

# Profit Margins Using Co-Measures of Risk

Mark J. Homan, FCAS, MAAA, CPCU

---

**Abstract.** Insurance policies cover multiple loss components. Lately, there is a move to determining the premium for a policy by combining the components. This has led to the desire to have profit margins that can be combined. This paper demonstrates that profit margins by component are not additive. Those wishing to introduce rating by peril will need to consider how they will determine profit margins as they combine the underlying loss costs. The Excel worksheet used in the examples will be available on the CAS website.

**Keywords.** Profit Loads; capital allocation; risk loads

---

## 1. INTRODUCTION

There is a trend towards rating multi-peril products (i.e., Homeowners and Business Owners) by peril, or splitting rates between catastrophe and noncatastrophe. So the issue of determining the required profit loads naturally arises. The desire is to have profit loads by component so premiums can be determined by component and added together to get the final rate.

For example, the Florida legislature, recognizing the need for an appropriate return on catastrophe risk, required the Office of Insurance Regulation to determine an appropriate profit margin for the catastrophe portion of the Homeowners rate. While it is important for the industry that the legislature recognizes that catastrophe exposed business requires an appropriate return for the risk, they also took the erroneous view that profit margins can be determined by component.

Clearly, the administration of rates, for both companies and state regulators, would be simpler if profit margins could be determined in an additive manner. However, reality once again is not as simple as we would like.

Unfortunately, additive profit margins by component cannot be developed. One may accept the compromise required to treat profit margins as additive. However, this involves significant compromise in some cases, creating significant differences in prices.

Loss costs, which are based on means, are additive. Profit loads are based on risk, reflecting additional moments of the distribution, and higher moments of distributions are not simply additive. Diversification and correlation impact the profit load for the aggregate risk.

The examples in this paper are based on splits between catastrophe and noncatastrophe portions of the risk. The two loss components are treated as independent. This is reasonable in the author's

experience, but the methodology can be applied to loss distributions that are correlated. All expenses are treated as variable with no volatility. This is a simplifying assumption to isolate and highlight the interplay of the two loss components. The analysis can be extended to reflect expense variability and the risk that expenses represent.

## **1.1 Research Context**

This paper deals primarily with the required profit margins. It also addresses related issues of capital allocation and ROE.

There are a number of papers in the CAS literature on setting required profit margins, or profit loads. These papers identify that catastrophe exposure is a key consideration in determining the required profit margin. These papers deal with how to determine the profit margin for an aggregate exposure, with all risks combined. No papers or presentations were found that addressed the issue of determining profit margins based on risk component.

## **1.2 Objective**

The paper will evaluate two different approaches for determining profit margins by component. Both approaches will show that profit margins by component are not linear, and as a result, they cannot be added together. The expectation is that the paper can refute the concept of additive component profit margins.

## **1.3 Outline**

The first part of the paper will demonstrate why profit loads cannot be determined by peril or component. Then it will demonstrate how to determine the overall required profit margin using a Risk Coverage Ratio (RCR) approach, and then how to allocate the required profit to component based on risk using an approach algorithm named after the developers Ruhm-Mango-Kreps (RMK). It will also demonstrate the limitations or compromises required in this approach.

## **2. BACKGROUND AND METHODS**

The two methods evaluated are RCR and the RMK algorithm. A brief overview of each method will be provided before using the method to evaluate profit margins for the components. Further information on each method is included in the Appendix..

## **2.1 Profit Margins using RCR**

The initial concept used in this paper to determine a profit margin is the RCR. RCR was introduced to the actuarial community in a paper by David Ruhm [1]. Although RCR does not require surplus, as implied by the title of the paper, it is easy to translate the required price into implied surplus to attribute capital. (More information is provided in the Appendix.)

As a reward-to-risk ratio, RCR balances the required return to the risk. In its basic application, RCR is calculated from the distribution of returns on operating cash flows. In this situation, a common adverse event, or minimum threshold is zero. This means that any scenario where the premium and investment income are insufficient to cover all expenses and losses is considered an adverse event. In other words, any operating loss is bad.

RCR has strong appeal for use in pricing as it includes all adverse events in its determination. The risk metric used in the denominator is related to TVAR (Tail Value at Risk), also known as CTE (Conditional Tail Expectation). The key difference is that TVAR is usually defined at a pre-determined percentile level. For RCR, that percentile is dynamic and will vary based on the shape of the distribution for the line.

Since the RCR is a ratio of reward to risk, each line will have the same cost per unit of risk. In other words, the dollars of return required for each dollar of risk will be uniform across all lines of business.

## **2.2 Using RMK to Allocate Profit Margin**

In Section 3.5 that follows, the RMK (Ruhm-Mango-Kreps) algorithm is used to allocate surplus and thus the profit margin to risk component. RMK is an approach that attributes surplus to risk component in proportion to the component's contribution to aggregate risk. It is solely a methodology to allocate surplus, it does not determine the amount of surplus that is required.

The derivation of this algorithm and its properties are covered in papers available through the CAS. An initial paper by Ruhm and Mango [2] provides the foundation and formulas. Another paper by David Clark [3] provides a practical application of the RMK algorithm. Neither paper will be covered in detail here.

RMK requires setting an initial vector outlining risk appetite. In this paper, all scenarios that generate a net loss are assigned the same weight. Depending on a company's risk appetite, there may be events that cause a more extreme loss that should get higher weight. For example, the

weight may be increased in situations where a company is forced to access the capital markets for additional funds. The simpler approach used in this paper works well in practice and adequately outlines the desired concepts. The initial weights then are normalized to average to 1.0, and this becomes the Z-vector discussed in the Mango-Ruhm paper [2].

For the first two examples, since there is only a single loss component, these calculations are uninteresting, but are included to demonstrate that they work in this situation

### **3. RESULTS AND DISCUSSION**

In this section, the required profit load is determined for various combinations of catastrophe and noncatastrophe losses using the Risk Coverage Ratio. Then for the same examples the surplus and profit loads are split to risk component using the RMK algorithm.

#### **3.1 Profit Loads**

This section provides a general overview of splitting profit margins into catastrophe loss and noncatastrophe loss components. The examples shown are simplified calculations. Only the volatility in the level of the catastrophe and noncatastrophe loss ratio is reflected. Additional sources of volatility (payment date, interest rate, expense ratio, etc.) are ignored. This allows for illustration of the concepts, without requiring too complex an Excel spreadsheet for the examples. The exhibits show the summary and first 20 scenarios for each simulation. The full Excel spreadsheet is available on the CAS website.

The assumptions used in all examples are shown below:

Expenses	30% (treated as all variable)
Loss Payment	1 year (for both catastrophe and noncatastrophe)
Yield	5.04% before-tax
Tax Rate	35% (ignore tax loss discount)
RCR Target	20

#### **3.2 Separate Profit loads by Component**

As a first step, let's look at the profit loads by component for catastrophe separate from noncatastrophe. Exhibit 1A shows the derivation of the required premium for \$35 catastrophe loss only with the base assumptions. The catastrophe loss distribution is a sample of 10,000 scenarios

### *Component Profit Margins Using Co-Measures of Risk*

from a vendor catastrophe model using a countrywide distribution. The required premium is \$90.85. The target combined ratio is 68.5%, or an underwriting profit margin of 31.5%

Exhibit 1B shows the derivation of the required premium for \$60 of noncatastrophe loss with the base assumptions. The noncatastrophe loss distribution is based on a lognormal distribution and also uses 10,000 scenarios. The required premium is \$97.32. The target combined ratio is 91.7%, or an underwriting profit margin of 8.3%. Adding the noncatastrophe premium to the catastrophe premium yields a total premium of \$188.18. (There is an additional cent from rounding in the Excel spreadsheets.)

Comparing the two combined ratios, or profit margins, it is clear that the higher risk represented by catastrophe losses requires a much higher price per dollar of loss. Since expenses are all variable, the required premium is scaleable with losses. So to more directly compare the two premiums, we can scale the noncatastrophe premium to \$35 of noncatastrophe loss. That premium would be \$56.77, or \$34.08 less than what is required for catastrophe losses.

### **3.3 Profit Load for Combined Components**

Now, let's combine the catastrophe and noncatastrophe distributions and create a single loss distribution and an aggregate return distribution. Exhibit 2 shows the resulting required premium (\$174.12) and combined ratio (84.6%) for \$35 of catastrophe loss and \$60 of noncatastrophe loss. Comparing this premium to the total premium of \$188.18 from Exhibits 1A and 1B, one can see that the required premium is less on an aggregate basis than the sum of the premiums from each risk separately. The difference in premium of \$14.05 is the diversification benefit. The diversification comes from the fact that a bad year on one distribution can be offset, completely or partially, by a lower than expected year on the other distribution. It should be noted that a low catastrophe year will more often offset a bad noncatastrophe result in the same year than the other way around. This is because the catastrophe distribution has a more extreme tail.

#### **3.3.1 Profit Load with a Different Mix**

To further illustrate the effect of looking at combined distributions to develop profit margins versus combining the components, let's look at some different splits between catastrophe and noncatastrophe losses.

Exhibit 3A shows the required premium if there is twice as much in catastrophe loss, or \$70. The required premium is \$264.07. Comparing this to twice the catastrophe premium plus the

## *Component Profit Margins Using Co-Measures of Risk*

noncatastrophe premium, which is \$279.03, we can see the diversification benefit is \$14.96. The diversification benefit is better than Exhibit 2, but only by a small amount. In addition, the target combined ratio is 79.2%, lower than in Exhibit 2.

Exhibit 3B shows the required premium if there is half the amount of catastrophe loss, or \$17.50. The required premium is now \$130.92. Comparing this to half the catastrophe premium plus the noncatastrophe premium, which is \$142.75, we can see the diversification benefit drop to \$11.83. Now the target combined ratio is 89.2%, up from Exhibit 2.

I will leave it to the curious reader to download the Excel file from the CAS website and verify the following statements. Clearly, as the catastrophe loss goes to zero, the diversification benefit will go to zero as we will only have the noncatastrophe premium as shown in Exhibit 1A. As the catastrophe loss increases, the diversification increases at a decreasing rate.

### **3.4 Diversification Benefit**

The key difference between separate profit margins and a combined profit margin is reflecting the diversification benefit between the components. From Exhibits 2, 3A and 3B, we can see that the diversification benefit is a nonlinear relationship between the two loss distributions. Since this is a nonlinear relationship, it is clear that one cannot determine separate profit margins for catastrophe and noncatastrophe components and then add them together. The diversification benefit must be considered, and it is not a single factor adjustment in all cases.

#### **3.4.1 Special Case – Complete Correlation**

There is a special case where component profit margins would be additive. If the two distributions are completely correlated, there is no diversification benefit from combining them. With no diversification benefit, then the profit margins are the sum of the parts.

### **3.5 Using RMK to allocate Surplus (and profit margin)**

The RMK algorithm is an alternate method for attributing surplus based on contribution to risk. From another perspective, it can be viewed as a method for allocating the diversification benefit.

#### **3.5.1 RMK – Still not a Solution to Component Profit Margins**

In Exhibit 2-2, the surplus allocation for the example in Exhibit 2 is derived. This shows that within this example, the surplus is needed predominately for the catastrophe risk. The

### *Component Profit Margins Using Co-Measures of Risk*

noncatastrophe risk contributes very little to the operating losses. So the diversification benefit that we discussed above is primarily seen in the noncatastrophe risk. Similar derivations of surplus attribution are shown in Exhibits 3A-2 and 3B-2.

Another way to look at the allocation of diversification benefit is to compare the surplus by component. The surplus indicated for the catastrophe risk only starts at \$168.02 (Exhibit 1A), which is reduced only to \$160.24 (Exhibit 2-2) in the combined example. In contrast, the required surplus for the noncatastrophe component starts at \$61.29 (Exhibit 1B), and this is reduced to \$14.53 in the combined example (Exhibit 2-2). This shows that the primary impact of diversification is to reduce the amount of surplus required to support the noncatastrophe risk.

Starting with the allocation of surplus and profit from Example 2, we can try to predict the required profit margins for Examples 3A and 3B. We do this by applying the leverage ratios from Exhibit 2-2 to the liabilities generated in Examples 3A and 3B. This is shown in Exhibit 4. This example shows we would come up short on our estimate of required surplus, and thus profit margin, for both cases.

It is interesting to understand why we are not predicting the correct answer. In both cases, it is because the level of diversification has changed. In Example 3A, we are not getting as much diversification from the noncatastrophe portion of the exposure. Since there is little surplus required for noncatastrophe, the difference in required surplus is moderate. In Example 3B, we have half the catastrophe loss level. Now, the noncatastrophe loss cannot be diversified away as much as in Example 2. In other words, the noncatastrophe risk has more impact on the bottom line, so we need to attribute more surplus to it.

RMK is considered one of the most sophisticated methods of attributing surplus and determining required profit, yet it still cannot provide correct answers for component profit margins that can be used as the mix of risk component changes.

#### **3.5.2 Materiality**

Let's shift from the theoretical to the practical. The RCR analysis was sufficient to demonstrate that component profit margins are not additive as risk varies, so why explore the application of RMK? It is because RMK can be used to flex profit margins within a reasonable range of changes in mix by component. The size of the range will depend on one's definition of materiality. Clearly, if there is a theoretical difference that does not translate to a difference in what a policyholder will actually pay (i.e. one that rounds away), then the difference would not be considered material.

## *Component Profit Margins Using Co-Measures of Risk*

For Example 3A, the shortfall is moderate, only \$0.78 on the true premium of \$264.07. This could very well be determined not to be material. And this is an example where we doubled the catastrophe losses, which is a fairly extreme change. If the increase in catastrophe losses was more moderate, like 10%, the difference would be even smaller and would be more likely to be considered immaterial by many companies.

The examples used in this paper were based on a split between catastrophe and noncatastrophe components. Also, the size of the differences in the split is extreme to more easily demonstrate the points in the paper. While the theoretical conclusions apply equally to more moderate splits, like Homeowners rating by peril, the differences in results are not as great. When the distributions are not as different in shape, then RMK can be used in a broader range without material bias. Or, if the range of changes anticipated in the mix of component is moderate, RMK can be reasonably used.

So, in the end, one may determine that the RMK algorithm creates a practical approach for addressing the component profit margins in certain situations.

## **4. CONCLUSIONS**

Profit margins are based on risk. Risk cannot be evaluated by component in isolation. Risk must be evaluated in the context of the whole, and how the various risks contribute diversification to each other. It is not theoretically possible to create additive component profit margins. However, it is possible using RMK to create profit margins that can be combined within reasonable ranges of mix change.

### **Acknowledgment**

The author acknowledges the foundational work for this analysis that was developed by Russ Bingham and David Ruhm, FCAS. He also acknowledges the efforts of those involved in reviewing this paper, in particular, Drew Golfín, FCAS, Dave Terné, FCAS, and David Ruhm, FCAS.

### **Supplementary Material**

The Excel workbook containing all the examples in the full 10,000 scenario detail is stored on the CAS website.

## **5. REFERENCES**

- [1] Ruhm, David L., "Risk Coverage Ratio: A Leverage-Independent Method of Pricing Based on Distribution of Return," *ASTIN Colloquium International Actuarial Association - Brussels, Belgium, 2001*: Washington.



### *Component Profit Margins Using Co-Measures of Risk*

- [2] Mango, Donald F., and Ruhm, David L., "A Risk Charge Calculation Based on Conditional Probability," *ASTIN Colloquium International Actuarial Association - Brussels, Belgium, 2003*: Berlin.
- [3] Clark, David R., "The Reinsurance Applications for the RMK Framework," *Casualty Actuarial Society Forum*, Spring **2005**, 353-366.

#### **Abbreviations and notations**

Collect here in alphabetical order all abbreviations and notations used in the paper

RCR, Rick Coverage Ratio

RMK, Ruhm Mango Kreps

ROE, Return on Equity

#### **Biography of the Author**

**Mark J. Homan** is an AVP and Actuary at The Hartford Financial Services Group in Hartford, CT. He is responsible for risk and return modeling, capital allocation support, and ratemaking compliance. He has a degree in Mathematics and Quantitative Methods from the University of St. Thomas, St. Paul, MN. He is a Fellow of the CAS and a Member of the American Academy of Actuaries. He currently is a member of the AAA Natural Catastrophe Subcommittee and has also recently participated on the CAS Professional Education Committee, ERM Subcommittee. He is a frequent presenter at industry symposia and has authored previous papers in the CAS literature dealing with Homeowners pricing.

## 6. APPENDIX

More information on the two approaches used in this paper is provided in this appendix. Recognizing that most readers will not be familiar with RCR or RMK, more information is provided here. This is not intended to replace reading the original papers, but should provide enough information to put this paper in context.

### 6.1 Risk Coverage Ratio (RCR)

RCR was introduced to the actuarial community in a paper by David Ruhm [1]. As stated in the title of Mr. Ruhm's article, RCR does not require leverage or surplus. The required price, and associated profit margin, is calculated to meet the target RCR. In addition, once the RCR and price are determined, you can use this information to attribute capital.

As a reward-to-risk ratio, RCR balances the required return to the risk. To calculate the RCR, one must first determine an adverse event threshold. This will define both the reward and the risk. The reward is the average return minus the adverse event. The risk is the probability of being below the adverse event threshold times the average amount below the threshold when it is below.

The basic formula is:

$$\text{RCR} = (R - m) / (\text{Pr}(x < m) * (m - T)) \quad (6.1)$$

where:

R is average return

m is the adverse event threshold, or minimum return

T is the average of all events below the adverse threshold, or the tail

RCR can be used to attribute capital. After solving for the required price to achieve the target RCR, the expected income from operating flows (O) is known. Given a target return on surplus (ROS) and the yield on surplus (y), it is merely algebra to solve for the surplus.

$$\text{Surplus} = O / (\text{ROS} - y) \quad (6.2)$$

The yield that is used in this case should be a risk-free or low-risk yield. There is additional investment risk in the actual investment portfolio for most companies, so the actual portfolio yield is usually higher, but also requires additional surplus to support that risk. The actual portfolio yield

can be reflected, but that will require additional modeling to solve for the RCR including portfolio yields and risk. The author recommends the use of LIBOR as a near risk-free yield in determining RCR. LIBOR is the standard rate used in the investment community for modeling, and is available at more durations and time points than Treasury yields.

One of the issues of working with RCR is determining the proper target value. There is no intuitive value that makes sense, nor are there any industry standards that can be used. A recommended approach is to use RCR to attribute surplus for all lines of business and solve for the RCR that attributes all of the company's carried surplus. This becomes the Target RCR value for use in pricing. Using this approach, the total surplus for the company will be attributed, and if all lines are at the target price determined by the RCR, the required return on surplus will be achieved.

## **6.2 Ruhm-Mango-Kreps (RMK) Algorithm**

RMK is a methodology designed to allocate risk charge, and thus capital.

There are some key issues in allocating risk charges, and attributing capital, that the RMK algorithm was created to address. As stated in the paper, "Accounting for aggregate portfolio effects in property-casualty insurance prices has historically created some difficult problems, including:

- 1) Additivity or sub-additivity of prices;
- 2) Measuring how much diversification efficiency actually exists;
- 3) Allocating the diversification benefits back to the individual risks; and
- 4) Order-dependence."

The authors of the paper go on to state, "The method begins at the aggregate level for evaluating risk, and ends by producing prices for individual risks, effectively allocating the total portfolio risk charge. The result is an internally consistent allocation of diversification benefits, avoiding the difficulties listed above. The method effectively extends any risk-valuation theory used at the aggregate portfolio level to the individual risks comprising the portfolio. The resulting prices are additive, with each risk's price reflecting the degree to which it contributes to total portfolio risk" [2].

RMK starts with an aggregate risk charge, or surplus, determined by some other methodology. RMK is used to distribute the risk charge to component in a consistent manner. Some of the key points from the paper are:

- 1) The aggregate risk charge is split to the individual risks based on the conditional relationship between the risks' outcomes and the aggregate results for the portfolio.

*Component Profit Margins Using Co-Measures of Risk*

- 2) All prices are determined solely by the portfolio-level and the probability structure, so that no other information is required.
- 3) Correlations between risks (and between each risk and the portfolio) are included in the prices in full detail, via the conditional probabilities. Since diversification is related to these correlations, it is also reflected in the risks through this calculation.
- 4) Prices produced by this method are additive. The price for each component is made up of its contribution to expected costs and its risk load, or profit margin.

The RMK algorithm requires that you assign a weight to each scenario based on the outcome that reflects the company's risk appetite. In this paper, any loss outcome gets the same weight. The RMK algorithm can handle more complex views on risk, such as assigning higher weights based on the size of the loss. Details on the calculations that are associated with this paper are provided in the Notes to Exhibits section 7.2.

## 7. NOTES TO EXHIBITS

This section provides more detailed information on the calculations of the various exhibits included in this paper. The Excel file used to develop the exhibits is available from the CAS website.

### 7.1 RCR Exhibits

The format and formulas in Exhibits 1A, 1B, 2, 3A and 3B are the same. It is just the inputs that vary. So they will be discussed together.

#### Items

- **Premium (solved)** – This is the premium required to meet the RCR target (below). It is solved for via iteration.
- **Combined (formula)** – Combined ratio determined from average loss and LAE dollars for catastrophe and noncatastrophe in total divided by premium plus the expense ratio.
- **Exp (assumption)** – Expense ratio. All expenses are treated as variable in these examples.
- **Loss (assumption)** – the expected loss and LAE dollars are shown as the average. For each scenario, a lognormal distribution was used to create a loss and LAE figure. The parameters for the lognormal are hypothetical used for these examples. The same distribution is used for all examples, with varying means.
- **Cat Loss (assumption)** – the expected catastrophe loss and LAE dollars are shown as the average. The scenarios for cat loss came from the output of a cat model, manipulated to not reveal any real information. Again, the distribution of cat losses is the same in all examples, just the mean has changed.
- **Loss Lag and Cat Lag (assumption)** – represent the average payment date for the two loss components. A common value of 1.0 years is used for both loss components in these examples.
- **Yield (assumption)** – The yield is the average LIBOR yield for the period of time and duration assumed for investing the flows. A complete discussion of interest rates to use in modeling is beyond the scope of this paper. Suffice it to say that the use of LIBOR to

### *Component Profit Margins Using Co-Measures of Risk*

represent risk-free rates of return is common in the investment and finance community. And that practice has been adopted here. This model can be expanded to look at portfolio yields, but additional capital would be needed to address the increased risk in such a portfolio.

- **Loss Liab and Cat Liab (formula)** – These are the present value of the balance sheet liabilities for noncat loss and LAE and catastrophe loss and LAE, respectively. The formula is shown below:

$$\text{Liabilities} = \text{Loss} * [1 / (1 + \text{Yield}(\text{after-tax}))^{\text{Lag}}] / (\text{Yield}(\text{after-tax})) \quad (7.1)$$

- **Tax Rate** – Not shown on the exhibits. A 35% tax rate is used in these examples.
- **Net Liab (formula)** – the sum of the loss and cat liabilities. This is the present value of the total balance sheet liabilities. Since both components are on a present value basis, they can be added even if the lags are different.
- **UW Inc (formula)** – The underwriting profit which is the premium minus the sum of expenses, noncatastrophe losses and catastrophe losses, adjusted for taxes.

- **UW Inv Inc (formula)** – this is the investment income on the operating cash flows.

$$\text{UW Inv Inc} = \text{Net Liab} * \text{Yield}(\text{After-tax}) \quad (7.2)$$

- **Tot Inc (Formula)** – Total income, the sum of UW Inc and UW Inv Inc
- **ESD (formula)** – expected surplus drawdown. If the total income is negative, this is the complement of the income. It is zero if the income is positive. In other words, it is the amount of the loss when there is a loss. The average ESD is the risk metric used in calculating RCR. It can also be determined as follows:

$$\text{Risk} = E(\text{ESD}) = -\text{Pr}(\text{Tot Inc} < 0) * E(\text{Tot Inc} \mid \text{Tot Inc} < 0) \quad (7.3)$$

- **RCR (formula)** – Risk Coverage Ratio. Ratio of Total Income to risk, or:

$$\text{RCR} = \text{Tot Inc} / E(\text{ESD}) \quad (7.4)$$

- **Target ROS (assumption)** – this is the return on surplus that the company is targeting.
- **Surplus (formula)** – This is the surplus required by the line to translate the operating return (Tot Inc) to the target ROS. It is determined using formula 6.2, restated below using the variable names from these exhibits.

### *Component Profit Margins Using Co-Measures of Risk*

$$\text{Surplus} = \text{Tot Inc} / (\text{Target ROS} - \text{Yield}(\text{after-tax})) \quad (7.5)$$

## **7.2 RMK Exhibits**

The exhibits that demonstrate the application of the RMK algorithm use a common format and set of formulas. They build on information in the corresponding RCR exhibit and are numbered as such. The three exhibits included are 2-2, 3A-2, and 3B-2.

RMK requires a set of weights that can be based on any underlying view of risk. The weights are normalized to sum to 1.0, termed the Z-vector in the paper. There is no requirement as to what sort of risk preference is used to determine the initial weights. In this paper, a simple set of weights is used so all operating losses get the same weight of  $1 + (1/\text{RCR})$ , and the positive results are assigned a small weight of  $(1/\text{RCR})$ .

To start, the premium is apportioned to the components of expense, catastrophe loss and noncatastrophe loss. Then the underwriting gain/loss from each component for each scenario is determined. (Note that since there is no expense volatility in these examples, the expense component drops out and is not shown.) Next the deviation of the investment income for the scenario from the expected is determined. These two pieces are combined to determine the operating gain contribution for the scenario from each component. These figures are used to allocate the risk charge and then the surplus to the components.

### **Items**

- **Ave ROE (assumption)** – This is the target ROS from the RCR exhibit.
- **Surplus II (assumption)** – Investment Income (II) on Surplus. This is the yield adjusted to after-tax..
- **Avg Op Rtn (formula)** – Average Operating Return. Viewed as either the Avg ROE minus the Surplus II, or can be calculated from the RCR exhibit as the average Tot Inc divided by the average Net Liab.
- **Surplus (assumption)** – this is the figure determined in the RCR exhibit.
- **Risk Chg (assumption)** – Risk Charge. This is average Tot Inc from the RCR exhibit.

Here it is being viewed as the amount of return required to cover the risk, or as the risk charge.

- **Total Op Gain (assumption)** – This is the Tot Inc from the RCR exhibit, reproduced here.

*Component Profit Margins Using Co-Measures of Risk*

- **Crude Weights (formula)** – This is the initial set of weights, before normalization, used in the RMK formula. In these examples, the weights are  $1/RCR$  if the income is positive, and  $(1 + 1/RCR)$  if the income is negative. This puts more weight on the loss scenarios. This represents a simple utility function.
- **Z (formula)** – this is the Z-vector referred to in the Ruhm-Mango paper. It is a normalized set of weights calculated as the Crude Weight /  $E(\text{Crude Weight})$ .
- **Prem Split (formula)** – In order to determine the contribution to the underwriting gain or loss, the premium needs to be split into component. The split here is based on the average cost for each item. Since expenses do not vary, they are not relevant and the portion of premium for expense is not shown. The calculation is very insensitive to the premium split. However, it is easier to interpret the calculations if a reasonable split of the premium is used initially.

The formula for loss, with cat loss being similar, is:

$$\text{Prem Split} = \text{Premium} * E(\text{Loss}) / (E(\text{exp}) + E(\text{loss}) + E(\text{cat loss})) \quad (7.6)$$

The next items are the six columns. Then the formulas used to calculate across each column will be covered.

- **Loss xCat UW Gain (formula)** – This the contribution of the noncatastrophe loss portion to the UW gain or loss. It is calculated as the difference between the premium split for loss and the loss for the scenario, adjusted for taxes.
- **Cat Loss UW Gain (formula)** – Similar to the above, this is the contribution of the cat loss to the UW gain or loss. It is calculated in the difference between the premium split for loss and the loss for the scenario, adjusted for taxes.
- **Loss xCat Inv Gain/Cat Loss Inv Gain (formula)** – This is the contribution to investment income from the scenario. It is the liability times the yield adjusted for taxes. When there is an underwriting loss, this serves as an offset.
- **Loss xCat Op Gain/Cat Loss Op Gain (formula)** – This is the sum of the UW gain or loss plus the investment income for the component.

The following items are calculated for each column, or component. There are two risk factors, noncatastrophe loss and catastrophe loss, in three different levels – underwriting gain/loss, investment gain and total gain/loss.



*Component Profit Margins Using Co-Measures of Risk*

- **E(R) (formula)** – Average value for the column, or the risk contribution for the component.
- **E(ZR) (formula)** – Average value of the product of the weight (*Z*) times the risk contribution, or contribution, for each component.
- **Risk Chg (formula)** – the risk charge for the component, which is E(R) minus E (ZR). This is also the contribution to the average operating return from the component, so the sum across all components will equal the average operating return.
- **Surplus (formula)** – The surplus required for that component. This is calculated using the following formula:

$$\text{Surplus} = \text{Risk Chg} / \text{Avg Op Return (total)} \quad (7.6)$$

- **Avg Op Rtn (formula)** – Average Operating Return, is calculated for each item as the risk charge, divided by surplus. Given the formula used to get surplus, it will be equal to the average operating return for the total. So, the average operating return for each component should be the same as the overall average operating return, and this acts as a cross check.
- **Surplus II (assumption)** – the investment income on surplus. This is the same as the yield that was used in the total.
- **Avg ROE (formula)** – average ROE is the sum of the Avg Op Rtn and Surplus II.
- **Tot Und/Tot Inv (formula)** – The sum of the surplus for the risk components, noncatastrophe loss and catastrophe loss, at the underwriting and investment level respectively. Note that investment surplus is negative, as it acts to offset the positive surplus for underwriting.
  - **Leverage Ratios (formula)** – The two leverage ratios are shown, which are ratios of liabilities to surplus. The liabilities from the underlying RCR exhibit for the component are divided by the total surplus for that component. These leverage ratios are then used in other models as the expected catastrophe and noncatastrophe losses vary.

The final section in these exhibits is a summary, and shows how the underwriting profit for the component would be derived.

- **Surplus, Yield and Op Income** – are repeats of the items from earlier columns, shown here to see what is used in the following calculations.
- **Op Inv Inc (formula)** – investment income on operating cash flows. This is the liabilities for the component times the yield.

*Component Profit Margins Using Co-Measures of Risk*

- **UW Income (formula)** – underwriting income, it is the operating income minus the operating investment income. This split shows the composition of the underwriting profit margin.

## Sample Calculations

### For Row 1 on Exhibit 2

Premium	174.12	
	<b>Dollars</b>	<b>Ratio to Premium</b>
Loss	57.65	33.1%
Cat Loss	19.62	11.3%
Expense		30.0%
Combined Ratio		74.4%
	<b>Pre-Tax</b>	<b>Post-Tax</b>
Yield	5.04%	3.28%
Loss Liabilities	55.83	Loss * (1 - 1/(1+ post-tax-yield))/post-tax-yield
Cat Loss Liabilities	19.00	Cat Loss * (1 - 1/(1+ post-tax-yield))/post-tax-yield
Net Liabilities	74.82	Sum of Loss Liabilities and Cat Loss Liabilities
Underwriting Income	29.00	[Premium - (Loss + Cat Loss + Expense)] * (1 - tax rate)
UW Investment Income	2.45	Net Liabilities * post-tax-yield
Total Income	31.45	Sum of UW Income and UW Investment Income

*Profit Margins Using Co-Measures of Risk*

**Exhibit 1A**

Premium Combined	90.85 68.5%	Loss Lag	1.000	RCR	20.00
		Cat Lag	1.000	Target ROS	15.0%
		Yield	5.04%	Surplus	168.02

	Exp	Loss	Cat Loss	Loss Liab	Cat Liab	Net Liab	UW Inc	UW Inv Inc	Tot Inc	ESD
<b>Averages</b>	<b>0.30</b>	<b>0.00</b>	<b>35.00</b>	<b>0.0</b>	<b>33.9</b>	<b>33.9</b>	<b>18.59</b>	<b>1.11</b>	<b>19.70</b>	<b>0.98</b>
1	0.30	0.00	19.62	0.0	19.0	19.0	28.59	0.62	29.21	0.00
2	0.30	0.00	90.84	0.0	88.0	88.0	-17.70	2.88	-14.82	14.82
3	0.30	0.00	32.87	0.0	31.8	31.8	19.97	1.04	21.02	0.00
4	0.30	0.00	25.02	0.0	24.2	24.2	25.08	0.79	25.87	0.00
5	0.30	0.00	38.92	0.0	37.7	37.7	16.04	1.23	17.28	0.00
6	0.30	0.00	24.34	0.0	23.6	23.6	25.52	0.77	26.29	0.00
7	0.30	0.00	25.60	0.0	24.8	24.8	24.70	0.81	25.51	0.00
8	0.30	0.00	35.69	0.0	34.6	34.6	18.14	1.13	19.27	0.00
9	0.30	0.00	59.80	0.0	57.9	57.9	2.47	1.90	4.37	0.00
10	0.30	0.00	10.41	0.0	10.1	10.1	34.57	0.33	34.90	0.00
11	0.30	0.00	17.18	0.0	16.6	16.6	30.17	0.54	30.72	0.00
12	0.30	0.00	43.38	0.0	42.0	42.0	13.14	1.38	14.52	0.00
13	0.30	0.00	28.26	0.0	27.4	27.4	22.97	0.90	23.87	0.00
14	0.30	0.00	29.70	0.0	28.8	28.8	22.03	0.94	22.98	0.00
15	0.30	0.00	67.40	0.0	65.3	65.3	-2.47	2.14	-0.34	0.34
16	0.30	0.00	25.96	0.0	25.1	25.1	24.47	0.82	25.29	0.00
17	0.30	0.00	23.56	0.0	22.8	22.8	26.02	0.75	26.77	0.00
18	0.30	0.00	94.33	0.0	91.3	91.3	-19.98	2.99	-16.98	16.98
19	0.30	0.00	15.15	0.0	14.7	14.7	31.49	0.48	31.97	0.00
20	0.30	0.00	55.90	0.0	54.1	54.1	5.01	1.77	6.78	0.00
21	0.30	0.00	20.10	0.0	19.5	19.5	28.27	0.64	28.91	0.00
22	0.30	0.00	46.00	0.0	44.5	44.5	11.44	1.46	12.90	0.00
23	0.30	0.00	36.39	0.0	35.2	35.2	17.69	1.15	18.84	0.00
24	0.30	0.00	34.91	0.0	33.8	33.8	18.65	1.11	19.76	0.00
25	0.30	0.00	19.65	0.0	19.0	19.0	28.56	0.62	29.19	0.00

Profit Margins Using Co-Measures of Risk

Exhibit 1B

Premium	97.32	Loss Lag	1.000	RCR	20.00
Combined	91.7%	Cat Lag	1.000	Target ROS	15.0%
Cat Premi	90.85	Yield	5.04%	Surplus	61.29
(From Exh. 1A)					
Total Premium	188.18				

	Exp	Loss	Cat Loss	Loss Liab	Cat Liab	Net Liab	UW Inc	UW Inv Inc	Tot Inc	ESD
<b>Averages</b>	<b>0.30</b>	<b>60.00</b>	<b>0.00</b>	<b>58.1</b>	<b>0.0</b>	<b>58.1</b>	<b>5.28</b>	<b>1.90</b>	<b>7.19</b>	<b>0.36</b>
1	0.30	57.65	0.00	55.8	0.0	55.8	6.81	1.83	8.64	0.00
2	0.30	56.99	0.00	55.2	0.0	55.2	7.24	1.81	9.04	0.00
3	0.30	58.41	0.00	56.6	0.0	56.6	6.32	1.85	8.17	0.00
4	0.30	49.37	0.00	47.8	0.0	47.8	12.19	1.57	13.76	0.00
5	0.30	46.47	0.00	45.0	0.0	45.0	14.08	1.47	15.55	0.00
6	0.30	69.27	0.00	67.1	0.0	67.1	-0.74	2.20	1.45	0.00
7	0.30	54.62	0.00	52.9	0.0	52.9	8.78	1.73	10.51	0.00
8	0.30	54.38	0.00	52.7	0.0	52.7	8.94	1.72	10.66	0.00
9	0.30	71.10	0.00	68.8	0.0	68.8	-1.94	2.26	0.32	0.00
10	0.30	57.52	0.00	55.7	0.0	55.7	6.89	1.82	8.72	0.00
11	0.30	75.92	0.00	73.5	0.0	73.5	-5.06	2.41	-2.66	2.66
12	0.30	64.95	0.00	62.9	0.0	62.9	2.07	2.06	4.13	0.00
13	0.30	58.29	0.00	56.4	0.0	56.4	6.40	1.85	8.25	0.00
14	0.30	63.31	0.00	61.3	0.0	61.3	3.13	2.01	5.14	0.00
15	0.30	68.40	0.00	66.2	0.0	66.2	-0.18	2.17	1.99	0.00
16	0.30	64.73	0.00	62.7	0.0	62.7	2.21	2.05	4.26	0.00
17	0.30	63.56	0.00	61.5	0.0	61.5	2.97	2.02	4.99	0.00
18	0.30	50.21	0.00	48.6	0.0	48.6	11.65	1.59	13.24	0.00
19	0.30	46.21	0.00	44.7	0.0	44.7	14.25	1.47	15.71	0.00
20	0.30	68.67	0.00	66.5	0.0	66.5	-0.36	2.18	1.82	0.00
21	0.30	38.50	0.00	37.3	0.0	37.3	19.26	1.22	20.48	0.00
22	0.30	71.73	0.00	69.5	0.0	69.5	-2.34	2.28	-0.06	0.06
23	0.30	56.71	0.00	54.9	0.0	54.9	7.42	1.80	9.22	0.00
24	0.30	57.03	0.00	55.2	0.0	55.2	7.21	1.81	9.02	0.00
25	0.30	79.04	0.00	76.5	0.0	76.5	-7.10	2.51	-4.59	4.59

*Profit Margins Using Co-Measures of Risk*

**Exhibit 2**

Premium	174.12	Loss Lag	1.000	RCR	20.00
Combined	84.6%	Cat Lag	1.000	Target ROS	15.0%
Prior Premium	188.18	Yield	5.04%	Surplus	174.77
Diversification Benefit	14.05				

	Exp	Loss	Cat Loss	Loss Liab	Cat Liab	Net Liab	UW Inc	UW Inv Inc	Tot Inc	ESD
<b>Averages</b>	<b>0.30</b>	<b>60.00</b>	<b>35.00</b>	<b>58.1</b>	<b>33.9</b>	<b>92.0</b>	<b>17.48</b>	<b>3.01</b>	<b>20.49</b>	<b>1.02</b>
1	0.30	57.65	19.62	55.8	19.0	74.8	29.00	2.45	31.45	0.00
2	0.30	56.99	90.84	55.2	88.0	143.1	-16.86	4.69	-12.17	12.17
3	0.30	58.41	32.87	56.6	31.8	88.4	19.90	2.90	22.79	0.00
4	0.30	49.37	25.02	47.8	24.2	72.0	30.88	2.36	33.24	0.00
5	0.30	46.47	38.92	45.0	37.7	82.7	23.73	2.71	26.44	0.00
6	0.30	69.27	24.34	67.1	23.6	90.6	18.38	2.97	21.35	0.00
7	0.30	54.62	25.60	52.9	24.8	77.7	27.08	2.54	29.63	0.00
8	0.30	54.38	35.69	52.7	34.6	87.2	20.68	2.86	23.54	0.00
9	0.30	71.10	59.80	68.8	57.9	126.8	-5.86	4.15	-1.71	1.71
10	0.30	57.52	10.41	55.7	10.1	65.8	35.07	2.15	37.23	0.00
11	0.30	75.92	17.18	73.5	16.6	90.1	18.72	2.95	21.67	0.00
12	0.30	64.95	43.38	62.9	42.0	104.9	8.81	3.44	12.25	0.00
13	0.30	58.29	28.26	56.4	27.4	83.8	22.97	2.75	25.72	0.00
14	0.30	63.31	29.70	61.3	28.8	90.1	18.77	2.95	21.72	0.00
15	0.30	68.40	67.40	66.2	65.3	131.5	-9.05	4.31	-4.74	4.74
16	0.30	64.73	25.96	62.7	25.1	87.8	20.28	2.88	23.16	0.00
17	0.30	63.56	23.56	61.5	22.8	84.4	22.60	2.76	25.36	0.00
18	0.30	50.21	94.33	48.6	91.3	140.0	-14.72	4.58	-10.14	10.14
19	0.30	46.21	15.15	44.7	14.7	59.4	39.34	1.95	41.29	0.00
20	0.30	68.67	55.90	66.5	54.1	120.6	-1.74	3.95	2.21	0.00
21	0.30	38.50	20.10	37.3	19.5	56.7	41.14	1.86	42.99	0.00
22	0.30	71.73	46.00	69.5	44.5	114.0	2.71	3.73	6.44	0.00
23	0.30	56.71	36.39	54.9	35.2	90.1	18.72	2.95	21.67	0.00
24	0.30	57.03	34.91	55.2	33.8	89.0	19.47	2.92	22.38	0.00
25	0.30	79.04	19.65	76.5	19.0	95.6	15.07	3.13	18.20	0.00

*Profit Margins Using Co-Measures of Risk*

**Exhibit 3A**

Premium Combined	264.07 79.2%	Loss Lag	1.000	RCR	20.00
Prior Premium Diversification	279.03 14.96	Cat Lag	1.000	Target ROS Surplus	15.0% 339.27
		Yield	5.04%		

	Exp	Loss	Cat Loss	Loss Liab	Cat Liab	Net Liab	UW Inc	UW Inv Inc	Tot Inc	ESD
<b>Averages</b>	<b>0.30</b>	<b>60.00</b>	<b>70.00</b>	<b>58.1</b>	<b>67.8</b>	<b>125.9</b>	<b>35.65</b>	<b>4.12</b>	<b>39.78</b>	<b>1.99</b>
1	0.30	57.65	39.24	55.8	38.0	93.8	57.17	3.07	60.24	0.00
2	0.30	56.99	181.67	55.2	175.9	231.1	-34.98	7.57	-27.41	27.41
3	0.30	58.41	65.74	56.6	63.7	120.2	39.46	3.94	43.40	0.00
4	0.30	49.37	50.03	47.8	48.4	96.2	55.54	3.15	58.69	0.00
5	0.30	46.47	77.83	45.0	75.4	120.4	39.36	3.94	43.30	0.00
6	0.30	69.27	48.67	67.1	47.1	114.2	43.49	3.74	47.23	0.00
7	0.30	54.62	51.20	52.9	49.6	102.5	51.37	3.36	54.73	0.00
8	0.30	54.38	71.38	52.7	69.1	121.8	38.41	3.99	42.40	0.00
9	0.30	71.10	119.60	68.8	115.8	184.7	-3.81	6.05	2.24	0.00
10	0.30	57.52	20.82	55.7	20.2	75.9	69.23	2.48	71.72	0.00
11	0.30	75.92	34.35	73.5	33.3	106.8	48.48	3.50	51.97	0.00
12	0.30	64.95	86.76	62.9	84.0	146.9	21.54	4.81	26.35	0.00
13	0.30	58.29	56.51	56.4	54.7	111.2	45.53	3.64	49.17	0.00
14	0.30	63.31	59.40	61.3	57.5	118.8	40.39	3.89	44.29	0.00
15	0.30	68.40	134.81	66.2	130.5	196.8	-11.94	6.45	-5.49	5.49
16	0.30	64.73	51.91	62.7	50.3	112.9	44.33	3.70	48.03	0.00
17	0.30	63.56	47.13	61.5	45.6	107.2	48.21	3.51	51.72	0.00
18	0.30	50.21	188.66	48.6	182.7	231.3	-35.11	7.58	-27.54	27.54
19	0.30	46.21	30.30	44.7	29.3	74.1	70.42	2.43	72.85	0.00
20	0.30	68.67	111.79	66.5	108.2	174.7	2.85	5.72	8.57	0.00
21	0.30	38.50	40.20	37.3	38.9	76.2	69.00	2.50	71.49	0.00
22	0.30	71.73	91.99	69.5	89.1	158.5	13.73	5.19	18.93	0.00
23	0.30	56.71	72.78	54.9	70.5	125.4	35.99	4.11	40.10	0.00
24	0.30	57.03	69.81	55.2	67.6	122.8	37.70	4.02	41.73	0.00
25	0.30	79.04	39.31	76.5	38.1	114.6	43.22	3.75	46.98	0.00

*Profit Margins Using Co-Measures of Risk*

**Exhibit 3B**

Premium	130.92	Loss Lag	1.000	RCR	20.00
Combined	89.2%	Cat Lag	1.000	Target ROS	15.0%
Prior Premium	142.75	Yield	5.04%	Surplus	99.39
Diversification	11.83				

	Exp	Loss	Cat Loss	Loss Liab	Cat Liab	Net Liab	UW Inc	UW Inv Inc	Tot Inc	ESD
<b>Averages</b>	<b>0.30</b>	<b>60.00</b>	<b>17.50</b>	<b>58.1</b>	<b>16.9</b>	<b>75.0</b>	<b>9.19</b>	<b>2.46</b>	<b>11.65</b>	<b>0.58</b>
1	0.30	57.65	9.81	55.8	9.5	65.3	15.72	2.14	17.86	0.00
2	0.30	56.99	45.42	55.2	44.0	99.2	-7.00	3.25	-3.75	3.75
3	0.30	58.41	16.43	56.6	15.9	72.5	10.92	2.37	13.30	0.00
4	0.30	49.37	12.51	47.8	12.1	59.9	19.35	1.96	21.31	0.00
5	0.30	46.47	19.46	45.0	18.8	63.8	16.72	2.09	18.81	0.00
6	0.30	69.27	12.17	67.1	11.8	78.9	6.63	2.58	9.22	0.00
7	0.30	54.62	12.80	52.9	12.4	65.3	15.74	2.14	17.88	0.00
8	0.30	54.38	17.85	52.7	17.3	69.9	12.62	2.29	14.91	0.00
9	0.30	71.10	29.90	68.8	29.0	97.8	-6.08	3.20	-2.88	2.88
10	0.30	57.52	5.20	55.7	5.0	60.7	18.80	1.99	20.79	0.00
11	0.30	75.92	8.59	73.5	8.3	81.8	4.64	2.68	7.32	0.00
12	0.30	64.95	21.69	62.9	21.0	83.9	3.25	2.75	6.00	0.00
13	0.30	58.29	14.13	56.4	13.7	70.1	12.50	2.30	14.80	0.00
14	0.30	63.31	14.85	61.3	14.4	75.7	8.77	2.48	11.25	0.00
15	0.30	68.40	33.70	66.2	32.6	98.9	-6.80	3.24	-3.56	3.56
16	0.30	64.73	12.98	62.7	12.6	75.2	9.06	2.47	11.52	0.00
17	0.30	63.56	11.78	61.5	11.4	73.0	10.60	2.39	12.99	0.00
18	0.30	50.21	47.17	48.6	45.7	94.3	-3.72	3.09	-0.63	0.63
19	0.30	46.21	7.58	44.7	7.3	52.1	24.61	1.71	26.32	0.00
20	0.30	68.67	27.95	66.5	27.1	93.6	-3.24	3.06	-0.17	0.17
21	0.30	38.50	10.05	37.3	9.7	47.0	28.01	1.54	29.55	0.00
22	0.30	71.73	23.00	69.5	22.3	91.7	-2.00	3.00	1.00	0.00
23	0.30	56.71	18.19	54.9	17.6	72.5	10.88	2.38	13.26	0.00
24	0.30	57.03	17.45	55.2	16.9	72.1	11.15	2.36	13.51	0.00
25	0.30	79.04	9.83	76.5	9.5	86.1	1.80	2.82	4.62	0.00



Profit Margins Using Co-Measures of Risk

		Tot Und		Tot Inv		Leverage Ratios					
		<b>183.74</b>		<b>(8.97)</b>		<b>4.00 0.2115</b>		Exhibit 2-2			
		Avg ROE	15.00%	15.00%	15.00%	15.00%	15.00%	Surplus	Non-Cat	Cat	
Avg ROE	15.00%	Avg ROE	15.00%	15.00%	15.00%	15.00%	15.00%	Surplus	14.53	160.24	
Surplus II	3.28%	Surplus II	3.28%	3.28%	3.28%	3.28%	3.28%	Yield	3.28%	3.28%	
Avg Op Rtn	11.72%	Avg Op Rtn	11.72%	11.72%	11.72%	11.72%	11.72%	Op Income	1.70	18.79	
<b>Surplus</b>	<b>174.77</b>	<b>Surplus</b>	<b>15.28</b>	<b>168.46</b>	<b>(0.75)</b>	<b>(8.22)</b>	<b>14.53</b>	<b>160.24</b>	Op Inv Inc	1.90	1.11
Risk Chg	20.49	Risk Chg	1.791	19.750	-0.087	-0.964	1.70	18.79	UW Income	(0.20)	17.68
		E(ZR)	5.331	-15.596	1.991	2.074	7.322	-13.522	Pre-tax Margin	(0.31)	27.19
		E(R)	7.122	4.154	1.903	1.110	9.025	5.265			
		Prem Split	70.957	41.391							
		<b>0.12280</b>									
	Total	Crude	Loss xCat	Cat Loss	Loss xCat	Cat Loss	Loss xCat	Cat Loss			
	Op Gain	Weights	Z	UW Gain	UW Gain	Inv Gain	Inv Gain	Op Gain	Op Gain		
1	31.4	0.050	0.407	8.647	14.152	1.829	0.622	10.475	14.774		
2	-12.2	1.050	8.550	9.075	-32.139	1.808	2.881	10.883	-29.258		
3	22.8	0.050	0.407	8.159	5.540	1.853	1.043	10.011	6.583		
4	33.2	0.050	0.407	14.034	10.643	1.566	0.794	15.600	11.437		
5	26.4	0.050	0.407	15.917	1.609	1.474	1.234	17.391	2.843		
6	21.4	0.050	0.407	1.095	11.085	2.197	0.772	3.292	11.857		
7	29.6	0.050	0.407	10.616	10.266	1.733	0.812	12.349	11.078		
8	23.5	0.050	0.407	10.776	3.705	1.725	1.132	12.501	4.837		
9	-1.7	1.050	8.550	-0.096	-11.966	2.255	1.897	2.160	-10.069		
10	37.2	0.050	0.407	8.733	20.139	1.825	0.330	10.558	20.469		
11	21.7	0.050	0.407	-3.225	15.740	2.408	0.545	-0.817	16.285		
12	12.2	0.050	0.407	3.905	-1.292	2.060	1.376	5.965	0.084		
13	25.7	0.050	0.407	8.236	8.537	1.849	0.896	10.085	9.434		
14	21.7	0.050	0.407	4.972	7.600	2.008	0.942	6.981	8.542		
15	-4.7	1.050	8.550	1.659	-16.908	2.170	2.138	3.829	-14.770		
16	23.2	0.050	0.407	4.045	10.033	2.053	0.823	6.099	10.857		
17	25.4	0.050	0.407	4.809	11.588	2.016	0.747	6.825	12.335		
18	-10.1	1.050	8.550	13.488	-34.412	1.593	2.992	15.080	-31.419		
19	41.3	0.050	0.407	16.087	17.056	1.466	0.481	17.553	17.536		
20	2.2	0.050	0.407	1.483	-9.427	2.178	1.773	3.662	-7.654		
21	43.0	0.050	0.407	21.096	13.839	1.221	0.638	22.317	14.477		
22	6.4	0.050	0.407	-0.500	-2.994	2.275	1.459	1.775	-1.535		
23	21.7	0.050	0.407	9.263	3.252	1.799	1.154	11.062	4.407		

Profit Margins Using Co-Measures of Risk

		Tot Und		Tot Inv		Leverage Ratios		Exhibit 3A-2	
			<b>356.67</b>		<b>(17.41)</b>	<b>9.24</b>	<b>0.2036</b>	Non-Cat	Cat
Avg ROE	15.00%	Avg ROE	15.00%	15.00%	15.00%	15.00%	15.00%	Surplus	6.28 332.98
Surplus II	3.28%	Surplus II	3.28%	3.28%	3.28%	3.28%	3.28%	Yield	3.28% 3.28%
Avg Op Rtn	11.72%	Avg Op Rtn	11.72%	11.72%	11.72%	11.72%	11.72%	Op Income	0.74 39.04
<b>Surplus</b>	<b>339.27</b>	<b>Surplus</b>	<b>6.61 350.07</b>	<b>(0.32) (17.08)</b>	<b>6.28 332.98</b>	<b>6.28 332.98</b>	<b>6.28 332.98</b>	Op Inv Inc	1.90 2.22
Risk Chg	39.78	Risk Chg	0.775 41.042	-0.038 -2.003	0.74 39.04	0.74 39.04	0.74 39.04	UW Income	(1.17) 36.82
		E(ZR)	9.450 -29.113	1.941 4.223	11.391 -24.890	11.391 -24.890	11.391 -24.890	Pre-tax Margin	(1.79) 56.64
		E(R)	10.224 11.928	1.903 2.220	12.127 14.149	12.127 14.149	12.127 14.149		
		Prem Split	75.729 88.351						
		<b>0.12190</b>							
	Total	Crude	LxC	Cat Loss	Loss xCat	Cat Loss	Loss xCat	Cat Loss	
	Op Gain	Weights	Z UW Gain	UW Gain	Inv Gain	Inv Gain	Op Gain	Op Gain	
1	60.2	0.050	0.410 11.749	31.923	1.829	1.245	13.578	33.167	
2	-27.4	1.050	8.614 12.178	-60.659	1.808	5.763	13.986	-54.896	
3	43.4	0.050	0.410 11.261	14.700	1.853	2.085	13.114	16.785	
4	58.7	0.050	0.410 17.136	24.906	1.566	1.587	18.702	26.493	
5	43.3	0.050	0.410 19.020	6.837	1.474	2.469	20.494	9.306	
6	47.2	0.050	0.410 4.197	25.790	2.197	1.544	6.395	27.334	
7	54.7	0.050	0.410 13.719	24.151	1.733	1.624	15.451	25.775	
8	42.4	0.050	0.410 13.879	11.029	1.725	2.264	15.603	13.293	
9	2.2	0.050	0.410 3.007	-20.312	2.255	3.794	5.262	-16.518	
10	71.7	0.050	0.410 11.835	43.897	1.825	0.660	13.660	44.557	
11	52.0	0.050	0.410 -0.122	35.100	2.408	1.090	2.286	36.189	
12	26.4	0.050	0.410 7.007	1.034	2.060	2.752	9.068	3.787	
13	49.2	0.050	0.410 11.339	20.694	1.849	1.793	13.188	22.487	
14	44.3	0.050	0.410 8.075	18.820	2.008	1.884	10.083	20.704	
15	-5.5	1.050	8.614 4.761	-30.197	2.170	4.276	6.931	-25.920	
16	48.0	0.050	0.410 7.148	23.686	2.053	1.647	9.201	25.333	
17	51.7	0.050	0.410 7.911	26.795	2.016	1.495	9.928	28.290	
18	-27.5	1.050	8.614 16.590	-65.204	1.593	5.985	18.183	-59.219	
19	72.8	0.050	0.410 19.190	37.731	1.466	0.961	20.656	38.692	
20	8.6	0.050	0.410 4.585	-15.235	2.178	3.546	6.764	-11.689	
21	71.5	0.050	0.410 24.198	31.298	1.221	1.275	25.420	32.573	
22	18.9	0.050	0.410 2.603	-2.368	2.275	2.918	4.878	0.550	
23	40.1	0.050	0.410 12.365	10.124	1.799	2.309	14.164	12.433	

Profit Margins Using Co-Measures of Risk

		Tot Und		Tot Inv		Leverage Ratios		Exhibit 3B-2			
		<b>104.49</b>		<b>(5.10)</b>		<b>1.72 0.2586</b>		Non-Cat	Cat		
Avg ROE	15.00%	Avg ROE	15.00%	15.00%	15.00%	15.00%	15.00%	Surplus	33.87	65.51	
Surplus II	3.28%	Surplus II	3.28%	3.28%	3.28%	3.28%	3.28%	Yield	3.28%	3.28%	
Avg Op Rtn	11.72%	Avg Op Rtn	11.72%	11.72%	11.72%	11.72%	11.72%	Op Income	3.97	7.68	
<b>Surplus</b>	<b>99.39</b>	<b>Surplus</b>	<b>35.61</b>	<b>68.87</b>	<b>(1.74)</b>	<b>(3.36)</b>	<b>33.87</b>	<b>65.51</b>	Op Inv Inc	1.90	0.56
Risk Chg	11.65	Risk Chg	4.175	8.075	-0.204	-0.394	3.97	7.68	UW Income	2.07	7.13
		E(ZR)	0.549	-6.697	2.107	0.949	2.656	-5.748	Pre-tax Margin	3.18	10.96
		E(R)	4.724	1.378	1.903	0.555	6.627	1.933			
		Prem Split	67.267	19.620							
		<b>0.13420</b>									
	Total	Crude	LxC	Cat Loss	Loss xCat	Cat Loss	Loss xCat	Cat Loss			
	Op Gain	Weights	Z	UW Gain	Inv Gain	Inv Gain	Op Gain	Op Gain			
1	17.9	0.050	0.373	6.249	6.376	1.829	0.311	8.077	6.688		
2	-3.8	1.050	7.824	6.677	-16.769	1.808	1.441	8.485	-15.328		
3	13.3	0.050	0.373	5.761	2.071	1.853	0.521	7.613	2.592		
4	21.3	0.050	0.373	11.636	4.622	1.566	0.397	13.202	5.019		
5	18.8	0.050	0.373	13.519	0.105	1.474	0.617	14.993	0.722		
6	9.2	0.050	0.373	-1.303	4.843	2.197	0.386	0.894	5.229		
7	17.9	0.050	0.373	8.218	4.434	1.733	0.406	9.951	4.840		
8	14.9	0.050	0.373	8.378	1.153	1.725	0.566	10.103	1.719		
9	-2.9	1.050	7.824	-2.494	-6.682	2.255	0.948	-0.238	-5.734		
10	20.8	0.050	0.373	6.335	9.370	1.825	0.165	8.160	9.535		
11	7.3	0.050	0.373	-5.623	7.171	2.408	0.272	-3.215	7.443		
12	6.0	0.050	0.373	1.507	-1.346	2.060	0.688	3.567	-0.658		
13	14.8	0.050	0.373	5.838	3.569	1.849	0.448	7.687	4.017		
14	11.2	0.050	0.373	2.574	3.101	2.008	0.471	4.583	3.572		
15	-3.6	1.050	7.824	-0.739	-9.153	2.170	1.069	1.431	-8.084		
16	11.5	0.050	0.373	1.647	4.317	2.053	0.412	3.701	4.729		
17	13.0	0.050	0.373	2.411	5.094	2.016	0.374	4.427	5.468		
18	-0.6	1.050	7.824	11.090	-17.905	1.593	1.496	12.682	-16.409		
19	26.3	0.050	0.373	13.689	7.828	1.466	0.240	15.155	8.069		
20	-0.2	1.050	7.824	-0.915	-5.413	2.178	0.887	1.264	-4.527		
21	29.6	0.050	0.373	18.698	6.220	1.221	0.319	19.919	6.539		
22	1.0	0.050	0.373	-2.898	-2.196	2.275	0.730	-0.623	-1.467		
23	13.3	0.050	0.373	6.865	0.927	1.799	0.577	8.664	1.504		

*Profit Margins Using Co-Measures of Risk*

**Exhibit 4**

	<b>Exh. 2</b>	<b>Exhibit 3A</b>			<b>Exhibit 3B</b>		
	<b>Base</b>	<b>Estimate</b>	<b>Actual</b>	<b>Difference</b>	<b>Estimate</b>	<b>Actual</b>	<b>Difference</b>
Surplus	174.77	335.01	339.27	-1.3%	94.65	99.39	-4.8%
Yield	3.28%	3.28%	3.28%		3.28%	3.28%	
Op Income	20.49	39.28	39.78	-1.3%	11.10	11.65	-4.8%
OP Inv Inc	3.01	4.12	4.12	0.0%	2.46	2.46	0.0%
UW Income	17.48	35.15	35.65	-1.4%	8.64	9.19	-6.0%
Pre-tax Margin	26.89	54.08	54.85		13.29	14.14	
ROE w/estimate		14.85%			14.44%		