Deductibles, Policy Limits, and Reinsurance: A Case Study in Malaysia

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Abstract
In developing countries such as Malaysia, the availability of reinsurance arrangements provides several advantages to primary insurers, such as keeping their risk exposures at prudent levels by having large risk exposures reinsured by another company, meeting client requests for larger insurance coverage by having their limited financial sources supported by another company, and acquiring another company's underwriting skills, experience and complex claim handling ability. These are essential considerations for primary insurers that wish to expand their insurance business and reduce the size of their loss exposure, especially in countries like Malaysia, where the number of primary insurers is large and the size of their resources is small. This paper aims to model the amount of insurance loss, to provide a range of deductibles and policy limits based on Loss Elimination Ratios (LER), to compute insolvency probabilities via linear loading and PH-Transform assumptions, to calculate Increased Limit Factors (ILF), to apply a frequency and severity approach to pricing excess-of-loss layers, and to assess the insolvency probability of a reinsurance treaty. In particular, the PH-Transform assumption is applied throughout as a means of incorporating a risk load, thus lowering the insolvency probability of a single excess-of-loss layer as well as multiple layers of a reinsurance treaty.

Keywords: Loss elimination ratio; insolvency probability; reinsurance; general insurance, PH-Transform.

1. INTRODUCTION

Reinsurance premiums in the Malaysian non-life insurance industry may be categorized into those ceded abroad and those ceded within Malaysia. In 1965 and 1975, for instance, reinsurance premiums ceded abroad were RM12 million and RM60 million, equivalent to 17% and 21% of written premiums respectively. These amounts increased to RM296 million and RM1223 million in 1985 and 1995, equivalent to 24% and 27% of written premiums respectively, but decreased to RM957 million in 2005, equivalent to 10% of written premiums (Lee [9], Bank Negara Malaysia [1], Bank Negara Malaysia [2]). Figures 1-2 show the reinsurance premiums ceded abroad (1965-2005) in terms of volume and proportion of written premium. It should be noted that the currency of Ringgit Malaysia (RM) was pegged at RM3.80=USD1 on 2 September 1998 and shifted to a managed float against a basket of currencies as of 21 July 2005.

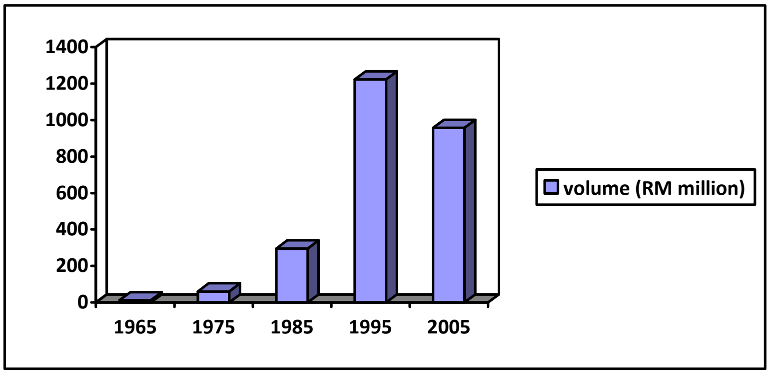

Figure 1: Volume of reinsurance premium ceded abroad (RM million)

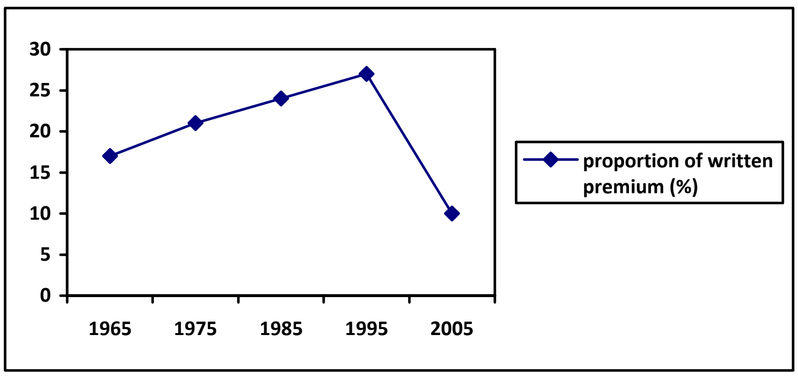

Figure 2: Proportion of reinsurance premium ceded abroad (% of written premium)

Based on the proportion of written premiums, there was a marked deterioration in 1985 and 1995 in terms of domestic retention compared to 1965 and 1975, due to the fact that Malaysia never imposed restrictions on foreign exchange outflows for reinsurance purposes. For most companies, their limited financial resources and expertise in underwriting and handling complex claims increased their dependence upon outside reinsurers, leading to the issue of unsatisfactory domestic retention of premium (Lee [9]). The level of retention improved in 2005, however, largely due to the continuous efforts taken by regulatory bodies and industry players, especially in encouraging domestic insurers and reinsurers to absorb higher proportions of large risks.

Over the past decade, there were many discussions on trade liberalization not only in Malaysia but also in the rest of the world, involving the removal of trade barriers or easing of regulations that inhibit the workings of the free market (Lau [8]). In March 2001, the central bank of

Malaysia, Bank Negara Malaysia (BNM), launched the Financial Sector Masterplan (FSMP). This fairly extensive ten-year road map for the banking and insurance sectors includes specific recommendations that are to be implemented in phases over a ten-year period to deregulate and liberalize the country's financial industry (Bank Negara Malaysia [3]). Even though the local tariff on motor and fire insurance has served its purpose well since its implementation, it is now considered outdated and not reflective of market realities (Lau [8]). The tariff mechanism specified floor rates for various risk classes, but sometimes resulted in cross-subsidization among risk classes, and also within risk classes, whereby better risks subsidized the worse ones (Cummins [9]). In addition, limitations on deductibles and limits have not been appropriately revised to reflect inflation and other economic changes (Rao [10]).

This study aims to model the amount of insurance loss, to provide a range of deductibles and policy limits based on Loss Elimination Ratios (LER), to compute insolvency probabilities via linear loading and PH-Transform assumptions, to calculate Increased Limit Factors (ILF), to apply a frequency and severity approach to pricing excess-of-loss layers, and to assess the insolvency probability of a reinsurance treaty. In particular, the PH-Transform assumption is applied throughout as a means of incorporating a risk load, thus lowering the insolvency probability of a single excess-of-loss layer as well as multiple layers of a reinsurance treaty.

Several studies focusing on reinsurance, deductibles and policy limits have been carried out in the insurance and actuarial literature. Zhuang [14] established orderings of optimal allocations of policy limits and deductibles with respect to the distortion of risk measures; Hua and Cheung [9] applied the equivalent utility premium principle and studied the worst allocations of policy limits and deductibles; Dimitriyadis and Oney [5] modeled loss distributions using the Allianz tool pack, derived premiums at different levels of deductibles, and computed ruin probabilities; and Wang [12] introduced the Proportional Hazard (PH) Transform and applied this method to price ambiguous risks, excess-of-loss coverage, increased limits, risk portfolios and reinsurance treaties.

In this study, the modeling of loss amount, the computation of insolvency probability and the pricing of excess-of-loss layers are based on loss data obtained from one of the leading insurers in Malaysia. The approach suggested in this study can be considered to be fair, as it serves to lower insolvency probability. The suggested approach can also be considered to be efficient, since it can be computed in a straightforward manner using R programming.

2. LOSS MODEL

2.1 Maximum Likelihood Method

Claims data on health insurance's critical illnesses was obtained from one of the leading insurers in Malaysia, providing information on gender (male and female) and age of policyholders (below 25, 25-50 and above 50) in year 2008. In particular, the loss data of sample size $n = 192$ for female aged 25-50 is fitted using a maximum likelihood method. Preliminary analysis has been conducted prior to the fitting procedure to ensure that the sample data is trended and does not contain any anomalies or outliers.

The likelihood function for complete individual data is

$$L(\underline{\theta}) = \prod_{i=1}^{n} f(x_i \mid \underline{\theta}), \tag{1}$$

where $f(x_i \mid \underline{\theta})$ denotes the probability density function (p.d.f.) with parameters $\underline{\theta} = \theta_1, \theta_2, ..., \theta_k$.

The maximum likelihood estimators are obtained by maximizing the log likelihood function:

t

$$\ln L(\underline{\theta}) = \sum_{i=1}^{n} \ln f(x_i \mid \underline{\theta}). \tag{2}$$

Table 1 shows the estimated parameters and the log likelihood of several parametric distributions fitted on the amount of loss, sorted by decreasing values of log-likelihood within the number of parameters. The best models for one-parameter, two-parameter and three-parameter distributions are selected by choosing the largest value of the log likelihood function, $\ln L(\underline{\theta})$.

2.2 Model Selection

The next step to select the best model is to perform the Kolmogorov-Smirnov (K-S) and Anderson-Darling (A-D) tests. The K-S statistical test is defined as (Klugman et al. [7])

$$D = \max_{t \le x \le u} \left| F_n(x_i) - F^*(x_i) \right|, \qquad i = 1, 2, \ldots, n \tag{3}$$

where $F^*(x_i)$ denotes the parametric cumulative distribution function (c.d.f.), and $F_n(x_i)$ the empirical c.d.f. evaluated at x_i respectively. The best model is chosen by selecting the lowest D.

Table 1: Estimated parameters

Parametric distribution	Number of parameters	Estimated parameters	$\ln L(\theta)$
Exponential	1	$\lambda = 0.000025$	-2,207 Best one-parameter model
Inverse exponential	1	$\theta = 8582.61$	-2,349
Gamma	2	$\alpha = 1.4637$ $\theta = 26{,}279.57$	-2199.9 Best two-parameter model
Weibull	2	$\theta = 41{,}256.46$ $\tau = 1.2401$	-2200.4
Loglogistic	2	$\theta = 29{,}628.99$ $\gamma = 1.9801$	-2,205
Pareto	2	$\theta = 350{,}026.3$ $\alpha = 9.8929$	-2,211
Inverse Paralogistic	2	$\theta = 20{,}728.54$ $\tau = 1.4871$	-2,219
Lognormal	2	$\mu = 10.1786$ $\sigma = 1.0639$	-2,227
Inverse Pareto	2	$\theta = 13{,}487.44$ $\tau = 1.8890$	-2,243
Inverse Weibull	2	$\theta = 14{,}301.71$ $\tau = 0.6626$	-2,291
Inverse Gamma	2	$\alpha = 0.5573$ $\theta = 4{,}782.70$	-2,321
Inverse Gaussian	2	$\theta = 8{,}607.16$ $\mu = 6{,}000{,}000$	-2,322
Burr	3	$\theta = 86{,}426.43$ $\gamma = 1.5169$ $\alpha = 3.7783$	-2,197 Best three-parameter model
Generalized Pareto	3	$\theta = 731{,}790.4$ $\tau = 1.5305$ $\alpha = 30.1434$	-2,200
Transformed Gamma	3	$\theta = 30{,}270.96$ $\tau = 1.0664$ $\alpha = 1.3183$	-2,200
Inverse Transformed Gamma	3	$\theta = 8 \times 10^{-12}$ $\tau = 0.1684$ $\alpha = 27.3012$	-2,238

The A-D statistical test, defined as the weighted average of the squared differences of the empirical and parametric c.d.f.s, emphasizes the goodness of fit of the tail over the middle of distribution (Klugman et al. [7]),

$$\begin{aligned} A^2 = -nF^*(u) + n\sum_{j=0}^{k}(1-F_n(y_j))^2\left[\ln(1-F^*(y_j)) - \ln(1-F^*(y_{j+1}))\right] \\ + n\sum_{j=1}^{k}F_n(y_j)^2\left[\ln(F^*(y_{j+1})) - \ln(F^*(y_j))\right], \end{aligned} \tag{4}$$

where $y_0 < y_1 < ... < y_k < y_{k+1} = u$ denote the unique non-censored data, $F^*(y_j)$ the parametric c.d.f. and $F_n(y_j)$ the empirical c.d.f. The best model is chosen by selecting the lowest A^2 .

Finally, the Schwarz Bayesian Criterion (SBC) penalizes models having a greater number of parameters. The SBC is defined as (Klugman et al. [7])

$$SBC = \ln L - \frac{r}{2}\ln n\,, \tag{5}$$

where r denotes the number of parameters and n the sample size. The best model is chosen by selecting the highest SBC. Table 2 shows the results of the K-S, A-D and SBC tests carried out on loss data. The best-fitting distribution for the loss amount is Burr with parameters $\theta = 86,426.43$, $\gamma = 1.5169$ and $\alpha = 3.7783$ and thus, the following discussion will use this distribution.

Table 2: Results of K-S, A-D and SBC tests

Parametric distribution	Numbers of parameters	K-S test	A-D test	SBC
Exponential	1	0.18655	389.31	-2209.63
Gamma	2	0.11098	384.68	-2205.16
Burr	3	0.09454	383.87	-2204.40

3. LOSS ELIMINATION RATIO (LER)

The Loss Elimination Ratio (LER) is the ratio of the decrease in expected loss for an insurer writing a policy with a deductible and/or policy limit to the expected loss for an insurer writing a full-coverage policy.

3.1 Deductible Policy

When an insurer introduces a deductible to a policy, say at the value of d, the loss retained by the insured may be represented by the random variable Y, where

$$Y = \begin{cases} X, & X < d \\ d, & X \geq d \end{cases}, \tag{6}$$

whereas the loss covered by the insurer and paid as claim may be represented by the random variable W, where

$$W = \begin{cases} 0, & X < d \\ X - d, & X \geq d \end{cases}, \tag{7}$$

so that $X = Y + W$.

Therefore, in terms of an insurer's perspective, the Loss Elimination Ratio (LER) is equal to

$$LER = \frac{E(X;d)}{E(X)}, \tag{8}$$

where

$$E(X;d) = \int_0^d xf(x)dx + d\int_d^\infty f(x)dx,$$

and

$$E(X) = \int_0^\infty xf(x)dx = \int_0^\infty S(x)dx,$$

where $S(x)$ denotes the survival function, which is equal to $1-F(x)$.

Table 3 shows the LER, written in the currency of Ringgit Malaysia (RM), for several deductible values, assuming individual losses follow a Burr distribution with parameters $\theta = 86{,}426.43$, $\gamma = 1.5169$ and $\alpha = 3.7783$. As an example, the LER at $d =$ RM10,000 is 0.25, implying that 25% of insurer's losses is eliminated by introducing a deductible of RM10,000. Appendix 1 shows the calculation of LER using R programming with the assistance of the *actuar* package.

Table 3: Values of d and LER

Burr distribution, $E(X) = RM\,38{,}131$			
d (RM)	$E(X;d)$ (RM)	LER	Δ LER
0	0	0.000	-
1000	998.27	0.026	0.026
2000	1990.13	0.052	0.026
3000	2972.73	0.078	0.026
4000	3944.03	0.103	0.025
5000	4902.40	0.129	0.026
6000	5846.51	0.153	0.024
7000	6775.27	0.178	0.025
8000	7687.74	0.202	0.024
9000	8583.16	0.225	0.023
10000	9460.91	0.248	0.023
11000	10320.45	0.271	0.023
12000	11161.40	0.293	0.022
13000	11983.42	0.314	0.021
14000	12786.30	0.335	0.021
15000	13569.87	0.356	0.021
16000	14334.05	0.376	0.020
17000	15078.82	0.395	0.019
18000	15804.21	0.414	0.019
19000	16510.29	0.433	0.019
20000	17197.19	0.451	0.018

The graph of LER vs. d is shown in Figure 3, indicating that the ratio of eliminated loss is directly proportional to the deductible. However, after a certain point, a higher deductible can no longer provide a significant proportion of eliminated loss to an insurer.

In practice, the criteria for deductible may differ depending on the requirements and preferences of each insured. Nevertheless, an insurer may use the values shown in Table 3 and the graph shown in Figure 3 to indicate whether the deductible proposed by the insured provides a significant proportion of eliminated losses to the insurer. The insurer should also recognize that a high deductible is not attractive to policyholders since they have to retain a large portion of losses on their own.

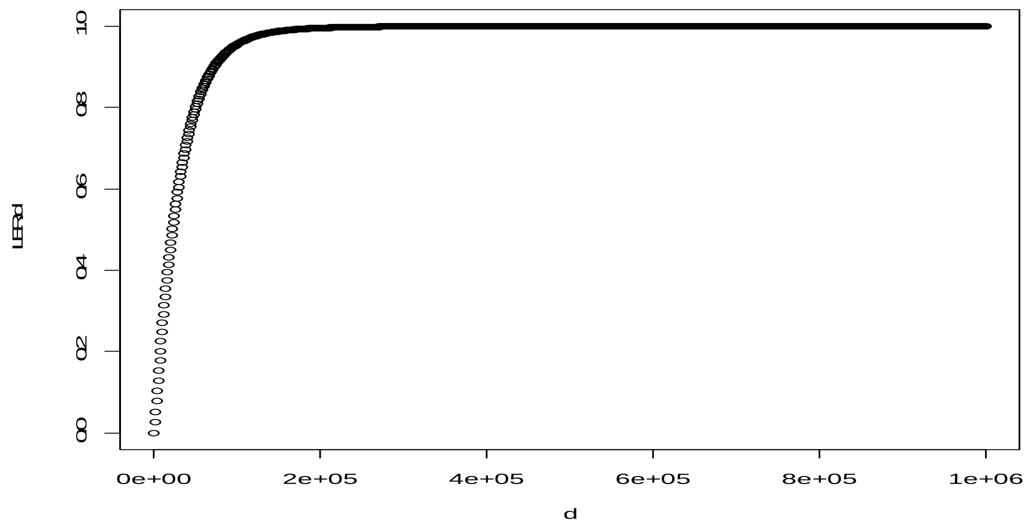

Figure 3: Graph of LER vs. deductible

3.2 Policy Limit

When an insurer introduces a policy limit in its coverage, say at the value of u, the loss covered by the insurer and paid as claim may be represented by the random variable K, where

$$K = \begin{cases} X, & X < u \\ u, & X \geq u \end{cases}, \tag{9}$$

whereas the loss covered by a reinsurer may be represented by the random variable L, where

$$L = \begin{cases} 0, & X < u \\ X - u, & X \geq u \end{cases}, \tag{10}$$

so that $X = K + L$.

Therefore, in terms of an insurer's perspective, the Loss Elimination Ratio (LER) is

$$LER = \frac{E(X) - E(X;u)}{E(X)}, \tag{11}$$

where

$$E(X;u) = \int_0^u xf(x)dx + u\int_u^\infty f(x)dx,$$

and

$$E(X) = \int_0^\infty xf(x)dx = \int_0^\infty S(x)dx \,.$$

Table 4 shows the LER for several policy limit values, assuming individual losses follow a Burr distribution with parameters $\theta = 86{,}426.43$, $\gamma = 1.5169$ and $\alpha = 3.7783$.

Table 4: Values of u and LER

Burr distribution, $E(X) = RM\,38{,}131$			
u (RM)	$E(X;u)$ (RM)	LER	Δ LER
40000	27332.77	0.283	-0.010
41000	27686.41	0.274	-0.009
42000	28028.2	0.265	-0.009
43000	28358.49	0.256	-0.009
44000	28677.63	0.248	-0.008
60000	32528.78	0.147	-0.005
61000	32705.46	0.142	-0.005
62000	32876.09	0.138	-0.004
63000	33040.89	0.133	-0.005
64000	33200.05	0.129	-0.004
80000	35123.51	0.079	-0.002
81000	35212.36	0.077	-0.002
82000	35298.28	0.074	-0.003
83000	35381.36	0.072	-0.002
84000	35461.7	0.07	-0.002

As an example, the LER at u =RM60,000 is 0.15, implying that 15% of losses can be eliminated by introducing a policy limit of RM60,000. The graph of LER vs. u is shown in Figure 4, indicating that the ratio of eliminated loss is inversely proportional to the limit. However, after a certain point, a higher limit can no longer provide a significant proportion of eliminated loss to an insurer.

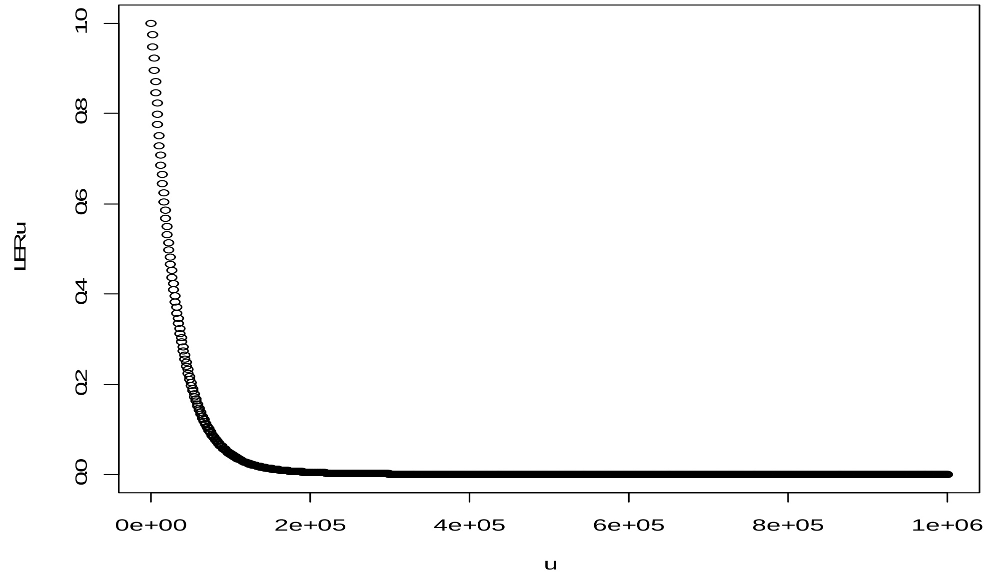

Figure 4: Graph of LER vs. policy limit

In practice, the criteria for policy limit may also differ depending on the requirements and preferences of both insurers and reinsurers. Nevertheless, an insurer may use the values shown in Table 4 and the graph illustrated in Figure 4 to indicate whether the proposed limit provides a significant proportion of eliminated losses.

4. LINEAR LOADING ASSUMPTION

4.1 Insolvency Probability of Deductible Policy

When an insurer introduces a policy with a deductible, at the value of d, the loss covered by insurer and paid as a claim may be represented by the random variable W as shown in equation (7). For an individual risk model, the aggregate claims of a deductible policy, with a deductible of d, may be defined as

$$S = W_1 + W_2 + \ldots + W_n , \tag{12}$$

where $W_1, W_2, \ldots, W_n$ denote independent and identically distributed (i.i.d.) random variables.

The conditional mean and variance of W_i, respectively, are

$$E(W_i \mid X > 0) = \mu_W \tag{13}$$

and

$$Var(W_i \mid X > 0) = \sigma_W^2, \tag{14}$$

where the probability of loss greater than zero or equivalently the probability of incurring a claim is equal to

$$\Pr(X > 0) = q. \tag{15}$$

Therefore, for a deductible policy, $E(W \mid X > 0)$ and $E(W^2 \mid X > 0)$ can be written as

$$\mu_W = E(W \mid X > 0) = \int_d^{\infty} (x-d) f(x) dx = E(X) - E(X;d) \tag{16}$$

and

$$E(W^2 \mid X > 0) = \int_d^{\infty} (x-d)^2 f(x) dx = E(X^2) - E((X;d)^2) - 2dE(X) + 2dE(X;d), \tag{17}$$

so that

$$\sigma_W{}^2 = E(W^2 \mid X > 0) - (E(W \mid X > 0))^2. \tag{18}$$

Finally, the distribution of aggregate claims, S, for a single portfolio of risk in an individual risk model may be estimated by applying Central Limit Theorem (CLT). In particular, if the number of policies, n, is large, the distribution of S may be estimated by a normal distribution with mean,

$$\mu_{S,W} = E(S) = n\mu_W q, \tag{19}$$

and variance,

$$\sigma_{S,W}^2 = Var(S) = n(\sigma_W^2 q + \mu_W^2 q(1-q)). \tag{20}$$

The same approach can also be applied to multiple portfolios of risks, whereby equation (19) is rewritten as $\mu_{S,W} = E(S) = \sum_i n_i \mu_{W,i} q_i$ where i denotes the i^{th} portfolio. Equivalently, equation (20) can be rewritten as $\sigma_{S,W}^2 = Var(S) = \sum_i n_i (\sigma_{W,i}^2 q_i + \mu_{W,i}^2 q_i (1-q_i))$.

If the premium is calculated using a linear loading assumption, i.e., premium= $\mu_{S,W}(1+\xi)$, where ξ denotes the relative loading, a simple definition of the probability of insolvency for a single portfolio of risk may be expressed as the probability of having aggregate claims larger than aggregate premiums, or, equivalently,

$$\Pr(S > (1+\xi)\mu_{S,W}) = \Pr\left(\frac{S-\mu_{S,W}}{\sigma_{S,W}} > \frac{(1+\xi)\mu_{S,W} - \mu_{S,W}}{\sigma_{S,W}}\right) = 1 - \Pr\left(Z < \frac{\mu_{S,W}}{\sigma_{S,W}}\xi\right). \tag{21}$$

It should be noted that when $\xi = 0$, the premium is equivalent to the expected aggregate claims of policies with a deductible at d. The linear loading assumption indicates that the relative loading, ξ, is fixed as a constant proportion of $\mu_{S,W}$ regardless of any values of d.

Tables 5-7 show the values of the insolvency probability for several values of ξ, n and q, assuming the amount of loss follows Burr with parameters $\theta = 86{,}426.43$, $\gamma = 1.5169$, and $\alpha = 3.7783$.

The graphs of insolvency probability vs. deductible for several values of d, ξ, n, and q are shown in Figures 5-7, indicating that under the assumption of linear loading, the insolvency probability increases as the deductible increases. One possible justification for this increase in the insolvency probability can be explained by observing the values of $\mu_{S,W}$ and $\sigma_{S,W}$ displayed in Table 5. Even though both $\mu_{S,W}$ and $\sigma_{S,W}$ decrease when the deductible increases, $\mu_{S,W}$ decreases faster than $\sigma_{S,W}$, causing the quantity $\mu_{S,W}(\sigma_{S,W})^{-1}$ to decrease. Based on equation (21), the probability of insolvency is therefore expected to increase.

In addition, the graphs in Figures 5-7 also show that the insolvency probability:

- decreases as the relative loading, ξ, increases
- decreases as the probability of incurring claim, q, increases
- decreases as the number of policies, n, increases

When the probability of incurring a claim or the number of policies increases, $\mu_{S,W}$ increases faster than $\sigma_{S,W}$, causing the quantity $\mu_{S,W}(\sigma_{S,W})^{-1}$ to increase. Therefore, based on equation (21), the probability of insolvency is expected to decrease.

Appendix 2 shows the calculation of the insolvency probability for a deductible policy using R programming with the assistance of the *actuar* package, assuming the amount of loss follows a Burr distribution.

Table 5: Values of d and insolvency probability ($n = 3000$, $q = 0.2$)

d (RM)	$\mu_{S,W}$ (RM)	$\sigma_{S,W}$ (RM)	$\xi = 0.25$ Insolvency probability	$\xi = 0.20$ Insolvency probability	$\xi = 0.15$ Insolvency probability	$\xi = 0.10$ Insolvency probability	$\xi = 0.05$ Insolvency probability	$\xi = 0.00$ Insolvency probability
5,000	19,937,056	1,071,492	0.000002	0.000099	0.002627	0.031395	0.176097	0.50
10,000	17,201,950	998,232	0.000008	0.000284	0.004871	0.042422	0.194448	0.50
15,000	14,736,570	929,117	0.000037	0.000757	0.008677	0.056360	0.213877	0.50
20,000	12,560,178	864,187	0.000140	0.001826	0.014624	0.073055	0.233703	0.50

Table 6: Values of d and insolvency probability ($n = 3000$, $\xi = 0.15$)

d	$q = 0.40$			$q = 0.30$			$q = 0.20$		
	$\mu_{S,W}$	$\sigma_{S,W}$	Insolvency probability	$\mu_{S,W}$	$\sigma_{S,W}$	Insolvency probability	$\mu_{S,W}$	$\sigma_{S,W}$	Insolvency probability
5,000	39,874,112	1,425,201	0.000014	29,905,584	1,273,880	0.000215	19,937,056	1,071,492	0.002627
10,000	34,403,900	1,340,023	0.000059	25,802,925	1,191,941	0.000583	17,201,950	998,232	0.004871
15,000	29,473,141	1,257,672	0.000220	22,104,856	1,113,820	0.001460	14,736,570	929,117	0.008677
20,000	25,120,356	1,178,332	0.000692	18,840,267	1,039,610	0.003280	12,560,178	864,187	0.014624

Table 7: Values of d and insolvency probability ($\xi = 0.15$, $q = 0.2$)

d	$n = 3000$			$n = 2000$			$n = 1000$		
	$\mu_{s,w}$	$\sigma_{s,w}$	Insolvency probability	$\mu_{s,w}$	$\sigma_{s,w}$	Insolvency probability	$\mu_{s,w}$	$\sigma_{s,w}$	Insolvency probability
5,000	19,937,056	1,071,492	0.002627	13,291,371	874,869	0.011338	6,645,685	618,626	0.053546
10,000	17,201,950	998,232	0.004871	11,467,967	815,053	0.017406	5,733,983	576,329	0.067801
15,000	14,736,570	929,117	0.008677	9,824,380	758,621	0.026035	4,912,190	536,426	0.084785
20,000	12,560,178	864,187	0.014624	8,373,452	705,606	0.037533	4,186,726	498,939	0.104071

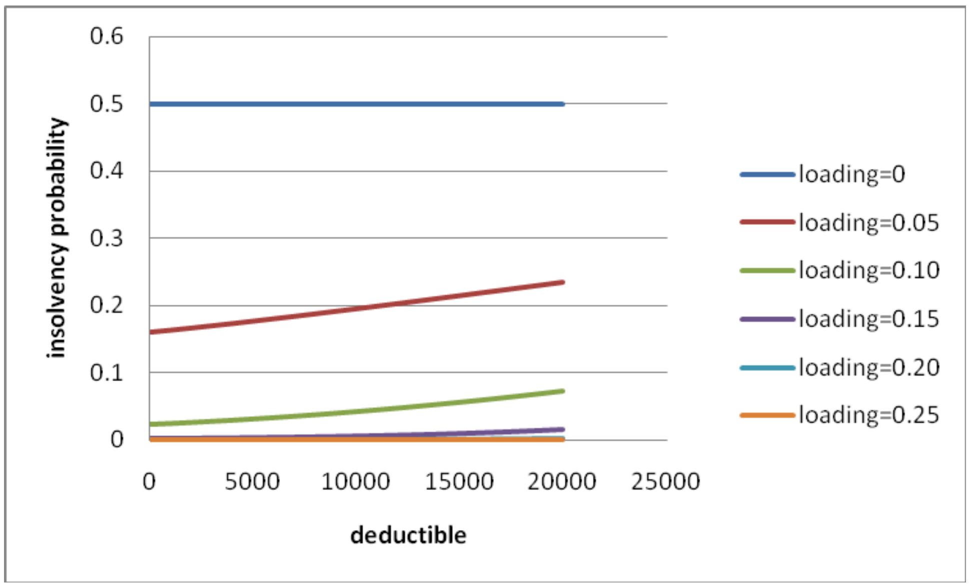

Figure 5: Graph of insolvency probability vs. deductible ($n = 3000$, $q = 0.2$)

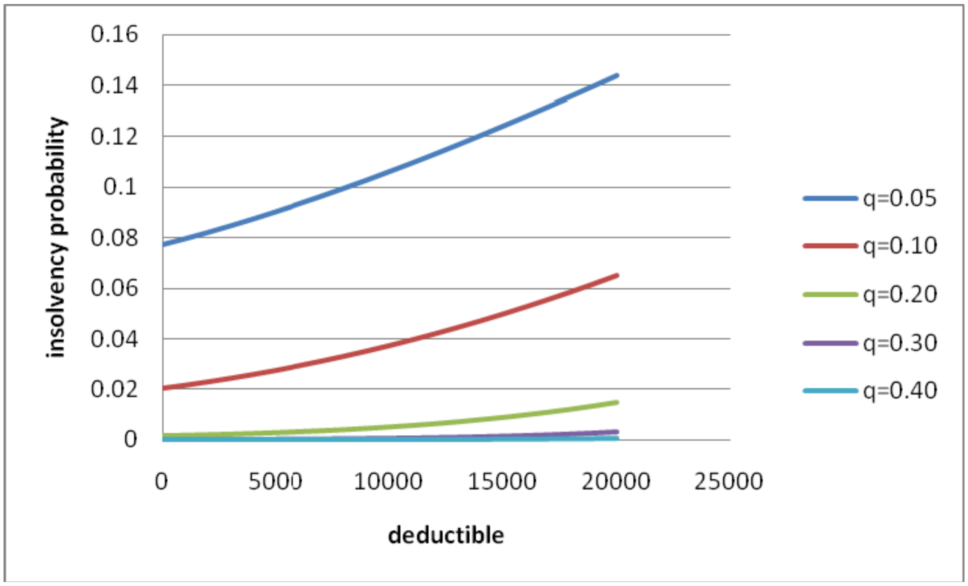

Figure 6: Graph of insolvency probability vs. deductible ($n = 3000$, $\xi = 0.15$)

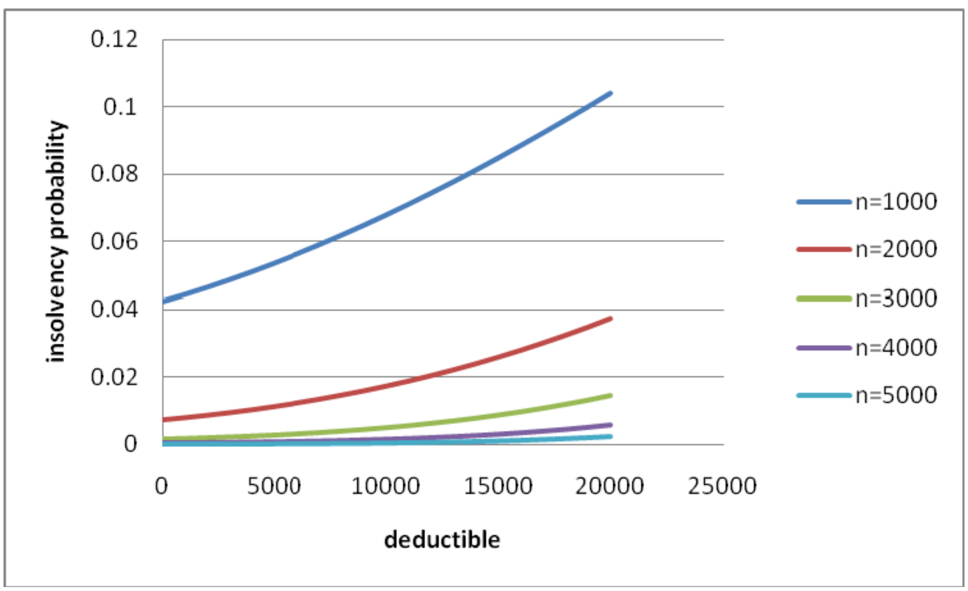

Figure 7: Graph of insolvency probability vs. deductible ($\xi = 0.15$, $q = 0.2$)

4.2 Insolvency Probability of Policy Limit

When an insurer introduces a policy limit, say at the value of u, the loss covered by insurer and paid as a claim may be represented by the random variable K as shown in equation (9). For an individual risk model, the aggregate claims of a policy with limit u may be defined as

$$S = K_1 + K_2 + ... + K_n, \tag{22}$$

where $K_1, K_2, ..., K_n$ denote independent and identically distributed (i.i.d.) random variables.

The conditional mean and variance of K_i respectively are

$$E(K_i \mid X > 0) = \mu_K, \tag{23}$$

and

$$Var(K_i \mid X > 0) = \sigma_K^2. \tag{24}$$

Therefore, for a policy limit, $E(K \mid X > 0)$ and $E(K^2 \mid X > 0)$ can be written as

$$\mu_K = E(K \mid X > 0) = \int_0^\infty xf(x)dx - \int_u^\infty (x-u) f(x)dx = E(X;u) \tag{25}$$

and

$$E(K^2 \mid X > 0) = \int_0^\infty x^2 f(x)dx - \int_u^\infty (x-u)^2 f(x)dx = E((X;u)^2), \tag{26}$$

so that

$$\sigma_K^2 = E(K^2 \mid X > 0) - (E(K \mid X > 0))^2. \tag{27}$$

The distribution of S, by applying Central Limit Theorem (CLT), may be estimated by normal distribution with mean,

$$\mu_{S,K} = E(S) = n\mu_K q, \tag{28}$$

and variance,

$$\sigma_{S,K}^2 = Var(S) = n(\sigma_K^2 q + \mu_K^2 q(1-q)). \tag{29}$$

If the premium is calculated using a linear loading assumption, i.e. premium= $\mu_{S,K}(1+\xi)$, the probability of insolvency for a single portfolio of risk may be equated as the probability of having aggregate claims larger than aggregate premiums, or, equivalently,

$$\Pr(S > \mu_{S,K}(1+\xi)) = \Pr\left(\frac{S - \mu_{S,K}}{\sigma_{S,K}} > \frac{\mu_{S,K}(1+\xi) - \mu_{S,K}}{\sigma_{S,K}}\right) = 1 - \Pr\left(Z < \frac{\mu_{S,K}}{\sigma_{S,K}}\xi\right). \tag{30}$$

It should be noted that when ξ =0, the premium is equivalent to the expected aggregate claims of policies with a policy limit at u. The linear loading assumption indicates that the relative loading, ξ, is fixed as a constant proportion of $\mu_{S,K}$ regardless of any values of u.

Tables 8-10 show the values of the insolvency probability for several values of u, ξ, n and q, assuming the amount of loss follows Burr with parameters $\theta = 86{,}426.43$, $\gamma = 1.5169$ and $\alpha = 3.7783$.

The graphs of insolvency probability vs. policy limit for several values of ξ, n and q are shown in Figures 8-10, indicating that under the assumption of linear loading, the insolvency probability increases as the policy limit increases. Based on values of $\mu_{S,K}$ and $\sigma_{S,K}$ displayed in Table 8, even though both $\mu_{S,K}$ and $\sigma_{S,K}$ increase when the limit increases, $\sigma_{S,K}$ increases faster than $\mu_{S,K}$ causing the quantity $\mu_{S,K}(\sigma_{S,K})^{-1}$ to decrease. Based on equation (30), the probability of insolvency is expected to increase.

In addition, the graphs in Figures 8-10 also show that insolvency probability

- decreases as the relative loading, ξ, increases;
- decreases as the probability of incurring claim, q, increases; and
- decreases as the number of policies, n, increases.

When the probability of incurring a claim or the number of policies increases, $\mu_{S,K}$ increases faster than $\sigma_{S,K}$ causing the quantity $\mu_{S,K}(\sigma_{S,K})^{-1}$ to increase. Therefore, based on equation (30), the probability of insolvency is expected to decrease.

Table 8: Values of u and insolvency probability ($n = 3000$, $q = 0.2$)

u (RM)	$\mu_{S,K}$ (RM)	$\sigma_{S,K}$ (RM)	$\xi = 0.25$ Insolvency probability	$\xi = 0.24$ Insolvency probability	$\xi = 0.23$ Insolvency probability	$\xi = 0.22$ Insolvency probability	$\xi = 0.21$ Insolvency probability
40,000	16,399,665	674,696	6.13E-10	2.71E-09	1.13E-08	4.46E-08	1.66E-07
60,000	19,517,266	849,996	4.72E-09	1.79E-08	6.42E-08	2.19E-07	7.11E-07
80,000	21,074,104	956,995	1.84E-08	6.28E-08	2.04E-07	6.34E-07	1.88E-06
100,000	21,866,758	1,022,471	4.48E-08	1.43E-07	4.35E-07	1.27E-06	3.54E-06

Table 9: Values of u and insolvency probability ($n = 3000$, $\xi = 0.15$)

u	$q = 0.40$			$q = 0.30$			$q = 0.20$		
	$\mu_{S,K}$	$\sigma_{S,K}$	Insolvency probability	$\mu_{S,K}$	$\sigma_{S,K}$	Insolvency probability	$\mu_{S,K}$	$\sigma_{S,K}$	Insolvency probability
40,000	32,799,329	855,061	0.0000000	24,599,497	784,592	0.0000013	16,399,665	674,696	0.0001332
60,000	39,034,532	1,091,347	0.0000000	29,275,899	994,238	0.0000050	19,517,266	849,996	0.0002863
80,000	42,148,208	1,239,193	0.0000002	31,611,156	1,123,712	0.0000122	21,074,104	956,995	0.0004780
100,000	43,733,516	1,331,212	0.0000004	32,800,137	1,203,592	0.0000218	21,866,758	1,022,471	0.0006685

Table 10: Values of u and insolvency probability ($\xi = 0.15$, $q = 0.2$)

u (RM)	$n = 3000$			$n = 2000$			$n = 1000$		
	$\mu_{S,K}$ (RM)	$\sigma_{S,K}$ (RM)	Insolvency probability	$\mu_{S,K}$	$\sigma_{S,K}$	Insolvency probability	$\mu_{S,K}$	$\sigma_{S,K}$	Insolvency probability
40,000	16,399,665	674,696	0.000133	10,933,110	550,887	0.001456	5,466,555	389,536	0.017644
60,000	19,517,266	849,996	0.000286	13,011,511	694,019	0.002460	6,505,755	490,746	0.023376
80,000	21,074,104	956,995	0.000478	14,049,403	781,383	0.003498	7,024,701	552,521	0.028255
100,000	21,866,758	1,022,471	0.000668	14,577,839	834,844	0.004406	7,288,919	590,324	0.032006

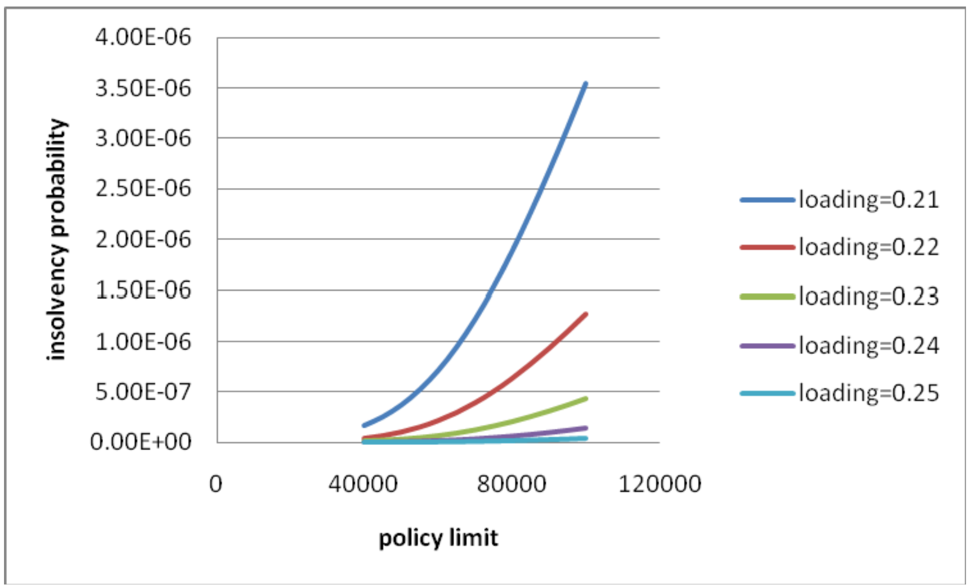

Figure 8: Graph of insolvency probability vs. policy limit ($n = 3000$, $q = 0.2$)

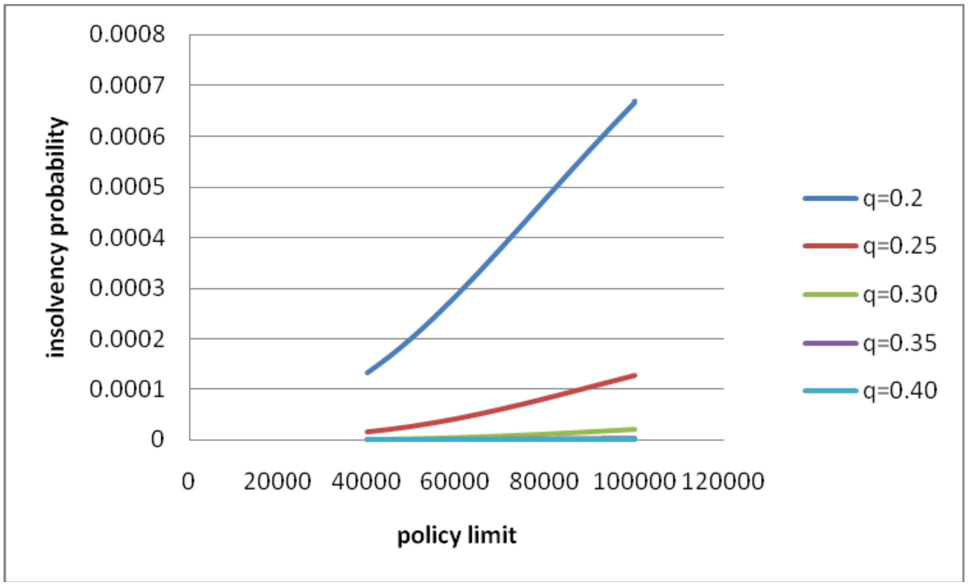

Figure 9: Graph of insolvency probability vs. policy limit ($n = 3000$, $\xi = 0.15$)

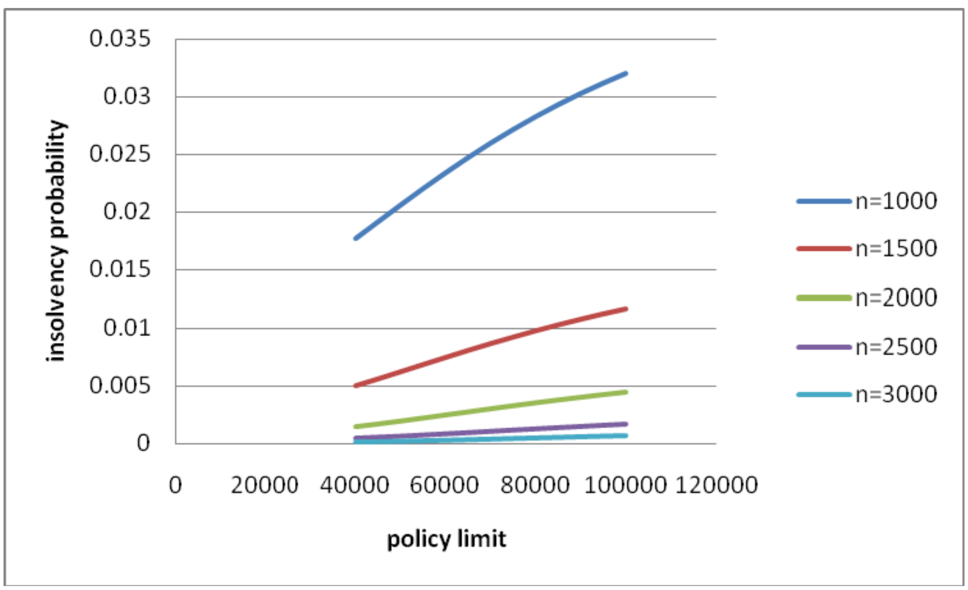

Figure 10: Graph of insolvency probability vs. policy limit ($\xi = 0.15$, $q = 0.2$)

5. PH-TRANSFORM ASSUMPTION

The determination of expected loss or mean severity based on the Proportional Hazard Transform (PH-Transform) assumption introduced by Wang [12] may be used as an alternative to reduce the probability of insolvency at a higher deductible or policy limit. In particular, the PH-Transform assumption incorporates an "appropriate" risk load in the severity distribution at a higher deductible or policy limit, and thus allows the probability of insolvency to be lower.

The mean severity under the PH-Transform assumption can be calculated as (Wang [12]-[13])

$$H(X) = \int_0^\infty (S(x))^r dx \,, \qquad 0 < r \leq 1, \tag{31}$$

where r denotes the index of ambiguity degree. The PH-mean shown in equation (31) represents a risk-adjusted premium and is quite sensitive to the choice of r. Index r can be assigned to the level of confidence in the estimation of loss, where a lower value of r implies a more ambiguous situation. For example, a non-ambiguous scenario for the best estimate could occur when there is little ambiguity regarding the best estimate of the severity distribution, such as when all experts agree with confidence in the estimate, whereas an ambiguous scenario could occur when there is considerable ambiguity regarding the best estimate of the severity distribution, such as when experts disagree and have little confidence in such estimate. From a broader perspective, examples of conditions contributing to greater ambiguity include uncertainty of the underlying loss distribution, incomplete information, insufficient data, changes in claim generating mechanisms, extra expenses associated with risk-sharing transactions, and difference in local market climates due to differences in geographic areas and/or lines of insurance (Wang [11]).

The PH-Transform can also be applied using subjective guidelines for the error of estimation; an actuary may construct his own table for index r to reflect different levels of ambiguity. One such example is given by Wang [11]:

Table 11: Ambiguity level and index r

Ambiguity level	Index r
Slightly ambiguous	0.96 – 1.00
Moderately ambiguous	0.90 – 0.95
Highly ambiguous	0.80 – 0.89
Extremely ambiguous	0.50 - 0.79

Source: Wang [11]

In addition to the severity distribution, the PH-Transform assumption can be applied on the frequency distribution where appropriate. As an example, in pricing a reinsurance contract, the PH-Transform can be applied separately on the severity and frequency distributions. The choice of r depends on the level of confidence in the estimate of claim severity and frequency. If the actuary has higher confidence in the estimate of claim frequency distribution but lower confidence in the estimate of claim severity distribution, he should chose a higher r for claim frequency, say 0.95, and a lower r for claim severity, say 0.85. For example, higher confidence for the frequency distribution and lower confidence for the severity distribution should be applied on types of insurance risks that provide considerable past data on the probability of occurrence but much uncertainty on the size of loss due to arbitrary court awards.

5.1 Insolvency Probability of Deductible Policy

The same approach may be used to find the expected loss of a deductible policy,

$$H(W) = \int_d^\infty (S(x))^r dx \,, \qquad 0 < r \leq 1. \tag{32}$$

where W is defined as equation (7).

For example, assume that the amount of loss follows a Burr distribution with parameters (α, θ, γ). The survival function is equal to

$$S(x) = \left(\frac{\theta^\gamma}{\theta^\gamma + x^\gamma} \right)^\alpha , \tag{33}$$

and if the PH-Transform assumption is applied, the survival function also follows a Burr distribution, but with parameters $(r\alpha, \theta, \gamma)$,

$$S(x)=\left(\frac{\theta^{\gamma}}{\theta^{\gamma}+x^{\gamma}}\right)^{r\alpha}. \tag{34}$$

Therefore, the equation of expected loss shown by equation (32) can also be rewritten as $H(W)=E(X)-E(X;d)$, this time assuming that the loss distribution follows a Burr distribution with parameters $(r\alpha,\theta,\gamma)$. In addition, $H(W)$ can be rewritten as a function of $E(W)$,

$$H(W)=(1+\psi)E(W) \tag{35}$$

where $E(W)=\int_d^{\infty} S(x)dx$, and ψ denotes the equivalent relative loading of a policy with deductible valued at d.

Table 12 shows the expected loss, $H(W)$, and the equivalent relative loading, ψ, under the PH-Transform assumption for several values of r. For example, the expected loss with no loading, i.e. the expected loss at $r=1$, for a deductible valued at RM5,000 is equivalent to RM33,228. If the PH-Transform assumption with $r=0.9$ is applied, the expected loss is RM36,804 and the equivalent relative loading, ψ, is equal to 0.11.

Table 12: Expected loss and relative loading (deductible policy)

d (RM)	Expected loss $r=1$ (RM)	Expected loss $r=0.9$ (RM)	Relative loading	Expected loss $r=0.7$ (RM)	Relative loading
5,000	33,228	36,804	0.11	47,426	0.43
10,000	28,670	32,203	0.12	42,740	0.49
15,000	24,561	28,013	0.14	38,382	0.56
20,000	20,934	24,267	0.16	34,389	0.64

Figure 11 shows the graph of expected loss vs. deductible for several values of r under the assumption of PH-Transform. It can be seen that the expected loss calculated under the PH-Transform ($r=0.9$ and $r=0.7$) is higher than the basic expected loss ($r=1$), implying that the expected loss is higher when the estimation of loss amount becomes more ambiguous.

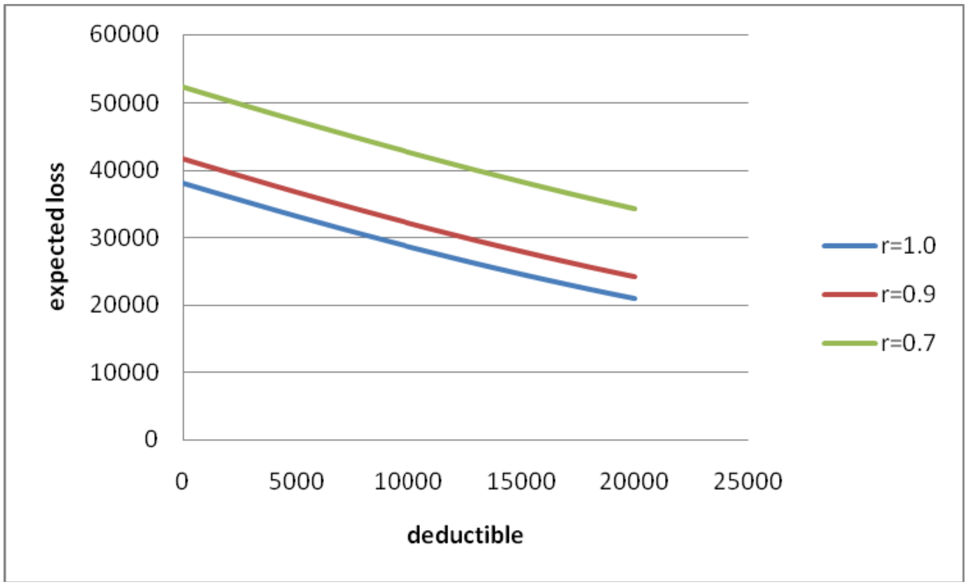

Figure 11: Graph of expected loss vs. deductible

If the probability of insolvency is calculated using equation (21), the linear loading assumption and PH-Transform assumption can be compared by using ξ as the relative loading for linear assumption and ψ as the relative loading for PH-Transform assumption. The main difference between the assumptions is that the relative loading for PH-Transform increases when d increases, whereas for linear loading, the relative loading remains fixed when d increases. Table 13 shows the values for insolvency probability for several values of ξ and ψ assuming $n = 3000$ and $q = 0.2$. Figure 12 shows the graph of insolvency probability vs. deductible under several linear loading and PH-Transform assumptions, also assuming $n = 3000$ and $q = 0.2$. It can be seen that the insolvency probability is lower for higher deductibles under the PH-Transform assumption. Thus, the PH-Transform can be used as an alternative to reduce the probability of insolvency at higher deductible values by incorporating an "appropriate" risk load in the severity distribution.

Appendix 3 uses R programming with the assistance of the *actuar* package to calculate the expected loss for a deductible policy under the PH-Transform assumption, assuming the amount of loss follows a Burr distribution.

Table 13: Insolvency probability for linear loading and PH-Transform

d (RM)	Linear loading				PH-Transform $r=0.9$	
	ξ	Insolvency probability	ξ	Insolvency probability	ψ	Insolvency probability
5,000	0.15	0.003	0.10	0.031	0.11	0.023
10,000	0.15	0.005	0.10	0.042	0.12	0.017
15,000	0.15	0.009	0.10	0.056	0.14	0.013
20,000	0.15	0.015	0.10	0.073	0.16	0.010

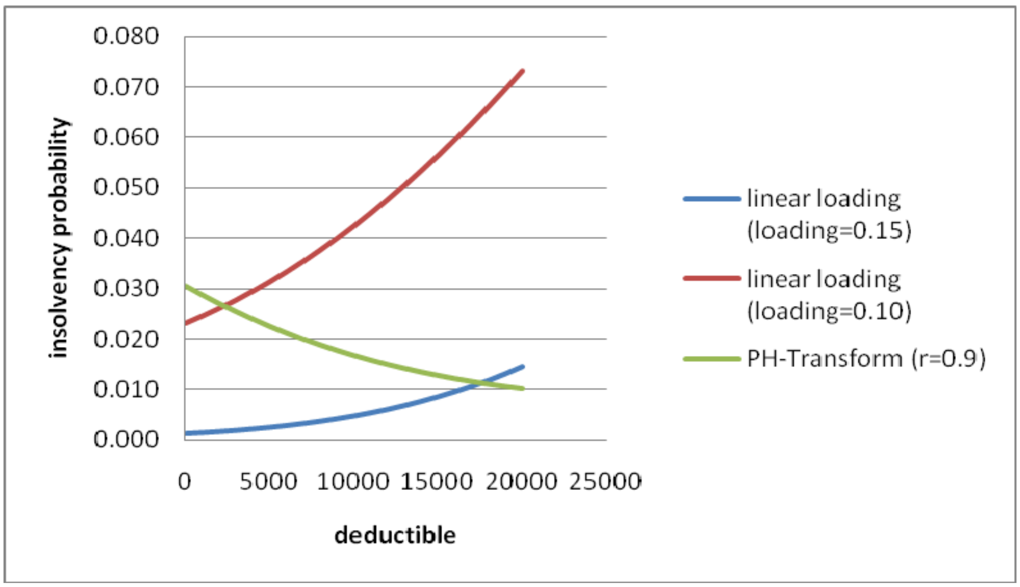

Figure 12: Graph of insolvency probability vs. deductible

5.2 Insolvency Probability of Policy Limit

Similar to a deductible policy, the expected loss of a policy limit under PH-Transform assumption can be calculated as

$$H(K)=\int_{o}^{u}(S(x))^{r}dx\ , \qquad 0<r\leq 1, \tag{36}$$

where K is defined as equation (9).

If the amount of loss follows a Burr distribution with parameters (α,θ,γ), the equation of expected loss shown by equation (36) can be rewritten as $H(K)=E(X;u)$, this time assuming that the loss distribution follows a Burr distribution with parameters $(r\alpha,\theta,\gamma)$. In addition, $H(K)$ can be rewritten as a function of $E(K)$,

$$H(K)=(1+\eta)E(K), \tag{37}$$

where $E(K)=\int_{o}^{u} S(x)dx$, and η denotes the equivalent relative loading of a policy with limit valued at u .

Table 14 provides the expected loss, $H(K)$, and the equivalent relative loading, η, under the PH-Transform assumption for several values of r assuming $n=3000$ and $q=0.2$. Figure 13 shows the graph of expected loss vs. policy limit for several values of r under the PH-Transform assumption, also assuming $n=3000$ and $q=0.2$. It can be seen that the expected loss calculated under the PH-Transform assumption ($r=0.8$ and $r=0.7$) is higher than the basic expected loss ($r=1$), also implying that the expected loss is higher when the estimation of loss amount becomes more ambiguous.

Table 14: Expected loss and relative loading (policy limit)

u (RM)	Expected loss $r=1$ (RM)	Expected loss $r=0.8$ (RM)	Relative loading	Expected loss $r=0.7$ (RM)	Relative loading
40,000	27,333	29,286	0.07	30,353	0.11
60,000	32,529	36,068	0.11	38,106	0.17
80,000	35,124	39,960	0.14	42,875	0.22
100,000	36,445	42,228	0.16	45,849	0.26

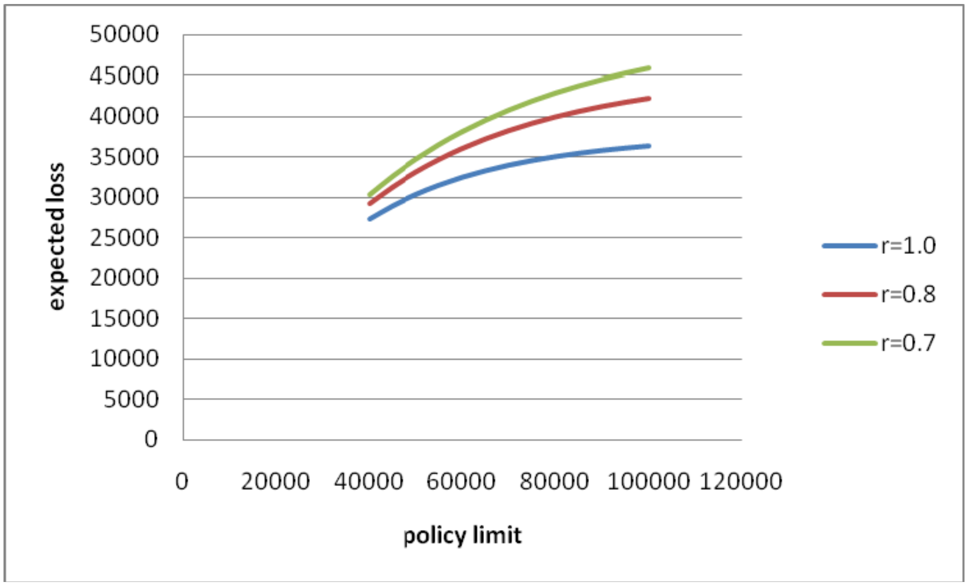

Figure 13: Graph of expected loss vs. policy limit

If the probability of insolvency is calculated using equation (30), the linear loading assumption and PH-Transform assumption can also be compared by using ξ as the relative loading for the linear assumption and η as the relative loading for PH-Transform assumption. The main difference between the assumptions is that the relative loading for PH-Transform increases when u increases, whereas for linear loading, the relative loading remains fixed when u increases. Table 15 shows the values for insolvency probability for several values of ξ and η.

Table 15: Insolvency probability for linear loading and PH-Transform

u (RM)	Linear loading				PH-Transform $r = 0.8$	
	ξ	Insolvency probability	ξ	Insolvency probability	η	Insolvency probability
40,000	0.15	0.0001	0.10	0.0075	0.07	0.0412
60,000	0.15	0.0003	0.10	0.0108	0.11	0.0062
80,000	0.15	0.0005	0.10	0.0138	0.14	0.0012
100,000	0.15	0.0007	0.10	0.0162	0.16	0.0003

Figure 14 shows the graph of insolvency probability vs. policy limit under several linear loading and PH-Transform assumptions. It can be seen that the insolvency probability is lower for higher limits under the PH-Transform assumption. Thus, the PH-Transform can be used as an alternative to reduce the probability of insolvency at higher limit values by incorporating an "appropriate" risk load in the severity distribution.

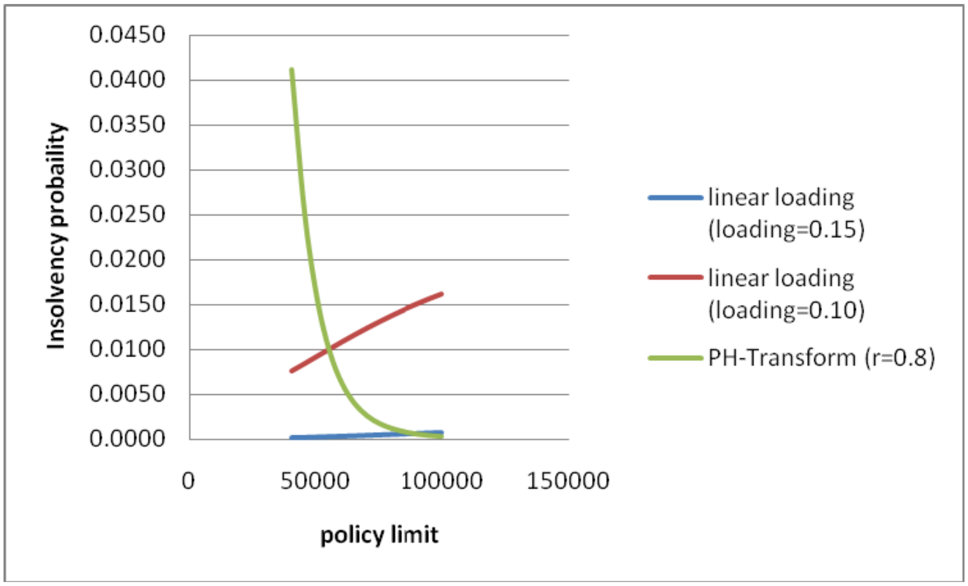

Figure 14: Graph of insolvency probability vs. policy limit

6. EXCESS LAYERS OF A SINGLE RISK

6.1 Pricing of Excess Layers

In an insurance contract containing both a deductible d and a policy limit u, the loss of a layer $(d, d+u]$ of a risk X can be defined by the random variable M, where

$$M = \begin{cases} 0, & X \le d \\ X - d, & d < X < d + u \\ u, & X \ge d + u \end{cases}. \tag{38}$$

Therefore, the average loss or mean severity of a layer $(d, d+u]$ may be written as

$$E(M) = \int_{d}^{d+u} S(x)\, dx\,, \tag{39}$$

whereas under the PH-Transform assumption, the average loss of the same layer is

$$H(M) = \int_{d}^{d+u} (S(x))^{r}\, dx\,. \tag{40}$$

If the amount of loss follows a Burr distribution with parameters (α,θ,γ), the equation of expected loss or mean severity shown by equation (39) can also be rewritten as

$$E(M) = E(X;d+u) - E(X;d)\,, \tag{41}$$

whereas under the PH-Transform assumption, equation (40) can also be rewritten as

$$H(M) = E(X;d+u) - E(X;d)\,, \tag{42}$$

this time assuming the amount of loss follows a Burr distribution with parameters $(r\alpha,\theta,\gamma)$.

For a single risk, the expected aggregate claims shown by equations (19) and (28) can be simplified into

$$E(S) = E(M)q\,, \tag{43}$$

i.e., assuming $n=1$.

Under the PH-Transform assumption, the expected aggregate claim amount can also be calculated, and it is equal to

$$E(S) = H(M)q\,. \tag{44}$$

$H(M)q$ can also be rewritten as a function of $E(M)q$,

$$H(M)q = (1+\xi)E(M)q\,, \tag{45}$$

where ξ denotes the equivalent relative loading of a policy with deductible d and limit u.

Table 16 shows the expected aggregate claims and equivalent relative loading, ξ, for several values of d and u under the PH-Transform assumption, where $n=1$, $q=0.1$ and the individual loss amount follows a Burr distribution with parameters $\theta = 86{,}426.43$, $\gamma = 1.5169$, and $\alpha = 3.7783$. For example, the expected aggregate claim amount or the premium with no loading, i.e., $r=1$, for layer (0, 5000], is equivalent to RM490.24. If the PH-Transform assumption with $r=0.92$ is applied, the premium is RM491.01 and the equivalent relative loading is ξ=0.002. It can be observed from the table that the relative loading, ξ, under the PH-Transform assumption increases as the layer, $(d, d+u]$, increases.

Figure 15 shows the graph of expected aggregate claim amount vs. layer for several values of the ambiguity index, r, assuming $q = 0.1$ for the same loss distribution assumption. The graph shows that the expected aggregate claim amount decreases when the value of the layer, $(d, d+u]$, increases. Equations (39) and (40) imply that the expected aggregate claim amount depends on the integrals of $S(x)$ and $S(x)^r$. Since $S(x)$ is a decreasing function, the areas under the curves of $S(x)$ and $S(x)^r$ are smaller as the value of $(d, d+u]$ is higher, which causes the expected aggregate claim amount to decrease. In addition, the graph also shows that the expected aggregate claim amount increases when the ambiguity index, r, decreases, indicating that the relative loading, ξ, is higher when the estimation of loss is more ambiguous.

Table 16: Expected aggregate claim amount and relative loading (single risk, PH Transform)

d (RM)	$d+u$ (RM)	Aggregate claims (RM) $(r=1)$	Aggregate claims (RM) $(r=0.92)$	Relative loading	Aggregate claim (RM) $(r=0.90)$	Relative loading
0	5,000	490.24	491.01	0.002	491.20	0.002
5,000	10,000	455.85	459.22	0.007	460.07	0.009
10,000	15,000	410.90	417.38	0.016	419.02	0.020
20,000	25,000	315.38	327.20	0.037	330.23	0.047
40,000	45,000	165.32	180.61	0.092	184.65	0.117
80,000	85,000	41.59	50.74	0.220	53.33	0.282
100,000	105,000	21.68	27.87	0.285	29.67	0.368
160,000	165,000	3.93	5.80	0.473	6.39	0.623

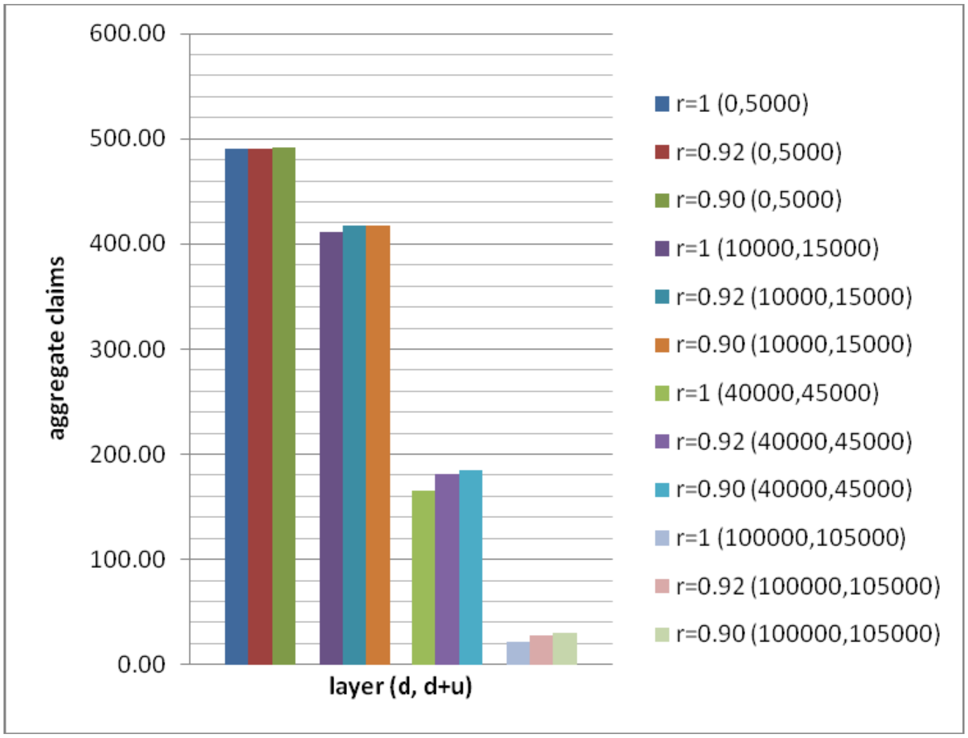

Figure 15: Graph of expected aggregate claim amount vs. layer (single risk, $n=1$, $q=0.1$)

Appendix 4 shows the calculation of the expected aggregate claim amount for a single risk and a single layer using R programming with the assistance of the *actuar* package, assuming that the severity follows a Burr distribution.

6.2 Increased Limit Factor (ILF)

In liability insurance, a policy generally provides coverage up to a specified maximum amount that will be paid on any individual loss. In the U.S., it is general practice to publish rates for some standard limit, the "basic limit" (for example, USD$100,000), to which rates the increased limit factors (ILF) are applied to calculate increased limit rates (Wang [11]). In Malaysia, however, the practice has not been implemented; therefore, the ILF calculated in this study may be used as some indication or basis for possible basic and increased rates.

If the basic limit is valued at RM100,000, the ILF can be calculated as the expected loss at the increased limit divided by the expected loss at the basic limit,

$$ILF(a) = \frac{E(X;a)}{E(X;100000)}. \tag{46}$$

If a risk load is to be included, equation (46) can be rewritten as

$$ILF(a) = \frac{E(X;a) + RL(a)}{E(X;100000) + RL(100000)}, \tag{47}$$

where $RL(a)$ and $RL(100000)$ denote the risk load.

Under the PH-Transform assumption, equation (47) can be rewritten as

$$ILF(a) = \frac{H(X;a)}{H(X;100000)}, \tag{48}$$

where $H(X;a)$ and $H(X;100000)$ denote the mean severity calculated under the PH-Transform assumption. Since $H(X;a) > E(X;a)$ and $H(X;100000) > E(X;100000)$, the equivalent risk load for the PH-Transform assumption can be calculated. Table 17 shows the ILFs under the PH-Transform assumption assuming that the loss distribution follows a Burr distribution with parameters $\theta = 86{,}426.43$, $\gamma = 1.5169$ and $\alpha = 3.7783$. However, the ILFs calculated appear to be extremely flat, indicating that larger claims may be under-represented by fitting a Burr distribution. Additional treatment is needed in this situation, such as considering a mixed distribution which may produce a more appropriate result for fitting large claims.

Figure 16 shows the graph of ILF vs. a under the PH-Transform assumption for the same severity distribution. The graph shows that the ILFs increase when a increases but remain at a fixed value for large values of a. In addition, the graph shows that the ILFs increase when the ambiguity index, r, decreases, implying that the risk load is higher when loss estimation is more ambiguous.

Table 17: ILF

a (RM)	$E(X;a)$ (RM)	ILF without RL	Risk Load (RM) ($r=0.9$)	ILF ($r=0.9$)	Risk Load (RM) ($r=0.85$)	ILF ($r=0.85$)
100,000	36,444.60	1.000000	2,678.91	1.000000	4,172.73	1.000000
200,000	37,960.89	1.041605	3,412.12	1.057497	5,401.38	1.067581
300,000	38,097.00	1.045340	3,535.89	1.064140	5,624.74	1.076431
400,000	38,120.88	1.045995	3,566.56	1.065534	5,683.37	1.078462
500,000	38,127.10	1.046166	3,576.64	1.065951	5,703.53	1.079112

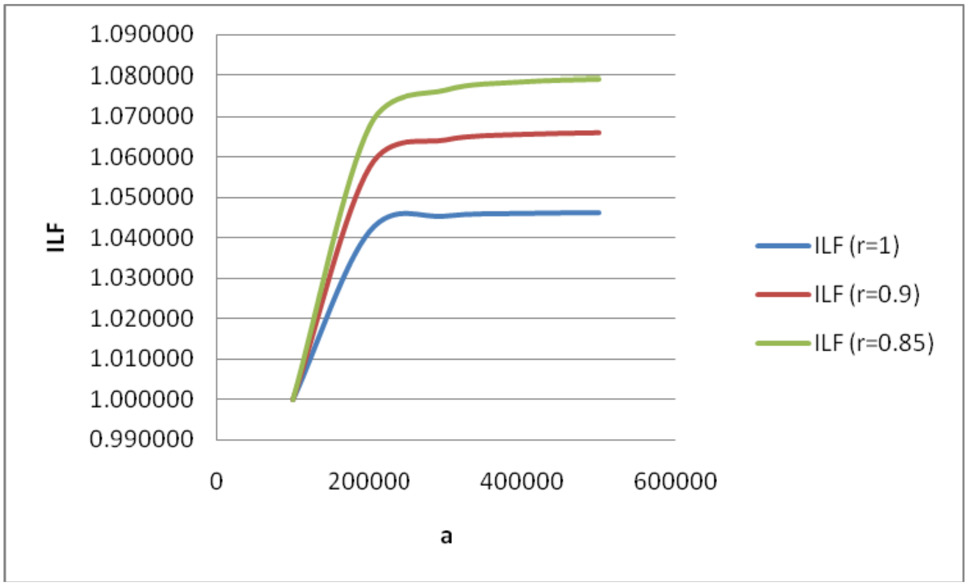

Figure 16: Graph of ILF vs. a

Appendix 5 shows the calculation of ILFs using R programming with the assistance of *actuar* package, assuming that the amount of loss follows a Burr distribution.

7. EXCESS-OF-LOSS FOR REINSURANCE TREATY

In a developing country such as Malaysia, we seldom have a single local insurer covering a single large risk, especially in non-life insurance businesses. In practice, a large risk is usually divided into several excess-of-loss layers shared and insured by several local or multinational insurers or reinsurers. The pricing of layers, therefore, is crucial, especially in the process of dividing risk and pricing risk fairly for each insurer. In this paper, we would like to introduce an approach which may be considered as fair and efficient for pricing excess-of-loss layers of a reinsurance treaty. The fairness in pricing may be achieved by implementing a PH-Transform assumption whereby the insolvency probability is lowered. In addition, the efficiency in pricing may be obtained by using R programming with the *actuar* package to allow the pricing by layer to be computed with less effort.

Let N denote the random variable for claim frequency. Hence, the expected frequency can be calculated as

$$E(N) = \sum_{k=0}^{\infty} S(k),\ k = 0,1,\ldots, \tag{49}$$

whereas under a PH-Transform assumption, the expected frequency is equivalent to (Wang [11],

$$H(N)=\sum_{k=0}^{\infty}(S(k))^{r}\ . \tag{50}$$

Let X denote the random variable for loss severity. The expected severity is $E(X)=\int_{0}^{\infty}S(x)dx$, whereas under the PH-Transform assumption, the expected severity is equal to $H(X)=\int_{0}^{\infty}(S(x))^{r}\,dx$.

By implementing both frequency and severity approaches, the expected aggregate claims can be calculated as

$$E(S)=E(N)E(X)\,, \tag{51}$$

whereas under the assumption of PH-Transform, the expected aggregate claims is equal to

$$H(N)H(X)\,. \tag{52}$$

The same approach may also be implemented for calculating the price of several excess-of-loss layers. The mean severity for layer $(d,d+u]$ is the same as equation (41) whereas under a PH-Transform assumption, the mean severity for the same layer is the same as equation (42). Therefore, the expected aggregate claims is

$$E(S)=E(N)E(M)\,, \tag{53}$$

whereas under a PH-Transform assumption, the expected aggregate claims is

$$H(N)H(M)\,. \tag{54}$$

If the amount of loss follows a Burr distribution with parameters (α,θ,γ), the calculation of $H(M)$ in equation (54) also follows a Burr distribution, this time with parameters $(r\alpha,\theta,\gamma)$.

If the claim frequency follows a Poisson distribution with parameter λ, the aggregate claims, S, follow a compound Poisson distribution whereby the variance of aggregate claims can be written as

$$Var(S)=\lambda E(M^{2})\,, \tag{55}$$

where $E(M^{2})=E((X;d+u)^{2})-E((X;d)^{2})-2dE(X;d+u)+2dE(X;d)$.

Table 18 shows the mean severity, mean frequency, burning cost, loaded rate, and relative loading under a PH-Transform assumption for several excess-of-loss layers, assuming N is Poisson with parameter $\lambda=100$, X is Burr with parameters $\theta=86{,}426.43$, $\gamma=1.5169$ and $\alpha=3.7783$, and $r=0.95$ for both frequency and severity distributions.

Table 18: Mean severity, mean frequency, burning cost, loaded rate and relative loading

Layer (RM)	$E(M)$ (RM)	$H(M)$ ($r=0.95$) (RM)	$E(N)$	$H(N)$ ($r=0.95$)	Burning Cost	Loaded Rate	Relative Loading
$(100k,300k]$	1,652.40	2,033.77	100	100.47	0.016524	0.020434	0.24
$(300k,500k]$	30.10	46.15	100	100.47	0.000301	0.000464	0.54
$(500k,700k]$	2.91	5.04	100	100.47	0.000029	0.000051	0.74
$(700k,900k]$	0.56	1.06	100	100.47	0.000006	0.000011	0.90
$(100k,900k]$	1,685.97	2,086.01	100	100.47	0.016860	0.020959	0.24

The burning cost is calculated as (Wang [11])

$$\frac{E(M)E(N)}{SEP}, \tag{56}$$

where SEP denotes the subject earned premium. In this study, the SEP is assumed to be RM10,000,000.

The loaded rate is calculated as (Wang [11])

$$\frac{H(M)H(N)}{SEP}, \tag{57}$$

whereby it can also be written as a function of the burning cost,

$$\frac{H(M)H(N)}{SEP} = (1+\xi)\frac{E(M)E(N)}{SEP}, \tag{58}$$

where ξ denotes the equivalent relative loading. Based on Table 18, the relative loading, ξ, under a PH-Transform assumption increase as the excess-of-loss layer, $(d, d+u]$, increase. In addition, the values of $E(M)$ and $H(M)$ decrease when the layer, $(d, d+u]$, increases.

The distribution of aggregate claims, S, by applying Central Limit Theorem, may be estimated by the Normal distribution with mean $E(S) = \lambda E(M)$ and variance $Var(S) = \lambda E(M^2)$. The probability of insolvency, i.e. the probability of having aggregate claims larger than aggregate premiums, for a PH-Transform assumption can be calculated as

$$\Pr(S > H(N)H(M)) = \Pr(S > (1+\xi)E(S)) = \Pr\left(Z > \frac{E(S)}{\sqrt{Var(S)}}\xi\right). \tag{59}$$

In terms of insolvency probability, the main difference between a linear loading assumption and a PH-Transform assumption is that the relative loading for a PH-Transform increases when the layer $(d, d+u]$ increases, whereas the relative loading remains fixed at ξ for all layers under the linear loading.

Table 19 provides the value of mean severity, mean frequency, mean aggregate claims, and variance aggregate claims. It should be noted that both $E(S)$ and $Var(S)$ decrease when excess-of-loss layer, $(d, d+u]$, increases.

Table 20 shows the values of premium and relative loading for several excess-of-loss layers under the PH-Transform assumptions ($r = 0.95$, $r = 0.90$ and $r = 0.85$). It should be noted that the lower the ambiguity index, r, the higher the premium layer, implying that the relative loading is higher when ambiguity increases. In addition, the premium is lower when the layer, $(d, d+u]$, increases. The relative loading is also higher when the layer, $(d, d+u]$, increases.

Table 21 shows the values of insolvency probability under a linear loading assumption for several values of relative loading ($\xi = 0.10$, $\xi = 0.15$ and $\xi = 0.20$), and a PH-Transform assumption for several values of ambiguity index ($r = 0.95$, $r = 0.90$ and $r = 0.85$). The table shows that the insolvency probability for the PH-Transform is lower than the linear loading for all layers, but the difference is lower when the layer of $(d, d+u]$ increases. Therefore, a PH-Transform assumption may be used as an alternative to reduce insolvency probability of excess-of-loss layers in reinsurance treaties by incorporating "appropriate" risk loads in the frequency and severity distributions of all layers.

Table 19: Mean severity, mean frequency, mean aggregate claims and variance aggregate claims

Layer	$E(M)$ (RM)	$E(N)$	$E(S)=\lambda E(M)$ (RM)	$Var(S)=\lambda E(M^2)$ (RM)
$(100k, 300k]$	1,652.40	100	165,240	12,596,760,695
$(300k, 500k]$	30.10	100	3,010	356,232,253
$(500k, 700k]$	2.91	100	291	41,096,487
$(700k, 900k]$	0.56	100	56	8,650,994
$(100k, 900k]$	1,685.97	100	168597	14,506,333,740

Table 20: Premium and relative loading (PH-Transform)

Layer	$H(M)H(N)$ (RM) $(r=0.95)$	Relative loading	$H(M)H(N)$ (RM) $(r=0.9)$	Relative loading	$H(M)H(N)$ (RM) $(r=0.85)$	Relative loading
$(100k, 300k]$	204,337	0.24	253,397	0.53	315,181	0.91
$(300k, 500k]$	4,637	0.54	7,154	1.38	11,055	2.67
$(500k, 700k]$	507	0.74	884	2.04	1,543	4.31
$(700k, 900k]$	106	0.90	201	2.60	383	5.84
$(100k, 900k]$	209,587	0.24	261,635	0.55	328,162	0.95

Table 21: Insolvency probability

Layer (RM)	Linear loading			PH-Transform		
	$\Pr(S > E(S)(1+\xi))$ $\xi = 0.1$	$\Pr(S > E(S)(1+\xi))$ $\xi = 0.15$	$\Pr(S > E(S)(1+\xi))$ $\xi = 0.2$	$\Pr(S > H(X)H(N))$ $(r = 0.95)$	$\Pr(S > H(X)H(N))$ $(r = 0.9)$	$\Pr(S > H(X)H(N))$ $(r = 0.85)$
$(100k, 300k]$	0.4415	0.4126	0.3842	0.3638	0.2161	0.0908
$(300k, 500k]$	0.4936	0.4905	0.4873	0.4657	0.4131	0.3350
$(500k, 700k]$	0.4982	0.4973	0.4964	0.4866	0.4632	0.4226
$(700k, 900k]$	0.4992	0.4989	0.4985	0.4932	0.4803	0.4558
$(100k, 900k]$	0.4443	0.4168	0.3898	0.3668	0.2199	0.0926

Figures 17-20 show the graphs of insolvency probability for several values of ξ (under a linear loading assumption) and r (under a PH-Transform assumption) for each layer of $(d, d+u]$. Figure 21 shows the graph of insolvency probability for all layers. The equivalent loading, ξ, for each r is also shown in the figures. As an example, when $r = 0.95$ under the PH-Transform, the equivalent ξ for layer $(100k, 300k]$ is $\xi = 0.24$, as shown in Figure 17.

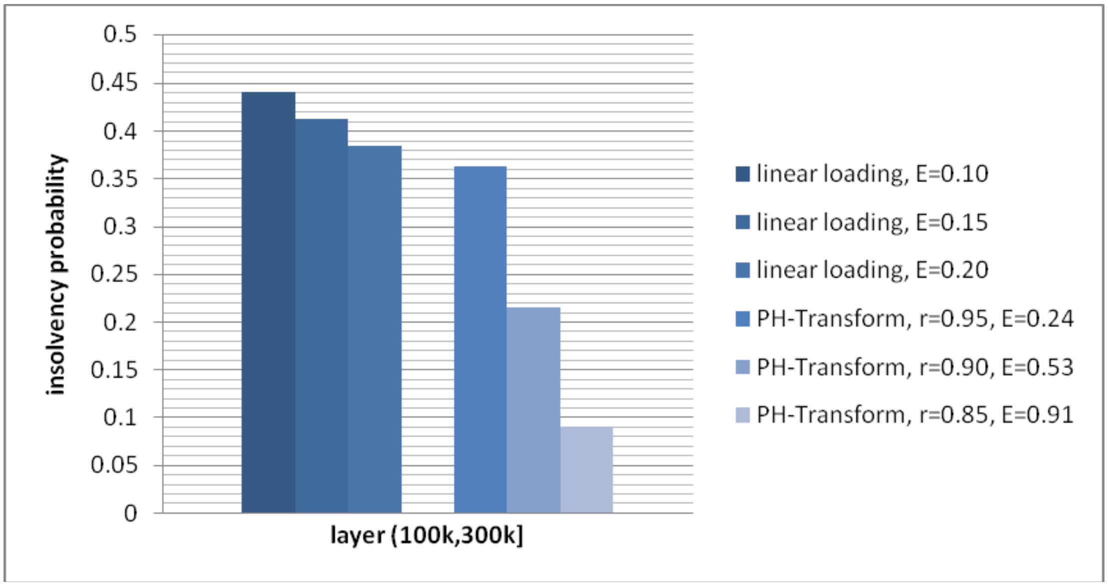

Figure 17: Graph of insolvency probability (layer $(100k, 300k]$)

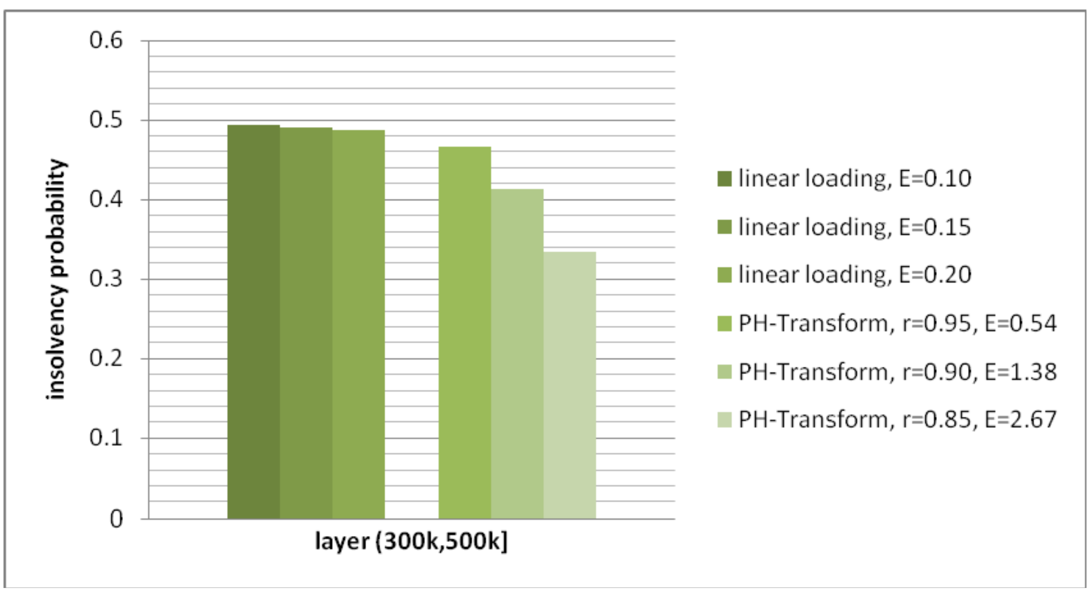

Figure 18: Graph of insolvency probability (layer $(300k, 500k]$)

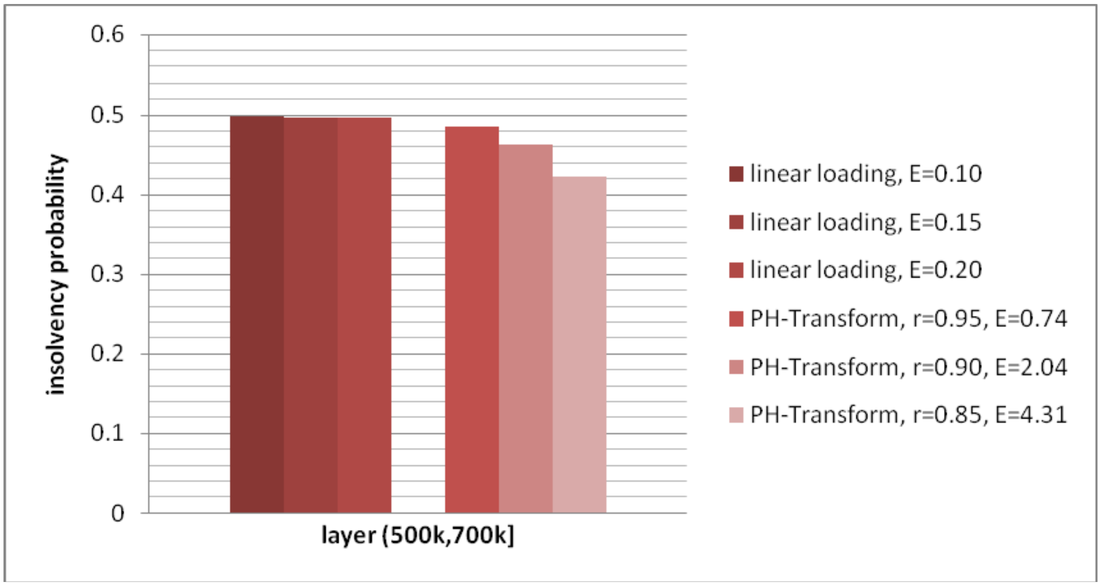

Figure 19: Graph of insolvency probability (layer $(500k,700k]$)

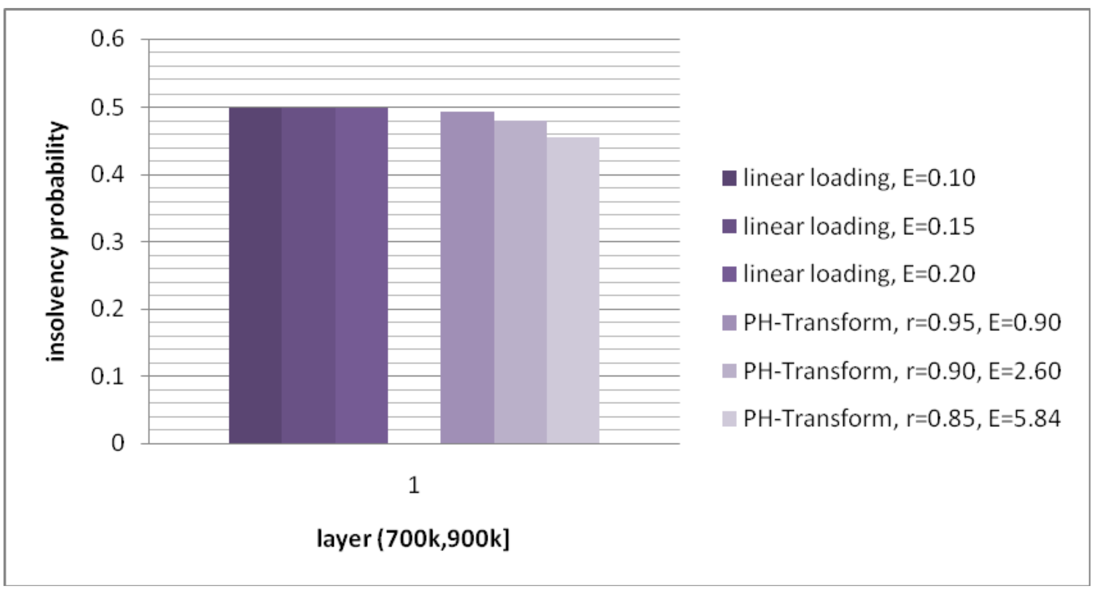

Figure 20: Graph of insolvency probability (layer $(700k,900k]$)

The graphs show that under the linear loading assumption, insolvency probability decreases when relative loading increases. When the PH-Transform assumption is applied, the insolvency probability is reduced to a lower level compared to the linear loading assumption, and the reason for this is that the equivalent risk load is higher under the PH-Transform.

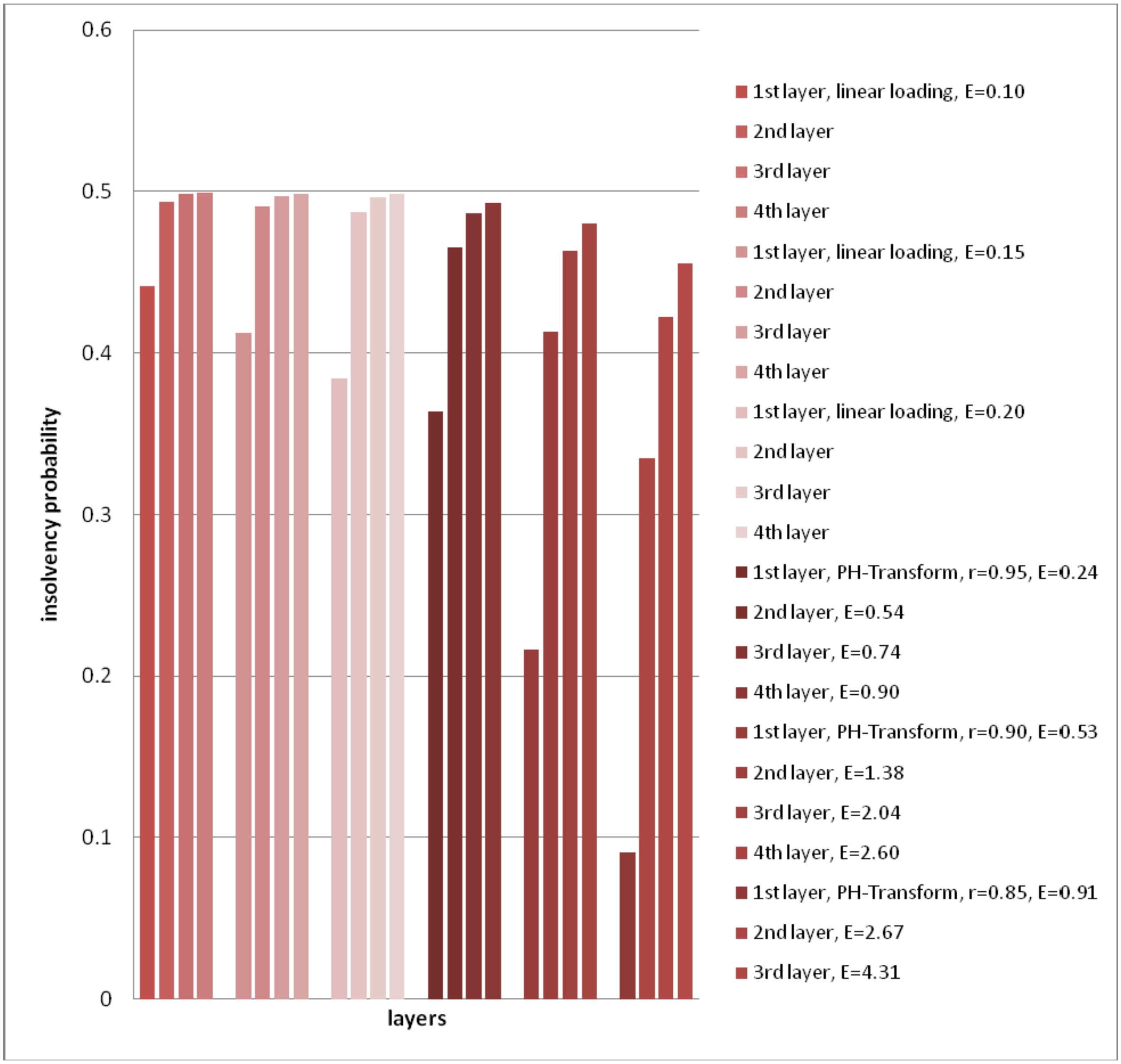

Figure 21: Graph of insolvency probability (all layers)

Appendix 6 shows the calculation of mean severity, mean frequency, mean aggregate claims, variance aggregate claims and insolvency probability under linear loading and PH-Transform assumptions, using R programming with the assistance of *actuar* package, assuming the severity distribution is Burr and the frequency distribution is Poisson.

8. CONCLUSION

In this paper, we have modeled individual loss amount, selected the best model using Kolmogorov-Smirnov, Anderson-Darling and Schwarz Bayesian Criterion, provided a range of deductible and policy limits based on Loss Elimination Ratio (LER), calculated insolvency probability under linear loading and PH-Transform assumptions, priced excess-of-loss of layer $(d, d+u]$ assuming a single risk, calculated increased limit factors (ILF), priced layers of a reinsurance treaty using a frequency and severity approach, and calculated the insolvency probability of a reinsurance treaty. Our proposed approach may be considered fair and efficient for two main reasons; the PH-Transform assumption may be implemented to lower the insolvency probability, and the R programming with the *actuar* package may be used for pricing excess-of-loss layers with less effort. In particular, the PH-Transform assumption is applied as a means of incorporating a risk load in the severity and/or frequency distributions and can be used to lower the insolvency probability of a single excess-of-loss layer as well as multiple layers of a reinsurance treaty. In addition, the ILF calculated in this study may be used as some indication or basis for possible basic and increased rates of the Malaysian insurance losses.

It is noteworthy that different distributions for loss severity and frequency can also be applied. Besides Burr distribution, Wang [12] showed that the PH-Transform assumption can be applied to several loss amount distributions such as exponential, uniform, Pareto and Weibull. The mean severity for a PH-Transform assumption, i.e., $H(M) = \int_{d}^{d+u} (S(x))^r dx$, can easily be computed using R programming with *actuar* package for such distributions. In addition, the computation of mean frequency for a PH-Transform assumption, i.e., $H(N) = \sum_{k=0}^{\infty} (S(k))^r$, for other frequency distributions such as binomial or negative binomial, can be also be implemented using R programming with the *actuar* package.

REFERENCES

[1] Bank Negara Malaysia, "Annual report of the Director General of Insurance," 1995.

[2] Bank Negara Malaysia, "Annual report of the Director General of Insurance," 2005.

[3] Bank Negara Malaysia, "The Financial Sector Masterplan," 2001.

[4] Cummins, J.D., "Property-Liability Insurance Price Regulation: The Last Bastion?", Chapter 1 in *Deregulating Property-Liability Insurance: Restoring Competition and Increasing Market Efficiency*, Washington, DC: AEI-Brookings Joint Center for Regulatory Studies, the American Enterprise Institute for Public Policy Research, 2002.

[5] Dimitriyadis, I., and U.N. Oney, "Deductibles in Health Insurance," *Journal of Computational and Applied Mathematics* 233(1), 2009, pp. 51-60.

[6] Hua, L. and K.C. Cheung, "Worst Allocation of Policy Limits and Deductibles," *Insurance: Mathematics and Economics* 40(1), 2008, pp. 93-98.
[7] Klugman, S., H. Panjer, and G. Willmot, *Loss Models: From Data to Decisions*, 3rd ed., Hoboken, NJ: John Wiley and Sons, 2008.
[8] Lau, S.W.,"Malaysia Reinsurance Market Environment: Where Are We Going?", *Insurance*, July-September 2009, pp. 19-24.
[9] Lee, H.L., *The Insurance Industry in Malaysia: A Study in Financial Development and Regulation*, Kuala Lumpur: Oxford University Press, 1997.
[10] Rao, K., "The Impact of Detariffing," *Newstrack: Corporate Newsletter of Bajaj Allianz General Insurance*, December, 6, 2006.
[11] Wang, S., "Implementation of Proportional Hazards Transforms in Ratemaking," *Proceedings of the Casualty Actuarial Society Casualty Actuarial Society* LXXXV, 1998, pp. 940-979.
[12] Wang, S., "Insurance Pricing and Increased Limits Ratemaking by Proportional Hazards Transforms," *Insurance: Mathematics and Economics* 17, 1995, pp. 43-54.
[13] Wang, S., "Premium Calculation by Transforming the Layer Premium Density," *ASTIN Bulletin* 26, 1996, pp. 71-92.
[14] Zhuang, W., Z. Chen, and T. Hu, "Optimal Allocation of Policy Limits and Deductibles Under Distortion Risk Measures," *Insurance: Mathematics and Economics* 44(3), 2008, pp. 409-414.

Appendix 1: R programming for LER (deductible policy, Burr distribution)

```
deduktibel <- function(alfa, gama, teta)
{
# to calculate E(X), d, E(X:d) and LER
EX <- mburr(1, alfa, gama, 1, teta)
d <- seq(0, 20000, by=1000)
EX.d <- levburr(d, alfa, gama, 1, teta, 1)
LER.d <- EX.d/EX
result.d <- cbind(d, EX.d, LER.d)
# to plot LER vs. d
plot.LERvsD <- plot(d, LER.d, type="p")
# to print result
list(EX=EX, result.d=result.d, plot.LERvsD)
}
deduktibel(alfa=3.778263226, gama=1.516886923, teta=86426.43339)
```

Appendix 2: R programming for insolvency probability (deductible policy, Burr distribution, linear loading)

```
insolvent.prob <- function(alfa, gama, teta, n, prob.claim, loading)
{
# to calculate d, E(W), E(W2), Var(W), E(S), Var(S) and insolvency probability
 d <- seq(0, 20000, by=1000)
```

```
 EW <- mburr(1,alfa,gama,1,teta) - levburr(d,alfa,gama,1,teta,1)
 EW2 <- mburr(2,alfa,gama,1,teta) - levburr(d,alfa,gama,1,teta,2) -
          2*d*mburr(1,alfa,gama,1,teta) + 2*d*levburr(d,alfa,gama,1,teta,1)
 VW <- EW2 - (EW^2)
 ES <- n*EW*prob.claim
 VS <- n*(VW*prob.claim+(EW^2)*prob.claim*(1-prob.claim))
 sigmaS <- VS^0.5
 insolven.prob <- pnorm(ES*loading/sigmaS, 0, 1, FALSE, FALSE)
 result <- cbind(d, ES, sigmaS, insolven.prob)
# to plot insolvency probability vs. deductible
plot.PROBvsD <- plot(d,insolven.prob,type="p")
# to print result
 list(n=n, prob.claim=prob.claim, loading=loading, result=result, plot.PROBvsD)
}
insolvent.prob(alfa=3.778263226, gama=1.516886923, teta=86426.43339, n=3000, prob.claim=0.2,
loading=0.25)
```

Appendix 3: R programming for expected loss (deductible policy, Burr distribution, PH Transform)

```
explossPH <- function(alfa, gama, teta, r)
 {
 # to compute d, E(X) and loading
  d <- seq(0, 20000, by=1000)
  EX.basic <- mburr(1,alfa,gama,1,teta) - levburr(d,alfa,gama,1,teta,1)
  EX.r <- mburr(1,r*alfa,gama,1,teta) - levburr(d,r*alfa,gama,1,teta,1)
  loading <- (EX.r-EX.basic)/EX.basic
  result <- cbind(d, EX.basic, EX.r, loading)
  # to plot E(X) vs. deductible
  plot.EXvsD <- plot(c(d,d), c(EX.basic, EX.r), type="p")
  # to print result
  list(r=r, result=result, plot.EXvsD)
 }
explossPH(alfa=3.778263226, gama=1.516886923, teta=86426.43339, r=0.8)
```

Appendix 4: R programming for expected aggregate premium (single layer, single risk, Burr distribution, PH Transform)

```
layer <- function(alfa, gama, teta, d, u, prob.claim, r)
{
# to compute E(S) and loading
ES <- prob.claim*(levburr(u,alfa,gama,1,teta,1) - levburr(d,alfa,gama,1,teta,1))
ESr <- prob.claim*(levburr(u,r*alfa,gama,1,teta,1) - levburr(d,r*alfa,gama,1,teta,1))
loading <- (ESr-ES)/ES
result <- cbind(d, u, ES, ESr, loading)
# to print result
list(prob.claim=prob.claim, r=r, result=result)
}
d<-scan(n=8)
0 5000 10000 20000 40000 80000 100000 160000
u<-d+5000
layer(alfa=3.778263226, gama=1.516886923, teta=86426.43339, d, u, prob.claim=0.1, r=0.92)
```

Appendix 5: R programming for ILF (Burr distribution)

```
ILF <- function(alfa, gama, teta, r)
{
# to calculate a, E(X), risk load and ILF
a <- seq(100000,2000000,by=100000)
EX.a <- levburr(a,alfa,gama,1,teta,1)
EX.ar <- levburr(a,alfa*r,gama,1,teta,1)
EX.100k <- levburr(100000,alfa,gama,1,teta,1)
EX.100kr <- levburr(100000,alfa*r,gama,1,teta,1)
riskload <- EX.ar - EX.a
ILF <- EX.a/EX.100k
ILF.r <- EX.ar/EX.100kr
result <- cbind(a, EX.a, ILF, riskload, ILF.r)
# to print result
list(r=r, result=result)
}
ILF(alfa=3.778263226, gama=1.516886923, teta=86426.43339, r=0.9)
```

Appendix 6: R programming for mean severity, mean frequency, mean aggregate claims, variance aggregate claims and insolvency probability (excess-of-loss layers, Burr and Poisson distributions)

```
reinsurans <- function(alfa, gama, teta, lamda, d, u, r, SEP, loading)
{
# to compute E(M), H(M), E(N) and H(N)
EM <- levburr(d+u,alfa,gama,1,teta,1) - levburr(d,alfa,gama,1,teta,1)
HM <- levburr(d+u,r*alfa,gama,1,teta,1) - levburr(d,r*alfa,gama,1,teta,1)
data.diskret <- 0:10000
EN <- lamda
HN <- sum((1-ppois(data.diskret,lamda))^r)
# to compute E(S), Var(S) and insolvency probability
ES <- EM*EN
VS <- lamda*(levburr(d+u,alfa,gama,1,teta,2)-levburr(d,alfa,gama,1,teta,2)-
    2*d*levburr(d+u,alfa,gama,1,teta,1)+2*d*levburr(d,alfa,gama,1,teta,1))
insolvency.prob <- pnorm(ES*loading/(VS^(0.5)),0,1,FALSE,FALSE)
insolvency.probr <- pnorm(((HM*HN)-ES)/(VS^(0.5)),0,1,FALSE,FALSE)
# to compute H(M)H(N), burning cost, loaded rate and relative loading
HMHN <- HM*HN
burning.cost <- (EM*EN)/SEP
loaded.rate <- (HM*HN)/SEP
relative.loading <- (loaded.rate-burning.cost)/burning.cost
result <- cbind(d, d+u, EM=EM, HM=HM, EN=EN, HN=HN, HMHN=HMHN,
burning.cost=burning.cost, loaded.rate=loaded.rate, relative.loading=relative.loading, ES=ES, VS=VS,
insolvency.prob=insolvency.prob, insolvency.probr=insolvency.probr)
# to print output
list(r=r, loading=loading, SEP=SEP, result=result)
}
d <- scan(n=5)
100000  300000  500000  700000  100000
u <- scan(n=5)
200000  200000   200000  200000  800000
reinsurans(alfa=3.778263226, gama=1.516886923, teta=86426.43339, lamda=6, d, u, r=0.9, SEP=10000000,
loading=0.1)
```