Revenue Management & Insurance Cycle

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Abstract: This paper investigates how an insurer's pricing strategy can be adapted to respond to market conditions, and in particular the insurance cycle. For this purpose, we explore the use of dynamic pricing strategies, such as the revenue management techniques used by other industries (e.g., airlines, car rentals, internet service providers) in an insurance context. We then compare these dynamic pricing techniques with the static ones currently used in the market, and demonstrate that they can prove very valuable to insurers looking to enhance their competitive strategy.

Keywords: Cycle management, dynamic pricing, profit optimization, revenue management, dynamic programming

1. INTRODUCTION

This paper is a reflection on the optimal strategy for deploying a fixed amount of insurance capacity over a period of time. In particular, we consider the following questions: Which pricing strategy maximizes the expected profits? Should it be based on market conditions or shareholders' expectations? Should it be static or dynamic? How does an insurer manage the insurance cycle? Can we expect to make a profit when market returns are negative?

To respond, we introduce the theory of revenue management, which integrates market conditions and fluctuations in demand into the decision-making process. We use this framework to develop an optimal pricing strategy and demonstrate how it can be a valuable tool to manage the insurance cycle. In order to exemplify our case in point, we can think of an insurer with a surplus, denoted S, of $ 1 billion ($US) and a capacity constraint driven by a 5:1 maximum written premiums-to-surplus ratio imposed by its regulator.

As a result, this insurer has a capital allocation of 20% of premiums written and it prices each policy based on a 15% charge on allocated capital, 15% being the target return on equity promised to its shareholders.[1]

We have therefore the following pricing formula:

$$PV[P_i] = PV[E(L_i) + E_i + r * K_i] = PV\left[\frac{(E(L_i) + E_i)}{(1 - 15\% * 20\%)}\right] \tag{1.1}$$

with

- P_i: price-bid[2] for policy i

[1] We assume in this example that the shareholders' equity is equal to the company's surplus.

[2] I.e., the minimum acceptable premium for policy i

- L_i: losses for policy i
- E_i: expenses for policy i
- K_i: capital allocated to policy i, here 20% * P_i
- r: required return on allocated capital (a.k.a. capacity charge)

As we can see, the insurer has a limited capacity for the underwriting year and each policy written "consumes" some of it.

This example reflects a fairly common approach to pricing and charge for capacity, and the object of our analysis is to explore the following questions: Does this pricing strategy maximize the expected profits? If not, what would be the best alternative?

2. BACKGROUND

2.1 Revenue Management

Revenue management techniques first appeared in the early 1980s in the airlines industry and have since been introduced progressively in other industries (e.g., hotels, car rentals, internet service providers, and others). Their objective is simple: maximizing the profits from a fixed supply of perishable goods and services over a period of time.

For instance, airlines use sophisticated revenue management systems based on historical booking patterns to estimate the likelihood of an empty seat at departure. They need to balance the risk of not selling that seat, with the opportunity cost of passing up a "premium customer" willing to pay a higher price. "If a plane is not filling up as rapidly as historically expected, the probability of an empty seat goes up and the opportunity cost of selling more discounted seats goes down, so the airline's management system may offer some tickets at an exceptionally low price. If, however, a group of seven business people suddenly books onto the flight, the probability of filling the flight jumps substantially, the opportunity cost goes up, and the airline's management system blocks additional sales of the cheapest tickets." [5]

2.2 Insurance Applications

From a practical viewpoint, revenue management techniques require the market to offer full flexibility in price setting. In an insurance context, this excludes lines of business where rates are subject to a tariff or to filing/approval by the regulator. There are, however, many insurance markets where rates are set freely and can be changed frequently by the market participants (e.g., excess & surplus lines, commercial lines, reinsurance, and personal lines in most European countries).

For those insurance markets with flexibility in price setting, it is fairly easy to see how these techniques can be applied:

- insurers have a fixed supply of insurance capacity over a period of time (more accurately, capacity can be increased or decreased at times but it is fixed in between these events).
- insurance capacity is perishable, in the sense that unused capital for an underwriting year can not be transferred to the next.[3]

While revenue management can take several forms,[4] the framework we present in this paper is purely price driven: we seek to set r over time so that it maximizes the expected profit based on market conditions and expected demand. The required return on allocated capital becomes a stochastic process r(t) and the pricing formula becomes:

$$PV\left[P_i(t)\right] = PV\left[E\left(L_i\right) + E_i + r(t) * K_i\right] \qquad (2.1)$$

We call pricing strategy a path r for r(t) over the underwriting period [0, T], r $=\{r(t), t \in [0, T]\}$. Our objective is to determine r^* which maximizes the expected profit process $\Pi^*(t, s_t)$[5].

We can describe the expected profit as the expected value of 1) the capacity sold (K) multiplied by 2) the price charged for that capacity (r), over the time period until all the capacity is exhausted. If we use exponential discounting for converting these cash flows to present value, we obtain the following formula for $\Pi^*(t, s_t)$:

$$\Pi^*(t, s_t) = \sup_r \Pi(t, s_t, r_t) = \sup_r E[\int_t^{\tau_{r,t,st}} e^{-\rho u} r(u) K(u, r(u))) du] \qquad (2.2)$$

with

- K(t, r(t)): capacity demand at time t for a given r(t),
- s_t: remaining capacity inventory at time t,
- $\tau_{r,t,st}$: time when the all the capacity is exhausted, and
- ϱ: discount rate.

Although our introductory example assumes a capital allocation based on premium writings over an underwriting year, our framework is more general and encompasses different capital allocation

[3] We work under the assumption that capital usage is triggered by underwriting decisions; it would be fairly easy to integrate other sources of capital usage, such as running-off of existing policies, by only considering the capacity available for writing new policies.

[4] E.g., managing the release of capacity between classes of customers, such as business vs. economy travellers.

[5] On a present value basis; note that if Π is not independent over time (e.g., Markovian processes), the expected value becomes conditional on history h_t; and we have $\Pi(t, s_t, h_t)$.

approaches (e.g., profit margin, rating agency or regulatory formulas, risk-based formulas, and so forth), time periods and definition of capacity (e.g., capital, resources).

We choose to use the required return on allocated capital r^* as our optimising variable, because prices are usually easier to adjust than capacity. It should be noted, however, that a similar revenue management framework could be derived to optimize capacity S^* for a given pricing policy r.

2.3 Insurance Cycle

Most insurers face fluctuations in demand over time, resulting from

- fluctuations in the flow of business shown to the insurer (e.g., changes in marketing/distribution strategy), and
- the insurance cycle: progressive or abrupt shifts in market "supply and demand" conditions, resulting in shifts in the insurer's demand function.

Our revenue management framework provides a tool to adapt to these fluctuations:

- it integrates expectations for market conditions (i.e., evolution of the insurance cycle), and
- it can be re-parameterized dynamically in light of the latest information on actual capacity usage and demand expectations; for instance, an insurer could decide to review its strategy and retune its revenue management model on a monthly basis.

An insurer can therefore manage the ups and downs of the cycle by adjusting its capacity charges so that its expected profits are maximized.

3. MODELING FRAMEWORK

We use the theory of revenue management to contend with our optimization problem: to "maximize the expected profits under the constraints of the capacity demand and capacity inventory processes."

In this section, we detail these two processes, formulate the optimization problem, and present methods to derive its optima.

3.1 Capacity Demand

The demand for the insurer's capacity $K(t, r(t))$ can be analysed in two parts: the business flow shown to and quoted by the insurer $N(t)$, and the demand function $d(t, r(t))$ which reflects the acceptance level of quotes by prospects.

The demand for capacity at time t for a given r(t) is therefore K(t, r(t)) = N(t) d(t, r(t)).

Business Flow

The business flow is the flow of requests for the insurer's capacity, i.e., demand for quotes. It is modelled by a stochastic process N(t) which varies over time according to

- the overall demand for the insurance products sold by the insurer,
- the effectiveness of the marketing and distribution network, and
- seasonal fluctuations (e.g. large renewal months).

N(t) is typically modelled with Poisson $\lambda(t)$, Mixed Poisson $\Lambda(t)$ (e.g., Negative Binomial), or Geometric Brownian (μ_t, σ_t) processes. The model formulation and estimation can be derived from historical observations, after allowing for anticipated trends and future changes in the business flow process. In practice, this calibration exercise yields more robust results when the volumes of business are large and the business flow is stable over time. For instance, a personal lines insurer quoting thousands of policies daily would be expected to have a better assessment of its business flow than a reinsurer quoting a handful of treaties each day.

Demand Function

The demand function d(t, r(t)) reflects the price-elasticity relationship between the level of required return r(t) and the quantity of capacity sold at that level. It can be described as the probability distribution for the market reservation price, which is the highest price at which a prospect is willing to accept a quote.

The demand function depends on

- the competitive forces in the market place, determined by supply and demand, and
- the prospects' utility function.

Commonly used families of demand functions are

- exponential survival functions $d(t, r(t)) = e^{-r(t)/v(t)}$, and other Weibull survival functions,

- Normal survival functions d(t, r(t))= 1-$\Phi_{\mu,\sigma}$(r(t)),

- iso-elastic functions d(t, r(t)) = $(1+r(t))^{-\nu(t)}$, and

- perfectly elastic functions, representing a single market clearing price.

The form and parameters for the demand function can be inferred from empirical observations of "hit ratios" and/or using the quotations systems available in some markets, such as UK Motor.

3.2 Capacity Inventory

Starting with a capacity of S, the capacity inventory process is defined as:

$$s(t) = S - \int_0^{\min(t,\tau_{r,t,st})} K(u, r(u))du \quad \text{and} \quad \frac{ds(t)}{dt} = -K(t, r(t)) \tag{3.1}$$

The capacity inventory is exhausted at a time $\tau_{r,t,st}$, at which point the demand process is turned off.

3.3 Optimal Pricing Strategy

Optimization Problem

As noted in the introduction section, our optimization problem is finding the pricing strategy r^* which maximizes the expected profits process $\Pi^*(t, s_t)$. This is summarized in Equation (3.2):

$$\Pi^*(t, s_t) = \sup_r E[\int_t^{\tau_{r,t,st}} e^{-\rho u} r(u) K(u, r(u))) du] \tag{3.2}$$

We will limit our range for r(t) to [0,+∞] as a negative required return strategy of selling below the expected marginal cost is always strictly dominated by abstaining from selling capacity. We note that, in practice, there may be instances where a negative required return strategy may be justified. For example, it may be more expensive to attract new clients when the market turns than it is to keep the current insureds at a loss.

Dynamic Programming

Dynamic programming is concerned with dynamic systems and their optimization over time, and we can use some of its classical results to find our optimal pricing strategy. Our optimization problem is an example of dynamic programming, with s(t) as the state variable, r(t) as the control variable and $\Pi(t, s_t)$ as the value function.

The key idea in dynamic programming is the Principle of Optimality is "An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision" [1].

This principle translates into the following recursive equation, known as the Optimality or Bellman Equation:

$$\forall \theta \in [t,T], \quad \Pi^*(t,s_t) = \sup_r \left[E[\int_t^\theta e^{-\rho t} r(u) K(u,r(u)))\, du] + e^{-\rho(\theta-t)} \Pi^*(\theta, s_\theta) \right] \tag{3.3}$$

Closed-form solutions have been derived for particular formulations of N(t) and d(t, r(t)). For instance,

- Gallego and van Ryzin [5]: time-invariant Poisson business flow with exponential demand functions,
- Zhao and Zheng [14] for time-variant Poisson business flow with iso-elastic demand functions, and
- Xu and Hopp [12] for Geometric Brownian business flow with iso-elastic demand functions.

But these solutions correspond only to a limited number of practical applications, and numerical solutions provide a more flexible alternative.

Numerical Solutions: Backward Recursion Algorithm

We compute our numerical solutions to the "discretized" optimization problem using the backward recursion algorithm. This approach consists in:

1. solving $\Pi^*(T, s_T)$ for each possible value of s_T,
2. solving $\Pi^*(T\text{-}1, s_{T-1})$ using the values computed for $\Pi^*(T, s_T)$: the principle of optimality states that the solution $r^*(T\text{-}1)$ for $\Pi^*(T\text{-}1, s_{T-1})$ will also maximize $\Pi^*(T, s_{T-1}\text{-}K(r^*(T\text{-}1), d(T\text{-}1, r^*(T\text{-}1))))$, and
3. solving $\Pi^*(t, s_t)$ for t=1…T-2 using the same iterative process.

The advantage of the backward recursion approach is its computational efficiency, resulting from the principle of optimality.

4. COMPARATIVE ANALYSIS

In this section, we apply the revenue management approach to a simple but realistic case study and compare the performance of different pricing strategies: one strategy based on shareholders' expectations, one strategy based on market returns, and three revenue management strategies[6] -- the first one static constant for the year, the second static but variable for each month (i.e., the pricing strategy for each month is set at the beginning of the year based on the initial anticipations), and the third one dynamic reparameterized each month (i.e., the pricing strategy is revised dynamically every month based on the revised anticipations for the rest of the year).

4.1 Case Study Scenario

The assumptions of our case study are as follows:

- Insurer:

  → Mono-line insurer

  → Capacity constraint based on underwriting decisions (the actual capital allocation formula is not relevant)

  → Shareholders' expectations: 15% return on equity

- Capacity:

  → Fixed capacity of $ 1billion

  → Capacity is sold by blocks of $ 1million.

- Time period:

  → One underwriting year with 12 monthly periods

- Business flow:

  → Business flow process is Negative Binomial ($450 million, 0.2) with an expected value of $1.8 billion and standard deviation of $95 million. The simulations are plotted in Graph 1.

  → Monthly business flows follow a seasonal pattern (cf. Graph 2), with each month simulated as a Negative Binomial variable

- Demand function:

[6] We call revenue management strategy any optimal strategy derived from the revenue management framework (i.e., integrating market conditions and expected demand); we get different "optimal strategies" depending on the context of the optimization problem. For instance:

- We can get the optimal fixed constant charge for the year, or allow the charge to vary monthly.
- We can get a static or dynamic strategy: a static strategy is set at the beginning of the period and remains unchanged, whereas a dynamic strategy is reset periodically using the latest information available.

→ Survival function of a Normal(μ(t), 3.5%) (cf. graph [3] with μ=5%); the Normal function has the advantage of being symmetrical and allowing negative capacity charges.

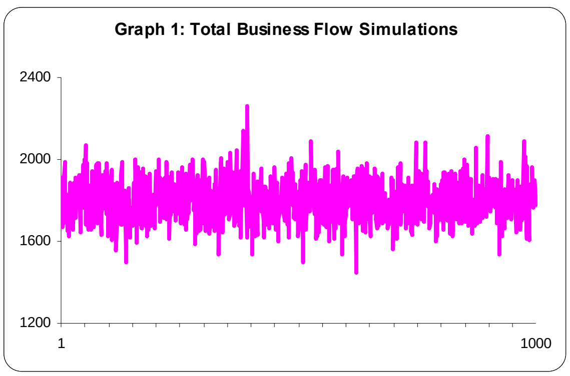
→ μ(t) is the average market reservation price; it can be interpreted as the market return in month t.

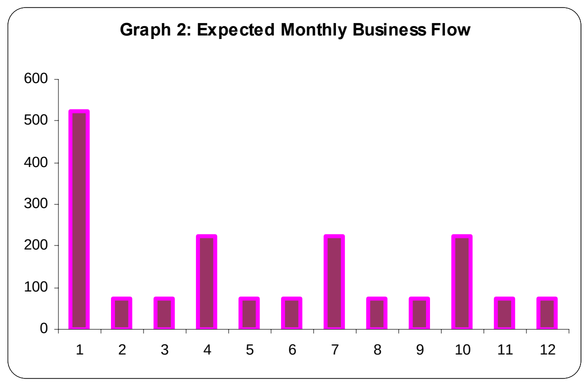
- Market conditions:

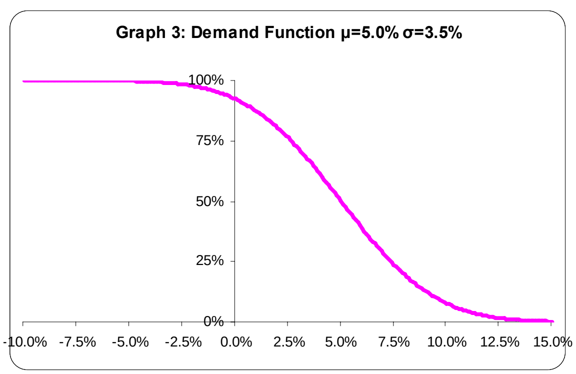
→ The market return μ(t) is decreasing linearly from 10% capacity charge in month 1 to 5% in month 12 (cf. graph [4]).[7]

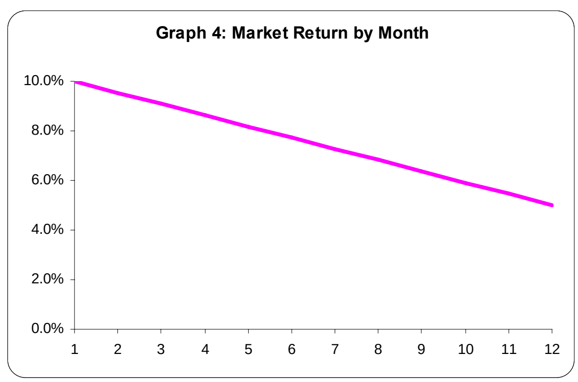
- Discount rate:

→ 5% per annum constant over the year.









4.2 Alternative Strategies

To respond to the questions specified in the introduction, we have compared the performance of the following five pricing strategies:

[7] This assumption has been made to gauge the responsiveness of each strategy to changes in market conditions. An annual drop of 5% in market returns is not inconsistent with the changes observed in financial analysts forecasts at certain stages of a softening market.

- **Strategy 1:** "Charge 15% return for the year."

  → A fixed charge r for the year, based on the ***target return to the shareholders*** of 15%.

  → r= 15% for t=1 to 12.

- **Strategy 2:** "Charge the market return each month."

  → A variable charge r(t) based on the ***anticipated market conditions*** for each month.

  → r(t)= μ(t) for t=1 to 12

- **Strategy 3:** "Charge the demand-driven price for the year."

  → A fixed charge r based on the anticipated market conditions and the expected demand for the year.

  → r is determined using ***static revenue management for the year***.

  → Market conditions are determined by the weighted average μ(t) for year.

  → Expected demand is determined by the expected total business flow of $ 1.8 billion and by the insurer's demand function.

- **Strategy 4:** "Charge the demand-driven price each month."

  → A variable charge r(t) based on the anticipated market conditions and the expected demand for each month.

  → r(t) is determined using ***static revenue management for each month***.

  → Market conditions are determined by μ(t) t=1 to 12.

  → Expected demand is determined by the expected business flow N(t) for each month and by the insurer's demand function.

- **Strategy 5:** "Charge the re-forecast demand-driven price each month."

  → A variable charge r(t), recomputed at the end of each month based on

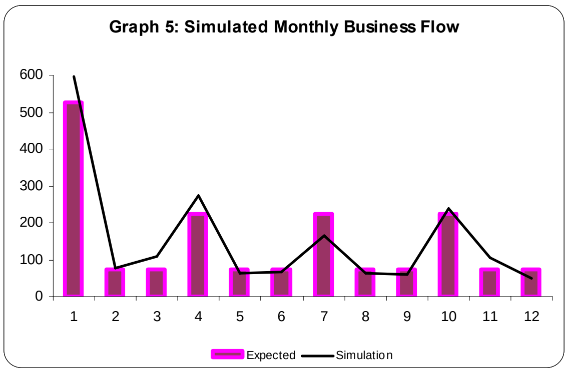
    1. actual writings to date and remaining capacity inventory, and
    2. anticipated market conditions and expected demand for rest of the year.

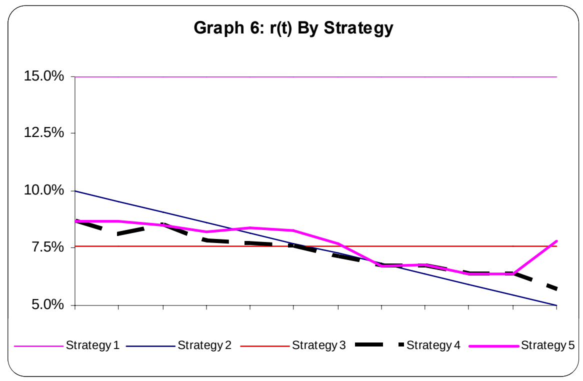
  → r(t) is determined using ***dynamic revenue management for each month***.

  → Market conditions are determined by μ(t) t=1 to 12.

→ Expected demand is determined by the expected business flow N(t) for each t=1 to 12 and by the insurer's demand function.

The behaviour of these 5 strategies is illustrated on the example detailed in Graphs 5 and 6. Graph 5 shows the simulated path for the business flow N(t); the total business flow is $1.869 million. Graph 6 plots the values of r(t) under the five strategies.





- **Strategy 1** is a flat r(t)=15% for the year.
- **Strategy 2** is a linear decrease in r(t) from 10% down to 5%, reflecting the evolution of μ(t) over the months.
- **Strategy 3** is a flat 7.6% for the year; 7.6% being the revenue management optimum for the year based on the weighted average μ(t) for year (which is 8.1%).
- **Strategy 4** is the revenue management optimum strategy based on the initial expectations for N(t).
- **Strategy 5** is also the revenue management optimum but reparameterized at time t based on the remaining capacity inventory (we assumed that the anticipations for the demand functions, business flow and market conditions are not changed over the year).

We can note that **Strategy 5** suggests higher r(t) than **Strategy 4**: this results from the higher than expected business flow, which translates into a lower capacity inventory sold at a higher price.

4.3 Results

Table 1 compares the results for the 5 strategies over 1,000 simulations. For each simulation, the business flow N(t) is the only stochastic variable as we have assumed that the demand function was deterministic.

Table 1	**Strategy 1**	**Strategy 2**	**Strategy 3**	**Strategy 4**	**Strategy 5**
$\bar{r}$	15.0%	8.1%	7.6%	7.6%	7.6%
K $m	65	920	973	977	995
Π $m	9.7	71.6	73.0	74.8	75.5

with:

- the average required return $\bar{r} = \left(\sum_{t=1}^{12} r(t)N(t)\right) / \left(\sum_{t=1}^{12} N(t)\right)$
- the total capacity sold $K = \sum_{t=1}^{12} K(t, d(t, r(t)))$
- the present value profit $\Pi = \sum_{t=1}^{12} \Pi(t, r(t))$

We note first that none of the strategies achieves the market return of 8.1% (i.e. weighted average μ(t)). This is due to the fact that the expected business flow of $ 1.8 billion is low in relation to the $ 1.0 billion capacity and the 5:1 premium-to-surplus permitted; the company has to provide a discount on the market return to sell more and maximize its expected profits.

Comparing the different strategies based on total profit, we can observe that:

- **Strategies 2-5** based on market conditions are superior to Strategy 1, which is based on shareholders' expectations over the cycle.
- **Strategies 3-5** based on market conditions and expected demand are superior to Strategy 2, which only integrates market conditions.
- **Strategies 4-5** are superior to Strategy 3, as they are refined to include the monthly patterns in capacity demand and market conditions.
- **Strategy 5** is superior to Strategy 4, because capacity charges are set dynamically to incorporate the latest capacity inventory information.

As could have been expected intuitively, the optimal pricing strategy is the dynamic revenue management approach.

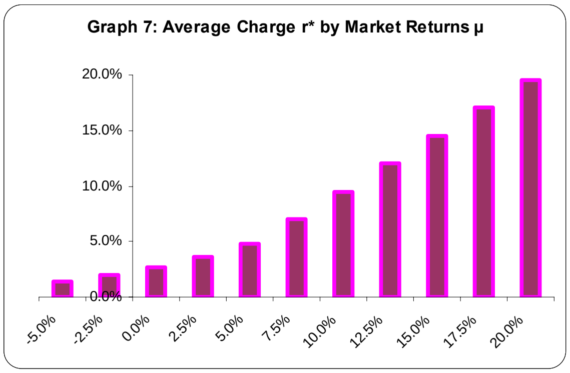
5. INSURANCE CYCLE APPLICATIONS

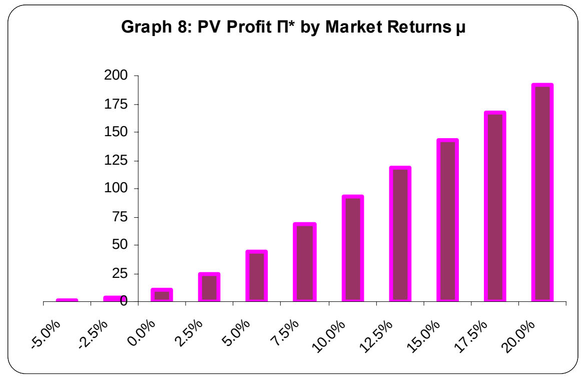
In this section, we detail practical applications of the revenue management approach to the management of the insurance cycle.

5.1 Optimal Pricing Strategy

We can use our model to investigate the optimal pricing strategy for the different stages of the insurance cycle. For this purpose, we have computed the optimal pricing strategy for various level of $\mu(t)$, kept constant for the year.[8]

Graph 7 plots the average required return r^* and Graph 8 the PV Profit Π^* in millions of dollars for different level of market returns $\mu(t)$. They illustrate how an insurer can adapt to the different market conditions over the insurance cycle, in order to maximize its expected profits.





We can observe that the optimal approach to negative market returns consists in setting r so that it captures and maximizes the returns on the few accounts with a positive return. In practice this means a low but positive capacity charge.

5.2 Optimal Capacity Strategy

As discussed in the introduction, a revenue management framework similar to the one presented can be utilized to optimize the insurer's amount of capacity to achieve a target return on equity for its shareholders over the cycle.[9] For this purpose, we have computed the optimal capacity strategy in order to achieve a 15% return on equity for various level of $\mu(t)$, kept constant for the year.[10]

Graph 9 plots the capacity S^* and Graph 10 the PV Profit Π^* in millions of dollars for different level of market returns $\mu(t)$.

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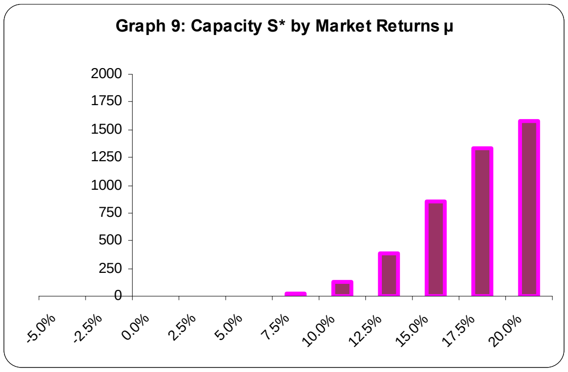
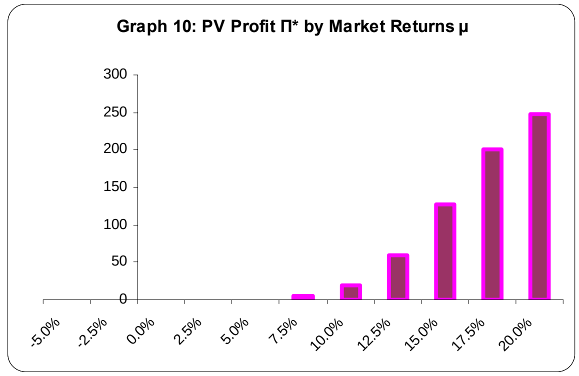
[8] all the other parameters as in the case study.
[9] all the other parameters as in the case study.
[10] all the other parameters as in the case study.





These graphs illustrate how an insurer can adapt to the different market conditions over the insurance cycle, in order to meet a target return on equity for its shareholders.

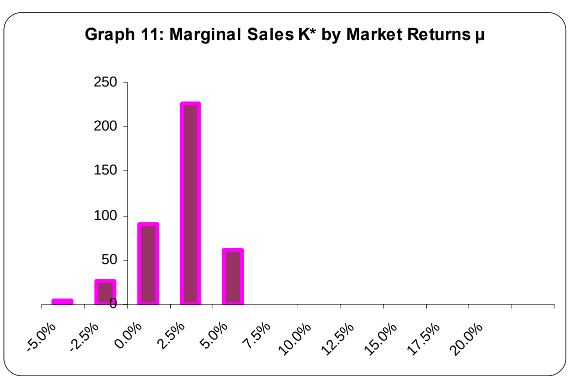
Although adjusting capacity is more problematic than adjusting capacity charges, one could envisage that this could be partially achieved through a flexible reinsurance programme and/or a proactive capital management policy (e.g. dividends, buybacks, flexible debt/equity arrangements…).

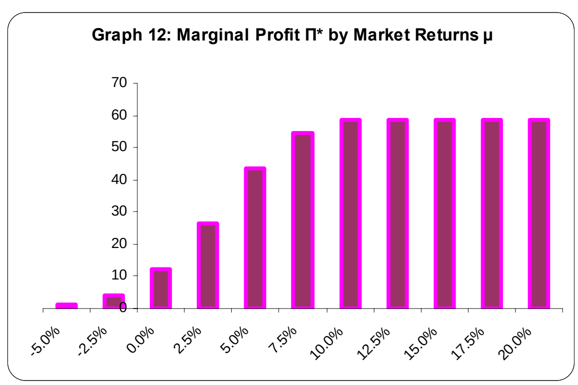
We can remark that the adjustments required to achieve the 15% target return on equity are fairly dramatic; and it becomes impossible for the insurer to achieve a 15% return on equity when the market returns are lower or equal to 5%.

5.3 Strategic Marketing Decision

We can also use our revenue management framework to assess the outcome of strategic decisions. For instance, we can compare the impact of a marketing campaign to increase business flow by 25% at different times in the insurance cycle.[11]

Graph 11 plots the marginal increase in sales and Graph 12 shows the marginal benefit of the campaign in millions of dollars for different levels of market returns μ(t).





[11] All the other parameters as in the case study.

We can note that the improvement in sales resulting from the increased business flow is the most significant when the market returns are between 0.0% and 5.0%, and nil above that level because the inventory would have been entirely sold without the marketing efforts.

The marginal profit, however, is most impacted for market returns greater than 5.0%, as the insurer is able to sell all its capacity and attract a higher return on it.

6. CONCLUSION

Our investigation has provided very insightful results, which challenge some of the current pricing practices. For an insurer deploying a fixed amount of insurance capacity over a period of time, we constructed revenue management strategies based on market conditions and expected demand, and observed that:

- *These strategies were superior to other strategies based on the target return to shareholders or market conditions alone.* As a result:

  → Companies should vary their capacity charge over time, as market conditions change.

  → Multi-line companies should adopt specific capacity charges for each business segment.

  → Pricing analyses should not be done independently of market conditions and expected demand; on the contrary, intelligence and research in these fields should be a key part of the pricing strategy.

- *Dynamic strategies delivered better results than static ones:*

  → Integrating anticipations of future market conditions helps maximize the return on a limited insurance capacity by ensuring that it is sold at the best rates.

  → Regular reparameterization helps integrate the latest information on capacity inventory and adjust the strategy accordingly.

- *An insurer can maximize its expected profits over the insurance cycle by adapting its capacity charge to market conditions, and expect a profit even when market returns are negative.*

- *Alternatively, this insurer can target a return on equity to its shareholders and adjust its capacity accordingly.*

To derive these conclusions we have used a revenue management framework, similar to those developed in other industries (e.g., airlines, hotels…). In these industries, revenue management is an essential piece of the pricing strategy. This framework proved very valuable and practical, and we are expecting that insurers will start implementing these techniques to enhance their competitive strategy.

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