

# Using Life Expectancy to Inform the Estimate of Tail Factors for Workers Compensation Liabilities

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## Abstract

Traditional accident year paid and/or reported loss development methods are often used to estimate liabilities for workers compensation claims by selecting age-to-age loss development factors which are then fitted to a curve.

This short paper shares a practical reserving technique that can inform traditional loss development methods to more accurately estimate the liabilities associated with a body of claims that has claimant mortality as the main driver of the length of the tail.

**Keywords:** Workers Compensation, Reserving, Reserving Methods.

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## 1. INTRODUCTION

In his paper “Overcoming Claims Inadequacies: A Mortality-Based Approach to Reserving for Old Workers’ Compensation Claims,” Brian Jones [1] provides a survey of how mortality can be used in workers compensation reserving. He specifically mentions its use in tail factor estimation before exploring the means to build a ground-up, mortality-based claims model. Our paper illustrates a practical technique for the aforementioned use of mortality in tail factor estimation.

### 1.1 Research Context

Richard Sherman and Gordon Diss [2] note in their award-winning paper, “Estimating the Workers’ Compensation Tail,” that the workers compensation tail largely consists of the medical component of permanent disability claims. Their paper then presents a fairly complex method for utilizing incremental payment data prior to the standard triangle to extend development factors beyond the end of the triangle. Frank Schmid [3] further analyzed aggregate workers compensation loss triangles to explain the drivers of tail development in another technical contribution to the literature, “The Workers Compensation Tails.”

In practice, however, we’ve found the above-referenced works difficult to put into use. These approaches require data that’s often unavailable and assumptions that can result in a highly-parameterized model that may not lend itself to easy explanation.

## **1.2 Objective**

We'd like to share a practical reserving technique that we've implemented and used to help more accurately reserve run-off books of workers compensation claims but that can be applied to any body of claims that has claimant mortality as the main driver of the length of the tail (e.g., unlimited PIP).

We believe it's a relatively simple and readily understandable extension of the traditional loss development techniques that many reserve practitioners use and can be scaled up or down in complexity based on the quality and availability of the underlying data.

Our method starts with traditional accident year paid and/or reported loss development triangles. Age-to-age factors are selected as far as the data reasonably allows. These selected factors are then fitted to a curve. This, we believe, is where many reserve practitioners stop or experience difficulty. Since most fitting techniques will allow development to go on indefinitely, the length of the tail is often selected based simply on actuarial judgment. Our method provides an actuary with a way to inform the length of the tail based on the underlying claim data.

The technique we use to inform the length of the tail begins with the determination and review of claimant life expectancy percentiles for all open claims. In its simplest application all one needs is an accident date and a date of birth for each claimant. We then try to answer the question, "If claimants in a given accident year or cohort group of accident years survive to some percentile of life expectancy, how do we expect to see their related losses develop to that point in time?"

## **1.3 Outline**

The remainder of the paper proceeds as follows:

Section 2 will discuss data considerations.

Section 3 will present loss development and curve fitting.

Section 4 will examine mortality and life expectancy.

Section 5 will describe the adjustment of tail factors for life expectancy.

## **2. DATA CONSIDERATIONS**

Ideally one would want to use this method for the medical component of paid losses only, as the indemnity component may be more heavily influenced by factors other than claimant mortality, such as statutory requirements or the payment of survivor benefits. However, we believe the approach has predictive power for almost any aggregation of workers compensation losses because claimant mortality is the main driver of the length of the tail regardless of the mix of indemnity versus medical or loss versus expense components, though a split of first-dollar exposures versus homogeneous groupings of excess coverages is important.

Similarly one would ideally want to match mortality tables as closely as possible to the characteristics of the underlying claimant population. In our experience though there is a declining return from increased precision unless the volume of underlying data is sufficiently credible.

## **3. LOSS DEVELOPMENT AND CURVE FITTING**

Age-to-age loss development factors are selected as one would normally for traditional loss development methods. These selected factors are then fitted to a closed-form inverse power curve as described in Richard Sherman's "Extrapolating, Smoothing, and Interpolating Development Factors" [4].

In our example we evaluate a hypothetical book of run-off workers compensation business. Given a triangle of cumulative paid loss and expense (combined) we select age-to-age factors, including the judgmental selection of a tail factor, yielding cumulative age-to-ultimate factors. The results of this analysis are as follows:

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**Workers Compensation as of 12/31/2012** **(\$000s)**

| Accident<br>Year | ITD<br>Paid | Selected Development Factors |            |
|------------------|-------------|------------------------------|------------|
|                  |             | Age-to-Age                   | Cumulative |
| 1993             | 62,574      | 1.034                        | 1.034      |
| 1994             | 92,671      | 1.002                        | 1.036      |
| 1995             | 103,027     | 1.003                        | 1.039      |
| 1996             | 119,457     | 1.003                        | 1.043      |
| 1997             | 169,521     | 1.003                        | 1.046      |
| 1998             | 165,049     | 1.003                        | 1.049      |
| 1999             | 206,325     | 1.004                        | 1.053      |
| 2000             | 260,194     | 1.005                        | 1.058      |
| 2001             | 279,992     | 1.005                        | 1.063      |
| 2002             | 312,353     | 1.006                        | 1.070      |
| 2003             | 362,792     | 1.007                        | 1.078      |
| 2004             | 375,976     | 1.009                        | 1.088      |
| 2005             | 294,499     | 1.013                        | 1.102      |
| 2006             | 237,595     | 1.022                        | 1.127      |
| 2007             | 168,798     | 1.031                        | 1.162      |
| 2008             | 135,238     | 1.051                        | 1.222      |
| 2009             | 125,394     | 1.089                        | 1.330      |
| 2010             | 94,536      | 1.174                        | 1.562      |
| 2011             | 67,674      | 1.378                        | 2.151      |
| 2012             | 16,920      | 2.340                        | 5.034      |

We then fit these selected age-to-age development factors, excluding the tail factor, to the inverse power curve.

$$\text{Inverse Power Curve} = f(t) = 1 + a \cdot t^{-b} \tag{3.1}$$

where  $t$  = the age,  $b$  = the slope,  $\ln(a)$  = the intercept, and  $\ln(f(t)-1) = \ln(a) + b \cdot \ln(1/t)$

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**Fit of Selected Development Factors to Inverse Power Curve**

| Accident Year | Age ( $t$ ) | Selected LDF | $x = \ln(1/t)$ | $y = \ln(\text{LDF}-1)$ |
|---------------|-------------|--------------|----------------|-------------------------|
| 2012          | 1           | 2.340        | 0.000          | 0.293                   |
| 2011          | 2           | 1.378        | (0.693)        | (0.974)                 |
| 2010          | 3           | 1.174        | (1.099)        | (1.748)                 |
| 2009          | 4           | 1.089        | (1.386)        | (2.420)                 |
| 2008          | 5           | 1.051        | (1.609)        | (2.978)                 |
| 2007          | 6           | 1.031        | (1.792)        | (3.466)                 |
| 2006          | 7           | 1.022        | (1.946)        | (3.799)                 |
| 2005          | 8           | 1.013        | (2.079)        | (4.321)                 |
| 2004          | 9           | 1.009        | (2.197)        | (4.681)                 |
| 2003          | 10          | 1.007        | (2.303)        | (4.893)                 |
| 2002          | 11          | 1.006        | (2.398)        | (5.078)                 |
| 2001          | 12          | 1.005        | (2.485)        | (5.290)                 |
| 2000          | 13          | 1.005        | (2.565)        | (5.358)                 |
| 1999          | 14          | 1.004        | (2.639)        | (5.603)                 |
| 1998          | 15          | 1.003        | (2.708)        | (5.783)                 |
| 1997          | 16          | 1.003        | (2.773)        | (5.702)                 |
| 1996          | 17          | 1.003        | (2.833)        | (5.659)                 |
| 1995          | 18          | 1.003        | (2.890)        | (5.963)                 |
| 1994          | 19          | 1.002        | (2.944)        | (6.107)                 |

The Excel functions for SLOPE and INTERCEPT are then populated with the array of  $x$  and  $y$  values from the above table yielding  $b = 2.28223156047852$  and  $\ln(a) = 0.539573651269289$  as the inputs into the inverse power curve function. The fitted values are then calculated as shown below:

**Fitted Development Factors**

| Accident<br>Year | Development Factors |        |            |
|------------------|---------------------|--------|------------|
|                  | Age-to-Age          | Fitted | Cumulative |
| 1994             | 1.002               | 1.002  | 1.028      |
| 1995             | 1.003               | 1.002  | 1.030      |
| 1996             | 1.003               | 1.003  | 1.033      |
| 1997             | 1.003               | 1.003  | 1.036      |
| 1998             | 1.003               | 1.004  | 1.040      |
| 1999             | 1.004               | 1.004  | 1.044      |
| 2000             | 1.005               | 1.005  | 1.049      |
| 2001             | 1.005               | 1.006  | 1.056      |
| 2002             | 1.006               | 1.007  | 1.063      |
| 2003             | 1.007               | 1.009  | 1.073      |
| 2004             | 1.009               | 1.011  | 1.085      |
| 2005             | 1.013               | 1.015  | 1.101      |
| 2006             | 1.022               | 1.020  | 1.123      |
| 2007             | 1.031               | 1.029  | 1.156      |
| 2008             | 1.051               | 1.044  | 1.206      |
| 2009             | 1.089               | 1.072  | 1.293      |
| 2010             | 1.174               | 1.140  | 1.474      |
| 2011             | 1.378               | 1.353  | 1.994      |
| 2012             | 2.340               | 2.715  | 5.414      |

However, since this fit generates loss development factors indefinitely out into time, using the calculated cumulative loss development factors directly would likely overstate development in the tail. Due to this we review projected life expectancies for all open claimants in the underlying data and then use this information to adjust the length of the tail.

#### **4. MORTALITY AND LIFE EXPECTANCY**

As discussed by Elizabeth Arias in “United States Life Tables,” 2004 [5] there are two types of mortality tables, the cohort life table and the period life table. A cohort life table presents the mortality experience of all persons born in a particular year. A period life table, which is what we use here, presents what would happen to a hypothetical cohort if it experienced throughout its entire life the mortality conditions of a particular period in time.

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There are many opinions regarding which life table would be the most appropriate to use in a loss reserving context, but this paper does not attempt to answer that question. The choice of the appropriate life table to use is left to the practitioner to determine.

For this example we use the CDC's 2004 U.S. period life table for males to determine our life expectancies. We then calculate "life expectancy percentiles" for each age at various intervals from 60% to 90% which represent the percentage of lives that have left the population from a given point in time up to another point in the future. The life expectancy for a given percentile is the number of years until the remaining lives drops by the given percentage. The determination of the appropriate percentiles to calculate requires judgment and may depend on the number of underlying claimants in the data. However, the selections should also consider how much the population will need to shrink before future development is no longer likely to occur.

In addition, in certain instances where it has a material impact, we have weighted the statistics from male and female life tables together based on the gender distribution of the claimants in a given set of data. For example, if we wanted a 75% male | 25% female mix, we'd calculate the number of lives,  $L(x)$ , as a weighted average =  $0.75 L_m(x) + 0.25 L_f(x)$  and then determine life expectancy percentiles using this weighted  $L(x)$ .

To find the  $p$ -percentile of mortality for age  $(x)$ , we find the first age  $(a)$  at which: (4.1)

$$L(a) \leq (1-p) * L(x)$$

Where  $L(x)$  = lives remaining at age  $x$ , and  
the life expectancy at that percentile is then  $(a-x)$ .

For example, to find the 75th percentile of life expectancy for a 40-year old male ( $x = 40$ ,  $p = 0.75$ ) we start by going to the period life table and determining that  $L(40) = 95,527$ . We then calculate that  $(1 - 0.75) * 95,527 = 23,882$ . Another review of the table shows that  $L(87) = 24,413$  and  $L(88) = 21,447$ . Therefore,  $a = 88$  is selected and the related life expectancy is  $88 - 40 = 48$  years.

Returning to our hypothetical book of run-off workers compensation business, we then determine the life expectancy for claimants by accident year at each year-end from 1993 to 2012 by starting with a table (partially displayed below) of calculated life expectancies at various percentiles.

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**Calculated Male Life Expectancies by Age at Various Percentiles**

| Age | Number Surviving to Age $x$ | Expectation of Life at Age $x$ | Life Expectancy Percentiles |     |     |
|-----|-----------------------------|--------------------------------|-----------------------------|-----|-----|
|     | $l(x)$                      | $e(x)$                         | 60%                         | 75% | 90% |
| 40  | 95,527                      | 37.6                           | 43                          | 48  | 53  |
| 41  | 95,294                      | 36.7                           | 42                          | 47  | 52  |
| 42  | 95,043                      | 35.8                           | 41                          | 46  | 51  |
| 43  | 94,772                      | 34.9                           | 40                          | 45  | 50  |
| 44  | 94,477                      | 34.0                           | 39                          | 44  | 49  |
| 45  | 94,154                      | 33.1                           | 38                          | 43  | 48  |
| 46  | 93,803                      | 32.3                           | 37                          | 42  | 47  |
| 47  | 93,421                      | 31.4                           | 36                          | 41  | 46  |
| 48  | 93,007                      | 30.5                           | 35                          | 40  | 45  |
| 49  | 92,560                      | 29.7                           | 34                          | 39  | 44  |
| 50  | 92,078                      | 28.8                           | 34                          | 38  | 43  |
| 51  | 91,558                      | 28.0                           | 33                          | 37  | 43  |
| 52  | 90,998                      | 27.2                           | 32                          | 36  | 42  |
| 53  | 90,398                      | 26.3                           | 31                          | 35  | 41  |
| 54  | 89,761                      | 25.5                           | 30                          | 34  | 40  |
| 55  | 89,089                      | 24.7                           | 29                          | 33  | 39  |
| 56  | 88,381                      | 23.9                           | 28                          | 32  | 38  |
| 57  | 87,633                      | 23.1                           | 27                          | 31  | 37  |
| 58  | 86,839                      | 22.3                           | 26                          | 30  | 36  |
| 59  | 85,987                      | 21.5                           | 25                          | 29  | 35  |
| 60  | 85,067                      | 20.8                           | 24                          | 29  | 34  |

Life expectancy percentiles for an accident year or cohort of accident years is subsequently calculated based on some weighting (e.g., the past three years of paid losses and/or open case reserves) of individual claimants:

**Selected 75th Percentile Life Expectancy by Accident Year Cohort**

| Accident Years | Open Claims | 3-Year Avg Paid | Average Case Reserve | Paid Weighted LE | Case Weighted LE | Selected LE |
|----------------|-------------|-----------------|----------------------|------------------|------------------|-------------|
| 1993-1997      | 62          | 39,074          | 121,657              | 24.2             | 25.8             | 25.0        |
| 1998-2002      | 164         | 23,831          | 93,113               | 29.4             | 27.6             | 28.0        |
| 2003-2007      | 334         | 27,552          | 125,519              | 32.4             | 33.1             | 33.0        |
| 2008-2012      | 564         | 34,162          | 165,989              | 35.0             | 36.2             | 36.0        |



## **5. ADJUSTMENT OF TAIL FACTORS FOR LIFE EXPECTANCY**

Now that we have the projected life expectancy for the claimant population the tail factor of the fitted age-to-ultimate development factors can be adjusted.

This is done by dividing the cumulative development factor (CDF) at the accident year's current age as of the evaluation period by the CDF at the accident year's current age plus the selected life expectancy percentile, or the accident year's selected terminal age. As mentioned previously, because each accident-year cohort of claims is made up of claimants with different ages, the weighted life expectancy of the cohort is used for the selected percentile.

For example: Assume the selected remaining life expectancy at the 75<sup>th</sup> percentile for Accident Year 2000 was determined to be 28 years. In addition, at the time of the analysis, Accident Year 2000 was 13 years old. In this example the, fitted cumulative loss development factor at time 13 is 1.049 and, based on the remaining life expectancy of 28 years, development is expected to end at time 41. Moving along the fitted values, the CDF at time 41 is 1.008, so the age-to-ultimate factor informed by the underlying life expectancy assumption is  $1.049/1.008 = 1.042$ .

The results for all years in our example are displayed in the following table:

**Fitted Paid Age-to-Ultimate Development Factors Adjusted for Life Expectancy**

| Accident Year | Evaluation Age | Selected 75th Life Expectancy Percentile | CDF at Accident Year's Current Age | CDF at Accident Year's Terminal Age | CDF Adjusted for Life Expectancy |
|---------------|----------------|--|------------------------------------|-------------------------------------|----------------------------------|
| 1993          | 20             | 25                                       | 1.026                              | 1.006                               | 1.020                            |
| 1994          | 19             | 25                                       | 1.028                              | 1.006                               | 1.021                            |
| 1995          | 18             | 25                                       | 1.030                              | 1.007                               | 1.023                            |
| 1996          | 17             | 25                                       | 1.033                              | 1.007                               | 1.026                            |
| 1997          | 16             | 25                                       | 1.036                              | 1.008                               | 1.029                            |
| 1998          | 15             | 28                                       | 1.040                              | 1.007                               | 1.033                            |
| 1999          | 14             | 28                                       | 1.044                              | 1.007                               | 1.037                            |
| 2000          | 13             | 28                                       | 1.049                              | 1.008                               | 1.042                            |
| 2001          | 12             | 28                                       | 1.056                              | 1.008                               | 1.047                            |
| 2002          | 11             | 28                                       | 1.063                              | 1.008                               | 1.054                            |
| 2003          | 10             | 33                                       | 1.073                              | 1.007                               | 1.065                            |
| 2004          | 9              | 33                                       | 1.085                              | 1.007                               | 1.077                            |
| 2005          | 8              | 33                                       | 1.101                              | 1.008                               | 1.093                            |
| 2006          | 7              | 33                                       | 1.123                              | 1.008                               | 1.115                            |
| 2007          | 6              | 33                                       | 1.156                              | 1.008                               | 1.146                            |
| 2008          | 5              | 36                                       | 1.206                              | 1.008                               | 1.197                            |
| 2009          | 4              | 36                                       | 1.293                              | 1.008                               | 1.283                            |
| 2010          | 3              | 36                                       | 1.474                              | 1.008                               | 1.462                            |
| 2011          | 2              | 36                                       | 1.994                              | 1.009                               | 1.977                            |
| 2012          | 1              | 36                                       | 5.414                              | 1.009                               | 5.365                            |

## 6. RESULTS AND DISCUSSION

We typically use the calculated life expectancies directly to modify the length of the fitted tail for paid development as described in this paper. However as we generally expect reported development to end sooner than paid development, our calculated life expectancies are often judgmentally adjusted (e.g., shortened by 10 years) to reflect, on average, how long before final payment accurate case reserves are expected to be recorded for an accident year or cohort of accident years. This judgment can be informed either through discussion with the claims adjusting staff or based on a hindsight review of case reserve development for closed claims.

## 7. CONCLUSIONS

The practicing actuary often relies upon judgment when selecting the length of the tail to be used in estimating liabilities for workers compensation claims with traditional accident year paid and/or reported loss development methods.

This paper shares a practical reserving technique that can inform traditional loss development methods as to the length of the tail using claimant mortality. In the example presented above, and in more detail in the accompanying tool in Excel, the impact of the tail assumption is material and materially different when the life expectancy of the underlying claimant population is considered:

**Comparison of Selected Gross Paid Loss and Expense Reserves (\$000s)**

| Accident Year | ITD Paid  | Selected CDFs |        |                  | Selected Total Reserves |         |                  |
|---------------|-----------|---------------|--------|------------------|-------------------------|---------|------------------|
|               |           | Traditional   | Fitted | Fitted w/ LE Adj | Traditional             | Fitted  | Fitted w/ LE Adj |
| 1993          | 62,574    | 1.034         | 1.026  | 1.020            | 2,123                   | 1,616   | 1,251            |
| 1994          | 92,671    | 1.036         | 1.028  | 1.021            | 3,357                   | 2,590   | 1,946            |
| 1995          | 103,027   | 1.039         | 1.030  | 1.023            | 4,006                   | 3,128   | 2,370            |
| 1996          | 119,457   | 1.043         | 1.033  | 1.026            | 5,078                   | 3,955   | 3,106            |
| 1997          | 169,521   | 1.046         | 1.036  | 1.029            | 7,796                   | 6,149   | 4,916            |
| 1998          | 165,049   | 1.049         | 1.040  | 1.033            | 8,122                   | 6,594   | 5,447            |
| 1999          | 206,325   | 1.053         | 1.044  | 1.037            | 10,952                  | 9,135   | 7,634            |
| 2000          | 260,194   | 1.058         | 1.049  | 1.042            | 15,102                  | 12,857  | 10,928           |
| 2001          | 279,992   | 1.063         | 1.056  | 1.047            | 17,744                  | 15,571  | 13,160           |
| 2002          | 312,353   | 1.070         | 1.063  | 1.054            | 21,865                  | 19,746  | 16,867           |
| 2003          | 362,792   | 1.078         | 1.073  | 1.065            | 28,307                  | 26,389  | 23,581           |
| 2004          | 375,976   | 1.088         | 1.085  | 1.077            | 33,092                  | 31,942  | 28,950           |
| 2005          | 294,499   | 1.102         | 1.101  | 1.093            | 30,180                  | 29,782  | 27,388           |
| 2006          | 237,595   | 1.127         | 1.123  | 1.115            | 30,212                  | 29,315  | 27,323           |
| 2007          | 168,798   | 1.162         | 1.156  | 1.146            | 27,409                  | 26,276  | 24,645           |
| 2008          | 135,238   | 1.222         | 1.206  | 1.197            | 29,961                  | 27,860  | 26,642           |
| 2009          | 125,394   | 1.330         | 1.293  | 1.283            | 41,403                  | 36,795  | 35,487           |
| 2010          | 94,536    | 1.562         | 1.474  | 1.462            | 53,103                  | 44,831  | 43,676           |
| 2011          | 67,674    | 2.151         | 1.994  | 1.977            | 77,913                  | 67,273  | 66,117           |
| 2012          | 16,920    | 5.034         | 5.414  | 5.365            | 68,253                  | 74,693  | 73,856           |
| Total         | 3,650,585 |               |        |                  | 515,978                 | 476,496 | 445,290          |

We hope the technique described in this paper proves useful to the traditional actuarial reserving practitioner and provides the foundation for further work in this area.

## 8. REFERENCES

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### Abbreviations and Notations

CDC, Centers for Disease Control and Prevention  
CDF, Cumulative Development Factor (Age-to-Ultimate)  
ITD, Inception to Date

LDF, Loss Development Factor (Age-to-Age)  
LE, Life Expectancy  
PIP, Personal Injury Protection

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