A Total Credibility Approach to Pool Reserving

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Abstract
Motivation. Among other services in the assigned risk market, NCCI provides actuarial services for the National Workers Compensation Reinsurance Pooling Mechanism (NWCRP), the Massachusetts Workers' Compensation Assigned Risk Pool, the Michigan Workers' Compensation Placement Facility, and the New Mexico Workers' Compensation Assigned Risk Pool. Pool reserving triangles pose specific challenges as they may be sparsely populated; this is because states may have left the NWCRP, recently joined it, or re-joined it after several years of absence. Furthermore, triangles of partial coverage states may have unpopulated cells due to not having experienced a claim in a given policy year.
Method. There are two credibility aspects to Pool reserving to be addressed. First, the degree of variability of the link ratios may differ across states, possibly (but not necessarily entirely) due to differences in size of the assigned risk market across states. Second, the number of link ratios available varies greatly across states, from fully populated diagonals to very few observations per diagonal or, for some partial coverage states, to no observations at all. To address these challenges, a comprehensive credibility approach has been developed, where the credibility-adjustment applies to the data-generating process as opposed to the outcome. This new concept, called Total Credibility, rests on multilevel (hierarchical) modeling, which implies that the Pool triangles of all states are estimated simultaneously.
Results. The model is applied to the logarithmic paid plus case link ratios of the latest five diagonals of the Pool triangles of 45 jurisdictions, some of which are partial coverage states. Diagnostic charts of in-sample fit show that the model is well suited for replicating the observed data. Further, diagnostic charts of forecast errors indicate that the structure of the model is a proper representation of the data-generating process.
Availability. The model was implemented in R (http://cran.r-project.org/) using the sampling platform JAGS (Just Another Gibbs Sampler, http://www-ice.iarc.fr/~martyn/software/jags/). JAGS was linked to R by means of the R package rjags (http://cran.r-project.org/web/packages/rjags/index.html).

Keywords. Pool Reserving, Growth Curve, Total Credibility

1. INTRODUCTION

Among other services in the assigned risk market, NCCI provides actuarial services for the National Workers Compensation Reinsurance Pooling Mechanism (NWCRP), the Massachusetts Workers' Compensation Assigned Risk Pool, the Michigan Workers' Compensation Placement Facility, and the New Mexico Workers' Compensation Assigned Risk Pool. Pool reserving triangles pose specific challenges as they may be sparsely populated; this is because states may have left the NWCRP, recently joined it, or re-joined it after several years of absence. Furthermore, triangles of partial coverage states may have unpopulated cells due to not having experienced a claim in a given policy year.

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1.1 Research Context

Gelman and Hill [3] offer a textbook introduction to multilevel (hierarchical) modeling. In multilevel modeling, credibility is implemented by means of partial pooling (or, synonymously, shrinkage). The concept of partial pooling is akin to (and, in specific instances equivalent to) Bühlmann credibility.

Let α be one of the parameters that govern the data-generating process. In partial pooling, the parameter α is allowed to vary across the units of observations; in NCCI Pool reserving, these units are U.S. states. With partial pooling, the state-specific α's are draws from the same, common distribution—the parameters that define this common distribution are called hyperparameters. Shrinkage is an adjustment toward the expected value of the α's (that is, the expected value of the common distribution).

In the normal linear model, partial pooling is equivalent to Bühlmann credibility. Following Gelman and Hill [3], let y be a normally distributed variable:

$$y_i \sim \mathrm{N}(\alpha_{j[i]}, \sigma_y^2) \ , \tag{1}$$

where i indicates the observation and j, for instance, indicates the jurisdiction in which this observation occurred. (In multilevel modeling, it is common to make use of double-indexing.)

Multilevel modeling assumes that the state-level parameter α_j is a draw from a distribution that is common to all states:

$$\alpha_j \sim \mathrm{N}(\mu_\alpha, \sigma_\alpha^2) \ . \tag{2}$$

It can be shown that the multilevel estimator for α_j reads (see Gelman and Hill [3]):

$$\hat{\alpha}_j = \omega_j \cdot \mu_\alpha + (1-\omega_j)\bar{y}_j \ , \ \omega_j = 1 - \frac{\sigma_\alpha^2}{\sigma_\alpha^2 + \frac{\sigma_y^2}{n_j}} \ , \tag{3}$$

where $\bar{y}_j$ is the sample mean for state j based on n_j observations. Clearly, Equation (3) is equivalent to Bühlmann credibility.

Guszcza [4], Zhang, Dukic, and Guszcza [8], and Meyers [6] discuss the use of multilevel modeling in reserving. Guszcza [4] fits growth curves to cumulative losses using frequentist methods with random effects in parameters. Zhang, Dukic, and Guszcza [8] take a Bayesian approach to fitting growth curves to cumulative losses—the authors estimate multiple triangles simultaneously, thus accounting for correlation across loss triangles within an industry. Meyers [6], in reference to Guszcza [4] and Zhang, Dukic, and Guszcza [8], fits an autoregressive process to loss ratios in a Bayesian model—yet, there is no concept of shrinking built into the model.

Neither the models discussed by Guszcza [4] nor the one suggested by Meyers [6] serve our purpose—whereas the former cannot handle missing values, the latter is not multilevel. The approach closest to the Total Credibility model presented below is the Bayesian framework developed by Zhang, Dukic, and Guszcza [8].

1.2 Objective

The objective of the Total Credibility Model (TCM) is to provide credibility-adjusted link ratios for the Pool reserving triangles of all jurisdictions that are serviced by NCCI, either directly or through NWCRP. Specifically, these link ratios are to be derived from the latest *n* (for instance, five) diagonals. Some of these diagonals are sparsely populated to the point of being devoid of any observations. Some states may have left the NWCRP, and some of these states may be partial coverage states (the exposure of which was very limited). Partial coverage states are jurisdictions in which the respective (competitive or monopolistic) state fund offered assigned risk coverage but was, under its charter, precluded from providing required Federal Act coverage; examples of Federal Act coverage are USL&H (United States Longshore and Harbor Workers' Compensation Act) coverage and occupational disease coverage related to the Federal Coal Mine Health and Safety Act. In certain policy years, this federal coverage was provided through the NWCRP.

1.3 Outline

The next section describes the data. Section 3 presents the model; Section 4 discusses the results. Section 5 concludes.

2. THE DATA

The data set consists of (paid plus case and, alternatively, paid) link ratios of the latest five diagonals (2005–2009) of the Pool triangles of 45 jurisdictions. The paid link ratio from time t to time $t+1$ is defined as the ratio of cumulative payments up to (and inclusive of) time $t+1$ to the cumulative payments up to (and inclusive of) time t; for the paid plus case link ratio, it is the cumulative payments plus the applicable case reserves. For research purposes, the policy year data are annual (instead of quarterly), which means that the data are as of the fourth quarter of the calendar year.

For many states, the data are sparse, particularly for the eight partial coverage states: CA, CO, MD, MT, OK, UT, WA, and WY. A total of 18 jurisdictions (or 40 percent) have a complete history of link ratios (AK, AL, AR, CT, DC, DE, GA, IA, IL, KS, MI, NC, NH, NJ, NM, SD, VA, and VT); there are four states for which all available link ratios are unity (between 4 and 13 unity link ratios per diagonal—CO, OK, WA, and WY); there are two states with one observation (CA and WV); and there are two states with no data (MT and UT). Of the 45 analyzed jurisdictions, 20 states (among which are the mentioned partial coverage states) are no longer in the NWCRP.

Charts 1 through 5 display box plots for the empirically observed logarithmic paid plus case link ratios of the set of 45 jurisdictions for the diagonals 2009 through 2005. The box comprises 50 percent of the data—its upper and lower hinges indicate the interquartile range (IQR). The horizontal bar inside the box represents the median. The whiskers at the end of the stems indicate the smallest (bottom) and largest (top) observed value that is within 1.5 IQRs from the box limits. Observations beyond the whiskers are plotted as dots and constitute outliers as judged by the normal distribution.

The boxplots shown in Charts 1 through 5 indicate that the (logarithmic) paid plus case link ratio distributions are heavy-tailed, yet not skewed. This is in contrast to the (logarithmic) paid link ratios, which are displayed in Charts 6 through 10. The (logarithmic) paid link ratios are highly skewed to the right, which implies that, despite the logarithmic transformation, there are more outliers on the upside than on the downside.

Chart 1: Logarithmic Paid Plus Case Link Ratio Boxplots, 45 Jurisdictions, 2009 Diagonal

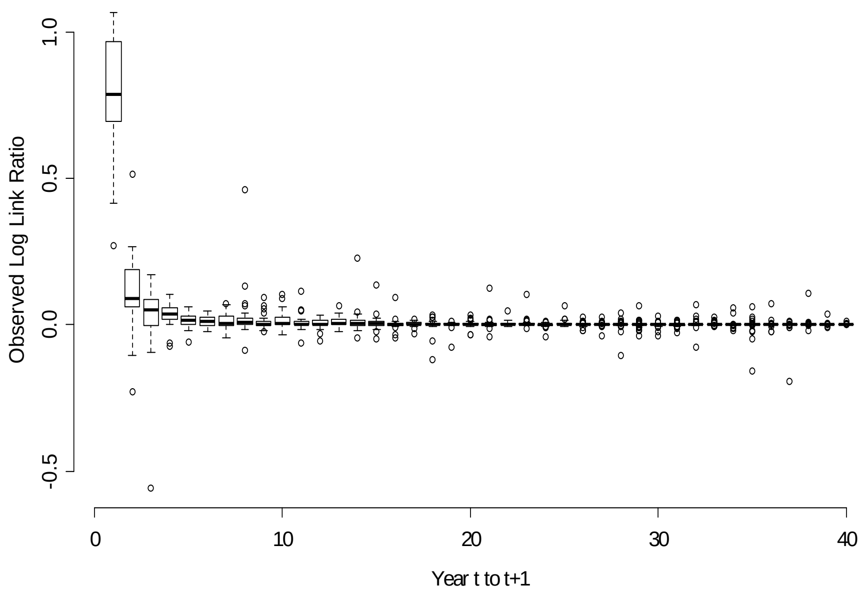


Chart 2: Logarithmic Paid Plus Case Link Ratio Boxplots, 45 Jurisdictions, 2008 Diagonal

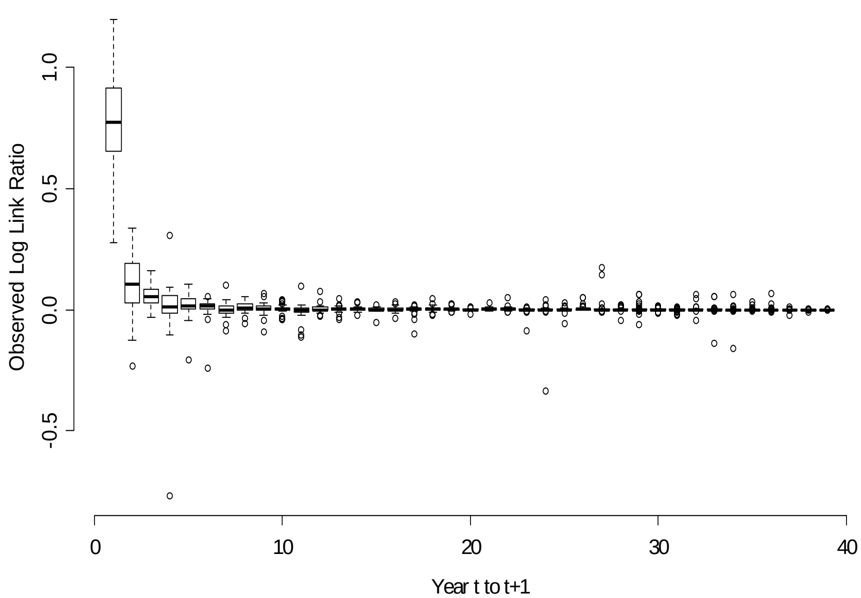


Chart 3: Logarithmic Paid Plus Case Link Ratio Boxplots, 45 Jurisdictions, 2007 Diagonal

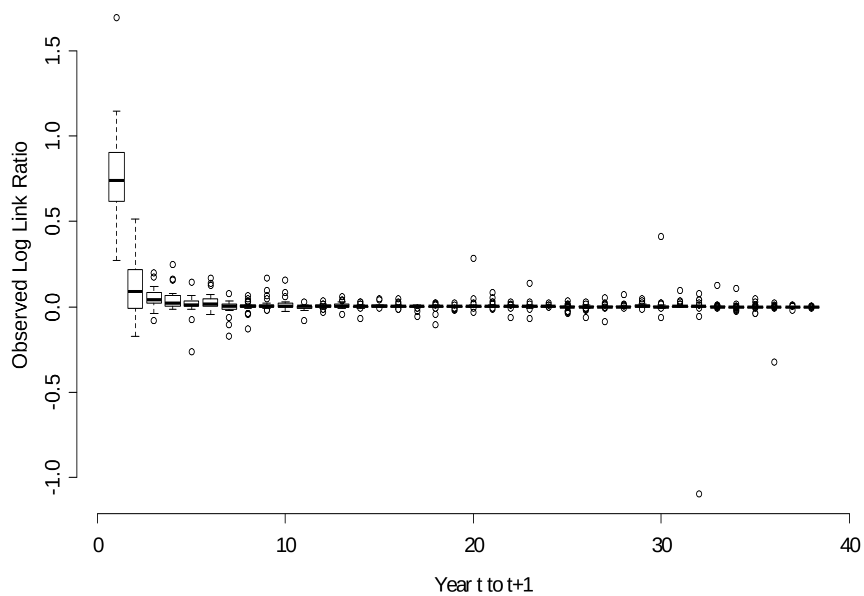


Chart 4: Logarithmic Paid Plus Case Link Ratio Boxplots, 45 Jurisdictions, 2006 Diagonal

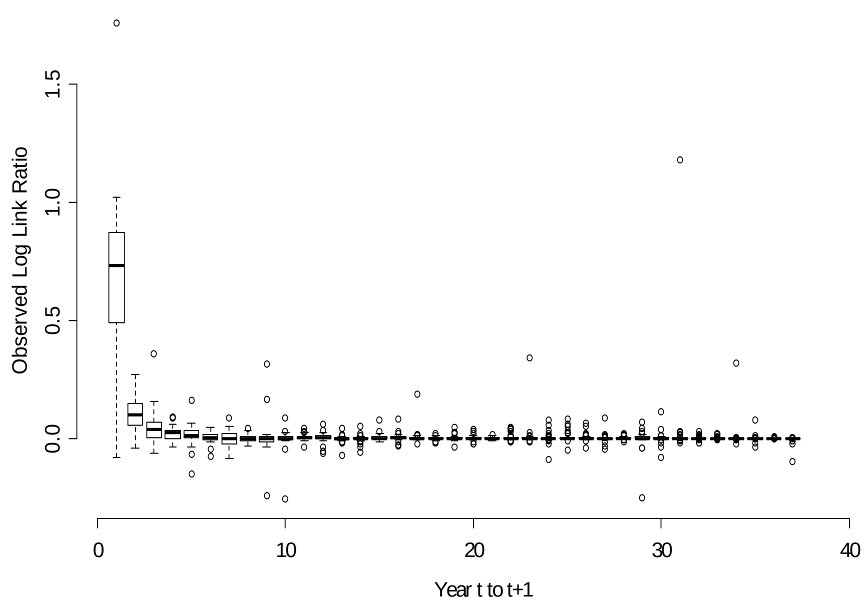


Chart 5: Logarithmic Paid Plus Case Link Ratio Boxplots, 45 Jurisdictions, 2005 Diagonal

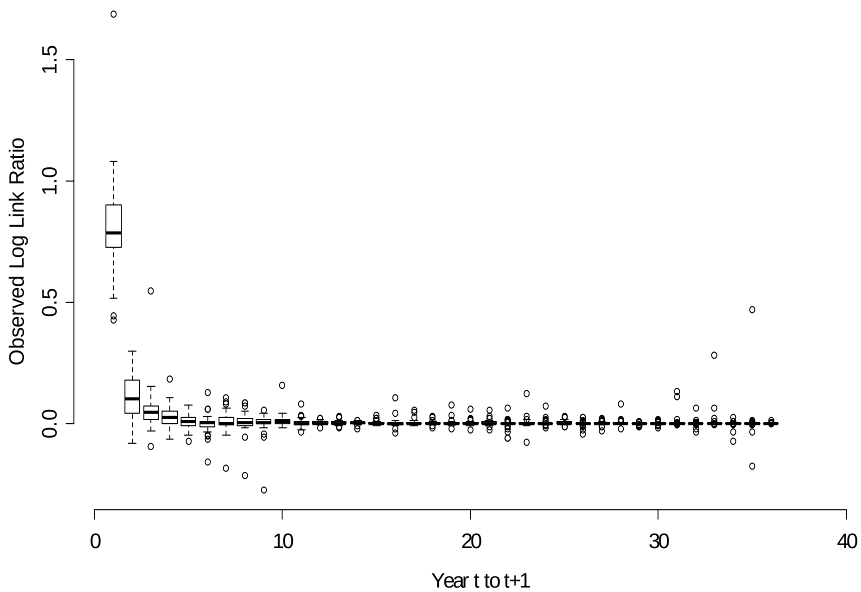


Chart 6: Logarithmic Paid Link Ratio Boxplots, 45 Jurisdictions, 2009 Diagonal

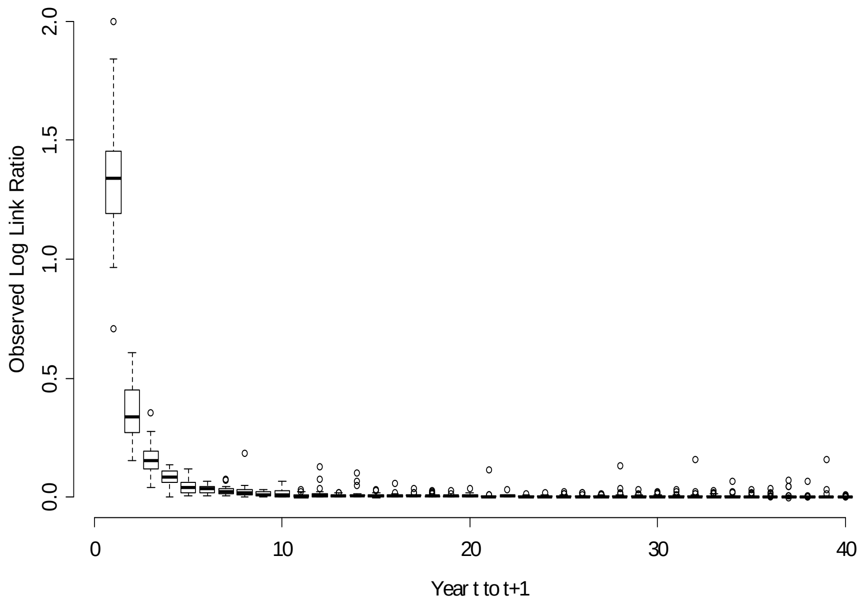


Chart 7: Logarithmic Paid Link Ratio Boxplots, 45 Jurisdictions, 2008 Diagonal

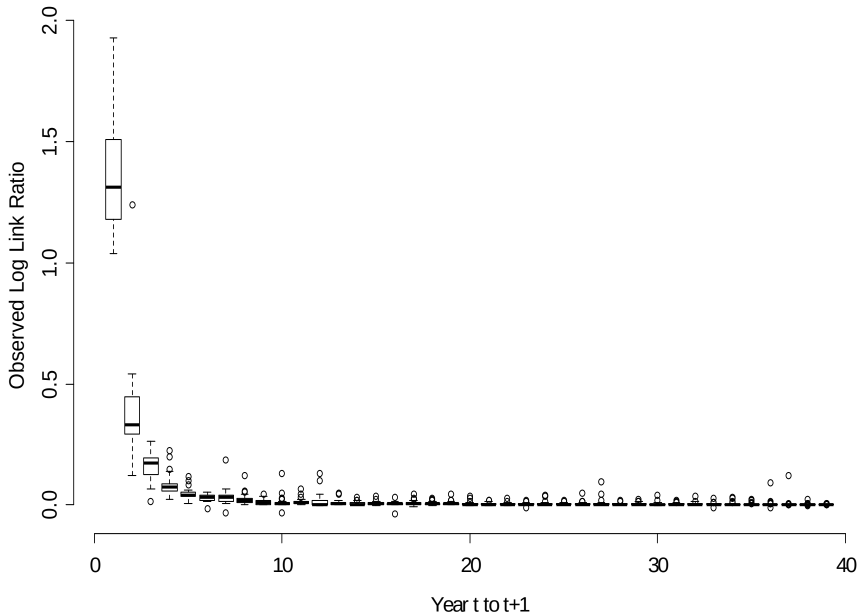


Chart 8: Logarithmic Paid Link Ratio Boxplots, 45 Jurisdictions, 2007 Diagonal

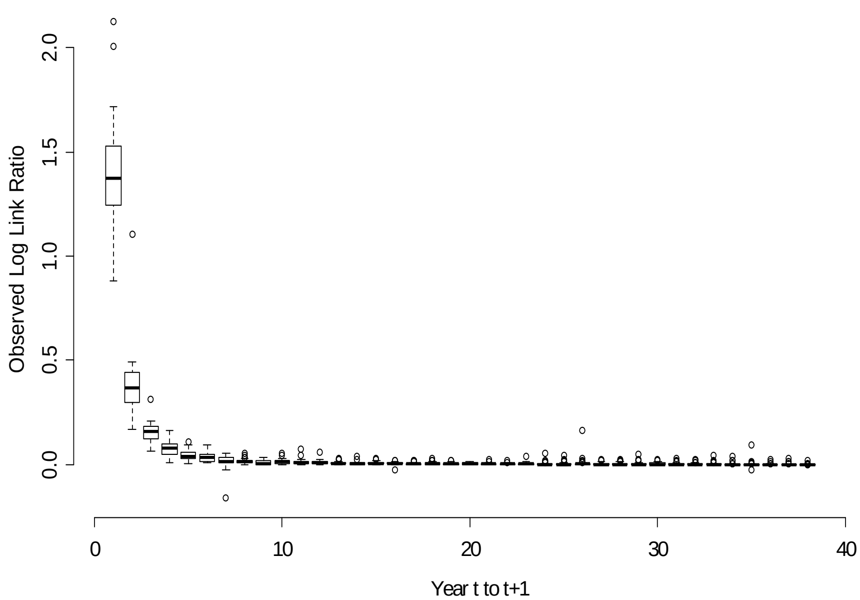


Chart 9: Logarithmic Paid Link Ratio Boxplots, 45 Jurisdictions, 2006 Diagonal

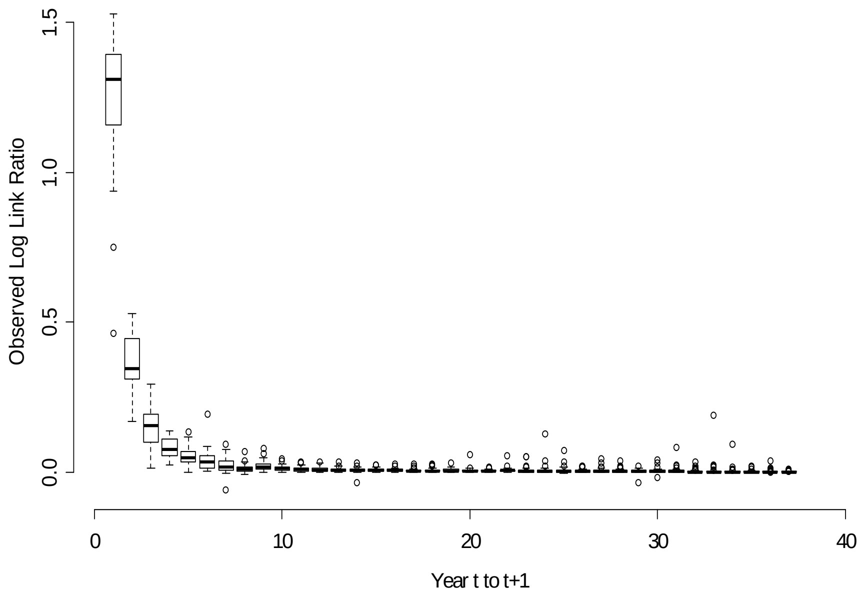
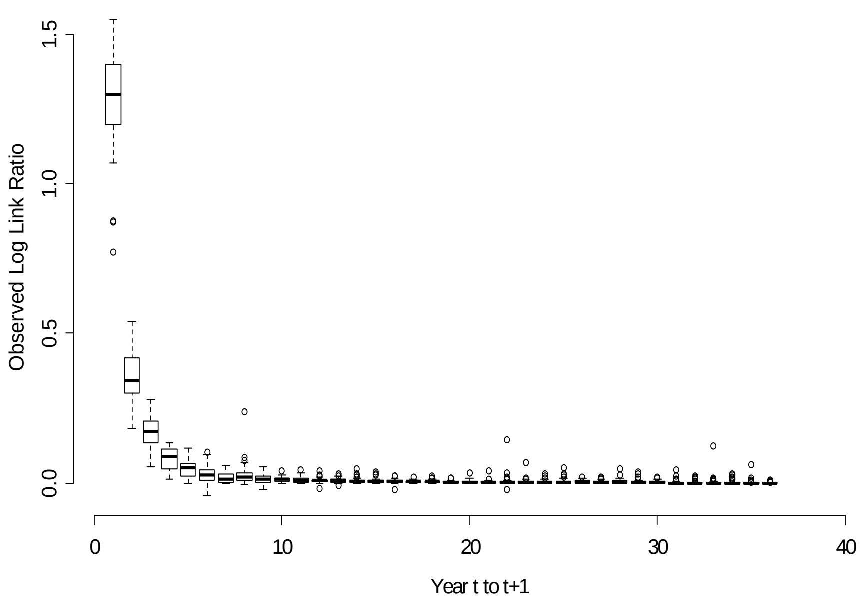


Chart 10: Logarithmic Paid Link Ratio Boxplots, 45 Jurisdictions, 2005 Diagonal



3. THE STATISTICAL MODEL

At the center of the TCM is a growth curve that feeds into a double exponential likelihood. The growth curve is motivated by the fact that logarithmic link ratios represent logarithmic rates of growth of cumulative losses, thus resembling a biological growth process.

The three-parameter growth curve employed in the model reads:

$$y_{i,j} = \beta_i \cdot \gamma_i^{\,q_i \cdot \log(j) + (1-q_i)\cdot(j-1)} \;,\; j = 1,\ldots,N,\; \beta_i > 0,\; 0 < \gamma_i < 1,\; 0 \le q_i \le 1 \;, \tag{1}$$

where i indicates the state and j indicates the maturity (year); $y_{i,j}$ is the (natural) logarithm of the link ratio, and N stands for the number of observations (per state) in the data set.

The parameter β delivers an estimate of the first-to-second link ratio; the parameter q is a weighting factor between log-linear and linear influences.

All three parameters of the growth curve are subject to partial pooling. For the parameter β, this is accomplished by means of a half-normal distribution; both the location parameter of the parent distribution and the draws for the individual states are generated by normal distributions that are truncated on the left at zero. For the parameter γ, the partial pooling of the location parameter is implemented using beta distributions, which implies that both the location parameter of the parent distribution and the draws for the individual states are generated by beta distributions. For the parameter q, the location parameter of the parent distribution is again a beta distribution, but the draws for the individual states are from a normal distribution that is left-truncated at zero and right-truncated at unity. The use of a truncated normal (instead of a beta) distribution is motivated by an easier convergence of the Markov chains in the Bayesian estimation process.

The growth curve stated in Equation (1) is a generalization of a functional form, the Bayesian estimation of which has first been discussed by Gelfand and Carlin [2].

The likelihood consists of a double exponential (or, equivalently, Laplace) distribution. The double exponential distribution is heavy-tailed and minimizes the sum of absolute errors (as opposed to the sum of squared errors), which makes this distribution robust to outliers. Minimizing the sum of absolute errors implies estimating the conditional median (as opposed to the traditional approach of modeling the conditional mean). The double exponential likelihood, in its standard form, does not account for skewness, which makes it unsuitable for studying (logarithmic) paid link ratios.

The precision (which is the reciprocal of the variance) of the double exponential is credibility-adjusted. The variance of the log link ratios is allowed to vary across states. Credibility-adjusting these variances is critical when it comes to simulating data for states that have no observations. Further, the likelihood allows for heteroskedasticity, as the variances of the first and second log link ratios are allowed to differ from each other and from the variance that applies to all subsequent link ratios.

Generally, the model can be extended to accommodate more complex error structures, especially with regards to heteroskedasticity but also with respect to the time series behavior of link ratios. For instance, the error variance in the likelihood can be modeled as a function of the number of open claims. Further, an autoregressive process similar to the one implemented by Meyers [6] could be considered. However, given the sparseness of the data, such additional complexity was not deemed desirable for annual data. Implementation of the model for quarterly data may offer additional modeling options due to the higher number of observations.

4. RESULTS

The model is estimated by means of Markov-chain Monte Carlo simulation (MCMC). The JAGS code of the core model is displayed in the appendix.

As mentioned, there are four states for which all link ratios are equal to unity. As a result of there being no variation in the data for these states, the sampling process breaks down. For this reason, the data set is jittered by adding a normally distributed error term to the logarithmic link ratios that are equal to zero (for any state). The standard deviation of the added error term equals 0.0001.

Although the added error term is close to zero (due to the small standard deviation), in order to have it average out to (approximately) zero, 30 jittered data sets are created and the model is run on all of them independently. For the purpose of obtaining the posterior distributions, the codas of the 30 runs are pooled.

Three Markov chains are employed in the estimation. After a burn-in phase of 20,000 draws, 200 samples are collected per chain (from 20,000 draws per chain with a thinning parameter of 100). The 200 samples from three chains of 30 jittered data sets then amount to 18,000 draws per parameter. The link ratios are obtained from the logarithmic link ratio by exponentiating draw by draw.

Charts 11 through 16 present the estimated (and observed) link ratios for New Jersey, Massachusetts, Michigan, New Mexico, Tennessee, and West Virginia. These states were selected to test the efficacy of the model for jurisdictions with differing characteristics. New Jersey is the largest member of the NWCRP by Reinsurance Pool Premiums Written [7]. Massachusetts left the NWCRP effective 1/1/1991 and formed its own pool, for which NCCI provides actuarial services. Similarly, Michigan left the NWCRP effective 1/1/1983 to form its own pool; here too, NCCI provides actuarial services. New Mexico has its own pool (without ever having been with the NWCRP); NCCI provides actuarial services. Tennessee is of interest because this state left the Pool effective 1/1/1998; for the post-NWCRP policy years, NCCI provides no actuarial services for the residual market, which creates an incomplete data set. Finally, West Virginia is of interest because it recently joined the NWCRP; only a single link ratio is available.

Charts 11 through 16 show that there is clearly more variance in the logarithmic link ratios at the first maturity (Year 1 on the horizontal axis) than there is at the second (Year 2); there is little variation in the variance thereafter, as assumed in the model. The states vary greatly by the degree of convexity (curvature) in the link ratio trajectory. For instance, for Michigan the decline is gentler than for New Mexico, where the link ratios drop precipitously from Year 1 to Year 2.

Tennessee (Chart 15) does not have its first link ratio before Year 9. Thus, prior to this maturity, the trajectory draws heavily on the common distributions of the growth curve parameters. This holds even more so for West Virginia (Chart 16), which sports only a single observation. As the chart shows, the estimated value falls short of the observed value, which is due to shrinkage.

Chart 11: Paid Plus Case Link Ratios, New Jersey, Five Diagonals

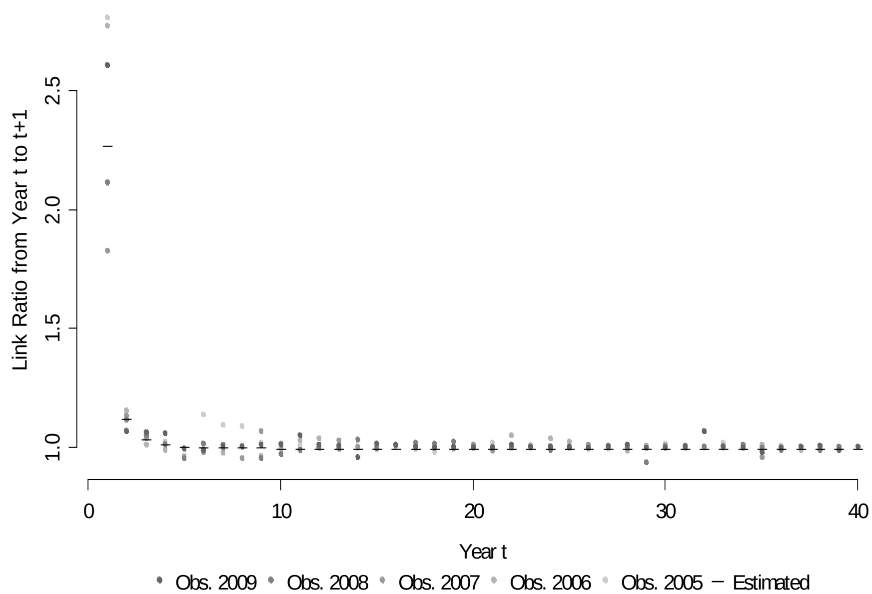


Chart 12: Paid Plus Case Link Ratios, Massachusetts, Five Diagonals

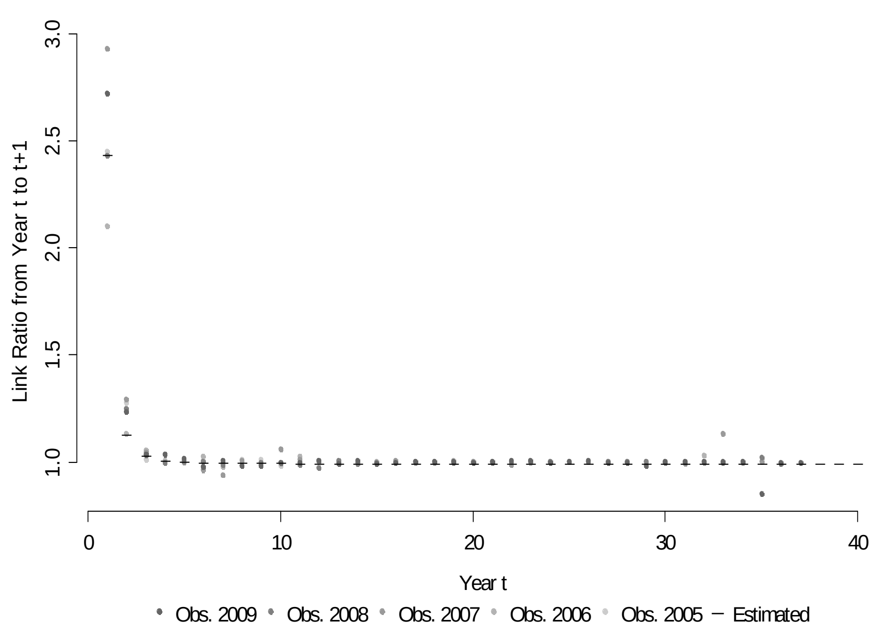


Chart 13: Paid Plus Case Link Ratios, Michigan, Five Diagonals

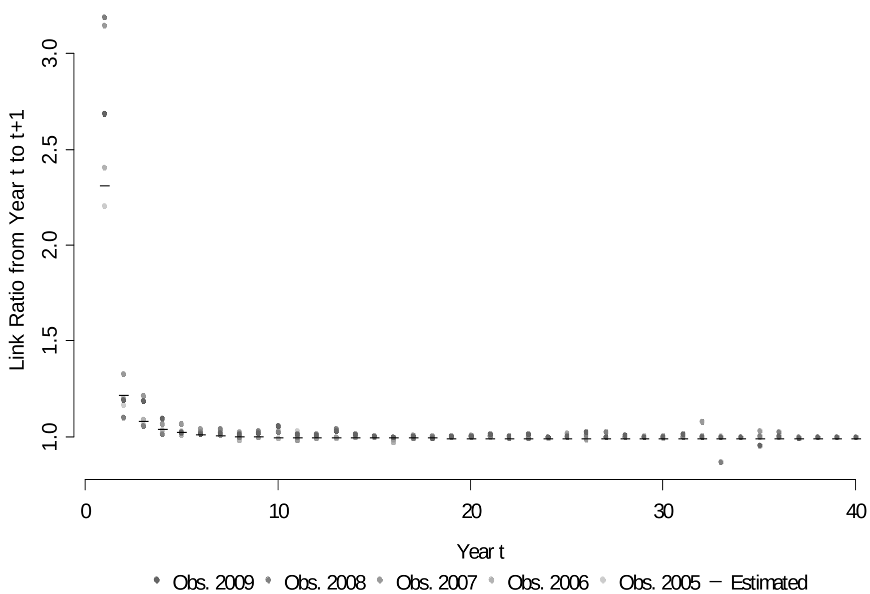


Chart 14: Paid Plus Case Link Ratios, New Mexico, Five Diagonals

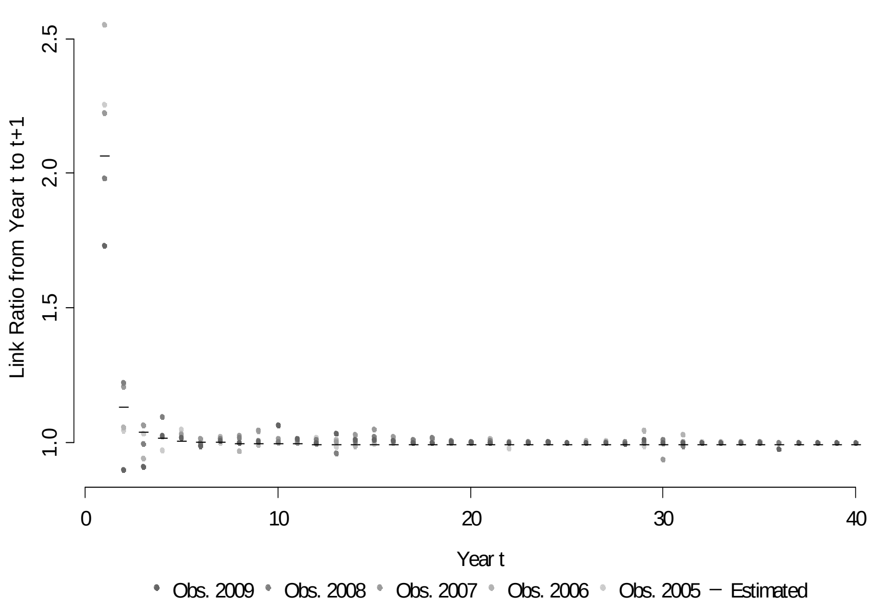


Chart 15: Paid Plus Case Link Ratios, Tennessee, Five Diagonals

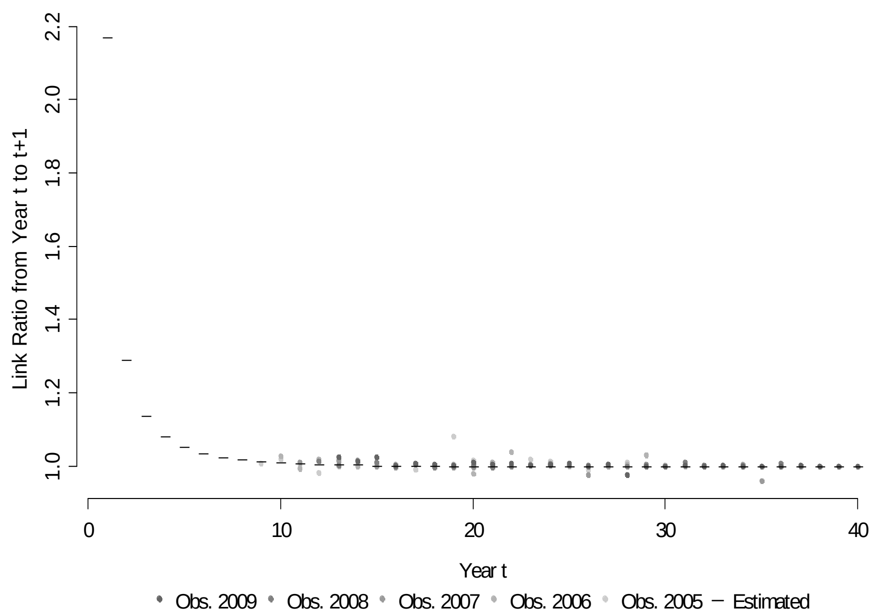
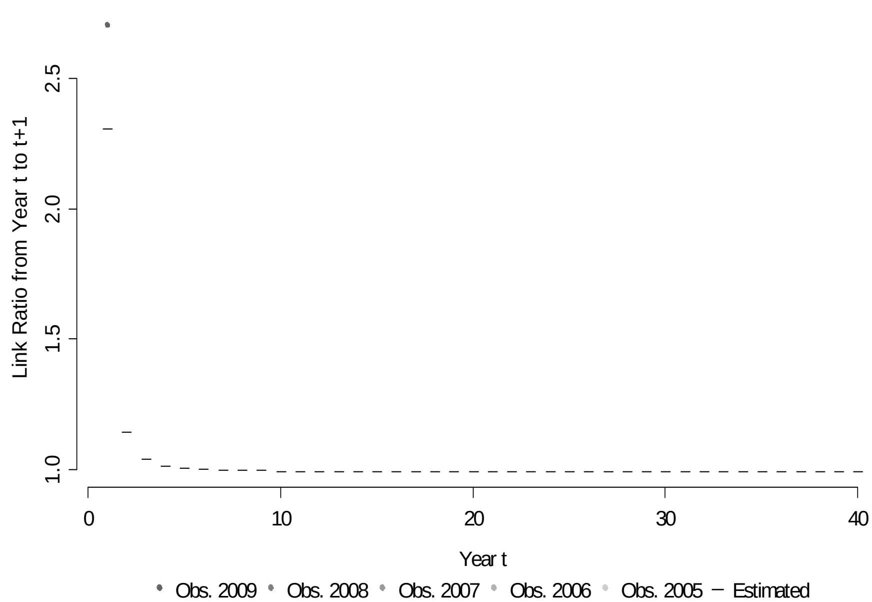


Chart 16: Paid Plus Case Link Ratios, West Virginia, Five Diagonals



Charts 17 through 21 provide in-sample error diagnostics for the diagonals 2009 through 2005. The displayed residuals are standardized, thus accounting for the heteroskedasticity that has been built into the model. There is no discernible pattern in these errors. Specifically, it appears that the growth curve is capable of accounting for the various degrees of convexity in the link ratio trajectories across states. Also, due to the errors being symmetric around zero along the entire horizontal axis, the model does not systematically underpredict or overpredict at certain maturities. Finally, the errors are not widening or narrowing in systematic ways with maturity (that is, along the horizontal axis).

Some of the residuals are comparatively large, thus pointing to outliers in the data. This supports the choice of a double exponential likelihood, which, due to the modeling of the conditional median (as opposed to the mean), shows little sensitivity to outliers.

Chart 17: In-Sample Diagnostics, Paid Plus Case, 2009 Diagonal

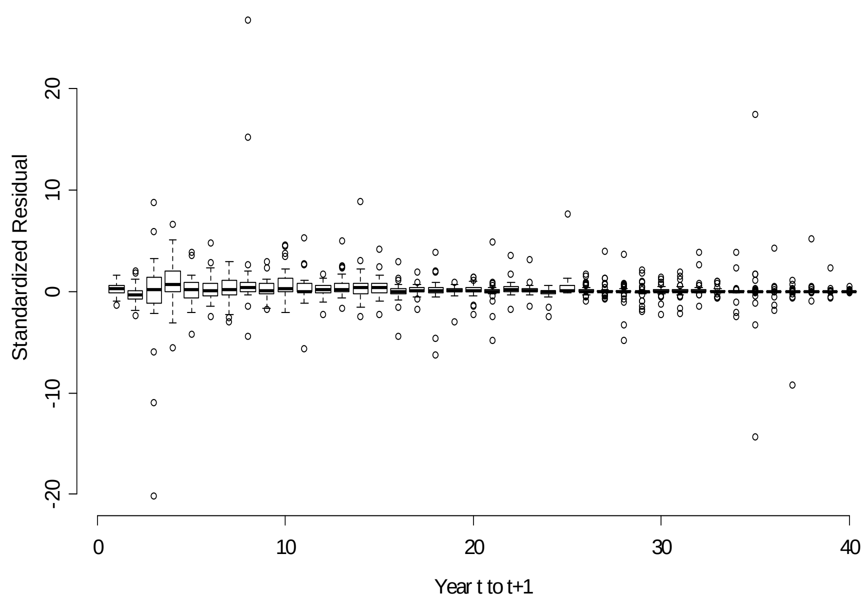


Chart 18: In-Sample Diagnostics, Paid Plus Case, 2008 Diagonal

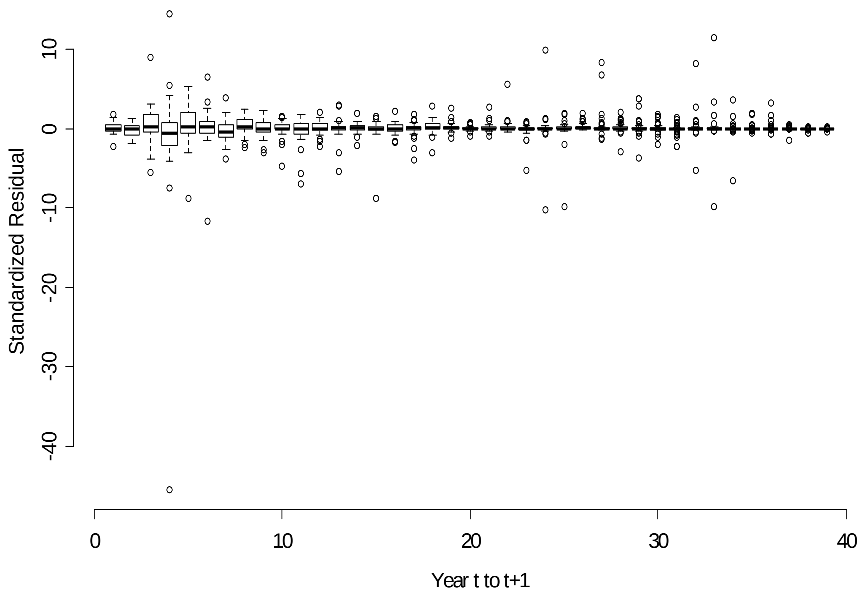


Chart 19: In-Sample Diagnostics, Paid Plus Case, 2007 Diagonal

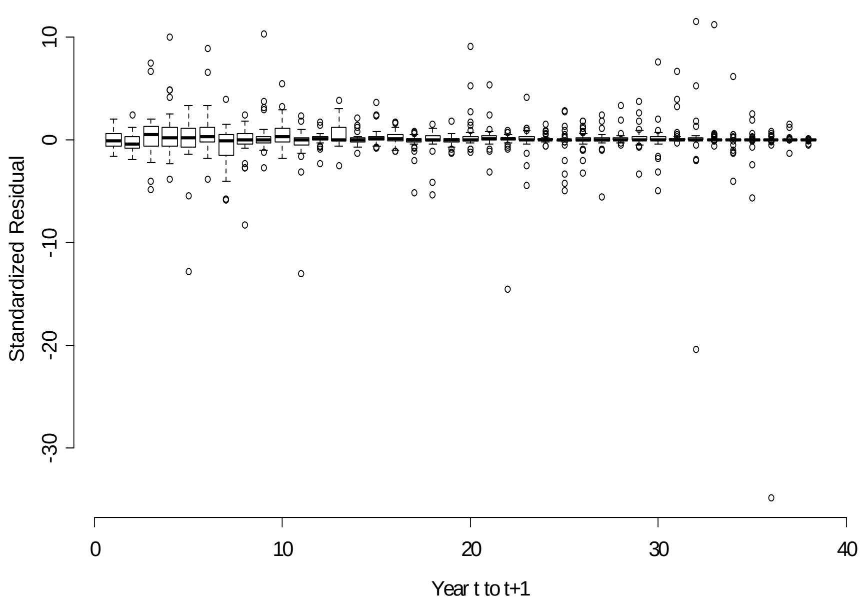


Chart 20: In-Sample Diagnostics, Paid Plus Case, 2006 Diagonal

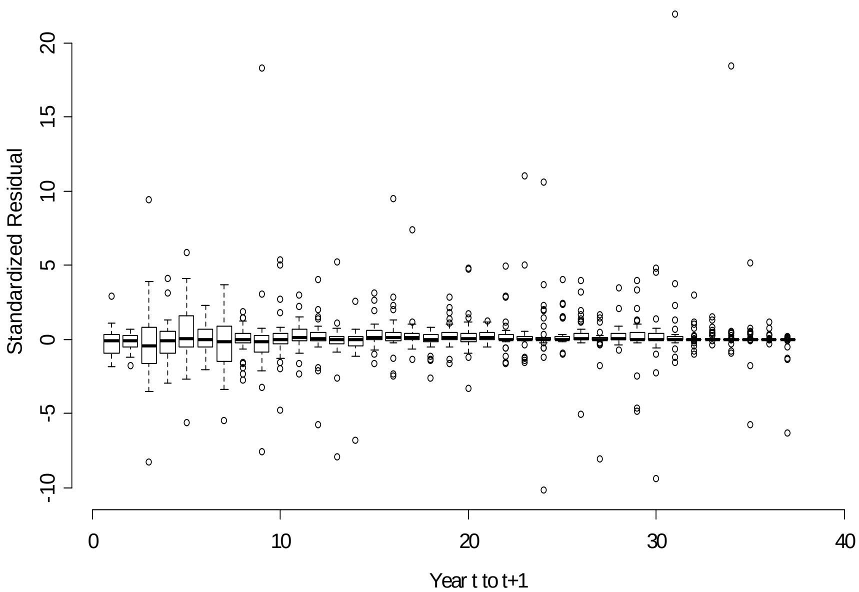
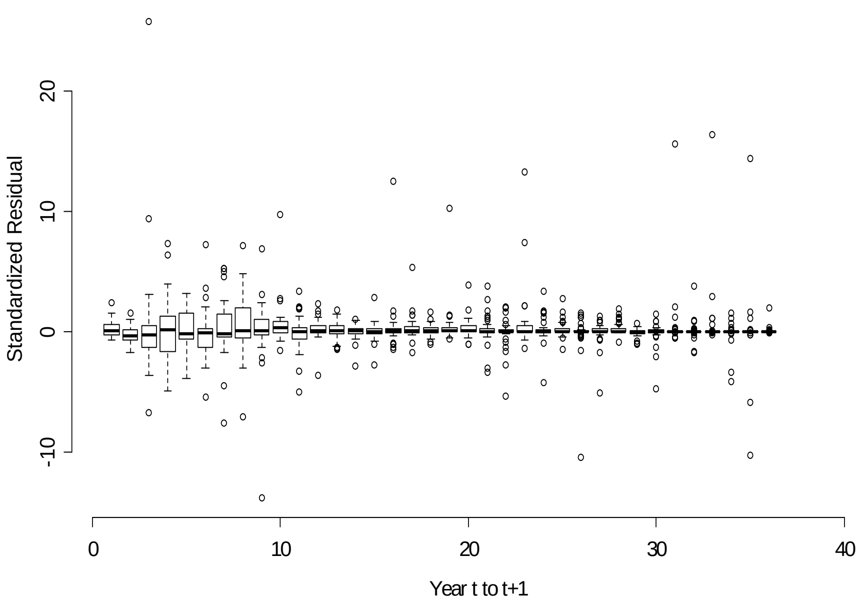


Chart 21: In-Sample Diagnostics, Paid Plus Case, 2005 Diagonal



A Total Credibility Approach to Pool Reserving

The model is validated using a one-year holdout period. The model is fit to the diagonals of Calendar Years 2004 through 2008. Based on the estimated model parameters, link ratios for Calendar Year 2009 are simulated. By comparing the simulated link ratios to the 2009 observed values, the mean absolute forecast error is calculated. The process of model validation is repeated using the diagonals of Calendar Years 2003 through 2007—the forecast errors are calculated based on the observed 2008 diagonal.

Chart 22 displays the forecast errors for the 2009 diagonal; Chart 23 provides this information for the 2008 diagonal. Clearly, in Chart 22, for the first maturity (Year 1 on the horizontal axis), the high degree of volatility in the link ratios in these maturities leaves the median forecast error noticeably greater than zero; this is because in that year, several states had considerably higher link ratios in Year 1 than usual. At the same time, in Chart 23, the forecast for the link ratio at maturities Year 1 performs considerably better. Beyond the first two maturities, the forecasts offer a high degree of accuracy; the forecast errors are clearly symmetric.

Chart 22: Forecast Diagnostics, Paid Plus Case, 2009 Diagonal

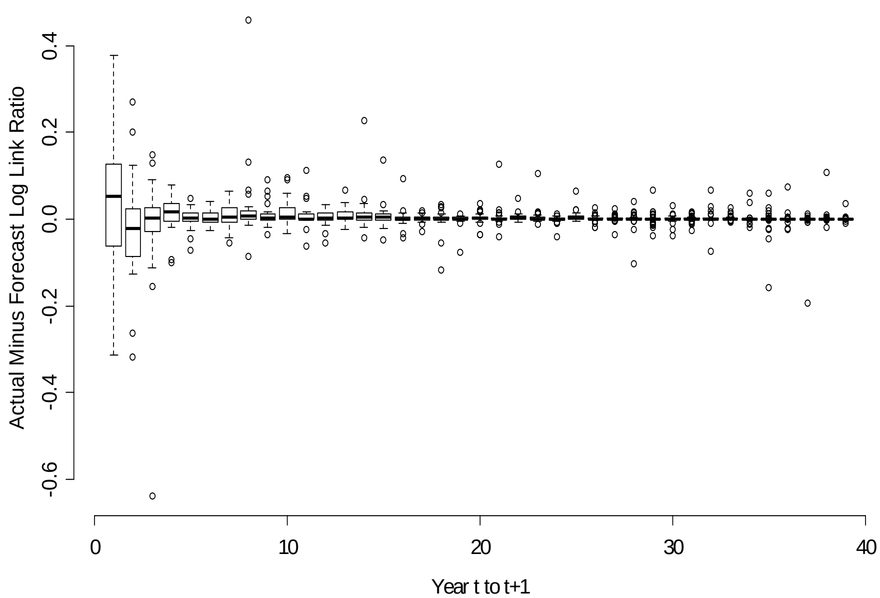
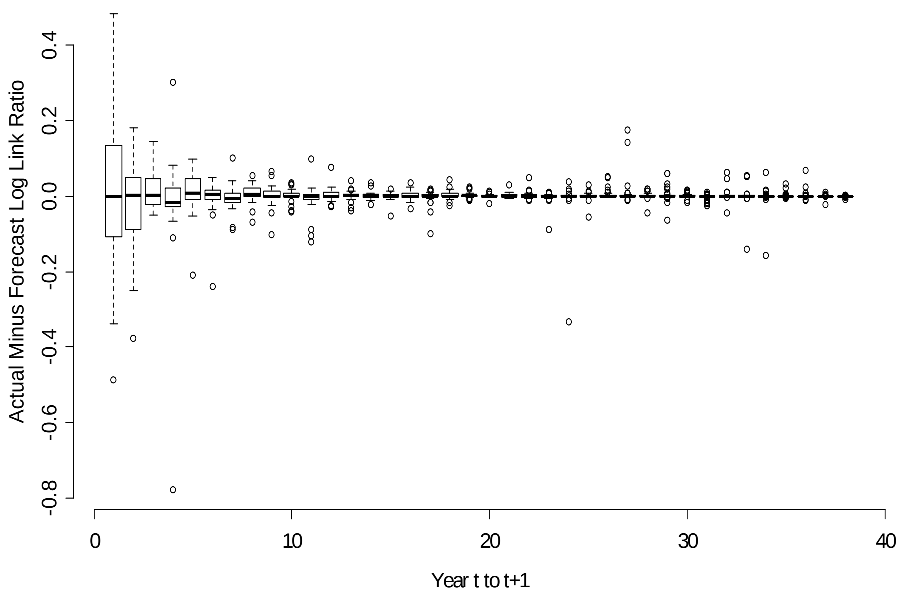


Chart 23: Forecast Diagnostics, Paid Plus Case, 2008 Diagonal



5. CONCLUSIONS

The TCM is part of a family of multilevel reserving models that have recently been discussed in actuarial literature. The model is capable of estimating not only link ratios but also delivers tail factors. Further, the model offers credible intervals for the estimated link ratios and tail factors.

The TCM, in its current version, is not built to replicate the skewness of logarithmic paid link ratios—thus, the model is not suitable for the analysis of paid link ratios. The use of a double-exponential likelihood makes the estimates robust to outliers. An alternative robust likelihood is one that rests on Student's *t* distribution. This likelihood was tested on (logarithmic) paid link ratios, using the skewed *t* approach developed by Kim and McCulloch [5]. The sparseness of the data however, did not allow for a reliable identification of skewness and heavy-tailedness.

For research purposes, only annual data was used in the analysis. The model can be extended to process quarterly (instead of annual) data. The use of quarterly data will allow for more complex error structures, both with respect to differences across states and variation over time.

6. APPENDIX

JAGS Code (Core Model)

```
model
{

#shrinkage
beta.mu ~ dnorm(0,1.E-2)T(0,)
beta.tau <- pow(beta.sigma,-2)
beta.sigma ~ dunif(0,2)

gamma.mu ~ dbeta(1,1)
gamma.sigma ~ dunif(0,1)
gamma.tau <- pow(gamma.sigma,-2)
gamma.alpha <- gamma.mu * gamma.tau
gamma.beta <- (1-gamma.mu) * gamma.tau

q.mu ~ dbeta(1,1)
q.sigma~ dunif(0,1)
q.tau <- pow(q.sigma,-2)
#q.alpha <- q.mu * q.tau
#q.beta <- (1-q.mu) * q.tau

for(m in 1:3){ #different variances for first two development years
    tau.alpha[m] ~ dexp(1.0)
    tau.beta[m] ~ dgamma(0.1,0.1)
    }

#likelihood

for(i in 1:L){ #rows (states)

   beta[i] ~ dnorm(beta.mu,beta.tau)T(0,)
   gamma[i] ~ dbeta(gamma.alpha,gamma.beta)
   q[i] ~ dnorm(q.mu,q.tau)T(0,1) #using normal instead of beta eases convergence

   for(m in 1:3){
       tau[i,m] ~ dgamma(tau.alpha[m],tau.beta[m])
       sigma[i,m] <- sqrt(2)/tau[i,m] #double exponential errors
       }

   for(j in 1:T){ #columns (development years)

        y.pred[i,j] ~ ddexp(mu[i,j],tau[i,tau.index[j]]) #double-indexing for tau
        cdf[i,j] <- sum(mu[i,j:T])
        mu[i,j] <- beta[i]*pow(gamma[i],q[i]*log(j)+(1-q[i])*(j-1))

        }

   for(j in 1:N){ #columns (development years)

        y.2009[i,j] ~ ddexp(mu[i,j],tau[i,tau.index[j]]) #double-indexing for tau
        y.2008[i,j] ~ ddexp(mu[i,j],tau[i,tau.index[j]])
        y.2007[i,j] ~ ddexp(mu[i,j],tau[i,tau.index[j]])
        y.2006[i,j] ~ ddexp(mu[i,j],tau[i,tau.index[j]])
        y.2005[i,j] ~ ddexp(mu[i,j],tau[i,tau.index[j]])

     }
   }
}
```

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7. REFERENCES

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Abbreviations and notations

IQR, Interquartile Range
JAGS, Just Another Gibbs Sampler
MCMC, Markov-Chain Monte Carlo Simulation
NCCI, National Council on Compensation Insurance
USL&H, United States Longshore and Harbor Workers
TCM, Total Credibility Model

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