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Claims Development by Layer: The Relationship between Claims Development Patterns, Trend and Claim Size Models

Rajesh Sahasrabuddhe, FCAS, MAAA

The purpose of Charles Cook's 1970 paper *Trend and Loss Development Factors* was to address the "overlap fallacy." That is, the focus of that paper was to demonstrate that trend and claims development were mutually exclusive adjustments. While this is certainly true, it should also be understood that there is a relationship between limited claims development patterns and trend factors. The "connector" between claims development patterns and trend is the claim size model. This relationship is critical to analyzing "real word" data which is rarely available on a ground-up, unlimited basis and where the implicit assumption of trend in a single direction may not be appropriate.

This paper presents a demonstration of that relationship and also provides an approach to adjust development patterns for a particular claim size layer in order to calculate a development pattern for any other layer. As importantly, the approach discussed is designed to produce models that are internally consistent with respect to development patterns, trend factors and size of loss models (increased / decreased limit factors).

Keywords development patterns, excess layer

1. INTRODUCTION

The purpose of this paper is to demonstrate the relationship between claims development, trend and claim size factors. Those relationships are then explored in order to provide a practical approach for adjusting a development pattern appropriate for any claim layer to produce a development pattern for any other layer. The approach also allows for adjustments related to cost level assumptions implicit in development patterns and ensures that assumptions related to claim size models, claims development and trend are internally consistent.

The procedure may be applied to either paid claims or reported claims. Additionally, although we use "claims" in the discussion, the procedure may also be applied to claims and allocated claim adjustment expenses (or only allocated claim adjustment expenses) assuming that all parameters and assumptions are defined consistently.

[1] A previous revision dated November 25, 2012 corrected minor typographical errors in Equations 2.3 and 3.6, and the cross reference for the calculation of item D1 in Examples 1 and 2.
This January 2, 2013 revision includes exhibits that were inadvertently excluded from the November 25, 2013 version. Those exhibits include a minor correction to Example 3.

1.1 Research Context

The current approach for estimating excess layer development is based on Emanuel Pinto and Daniel Gogol's paper, "*An Analysis of Excess Loss Development.*" The focus of that paper is the fitting of observed development factors as a function of retentions. The observed factors were developed using an analysis of a large industry database. Pinto/Gogol then present an approach for calculating excess layer development in Section 5 and this approach is explored further in George M. Levine's review. However, this approach requires that the actuary first calculate excess layer development using their fitting approach.

Many actuaries would not have access to such industry data and as such the Pinto/Gogol approach would not be practical. In addition to this issue, the methodology does not use the inherent relationship of claims size models, trend and claims development patterns.

1.2 Scope and Objective

This paper includes comments related to assumptions implicit in the determination of development patterns, trend and claim size distributions in practice. However, the development of these actuarial models and their parameters is beyond the scope of this paper. The objective of this paper is to provide a methodology to calculate development factors by layer once the actuary has already determined his/her assumptions with respect to a "base" development pattern, trend and claim size models.

1.3 Outline

The paper presents a discussion of a robust approach and then provides an example that incorporates simplifying assumptions that are common in actuarial practice. The remainder of the paper proceeds as follows. Section 2 will provide notation and define important algebraic definitions of model factors. Section 3 provides the discussion of the inter-relationship between claims development, trend and claim size models. Section 4 will provide implementation examples to the oft-studied *Mack* triangle and a simpler approach that may be sufficient for many analyses.

2. BACKGROUND

We begin by examining the implicit and explicit assumptions of claims development, trend and claim size models.

The discussion will assume that we are analyzing an $n \times n$ claims triangle. We generalize our discussion to allow for data that is truncated from below at d and censored from above at p. This is typical of data subject to deductibles and policy limits. Of course, if $d = 0$ and $p = \infty$, then the claims data is provided on a ground-up, unlimited (GUU) basis. The notation used in this paper is as follows:

$C_{i,j}^{\boldsymbol{L}}$ = Cumulative claims in the layer *L*, for exposure period *i* as of the end of development interval *j*

$C_{i,\infty}^{\boldsymbol{L}}$ = Ultimate claims in the layer *L*, for exposure period *i* ($j = \infty$)

$L(d, p)$ = Claims layer truncated from below at *d* and censored from above at *p* where $0 \leq d < p \leq \infty$

Though it will be obvious that this is not a necessary assumption, in order to simplify notation, we will assume claims layer $\boldsymbol{L}$ is consistent throughout the data triangle. Claims data is typically organized as presented in Table 1.

TABLE 1
CUMULATIVE CLAIMS DATA

		Development Interval (*j*)				
		1	2	3	…	*n*
Exposure Period (*i*)	1	$C_{1,1}^{\boldsymbol{L}}$	$C_{1,2}^{\boldsymbol{L}}$	$C_{1,3}^{\boldsymbol{L}}$	…	$C_{1,n}^{\boldsymbol{L}}$
	2	$C_{2,1}^{\boldsymbol{L}}$	$C_{2,2}^{\boldsymbol{L}}$	$C_{2,3}^{\boldsymbol{L}}$	…	
	3	$C_{3,1}^{\boldsymbol{L}}$	$C_{3,2}^{\boldsymbol{L}}$	$C_{3,3}^{\boldsymbol{L}}$	…	
	…	…				
	n	$C_{n,1}^{\boldsymbol{L}}$				

Below we first discuss trend, claims size models and development patterns separately and then discuss their relationships.

2.1 Trend Factors

Trend rates typically refer to the annual change in cost level for a particular claims layer. In practice, trend rates often do not vary between accident periods. In addition, trend that acts in the development period or calendar period direction is often not considered. Finally, the consideration of the varying effects of trend applicable to different claims layer is often nonexistent.

Rather than using annual rates of change, we will use cost level indices, $\boldsymbol{T}$. Cost level indices are determined so as to apply to cumulative claims for accident year *i* as of development maturity *j*. The indices are an accumulation of the incremental changes relative to a "base cost level." Any accident year and maturity combination can be considered the "base." In practice, the base cost level will typically be defined as the cost level associated with ultimate claims for the oldest exposure period.

Our trend is explicitly defined to apply to the ground-up, unlimited claims layer. This is consistent with approaches in practice where the trend assumption is based on external cost information such as the Consumer Price Index. If trend is estimated from claims data that is subject to policy limits or deductibles then we will first need to adjust the data to a ground-up, unlimited basis using the claim size model.

Our model allows for trend that acts in multiple directions. We use the following notation for cost level indices.

$T_{i,j}$ = Trend indices for cumulative GUU claims for exposure period *i* at the end of development interval *j*

TABLE 2
COST LEVEL INDICES

Exposure Period (*i*)	Development Interval (*j*): 1	2	3	…	*n*
1	$T_{1,1}$	$T_{1,2}$	$T_{1,3}$	…	$T_{1,n}$
2	$T_{2,1}$	$T_{2,2}$	$T_{2,3}$	…	
3	$T_{3,1}$	$T_{3,2}$	$T_{3,3}$	…	
…	…				
n	$T_{n,1}$				

2.2 Claim Size Model

The claim size model describes the distribution of claim sizes. Though we do not restrict claim size models with respect to complexity, for practicality we require the following:

- that claims size model parameters can be adjusted for the impact of inflation (includes most common claim size models such as the lognormal and exponential)
- that limited expected values and unlimited means (first moments) can be calculated with reasonable effort.

2.2.1 Limit Adjustment Factors

The limit adjustment factors, $S(a,b)$, represents the ratio of expectations of claims between layer L_a and L_b.

$$S_{i,\infty}(\boldsymbol{L}_a, \boldsymbol{L}_b) = \{LEV(p_a; \boldsymbol{\Phi}_{i,\infty}) - LEV(d_a; \boldsymbol{\Phi}_{i,\infty})\} \;/\; \{LEV(p_b; \boldsymbol{\Phi}_{i,\infty}) - LEV(d_b; \boldsymbol{\Phi}_{i,\infty})\} \tag{2.1}$$

$$S_{i,j}(\boldsymbol{L}_a, \boldsymbol{L}_b) = \{LEV(p_a; \boldsymbol{\Phi}_{i,j}) - LEV(d_a; \boldsymbol{\Phi}_{i,j})\} \;/\; \{LEV(p_b; \boldsymbol{\Phi}_{i,j}) - LEV(d_b; \boldsymbol{\Phi}_{i,j})\} \tag{2.2}$$

$$S_{i,j}(L_a, L_b) = \mathrm{E}[C_{i,j}^{\boldsymbol{L}_a} / C_{i,j}^{\boldsymbol{L}_b}] \tag{2.3}$$

where LEV is the characteristic limited expected value function for the claim size model and $\boldsymbol{\Phi}$ represents the "name" (e.g. lognormal, Pareto, exponential) and parameters of the claim size model. We also acknowledge that the parameters of the claims size model, $\boldsymbol{\Phi}$, will vary by exposure period i and development interval j as a result of differences in cost level.

In later sections, we will use the notation $LEV(\boldsymbol{L}; \boldsymbol{\Phi})$ to refer to the limited expected value for the layer $L(d, p)$. This is calculated as follows:

$$LEV(\boldsymbol{L}; \boldsymbol{\Phi}) = LEV(p; \boldsymbol{\Phi}) - LEV(d; \boldsymbol{\Phi}) \tag{2.4}$$

2.2.2 Gross-up Factors

In the special case where p_a=∞ and d_a=0, $S(a,b)$ simplifies to a factor to gross-up claims to a GUU basis. We can then use the characteristic first moment (mean) function, M, in the numerator rather than the limited expected value function.

$$G_{i,.}(b) = M(\boldsymbol{\Phi}_{i,\infty}) \;/\; \{LEV(p_b; \boldsymbol{\Phi}_{i,\infty}) - LEV(d_b; \boldsymbol{\Phi}_{i,\infty})\} \tag{2.4}$$

$$G_{i,j}(b) = M(\boldsymbol{\Phi}_{i,j}) \;/\; \{LEV(p_b; \boldsymbol{\Phi}_{i,j}) - LEV(d_b; \boldsymbol{\Phi}_{i,j})\} \tag{2.5}$$

2.3 Claims Development

Claims development factors, F, represent the expected ratios of ultimate claims to claims at maturities prior to ultimate. That is:

$$F_{i,j}^{\boldsymbol{L}} = \mathrm{E}[\, C_{i,\infty}^{\boldsymbol{L}} \;/\; C_{i,j}^{\boldsymbol{L}} \,] \tag{2.6}$$

3. RESULTS AND DISCUSSION

We can now explore the relationships between claims development, trend, and claim size models. The discussion assumes that we have been provided with unlimited claims trend factors and that we have developed the cost level indices as presented in Table 2.

3.1 Claim Size and Trend

As per the requirements of Section 2.2, for our selected claim size model, we can calculate model parameters for prior or future exposure periods using the trend indices.

$$\boldsymbol{\Phi}_{i,j} \sim f\left(\boldsymbol{\Phi}_{n,j}, T_{i,j}, T_{n,j}\right) \tag{3.1}$$

3.2 Claim Development Patterns, Claim Size and Trend

In practice, claims development patterns are estimated from unadjusted data and are applied to claims for all exposure periods. We should acknowledge that this is not appropriate unless (i) claims data are provided on a GUU basis and (ii) trend acts only in the accident year direction. Since this is oftentimes not the case, we address these issues by adjusting the triangle of claims data prior to analysis. Specifically, we adjust observed claim amounts for differences in cost level and limit using the limited expected value function.

3.2.1 Development of Basic Limit Claims Development Pattern, Exposure Year *n* Cost Level

We first select a Basic Limit, $\boldsymbol{B}$, which is the threshold at which we believe the data is sufficiently credible for the purpose of estimating claims development patterns. Recall from Table 1 that $\boldsymbol{L}$ represents the layer for which data is available. We then adjust each observation of cumulative claims as follows[2]:

$$E\left[\acute{C}_{i,j}^{\boldsymbol{B}} | C_{i,j}^{\boldsymbol{L}}\right] = C_{i,j}^{\boldsymbol{L}} \times LEV(\boldsymbol{B}; \boldsymbol{\Phi}_{n,j}) / LEV(\boldsymbol{L}; \boldsymbol{\Phi}_{i,j}) \tag{3.2}$$

We note that there is no restriction that $\boldsymbol{B} \neq \boldsymbol{L}$. We should recognize that if $\boldsymbol{B} = \boldsymbol{L}$, then we are simply adjusting the data for differences due to the impact of trend in the layer. (Note the difference between the first subscript of $\boldsymbol{\Phi}$ in the numerator and denominator of Equation 3.2).

We then analyze this adjusted data, $\acute{C}_{i,j}^{\boldsymbol{B}}$, in order to estimate development patterns at a common (basic) limit and an exposure period i=n cost level. This pattern is denoted $F_{n,j}^{\boldsymbol{B}}$ and we have the following relationship:

$$F_{n,j}^{\boldsymbol{B}} = E\left[\acute{C}_{n,\infty}^{\boldsymbol{B}} / \acute{C}_{n,j}^{\boldsymbol{B}}\right] \tag{3.3}$$

As you review the following sections, keep in mind that this basic limit development pattern at exposure year *n* cost level will now be used to calculate basic limit development for any other layer and exposure period (cost level).

[2] We presume that a triangle at the basic limit is not readily available.

3.2.2 Calculation of Claims Development Pattern for Any Layer and Cost Level

Equation 3.2 also provides an important general relationship applicable to any layer $\boldsymbol{X}$ if we have data for layer $\boldsymbol{L}$.

$$E\left[C_{i,j}^{\boldsymbol{X}} | C_{i,j}^{\boldsymbol{L}}\right] = C_{i,j}^{\boldsymbol{L}} \times LEV(\boldsymbol{X}; \boldsymbol{\Phi}_{i,j}) / LEV(\boldsymbol{L}; \boldsymbol{\Phi}_{i,j}) \tag{3.4}$$

$$C_{i,j}^{\boldsymbol{L}} \times S_{i,j}(\boldsymbol{X}, \boldsymbol{L}) \tag{3.5}$$

Using this general relationship, we can calculate basic limit development factors for any exposure period for any layer $\boldsymbol{X}$ from the development factor for $\boldsymbol{B}$ at exposure year n cost levels:

$$F_{i,j}^{\boldsymbol{X}} = E\left[\frac{C_{i,\infty}^{\boldsymbol{X}}}{C_{i,j}^{\boldsymbol{X}}}\right] = E\left[\frac{C_{n,\infty}^{B}}{C_{n,j}^{B}} \times \frac{LEV(\boldsymbol{X}; \boldsymbol{\Phi}_{i,\infty}) / LEV(\boldsymbol{B}; \boldsymbol{\Phi}_{n,\infty})}{LEV(\boldsymbol{X}; \boldsymbol{\Phi}_{i,j}) / LEV(\boldsymbol{B}; \boldsymbol{\Phi}_{n,j})}\right] \tag{3.6}$$

$$F_{i,j}^{\boldsymbol{X}} = F_{n,j}^{\boldsymbol{B}} \times \frac{LEV(\boldsymbol{X}; \boldsymbol{\Phi}_{i,\infty}) / LEV(\boldsymbol{B}; \boldsymbol{\Phi}_{n,\infty})}{LEV(\boldsymbol{X}; \boldsymbol{\Phi}_{i,j}) / LEV(\boldsymbol{B}; \boldsymbol{\Phi}_{n,j})} \tag{3.7}$$

$$F_{i,j}^{\boldsymbol{X}} = F_{n,j}^{\boldsymbol{B}} \times \frac{S_{i,\infty}(\boldsymbol{X}, \boldsymbol{B})}{S_{i,j}(\boldsymbol{X}, \boldsymbol{B})} \tag{3.8}$$

However, as we demonstrated in Equation 3.1, $\boldsymbol{\Phi}_{i,j}$ is a function of trend indices and $\boldsymbol{\Phi}_{n,j}$. So, substituting Equation 3.1 into Equation 3.7, we have:

$$\boxed{F_{i,j}^{\boldsymbol{X}} = F_{n,j}^{\boldsymbol{B}} \times \frac{LEV(\boldsymbol{X}; T_{i,\infty}, T_{n,\infty}, \boldsymbol{\Phi}_{n,\infty}) / LEV(\boldsymbol{B}; \boldsymbol{\Phi}_{n,\infty})}{LEV(\boldsymbol{X}; T_{i,j}, T_{n,j}, \boldsymbol{\Phi}_{n,j}) / LEV(\boldsymbol{B}; \boldsymbol{\Phi}_{n,j})}} \tag{3.9}$$

Equations 3.8 and 3.9 are the primary findings of this research: **Development factors at different cost levels and different layers are related to each other based on claim size models and trend.**

3.3 Other Practical Uses

Oftentimes, we are simply provided with a development pattern. Although we are typically aware of the limits associated with the triangle and/or pattern, it is not stated at any particular cost level.

In Equation 3.9, we demonstrated that, for limited claims data, development patterns will vary with cost level. However, this relationship is often ignored usually because it is presumed immaterial. For convenience, we will simply assert that the cost level is that of the latest exposure period.

We also typically have a claim size model at ultimate (e.g. increased limit factors), but size models by age are usually not available. Let us also assume that we are only concerned with estimating development factors applicable to claims at the latest valuation date.

We can use a variation of Equation 3.6 to develop claims development patterns:

$$F_{i,j}^{\boldsymbol{X}} = F_{n,j}^{B} \times \frac{LEV(\boldsymbol{X}; \boldsymbol{\Phi}_{i,\infty}) / LEV(\boldsymbol{B}; \boldsymbol{\Phi}_{n,\infty})}{R_j\ (X, B)} \tag{3.10}$$

The primary difference between Equations 3.8 and Equation 3.6 is that rather than using claim size models by age in the denominator, we use a quantity, $R_j\ (X, B)$, that is simpler to estimate approximately.

$R_j\ (X, B)$ is the ratio between limited expected values for layer $\boldsymbol{X}$ and $\boldsymbol{B}$ at the end of development interval *j*. $R_j\ (X, B)$ is only evaluated along a single diagonal since we typically have at least one diagonal (usually the current diagonal) where we can observe ratios of claims at various limits. It should be noted that *R* carries only one subscript, that for maturity. In using this latter approach, we assume that differences in cost level are immaterial to the calculation of **ratios** of claims by layer[3].

For the moment, we will ignore the possibility of negative development and assume that $R_j\ (X, B)$<1. The latter assumption indicates that we are trying to develop an estimate for a pattern at a lower layer given a pattern at a higher layer. We should recognize that $\boldsymbol{R}$ will have the following properties:

i. $\boldsymbol{R}_a > \boldsymbol{R}_b$ for a<b - At early maturities, there will be less development in the excess layer than at later maturities.

ii. $\boldsymbol{R}_a \geq U$, where $\boldsymbol{U} = \lim_{a\to\infty} \boldsymbol{R}_a$ - We should recognize that *U* can be calculated as the product of $\boldsymbol{R}$ and the ratio of ultimate claim development factors layer $\boldsymbol{X}$ and $\boldsymbol{B}$. Until we reach ultimate, the reported ratio will always be greater than ultimate ratio. This is because the there is more development associated with the denominator of $\boldsymbol{R}$ (claims in layer $\boldsymbol{B}$, the higher limit) than the numerator of $\boldsymbol{R}$ (claims in layer $\boldsymbol{X}$, the lower limit) and at ultimate $\boldsymbol{R} = \boldsymbol{U}$.

iii. If our base development pattern is provided on an unlimited basis (i.e. $\boldsymbol{B}$=GUU), then the maximum value for $\boldsymbol{R}$ may be calculated as *U*Claims Development Factor*. The derivation of this maximum is presented in Appendix A.

It should be recognized that these conditions will be violated if there is negative development or if we assume that an excess layer might develop more quickly than a working layer. These conditions are not necessary for application of this approach. However, it is useful to review the results under the typical considerations described above to provide a more intuitive understanding of the dynamics of the calculation.

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[3] Note that we are not asserting that they are immaterial with respect to absolute limited expected values.

In the third example presented in Section 4, we use a simpler approach to calculating $\boldsymbol{R}$[4] which is then used to calculate development factors for a layer other than the layer associated with the development pattern provided.

3.4 Issues

Relative to common development method projections, the procedure described above requires additional assumptions and calculations. The use of certain assumptions and calculations would not appear to be overly onerous:

1. The procedure requires that the actuary select a basic limit. However, actuaries either explicitly or implicitly select a basic limit in applying the development method. That is, whenever a development triangle is analyzed there is an implicit assumption that the limit associated with that triangle is sufficiently credible to produce development factors.

2. The procedure requires the use of a(n ultimate) claim size model in order to implement a development method analysis. This may or may not result in an additional burden on the actuary. Oftentimes, claim size information (such as increased limit factors) or a claim size model is already available to the actuary. If not, we would submit that knowledge of the distribution of claim sizes is important in understanding the dynamics of claims development.

   We should also recognize that we use the claim size model only to calculate relative limited expected values near the deductible, basic limit, policy limit and limit underlying the development data. Deductibles generally would not be an issue for the types of exposures for which the actuary would be willing to invest the effort required of this approach. As such, what is important is that our claim size model produces reasonable ratios of limited expectations to unlimited means at higher values. It is less important that the absolute limited expected values are accurate and therefore a simpler size of loss model may be sufficient though we need to recognize its shortcomings and not use that model out of context.

3. The procedure requires that the data triangle be adjusted to a basic limit and common cost level. As demonstrated in Examples 1 & 2 of Section 4, given claim size and trend information, the calculation and application of adjustment factors would not seem to create a significant additional burden.

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[4] Simpler than calculating claim size models by age.

There are however two sets of assumptions that could be perceived as resulting in a significant additional burden.

1. Claim size models at maturities prior to ultimate are generally not available. In addition, these models would have limited application outside of this context. However, understanding changes in claims size models over time would be a significant benefit for actuaries to understand excess layer development.

   With an insurance company database or even a self-insured risk of sufficient size, we believe that an algorithm could be reasonably programmed to calculate these claim size models.

   Although a robust claim size model is required for full implementation of this approach (Examples 1 & 2), it should be recognized that only the ratio of expected values is required to adjust development patterns from one layer to another. This is a significantly reduced burden as will be demonstrated in Example 3 in the next section.

2. The procedure requires the calculation of a triangle of trend indices in order to implement a development method analysis. We would expect that a trend assumption exists in the analysis. The trend indices specify the cost level associated with cumulative claim observations. This becomes somewhat difficult to conceptualize in two respects:

   a. Trend typically acts on incremental activity.

   b. The impact of trend on reported incurred claims and, more specifically, the timing of the effect of trend on case basis reserves, is difficult to ascertain.

   These difficulties are not an issue if we assume that development only acts in the exposure period direction. Even if we have trend also acting across calendar periods, we would submit that this will require the actuary to confront the assumption with respect to the direction(s) in which trend acts or (more importantly) does not act. In addition documenting this assumption produces greater transparency and better informs the consumer of actuarial information.

4. EXAMPLES

We now present three examples that implement the concepts described in Section 3. The first two examples are based on the oft-studied claims triangle included in the *Distribution-Free Calculation* *of the Standard Error of Chain Ladder Reserve Estimates* by Thomas Mack. Example 1 and Example 2 are identical except that in Example 1, the Basic Limit is well above the working claims layer; in Example 2, the Basic Limit is within the working layer. The third example presents the approach discussed in Section 3.3 where we adjust a development pattern provided to us to determine patterns for other layers.

4.1 Example 1 & 2

For Examples 1 & 2, we provide the following additional (contrived) information about the Mack triangle. This information is intended to be typical of that which might apply to actual data:

- We have selected a basic limit of $500 thousand
- The policy limit is $2 million
- The data in the triangle is for the ground-up layer to $1 million
- Trend acts at a rate of 2% each exposure period; but there was a one-time increase to 5% between exposure period 6 and 7.
- Trend acts at a rate of 1% each calendar period; but there was a one-time decrease of 5% between calendar period 2 and 3.

The calculations in the examples are presented as follows:

- In Section A, we present the claims data and relevant information. Both exposure periods and development intervals are annual. However, since this is not a strict requirement of our approach, we have retained the more generic labels: "Exposure Period" and "Development Interval."
- In Section B, we present the calculation of trend indices.
- In Section C, we present the claim size model. Section C1 provides the claim size model at Exposure Period 10 cost level. We use an exponential model for simplicity of presentation; however any model that meets the requirements of Section 2.2 could be used.

  In Section C2, we present the calculation of adjusted exponential parameters based on the Exposure Year 10 parameters and trend indices.

  In Sections C4 through C6, we present the calculation of limited expected values using the characteristic function of the exponential model.
- In Section D1, we present the adjusted cumulative claims triangle. This triangle adjusts all historical observations to the basic limit at Exposure Period 10 cost levels. The

adjustments are based on ratios of limited expected values. In Sections D2 and D3, we calculate the incremental and cumulative development patterns.

- In Section E, we apply Equation 3.7 to calculate development factors for various layers at appropriate exposure year cost levels. In Section E7, we present the differences between factors calculated through examination of the (unadjusted) triangle in Section A1 and the factors resulting from our approach.

  Factors for certain excess layers are presented as "very large." This occurs since the expectation of claim in the layer at early maturities is very small.

  We note that the differences presented in Section of E7 of Example 1 are quite small. The differences will grow with the expectation of claims in the layer between the basic limit and layer under review. This is demonstrated in Example 2, where the resulting differences are quite a bit greater. We should also recognize that layers that are excess layers for an insurer (or self-insured) become working layers for reinsurers (excess insurers).

  It will also grow in situations where trend and/or development act over longer periods or at higher rates.

4.2 Example 3

The third example presents the approach described in Section 3.3. This approach is intended to provide a simpler application of the theory in Section 3. As presented in Example 1, if the basic limit is sufficiently high and trend is contained, the impact of data adjustments is minimal.

The calculations in Example 3 are reasonably self-explanatory. However, readers should note the following:

- At ultimate, all claims development factors equal unity and the ratio at age (col. 9) equals the ratio at ultimate (col. 8).
- The x axis is labeled "maturity," not exposure period. The observed pattern should be viewed as one observation of a random process at a particular maturity and not viewed as the ratio applicable to an exposure period.
- We use an algorithm to select ratios by age. At the earliest maturity, we know that the ratio should be "high." That is because claims emergence in excess layers is still "low."

  Our selected ratios are calculated as follows:

Selected Ratio = Ultimate Ratio + (1-Ultimate Ratio) * Decay Factor

This approach recognizes that we want to "keep" a portion of the distance between the ultimate ratio and the maximum ratio (unity). This portion is determined through the use of a decay model where we keep most of the difference at the earliest maturity and none at ultimate.

In practice, assuming we are analyzing development patterns at limits at or above the working layer, the ratios will be close to unity and the amount of error that could possibly be created by this approach is minimal.

5. CONCLUSION

In this paper we have demonstrated that there is a relationship between claim development patterns by layer and that that relationship is a function of trend and claim size models. This relationship can be used to calculate development patterns for a claims layer from a development pattern for any other claims layer.

These relationships also demonstrate that limited development factors are a function of not only maturity but also cost level. Therefore, the same pattern of limited factors should not always be applied to all exposure periods under review.

With short development patterns, low trend rates and limits above the working layer, the adjustment is small and often immaterial. Not all exposures exhibit these characteristics and for these exposures, the adjustments may be meaningful. For exposures where the adjustment may not be meaningful, we provided an alternative simpler approach to adjust development patterns.

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Appendix A: Calculation of Maximum Ratios of Basic Limit to Unlimited Claims

The maximum ratio is represented by the limiting case where all development in the unlimited layer occurs above the basic limit. The maximum ratio is calculated as follows:

Notation:

R = Ultimate ratio of basic limit to unlimited claims
A = Ratio of basic limit to unlimited claims prior to ultimate
D = Unlimited claim development factor

Claims

	Prior to Ultimate	At Ultimate
Limited to Basic Limit	B_a	B_r
Excess of Basic Limit	X_a	X_r
Unlimited	C_a	C_r

Identities:

I1: $B_a = B_r$ (All development in excess layer; basic limit layer at ultimate)
I2: $R = B_r / C_r$
I3: $C_r = C_a * D$

Then under maximum conditions:

$A_{\max} = B_a / C_a$
$A_{\max} = B_a / (C_r/D)$ « per I3 »
$A_{\max} = D * B_a / C_r$
$A_{\max} = D * B_r / C_r$ « per I1 »
$A_{\max} = D * R$ « per I2 »

REFERENCES

C Cook, "Trend and Loss Development Factors," *PCAS* **1970**, Vol. LVII, 1-14.
Gogol, Daniel F, and Emanuel Pinto, "An Analysis of Excess Loss Development," *PCAS* **1987**, Vol. LXXIV, 227-255.
G Levine, "An Analysis of Excess Loss Development (Discussion)," *PCAS* **1987**, Vol. LXXIV, 256-271.
T Mack, "Distribution-Free Calculation of the Standard Error of Chain Ladder Reserve Estimates," *ASTIN Bulletin* **1993**, Vol. 23:2, 213-225.

Biography of the Author

Rajesh Sahasrabuddhe is a Principal with Oliver Wyman Actuarial Consulting in Philadelphia. He is responsible for providing actuarial consulting services to a variety of clients. He graduated *summa cum laude* with a BS degree in Mathematics-Actuarial Science from the University of Connecticut. He is a Fellow of the CAS and a Member of the American Academy of Actuaries. He participates on the CAS Syllabus Committee in a leadership role.

Current contact information is available to members of the CAS through its website (www.casact.org).

Claims Development by Layer

Example 1

A. Data and Information

1 Cumulative Development Triangle (C_{ij})

Exposure Period (i)	Development Interval (j) 1	2	3	4	5	6	7	8	9	10
1	357,848	1,124,788	1,735,330	2,218,270	2,745,596	3,319,994	3,466,336	3,606,286	3,833,515	3,901,463
2	352,118	1,236,139	2,170,033	3,353,322	3,799,067	4,120,063	4,647,867	4,914,039	5,339,085	
3	290,507	1,292,306	2,218,525	3,235,179	3,985,995	4,132,918	4,628,910	4,909,315		
4	310,608	1,418,858	2,195,047	3,757,447	4,029,929	4,381,982	4,588,268			
5	443,160	1,136,350	2,128,333	2,897,821	3,402,672	3,873,311				
6	396,132	1,333,217	2,180,715	2,985,752	3,691,712					
7	440,832	1,288,463	2,419,861	3,483,130						
8	359,840	1,421,128	2,864,498							
9	376,686	1,363,294								
10	344,014									

2 Limit of Data in Triangle 1,000,000

3 Selected Basic Limit 500,000

4 Policy Limit 2,000,000

B. Trend Indices

1 Exposure Period Trend Index [2% EP Trend; 5% between EP 6 and 7]

Exposure Period (i)	Development Interval (j) 1	2	3	4	5	6	7	8	9	10
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.020	1.020	1.020	1.020	1.020	1.020	1.020	1.020	1.020	1.020
3	1.040	1.040	1.040	1.040	1.040	1.040	1.040	1.040	1.040	1.040
4	1.061	1.061	1.061	1.061	1.061	1.061	1.061	1.061	1.061	1.061
5	1.082	1.082	1.082	1.082	1.082	1.082	1.082	1.082	1.082	1.082
6	1.104	1.104	1.104	1.104	1.104	1.104	1.104	1.104	1.104	1.104
7	1.159	1.159	1.159	1.159	1.159	1.159	1.159	1.159	1.159	1.159
8	1.182	1.182	1.182	1.182	1.182	1.182	1.182	1.182	1.182	1.182
9	1.206	1.206	1.206	1.206	1.206	1.206	1.206	1.206	1.206	1.206
10	1.230	1.230	1.230	1.230	1.230	1.230	1.230	1.230	1.230	1.230

2 Calendar Period Trend Index [1% Calendar Period Trend; -5% between CP 2 and 3]

Exposure Period (i)	Development Interval (j) 1	2	3	4	5	6	7	8	9	10
1	1.000	1.010	0.960	0.969	0.979	0.989	0.998	1.008	1.019	1.029
2	1.010	0.960	0.969	0.979	0.989	0.998	1.008	1.019	1.029	1.039
3	0.960	0.969	0.979	0.989	0.998	1.008	1.019	1.029	1.039	1.049
4	0.969	0.979	0.989	0.998	1.008	1.019	1.029	1.039	1.049	1.060
5	0.979	0.989	0.998	1.008	1.019	1.029	1.039	1.049	1.060	1.070
6	0.989	0.998	1.008	1.019	1.029	1.039	1.049	1.060	1.070	1.081
7	0.998	1.008	1.019	1.029	1.039	1.049	1.060	1.070	1.081	1.092
8	1.008	1.019	1.029	1.039	1.049	1.060	1.070	1.081	1.092	1.103
9	1.019	1.029	1.039	1.049	1.060	1.070	1.081	1.092	1.103	1.114
10	1.029	1.039	1.049	1.060	1.070	1.081	1.092	1.103	1.114	1.125

3 Combined Trend Index [B1 * B2]

Exposure Period (i)	Development Interval (j) 1	2	3	4	5	6	7	8	9	10
1	1.000	1.010	0.960	0.969	0.979	0.989	0.998	1.008	1.019	1.029
2	1.030	0.979	0.988	0.998	1.008	1.018	1.029	1.039	1.049	1.060
3	0.998	1.008	1.018	1.029	1.039	1.049	1.060	1.070	1.081	1.092
4	1.028	1.039	1.049	1.060	1.070	1.081	1.092	1.103	1.114	1.125
5	1.059	1.070	1.081	1.092	1.102	1.114	1.125	1.136	1.147	1.159
6	1.091	1.102	1.113	1.125	1.136	1.147	1.159	1.170	1.182	1.194
7	1.157	1.169	1.181	1.193	1.204	1.217	1.229	1.241	1.253	1.266
8	1.192	1.204	1.216	1.229	1.241	1.253	1.266	1.278	1.291	1.304
9	1.228	1.241	1.253	1.266	1.278	1.291	1.304	1.317	1.330	1.344
10	1.266	1.278	1.291	1.304	1.317	1.330	1.343	1.357	1.370	1.384

Claims Development by Layer

Example 1

C. Claim Size Model (Apply to Cumulative Claims)

1 Claims Size Model Parameters at Exposure Year 10 Cost Level [via claim size modeling]

Development Interval (*j*)

Exponential (θ)	1	2	3	4	5	6	7	8	9	10
i =10	28,138	84,242	133,998	182,460	204,649	228,245	252,830	265,063	275,707	280,000

2 Claims Size Model Parameters [C1 * $B3_{i,j}$ / $B3_{10,j}$]

Development Interval (*j*)

Exposure Period (*i*) / Exponential (θ)	1	2	3	4	5	6	7	8	9	10
1	22,233	66,564	99,590	135,608	152,099	169,636	187,908	197,000	204,911	208,101
2	22,905	64,501	102,598	139,703	156,693	174,759	193,583	202,949	211,099	214,386
3	22,195	66,449	105,696	143,922	161,425	180,037	199,429	209,078	217,475	220,861
4	22,865	68,455	108,888	148,268	166,300	185,474	205,452	215,392	224,042	227,531
5	23,555	70,523	112,177	152,746	171,322	191,075	211,657	221,897	230,808	234,402
6	24,267	72,653	115,564	157,359	176,496	196,846	218,049	228,598	237,779	241,481
7	25,735	77,048	122,556	166,879	187,174	208,755	231,241	242,429	252,164	256,090
8	26,512	79,375	126,257	171,919	192,827	215,059	238,224	249,750	259,780	263,824
9	27,313	81,772	130,070	177,111	198,650	221,554	245,418	257,292	267,625	271,792
10	28,138	84,242	133,998	182,460	204,649	228,245	252,830	265,063	275,707	280,000

3 Unlimited Means

Development Interval (*j*)

Exposure Period (*i*)	1	2	3	4	5	6	7	8	9	10
1	22,233	66,564	99,590	135,608	152,099	169,636	187,908	197,000	204,911	208,101
2	22,905	64,501	102,598	139,703	156,693	174,759	193,583	202,949	211,099	214,386
3	22,195	66,449	105,696	143,922	161,425	180,037	199,429	209,078	217,475	220,861
4	22,865	68,455	108,888	148,268	166,300	185,474	205,452	215,392	224,042	227,531
5	23,555	70,523	112,177	152,746	171,322	191,075	211,657	221,897	230,808	234,402
6	24,267	72,653	115,564	157,359	176,496	196,846	218,049	228,598	237,779	241,481
7	25,735	77,048	122,556	166,879	187,174	208,755	231,241	242,429	252,164	256,090
8	26,512	79,375	126,257	171,919	192,827	215,059	238,224	249,750	259,780	263,824
9	27,313	81,772	130,070	177,111	198,650	221,554	245,418	257,292	267,625	271,792
10	28,138	84,242	133,998	182,460	204,649	228,245	252,830	265,063	275,707	280,000

4 Limited Expected Values at Policy Limits

Development Interval (*j*)

Exposure Period (*i*)	1	2	3	4	5	6	7	8	9	10
1	22,233	66,564	99,590	135,607	152,099	169,635	187,904	196,992	204,899	208,087
2	22,905	64,501	102,598	139,703	156,692	174,757	193,577	202,938	211,083	214,367
3	22,195	66,449	105,696	143,922	161,424	180,034	199,420	209,063	217,453	220,835
4	22,865	68,455	108,888	148,268	166,299	185,470	205,440	215,372	224,013	227,496
5	23,555	70,523	112,177	152,746	171,321	191,070	211,640	221,870	230,769	234,356
6	24,267	72,653	115,564	157,358	176,494	196,838	218,026	228,562	237,726	241,420
7	25,735	77,048	122,556	166,878	187,170	208,740	231,200	242,365	252,074	255,987
8	26,512	79,375	126,257	171,917	192,821	215,039	238,170	249,667	259,662	263,690
9	27,313	81,772	130,070	177,109	198,642	221,527	245,348	257,184	267,473	271,619
10	28,138	84,242	133,998	182,456	204,638	228,209	252,737	264,922	275,512	279,779

5 Limited Expected Values at Limits of Data Triangle

Development Interval (*j*)

Exposure Period (*i*)	1	2	3	4	5	6	7	8	9	10
1	22,233	66,564	99,586	135,522	151,887	169,169	186,990	195,770	203,355	206,398
2	22,905	64,501	102,592	139,594	156,428	174,187	192,478	201,479	209,249	212,366
3	22,195	66,449	105,688	143,784	161,096	179,340	198,105	207,328	215,285	218,474
4	22,865	68,455	108,877	148,094	165,893	184,629	203,871	213,318	221,461	224,723
5	23,555	70,523	112,161	152,527	170,822	190,056	209,778	219,448	227,777	231,112
6	24,267	72,652	115,544	157,085	175,885	195,621	215,826	225,719	234,233	237,640
7	25,735	77,048	122,521	166,462	186,279	207,020	228,179	238,510	247,385	250,932
8	26,512	79,375	126,211	171,407	191,748	213,003	234,644	245,194	254,248	257,866
9	27,313	81,772	130,010	176,486	197,356	219,126	241,247	252,014	261,246	264,932
10	28,138	84,241	133,921	181,699	203,105	225,390	247,987	258,969	268,375	272,128

6 Limited Expected Values at Basic Limit

Development Interval (*j*)

Exposure Period (*i*)	1	2	3	4	5	6	7	8	9	10
1	22,233	66,528	98,933	132,211	146,418	160,735	174,776	181,433	187,052	189,273
2	22,905	64,473	101,813	135,805	150,248	164,762	178,957	185,674	191,337	193,574
3	22,195	66,413	104,764	139,462	154,134	168,836	183,176	189,948	195,652	197,903
4	22,865	68,409	107,785	143,181	158,075	172,956	187,431	194,253	199,993	202,256
5	23,555	70,464	110,876	146,960	162,068	177,119	191,718	198,586	204,358	206,632
6	24,267	72,578	114,037	150,798	166,111	181,322	196,035	202,944	208,743	211,026
7	25,735	76,931	120,483	158,539	174,229	189,725	204,633	211,606	217,447	219,744
8	26,512	79,229	123,851	162,538	178,404	194,029	209,019	216,018	221,873	224,175
9	27,313	81,591	127,286	166,587	182,619	198,360	213,422	220,441	226,307	228,611
10	28,138	84,019	130,788	170,682	186,869	202,716	217,839	224,873	230,745	233,050

Claims Development by Layer

Example 1

D. Calculation of Development Factors at Basic Limit

1 Cumulative Triangle Exposure Year 10 Cost Levels and Basic Limit ($C_{i,j}$) [$A1_{i,j}$ * $C6_{10,j}$ / $C5_{i,j}$]

Exposure Period (i)	Development Interval (j) 1	2	3	4	5	6	7	8	9	10
1	452,881	1,419,731	2,279,040	2,793,772	3,377,947	3,978,376	4,038,185	4,142,394	4,349,865	4,405,265
2	432,566	1,610,197	2,766,439	4,100,116	4,538,378	4,794,873	5,260,266	5,484,617	5,887,561	
3	368,296	1,634,013	2,745,402	3,840,401	4,623,708	4,671,636	5,090,015	5,324,759		
4	382,236	1,741,436	2,636,785	4,330,559	4,539,482	4,811,270	4,902,613			
5	529,368	1,353,815	2,481,777	3,242,747	3,722,312	4,131,335				
6	459,320	1,541,795	2,468,413	3,244,186	3,922,258					
7	481,990	1,405,037	2,583,135	3,571,425						
8	381,903	1,504,277	2,968,365							
9	388,062	1,400,758								
10	344,014									

2 Exposure Year 10 Incremental Basic Limit Development Factors [per D1; Volume Weighted Averages]

	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	8 to 9	9 to 10
i=10	3.511	1.714	1.399	1.147	1.076	1.057	1.039	1.063	1.013

3 Exposure Year 10 Cumulative Development Factors [per D2]

	1 to ult	2 to ult	3 to ult	4 to ult	5 to ult	6 to ult	7 to ult	8 to ult	9 to ult	10 to ult
i=10	12.291	3.501	2.042	1.460	1.273	1.183	1.119	1.077	1.013	1.000

E. Calculation of Development Factors by Layer

1 Basic Limit [D_{3j} * ($C6_{i,10}$/$C6_{10,10}$) / ($C6_{i,j}$/$C6_{10,j}$)]

Exposure Period (i)	Development Interval (j) 1 to ult	2 to ult	3 to ult	4 to ult	5 to ult	6 to ult	7 to ult	8 to ult	9 to ult	10 to ult
1	12.633	3.590	2.193	1.531	1.319	1.211	1.133	1.084	1.015	1.000
2	12.541	3.789	2.179	1.524	1.315	1.209	1.132	1.083	1.014	
3	13.232	3.761	2.165	1.517	1.310	1.206	1.130	1.083		
4	13.126	3.731	2.151	1.510	1.306	1.203	1.129			
5	13.017	3.701	2.136	1.503	1.301	1.200				
6	12.904	3.669	2.121	1.496	1.296					
7	12.671	3.605	2.090	1.482						
8	12.547	3.571	2.074							
9	12.421	3.536								
10	12.291									

2 Basic Limit to Policy Limit [D_{3j} * (($C4_{i,10}$-$C6_{i,10}$) / $C6_{10,10}$) / (($C4_{i,j}$-$C6_{i,j}$/$C6_{10,j}$)]

Exposure Period (i)	Development Interval (j) 1 to ult	2 to ult	3 to ult	4 to ult	5 to ult	6 to ult	7 to ult	8 to ult	9 to ult	10 to ult
1	very large	652.420	32.802	5.924	3.380	2.175	1.499	1.257	1.057	1.000
2	very large	946.242	30.374	5.704	3.293	2.140	1.488	1.252	1.056	
3	very large	807.075	28.187	5.498	3.210	2.107	1.477	1.247		
4	very large	691.561	26.215	5.305	3.132	2.075	1.466			
5	very large	595.239	24.431	5.124	3.058	2.044				
6	very large	514.560	22.814	4.954	2.987					
7	very large	390.710	20.042	4.647						
8	very large	341.869	18.820							
9	very large	300.278								
10	very large									

3 Policy Limit to Unlimited [D_{3j} * (($C3_{i,10}$-$C4_{i,10}$) / $C6_{10,10}$) / (($C3_{i,j}$-$C4_{i,j}$) / $C6_{10,j}$)]

Exposure Period (i)	Development Interval (j) 1 to ult	2 to ult	3 to ult	4 to ult	5 to ult	6 to ult	7 to ult	8 to ult	9 to ult	10 to ult
1	very large	very large	84,538.278	279.503	48.056	11.155	3.254	1.887	1.183	1.000
2	very large	very large	62,192.336	240.423	43.321	10.464	3.157	1.857	1.178	
3	very large	very large	46,166.664	207.723	39.172	9.835	3.066	1.829		
4	very large	very large	34,571.138	180.241	35.524	9.261	2.979			
5	very large	very large	26,108.458	157.047	32.309	8.735				
6	very large	very large	19,880.311	137.391	29.466					
7	very large	very large	11,880.695	106.724						
8	very large	very large	9,257.797							
9	very large	very large								
10	very large									

Claims Development by Layer

Example 1

4 Limit of Data in Triangle [D_{3j} * ($C5_{i,10}$ / $C6_{10,10}$) / ($C5_{i,j}$/$C6_{10,j}$)]

Development Interval (*j*)

Exposure Period (*i*)	1 to ult	2 to ult	3 to ult	4 to ult	5 to ult	6 to ult	7 to ult	8 to ult	9 to ult	10 to ult
1	13.776	3.913	2.375	1.629	1.387	1.255	1.155	1.096	1.018	1.000
2	13.759	4.155	2.372	1.627	1.385	1.254	1.154	1.095	1.018	
3	14.607	4.149	2.369	1.625	1.384	1.253	1.154	1.095		
4	14.584	4.143	2.366	1.623	1.382	1.252	1.153			
5	14.559	4.136	2.362	1.620	1.381	1.251				
6	14.532	4.128	2.357	1.618	1.379					
7	14.469	4.110	2.347	1.612						
8	14.433	4.100	2.342							
9	14.394	4.089								
10	14.352									

5 Unadjusted Incremental Development Factors at Limits of Data Triangle[per A1; Volume Weighted Averages]

1 to 2	2 to 3	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	8 to 9	9 to 10
3.490	1.747	1.457	1.174	1.104	1.086	1.054	1.077	1.018

6 Unadjusted Cumulative Development Factors [per E5]

1 to ult	2 to ult	3 to ult	4 to ult	5 to ult	6 to ult	7 to ult	8 to ult	9 to ult	10 to ult
14.445	4.139	2.369	1.625	1.384	1.254	1.155	1.096	1.018	1.000

Differences [E6 / E4, last diagonal -1]

7	+0.7%	+1.2%	+1.2%	+0.8%	+0.4%	+0.3%	+0.1%	+0.1%	+0.0%	+0.0%

Claims Development by Layer

Example 2

A. Data and Information

1 Cumulative Development Triangle (C_{ij})

Exposure Period (i)	Development Interval (j) 1	2	3	4	5	6	7	8	9	10
1	357,848	1,124,788	1,735,330	2,218,270	2,745,596	3,319,994	3,466,336	3,606,286	3,833,515	3,901,463
2	352,118	1,236,139	2,170,033	3,353,322	3,799,067	4,120,063	4,647,867	4,914,039	5,339,085	
3	290,507	1,292,306	2,218,525	3,235,179	3,985,995	4,132,918	4,628,910	4,909,315		
4	310,608	1,418,858	2,195,047	3,757,447	4,029,929	4,381,982	4,588,268			
5	443,160	1,136,350	2,128,333	2,897,821	3,402,672	3,873,311				
6	396,132	1,333,217	2,180,715	2,985,752	3,691,712					
7	440,832	1,288,463	2,419,861	3,483,130						
8	359,840	1,421,128	2,864,498							
9	376,686	1,363,294								
10	344,014									

2 Limit of Data in Triangle 1,000,000

3 Selected Basic Limit 500,000

4 Policy Limit 2,000,000

B. Trend Indices

1 Exposure Period Trend Index [2% EP Trend; 5% between EP 6 and 7]

Exposure Period (i)	Development Interval (j) 1	2	3	4	5	6	7	8	9	10
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.020	1.020	1.020	1.020	1.020	1.020	1.020	1.020	1.020	1.020
3	1.040	1.040	1.040	1.040	1.040	1.040	1.040	1.040	1.040	1.040
4	1.061	1.061	1.061	1.061	1.061	1.061	1.061	1.061	1.061	1.061
5	1.082	1.082	1.082	1.082	1.082	1.082	1.082	1.082	1.082	1.082
6	1.104	1.104	1.104	1.104	1.104	1.104	1.104	1.104	1.104	1.104
7	1.159	1.159	1.159	1.159	1.159	1.159	1.159	1.159	1.159	1.159
8	1.182	1.182	1.182	1.182	1.182	1.182	1.182	1.182	1.182	1.182
9	1.206	1.206	1.206	1.206	1.206	1.206	1.206	1.206	1.206	1.206
10	1.230	1.230	1.230	1.230	1.230	1.230	1.230	1.230	1.230	1.230

2 Calendar Period Trend Index [1% Calendar Period Trend; -5% between CP 2 and 3]

Exposure Period (i)	Development Interval (j) 1	2	3	4	5	6	7	8	9	10
1	1.000	1.010	0.960	0.969	0.979	0.989	0.998	1.008	1.019	1.029
2	1.010	0.960	0.969	0.979	0.989	0.998	1.008	1.019	1.029	1.039
3	0.960	0.969	0.979	0.989	0.998	1.008	1.019	1.029	1.039	1.049
4	0.969	0.979	0.989	0.998	1.008	1.019	1.029	1.039	1.049	1.060
5	0.979	0.989	0.998	1.008	1.019	1.029	1.039	1.049	1.060	1.070
6	0.989	0.998	1.008	1.019	1.029	1.039	1.049	1.060	1.070	1.081
7	0.998	1.008	1.019	1.029	1.039	1.049	1.060	1.070	1.081	1.092
8	1.008	1.019	1.029	1.039	1.049	1.060	1.070	1.081	1.092	1.103
9	1.019	1.029	1.039	1.049	1.060	1.070	1.081	1.092	1.103	1.114
10	1.029	1.039	1.049	1.060	1.070	1.081	1.092	1.103	1.114	1.125

3 Combined Trend Index [B1 * B2]

Exposure Period (i)	Development Interval (j) 1	2	3	4	5	6	7	8	9	10
1	1.000	1.010	0.960	0.969	0.979	0.989	0.998	1.008	1.019	1.029
2	1.030	0.979	0.988	0.998	1.008	1.018	1.029	1.039	1.049	1.060
3	0.998	1.008	1.018	1.029	1.039	1.049	1.060	1.070	1.081	1.092
4	1.028	1.039	1.049	1.060	1.070	1.081	1.092	1.103	1.114	1.125
5	1.059	1.070	1.081	1.092	1.102	1.114	1.125	1.136	1.147	1.159
6	1.091	1.102	1.113	1.125	1.136	1.147	1.159	1.170	1.182	1.194
7	1.157	1.169	1.181	1.193	1.204	1.217	1.229	1.241	1.253	1.266
8	1.192	1.204	1.216	1.229	1.241	1.253	1.266	1.278	1.291	1.304
9	1.228	1.241	1.253	1.266	1.278	1.291	1.304	1.317	1.330	1.344
10	1.266	1.278	1.291	1.304	1.317	1.330	1.343	1.357	1.370	1.384

Claims Development by Layer

Example 2

C. Claim Size Model (Apply to Cumulative Claims)

1 Claims Size Model Parameters at Exposure Year 10 Cost Level [via claim size modeling]

Exponential (θ) \ Development Interval (j)	1	2	3	4	5	6	7	8	9	10
i=10	60,295	168,483	267,996	348,332	409,299	456,490	505,660	530,125	551,415	565,000

2 Claims Size Model Parameters [C1 * $B3_{i,j}$ / $B3_{10,j}$]

Exponential (θ) / Exposure Period (i) \ Development Interval (j)	1	2	3	4	5	6	7	8	9	10
1	47,642	133,128	199,180	258,887	304,199	339,272	375,816	393,999	409,822	419,919
2	49,081	129,001	205,195	266,705	313,386	349,518	387,166	405,898	422,199	432,600
3	47,560	132,897	211,392	274,760	322,850	360,073	398,858	418,156	434,949	445,665
4	48,996	136,911	217,776	283,058	332,600	370,948	410,904	430,784	448,085	459,124
5	50,476	141,046	224,353	291,606	342,644	382,150	423,313	443,794	461,617	472,990
6	52,000	145,305	231,129	300,413	352,992	393,691	436,097	457,197	475,558	487,274
7	55,146	154,096	245,112	318,588	374,348	417,509	462,481	484,857	504,329	516,754
8	56,812	158,750	252,514	328,209	385,654	430,118	476,448	499,500	519,560	532,360
9	58,527	163,544	260,140	338,121	397,300	443,108	490,837	514,585	535,250	548,437
10	60,295	168,483	267,996	348,332	409,299	456,490	505,660	530,125	551,415	565,000

3 Unlimited Means

Exposure Period (i) \ Development Interval (j)	1	2	3	4	5	6	7	8	9	10
1	47,642	133,128	199,180	258,887	304,199	339,272	375,816	393,999	409,822	419,919
2	49,081	129,001	205,195	266,705	313,386	349,518	387,166	405,898	422,199	432,600
3	47,560	132,897	211,392	274,760	322,850	360,073	398,858	418,156	434,949	445,665
4	48,996	136,911	217,776	283,058	332,600	370,948	410,904	430,784	448,085	459,124
5	50,476	141,046	224,353	291,606	342,644	382,150	423,313	443,794	461,617	472,990
6	52,000	145,305	231,129	300,413	352,992	393,691	436,097	457,197	475,558	487,274
7	55,146	154,096	245,112	318,588	374,348	417,509	462,481	484,857	504,329	516,754
8	56,812	158,750	252,514	328,209	385,654	430,118	476,448	499,500	519,560	532,360
9	58,527	163,544	260,140	338,121	397,300	443,108	490,837	514,585	535,250	548,437
10	60,295	168,483	267,996	348,332	409,299	456,490	505,660	530,125	551,415	565,000

4 Limited Expected Values at Policy Limits

Exposure Period (i) \ Development Interval (j)	1	2	3	4	5	6	7	8	9	10
1	47,642	133,128	199,171	258,773	303,774	338,338	373,981	391,539	406,709	416,332
2	49,081	129,001	205,183	266,558	312,855	348,374	384,956	402,957	418,499	428,352
3	47,560	132,897	211,376	274,570	322,191	358,680	396,209	414,655	430,569	440,653
4	48,996	136,911	217,754	282,816	331,786	369,258	407,742	426,635	442,921	453,234
5	50,476	141,045	224,323	291,300	341,645	380,112	419,557	438,896	455,554	466,096
6	52,000	145,305	231,088	300,027	351,770	391,243	431,653	451,439	468,466	479,234
7	55,146	154,096	245,042	317,989	372,558	414,040	456,358	477,020	494,769	505,979
8	56,812	158,749	252,423	327,468	383,496	426,005	469,287	490,388	508,497	519,926
9	58,527	163,543	260,021	337,208	394,713	438,252	482,494	504,028	522,492	534,136
10	60,295	168,482	267,843	347,214	406,209	450,779	495,975	517,938	536,750	548,605

5 Limited Expected Values at Limits of Data Triangle

Exposure Period (i) \ Development Interval (j)	1	2	3	4	5	6	7	8	9	10
1	47,642	133,056	197,865	253,447	292,836	321,470	349,552	362,866	374,105	381,110
2	49,081	128,946	203,626	260,430	300,495	329,523	357,913	371,347	382,674	389,729
3	47,560	132,826	209,527	267,544	308,268	337,673	366,352	379,896	391,303	398,402
4	48,996	136,819	215,569	274,786	316,150	345,913	374,861	388,507	399,986	407,123
5	50,476	140,928	221,752	282,155	324,136	354,239	383,436	397,173	408,715	415,886
6	52,000	145,156	228,075	289,646	332,222	362,644	392,070	405,887	417,485	424,685
7	55,146	153,862	240,967	304,782	348,458	379,450	409,265	423,212	434,894	442,134
8	56,812	158,458	247,701	312,616	356,809	388,057	418,038	432,035	443,746	450,999
9	58,527	163,183	254,572	320,556	365,237	396,720	426,844	440,882	452,614	459,875
10	60,295	168,038	261,575	328,598	373,738	405,433	435,677	449,745	461,490	468,753

6 Limited Expected Values at Basic Limit

Exposure Period (i) \ Development Interval (j)	1	2	3	4	5	6	7	8	9	10
1	47,641	130,016	182,998	221,361	245,406	261,556	276,465	283,245	288,835	292,261
2	49,079	126,327	187,251	225,794	249,827	265,920	280,743	287,475	293,020	296,416
3	47,558	129,810	191,537	230,232	254,237	270,263	284,992	291,670	297,168	300,532
4	48,994	133,360	195,853	234,671	258,632	274,581	289,207	295,830	301,277	304,609
5	50,473	136,974	200,196	239,109	263,009	278,872	293,389	299,953	305,347	308,645
6	51,997	140,651	204,561	243,542	267,367	283,134	297,533	304,035	309,374	312,637
7	55,140	148,090	213,237	252,269	275,900	291,453	305,601	311,973	317,197	320,386
8	56,803	151,944	217,653	256,671	280,182	295,615	309,626	315,928	321,092	324,242
9	58,516	155,855	222,080	261,056	284,435	299,739	313,608	319,837	324,938	328,049
10	60,280	159,819	226,514	265,423	288,655	303,824	317,544	323,700	328,736	331,806

Claims Development by Layer

Example 2

D. Calculation of Development Factors at Basic Limit

1 Cumulative Triangle Exposure Year 10 Cost Levels and Basic Limit ($C_{i,j}$) [$A1_{i,j}$ * $C6_{10,j}$ / $C5_{i,j}$]

Exposure Period (i)	Development Interval (j) 1	2	3	4	5	6	7	8	9	10
1	452,767	1,351,032	1,986,583	2,323,084	2,706,395	3,137,755	3,148,934	3,217,037	3,368,618	3,396,735
2	432,457	1,532,103	2,413,944	3,417,616	3,649,372	3,798,743	4,123,637	4,283,518	4,586,542	
3	368,203	1,554,934	2,398,380	3,209,537	3,732,390	3,718,630	4,012,222	4,183,099		
4	382,140	1,657,379	2,306,488	3,629,412	3,679,452	3,848,804	3,886,711			
5	529,235	1,288,675	2,174,035	2,725,977	3,030,200	3,322,067				
6	459,205	1,467,893	2,165,788	2,736,050	3,207,585					
7	481,869	1,338,349	2,274,720	3,033,321						
8	381,807	1,433,334	2,619,477							
9	387,965	1,335,194								
10	343,928									

2 Exposure Year 10 Incremental Basic Limit Development Factors [per D1; Volume Weighted Averages]

	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	8 to 9	9 to 10
i=10	3.344	1.578	1.341	1.109	1.061	1.046	1.035	1.061	1.008

3 Exposure Year 10 Cumulative Development Factors [per D2]

	1 to ult	2 to ult	3 to ult	4 to ult	5 to ult	6 to ult	7 to ult	8 to ult	9 to ult	10 to ult
i=10	9.639	2.883	1.827	1.363	1.229	1.158	1.107	1.069	1.008	1.000

E. Calculation of Development Factors by Layer

1 Basic Limit [D_{3j} * ($C6_{i,10}$/$C6_{10,10}$) / ($C6_{i,j}$/$C6_{10,j}$)]

Exposure Period (i)	Development Interval (j) 1 to ult	2 to ult	3 to ult	4 to ult	5 to ult	6 to ult	7 to ult	8 to ult	9 to ult	10 to ult
1	10.743	3.121	1.992	1.439	1.273	1.185	1.120	1.077	1.011	1.000
2	10.576	3.258	1.975	1.431	1.269	1.182	1.119	1.076	1.011	
3	11.066	3.215	1.957	1.423	1.264	1.179	1.117	1.075		
4	10.888	3.172	1.940	1.415	1.259	1.177	1.116			
5	10.709	3.129	1.923	1.407	1.255	1.174				
6	10.529	3.086	1.906	1.400	1.250					
7	10.175	3.004	1.874	1.385						
8	9.996	2.963	1.858							
9	9.817	2.923								
10	9.639									

2 Basic Limit to Policy Limit [D_{3j} * (($C4_{i,10}$-$C6_{i,10}$) / $C6_{10,10}$) / (($C4_{i,j}$-$C6_{i,j}$/$C6_{10,j}$)]

Exposure Period (i)	Development Interval (j) 1 to ult	2 to ult	3 to ult	4 to ult	5 to ult	6 to ult	7 to ult	8 to ult	9 to ult	10 to ult
1	very large	55.346	9.569	3.616	2.273	1.714	1.348	1.195	1.052	1.000
2	very large	68.491	9.178	3.529	2.238	1.697	1.342	1.192	1.050	
3	very large	63.024	8.810	3.445	2.205	1.681	1.335	1.189		
4	very large	58.114	8.465	3.366	2.172	1.665	1.329			
5	very large	53.693	8.140	3.289	2.141	1.649				
6	very large	49.704	7.834	3.216	2.111					
7	very large	42.907	7.279	3.079						
8	very large	39.930	7.020							
9	very large	37.221								
10	very large									

3 Policy Limit to Unlimited [D_{3j} * (($C3_{i,10}$-$C4_{i,10}$) / $C6_{10,10}$) / (($C3_{i,j}$-$C4_{i,j}$) / $C6_{10,j}$)]

Exposure Period (i)	Development Interval (j) 1 to ult	2 to ult	3 to ult	4 to ult	5 to ult	6 to ult	7 to ult	8 to ult	9 to ult	10 to ult
1	very large	very large	515.543	34.213	9.036	4.072	2.071	1.521	1.151	1.000
2	very large	very large	441.637	31.367	8.568	3.939	2.037	1.507	1.147	
3	very large	very large	380.045	28.831	8.138	3.814	2.005	1.494		
4	very large	very large	328.487	26.566	7.741	3.697	1.974			
5	very large	very large	285.139	24.538	7.373	3.586				
6	very large	very large	248.540	22.717	7.034					
7	very large	very large	191.734	19.638						
8	very large	very large	169.080							
9	very large	very large								
10	very large									

Claims Development by Layer

Example 2

4 Limit of Data in Triangle [D_{3j} * ($C5_{i,10}$ / $C6_{10,10}$) / ($C5_{i,j}$/$C6_{10,j}$)]

Exposure Period (i)	Development Interval (j)									
	1 to ult	2 to ult	3 to ult	4 to ult	5 to ult	6 to ult	7 to ult	8 to ult	9 to ult	10 to ult
1	14.008	3.977	2.403	1.639	1.392	1.257	1.155	1.096	1.018	1.000
2	13.905	4.197	2.387	1.632	1.387	1.254	1.154	1.095	1.017	
3	14.669	4.165	2.372	1.623	1.382	1.251	1.152	1.094		
4	14.551	4.132	2.356	1.615	1.377	1.248	1.151			
5	14.429	4.098	2.339	1.607	1.372	1.245				
6	14.302	4.063	2.323	1.599	1.367					
7	14.040	3.990	2.289	1.582						
8	13.902	3.952	2.271							
9	13.760	3.913								
10	13.614									

5 Unadjusted Incremental Development Factors at Limits of Data Triangle[per A1; Volume Weighted Averages]

1 to 2	2 to 3	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	8 to 9	9 to 10
3.490	1.747	1.457	1.174	1.104	1.086	1.054	1.077	1.018

6 Unadjusted Cumulative Development Factors [per E5]

1 to ult	2 to ult	3 to ult	4 to ult	5 to ult	6 to ult	7 to ult	8 to ult	9 to ult	10 to ult
14.445	4.139	2.369	1.625	1.384	1.254	1.155	1.096	1.018	1.000

Differences [E6 / E4, last diagonal -1]

7	+6.1%	+5.8%	+4.3%	+2.8%	+1.3%	+0.7%	+0.3%	+0.1%	+0.0%	+0.0%

Claims Development by Layer

Example 3

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
								$500K to $1m				
Exposure Period (*i*)	Maturity	Claims, Limited to $1m, as of End of EP 10	Claims, Limited to Basic Limit ($500K), as of End of EP 10	Observed Ratio	Exponential Claim Size Model Parameter (θ)	Limited Expected Value at Basic Limit at Ultimate	Limited Expected Value at $1m Limit at Ultimate	Ratio at Ultimate	Selected Ratio at Age	Ultimate Claims Development Factor at $1m	Ultimate Claims Development Factor at $500K	Ultimate Claims Development Factor $500K to $1m
1	10	3,901,463	3,846,592	0.986	208,000	189,203	206,301	0.917	0.917	1.000	1.000	1.000
2	9	5,339,085	4,692,053	0.879	214,985	193,978	212,932	0.911	0.917	1.018	1.011	1.094
3	8	4,909,315	4,695,780	0.957	222,204	198,788	219,736	0.905	0.914	1.096	1.085	1.213
4	7	4,588,268	3,795,644	0.827	229,665	203,628	226,713	0.898	0.912	1.155	1.137	1.335
5	6	3,873,311	3,873,311	1.000	237,377	208,493	233,862	0.892	0.912	1.254	1.226	1.546
6	5	3,691,712	3,670,631	0.994	245,348	213,379	241,183	0.885	0.915	1.384	1.339	1.878
7	4	3,483,130	2,750,008	0.790	253,587	218,283	248,672	0.878	0.923	1.625	1.546	2.563
8	3	2,864,498	1,771,896	0.619	262,102	223,199	256,328	0.871	0.937	2.369	2.202	4.833
9	2	1,363,294	1,363,294	1.000	270,903	228,123	264,147	0.864	0.961	4.139	3.721	14.296
10	1	344,014	344,014	1.000	280,000	233,050	272,128	0.856	0.999	14.445	12.389	1,444.501

(5)	= (4) / (3)
(6)	Via claim size model
(7)	LEV [exponential(θ);x]= q * (1 - exp (x/θ))
(8)	LEV [exponential(θ);x]= q * (1 - exp (x/θ))
(9)	= (7) / (8)
(10)	See Section 4.2
(11)	Provided
(12)	= (11) * (9) / (10)
(13)	= (11) * (1- (9)) / (1- (10))