.406	8.676	8.739	8.804	9.127	9.245	8.844	8.286
2014	9	47.131	40.685	45.744	46.044	46.345	56.352	56.912	52.049	40.113

As seen below, the curve of percent reported between 3 and 15 months shows a reverse arc with actual results falling between this arc and the linear method:



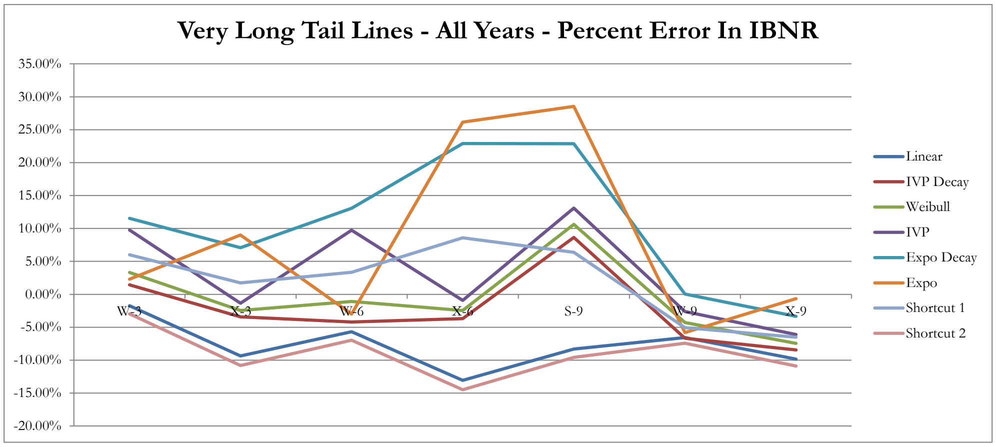
Paid Factors are as follows:

Very Long Tailed Lines - Paid Development Factors

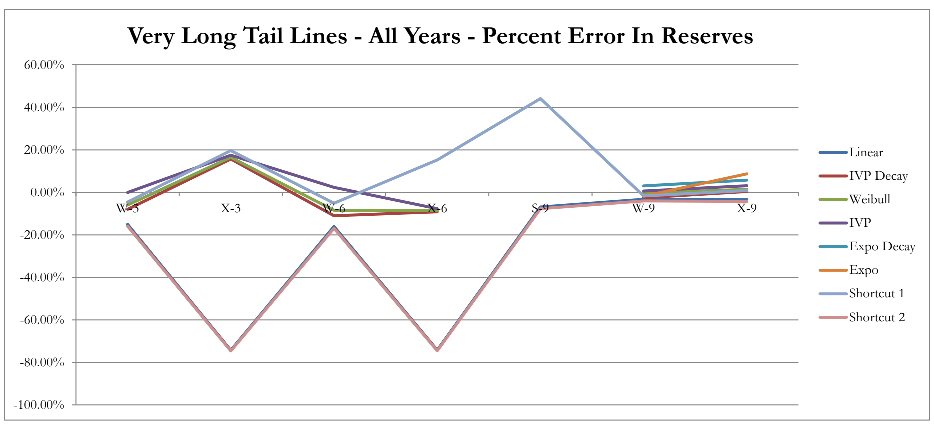
Accident Year	Age	From Quarterly "Actual"	Linear	IVP Decay	Weibull	IVP	Expo Decay	Expo	Shortcut 1	Shortcut 2
				Using factors interpolated from 9 months prior to date						
2006	105	1.105	1.123	1.116	1.118	1.123	1.117	1.124	1.116	1.122
2007	93	1.225	1.244	1.239	1.241	1.243	1.241	1.245	1.240	1.243
2008	81	1.371	1.397	1.390	1.393	1.398	1.393	1.401	1.391	1.395
2009	69	1.684	1.694	1.683	1.689	1.697	1.690	1.705	1.686	1.688
2010	57	2.265	2.280	2.272	2.282	2.293	2.289	2.312	2.277	2.269
2011	45	3.561	3.497	3.512	3.528	3.547	3.563	3.601	3.522	3.475
2012	33	6.944	6.549	6.682	6.711	6.743	6.883	6.952	6.735	6.507
2013	21	23.245	16.879	18.972	19.078	19.186	21.128	21.404	19.488	16.771
2014	9	369.532	-	-	-	-	-	-	-	-
				Using factors interpolated from 6 months prior to date						
2006	105	1.105	1.126	1.118	1.120	1.126	1.119	1.127	1.119	1.126
2007	93	1.225	1.231	1.225	1.227	1.231	1.227	1.232	1.226	1.230
2008	81	1.371	1.386	1.376	1.380	1.386	1.380	1.390	1.378	1.383
2009	69	1.684	1.670	1.656	1.664	1.673	1.665	1.682	1.660	1.664
2010	57	2.265	2.260	2.246	2.261	2.280	2.269	2.304	2.250	2.241
2011	45	3.561	3.489	3.496	3.516	3.538	3.554	3.597	3.511	3.460
2012	33	6.944	6.343	6.512	6.556	6.602	6.752	6.845	6.570	6.273
2013	21	23.245	17.116	20.421	20.595	20.772	23.056	23.445	20.948	16.886
2014	9	369.532	139.914	217.088	217.822	218.561	411.127	413.753	315.033	139.326
				Using factors interpolated from 3 months prior to date						
2006	105	1.105	1.108	1.095	1.099	1.108	1.096	1.109	1.096	1.107
2007	93	1.225	1.229	1.221	1.224	1.228	1.223	1.230	1.222	1.227
2008	81	1.371	1.372	1.366	1.368	1.371	1.368	1.373	1.368	1.370
2009	69	1.684	1.664	1.651	1.658	1.666	1.658	1.672	1.654	1.658
2010	57	2.265	2.272	2.259	2.271	2.284	2.276	2.300	2.263	2.258
2011	45	3.561	3.496	3.499	3.516	3.535	3.542	3.576	3.509	3.470
2012	33	6.944	6.612	6.752	6.790	6.829	6.927	6.999	6.789	6.542
2013	21	23.245	19.497	22.361	22.504	22.649	24.103	24.374	22.717	19.208
2014	9	369.532	157.185	272.604	273.555	274.508	382.104	384.055	326.841	155.917

6.4.2 The Errors

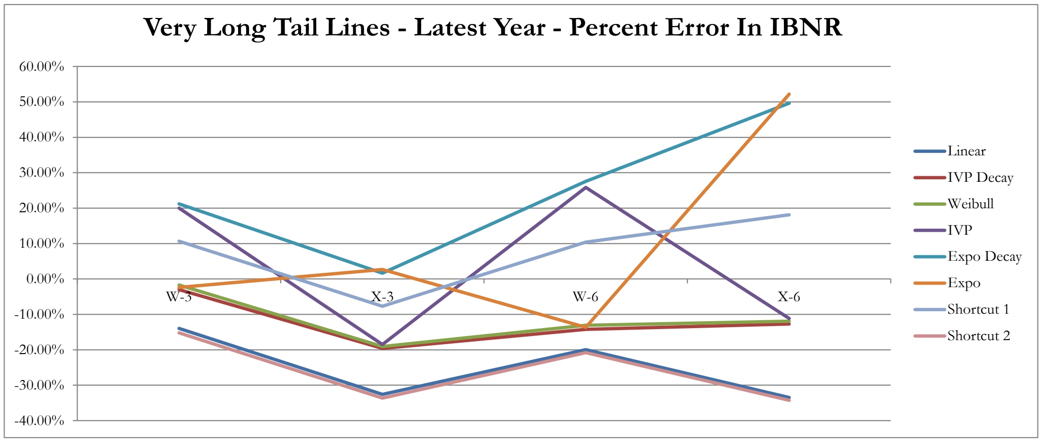
The error pattern is similar to long tailed lines for incurred losses:



Paid results are more erratic particularly for simple averages, which do not seem to perform well with interpolation:



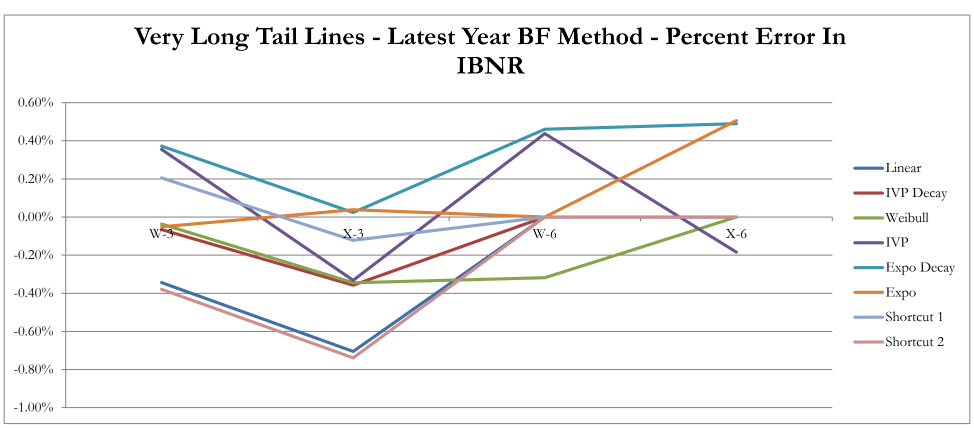
A graph of the latest accident year shows that only Shortcut 1 performs reasonably well in terms of being close to zero error and not underestimating amounts:

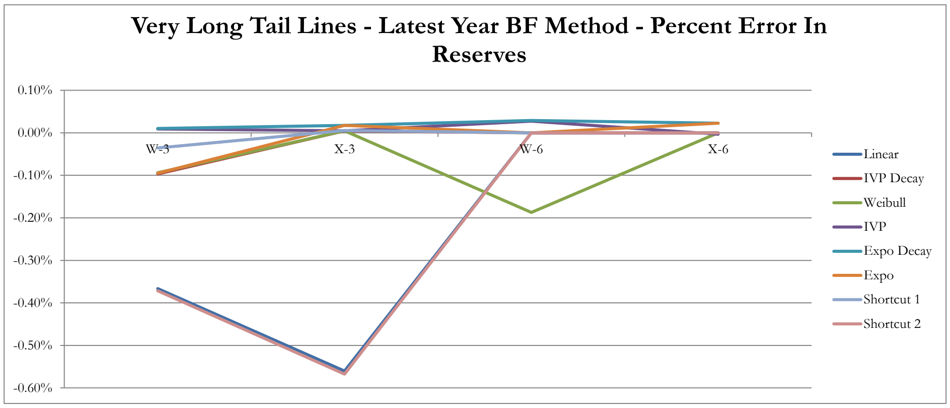


6.5 The Impact of using Exposure Based Methods

It is well known that many actuaries use exposure based methods, such as Bornhuetter Ferguson (BF) in the more recent accident years. To the extent that this is true, the magnitude of errors might be less significant. To test this effect, I used an all year initial expected loss ratio for my data based on a Cape Cod calculation and tested the differences between using this method with the quarterly data versus using it with the interpolated annual data. Note that I used the same initial expected loss ratio for both quarterly and annual data in order to isolate the reserves changes that would be caused by interpolation alone. It is assumed that the practitioner has a reasonable initial expected loss ratio estimate that does not rely on interpolation.

As can be expected the use of the BF method has very little impact on the shorted tailed and medium tailed lines of business. Percent errors decreased very little and sometimes even increased for all years since the latest year had the most impact. I tested the BF on the very long tailed lines of business since there was no exposure data for the long tailed line. The BF method reduced the errors to nearly zero for both paid and incurred data:





7. TESTING OF METHODS AND RESULTS FOR EXPTRAPOLATION

Using the annual triangles as described above, I extrapolated factors from the 6, 9 and 12 month triangles using the earliest CDF. Each one was used to estimate earlier quarters such that the 12 month factor was used to estimate a 3, 6 and 9 month factors and so forth.

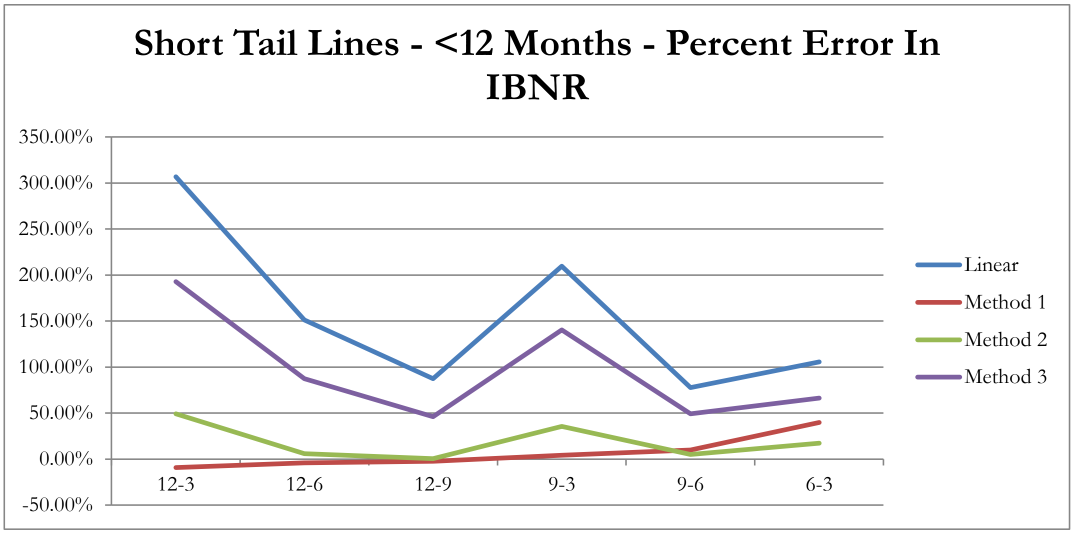
I applied the interpolation methods as described in Section 4. Then I projected the quarterly results by accident year using the most recent data and quarterly factors. Each result was then compared to the extrapolated estimate. The errors were also calculated on a paid and incurred basis and for all three averages.

Error was measured in terms of IBNR for the incurred triangles and total reserves for the paid triangles. The percent error was calculated as a percent of total IBNR or reserve. Therefore percent error for paid losses would equal:

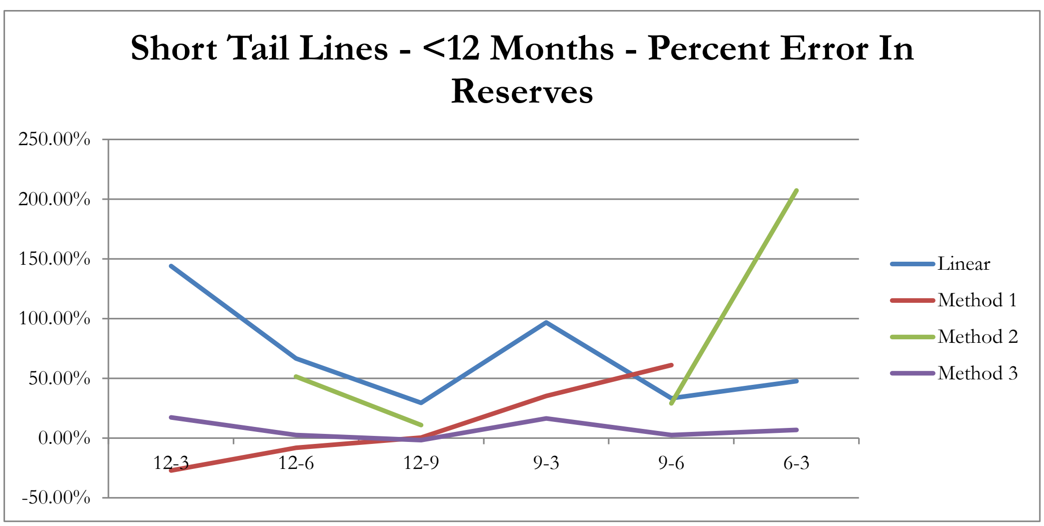
[Ultimate losses derived from Extrapolated method – Ultimate losses derived from quarterly triangle factors] / [Ultimate losses derived from quarterly triangle factors – Paid losses at latest evaluation].

7.1 Results for Short Tailed Lines of Business

Results were extremely volatile, particularly for the Linear method and Method 3 (which tends toward linear). Note in the graph below "12-3" indicates a 3 month factor estimated from a 12 month factor and "6-3" indicates a 3 month factor estimated from a 6 month factor and so on.



The 12-9 factors in general tend to be more accurate across all methods. Paid results are more volatile (results over +500% not shown) but once again the 12-9 gives better results than other maturities:

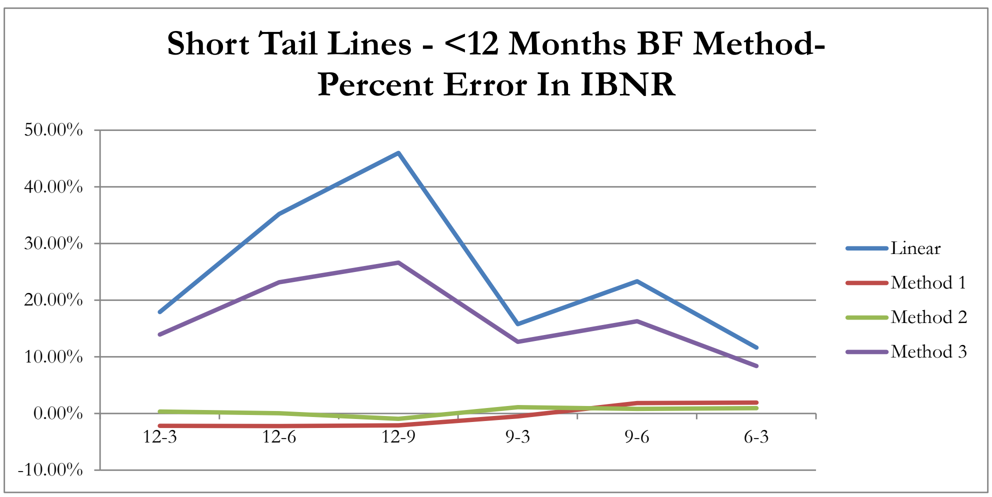


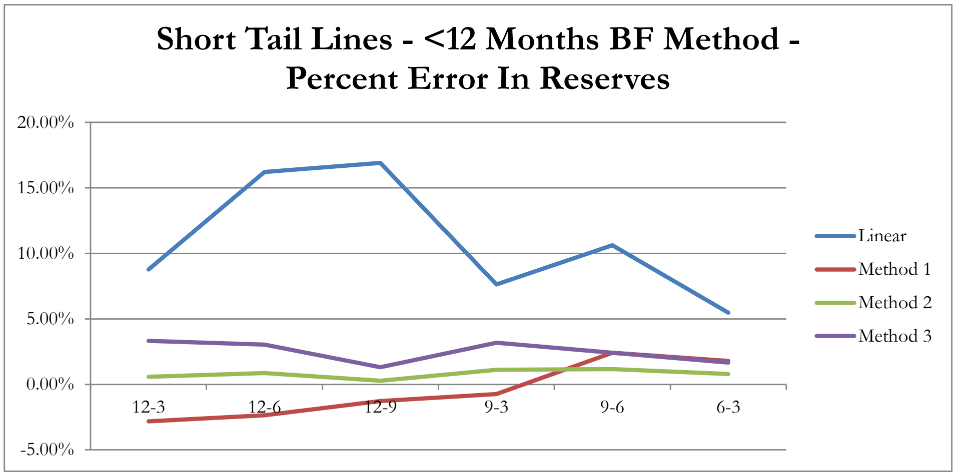
The overall factors for weighted averages looked as follows:

Short Tailed Lines

	Paid Development Factors					***Incurred Development Factors***				
Accident Month	From Quarterly "Actual"	Linear	Method 1	Method 2	Method 3	From Quarterly "Actual"	Linear	Method 1	Method 2	Method 3
					Using factors interpolated from 12 months					
9	2.163	2.508	2.168	2.290	2.143	1.497	1.931	1.486	1.499	1.726
6	3.523	5.205	3.321	4.827	3.588	2.319	4.314	2.264	2.398	3.470
3	8.458	19.204	6.432	127.700	9.753	4.975	17.168	4.615	6.930	12.642
					Using factors interpolated from 9 months					
6	3.523	4.368	5.065	4.261	3.588	2.319	3.342	2.448	2.386	2.966
3	8.458	15.684	11.087	62.655	9.689	4.975	13.310	5.138	6.390	10.550
					Using factors interpolated from 6 months					
3	8.458	12.008	86.524	23.925	8.977	4.975	9.179	6.562	5.661	7.614

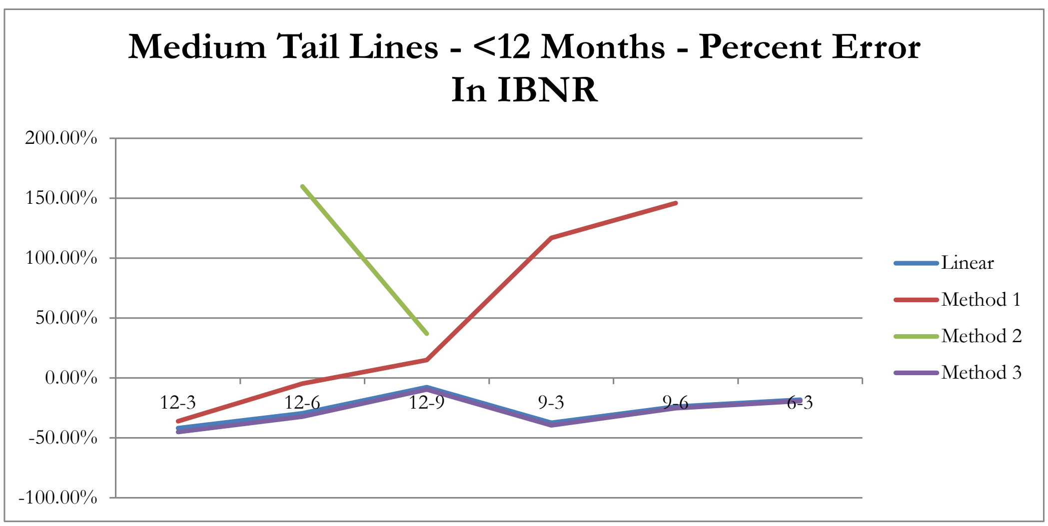
Using the BF approach as described in Section 6, incurred and paid results become much more stable with Methods 1 and 2 having error close to zero:

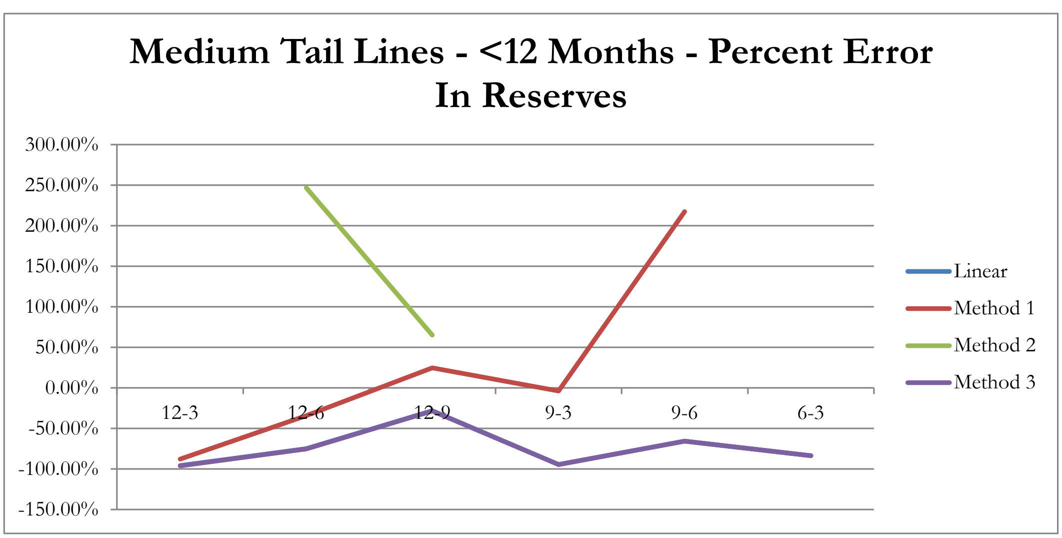




7.2 Results for Medium Tailed Lines of Business

Results are more volatile on paid and incurred bases as the development factors increase. In this case the Linear method and Method 3 perform better but underestimate the liability.





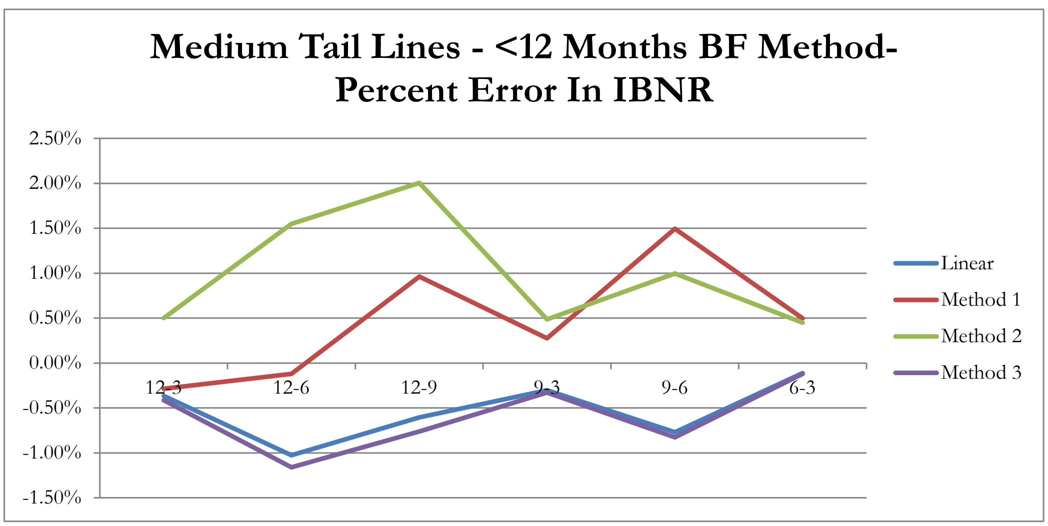
Once again, the 12-9 factors generally tend to be more accurate across all methods.

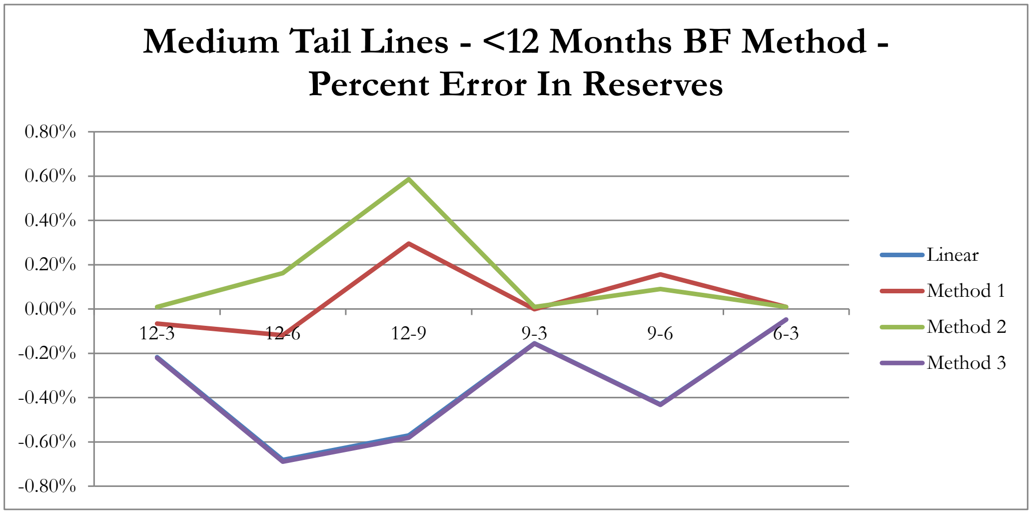
The overall factors looked as follows:

Medium Tailed Lines

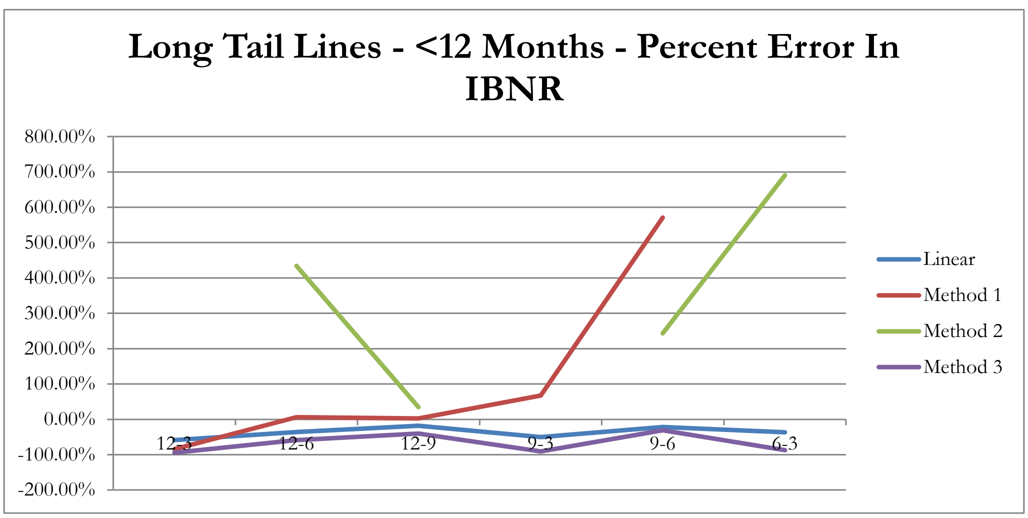
	Paid Development Factors					*Incurred Development Factors*				
Accident Month	From Quarterly "Actual"	Linear	Method 1	Method 2	Method 3	From Quarterly "Actual"	Linear	Method 1	Method 2	Method 3
					Using factors interpolated from 12 months					
9	67.597	48.971	84.143	110.923	48.746	13.753	12.768	15.676	18.475	12.536
6	438.308	110.185	289.149	1,517.584	109.175	40.322	28.727	38.493	103.157	27.690
3	10,674.545	440.739	1,324.852	2,303,061.379	434.693	196.797	114.909	126.029	10,641.442	108.722
					Using factors interpolated from 9 months					
6	438.308	152.094	1,388.929	721.963	151.591	40.322	30.944	97.749	66.253	30.429
3	10,674.545	608.375	10,281.108	521,230.571	604.357	196.797	123.774	425.557	4,389.411	119.684
					Using factors interpolated from 6 months					
3	10,674.545	1,753.233	2,844,029.625	192,114.122	1,751.231	196.797	161.289	7,300.351	1,625.874	159.263

Using the BF approach as described in Section 6, incurred and paid results become much more stable with all methods having error close to zero:





7.3 Results for Long Tailed Lines of Business



Results are very similar to medium tailed lines of business with a higher degree of error.

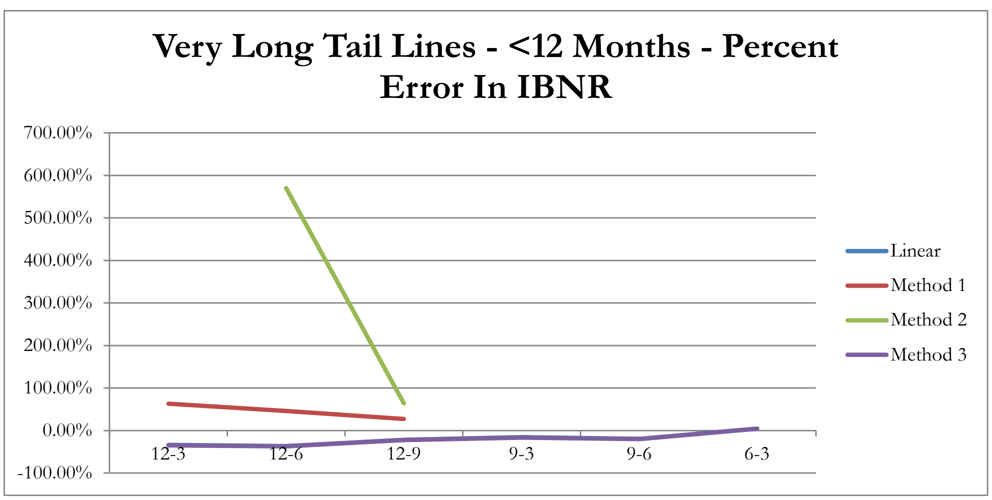
The factors for weighted averages looked as follows:

Long Tailed Lines
Incurred Development Factors

Accident Month	From Quarterly "Actual"	Linear	Method 1	Method 2	Method 3
		Using factors interpolated from 12 months			
9	32.831	27.002	39.980	50.154	26.776
6	94.059	60.756	118.391	461.404	59.739
3	590.278	243.022	467.445	212,894.074	236.937
		Using factors interpolated from 9 months			
6	94.059	73.869	416.804	244.366	73.363
3	590.278	295.476	2,425.167	59,714.879	291.439
		Using factors interpolated from 6 months			
3	590.278	376.236	60,671.471	8,847.073	374.225

7.4 Results for Very Long Tailed Lines of Business

Results are very similar to medium and long tailed lines of business with a higher degree of error. In fact, the paid graph has very few points. All Graphs are shown in Appendix A.

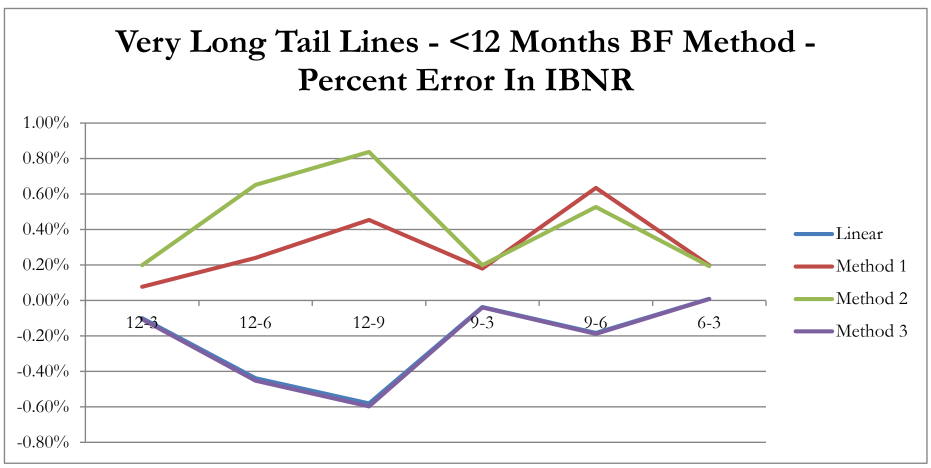


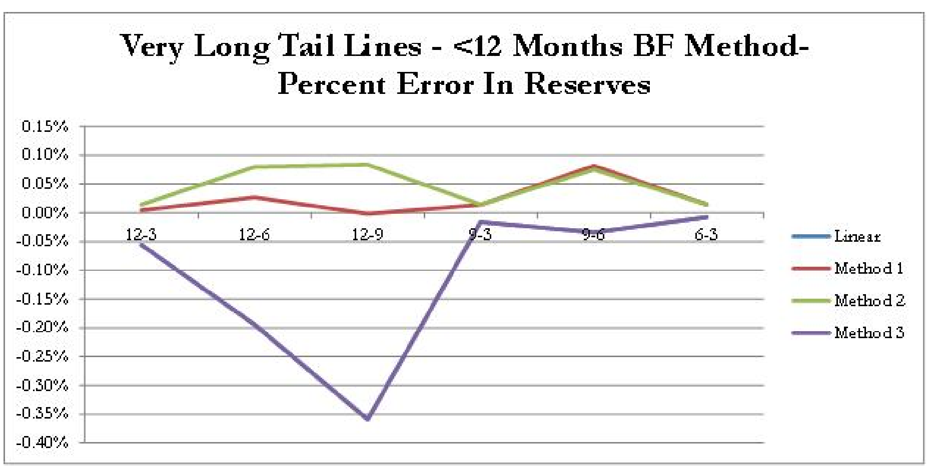
Factors for weighted averages were as follows:

Very Long Tailed Lines

	Paid Development Factors					*Incurred Development Factors*				
Accident Month	From Quarterly "Actual"	Linear	Method 1	Method 2	Method 3	From Quarterly "Actual"	Linear	Method 1	Method 2	Method 3
Using factors interpolated from 12 months										
9	369.532	159.124	367.080	533.846	158.901	47.131	37.181	59.634	76.829	36.955
6	1,165.974	358.029	1,693.623	16,023.066	357.026	131.394	83.657	191.290	874.810	82.645
3	7,125.067	1,432.115	10,418.643	256,738,649.772	1,426.101	502.602	334.628	818.151	765,292.557	328.566
Using factors interpolated from 9 months										
6	1,165.974	831.447	23,562.370	9,227.840	830.947	131.394	106.045	761.457	420.324	105.541
3	7,125.067	3,325.789	307,246.366	85,153,035.921	3,321.785	502.602	424.180	4,998.025	176,671.942	420.155
Using factors interpolated from 6 months										
3	7,125.067	4,663.895	32,825,164.845	1,359,494.738	4,661.894	502.602	525.578	139,935.418	17,264.505	523.570

Once again, BF methods render the errors in development factors immaterial:





8. SEASONAL ADJUSTMENT METHOD

This section deals with the situation where the actuary has specific knowledge of company practices which may change the view of how interpolation should occur. For this example we will assume that the company has unusually high payments during the fourth quarter of every year due to extra efforts to close claims in that quarter. To start we will assume that the company knows that payments are 50% higher in Q4 than they would be without such efforts. An alternative assumption will be addressed following the main scenario.

Any of our interpolation methods can be used and adapted for this situation. In this example, I use Shortcut 1. I start by interpolating factors to each quarter as usual, but I extend the calculations to each quarter of the year even though I am most interested in the CDF at fourth quarter after the unusually high payments since I don't want ultimate losses to be overstated or fluctuate wildly from quarter to quarter.

Using the selected interpolation method I set up a table (more detail given in Appendix B):

			3Q 2014			4Q 2014		
Accident Year	Maturity in Months	Paid CDF 2nd Quarter	Maturity	Interpolated Paid CDF	Incremental Percent Paid	Maturity	Interpolated Paid CDF	Incremental Percent Paid
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
					1/(4) - 1/(2)			1/(7) - 1/(4)
2004	126	1.001	129	1.000	0.001	132	1.000	-
2005	114	1.003	117	1.002	0.001	120	1.002	0.001
2006	102	1.008	105	1.006	0.002	108	1.005	0.001
2007	90	1.018	93	1.015	0.003	96	1.012	0.003
2008	78	1.038	81	1.032	0.006	84	1.026	0.005
2009	66	1.064	69	1.057	0.007	72	1.050	0.006
2010	54	1.108	57	1.095	0.011	60	1.083	0.010
2011	42	1.191	45	1.165	0.019	48	1.143	0.017
2012	30	1.334	33	1.290	0.026	36	1.252	0.024
2013	18	2.068	21	1.780	0.078	24	1.580	0.071
2014	6	2.843	9	2.596	0.033	12	2.390	0.033
2014 Full Year	6	5.686	9	3.462	0.113		2.390	0.130

For the next step, I calculate the percentage of total yearly payments assumed paid in each quarter by the selected interpolation method. I use the relative values from columns (5), (8), (11) and (14) above.

	1Q 2015			2Q 2015		
Accident Year	Maturity	Interpolated Paid CDF	Incremental Percent Paid	Maturity	Interpolated Paid CDF	Incremental Percent Paid
	(9)	(10)	(11)	(12)	(13)	(14)
			1/(10) - 1/(7)			1/(13) - 1/(10)
2004	135	1.000	-	138	1.000	-
2005	123	1.001	0.000	126	1.001	0.000
2006	111	1.004	0.001	114	1.003	0.001
2007	99	1.010	0.002	102	1.008	0.002
2008	87	1.022	0.004	90	1.018	0.004
2009	75	1.044	0.005	78	1.038	0.005
2010	63	1.073	0.009	66	1.064	0.008
2011	51	1.124	0.015	54	1.108	0.013
2012	39	1.219	0.021	42	1.191	0.019
2013	27	1.438	0.063	30	1.334	0.054
2014	15	2.216	0.033	18	2.068	0.032
2014 Full Year		2.216	0.033		2.068	0.032

I then "restate" these percentages by assuming that the 4th quarter will be 50% higher than what is shown above. The other three quarters are renormalized to the new remainder. For example

	Percent of Year Paid in				
Accident Year	3Q2014	4Q 2014	1Q 2015	2Q 2015	Total
	(15)	(16)	(17)	(18)	(19)
2004					
2005	36.0%	27.4%	20.8%	15.8%	100.0%
2006	34.7%	27.2%	21.3%	16.7%	100.0%
2007	32.9%	27.0%	22.1%	18.1%	100.0%
2008	32.1%	26.8%	22.4%	18.7%	100.0%
2009	29.6%	26.3%	23.3%	20.7%	100.0%
2010	29.4%	26.3%	23.4%	20.9%	100.0%
2011	29.4%	26.3%	23.4%	20.8%	100.0%
2012	28.7%	26.1%	23.7%	21.5%	100.0%
2013	29.4%	26.7%	23.6%	20.3%	100.0%
2014	25.3%	25.2%	24.9%	24.5%	100.0%

Column 15 would be restated as follows.

(15)/ [(15)+(17)+(18)]*[1-(16)*1.5]

The restated percent paid for Q4 would be simply:

(16) *1.5.

Finally, the restated CDF for Q4 is given as:

1/[Percent paid at 2Q 2014 +sum(restated percent in 3Q and 4Q 2014)*[expected paid for full calendar year]]

This method can be adapted for other seasonal situations using paid or incurred losses. In addition, in the situation where the percent increase is a rough estimate, the company's own Q4 data can be used to calibrate a percentage that fits. More detail is provided in Appendix B.

However, if the knowledge about fourth quarter payments reflects a percentage higher than the payments *in other quarters* as opposed to simply a percentage higher than *it would be otherwise*, the last restated percent paid for Q4 should be given as:

(16)/[(16)*1.5 + 1 – (16)] x (1.5)

In reality, the percentage increase and the choice of which assumption is more appropriate will be very hard to ascertain. However, using actual emergence to calibrate the adjustment over time in the absence of full quarterly triangles should add more value to the interpolated factors.

9. CONCLUSIONS

The appropriateness and accuracy of various interpolation and extrapolation methods varies greatly with the development characteristics of the line of business. Sophisticated methods don't seem to provide much advantage over simple shortcuts. For short tailed lines or lines with development factors less than 2.00 at 12 months, Shortcut 2 seems to perform relatively well, whereas Shortcut 1 seems to perform better on paid data or once development is greater than 2.00 at 12 months. Shortcut 1 also seems to perform well once the second year of development is reached. Exponential curves seem to regularly overstate reserves by large amounts.

Weighted average development factors also seem to work much better and are not prone to unusual swings which may distort interpolation methods. However, in practice development factors are often selected judgmentally so this may be hard to follow when interpolating.

Extrapolated values, especially for long tailed lines, are predictability overstated and distorted. However the BF method seems to mitigate this risk almost entirely. For the Development method, Methods 1 and 2 seem to perform the best without understating reserves on shorter tailed lines whereas Method 3 performs well on longer tailed lines.

Since the data used is not exhaustive but more a sampling of typical quarterly triangles, the practitioner can use this paper to decide how each of these formulae are applicable to the underlying characteristics of individual company data.

Acknowledgment

The author acknowledges Stephanie Celona for extensive spreadsheet work and fantastic editorial review.

Supplementary Material

The Appendices to this paper and a practical tool are available electronically at the CAS website at (fill in later). The practical tool demonstrates interpolation and extrapolation methods as well as the seasonal adjustment method.

Appendix A

The graphs in the body of the paper as well as some additional graphs are included in this appendix.

Appendix B

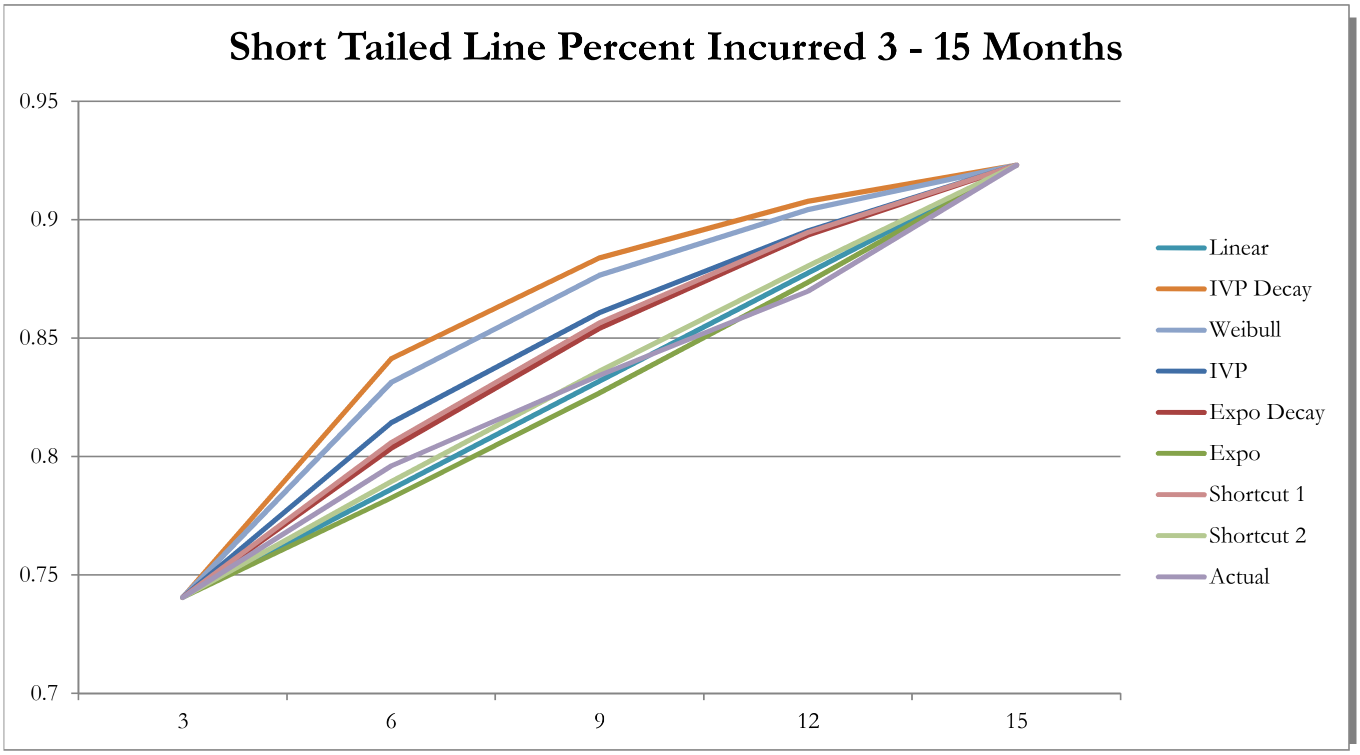
Details of the Seasonal Adjustment Method are provided here.

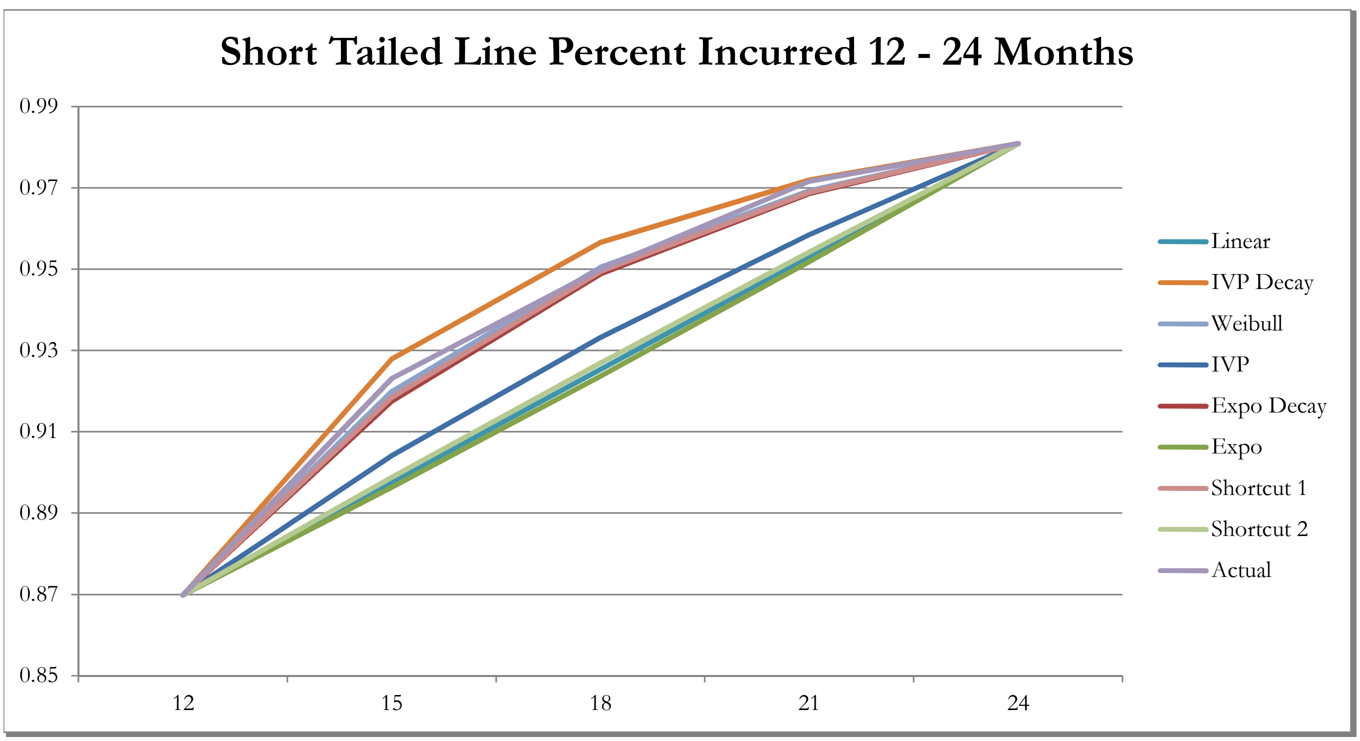
Biography of the Author

Lynne Bloom, FCAS, MAAA, is a Director at PwC in Philadelphia, PA. She has a B.B.A. in Finance from the Wharton Business School at the University of Pennsylvania. She is a Fellow of the CAS and a Member of the American Academy of Actuaries. Lynne is the chairman of the CAS Research Oversight Committee and Vice President of CAMAR.

Short Tail Line Incurred Graphs

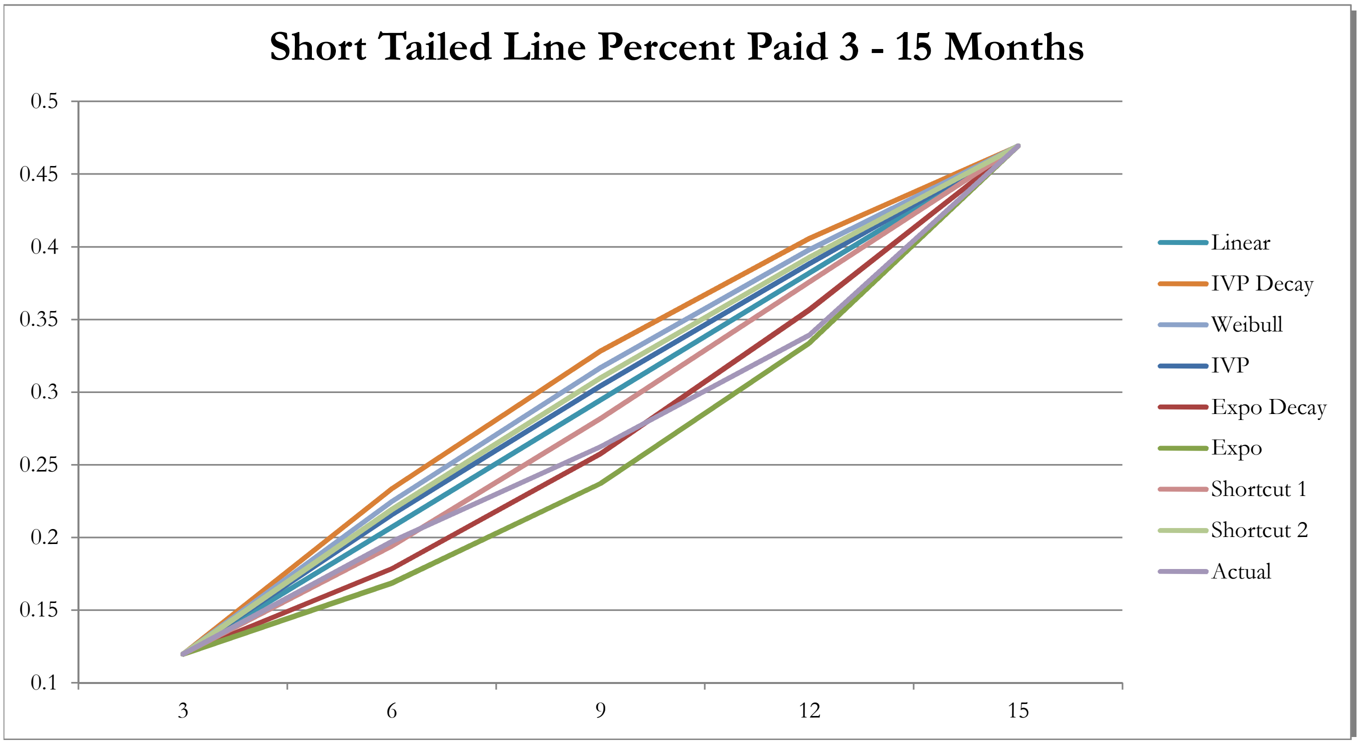
Appendix A
Sheet 1

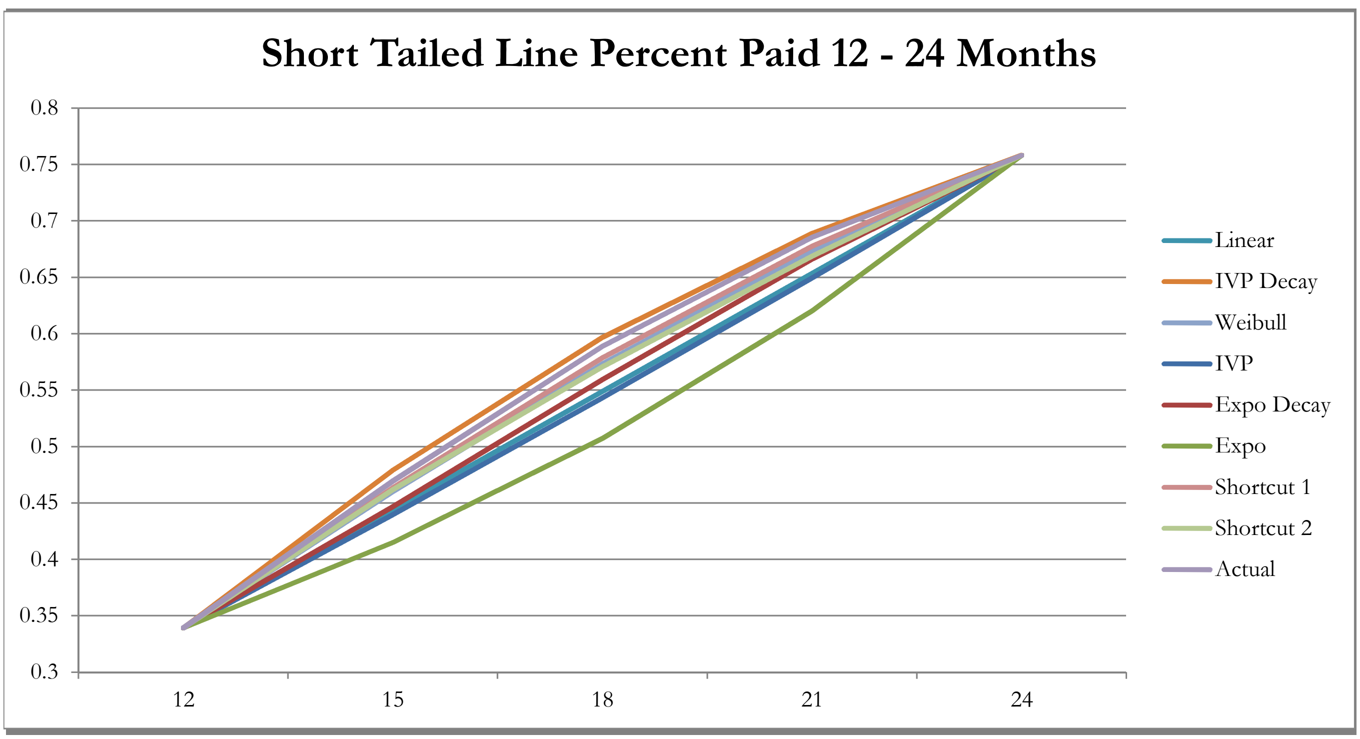




Short Tail Line Paid Graphs

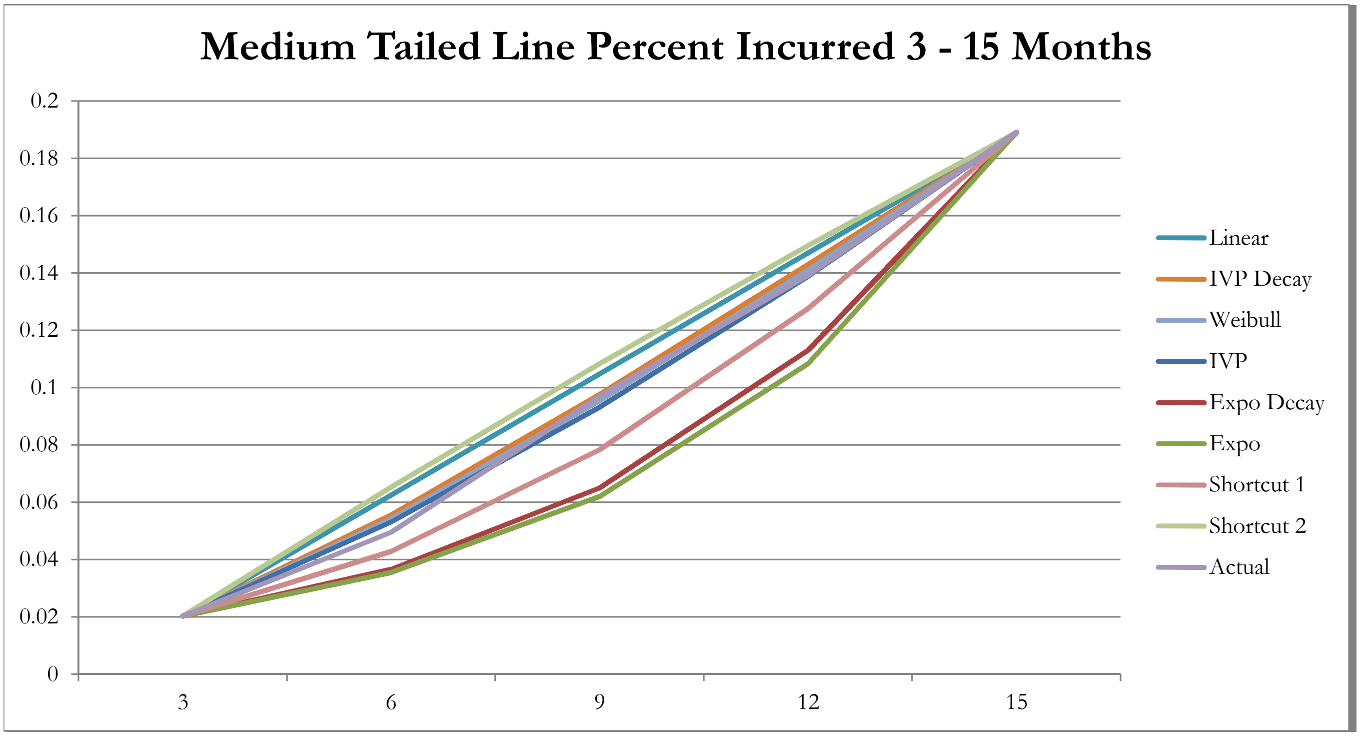
Appendix A
Sheet 2

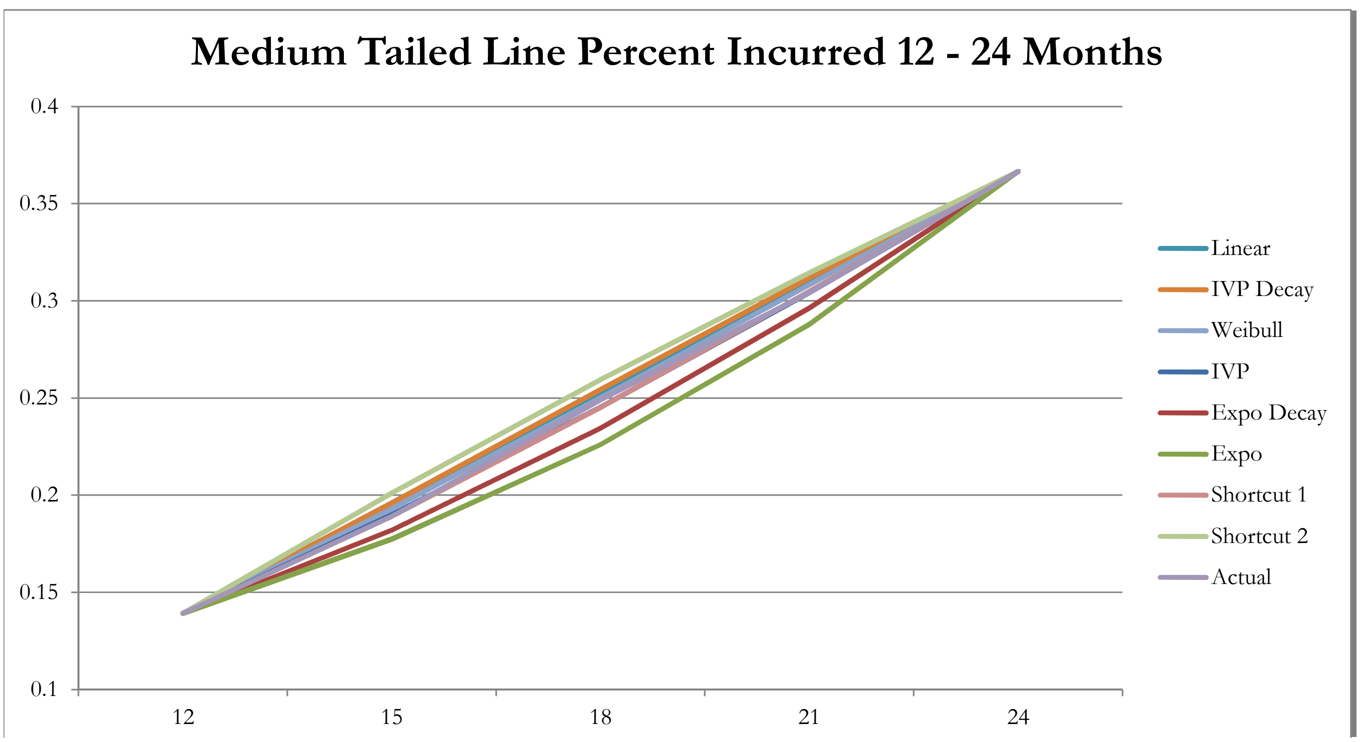




Medium Tail Line Incurred Graphs

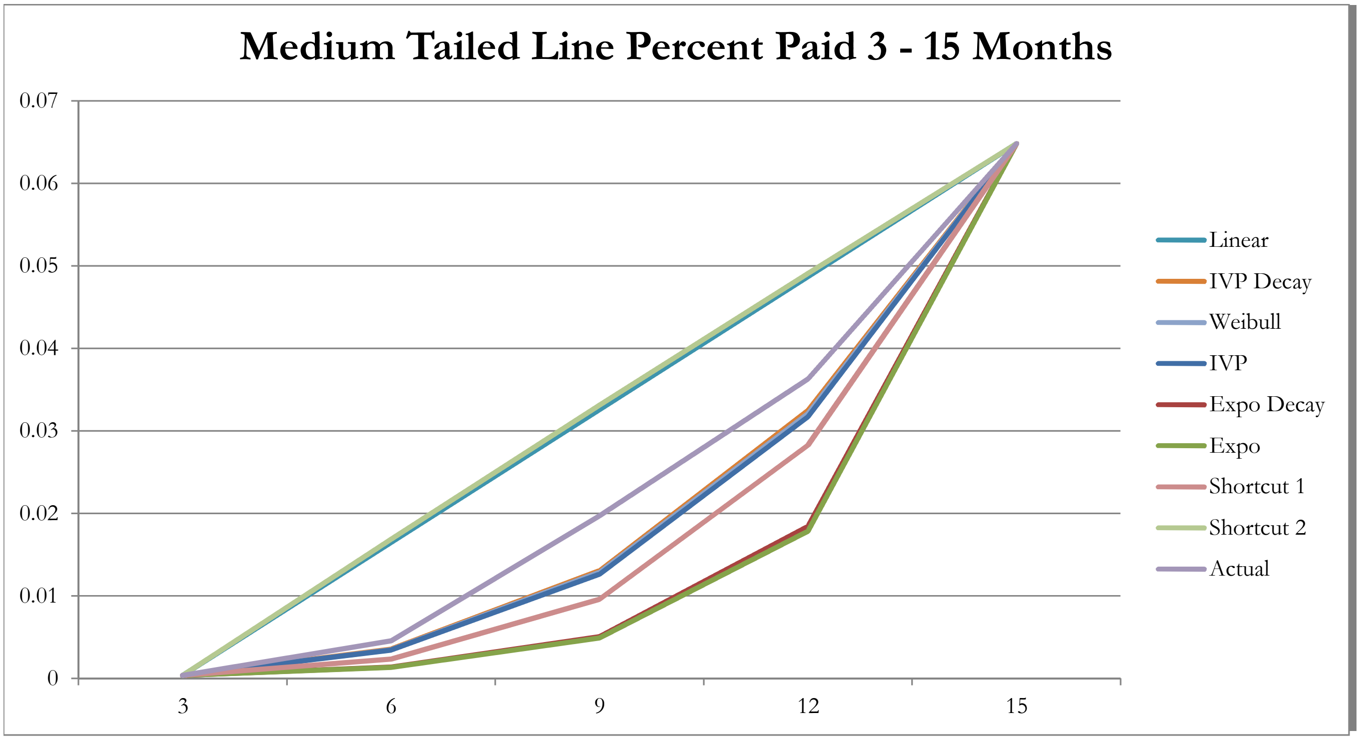
Appendix A
Sheet 3

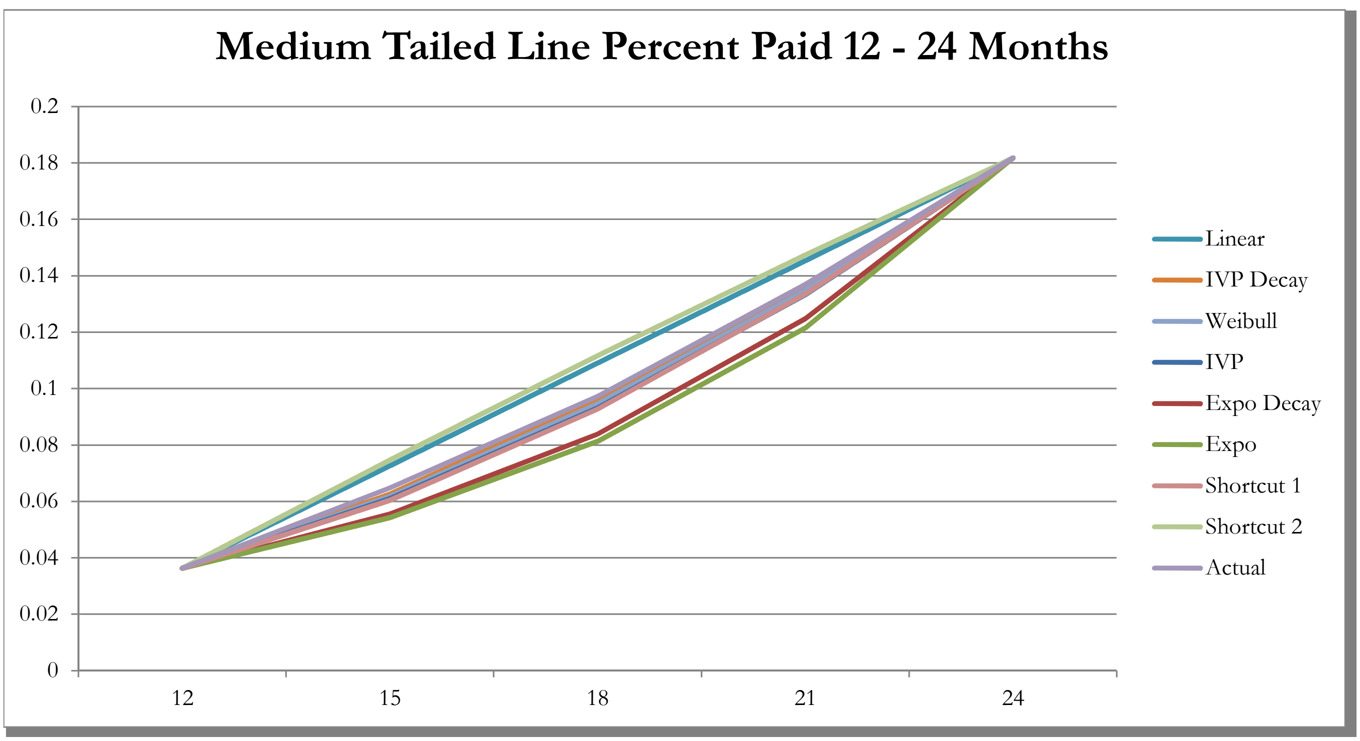




Medium Tail Line Paid Graphs

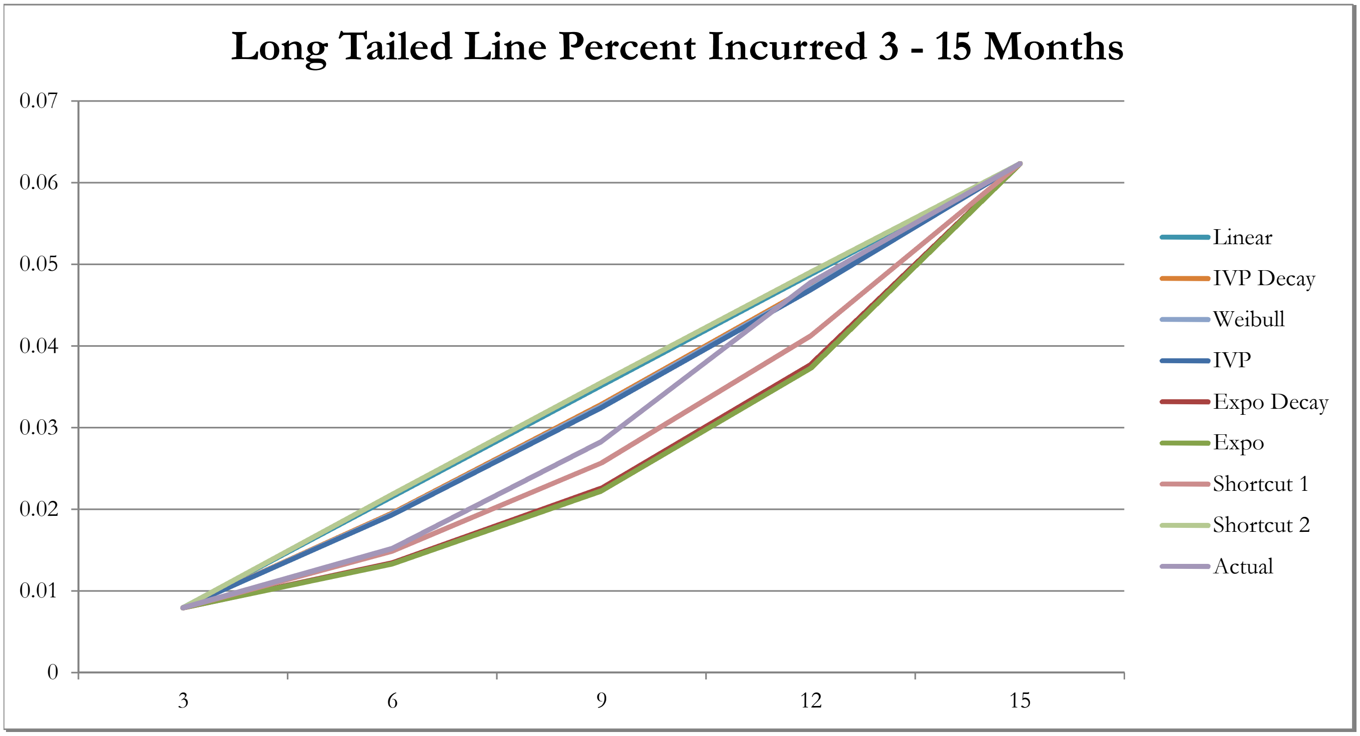
Appendix A
Sheet 4

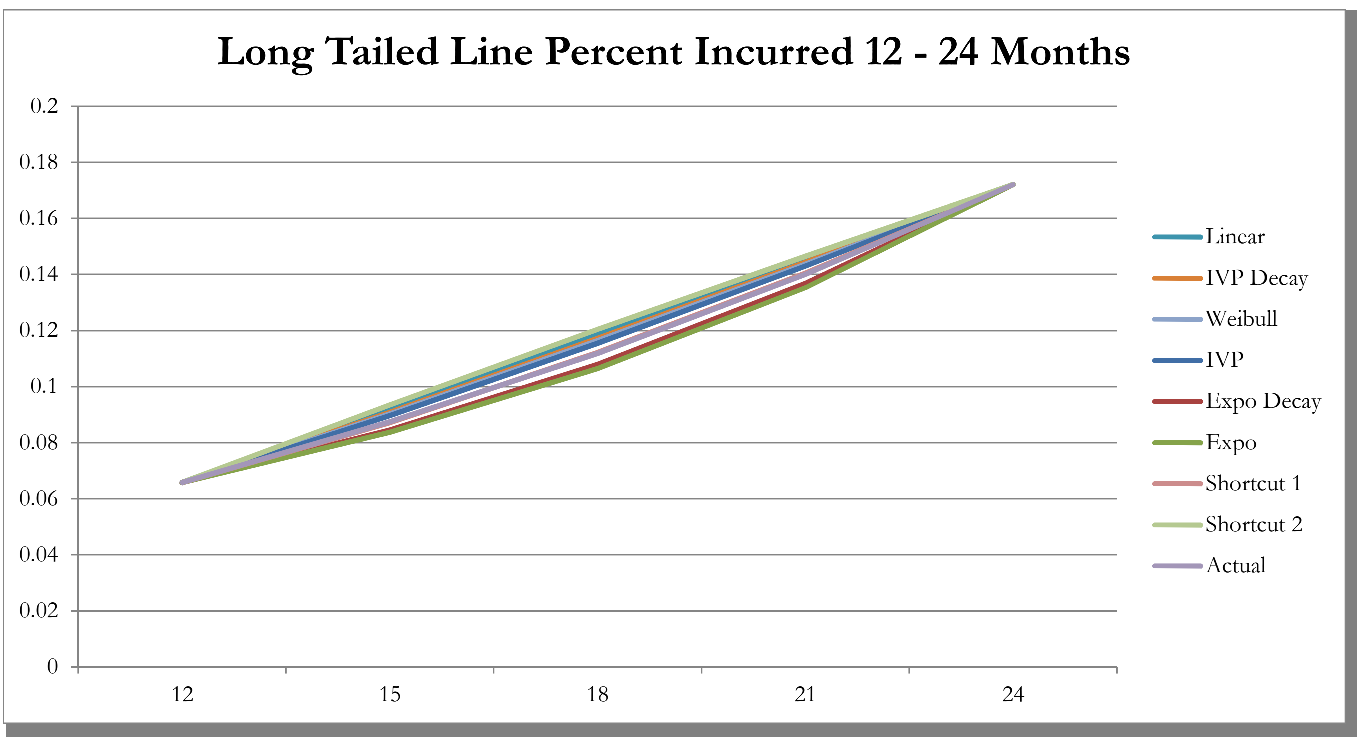




Long Tail Line Incurred Graphs

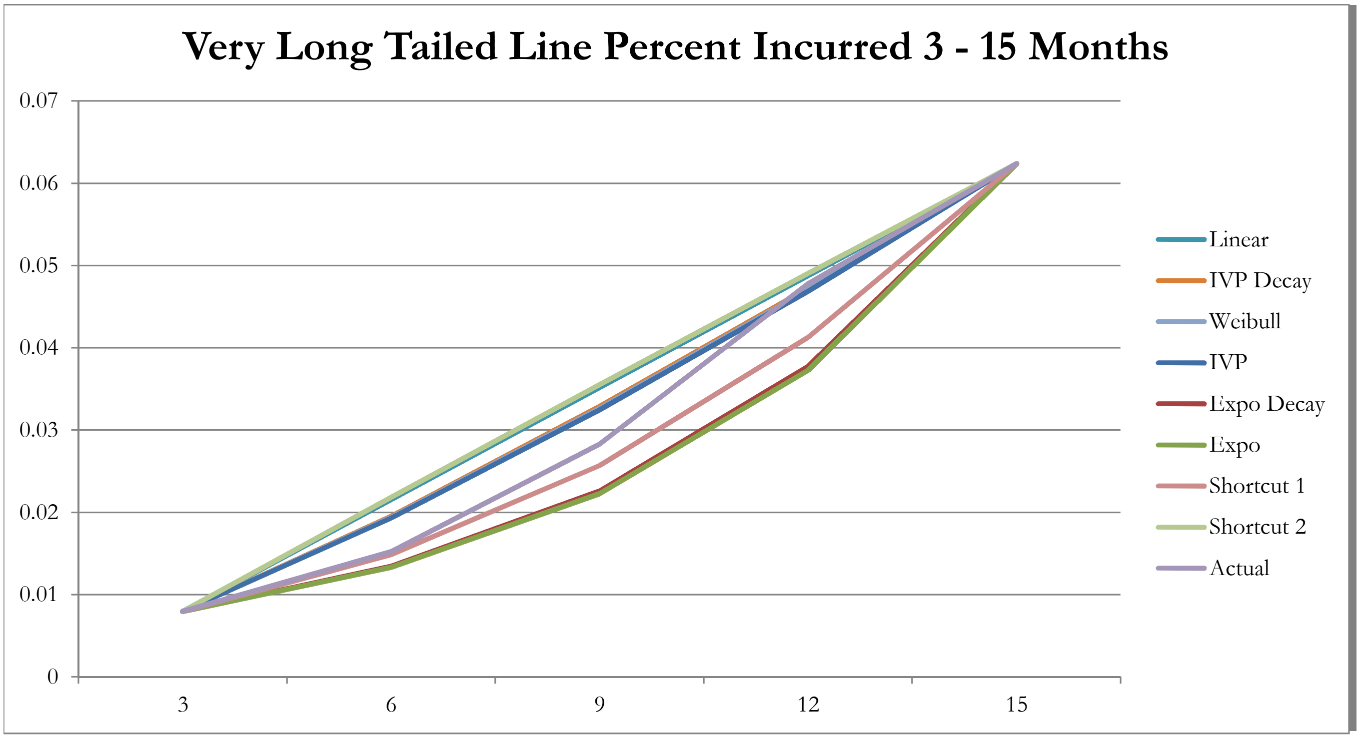
Appendix A
Sheet 5

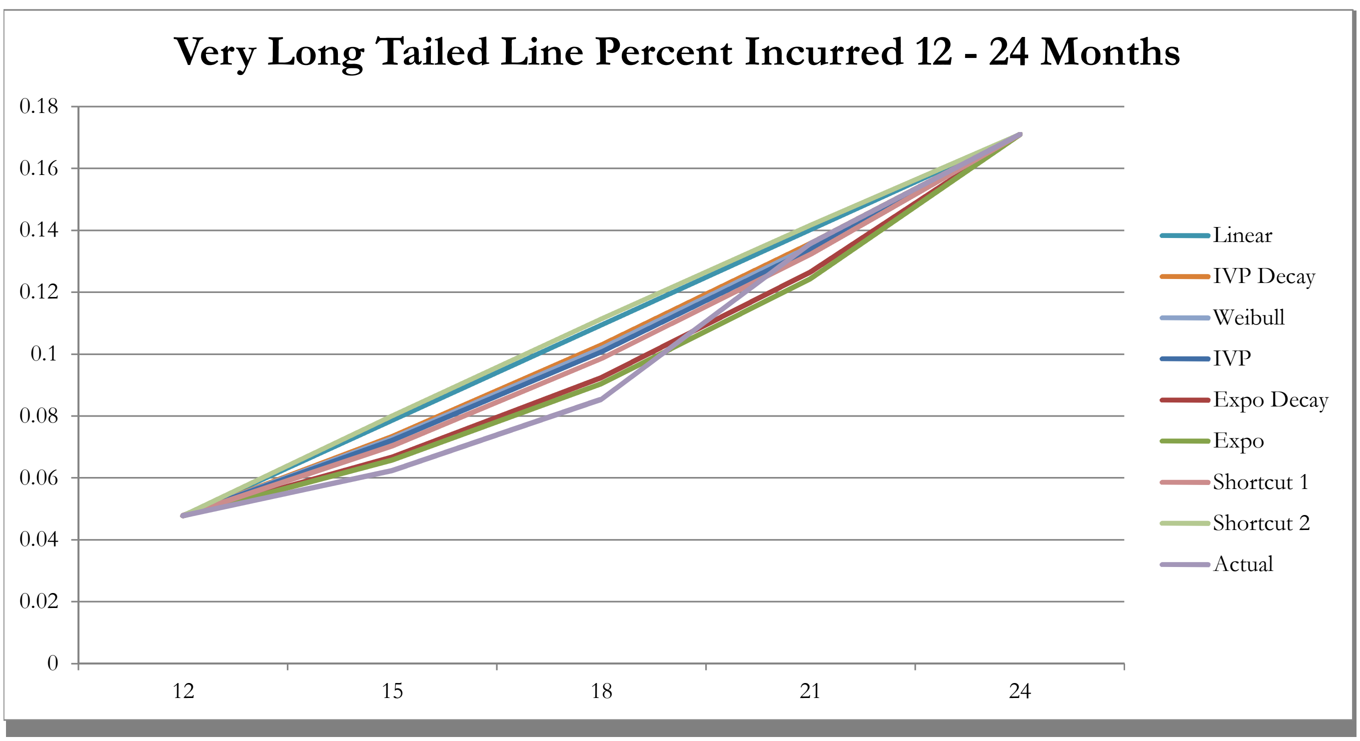




Very Long Tail Line Incurred Graphs

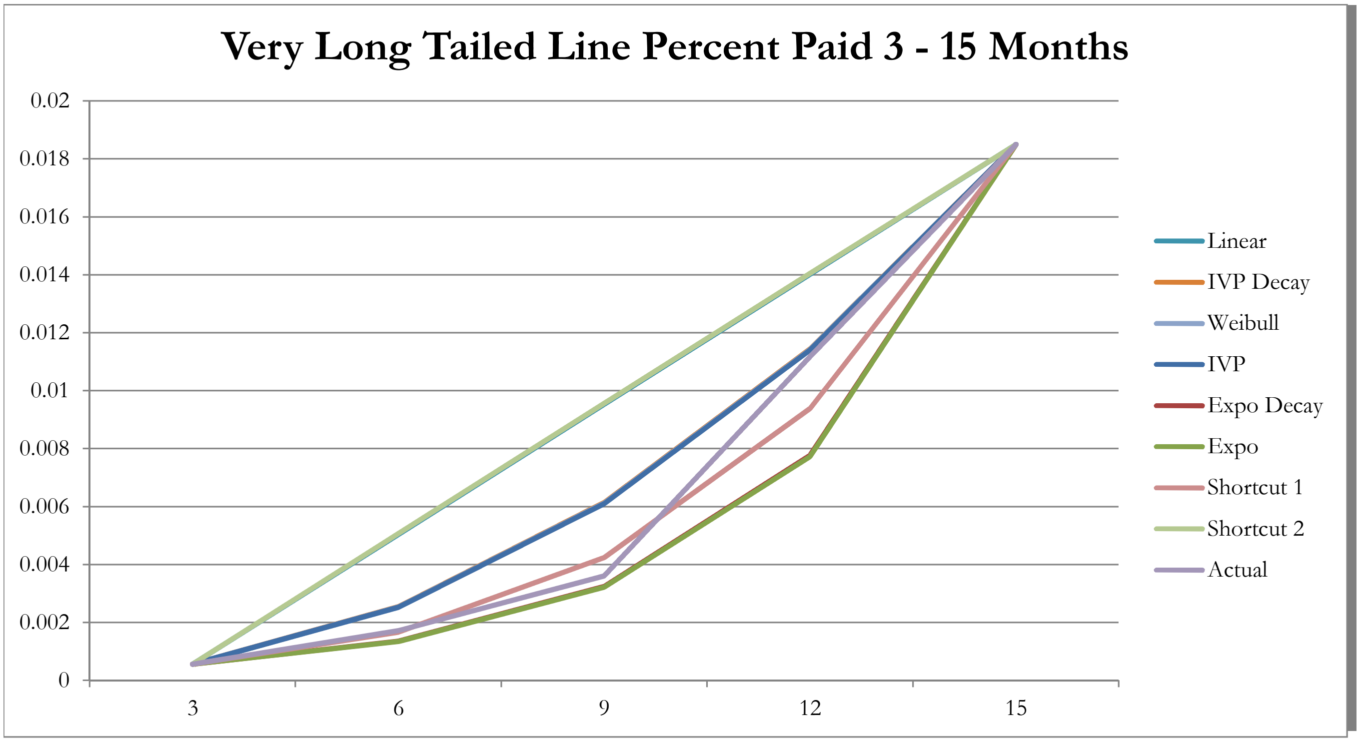
Appendix A
Sheet 6

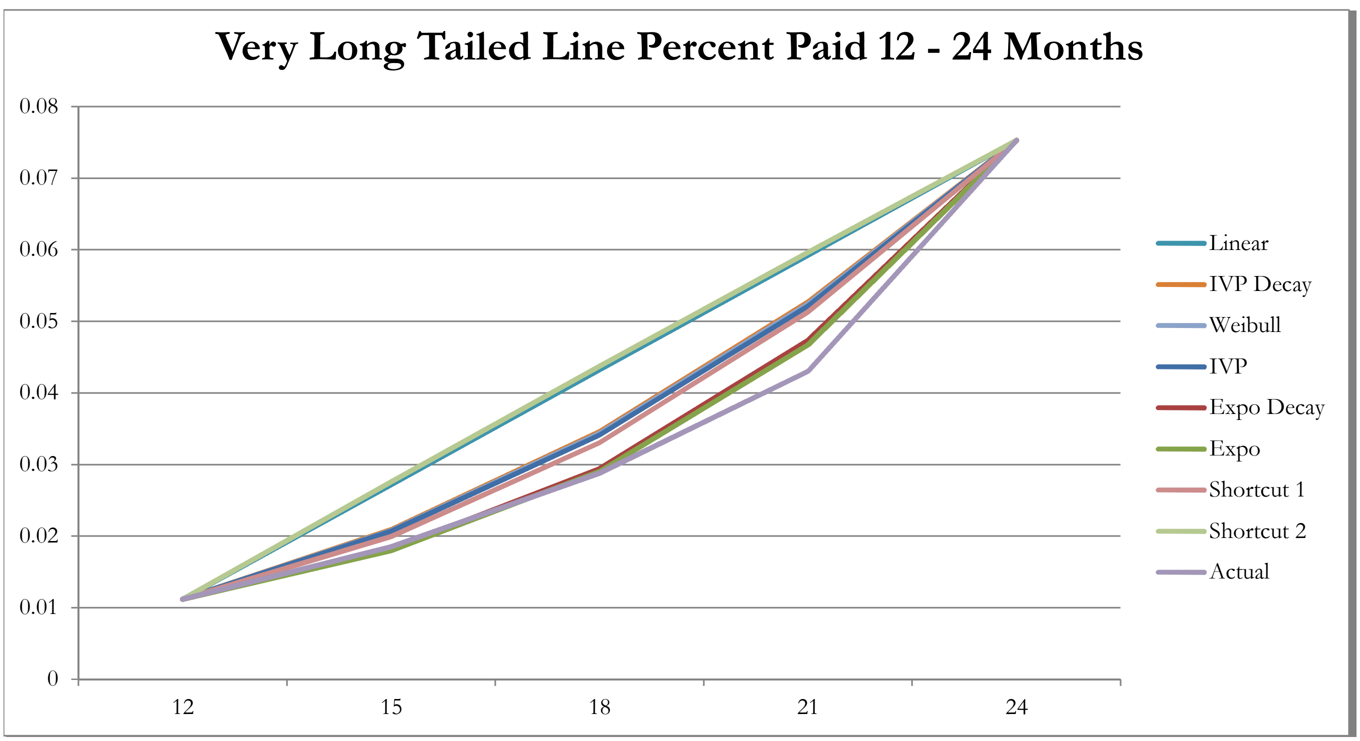




Very Long Tail Line Paid Graphs

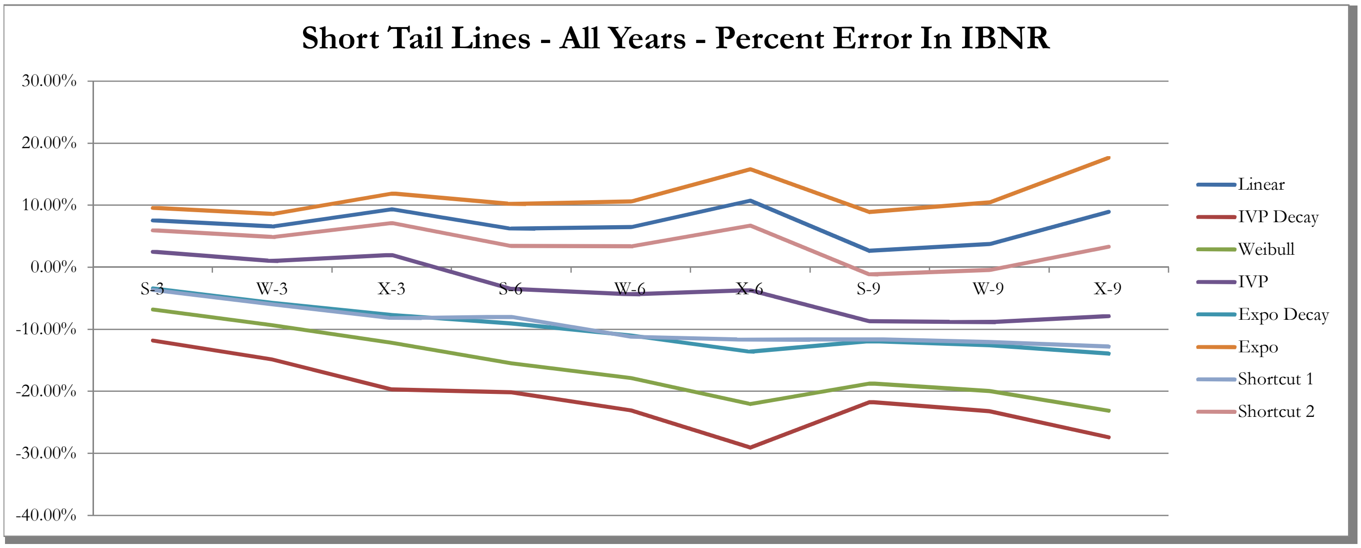
Appendix A
Sheet 7

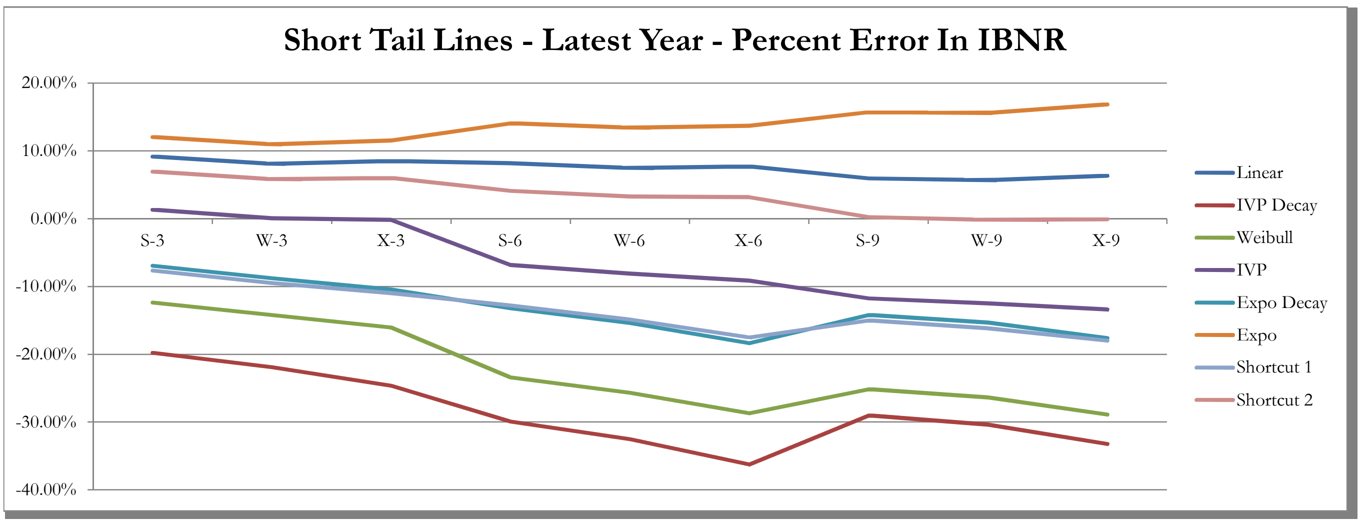


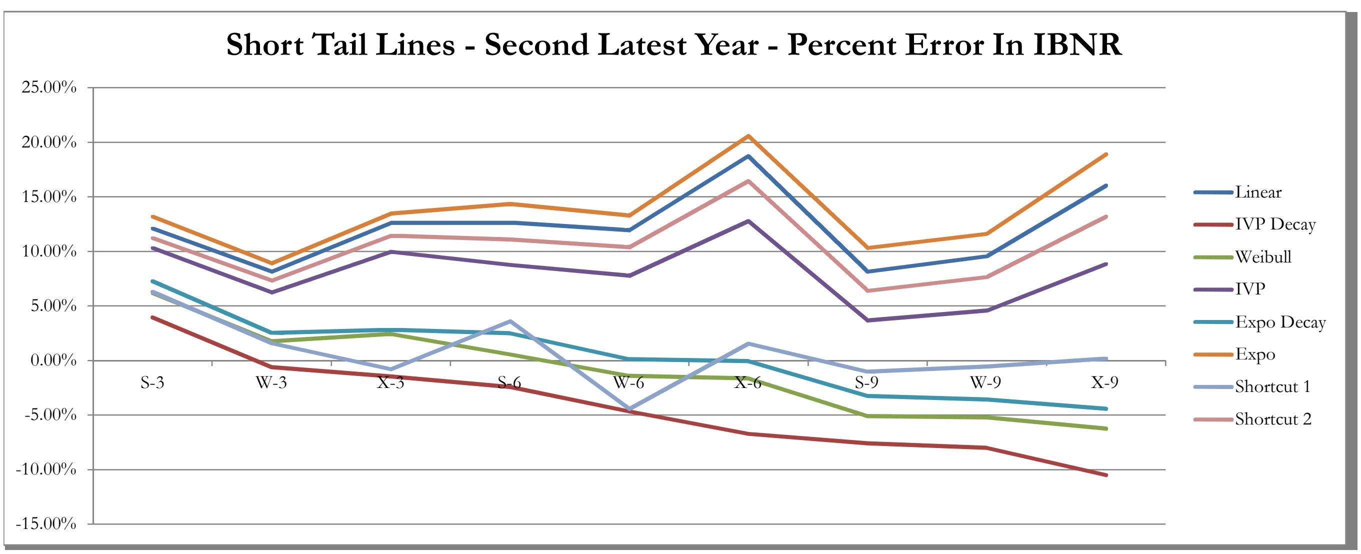


Short Tail Lines Incurred Error

Appendix A
Sheet 8

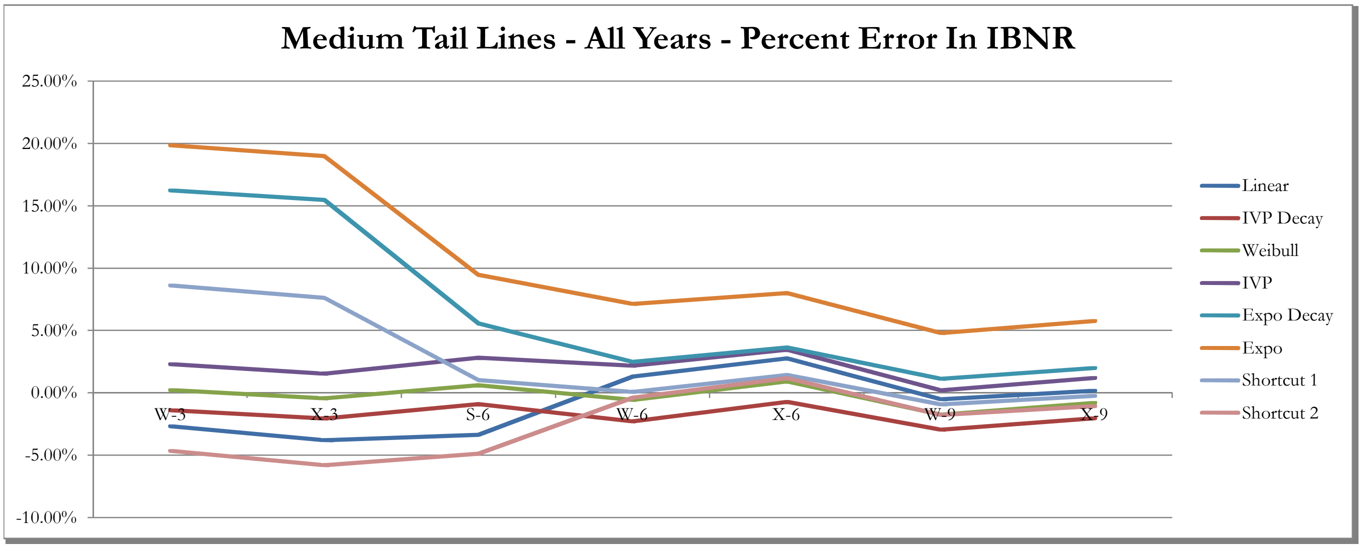


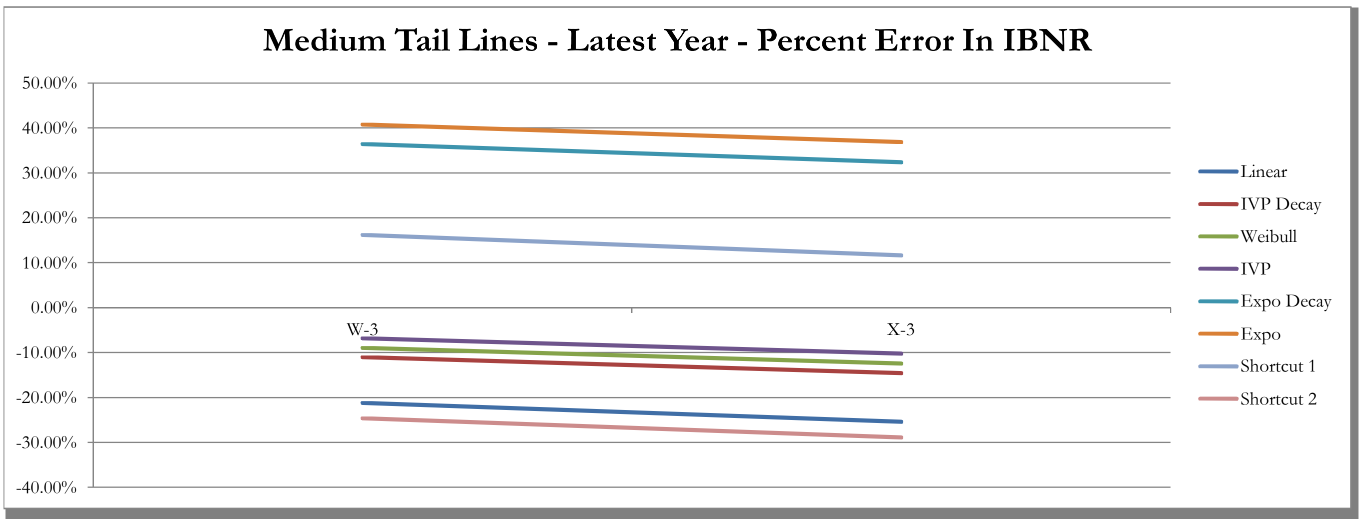


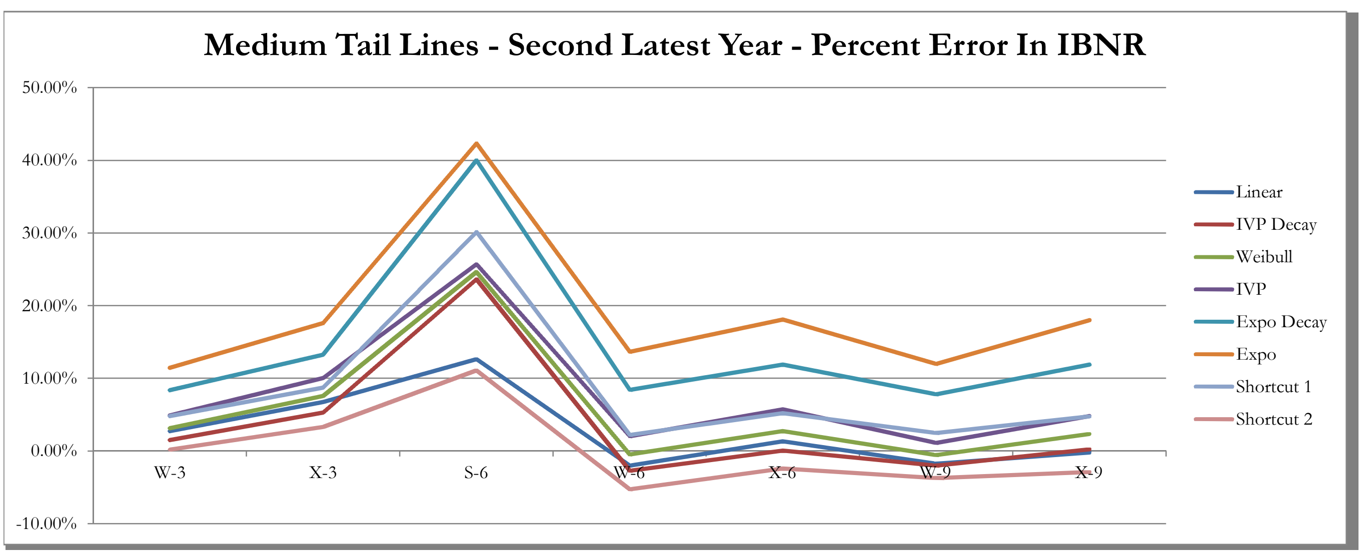


Medium Tail Lines Incurred Error

Appendix A
Sheet 9

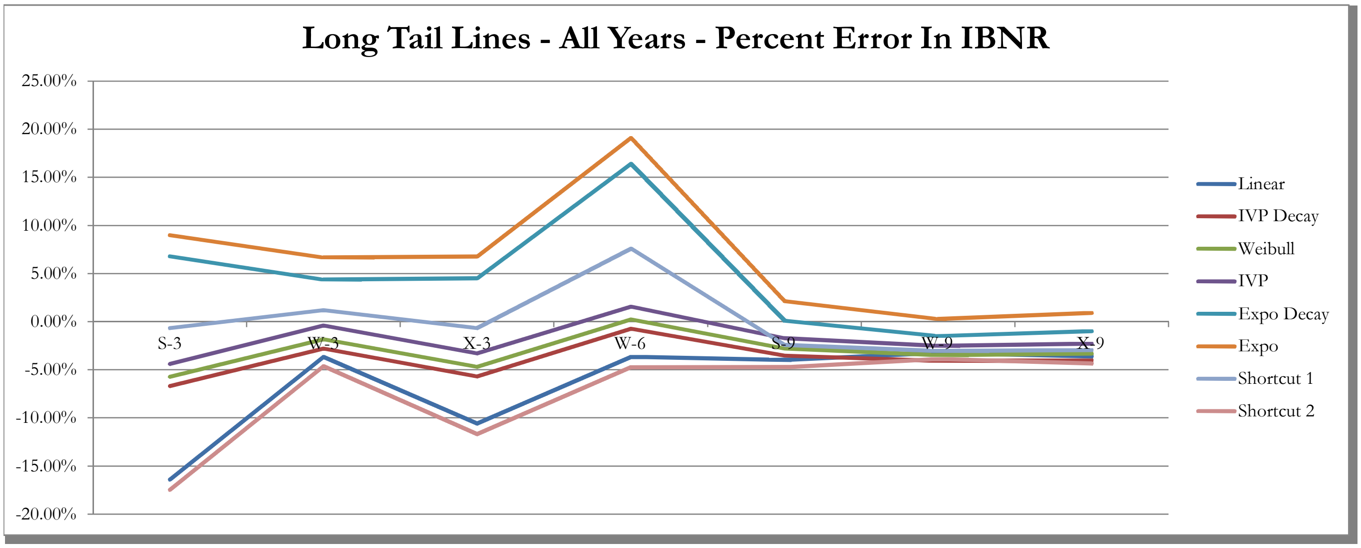


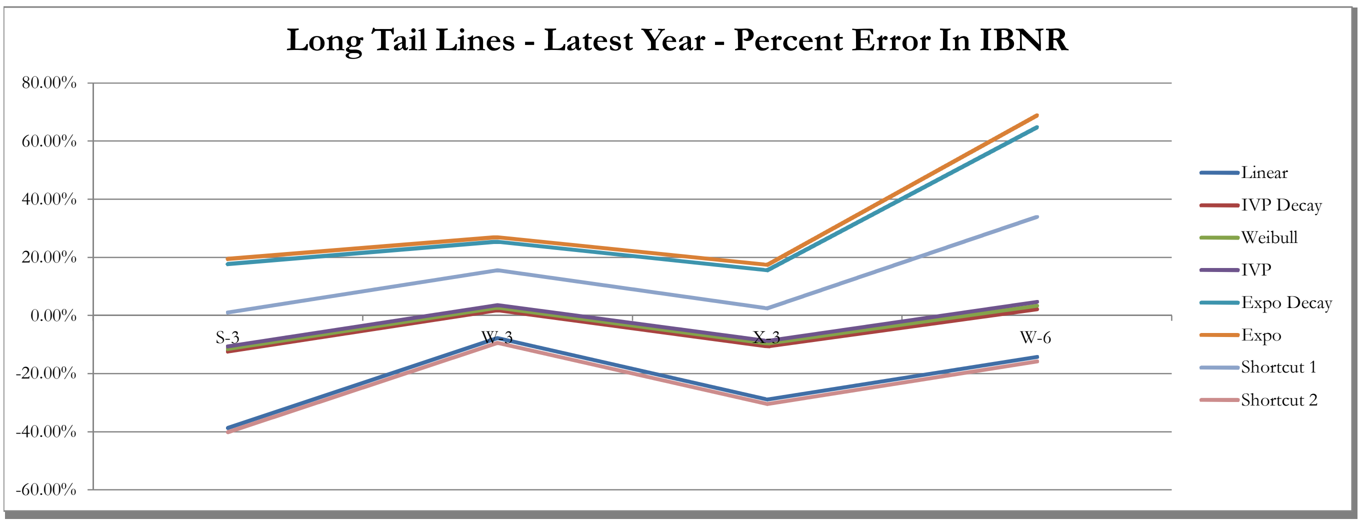


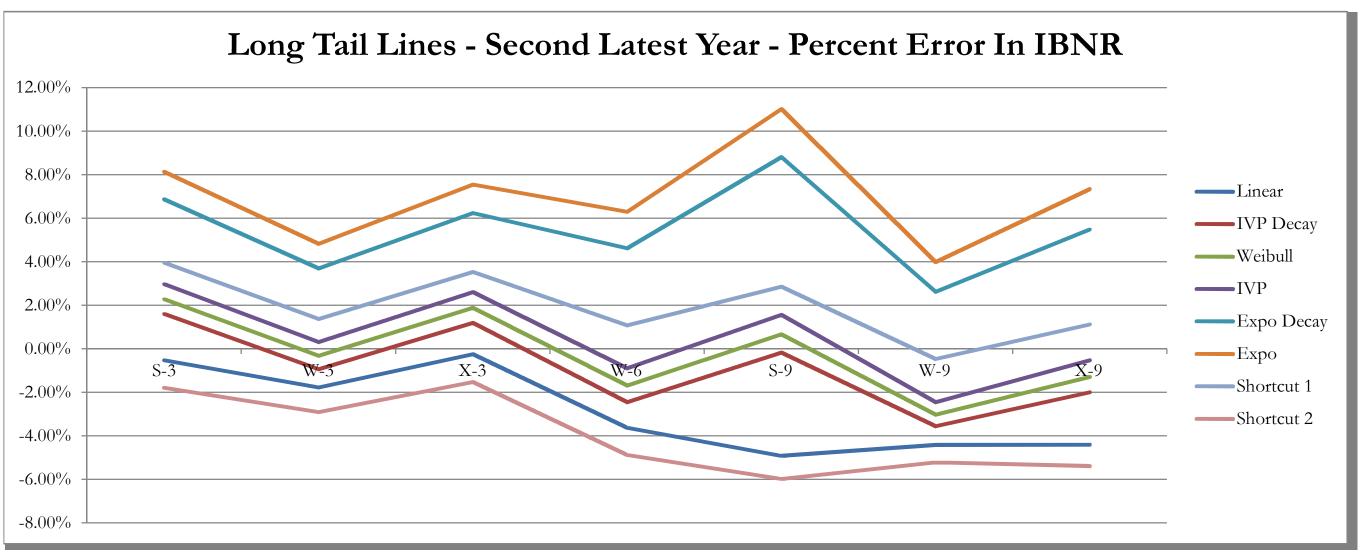


Long Tail Lines Incurred Error

Appendix A
Sheet 10

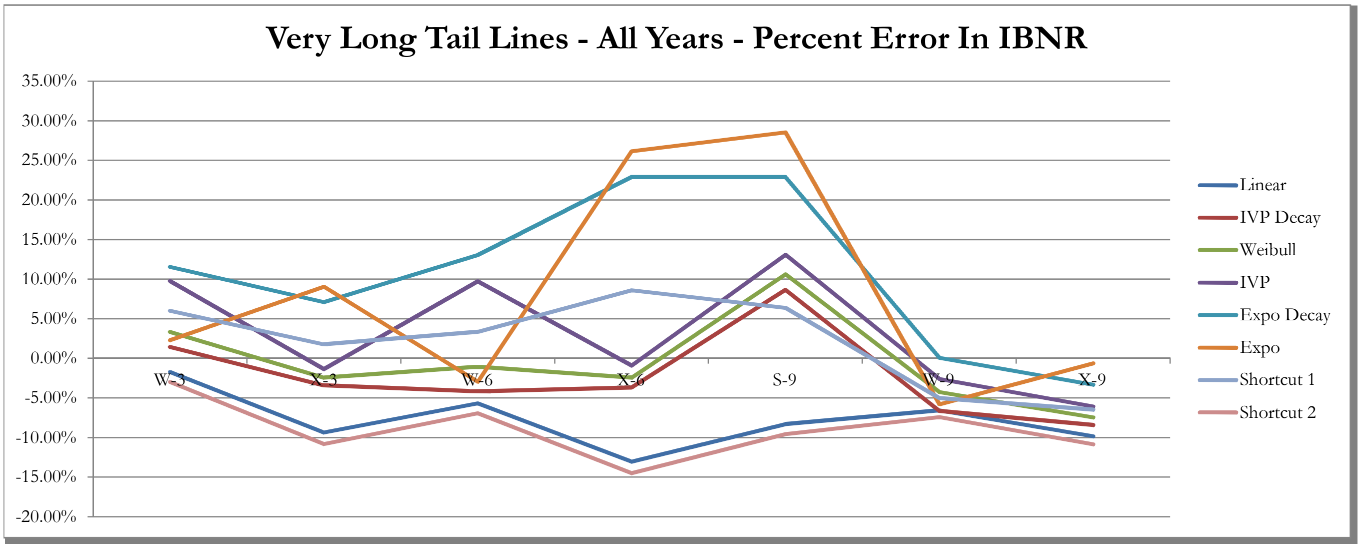


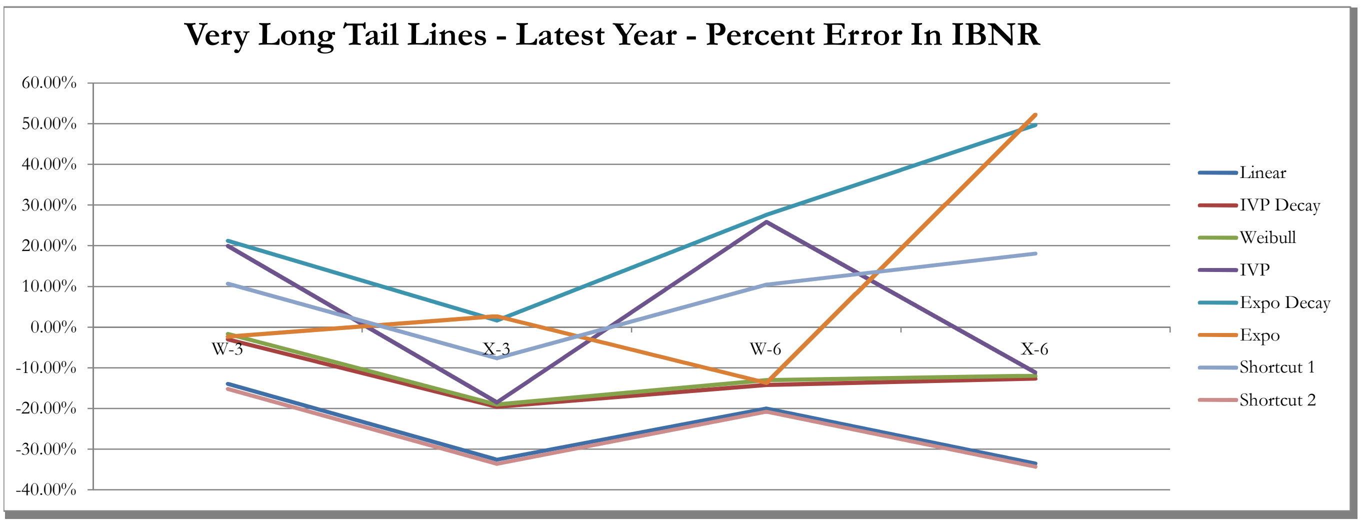


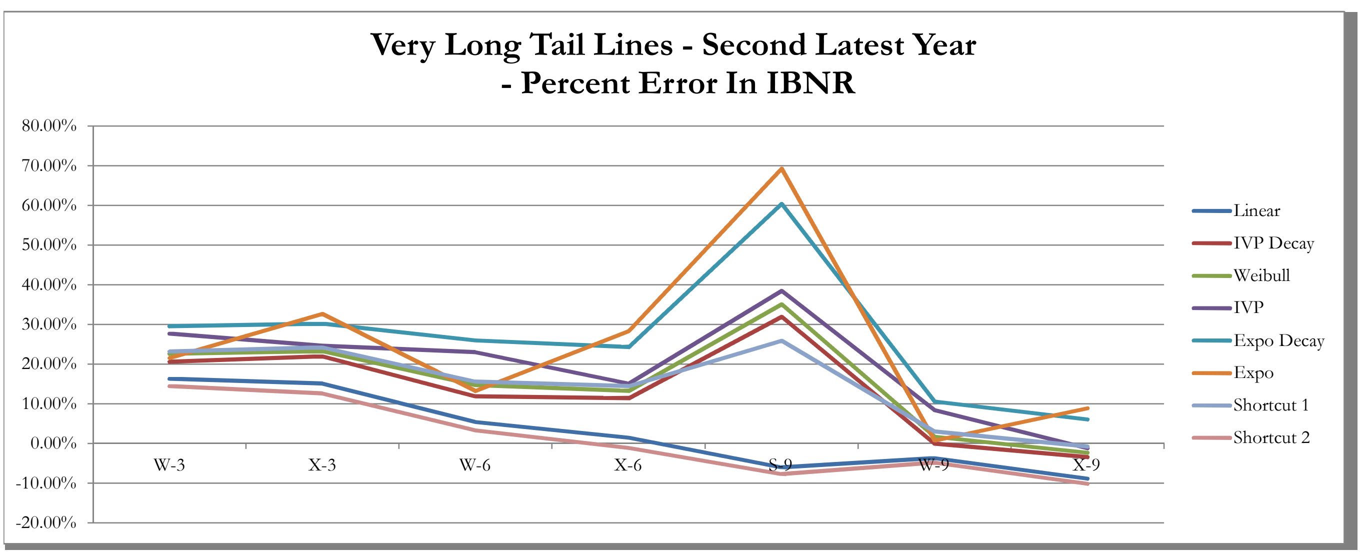


Very Long Tail Lines Incurred Error

Appendix A
Sheet 11

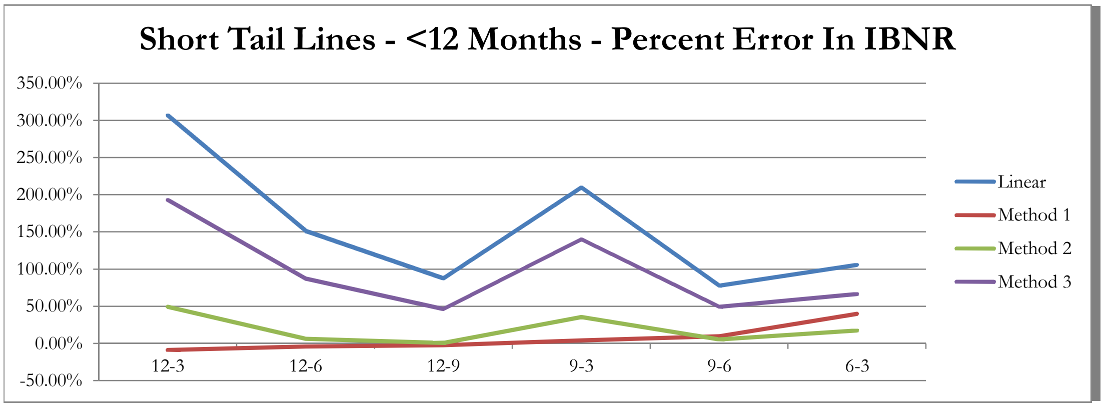


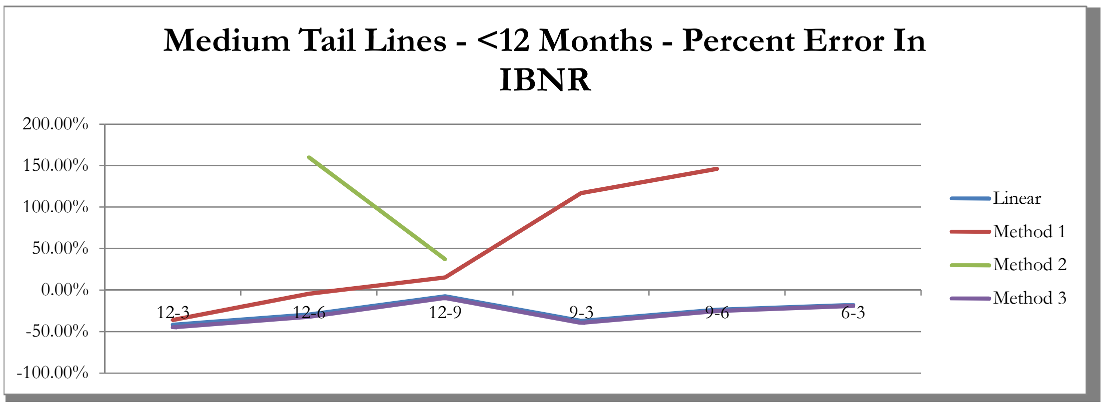


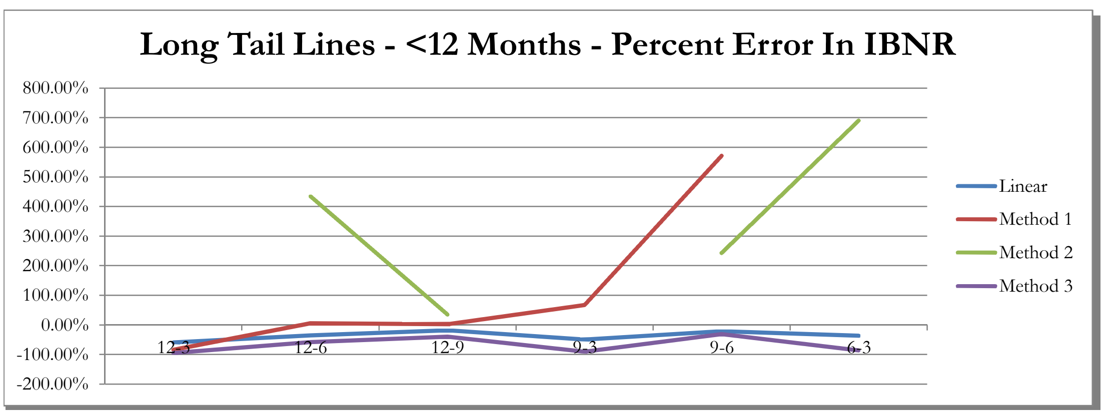


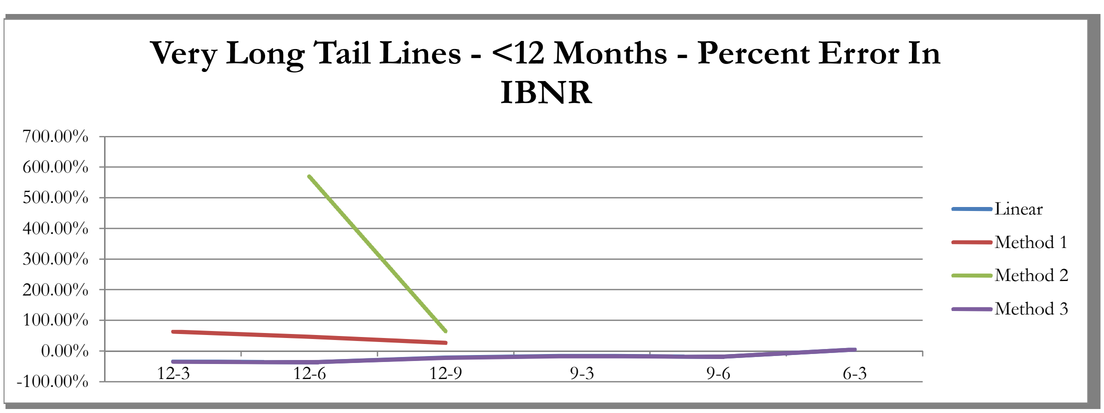
Incurred Extrapolation Errors

Appendix A
Sheet 12



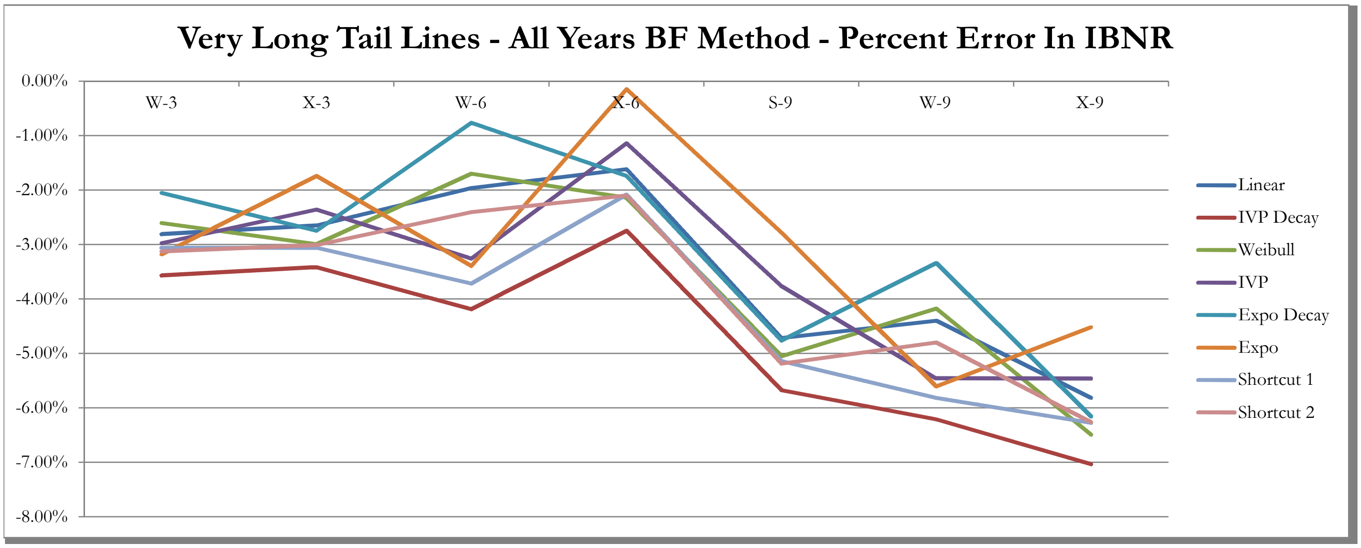


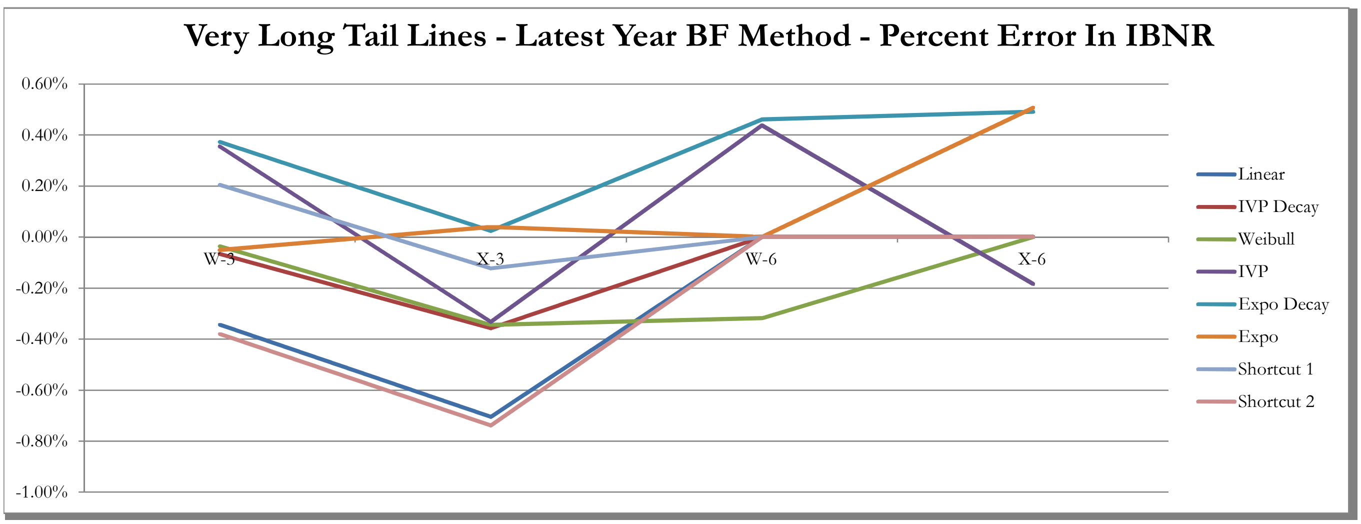


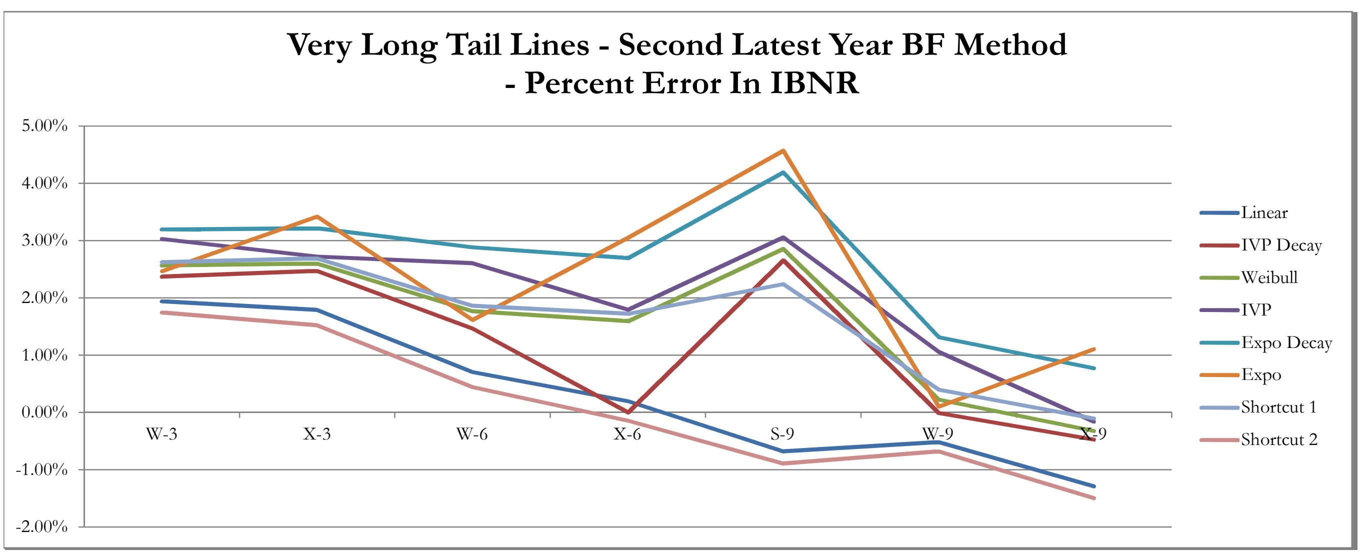


Very Long Tail Lines BF Error

Appendix A
Sheet 13

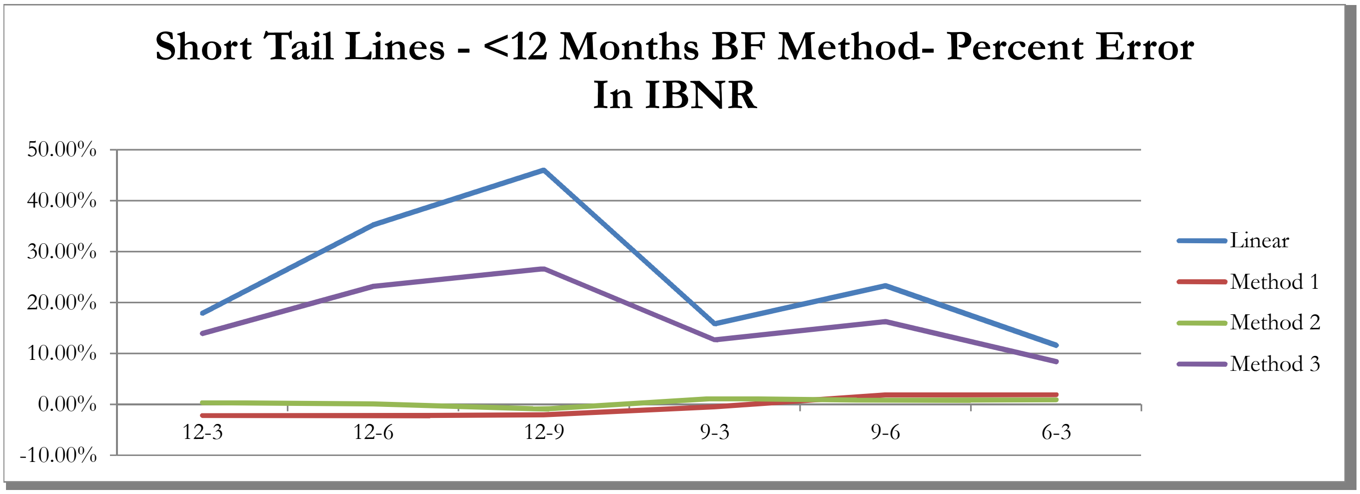


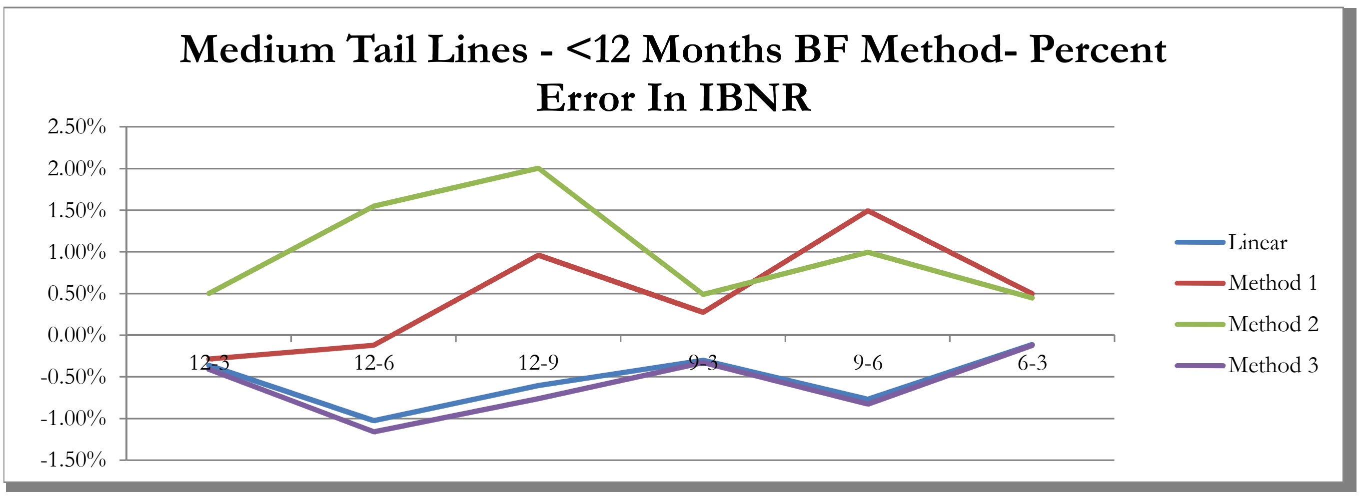


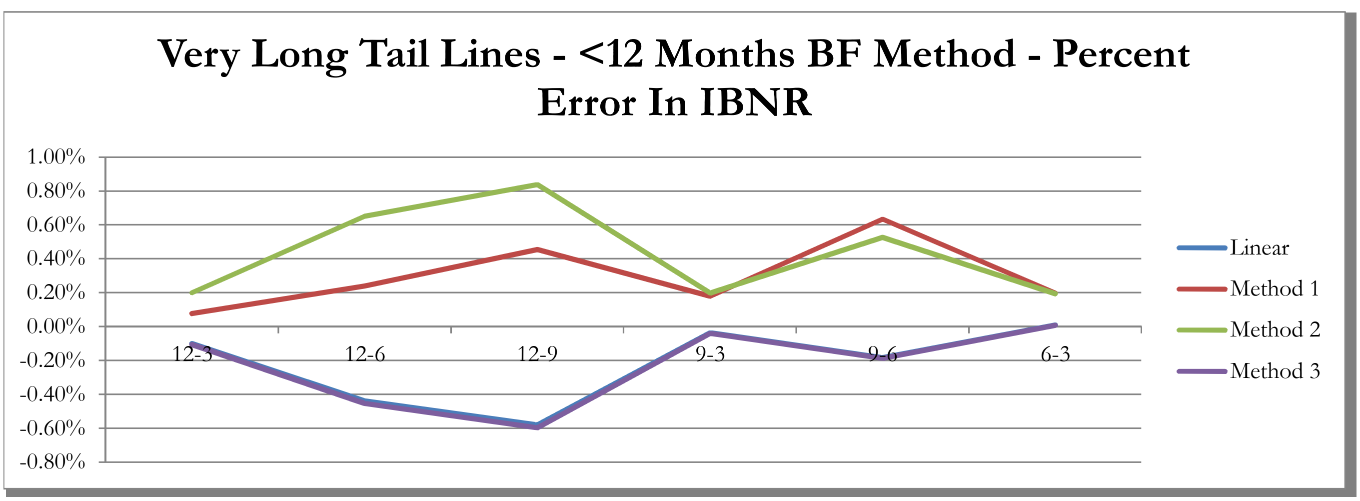


Incurred BF Extrapolation Errors

Appendix A
Sheet 14

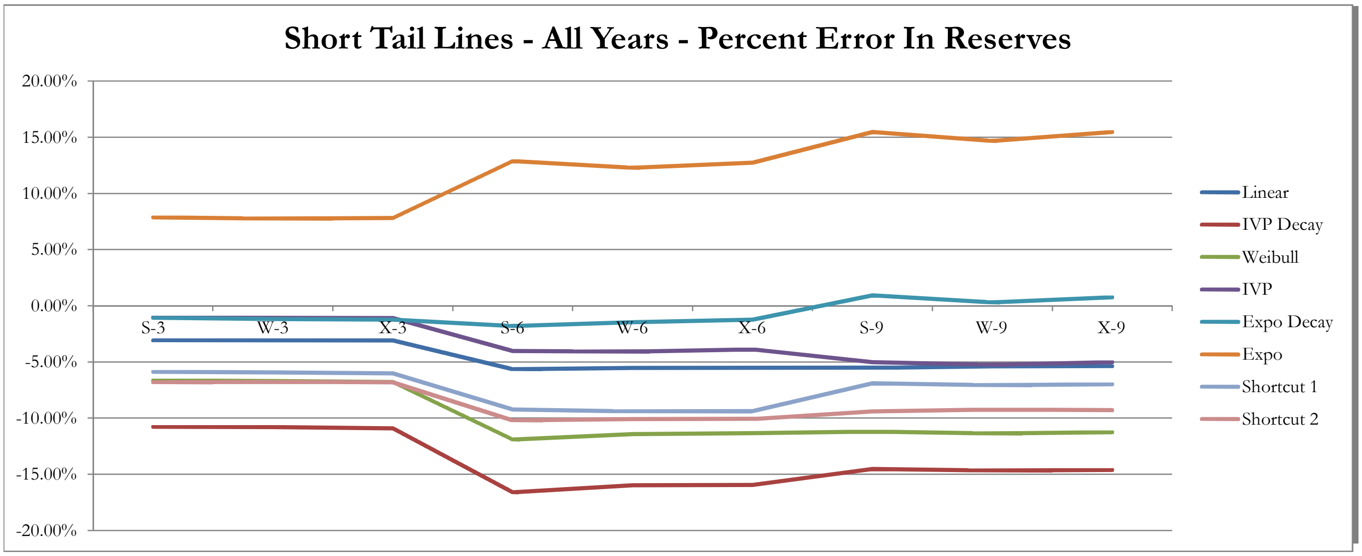


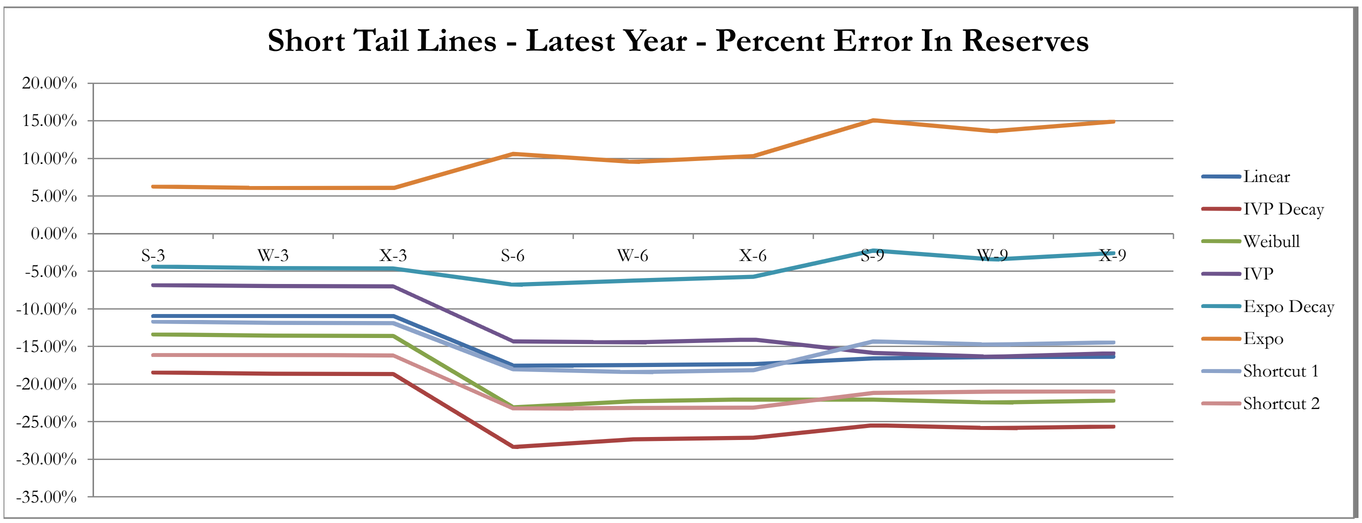


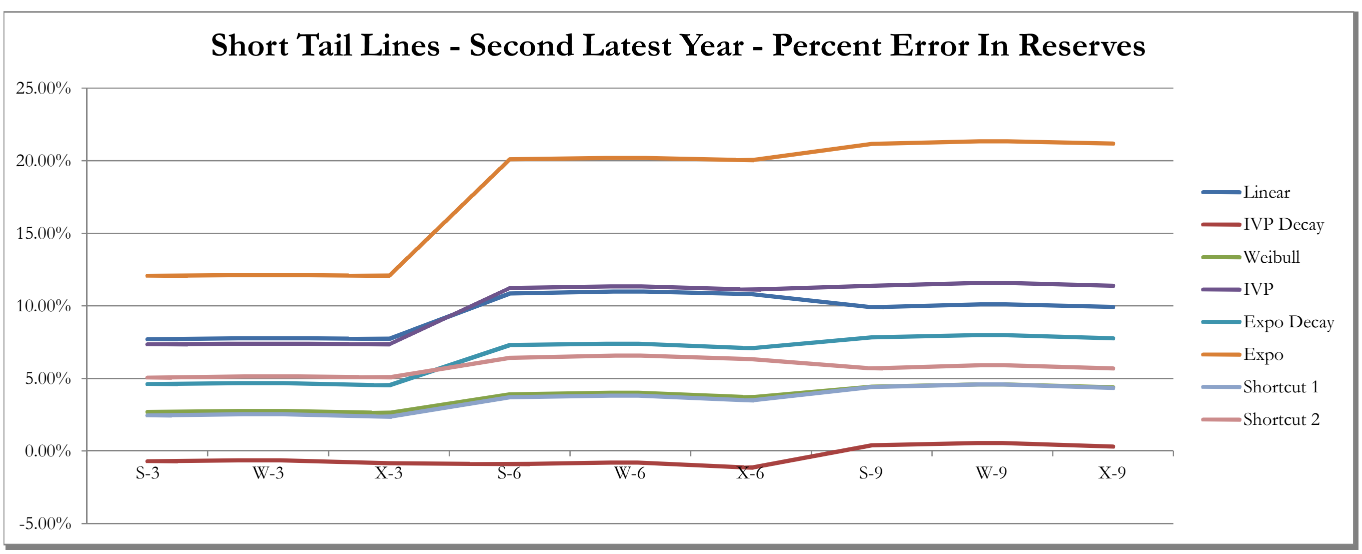


Short Tail Lines Paid Error

Appendix A
Sheet 15

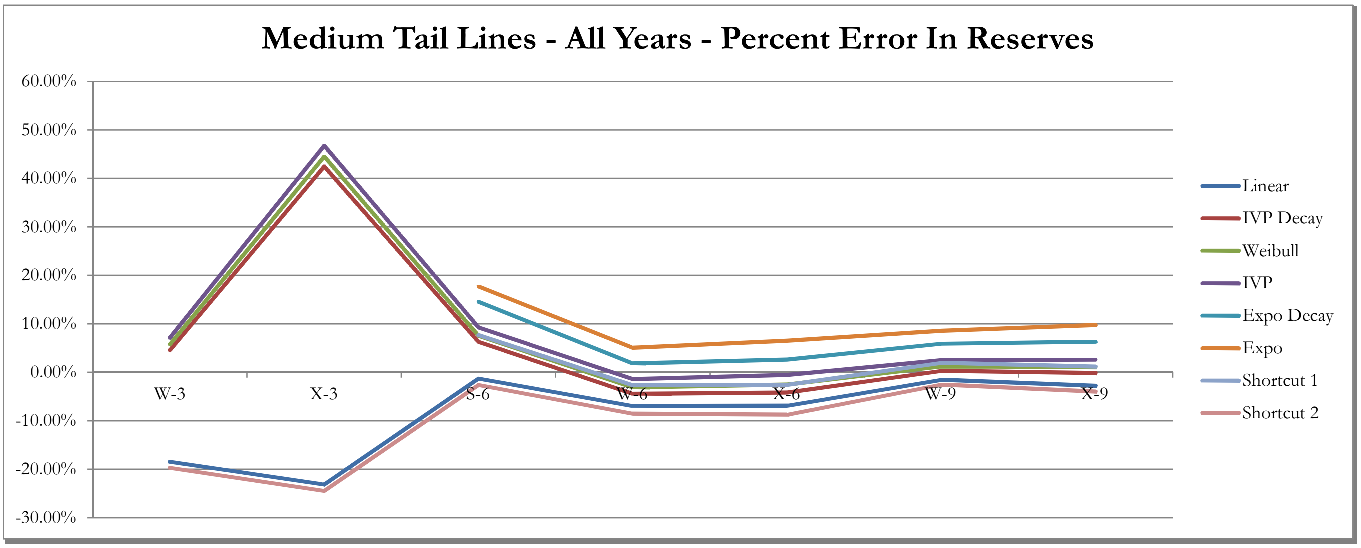


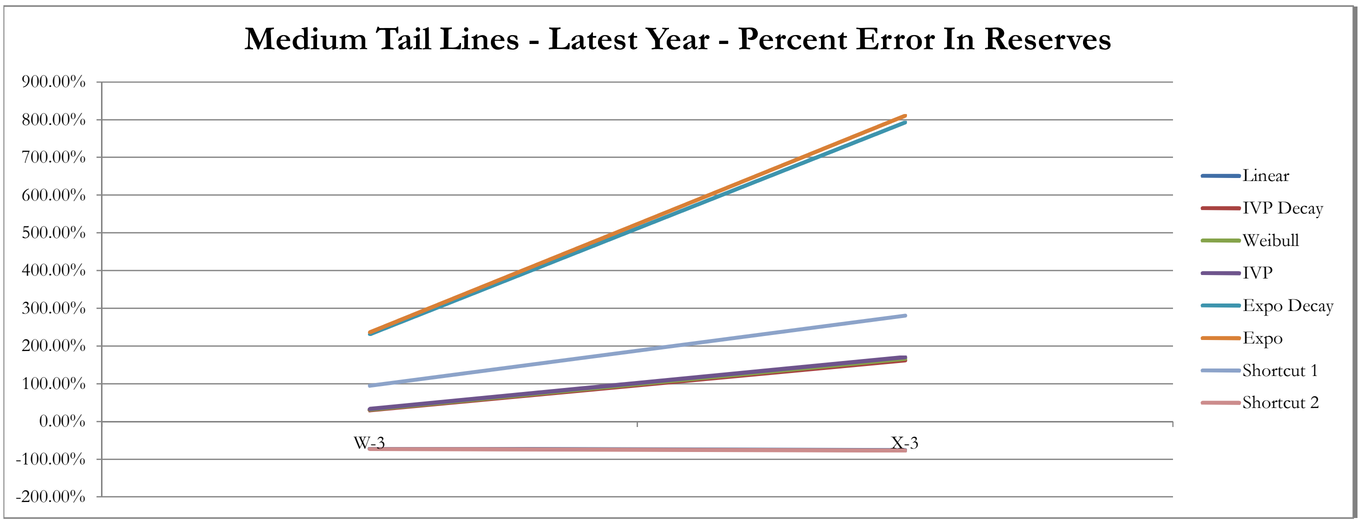


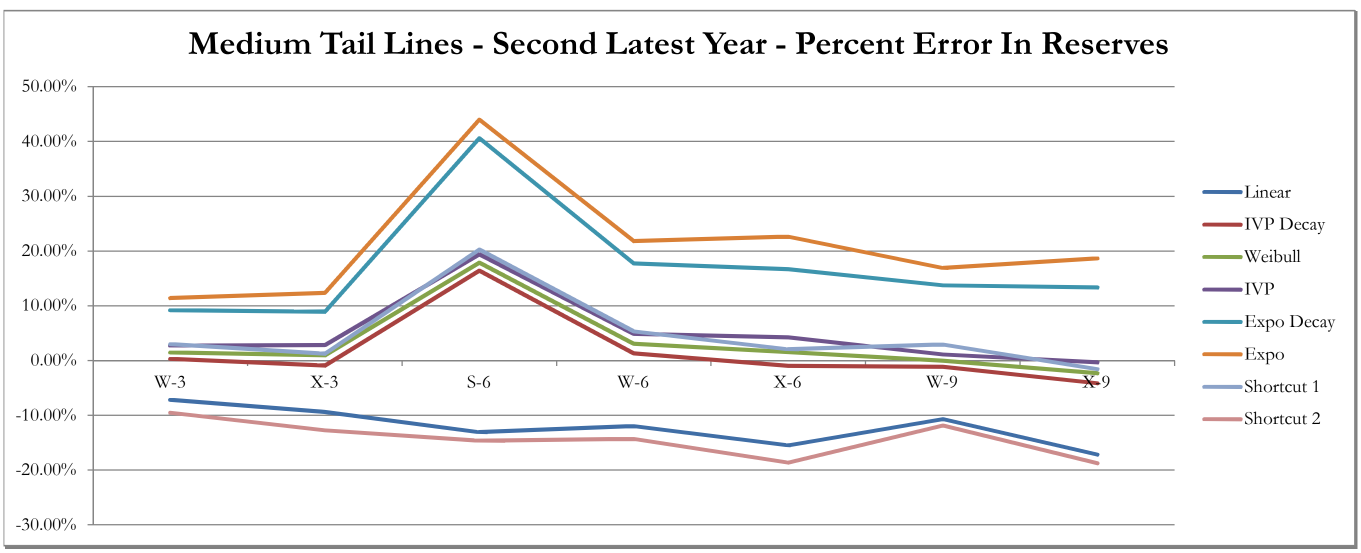


Medium Tail Lines Paid Error

Appendix A
Sheet 16

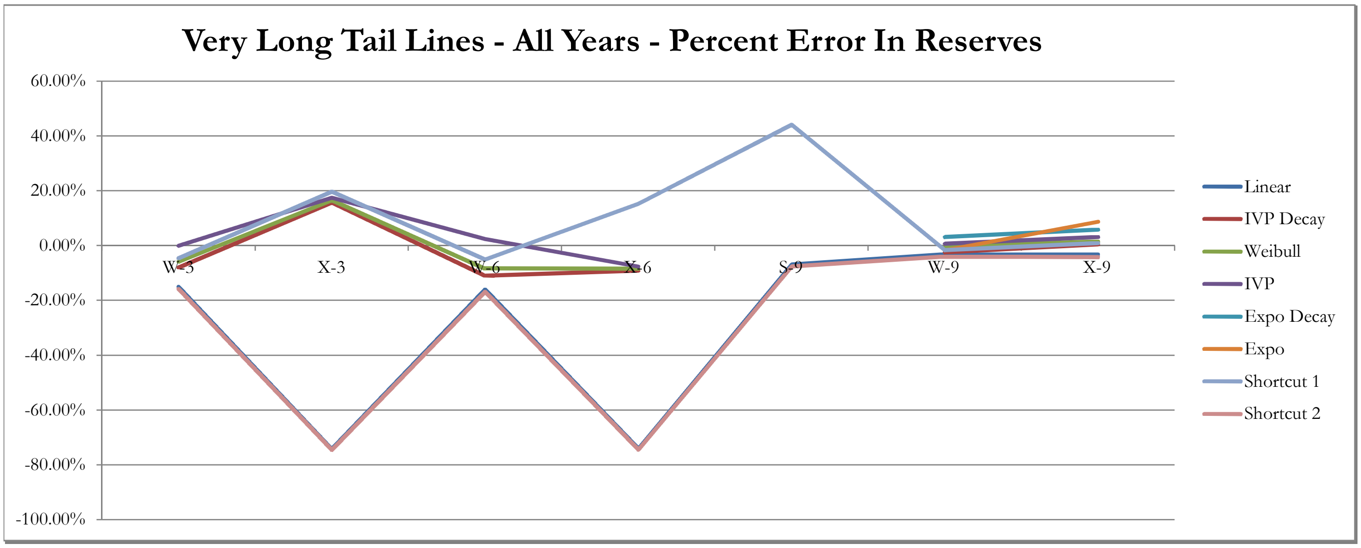


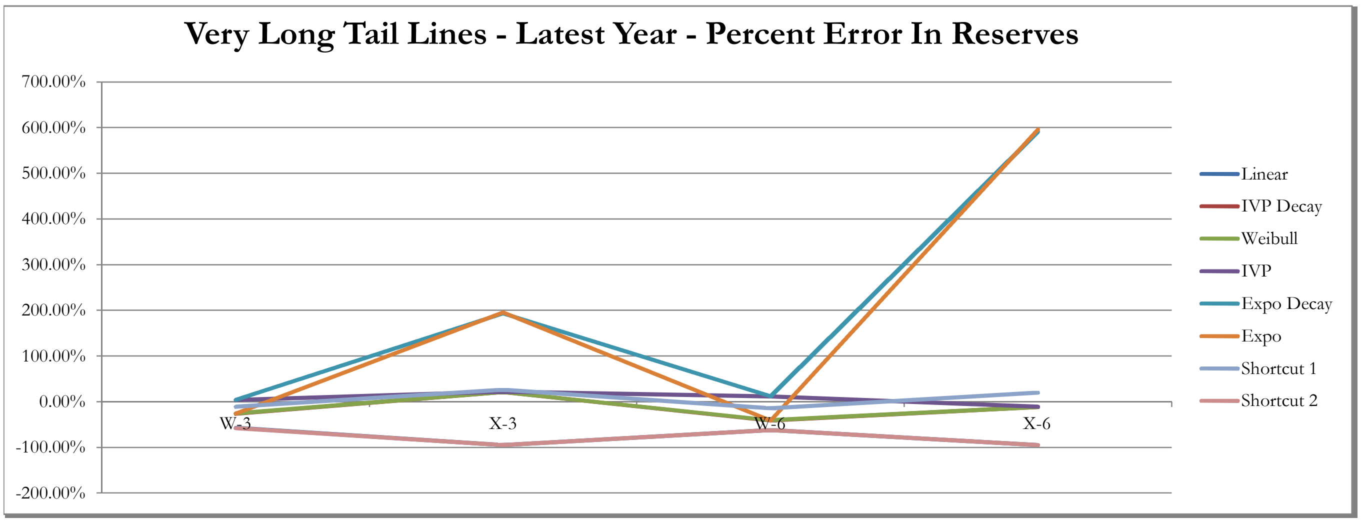


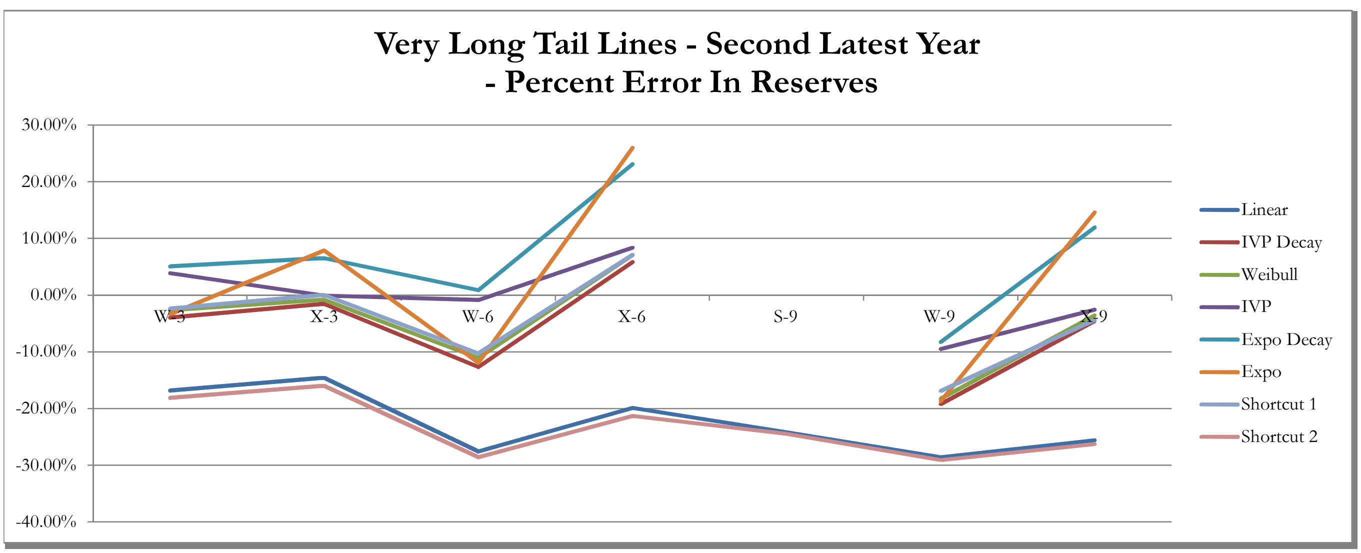


Very Long Tail Lines Paid Error

Appendix A
Sheet 17

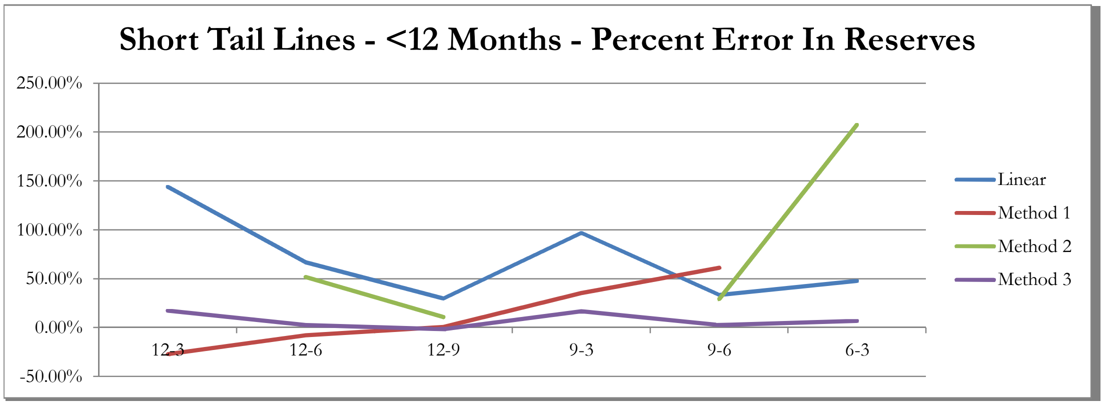


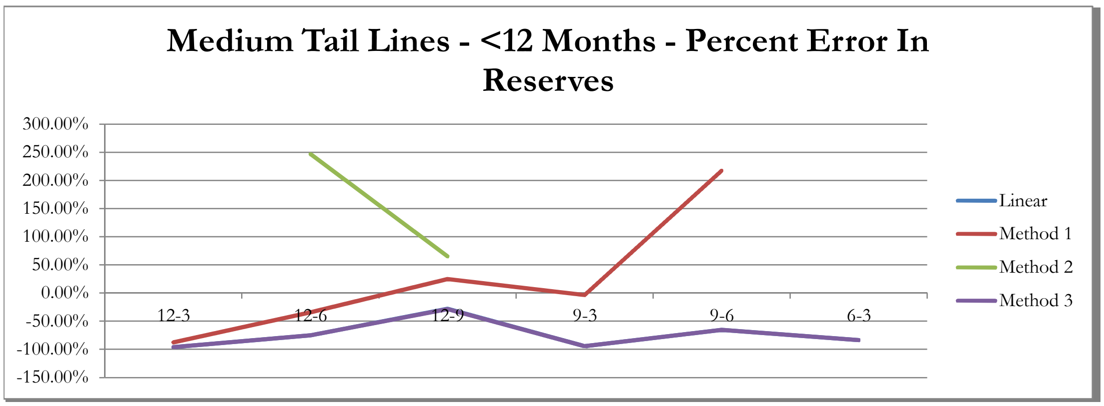


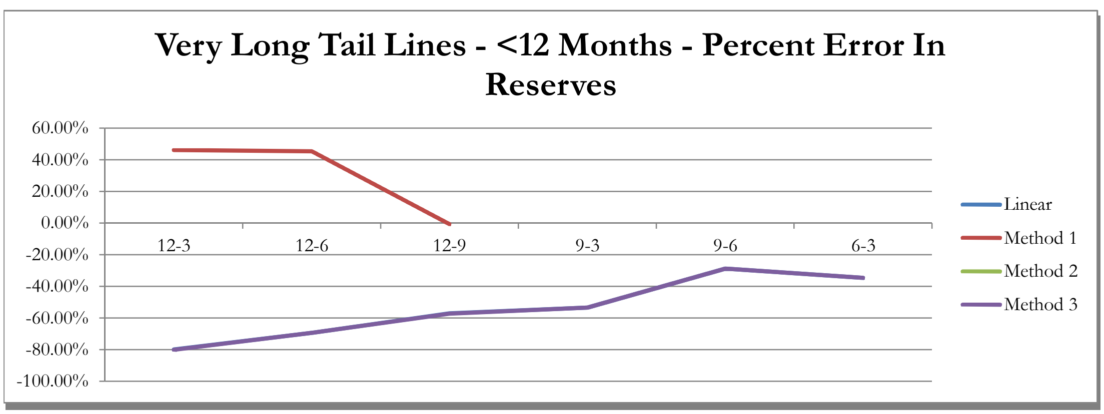


Paid Extrapolation Errors

Appendix A
Sheet 18

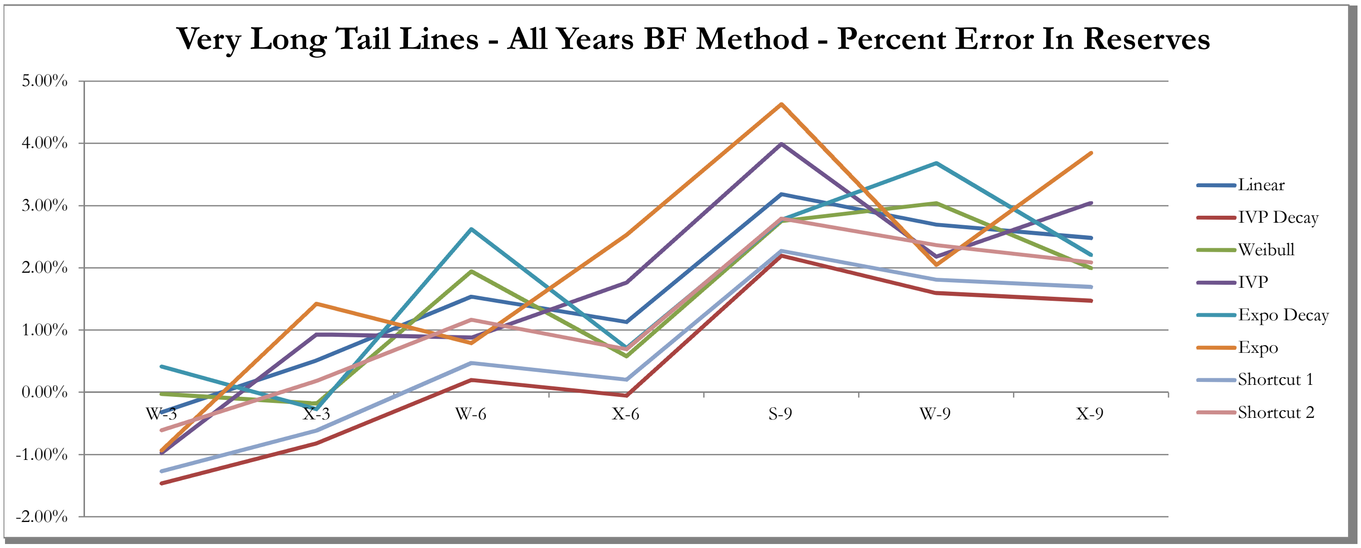


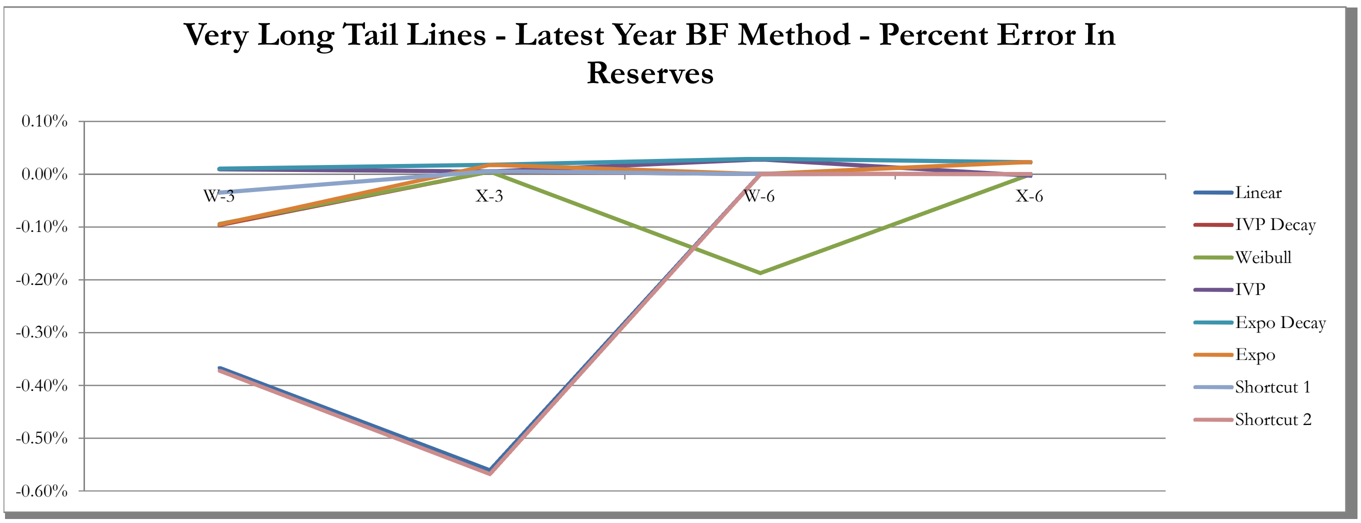


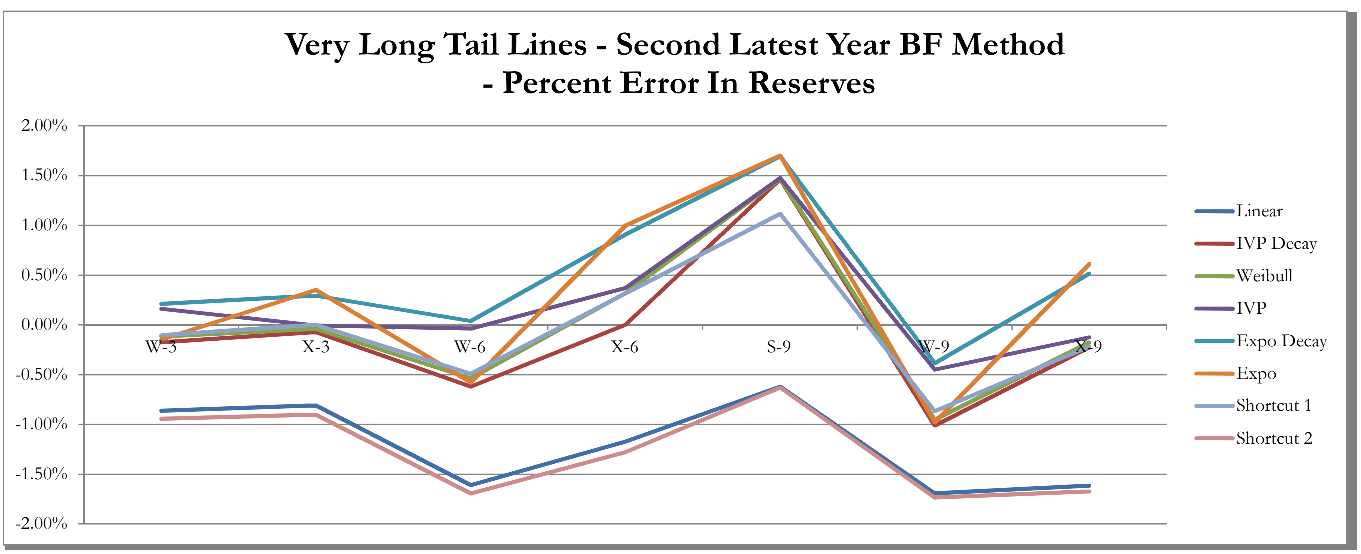


Very Long Tail Lines BF Error

Appendix A
Sheet 19

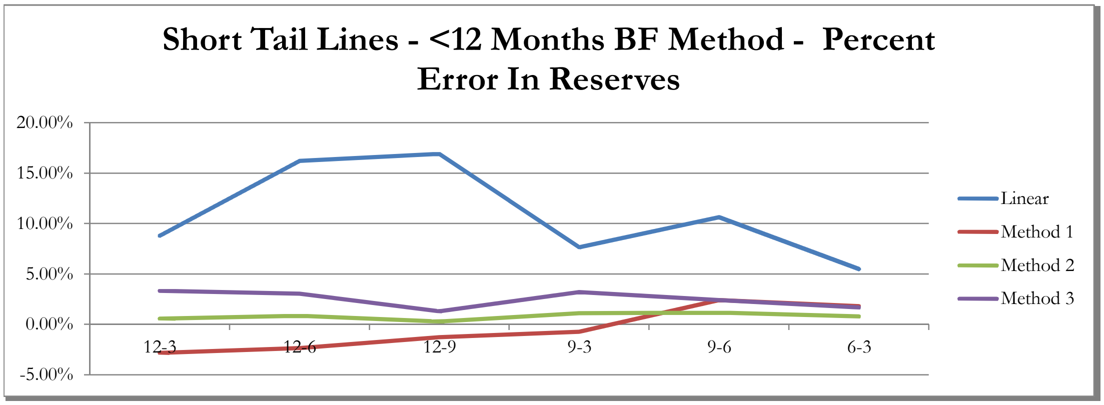


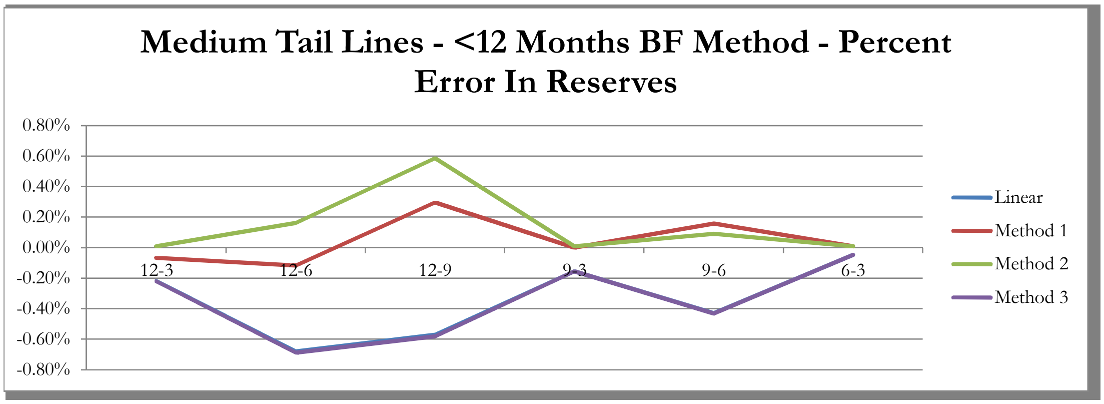


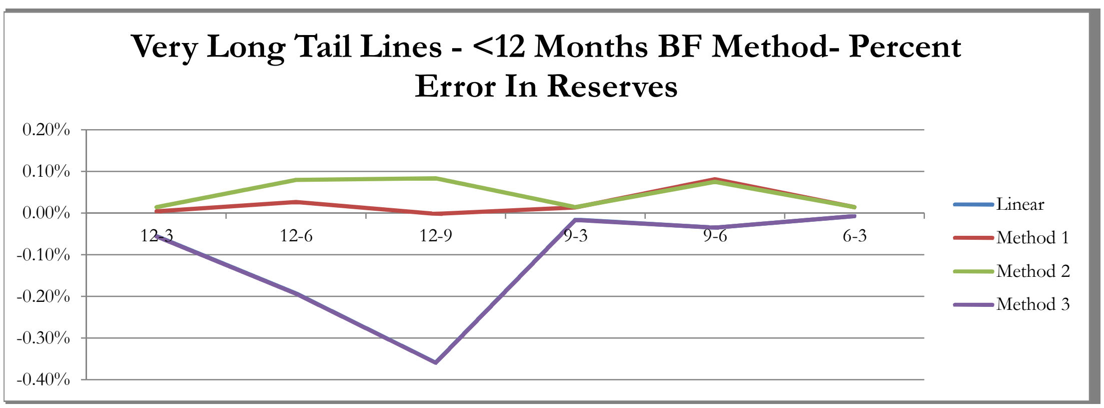


Paid BF Extrapolation Errors

Appendix A
Sheet 20







Appendix B
Sheet 1

			3Q 2014			4Q 2014			1Q 2015			2Q 2015		
Accident Year	Maturity in Months	Paid CDF 2nd Quarter	Maturity	Interpolated Paid CDF	Incremental Percent Paid	Maturity	Interpolated Paid CDF	Incremental Percent Paid	Maturity	Interpolated Paid CDF	Incremental Percent Paid	Maturity	Interpolated Paid CDF	Incremental Percent Paid
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
					1/(4) - 1/(2)			1/(7) - 1/(4)			1/(10) - 1/(7)			1/(13) - 1/(10)
2004	126	1.001	129	1.000	0.001	132	1.000	-	135	1.000	-	138	1.000	-
2005	114	1.003	117	1.002	0.001	120	1.002	0.001	123	1.001	0.000	126	1.001	0.000
2006	102	1.008	105	1.006	0.002	108	1.005	0.001	111	1.004	0.001	114	1.003	0.001
2007	90	1.018	93	1.015	0.003	96	1.012	0.003	99	1.010	0.002	102	1.008	0.002
2008	78	1.038	81	1.032	0.006	84	1.026	0.005	87	1.022	0.004	90	1.018	0.004
2009	66	1.064	69	1.057	0.007	72	1.050	0.006	75	1.044	0.005	78	1.038	0.005
2010	54	1.108	57	1.095	0.011	60	1.083	0.010	63	1.073	0.009	66	1.064	0.008
2011	42	1.191	45	1.165	0.019	48	1.143	0.017	51	1.124	0.015	54	1.108	0.013
2012	30	1.334	33	1.290	0.026	36	1.252	0.024	39	1.219	0.021	42	1.191	0.019
2013	18	2.068	21	1.780	0.078	24	1.580	0.071	27	1.438	0.063	30	1.334	0.054
2014	6	2.843	9	2.596	0.033	12	2.390	0.033	15	2.216	0.033	18	2.068	0.032
2014 Full Year	6	5.686	9	3.462	0.113		2.390	0.130		2.216	0.033		2.068	0.032

Appendix B
Sheet 2

(a) 4th quarter increase factor **50%**

	Percent of Year Paid in					Restated Percent				Restated Pattern			
Accident Year	3Q2014	4Q 2014	1Q 2015	2Q 2015	Total	3Q2014	4Q 2014	1Q 2015	2Q 2015	3Q2014	4Q 2014	1Q 2015	2Q 2015
	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)
2004													
2005	36.0%	27.4%	20.8%	15.8%	100.0%	29.2%	41.1%	16.9%	12.8%	1.002	1.002	1.001	1.001
2006	34.7%	27.2%	21.3%	16.7%	100.0%	28.2%	40.8%	17.3%	13.6%	1.007	1.005	1.004	1.003
2007	32.9%	27.0%	22.1%	18.1%	100.0%	26.8%	40.4%	18.0%	14.7%	1.015	1.011	1.009	1.008
2008	32.1%	26.8%	22.4%	18.7%	100.0%	26.2%	40.2%	18.3%	15.2%	1.033	1.025	1.021	1.018
2009	29.6%	26.3%	23.3%	20.7%	100.0%	24.3%	39.5%	19.2%	17.0%	1.058	1.048	1.043	1.038
2010	29.4%	26.3%	23.4%	20.9%	100.0%	24.2%	39.4%	19.3%	17.2%	1.097	1.080	1.072	1.064
2011	29.4%	26.3%	23.4%	20.8%	100.0%	24.2%	39.4%	19.2%	17.1%	1.170	1.137	1.121	1.108
2012	28.7%	26.1%	23.7%	21.5%	100.0%	23.6%	39.2%	19.5%	17.7%	1.297	1.241	1.214	1.191
2013	29.4%	26.7%	23.6%	20.3%	100.0%	24.1%	40.0%	19.3%	16.6%	1.826	1.529	1.418	1.334
2014	25.3%	25.2%	24.9%	24.5%	100.0%	21.1%	37.9%	20.7%	20.4%	2.635	2.329	2.189	2.068
2014 Full Year										3.513	2.329	2.189	2.068

Notes:

(15) (5)/ [(5)+(8)+(11)+(14)]
(16) (8)/ [(5)+(8)+(11)+(14)]
(17) (11)/ [(5)+(8)+(11)+(14)]
(18) (14)/ [(5)+(8)+(11)+(14)]
(20) (15)/ [(15)+(17)+(18)]*[1-(21)]
(21) (16)/ [1 + (a)]
(22) (17)/ [(15)+(17)+(18)]*[1-(21)]
(23) (17)/ [(15)+(17)+(18)]*[1-(21)]

(24) 1/[1/(2)+(20)*{1/Prior(2)-1/(2}]
(25) 1/[1/(2)+sum(20:21)*{1/Prior(2)-1/(2}]
(26) 1/[1/(2)+sum(20:22)*{1/Prior(2)-1/(2}]
(27) 1/[1/(2)+sum(20:23)*{1/Prior(2)-1/(2}]