An Experience Rating Approach to Insurer Projected Loss Ratios

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Abstract: The traditional approach to Property/Casualty rate indications starts with a methodology that uses internal data to forecast the Ultimate Loss Ratio, with losses making up about half of the expenses. For parties that are external to the insurer, this approach to forecasting a key component of future profitability is impractical as they generally do not have access to the necessary data. Using publicly available information, that is, the National Association of Insurance Commissioners Schedule P of the statutory financial statements from 1992 to 2010, we develop by line of business forecasts of the relativity to the industry Loss Ratio. To develop these forecasts, we use a weighted regression methodology that incorporates key ideas from fixed-effects regression, instrumental variables regression, credibility theory, as well as a flexible covariance structure for the residuals. Results indicate that the proposed approach of using lagged relativities from insurer own and other lines of business can provide adequate fits for many lines of business and for the combined results of the insurer as a whole.

Keywords. Experience Rating, Panel Data, Fixed-Effects Regression, Instrumental Variable Regression, Credibility Theory

1. INTRODUCTION

The traditional approach to Property/Casualty rate indications[1] (Werner and Modlin 2010) starts with a methodology that uses internal data to forecast the Ultimate Loss Ratio, with losses making up about half of the expenses. For parties that are external to the insurer, this approach to forecasting a key component of future profitability is impractical as they generally do not have access to the necessary data. External parties that are tasked with solvency surveillance, stock pricing, bond pricing, reinsurance underwriting, *etc*. need a Loss Ratio forecasting approach that relies on publicly available data. Even for the internal actuaries, using an alternate forecasting method can provide the actuary with a point of comparison that can supplement and complement forecasts supported by internal data.

Using publicly available information, that is, the National Association of Insurance Commissioners Schedule P of the statutory financial statements from 1992 to 2010, we develop by line of business forecasts of the relativity[2] to the industry Loss Ratio. To develop these forecasts, we

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[1] Rate indications refers to approaches to the overall costing of a P/C insurance portfolio that rely mostly on the insurers own premium/exposure and loss data. Rate indications can be done using the Loss Ratio approach, where past LR are adjusted to be at the level of when the matching rates would be in-force, averaged out and compared with a Permissible Loss Ratio to attain a given level of profitability, or using the Loss Cost approach, where past insurance unit cost are adjusted to be at the level of when the matching rates would be in-force, average out and inflated for expected fixed and variable expenses.

[2] Relativity is a commonly used actuarial measure where a value of interest is compared to the same value of interest but for a larger set. For example, in ratemaking, it is common practice to breakdown manual rates into base rates and relativities. The said relativities can be calibrated by comparing the

use a weighted regression methodology that incorporates key ideas from fixed-effects regression, instrumental variables regression, credibility theory, as well as a flexible covariance structure for the residuals. From fixed-effects regression (Frees, Longitudinal and panel data: analysis and applications in the social sciences 2004, 51), we borrow the idea that the forecasts incorporate a (weighted) average of past results. From instrumental variables regression (Frees, Meyers and Cummings, Predictive Modeling of Multi-Peril Homeowners 2011, 3), we borrow the idea that other lines of business can share result-drivers in common, like similar strategies, similar clients or similar perils. From credibility theory, we borrow the idea that the experience rating values vary with the size of the individual. We also use a Toeplitz, or Moving Average, intra-insurer/line of business structure for the residuals over time (Frees, Longitudinal and panel data: analysis and applications in the social sciences 2004, 281).

Given that "[e]xperience rating recognizes the differences among individuals (…) by comparing the experience of individual (…) with the average (…) in the same classification" (National Council on Compensation Insurance 2007, R2), the proposed modeling approach can be thought of a form of experience rating. In line with more traditional experience rating methodologies, the forecasted relativities can be thought of as a modifier to a base rate, which is here the forecast of the by line industry Loss Ratio. These forecasts can reflect outlooks concerning the economy as a whole, the softness/hardness of the market, *etc*. We do not address the issue of how to forecast the state of the P/C industry market [by line of business] as a whole and instead presume that parties that may wish to follow our approach have developed an expertise in making these types of forecasts[3].

Contrary to the traditional use made of experience rating, our approach is not aimed at increasing incentive alignment between an insured and an insurer, decreasing the potential for adverse selection, or increasing fairness (Venter 1987, 1-2); instead, the main goal that our approach shares with traditional experience rating is predictive accuracy. These differences in goals make it such that, while we will have the chance to comment on modeling choices that also have to be made when calibrating an experience rating scheme, we will not comment on the potential micro-economic

actual Loss Ratio for a given value of a rating variable, in the numerator, to the overall actual Loss Ratio across all values of the variable, in the denominator.

[3] The author does not have specific expertise on that topic; nonetheless, the Loss Ratio projection methodology of the Loss Ratio approach to rate indications should be applicable to the industry as a whole, as long as the user can make assumptions about the future rate changes of the P/C insurance industry as whole, as well as future catastrophic loss activity.

importance of experience rating, like the rate at which insureds and insurers learn about the underlying riskiness of the insureds, the self-censoring of losses, and moral hazard avoidance.

Results indicate that the proposed approach of using lagged relativities from insurer own and other lines of business can provide adequate fits for many lines of business and for the combined results of the insurer as a whole. For solvency surveillance usage, we recommend that a regulator or a rating agency supplement the model with measured rate changes so as to better anticipate large changes in the Loss Ratio than are not due to smooth changes.

The rest of the paper will go as follows: section 2 will cover a short history of the actuarial development of experience rating, section 3 will cover a summarized version of elements that are normally included in an experience rating plan, section 4 will cover the modern statistical foundation of experience rating, section 5 will describe the data that was used for our current analysis, section 5.2 will cover the descriptive statistics, section 6 will cover the statistical analysis as such, including model selection and fit analysis, and section 7 will look back at practical choices that need to be made to calibrate an experience rating plan and we will be able to comment how our modeling choices can apply to such an exercise.

2. ACTUARIAL HISTORY OF EXPERIENCE RATING

Experience rating has been at the heart of Property/Casualty actuarial science ever since P/C actuarial science has developed has a separate sub-field of actuarial science. Early on, the foundation of what will come to be known as American credibility was developed by Mowbray (How Extensive a Payroll Exposure is Necessary to Give a Dependable Pure Premium 1914) who was attempting to answer the question of just how large an insured needed to be to generate, without using data related to other insureds, a forecast of future losses that had a given level of precision. To this day, P/C actuaries around the world know of the 1082 claims for full credibility rule-of-thumb (Hansen 1972) that can be derived using this approach.

As early as 1918, Whitney (The Theory of Experience Rating) used Bayesian and approximation arguments to derive the $Pn/(Pn+K)$ formula for credibility (Whitney 1918, 288), which is reminiscent of the traditional one-way random-effects analysis of variance models (Frees, Longitudinal and panel data: analysis and applications in the social sciences 2004, 126). This formula is still at the heart of many experience rating plans today (Gillam and Snader, Fundamentals of Individual Risk Rating, Part I 1992, 1-4).

In his 1934 Casualty Actuarial Society Presidential address, Dorweiler (A Survey of Risk Credibility in Experience Rating) presented the rating plan performance principle that was to become the foundation of what is known as the quintile test (Couret and Venter 2008, 82).

> A necessary condition for proper credibility is that the credit risks and debit risks equally reproduce the permissible loss ratio. Also, if the proper credibility has been attained, each sub-group of the credit and debit risks, provided it has adequate volume, should give the permissible loss ratio. While these conditions are necessary for a proper credibility of the experience rating plan, it does not follow that they are also sufficient. For a sufficient condition it would be required to establish that the risks within a group cannot be subdivided on any experience basis so as to give different loss ratios for the subdivisions, assuming the latter have adequate volume. (Dorweiler 1934, 100)

In 1959, Bailey and Simon (An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car) demonstrated[4] that experience rating was also pertinent for lines of business other than Workers' Compensation. Even when risks are fairly homogeneous to start with, the claiming history of an individual insured has predictive value that allows for rating that is more precise than that implied by classification rates.

The American history of credibility theory was complemented by what is sometimes called European credibility, as exemplified by the developments of Bühlmann, Bühlmann-Straub, Hachemeister, Jewell, (Frees 2004, 155) Dannenburg, and Goulet (Goulet 2001, 205-206). As is demonstrated by (Goulet 2001, 207), European credibility formulas can be interpreted as Best Linear Unbiased Predictors. As such, what is known as European credibility can be thought of as theoretical and practical developments that paralleled those made in North America by the econometrician Goldberger and associates (Frees 2004, 130).

3. CONCRETE EXAMPLES OF EXPERIENCE RATING PLANS

One of our aims with this proposal is to address practical modeling choices that would need to be made in the calibration of a more traditional experience rating algorithm; therefore, before going any further, we'll discuss elements that are traditionally included in an experience rating plan. For readers that are not already familiar with experience rating plans or with the material presented in the

[4] Using a non-parametric approach.

Advanced Ratemaking exam of the Casualty Actuarial Society, this section can safely be foregone at a first reading.

We will focus on the National Council on Compensation Insurance Workers' Compensation experience rating plan (2007) and on the Insurance Services Office's Commercial General Liability experience rating plan (2006). So doing, we won't be directly addressing other types of experience rating plans like driving records in Personal Automobile, fleet rating in Commercial Automobile, claims rating in Property insurance, *etc.*

A starting but key element of any experience rating plan is the definition of what counts as an 'individual' under the plan. Generally speaking, an 'individual' will be an insured, but there can be exceptions. For example, under the NCCI plan, an entity is defined with reference to ownership rules (R13) while, under the ISO plan, the definition of risk also refers to considerations relating to franchising (1).

Another key strategic rating consideration is the number of years of experience used. This can affect the way that the rating information is accumulated. Depending on the distribution channel used (*e.g.* direct or brokerage), the number of years of experience considered can also affect the burden put on parties involved in the distribution of insurance, especially if the used plan differs from industry standards. Under the NCCI plan, up to about four years of experience can be used (R10-R11) while, under the ISO plan, up to three years of experience are used (1). The use of the optimal quantity of experience implies that the plan must include rules about how to deal with the experience with other insurers: for example, under the NCCI plan, experience with other insurers can be included but is subject to verification (R11) and, under the ISO plan, special rules are formulated to deal with the fact that losses that occurred with another insurer are not revalued (10-11).

Properly actuarial elements also need to be grounded in rules. In particular, the losses and premium need to be put on-level[5] to ensure the comparability of the experience from multiple periods; therefore, commonly addressed elements include loss development and trends. Under the NCCI plan, the losses are extracted from the appropriate statistical plans (R6) while, under the ISO plan, factors are specifically provided to develop and detrend the losses (12-13). To properly address

[5] That is, in dollars of the forecasted-to period. The first rule that makes the experience on-level under a Loss Ratio based experience rating plan is the use of premium set at current rates in the denominator of the Loss Ratio.

the predictiveness of large losses, the plan can specify rules relating to the capping of losses and provide a way to compute an Expected Loss Ratio that covers only the lower layer of losses. For example, under the ISO plan, losses are capped at a rule-determined Maximum Single Loss (3) and the capped Loss Ratio is compared to an Expected Experience Ratio that reflects losses that are expected under the MSL. Under the NCCI plan, the actual experience of large losses is partially reflected in the rating modification m to manual rates: in this case,

$$m = \frac{\mathrm{A_p} + z_\mathrm{e}\mathrm{A_e} + (1 - z_\mathrm{e})\mathrm{E_e} + \mathrm{B}}{\mathrm{E_p} + z_\mathrm{e}\mathrm{E_e} + (1 - z_\mathrm{e})\mathrm{E_e} + \mathrm{B}}, \text{ where}$$

$\mathrm{A_p}$ refers to actual primary losses,
$\mathrm{A_e}$ refers to actual excess losses,
$\mathrm{E_p}$ refers to expected primary losses,
$\mathrm{E_e}$ refers to expected excess losses,
z_e refers to the credibility of actual excess losses, and
B is a ballast value. (R10)[6].

Other rules that can be included in an experience plan can include: rules relating to types of policies (*e.g.* rules to convert the experience of claims-made and occurrence-based[7] Commercial General Liability policies that have different development patterns (11-12)), schedule rating that relates to softer characteristics of the risk that may not be fully reflected in the experience as such (9), and rules relating to corrections of previously available information (R17).

Even though the context in which we want to apply the experience rating framework is different from a traditional experience rating application, it is our hope that we can comment on modeling choices that would be encountered in the calibration of a traditional experience rating plan. In particular, we hope to address how to handle the selection of the number of years of experience and the development of losses.

4. STATISTICAL FOUNDATIONS

The purpose of the section is to familiarize practicing actuaries with the statistical methods and hypothesis that will underlie our proposed models. Readers familiar with modern statistical

[6] The value that comes after the $(1 - z.)$ term is commonly called a complement of credibility. (Boor 1996) has documented commonly used complements of credibility. Moreover, at pp.36-37, he shows how to determine the optimal credibility weight as a function of the correlation between two unbiased estimators of the same parameter. This result can also be proven using a Generalized Method of Moments approach.
[7] More on that topic below.

techniques or readers that are mainly interested in the data and the results can safely skip this section at a first reading.

As was demonstrated in (Frees, Young and Luo, A Longitudinal Data Analysis Interpretation of Credibility Models 1999), many credibility models, like those that are used in experience rating, can be interpreted in terms of estimation in a longitudinal data context. It is not uncommon for credibility theory to be cast in terms of random-effects models. For example, one could write a model the following way:

$$y_{i,t} = \boldsymbol{x}_{i,t}\boldsymbol{\beta} + \boldsymbol{z}_{i,t}\boldsymbol{\alpha}_i + \varepsilon_{i,t} \text{ for } 1 \leq i \leq n, 1 \leq t \leq T \text{where } E\left[\varepsilon_{i,t}\right] = 0, V[\boldsymbol{\varepsilon}_i] = \boldsymbol{R}_i,$$

$$E[\boldsymbol{\alpha}_i] = \boldsymbol{0} \text{ and } V[\boldsymbol{\alpha}_i] = \boldsymbol{D} \text{ with } \boldsymbol{\alpha}_i \text{ independent and identically distributed.}$$

One possible way to interpret this model is to think of it as a mixture of fixed-effects $\boldsymbol{x}_{i,t}\boldsymbol{\beta}$ due to the observable variables $\boldsymbol{x}_{i,t}$ and random-effects due to unobserved individual heterogeneity $\boldsymbol{z}_{i,t}\boldsymbol{\alpha}_i$. Take Private Passenger Automobile insurance as an example. In this case, we can imagine that each driver is receiving a random draw that fixes the individual's 'driving abilities'. We then assume that this 'driving ability' is not directly observable but remains constant through time. Observing drivers that are consistently better/worse than average, we can infer that it is likely that these drivers were given better/worse driving ability draws. In effect, the unobserved 'driving ability' is inducing serial correlation between the observations made of the drivers: a better than average driver will tend to remain better than average and a worse than average driver will tend to remain worse than average. Going back to the mathematical formulation of the model, we can further interpret it as saying: (1) the expected observed average given the observable variables $\boldsymbol{x}_{i,t}$ is $\boldsymbol{x}_{i,t}\boldsymbol{\beta}$, (2) if one knew the values of the unobserved heterogeneity terms $\boldsymbol{\alpha}_i$ and the observable variables $\boldsymbol{z}_{i,t}$, then the unexplained portion the observations would form a potentially auto-correlated and heteroskedastic sample, and (3) there exists unobserved heterogeneity that drives serial intra-individual correlation and this unobserved heterogeneity $\boldsymbol{\alpha}_i$ forms a random sample.

For our purposes, however, we will instead anchor ourselves in a fixed-effects model. One way to think about fixed-effects models is as a classical regression that includes an indicator function for each of the included 'individual'[8]. As such, under traditional fixed-effects models, there is a unique

[8] Contrasting this with the random-effects models: under a fixed-effects model, other considered individuals do not provide information about the coefficients that need to be attached to time constant covariates and all time constant covariates become collinear with the individual-specific indicator covariate.

intercept terms for each of the considered 'individual'. Another way to think about fixed-effects models is as a regression that includes a straight average of the residuals (from the regression of $y_{i,t} - \bar{y}_i$ on the covariates $\boldsymbol{x}_{i,t} - \bar{\boldsymbol{x}}_i$) as a covariate that is affected with a slope of unity.

Leaping with this idea, we can think of more traditional Auto-Regressive time series models (Wikipedia n.d.)[9] as fixed-effects models that include only the individual specific intercepts, but that uses a weighted instead of a straight average of the residuals to estimate the individual specific intercepts. The connection with time series is particularly relevant for rating and forecasting purposes. It is critical in rating and forecasting applications that the used covariates constitute available information at the time of the forecast. In the probability literature, this has been captured by the filtration concept[10].

Pushing even further the connection with time series models, it is also possible to include a general structure for the correlation of the residuals. Of particular interest is the inter-temporal intra-individual covariance structure for the residuals. In our case, we will consider a flexible Moving Average model (Wikipedia n.d.) called a Toeplitz specification for $\boldsymbol{R}_i$ (Frees, Longitudinal and panel data: analysis and applications in the social sciences 2004, 281). One of the advantages of the Toeplitz specification is that it presumes homoskedasticity. For our purposes, this will greatly simplify the forecasting process, as the variance of the residuals will not first have to be forecasted for future periods before the forecasts can be computed. The hypothesis is far from perfect[11]. Take, for example, the case of Homeowners insurance that can be greatly affected by natural catastrophes. In a year where a great hurricane or earthquake hits, some insurer will have exposures in the affected region and have poor underwriting results, but insurers that do not have any exposures in the region will only be affected by 'normal' noise. Given that it is next to impossible to forecast these great catastrophes, the forecast of the future variance of the residuals is also very difficult. That is why we will focus on a covariance structure for the residuals that does not imply that we need to forecast the variance of the residuals before computing the forecasts as such.

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[9] The author understands that many academic parties are uncomfortable with Wikipedia as a reference source. One traditional argument against Wikipedia is the non-certification of the source. As an actuary, the author is effectively endorsing any cited source as professional standards generally require that an actuary cannot cite references to other work for why the actuarial work product is not adequate. Another reason to support the resistance to the use of Wikipedia in academic work is it relative instability, in as much as this is a source that gets constantly updated. Here, the author is effectively making the practical assumption that ease of accessibility is more important than the stability of the source. Wikipedia, being a free web reference source, is imminently accessible to academic and professional populations.

[10] (Steele 2000, 50)

[11] Preliminary testing of the models indicates that models incorporating heteroskedasticity do better than models that imply homoskedasticity.

For our considered covariates, we will introduce the past results[12] in similar lines of business. An alternate modeling choice could have been to introduce a current period forecast of the results of similar lines of business as statistical instruments (Frees, Meyers and Cummings, Predictive Modeling of Multi-Peril Homeowners 2011)[13]. Given that we are working in Accident Year[14] and that about half of the results of the following Accident Year are driven by the same contracts as the current Accident Year, we believe that including the latest available Accident Year results serves substantially the same purpose. In effect, we are saying that if one wanted to make a 'back-of-the-envelop' forecast of the current Accident Year Ultimate Loss Ratio for a given line of business for a given insurer, one could use only the prior Accident Year Ultimate Loss Ratio as a covariate and come up with a good initial value for the forecast.

Because, under most experience rating formulas, the credibility factor z changes as the size of the account changes, we have included interaction terms that cross past insurer line of business past paid Ultimate Loss Ratio relativity with current insurer size, measured by a non-linear increasing concave down function of Earned Premium. This accomplishes the goal of varying the models for different insurer size. Further comments will be presented in section 6.

5. NAIC DATA

Before moving on to the selected models and the assessment of their predictiveness, let's first discuss the publicly available data that supports our methodology.

5.1 Data Preparation

At the heart of our analysis lies the National Association of Insurance Commissioners Schedule P of the statutory financial statements[15] from 1992 to 2010. As such, we are only focusing on

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[12] Notice that past results are part of the information set of the person applying the rating or forecasting algorithm.

[13] The instrumental variable approach to dependency between lines of business can be thought of as an alternative to the copula approach (Frees, Meyers and Cummings, Dependent Multi-Peril Ratemaking 2009). The copula approach would be especially relevant for capital adequacy testing. For ratemaking purposes, because copula regressions preserve the conditional on the covariates models and we are only interested in the expected values, the only place where the copula could affect the results is in the joint estimation of the conditional on the covariates and copula models. A natural way to approach this is through Maximum Likelihood estimation that requires parametric modeling. Given model uncertainty that is inherent in the selection of the distribution of Ultimate Loss Ratio, this approach is not preferred here.

[14] More on what we mean by Accident Year below.

[15] Academic works that explored the relative efficiency of different P/C insurers also made reference directly or indirectly to the NAIC data. For example, (A Note on the Relative Efficiency of Property-Liability Insurance Distribution Systems 1979) (Independent and Exclusive Agency Insurers: A Reexamination of the Cost Differential 1992), (The Coexistence of Multiple Distribution Systems for Financial Services: The Case of Property-Liability Insurance 1997), and (Long-tail Longitudinal Modeling of Insurance Company Expenses 2010).

American Property/Casualty insurance exposures. For our purposes, one interesting feature of the Schedule P is its relative stability through time.

From the Schedule P Part 1, we extracted 'Premium Earned Direct and Assumed'[16], hereafter referred to as Earned Premium or EP. We extracted Part 3, which covers Paid Loss and ALAE[17]. This information is particularly useful because the key determinant of the loss development pattern is driven by the line of business and not by the insurer. As a consequence, it was possible for us to compute[18] Loss Development Factors (to Ultimate, or ULDFs), by maturity, by line of business, for the industry as a whole.

We also extracted Part 2, which refers to incurred loss and ALAE[19]; however, early tests demonstrated that, for different insurers, for a given line of business, the development patterns could be qualitatively and materially different: therefore, we chose ultimately to not use this information. Finally, we extracted Part 5 Section 3 'Cumulative Number of Claims Reported Direct and Assumed at Year End' and Part 6 Section 1 'Cumulative Premiums Earned Direct and Assumed at Year End'. Again, we ultimately chose not to use the information. For the claim counts, we chose not to use the information because it was not available for all the lines of business that were of interest to us[20]. As for the Earned Premium triangle, we are content in using the latest valuation of the Earned Premium.

We chose to work with insurer groups instead of the individual entities that report to the NAIC. One motivation for doing so was that internal strategic considerations can lead insurer groups to selectively assign risks to different insurers and this assignment can vary through time for endogenous reasons. Another motivation for this choice is that there should be fewer insurers entering and exiting when looking at the industry at the insurer group level. Note that no specific

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[16] Earned Premium refers to main revenue source of P/C insurers. Written Premium corresponds to the value of policy sold, while Earned Premium refers to the accrual of revenues relating to sold policies. 'Direct and assumed' refers that the said sold insurance policies can have been sold to the public directly (direct) or to another insurer (assumed).
[17] 'Cumulative Paid Net Losses and Defense and Cost Containment Expenses Reported at Year End'. ALAE refers to Allocated Loss Adjustment Expenses.
[18] Using the Chain Ladder method (Werner and Modlin 2010, 105-109). One unfortunate aspect of the Chain Ladder method for Loss Development is the induced serial correlation of the residuals that results from the use of cumulative loss triangles. Generally, unaccounted for serial correlation of the residuals can lead to biased regression estimates. That being said, given that we are using the Chain Ladder method on the loss triangle generated by the industry as a whole and given that our covariates are more driven by the line of business than by the insurer, our Ultimate Loss estimates should not be materially inaccurate (taking into account the available information set). Also, whenever possible, for a given Accident Year, we use the latest available valuation, which is after 10 years for most lines of business (except Auto Physical Damage for which only 2 years of development is available.). In determining the latest valuation year, we have included a test that checks that the by insurer / line of business / accident year EP is not materially changing with new valuation. We have introduced this test because financial statements appear to be re-stated when the entities that form an insurer group change.
[19] 'Incurred Net Losses and Defense and Cost Containment Expense Reported at Year End'
[20] More on that topic is to come.

treatment was made for entering or exiting insurers but, as will be seen below, we do indirectly account for some forms of entries.

We excluded insurer/line of business/Accident Year, on a per-observation basis, where the Earned Premium was less than 1M nominal USD. The net effect of that exclusion was measured to be in the order of the one tenth of a percent. We did so because these records generate missing, negative or highly volatile measured Loss Ratios.

We chose to focus on selected lines of business found in the Table 1 below. Another party could easily extend our results to include all available lines of business.

Line of Business		
Reference Letter	Description	both Occurrence and Claims-Made
A	Homeowners/Farmowners	
B	Private Passenger Auto Liability/Medical	
C	Commercial Auto/Truck Liability Medical	
D	Workers' Compensation	
E	Commercial Multi-Peril	
F	Medical Professional Liability	Y
G	Special Liability (Ocean Marine, Aircraft (All Perils), Boiler and Machinery)	
H	Other Liability	Y
J	Auto Physical Damage	
R	Products Liability	Y

Occurrence liability policies refer to liability policies where coverage is determined as a function of the **occurrence dates** of the alleged wrong-doing of the insured.
Claims-made liability policies refer to liability policies where coverage is determined as a function of the **reporting date** of the alleged wrong-doing of the insured.

Table 1: Line of Business Listing

For readers that are familiar with the Rate Indications methodology (Werner and Modlin 2010, 71-80), please note that no rate or exposure changes were available for extraction. The absence of rate changes creates a less than ideal environment for forecasting. Falling back on the internal Projected Loss Ratio methodology, the change in Ultimate Loss Ratio can generally be thought of to be the result of a loss trend[21], a premium trend, rate changes and mix changes[22]. Contrary to the other effects, rate changes are primarily the result of overt actions taken by the insurer and are not as much subject to momentum effects[23]. If an insurer decided to pass a rate increase of +25%, we would expect the Ultimate Loss Ratio to immediately begin to fall; *vice versa* for a rate decrease. Therefore, we expect that we will encounter instances where our predicted Loss Ratios would be off because they will not reflect rate change information.

Note that, in using the 'simple' industry-wide by line of business Chain Ladder methodology[24] for loss development, we are putting ourselves in a situation similar to an actuary that was calibrating a traditional Experience Rating algorithm: a traditional Experience Rating algorithm will generally not have rating factors that change insured by insured. Rating algorithms cannot generally be calibrated to the individual insured, unless the account is so large as to be able to be self-ratable. Even so, our practical assumption here is that no individual insurer group is so large that no information from the other insurers is necessary to forecast its Projected Loss Ratio.

5.2 Descriptive Statistics

Before justifying the exact nature of our modeling choices, let's explore the data. Note that 'Year' always refers to Accident Year. This can be contrasted with Policy Year, that refers to the inception year of the insurance contract, and with Accounting Year, that refers to the year in which the revenue and losses were recognized for accounting purposes. Accident Year is generally preferred for most P/C actuarial purposes because many factors that affect losses are best accounted for using the date of the accident: *e.g.* seasonality relating to natural catastrophes or driving conditions. If Policy Year was available in the NAIC data, it might be appropriate for our uses, but Loss Development requires extra care. Accounting Year is not suitable for our purposes as the year in

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[21] That can be decomposed into a frequency and a severity trend.

[22] Mix changes sometimes refer to the effect of the change of the proportion of different types of insureds, instead here refers to 'other changes'.

[23] Contrast with loss trends that are largely due to the direct inflation associated with insured 'objects' and the indirect inflation of changing insured 'objects'.

[24] "The distinguishing characteristic of the development method is that ultimate claims for each accident year are produced from recorded values assuming that future claims' development is similar to prior years' development. In this method, the actuary uses the development triangles to track the development history of a specific group of claims. The underlying assumption in the development technique is that claims recorded to date will continue to develop in a similar manner in the future – that the past is indicative of the future. That is, the development technique assumes that the relative change in a given year's claims from one evaluation point to the next is similar to the relative change in prior years' claims at similar evaluation points." (Friedland 2010, 84)

which losses get recognized may have only to do with the timing of reserve changes and little with current policy wording, legal environment or general market conditions of the P/C insurance market.

First, as Figure 1 demonstrates, a key driver of an insurer Loss Ratio is the line of business mix[25], as different lines of business tend to have materially different Loss Ratios. With Figure 2, we can see that these differences are persistent through time.

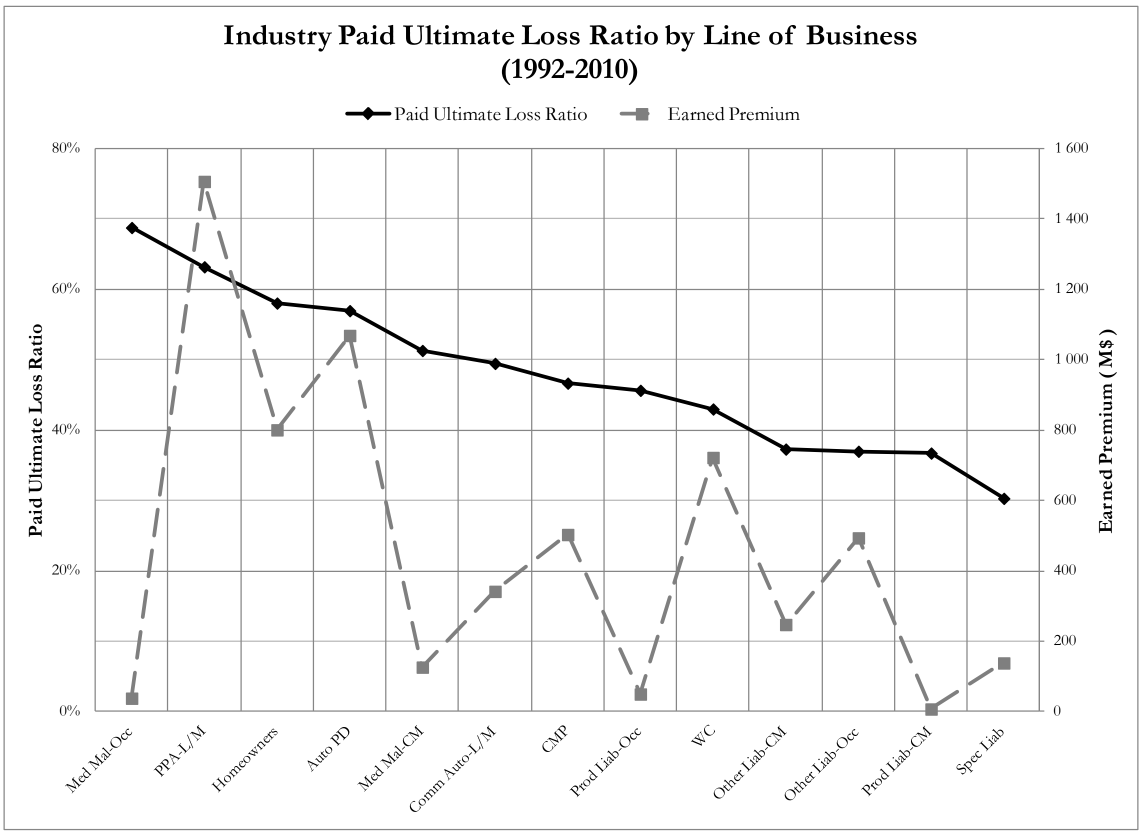

Figure 1: Industry Paid Ultimate Loss Ratio by Line of Business

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[25] Our modeling presumption is that the line of business mix of an insurer is either stable or predictably changing.

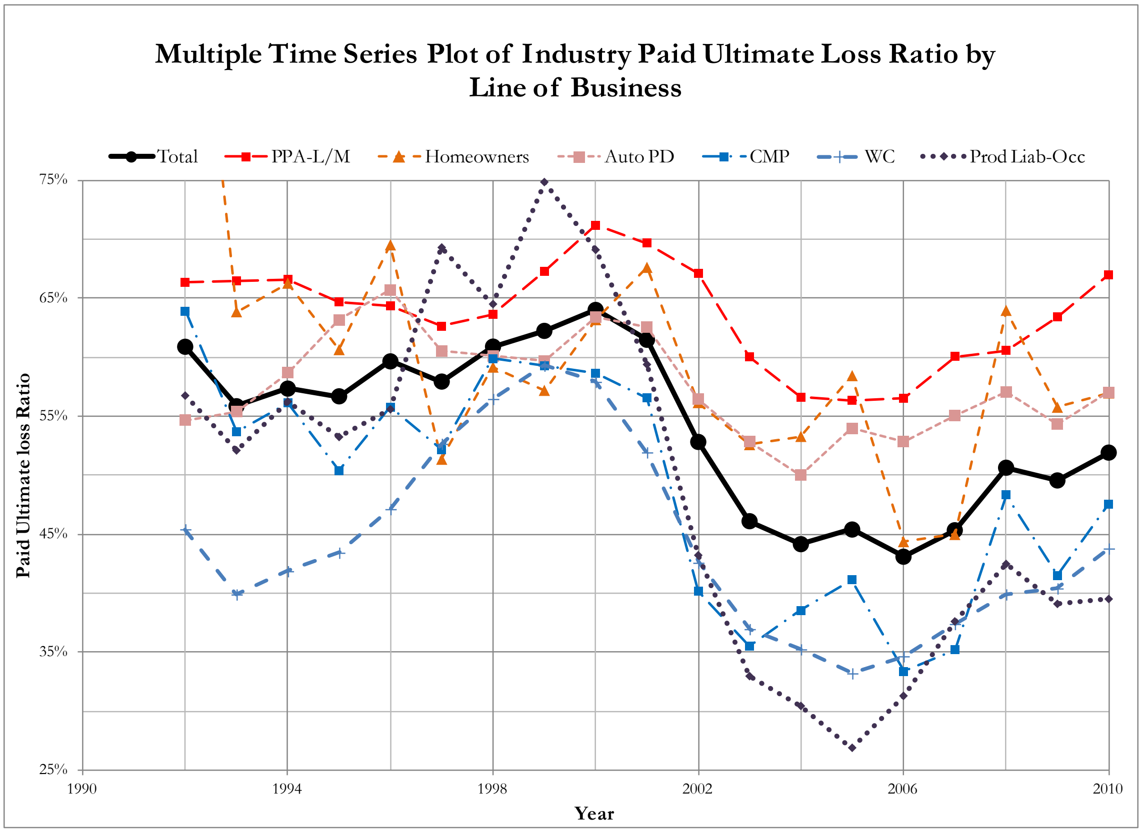


Figure 2: Multiple Time Series Plot of Industry Paid ULR by LOB

As the selected[26] lines of business (EP weighted) quantile and mean time series plots in Figures 3 and 4 demonstrate, the distribution of Ultimate Loss Ratio is fairly symmetric (if a little right-skewed) but heavier tailed than a Normal distribution in some years. For the Property lines of business, skewness and heaviness of the right tail can be affected by natural catastrophe like, for example, Hurricane Andrews.

[26] The plots for all lines of business were produced and can be presented upon request to the author. This applies for other presented plots also.

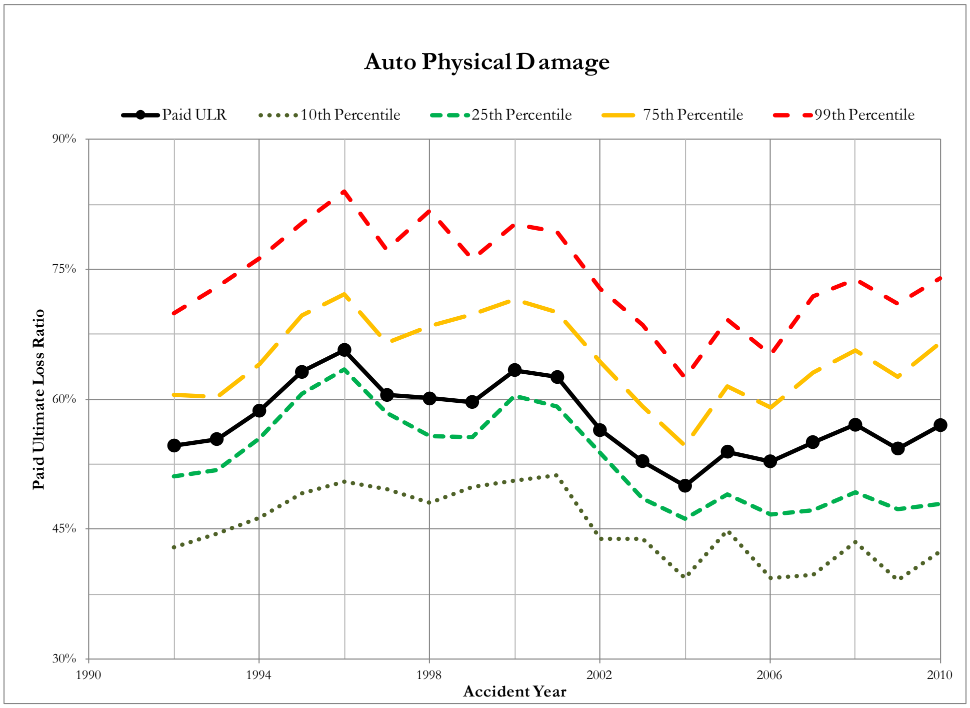


Figure 3: Multiple Time Series Plot of Features of the Auto PD Paid ULR Distribution

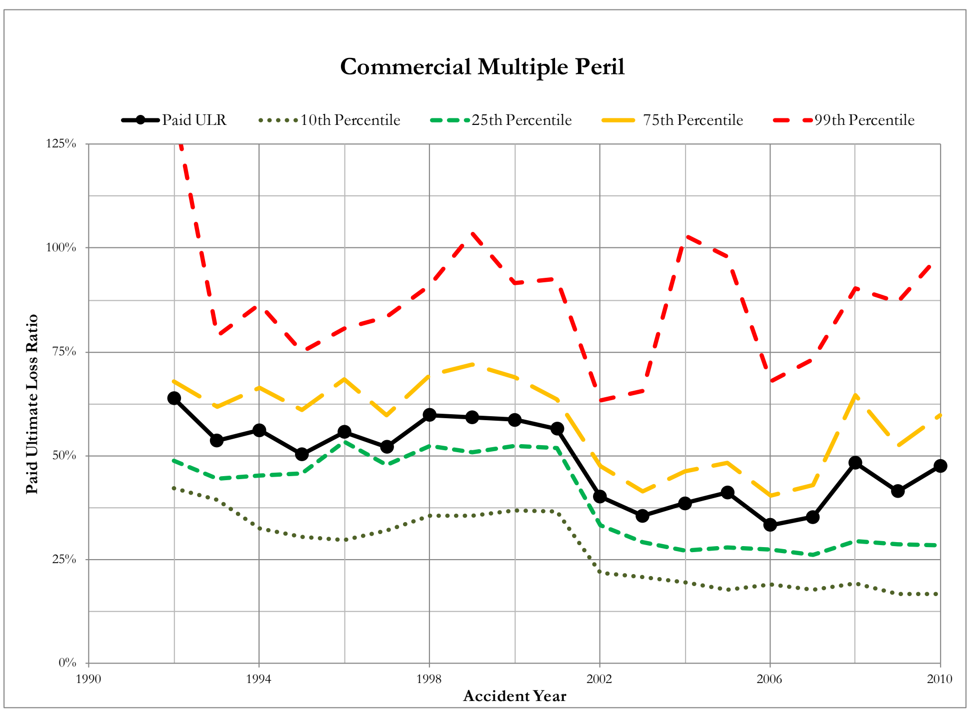


Figure 4: Multiple Time Series Plot of Features of the CMP Paid ULR Distribution

From the charts for the selected lines of business found in Figures 5 and 6, we can see that insurer Loss Ratio rankings are persistent through time, as the relative positions of the lines remain fairly stable.

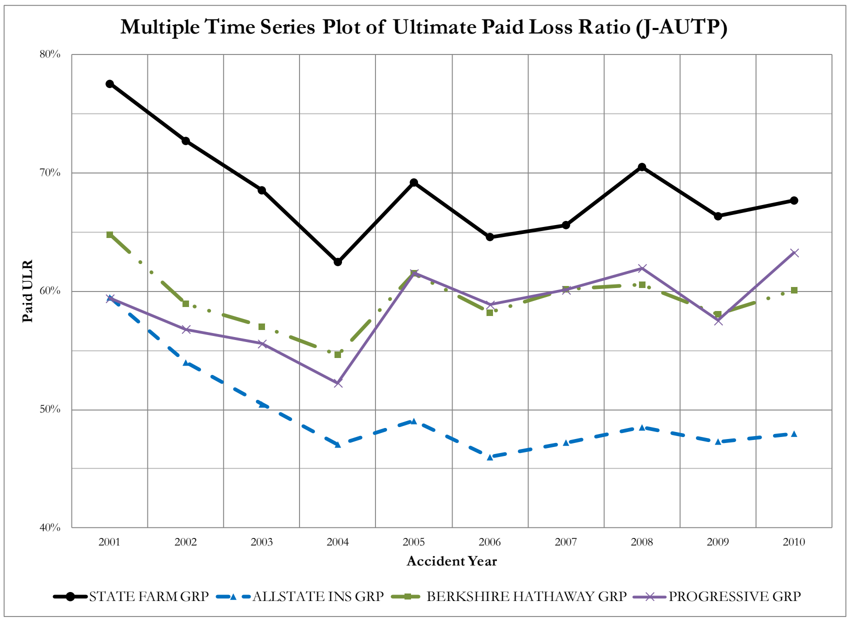


Figure 5: Multiple Time Series Plot of Paid ULR of Large Insurers: Auto PD

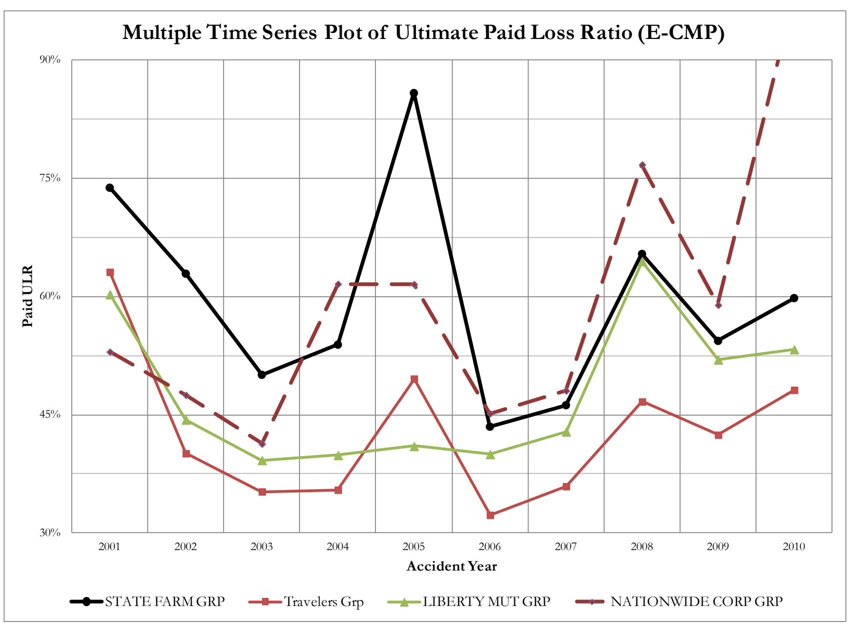


Figure 6: Multiple Time Series Plot of Paid ULR of Large Insurers: CMP

For the selected insurers found in Table 2, the tables below show summary statistics relating to the relativity of the ULR for the insurer group/line of business/Accident Year to the industry/line of business/Accident Year. We will model the Loss Ratio relativity instead of the Loss Ratio directly because we believe that a view on the future state of the P/C industry is generally easier to develop than a particular view for a given insurer group: this is analogous to why Experience Rating is generally calibrated with the practical assumption that Classification Rating has already appropriately reflected all factors other than claiming history. In that sense, Experience Rating can be thought of as the predictive modeling of the future profitability of an insurance account that uses the history of the 'individual' that has not already been accounted for by other known effects. In these cases, the Loss Ratio relativities of different lines of business appear quite linked: either because they have similar values or because the movements are correlated. The effect is quite general and applies to many other insurers, especially for the more important lines of business of larger insurers.

STATE FARM GRP

LOB	B_PPAL	J_AUTP	A_HMOW	TOTAL
EP (M$)	298 702	207 051	180 725	731 259
EP %	40.8%	28.3%	24.7%	100.0%
Avg Relativity	1.114	1.195	1.162	1.168

correlation with	J_AUTP	A_HMOW	EP
B_PPAL	16.0%	53.0%	298 702
J_AUTP		21.6%	207 051
EP	207 051	180 725	

PROGRESSIVE GRP

LOB	B_PPAL	J_AUTP	C_CA_L	TOTAL
EP (M$)	86 560	56 671	14 133	159 141
EP %	54.4%	35.6%	8.9%	100.0%
Avg Relativity	0.977	1.049	1.182	1.022

correlation with	J_AUTP	C_CA_L	EP
B_PPAL	64.4%	53.2%	86 560
J_AUTP		79.8%	56 671
EP	56 671	14 133	

Table 2: Typical Cross-Lines of Business Correlations
'B_PPAL' refers to line of business B 'Private Passenger Auto Liability/Medical', 'J_AUTP' refers to line of business J 'Auto Physical Damage', 'A_HMOW' refers to line of business A 'Homeowners/Farmowners', and 'C_CA_L' refers to lines of business C 'Commercial Auto/Truck Liability Medical'

In Table 3, we show (EP weighted) descriptive statistics by line of business for the Loss Ratio relativity for Accident Year 2006 as of 2010. Notice how the mean relativity is always 1.00: it is so by definition of a relativity. Notice also that comments made above about skewness and heavy-tails also apply here: the distributions are fairly symmetric but not quite Normal.

		Relativity					
			Percentile				
Line of Business Short-Hand	Line of Business Long Description	Avg	10th	25th	50th	75th	90th
B_PPAL	Private Passenger Auto Liability/Medical	1.00	0.83	0.98	1.03	1.10	1.12
J_AUTP	Auto Physical Damage	1.00	0.82	0.88	1.04	1.15	1.21
A_HMOW	Homeowners/Farmowners	1.00	0.77	0.88	1.01	1.09	1.32
D_WC__	Workers' Compensation	1.00	0.57	0.80	0.97	1.34	1.37
E_CMP_	Commercial Multi-Peril	1.00	0.56	0.90	1.08	1.23	1.32
H_OL_O	Other Liability - Occurrence	1.00	0.39	0.64	1.03	1.23	1.43
C_CA_L	Commercial Auto/Truck Liability Medical	1.00	0.57	0.93	1.03	1.17	1.31
H_OL_C	Other Liability - Claims-Made	1.00	0.28	0.59	1.31	1.34	1.59
G_SL__	Special Liability (Ocean Marine, Aircraft (All Perils), Boiler and Machinery)	1.00	0.31	0.70	0.87	1.32	1.77
R_PL_O	Products Liability - Occurrence	1.00	0.17	0.50	0.95	1.09	1.90
R_PL_C	Products Liability - Claims-Made	1.00	0.11	1.06	1.09	1.25	1.25
F_MM_C	Medical Professional Liability - Claims-Made	1.00	0.39	0.84	0.90	1.24	1.59
F_MM_O	Medical Professional Liability - Occurrence	1.00	0.43	0.67	1.11	1.29	1.66

Table 3: Summary Statistics, by Line of Business, of the Relativity Variables (AY 2006, as of 2010)

6. STATISTICAL ANALYSIS

We are finally at the point where we can discuss the fitted models. Our modeling approach was not entirely dissimilar to data mining, in as much as many models were fitted and compared. To fit our models, we used the SAS `proc mixed` procedure. The procedure was used on a line by line basis: that is, a model was fitted for each line of business. Only Accident Years 1997 to 2006 were used for the purposes of model selection. Even if the 1992 to 1996 Accident Years are known to us, given that we want to preserve inter-model comparability and that we will allow the use of up to 5 years of same line prior relativities, we need to start using our data starting in the 1997 Accident Year. We chose to stop using data past the 2006 Accident Year in an effort to balance responsiveness and stability (Werner and Modlin 2010, 80): using more recent data would increase responsiveness to current conditions but would be counter-balanced by the fact that more recent Ultimate Loss Ratios have a much greater portion that is estimated, as opposed to realized[27].

For model selection purposes, we used the empirical estimates of the standard errors (SAS n.d.), to allow for model misspecification. The model was used using the Residual Maximum Likelihood approach. The Bayesian Information Criterion was used for model selection as, among the common information criteria, it is the one that most penalizes for extra variables. For our purposes, it was particularly important to favor parsimony, as many hundreds of models were attempted for each line of business.

Again, to ensure comparability of models, we ensured that none of the covariates were missing by initializing them to a neutral value if they were otherwise missing and adding an indicator variable to indicate that the value was missing. From the fitted line of business, we included up to 5 prior realizations of the Loss Ratio relativity. We selected 3 lines of business that have similar strategies, clients or perils and included up to 2 years of prior realized relativities for those selected lines of business. As mentioned above, we have included EP-based interaction terms[28]. Interaction terms relating to older data were not included in the attempted models unless including the interaction term from more recent data improved the model. We allowed for serial correlation between intra-insurer residuals using a `repeated` statement (SAS n.d.). A Toeplitz specification was used because of the implied homoskedasticity and the flexible correlation structure. For forecasting, the implied homoskedasticity is particularly convenient, because we would otherwise first have to forecast the variability of the future relativities and that variability is largely determined by random factors, especially for Property lines of business. For the Toeplitz specification, we allowed ourselves a window of up to 5 years, consistent with our modeling choice for the Auto-Regressive component of the model. The regression was a weighted regression: with Earned Premium used as weights[29]. The choice of weights was not due to statistical efficiency considerations, but rather due to economic relevance considerations. As such, even if weights are used, the regression should not be

[27] Relating to footnote 10, we have chosen to always use the latest available maturity rather than demand that a minimum maturity. Doing so, we may create a bias because the ULDFs may not be as equally appropriate for all insurers, especially at early durations. We could have demanded a minimum maturity, but that would imply dropping a material quantity of data from the analysis. We could also use the always available earliest maturity, but that would imply that our modeled quantity is basically always an estimate and never materially realized.

[28] We used interaction terms based on $\log_{10} EP$, which is non-linear in EP.

[29] One may wonder at our modeling choice of including EP in both the interaction terms and the weights. Both uses have different rationales that are not mutually inconsistent. As mentioned above, the use of EP in the weights is aimed at reflecting the economic importance of the fit. The use of EP in the interaction terms is aimed at varying the models for different insurer size, just like in traditional credibility models. In effect, this implies that different models are fitted for different insurer sizes, but also that, among insurers of similar size, the bigger ones count more towards model fitting.

construed as a first approximation to a Feasible Generalized Least Squares estimator, but rather as an Ordinary Least Squares regression that puts equal weight on all dollars of Earned Premium[30]. A table summarizing the selected best fitting models is presented in Table 4. The fitted values for the parameters are also presented in Appendix 1.

Line of Business Short-Hand	B_PPAL	J_AUTP	A_HMOW	D_WC__	E_CMP_	H_OL_O	C_CA_L	H_OL_C	G_SL__	R_PL_O	R_PL_C	F_MM_C	F_MM_O
Line of Business Long Description	Private Passenger Auto Liability/Medical	Auto Physical Damage	Homeowners/Farmowners	Workers' Compensation	Commercial Multi-Peril	Other Liability - Occurrence	Commercial Auto/Truck Liability Medical	Other Liability - Claims-Made	Special Liability (Ocean Marine, Aircraft (All Perils), Boiler and Machinery)	Products Liability - Occurrence	Products Liability - Claims-Made	Medical Professional Liability - Claims-Made	Medical Professional Liability - Occurrence
Covariates	intercept	intercept	intercept	intercept	intercept	intercept	intercept	intercept	intercept	intercept	intercept	intercept	intercept
	same line - lag 1	same line - lag 1	interaction term - intercept	interaction term - intercept	same line - lag 1	interaction term - intercept	interaction term - intercept	same line - lag 1	interaction term - intercept	same line - lag 1	interaction term - intercept	same line - lag 1	interaction term - intercept
	interaction term - same line - lag 1	interaction term - same line - lag 1	same line - lag 1	same line - lag 1	same line - lag 2	same line - lag 1	same line - lag 1	same line - lag 2	same line - lag 1	interaction term - same line - lag 1	line H_OL_O - lag 1	interaction term - same line - lag 1	same line - lag 1
	same line - lag 2	same line - lag 2	interaction term - same line - lag 1	interaction term - same line - lag 1	same line - lag 3	interaction term - same line - lag 1	interaction term - same line - lag 1	line E_CMP_ - lag 1	interaction term - same line - lag 1	same line - lag 2	interaction term - line H_OL_O - lag 1	same line - lag 2	interaction term - same line - lag 1
	same line - lag 3	interaction term - same line - lag 2	same line - lag 2	line C_CA_L - lag 1	line C_CA_L - lag 1	same line - lag 2	same line - lag 2	interaction term - line E_CMP_ - lag 1	same line - lag 2	same line - lag 3	line H_OL_O - lag 2	same line - lag 3	same line - lag 2
	same line - lag 4	same line - lag 3	interaction term - same line - lag 2	interaction term - line C_CA_L - lag 1	interaction term - line C_CA_L - lag 1	interaction term - same line - lag 2	interaction term - same line - lag 2	line D_WC__ - lag 1	interaction term - same line - lag 2	same line - lag 4	line D_WC__ - lag 1	same line - lag 4	same line - lag 3
	same line - lag 5	line B_PPAL - lag 1	same line - lag 3	line E_CMP_ - lag 1	line D_WC__ - lag 1	same line - lag 3	same line - lag 3		same line - lag 3		interaction term - line D_WC__ - lag 1	same line - lag 5	same line - lag 4
	line J_AUTP - lag 1	line C_CA_L - lag 1	same line - lag 4		line D_WC__ - lag 2	interaction term - same line - lag 3	same line - lag 4		interaction term - same line - lag 3		line C_CA_L - lag 1	line F_MM_O - lag 1	line F_MM_C - lag 1
	interaction term - line J_AUTP - lag 1	interaction term - line C_CA_L - lag 1	line B_PPAL - lag 1			same line - lag 4	same line - lag 5		same line - lag 4		interaction term - line C_CA_L - lag 1	line F_MM_O - lag 2	interaction term - line F_MM_C - lag 1
	line J_AUTP - lag 2		interaction term - line B_PPAL - lag 1			interaction term - same line - lag 4	line D_WC__ - lag 1		interaction term - same line - lag 4		line C_CA_L - lag 2	line G_SL__ - lag 1	line F_MM_C - lag 2
			line J_AUTP - lag 1			same line - lag 5	interaction term - line D_WC__ - lag 1		same line - lag 5			interaction term - line G_SL__ - lag 1	interaction term - line F_MM_C - lag 2
			interaction term - line J_AUTP - lag 1			interaction term - same line - lag 5	line D_WC__ - lag 2		interaction term - same line - lag 5			line G_SL__ - lag 2	line G_SL__ - lag 1
			line E_CMP_ - lag 1			line E_CMP_ - lag 1	line J_AUTP - lag 1		line D_WC__ - lag 1			interaction term - line G_SL__ - lag 2	interaction term - line G_SL__ - lag 1
			interaction term - line E_CMP_ - lag 1			interaction term - line E_CMP_ - lag 1	line J_AUTP - lag 2		interaction term - line D_WC__ - lag 1			line H_OL_C - lag 1	line G_SL__ - lag 2
			line E_CMP_ - lag 2			line D_WC__ - lag 1			line C_CA_L - lag 1			line H_OL_C - lag 2	interaction term - line G_SL__ - lag 2
			interaction term - line E_CMP_ - lag 2			interaction term - line D_WC__ - lag 1			interaction term - line C_CA_L - lag 1				line H_OL_O - lag 1
Intra-insurer Group Covariance Structure for the Residuals	TOEP(5)	TOEP(2)	TOEP(2)	TOEP(5)	TOEP(5)	TOEP(5)	TOEP(4)	TOEP(5)	TOEP(5)	TOEP(4)	TOEP(3)	TOEP(5)	TOEP(2)

Table 4: Table of Best Fitting Models

Add text: notice that many models include terms for other LOBs as well as interaction terms

At this point, it is worthwhile to mention that (1) the past (relative) results of other lines of business generally are significantly influential and (2) many coefficients statistically vary with insurer size. That other lines of business are predictive supports our expectations that lines with similar clients, perils or strategies should move together. That coefficients vary with insurer size is consistent with the expectations formed by a century of developments in credibility theory.

We are now in a position to comment on the quality of the best fitting models. As can be seen in the selected exhibits in Figures 7 and 8, the fitted Loss Ratio relativities generally preserve the relativity for the insurer in a given line of business. Given that the estimators are of the Auto-Regressive Moving Average family, they suffer from the same defect: the predicted values lag behind

[30] Which are hopefully roughly proportional to the underlying insurance exposure. Another alternative might have been to put equal weight on all insureds. This measure would be even less perfect than our chosen weights as, even though the Loss Ratio varies by line of business, it does so materially less than the loss cost does across lines of business.

if a trend is present. Again, if rate changes were known, the hope is that this particular shortcoming could be dampened.

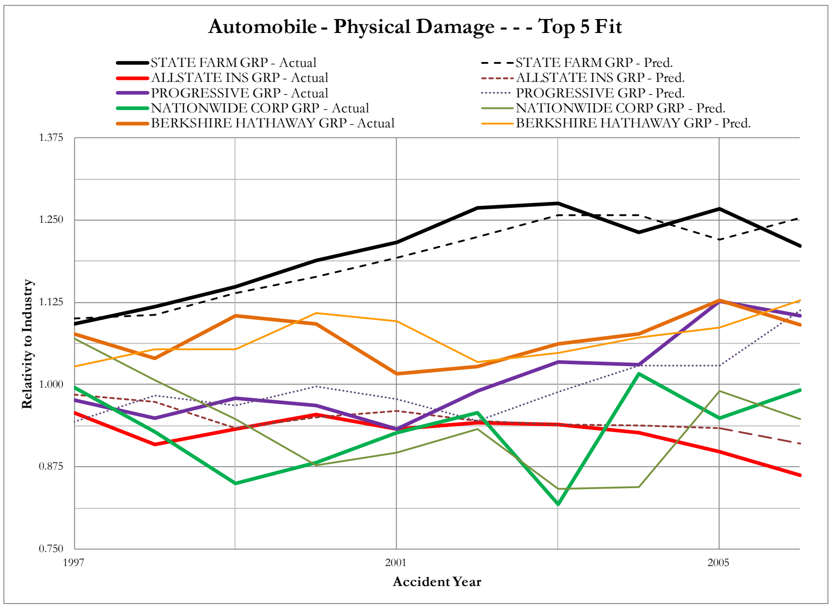


Figure 7: Multiple Comparative Time Series Plot of Large Insurer Actual *vs*. Predicted Relativities (AY 1997-2006, Auto PD)

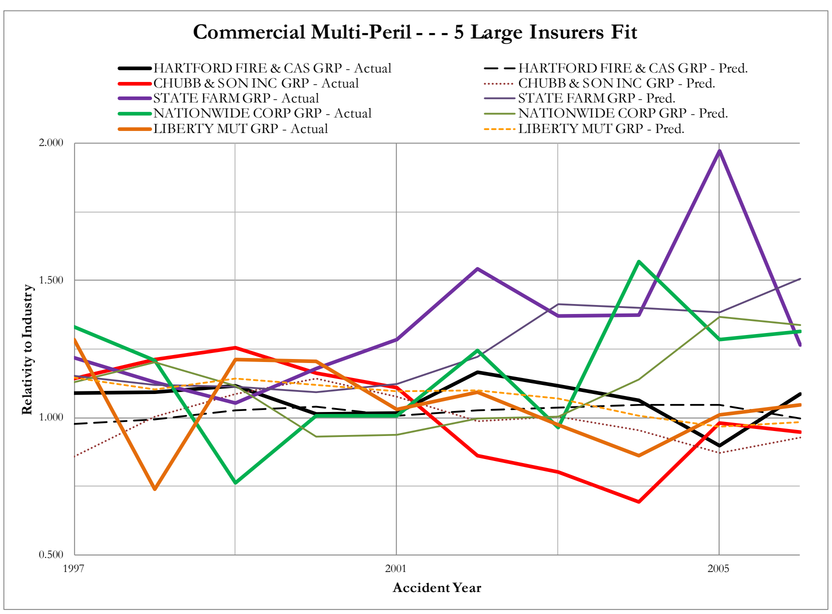
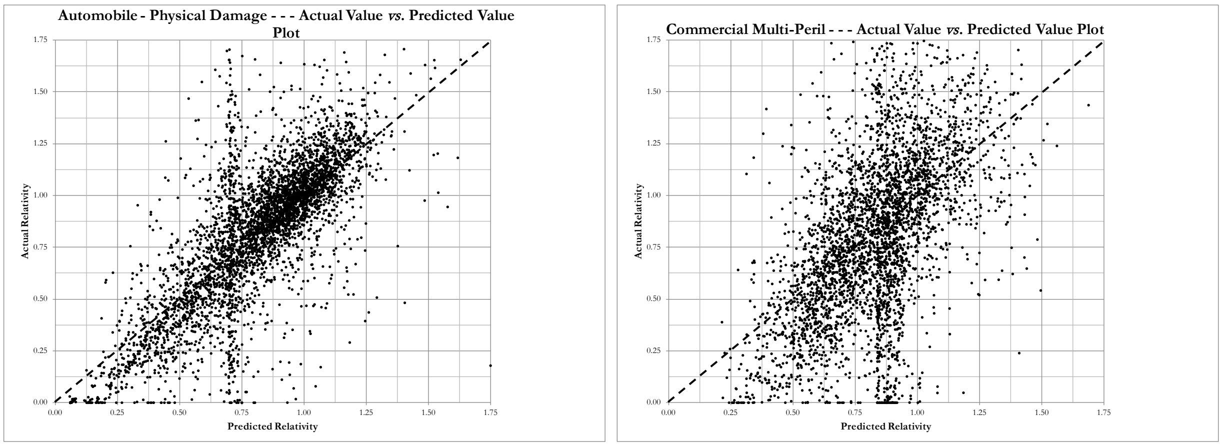
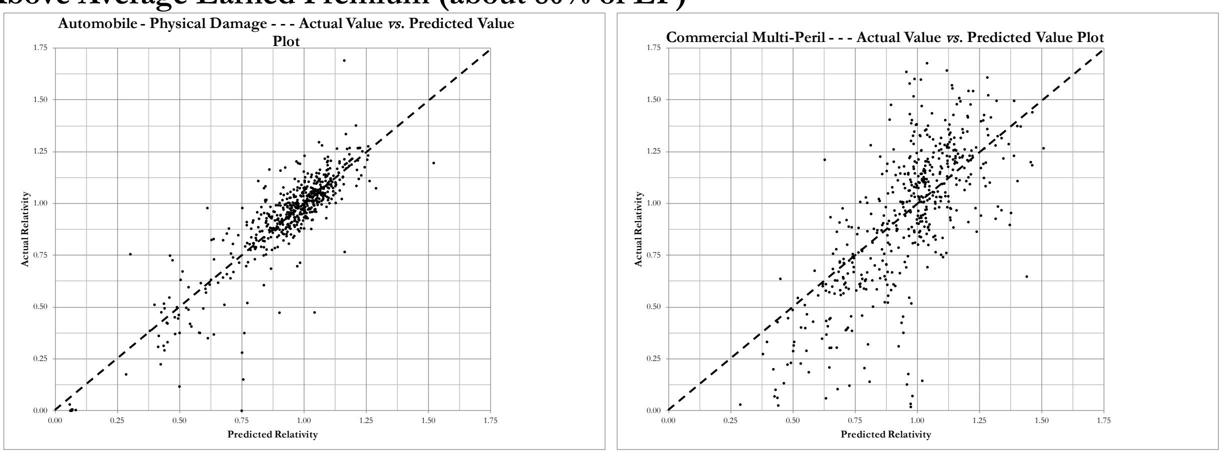


Figure 8: Multiple Comparative Time Series Plot of Large Insurer Actual *vs*. Predicted Relativities (AY 1997-2006, CMP)



Above Average Earned Premium (about 80% of EP)



Below Average Earned Premium

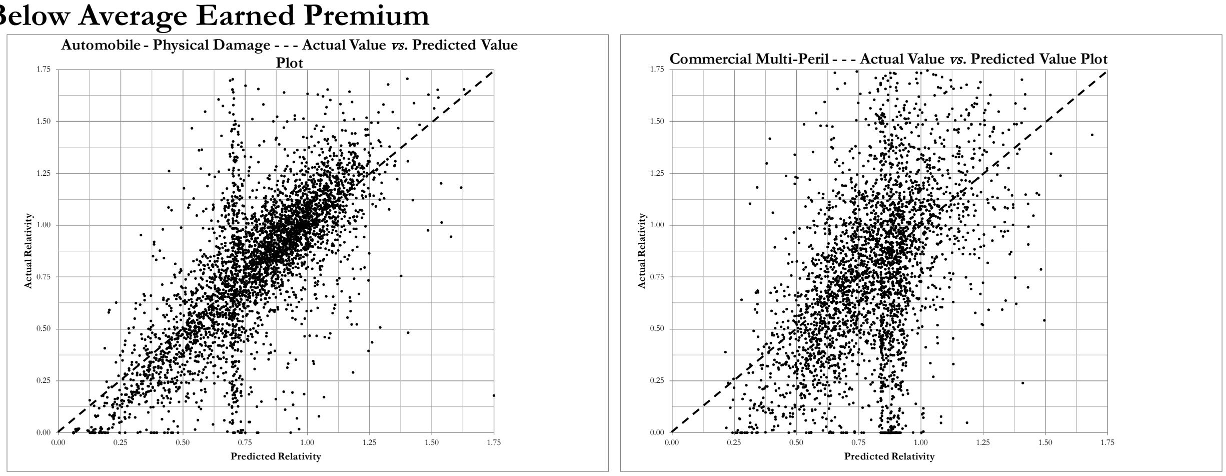
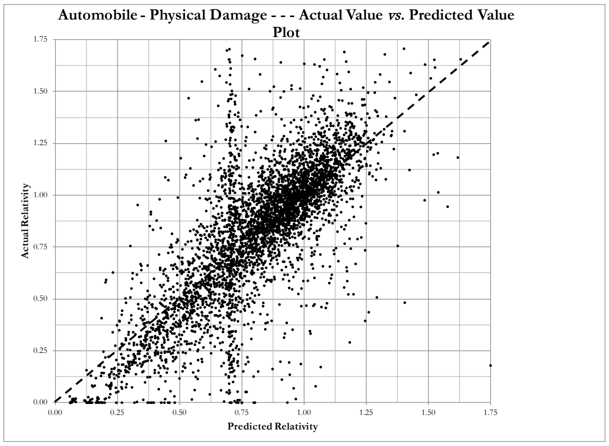
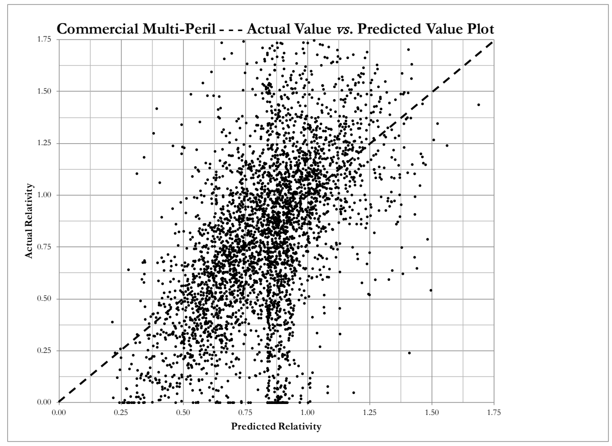
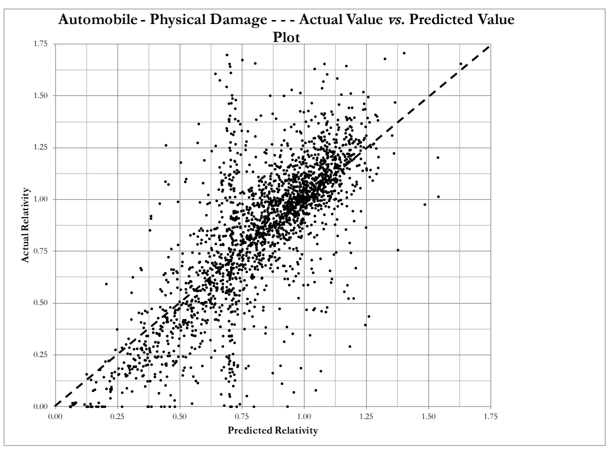


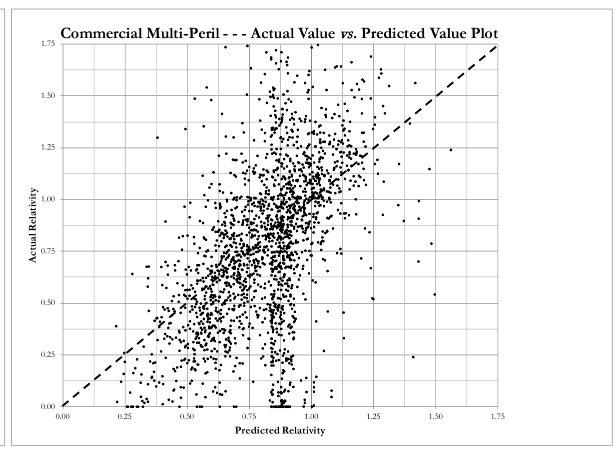
Figure 9: Actual *vs.* Predicted Relativity Plots (AY 1997-2006, Auto PD and CMP)



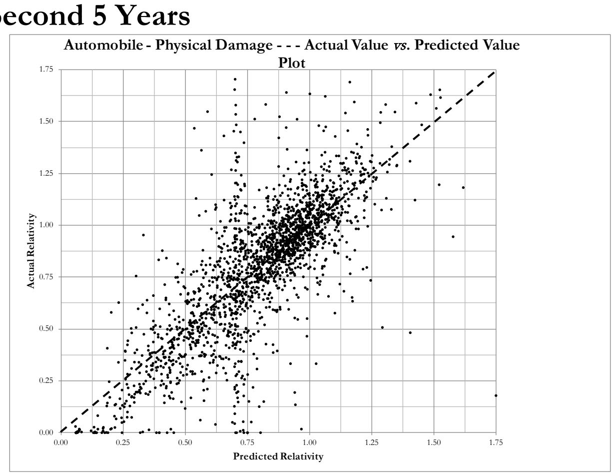


First 5 Years





Second 5 Years



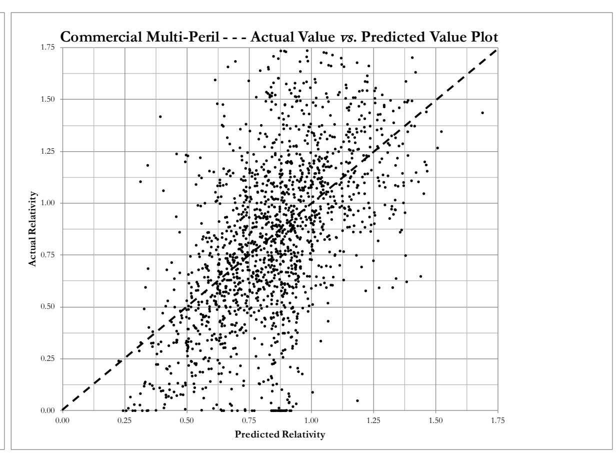
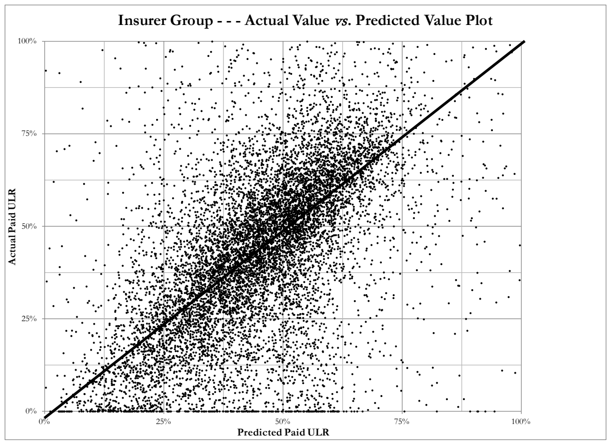


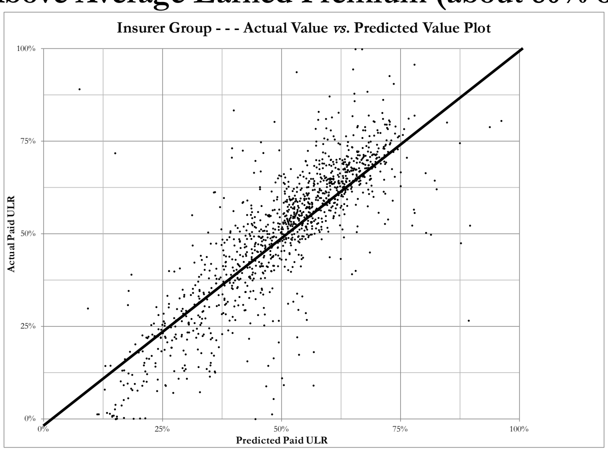
Figure 10: Actual *vs.* Predicted Relativity Plots (AY 1997-2006, Auto PD and CMP)

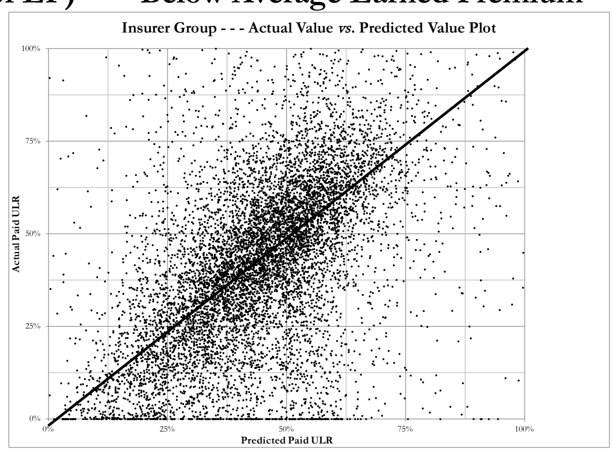
Insurer Group	2004	2005	2006	Grand Total	Last 5 Years	Last 3 Years	Last 2 Years
STATE FARM GRP							
Earned Premium	45 940 231	46 249 303	47 103 431	392 096 073	223 009 968	139 292 965	93 352 734
Actual ULR	62.6%	71.2%	60.2%	69.3%	66.6%	64.7%	65.7%
Pred. ULR	62.9%	66.4%	62.5%	68.5%	66.0%	63.9%	64.4%
ULR Diff.	**-0.2%**	4.8%	-2.3%	**0.7%**	**0.6%**	**0.7%**	1.2%
ALLSTATE INS GRP							
Earned Premium	25 066 983	26 263 811	26 899 116	223 012 544	124 437 058	78 229 910	53 162 927
Actual ULR	57.1%	64.9%	50.9%	61.9%	58.1%	57.6%	57.8%
Pred. ULR	55.6%	60.5%	52.3%	61.2%	57.6%	56.1%	56.4%
ULR Diff.	1.5%	4.3%	-1.4%	**0.7%**	**0.4%**	1.4%	1.4%
AMERICAN INTL GRP							
Earned Premium	31 729 086	31 795 842	32 810 846	193 972 203	135 646 692	96 335 774	64 606 688
Actual ULR	39.0%	42.4%	46.4%	47.1%	43.0%	42.6%	44.4%
Pred. ULR	39.3%	42.0%	42.2%	45.3%	40.8%	41.2%	42.1%
ULR Diff.	**-0.4%**	**0.4%**	4.1%	1.9%	2.3%	1.4%	2.3%
NATIONWIDE CORP GRP							
Earned Premium	13 443 304	14 516 434	15 245 735	124 270 757	72 470 460	43 205 473	29 762 169
Actual ULR	56.4%	52.9%	51.3%	54.8%	51.5%	53.4%	52.1%
Pred. ULR	48.1%	54.4%	50.3%	55.5%	51.5%	51.0%	52.3%
ULR Diff.	**8.3%**	-1.6%	1.0%	**-0.7%**	**-0.1%**	2.4%	**-0.2%**
LIBERTY MUT GRP							
Earned Premium	15 387 164	15 804 196	16 817 465	114 148 362	74 317 891	48 008 825	32 621 661
Actual ULR	41.0%	42.4%	40.3%	51.1%	43.0%	41.2%	41.3%
Pred. ULR	43.3%	43.2%	41.6%	53.0%	45.3%	42.7%	42.4%
ULR Diff.	-2.3%	**-0.8%**	-1.3%	-1.9%	-2.3%	-1.4%	-1.0%
Travelers Grp							
Earned Premium	19 350 200	18 734 052	18 869 201	110 119 067	73 768 428	56 953 453	37 603 253
Actual ULR	36.8%	41.2%	36.0%	48.7%	40.0%	38.0%	38.6%
Pred. ULR	39.1%	41.2%	39.6%	47.9%	41.0%	39.9%	40.4%
ULR Diff.	-2.3%	**0.1%**	-3.6%	**0.8%**	-1.0%	-1.9%	-1.8%
BERKSHIRE HATHAWAY GRP							
Earned Premium	12 243 116	14 124 855	15 327 065	91 477 201	62 223 013	41 695 036	29 451 920
Actual ULR	52.2%	51.2%	52.7%	58.7%	53.6%	52.0%	52.0%
Pred. ULR	51.9%	55.3%	52.7%	59.7%	54.9%	53.4%	54.0%
ULR Diff.	**0.3%**	-4.2%	**0.0%**	-1.0%	-1.3%	-1.3%	-2.0%
PROGRESSIVE GRP							
Earned Premium	13 386 629	13 959 011	14 233 252	91 080 785	62 134 914	41 578 892	28 192 263
Actual ULR	52.0%	57.3%	57.4%	57.9%	55.8%	55.6%	57.4%
Pred. ULR	55.0%	56.4%	58.3%	59.7%	56.9%	56.6%	57.4%
ULR Diff.	-3.0%	**0.9%**	**-0.9%**	-1.8%	-1.1%	**-1.0%**	**0.0%**
HARTFORD FIRE & CAS GRP							
Earned Premium	9 541 892	10 317 618	10 714 211	74 954 966	46 764 468	30 573 721	21 031 829
Actual ULR	43.7%	42.4%	43.0%	52.1%	45.0%	43.0%	42.7%
Pred. ULR	43.7%	45.2%	40.3%	50.7%	44.6%	43.0%	42.7%
ULR Diff.	**0.1%**	-2.8%	2.7%	1.3%	**0.4%**	**0.0%**	**0.0%**
Ace Ltd Grp							
Earned Premium	5 123 521	6 210 710	6 340 060	34 039 948	25 736 712	17 674 291	12 550 770
Actual ULR	22.3%	23.2%	20.7%	28.4%	23.3%	22.0%	22.0%
Pred. ULR	22.0%	25.0%	26.0%	31.7%	25.9%	24.5%	25.5%
ULR Diff.	**0.3%**	-1.8%	**-5.2%**	-3.3%	-2.6%	-2.4%	-3.5%

Table 5: Overall Insurer Back Testing of Actual *vs*. Predicted Relativity

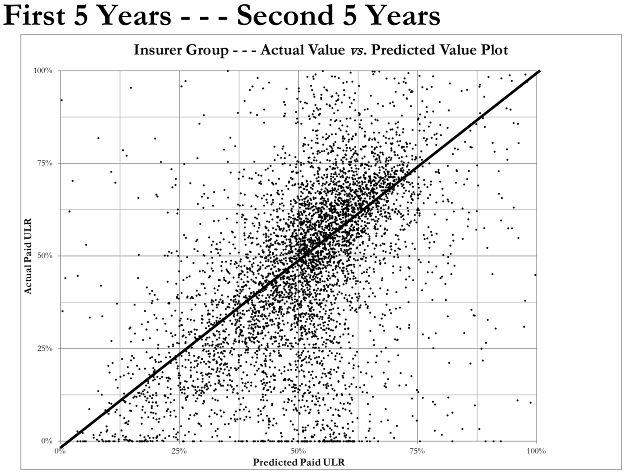


Above Average Earned Premium (about 80% of EP) - - - Below Average Earned Premium





First 5 Years - - - Second 5 Years



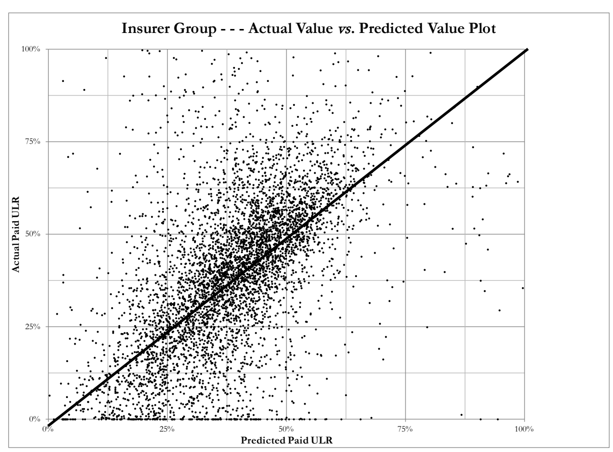


Figure 11: Actual *vs.* Predicted Paid ULR Plots AY 1997-2006, all LOB combined)

As can be seen in Figure 9, we show the plot the actual values against the predicted values. An ideal model would have all of its points lining up on the 45° line. We also show the residual plot for larger and smaller insurer separately. As can be seen from the graphs, the models seem equally valuable for both larger and smaller insurers. For smaller insurers, there is a cluster for predicted value that is apparent in both graphs. These clusters are due to the way missing covariates were treated. Clearly, the variance of the residuals is affected by the size of the insurers but, other than for the cluster of missing covariates, the conditional variance seems unaffected by the predicted value. Given that the predicted values are a form of an average, the increased variability of the residuals for smaller insurers is not unexpected[31]. Note also the quality of the fit seems equally good for the first 5 and second 5 years of the 10 year horizon that is considered.

As can be seen in Figure 10, we show the actual *versus* predicted relativity graphs for two sub-periods (the first 5 years and the second 5 years) and we find that the models appear equally valid for each sub-periods.

In Table 5, we also present the actual *versus* predicted Loss Ratios at the insurer-group level. Contrary to a future forecast, this back-testing exercise starts with the realized industry/line of business/Accident Year Ultimate Loss Ratio as its basis[32]. As can be seen, the fits are generally good. The author is unaware of a statistical study that would allow for a comparison with the performance of Loss Ratio projection methods that rely only on internal data. The author conjectures that internal budgets can be missed by several Loss Ratio points and so not only because of undue aggressiveness or conservatism. Interestingly, the estimator seems to perform even better when several years are compared together. Therefore, for most of the purposes mentioned in the introduction, the proposed forecasting methodology seems particularly relevant.

Figure 11 presents the actual *versus* predicted paid ULR by insurer group and Accident Year. We present it overall, for smaller and larger insurers separately, and for the first and last 5 years. The findings are similar to those found by line of business.

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[31] In a follow-up to this project, this will be pursued.

[32] This leads us to schedule more appropriate out-of-sample performance testing for a further phase of the project. We intend to use (Frees, Meyers and Cummings, Summarizing Insurance Scores Using a Gini Index 2011) as a basis for that research.

7. ANALOGY TO CALIBRATION OF EXPERIENCE RATING PLANS

With these promising results in hand, we can now come back and comment on some guidance that can be given for the calibration of a more traditional Experience Rating plan.

Regarding the definition of an individual under the rating plan, we have proposed to use a definition of an individual as an entity that exercises control over activities that influence the loss potential: the insurer-group in our example.

Regarding the selection of the used number of years of experience, we have proposed to use semi-parametric predictive modeling[33] to make that selection but, just like would be done in practice, we have chosen not to use information past a certain age for practical reasons. Also, like in practice, we developed some rules to allow us to deal with missing information.

Regarding the issue of how to best reflect Loss Development, we proposed to always use the latest available valuation and use a definition of claim that makes the Loss Development pattern most similar across individuals.

Regarding the most adequate formula for the Experience Rating modification, we have departed from tradition to the extent that we have proposed a formula that did not incorporate explicit credibility considerations. We feel that, while credibility-type formulas can have the advantage of parsimony of the rating factors that need to vary by size of account, our coefficient-based approach can be quite parsimonious and has the benefit that it can be explained in terms of the more widely known regression framework. Nonetheless, even though our proposed models were inspired in part by fixed-effects regression, a traditional credibility interpretation is possible because an intercept term was always included. In this case, the complement of credibility is effectively always 1.0: that is, the overall average relativity. All the models can then always be re-written as $z \cdot \overline{rel_{i,t}} + (1 - z) \cdot 1$, where $\overline{rel_{i,t}}$ refers to a weighted[34] average of past own line and other lines past Loss Ratio relativities and z is the credibility.

We have forgone commenting on the issue of trending and on-leveling[35], as well as on the issue of loss capping. Regarding the issue of trending and on-leveling, we recognize the value of creating an estimate of the losses and premium as if they were experienced in the current period.

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[33] In that sense, our approach is not entirely unlike the one proposed by (Bailey and Simon 1959).
[34] Where the weights can vary with the size of the insurer.
[35] Because we used a relativity approach and because we allowed for a flexible structure for the residuals.

Regarding loss capping, we believe that the proposed predictive framework would be as valuable in the selection of the appropriate loss capping as it was to us in the selection of the loss experience horizon.

We have also proposed that the experience from other lines of business could potentially carry information. To our knowledge, that has not been commonly been incorporated in Experience Rating algorithms.

There remains the issue of whether an Experience Rating algorithm needs to (approximately) balance to a 1.0 relativity. The author mentions the issue because he is aware of many plans where the average debit/credit is not 0.0%. From a logical point of view, an Experience Rating scheme that does not balance to a 1.0 relativity implies that the classification rates are not adequate. Although this is not inconsistent as such, it implies incoherence in the rating algorithm. A non-balanced Experience Rating plan is more likely to occur if the when the credibility/size of account is correlated with the bias in the classification rates.

8. CONCLUSION

For this research project, we have chosen to present a close simile to the calibration of an experience rating scheme that could be used for Loss Ratio projection purposes for a party external to an insurer. Doing so allowed us to comment on practical modeling choices that would need to be made by a practicing actuary calibrating an experience rating scheme. We have departed from the traditional credibility-type approach to experience rating to instead anchor ourselves in a predictive modeling approach. We modeled the relativity to the industry Loss Ratio by Accident Year and Line of Business and found that, generally, (1) the own line of business past results were relevant predictors with factors varying by size of insurer, and (2) that past results in lines of business with similar clients, perils or strategies were also relevant predictors, again with factors potentially varying with the size of the insurer.

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9. REFERENCES

Bailey, Robert A., and LeRoy J. Simon. "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car." *Proceedings of the Casualty Actuarial Society* XLVI (1959): 159-164.

Barrese, James, and Jack M. Nelson. "Independent and Exclusive Agency Insurers: A Reexamination of the Cost Differential." *Journal of Risk and Insurance* 59, no. 3 (1992): 375-397.

Berger, Allen N., J. David Cummins, and Mary A. Weiss. "The Coexistence of Multiple Distribution Systems for Financial Services: The Case of Property-Liability Insurance." *Journal of Business* 70, no. 4 (October 1997): 515-546.

Boor, Joseph A. "The Complement of Credibility." *Proceedings of the Casualty Actuarial Society* LXXXIII, no. 158 (May 1996): 1-32.

Couret, Jose, and Gary Venter. "Using Multi-Dimensional Credibilityto Estimate Class Frequency Vectors in Workers Compensation." *ASTIN Bulletin* 38, no. 1 (May 2008): 72-85.

Cummins, J. David, and Jack VanDerhei. "A Note on the Relative Efficiency of Property-Liability Insurance Distribution Systems." *Bell Journal of Economics* 10, no. 2 (Autumn 1979): 708-719.

Dorweiler, Paul. "A Survey of Risk Credibility in Experience Rating." *Proceedings of the Casualty Actuarial Society* XXI, no. I (November 1934): 90-114.

Frees, Edward W. *Longitudinal and panel data: analysis and applications in the social sciences.* New York, New York: Cambridge Univ Press, 2004.

Frees, Edward W., and Ping Wang. "Credibility Using Copulas." *North American Actuarial Journal* 9, no. 2 (2005): 31-48.

Frees, Edward W., Glenn Meyers, and A. David Cummings. "Dependent Multi-Peril Ratemaking." 2009.

Frees, Edward W., Glenn Meyers, and A. David Cummings. "Predictive Modeling of Multi-Peril Homeowners." 2011: 33.

Frees, Edward W., Glenn Meyers, and A. David Cummings. "Summarizing Insurance Scores Using a Gini Index." *Journal of the American Statistical Association* 106 (2011): 1085-1098.

Frees, Edward W., Virginia R. Young, and Yu Luo. "A Longitudinal Data Analysis Interpretation of Credibility Models." *Insurance: Mathematics and Economics* 24 (1999): 229-247.

Friedland, J. F. *Estimating Unpaid Claims Using Basic Techniques.* 3rd. Casualty Actuarial Society, 2010.

Gillam, William R. "Workers' Compensation Experience Rating: What Every Actuary Should Know." *Proceedings of the Casualty Actuarial Society* LXXIX (1992): 215-239.

Gillam, William R., and R. H. Snader. "Fundamentals of Individual Risk Rating, Part I." Study Note, National Council on Compensation Insurance, 1992.

Goulet, Vincent. "A Generalized Crossed Classification Credibility Model." *Insurance. Mathematics & Economics* 28, no. 2 (2001): 205-216.

Hansen, J. Ernest. "A Note on Full Credibility for Estimating Claim Frequency." *Proceedings of the Casualty Actuarial Society*, 1972: 51-56.

Insurance Services Office, Inc. "Commercial General Liability Experience and Schedule Rating Plan." 2006.

Klugman, Stuart A., Harry H. Panjer, and Gordon E. Willmot. *Loss Models: From Data to Decisions.* New York, New York: John Wiley and Sons, 1998.

Mahler, Howard C., and Curtis Gary Dean. "Credibility." In *Foundations of Casualty Actuarial Science*. Casualty Actuarial Society.

Mowbray, Albert H. "How Extensive a Payroll Exposure is Necessary to Give a Dependable Pure Premium." *Proceedings of the Casualty Actuarial Society* I, no. 1 (1914): 24-30.

National Council on Compensation Insurance. "Experience Rating Plan Manual for Workers Compensation and Employers Liability Insurance." 2007.

SAS. *PROC MIXED Statement.* http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/viewer.htm#statug_mixed_sect008.htm (accessed 05 21, 2012).

—. *REPEATED Statement.* http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/viewer.htm#statug_mixed_sect019.htm (accessed 05 21, 2012).

Shi, Peng, and Edward W. Frees. "Long-tail Longitudinal Modeling of Insurance Company Expenses." *Insurance: Mathematics and Economics* 47 (2010): 303-314.

Steele, J. Michael. *Stochastic Calculus and Financial Applications.* New York, NY: Springer, 2000.

Venter, Gary. "Experience Rating—Equity and Predictive Accuracy." *NCCI Digest* II, no. I (April 1987): 27-35.

Werner, Geoff, and Claudine Modlin. *Basic Ratemaking.* Casualty Actuarial Society, 2010.

Whitney, Albert W. "The Theory of Experience Rating." *Proceedings of the Casualty Actuarial Society* IV, no. 10 (May 1918): 274-292.

Wikipedia. *Autoregressive model.* http://en.wikipedia.org/wiki/Auto-regressive_process (accessed 05 20, 2012).

—. *Moving-average model.* http://en.wikipedia.org/wiki/Moving_average_model (accessed 05 20, 2012).

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Marc-André Desrosiers is a Ph.D. candidate at UW-Madison in the Actuarial Science, Risk Management and Insurance program. He also completed his MBA at University of Calgary, after receiving his FCAS. Marc-André has studied Actuarial Mathematics and Philosophy at Concordia University, Montréal. The author also keeps contact with the industry as he is currently working as an external consultant for Intact Financial Corporation Actuarial Commercial Lines department. He is interested in pricing optimization, behavioral economics, customer behavior, and demand modeling. He can be joined at mdesrosiers@bus.wisc.edu.

APPENDIX 1. COEFFICIENTS OF THE PREDICTIVE MODELS

B_PPAL				
Predictors	Estimate	Standard Error	t Value	Pr > |t|
intercept	0.35	0.06	6.05	< 0.01%
same line - lag 1	-0.26	0.16	-1.57	11.59%
M.V. Ind. - same line - lag 1	-0.05	0.09	-0.52	60.44%
interaction term - same line - lag 1	0.11	0.03	3.78	0.02%
same line - lag 2	-0.03	0.03	-0.98	32.65%
M.V. Ind. - same line - lag 2	0.05	0.06	0.89	37.45%
same line - lag 3	-0.01	0.01	-0.56	57.42%
M.V. Ind. - same line - lag 3	0.07	0.04	1.70	8.92%
same line - lag 4	0.03	0.02	1.11	26.62%
M.V. Ind. - same line - lag 4	-0.04	0.05	-0.83	40.90%
same line - lag 5	0.10	0.04	2.78	0.54%
M.V. Ind. - same line - lag 5	-0.08	0.04	-2.01	4.47%
line J_AUTP - lag 1	0.50	0.14	3.50	0.05%
M.V. Ind. - line J_AUTP - lag 1	-0.03	0.06	-0.52	60.05%
interaction term - line J_AUTP - lag 1	-0.08	0.02	-3.03	0.25%
line J_AUTP - lag 2	0.06	0.04	1.75	8.00%
M.V. Ind. - line J_AUTP - lag 2	0.00	0.03	-0.17	86.71%
Toeplitz(2)	2 239.29	111.22	20.13	< 0.01%
Toeplitz(3)	1 497.42	94.71	15.81	< 0.01%
Toeplitz(4)	1 108.23	83.62	13.25	< 0.01%
Toeplitz(5)	565.27	74.92	7.54	< 0.01%
Residual	3 808.87	115.84	32.88	< 0.01%

J_AUTP				
Predictors	Estimate	Standard Error	t Value	Pr > |t|
intercept	0.06	0.02	3.44	0.06%
same line - lag 1	0.01	0.13	0.05	96.05%
M.V. Ind. - same line - lag 1	-0.15	0.05	-2.78	0.54%
interaction term - same line - lag 1	0.13	0.03	4.95	< 0.01%
same line - lag 2	0.52	0.13	3.92	< 0.01%
M.V. Ind. - same line - lag 2	-0.07	0.03	-2.21	2.69%
interaction term - same line - lag 2	-0.08	0.02	-3.21	0.14%
same line - lag 3	0.05	0.02	2.26	2.40%
M.V. Ind. - same line - lag 3	0.00	0.02	-0.09	92.53%
line B_PPAL - lag 1	0.05	0.02	2.33	1.97%
M.V. Ind. - line B_PPAL - lag 1	-0.02	0.02	-0.92	35.69%
line C_CA_L - lag 1	0.16	0.06	2.51	1.21%
M.V. Ind. - line C_CA_L - lag 1	0.01	0.01	2.15	3.20%
interaction term - line C_CA_L - lag 1	-0.03	0.01	-2.51	1.20%
Toeplitz(2)	148.40	30.99	4.79	< 0.01%
Residual	1 253.44	27.72	45.21	< 0.01%

A_HMOW				
Predictors	Estimate	Standard Error	t Value	Pr > |t|
intercept	-0.21	0.18	-1.16	24.46%
interaction term - intercept	0.07	0.03	2.00	4.51%
same line - lag 1	0.28	0.32	0.87	38.70%
M.V. Ind. - same line - lag 1	0.27	0.07	4.18	< 0.01%
interaction term - same line - lag 1	-0.03	0.06	-0.45	65.07%
same line - lag 2	0.55	0.25	2.14	3.22%
M.V. Ind. - same line - lag 2	-0.04	0.06	-0.75	45.63%
interaction term - same line - lag 2	-0.08	0.05	-1.57	11.76%
same line - lag 3	0.19	0.05	3.47	0.05%
M.V. Ind. - same line - lag 3	-0.26	0.07	-3.94	< 0.01%
same line - lag 4	0.25	0.07	3.67	0.02%
M.V. Ind. - same line - lag 4	-0.15	0.05	-2.78	0.55%
line B_PPAL - lag 1	-0.92	0.35	-2.60	0.93%
M.V. Ind. - line B_PPAL - lag 1	-0.11	0.08	-1.52	12.97%
interaction term - line B_PPAL - lag 1	0.20	0.07	2.62	0.88%
line J_AUTP - lag 1	0.51	0.20	2.51	1.22%
M.V. Ind. - line J_AUTP - lag 1	0.11	0.08	1.38	16.92%
interaction term - line J_AUTP - lag 1	-0.09	0.04	-2.36	1.81%
line E_CMP_ - lag 1	-0.29	0.34	-0.85	39.51%
M.V. Ind. - line E_CMP_ - lag 1	0.08	0.05	1.48	14.03%
interaction term - line E_CMP_ - lag 1	0.06	0.07	0.90	36.73%
line E_CMP_ - lag 2	0.46	0.30	1.51	13.14%
M.V. Ind. - line E_CMP_ - lag 2	-0.10	0.05	-1.85	6.39%
interaction term - line E_CMP_ - lag 2	-0.09	0.06	-1.53	12.63%
Toeplitz(2)	728.22	111.06	6.56	< 0.01%
Residual	4 452.56	98.64	45.14	< 0.01%

D_WC__				
Predictors	Estimate	Standard Error	t Value	Pr > |t|
intercept	1.70	0.39	4.40	< 0.01%
interaction term - intercept	-0.26	0.07	-3.44	0.06%
same line - lag 1	-0.44	0.26	-1.69	9.15%
M.V. Ind. - same line - lag 1	-0.02	0.06	-0.34	73.04%
interaction term - same line - lag 1	0.20	0.06	3.43	0.06%
line C_CA_L - lag 1	-0.25	0.31	-0.80	42.22%
M.V. Ind. - line C_CA_L - lag 1	-0.11	0.04	-2.92	0.35%
interaction term - line C_CA_L - lag 1	0.04	0.06	0.65	51.89%
line E_CMP_ - lag 1	0.09	0.04	2.17	2.98%
M.V. Ind. - line E_CMP_ - lag 1	0.07	0.05	1.61	10.84%
Toeplitz(2)	2 968.58	219.98	13.49	< 0.01%
Toeplitz(3)	2 195.32	201.70	10.88	< 0.01%
Toeplitz(4)	1 841.19	180.14	10.22	< 0.01%
Toeplitz(5)	854.54	172.21	4.96	< 0.01%
Residual	8 365.16	235.52	35.52	< 0.01%

E_CMP_				
Predictors	Estimate	Standard Error	t Value	Pr > |t|
intercept	0.27	0.05	5.63	< 0.01%
same line - lag 1	0.24	0.06	3.95	< 0.01%
M.V. Ind. - same line - lag 1	0.07	0.10	0.68	49.38%
same line - lag 2	0.15	0.07	2.06	3.93%
M.V. Ind. - same line - lag 2	-0.09	0.10	-0.87	38.41%
same line - lag 3	0.16	0.07	2.34	1.94%
M.V. Ind. - same line - lag 3	0.06	0.09	0.68	49.75%
line C_CA_L - lag 1	-0.29	0.06	-4.61	< 0.01%
M.V. Ind. - line C_CA_L - lag 1	-0.05	0.04	-1.20	22.83%
interaction term - line C_CA_L - lag 1	0.06	0.01	5.30	< 0.01%
line D_WC__ - lag 1	0.12	0.04	3.42	0.06%
M.V. Ind. - line D_WC__ - lag 1	-0.09	0.03	-2.87	0.41%
line D_WC__ - lag 2	0.01	0.03	0.27	78.91%
M.V. Ind. - line D_WC__ - lag 2	0.11	0.06	1.86	6.36%
Toeplitz(2)	1 364.39	113.47	12.02	< 0.01%
Toeplitz(3)	246.03	115.11	2.14	3.26%
Toeplitz(4)	472.77	121.62	3.89	0.01%
Toeplitz(5)	502.16	100.27	5.01	< 0.01%
Residual	4 678.28	120.95	38.68	< 0.01%

H_OL_O				
Predictors	Estimate	Standard Error	t Value	Pr > |t|
intercept	-0.99	0.59	-1.66	9.74%
interaction term - intercept	0.24	0.11	2.12	3.44%
same line - lag 1	0.47	0.24	1.99	4.68%
M.V. Ind. - same line - lag 1	0.12	0.08	1.41	15.95%
interaction term - same line - lag 1	-0.05	0.04	-1.15	25.18%
same line - lag 2	0.27	0.26	1.03	30.43%
M.V. Ind. - same line - lag 2	0.01	0.07	0.20	84.13%
interaction term - same line - lag 2	-0.02	0.06	-0.41	68.47%
same line - lag 3	0.22	0.54	0.41	68.33%
M.V. Ind. - same line - lag 3	0.06	0.12	0.48	63.23%
interaction term - same line - lag 3	-0.03	0.11	-0.30	76.68%
same line - lag 4	0.29	0.41	0.72	47.30%
M.V. Ind. - same line - lag 4	-0.05	0.08	-0.59	55.41%
interaction term - same line - lag 4	-0.05	0.08	-0.61	53.90%
same line - lag 5	1.45	0.42	3.48	0.05%
M.V. Ind. - same line - lag 5	-0.37	0.26	-1.40	16.10%
interaction term - same line - lag 5	-0.28	0.08	-3.66	0.03%
line E_CMP_ - lag 1	0.29	0.50	0.58	56.24%
M.V. Ind. - line E_CMP_ - lag 1	0.21	0.13	1.58	11.37%
interaction term - line E_CMP_ - lag 1	-0.03	0.10	-0.35	72.43%
line D_WC__ - lag 1	-0.60	0.25	-2.43	1.51%
M.V. Ind. - line D_WC__ - lag 1	0.00	0.08	-0.02	98.29%
interaction term - line D_WC__ - lag 1	0.14	0.05	2.97	0.30%
Toeplitz(2)	5 631.71	473.06	11.90	< 0.01%
Toeplitz(3)	4 511.05	432.00	10.44	< 0.01%
Toeplitz(4)	2 531.73	406.25	6.23	< 0.01%
Toeplitz(5)	1 783.30	459.55	3.88	0.01%
Residual	20 914.00	506.42	41.30	< 0.01%

C_CA_L				
Predictors	Estimate	Standard Error	t Value	Pr > |t|
intercept	1.44	0.42	3.44	0.06%
interaction term - intercept	-0.21	0.08	-2.77	0.56%
same line - lag 1	-0.60	0.19	-3.22	0.13%
M.V. Ind. - same line - lag 1	0.08	0.11	0.71	47.66%
interaction term - same line - lag 1	0.12	0.04	3.07	0.22%
same line - lag 2	-0.61	0.20	-3.05	0.23%
M.V. Ind. - same line - lag 2	-0.12	0.06	-1.88	5.96%
interaction term - same line - lag 2	0.16	0.04	3.63	0.03%
same line - lag 3	0.07	0.03	2.27	2.35%
M.V. Ind. - same line - lag 3	-0.15	0.06	-2.42	1.54%
same line - lag 4	-0.02	0.02	-1.39	16.36%
M.V. Ind. - same line - lag 4	0.00	0.07	-0.02	98.18%
same line - lag 5	0.02	0.04	0.45	65.34%
M.V. Ind. - same line - lag 5	0.14	0.05	2.71	0.69%
line D_WC__ - lag 1	0.49	0.28	1.74	8.22%
M.V. Ind. - line D_WC__ - lag 1	-0.06	0.04	-1.28	19.94%
interaction term - line D_WC__ - lag 1	-0.09	0.06	-1.61	10.71%
line D_WC__ - lag 2	0.03	0.03	0.88	37.64%
M.V. Ind. - line D_WC__ - lag 2	0.02	0.04	0.63	52.76%
line J_AUTP - lag 1	0.22	0.05	4.78	< 0.01%
M.V. Ind. - line J_AUTP - lag 1	-0.03	0.06	-0.54	58.65%
line J_AUTP - lag 2	0.10	0.03	3.20	0.14%
M.V. Ind. - line J_AUTP - lag 2	-0.06	0.04	-1.66	9.63%
Toeplitz(2)	2 878.02	143.25	20.09	< 0.01%
Toeplitz(3)	1 611.96	110.61	14.57	< 0.01%
Toeplitz(4)	724.10	72.07	10.05	< 0.01%
Residual	4 738.42	155.58	30.46	< 0.01%

H_OL_C				
Predictors	Estimate	Standard Error	t Value	Pr > |t|
intercept	0.40	0.13	3.01	0.29%
same line - lag 1	0.11	0.07	1.59	11.10%
M.V. Ind. - same line - lag 1	0.43	0.14	3.00	0.28%
same line - lag 2	0.21	0.06	3.65	0.03%
M.V. Ind. - same line - lag 2	-0.23	0.14	-1.68	9.39%
line E_CMP_ - lag 1	-0.46	0.28	-1.69	9.19%
M.V. Ind. - line E_CMP_ - lag 1	-0.32	0.10	-3.08	0.21%
interaction term - line E_CMP_ - lag 1	0.11	0.05	2.15	3.20%
line D_WC__ - lag 1	0.13	0.08	1.51	13.16%
M.V. Ind. - line D_WC__ - lag 1	0.02	0.08	0.22	82.98%
Toeplitz(2)	7 970.58	631.92	12.61	< 0.01%
Toeplitz(3)	4 073.46	597.59	6.82	< 0.01%
Toeplitz(4)	3 048.24	499.47	6.10	< 0.01%
Toeplitz(5)	1 679.18	413.00	4.07	< 0.01%
Residual	16 577.00	689.27	24.05	< 0.01%

G_SL__				
Predictors	Estimate	Standard Error	t Value	Pr > |t|
intercept	-0.52	0.99	-0.53	59.94%
interaction term - intercept	0.21	0.22	0.98	32.77%
same line - lag 1	1.26	0.85	1.49	13.75%
M.V. Ind. - same line - lag 1	0.22	0.15	1.47	14.14%
interaction term - same line - lag 1	-0.22	0.19	-1.14	25.50%
same line - lag 2	-1.93	1.00	-1.93	5.41%
M.V. Ind. - same line - lag 2	-0.18	0.14	-1.29	19.86%
interaction term - same line - lag 2	0.44	0.22	1.95	5.12%
same line - lag 3	1.37	0.50	2.72	0.67%
M.V. Ind. - same line - lag 3	0.06	0.09	0.72	47.27%
interaction term - same line - lag 3	-0.26	0.11	-2.47	1.38%
same line - lag 4	0.97	0.36	2.68	0.75%
M.V. Ind. - same line - lag 4	-0.10	0.10	-1.05	29.22%
interaction term - same line - lag 4	-0.19	0.07	-2.64	0.84%
same line - lag 5	-0.65	0.52	-1.23	21.79%
M.V. Ind. - same line - lag 5	-0.03	0.12	-0.26	79.73%
interaction term - same line - lag 5	0.12	0.10	1.16	24.51%
line D_WC__ - lag 1	0.58	0.52	1.12	26.26%
M.V. Ind. - line D_WC__ - lag 1	-0.01	0.08	-0.14	88.87%
interaction term - line D_WC__ - lag 1	-0.10	0.11	-0.93	35.11%
line C_CA_L - lag 1	0.19	0.36	0.52	60.50%
M.V. Ind. - line C_CA_L - lag 1	0.11	0.07	1.49	13.65%
interaction term - line C_CA_L - lag 1	-0.04	0.07	-0.63	53.02%
Toeplitz(2)	5 108.87	638.02	8.01	< 0.01%
Toeplitz(3)	3 912.92	574.73	6.81	< 0.01%
Toeplitz(4)	2 129.37	529.33	4.02	< 0.01%
Toeplitz(5)	1 478.48	486.34	3.04	0.24%
Residual	14 068.00	659.67	21.33	< 0.01%

R_PL_O				
Predictors	Estimate	Standard Error	t Value	Pr > |t|
intercept	0.38	0.10	3.87	0.02%
same line - lag 1	0.49	0.39	1.26	20.78%
M.V. Ind. - same line - lag 1	0.21	0.17	1.25	21.25%
interaction term - same line - lag 1	-0.09	0.08	-1.13	26.06%
same line - lag 2	0.13	0.05	2.32	2.08%
M.V. Ind. - same line - lag 2	-0.30	0.17	-1.75	8.03%
same line - lag 3	0.23	0.06	3.58	0.04%
M.V. Ind. - same line - lag 3	-0.18	0.23	-0.80	42.66%
same line - lag 4	0.16	0.04	4.13	< 0.01%
M.V. Ind. - same line - lag 4	0.34	0.37	0.94	34.74%
Toeplitz(2)	9 623.39	1 043.54	9.22	< 0.01%
Toeplitz(3)	5 925.52	875.16	6.77	< 0.01%
Toeplitz(4)	1 632.53	684.78	2.38	1.71%
Residual	19 328.00	1 091.68	17.70	< 0.01%

R_PL_C				
Predictors	Estimate	Standard Error	t Value	Pr > |t|
intercept	3.59	2.05	1.75	8.69%
interaction term - intercept	-0.76	0.47	-1.62	10.80%
line H_OL_O - lag 1	1.81	0.85	2.13	3.47%
M.V. Ind. - line H_OL_O - lag 1	0.71	0.17	4.07	< 0.01%
interaction term - line H_OL_O - lag 1	-0.37	0.18	-2.06	4.08%
line H_OL_O - lag 2	0.21	0.13	1.57	11.81%
M.V. Ind. - line H_OL_O - lag 2	-0.54	0.13	-4.31	< 0.01%
line D_WC__ - lag 1	2.07	1.31	1.58	11.64%
M.V. Ind. - line D_WC__ - lag 1	-0.15	0.16	-0.93	35.13%
interaction term - line D_WC__ - lag 1	-0.41	0.29	-1.40	16.34%
line C_CA_L - lag 1	-6.59	2.02	-3.26	0.13%
M.V. Ind. - line C_CA_L - lag 1	0.62	1.21	0.51	60.73%
interaction term - line C_CA_L - lag 1	1.62	0.50	3.22	0.15%
line C_CA_L - lag 2	-0.07	0.03	-2.15	3.32%
M.V. Ind. - line C_CA_L - lag 2	-0.92	1.15	-0.79	42.79%
Toeplitz(2)	3 267.34	802.27	4.07	< 0.01%
Toeplitz(3)	2 923.03	608.49	4.80	< 0.01%
Residual	9 664.57	975.72	9.91	< 0.01%

F_MM_C				
Predictors	Estimate	Standard Error	t Value	Pr > |t|
intercept	0.17	0.20	0.84	40.07%
same line - lag 1	-1.35	0.42	-3.19	0.15%
M.V. Ind. - same line - lag 1	-0.01	0.06	-0.11	91.41%
interaction term - same line - lag 1	0.33	0.10	3.38	0.08%
same line - lag 2	0.07	0.06	1.20	23.22%
M.V. Ind. - same line - lag 2	-0.14	0.08	-1.75	8.13%
same line - lag 3	0.14	0.05	2.94	0.33%
M.V. Ind. - same line - lag 3	0.08	0.08	0.95	34.01%
same line - lag 4	0.16	0.07	2.43	1.51%
M.V. Ind. - same line - lag 4	-0.06	0.08	-0.76	44.78%
same line - lag 5	0.11	0.08	1.36	17.55%
M.V. Ind. - same line - lag 5	0.04	0.09	0.50	61.97%
line F_MM_O - lag 1	0.00	0.01	0.49	62.59%
M.V. Ind. - line F_MM_O - lag 1	-0.09	0.09	-0.99	32.39%
line F_MM_O - lag 2	-0.04	0.01	-2.78	0.55%
M.V. Ind. - line F_MM_O - lag 2	0.00	0.07	0.00	99.62%
line G_SL__ - lag 1	3.11	1.50	2.07	3.90%
M.V. Ind. - line G_SL__ - lag 1	0.18	0.15	1.19	23.39%
interaction term - line G_SL__ - lag 1	-0.72	0.35	-2.07	3.90%
line G_SL__ - lag 2	-2.31	1.42	-1.62	10.48%
M.V. Ind. - line G_SL__ - lag 2	-0.28	0.22	-1.28	20.24%
interaction term - line G_SL__ - lag 2	0.56	0.34	1.66	9.80%
line H_OL_C - lag 1	0.03	0.04	0.87	38.19%
M.V. Ind. - line H_OL_C - lag 1	0.09	0.05	1.77	7.70%
line H_OL_C - lag 2	0.06	0.03	2.09	3.68%
M.V. Ind. - line H_OL_C - lag 2	0.11	0.06	1.63	10.27%
Toeplitz(2)	7 865.46	671.63	11.71	< 0.01%
Toeplitz(3)	4 580.28	597.73	7.66	< 0.01%
Toeplitz(4)	2 637.96	485.50	5.43	< 0.01%
Toeplitz(5)	1 310.39	331.46	3.95	< 0.01%
Residual	12 901.00	697.13	18.51	< 0.01%

F_MM_O				
Predictors	Estimate	Standard Error	t Value	Pr > |t|
intercept	3.57	3.39	1.05	29.37%
interaction term - intercept	-1.05	0.74	-1.43	15.26%
same line - lag 1	7.34	1.79	4.09	< 0.01%
M.V. Ind. - same line - lag 1	0.93	0.50	1.85	6.45%
interaction term - same line - lag 1	-1.99	0.50	-4.00	< 0.01%
same line - lag 2	0.50	0.17	2.97	0.31%
M.V. Ind. - same line - lag 2	-1.28	0.50	-2.57	1.04%
same line - lag 3	0.45	0.25	1.81	7.14%
M.V. Ind. - same line - lag 3	0.32	0.38	0.84	40.31%
same line - lag 4	0.68	0.31	2.19	2.91%
M.V. Ind. - same line - lag 4	-0.54	0.35	-1.56	12.04%
line F_MM_C - lag 1	-7.76	2.23	-3.49	0.05%
M.V. Ind. - line F_MM_C - lag 1	0.90	0.53	1.70	9.03%
interaction term - line F_MM_C - lag 1	1.96	0.54	3.66	0.03%
line F_MM_C - lag 2	4.85	1.90	2.55	1.10%
M.V. Ind. - line F_MM_C - lag 2	0.14	0.37	0.39	69.89%
interaction term - line F_MM_C - lag 2	-1.17	0.42	-2.79	0.54%
line G_SL__ - lag 1	0.92	3.61	0.25	79.89%
M.V. Ind. - line G_SL__ - lag 1	0.72	0.38	1.90	5.82%
interaction term - line G_SL__ - lag 1	-0.12	0.90	-0.13	89.34%
line G_SL__ - lag 2	-11.22	3.62	-3.10	0.20%
M.V. Ind. - line G_SL__ - lag 2	-1.57	0.64	-2.46	1.42%
interaction term - line G_SL__ - lag 2	2.65	0.89	2.98	0.30%
line H_OL_O - lag 1	0.16	0.10	1.67	9.60%
M.V. Ind. - line H_OL_O - lag 1	1.13	0.40	2.84	0.47%
Toeplitz(2)	11 553.00	5 332.77	2.17	3.03%
Residual	115 396.00	5 959.67	19.36	< 0.01%