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Including the 2009 Ratemaking Call Papers

The 2009 CAS Ratemaking Call Papers

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The Winter 2009 Edition of the CAS *E-Forum* is a cooperative effort between the Committee for the CAS *E-Forum* and the Committee on Ratemaking.

The CAS Committee on Ratemaking presents for discussion twelve papers prepared in response to their 2009 Call for Papers.

This *E-Forum* includes papers that will be discussed by the authors at the 2009 Ratemaking and Product Management Seminar March 9-11, 2009, in Las Vegas, Nevada.

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Measuring Rate Change

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Abstract

Motivation. Calculated rate changes can substantially affect loss ratio forecasts and thus are critical parameters for ratemaking. However, current methods are not well suited to a changing book of business.

Method. The analysis first explores the conceptual underpinnings of rate change and then applies the conclusions of this analysis to several practical problems.

Results. The proposed approach shows improved accuracy as compared to other methods, with particular significance for a nonstatic book of business.

Conclusions. I conclude that "rate change" measures the change in premium *relative to loss potential*. One can then apply this conceptual formulation in order to solve several problems that one confronts in practice: how to adjust for shifts in limits and deductibles, how to blend together changes in exposures when the portfolio uses several different exposure bases, and how to properly weight together granular measures of rate change (e.g., for each policy, subline, etc.) into an overall rate change for the entire portfolio.

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Keywords. Rate change, rate change factors, on-level adjustments, adjusted premium, exposure bases.

1. INTRODUCTION

In theory, measuring rate change[1] ought to be straightforward: using the company's "manual," one can simply find the rates in effect during one time period and compare them to rates in effect during another period. Or, similarly, one can track over time the rate changes the company achieves through its periodic rate filings. In practice, however, measuring rate change is not this simple, for a variety of reasons. Some of these reasons are:

1. Some policies, such as "excess" policies (including "umbrella"), attach above an underlying policy. Rates for such policies often derive from the premium charged for the underlying policy, thus complicating the notion of a clearly defined rate for such business. Moreover, the factors used for excess policies often have a wide range of filed rates; the actual charged rate can vary quite significantly over time without any change to the rating plan.

2. More generally, the rating plans for commercial lines also incorporate a significant amount of underwriting judgment in the final rate that can be charged.[2] Therefore,

[1] In this paper, the terms "rate change" and "rate change factors" relate to the actual rate changes achieved by the company; they relate to the historical period and are descriptive. They do not refer to "indicated rate changes" or "required rate changes," which are both prospective and prescriptive.

[2] See Vaughn [5], pp. 498-502.

tracking the changes to the company's filed rates will provide an inaccurate picture of rate movements.

3. Even when dealing with rating plans that do not allow for judgmental rates, one can encounter other complications. For example, if one simply tracks over time the rate increases and decreases that a company files on any particular date, one may overlook the resulting shift in the company's mix of business.[3]

One approach to overcoming these problems is to discard the measuring of filed, manual rates and to focus instead on measuring changes in the premium the company actually charges. Under this approach, one matches each renewing policy to its corresponding expiring policy and measures the rate change for each policy.[4,5] Such an approach is often referred to as measuring "renewal rate change."

Measuring renewal rate change can introduce more granularity and precision to the measuring of rate change. Still, many questions persist, such as:

1. How do I account for changes to a policy's limit and deductible when measuring the renewal policy's rate change?
2. When I measure rate change for excess casualty policies, which cover auto liability and also general liability claims, how do I combine rate changes for these two sublines, which have different exposure bases? More generally, how do I combine any two sublines that have different exposure bases? Is it possible to obtain one overall number for "exposure change" when the sublines have different exposure bases?
3. When I measure rate changes for several different sublines or multiple individual policies, how do I weight them together to obtain one blended rate change factor for the overall portfolio?
4. When my firm implements rate increases and rate decreases for various classes of business, volume tends to grow in those classes that received rate decreases and volume

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[3] See McCarthy [2], who notes this problem and provides an alternative solution.

[4] New policies, by definition, must be excluded and measured separately; measuring rate change for new policies is outside the scope of this paper.

[5] When premium rates are not unique for each individual policy but do vary by subline, then one need not measure the rate change of each policy but rather each subline. In such a situation, the only "new" business that would need to be excluded would be a new subline of business that did not exist in the prior rating plan. In contradistinction, new individual policies within existing sublines would not need to be excluded as "new" business but rather should be included as exposure growth within existing sublines.

tends to decline in those classes that received rate increases. Thus, rate changes tend to generate additional shifts in the mix of business in the firm's portfolio; how do I properly reflect this shift when calculating rate change for the total book of business?

2. THE THEORY AND PURPOSE OF RATE CHANGE FACTORS

In order to answer these detailed questions, we need to first examine the fundamental principles underlying the theory of rate change. How should one calculate a company's rate change factors? The answer to this question depends upon the answer to the following question: for what purpose will we use these rate change factors?

In theory, rate change factors can be used for several different purposes. For example, one potential use of rate change factors is to enable management to better run the company. Under this approach, rate change factors indicate how the company is performing: they tell management where performance is improving and where it is slipping, thus allowing for better steering of the business and better implementation of strategy. If in fact this is the purpose of the rate change factors, then consider the dynamic situation in which policies currently issued by the company have higher deductibles than policies issued in the past. As the deductibles increase, the stable volume of losses in the deductible layer disappears and the company covers policies that have more variability, lower premium volume, and (because of fixed costs) higher expense ratios. Therefore, if the goal of the company is to understand the true nature of its performance, then traditional rate change factors, which ignore shifts in required risk load and shifts in expense ratios, will fall short of the desired goal. Rather, the company must implement an approach whereby each policy in the portfolio, accounting for risk load and fixed expenses, is priced to a target premium; then, the company can evaluate how the actual premium compares to the target premium and how this ratio of "actual to target" changes over time.

Table 1

	1	2	3	4	5	6	7 = (3+4+5) / (1-6)	8 = 3 / 7	9	10 = 3 / 9	11 = 9 / 7
	Limit	Deductible	Expected Loss	Target Risk Load	Fixed Expenses	Variable Expenses	Target Premium	ELR to Target Premium	Actual Premium	ELR to Actual Premium	Actual / Target
Expiring	2,000,000	1,000	7,601	1,383	1,000	15%	11,746	65%	12,500	61%	1.064
Renewing	2,000,000	100,000	3,045	1,133	1,000	15%	6,091	50%	5,900	52%	0.969

"Rate Adequacy Change" (Change in Ratio of Actual Premium to Target Premium) -9.0%

Table 1 shows an example in which the company's expected loss ratio (ELR) improves. By measuring the change in the ratio of Actual to Target, however, one can determine that rate adequacy has actually deteriorated. In a dynamic environment with changing policy provisions, only such an approach can give complete information to management about the company's "rate adequacy change."

Given that most rate change factors do not typically account for all the aspects of shifts in target risk load and shifts in expense ratios, the question persists: what good are rate change factors, for what purpose can we use them, and how does this affect how we ought to calculate them?

Traditional rate change factors therefore appear to be much more relevant to a second purpose: formulating a loss ratio projection for a book of business. Such a projection is often helpful for operational needs, such as estimating initial loss reserves, or for transactional purposes, such as effecting reinsurance treaties. In order to forecast the projected loss ratio, the actuary often begins by looking at historical experience data; in order to make the data relevant to the projected period, the losses and premium are adjusted to current level.

Therefore, in order to understand the role of rate change factors, we must investigate the nature of the traditional loss ratio projection and articulate its assumptions.

3. PROJECTING LOSS RATIO USING ADJUSTED HISTORICAL DATA

What is the nature of the loss ratio projection framework? Losses (in aggregate for any given historical year) are simply adjusted to current cost level; they are typically not adjusted in any way to incorporate changes in mix of business or changes in policy provisions such as deductibles and limits.[6] Premium is adjusted to what it "would be" had the historical policies been written today (or, more precisely, during the projected period).[7] Just as with losses, there seem to be no adjustments for shifts in the mix of business or in policy features. Thus traditional methods appear to be relevant only for the limited situations of a static book of business or one that changes only glacially.

How can traditional loss ratio projection be appropriate, then, for the many books of business that sustain significant changes in policies, classes of business, exposures, limits, and deductibles?

One response to this challenge is simply to concede: using historical data to project the future only makes sense when the portfolio is reasonably static, but not when it undergoes significant changes. This conclusion appears especially relevant to the "extended exposures" method for adjusting premium to current level. After all, the extended exposures approach takes historical policies and simply re-rates the policies at today's rates;[8] but if the types of policies in the portfolio have changed, the mix of business has shifted, and the limits and deductibles are different, what is the relevance of re-rating the policies of the historical portfolio?

Nevertheless, I believe that one can defend the use of historical data and adjusting for rate change by advancing the following reasoning. The goal of analyzing adjusted historical data is not to measure the **amount** of losses and premium that would occur from the historical portfolio, adjusted to today's dollars; rather, the goal is to measure premium and losses with respect to each other, i.e., the **interrelationship** of premiums to losses, and to measure what this relationship from the historical period would be in today's environment. Thus, even when the insurer's portfolio of policies undergoes significant change, when traditional adjustments to historical data do not accurately measure the projected amounts of losses and premium, the loss ratio projection can still be quite relevant; its relevance is rooted in its focus on measuring the relationship between premium and losses. This understanding of the purpose of using adjusted historical premium and losses, in turn, has ramifications for our understanding of what rate change factors should do and how we should calculate them, as we shall see in the following section.

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[6] Patrik [4] recommends that trending reflect all changes "that might affect the loss potential"; however, this step is difficult to implement and is often not done in practice.
[7] McClenahan [3], p. 88, describes the on-level premium as the premium "that would have resulted for the experience period had the current rates been in effect for the entire period." Thus we see that on-level premium is defined as historical premium adjusted solely for changes in rate level; apparently, no adjustments are made for changes in the portfolio's composition.
[8] See McClenahan [3], p. 94.

4. MEASURING RATE CHANGE FOR USE IN LOSS RATIO PROJECTION

Before proceeding to the derivation of the relevant formulas, let us articulate several observations, caveats, and limitations of scope.

1. Nothing in this paper intends to relate to the question of converting rate changes from a policy year, written premium basis to an accident year, earned premium basis; nor does this paper have any connection to rate level calculations based upon geometric techniques that rely on parallelograms and rectangles. These issues are addressed extensively elsewhere in the actuarial literature and are outside the scope of this paper.[9] Therefore, one should interpret all references to premium as references to policy year, written premium.
2. As noted in Section 2, how one ought to calculate rate change factors depends upon their intended purpose. Our discussion in this section presupposes that one will use the rate change factors in the context of projecting a loss ratio. However, if one were to use these factors for a different purpose, then the procedure of calculating the rate change factors may very well need to be different.
3. This paper does not intend to address the issue of inflation-sensitive exposure bases. Therefore, the reader should interpret the exposure base information as having already been converted from a nominal basis to a real (i.e., inflation-adjusted) basis.
4. When using historical data to project a loss ratio, actuaries often use multiple years of data; for simplicity, we will discuss the case of using data of one historical year (period t). In addition, we will simplify by discussing the procedure of adjusting this data one year forward (to period t+1).

4.1 Algebraic Representation

Let:

- Premium(observation(t), portfolio(t), rates(t)) = premium for historical period t, reflecting the portfolio in force and rates in effect during period t
- Loss(observation(t), portfolio(t), cost(t)) = losses for historical period t, reflecting the

[9] See McClenahan [3].

portfolio in force and claim cost inflation level in force during period t

- LP(portfolio(t)) = loss potential for the portfolio for historical period t; reflects the portfolio's propensity for loss given its limits, deductibles, and exposure base units, but does not reflect claim cost inflation[10]
- LP(portfolio(t+1)) = loss potential for portfolio for projected period t+1; reflects the portfolio's propensity for loss given its limits, deductibles, and exposure base units, but does not reflect claim cost inflation
- LP(portfolio(t+1))/ LP(portfolio(t)) = "shift in loss potential" = multiplier that adjusts the loss potential for the portfolio at time t to the loss potential for the portfolio at time t+1
- Trend(t, t+1) = claim cost inflation level during period t+1 / claim cost inflation level during period t = cost(t+1) / cost(t)

Let's assume that there are changes in the book of business relating to exposures, limits, and deductibles.

We want to take observed premium and losses from historical period t and to adjust them to the basis of period t+1, so we must calculate:

$$\begin{aligned}
&\text{Fully Adjusted Losses}(\text{t} \rightarrow \text{t}+1) = \\
&\text{Loss}(\text{observation(t)}, \text{portfolio}(\text{t}+1), \text{cost}(\text{t}+1)) = \\
&\text{Loss}(\text{observation(t)}, \text{portfolio(t)}, \text{cost(t)}) * \frac{\text{LP}(\text{portfolio}(\text{t}+1))}{\text{LP}(\text{portfolio(t)})} * \text{Trend}(\text{t}, \text{t}+1)
\end{aligned} \tag{4.1}$$

And

[10] Loss potential is essentially the expected loss cost. However, "loss cost" is usually measured in dollar units and thus tends to emphasize a particular numerical dollar value. In contrast, "loss potential" emphasizes the underlying real exposure to loss (and, as a result, changes to dollars of loss cost arising from inflation will not here be classified as a change in loss potential).

$$\text{Fully Adjusted Premium}(t \to t+1) = \text{Premium}(\text{observation}(t), \text{portfolio}(t+1), \text{rates}(t+1)) \tag{4.2a}$$

Multiplying both the numerator and denominator by equal quantities, we derive:

$$\text{Fully Adjusted Premium}(t \to t+1) = \text{Premium}(\text{observation}(t), \text{portfolio}(t), \text{rates}(t)) * \frac{\text{LP}(\text{portfolio}(t+1))}{\text{LP}(\text{portfolio}(t))} * \frac{\text{Premium}(\text{observation}(t), \text{portfolio}(t+1), \text{rates}(t+1))}{\text{Premium}(\text{observation}(t), \text{portfolio}(t), \text{rates}(t)) * \frac{\text{LP}(\text{portfolio}(t+1))}{\text{LP}(\text{portfolio}(t))}} \tag{4.2b}$$

Then dividing losses by premium, we derive:

$$\text{Fully Adjusted Loss Ratio}(t \to t+1) = \frac{\text{Fully Adjusted Losses}(t \to t+1)}{\text{Fully Adjusted Premium}(t \to t+1)} \tag{4.3a}$$

As stated above, and as implied by Equation (4.1), in theory the losses should be adjusted to reflect all changes in loss potential, whether from changes in exposures, mix of business, limits, deductibles, etc. Nevertheless, if we focus on the interrelationship of losses and premium, we note that the shift in loss potential [i.e., LP(portfolio(t+1)) / LP(portfolio(t))] appears both in Equation (4.1) for Fully Adjusted Losses and in Equation (4.2b) for Fully Adjusted Premium. Dividing Equation (4.1) by Equation (4.2b) and canceling the factor for shift in loss potential, we derive:

$$\text{Fully Adjusted Loss Ratio}(t \to t+1) = \frac{\text{Fully Adjusted Losses}(t \to t+1)}{\text{Fully Adjusted Premium}(t \to t+1)}$$

$$= \text{Adjusted Loss Ratio}(t \to t+1) = \frac{\text{Adjusted Losses}(t \to t+1)}{\text{Adjusted Premium}(t \to t+1)} \tag{4.3b}$$

Such that:

$$\text{Adjusted Losses}(t \to t+1) = \text{Loss}(\text{observation}(t), \text{portfolio}(t), \text{cost}(t)) * \text{Trend}(t, t+1) \tag{4.4}$$

And:

$$\text{Adjusted Premium}(t \to t+1) =$$

$$\text{Premium}(\text{observation}(t), \text{portfolio}(t), \text{rates}(t)) *$$

$$\frac{\text{Premium}(\text{observation}(t), \text{portfolio}(t+1), \text{rates}(t+1))}{\text{Premium}(\text{observation}(t), \text{portfolio}(t), \text{rates}(t)) * \dfrac{\text{LP}(\text{portfolio}(t+1))}{\text{LP}(\text{portfolio}(t))}} \tag{4.5}$$

Note that Equation (4.4) for adjusted losses is similar to Equation (4.1) for fully adjusted losses; however, it no longer has any factor for changes in loss potential from exposures, limits, and deductibles. Therefore, the practice of not adjusting losses for these shifts in loss potential is sustainable, but only if one simultaneously defines adjusted premium in a corresponding fashion, per Equation (4.5).

Now, let us define the Rate Change Factor as the multiplier which converts historical premium to adjusted premium.

Therefore:

$$\text{Adjusted Premium}(t \to t+1) = \text{Premium}(\text{observation}(t), \text{portfolio}(t), \text{rates}(t)) * \text{Rate Change Factor}(t \to t+1) \tag{4.6a}$$

And:

$$\text{Rate Change Factor}(t \to t+1) = \frac{\text{Adjusted Premium}(t \to t+1)}{\text{Premium}(\text{observation}(t), \text{portfolio}(t), \text{rates}(t))} \tag{4.6b}$$

Then combining Equations (4.5) and (4.6a), we derive:

$$\text{Rate Change Factor}(t \to t+1) = \frac{\text{Premium}(\text{observation}(t), \text{portfolio}(t+1), \text{rates}(t+1))}{\text{Premium}(\text{observation}(t), \text{portfolio}(t), \text{rates}(t)) * \dfrac{\text{LP}(\text{portfolio}(t+1))}{\text{LP}(\text{portfolio}(t))}} \tag{4.7a}$$

The premium observed during any period reflects the portfolio and rates in effect at the time; however, in contradistinction to losses, premium is not a stochastic process and is not subject to random observation.[11] Therefore, we can drop the reference to "observation(t)" from Equation 4.7a and write:

$$\text{Rate Change Factor}(t \to t+1) = \frac{\text{Premium}(\text{portfolio}(t+1), \text{rates}(t+1))}{\text{Premium}(\text{portfolio}(t), \text{rates}(t)) * \dfrac{\text{LP}(\text{portfolio}(t+1))}{\text{LP}(\text{portfolio}(t))}} \tag{4.7b}$$

Or, equivalently,

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[11] One exception to this general rule occurs if a policy's premium is "loss sensitive": then the observed premium is a function of the observed losses. Policies with loss sensitive premium are outside the scope of this analysis.

$$\text{Rate Change Factor}(t \rightarrow t+1) = \frac{\text{Premium}(t+1)}{\text{Premium}(t) * \text{Shift in Loss Potential}} \tag{4.8}$$

Equation (4.8) demonstrates that one must calculate the rate change factor using the ratio of 2 quantities:

1) Actual premium in period (t+1)

2) Actual premium in period (t) "restated" for all shifts in loss potential, reflecting changes in exposures, limits, deductibles, etc.

To summarize, we have demonstrated three points:

1) To obtain an Adjusted Loss Ratio, the losses in the numerator do not need to be adjusted for changes in loss potential, thus somewhat exonerating current practice.

2) The Rate Change Factor is defined by Equation (4.8), which shows that when measuring rate change, one must first restate premium from the prior period for changes in loss potential.

3) Per Equation (4.6a), Adjusted Premium for use in loss ratio projection equals actual historical premium multiplied by the Rate Change Factor.

An important consequence of these results relates to <u>when</u> one can accurately measure the true rate change from period t ("the expiring period") to period t+1 ("the renewing period"). Formula (4.8) makes clear that one must take the premium from the expiring period and restate it based upon the shift in loss potential in the renewing period; however, the shift in loss potential cannot be known until the end of the renewing period. Therefore, when one implements rate changes to various segments of the portfolio at the beginning of a period, one can only estimate the rate change; the true rate change cannot be precisely calculated until the end of the period.

5. APPLICATIONS

We will now apply the conclusions of the discussion above to solve the problems raised at the beginning of this paper.

5.1 Weighting Together Multiple Rate Changes

This section will discuss how to measure the rate change for an entire portfolio in light of the rate changes of the portfolio's individual components.

Exhibit 1A: Change in Exposures

Expiring Period

	Premium	Exposures	Premium per Exposure
Red Trucks	12,000,000	600	20,000
Green Trucks	4,000,000	400	10,000
Total	16,000,000	1,000	16,000

Renewing Period

	Premium	Exposures	Premium per Exposure
Red Trucks	8,640,000	360	24,000
Green Trucks	4,480,000	560	8,000
Total	13,120,000	920	14,261

Exhibit 1B: Traditional Rate Change Calculations

Method 1: Average Rate per Exposure Unit

[1]	[2] Expiring Premium Per Exposure	[3] Renewing Premium Per Exposure	[4] = [3] / [2] -1 Change
Red Trucks	20,000	24,000	20.00%
Green Trucks	10,000	8,000	-20.00%
Total	16,000	14,261	-10.87%

Methods 2 and 3: Weighted Average of Rate Changes

[1]	[2] Change	[3] Expiring Premium Weight	[4] Renewing Premium Weight
Red Trucks	20.00%	75.00%	65.85%
Green Trucks	-20.00%	25.00%	34.15%
Weighted Average		10.00%	6.34%

In this example, we show three traditional methods of measuring rate change:

1) Calculate the weighted average premium per exposure; measure this quantity for the renewal portfolio relative to the expiring portfolio for the rate change.[12]

2) Measure the rate change of each class or policy in the portfolio; blend these rate changes together using a weighted average; use expiring premium as the weights.[13]

3) Measure the rate change of each class or policy in the portfolio; blend these rate changes together using a weighted average; use renewing premium as the weights.[14]

Note that all of the traditional methods produce different answers; all of them measure the rate change approximately, but not one of them measures the rate change precisely.

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[12] See Jones [1], pp. 9 – 10, who focuses on average premium per exposure as a measure of rate change.

[13] See http://www.casact.org/education/reinsure/2008/handouts/schober.ppt. On slide 33, discussing commercial property, Schober suggests one "re-rate to expiring," which refers to taking renewal policies and re-rating them on the basis of the expiring coverage. The wording appears to imply that one should use expiring premium as the weighting basis.

[14] Vaughn [5], p. 503.

The exhibit below shows the proposed approach.

Exhibit 1C: Proposed Approach to Calculating Rate Change

[1]	[2]	[3]	[4] = [3] * [2]	[5]	[6] = [5]/[4] - 1
	Expiring Premium	Renewing Exposures / Expiring Exposures	Expiring Premium Restated For Change in Exposure	Renewing Premiums	Rate Change
Red Trucks	12,000,000	0.60	7,200,000	8,640,000	20.00%
Green Trucks	4,000,000	1.40	5,600,000	4,480,000	-20.00%
Total	16,000,000		12,800,000	13,120,000	2.50%

Exhibit 1D: Comparison Exhibit

Method	Description	Calculated Rate Change
1	Ratio of Average Rate per Exposure Unit	-10.87%
2	Expiring Premium Weighted Average of Rate Changes	10.00%
3	Renewing Premium Weighted Average of Rate Changes	6.34%
Proposed	Restate Expiring Premium for Change in Loss Potential	2.50%

The proposed approach builds upon the prior conceptual discussion and Equation (4.8); thus, expiring premium must be "restated" for all shifts in loss potential before measuring rate change.[15] In Exhibit 1D, we see that the proposed approach can generate significantly different rate change factors than other methods.

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[15] For the total portfolio, the premium must be restated for the shift in the total loss potential, which in turn depends upon the expected loss ratios of the various components of the portfolio. Here, however, we do not use any explicit assumptions about the components' loss ratios. Thus, implicitly, we presume that the expiring expected loss ratios for all the components are equal. Given that one has chosen to combine the various components into one portfolio for measuring loss ratio, the assumption of equal loss ratios by component is usually reasonable. However, if one were to combine different segments of business with clearly different expected loss ratios, one would need to explicitly reflect the different loss ratios by component when measuring the "shift in loss potential" for the total portfolio.

5.2 Dealing with a Portfolio of Heterogeneous Exposure Bases

The proposed framework for measuring rate change also allows us to solve the problem of how to deal with a portfolio with multiple, dissimilar exposure bases.

The exhibits below demonstrate the proposed approach.

Exhibit 2A: Dissimilar Exposure Bases

Expiring

	Premium	Exposure Base	Exposures	Premium per Exposure
Jane's Contracting	12,000,000	sales (000s)	600	20,000
Jill's Stores	4,000,000	square feet (000s)	400	10,000
Total	16,000,000	undefined	undefined	undefined

Renewing

	Premium	Exposure Base	Exposures	Premium per Exposure
Jane's Contracting	8,640,000	sales (000s)	360	24,000
Jill's Stores	4,480,000	square feet (000s)	560	8,000
Total	13,120,000	undefined	undefined	undefined

Exhibit 2B: Measuring "Change in Premium from Change in Exposure Base Units"

Proposed Approach to Measuring Rate Change

[1]	[2]	[3]	[4]	[5] = [4] * [2]	[6]	[7] = [6] / [5] - 1
	Expiring Premium	Exposure Base	Renewing Exposures / Expiring Exposures	Expiring Premium Restated For Change in Exposure	Renewing Premiums	Rate Change
Jane's Contracting	12,000,000	sales (000s)	0.600	7,200,000	8,640,000	20.00%
Jill's Stores	4,000,000	square feet (000s)	1.400	5,600,000	4,480,000	-20.00%
Total	16,000,000	loss potential	0.800	12,800,000	13,120,000	2.50%

Measuring Exposure Change for Total Book

[1]	[2]	[3]	[4] = [3] / [2]	[5] = [3] / [2] -1
	Expiring Premium	Expiring Premium Restated For Change in Exposure	Ratio	Change in Premium from Changes in Exposure Base Units
Total	16,000,000	12,800,000	0.800	-20.00%

Initially, the disparate exposure bases of the classes of business prevent us from measuring the exposure base change for the total book. However, by restating the expiring premium for shifts in exposure bases, we create a new way to measure total exposure base change; we simply measure the total change in premium arising from changes in exposure bases. Thus, the proposed procedure of restating expiring premium for shifts in loss potential provides a framework for measuring the total exposure base change for a portfolio that has multiple, incongruous exposure bases.

5.3 Measuring Rate Change When Limits and Deductibles Change

The proposed framework for measuring rate change also allows us to solve the problem of how to measure rate change when values of the limit and deductible of a renewing policy change from their values under an expiring policy, as demonstrated in the exhibits below:

Exhibit 3A: Change in Deductibles

Expiring

	Premium	Square Feet (000s)	Limit	Deductible	ILF Index = ILF(Limit) - ILF(Deductible)	Premium per Exposure
Joe's Stores	13,500,000	900	1,000,000	-	1.00	15,000
Bill's Stores	9,000,000	900	1,000,000	250,000	0.50	10,000
Total	22,500,000	1,800				12,500

Renewing

	Premium	Square Feet (000s)	Limit	Deductible	ILF Index = ILF(Limit) - ILF(Deductible)	Premium per Exposure
Joe's Stores	8,977,500	800	1,000,000	250,000	0.50	11,222
Bill's Stores	14,400,000	1,000	1,000,000	-	1.00	14,400
Total	23,377,500	1,800				12,988

Exhibit 3B: Traditional Rate Change Calculations

Class	Change	Expiring Premium Weight	Renewing Premium Weight
Joe's Stores	49.6%	60.0%	38.4%
Bill's Stores	-28.0%	40.0%	61.6%
Weighted Average		18.6%	1.8%

Exhibit 3C: Proposed Approach to Calculating Rate Change

	[1]	[2]	[3] = [1] * [2]	[4]	[5] = [3] * [4]	[6]	[7] = [6] / [5]	[8] = [6] / [5] - 1
	Expiring Premium	Renewing Exposures / Expiring Exposures	Expiring Premium Restated For Change in Exposure	Renewing ILF Index / Expiring ILF Index	Expiring Premium Restated For Change in Exposure **and** Change in Limits & Deductibles	Renewing Premium	Rate Change Factor	Rate Change
Joe's Stores	13,500,000	0.889	12,000,000	0.50	6,000,000	8,977,500	1.496	49.6%
Bill's Stores	9,000,000	1.111	10,000,000	2.00	20,000,000	14,400,000	0.720	-28.0%
Total	22,500,000		22,000,000		26,000,000	23,377,500	0.899	-10.1%

Change in Premium from Change in Exposure (= [3] total / [1] total - 1)	-2.2%
Change in Premium from Change in Limits, Deductibles (= [5] total / [3] total - 1)	18.2%
Change in Premium from Rate Change (= [6] total / [5] total - 1)	-10.1%

Exhibit 3D: Comparison Exhibit

Method	Description	Calculated Rate Change
1	Expiring Premium Weighted Average of Rate Changes	18.6%
2	Renewing Premium Weighted Average of Rate Changes	1.8%
Proposed Approach	Adjust Expiring Premium for Change in Loss Potential	-10.1%

Again, we see the importance of measuring rate change only after restating expiring premium for changes in loss potential.

5.3.1 Clarifying Which ILFs to Use

In the numerical example above (Exhibits 3A through 3D), we use ILFs (increased limits factors) to measure the change in loss potential from changing limits and deductibles. However, there is more than one type of ILF. "Loss ILFs" measure the relationship of loss costs at different limits and deductibles; they derive from measures of Limited Expected Value (LEV, aka LAS or Limited Average Severity). "Premium ILFs," however, measure the relationship of the premium the company charges for different limits and deductibles; they incorporate LEVs, risk load, and expenses. So when measuring rate change and restating premium for changes to limits and deductibles, which ILFs should one use?

Equation (4.8) demonstrates that when measuring rate change one must restate expiring premium for changes in loss potential. Therefore, when measuring rate change, it is more precise to restate expiring premium via Loss ILFs than via Premium ILFs; after one has used Loss ILFs to restate the expiring premium, one can then calculate the rate change factor as the ratio of renewing premium to restated expiring premium.

5.3.2 Tracking All Sources of Change

Exhibit 3C highlights another benefit of the proposed approach: the ability to completely track all changes to premium. Other methods for measuring rate change do not necessarily provide the framework to fully track the changes in premium and to connect the expiring premium to the renewing premium in a comprehensive way; nor do they identify the catalysts that are driving the changes in premium.

In contrast, the proposed approach allows one (as in Exhibit 3C) to measure all changes of premium, properly weighting together the changes of each policy or segment of the portfolio. In addition, applying all sources of change to the expiring premium will actually balance to the renewing premium. In other words, one can begin with expiring premium and then calculate:

*Expiring premium * (1+change in premium from exposure change) * (1+change in premium from change in limits & deductibles) * … * (1+ rate change) = Renewing premium [excluding new business]*

5.4 Change in Share

Sometimes a company writes a portion of a policy; for example, one company might take only a 50% "share" or "participation" in a given excess policy. The following exhibit describes such a situation:

Exhibit 4A: Change in Share

Expiring

	Premium @100% share	Square Feet (000s)	Limit	Deductible	ILF Index = ILF(Limit) - ILF(Deductible)	Premium per Exposure	Company Share	Premium @Company share
Joe's Stores	13,500,000	900	1,000,000	-	1.00	15,000	50%	6,750,000
Bill's Stores	9,000,000	900	1,000,000	250,000	0.50	10,000	50%	4,500,000
Total	22,500,000	1,800				12,500		11,250,000

Renewing

	Premium @100% share	Square Feet (000s)	Limit	Deductible	ILF Index = ILF(Limit) - ILF(Deductible)	Premium per Exposure	Company Share	Premium @Company share
Joe's Stores	8,977,500	800	1,000,000	250,000	0.50	11,222	25%	2,244,375
Bill's Stores	14,400,000	1,000	1,000,000	-	1.00	14,400	75%	10,800,000
Total	23,377,500	1,800				12,988		13,044,375

In Exhibit 4A, the values are the same as in Exhibit 3A, but with one important change: the company's share declines for the policy that receives a rate increase, whereas the company's share increases for the policy that receives a rate decrease. The following exhibit demonstrates the proposed approach of measuring rate change in such a situation:

Exhibit 4B: Proposed Approach to Calculating Rate Change

	[1]	[2]	[3]	[4]	[5] = [3] * [4]	[6]	[7] = [6] / [5]	[8] = [7]-1
	Expiring Premium @Company Share	Expiring Premium Restated For Change in Exposure	Expiring Premium Restated For Change in Exposure **and** Change in Limits & Deductibles	Renewing Share / Expiring Share	Expiring Premium Restated For Change in Exposure **and** Change in Limits & Deductibles **and** Change in Share	Renewing Premium	Rate Change Factor	Rate Change
Joe's Stores	6,750,000	6,000,000	3,000,000	0.50	1,500,000	2,244,375	1.496	49.6%
Bill's Stores	4,500,000	5,000,000	10,000,000	1.50	15,000,000	10,800,000	0.720	-28.0%
Total	11,250,000	11,000,000	13,000,000		16,500,000	13,044,375	0.791	-20.9%

Change in Premium from Change in Exposure (= [2] total / [1] total - 1)	-2.2%
Change in Premium from Change in Limits, Deductibles (= [3] total / [2] total - 1)	18.2%
Change in Premium from Change in Company Share (= [5] total / [3] total - 1)	26.9%
Change in Premium from Rate Change (= [6] total / [5] total - 1)	-20.9%

Note that the rate change for each individual policy is unaffected by the change in company share; thus, each policy's rate change in Exhibit 4B is exactly equal to the value calculated in Exhibit 3C. However, there is now a significant difference in the rate change for the overall portfolio. Thus accurately measuring rate change for the portfolio requires that one use information about each policy's share; conversely, measuring rate change by first "grossing up" each policy's share to a common 100% basis can potentially lead to an imprecise rate change calculation for the portfolio.

6. SUMMARY

Quantitative analysis that projects an expected loss ratio often makes use of historical experience data and rate change factors. The appropriate application of such an analysis and the accurate calculation of rate change factors require a clear understanding of the conceptual foundations that undergird these methods. Having explored these foundational concepts, we conclude that the key goal of analyzing historical data is to forecast the interrelationship of losses and premiums for the projected book of business. Thus, when calculating rate change factors, one must first restate expiring premium for changes in all sources of loss potential, including changes in exposure base units, limits and deductibles, company share, etc. As a result, one can take the theory of measuring rate change factors and apply it towards solving problems in practice.

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Incorporating a Primary Insurer's Risk Load into the Property Rate

Kevin Burke

Abstract: There have been numerous articles giving guidance on how to include the cost of reinsurance in rate indications. What has been missing from the discussion is a method to account for the risk assumed by the primary insurer at the higher layers of the reinsurance program. This note provides such a method, using information from a catastrophe model and a company's reinsurance program.

Keywords: Large loss and extreme event loading, traditional risk load (profit margin)

1. INTRODUCTION

Subsequent to the hurricanes of 2003–2004 came large increases in the cost of catastrophe reinsurance. Insurers responded to these costs in some combination of three ways: by (1) passing the costs along to the consumer, (2) restricting their business in areas prone to hurricanes, or (3) retaining more risk, most likely with the same risk load as the noncatastrophe portion of the homeowner rate. The purpose of this note is to present an elementary method for including a charge for this additional risk in the catastrophe premium and incorporating that charge in the rate indication.

2. BACKGROUND

Assume that an actuary computes the following indication:

[A]	Average Loss and Expense Ratio	70%
[B]	Fixed Underwriting Expenses	5%
[C]	Variable Underwriting Expenses	22%
[D]	(Variable) Profit and Contingency Factor	3%
[E]	Indicated Rate Change	0%

[E]=([A]+[B])/(1-[C]-[D])-1

(Here we assume that all loss adjustment expenses are contained in [A].) Suppose that the company has $10,000,000 in average annual catastrophic loss that the rating agencies and CEO are concerned about. The CEO decides to reinsure 100% of this amount at a cost of $15,000,000. The actuary includes a provision for fixed reinsurance costs as in [6] (see also [3], [2].)

[1]	CY Direct Earned Premium	50,000,000
[2]	Modeled Loss Cost	10,000,000
[3]	Reinsurance Premium	15,000,000
[4]	Reinsured Portion of Loss Cost	10,000,000
[5]=[3]-[4]	Implied Reinsurance Expenses	5,000,000
[6]=[5]/[1]	Provision for Fixed Reinsurance Costs	10%

The actuary then computes the rate indication:

[A]	Average Loss and Expense Ratio	70%
[B1]	Fixed Underwriting Expenses	5%
[B2]	Provision for Fixed Reinsurance Costs	10%
[C]	Variable Underwriting Expenses	22%
[D]	(Variable) Profit and Contingency Factor	3%
[E]	Indicated Rate Change	13.3%

[E]=([A]+[B1]+[B2])/(1-[C]-[D])-1

Practically, the reinsurer will require a retention and a coparticipation of 10%, with a reduction in premium, so assume that the following program is in place.[1]

Layer	Modeled Loss Cost	Reinsurance Premium
$3,000,000 Retention	1,750,000	0
Excess of $3,000,000	8,250,000	11,375,000

The actuary again computes

[1]	CY Direct Earned Premium	50,000,000
[2]	Modeled Loss Cost	10,000,000
[3]	Reinsurance Premium	11,375,000
[4]	Reinsured Portion of Loss Cost	7,425,000
[5]=[3]-[4]	Implied Reinsurance Expenses	3,950,000
[6]=[5]/[1]	Provision for Fixed Reinsurance Costs	7.9%

The actuary then computes the rate indication:

[A]	Average Loss and Expense Ratio	70%

[1] The examples and numbers here are designed to be illustrative.

[B1]	Fixed Underwriting Expenses	5%
[B2]	Provision for Fixed Reinsurance Costs	7.9%
[C]	Variable Underwriting Expenses	22%
[D]	(Variable) Profit and Contingency Factor	3%
[E]	Indicated Rate Change	10.5%

The CEO then makes the following observation to the actuary: "If a reinsurer assumes all the catastrophic risk the cost of assuming that risk is transferred to the policyholders but if the primary insurer assumes some (or all) of that risk, the current methodology doesn't allow us to collect additional premium for the assumption of additional risk."

The actuary knows that the CEO is right. As the underlying risk changes, the profit load should change. The 3% profit and contingencies factor is computed using standard actuarial methods that take into account the short-tailed nature of property lines (see, for example, [5]) but does not properly take into account the catastrophic risk that his company faces. With a base premium of $500 and a catastrophic loss cost of $100, the actuary decomposes the premium.

	Scenario 1 (no reinsurance)	Scenario 2 (100% reinsurance)	Scenario 3 ($3M retention)
Hurricane Loss Cost	100	100	100
Fixed Expenses	25	25	25
Variable Expenses	110	125	122
Profit & Contingencies	15.0	17.0	16.7
Provision for Reinsurance	0	50	41
Other Perils Loss Cost	250	250	250
Indicated Premium	500	567	553

The CEO's complaint is more fully illustrated here. An increase in the company's catastrophic exposure results in a decreased reinsurance premium but doesn't result in a corresponding increase in profit.

3. RESULTS AND DISCUSSION

3.1 Incorporating the Risk Load

The approach taken in addressing this issue is nontheoretic and may not pass the scrutiny of those wishing to view risk transfer within a larger economic framework. The approach is, however, practical and easy to implement. An additional drawback is that it may not pass the review of regulators.[2]

We begin by examining a typical catastrophe reinsurance program. Such a program is divided into layers $L_0, L_1, \ldots, L_n$ and corresponding retained percentages $p_0, p_1, \ldots, p_n$. The expected hurricane loss E[L] is given by $E[L] = \sum_{j=1}^{n} E[L_j]$, the expected retained loss is given by $\sum_{j=1}^{n} p_j E[L_j]$, and the expected ceded portion is $\sum_{j=1}^{n} (1-p_j)E[L_j]$. Each layer has a reinsurance premium R_j. Let λ_j denote the risk premium for layer j, so that

$$1+\lambda_j = \frac{R_j}{(1-p_j)E[L_j]}.$$

We may then formalize the computation the provision for fixed reinsurance costs as follows.

[1] Calendar Year Direct Earned Premium	P
[2] Modeled Loss Cost	$\sum E[L_j]$
[3] Reinsurance Premium	$\sum (1+\lambda_j)(1-p_j)E[L_j]$
[4] Reinsured Portion of Loss Cost	$\sum (1-p_j)E[L_j]$
[5] Implied Reinsurance Expense	$\sum \lambda_j (1-p_j)E[L_j]$
[6] Provision for Fixed Reinsurance Costs	$\frac{1}{P}\sum \lambda_j (1-p_j)E[L_j]$

Note that, as observed earlier, the retained portion of the hurricane losses, $\sum_{j=0}^{n} p_j E[L_j]$, has no corresponding risk load and the additional risk taken on by the primary insurer is not reflected in the indication. To rectify this, we choose π_j with $0 \le \pi_j < \lambda_j$. It is at this point where actuarial

[2] Note that Florida specifically addresses this issue in 627.062, F.S. which states that "…For that portion of the rate covering the risk of hurricanes and other catastrophic losses for which the insurer has not purchased reinsurance and has exposed its capital and surplus to such risk, the office must approve a rating factor that provides the insurer a reasonable rate of return that is commensurate with such risk."

judgment or other analysis is used to select the primary company's risk load. Our only constraint is that the reinsurer's risk load is an upper bound for the primary company's risk load. The selection could be guided, for example, by a desire to reach a target risk-adjusted return.[3] The corresponding risk load for layer *j* is then given by

$$\pi_j p_j E[L_j]$$

The total company risk load $\sum \pi_j p_j E[L_j]$ is then built into the indication in the same manner as the provision for fixed reinsurance costs.

Continuing with scenario 3 we see that

$$\begin{array}{lll} E[L_0]=1{,}750{,}000 & p_0=1.00 & R_0=0 \\ E[L_1]=8{,}250{,}000 & p_1=0.10 & R_1=11{,}375{,}000 \end{array}$$

Clearly $\lambda_0 = 0$ and

$$1+\lambda_1 = \frac{11{,}375{,}000}{7{,}425{,}000} = 1.532\,.$$

We choose $\pi_0 = 0$ and $0 \le \pi_1 < 0.532$ judgmentally selecting $\pi_1 = 0.25$ gives a risk load for layer 1 of 0.25(825,000)=206,250 and a provision for primary company risk load of 206,250/50,000,000=0.004. Incorporating this into the indication gives us the following adjusted indication.

[A]	Average Loss and Expense Ratio	70%
[B1]	Fixed Underwriting Expenses	5%
[B2]	Provision for Fixed Reinsurance Costs	7.9%
[B3]	Provision for Primary Company Risk	0.4%
[C]	Variable Underwriting Expenses	22%
[D]	(Variable) Profit and Contingency Factor	3%
[E]	Indicated Rate Change	11.1%

[E]=([A]+[B1]+[B2]+[B3])/(1-[C]-[D])-1

3 A discussion of the computation of a line of business' risk-adjusted return on capital is beyond the scope of this note. The reader is directed to [1] for an introduction.

3.2 Allocation of Loading to Territory

Once we arrive at the appropriate primary insurer risk load we allocate it to territory in the same way that Rollins allocates the Reinsurance Risk Load. For completeness, we illustrate the procedure (see Exhibit 9 of [6].) Let σ_i and e_i denote the standard deviation of modeled losses and exposures for territory *i*, respectively. Let $E[T_i]$ denote the average modeled hurricane loss cost per exposure in territory *i*. The risk load for each territory is given by $k\sigma_i$ where k is chosen so that the total risk load is equal to the sum of the reinsurance risk load. We summarize Rollins' result using the notation from Section 3.1.

The reinsurance risk load (as a percent of gross loss cost) is given by

$$\frac{\sum_{j=0}^{n} \lambda_j (1-p_j) E[L_j]}{\sum_{j=0}^{n} E[L_j]} \tag{3.1}$$

If there are *m* territories then the total risk load is given by

$$\sum_{i=1}^{m} k\sigma_i e_i \tag{3.2}$$

and the total modeled gross loss cost is given by

$$\sum_{i=1}^{m} E[T_i] e_i \tag{3.3}$$

The total risk load (3.2) must equal the product of (3.1) and (3.3) so that

$$k = \frac{\sum_{j=0}^{n} \lambda_j (1-p_j) E[L_j]}{\sum_{j=0}^{n} E[L_j]} \frac{\sum_{i=1}^{m} E[T_i] e_i}{\sum_{i=1}^{m} \sigma_i e_i}$$

It will not, in general, be true that the total expected modeled loss costs in the territorial analysis is equal to the total expected modeled loss costs from the indication. This is because the territorial analysis will generally involve a subset of the risks used in the overall indication.

In order to extend this relationship to include the primary insurer's risk load, we observe that, as a percentage of total modeled hurricane losses, the primary insurer's risk load is given by

$$\frac{\sum_{j=0}^{n} \pi_j p_j E[L_j]}{\sum_{j=0}^{n} E[L_j]}$$

We add this amount to the total reinsurance risk load and get

$$k = \frac{\sum_{j=0}^{n} \left(\lambda_j (1-p_j) + \pi_j p_j\right) E[L_j]}{\sum_{j=0}^{n} E[L_j]} \frac{\sum_{i=1}^{m} E[T_i] e_i}{\sum_{i=1}^{m} \sigma_i e_i}$$

Returning to the illustrative example, suppose that our company has three territories, Inland, Seacoast, and Beach, and that we have the following information.

Territory	Exposures	Modeled Hurricane Loss Cost	Modeled Standard Deviation
Inland	175,000	65	357.5
Seacoast	160,000	225	1,462.5
Beach	100,000	450	3,375.0
Total	435,000	212.36	

We then compute the following.

$\sum_{i=1}^{m} E[T_i] e_i = 92{,}375{,}000$	$\sum \lambda_j (1-p_j) E[L_j] = 3{,}950{,}000$
$\sum_{i=1}^{m} \sigma_i e_i = 634{,}062{,}500$	$\sum \pi_j p_j E[L_j] = 206{,}250$
$\sum E[L_j] = 10{,}000{,}000$	

$$\begin{aligned} k &= \frac{92{,}375{,}000}{10{,}000{,}000} \frac{3{,}950{,}000 + 206{,}250}{634{,}062{,}500} \\ &= 0.061 \end{aligned}$$

The allocated risk load is then added to the modeled loss cost to obtain the risk adjusted hurricane loss cost.

Territory	Exposures	Modeled	Modeled	Allocated	Risk-Adjusted

		Hurricane Loss Cost	Standard Deviation	Risk Load	Modeled Loss Cost
Inland	175,000	65	357.5	21.65	86.65
Seacoast	160,000	225	1,462.5	88.56	313.56
Beach	100,000	450	3,375.0	204.36	654.36
Total	435,000	212.36		88.26	300.62

The risk-adjusted modeled loss cost can then be built into the territorial indication using standard actuarial techniques.

3.3 Relationship to Standards of Practice and Statement of Principles

Actuarial Standards of Practice 38 & 39, "Using Models Outside the Actuary's Area of Expertise" and "Treatment of Catastrophe Losses in Property/Casualty Insurance Ratemaking," respectively, provide guidance to the actuary when dealing with catastrophe losses, both actual and modeled. While there are regulatory hurdles and misconceptions concerning the use of the model (see [4] for a discussion of some of these issues), the use of catastrophe models in pricing is by now a standard pricing technique. In fact, projected climatic changes practically mandate the use of a model.

The inclusion of reinsurance costs in the property rate and its allocation to territory are required by the CAS Statement of Principles Regarding Property and Casualty Insurance Ratemaking. The relevant principles are

"A rate provides for all costs associated with the transfer of risk"
"A rate provides for the costs associated with an individual risk transfer."

These costs must include a risk load. Standard of Practice Number 30: "Treatment of Profit and Contingency Provisions and the Cost of Capital in Property/Casualty Insurance Ratemaking," tells us that

"Property/casualty insurance rates should provide for all expected costs, including an appropriate cost of capital associated with the specific risk transfer."

By choosing to retain a portion of the catastrophic risk, a company is putting its surplus at risk. In return for putting that capital at risk, the insurer is entitled to a return commensurate with that risk. Using the reinsurer's risk load as a proxy for an actual market return allows the actuary to incorporate that risk into the rate indication. Performing all of these steps is supported and required by the relevant Standards of Practice and Statements of Principles.

4. CONCLUSION

The primary appeal of this method is its simplicity. We need only do the following:

- Allocate reinsurance premium to state and line of business (this is done, e.g., in [2]).
- Partition the modeled catastrophe loss cost by layer of reinsurance.
- Compute the reinsurer's risk load and select an appropriate company risk load.
- Allocate the company risk load to territory.

The numeric example shown was created to highlight the steps involved. There are no barriers to applying these principles to a more complicated reinsurance program. Finally, while there may be both institutional and regulatory objections to the inclusion of these costs, these objections must be addressed on an individual basis.

Acknowledgment

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Revenue Management & Insurance Cycle

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Abstract: This paper investigates how an insurer's pricing strategy can be adapted to respond to market conditions, and in particular the insurance cycle. For this purpose, we explore the use of dynamic pricing strategies, such as the revenue management techniques used by other industries (e.g., airlines, car rentals, internet service providers) in an insurance context. We then compare these dynamic pricing techniques with the static ones currently used in the market, and demonstrate that they can prove very valuable to insurers looking to enhance their competitive strategy.

Keywords: Cycle management, dynamic pricing, profit optimization, revenue management, dynamic programming

1. INTRODUCTION

This paper is a reflection on the optimal strategy for deploying a fixed amount of insurance capacity over a period of time. In particular, we consider the following questions: Which pricing strategy maximizes the expected profits? Should it be based on market conditions or shareholders' expectations? Should it be static or dynamic? How does an insurer manage the insurance cycle? Can we expect to make a profit when market returns are negative?

To respond, we introduce the theory of revenue management, which integrates market conditions and fluctuations in demand into the decision-making process. We use this framework to develop an optimal pricing strategy and demonstrate how it can be a valuable tool to manage the insurance cycle. In order to exemplify our case in point, we can think of an insurer with a surplus, denoted S, of $ 1 billion ($US) and a capacity constraint driven by a 5:1 maximum written premiums-to-surplus ratio imposed by its regulator.

As a result, this insurer has a capital allocation of 20% of premiums written and it prices each policy based on a 15% charge on allocated capital, 15% being the target return on equity promised to its shareholders.[1]

We have therefore the following pricing formula:

$$PV[P_i] = PV[E(L_i) + E_i + r * K_i] = PV\left[\frac{(E(L_i) + E_i)}{(1 - 15\% * 20\%)}\right] \qquad (1.1)$$

with

- P_i: price-bid[2] for policy i

[1] We assume in this example that the shareholders' equity is equal to the company's surplus.

[2] I.e., the minimum acceptable premium for policy i

- L_i: losses for policy i
- E_i: expenses for policy i
- K_i: capital allocated to policy i, here 20% * P_i
- r: required return on allocated capital (a.k.a. capacity charge)

As we can see, the insurer has a limited capacity for the underwriting year and each policy written "consumes" some of it.

This example reflects a fairly common approach to pricing and charge for capacity, and the object of our analysis is to explore the following questions: Does this pricing strategy maximize the expected profits? If not, what would be the best alternative?

2. BACKGROUND

2.1 Revenue Management

Revenue management techniques first appeared in the early 1980s in the airlines industry and have since been introduced progressively in other industries (e.g., hotels, car rentals, internet service providers, and others). Their objective is simple: maximizing the profits from a fixed supply of perishable goods and services over a period of time.

For instance, airlines use sophisticated revenue management systems based on historical booking patterns to estimate the likelihood of an empty seat at departure. They need to balance the risk of not selling that seat, with the opportunity cost of passing up a "premium customer" willing to pay a higher price. "If a plane is not filling up as rapidly as historically expected, the probability of an empty seat goes up and the opportunity cost of selling more discounted seats goes down, so the airline's management system may offer some tickets at an exceptionally low price. If, however, a group of seven business people suddenly books onto the flight, the probability of filling the flight jumps substantially, the opportunity cost goes up, and the airline's management system blocks additional sales of the cheapest tickets." [5]

2.2 Insurance Applications

From a practical viewpoint, revenue management techniques require the market to offer full flexibility in price setting. In an insurance context, this excludes lines of business where rates are subject to a tariff or to filing/approval by the regulator. There are, however, many insurance markets where rates are set freely and can be changed frequently by the market participants (e.g., excess & surplus lines, commercial lines, reinsurance, and personal lines in most European countries).

For those insurance markets with flexibility in price setting, it is fairly easy to see how these techniques can be applied:

- insurers have a fixed supply of insurance capacity over a period of time (more accurately, capacity can be increased or decreased at times but it is fixed in between these events).
- insurance capacity is perishable, in the sense that unused capital for an underwriting year can not be transferred to the next.[3]

While revenue management can take several forms,[4] the framework we present in this paper is purely price driven: we seek to set r over time so that it maximizes the expected profit based on market conditions and expected demand. The required return on allocated capital becomes a stochastic process r(t) and the pricing formula becomes:

$$PV\left[P_i(t)\right] = PV\left[E\left(L_i\right) + E_i + r(t) * K_i\right] \tag{2.1}$$

We call pricing strategy a path r for r(t) over the underwriting period [0, T], r $= \{r(t), t \in [0, T]\}$. Our objective is to determine r^* which maximizes the expected profit process $\Pi^*(t, s_t)$[5].

We can describe the expected profit as the expected value of 1) the capacity sold (K) multiplied by 2) the price charged for that capacity (r), over the time period until all the capacity is exhausted. If we use exponential discounting for converting these cash flows to present value, we obtain the following formula for $\Pi^*(t, s_t)$:

$$\Pi^*(t, s_t) = \sup_r \Pi(t, s_t, r_t) = \sup_r E[\int_t^{\tau_{r,t,st}} e^{-\rho u} r(u)\, K(u, r(u)))\, du] \tag{2.2}$$

with

- K(t, r(t)): capacity demand at time t for a given r(t),
- s_t: remaining capacity inventory at time t,
- $\tau_{r,t,st}$: time when the all the capacity is exhausted, and
- ϱ: discount rate.

Although our introductory example assumes a capital allocation based on premium writings over an underwriting year, our framework is more general and encompasses different capital allocation

[3] We work under the assumption that capital usage is triggered by underwriting decisions; it would be fairly easy to integrate other sources of capital usage, such as running-off of existing policies, by only considering the capacity available for writing new policies.

[4] E.g., managing the release of capacity between classes of customers, such as business vs. economy travellers.

[5] On a present value basis; note that if Π is not independent over time (e.g., Markovian processes), the expected value becomes conditional on history h_t; and we have $\Pi(t, s_t, h_t)$.

approaches (e.g., profit margin, rating agency or regulatory formulas, risk-based formulas, and so forth), time periods and definition of capacity (e.g., capital, resources).

We choose to use the required return on allocated capital r^* as our optimising variable, because prices are usually easier to adjust than capacity. It should be noted, however, that a similar revenue management framework could be derived to optimize capacity S^* for a given pricing policy r.

2.3 Insurance Cycle

Most insurers face fluctuations in demand over time, resulting from

- fluctuations in the flow of business shown to the insurer (e.g., changes in marketing/distribution strategy), and
- the insurance cycle: progressive or abrupt shifts in market "supply and demand" conditions, resulting in shifts in the insurer's demand function.

Our revenue management framework provides a tool to adapt to these fluctuations:

- it integrates expectations for market conditions (i.e., evolution of the insurance cycle), and
- it can be re-parameterized dynamically in light of the latest information on actual capacity usage and demand expectations; for instance, an insurer could decide to review its strategy and retune its revenue management model on a monthly basis.

An insurer can therefore manage the ups and downs of the cycle by adjusting its capacity charges so that its expected profits are maximized.

3. MODELING FRAMEWORK

We use the theory of revenue management to contend with our optimization problem: to "maximize the expected profits under the constraints of the capacity demand and capacity inventory processes."

In this section, we detail these two processes, formulate the optimization problem, and present methods to derive its optima.

3.1 Capacity Demand

The demand for the insurer's capacity K(t, r(t)) can be analysed in two parts: the business flow shown to and quoted by the insurer N(t), and the demand function d(t, r(t)) which reflects the acceptance level of quotes by prospects.

The demand for capacity at time t for a given r(t) is therefore K(t, r(t)) = N(t) d(t, r(t)).

Business Flow

The business flow is the flow of requests for the insurer's capacity, i.e., demand for quotes. It is modelled by a stochastic process N(t) which varies over time according to

- the overall demand for the insurance products sold by the insurer,
- the effectiveness of the marketing and distribution network, and
- seasonal fluctuations (e.g. large renewal months).

N(t) is typically modelled with Poisson $\lambda(t)$, Mixed Poisson $\Lambda(t)$ (e.g., Negative Binomial), or Geometric Brownian (μ_t, σ_t) processes. The model formulation and estimation can be derived from historical observations, after allowing for anticipated trends and future changes in the business flow process. In practice, this calibration exercise yields more robust results when the volumes of business are large and the business flow is stable over time. For instance, a personal lines insurer quoting thousands of policies daily would be expected to have a better assessment of its business flow than a reinsurer quoting a handful of treaties each day.

Demand Function

The demand function d(t, r(t)) reflects the price-elasticity relationship between the level of required return r(t) and the quantity of capacity sold at that level. It can be described as the probability distribution for the market reservation price, which is the highest price at which a prospect is willing to accept a quote.

The demand function depends on

- the competitive forces in the market place, determined by supply and demand, and
- the prospects' utility function.

Commonly used families of demand functions are

- exponential survival functions $d(t, r(t)) = e^{-r(t)/v(t)}$, and other Weibull survival functions,

- Normal survival functions $d(t, r(t)) = 1-\Phi_{\mu,\sigma}(r(t))$,
- iso-elastic functions $d(t, r(t)) = (1+r(t))^{-\nu(t)}$, and
- perfectly elastic functions, representing a single market clearing price.

The form and parameters for the demand function can be inferred from empirical observations of "hit ratios" and/or using the quotations systems available in some markets, such as UK Motor.

3.2 Capacity Inventory

Starting with a capacity of S, the capacity inventory process is defined as:

$$s(t) = S - \int_0^{\min(t,\tau_{r,t,st})} K(u, r(u))du \quad \text{and} \quad \frac{ds(t)}{dt} = -K(t, r(t)) \tag{3.1}$$

The capacity inventory is exhausted at a time $\tau_{r,t,st}$, at which point the demand process is turned off.

3.3 Optimal Pricing Strategy

Optimization Problem

As noted in the introduction section, our optimization problem is finding the pricing strategy r^* which maximizes the expected profits process $\Pi^*(t, s_t)$. This is summarized in Equation (3.2):

$$\Pi^*(t, s_t) = \sup_r E[\int_t^{\tau_{r,t,st}} e^{-\rho u} r(u) K(u, r(u))) du] \tag{3.2}$$

We will limit our range for r(t) to $[0,+\infty]$ as a negative required return strategy of selling below the expected marginal cost is always strictly dominated by abstaining from selling capacity. We note that, in practice, there may be instances where a negative required return strategy may be justified. For example, it may be more expensive to attract new clients when the market turns than it is to keep the current insureds at a loss.

Dynamic Programming

Dynamic programming is concerned with dynamic systems and their optimization over time, and we can use some of its classical results to find our optimal pricing strategy. Our optimization problem is an example of dynamic programming, with s(t) as the state variable, r(t) as the control variable and $\Pi(t, s_t)$ as the value function.

The key idea in dynamic programming is the Principle of Optimality is "An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision" [1].

This principle translates into the following recursive equation, known as the Optimality or Bellman Equation:

$$\forall \theta \in [t,T], \quad \Pi^*(t,s_t) = \sup_r \left[E[\int_t^\theta e^{-\rho t} r(u)\, K(u, r(u)))\, du] + e^{-\rho(\theta - t)} \Pi^*(\theta, s_\theta) \right] \tag{3.3}$$

Closed-form solutions have been derived for particular formulations of N(t) and d(t, r(t)). For instance,

- Gallego and van Ryzin [5]: time-invariant Poisson business flow with exponential demand functions,
- Zhao and Zheng [14] for time-variant Poisson business flow with iso-elastic demand functions, and
- Xu and Hopp [12] for Geometric Brownian business flow with iso-elastic demand functions.

But these solutions correspond only to a limited number of practical applications, and numerical solutions provide a more flexible alternative.

Numerical Solutions: Backward Recursion Algorithm

We compute our numerical solutions to the "discretized" optimization problem using the backward recursion algorithm. This approach consists in:

1. solving $\Pi^*(T, s_T)$ for each possible value of s_T,
2. solving $\Pi^*(T\text{-}1, s_{T-1})$ using the values computed for $\Pi^*(T, s_T)$: the principle of optimality states that the solution $r^*(T\text{-}1)$ for $\Pi^*(T\text{-}1, s_{T-1})$ will also maximize $\Pi^*(T, s_{T-1}\text{-}K(r^*(T\text{-}1), d(T\text{-}1, r^*(T\text{-}1))))$, and
3. solving $\Pi^*(t, s_t)$ for t=1…T-2 using the same iterative process.

The advantage of the backward recursion approach is its computational efficiency, resulting from the principle of optimality.

4. COMPARATIVE ANALYSIS

In this section, we apply the revenue management approach to a simple but realistic case study and compare the performance of different pricing strategies: one strategy based on shareholders' expectations, one strategy based on market returns, and three revenue management strategies[6] -- the first one static constant for the year, the second static but variable for each month (i.e., the pricing strategy for each month is set at the beginning of the year based on the initial anticipations), and the third one dynamic reparameterized each month (i.e., the pricing strategy is revised dynamically every month based on the revised anticipations for the rest of the year).

4.1 Case Study Scenario

The assumptions of our case study are as follows:

- Insurer:
  - → Mono-line insurer
  - → Capacity constraint based on underwriting decisions (the actual capital allocation formula is not relevant)
  - → Shareholders' expectations: 15% return on equity
- Capacity:
  - → Fixed capacity of $ 1billion
  - → Capacity is sold by blocks of $ 1million.
- Time period:
  - → One underwriting year with 12 monthly periods
- Business flow:
  - → Business flow process is Negative Binomial ($450 million, 0.2) with an expected value of $1.8 billion and standard deviation of $95 million. The simulations are plotted in Graph 1.
  - → Monthly business flows follow a seasonal pattern (cf. Graph 2), with each month simulated as a Negative Binomial variable
- Demand function:

[6] We call revenue management strategy any optimal strategy derived from the revenue management framework (i.e., integrating market conditions and expected demand); we get different "optimal strategies" depending on the context of the optimization problem. For instance:

- We can get the optimal fixed constant charge for the year, or allow the charge to vary monthly.
- We can get a static or dynamic strategy: a static strategy is set at the beginning of the period and remains unchanged, whereas a dynamic strategy is reset periodically using the latest information available.

→ Survival function of a Normal(μ(t), 3.5%) (cf. graph [3] with μ=5%); the Normal function has the advantage of being symmetrical and allowing negative capacity charges.

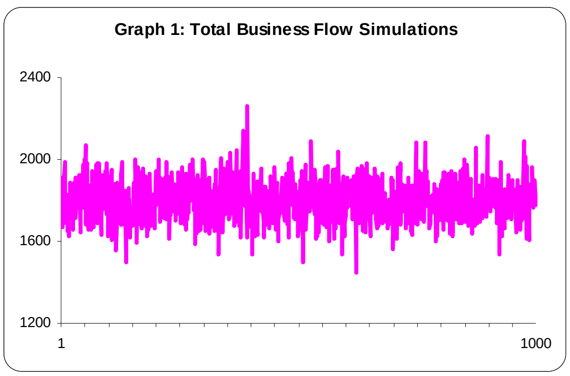
→ μ(t) is the average market reservation price; it can be interpreted as the market return in month t.

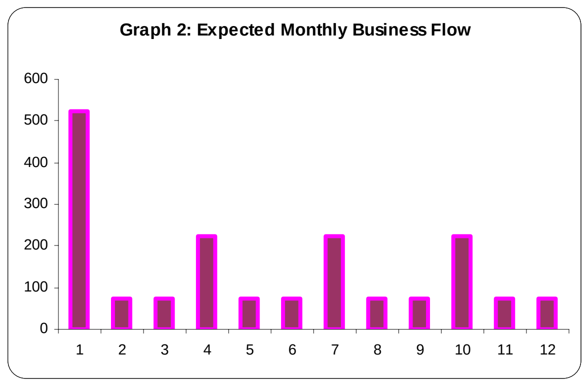
- Market conditions:

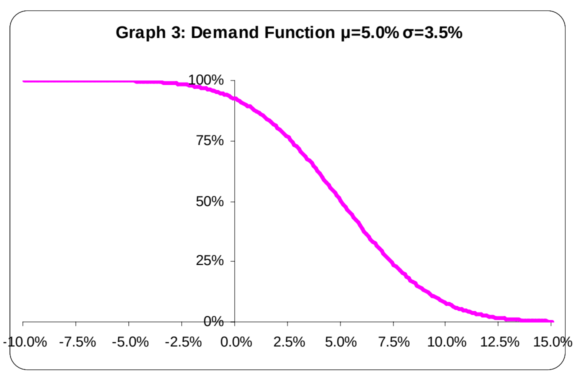
→ The market return μ(t) is decreasing linearly from 10% capacity charge in month 1 to 5% in month 12 (cf. graph [4]).[7]

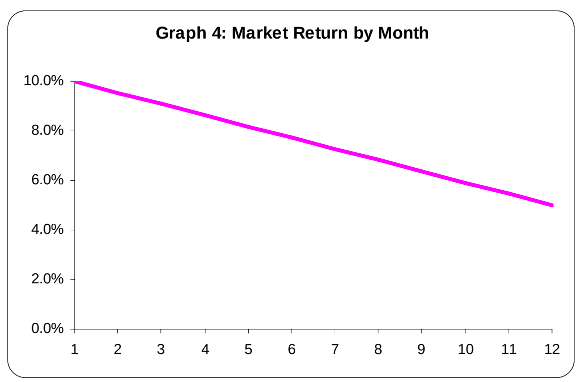
- Discount rate:

→ 5% per annum constant over the year.









4.2 Alternative Strategies

To respond to the questions specified in the introduction, we have compared the performance of the following five pricing strategies:

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[7] This assumption has been made to gauge the responsiveness of each strategy to changes in market conditions. An annual drop of 5% in market returns is not inconsistent with the changes observed in financial analysts forecasts at certain stages of a softening market.

- **Strategy 1:** "Charge 15% return for the year."

  → A fixed charge r for the year, based on the ***target return to the shareholders*** of 15%.

  → r= 15% for t=1 to 12.

- **Strategy 2:** "Charge the market return each month."

  → A variable charge r(t) based on the ***anticipated market conditions*** for each month.

  → r(t)= μ(t) for t=1 to 12

- **Strategy 3:** "Charge the demand-driven price for the year."

  → A fixed charge r based on the anticipated market conditions and the expected demand for the year.

  → r is determined using ***static revenue management for the yea***r.

  → Market conditions are determined by the weighted average μ(t) for year.

  → Expected demand is determined by the expected total business flow of $ 1.8 billion and by the insurer's demand function.

- **Strategy 4:** "Charge the demand-driven price each month."

  → A variable charge r(t) based on the anticipated market conditions and the expected demand for each month.

  → r(t) is determined using ***static revenue management for each month***.

  → Market conditions are determined by μ(t) t=1 to 12.

  → Expected demand is determined by the expected business flow N(t) for each month and by the insurer's demand function.

- **Strategy 5:** "Charge the re-forecast demand-driven price each month."

  → A variable charge r(t), recomputed at the end of each month based on

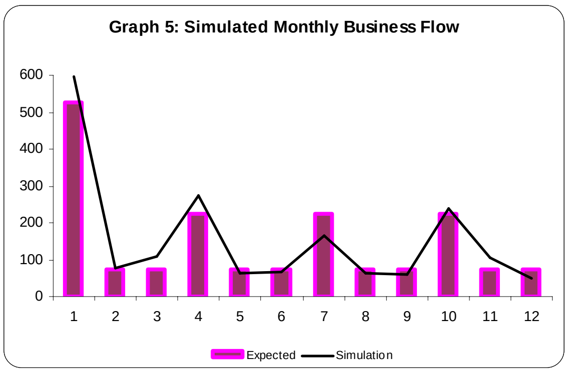
  1. actual writings to date and remaining capacity inventory, and
  2. anticipated market conditions and expected demand for rest of the year.

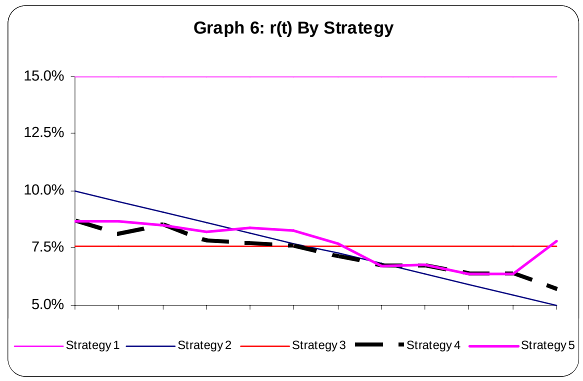
  → r(t) is determined using ***dynamic revenue management for each month***.

  → Market conditions are determined by μ(t) t=1 to 12.

→ Expected demand is determined by the expected business flow N(t) for each t=1 to 12 and by the insurer's demand function.

The behaviour of these 5 strategies is illustrated on the example detailed in Graphs 5 and 6. Graph 5 shows the simulated path for the business flow N(t); the total business flow is \$1.869 million. Graph 6 plots the values of r(t) under the five strategies.





- **Strategy 1** is a flat r(t)=15% for the year.
- **Strategy 2** is a linear decrease in r(t) from 10% down to 5%, reflecting the evolution of μ(t) over the months.
- **Strategy 3** is a flat 7.6% for the year; 7.6% being the revenue management optimum for the year based on the weighted average μ(t) for year (which is 8.1%).
- **Strategy 4** is the revenue management optimum strategy based on the initial expectations for N(t).
- **Strategy 5** is also the revenue management optimum but reparameterized at time t based on the remaining capacity inventory (we assumed that the anticipations for the demand functions, business flow and market conditions are not changed over the year).

We can note that **Strategy 5** suggests higher r(t) than **Strategy 4**: this results from the higher than expected business flow, which translates into a lower capacity inventory sold at a higher price.

4.3 Results

Table 1 compares the results for the 5 strategies over 1,000 simulations. For each simulation, the business flow N(t) is the only stochastic variable as we have assumed that the demand function was deterministic.

Table 1	Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5
$\bar{r}$	15.0%	8.1%	7.6%	7.6%	7.6%
K $m	65	920	973	977	995
Π $m	9.7	71.6	73.0	74.8	75.5

with:

- the average required return $\bar{r} = \left(\sum_{t=1}^{12} r(t)N(t)\right) / \left(\sum_{t=1}^{12} N(t)\right)$
- the total capacity sold $K = \sum_{t=1}^{12} K(t, d(t, r(t)))$
- the present value profit $\Pi = \sum_{t=1}^{12} \Pi(t, r(t))$

We note first that none of the strategies achieves the market return of 8.1% (i.e. weighted average $\mu(t)$). This is due to the fact that the expected business flow of $ 1.8 billion is low in relation to the $ 1.0 billion capacity and the 5:1 premium-to-surplus permitted; the company has to provide a discount on the market return to sell more and maximize its expected profits.

Comparing the different strategies based on total profit, we can observe that:

- **Strategies 2-5** based on market conditions are superior to Strategy 1, which is based on shareholders' expectations over the cycle.
- **Strategies 3-5** based on market conditions and expected demand are superior to Strategy 2, which only integrates market conditions.
- **Strategies 4-5** are superior to Strategy 3, as they are refined to include the monthly patterns in capacity demand and market conditions.
- **Strategy 5** is superior to Strategy 4, because capacity charges are set dynamically to incorporate the latest capacity inventory information.

As could have been expected intuitively, the optimal pricing strategy is the dynamic revenue management approach.

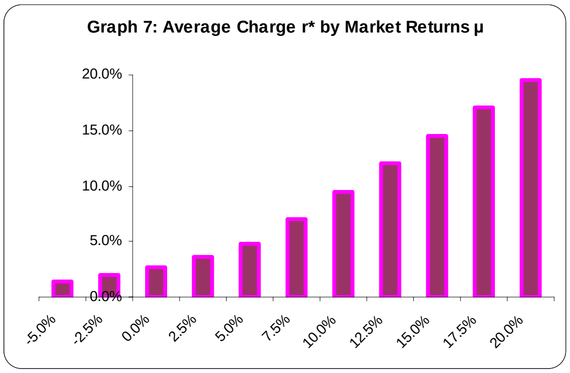
5. INSURANCE CYCLE APPLICATIONS

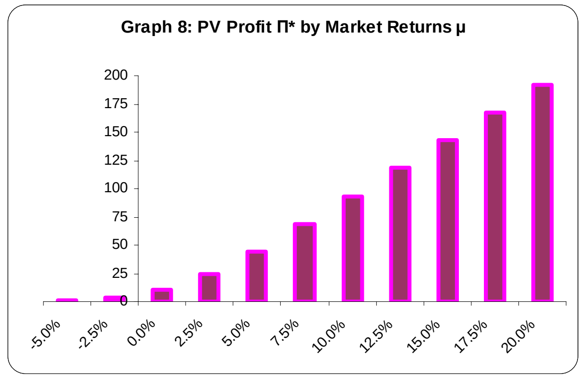
In this section, we detail practical applications of the revenue management approach to the management of the insurance cycle.

5.1 Optimal Pricing Strategy

We can use our model to investigate the optimal pricing strategy for the different stages of the insurance cycle. For this purpose, we have computed the optimal pricing strategy for various level of $\mu(t)$, kept constant for the year.[8]

Graph 7 plots the average required return r^* and Graph 8 the PV Profit Π^* in millions of dollars for different level of market returns $\mu(t)$. They illustrate how an insurer can adapt to the different market conditions over the insurance cycle, in order to maximize its expected profits.





We can observe that the optimal approach to negative market returns consists in setting r so that it captures and maximizes the returns on the few accounts with a positive return. In practice this means a low but positive capacity charge.

5.2 Optimal Capacity Strategy

As discussed in the introduction, a revenue management framework similar to the one presented can be utilized to optimize the insurer's amount of capacity to achieve a target return on equity for its shareholders over the cycle.[9] For this purpose, we have computed the optimal capacity strategy in order to achieve a 15% return on equity for various level of $\mu(t)$, kept constant for the year.[10]

Graph 9 plots the capacity S^* and Graph 10 the PV Profit Π^* in millions of dollars for different level of market returns $\mu(t)$.

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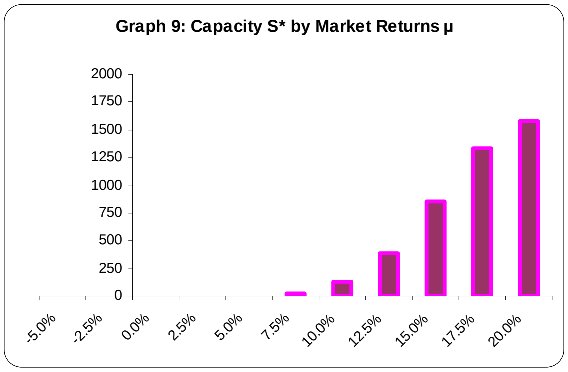
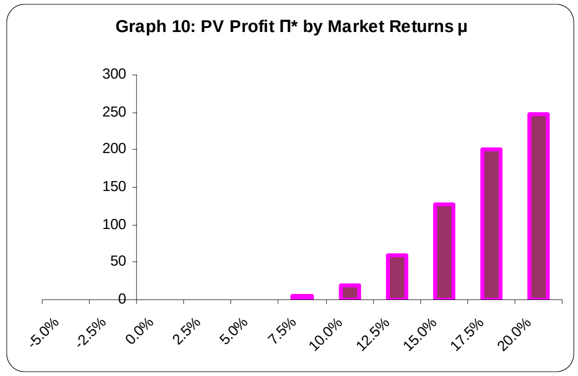
[8] all the other parameters as in the case study.
[9] all the other parameters as in the case study.
[10] all the other parameters as in the case study.





These graphs illustrate how an insurer can adapt to the different market conditions over the insurance cycle, in order to meet a target return on equity for its shareholders.

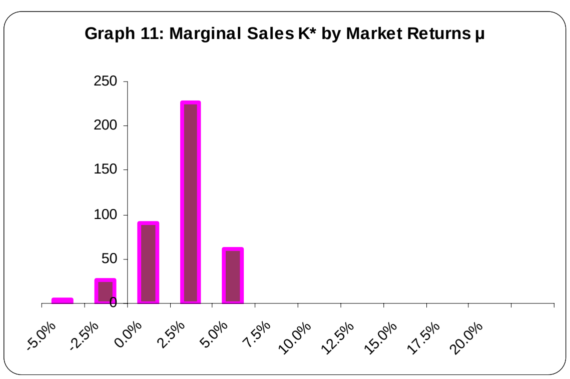
Although adjusting capacity is more problematic than adjusting capacity charges, one could envisage that this could be partially achieved through a flexible reinsurance programme and/or a proactive capital management policy (e.g. dividends, buybacks, flexible debt/equity arrangements…).

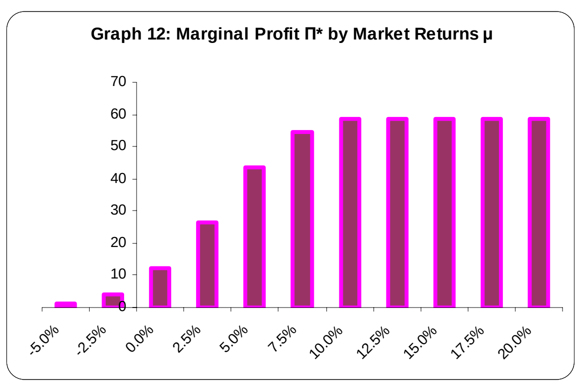
We can remark that the adjustments required to achieve the 15% target return on equity are fairly dramatic; and it becomes impossible for the insurer to achieve a 15% return on equity when the market returns are lower or equal to 5%.

5.3 Strategic Marketing Decision

We can also use our revenue management framework to assess the outcome of strategic decisions. For instance, we can compare the impact of a marketing campaign to increase business flow by 25% at different times in the insurance cycle.[11]

Graph 11 plots the marginal increase in sales and Graph 12 shows the marginal benefit of the campaign in millions of dollars for different levels of market returns $\mu(t)$.





[11] All the other parameters as in the case study.

We can note that the improvement in sales resulting from the increased business flow is the most significant when the market returns are between 0.0% and 5.0%, and nil above that level because the inventory would have been entirely sold without the marketing efforts.

The marginal profit, however, is most impacted for market returns greater than 5.0%, as the insurer is able to sell all its capacity and attract a higher return on it.

6. CONCLUSION

Our investigation has provided very insightful results, which challenge some of the current pricing practices. For an insurer deploying a fixed amount of insurance capacity over a period of time, we constructed revenue management strategies based on market conditions and expected demand, and observed that:

- *These strategies were superior to other strategies based on the target return to shareholders or market conditions alone.* As a result:

  → Companies should vary their capacity charge over time, as market conditions change.

  → Multi-line companies should adopt specific capacity charges for each business segment.

  → Pricing analyses should not be done independently of market conditions and expected demand; on the contrary, intelligence and research in these fields should be a key part of the pricing strategy.

- *Dynamic strategies delivered better results than static ones:*

  → Integrating anticipations of future market conditions helps maximize the return on a limited insurance capacity by ensuring that it is sold at the best rates.

  → Regular reparameterization helps integrate the latest information on capacity inventory and adjust the strategy accordingly.

- *An insurer can maximize its expected profits over the insurance cycle by adapting its capacity charge to market conditions, and expect a profit even when market returns are negative.*

- *Alternatively, this insurer can target a return on equity to its shareholders and adjust its capacity accordingly.*

To derive these conclusions we have used a revenue management framework, similar to those developed in other industries (e.g., airlines, hotels…). In these industries, revenue management is an essential piece of the pricing strategy. This framework proved very valuable and practical, and we are expecting that insurers will start implementing these techniques to enhance their competitive strategy.

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Class Ratemaking for Workers Compensation: NCCI's New Methodology

Tom Daley, ACAS, MAAA

Abstract:

For the first time in many years, NCCI is revising the methodology used to determine class relativities in workers compensation loss cost filings.

This paper will describe the new methodology NCCI has developed, and reveal the research approach and analyses underlying the modifications NCCI will be implementing to several key class ratemaking components. The paper will discuss in detail how the traditional areas of class ratemaking were modified, namely loss development, limiting large claims and applying expected excess provisions, updating credibility standards, and the derivation of industry group differentials.

The paper will also focus on the new NCCI class ratemaking approach from an educational perspective for actuaries who are just becoming familiar with workers compensation. Exhibits are provided in Appendix B illustrating the stepwise derivation of a loss cost for a classification from beginning to end.

Keywords: workers compensation; NCCI ratemaking; NCCI loss cost filings; class ratemaking; loss development by part of body; expected excess by hazard group.

1. INTRODUCTION

NCCI has recently modified the methodology used to determine class relativities for workers compensation insurance. The last time the class relativity methodology was modified took place in 1993. At that time, NCCI implemented the following changes: a) the number of policy periods used in determining pure premiums for each class was increased from three to five, b) the underlying class credibility formulas were modified, and c) the number of industry groups used for targeting class loss cost changes was increased from three to five.

Some of my colleagues would jokingly quip that the number of people who understood these changes increased from three to five. So the primary motivation of this paper is to document the new NCCI class ratemaking methodology and the research analyses supporting it. Many of my colleagues at NCCI made very significant contributions to the overall success of this huge undertaking, and are duly mentioned in the acknowledgement. This could not have been possible without their valuable insights and support.

1.1 Research Context

The focus of this research is to document the various analyses and research approach used to support the modifications being implemented within the NCCI class ratemaking methodology. Current CAS literature that addresses some of the same issues include "Workers Compensation Ratemaking" by Sholom Feldblum, and "Workers Compensation Classification Credibilities" by Howard C. Mahler.

1.2 Objective

This paper updates the CAS literature on workers compensation ratemaking techniques, with particular attention to recent modifications in the NCCI class ratemaking methodology for handling large claims, improving the predictive ability of class loss development factors, and the approach used for updating certain other important components such as industry group differentials and credibility standards. To address its absence in the current CAS literature, this paper also provides a detailed stepwise illustration of the new workers compensation class ratemaking methodology. The methodology supporting the aggregate change in a state's overall indicated loss cost level will not be addressed in this paper. The new methodology for determining the seven hazard groups and the methodology for determining the expected excess loss factors also will not be addressed in this paper.

1.3 Outline

The remainder of the paper proceeds as follows. Section 2 will discuss the reasons and impetus for the changes made, the thought process NCCI has followed, the specific class ratemaking methodology changes being implemented, and the supporting research analyses and results. Section 3 contains two appendices of exhibits: Appendix A contains the supporting research exhibits and Appendix B contains exhibits that illustrate the new methodology for calculating the loss cost for a class code from beginning to end.

2. BACKGROUND AND METHODS

There were three motivational factors underlying the research approach that NCCI followed in making some recent significant changes to its class ratemaking methodologies. They were:

- To improve the predictive ability and adequacy of loss costs by class code.

- To provide year-to-year stability of loss cost changes by class code.
- To explore the potential of new data elements that NCCI began collecting in the 1996 Unit Report Expansion (URE), and try to utilize them accordingly.

2.1 Availability of New URE Data Elements

Many of the NCCI states approved the collection of the URE data elements in 1996. Thus, the first complete policy period available in most states is policy year 1997. Furthermore, some states did not approve the collection of URE data in their state for a few more years (the last state was approved as late as January 1, 2002). Thus, in a few states, the database is less complete historically, adding further to the challenges of our research agenda.

The following is a list of some, but not all, of the new URE data elements to be reported to NCCI by carriers and available to the NCCI actuaries:

- Paid ALAE (case reserves were optional)
- Paid losses separate from "paid + case" losses
- Injured part of body
- Nature of injury
- Cause of injury
- Deductible reimbursement amounts
- Lump sum indicator
- Etc.

More recently, effective with policy period 1999 and subsequent, carriers began mandatory reporting beyond a 5th report for all WCSP unit data and URE data elements on all open claims, up to and including a 10th report.

2.1 Overview of the Methodology Changes

Several significant changes to the NCCI class ratemaking are currently being targeted for implementation in 2009. The majority of changes are contained in the following six areas:

1. Loss development factors (LDF) will be derived using claim characteristics such as injured body part, the open and closed claim status at 1st report, and the injury type category.

2. The loss development triangles are being expanded from five reports out to 10 reports (eventually).
3. Large claims will be capped at $500,000 and expected excess factors (derived from the new seven hazard group mapping by class code) will be used to calculate ultimate losses.
4. Serious and non-serious pure premium components will no longer exist. There will only be indemnity and medical components.
5. The computation of the industry group differentials was slightly modified.
6. The full credibility standards for indicated and national pure premiums were slightly modified.

Each of these six major areas will be discussed in this paper, some in much more depth than others, and a summary of the analyses underlying the decisions will be presented.

2.2 Background: The Current Loss Development Approach

It is important to understand the nuances of the former approach to gain a better appreciation for the changes NCCI is now implementing and the reasoning behind the changes being made. The source data used is the NCCI Workers Compensation Statistical Plan (WCSP) data. The previous approach used by NCCI to generate loss development factors for class ratemaking was to segregate the dollars of loss generated from claims into two loss development categories. They were a) the serious grouping and b) the non-serious grouping. An arbitrary dollar value, referred to as the critical value, which varied significantly by state, was determined for each loss cost filing. All permanent partial claims whose indemnity dollar amount, as measured on a "paid + case" basis, exceeded the critical value were categorized to be included in the serious grouping, and referred to as major permanent partial claims. Four loss development triangles were compiled from the dollars of losses associated with these claims. The four triangles compiled were indemnity and medical, and each had a serious and non-serious component. The serious grouping consisted of all fatalities, all permanent total claims, and the major permanent partial claims (i.e. those claims whose indemnity dollar amounts exceeded the critical value). The non-serious grouping consisted of all temporary total claims, the remaining minor permanent partial claims, and the medical-only claims. Examples of each of the serious and non-serious loss development triangles for a large state are shown in Exhibit 1.

WCSP "paid + case" loss data is reported by carriers to NCCI at five different reports for open claims. The losses are evaluated @18, @30, @42, @54, and @66 months, respectively. A tail factor was applied to the serious loss development triangles only, and was derived from NCCI financial call data used in the overall aggregate loss cost indication for the state. It was assumed that all loss development beyond the 5th report was inherently due to serious claims only. In order to balance to the financial data tail, a significantly large tail factor was applied to the serious losses to generate a 5th-to-ultimate, while a tail factor of unity was applied to the non-serious losses. An illustration of the derivation of the tail factor is also found in Exhibit 1.

The current loss development approach had four shortcomings, which made its serious and non-serious loss development groupings less than optimal. The key shortcomings were:

1. As claims matured, many claims would "cross over" the critical value at subsequent reports, and therefore be reassigned into the serious grouping, and thus, distort the predictive ability of the loss development factors in the serious and non-serious triangles.
2. Severity was not a good indicator of the propensity of a claim to develop in the future.
3. The medical dollar amount was ignored in determining whether or not a claim was categorized as serious or non-serious.
4. No distinction between serious and non-serious loss dollars was made within the medical loss triangles from 1st through 5th report. The only distinction between serious and non-serious medical was that a 5th-to-ultimate medical tail factor was applied to the medical loss dollars associated with the serious lost-time claims.

2.3 The Problem of Critical Value Crossover

The research approach began as a review of the critical value methodology, which had begun to be used in class ratemaking at NCCI in 1966. A previous attempt years earlier at improving the critical value methodology involved the idea of using an open and closed claim indicator, and only applying loss development to open claims. Although that idea was not embraced at the time, a better variation of it will be introduced to the reader later in the paper.

Exhibit 2 demonstrates the distorting impact that critical value "crossover" inflicts on a dataset of permanent partial claims countrywide. Claims below the critical value are deemed minor while those that exceed it are deemed major. Various link ratios were computed for comparison from 1st report to 4th report. The true distortion of critical value "crossover" is illustrated by the second and third rows of the indemnity and medical sections of Exhibit 2. These rows consist of claims where the status changed from major to minor, and vice versa, between the 1st and 4th reports. Columns (4) and (5) on Exhibit 2 provide a stark contrast of the distortion critical value crossover can inflict on the predictive nature of a link ratio.

Although not illustrated in Exhibit 2, a "natural crossover" of claims moving between injury types may provide similar distortions to link ratios as claims evolve over time. It is common in workers compensation for a temporary total claim to eventually evolve into a permanent partial claim, or a medical-only claim at 1st report to potentially become a lost-time claim at subsequent reports. The manner in which NCCI's actuaries address natural crossover will be presented later in this paper. One of the goals of the new methodology was to try to mitigate "crossover" in order to generate loss development factors that were more predictive.

2.4 How We Solved the Crossover Problem

A fresh approach was begun by investigating a new field, the injured part of body, that NCCI began collecting on its Unit Report Expansion starting with policies effective in 1996. NCCI actuaries soon began researching to see if the injured body part provided any causal relationship upon predicting whether or not a claim's loss dollar amount developed upward at later reports. The initial approach NCCI took to research its loss development methodology proceeded as follows:

1. Extract a large volume of claims containing claim-specific information such as injury type, report level, injured body part, and associated dollars of incurred loss.
2. Review the impact that critical value "crossover" (illustrated earlier) and injury type "crossover" may have upon loss development factors.
3. Determine if claim severity is an indicator of the propensity of a claim to develop.
4. Analyze the injured body part to determine if it could provide value as a predictor of a claims' propensity to develop (or not develop).
5. Group the body part and injury type combinations into those more likely to develop and those not likely to develop so that the groupings are more predictive than the serious and non-serious groupings.
6. Update NCCI's Actuarial Committee and incorporate their feedback.

Note that at the outset, the impact of the claim status (open or closed) was not considered. As the main thrust of the initial research was analyzing body part and injury type combinations, and mitigating the crossover problem, claim status was not incorporated until much later. How claim status was incorporated into the research will be described later on in the paper. Exhibits 2 through 9 reveal the initial research findings outlined in the steps above.

The analysis of the distortion to link ratios that "cross over" caused provided valuable insight. True loss development can best be determined if claims are not allowed to migrate across different development groups to the extent possible. As claims were moving over the critical value and across the injury types, a solution was posed as to how to research whether or not the injured body part was a determining characteristic of loss development. The solution was to "lock down" the entire dataset of claims being studied at each link ratio. Thus, the exact same set of claims were observed at adjacent reports, such as 1st to 2nd, and the loss development measured accordingly. Note, the set of claims used to observe the loss development from 2nd to 3rd report could be a different set of claims than those observed at 1st to 2nd report.

This approach was the key to determining which injured body parts developed more or less than others, and as you will later read, it also helped NCCI determine that two other key claim characteristics (claim status at 1st report and injury type) can also be associated with more or less dollars of loss development.

2.4.1 How Was the Injured Body Part Approach Determined?

Two new loss development triangle groupings were envisioned. The first was a grouping of claims whose injured body parts, and associated dollars of loss, were likely to develop upwards over time. The second grouping would consist of claims whose injured body parts, and associated dollars of loss, were not-likely-to-develop upwards over time. Grouping body parts together made sense as there were 55 body part codes in the WCSP, and credible volume at a state level by injured part of body was a concern. Loss development between the two groups would have to be compared relative to one another, as the losses in some states develop significantly more than others. For example, a back claim filed in a state having a lot of attorney involvement and longer durations would be expected to develop more than a similar back claim in a state with little or no attorney involvement and shorter durations. (As an example of duration, many states have time limits for benefits, such as 300 weeks or 425 weeks for permanent partial claims.)

The next step was to determine which of the 55 body part categories would be mapped into the likely-to-develop and not-likely-to-develop. A listing of all the body part codes and the grouping to which they were mapped is shown in Exhibit 6. One drawback in using the NCCI WCSP data for determining loss development in a long-term line of insurance like workers compensation is that only five reports of losses are provided to NCCI by carriers, yet much of the loss development can and does take place beyond the 5th report. At times, certain analyses only used four reports of data simply because the 5th and final report was not yet reported to NCCI as the body part code was introduced in 1996 for the first time.

Two different analyses were completed for body part grouping. The result of the first analysis is shown in Exhibit 3. This analysis measured loss development dollars by fixing the set of claims from 1st through 4th report (at the time, 5th report was unavailable), quantifying the observed loss development per claim as follows:

$$\frac{(\text{Reported Losses @}4^{th}\text{ - Reported Losses @}1^{st})}{\text{Number of claims}}$$

This approach provided an initial insight into which body parts developed more than others. Exhibit 3 shows that the following general areas of body parts contributed the largest amount of development per case: back, head, neck, multiple body, and internal organs. The downside of using this approach as the only measure for making body part decisions is that much loss development in workers compensation happens beyond 5th report, and until recently, carriers did not report WCSP data beyond the 5th. (Starting in 2005, NCCI began collecting 6th reports of open claims, and will eventually collect up to a 10th report. This expansion will be used to extend the class loss development triangles out beyond the 5th report, and eventually to a 10th).

Thus, a second measure was considered to fine-tune the decision making for determining groupings of body parts into likely-to-develop and not-likely-to-develop categories. The second measure was to determine what percentage of claims, sorted by body part, remained open at 5th report. Exhibits 4 and 5 illustrate these results for countrywide permanent partial and temporary total claims, respectively. Those body parts having a higher percentage of open claims at the 5th report were assumed to be more likely to develop.

Actuarial judgment also played a role in the final decisions to determine into which groupings the various body parts were ultimately placed. Some consideration was given to the fact that certain body parts are considered scheduled injuries in states having scheduled permanent partial injuries. Body parts like toes, fingers, hands, feet, arms, and legs are often mandated a pre-determined dollar amount in statutory benefit schedules, and therefore, are not likely to develop upward. Exhibit 6 summarizes the grouping to which each body part has been mapped.

2.4.2 How Was the Injury Type Considered?

More refinements to the grouping logic were researched after the body part mappings were completed. The first characteristic considered was the claim's injury type. In workers compensation, different levels of indemnity benefits are paid based upon the injury type. The injury types are: fatal (F), permanent total (PT), permanent partial (PP), temporary total (TT), and medical only (MO).

Two injury types initially examined in depth were TT and PP, as this is where the majority of claims and dollars of loss resides. Once the body parts were mapped to the likely-to-develop (L) and not-likely-to-develop (N) groupings, a few different tests were performed. The first was whether or not severity was a good indicator of the likelihood of a claim developing and the second was a test to see if the groupings of body parts produced link ratios that were larger for the L grouping than the N grouping. The second test would substantiate the mapping of body parts to the L and N groupings.

Exhibit 7 shows results for both tests, again on a countrywide basis. A critical value of $26,000 was selected.[1] The claims were fixed at each adjacent link ratio to eliminate both critical value and natural "crossover" and to allow us to observe the development pattern that resulted. The results shown on Exhibit 7 clearly illustrated three key observations: 1) claim severity itself is not a predictor of higher loss development, as evidenced that claims <u>below</u> $26,000 developed much greater for TT than those which began at a value <u>greater</u> than $26,000, 2) the medical pattern behaved differently than indemnity, in that the ldf from 1st to 5th was about the same whether above or below the $26,000, and 3) claims within the L grouping developed significantly more than claims in the N grouping for both PP and TT, as evidenced by the much higher link ratios.

At this point in the research process, the feedback from NCCI's Actuarial Committee was positive, and the Committee requested to see what the new groupings and their new development pattern would look like by state as compared to serious and non-serious loss development factors (LDF). Exhibits 8 (indemnity) and 9 (medical) provide the support for LDF comparisons for two states, identified only as a large state and a small state. Note the characteristics of the serious and non-serious development factors: "crossover" generates very large serious factors and very low non-serious factors. At the same time, relative to the serious and non-serious LDF, the likely-to-develop (L) and not-likely-to-develop (N) are much different: L produces LDF patterns that are much lower

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[1] $26,000 was an indemnity dollar amount determined arbitrarily assuming a typical weekly indemnity benefit of $500 per week for 52 weeks.

than serious while N is much higher than the non-serious LDF. As will be shown later on in this paper, class equity is affected in that class codes with more serious losses, such as contracting codes, will experience reduced loss costs under the new loss development methodology while classes with more non-serious losses (office and clerical) will experience increased loss costs due solely to the change in loss development methodology. (Other components of the new methodology do provide an offsetting impact. The expected excess provision is a good example as it is greater for contracting codes than it is for office and clerical.)

Also note that for Exhibit 9, the previous methodology only provided a total LDF for medical from 1st to 5th. Under the new methodology, an improvement is generated in that LDFs are bifurcated into two homogeneous groupings with distinctly different loss development patterns; that is, L and N. This refinement should improve class equity.

Exhibits 8 and 9 show LDFs on an unlimited basis and on a limited (@ $500K) basis. This is because unlimited factors are used in the previous class ratemaking methodology. The new class ratemaking enhancements include limiting individual claims at $500K. Thus, a portion of the difference in the magnitude of LDF from previous to new methodology is due solely to a loss limitation being applied to the new NCCI class loss development methodology.

It is important to note that Exhibits 8 and 9 are illustrating LDF patterns using the following loss development groupings of claims:

$$\text{Likely (L)} = \text{Fatal} + \text{PT} + \text{PP-L} + \text{TT-L} \tag{2.1}$$

$$\text{Not Likely (N)} = \text{PP-N} + \text{TT-N} + \text{MO} \tag{2.2}$$

$$\text{Serious} = \text{Fatal} + \text{PT} + \text{Major PP} \tag{2.3}$$

$$\text{Non-Serious} = \text{Minor PP} + \text{TT} + \text{MO} \tag{2.4}$$

Because most fatal and permanent total claims are open at the 5th report, it was quickly decided to put them into the L grouping. This also coincided well with the previous serious grouping. The reasoning used for assigning medical-only claims to the N grouping was that almost all of them close out quickly, and thus, are unlikely to develop further.

The injury types that provided the NCCI actuaries with the biggest challenges were the permanent partial and temporary total claims. In most states, these two injury types comprise between 70% and 80% of all loss dollars incurred. These claims also are intricately connected as many temporary total claims evolve into permanent partial claims as injured workers reach a point in time referred to as maximum medical improvement. It was for these reasons that the research on injured body part focused on these two injury types for the most part. The L and N groupings would also benefit from a fairly even distribution of loss volume if each of these injury types were assigned to either the L or the N, based on injured body part.

It was at this point in the research that some other NCCI colleagues were becoming heavily involved in the class ratemaking research, and began asking questions and probing into the details underlying the assignment of claims into the L and N groupings. The team started investigating injury type loss development patterns closely for the large state/small state analysis, and started questioning if other URE data elements could be used to further refine the L and N groupings. Some NCCI actuaries thought the fatal claims should be N and not L. Others thought temporary total claims should all be assigned to the N grouping. Others felt the disparity between the magnitude of the LDF for the L and N groupings was not large enough. So more research was conducted to try to resolve the issue of what is the optimal loss development grouping.

2.4.3 The Final Refinements to the Loss Development Groupings

Staff explored other URE data elements to determine if their presence could better determine the likelihood that a claim might develop upward. Some of the data elements explored were claims including ALAE, the nature of injury, and the cause of injury. None of these provided any solutions. However, there was one data element that was clearly connected with the propensity of a claim to develop (or not). And that was the open or closed claim status. The majority of development was coming from claims that were open at 1st report. It seemed so logical. Almost all actuaries, and non-actuaries, would agree that closed claims are not likely to develop (note that there are a small percentage of claims that do close and reopen in workers compensation). So the research continued.

A new countrywide (all NCCI states) data extract was created for policy years 1999 through 2002 at each available report level, and for 1999, that now encompassed six reports of data. Dollars of loss were compiled for each policy year and state as follows:

- By injury type at each report level
- By the claim status open (O) or closed (C) at first report and each subsequent report level

- By the body part category L or N
- Losses were limited at $500,000
- Indemnity and medical aggregated separately
- Only states and years in URE format (Oregon did not approve URE until 1-1-02.)

The loss dollars were aggregated countrywide. Claims having an injured body part that was assigned to the L grouping were referred to as "likely" body parts. Similarly, claims having an injured body part that was assigned to the N grouping were referred to as "not-likely" body parts.[2] All claims were "locked down" at each report level to examine the impact of true loss development, and therefore, not allowed to move across subcategories. Once "locked down" at the initial report, no claims were allowed to enter or leave the group throughout the entire observed development timeframe (i.e. 1st through 6th report or 2nd through 6th report). This is a different variation of the "lock down" than that used earlier in the initial research of injured part of body, where the set of claims was the same only for adjacent reports for determining a link ratio. The loss dollars were segregated into the following four subcategories and the LDF were computed:

- LO — "likely" body part and claim open at 1st report
- LC — "likely" body part and claim closed at 1st report
- NO — "not-likely" body part and claim open at 1st report
- NC — "not-likely" body part and claim closed at 1st report

Exhibits 10 and 11 display a myriad of LDF combinations that have become the heart and soul of the new loss development proposal. Every injury type is broken out into the four subcategories and for policy years 1999 and 2000, the LDF are illustrated from 1st – 6th and 1st - 5th, respectively. The LDF patterns provided NCCI with remarkable evidence suggesting further refinements to the loss develop groupings should be made. Several key observations and conclusions generated from the analysis illustrated on Exhibits 10 and 11 follow. Specifically, for permanent partial (PP), temporary total (TT), and medical-only (MO) claims:

1. Losses from claims in the L body part categories consistently develop much more than

[2] In the future, NCCI may rename the "likely" body parts as Part of Body Group A and the "not-likely" body parts as Part of Body Group B to differentiate the body part assignments from the loss development groupings.

its N counterpart. Thus, the body part assignments are sound.

2. Claims that were open (O) at 1st report develop much more than the closed (C) claims do. Thus, the combination of L and O at 1st report generates the largest LDF by far.

3. Focus on the arrows on Exhibit 10 for TTLC and PPLC. Claims that were L and closed (C) at 1st report align more closely with the TT-N and PP-N grouping. Thus, by moving claims having the combination of L and C at 1st report into the N grouping further refines the LDF patterns.

4. Exhibit 11, Option 1 demonstrates that a greater differentiation in LDF magnitude occurs when the likely closed (LC) claims were removed from PP and TT and placed in the N grouping. This is seen by a comparison of Option 1 relative to the grouping labeled "current" in the row above it. (Thus, within option 1, L = Fatal + PT + PPLO + TTLO.)

5. Although similar LDF patterns were observed for MO, it was decided to keep all MO claims in the N grouping for two reasons: a) only 1% of all losses shift, and b) some carriers may report their entire inventory of MO claims as closed claims when reporting WCSP data to NCCI, which could be problematic.

6. Claims from the permanent total (PT) and fatal injury types do <u>not</u> demonstrate the same pattern of loss development characteristics. That is, the L and N body part categories do not discern loss development patterns as it does in other injury types. The LDF behave in the opposite manner (i.e., L < N). Also, the opposite behavior happens with the open and closed claim status LDF (C > O).

The results of the last observation suggested that even more research should be conducted on the development patterns of fatal and PT claims. Natural "crossover" across injury types further complicates the analysis so three groups of fatal and PT claims were created and the LDF observed:

- Those claims which remained within the injury type across all report levels
- Those claims that moved into the fatal and PT injury types after initially being reported as another injury type at 1st report
- Those claims that migrated out of the injury type at later reports after initially being reported as fatal and PT at 1st report

In this analysis, the injury type of claims were observed at 6th report for PY 1999 and 5th report for PY 2000. Assuming the most recent reported injury type is the best observation for these PT and fatal claims, we then observed the injury type of these claims at 1st report. Exhibit 12 shows the loss development patterns for the three groups of fatal claims while Exhibit 13 shows the same for the PT claims. Several key observations, conclusions, and reasoning follow that were generated from the analysis illustrated on Exhibits 12 and 13. And, importantly, the debate over whether fatal claims should be placed in the L or N grouping was resolved.

1st observation: Fatal claims (at 6th or 5th report), which were reported initially as a fatality at 1st report, distinctly developed downward from 1st through 6th (and 5th) report (see top section of Exhibit 12).

Conclusion #1: Move fatal claims at 1st report into the N grouping, and no longer assign them as likely-to-develop.

Reasoning: This one makes practical sense because only the dependents, if any, of the deceased worker receive benefits and these benefits are defined streams of payments over time in most states. A few states pay a predetermined lump sum of money to beneficiaries. Also, there is no need for carriers to estimate case reserves for future medical costs when the injured worker dies.

2nd observation: Claims that become fatalities at subsequent reports (2nd through 6th) developed significantly upward from 1st to 6th (and 5th) report (see middle section of Exhibit 12).

Conclusion #2: Claims that become fatalities at subsequent reports (2nd through 6th and eventually 10th report) will continue to be categorized in the L grouping.

Reasoning: Claims of this nature were observed within all injury types, and conditions subsequently worsened to the point where the injured worker died. Large amounts of upward loss development dollars were observed, and medical costs become very large in many of these claims over time.

Note at the bottom of Exhibit 12 a small amount of claims reported as fatalities at the 1st report actually moved to other injury types at subsequent reports. Upon investigating several of them, it was concluded that compensability was actually an issue. In other words, some claims were contested as to whether or not the death was due to work-related causes. In a few other instances, the initial injury type was simply misreported and corrected. As a group, this small number of claims did develop downwards and will be assigned to the N grouping.

Now refer back to Exhibit 11, Option 2. It demonstrates that a greater differentiation in

LDF magnitude occurs, particularly for indemnity, when the fatal claims at 1st report were removed from the L grouping and placed in the N grouping. This is seen by a comparison of Option 2 relative to the groupings labeled "current" and Option 1 in the rows above it. (Thus, within option 2, L = Fatal – Fatal @1st + PT + PPLO + TTLO.)

3rd observation: An overwhelming number of PT claims (at 6th or 5th report), which were reported initially as other injury types at 1st report, developed significantly upward from 1st through 6th (and 5th) report (see middle section of Exhibit 13).

Conclusion #3: Categorize all PT claims, regardless of the report, into the L grouping.

Reasoning: Many PT claims were observed whereby they were initially reported as another injury type, and conditions subsequently worsened to the point where the injured worker became permanently totally disabled. Large amounts of upward loss development dollars were observed, and the medical costs become very large in many of these claims over time. Also, almost all PT claims were open at 1st report and were comprised mainly of Group A parts of body (i.e., likely).

It should be noted that a subset of PT claims that stayed within the PT injury type at all reports had a slight downward development (see top of Exhibit 13). After considering moving those out of the L grouping, similar to fatal at 1st report, it was decided to be appropriate to keep assigning them to the L grouping, as most were still open at a 6th report, and could eventually develop upwards out in the tail if the claimant's condition worsened in the future.

Thus, Option 2 on Exhibit 11 represents the proposed final L grouping, which excludes fatalities at 1st report, and includes all PT claims. The equation is as follows:

$$L = \text{Fatal} - \text{Fatal @}1^{\text{st}} + \text{PT} + \text{PPLO} + \text{TTLO}. \tag{2.5}$$

2.4.4 What about the Tail Factor?

The tail factor in workers compensation presents a formidable challenge to NCCI actuaries. In aggregate ratemaking, in order to determine a state's overall indicated change in loss cost or rate level, a tail factor is estimated separately for indemnity and medical and attached currently at a 19th report. NCCI financial call data is used as the source. However, only five reports of the WCSP data, which is the basis for class relativities, was required to be reported to NCCI by its affiliated carriers. This has changed recently. Beginning with policy year 1999, NCCI is now collecting up to 10 reports of open claims.

For class ratemaking, in order to maintain consistency for a state's class relativities, the financial tail factor is used as a starting point. NCCI actuaries assume that 100% of loss development beyond the 5th report is due to development on the serious claims, and 0% due to development on non-serious claims. A 5th – ultimate LDF is computed from the state financial data, referred to below as Fin5U. Thus, the following formula is used for indemnity losses to determine the class ratemaking 5th – ultimate LDF, referred to below as $Class5U_I$. It is applied to serious losses at 5th report.

$$\text{Class5U}_I = [\text{SER\$}_I + (\text{SER\$}_I + \text{NS\$}_I) * (\text{Fin5U}_I - 1.000)] \,/\, \text{SER\$}_I. \tag{2.6}$$

Where,

$SER\$_I$ = two years of limited "paid+case" serious indemnity loss dollars on-leveled and developed to 5th report for the state;

$NS\$_I$ = two years of limited "paid+case" non-serious indemnity loss dollars on-leveled and developed to 5th report for the state;

$Class5U_I$ = unlimited 5th – ultimate indemnity (I) tail factor applied to <u>serious</u> losses at 5th report for each class code. No tail is applied to non-serious losses;

$Fin5U_I$ = Unlimited statewide financial data 5th – ultimate tail factor for indemnity (I).

The same exact approach is also used to determine a 5th – ultimate tail factor for medical losses, but is not shown here. Only the subscript would change from (I) to (M). Also note that although individual claims are limited in the current NCCI ratemaking at five times the state's serious average cost per case, loss development factors are unlimited. By rearranging the formula, the following is derived:

$$\text{Class5U}_I = \text{Fin5U}_I + [(\text{NS\$}_I \,/\, \text{SER\$}_I) * (\text{Fin5U}_I - 1.000)] \tag{2.7}$$

Note that the magnitude of the class tail factor is inversely proportional to the percentage of serious losses in a state relative to the non-serious losses. The lower the percentage of serious losses, the higher the class ratemaking tail factor that is applied to the serious losses. Again, recall Exhibits 8 and 9, and how much higher the serious LDF-to-ultimate towered above the likely-to-develop LDF-to-ultimate in the bar charts. A good portion of that phenomenon is due to the pro rata share of serious and non-serious losses in a state. States with lower percentages of serious losses relative to non-serious generally have a much higher serious tail factor applied, all else equal. As you will see shortly, the new class ratemaking loss development methodology will modify that phenomenon of a highly leveraged tail factor.

The tail factor under the new methodology starts with a similar formula to determine the class ratemaking 5^{th} – ultimate LDF, referred to below as Class5U. The notation is analogous except the likely-to-develop (L) and the not-likely-to-develop (N) groupings are substituted for serious and non-serious. From an analysis of other states, initial indications are that the pro rata share for L and N is closer to 50% than for serious and non-serious.

The previous methodology assumed that all loss development in the tail beyond 5^{th} report is due to serious claims only. This implies that 100% of the tail loss dollars were applied to serious and 0% applied to non-serious. NCCI is modifying this assumption to be that a percentage of tail development, *y*, will be applied to the N grouping dollars of loss and (1-*y*) will be applied to the L grouping dollars of loss. This practicality allows a portion of tail development to be applied to the not-likely-to-develop losses. Thus, two new class ratemaking tail factors could be applied at 5^{th} report, one for L and one for N. The formulas are as follows:

$$\text{Class5U}_{L,\,I} = [\text{L\$}_I + (1\text{-y})*(\text{L\$}_I + \text{NL\$}_I)*\ (\text{Fin5U}_I - 1.000)]\ /\ \text{L\$}_I. \tag{2.8}$$

$$\text{Class5U}_{N,\,I} = [\text{NL\$}_I + \text{y}*(\text{L\$}_I + \text{NL\$}_I)*\ (\text{Fin5U}_I - 1.000)]\ /\ \text{NL\$}_I. \tag{2.9}$$

Where,

$L\$_I$ = two years of limited likely-to-develop "paid+case" indemnity loss dollars on-leveled and developed to 5^{th} report for the state.

$NL\$_I$ = two years of limited not-likely-to-develop "paid+case" indemnity loss dollars on-leveled and developed to 5^{th} report for the state.

$Class5UL_{L,\ I}$ = a likely-to-develop 5^{th} – ultimate indemnity (I) tail factor applied to likely-to-develop losses at 5th report for each class code. It is limited at state threshold T.

Class5UL$_{N,\ I}$ = A not-likely-to-develop 5th – ultimate indemnity (I) tail factor applied to not-likely-to-develop losses at 5th report for each class code. It is limited at state threshold T.

Fin5U$_I$ = Limited (at T) statewide financial data 5th – ultimate tail factor for indemnity (I).

y = percentage between 0% and 100% used to allocate a portion of tail development dollars to the not-likely-to-develop grouping.

Note that the new methodology uses limited loss development dollars (all claims are limited at \$500K at all report levels). The previous methodology only limited loss dollars at the latest report, while LDF were unlimited. The same exact approach is also used to determine a 5th – ultimate tail factor for medical losses, but is not shown here. Only the subscript would change from (I) to (M).

As NCCI collects URE data out to a 10th report, *y* could vary in magnitude as the tail attachment moves out toward 10th report. For example, at 5th report, *y* may be a higher percentage than what *y* would be at 10th report. It is also a consideration worth noting that *y* could vary between indemnity and medical. Based on very recent research observing actual WCSP loss development patterns through 7th report, NCCI is initially using a value of 20% for *y* for both indemnity and medical for all tail attachment points out to 10th report. Thus, 80% of the total dollars of tail development will be assigned to the likely-to-develop loss triangle, and 20% of the dollars to the not-likely triangle. NCCI will revisit this assumption when more WCSP unit reports are available through 10th report.

The formulas above may be written in a more general form to account for the various tail attachment points that may be used in the future. Let each tail attachment point be time *t*, *t* = 5,6,7,8,9,10. Then the formulas above may be rewritten as follows:

$$\text{Class } tU_{L,I} = [L\$_I + (1-y)*(L\$_I + NL\$_I)*(\text{Fin } tU_I - 1.000)] \,/\, L\$_I \tag{2.10}$$

$$\text{Class } tU_{N,I} = [NL\$_I + y*(L\$_I + NL\$_I)*(\text{Fin } tU_I - 1.000)] \,/\, NL\$_I \tag{2.11}$$

Where,

L\$$_I$ = two years of limited likely-to-develop "paid+case" indemnity loss dollars on-leveled and developed to *t*th report for the state.

$NL\$_I$ = two years of limited not-likely-to-develop "paid+case" indemnity loss dollars on-leveled and developed to t^{th} report for the state.

Class $tU_{L,\ I}$ = A likely-to-develop t^{th} – ultimate indemnity (I) tail factor applied to likely-to-develop losses at t^{th} report for each class code. It is limited at state threshold T.

Class $tU_{N,\ I}$ = A not-likely-to-develop t^{th} – ultimate indemnity (I) tail factor applied to not-likely-to-develop losses at t^{th} report for each class code. It is limited at state threshold T.

Fin tU_I = Limited (at T) statewide financial data t^{th} – ultimate tail factor for indemnity (I).

y = percentage between 0% and 100% used to allocate a portion of tail development dollars to the not-likely-to-develop grouping.

t = time t representing the report level of WCSP data at which the attachment point for the class ratemaking tail is applied. t = 5,6,7,8,9,10

One improvement in the revised tail factor is the distribution of losses between L and N are more evenly distributed than the previous serious and non-serious distribution. This should help temper the leverage on the LDF in the new methodology. The tail factor is an area that warrants continued research, and should improve as 10 reports of data are analyzed.

2.4.5 Summary of the New Loss Development Proposal

Table 1 summarizes all of the decisions that were researched, discussed, and made by NCCI up to this point in the paper. It introduces the Part of Body Group A and Group B terminology to refer to parts of body that are assigned to the likely-to-develop (L) and the not-likely-to-develop groupings (N), respectively. POB Group A consists of claims that have a greater potential to develop upward over time such as injuries to the back, head, shoulders, trunk, and multiple body parts. POB Group B consists of all others.

Under NCCI's new loss development methodology, claim dollars will be assigned to one of four development categories (listed below). The assignment will be a function of three claim characteristics: (1) injury type, (2) part of body, and (3) claim status (open vs. closed).

- Medical — Likely-to-develop
- Medical — Not-Likely-to-Develop
- Indemnity — Likely-to-Develop
- Indemnity — Not-Likely-to-Develop

Table 1

Injury Type	Claim Status	Part of Body	LDF Grouping
1st Report			
Fatal	Open	Group A	Not Likely
"	Open	Group B	Not Likely
"	Closed	Group A	Not Likely
"	Closed	Group B	Not Likely
Permanent Total (PT)	Open	Group A	Likely
"	Open	Group B	Likely
"	Closed	Group A	Likely
"	Closed	Group B	Likely
Permanent Partial (PPD)	Open	Group A	Likely
"	Open	Group B	Not Likely
"	Closed	Group A	Not Likely
"	Closed	Group B	Not Likely
Temporary Total (TT)	Open	Group A	Likely
"	Open	Group B	Not Likely
"	Closed	Group A	Not Likely
"	Closed	Group B	Not Likely
Medical Only (MO)	Open	Group A	Not Likely
"	Open	Group B	Not Likely
"	Closed	Group A	Not Likely
"	Closed	Group B	Not Likely

At subsequent reports (2nd through 10th), as noted above, only changes in injury type will be monitored for the purpose of assigning claims to development grouping. The claim status (open vs. closed) and body part, both evaluated at 1st report, will be used for the purpose of determining the development category, regardless of what is reported on a subsequent report.

The term "arising" refers to claims for which there is no 1st report that are reported as of 2nd report or subsequent. For the purpose of assigning claims to a development category, these claims will be assumed to be open at 1st report. The body part will be based upon the initial report submitted to NCCI. The injury type will be monitored at all reports.

Note the new loss development methodology will significantly reduce, but not completely eliminate, instances of crossover. The following list provides a few common examples of how crossover may still occur under the new methodology in certain injury types:

- Medical Only (MO) — MO claims in POB Group A, open at 1st report, which become any other injury type at a later report, will move from N to L. Another example is a lost-time claim, open at 1 st report and in POB Group A, which closes as a medical only. This claim would move from L to N.
- Temporary Total (TT) — Crossover would occur on TT claims that evolve into a PT or fatality at a later report that were originally categorized in the N grouping.
- Permanent Partial (PP) — Crossover would occur on PP claims that evolve into a PT or fatality at a later report that were originally categorized in the N grouping.

These examples represent the most common crossover examples. A few other less likely (no pun intended) cases could be conjured as well.

Exhibits 23a through 23f illustrate the loss development pattern of the new loss development methodology for a "test" state. Note this is a different state than the triangles illustrated in exhibit 1 for a "large" state. The reader should be able to discern the differences in the loss development patterns and the magnitude and derivation of the tail factor.

2.4.6 Advantages and Disadvantages of the New Loss Development Groupings

The most important advantage the new loss development methodology provides is better, more predictive loss development factors. Expanding the triangles out to 10th report should also improve the predictive ability. Much crossover has been mitigated due to the elimination of the critical value, and the new data element combination of body part, injury type, and claim status has improved the LDF groupings. Most importantly, class equity should improve as the class codes with more head, back, trunk, multiple body, etc., types of injuries will be charged more than class codes with other less complex injuries, all else equal. Thus, loss costs should be more predictive in the future.

The use of injured body part in conjunction with the open and closed claim status also adds a practical sense of logic to it all that most regulators and insurance industry actuaries and non-actuaries should readily understand.

About the only disadvantage the new methodology has is that as claims evolve over time, and change injury types, some crossover from one grouping to another can still occur on occasion.

2.5 Lower Loss Limits, Expected Excess, and the New Seven Hazard Groups

The previous class ratemaking methodology limited large claims for a class code at a loss limit equal to five times the state's serious average cost per case. For the NCCI states, these limits ranged from $300,000 to about $1M during the 2006 filing season. A multi-claim occurrence was capped at twice the single claim limit. The claims underlying the loss development factors were unlimited. It should also be noted that the excess dollars removed from the individual class codes were distributed to the industry group to which the class code belonged. Thus, the indicated losses used within the industry group differential calculations were put back on an unlimited basis by deriving an unlimited-to-limited ratio for each industry group. In summary, the previous class ratemaking methodology limited large claims on a class code basis and in most other aspects of the ratemaking, unlimited loss dollars were used.

The new ratemaking methodology is changing much of that. The most noteworthy changes are as follows:

1. Standardizing the single claim loss limit for class codes across NCCI states to be $500,000 (and the multi-claim occurrence to be three times the single claim limit).
2. Basing loss development factors on claims limited at $500,000.
3. Use of a multiplicative factor based on excess ratios to estimate the expected losses excess of $500K using excess ratios from the new seven hazard group mapping.
4. Removing the unlimited-to-limited ratio from the class and industry group differential calculations, and replacing it with expected excess.

This section of the paper will discuss and summarize the analyses and reasoning underlying these decisions.

2.5.1 Applying the Loss Limitations to Individual Claims

In workers compensation ratemaking, losses are separately analyzed by type of benefit; namely, indemnity and medical losses. NCCI uses proportional capping to allocate limited claim amounts. This method of capping large claims for class ratemaking remains similar under the new methodology. The WCSP losses used in class ratemaking are "paid+case". Limited loss amounts for claims above the threshold will be allocated to indemnity and medical in the proportion that their values contribute to the total unlimited value of the claim and the threshold. In order to understand the mechanics of how claims are limited, the following hypothetical illustrative example is included:

Illustration 1: A $1.5M single claim has pierced the threshold on a "paid+case" basis; State threshold = $500K:

UNLIMITED LOSSES ($000s)	**Paid**	**Case**	**Total**
Indemnity	100	200	300
Medical	300	900	1,200
Total	400	1,100	1,500

In this situation, the resultant limited amounts are as follows:

LIMITED LOSSES ($000s)	**Total**
Indemnity	100
Medical	400
Total	500

In Illustration 1, the formula for limited "paid+case" amounts for indemnity and medical are:

Limited Indemnity = (300 / 1,500) x 0.5M = 100.

Limited Medical = (1,200 / 1,500) x 0.5M = 400.

Note that the NCCI procedure for capping large claims in the financial data is different than for class ratemaking. The financial data procedure uses a "paid first-case reserve second" approach that uses proportional capping. Although an illustration of the multi-claim occurrence capping is not included here, proportional capping amongst the claims is applied. The threshold was changed to be three times the single claim limit mainly because the previous single claim limit (i.e., five times the state's serious average cost per case) times two is about $1.5M on average across NCCI states. With the change of the single claim limit to 0.5M, the choice of three times the single claim limit kept the multi-claim cap approximately the same as in the past.

2.5.2 Application of the Excess Ratios

Adjusted per claim excess ratios will be used in calculating unlimited ultimate losses from limited ultimate losses. Excess losses are defined as the sum of the excess portion of claims above a given per claim threshold. NCCI produces proposed excess ratios with each loss cost or rate filing.

The excess ratio, XS_T, for a given threshold T, is defined as:

$$XS_T = \frac{\text{Expected Excess Losses Above Threshold } T}{\text{Expected Total Unlimited Losses}} \tag{2.12}$$

The threshold T is proposed to be \$500,000 in all states for class ratemaking claim limitations. The ratio of excess losses to total unlimited losses is at an ultimate value. The excess ratio applied is on a per claim basis and varies by state. This differs from an excess loss factor as excess loss factors are on a per occurrence basis, and also may include a provision for expenses. For a more detailed discussion of the methodology underlying NCCI excess ratios, see the Fall 2006 CAS *Forum* paper by Engl and Corro titled, "The 2004 NCCI Excess Loss Factors" [1].

The adjusted, per claim excess ratio is applied as a factor, $1/(1 - XS_{500K})$, to limited (@500K) ultimate losses that have been developed, on-leveled, and trended to the midpoint of the proposed filing effective period. Similarly, the excess ratio applied has also been trended to the midpoint of the proposed filing effective period. Within each policy period in the experience period, the same factor $1/(1 - XS_{500K})$ is applied to both indemnity and medical losses, since the size-of-loss distributions are on a combined indemnity and medical basis.

NCCI uses five policy periods as the experience period for each class code. Excess ratios are <u>not</u> adjusted when applied to different experience period years for purposes of calculating pure premiums for class ratemaking. Therefore, in a given filing, the same excess ratio factor is applied to each of the five years in the experience period. NCCI considered de-trending the threshold as is done in the state's overall indicated loss cost level change. By de-trending the threshold in the loss development history, the proportion of losses above the threshold is preserved. But unlike the aggregate ratemaking, where thresholds are de-trended 20 years, and the impact of de-trending can be significant, the impact of de-trending across the five policy periods used in class ratemaking resulted in a negligible impact on class relativities. Practically speaking, it would add a lot of complication to de-trend the five policy periods for little or no added value. For this reason, NCCI chose not to de-trend in its class ratemaking.

For many years, the excess ratios were determined for each of the four hazard groups in each state: that is, hazard groups I, II, III, and IV. The vast majority of classes were assigned to HG II and III. In 2006, NCCI filed a countrywide item-filing, B-1403, which was successfully approved in all NCCI states and adopted by other independent bureau states as well. Based upon an analysis of countrywide excess ratios by class code, an entirely new mapping of class codes to seven hazard groups was implemented in 2007. This item-filing is referred to as the NCCI Hazard Group Remapping. One of the advantages that the new mapping provides is a much more uniform distribution of class codes across the hazard groups.

The seven new hazard groups are referred to as A, B, C, D, E, F, and G. Class codes having the highest excess ratios were mapped to G and may be considered the most hazardous classes. Class codes having the lowest excess ratios were mapped to hazard group A and may be considered the least hazardous classes. As you will soon see, the new hazard groups will be used to provide an excellent refinement for use in the future class ratemaking. This is because excess ratios are now produced for every state for all seven hazard groups. For a more detailed discussion of the methodology underlying the NCCI hazard group mapping, see the paper by NCCI staff titled, "NCCI's 2007 Hazard Group Mapping" submitted for publication [2].

2.5.3 Simulation and Expected Excess

The factor $1/\ (1 - XS_{500K})$ was selected by NCCI for use in the new class ratemaking to derive expected unlimited ultimate losses by class code based on limited (@500K) ultimate losses. It was selected after reviewing results from 16 different potential capping and excess spreading alternatives analyzed using a Monte Carlo simulation technique. Some alternatives used expected excess while others used actual excess. Other alternatives capped individual claims at three different loss limits: $300K, $500K, and $1.0M. One alternative used unlimited losses. Exhibit 14 illustrates all of the options considered and analyzed.

The simulation approach of testing the alternatives was completed as follows:

1. Five years of simulated losses were produced for every class code in two large states and two small states.

2. The claim counts were based on actual national incidence rates for the class code. A Poisson distribution was assumed with lambda equal to the national incidence rate by injury type multiplied by actual payroll for the class in each state.

3. The new excess ratio loss distributions by injury type by state (per Corro and Engl) were used for determining the average cost per case. In determining the state distributions, each class was scaled to the state's average cost per case adjusted for hazard group.

4. One hundred different simulation trials by class code were produced. Each simulation generates five years of unlimited loss data for the given class.

5. The simulated claims' loss data was then modified by the specific capping alternative to provide modified expected unlimited losses.

6. The performance of each alternative was assessed using four overall metrics. Two of the metrics measured loss cost adequacy and two measured loss cost stability across the 100 simulation trials.

The following are the four metrics that were used to assess the success of the various alternatives for limiting claims and allocating the excess.

Adequacy Metric 1: Desired range [-0.25, +0.25]

$$\frac{\overline{L}^{(k)} - \mu}{\mu} \tag{2.13}$$

Where,

L_n = 5 years of simulated losses for the *n*th trial whereby *n* = [1, 2, …100]

$L_n^{(k)}$ = 5 years of simulated losses for the *n*th trial whereby the losses were capped as in alternative *k* for limiting losses and allocating the excess (see Exhibit 14 for alternatives).

μ = hypothetical mean expected losses for a class code based on simulated frequency and actual severity times actual class payroll for that state.

$\overline{L}^{(k)}$ = the average losses for a specific class code over *N* simulations for alternative *k*.

Mathematically, it equals:

$$\overline{L}^{(k)} = (\sum_{n=1}^{N} L_n^{(k)}) \,/\, N. \tag{2.14}$$

Adequacy Metric 2: Desired range [0, +0.50]

$$= \sum_{n=1}^{N} \left| L_n^{(k)} \; - u \right| /100u. \tag{2.15}$$

This metric differs from the first in that the high and low values cannot cancel out due to the absolute value.

Stability Metric 1: Desired range [0, +0.10]

$$CV_c^{(k)} = \frac{\text{standard deviation}}{\text{mean}} = \frac{\sqrt{\dfrac{\sum_{n=1}^{N} (L_{n,c}^{(k)} - \overline{L}_c^{(k)})^2}{N}}}{\overline{L}_c^{(k)}} \qquad (2.16)$$

Where,

$CV_c^{(k)}$ = the coefficient of variation for class code *c* under alternative *k*.

$L_{n,c}^{(k)}$ = 5 years of simulated losses for the *n*th trial for class *c* whereby the losses were capped as in alternative *k* for limiting losses and allocating the excess.

$\overline{L}_c^{(k)}$ = average of simulated losses for alternative *k* over all simulations

Thus, stability metric 1 is the coefficient of variation for a specific class under the conditions of alternative *k* for capping claims and allocating the excess.

Stability Metric 2: Desired range [0, +0.50]

$$\frac{\sum_{n,m} \dfrac{\left| L_{n,c}^{(k)} - L_{m,c}^{(k)} \right|}{u_c}}{N(N-1)} \qquad (2.17)$$

Where,

$L_{n,c}^{(k)}$ = 5 years of simulated losses for the *n*th trial for class c whereby the losses were capped as in alternative *k* for limiting losses and allocating the excess.

$L_{m,c}^{(k)}$ = 5 years of simulated losses for the *m*th trial for class *c* whereby the losses were capped as in alternative *k* for limiting losses and allocating the excess.

μ_c = hypothetical mean expected losses for a class code based on simulated frequency and actual severity times actual class payroll for that state.

For the performance measurement of stability metric 2, the average absolute change in losses for a class is computed across all combinations of the 100 simulations for each alternative *k*.

2.5.4 Choosing the Final Alternative

Exhibits 15a) and 15b) were included to provide an illustrative example of the type of exhibits that were generated and observed for all four of the metrics for each state studied. Several statistics were analyzed such as minimum and maximum values, the classes which comprised these outliers, and various different percentile levels such as the 90th, 10th, and the median. It was noted which capping and excess-spreading alternatives were succeeding the most and which ones were not succeeding. For example, on Exhibit 14a) alternative *k* = 0, which uses unlimited losses, performed most poorly as measured by the stability metric 1. Alternatives 11 and 12, which use expected excess, performed the best. Exhibits similar to 15b) were produced for each alternative so that we understood how many classes were changing within an industry group and by how much. This exhibit shows a drill down on Alternative 12 for adequacy metric 1. Outlier classes were sometimes reviewed, and often a class that performed poorly was a very small volume class. Typically, the outlier class had no losses for almost all of the simulation trials but a few. This is a real-life challenge that the various credibility formulae attempt to address. For the sake of brevity, the author has only chosen but a few examples simply to illustrate for the reader the type of analyses that were completed to select between alternatives for capping and allocating excess.

The final two loss limits NCCI considered were $300,000 and $500,000. The $1M loss limit was eliminated based on class stability considerations. It would have increased the loss limit significantly in most states. The expected excess at $300,000 was very significant upon reviewing the results of indicated pure premiums by class code in states with high excess ratios. The choice of the $500,000 limit provided a nice balance between allowing a significant amount of actual loss experience of the class code into the pure premium calculation combined with less reliance on the expected excess provision. It was significantly lower than the loss limit used today, namely that based on five times the state serious average cost per case (SACC). Test results also revealed that actual excess losses were closer to expected excess losses at $500,000 than the lower loss limit. It also aligned well with the fact that the 95th percentile of all countrywide large claims over a five year period was 2.5 times the SACC, one-half of the previous loss limitation. NCCI decided to target the 95th percentile, or approximately $500,000. Another practical consideration was that the loss limit coincides with the loss limit on the NCCI Large Loss Call #31. The choice of loss limit will be reviewed in the future upon review of the results of the new methodology, and may be updated for inflation periodically.

After reviewing the results of indicated pure premiums derived under the best performing alternatives for several states, Alternative 11 was chosen by NCCI to be the methodology for allocating the excess losses (over $500K) on a class code basis. The main reasons for this decision were:

1. Alternative 11 performed very well on the four metrics.
2. The use of the multiplicative excess factor, $1/(1 - XS_{500K})$, is consistent with the methodology used for determining the overall statewide indicated loss cost change.
3. Given two class codes of similar size within the same hazard group in a state, the class with greater primary losses would receive a greater proportional share of excess losses under alternative 11.
4. After application of the three-way credibility procedure, alternative 11 produced very similar results compared to the other leading alternatives.

One final adjustment was made to the multiplicative excess factor methodology. Recall that the NCCI excess ratios are produced on a combined indemnity and medical basis. This implies that the excess split of losses into indemnity and medical under Alternative 11 is equivalent to the primary split of indemnity and medical. As varying amounts of credibility will be applied separately for indemnity and medical in the new class ratemaking methodology, a refinement was needed to account for the fact that the majority of excess loss in workers compensation is due to the medical component.

One more analysis was prepared to study this and the result is shown in Exhibit 16. This analysis shows only claims excess of $500,000 and the indemnity and medical split of primary and excess dollars. Note the results show an approximate split of the excess dollars to be around 71% medical. Similar results were derived using WCSP data.

NCCI decided it was desirable to apply the $1/\ (1 - XS_{500K})$ factor to indemnity and medical primary losses by class code initially to preserve the correct total excess dollar amount. An adjustment is then made to transfer 40% of the total excess dollars produced within the indemnity pure premium component to the medical pure premium component. The practical reasons for transferring 40% of the indemnity excess dollars include the following considerations:

- It preserves state and class differences as it is a function of the actual primary indemnity and medical split.
- It achieves the desired higher proportion of medical excess (i.e., close to the 70% figure across all states combined).
- It never results in a medical excess provision percentage that falls below the medical primary provision percentage for any class or state.
- It mirrors the reality that more of the excess dollars are medical.

2.5.5 Implications on the Industry Group (IG) Differential Methodology

NCCI is maintaining its IG differential methodology, and it will look very similar to how it is done under the current methodology. The value that the IG differential calculation adds to class ratemaking is:

- It reflects wage trend differences by industry group.
- The industry group rate change is applied to determine the present-on-rate-level pure premiums, which are important for low credibility class codes.
- It was the point where losses were brought to an unlimited basis in the previous methodology.

The majority of the calculation will look the same as before. Oversimplified, the IG differential is a ratio of five years of indicated losses from WCSP data to five years of expected losses, both brought to the proposed level. As a result of the methodology changes discussed to this point, a few changes had to be addressed within the calculations. They were:

1. The unlimited-to-limited ratio by IG was removed.
2. The new loss development groupings were applied to bring indicated losses to an ultimate level limited at $500K.
3. The ultimate losses limited at $500K will be brought to an expected unlimited level via the multiplicative excess factor and transfer of 40% of the indemnity excess to medical.
4. The full credibility standard was changed to 12,000 lost-time cases. It previously ranged from 7,000 to 11,000 by IG. This will be discussed further in the credibility section of the paper.

An example of the IG differential exhibit is found in Appendix B, which displays the calculation of a loss cost for a class code under the new methodology.

2.6 New Credibility Standards

It was mentioned early in the paper that in 1993 NCCI modified the credibility formulas used in the class ratemaking. This is because changes were made to the experience period and number of industry groups, both going from three to five. The past formulas were derived using a limited fluctuation approach. The full approach is quite involved and a full expose may be found in NCCI Actuarial Committee Agenda, dated June 7, 1993 (ACT-93-7) [3]. This paper will present a very high-level overview of the past approach, and the challenges NCCI faced updating the credibility standards this time around.

The new class ratemaking approach is adding stabilizing features that, all else equal, suggest the full credibility standards should be modified to provide more credibility on pure premiums. Those features include:

- a lower loss limit of $500K should reduce class fluctuations
- less volatile loss development factors due to reduced crossover and the introduction of a $500K loss limit
- less variance in excess losses by using expected excess factors

There was also a change within the new class ratemaking that may suggest reduced credibility on pure premiums:

- Eliminating the serious and non-serious pure premiums and creating a more heterogeneous indemnity pure premium.

The challenge NCCI faced was how to modify the full credibility standards, and by how much, for the changes being made without having the benefit of being able to observe the results of the new methodology over a substantial period of time.

2.6.1 Background of Previous Class Credibility Formulae

The previous methodology determined full credibility standards in 1993 based on the actual variability of indicated pure premiums over five successive rate revisions as measured by a coefficient of variation (*CV*). The rate revisions were all brought to a common level of the latest revision. An average of the expected number of claims (*N*) for each class over the five revisions was computed by dividing its expected losses by its average cost per case in that revision.

Next, the expected number of claims by class was plotted on the *x*-axis versus the *CV* on the *y*-axis and regression statistics observed for several states. At the end, the following model was used:

$$\ln CV = a \ln N + b \tag{2.18}$$

Where,

CV = coefficient of variation of indicated pure premiums over five rate revisions

N = expected number of claims

N_f = full credibility standard

Rearranging the formula and exponentiating, the partial credibility *z*, assigned to the indicated pure premium in order to limit variability to an acceptable amount is:

$$Z = CV \text{ acceptable}/CV \text{ actual} = (N_f^{\,a}\, e^{b} \,/\, N^{a}\, e^{b}) = [N/\, N_f]^{-a} \tag{2.19}$$

The acceptable value for the *CV* was .10, chosen so as to limit the fluctuation of the pure premiums to within +/- 25% (NCCI swing limits) 95% of the time. The exponent, *a*, was computed as the slope of the regression line, and was determined to be approximately -0.4 using 95% confidence intervals. Thus, the final formula used today for all NCCI states is:

$$Z = [N/\, N_f]^{\,0.4} \tag{2.20}$$

The table below provides the full credibility standards previously in effect for the state class indicated pure premiums.

Table 1: Indicated Pure Premiums

Partial Pure Premium	**Full Credibility Standard N_f**
Serious	125
Non-Serious	350
Medical	750

The value N_f is applied to the average cost per case for each partial pure premium to derive a full credibility standard (FCS) of expected losses used across all class codes in each state's loss cost filing. The numerator of the class credibility formula is the class expected losses determined by the payroll for a class times its underlying pure premium. One unusual nuance was that the medical partial pure premium FCS used the non-serious indemnity average cost per case. This is being changed, as will soon be described in this paper.

2.6.3 Class Credibility Changes for the State Indicated Pure Premium

The new methodology is eliminating the critical value which helped determine the serious and non-serious partial pure premiums. The new methodology is reducing the number of pure premiums to two: indemnity and medical. So the question was raised as to what credibility to assign to each, given the observed results of the new methodology were not available.

As mentioned earlier, there were stabilizing changes being put in place for the new ratemaking, and a countering influence from the added heterogeneity of the indemnity pure premium. Thus, the decision was made to compute new credibility standards that maintained approximately the same credibility as was applied in the previous ratemaking. Longer term, after five years of the new methodology can be observed, new regressions of the fluctuations of indicated pure premiums can be calculated.

NCCI ran the same regression methodology for six states of various sizes using more recent data under the previous ratemaking methodology. But a new twist was added. Revised indicated full credibility standards were derived for serious, non-serious, and for the new combined indemnity pure premium. Over time, the indicated FCS was significantly higher using the recent data. See Exhibit 17 for the results. NCCI actuaries then assumed that the stabilizing forces of the new methodology would offset the need to move to the higher indicated FCS of the regressions. From Exhibit 17, new indicated standards were derived and a ratio of current to indicated was computed. For indemnity, that ratio was 61%, which was then applied to the indication of 1,397 to derive 850 after rounding. For medical, the ratio of 56% was applied to 719 to derive a rounded value of 400.

Table 2: Indicated Pure Premiums-New Methodology

Partial Pure Premium	**New Full Credibility Standard N_f**
Indemnity	850
Medical	400

Note that N_f will still be multiplied by the state average cost per case to determine expected losses. However, for medical, the medical average cost per case will be used in lieu of the non-serious average cost per case. This more appropriately indexes the medical FCS over time. The medical average cost per case is computed using total medical dollars of loss (including medical-only losses) divided by lost-time claim counts, similar to the calculations NCCI computes in most other areas.

The regressions indicated that the 0.4 power rule is still appropriate. The remaining credibility decisions include maintaining the 0.4 power rule shown earlier and the three-way credibility weighting procedure between the indicated, national, and present-on-rate-level pure premiums. In no case is the national credibility permitted to exceed 50% of the complement of the state credibility.

2.6.4 Class Credibility Changes for the National Pure Premium

The credibility decisions for national pure premiums followed a very similar path. As background, the FCS for national pure premiums, also derived in 1993, use the actual number of lost-time claims, not expected claims and expected losses.

Without going through more detailed calculations, the table below provides the full credibility standards previously in effect for the national pure premiums.

Table 3: National Pure Premiums

Partial Pure Premium	**Full Credibility Standard N_f** **Actual # of Lost-Time Claims**
Serious	175
Non-Serious	500
Medical	1,000

Table 4: National Pure Premiums: New Methodology

Partial Pure Premium	**Full Credibility Standard N_f** **Actual # of Lost-Time Claims**
Indemnity	1,150
Medical	1,000

Revised national pure premium full credibility standards were derived for serious, non-serious, and a combined indemnity pure premium. The indicated FCS for the national using the regressions was significantly higher using the recent data, just as with the state indicated pure premium. See Exhibit 18 for the results. Similarly, NCCI actuaries then assumed that the stabilizing forces of the new methodology would offset the need to move to the higher indicated FCS of the regressions. From Exhibit 18, the new indicated national standards were derived and a ratio of current to indicated was computed. For indemnity, that ratio was 54%, which was then applied to the indication of 2,127 to derive 1,150 after rounding. For medical, the ratio of 65% was applied to 1548 to derive a rounded value of 1,000.

The final step was to ensure that on average, a state's overall credibility was remaining similar in magnitude after the changes to the new FCS. Exhibits 19 and 20 show the average credibility across the six states tested for indemnity and medical, respectively. The top 50 classes were also excluded to ensure the credibility of small volume classes was not changing much as well. These results showed that both state indicated pure premium and national pure premium credibility were approximately the same, which was the objective.

2.6.5 Industry Group Credibility Changes

The full credibility standard was changed to 12,000 lost-time cases in the new methodology, uniform for all industry groups. It previously ranged from 7,000 to 11,000 by IG. The previous FCS was based on the following square root rule where the probability, p, of the IG differential being within k= +/-.075 was 95%:

$$Z = \text{Min}\ [(N_i / N_{f,i})^{0.5}, 100\%] \tag{2.21}$$

Where,

$Z_{i,s}$ = the credibility assigned to industry group i within state s

N_i = the actual number of lost-time claims for industry group i

$N_{f,i}$ = full credibility standard for industry group i

Much of the theory underlying the square root rule is described in Gary Venter's "Limited Fluctuations" approach, found in the "Credibility" chapter of *Foundations of Casualty Actuarial Science* [4]. The previous full credibility standards are in the table below.

Table 5: Previous FCS for IG Differentials

Industry Group	**Full Credibility Standard N_f**
Manufacturing	10,000
Contracting	8,000
Office & Clerical	7,000
Goods & Services	9,000
Miscellaneous	11,000

To the extent that an industry group's number of lost-time claims was less than the FCS, a value for $z_{i,s}$ is computed using the square root rule, whereby $0 \le z_{i,s} \le 1$. The complement of credibility, $1-z_{i,s}$, is assigned unity, or no change. In practice, the IG differential is judgmentally tempered to be between [.90, 1.10].

The new FCS of 12,000 was based on an analysis of five successive years of five IG differential fluctuations across 36 states. Exhibit 21 displays the results of applying various values of p and k, and the FCS that was indicated within each combination. The final selection by NCCI was to continue to use the same p and k (i.e., 95% and k=+/-.075). This resulted in 12,000 lost-time claims. Although this put a little less weight on the state's IG differentials than the past methodology did, this was deemed appropriate given the volatility observed within an industry group in successive filings in the sample of data.

2.7 The Impact of the Methodology Changes

One of the last steps in the process was to test the results of two states, a large and a small state, to determine the impact that all of the methodology changes had on class loss costs. Each major change was measured individually and naturally, the final results were observed in a cumulative manner. The results were determined by class and by industry group. The targeted aggregate statewide overall change was the same for both the previous and new methodologies. The national and present on-rate-level pure premiums were based on the previous methodology. Only the indicated pure premiums reflected the new methodology because at this time, it was not possible to construct national pure premiums using the new methodology.

Exhibit 22a illustrates the observed results for the large state, which has many class codes receiving 100% credibility for the indicated pure premium. Focus on the two industry groups contracting and office and clerical. Key observations include:

- Column (2) of Exhibit 22a shows the new loss development methodology produced lower LDFs for classes that have a propensity to have serious claims, such as contractors, than for office and clerical.

- The expected excess provision in column (3) offsets the loss development to a degree by applying a higher multiplicative expected excess provision to contractors than the provision applied to office and clerical.

- The count of class codes in Exhibit 22b shows that the majority of all classes in the large state changed between +/- 7.5% from the previous to new methodologies.

- The change in credibility methodology had a very small impact.

These were only a few of many other results which were explored. Other analyses included a review of the change in indicated pure premiums only, which were more volatile than final loss costs, the imposition of swing limits, and a drill down on class codes with larger changes than normal. NCCI plans on testing more states in the future prior to implementation.

2.8 The Pros and Cons of the Methodology Changes

Implementing large modifications to class ratemaking brings with it many positive enhancements including more stability from year to year on a class code level. Long-term loss cost adequacy should also be improved by some of the innovations leveraged from the expected excess from the new seven hazard group mapping and the new loss development methodology. The use of new data elements like injured body part helps to invigorate the methodologies.

The cons to making such a large number of changes will be the challenge of explaining the new methodology to regulatory entities, and obtaining their buy-in, as the loss costs underlying the new methodology, although very much improved, may generate unpalatable premium changes in the year of implementation for certain regulators and the employers within their jurisdictions.

2.9 Possible Future Enhancements to NCCI Class Ratemaking

The credibility formulas are a ripe area for further research. Once several rate revisions have been observed under the new methodology, much more work can be done to derive new standards and revisit the three-way credibility formula. Other areas include revisiting the body part mappings after NCCI collects 10 reports of WCSP data, as well as the tail factor. Other areas that will need continuous monitoring over time include the loss limit, and the transfer of a percentage of excess dollars from indemnity to medical, and the groupings of likely-to-develop and not- likely-to-develop.

Although the analysis is not presented in this paper, the potential use of allocated loss adjustment expense (defense and cost containment expense) was explored thoroughly. The observed result was that the value that it would add to class relativities was minimal relative to issues its inclusion may create, particularly with experience rating modifier calculations.

3. CONCLUSIONS

This paper documents several important changes that are being implemented in the class ratemaking process used to determine workers compensation loss cost and rate changes by class. The changes NCCI is implementing support the long-term goals of adequacy and stability of loss costs and rates, and help to consistently estimate class relativities from state to state in the ratemaking methodology.

This paper also serves to provide an illustration of the derivation of a loss cost for a class code in workers compensation using NCCI' s new methodology.

Acknowledgment

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5. REFERENCES

[1] Engl, and Dan Corro, "The 2004 NCCI Excess Loss Factors," Casualty Actuarial Society Fall *Forum,* **2006**.

[2] NCCI Staff, "NCCI's 2007 Hazard Group Mapping" submitted for publication.

[3] NCCI Actuarial Committee Agenda, ACT-93-7, June 7, 1993.

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Abbreviations and notations

AY — accident year
C — refers to a claim closed at 1st report
CAS — Casualty Actuarial Society
Class $tU_{L,I}$ = A likely-to-develop t^{th} – ultimate indemnity (I) tail factor applied to likely-to-develop losses at t^{th} report for each class code. It is limited at state threshold T.
Class $tU_{N,I}$ = A not-likely-to-develop t^{th} – ultimate indemnity (I) tail factor applied to not-likely-to-develop losses at t^{th} report for each class code. It is limited at state threshold T.
$CV_c^{(k)}$ – the coefficient of variation for class c under alternative (k)
CV — coefficient of variation of indicated pure premiums based upon five successive rate revisions
DSR — Designated Statistical Reporting level of NCCI
FCS — full credibility standard
Fa — claims reported within the fatal injury type
Fin tU_I = Limited (at T) statewide financial data t^{th} – ultimate tail factor for indemnity (I)
HG — hazard group
IG — industry group
k — acceptable tolerance around a mean value
L — reference to likely-to-develop grouping
LC — "likely" body part and claim closed at 1st report
LDF — loss development factors
LO — "likely" body part and claim open at 1st report
$L\$_I$ = two years of limited likely-to-develop "paid+case" indemnity loss dollars on-leveled and developed to t^{th} report for the state
L_n = five years of simulated losses for the nth trial whereby n = [1, 2, …, 100]
$\overline{L}^{(k)}$ = the average losses for a specific class code over N simulations for alternative k
$\overline{L}^{(k)}{}_c$ = the average of simulated losses for alternative k over all simulations
$L_{n},{}^{(k)}$ — five years of simulated losses for the nth trial for class c whereby the losses were capped as in alternative k
$L_{m},{}^{(k)}{}_c$ — five years of simulated losses for the m^{th} trial for class c whereby the losses were capped as in alternative k.
M — $millions
MO — claims reported within the medical-only injury type
N = the expected number of lost-time claims for a class
N_f = full credibility standard
N_{f}, i = full credibility standard for industry group i
N_i = the actual number of lost-time claims for industry group i
N — reference to the not-likely-to-develop grouping in terms of loss development.
NC — "not-likely" body part and claim closed at 1st report
NCCI — National Council on Compensation Insurance, Inc.
$NL\$_I$ = two years of limited not-likely-to-develop "paid+case" indemnity loss dollars on-leveled and developed to t^{th} report for the state
NO — "not-likely" body part and claim open at 1st report
$NS\$_I$ = two years of limited "paid+case" non-serious indemnity loss dollars on-leveled and developed to 5th report for the state

O — refers to a claim open at 1st report
p — probability
POB — the injured part of body as reported on the claim
POB Group A — claims with a greater potential to develop upward
POB Group B — claims with less potential to develop upward
PP — claims reported within the permanent partial injury type
PT — claims reported within the permanent total injury type
PY — policy year
SACC — state serious average cost per case
$SER\$_I$ = two years of limited "paid+case" serious indemnity loss dollars on-leveled and developed to the 5th report for the state
t = time t representing the report level of WCSP data at which the attachment point for the class ratemaking tail is applied. t = 5,6,7,8,9,10
T — dollar threshold for capping large claims
TT —claims reported within the temporary total injury type
μ = hypothetical mean expected losses for a class code based on simulated frequency and severity times actual class payroll for that state
URE — Unit Report Expansion
WCSP — NCCI's Workers Compensation Statistical Plan
XS_T — Per Claim adjusted excess ratio at threshold T
y = percentage between 0% and 100% used to allocate a portion of tail development dollars to the not-likely-to-develop grouping
z — partial credibility assigned to a pure premium
$Z_{i,s}$ = the credibility assigned to industry group i within state s

Biography of the Author

Tom Daley is Director and Actuary at NCCI, Inc. He is currently responsible for both applied research and implementation of the new methodologies in class ratemaking for all NCCI states, as well as handling state actuary loss cost and rate filing duties in several other states. He has a B.S. degree in Mathematics from the Pennsylvania State University. He is an Associate of the CAS and a Member of the American Academy of Actuaries.

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Appendix A: Research Exhibits 1- 23

Appendix B: Derivation of a Loss Cost for a Class Code

UNLIMITED INDEMNITY LOSS DEVELOPMENT

Exhibit 1a

Serious Large State

1st Report Start: 1/1/2003
1st Report End: 12/31/2003

PY Data	1st Report	2nd Report	3rd Report	4th Report	5th Report
1/98-12/98			460,401,442	535,321,008	574,106,684
1/99-12/99		340,191,451	489,175,745	560,465,442	592,806,690
1/00-12/00	141,410,721	312,882,740	450,176,823	526,656,041	
1/1-12/1	128,481,157	295,773,844	438,063,233		
1/2-12/2	108,611,922	260,153,546			
1/3-12/3	105,915,019				

Link Ratios	1:2	2:3	3:4	4:5
1/98-12/98			1.163	1.072
1/99-12/99		1.438	1.146	1.058
1/00-12/00	2.213	1.439	1.170	
1/1-12/1	2.302	1.481		
1/2-12/2	2.395			

AVERAGE DEV.	1:2	2:3	3:4	4:5
2 Year Averages	2.349	1.460	1.158	1.065

2 YR. DEV. TO ULT.	1:U	2:U	3:U	4:U	5:U
Unadjusted	5.082	2.164	1.482	1.280	1.202

UNLIMITED INDEMNITY LOSS DEVELOPMENT

Non-Serious Large State

1st Report Start: 1/1/2003
1st Report End: 12/31/2003

PY Data	1st Report	2nd Report	3rd Report	4th Report	5th Report
1/98-12/98			437,508,261	432,646,920	431,589,463
1/99-12/99		507,462,094	503,838,453	499,819,176	498,146,055
1/00-12/00	513,724,388	580,792,681	577,827,036	573,577,900	
1/1-12/1	491,994,692	545,990,644	542,748,392		
1/2-12/2	484,992,408	535,107,606			
1/3-12/3	454,969,833				

Link Ratios	1:2	2:3	3:4	4:5
1/98-12/98			0.989	0.998
1/99-12/99		0.993	0.992	0.997
1/00-12/00	1.131	0.995	0.993	
1/1-12/1	1.110	0.994		
1/2-12/2	1.103			

AVERAGE DEV.	1:2	2:3	3:4	4:5
2 Year Averages	1.107	0.995	0.993	0.998

2 YR. DEV. TO ULT.	1:U	2:U	3:U	4:U	5:U
Unadjusted	1.092	0.986	0.991	0.998	1.000

Source: NCCI WCSP Data

SERIOUS DEVELOPMENT TO ULTIMATE — Large State

Exhibit 1b

Unlimited Indemnity

(using 2-year average development)

FIRST REPOR 1/3-12/3	(1) Incurred Losses	(2) Development 1:5	(3) Amendment Factor	(4) Modified Losses (1)x((2)x(3))
Fatal	13,262,549	4.228	1.098	61,564,752
Permanent To	22,327,493	4.228	0.752	70,979,100
Major	70,324,977	4.228	0.907	269,696,287
Minor	135,337,672	1.092	0.907	133,984,295
Temporary To	319,632,161	1.092	0.983	342,965,309
Medical Only				
Contract Medical				

SECOND REP 1/2-12/2	(5) Incurred Losses	(6) Development 2:5	(7) Amendment Factor	(8) Modified Losses (5)x((6)x(7))
Fatal	11,800,628	1.800	1.258	26,716,622
Permanent To	57,888,155	1.800	0.569	59,277,471
Major	190,464,763	1.800	0.807	276,745,301
Minor	182,412,684	0.986	0.807	145,200,496
Temporary To	352,694,922	0.986	0.960	334,002,091
Medical Only				
Contract Medical				

CALCULATION OF SERIOUS FIFTH-TO-ULTIMATE

(9) Combined Serious Losses	764,979,533
(10) Combined Non-Serious Losses	956,152,191
(11) Combined Total Losses	1,721,131,724
(12) Financial Data Fifth-to-Ultimate Development Factors	1.090
(13) Fifth-to-Ultimate Loss Development **(13) = ((12)-1)x(11)**	154,901,855
(14) Fifth-to-Ultimate Serious Loss Development Factors **(14) = ((9)+(13))/(9)**	1.202

Source: NCCI WCSP Data

Exhibit 1c

UNLIMITED MEDICAL LOSS DEVELOPMENT

Total Medical — Large State

1st Report Start: 1/1/2003
1st Report End: 12/31/2003

PY Data	1st Report	2nd Report	3rd Report	4th Report	5th Report
1/98-12/98			1,074,507,205	1,121,412,973	1,151,169,235
1/99-12/99		1,079,216,508	1,170,231,395	1,227,727,033	1,264,629,064
1/00-12/00	970,315,928	1,161,418,120	1,243,492,848	1,303,639,595	
1/1-12/1	977,360,304	1,142,236,135	1,243,998,714		
1/2-12/2	1,016,625,606	1,187,960,564			
1/3-12/3	1,037,743,388				

Link Ratios	1:2	2:3	3:4	4:5
1/98-12/98			1.044	1.027
1/99-12/99		1.084	1.049	1.030
1/00-12/00	1.197	1.071	1.048	
1/1-12/1	1.169	1.089		
1/2-12/2	1.169			

AVERAGE DEV.	1:2	2:3	3:4	4:5
2 Year Averages	1.169	1.080	1.049	1.029

Serious Development for Ratemaking	1:U	2:U	3:U	4:U	5:U
2-Year Unadjusted	3.431	2.935	2.718	2.592	2.519

Serious = Total Medical development to 5th report x Serious Medical 5th to Ultimate Tail Factor

NonSerious Development for Ratemaking	1:U	2:U	3:U	4:U	5:U
2-Year Unadjusted	1.362	1.165	1.079	1.029	1.000

Non-Serious = Total Medical development to 5th report

Source: NCCI WCSP Data

Exhibit 1d

SERIOUS DEVELOPMENT TO ULTIMATE

Unlimited Medical **Large State**

(using 2-year average development)

FIRST REPORT 1/3-12/3	(1) Incurred Losses	(2) Development 1:5	(3) Amendment Factor	(4) Modified Losses (1)x((2)x(3))
Fatal	3,769,846	1.362	1.008	5,175,999
Permanent Total	56,418,886	1.362	1.008	77,463,130
Major	92,132,869	1.362	1.008	126,498,429
Minor	202,853,463	1.362	1.008	278,517,805
Temporary Total	520,564,524	1.362	1.008	714,735,091
Medical Only	161,960,455	1.362	1.008	222,371,705
Contract Medical	43,345	1.362	1.008	59,513

SECOND REPORT 1/2-12/2	(5) Incurred Losses	(6) Development 2:5	(7) Amendment Factor	(8) Modified Losses (5)x((6)x(7))
Fatal	4,270,256	1.165	0.973	4,842,470
Permanent Total	91,136,323	1.165	0.973	103,348,590
Major	185,339,531	1.165	0.973	210,175,028
Minor	248,061,494	1.165	0.973	281,301,734
Temporary Total	507,060,323	1.165	0.973	575,006,406
Medical Only	152,090,873	1.165	0.973	172,471,050
Contract Medical	1,764	1.165	0.973	2,000

CALCULATION OF SERIOUS FIFTH-TO-ULTIMATE

(9) Combined Serious Losses	527,503,646
(10) Combined Non-Serious Losses	2,244,465,304
(11) Combined Total Losses	2,771,968,950
(12) Financial Data Fifth-to-Ultimate Development Factors	1.289
(13) Fifth-to-Ultimate Loss Development (13) = ((12)-1)x(11)	801,099,027
(14) Fifth-to-Ultimate Serious Loss Development Factors (14) = ((9)+(13))/(9)	2.519

Source: NCCI WCSP Data

Exhibit 2

Illustration of Critical Value "Crossover"
Permanent Partial Claims Only

Policy Year 1997
Countrywide - NCCI States

		(1)	(2)	(3)	(4)	(5)	
Status of Claim @ 1st	Status of Claim @ 4th	**Indemnity $** @ 1st	**Indemnity $** @ 4th	(2)/(1) **Indemnity** Link Ratio	Link Ratio Based on Status @ 1st	Link Ratio Incl. Crossover	
Major	Major	613,981,619	820,980,453	1.337	1.156	2.033	*
Major	minor	149,179,971	60,947,235	0.409			
minor	Major	207,820,368	730,279,392	3.514	1.339		
minor	minor	1,186,650,173	1,137,543,165	0.959		0.859	**

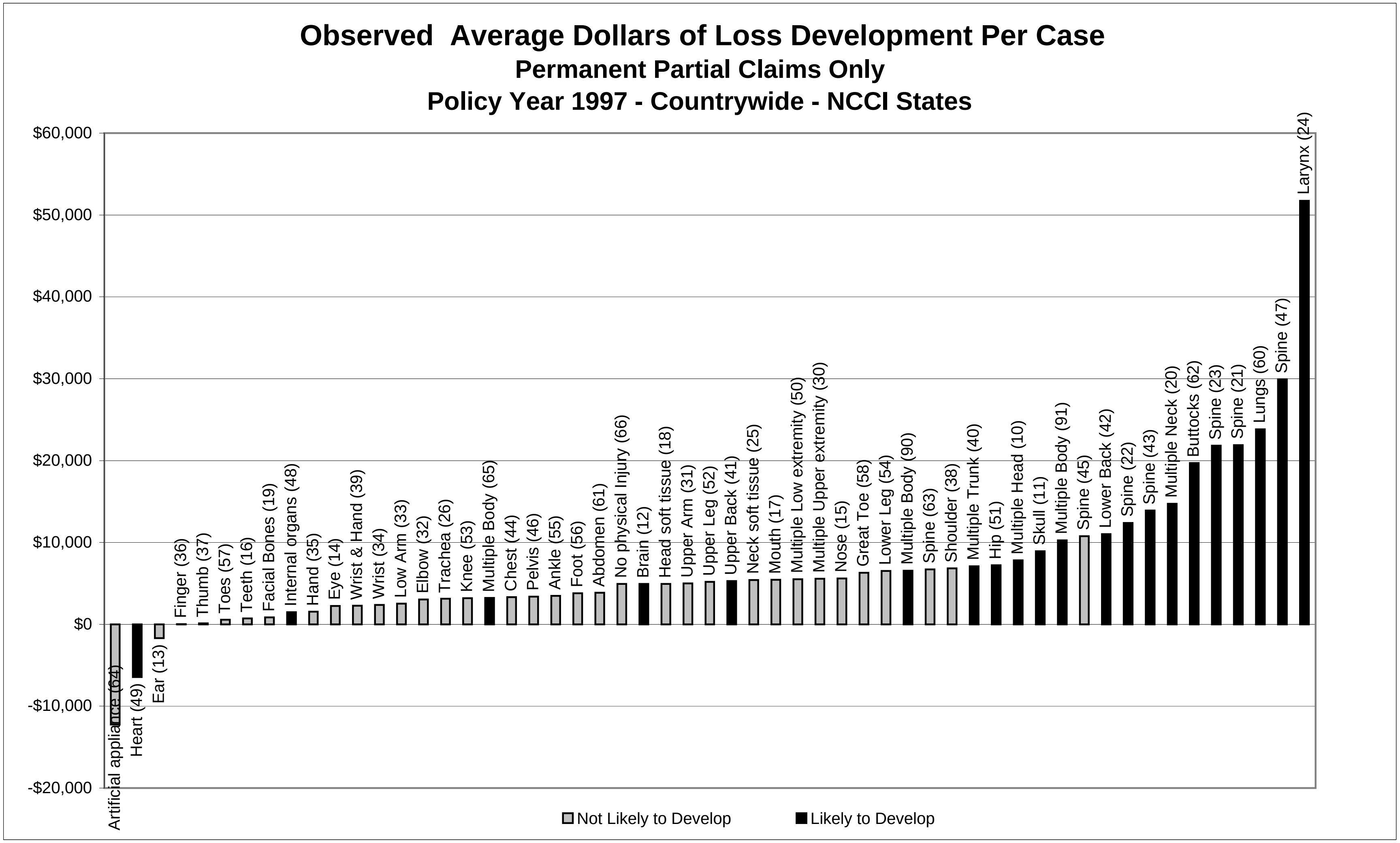
		(1)	(2)	(3)	(4)	(5)
Status of Claim @ 1st	Status of Claim @ 4th	**Medical $** @ 1st	**Medical $** @ 4th	(2)/(1) **Medical** Link Ratio	Link Ratio Based on Status @ 1st	Link Ratio Incl. Crossover
Major	Major	420,359,014	500,436,333	1.190	1.100	1.743
Major	minor	92,457,889	63,417,464	0.686		
minor	Major	211,613,060	393,182,703	1.858	1.075	
minor	minor	1,154,460,758	1,074,742,398	0.931		0.833

* 2.033=(820,980,453+730,279,392)/(613,981,619+149,179,971)
** 0.859=(60,947,235+1,137,543,165)/(207,820,368+1,186,650,173)

Range of Critical Values across NCCI states = [$20K, $90K]

Source: NCCI WCSP Data

Exhibit 3



Source: NCCI WCSP Data

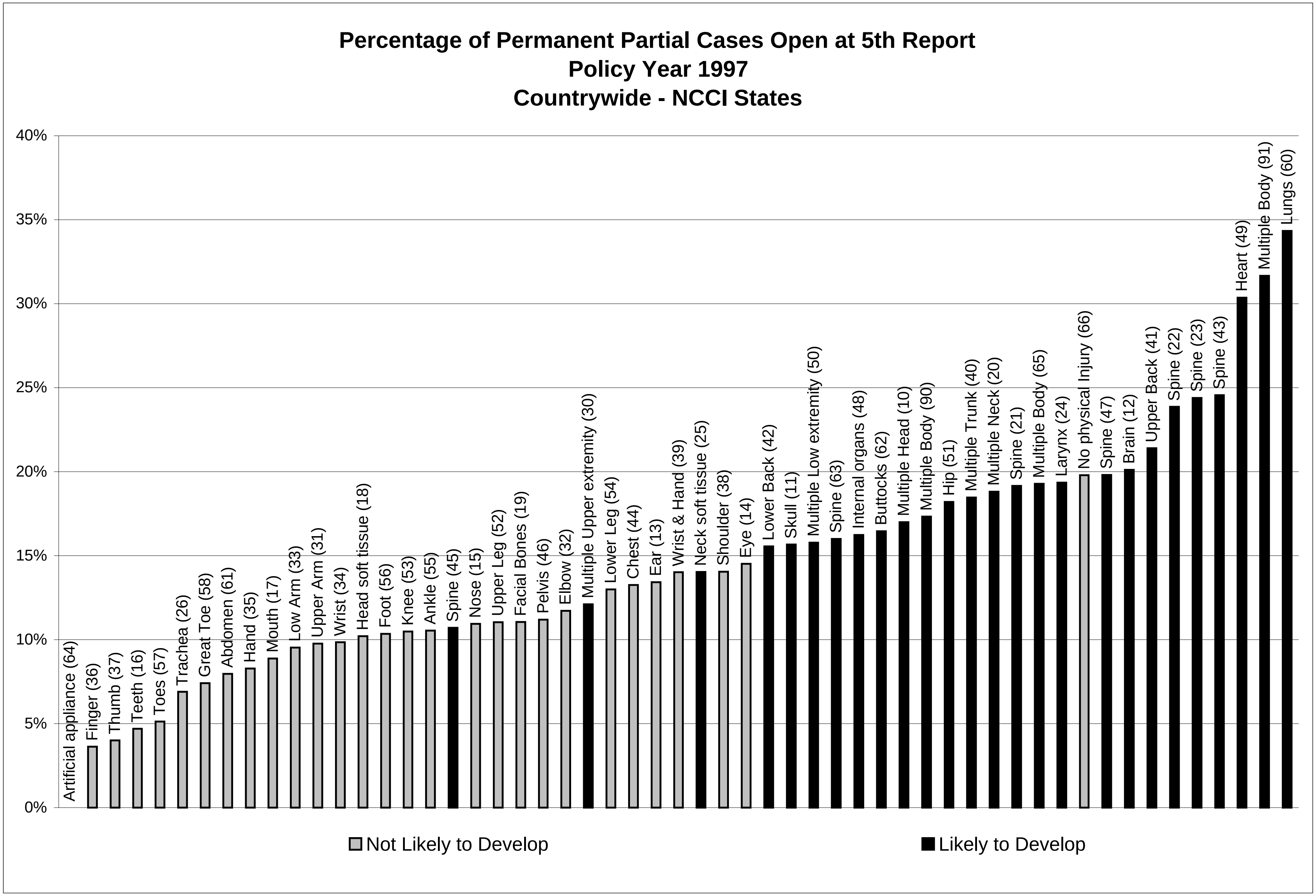
(##) - Part of Body code

Exhibit 4a

Permanent Partial Cases Open at 5th Report Policy Year 1997 Countrywide		
Injured Body Part (code)	Cases	% Open @ 5th Report
Artificial appliance (64)	4	0.0%
Finger (36)	14,638	3.6%
Thumb (37)	3,427	4.0%
Teeth (16)	234	4.7%
Toes (57)	975	5.1%
Trachea (26)	29	6.9%
Great Toe (58)	108	7.4%
Abdomen (61)	640	8.0%
Hand (35)	9,314	8.3%
Mouth (17)	214	8.9%
Low Arm (33)	4,502	9.5%
Upper Arm (31)	11,259	9.8%
Wrist (34)	12,583	9.9%
Head soft tissue (18)	1,009	10.2%
Foot (56)	5,073	10.3%
Knee (53)	20,363	10.5%
Ankle (55)	5,680	10.5%
Spine (45)	299	10.7%
Nose (15)	265	10.9%
Upper Leg (52)	797	11.0%
Facial Bones (19)	344	11.0%
Pelvis (46)	1,072	11.2%
Elbow (32)	4,095	11.7%
Multiple Upper extremity (30)	5,123	12.1%
Lower Leg (54)	3,787	13.0%
Chest (44)	1,245	13.3%
Ear (13)	939	13.4%
Wrist & Hand (39)	778	14.0%
Neck soft tissue (25)	798	14.0%
Shoulder (38)	4,843	14.0%
Eye (14)	1,240	14.5%
Lower Back (42)	32,287	15.6%
Skull (11)	862	15.7%
Multiple Low extremity (50)	1,914	15.8%
Spine (63)	150	16.0%
Internal organs (48)	930	16.2%
Buttocks (62)	79	16.5%
Multiple Head (10)	1,607	17.0%
Multiple Body (90)	17,372	17.3%
Hip (51)	1,505	18.2%
Multiple Trunk (40)	2,768	18.5%
Multiple Neck (20)	1,930	18.8%
Spine (21)	402	19.2%
Multiple Body (65)	830	19.3%
Larynx (24)	31	19.4%
No physical Injury (66)	96	19.8%
Spine (47)	202	19.8%
Brain (12)	348	20.1%
Upper Back (41)	3,309	21.4%
Spine (22)	905	23.9%
Spine (23)	82	24.4%
Spine (43)	2,154	24.6%
Heart (49)	168	30.4%
Multiple Body (91)	300	31.7%
Lungs (60)	201	34.3%
TOTAL	**186,109**	

Source: NCCI WCSP Data

Exhibit 4b



Source: NCCI WCSP Data

Exhibit 5a

Temporary Total Cases Open at 5th Report
Policy Year 1997
Countrywide

Injured Body Part	Cases	% Open @ 5th Report
Artificial appliance (64)	5	0.0%
Great Toe (58)	320	1.3%
Abdomen (61)	5,100	2.4%
Pelvis (46)	6,792	2.8%
Thumb (37)	8,026	3.0%
Finger (36)	28,496	3.1%
Ankle (55)	22,323	3.3%
Toes (57)	5,030	3.4%
Foot (56)	17,922	3.9%
Upper Leg (52)	3,402	4.0%
Spine (45)	1,355	4.4%
Chest (44)	7,675	4.7%
Hand (35)	23,433	4.8%
Internal organs (48)	7,210	5.0%
Elbow (32)	8,709	5.1%
Lower Leg (54)	10,306	5.6%
Eye (14)	4,550	5.6%
Low Arm (33)	10,379	5.7%
Knee (53)	38,104	5.7%
Upper Arm (31)	24,281	5.8%
Nose (15)	895	5.9%
Wrist (34)	23,847	6.0%
Multiple Trunk (40)	9,064	6.0%
Lower Back (42)	107,245	6.2%
Head soft tissue (18)	2,532	6.3%
Shoulder (38)	6,514	6.6%
Facial Bones (19)	683	6.9%
Skull (11)	2,495	7.1%
Spine (63)	465	7.3%
Trachea (26)	94	7.4%
Mouth (17)	468	7.7%
Multiple Upper extremity (30)	8,815	7.7%
Neck soft tissue (25)	2,337	7.7%
Hip (51)	3,966	7.8%
Upper Back (41)	10,139	7.8%
Wrist & Hand (39)	924	8.3%
Multiple Neck (20)	4,544	8.7%
Spine (23)	171	9.4%
Teeth (16)	361	9.4%
Multiple Body (90)	47,079	9.8%
Multiple Low extremity (50)	5,011	10.3%
Multiple Body (65)	2,484	10.5%
Spine (43)	1,268	11.0%
Buttocks (62)	255	11.0%
Ear (13)	953	11.1%
Larynx (24)	132	11.4%
Spine (21)	866	12.4%
Multiple Body (91)	993	12.6%
Spine (22)	1,027	13.2%
No physical Injury (66)	222	14.0%
Spine (47)	300	14.3%
Brain (12)	675	16.3%
Multiple Head (10)	5,108	19.3%
Lungs (60)	340	22.9%
Heart (49)	352	25.3%
TOTAL	**486,042**	

Source: NCCI WCSP Data

Exhibit 5b

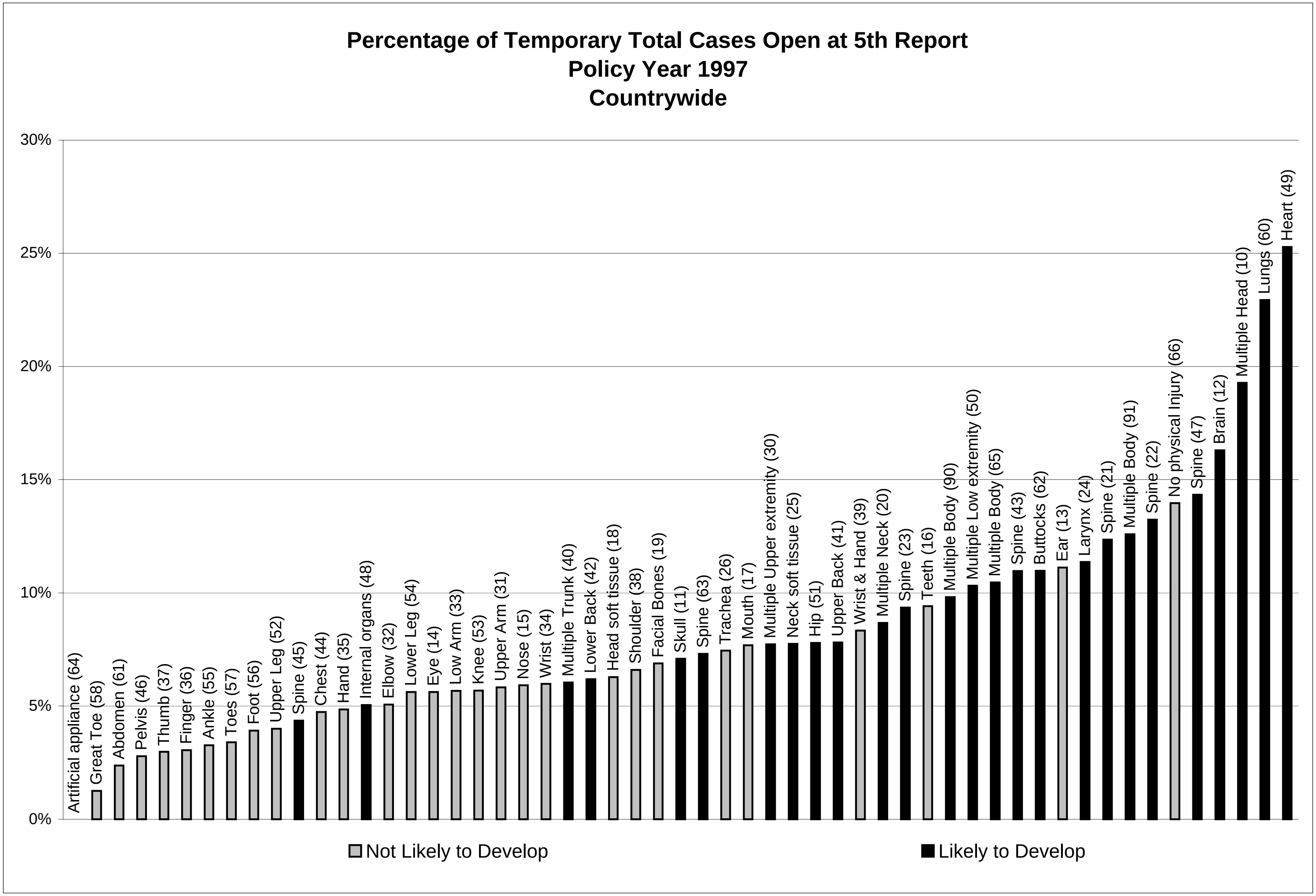

Percentage of Temporary Total Cases Open at 5th Report
Policy Year 1997
Countrywide
30%
25%
20%
15%
10%
5%
0%
Artificial appliance (64)
Great Toe (58)
Abdomen (61)
Pelvis (46)
Thumb (37)
Finger (36)
Ankle (55)
Toes (57)
Foot (56)
Upper Leg (52)
Spine (45)
Chest (44)
Hand (35)
Internal organs (48)
Elbow (32)
Lower Leg (54)
Eye (14)
Low Arm (33)
Knee (53)
Upper Arm (31)
Nose (15)
Wrist (34)
Multiple Trunk (40)
Lower Back (42)
Head soft tissue (18)
Shoulder (38)
Facial Bones (19)
Skull (11)
Spine (63)
Trachea (26)
Mouth (17)
Multiple Upper extremity (30)
Neck soft tissue (25)
Hip (51)
Upper Back (41)
Wrist & Hand (39)
Multiple Neck (20)
Spine (23)
Teeth (16)
Multiple Body (90)
Multiple Low extremity (50)
Multiple Body (65)
Spine (43)
Buttocks (62)
Ear (13)
Larynx (24)
Spine (21)
Multiple Body (91)
Spine (22)
No physical Injury (66)
Spine (47)
Brain (12)
Multiple Head (10)
Lungs (60)
Heart (49)
Not Likely to Develop
Likely to Develop

Exhibit 6

URE WORKERS COMPENSATION STATISTICAL PLAN
Part of Body—Injury Codes and Descriptions

Code*		Narrative Description
I.	**Head**	
10	Multiple Head Injury	Any combination of Head injuries
11	Skull	
12	Brain	
13	Ear(s)	Includes: Hearing, Inside Eardrum
14	Eye(s)	Includes: Optic Nerves, Vision, Eyelids
15	Nose	Includes: Nasal Passage, Sinus, Sense of Smell
16	Teeth	
17	Mouth	Includes: Lips, Tongue, Throat, Taste
18	Soft tissue	
19	Facial Bones	Includes: Jaw
II.	**Neck**	
20	Multiple Neck Injury	Any combination of Neck injuries
21	Vertebrae	Includes: Spinal Column Bone, "Cervical Segment"
22	Disc	Includes: Spinal Column cartilage, "Cervical Segment"
23	Spinal Cord	Includes: Nerve Tissue, "Cervical Segment"
24	Larynx	Includes: Cartilage and Vocal Cords
25	Soft Tissue	Other than Larynx or Trachea
26	Trachea	
III.	**Upper Extremities**	
30	Multiple Upper Extremities	Any combination of Upper Extremity injuries, excluding Hands and Wrists combined
31	Upper Arm	Humerus and Corresponding Muscles, excluding Clavicle and Scapula
32	Elbow	Radial Head
33	Lower Arm	Forearm—Radius, Ulna and Corresponding Muscles
34	Wrist	Carpals and Corresponding Muscles
35	Hand	Metacarpals and Corresponding Muscles - excluding Wrist or Fingers
36	Finger(s)	Other than Thumb and Corresponding Muscles
37	Thumb	
38	Shoulder(s)	Armpit, Rotator Cuff, Trapezius, Clavicle, Scapula
39	Wrist(s) & Hand(s)	
IV.	**Trunk**	
40	Multiple Trunk	Any combination of Trunk injuries
41	Upper Back Area	(Thoracic Area) Upper Back Muscles, excluding Vertebrae, Disc, Spinal Cord
42	Lower Back Area	(Lumbar Area and Lumbo Sacral) Lower Back Muscles, excluding Sacrum, Coccyx, Pelvis, Vertebrae, Disc, Spinal

* Shaded areas are part of body codes considered "likely to develop."

Exhibit 6

URE WORKERS COMPENSATION STATISTICAL PLAN
Part of Body—Injury Codes and Descriptions

Code*		Narrative Description
		Cord
43	Disc	Spinal Column Cartilage other than Cervical Segment
44	Chest	Including Ribs, Sternum, Soft Tissue
45	Sacrum and Coccyx	Final Nine Vertebrae - Fused
46	Pelvis	
47	Spinal Cord	Nerve Tissue other than Cervical Segment
48	Internal Organs	Other than Heart and Lungs
49	Heart	
60	Lungs	
61	Abdomen	Excluding Injury to Internal Organs Including Groin
62	Buttocks	Soft Tissue
63	Lumbar and/or Sacral Vertebrae (Vertebra NOC Trunk	Bone Portion of the Spinal Column
V.	**Lower Extremities**	
50	Multiple Lower Extremities	Any combination of Lower Extremity injuries
51	Hip	
52	Upper Leg	Femur and Corresponding Muscles
53	Knee	Patella
54	Lower Leg	Tibia, Fibula and Corresponding Muscles
55	Ankle	Tarsals
56	Foot	Metatarsals, Heel, Achilles Tendon and Corresponding Muscles - excluding Ankle or Toes
57	Toes	
58	Great Toe	
VI.	**Multiple Body Parts**	
64	Artificial Appliance	Braces, etc.
65	Insufficient Info to Properly Identify - Unclassified	Insufficient information to identify part affected
66	No Physical Injury	Mental Disorder
90	Multiple Body Parts (Including Body Systems & Body Parts)	Applies when more than one Major Body Part has been affected, such as an Arm and a Leg and Multiple Internal Organs
91	Body Systems and Multiple Body Systems	Applies when functioning of an Entire Body System has been affected without specific injury to any other part, as in the case of Poisoning, Corrosive Action, Inflammation, Affecting Internal Organs, Damage to Nerve Centers, etc.; does NOT apply when the systemic damage results from an External Injury affecting an External Part such as a Back Injury that includes damage to the Nerves of the Spinal Cord

* Shaded areas are part of body codes considered "likely to develop."

Exhibit 7

Loss Development on a fixed set of claims
Policy Year 1997
Countrywide

Cumulative 1st to 5th report *

	ind_dev	med_dev
PP-L	1.387	1.183
PP-N	1.234	1.028
TT-L<=26K	1.797	1.170
TT-N<=26K	1.373	1.014
TT-L>26K	1.226	1.168
TT-N>26K	1.084	0.953
TT-L	1.522	1.170
TT-N	1.271	1.001
TT<=26K	1.548	1.080
TT>26K	1.162	1.083

* Loss devlopment factors exclude all crossover.

Exhibit 8a

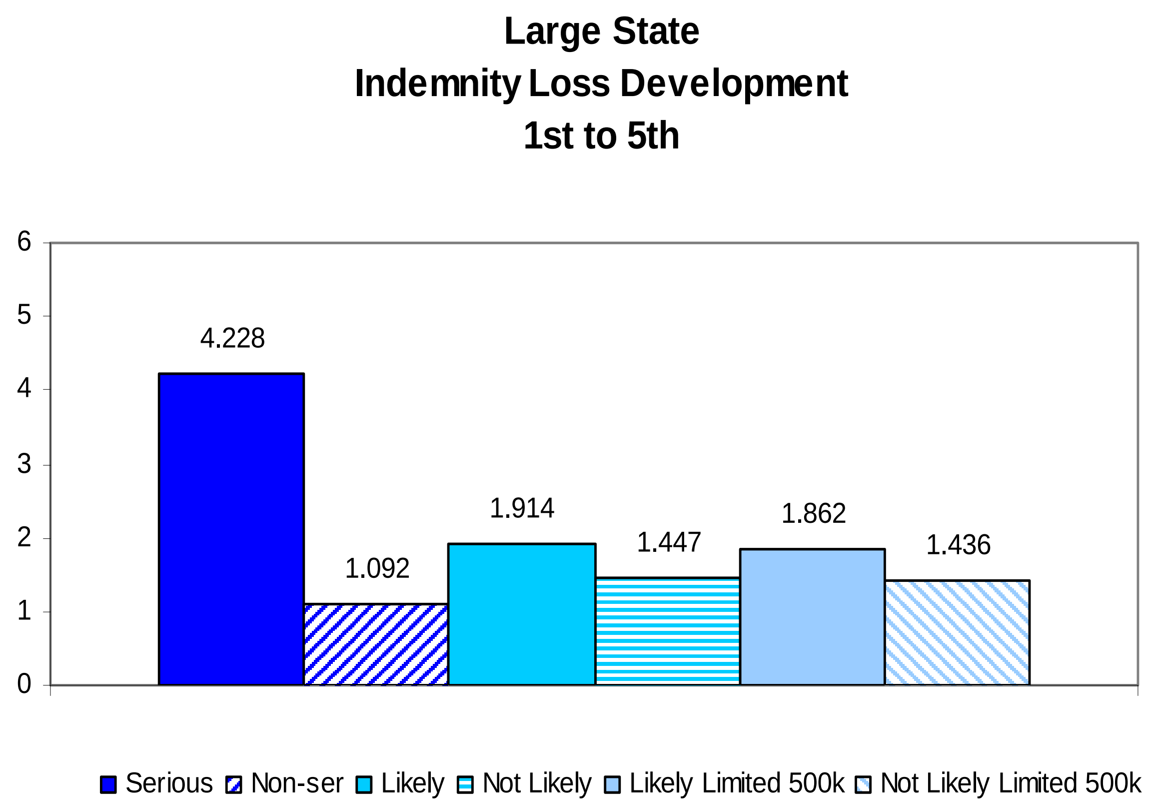

Large State
Indemnity Loss Development
1st to 5th
6
5
4
3
2
1
0
4.228
1.092
1.914
1.447
1.862
1.436
Serious
Non-ser
Likely
Not Likely
Likely Limited 500k
Not Likely Limited 500k

Exhibit 8b

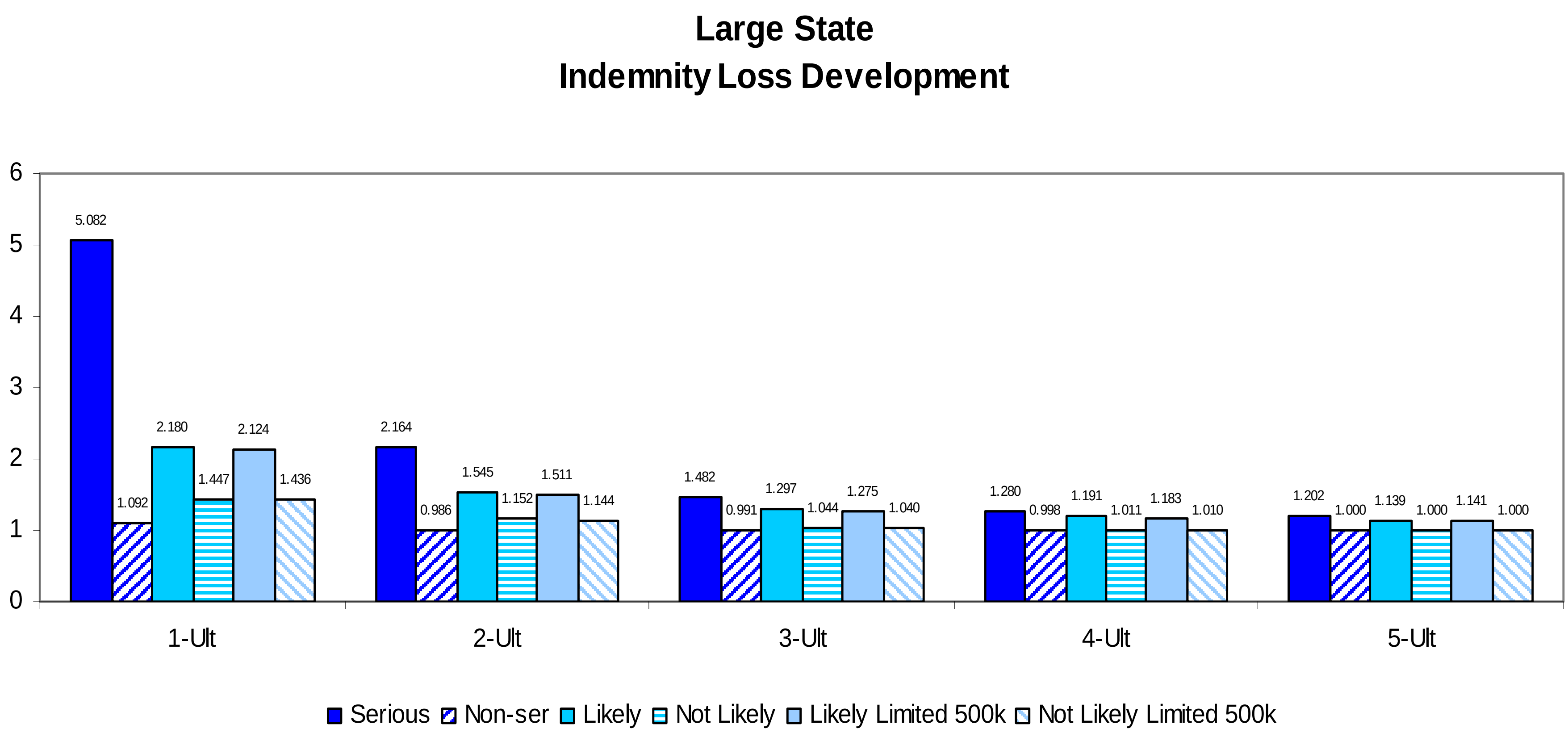

Large State
Indemnity Loss Development
6
5
4
3
2
1
0
5.082
1.092
2.180
1.447
2.124
1.436
2.164
0.986
1.545
1.152
1.511
1.144
1.482
0.991
1.297
1.044
1.275
1.040
1.280
0.998
1.191
1.011
1.183
1.010
1.202
1.000
1.139
1.000
1.141
1.000
1-Ult
2-Ult
3-Ult
4-Ult
5-Ult
Serious
Non-ser
Likely
Not Likely
Likely Limited 500k
Not Likely Limited 500k

Exhibit 8c

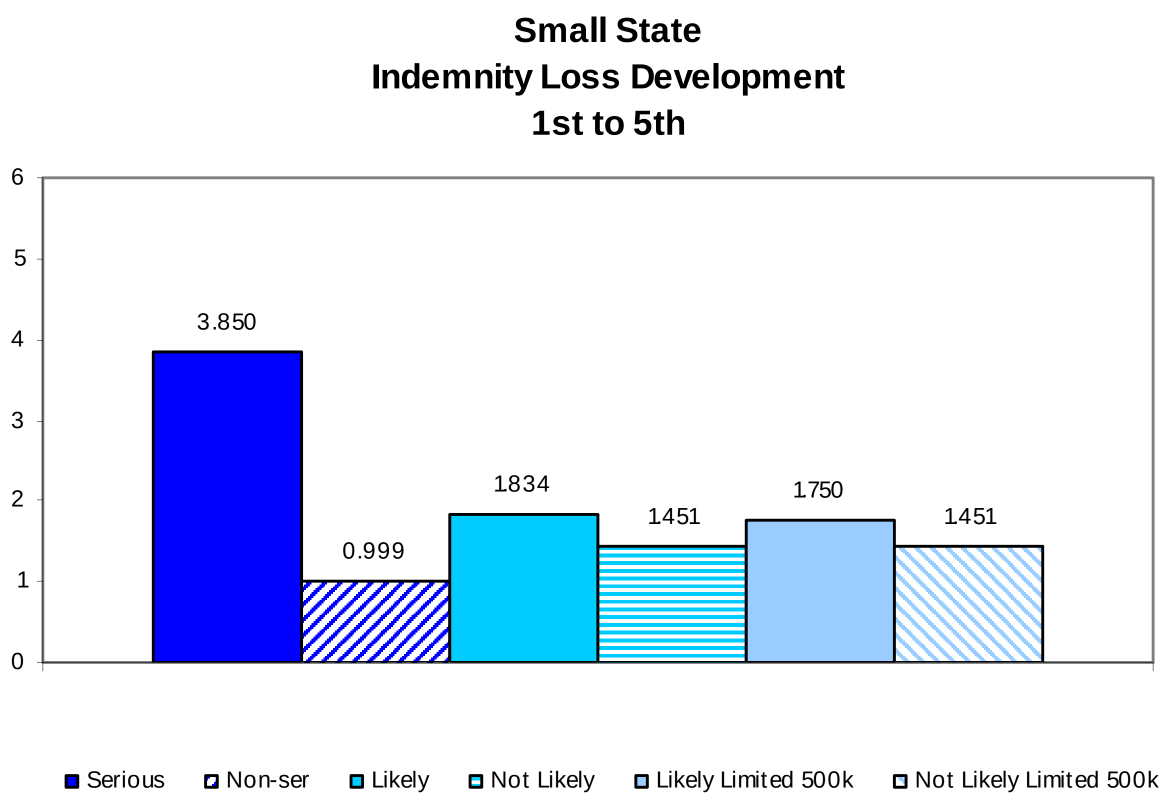

Small State
Indemnity Loss Development
1st to 5th
6
5
4
3
2
1
0
3.850
0.999
1834
1451
1750
1451
Serious
Non-ser
Likely
Not Likely
Likely Limited 500k
Not Likely Limited 500k

Exhibit 8d

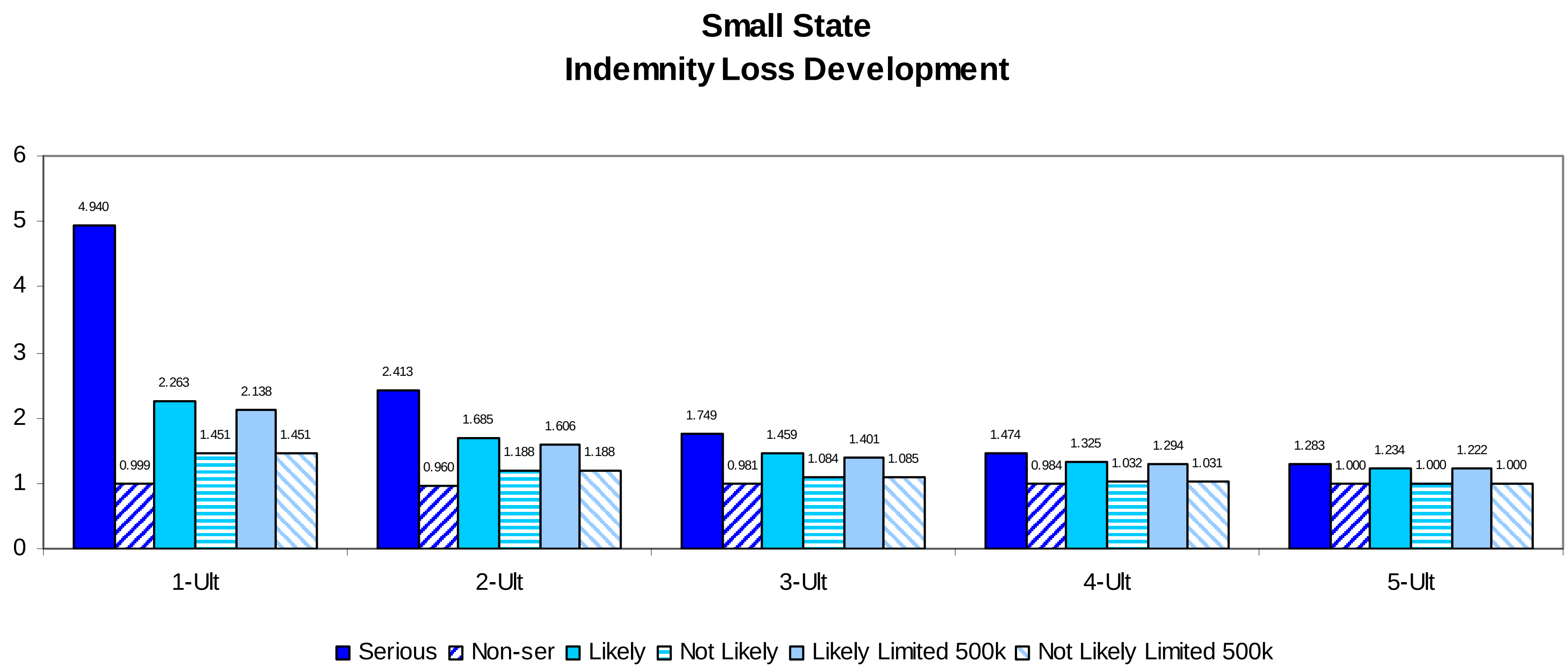

Small State
Indemnity Loss Development
6
5
4
3
2
1
0
4.940
0.999
2.263
1.451
2.138
1.451
2.413
0.960
1.685
1.188
1.606
1.188
1.749
0.981
1.459
1.084
1.401
1.085
1.474
0.984
1.325
1.032
1.294
1.031
1.283
1.000
1.234
1.000
1.222
1.000
1-Ult
2-Ult
3-Ult
4-Ult
5-Ult
Serious
Non-ser
Likely
Not Likely
Likely Limited 500k
Not Likely Limited 500k

Exhibit 9a

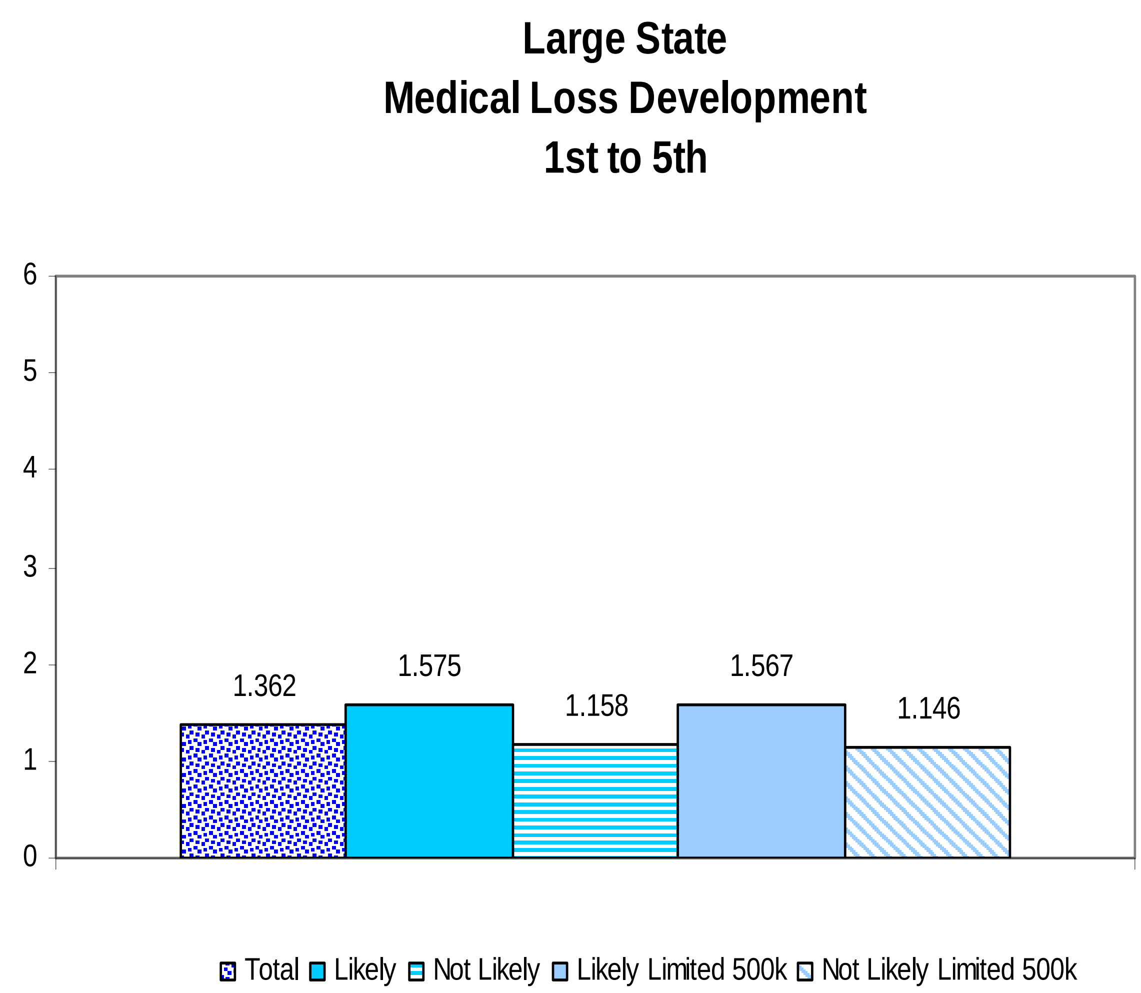

Large State
Medical Loss Development
1st to 5th
6
5
4
3
2
1
0
1.362
1.575
1.158
1.567
1.146
Total
Likely
Not Likely
Likely Limited 500k
Not Likely Limited 500k

Exhibit 9b

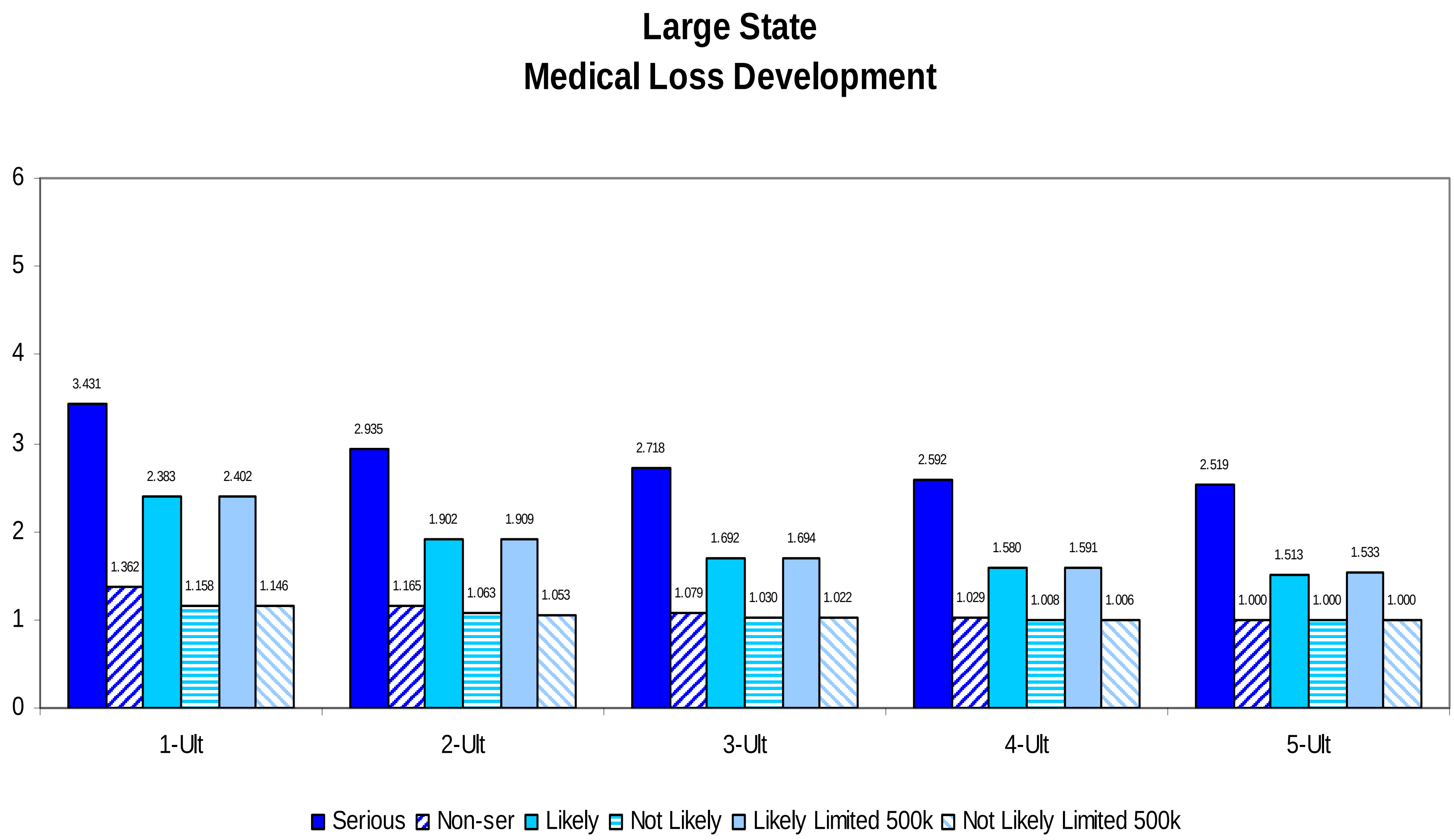

Large State
Medical Loss Development
6
5
4
3
2
1
0
3.431
1.362
2.383
1.158
2.402
1.146
2.935
1.165
1.902
1.063
1.909
1.053
2.718
1.079
1.692
1.030
1.694
1.022
2.592
1.029
1.580
1.008
1.591
1.006
2.519
1.000
1.513
1.000
1.533
1.000
1-Ult
2-Ult
3-Ult
4-Ult
5-Ult
Serious
Non-ser
Likely
Not Likely
Likely Limited 500k
Not Likely Limited 500k

Exhibit 9c

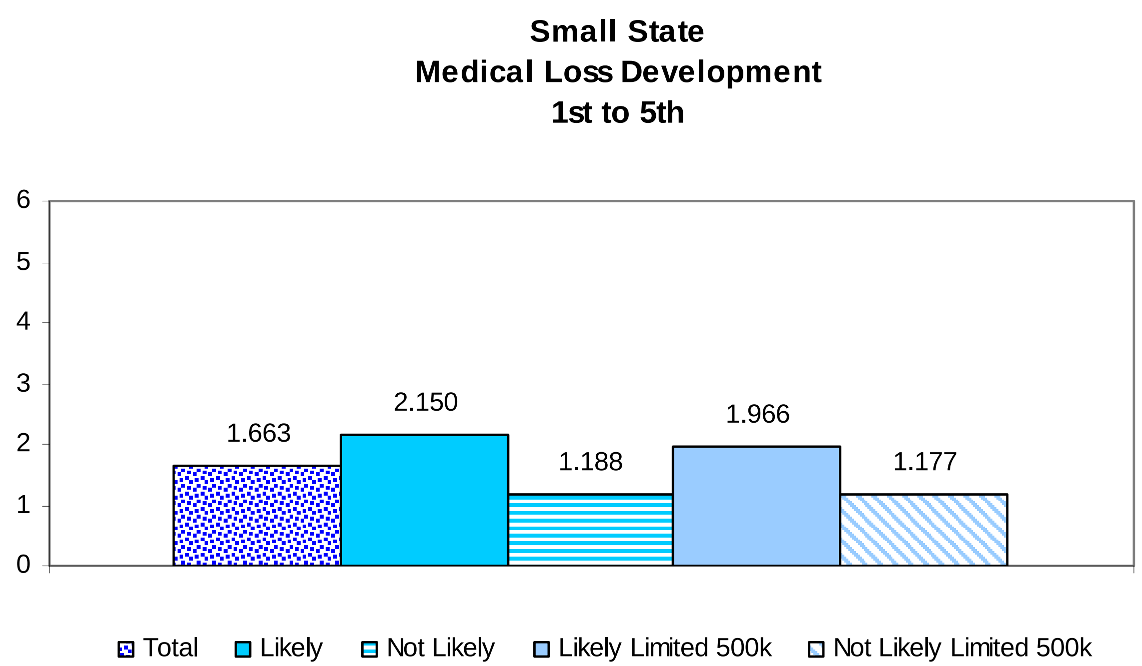

Small State
Medical Loss Development
1st to 5th
6
5
4
3
2
1
0
1.663
2.150
1.188
1.966
1.177
Total
Likely
Not Likely
Likely Limited 500k
Not Likely Limited 500k

Exhibit 9d

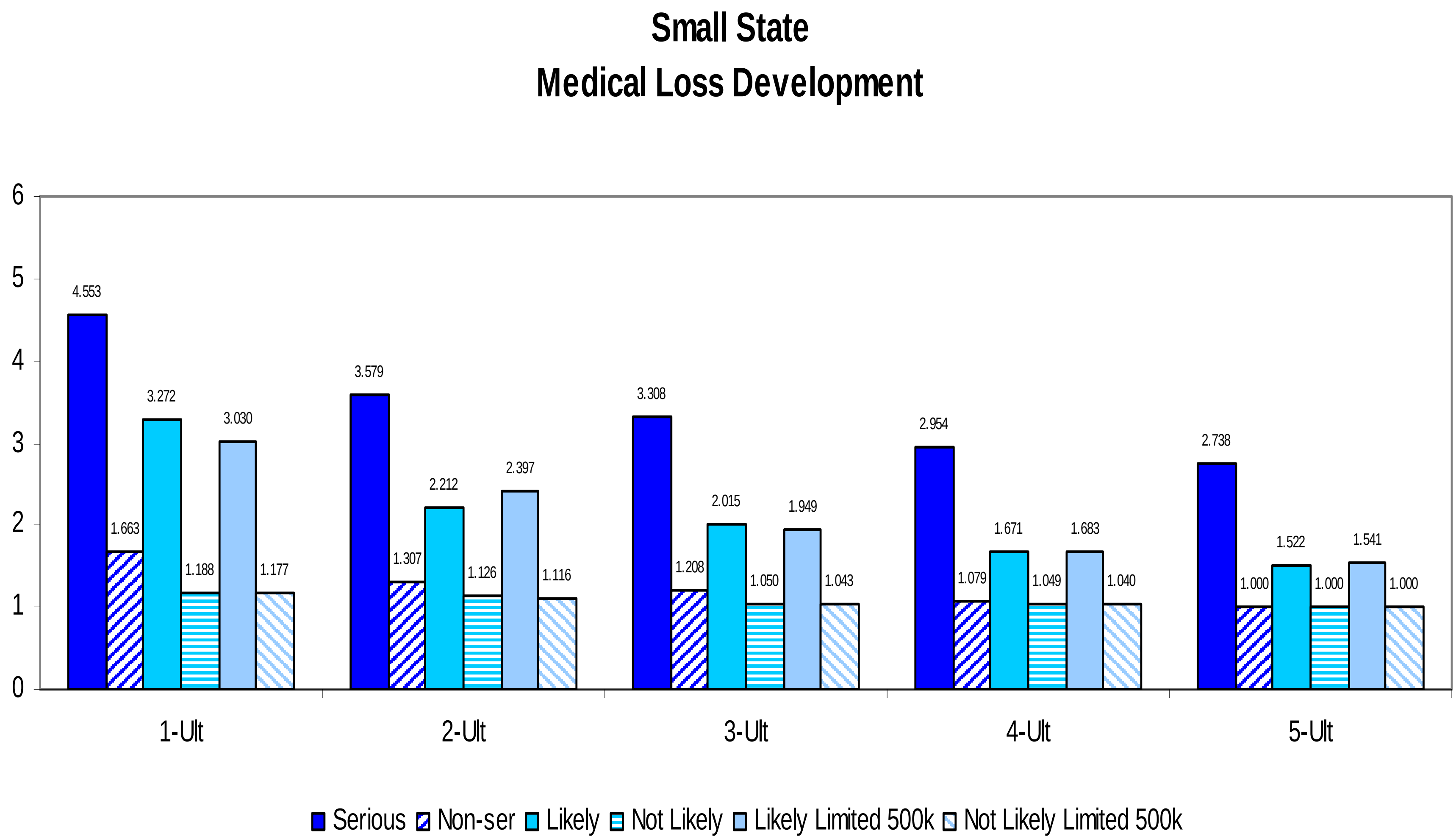

Small State
Medical Loss Development
6
5
4
3
2
1
0
4.553
1.663
3.272
1.188
3.030
1.177
3.579
1.307
2.212
1.126
2.397
1.116
3.308
1.208
2.015
1.050
1.949
1.043
2.954
1.079
1.671
1.049
1.683
1.040
2.738
1.000
1.522
1.000
1.541
1.000
1-Ult
2-Ult
3-Ult
4-Ult
5-Ult
Serious
Non-ser
Likely
Not Likely
Likely Limited 500k
Not Likely Limited 500k

Exhibit 10

ANALYSIS OF LOSS DEVELOPMENT
BY INJURY TYPE, PART OF BODY TYPE, AND OPEN/CLOSED (at 1st report)
STATISTICAL PLAN DATA - CLASS RATEMAKING

Summary Results

Analysis of Claim Status:

Claims Locked at 1st — Development 1st to x (1:x) -- Limited Incurred Losses

Injury Type Category	Description	Ind 1999 1:6	Ind 2000 1:5	Med 1999 1:6	Med 2000 1:5	I+M 1999 1:6	I+M 2000 1:5
TTLO	TT Likely Body Part, Open at 1st	1.680	1.670	1.268	1.277	1.467	1.467
TTLC	TT Likely Body Part, Closed at 1st	1.196	1.184	1.125	1.116	1.158	1.147
TTNO	TT Not Likely Body Part, Open at 1st	1.475	1.465	1.089	1.085	1.267	1.260
TTNC	TT Not Likely Body Part, Closed at 1st	1.140	1.124	1.085	1.074	1.108	1.095
TTL	TT Likely Body Part	1.577	1.569	1.235	1.241	1.398	1.398
TTN	TT Not Likely Body Part	1.378	1.370	1.088	1.082	1.218	1.212
PPLO	PP Likely Body Part, Open at 1st	1.483	1.494	1.252	1.266	1.375	1.388
PPLC	PP Likely Body Part, Closed at 1st	1.100	1.064	1.125	1.101	1.110	1.078
PPNO	PP Not Likely Body Part, Open at 1st	1.325	1.324	1.047	1.058	1.188	1.192
PPNC	PP Not Likely Body Part, Closed at 1st	1.068	1.061	1.071	1.063	1.069	1.062
PPL	PP Likely Body Part	1.425	1.428	1.237	1.246	1.339	1.345
PPN	PP Not Likely Body Part	1.270	1.268	1.052	1.059	1.163	1.165
MoLO	Mo Likely Body Part, Open at 1st	---	---	1.552	1.592	2.629	2.822
MoLC	Mo Likely Body Part, Closed at 1st	---	---	1.204	1.175	1.428	1.379
MoNO	Mo Not Likely Body Part, Open at 1st	---	---	1.188	1.265	1.914	2.082
MoNC	Mo Not Likely Body Part, Closed at 1st	---	---	1.111	1.118	1.232	1.230
MoL	Mo Likely Body Part	---	---	1.270	1.252	1.668	1.645
MoN	Mo Not Likely Body Part	---	---	1.120	1.135	1.318	1.333
FaLO	Fa Likely Body Part, Open at 1st	0.884	0.914	0.710	0.829	0.868	0.906
FaLC	Fa Likely Body Part, Closed at 1st	1.047	1.089	1.051	0.997	1.047	1.073
FaNO	Fa Not Likely Body Part, Open at 1st	0.948	0.933	0.847	0.994	0.928	0.942
FaNC	Fa Not Likely Body Part, Closed at 1st	1.018	0.999	1.095	1.018	1.040	1.004
FaL	Fa Likely Body Part	0.899	0.926	0.750	0.852	0.885	0.919
FaN	Fa Not Likely Body Part	0.953	0.937	0.875	0.996	0.937	0.947
PTLO	PT Likely Body Part, Open at 1st	0.895	0.900	0.942	0.966	0.922	0.937
PTLC	PT Likely Body Part, Closed at 1st	0.994	0.989	0.984	1.008	0.990	0.997
PTNO	PT Not Likely Body Part, Open at 1st	0.937	0.960	0.957	0.873	0.948	0.915
PTNC	PT Not Likely Body Part, Closed at 1st	1.015	0.981	1.048	0.985	1.029	0.983
PTL	PT Likely Body Part	0.904	0.906	0.944	0.967	0.926	0.940
PTN	PT Not Likely Body Part	0.949	0.962	0.967	0.882	0.958	0.920

Notes: Injury Type Category = Injury Type + Body Part category + Claim Status at 1st

Injury Types:
- Fa = Fatal
- PT = Permanent Total
- PP = Permanent Partial
- TT = Temporary Total
- Mo = Medical Only

Body Part Categories:
- N = Not likely body part
- L = Likely body part

Claim Status:
- C = Closed at 1st
- O = Open at 1st

Data: All NCCI ratemaking states
Excludes carriers not reporting in URE format
Applies the single claim loss limitation at $500K

Exhibit 11

ANALYSIS OF LOSS DEVELOPMENT
BY INJURY TYPE, PART OF BODY TYPE, AND OPEN/CLOSED (at 1st report)
STATISTICAL PLAN DATA - CLASS RATEMAKING

Summary Results

Analysis of Development Combinations:

Claims Locked at 1st

Development 1st to x (1:x) -- Limited Incurred Losses

Options	Devel. Category	Injury Type Categories Included	Ind 1999 1:6	Ind 2000 1:5	Med 1999 1:6	Med 2000 1:5	I+M 1999 1:6	I+M 2000 1:5	I+M 1999 $M	% Total $ Move	I+M 2000 $M	% Total $ Move
Current	Likely	Fa + PT + PPL + TTL	1.443	1.444	1.221	1.229	1.337	1.341	4,977		5,128	
	Not Likely	All Other	1.459	1.454	1.105	1.106	1.242	1.241	5,594		5,771	
Opt 1	Likely	Fa + PT + PPLO + TTLO	1.500	1.504	1.242	1.253	1.377	1.385	4,127	-8%	4,264	-8%
	Not Likely	All Other	1.409	1.402	1.107	1.106	1.229	1.225	6,444		6,635	
Opt 2	Likely	Fa -Fa1 + PT + PPLO + TTLO	1.556	1.561	1.247	1.258	1.403	1.411	3,918	-10%	4,041	-10%
	Not Likely	All Other	1.375	1.369	1.105	1.105	1.218	1.216	6,654		6,859	
Opt 3	Likely	Fa -Fa1 + PT + PPLO + TTLO + MoLO	1.601	1.609	1.259	1.270	1.428	1.438	4,000	-9%	4,120	-9%
	Not Likely	All Other	1.343	1.335	1.096	1.095	1.201	1.197	6,572		6,780	
Opt 4	Likely	All Injury Types LO	1.554	1.560	1.258	1.271	1.410	1.420	4,130	-8%	4,263	-8%
	Not Likely	All Other	1.367	1.359	1.097	1.095	1.208	1.203	6,442		6,637	
Opt 5	Likely	All injury types LO, -Fa1	1.609	1.616	1.262	1.275	1.434	1.444	3,952	-10%	4,071	-10%
	Not Likely	All Other	1.340	1.332	1.095	1.094	1.199	1.195	6,619		6,829	

Claims Not Locked

-- Includes Crossover and Arisings on Subs

Development 1st to x (1:x) -- Limited Incurred Losses

Options	Devel. Category	Injury Type Categories Included	Ind 1999 1:6	Ind 2000 1:5	Med 1999 1:6	Med 2000 1:5	I+M 1999 1:6	I+M 2000 1:5	I+M 1999 $M	I+M 2000 $M
Current	Likely	Fa + PT + PPL + TTL	1.694	1.678	1.394	1.384	1.550	1.538	4,977	5,128
	Not Likely	All Other	1.411	1.426	1.096	1.106	1.218	1.230	5,594	5,771
Opt 1	Likely	Fa + PT + PPLO + TTLO	1.771	1.757	1.426	1.419	1.607	1.596	4,127	4,264
	Not Likely	All Other	1.394	1.402	1.113	1.118	1.226	1.233	6,444	6,635
Opt 2	Likely	Fa -Fa1 + PT + PPLO + TTLO	1.832	1.826	1.426	1.421	1.631	1.625	3,918	4,041
	Not Likely	All Other	1.375	1.377	1.114	1.119	1.223	1.227	6,654	6,859

Notes: Injury Type Category = Injury Type + Body Part category + Claim Status at 1st

Injury Types:
- Fa = Fatal
- PT = Permanent Total
- PP = Permanent Partial
- TT = Temporary Total
- Mo = Medical Only

Fa1 = Fatal at 1st
LO = Likely body part, open at 1st

Body Part Categories:
- N = Not likely body part
- L = Likely body part

Claim Status:
- C = Closed at 1st
- O = Open at 1st

Data: All NCCI ratemaking states
Excludes carriers not reporting in URE format
Applies the single claim loss limitation at $500K

Exhibit 12

ANALYSIS OF LOSS DEVELOPMENT
BY INJURY TYPE, PART OF BODY, AND OPEN/CLOSED (at 1st report)
STATISTICAL PLAN DATA - CLASS RATEMAKING

Summary - Fatal and PT Development

FATAL
PY 1999

Category at 1st	Category at 6th	Ind+Med at 1st	Ind+Med at 6th	Ind+Med Development	Ind+Med Injury Type Development
Stays in Injury Type					
FaLO	FaL	172,831,898	149,614,886	-23,217,012	-23,217,012
FaLC	FaL	18,522,409	19,393,256	870,847	870,847
FaNO	FaN	11,940,325	11,054,696	-885,629	-885,629
FaNC	FaN	1,101,646	1,140,701	39,055	39,055
Total Fa to Fa		204,396,278	181,203,539	-23,192,739	**-23,192,739**
Moves into Injury Type					
PTLO	FaL	5,948,628	5,110,187	-838,441	5,110,187
TTLO	FaL	12,678,675	16,950,679	4,272,004	16,950,679
MoLO	FaL	213,376	1,369,657	1,156,281	1,369,657
PPLO	FaL	13,519,287	15,965,816	2,446,529	15,965,816
----	FaL	0	12,583,421	12,583,421	12,583,421
----	FaN	0	1,366,094	1,366,094	1,366,094
TTLC	FaL	339,998	520,788	180,790	520,788
MoLC	FaL	23,952	979,881	955,929	979,881
PPLC	FaL	265,121	226,770	-38,351	226,770
TTNO	FaN	1,265,655	1,498,696	233,041	1,498,696
MoNO	FaN	30,028	938,698	908,670	938,698
PPNO	FaN	1,321,388	2,025,430	704,042	2,025,430
TTNC	FaN	37,411	55,284	17,873	55,284
MoNC	FaN	6,722	38,582	31,860	38,582
PPNC	FaN	53,961	71,656	17,695	71,656
Total Other to Fa		35,704,202	59,701,639	23,997,437	**59,701,639**
Other LO to Fa		32,359,966	39,396,339	7,036,373	39,396,339
Arising to Fa		0	13,949,515	13,949,515	13,949,515
All other to Fa		3,344,236	6,355,785	3,011,549	6,355,785
Moves out of Injury Type					
FaLO	PTL	597,761	954,391	356,630	-597,761
FaLO	TTL	3,373,971	3,034,733	-339,238	-3,373,971
FaLO	MoL	254,232	32,821	-221,411	-254,232
FaLO	PPL	208,664	205,713	-2,951	-208,664
FaLC	PTL	2,888	2,888	0	-2,888
FaLC	TTL	33,168	58,565	25,397	-33,168
FaLC	MoL	21,257	1,255	-20,002	-21,257
FaNO	TTN	383,884	403,289	19,405	-383,884
FaNO	MoN	6,500	0	-6,500	-6,500
FaNO	PPN	183,853	155,381	-28,472	-183,853
FaNC	TTN	18,617	22,089	3,472	-18,617
FaNC	PPN	5,650	8,364	2,714	-5,650
Total Fa to Other		5,090,445	4,879,489	-210,956	-5,090,445
Locked Injury Type Development					
		209,486,723	**186,083,028**		**-23,403,695**
					0.888
Cross Over Injury Type Development					
		209,486,723	**240,905,178**		**31,418,454**
					1.150

FATAL
PY 2000

Category at 1st	Category at 5th	Ind+Med at 1st	Ind+Med at 5th	Ind+Med Development	Ind+Med Injury Type Development
Stays in Injury Type					
FaLO	FaL	186,762,580	169,522,620	-17,239,960	-17,239,960
FaLC	FaL	15,910,070	17,079,627	1,169,557	1,169,557
FaNO	FaN	13,298,124	12,123,076	-1,175,048	-1,175,048
FaNC	FaN	1,058,062	1,060,164	2,102	2,102
Total Fa to Fa		217,028,836	199,785,487	-17,243,349	**-17,243,349**
Moves into Injury Type					
PTLO	FaL	3,966,358	3,819,262	-147,096	3,819,262
TTLO	FaL	6,169,446	7,229,697	1,060,251	7,229,697
MoLO	FaL	61,485	1,180,318	1,118,833	1,180,318
PPLO	FaL	8,128,178	7,374,389	-753,789	7,374,389
----	FaL	0	11,867,836	11,867,836	11,867,836
----	FaN	0	1,623,951	1,623,951	1,623,951
TTLC	FaL	583,054	826,786	243,732	826,786
MoLC	FaL	92,349	463,387	371,038	463,387
PPLC	FaL	5,000	5,000	0	5,000
PTNO	FaN	1,390,720	1,000,000	-390,720	1,000,000
TTNO	FaN	1,431,739	1,560,824	129,085	1,560,824
MoNO	FaN	45,319	961,661	916,342	961,661
PPNO	FaN	768,723	1,238,737	470,014	1,238,737
TTNC	FaN	54,585	235,528	180,943	235,528
MoNC	FaN	15,095	78,620	63,525	78,620
Total Other to Fa		22,712,051	39,465,996	16,753,945	**39,465,996**
Other LO to Fa		18,325,467	19,603,666	1,278,199	19,603,666
Arising to Fa		0	13,491,787	13,491,787	13,491,787
All other to Fa		4,386,584	6,370,543	1,983,959	6,370,543
Moves out of Injury Type					
FaLO	PTL	1,238,018	1,105,647	-132,371	-1,238,018
FaLO	TTL	1,500,046	1,697,990	197,944	-1,500,046
FaLO	MoL	579,712	25,285	-554,427	-579,712
FaLO	PPL	2,088,289	1,717,273	-371,016	-2,088,289
FaLC	TTL	5,536	11,236	5,700	-5,536
FaLC	MoL	2,591	2,553	-38	-2,591
FaLC	PPL	239,984	240,284	300	-239,984
FaNO	TTN	292,945	687,510	394,565	-292,945
FaNO	MoN	55,505	3,478	-52,027	-55,505
FaNO	PPN	306,629	334,433	27,804	-306,629
FaNC	TTN	23,176	25,341	2,165	-23,176
Total Fa to Other		6,332,431	5,851,030	-481,401	-6,332,431
Locked Injury Type Development					
		223,361,267	**205,636,517**		**-17,724,750**
					0.921
Cross Over Injury Type Development					
		223,361,267	**239,251,483**		**15,890,216**
					1.071

Notes: Injury Type Category = Injury Type + Body Part category + Claim Status at 1st

Injury Types:
- Fa = Fatal
- PT = Permanent Total
- PP = Permanent Partial
- TT = Temporary Total
- Mo = Medical Only

Body Part Categories:
- N = Not likely body part
- L = Likely body part

Claim Status:
- C = Closed at 1st
- O = Open at 1st

LO = Likely body part, open at 1st

Data: All NCCI ratemaking states
Excludes carriers not reporting in URE format
Applies the single claim loss limitation at $500K

Exhibit 13

ANALYSIS OF LOSS DEVELOPMENT
BY INJURY TYPE, PART OF BODY, AND OPEN/CLOSED (at 1st report)
STATISTICAL PLAN DATA - CLASS RATEMAKING

Summary - Fatal and PT Development

PERMANENT TOTAL
PY 1999

Category at 1st	Category at 6th	Ind+Med at 1st	Ind+Med at 6th	Ind+Med Development	Ind+Med Injury Type Development
Stays in Injury Type					
PTLO	PTL	83,291,688	81,330,910	-1,960,778	-1,960,778
PTLC	PTL	7,474,144	7,494,083	19,939	19,939
PTNO	PTN	24,646,111	24,954,711	308,600	308,600
PTNC	PTN	4,647,731	4,751,566	103,835	103,835
Total PT to PT		120,059,674	118,531,270	-1,528,404	**-1,528,404**
Moves into Injury Type					
FaLO	PTL	597,761	954,391	356,630	954,391
TTLO	PTL	58,296,074	180,949,875	122,653,801	180,949,875
MoLO	PTL	1,350,219	12,210,680	10,860,461	12,210,680
PPLO	PTL	90,115,408	271,054,396	180,938,988	271,054,396
----	PTL	0	38,971,217	38,971,217	38,971,217
----	PTN	0	17,518,210	17,518,210	17,518,210
FaLC	PTL	2,888	2,888	0	2,888
TTLC	PTL	779,756	4,115,315	3,335,559	4,115,315
MoLC	PTL	50,962	6,193,259	6,142,297	6,193,259
PPLC	PTL	503,709	3,619,431	3,115,722	3,619,431
TTNO	PTN	27,315,144	94,919,491	67,604,347	94,919,491
MoNO	PTN	218,603	4,241,846	4,023,243	4,241,846
PPNO	PTN	35,247,948	111,113,846	75,865,898	111,113,846
TTNC	PTN	452,656	2,778,277	2,325,621	2,778,277
MoNC	PTN	38,566	2,329,502	2,290,936	2,329,502
PPNC	PTN	679,326	3,250,047	2,570,721	3,250,047
Total Other to PT		215,649,020	754,222,671	538,573,651	**754,222,671**
Other LO to PT		150,359,462	465,169,342	314,809,880	465,169,342
Arising to PT		0	56,489,427	56,489,427	56,489,427
All other to PT		65,289,558	232,563,902	167,274,344	232,563,902
Moves out of Injury Type					
PTLO	FaL	5,948,628	5,110,187	-838,441	-5,948,628
PTLO	TTL	4,953,081	3,961,440	-991,641	-4,953,081
PTLO	MoL	1,363,527	27,013	-1,336,514	-1,363,527
PTLO	PPL	16,564,439	12,898,826	-3,665,613	-16,564,439
PTLC	TTL	336,156	193,243	-142,913	-336,156
PTLC	MoL	5,760	5,756	-4	-5,760
PTLC	PPL	237,237	278,250	41,013	-237,237
PTNO	TTN	2,660,188	2,452,291	-207,897	-2,660,188
PTNO	MoN	155,104	51,436	-103,668	-155,104
PTNO	PPN	7,046,099	5,241,309	-1,804,790	-7,046,099
PTNC	TTN	187,636	201,725	14,089	-187,636
PTNC	MoN	9,936	7,836	-2,100	-9,936
PTNC	PPN	19,788	44,490	24,702	-19,788
Total PT to Other		39,487,579	30,473,802	-9,013,777	-39,487,579
Locked Injury Type Development					
		159,547,253	**149,005,072**		**-10,542,181**
					0.934
Cross Over Injury Type Development					
		159,547,253	**872,753,941**		**713,206,688**
					5.470

PERMANENT TOTAL
PY2000

Category at 1st	Category at 5th	Ind+Med @1st	Ind+Med @5th	Ind+Med Development	Ind+Med Injury Type Development
Stays in Injury Type					
PTLO	PTL	76,210,396	80,732,133	4,521,737	4,521,737
PTLC	PTL	5,739,227	5,690,608	-48,619	-48,619
PTNO	PTN	25,918,704	25,534,009	-384,695	-384,695
PTNC	PTN	3,480,736	3,415,996	-64,740	-64,740
Total PT to PT		111,349,063	115,372,746	4,023,683	**4,023,683**
Moves into Injury Type					
FaLO	PTL	1,238,018	1,105,647	-132,371	1,105,647
TTLO	PTL	54,534,730	162,970,263	108,435,533	162,970,263
MoLO	PTL	271,881	9,962,765	9,690,884	9,962,765
PPLO	PTL	81,150,562	241,413,195	160,262,633	241,413,195
----	PTL	0	35,827,796	35,827,796	35,827,796
----	PTN	0	17,544,736	17,544,736	17,544,736
TTLC	PTL	679,688	3,055,005	2,375,317	3,055,005
MoLC	PTL	57,968	5,425,783	5,367,815	5,425,783
PPLC	PTL	1,045,229	1,387,697	342,468	1,387,697
TTNO	PTN	21,615,791	77,574,737	55,958,946	77,574,737
MoNO	PTN	65,919	3,759,852	3,693,933	3,759,852
PPNO	PTN	35,776,820	109,426,819	73,649,999	109,426,819
TTNC	PTN	919,653	2,442,629	1,522,976	2,442,629
MoNC	PTN	25,279	1,874,535	1,849,256	1,874,535
PPNC	PTN	162,571	1,232,692	1,070,121	1,232,692
Total Other to PT		197,544,109	675,004,151	477,460,042	**675,004,151**
Other LO to PT		137,195,191	415,451,870	278,256,679	415,451,870
Arising to PT		0	53,372,532	53,372,532	53,372,532
All other to PT		60,348,918	206,179,749	145,830,831	206,179,749
Moves out of Injury Type					
PTLO	FaL	3,966,358	3,819,262	-147,096	-3,966,358
PTLO	TTL	6,349,440	4,749,108	-1,600,332	-6,349,440
PTLO	MoL	1,892,978	1,092,058	-800,920	-1,892,978
PTLO	PPL	27,503,897	18,239,496	-9,264,401	-27,503,897
PTLC	TTL	210,089	243,128	33,039	-210,089
PTLC	PPL	151,118	148,666	-2,452	-151,118
PTNO	FaN	1,390,720	1,000,000	-390,720	-1,390,720
PTNO	TTN	3,649,969	2,458,285	-1,191,684	-3,649,969
PTNO	MoN	605,065	81,494	-523,571	-605,065
PTNO	PPN	7,684,122	6,825,909	-858,213	-7,684,122
PTNC	TTN	126,692	128,622	1,930	-126,692
PTNC	MoN	934	417	-517	-934
PTNC	PPN	65,145	64,953	-192	-65,145
Total PT to Other		53,596,527	38,851,398	-14,745,129	-53,596,527
Locked Injury Type Development					
		164,945,590	**154,224,144**		**-10,721,446**
					0.935
Cross Over Injury Type Development					
		164,945,590	**790,376,898**		**625,431,308**
					4.792

Notes: Injury Type Category = Injury Type + Body Part category + Claim Status at 1st

Injury Types:
- Fa = Fatal
- PT = Permanent Total
- PP = Permanent Partial
- TT = Temporary Total
- Mo = Medical Only

Body Part Categories:
- N = Not likely body part
- L = Likely body part

Claim Status:
- C = Closed at 1st
- O = Open at 1st

LO = Likely body part, open at 1st

Data: All NCCI ratemaking states
Excludes carriers not reporting in URE format
Applies the single claim loss limitation at $500K

Alternatives for Limiting Losses and Allocating Excess

Exhibit 14

Alt k =	Limit	Using Actual Excess
0	Unlimited	
1	$1M	Allocates Actual IG Excess Uniformly by Class Within the IG
2	$300k	Allocates Actual IG Excess Uniformly by Class Within the IG
3	$300k	Allocates Actual HG Excess Uniformly by Class Within the HG
4	$300k	Same as k=3 with factor to balance to IG Unlimited Losses
5	$300k	Allocates Actual IG Excess by Class Within IG Using Limited Losses x XS% / (1-XS%)
6	$300k	Allocates HG Actual Excess by Class Within HG Using Limited Losses x XS% / (1-XS%)
7	$300k	Allocates Actual State Excess by Class Using Limited Losses x XS% / (1-XS%)

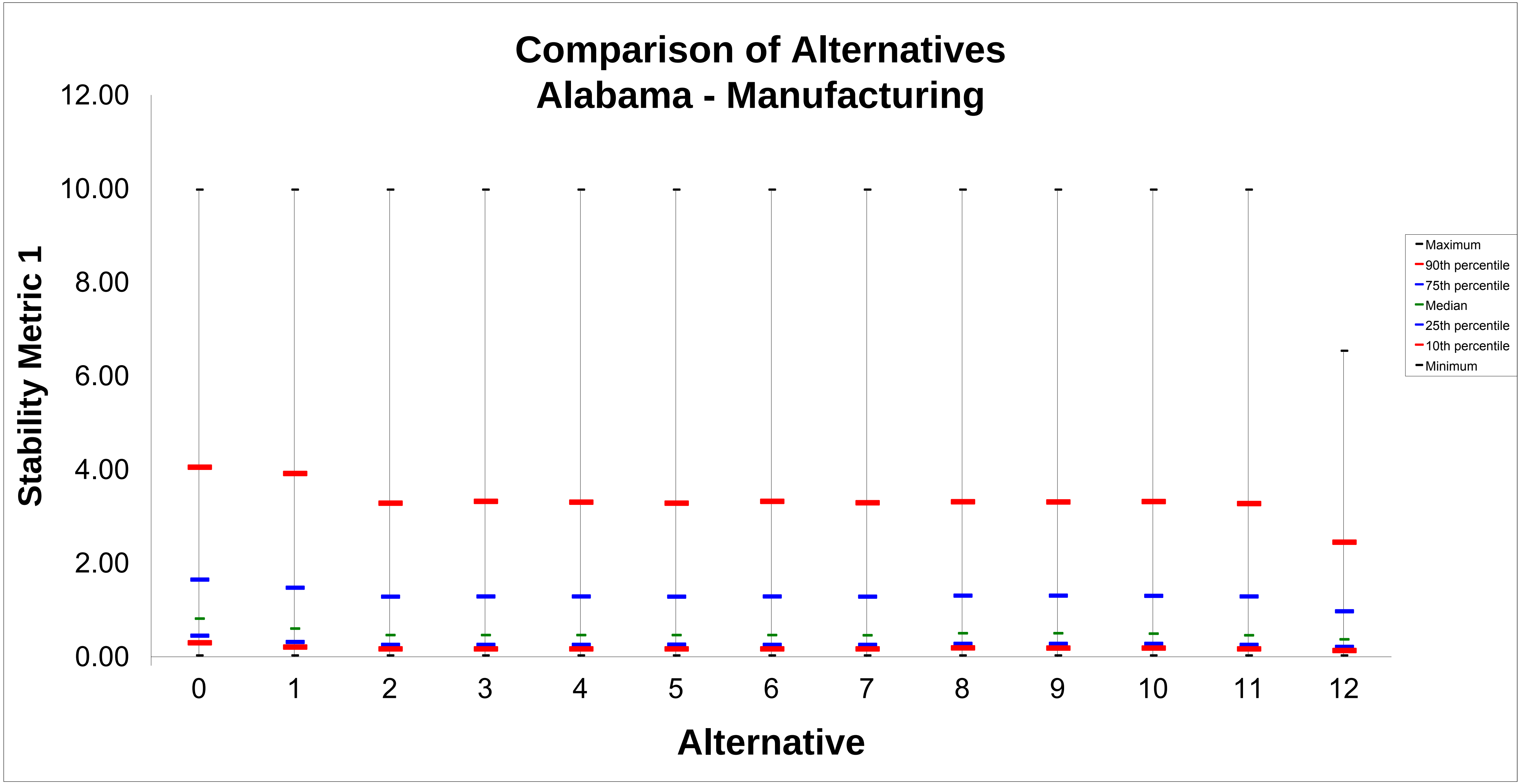
Alt k =	Limit	Using Actual Excess
8	$300k	Allocates Actual IG Excess by Class Within IG Using Unlimited Losses x XS%
9	$300k	Allocates Actual HG Excess by Class Within HG Using Unlimited Losses x XS%
10	$300k	Allocates Actual State Excess by Class Using Unlimited Losses x XS%
13	Vary by Class* $100k, $300k, $1M	Allocates Actual State Excess by Class Using Unlimited Losses x XS%

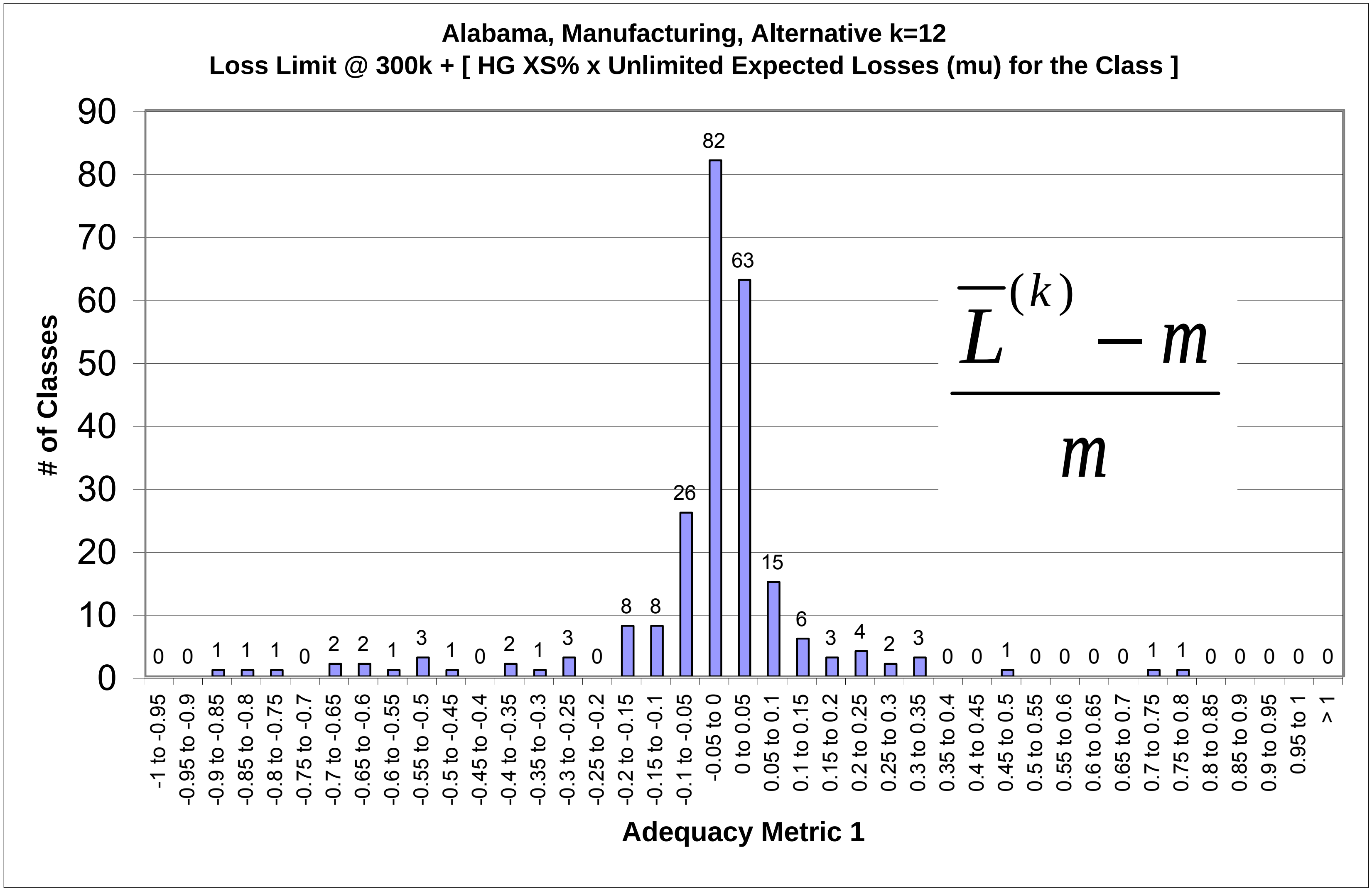
Alt k =	Limit	Using Expected Excess
11	$300k	Limited Actual Losses x 1 / (1- XS%)
12	$300k	Limited Losses + HG XS% x Unlimited Expected Losses (i.e. Mu)
14	Vary by Class* $100k, $300k, $1M	Limited Actual Losses x 1 / (1- XS%)
15	Vary by Class* $100k, $300k, $1M	Limited Losses + HG XS% x Unlimited Expected Losses (i.e. Mu)

Note: Alt 3 and Alt 6 are equivalent

Legend: IG - Industry Group
HG - Hazard Group
XS - per claim adjusted excess ratio

* Alts 13 - 15 proposed three loss limits: 100K for small classes, 300K for medium-size classes & $1M for large classes


Comparison of Alternatives
Alabama - Manufacturing
Stability Metric 1
12.00
10.00
8.00
6.00
4.00
2.00
0.00
0
1
2
3
4
5
6
7
8
9
10
11
12
Alternative
Maximum
90th percentile
75th percentile
Median
25th percentile
10th percentile
Minimum



desired range (-.25, 0, .25)

min	-0.864
unweighted average	-0.031
weighted average	-0.007
max	0.766

Exhibit 16

Analysis of the Indemnity and Medical Excess (XS) Split
Call #31 Data as of 12-31-04 for all NCCI States

PY	Primary Ind.	Primary Med.	XS Ind.	XS Med.	XS Split Ind.	XS Split Med.	Claim Count
Prior	169,816,166	139,683,834	70,279,751	182,840,412	**27.8%**	**72.2%**	619
1982	13,995,376	14,004,624	8,619,005	13,163,124	**39.6%**	**60.4%**	56
1983	62,784,206	65,715,794	39,612,997	119,661,433	**24.9%**	**75.1%**	257
1984	174,613,643	172,386,357	124,779,440	258,040,159	**32.6%**	**67.4%**	694
1985	189,175,924	179,824,076	127,426,362	269,832,016	**32.1%**	**67.9%**	738
1986	231,461,635	228,538,365	139,680,186	284,920,944	**32.9%**	**67.1%**	920
1987	251,592,143	251,907,857	164,604,780	340,891,816	**32.6%**	**67.4%**	1,007
1988	266,459,073	290,040,927	183,851,067	409,875,986	**31.0%**	**69.0%**	1,113
1989	263,077,846	281,922,154	177,611,338	370,323,060	**32.4%**	**67.6%**	1,090
1990	250,790,173	284,709,827	155,166,315	386,803,697	**28.6%**	**71.4%**	1,071
1991	211,153,813	258,346,187	126,792,831	374,456,842	**25.3%**	**74.7%**	939
1992	213,866,898	244,633,102	120,914,845	331,330,704	**26.7%**	**73.3%**	917
1993	177,959,200	215,040,800	112,139,690	298,596,771	**27.3%**	**72.7%**	786
1994	178,857,458	212,142,542	108,183,294	324,031,769	**25.0%**	**75.0%**	782
1995	166,982,566	223,017,434	101,467,025	308,899,713	**24.7%**	**75.3%**	780
1996	211,737,505	248,762,495	128,285,636	313,703,083	**29.0%**	**71.0%**	921
1997	235,761,313	279,738,687	148,148,864	430,926,448	**25.6%**	**74.4%**	1,031
1998	270,545,487	311,954,513	174,634,718	435,518,901	**28.6%**	**71.4%**	1,165
1999	279,735,890	312,764,110	183,062,743	429,648,973	**29.9%**	**70.1%**	1,185
2000	282,319,912	306,680,088	181,576,108	365,938,213	**33.2%**	**66.8%**	1,178
2001	244,889,269	281,610,731	161,786,219	410,967,203	**28.2%**	**71.8%**	1,053
2002	177,579,023	249,920,977	129,992,203	404,432,908	**24.3%**	**75.7%**	855
2003	133,019,301	215,480,699	112,305,448	425,846,975	**20.9%**	**79.1%**	697
2004	56,015,676	79,484,324	44,337,439	111,049,945	**28.5%**	**71.5%**	271
Total	**4,714,189,497**	**5,348,310,503**	**3,025,258,303**	**7,601,701,096**	**28.5%**	**71.5%**	**20,125**

Note: Claims < $500,000 are excluded from the analysis.

Exhibit 17

Proposal for State Indicated Pure Premium Full Credibility Standards

	Current	Indicated[1]	Current/ Indicated
Serious	125	244	51%
Non-serious	350	491	71%
Combined Indemnity	--	1,397	--
Medical (non-serious severity)	750	1,341	56%
Medical (medical severity)	--	719	--

	Selection Proposal	Selection Proposal / Indication
Combined Indemnity	**850**	61%
Medical (non-serious severity)	750	56%
Medical (medical severity)	**400**	56%

Note: 1. From p=95%, k=25% regression results averaged across all 6 states.

Exhibit 18

Proposal for National Pure Premium Full Credibility Standards (actual lost-time claims)

	Current	Indicated[1]	Current/ Indicated
Serious	175	271	65%
Non-serious	500	1,132	44%
Combined Indemnity	--	2,127	--
Medical	1000	1,548	65%

	Selection Proposal	Selection Proposal / Indication
Combined Indemnity	**1,150**	54%
Medical	**1,000**	65%

Note: 1. From p=95%, k=25% regression results averaged across all 6 states.

Exhibit 19

Comparison Of Current And Proposed Indemnity Statewide Credibility

Comparison Of Current And Proposed Indemnity Statewide Credibility

	Statewide Credibility		Statewide Credibility (excluding 50 largest classes)	
	Current Average	Proposal Average[1]	Current Average	Proposal Average[1]
State				
IA	73%	78%	53%	51%
IL	90%	93%	75%	80%
NC	80%	81%	59%	56%
CO	77%	82%	58%	55%
MO	83%	85%	67%	62%
TN	82%	82%	60%	55%
National				
IA	13%	11%	23%	24%
IL	6%	4%	14%	10%
NC	10%	10%	21%	22%
CO	11%	9%	21%	22%
MO	9%	8%	17%	19%
TN	10%	10%	21%	23%

Note: 1. Assuming state Nf = 850 and national Nf = 1150.

Exhibit 20

Comparison Of Current And Proposed Medical Statewide Credibility

Comparison Of Current And Proposed Medical Statewide Credibility

	Statewide Credibility		Statewide Credibility (excluding 50 largest classes)	
	Current Average	Proposal Average[1]	Current Average	Proposal Average[1]
State				
IA	90%	88%	74%	68%
IL	96%	96%	89%	91%
NC	93%	89%	83%	73%
CO	90%	90%	72%	73%
MO	91%	92%	76%	80%
TN	91%	89%	76%	72%
National				
IA	5%	6%	13%	16%
IL	2%	2%	5%	5%
NC	4%	6%	9%	14%
CO	5%	5%	14%	14%
MO	5%	4%	12%	10%
TN	5%	6%	13%	15%

Note: 1. Assuming state Nf = 400 and national Nf = 1000.

Exhibit 21

Selection for IG Differential
FCS = 12,000

Industry Group Claim Counts		Updated Square Root								
	p	90%	90%	90%	98%	98%	98%	**95%**	95%	95%
	k	0.075	0.050	0.025	0.075	0.050	0.025	**0.075**	0.050	0.025
	Nf	8,417	18,939	75,755	16,837	37,883	151,533	**11,951**	26,890	107,561
1,000		34%	23%	11%	24%	16%	8%	29%	19%	10%
2,000		49%	32%	16%	34%	23%	11%	41%	27%	14%
4,000		69%	46%	23%	49%	32%	16%	58%	39%	19%
8,000		97%	65%	32%	69%	46%	23%	82%	55%	27%
16,000		100%	92%	46%	97%	65%	32%	100%	77%	39%
32,000		100%	100%	65%	100%	92%	46%	100%	100%	55%
64,000		100%	100%	92%	100%	100%	65%	100%	100%	77%
State		Using Typical Average Industry Group Claim Counts								
Maine		69%	46%	23%	49%	32%	16%	58%	39%	19%
Vermont		73%	49%	24%	52%	34%	17%	61%	41%	20%
Alabama		100%	92%	46%	97%	65%	32%	100%	77%	39%
Illinois		100%	100%	95%	100%	100%	67%	100%	100%	80%

Exhibit 22a

Large State

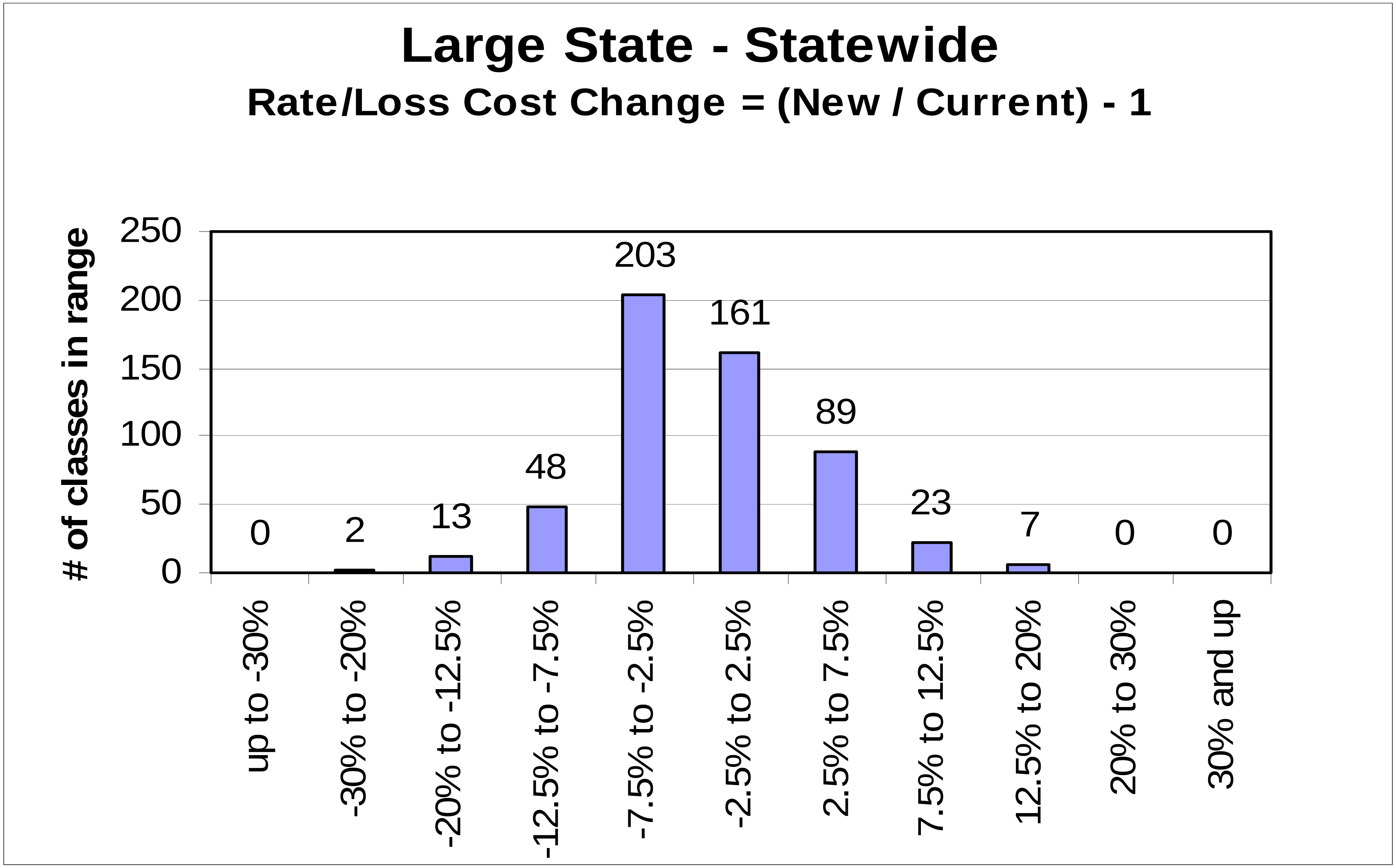
Impact of New Loss Development and Expected Excess by Industry Group

Industry Group	(1) Wtd Change in LDFs	(2) Wtd Change in Excess	(3) (1)x(2) Predicted Chg by IG	(4) (3) / Tot(3) Normalized Predicted Chg by IG	(5) (4)-1 Predicted %Chg by IG
Manufacturing	0.923	1.090	1.006	0.949	-5.1%
Contracting	0.892	1.170	1.043	0.984	-1.6%
Office & Clerical	0.975	1.101	1.073	1.012	1.2%
Goods & Services	0.988	1.096	1.083	1.021	2.1%
Miscellaneous	0.933	1.134	1.059	0.998	-0.2%
State Total	0.949	1.118	1.060	1.0000	0.0%

All five industry groups received IG credibility equal to 100% for Large state.

Exhibit 22b

All Classes – Final After Swing Limits



LIMITED INDEMNITY LOSS DEVELOPMENT Test State **04/01/08** Exhibit 23a

Likely

1st Report Start: 2/1/2004
1st Report End: 1/31/2005

PY Data	1st Report	2nd Report	3rd Report	4th Report	5th Report	6th Report	7th Report	8th Report	9th Report	10th Report
2/91-1/92	0	0	0	0	0	0	0	0	0	0
2/92-1/93	0	0	0	0	0	0	0	0	0	0
2/93-1/94	0	0	0	0	0	0	0	0	0	0
2/94-1/95	0	0	0	0	0	0	0	0	0	0
2/95-1/96	0	0	0	0	0	0	0	0	0	0
2/96-1/97	0	0	0	0	0	0	0	0	0	
2/97-1/98	0	0	0	0	0	0	0	0		
2/98-1/99	0	0	0	0	0	0	0			
2/99-1/00	60,996,530	92,218,325	117,039,686	132,422,739	140,802,919	144,380,191				
2/00-1/01	61,249,048	95,369,132	116,456,223	128,043,912	132,466,081					
2/01-1/02	63,330,710	101,332,628	120,907,703	131,065,662						
2/02-1/03	64,002,100	96,832,704	115,101,791							
2/03-1/04	62,441,089	97,204,707								
2/04-1/05	63,908,035									

Link Ratios	1:2	2:3	3:4	4:5	5:6	6:7	7:8	8:9	9:10
2/91-1/92									
2/92-1/93									
2/93-1/94									
2/94-1/95									
2/95-1/96									
2/96-1/97									
2/97-1/98									
2/98-1/99									
2/99-1/00	1.512	1.269	1.131	1.063	1.025				
2/00-1/01	1.557	1.221	1.100	1.035					
2/01-1/02	1.600	1.193	1.084						
2/02-1/03	1.513	1.189							
2/03-1/04	1.557								

AVERAGE DEV.	1:2	2:3	3:4	4:5	5:6	6:7	7:8	8:9	9:10
2 Year Averages	1.535	1.191	1.092	1.049					
3 Year Averages	1.557	1.201	1.105						
4 Year Averages	1.557	1.218							
5 Year Averages	1.548								
5 Yr Ex-Hi Lo Avgs	1.542								

AVG DEV. TO 5TH	1:5	2:5	3:5	4:5	5th:Ult
2 Year Averages	2.095	1.365	1.146	1.049	1.090
3 Year Averages					
4 Year Averages					
5 Year Averages					
5 Yr Ex-Hi Lo Avgs					

AVG DEV. TO ULT.	1:U	2:U	3:U	4:U	5:U
2 Year Averages	2.281	1.486	1.248	1.143	1.090
3 Year Averages					
4 Year Averages					
5 Year Averages					
5 Yr Ex-Hi Lo Avgs					

Averaging Method (Use '6' for 5 Yr Ex-HiLo)										
	2	2	2	2	2	2	2	2	2	2

Selected Average Development	1:2	2:3	3:4	4:5	5th:Ult
	1.535	1.191	1.092	1.049	1.090

LIMITED INDEMNITY LOSS DEVELOPMENT Test State **04/01/08** Exhibit 23b

Not-Likely

1st Report Start: 2/1/2004
1st Report End: 1/31/2005

PY Data	1st Report	2nd Report	3rd Report	4th Report	5th Report	6th Report	7th Report	8th Report	9th Report	10th Report
2/91-1/92	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	0	0
2/92-1/93	*0*	*0*	*0*	*0*	*0*	*0*	*0*	0	0	0
2/93-1/94	*0*	*0*	*0*	*0*	*0*	*0*	0	0	0	0
2/94-1/95	*0*	*0*	*0*	*0*	*0*	0	0	0	0	0
2/95-1/96	*0*	*0*	*0*	*0*	0	0	0	0	0	0
2/96-1/97	*0*	*0*	*0*	0	0	0	0	0	0	
2/97-1/98	*0*	*0*	0	0	0	0	0	0		
2/98-1/99	*0*	0	0	0	0	0	0			
2/99-1/00	92,600,271	117,560,761	133,963,360	141,479,415	146,067,806	147,360,667				
2/00-1/01	95,374,095	116,001,514	131,138,809	138,472,791	141,250,877					
2/01-1/02	97,129,731	122,748,458	135,195,013	139,317,464						
2/02-1/03	95,563,495	115,415,827	126,594,218							
2/03-1/04	90,843,197	109,219,366								
2/04-1/05	96,958,872									

Link Ratios	1:2	2:3	3:4	4:5	5:6	6:7	7:8	8:9	9:10
2/91-1/92									
2/92-1/93									
2/93-1/94									
2/94-1/95									
2/95-1/96									
2/96-1/97									
2/97-1/98									
2/98-1/99									
2/99-1/00	1.270	1.140	1.056	1.032	1.009				
2/00-1/01	1.216	1.130	1.056	1.020					
2/01-1/02	1.264	1.101	1.030						
2/02-1/03	1.208	1.097							
2/03-1/04	1.202								

AVERAGE DEV.	1:2	2:3	3:4	4:5	5:6	6:7	7:8	8:9	9:10
2 Year Averages	1.205	1.099	1.043	1.026					
3 Year Averages	1.225	1.109	1.047						
4 Year Averages	1.223	1.117							
5 Year Averages	1.232								
5 Yr Ex-Hi Lo Avgs	1.229								

AVG DEV. TO 5TH	1:5	2:5	3:5	4:5	5th:Ult
2 Year Averages	1.417	1.176	1.070	1.026	1.030
3 Year Averages					
4 Year Averages					
5 Year Averages					
5 Yr Ex-Hi Lo Avgs					

AVG DEV. TO ULT.	1:U	2:U	3:U	4:U	5:U
2 Year Averages	1.459	1.211	1.102	1.057	1.030
3 Year Averages					
4 Year Averages					
5 Year Averages					
5 Yr Ex-Hi Lo Avgs					

Averaging Method (Use '6' for 5 Yr Ex-HiLo)										
	2	2	2	2	2	2	2	2	2	2

Selected Average Development	1:2	2:3	3:4	4:5	5th:Ult
	1.205	1.099	1.043	1.026	1.030

LIKELY DEVELOPMENT TO ULTIMATE — Test State — Exhibit 23c

04/01/08

Limited Indemnity - Combined

(using 2-year average development)

FIRST REPORT 2/04-1/05	(1) Limited Incurred Losses	(2) Development 1:5	(3) Amendment Factor	(4) Modified Losses (1)x((2)x(3))
Fatal-Likely	0	2.095	1.045	0
Fatal-Not Likely	10,269,396	1.417	1.045	15,208,975
Permanent Total	4,413,333	2.095	1.046	9,669,613
Perm. Partial-Likely	42,468,001	2.095	1.025	91,178,798
Perm. Partial-Not Likely	53,963,071	1.417	1.025	78,354,379
Temp. Total-Likely	17,026,701	2.095	1.046	37,305,502
Temp. Total-Not Likely	32,726,405	1.417	1.046	48,500,532

SECOND REPORT 2/03-1/04	(5) Limited Incurred Losses	(6) Development 2:5	(7) Amendment Factor	(8) Modified Losses (5)x((6)x(7))
Fatal-Likely	1,299,643	1.365	1.051	1,864,988
Fatal-Not Likely	8,017,542	1.176	1.051	9,909,682
Permanent Total	11,441,423	1.365	1.052	16,429,883
Perm. Partial-Likely	71,430,014	1.365	1.029	100,359,170
Perm. Partial-Not Likely	70,091,621	1.176	1.029	84,810,861
Temp. Total-Likely	13,033,627	1.365	1.052	18,716,288
Temp. Total-Not Likely	31,110,203	1.176	1.052	38,483,321

CALCULATION OF LIKELY 5TH-TO-ULTIMATE	
(9) Combined Likely Losses	275,524,242
(10) Combined Not-Likely Losses	275,267,750
(11) Combined Total Losses	550,791,992
(12) Financial Data 5th-to-Ultimate Development Factors	1.060
(13) 5th-to-Ultimate Loss Development (13) = {(12)-1}x(11)	33,047,520
(14) % of Loss Development attributable to Not-Likely Losses at 5th rpt	0.250
(15) 5th-to-Ultimate Likely Loss Development Factors (15) = {(9)+ [1-(14)]x(13)}/(9)	1.090
(16) 5th-to-Ultimate Not-Likely Loss Development Factors (16) = {(10)+ (14)x(13)}/(10)	1.030

LIMITED MEDICAL LOSS DEVELOPMENT Test State **04/01/08** Exhibit 23d

Likely

1st Report Start:	2/1/2004
1st Report End:	1/31/2005

PY Data	1st Report	2nd Report	3rd Report	4th Report	5th Report	6th Report	7th Report	8th Report	9th Report	10th Report
2/91-1/92	0	0	0	0	0	0	0	0	0	0
2/92-1/93	0	0	0	0	0	0	0	0	0	0
2/93-1/94	0	0	0	0	0	0	0	0	0	0
2/94-1/95	0	0	0	0	0	0	0	0	0	0
2/95-1/96	0	0	0	0	0	0	0	0	0	0
2/96-1/97	0	0	0	0	0	0	0	0	0	
2/97-1/98	0	0	0	0	0	0	0	0		
2/98-1/99	0	0	0	0	0	0	0			
2/99-1/00	75,200,873	90,059,436	100,912,427	109,486,363	115,848,096	120,187,414				
2/00-1/01	71,384,912	88,432,334	97,351,469	102,016,362	104,712,638					
2/01-1/02	82,626,918	100,990,563	107,850,140	114,019,998						
2/02-1/03	86,723,140	101,434,110	109,735,237							
2/03-1/04	88,194,204	104,765,903								
2/04-1/05	97,105,237									

Link Ratios	1:2	2:3	3:4	4:5	5:6	6:7	7:8	8:9	9:10
2/91-1/92									
2/92-1/93									
2/93-1/94									
2/94-1/95									
2/95-1/96									
2/96-1/97									
2/97-1/98									
2/98-1/99									
2/99-1/00	1.198	1.121	1.085	1.058	1.037				
2/00-1/01	1.239	1.101	1.048	1.026					
2/01-1/02	1.222	1.068	1.057						
2/02-1/03	1.170	1.082							
2/03-1/04	1.188								

AVERAGE DEV.	1:2	2:3	3:4	4:5	5:6	6:7	7:8	8:9	9:10
2 Year Averages	1.179	1.075	1.053	1.042					
3 Year Averages	1.193	1.084	1.063						
4 Year Averages	1.205	1.093							
5 Year Averages	1.203								
5 Yr Ex-Hi Lo Avgs	1.203								

AVG DEV. TO 5TH	1:5	2:5	3:5	4:5	5th:Ult
2 Year Averages	1.390	1.179	1.097	1.042	1.647
3 Year Averages					
4 Year Averages					
5 Year Averages					
5 Yr Ex-Hi Lo Avgs					

AVG DEV. TO ULT.	1:U	2:U	3:U	4:U	5:U
2 Year Averages	2.291	1.943	1.807	1.716	1.647
3 Year Averages					
4 Year Averages					
5 Year Averages					
5 Yr Ex-Hi Lo Avgs					

Averaging Method (Use '6' for 5 Yr Ex-HiLo)										
	2	2	2	2	2	2	2	2	2	2

Selected Average Development	1:2	2:3	3:4	4:5	5th:Ult
	1.179	1.075	1.053	1.042	1.647

LIMITED MEDICAL LOSS DEVELOPMENT Test State **04/01/08** Exhibit 23e

Not-Likely

1st Report Start:	2/1/2004
1st Report End:	1/31/2005

PY Data	1st Report	2nd Report	3rd Report	4th Report	5th Report	6th Report	7th Report	8th Report	9th Report	10th Report
2/91-1/92	0	0	0	0	0	0	0	0	0	0
2/92-1/93	0	0	0	0	0	0	0	0	0	0
2/93-1/94	0	0	0	0	0	0	0	0	0	0
2/94-1/95	0	0	0	0	0	0	0	0	0	0
2/95-1/96	0	0	0	0	0	0	0	0	0	0
2/96-1/97	0	0	0	0	0	0	0	0	0	
2/97-1/98	0	0	0	0	0	0	0	0		
2/98-1/99	0	0	0	0	0	0	0			
2/99-1/00	153,833,071	168,754,862	175,377,809	179,794,298	181,687,652	185,095,079				
2/00-1/01	161,733,484	171,565,896	177,139,040	179,298,295	181,208,310					
2/01-1/02	172,959,433	185,061,442	188,700,978	189,239,144						
2/02-1/03	184,611,262	192,918,145	195,760,677							
2/03-1/04	181,237,908	188,403,055								
2/04-1/05	193,744,461									

Link Ratios	1:2	2:3	3:4	4:5	5:6	6:7	7:8	8:9	9:10
2/91-1/92									
2/92-1/93									
2/93-1/94									
2/94-1/95									
2/95-1/96									
2/96-1/97									
2/97-1/98									
2/98-1/99									
2/99-1/00	1.097	1.039	1.025	1.011	1.019				
2/00-1/01	1.061	1.032	1.012	1.011					
2/01-1/02	1.070	1.020	1.003						
2/02-1/03	1.045	1.015							
2/03-1/04	1.040								

AVERAGE DEV.	1:2	2:3	3:4	4:5	5:6	6:7	7:8	8:9	9:10
2 Year Averages	1.043	1.018	1.008	1.011					
3 Year Averages	1.052	1.022	1.013						
4 Year Averages	1.054	1.027							
5 Year Averages	1.063								
5 Yr Ex-Hi Lo Avgs	1.059								

AVG DEV. TO 5TH	1:5	2:5	3:5	4:5	5th:Ult
2 Year Averages	1.082	1.037	1.019	1.011	1.138
3 Year Averages					
4 Year Averages					
5 Year Averages					
5 Yr Ex-Hi Lo Avgs					

AVG DEV. TO ULT.	1:U	2:U	3:U	4:U	5:U
2 Year Averages	1.232	1.181	1.160	1.151	1.138
3 Year Averages					
4 Year Averages					
5 Year Averages					
5 Yr Ex-Hi Lo Avgs					

Averaging Method (Use '6' for 5 Yr Ex-HiLo)										
	2	2	2	2	2	2	2	2	2	2

Selected Average Development	1:2	2:3	3:4	4:5	5th:Ult
	1.043	1.018	1.008	1.011	1.138

LIKELY DEVELOPMENT TO ULTIMATE — Test State — Exhibit 23f

04/01/08

Limited Medical - Combined

(using 2-year average development)

FIRST REPORT 2/04-1/05	(1) Limited Incurred Losses	(2) Development 1:5	(3) Amendment Factor	(4) Modified Losses (1)x((2)x(3))
Fatal-Likely	0	1.390	1.000	0
Fatal-Not Likely	1,434,476	1.082	1.000	1,552,103
Permanent Total	7,075,471	1.390	1.000	9,834,905
Perm. Partial-Likely	53,804,199	1.390	1.000	74,787,837
Perm. Partial-Not Likely	66,844,773	1.082	1.000	72,326,044
Temp. Total-Likely	36,225,567	1.390	1.000	50,353,538
Temp. Total-Not Likely	68,052,815	1.082	1.000	73,633,146
Medical Only	57,388,896	1.082	1.000	62,094,785
Contract Medical	23,501	1.082	1.000	25,428

SECOND REPORT 2/03-1/04	(5) Limited Incurred Losses	(6) Development 2:5	(7) Amendment Factor	(8) Modified Losses (5)x((6)x(7))
Fatal-Likely	225,002	1.179	1.000	265,277
Fatal-Not Likely	660,108	1.037	1.000	684,532
Permanent Total	10,473,697	1.179	1.000	12,348,489
Perm. Partial-Likely	66,975,353	1.179	1.000	78,963,941
Perm. Partial-Not Likely	67,728,416	1.037	1.000	70,234,367
Temp. Total-Likely	27,091,851	1.179	1.000	31,941,292
Temp. Total-Not Likely	62,029,074	1.037	1.000	64,324,150
Medical Only	57,984,300	1.037	1.000	60,129,719
Contract Medical	1,157	1.037	1.000	1,200

CALCULATION OF LIKELY 5TH-TO-ULTIMATE	
(9) Combined Likely Losses	258,495,279
(10) Combined Not-Likely Losses	405,005,474
(11) Combined Total Losses	663,500,753

(12) Financial Data 5th-to-Ultimate Development Factors 1.336

(13) 5th-to-Ultimate Loss Development 222,936,253

(13) = {(12)-1}x(11)

(14) % of Loss Development attributable to Not-Likely Losses at 5th rpt 0.250

(15) 5th-to-Ultimate Likely Loss Development Factors 1.647

(15) = {(9)+ [1-(14)]x(13)}/(9)

(16) 5th-to-Ultimate Not-Likely Loss Development Factors 1.138

(16) = {(10)+ (14)x(13)}/(10)

Exhibit 1

Appendix B
New Class Ratemaking: Indicated Pure Premiums
NCCI State

Step 1: Start with 5 policy periods of Limited Losses and Payroll (00's)

Class Code 1234 Hazard Group C IG: Goods & Services

Current Loss Cost = 4.00

PY	Report	Payroll	Actual Limited Losses
1/00 thru 12/00	5	50,000,000	800,000
1/01 thru 12/01	4	53,200,000	690,000
1/02 thru 12/02	3	57,700,000	750,000
1/03 thru 12/03	2	61,000,000	730,000
1/04 thru 12/04	1	64,995,000	700,000

Notes:

a) The losses for each policy period are comprised of finer subcategories (see Step 2)

b) Individual claims are limited at $500,000.

c) The loss cost in this NCCI state includes loss adjustment expense (LAE).

Exhibit 2

Appendix B
New Class Ratemaking: Indicated Pure Premiums
NCCI State

Step 2: Adjust Limited Losses to Midpoint of Proposed Effective Date
Use Primary Conversion Factors (PCF varies by report)

Class	HG	Report	Actual Limited Losses	Dev't Group	LDF	Other PCF	Adjusted Limited Losses
1234	C	5	75,000	Fatal-L	1.400	0.95	99,750
1234	C	5	45,000	Fatal-N	1.100	0.95	47,025
1234	C	5	200,000	Permanent Total	1.400	0.99	277,200
1234	C	5	40,000	Permanent Partial-L	1.400	1.01	56,560
1234	C	5	20,000	Permanent Partial-N	1.100	1.01	22,220
1234	C	5	10,000	Temporary Total-L	1.400	0.94	13,160
1234	C	5	9,000	Temporary Total-N	1.100	0.94	9,306
1234	C	5	360,000	Medical-L	1.750	1.15	724,500
1234	C	5	41,000	Medical-N	1.250	1.15	58,938
			800,000				
1234	C	4	40,000	Fatal-L	1.480	0.96	56,832
1234	C	4	30,000	Fatal-N	1.125	0.96	32,400
1234	C	4	170,000	Permanent Total	1.480	0.98	246,568
1234	C	4	40,000	Permanent Partial-L	1.125	1.02	45,900
1234	C	4	45,000	Permanent Partial-N	1.125	1.02	51,638
1234	C	4	40,000	Temporary Total-L	1.480	0.94	55,648
1234	C	4	27,000	Temporary Total-N	1.125	0.94	28,553
1234	C	4	222,000	Medical-L	1.900	1.15	485,070
1234	C	4	76,000	Medical-N	1.300	1.15	113,620
			690,000				
1234	C	3	5,000	Fatal-L	1.550	0.97	7,518
.	.	.	.	.	.		
.	.	.	.	.	.		
.	.	.	.	.	.		

Notes:

a) The LDF is shown separately from the PCF for illustrative purposes only, and will be included in the PCF.

b) The Other PCF includes the LR trend to proposed level midpoint and benefit on-level factors.

c) Adjusted Limited Losses = Actual Limited Losses * LDF * Other PCF

d) The medical has similar injury type components plus medical only and contract medical, and is condensed simply for illustrative purposes.

Exhibit 3

Appendix B
New Class Ratemaking: Indicated Pure Premiums
NCCI State

Step 3: Compute Expected Excess Losses @$500,000
Use State Adjusted Per Claim Excess Ratios (vary by hazard group)

Class	HG	Report	Dev't Group	Adjusted Limited Losses	XS Ratio	XS Factor	Unadjusted XS Losses
1234	C	5	Fatal-L	99,750	0.194	1.241	24,009
1234	C	5	Fatal-N	47,025	0.194	1.241	11,319
1234	C	5	Permanent Total	277,200	0.194	1.241	66,721
1234	C	5	Permanent Partial-L	56,560	0.194	1.241	13,614
1234	C	5	Permanent Partial-N	22,220	0.194	1.241	5,348
1234	C	5	Temporary Total-L	13,160	0.194	1.241	3,168
1234	C	5	Temporary Total-N	9,306	0.194	1.241	2,240
1234	C	5	Medical-L	724,500	0.194	1.241	174,383
1234	C	5	Medical-N	58,938	0.194	1.241	14,186
1234	C	4	Fatal-L	56,832	0.194	1.241	13,679
1234	C	4	Fatal-N	32,400	0.194	1.241	7,799
1234	C	4	Permanent Total	246,568	0.194	1.241	59,348
1234	C	4	Permanent Partial-L	45,900	0.194	1.241	11,048
1234	C	4	Permanent Partial-N	51,638	0.194	1.241	12,429
1234	C	4	Temporary Total-L	55,648	0.194	1.241	13,394
1234	C	4	Temporary Total-N	28,553	0.194	1.241	6,872
1234	C	4	Medical-L	485,070	0.194	1.241	116,754
1234	C	4	Medical-N	113,620	0.194	1.241	27,348
1234	C	3	Fatal-L	7,518	0.194	1.241	1,809
.	.	.		.	.	.	.
.	.	.		.	.	.	.
.	.	.		.	.	.	.

Notes:

a) The adjusted per claim excess ratio (XS ratio) is indemnity and medical combined.

b) The XS Factor = [1.0 / (1.0 - XS Ratio)]

c) Unadjusted XS Losses = (XS factor -1.0) * Adjusted Limited Losses

d) The medical has similar injury type components plus medical only and contract medical, and is

Exhibit 4

Appendix B
New Class Ratemaking: Indicated Pure Premiums
NCCI State

Step 4: Transfer 40% of Expected Excess Losses from Indemnity to Medical

Class	HG	Report	Unadjusted XS Losses	Dev't Group	Adjusted XS Losses
			Indemnity	Indemnity	Indemnity
1234	C	5	107,511	Likely	64,507
1234	C	5	18,907	Not Likely	11,344
Total	C	5	126,418	Total	75,851
			Medical	Medical	Medical
1234	C	5	174,383	Likely	217,388
1234	C	5	14,186	Not Likely	21,749
Total	C	5	188,569	Total	239,137
			314,987		314,987
			Indemnity	Indemnity	Indemnity
1234	C	4	97,469	Likely	58,481
1234	C	4	27,100	Not Likely	16,260
Total	C	4	124,569	Total	74,741
			Medical	Medical	Medical
1234	C	4	116,754	Likely	155,741
1234	C	4	27,348	Not Likely	38,188
Total	C	4	144,102	Total	193,929
			268,670		268,670
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.

<u>Notes:</u>

a) The indemnity adjusted XS losses = .60 * unadjusted XS losses for indemnity (see exception in (c)).

b) The medical adjusted XS losses = unadjusted med. XS loss + 40% unadjusted indemnity XS loss (exception in (c)

c) If the unadjusted medical XS losses = $0 (for L or NL), transfer $0 excess to medical (L or NL).

d) At each report for each class code, adjusted XS loss = unadjusted XS loss for indemnity and medical combined.

e) Adjusted XS loss is allocated to all remaining non-zero injury type/dev't group combinations
based on its share of adjusted losses at each report.

Appendix B

Exhibit 5

New Class Ratemaking: Indicated Pure Premiums

NCCI State

Step 5: Combine Adjusted Limited Losses with Adjusted XS Losses

Then Apply Secondary Conversion Factors (SCF vary by report)

Class	Report	Dev't Group	Adjusted Limited Losses	Adjusted XS Losses	SCF	Expected Unlimited Losses
1234	5	Fatal-L	99,750	14,406	1.220	139,270
1234	5	Fatal-N	47,025	6,791	1.220	65,656
1234	5	Permanent Total	277,200	40,032	1.220	387,023
1234	5	Permanent Partial-L	56,560	8,168	1.220	78,968
1234	5	Permanent Partial-N	22,220	3,209	1.220	31,023
1234	5	Temporary Total-L	13,160	1,901	1.220	18,374
1234	5	Temporary Total-N	9,306	1,344	1.220	12,993
1234	5	Medical-L	724,500	217,388	1.220	1,149,103
1234	5	Medical-N	58,938	21,749	1.220	98,437
				314,987		
1234	4	Fatal-L	56,832	8,207	1.180	76,747
1234	4	Fatal-N	32,400	4,679	1.180	43,753
1234	4	Permanent Total	246,568	35,609	1.180	332,968
1234	4	Permanent Partial-L	45,900	6,629	1.180	61,984
1234	4	Permanent Partial-N	51,638	7,457	1.180	69,732
1234	4	Temporary Total-L	55,648	8,037	1.180	75,148
1234	4	Temporary Total-N	28,553	4,123	1.180	38,558
1234	4	Medical-L	485,070	155,741	1.180	756,157
1234	4	Medical-N	113,620	38,188	1.180	179,133
				268,670		
.	.	.	.	.	.	
.	.	.	.	.	.	
.	.	.	.	.	.	

Notes:

a) The medical is condensed for illustrative purposes, but has similar injury type components plus medical only and contract medical.

b) The SCF includes: the aggregate statewide loss cost change, the factor to adjust for proposed IG differential, proposed loss-based expense the balancing of indicated to expected losses, and misc. premium adjustments (a few states).

c) Secondary conversion factors vary by report and industry group.

d) Expected Unlimited Losses = (Adjusted Limited Losses + Adjusted XS Losses) * SCF

Exhibit 6

Appendix B
New Class Ratemaking: Indicated Pure Premiums
NCCI State

Step 6: Organize Expected Unlimited Losses (a.k.a. Converted Losses) into Indemnity and Medical Components
Compute Indicated Pure Premiums

CLASS 1234	Class Code Description								
Industry Group: Goods and Services		CONVERTED LOSSES							
Hazard Group C		INDEMNITY LIKELY		INDEMNITY NOT-LIKELY		MED LIKELY	MED NOT-LIKELY	TOTAL	TOTAL
POLICY PERIOD	PAYROLL	CASES	AMOUNT	CASES	AMOUNT	AMOUNT	AMOUNT	AMOUNT	PURE PREM.
1/00 thru 12/00	50,000,000	19	623,636	40	109,672	1,149,103	98,437	1,980,848	3.96
1/01 thru 12/01	53,200,000	17	546,847	50	152,043	756,157	179,133	1,634,180	3.07
1/02 thru 12/02	57,700,000	20	500,000	65	400,000	800,000	700,000	2,400,000	4.16
1/03 thru 12/03	61,000,000	18	310,000	57	300,000	1,150,000	850,000	2,610,000	4.28
1/04 thru 12/04	64,995,000	12	300,000	60	450,000	720,000	1,100,000	2,570,000	3.95
5 YR. TOTAL	286,895,000	86	2,280,482	272	1,411,715	4,575,261	2,927,570	11,195,028	3.90
		INDEMNITY		MEDICAL					
		CRED.	PURE PREM.*	CRED.	PURE PREM.*				PURE PREM.*
Indicated Pure Premium		66%	1.287	77%	2.615				3.90

Notes:

a) The indemnity and medical components replace the former serious, non-serious, and medical partial pure premiums.

b) Indemnity and Medical credibilities are derived in Step 7.

Appendix B
New Class Ratemaking: Indicated Pure Premiums
NCCI State

Exhibit 7

Step 7: Derive Expected Losses for Class 1234 for the Indemnity and Medical Components
Compute Credibility for each Component

Background:

Credibility Formula used for all classes

$Z = [N/ N_f]^{0.4}$

Full Credibility Standards (all classes)

N_f :	850	Indemnity
N_f :	400	Medical

*Pure premium underlying current loss cost for 1234 = 1.70 Indemnity

*Pure premium underlying current loss cost for 1234 = 2.10 Medical

* Adjusted to proposed level via changes in trend, benefits, and experience.

Average Cost per Case for NCCI state:	SACC =	$16,000	Indemnity
	SACC =	$28,500	Medical

Expected Losses for class code 1234= (5 years payroll in '00s) x Underlying PP

[c)] Full Credibility Standard (all classes) expected losses = N_f x SACC

Calculations:

Indemnity

N= Expected Losses for class code 1234=	2,868,950	X	1.70
N= Expected Losses for class code 1234=	4,877,215		

Medical

N=Expected Losses for class code 1234=	2,868,950	X	2.10
N=Expected Losses for class code 1234=	6,024,795		

[c)] Indemnity N_f	=	16,000	X	850	=	13,600,000
[c)] Medical N_f	=	28,500	X	400	=	11,400,000
Indemnity Z	=	66.35%				
Medical Z	=	77.48%				

Notes:

a) Credibility is rounded to nearest whole number percentage.
b) The SACC for medical includes all medical loss dollars (i.e. incl. med.-only dollars) divided by lost-time claims.
c) The Full Credibility Standard is also adjusted by a statewide balancing factor of 5 years of indicated-to-expected losses. This calculation assumes that the statewide balancing factor is unity for this state.

Exhibit 8

Appendix B
New Class Ratemaking: National Pure Premiums
NCCI State

Step 8: Derive National Pure Premiums for Class 1234 *

1. Compute a payroll-weighted average of the new revision indicated pure premiums across all classes in the reviewed state using reviewed states' payroll (5 years).

2. Compute Step 1 for indemnity and medical separately.

3. For all other NCCI states, compute a payroll-weighted average of derived-by-formula pure premiums (3 years) for each state using all classes in common with the reviewed state.

4. Compute Step 3 using the reviewed state payroll (5 years) for indemnity and medical separately.

5. Compute adjustment factor k for each state for indemnity and medical: k = (step 2 / step 4).

6. Adjust each state's losses by class code (3 years) to reviewed state level by multiplying by k.

7. Compute national pure premiums n_c (for indemnity and medical separately) for each class code c.

$$n_c = \frac{\text{3 years other states' losses (adjusted to reviewed state level)}}{\text{3 years other states' payroll (in 00's) for class c}}$$

Final adjustment: balance the national pure premiums to the indicated pure premiums in the reviewed state.

8. For each industry group (IG), compute the total indicated pure premium. Do this by extending 5 years of reviewed state payroll by the reviewed state indicated pure premiums.

9. For each industry group (IG), compute the total adjusted unbalanced national pure premium. Do this by extending 5 years of reviewed state payroll by the adjusted unbalanced national pure premiums.

10. Compute balancing factor B_{IG} for each IG, where B_{IG} = (step 8 / step 9).

11. Compute final balanced national pure premiums for reviewed state for each class c: $N_c = B_{IG} \times n_c$

* For a numerical illustration of the national pure premium calculation, refer to:

Boor, J.A. , "The Complement of Credibility," PCAS LXXXIII, 1996, pp 14-18

Appendix B
New Class Ratemaking: Present-On-Rate-Level Pure Premiums
NCCI State

Exhibit 9

Step 9: Derive Present-On-Rate-Level (PORL) Pure Premium for Class 1234

Pure premium underlying current loss cost for 1234 = $1.70 Indemnity

Pure premium underlying current loss cost for 1234 = $2.10 Medical

Apply separate adjustment factors for the indemnity and medical components to adjust to the proposed level of the loss cost filing.

	Indemnity	Medical	Total
1. PP underlying current loss cost:	$1.70	$2.10	
Adjustments:			
2. Change in Proposed LR Trend:	0.990	1.010	
3. Proposed Change in Benefits:	1.005	0.980	
4. Proposed Change in LBE:	1.000	1.000	
5. Proposed Change in Off-Balance:	0.990	0.990	
6. Proposed SW Experience Change	1.010	1.010	
7. Adjusted IG Differential:	1.021	1.021	
8. Miscellaneous factors	1.000	1.000	
9. Product of Step 2. through Step 8.	1.016	1.010	
10. Present On-Rate-Level Pure Premium: (The Product of Step 1 and Step 9)	1.727	2.122	3.849

Notes:

a) The PP underlying the current loss cost includes LAE (if any), the test correction factor, and applied swing limits

b) The PP underlying the current loss cost excludes the manual to standard premium ratio.

c) No loss development adjustment is necessary as the value is already at an ultimate level.

d) All adjustments are for a one-year timespan

e) Change in loss-based expenses (LBE) is change in LAE and change in any other Loss based assessments.

f) Proposed change in off-balance is current M/E / proposed M/E for the IG where class 1234 resides.

Exhibit 10

Appendix B
New Class Ratemaking: Remaining Credibility Steps
NCCI State

Step 10: Compute Credibility for National and PORL Pure Premiums for Class 1234

<u>Background: National Pure Premium</u>

Credibility Formula used for all classes

$Z = \min \{ 0.5*(1\text{- State Z}), [N/N_f]^{0.4} \}$

Full Credibility Standards (all classes)

N_f :	1,150	Indemnity
N_f :	1,000	Medical

N_f and N for the national pure premiums are based on <u>actual</u> number of lost-time claims based upon the latest three years of national data for the class code.

<u>Calculations: National Pure Premium</u>

Indemnity and Medical

N= Actual # of lost-time claims (all states) for class code 1234 = 1,025

National Z for Indemnity	=	$Z = \min \{ 0.5*(1\text{- State Z}), [N/N_f]^{0.4} \}$						
National Z for Indemnity	=	min {	17%	or	96%	}	=	17%
National Z for Medical	=	$Z = \min \{ 0.5*(1\text{- State Z}), [N/N_f]^{0.4} \}$						
National Z for Medical	=	min {	11%	or	100%	}	=	11%

<u>Background: Present On-Rate Level Pure Premium</u>

$Z = (1 - \text{State Z} - \text{National Z})$

<u>Calculations: Present On-Rate Level Pure Premium</u>

PORL Z for Indemnity	=	(1	-	66%	-	17%)	=	17%
PORL Z for Medical	=	(1	-	77%	-	11%)	=	12%

<u>Notes:</u>

a) Credibility is rounded to nearest whole number percentage.

Exhibit 11

Appendix B
New Class Ratemaking: Derived By Formula Pure Premiums
NCCI State

Step 11: Apply Three-way Credibility Formula to the Indemnity and Medical Components
Compute the Derived By Formula Pure Premium

CLASS 1234	Class Code Description								
Industry Group: Goods and Services		CONVERTED LOSSES							
Hazard Group C		INDEMNITY LIKELY		INDEMNITY NOT-LIKELY		MED LIKELY	MED NOT-LIKELY	TOTAL	TOTAL
POLICY PERIOD	PAYROLL	CASES	AMOUNT	CASES	AMOUNT	AMOUNT	AMOUNT	AMOUNT	PURE PREM.
5	50,000,000	19	623,636	40	109,672	1,149,103	98,437	1,980,848	3.96
4	53,200,000	17	546,847	50	152,043	756,157	179,133	1,634,180	3.07
3	57,700,000	20	500,000	65	400,000	800,000	700,000	2,400,000	4.16
2	61,000,000	18	310,000	57	300,000	1,150,000	850,000	2,610,000	4.28
1	64,995,000	12	300,000	60	450,000	720,000	1,100,000	2,570,000	3.95
5 YR. TOTAL	286,895,000	86	2,280,482	272	1,411,715	4,575,261	2,927,570	11,195,028	3.90

	INDEMNITY CRED.	INDEMNITY PURE PREM.*	MEDICAL CRED.	MEDICAL PURE PREM.*	TOTAL PURE PREM.*
Indicated Pure Premium	66%	1.287	77%	2.615	3.90
Pure Premium Indicated by National	17%	1.200	11%	2.800	4.00
Pure Premium Present on Rate Level	17%	1.727	12%	2.122	3.85
Pure Premium Derived by Formula		1.345		2.579	3.92

Appendix B

Exhibit 12

New Class Ratemaking: Final Loss Cost / Rate Calculation

NCCI State

Step 12: Compute the final proposed loss cost by adjusting pure premium derived by formula

Current Loss Cost for Class 1234 = 4.00

		Indemnity	Medical	Total
1	Indicated Pure Premium	1.287	2.615	3.90
2	Pure Premium Indicated by National Relativity	1.200	2.800	4.00
3	Pure Premium Present on Rate Level	1.727	2.122	3.85
4	State Credibilities	66%	77%	xxx
5	National Credibilities	17%	11%	xxx
6	Residual Credibilities = 100% - (4) - (5)	17%	12%	xxx
7	Derived by Formula Pure Premiums = (1) x (4) + (2) x (5) + (3) x (6)	1.345	2.579	3.92
8	Test Correction Factor	0.9963	0.9963	xxx
9	Underlying Pure Premiums = (7) x (8) *	1.341	2.569	3.91
10	Ratio of Manual to Standard Premium			1.063
11	Target Cost Ratio (TCR)			1.00
12	Loss Cost = (9) x (10) / (11)			4.16
13	Loss Cost Within Swing Limits			4.16
	Current Loss Cost x Swing Limits a) Lower bound = .75 x 4.00 b) Upper bound = 1.25 x 4.00			
14	Pure Premiums Underlying Proposed Loss Cost* ((14TOT) / (9TOT)) x (9) ; (14TOT) = (13) x (11) / (10))	1.341	2.569	3.91
15	Disease, PAP, and/or Miscellaneous Loadings			0.00
16	Final Proposed Loss Cost			4.16

* Indemnity pure premium is adjusted for the rounded total pure premium:

Notes:

a) The swing limits are applied as +/- 25% change around the IG change in most states.
b) The test correction factor is computed by IG to redistribute premium for classes exceeding swing limits.
c) The TCR is the fraction of the adequate premium dollar accounting for losses and loss-based expenses.

Appendix B
New Class Ratemaking: Derivation of Industry Group Differentials
NCCI State

Exhibit 13

II. Derivation Of Industry Group Differentials

a) INDUSTRY GROUP WAGE TREND ADJUSTMENT

Industry Group	(1) Converted Indicated Indemnity Losses*	(2) Converted Indicated Medical Losses*	(3) Converted Indicated Total Losses*	(4) CPS Average Weekly Wage Trends**	(5) Wage Trend Differential	(6) Medical Loss Wage Trend Adjustments	(7) Normalized Medical Loss Wage Trend Adjustments
I	321,604,662	247,834,851	569,439,513	1.130	0.979	0.991	0.990
II	542,740,889	488,814,443	1,031,555,332	1.100	1.005	1.002	1.001
III	242,098,488	248,912,602	491,011,090	1.099	1.006	1.003	1.002
IV	488,290,147	467,545,456	955,835,603	1.091	1.014	1.007	1.006
V	361,406,704	241,602,904	603,009,608	1.123	0.985	0.994	0.993
VI	0	0	0	0.000	1.000	1.000	0.999
ALL	1,956,140,889	1,694,710,257	3,650,851,146	1.106		1.001	1.000

* These expected unlimited losses are at ultimate, on-level, include the proposed experience and loss based expense changes and any miscellaneous premium adjustments (excludes trend).
** These CPS average weekly wage trends were fit to CPS average weekly wages based on the $150k payroll cap.

b) EXPECTED LOSSES

Industry Group	(8) Latest Year CURRENT Manual Pure Premium*	(9) Five Year CURRENT Manual Pure Premium*	(10) Five Year PROPOSED Manual Pure Premium*	(11) Current Ratio of Manual to Standard Premium	(12) Proposed Ratio of Manual to Standard Premium	(13) Latest Year CURRENT Expected Losses** (8) x (11) / (12)	(14) Five Year CURRENT Expected Losses** (9) x (11) / (12)	(15) Five Year PROPOSED Expected Losses** (10) x (11) / (12)	(16) Current / Proposed (9) / (10)	(17) Adjustment to Proposed for Current Relativities (16) / 0.975
I	119,092,461	559,793,421	574,558,035	1.088	1.072	120,869,960	568,148,546	583,133,528	0.974	0.999
II	240,949,465	1,051,366,791	1,076,899,697	1.096	1.077	245,200,198	1,069,914,580	1,095,897,928	0.976	1.001
III	104,805,551	464,202,966	476,975,093	1.109	1.086	107,025,190	474,034,153	487,076,776	0.973	0.998
IV	240,216,710	1,035,955,411	1,065,551,933	1.052	1.063	237,730,930	1,025,235,271	1,054,525,525	0.972	0.997
V	145,206,659	614,567,457	627,887,892	1.092	1.089	145,606,677	616,260,480	629,617,611	0.979	1.004
VI	0	0	0	1.000	1.000	0	0	0	0.000	0.000
ALL	850,270,846	3,725,886,046	3,821,872,651			856,432,955	3,753,593,030	3,850,251,368	0.975	1.000

* The CURRENT pure premiums are payroll extended underlying pure premiums. The PROPOSED pure premiums are adjusted to include the proposed experience, trend, benefit and loss based expense changes as well as any miscellaneous premium adjustments.
** The CURRENT expected losses are payroll extended underlying pure premium adjusted by the change in off-balance by industry group. The PROPOSED pure premiums are further adjusted to include the proposed experience, trend, benefit and loss based expense changes as well as any miscellaneous premium adjustments.

c) INDUSTRY GROUP DIFFERENTIALS

Industry Group	(18) Converted Indicated Balanced Losses*	(19) Five Year Ind to Exp Ratios (w/o Wage Trend) (18) / [(15) x (17)]	(20) Indicated Differentials (w/o Wage Trend) (19) / 0.818	(21) Five Year Ind to Exp Ratios (w/ Wage Trend) (19) x (7)	(22) Lost-Time Cases	(23) Full Standard for Credibility Lost-Time Cases	(24) Credibility Minimum of 1.00 and [(23) / (24)] ^ 0.50	(25) Credibility Weighted Ind to Exp Ratios (25) x (21) + [1 - (25)] x (21) Total	(26) Normalized Credibility Weighted Ind to Exp Ratios (aka IG Differentials)	(27) Final Industry Group Differentials
I	478,573,006	0.822	1.005	0.814	12,088	12,000	1.00	0.814	0.996	0.996
II	877,674,956	0.800	0.978	0.801	15,366	12,000	1.00	0.801	0.980	0.980
III	424,625,861	0.874	1.068	0.876	9,648	12,000	0.90	0.870	1.065	1.065
IV	869,514,555	0.827	1.011	0.832	27,209	12,000	1.00	0.832	1.018	1.018
V	497,494,959	0.787	0.962	0.781	10,494	12,000	0.94	0.783	0.958	0.958
VI	0	1.000	1.000	1.000	0	12,000	0.00	1.000	1.000	1.000
ALL	3,147,883,337	0.818		0.817				0.817		1.000

* These expected unlimited losses are at ultimate, on-level, trended, and include the proposed experience and loss based expense changes as well as any miscellaneous premium adjustments. These losses have also been balanced to the proposed level via the balancing factors.

More Flexible GLMs
Zero-Inflated Models and Hybrid Models

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Motivation: GLMs are widely used in insurance modeling applications. Claim or frequency models are a key component of many GLM ratemaking models. Enhancements to the traditional GLM that are described in this paper may by able to address practical issues that arise when fitting count models to insurance claims data.

For modeling claims within the GLM framework, the Poisson distribution is a popular distribution choice. In the presence of overdispersion, the negative binomial is also sometimes used. The statistical literature has suggested that taking excess zeros into account can improve the fit of count models when overdispersion is present. In insurance excess zeros may arise when claims near the deductible are not reported to the insurer, thus inflating the number of zero policies when compared to the predictions of a Poisson or Negative Binomial distribution.

In predictive modeling practice, data mining techniques such as neural networks and decision trees are often used to handle data complexities such as nonlinearities and interactions. Data mining techniques are sometimes combined with GLMs to improve the performance and/or efficiency of the predictive modeling analysis. One augmentation of GLMs uses decision tree methods in the data preprocessing step. An important preprocessing task reduces the number of levels on categorical variables so that sparse cells are eliminated and only significant groupings of the categories remain.

Method: This paper addresses some common problems in fitting count models to data. These are:

- Excess zeros
- Parsimonious reduction of category levels
- Nonlinearity

Results: The research described in this paper applied zero-inflated and hybrid models to claim frequency data. The research suggests that mixtures of GLM models incorporating adjustments for excess zeros improves the fit of the model compared to single distribution count models on some count data. The analysis also indicates that variable preprocessing using the CHAID tree technique can help reduce the complexity of models by retaining only category groupings that are significant with respect to their impact on the dependant variable.

Conclusions: By incorporating greater flexibility into GLM count models, practitioners may be able to improve the fit of models and increase the efficiency of the modeling effort. Use of the ZIP or ZINB improves the model fit for an illustrative automobile insurance database. The ZIP or ZINB distributions also provided a better overall approximation to the unconditional distribution of the data for the fit of a few additional insurance and non-insurance database. While the categorical variables in our illustrative data contained only a few categories compared to most realistic applications databases encountered in insurance, the fit of several predictive models. We also illustrate how the procedure can be applied to efficiently preprocess categorical variables with large numbers of categories.

Availability: Excel spreadsheets comparing the Poisson, negative binomial, zero-inflated Poisson and zero-inflated negative binomials well as R code for reproducing many models used in this paper will be available on the CAS Web Site.

Keywords: Predictive modeling, automobile ratemaking, generalized linear models, data mining

1. INTRODUCTION

Generalized linear models (GLMs) use a regression procedure to fit relationships between predictor and target variables. Unlike classical ordinary least squares regression where the random component (i.e., the error term) is assumed to follow a normal distribution, the random component in a GLM is assumed to belong to the exponential family of distributions. This family includes, along with the normal, the Poisson, the gamma and others commonly encountered in statistical analysis. GLMs are widely used in insurance modeling applications. In both the classical statistical literature (McCullagh and Nelder, 1989) and insurance-specific literature (de Jong and Heller, 2008) GLM techniques are applied to modeling insurance frequency and severity data. GLMs are a linear modeling procedure, since the relationship between a suitable transform of the dependent variable and the independent variables is assumed to be linear.

Commonly used data mining techniques employ automated procedures to efficiently address some limitations of linear modeling approaches, such as nonlinear relationships that are not adequately modeled by common transformations of variables. The group of procedures that includes GLMs and data mining techniques are often referred to as predictive models by insurance actuaries. In this paper we will show how data mining techniques and GLMs can be combined to take advantage of the strengths of each approach. In addition, we will present a common problem that arises in the modeling of count data: excess zeros. That is, sometimes, when actual instances of zero counts are compared to the theoretical values under the Poisson assumptions, there are significantly more zeros than the fitted distribution predicts. In the insurance context, this is believed to be due to the underreporting of small claims (Yip and Yau, 2005).

One of the symptoms of zero-inflated distributions is overdispersion. That is, under the Poisson assumption; the variance of the distribution is equal to its mean. Table 1.1 presents some automobile insurance count data from Yip and Yau that will be used throughout this paper to illustrate techniques and concepts. For this data the variance exceeds the mean. When the variance exceeds the mean, the situation is referred to as overdispersion, and a number of approaches are used to address it. One approach is to use a negative binomial model rather than a Poisson, as the variance of the negative binomial distribution exceeds the mean.

Table 1.1

Example of Overdispersion

K	Count	P(X=x)
0	1,706	0.607
1	351	0.125
2	408	0.145
3	268	0.095
4	74	0.026
5	5	0.002
Total	2,812	
Mean	0.815	
Variance	1.364	

Figure 1.1 displays a comparison of actual and theoretical probabilities at each value of K (or the five-year frequency) for the auto data. Note the actual data contains more zeros and fewer ones than predicted by the Poisson.

Figure 1.1

Actual Frequencies vs. Poisson Theoretical Frequencies

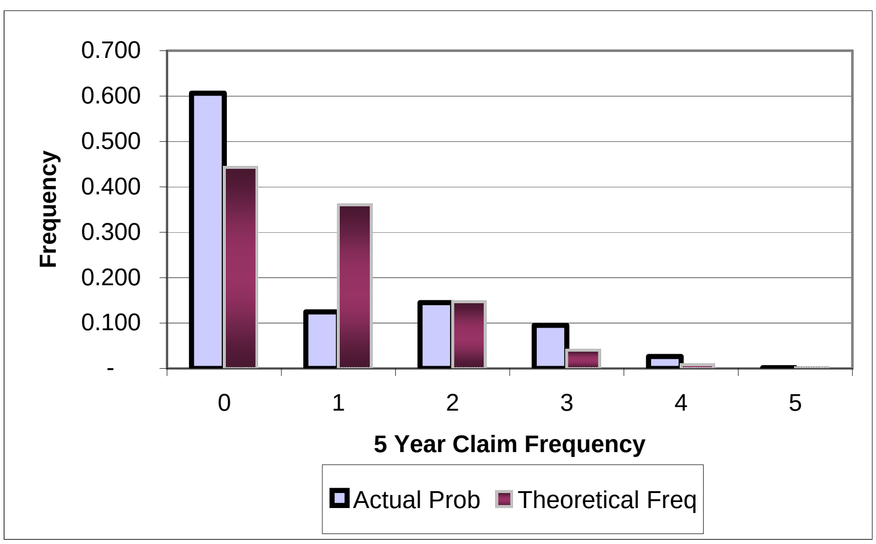


Table 1.2 displays the average claim frequency for the six car type categories in the data. The table indicates that some of the types such as Pickup and Van have similar frequencies. Might we be able to combine some of these categories and reduce the number of parameters in a regression model that uses categorical predictors? What procedures will facilitate efficiently combining of

categories that are not statistically different with respect to their effect on the dependent variable?

Table 1.2

Type of Car	Avg #Claims (Past 5 Years)
Panel Truck	**0.9133**
Pickup	**0.8262**
Sedan	**0.6674**
Sports Car	**0.9296**
SUV	**0.8092**
Van	**0.8449**
Total	**0.8006**

1.1 Research Context

As can be seen in some of the early literature on the subject (Bailey and Simon, 1959; Simon, 1962), the Poisson distribution has long been used in actuarial science as a stochastic model for claim count data. The negative binomial distribution is a key alternative when the variance of the count data exceeds the mean (Simon, 1962). Both distributions are members of the exponential family of distributions and have become popular for modeling frequency data in predictive modeling applications. Thus, the Poisson and negative binomial can be used within the GLM framework to fit regression models to insurance/claim frequency data.

Anderson et al. (2005) mention the problem of overdispersion that frequently occurs when using the Poisson distribution. Their suggested remedy follows that of the classic reference by McCulloch and Nelder (1989). The classical approach to overdispersion involves estimating an additional scale parameter for the Poisson distribution. This scale parameter has no effect on the estimated coefficients of the independent variables used in the regression model but does affect tests of significance for the variables. Ismail and Jemain (2007) extend the classical treatment of overdispersion using generalized Poisson and negative binomial models.

In Hilbe's recent book (Hilbe, 2007) points out that excess variability in Poisson regression can be due to a number of additional factors not remedied by using an overdispersion parameter or the negative binomial distribution including:

- Missing independent variables
- Interactions not included in the model

- Excess zeros

Yip and Yau (2005) illustrate how to apply zero-inflated Poisson (ZIP) and zero-inflated negative binomial (ZINB) models to claims data, when overdispersion exists and excess zeros are indicated. They also present another alternative, hurdle models, to approximate distributions with excess zeros. Jackman (2007) describes functions implemented in the statistical software R that can be used to implement ZIP, ZINB, and hurdle models. In this paper we will extend the work of these authors by combining ZIP, ZINB, and hurdle models with data mining procedures that efficiently search for significant terms in the data and reduce the dimensionality of categorical variables by clustering together categories of categorical dependent variables.

1.2 Objective

The paper attempts to improve the application of GLM procedures to claim prediction in property casualty insurance.

In this paper we will:

- Illustrate the problem of excess zeros in claim count data and then show how to remedy it with zero-adjusted mixture models
- Show how GLM models for count data can be combined with traditional data mining approaches to produce more robust models
- Apply the procedures to an insurance database as an illustration

1.3 Outline

The remainder of the paper proceeds as follows. Section 2 will present the problems of excess zeros in count data and show how to address it with zero-inflated models. In Section 3 we show how to augment GLM models with traditional data mining approaches to efficiently model nonlinear relationships and reduce the number of parameters contributed by categorical variables. In Section 4 we present overall conclusions. We have provided code in SAS for implementing some of the models in Appendices but numerous statistical tools contain the technology for implementing the models in this paper. Additional Code using R will be made available on the CAS's Web Site.

2. ZERO-INFLATED AND HURDLE MODELS

2.1 The Data

We will illustrate many of our key concepts using the auto data from Yip and Yau (2001). Yip and Yau supplied a frequency table of personal automobile claims that we use to illustrate Univariate distribution fitting methods. An additional database from Yip and Yau of personal automobile policy level information contains approximately 10,000 records and is used to illustrate multivariate regression models. Table 2.1 displays the variables in the data. The first variable on the list, claim frequency, is used as a dependent variable in the GLM, ZIP, and hybrid models. All other variables when used are used as predictor variables.

Table 2.1

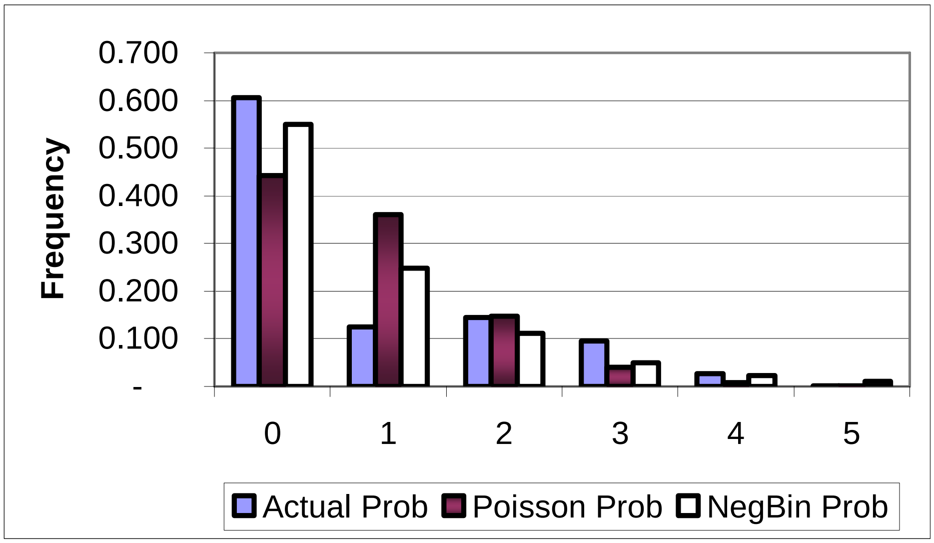
Variables in Automobile Database

Variable	Description
CLM_FREQ	No. of claims in 5 years
AGE	Policyholder age
BLUEBOOK	Blue book value of car
CAR_TYPE	Type of car: sedan, SUV, etc.
CAR_USE	Private or Commercial use
CLM_DATE	Accident Date
DENSITY	Population Density (rural, urban)
GENDER	Gender
HOME_VALUE	House value
HOMEKIDS	No. of children at home
INCOME	Policyholder income
JOBCLASS	Job category
KIDSDRIVE	No. of children that drive
MARRIED	Marital status
MAX_EDUC	Highest education
MVR_PTS	Motor Vehicle Points
NPOLICY	Number of policies
PARENT1	Single Parent?
PLCYDATE	Policy Inception Data
RETAINED	Number of years policy renewed
REVOKED	Licensed revoked?
SAMEHOME	How many years in current house?
TRAVTIME	Travel time to work
YOJ	Years on current job

Before fitting a conditional model of claim frequency using the predictor variables in the auto data, we first investigate the distribution of marginal claims (displayed in Table 1.1). Figure 1.1 presented a comparison of actual and fitted Poisson claim frequencies for this data and indicated that the actual number of zero claims exceeds those that would be expected if the data were Poisson distributed. A negative binomial distribution was fit next. A larger number of zeros (as well as larger frequencies) could be expected under a negative binomial model.

Figure 2.1

Comparison of Actual, Poisson, and Negative Binomial Frequencies



From Figure 2.1 it is apparent that the negative binomial distribution approximates the data better than the Poisson distribution. However, the actual data compared to the negative binomial shows an excess probability of zero claims and a significantly lower probability at a count of one.

2.2.1. Introduction to Zero-Inflated and Hurdle Probability Distributions

An alternative probability distribution when "excess" zeros appear to be present is the zero-inflated Poisson. The zero-inflated Poisson assumes the observed claim volumes are the result of a two-part process 1) a process that generates "structural zeros" and 2) a process that generates random claim counts. In insurance the "structural zeros" may be due to underreporting of small claims. Especially when claims are near or less than the policy deductible, a policyholder may not report the claim because 1) there may be no expected payment under the policy and 2) the policyholder may wish to avoid premium increases under an experience rating or merit rating system. The ZIP distribution is a mixture of exponential family distributions. Under the zero-inflated Poisson, the structural zeros are assumed to follow a Bernoulli process with parameter φ, denoting the probability of a zero and the random counts are assumed to follow a Poisson with parameter λ, the mean of the distribution. The distribution of the zero-inflated Poisson is:

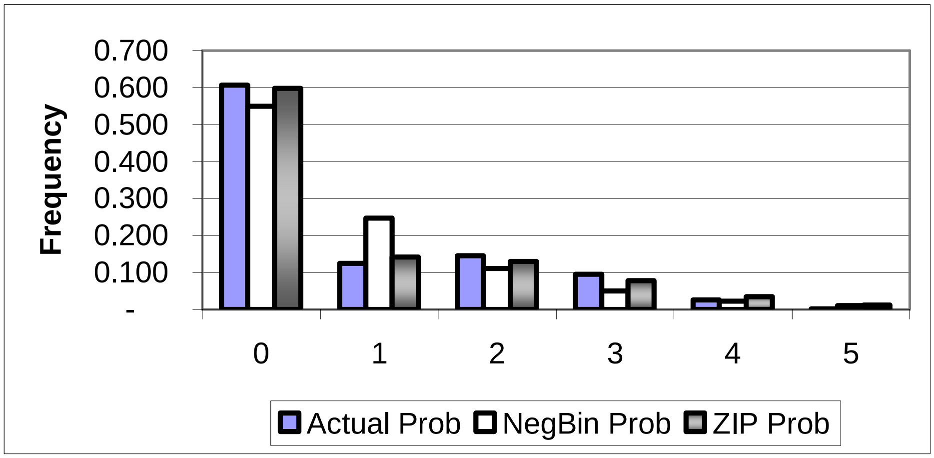
$$\begin{array}{ll} \varphi + (1-\varphi)e^{-\lambda} & x=0 \\ (1\text{-}\varphi)\dfrac{\lambda^{x}}{x!}e^{-\lambda} & x>0 \end{array} \tag{2.1}$$

The theoretical mean of the ZIP model is φ+ (1-φ)λ. The variance is (1-φ)λ(1+$\varphi\lambda$).

The parameters of the Poisson and negative binomial distribution can be estimated from the sample mean (Poisson) and the sample mean and variance (negative binomial). However, a numerical optimization procedure must be used to estimate the parameters of zero-inflated models. A description of the specific procedure we implemented in Excel is provided in Appendix G.

The parameters fit with Excel solver are displayed in Appendix G, Table G-2. The table indicates that on average, 54% of the records have structural zeros. For the remaining policyholders, the mean claim frequency over a five-year period is approximately 1.9. Figure 2.3 compares the negative binomial to the zero-inflated Poisson. The ZIP model appears to provide a better fit to the data.

Figure 2.3
Actual, Negative Binomial, and Zero-Inflated Poisson Frequencies



The Chi-Squared test can be used to test whether the ZIP model is a significantly better fit to the data than the negative binomial or Poisson models. The Chi-Squared statistic is:

(2.3) $$\chi_{k-1}{}^{2} = \frac{(Observed - Fitted)^2}{Fitted}$$

The Chi-Squared statistic compares the observed and fitted claim counts. It has degrees of freedom equal to *k*-1, where *k* is the number of categories (here equal to six).

Table 2.2
Chi Squared Statistic for Poisson, Negative Binomial and ZIP Models

	Chi-Squared
Model	**Statistic**
Poisson	**935.3**
Negative Binomial	**351.9**
ZIP	**60.2**

Note that the critical value for the Chi-Squared statistic at the 5% level is about 11, so that all three fitted models would be deemed significantly different from the data by this statistic.[1]

[1] It should be noted that a well-known limitation of the Chi-Square statistic is that it is very conservative when comparing actual to fitted distributions. That is, it is common for the distribution to be significantly different from the actual empirical distribution according to this measure.

However, it can be seen that the ZIP model provides a much better fit to the data.

Another mixed probability distribution related to the ZIP model is the zero-inflated negative binomial (ZINB) model. The ZINB is a mixture of a Bernoulli variable (for the structural zeros) and a negative binomial for the random counts. The distribution's formula is:

$$\begin{aligned}
&\varphi+(1-\varphi)NB(0,r,p), k=0\\
&(1-\varphi)NB(k,r,p), k>0 \qquad (2.4)\\
&NB(k,r,p)=\binom{k+r-1}{k}p^k(1-p)^r
\end{aligned}$$

The mean of the negative binomial is $r\,(1\text{-}p)/p$. The variance is $r\,(1\text{-}p)/p^2$. As with the ZIP model, the ZINB model can be fit in Microsoft Excel. Table G.3 in Appendix G shows the values of the estimated parameters.

In the example, the estimated parameters for ϕ is zero, indicating that the single negative binomial model is a better fit than the ZINB mixed model. Note the chi-square statistic for this model (397) was higher than that of the negative binomial fitted using the first two moments of the data.

A model related to the zero-inflated models is the hurdle model. The hurdle models assume two processes: 1) a process that generates no claim or at least one claim and 2) a process that generates the number of claims given that at least one claim occurs. A Bernoulli process is used to model the occurrence/nonoccurrence of a claim while a truncated Poisson or negative binomial is used to model positive claim counts. The formula for the hurdle Poisson model is shown in (2.5) and the fitted parameters are shown in Table G.4 of Appendix G. For this data the hurdle Poisson does not fit the data as well as the ZIP model, as it has a larger weighted squared deviation and its Chi-Square statistic of 97 is larger than that of the ZIP model.

(2.5)[2]

$$\begin{aligned}
&\varphi, k=0\\
&\frac{1-\varphi}{1-e^{-\lambda}}\frac{\lambda^k}{k!}e^{-\lambda}, k>0
\end{aligned}$$

[2] The mean of the hurdle Poisson is λ/ (1-exp(-λ)). The variance of the hurdle Poisson is λ/ (1-exp(-λ)) (1- λ exp(-λ))/ (1-exp(-λ)).

A negative binomial hurdle model was also fit to the data, but as with the ZINB model, the fitted model contained no Bernoulli parameter.

2.2.1.a Zero-Adjusted Models for Other Data Sets

Since the Yip and Yau data in our illustrations were used in their paper advocating the use of ZIP and ZINB models, one is not surprised when a zero-adjusted mixed model fits the data better than single count distribution models. In order to explore the broader applicability of zero-adjusted models, several other sample datasets were tested to determine if the ZIP or ZINB provided a better fit than simpler models:

- The Bailey and Simon credibility study (Bailey and Simon, 1959) used the experience from 1957 and 1958 for Canadian Private Passenger automobile exposure excluding Saskatchewan. The data is shown in Table 1 of their paper. This data is reorganized and displayed in Table F.1 of Appendix F. The data displayed was aggregated to the class level. For this data the negative binomial is a much better fit than the Poisson (illustrating the need to test for the negative binomial as an alternative to the Poisson), as well as the ZIP model. The ZINB, however, fits the data better than the negative binomial but the difference is not of the same magnitude as that between the negative binomial and Poisson. For this data, under the Poisson and ZIP assumptions observations are expected to be much closer to the distribution's mean value, while many of the actual observations are far from the mean, causing a very high chi-square values under Poisson and ZIP assumptions.
- Zero-inflated count data are also found in non-insurance applications. Five different datasets from various non-insurance analyses are displayed in Appendix F. Most of the examples tested displayed a very large variation in the goodness of fit. This wide variation indicates it may be prudent to test a number of possible alternatives before selecting a distribution to incorporate into a predictive model.
    - Hospital visit data from Deb and Trevedi (1997). The data contain the number of visits and hospital stays for a sample of United States residents aged 66 and over. For this data the ZINB was the best fit and the Poisson was a very poor fit.

- Doctor office visit data from Deb and Trevedi (1997). For this data the negative binomial was the best fit and the Poisson and ZIP were very poor fits.
- Patents data from Wang, Cockburn and Puterman (1998). The data contain the number of patents for a sample of pharmaceutical and biomedical companies. For this data the ZINB was the best fit and the Poisson and ZIP were very poor fits.
- Apple tree root cultivar[3] count data from Ridout and Demetrio (1998). For each cultivar, the number of roots produced during different experimental protocols was tabulated. For this data the ZINB was the best fit.

2.2.2. Poisson, Negative Binomial, ZIP, ZINB, and Hurdle Models with SAS

For simplicity of exposition, we have shown how to fit univariate zero-inflated and hurdle models in Microsoft Excel. However, nonlinear curve-fitting applications are typically performed in statistical or mathematical programming languages such as SAS, MATLAB, and R. For certain other distributions, specifically those that are members of the exponential family of distributions, a generalized linear model (GLM) can be use to fit the parameters of the distribution. For example an intercept-only GLM model with a Poisson distribution and log link can be used to estimate Poisson parameters. While this is a trivial example because the Poisson parameter equals its mean, it illustrates how common statistical software can be used to parameterize probability distributions. The model fit is:

(2.5) $Y = a + e$, where e is a random error term.

That is, a GLM procedure is used to fit a model that only has an intercept term, but no independent variables. For the Poisson, the intercept will equal the Poisson parameter. See Appendix B for an example of SAS code that can be used to fit the Poisson parameters.

For more complicated probability distributions such as zero-adjusted distributions, the analyst will want to use an approach that solves for parameters, given a function of the parameters to optimize. For instance in Appendix G the distance between an actual and fitted distribution is minimized when estimating the parameters of distributions using the Excel solver. It is common in distribution fitting to maximize the log of the likelihood function. For many common claim count

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[3] A cultivar (short for cultivated variety) is a cultivated plant with unique characteristics that separate from other similar cultivated plants.

distributions, the log-likelihood function is readily specified, either from first principles or from one of the many references on probability distributions (Hogg and Klugman, 1982). In Appendix A we present the PDF and log-likelihood function for the Poisson. Once a likelihood function has been specified, an optimization procedure is used to solve for the distribution's parameters. For common one and two parameter distributions, it is often unnecessary to specify a likelihood function, as these functions are prepackaged in statistical fitting software.

For more complex models, many software packages offer the user a procedure that fits nonlinear mixed models using a nonlinear fitting procedure. This is appropriate for the zero-adjusted models, which do not have a closed-form solution for the parameters, but such procedures can often be used to fit more familiar distributions (i.e., Poisson, logNormal) as well (ignoring any "mixed" model structure). Appendix A presents an example using SAS code to generate fitted distributions and predicted probabilities.

Figures 2.1 and 2.2 suggest that the actual claim data contain excess zeros compared to those expected under both the Poisson and negative binomial distribution approximations. Prior to fitting a zero-inflated distribution, we can formally test for zero inflation. Van den Broek (1995) provides a score test for zero inflation relative to a Poisson distribution. The statistic is based on a comparison of actual zeros to those predicted by the model:

$$S = \frac{\left\{ \sum_{i=1}^{n} (I(x_i = 0) - p_{0i}) / p_{0i} \right\}^2}{\sum_{i=1}^{n} (I(x_i = 0) - p_{0i}) / p_{0i} - n\overline{x}} \tag{2.6}$$

In formula (2.6) S is the score, $I(x_i=0)$ is an indicator function that is 1 if a given observation equals zero, and 0 otherwise. Denoting the probability, p_{0i} does so under the assumed distribution (typically Poisson) of a zero observation for observation *i*. Note that the probability is allowed to vary by observation. The score is assumed to follow a chi-squared distribution with one degree of freedom. Appendix C presents sample code that can be used to apply the score test. As seen in Appendix C, the score for our automobile count data was an 869, which is significant at the 0.001 level.

As the score statistic supports the possibility of a zero-inflated distribution, we proceed with fitting zero-inflated distribution using statistical software. Appendix D presents an example of fitting a zero-inflated distribution using a nonlinear mixed models procedure.

As discussed in section 2.1.1, in the presence of excess zeros, a hurdle model rather than a zero-inflated model may be more appropriate. Hurdle (Mullahy,1986) or two-part (Heilbron, 1994) models are so-called because the likelihood function is constructed to be separable, that is, the zero/positive component is typically handled with a logistic or Probit model, whereas the model for positive counts can include or exclude zeros. The count portion of the hurdle model may be Poisson, negative binomial, or other count model. Appendix D presents SAS procedures that can be used to fit these hurdle models.

If zeros are excluded from the count portion of the model, then the positive portion can be modeled via a zero-truncated Poisson, for example. (The formula was given earlier in equation 2.5). Additional applications of truncated count models include Grogger and Carson (1991), Shaw (1988), and Winkelmann and Zimmerman (1995). Alternatives to the truncated Poisson include subtracting one from the dependent count variable. This has been described as a shifted or positive Poisson distribution (Shaw, 1988). Johnson and Kotz (1969) refer to this as a displaced Poisson distribution.

2.3 Regression Models

In this section, the zero-inflated and hurdle models are generalized to regression applications. We will use the 10,000 record Yip and Yau automobile insurance dataset to develop a model to predict claim frequency. This section will show how to augment the Poisson and negative binomial models commonly used for count predictions with zero-inflated and hurdle capabilities.

We first review the basic assumptions of generalized linear models. See Anderson et al. (2005) for a more complete introduction to GLMs.

A generalized linear model is denoted: $Y = \eta + e = \mathbf{x}'\boldsymbol{\beta} + \mathbf{e}.$

It has the following components:

- a random component, denoted e
- a linear relationship between a dependent variable and its predictors. The estimate or expected value of the prediction is denoted η.
- $\eta = a + b_1 x_1 + b_2 x_{2+\dots} b_n x_n$
- a link function captures the form of the relationship between the dependent variable and the

regression expected value. Two common link functions used when applying GLMs to ratemaking are:

- the identity link $\mu=\eta$
- the log link $\mu=\exp(\eta)$ or $\eta=\log(\mu)$.

Under the log link, each predictor variable's impact on the estimate is multiplicative. That is: $Y = A\exp(b_1x_1)\exp(b_2x_2)...\exp(b_nx_n)$. In ratemaking applications it is common for the classification variables to raise or lower a rate by a percentage. Hence, the log link is intuitive for the ratemaking models being presented in this paper.

Another common link function is the logit link: $\eta=\log(p/(1-p))$, where p denotes a probability between zero and one and $p/(1-p)$ is the odds ratio or the odds of observing the target variable. The logit link is commonly used with a Bernoulli (binary) dependent variable.

In claim frequency modeling, it is common for the random component of the GLM to be the Poisson or negative binomial distribution. The Poisson and under certain assumptions, the negative binomial (i.e., when the scale parameter is known) are members of the exponential family of distributions that also includes the normal and gamma. The zero-inflated and hurdle models generalize the GLM to include mixture models. For instance, the ZIP model is a mixture of two distributions from the exponential family: the Bernoulli and the Poisson. The hurdle Poisson model is also a mixture of a Bernoulli and a Poisson random variable, but with the hurdle model, the Poisson is a truncated Poisson that models only positive claim counts and the zeros are modeled exclusively with the Bernoulli distribution.

This paper's first predictive modeling illustration will use four variables to predict claim frequency. The four variables are car use, marital status, density, and gender. Each of the predictor variables is categorical. Thus the model is:

(2.7)1. $Y = f(\text{car use, marital status, density, gender}) + e$.

Where Y denotes the dependent variable, number of claims reported within a five-year period. In the Poisson and negative binomial regressions, the log link will be used.

In this section, classical GLM count regression models are compared to zero-inflated and hurdle alternatives. As discussed in Section 1, overdispersion in count models is commonly handled by fitting an over-dispersed Poisson, which allows the variance to exceed the mean by a constant factor. We also present results for a geometric as well as a negative binomial model as the negative binomial becomes a geometric when the size parameter *r* is 1. Poisson and negative binomial regressions will be compared to ZIP, ZINB, and hurdle Poisson and hurdle negative binomial models. Under the zero-inflated and hurdle model there are two components denoted *Y* and *Z*:

Y = *f* (car use, marital status, density, gender) + e, Z = f(car use, marital status, density, gender) + *e*.

Thus, the predictor variables are used both to estimate the Bernoulli parameter *p* (the *Z* component) and are also used to estimate the Poisson expected claim count (the *Y* component). It is likely that the different variables will have a different importance in each component of the model. A nonlinear mixed models procedure can be used to estimate the parameters of the ZIP model. When using nonlinear mixed models procedures (or any other nonlinear optimization software) it is typically necessary to specify the log-likelihood function. For the ZIP regression the log-likelihood (denoted *ll*) is straightforward:

$$\begin{aligned} &\text{if } (\mathrm{Y}=0)\ ll = \log(p_0 + (1 - p_0)\exp(-\lambda)) \\ &\text{if } (\mathrm{Y}>0)\ ll = \log(1 - p_0) + Y * \log(\lambda) - \lambda - \log(x!) \end{aligned} \tag{2.8}$$

Appendix E presents code for fitting these models. In the particular example in Appendix E, the Bernoulli parameter *p* enters the function as a constant; that is, it is the same for every record, regardless of the value of the predictor variables, while the Poisson parameter is estimated from the regression function. It is straightforward to add a regression function for the Bernoulli parameter. To assess the goodness of fit of the models we compute the negative log-likelihoods (actually -2* the log-likelihood). In Table 2.3 the log-likelihood statistics from the different model fits are presented. It can be seen that the ZIP fits the data best while the simple Poisson regression provides the worst fit. Moreover, there is a significant improvement in fit when moving from the Poisson the ZIP.

The results indicate that the model fit to our sample auto claim counts was improved by using a zero-adjusted model. In the next section, we will compare and contrast a GLM and a zero-adjusted model with models augmented using hybrid techniques that employ a decision tree method to preprocess data. To keep the kinds of models to a manageable number we will only use the simple

Poisson and ZIP (the best performing model in Table 2.3) models in the next section.

Table 2.3

Model	-2*log-likelihood
Poisson	7,141.9
Overdispersed Poisson	6,843.9
Geometric	6,764.1
Negative Binomial	6,764.1
ZINB	6,541.2
ZIP	6,404.0

3. CHAID HYBRID MODELS

3.1 The CHAID method

The term "data mining" is loosely refers to a number of very different methods that apply computationally intensive nonparametric procedures, typically to large databases with many potential predictor variables and many records. Among the common data mining techniques used for prediction are neural networks and tree models. Trees fit a model by recursively partitioning the data into two or more groups, where data for each partition are more homogenous than the pre-partitioned data. The different groups are statistically determined to have significantly different values for the dependent variable. In the most common tree method, Classification and Regression Trees (C&RT), the data is split into two groups, one with a high average value for the dependent variable and the other group with a lower average value on the dependent variable. Each partition of the data in a tree model is referred to as a node.

The CHAID tree method is one of the oldest tree-based data mining methods and one of the earliest to appear in the casualty actuarial literature. The method was applied to classification ratemaking by Fish et al. (1990) following the passage of Proposition 103 in California.[4] Unlike C&RT, CHAID can partition data into more than two groups. CHAID is an acronym for chi-squared automatic interaction detection. As the name implies, CHAID relies heavily on the chi-squared statistic (Formula 2.3 in section 2) to partition data. In classical statistics the chi-squared statistic is typically used to assess whether discrete categorical variables are independent or whether a relationship exists between the variables (Faraway, 2006).

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[4] Proposition 103 constrained how variables could be used in automobile ratemaking.

One of the data preparation steps that is often applied prior to fitting of predictive models is cardinality reduction. Cardinality reduction refers to reduction of the number of categories in nominal and ordinal variables (Refaat, 2007). The CHAID procedure is a procedure that can be used to preprocess categorical variables and to group like categories of the independent variables together. A problem with nominal and ordinal variables with many categories is that some of the categories are sparsely populated and some of the categories are very similar with respect to their effect on the dependent variable. Inclusion of all the levels of a categorical variable can lead to overfit/overparameterized models that fit parameters to noise rather than legitimate patterns in the data. Using the chi-squared statistic, categories that are not significantly different with respect to their effect on a dependant variable can be combined and the total number of categories reduced.

For instance, the categorical variable density from the automobile database has four levels or categories: highly urban, urban, rural, and highly rural. Suppose the analyst is interested in knowing whether a relationship exists between population density and the likelihood of having at least one claim. Let the likelihood of having a claim be denoted by a binary categorical indicator variable that is 1 if the policyholder has had at least 1 claim and 0 otherwise. Table 3.1 displays a crosstabulation of density and the indicator variable based on data from the automobile database. The bottom section of the table shows that urban and highly urban policyholders have a significantly higher frequency of claims than do rural and highly rural policyholders. The chi-squared statistic can be used to test whether this apparent relationship is significant.

Table 3.1

Crosstabulation of Population Density vs. Binary Claim Indicator

Home/Work Area * Claim Indicator Crosstabulation

		Claim Indicator		
		No Claim	Claims	Total
Home/Work	Highly Rural	4,52	56	508
	Highly Urban	1,732	1,867	3,599
	Rural	1,369	196	1,565
	Urban	2,740	1,891	4,631
	Total	6,293	4,010	10,303

Percent of Policies With Claims

		Claim Indicator		
		No Claim	Claims	Total
Home/Work	Highly Rural	89%	11%	100%
	Highly Urban	48%	52%	100%
	Rural	87%	13%	100%
	Urban	59%	41%	100%
	Total	61%	39%	

The chi-squared statistic requires both an observed and expected record count for each of the cells in the crosstabulation. An expected count can be computed by applying the marginal proportions shown at the bottom of Table 3.1 (61% no claim, 29% at least one claim) to the total number of policyholders in each density category. This is shown in Table 3.2. For instance, the expected number of highly rural drivers with no claims is 310.3 (0.89*508). The expected count is then used in the computation of the chi-squared statistic, shown also in Table 3.2. This statistic has degrees of freedom equal to the number (*c*-1)*(r- 1) (here 6) where *c* denotes the number of columns and *r* denotes the number of rows. Its value as shown at the bottom of Table 3.2, 886, is significant at (less than) the .1% level, suggesting a relationship between density and propensity for an automobile claim.

Table 3.2 Expected Count & Chi-Squared Statistic

Expected Count				
		Claim Indicator		Total
		No Claim	Claims	
Home/Work	Highly Rural	3,10.30	197.70	508
	Highly Urban	2,198.20	1,400.80	3,599
	Rural	955.90	609.10	1,565
	Urban	2,828.60	1,802.40	4,631

Chi-Squared Statistic: (O-E)^2/E			
		Claim Indicator	
		No Claim	Claims
Home/Work	Highly Rural	64.70	101.60
	Highly Urban	98.90	155.20
	Rural	178.50	280.20
	Urban	2.80	4.40
			886.20

Suppose the claims are sorted in ascending order by proportion of policies with a claim. This is shown in Table 3.3. The table suggests that some of the categories of the density variable may not be significantly different from each other and therefore could be combined. For instance, the highly rural and rural categories at positive claim proportions of 11% and 13%, respectively, could perhaps be combined into a "rural" category, if the difference (in likelihood of having a claim) is not significant.

Table 3.3

Percent of Policies With Claims			
		Claim Indicator	
		No Claim	Claims
Home/Work	Highly Rural	89%	11%
	Rural	87%	13%
	Urban	59%	41%
	Highly Urban	48%	52%
	Total	61%	39%

Table 3.4 displays the calculation of the chi-squared statistic, including the calculation of expected counts, for the highly rural and rural categories. The chi-squared statistic of 0.81 (see bottom row of Table 3.4) is not significant, indicating the two categories can be combined.

Table 3.4
Comparison of Rural and Highly Rural Categories Using Chi-Squared Statistic

Observed			
	No Claim	Claim	Total
Highly Rural	452	56	508
Rural	1,369	196	1,565
Total	1,821	252	2,073
Expected			
	No Claim	Claims	Total
Highly Rural	446.25	61.75	508
Rural	1,374.75	190.25	1,565
Chi Squared			
	No Claim	Claims	
Highly Rural	0.07	0.54	
Rural	0.02	0.17	
Total	0.09	0.81	

The chi-squared statistic can be computed for all other pairs of combinations (actually it only makes sense to compare pairs of categories that are contiguous in a sorted table such as Table 3.3). Once the chi-squared statistic has been computed for the pair-wise comparisons, the two categories with the lowest chi-squared values can be combined, provided the chi-square statistic is not significant.[5] In this example, the rural and highly rural categories have the lowest chi-squared statistics, so they are combined, resulting in three density groupings.[6] Table 3.5 shows the new table that is created when the categories are combined. Using the new crosstabulation, the chi-squared

[5] It is common to use the 5% level as the threshold for significance, though other levels can be chosen. Thus categories where the significance levels below the threshold can be combined. If the chi-squared statistic is significant, the two categories should not be combined, as the null hypothesis that there is no difference between the categories in their effect on the dependent variable is rejected.

[6] The chi-squared for all other comparisons was more than 99.0, which is significant at the 5% level.

statistic can be recomputed for the new table and the categories with the lowest chi-squared statistic can be combined. The recursive process of combining categories continues until no more significant differences between the categories can be found.

Table 3.5

Crosstabulation after Combining Two Categories

Area * Claim Indicator Crosstabulation			
	Claim Indicator		
	No Claim	Claims	Total
Rural	1,821	252	2,073
Urban	2,740	1,891	4,631
Highly Urban	1,732	1,867	3,599
Total	6,293	4,010	10,303

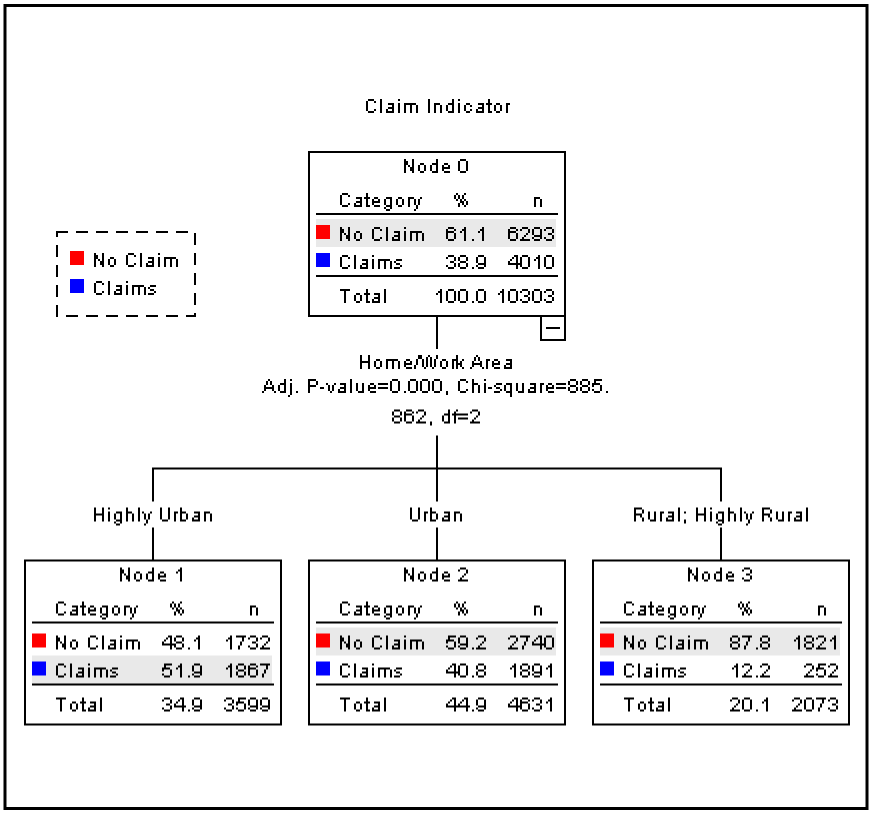
Percent of Policies With A Claim			
	Claim Indicator		
	No Claim	Claims	Total
Rural	88%	12%	508
Urban	59%	41%	3,599
Highly Urban	48%	52%	1,565

The results of the partitioning of the variables can be displayed graphically in a tree diagram. The tree diagram for the car density example is shown in Figure 3.1. The top box or "node" is a "parent" node. It displays the overall claim indicator statistics for all records before any partitioning occurs. Below the parent node are the "child" nodes resulting from the partitioning of the density variable using CHAID.[7] The nodes in this layer are also "terminal" nodes, as there is no further partitioning of the data. The terminal nodes contain the model's final prediction, which is typically the overall proportion of target variable records in the node.

[7] The CHAID models used in this paper were fit with SPSS Classification Trees. We are not aware of either SAS Stat or R functions for CHAID.

Figure 3.1

Tree for Population Density (Independent Variable) and Claim Indicator (Dependent Variable)



By adding a second variable to the model, say car use, it is possible to add another layer to the tree, however. To create a tree with two layers of nodes, it is necessary to partition the data on a second variable, after the partitions on the first variable, (density), have been completed. An example of partitioning using two variables is shown in Figure 3.2. As can be seen from Figure 3.2, not all nodes from the first layer can be further partitioned. When two variables are included in the model, CHAID performs the following process:

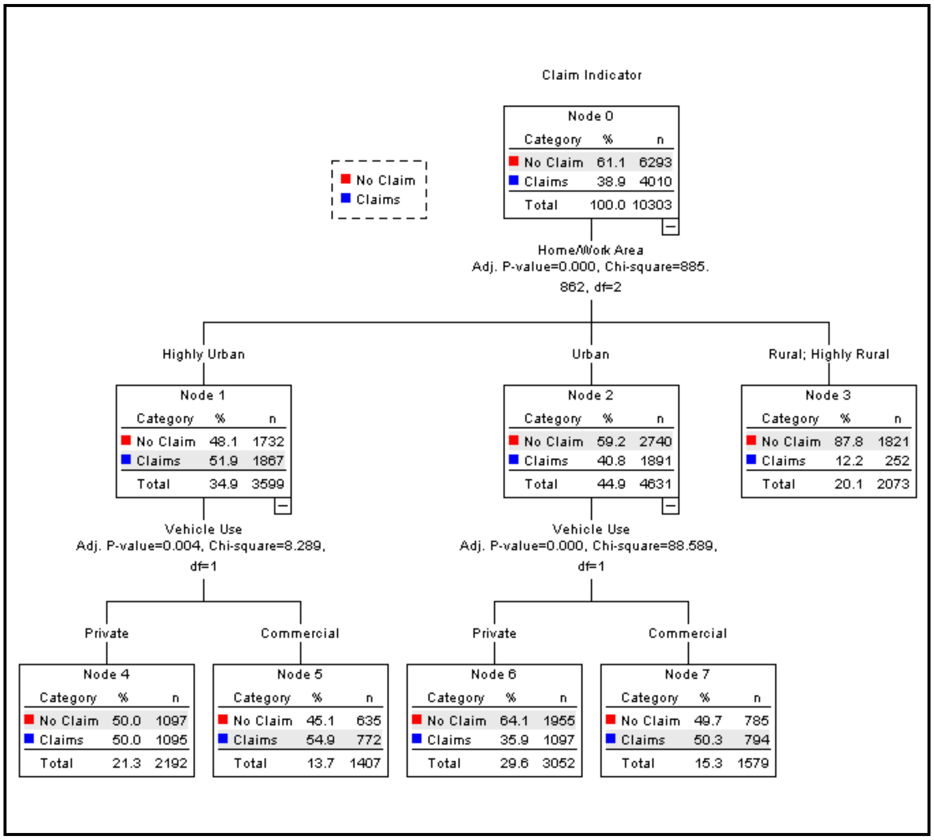
- Compute the beat partitioning of the data for the first variable and compute the chi-squared statistic for the partitioned data after categories that are not significantly different have been combined
- Compute the best partitioning of the data for the second variable and compute its chi-squared statistic. (In this very simple example, the car use variable has only two

categories, so no further combining of categories can be done)

- Select the variable that produces the highest chi-squared statistic to partition the data first
- Repeat the process for each of the nodes from the first partitioning. If none of the nodes can be further partitioned, stop.

Since, the focus of the current discussion is on the use of CHAID for cardinality reduction of categorical variables before fitting a GLM or other predictive model, further discussion of the CHAID for multivariable models is outside the scope of this paper. However, complete predictive models can be built using CHAID and other decision tree techniques.

Figure 3.2
CHAID Tree with Two Variables



It should be noted that the example of category reduction for the density variable is a relatively trivial one, as inspection of the statistical output from the fitted GLM, ZIP, and hurdle models could probably be used to reduce the number of categories. However, fast and computationally efficient procedures are needed for variables containing a large number of levels. Such variables occur frequently in insurance predictive models.

As a more realistic example, consider the car-type variable, which has six levels (Table 3.6). With six levels for a variable, there are hundreds of possible ways to combine categories.[8] In a typical automobile ratemaking database, there would likely be many more than six levels on a car-type variable. Figure 3.3 presents the CHAID tree that was fit using the car-type variable.

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[8] The number of all possible combinations is $\sum_k \binom{x}{k}$, but when the categories are ordered based on the proportion of policies with claims, the number goes down.

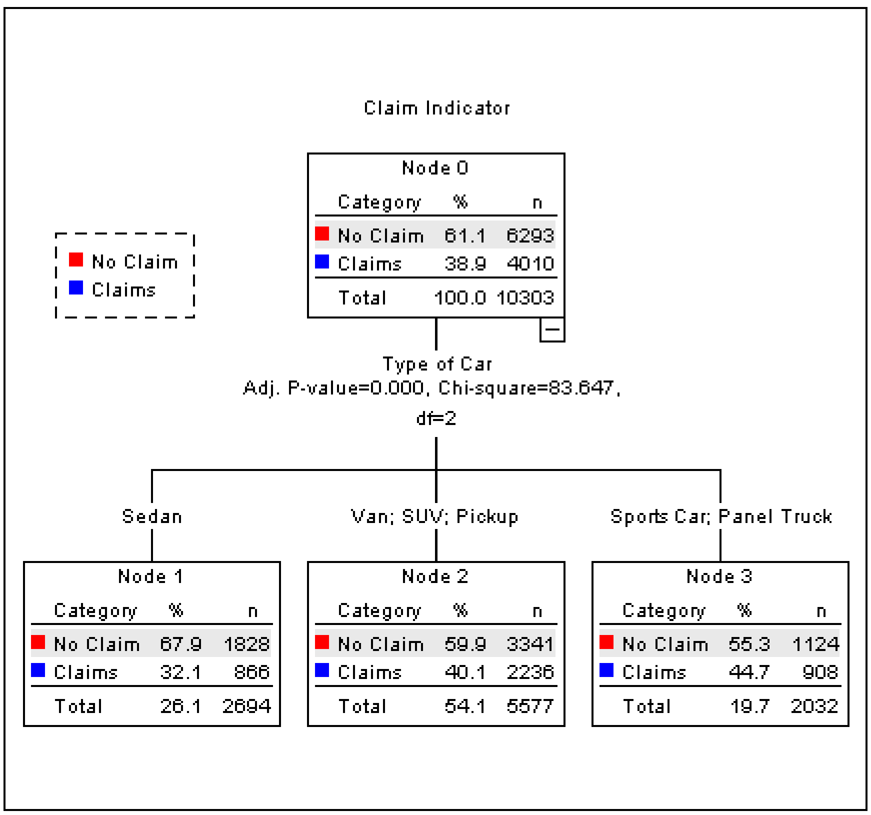
Table 3.6

Car Type Frequency Tabulation

Car Type	Frequency	Percent
Panel Truck	853	8%
Pickup	1,772	17%
Sedan	2,694	26%
Sports Car	1,179	11%
SUV	2,883	28%
Van	922	9%
Total	10,303	100%

Figure 3.3

CHAID Model for Car-type Variable



From Figure 3.3, the number of groupings is reduced from six to three when CHAID is used to preprocess the car-type variable.

When the dependent variable in the model is numeric, rather than categorical, most CHAID procedures use the *F*-statistic rather than the chi-squared statistic to partition data.

$$F = \frac{(RSS_1 - RSS_2)/(p_2 - p_1)}{RSS_2 / p_2} \tag{3.1}$$

RSS = residual sum of squares

p_1 = degrees of freedom for model 1

p_2 is degrees of freedom for model 2

When only two categories are compared, the *F*-test reduces to a *T* test.[9] Thus the categories can be compared using the *F* (or *T*) statistic and the categories that are not significantly different can be merged. The first two categories merged are the categories with the lowest *T* statistic.

Suppose, instead of using a binary categorical dependent variable, we treat claim frequency (number of claims in the past five years) as a numeric variable and use the *T* test to merge categories. Table 3.7 displays the mean claim frequency, along with standard deviations and confidence intervals for the density variable. It is clear that the rural and highly rural categories can be merged, as their claim frequencies are the same.

Table 3.7

Mean Five-Year Claim Frequency by Density

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean	
					Lower Bound	Upper Bound
Highly Rural	508	.24	.758	.034	.18	.31
Highly Urban	3,599	1.07	1.223	.020	1.03	1.11
Rural	1,565	.24	.707	.018	.21	.28
Urban	4,631	.84	1.171	.017	.81	.88
Total	10,303	.80	1.154	.011	.78	.82

Figure 3.4 shows that if claim frequency is treated as a numeric variable, and is used to group the

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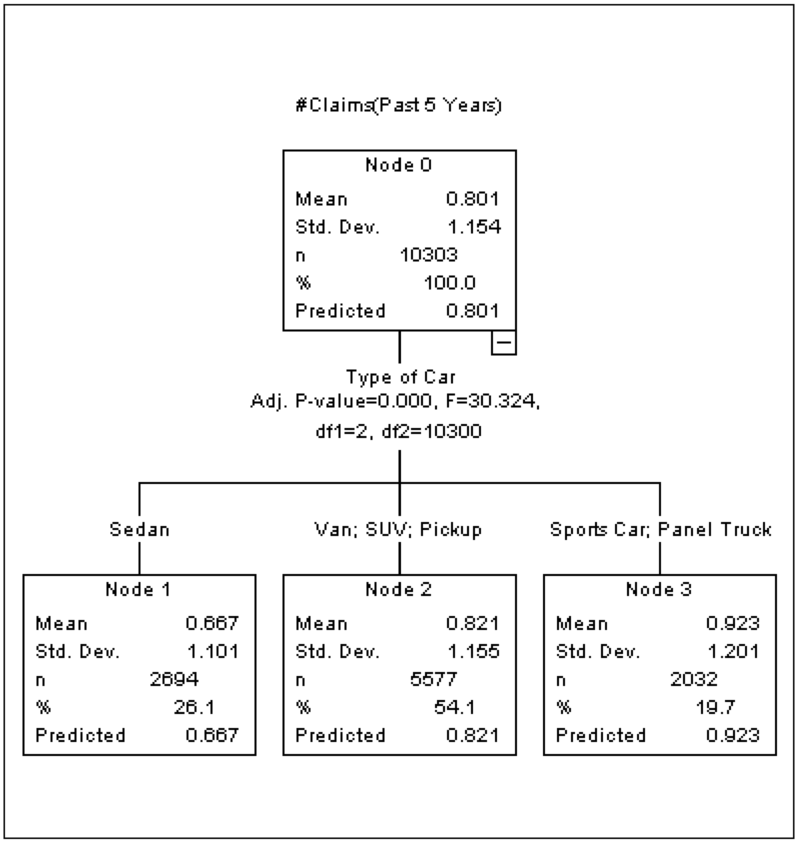
[9] T= $(\bar{x}_1 - \bar{x}_2)/s_{\bar{x}}$, $\bar{x}$ is mean of group and $s_{\bar{x}}$ is sd of difference between means

categories of the car-type variable, the same grouping is created as for the binary claim indicator variable in Figure 3.3, which used the chi-squared statistic to partition data.

Figure 3.4

CHAID Tree for Car Type with Numeric Dependent Variable



A new categorical variable can be created using the results of the CHAID analysis. The new car-type variable has three rather than six categories. Two predictive models were than fitted, using the new variables 1) a Poisson regression and 2) a ZIP regression. As a measure of goodness of fit, we use the Akaike Information Criterion (AIC) statistic. This statistic penalizes the log of the likelihood function when degrees of freedom, i.e., additional parameters, are incorporated into the model. Each variable in the model adds to its degrees of freedom. A model with a categorical variable having six levels adds five degrees of freedom[10] to the model, while a variable having three levels adds only two degrees of freedom. The formula for the AIC is:

[10] One degree of freedom for each binary dummy variable created which is *k*-1, where *k* is the number of categories

(3.2) $$AIC = 2 * df - 2 * \log \ likeihood$$

From Table 3.8, the AIC statistic indicated a better fit for both the Poisson regression and the ZIP regression when the car-type variable has been preprocessed to reduce the number of categories.

Table 3.8

Akaike Information Criterion, Car Type and Grouped Car Type

	Original Variables	Reduced Variables
Poisson Regression	12,066	12,026
ZIP	12,006	12,020

The CHAID procedure can also be used to preprocess numeric variables. The relationship between continuous independent variables and a dependent variable is frequently nonlinear. One way to model the nonlinearities is to bin the numeric variables. When a variable is binned, ranges of the variable are grouped together and treated as a level of a categorical variable. Thus, claimant ages can be binned into 0 – 10, 11 – 20, etc. Tree procedures such as CHAID can be used to optimally bin numeric variables (Refaat, 2007). To illustrate how this can be done, the CHAID procedure will be used to bin the motor vehicle record (i.e., the number of points on the policyholder's record) variable from the automobile data. Table 3.9 displays a frequency distribution for the motor vehicle record variable. It can be seen that the number of points ranges from 0 to 13.

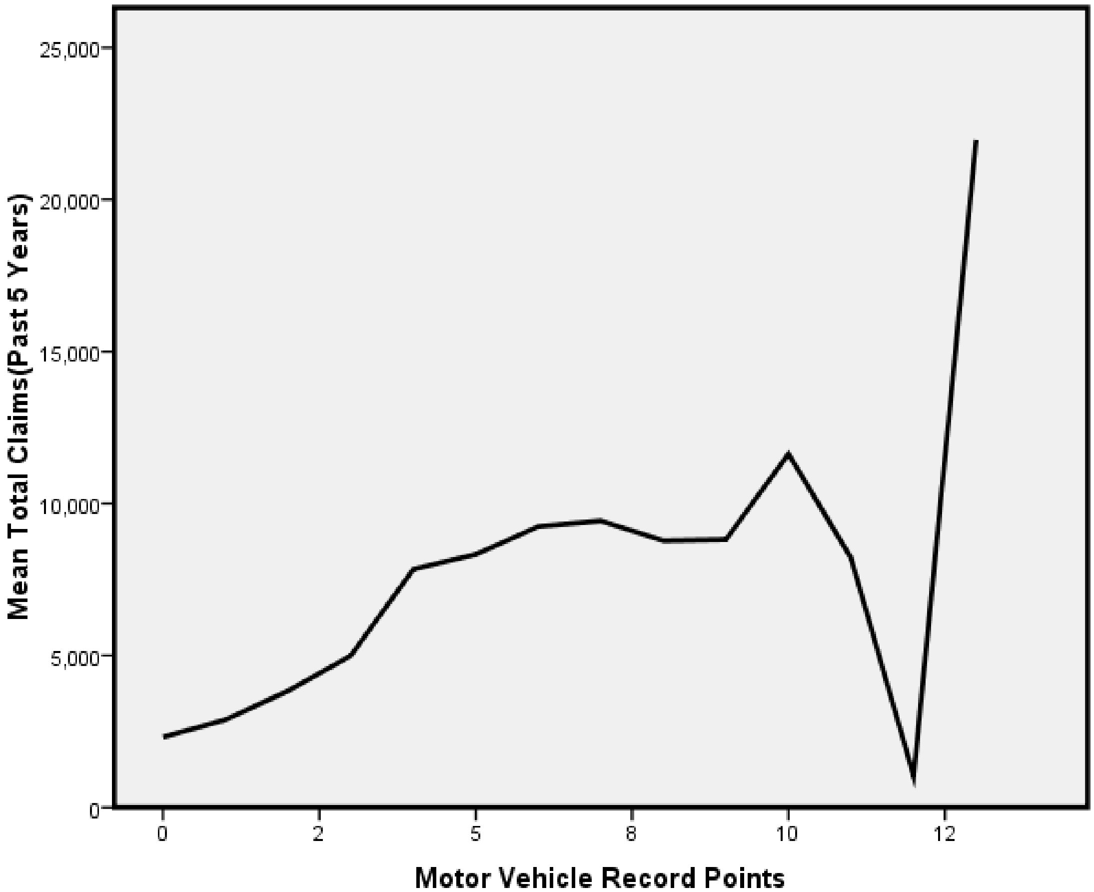
The distribution is a skewed distribution. That is, most of the values exceed the distribution's median. About 45% of policyholders have no points and 60% have one or fewer points. Figure 3.5, which displays the average frequency by motor vehicle record, indicates that the relationship between motor vehicle record and frequency is nonlinear. Claim frequency increases between zero and about five points and then (ignoring the inherent variability at high point values due to the sparseness of the data) appears to level off.

Table 3.9

Frequency Distribution for Motor Vehicle Record

Motor Vehicle Record Points			
Liscense Points	Frequency	Percent	Cumulative Percent
0	4,659	45.2	45.2
1	1,467	14.2	59.5
2	1,199	11.6	71.1
3	966	9.4	80.5
4	727	7.1	87.5
5	528	5.1	92.7
6	341	3.3	96
7	213	2.1	98
8	114	1.1	99.1
9	53	0.5	99.7
10	20	0.2	99.8
11	13	0.1	100
12	1	0	100
13	2	0	100
Total	10,303	100	

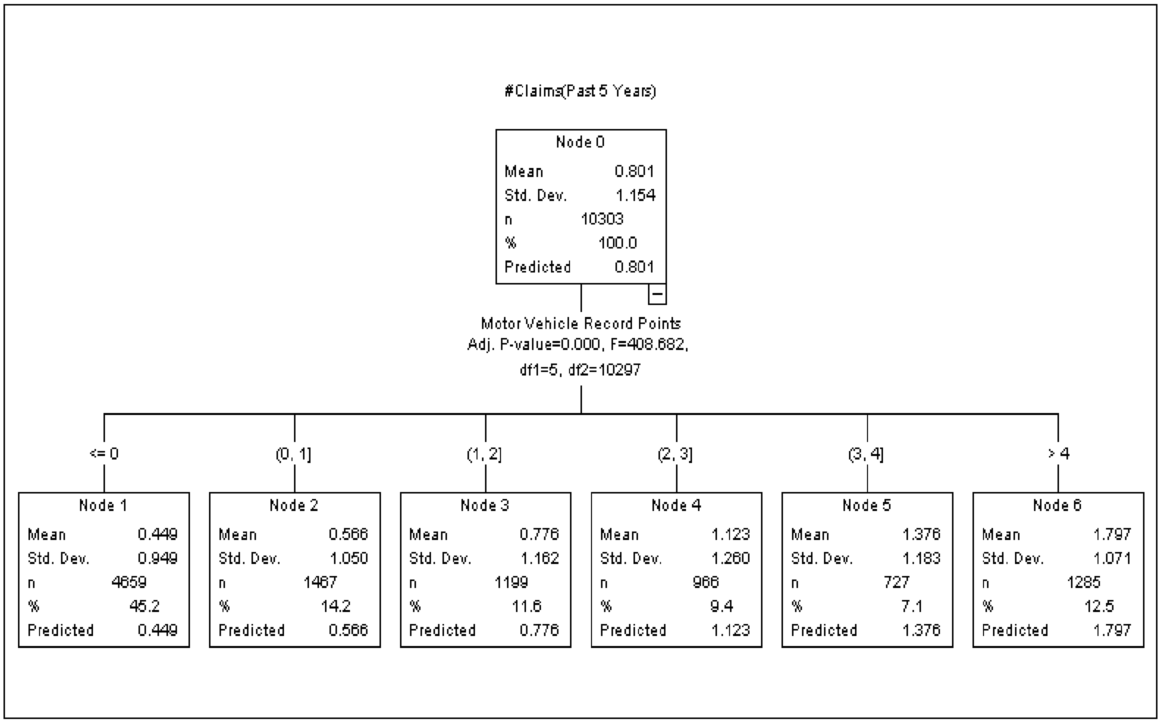
Figure 3.5
Average Claim Frequency by Motor Vehicle Record



How can the analyst best bin the motor vehicle record variable to approximate the relationship between motor vehicle points and claim frequency? One approach is to use the CHAID procedure to group together values of motor vehicle record with similar values for average claim frequency. Figure 3.6 displays the output of the CHAID procedure for motor vehicle record. Figure 3.6 indicates that each value from one through four is significantly different from other values and that it should stand alone as a bin. In predictive modeling, once the motor vehicle records have been binned, the new variable containing the binned categories can be used as a nominal variable in a regression. Alternatively, Figure 3.5 suggests that the relationship between motor vehicle record and claim frequency may be linear until about a value of 5 and then level off.

Figure 3.6

Tree Displaying Bin for Motor Vehicle Record



To test which treatment of the motor vehicle variable might work best, both a Poisson and ZIP regression were fit using the original variable, the variable capped at a value of 5 and the binned variable. For both the Poisson and the ZIP model, the binned variable performed better than the capped or original variable when AIC is used a goodness-of-fit measure. The lowest AIC was for the ZIP model with MVR binned.

Table 3.10

AIC for Original Variable, Capped Variable and Binned Variable

Treatment of Variable	Poisson	ZIP
MV Points	12,593	11,022
Capped MV Points	12,502	11,066
Binned MV Points	12,496	10,946

3.2 Results for Multi-Variable Model

To test the different methods a model that contained six variables (car use, gender, marital status, density, car type, and motor vehicle points) was fit. The number of categories for the density and car-type variables was reduced using CHAID. The motor vehicle record variable had two scenarios: MVR capped and MVR binned. The results for the Poisson regression and the ZIP regression are displayed in Table 3.11. Table 3.11 indicates that preprocessing improves the fit of the Poisson regression. The improvement was approximately the same whether motor vehicle record was capped or binned. On the other hand, the fit of the ZIP model declined when motor vehicle record was capped, but improved when it was binned. The AIC statistics in Table 3.11 also indicate that the ZIP model provides a significantly better fit than the Poisson model.

Table 3.11

AIC for Full Regression, Original Data and Preprocessed Data

Treatment of Variable	Poisson	ZIP
Original Variables	12,066	10,622
CHAID, MV Capped	12,009	10,676
CHAID, MV Binned	12,003	10,546

3.2.1 Out of Sample Goodness-of-Fit Measures

In predictive modeling, it is customary to test models on a sample of data that was set aside specifically for that purpose. The data used in this paper was split into two samples: a "training" sample used to fit the model's parameters and a "testing" sample used to test the fit of the model, once parameters have been estimated using the "training" sample.

In typical insurance databases, traditional measures of goodness of fit often perform poorly. For instance, the R^2 for the zip model applied to the test sample is 0.22, a number which, although low, is higher than what can be obtained in most data bases, likely because the frequencies in the data are based on five years of experience. In an automobile insurance data base where frequencies are based on annual experience, perhaps 90% of policyholders will not have experienced a claim, even though all policyholders have some positive expectation of a claim. Thus the actual value for most records will be "0" while the predicted value will be greater than "0" resulting in an R^2 statistic that tends to be low. To provide a useful test, comparisons must be based on aggregates of the data that are sufficiently large that the mean frequency in a group will be greater than zero. One way to aggregate the data is to create groups based on the value of the model's predicted value. The predicted value is sometimes referred to as the model's "score." All records can be sorted based on their score. The test data can then be grouped into quantiles based on the model score. For instance, the data can be split into ten groups based on the model score assigned to each record. For each decile, the actual frequency from the data can be computed. A graph comparing the actual to the predicted values within each decile can be created and used to visually evaluate the fit.

Figure 3.7

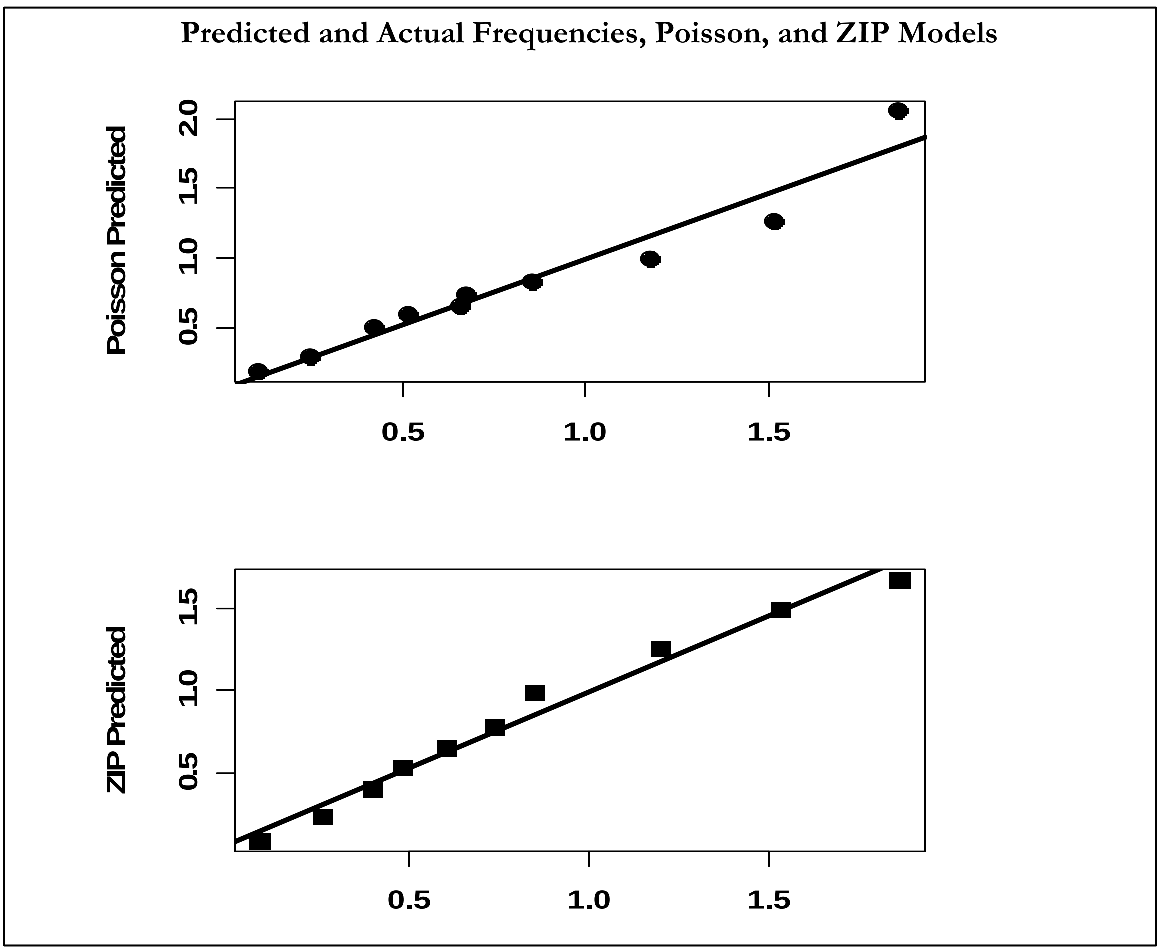


Figure 3.7 displays a comparison of actual and predicted frequencies for test data grouped by decile of the models score. A model with good predictive ability should be upward sloping; for each

increase in decile based on model score, the actual frequency should increase. Also, a high correlation between actual and predicted frequency indicates a good fit. In Figure 3.7 the best fitting line based on ordinary least squares regression is shown. A high correlation between actual and predicted values is indicated by a small scatter of points around the line. In table 3.12, the correlation coefficient of the six models on the test data is shown.

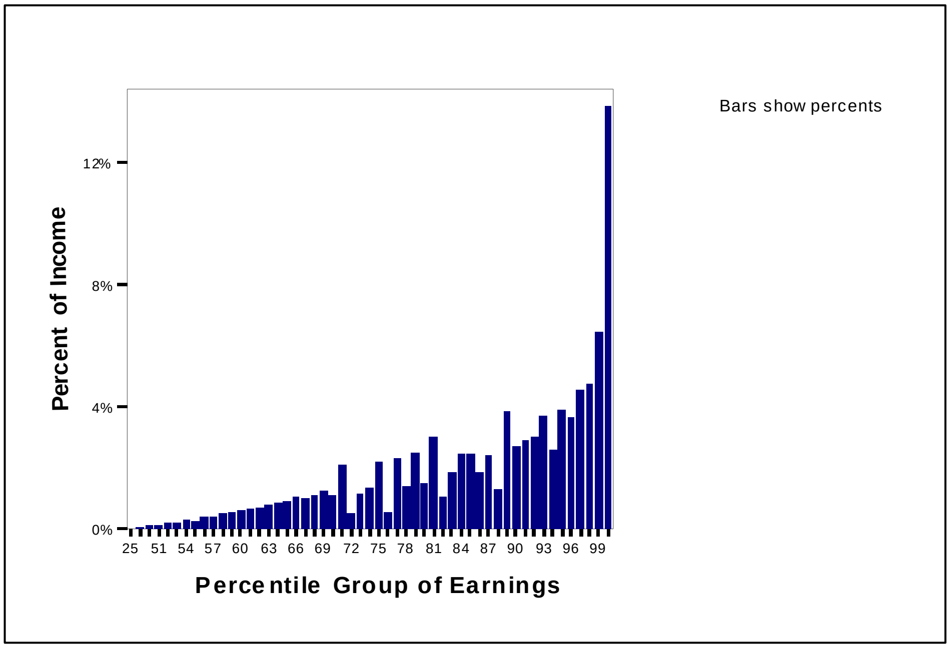
Table 3.12

Treatment of Variable	Poisson	ZIP
Original Variables	0.9720	0.9900
CHAID, MV Capped	0.9860	0.9900
CHAID, MV Binned	0.9810	0.9940

The correlations in Table 3.12 indicate that the ZIP models fit the out of sample data better than the Poisson models. It also indicates that preprocessing of variables with CHAID improves the fit of the Poisson regression models, but appears to have minimal effect on the ZIP models.

Meyers (2006) presented another curve that can be used to visualize the fit of models on out of sample data. The curve is based on the Lorenz curve. The Lorenz curve arose out of studies of income inequality by 19th and 20th century economists (Arnold, 1983). For example, Figure 3.8 displays a distribution of incomes from the 2000 census for the state of Pennsylvania. From this graph, it can be seen that earners in the highest percentiles earned a disproportionate share of incomes. The top 1% of individuals earned 13% of the state of Pennsylvania's income.

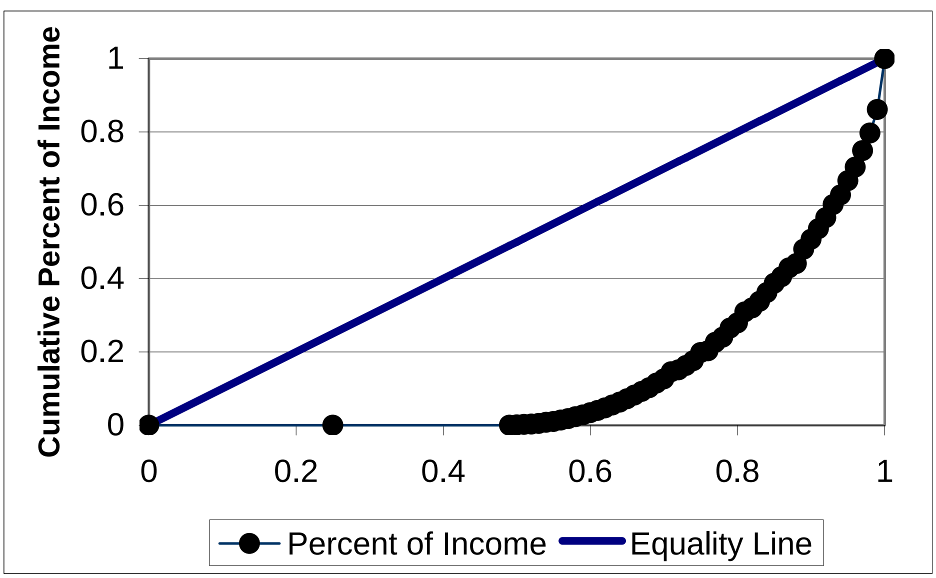
Figure 3.8: Income Distribution from 2000 Census



By cumulating the data from Figure 3.8, i.e., computing the cumulative percent of all income earned by a given percent of the population, a Lorenz curve can be created. This is shown in Figure 3.9.

Figure 3.9

Lorenz Curve for Income from 2000 Census



If income distribution were perfectly equal, incomes would be distributed according to the diagonal line. The area between this line and the curve for the income distribution is a measure of income known as the Gini Index. The greater the income inequality, the larger the Gini Index should be. A simple formula for this area based on the trapezoidal rule for numerical integration (Press et al., 1989) is:

$$A = \left(\frac{1}{2} F_1 + F_{2\ldots} + F_{n-1} + \frac{1}{2} F_n \right) * \Delta x, \tag{3.5}$$

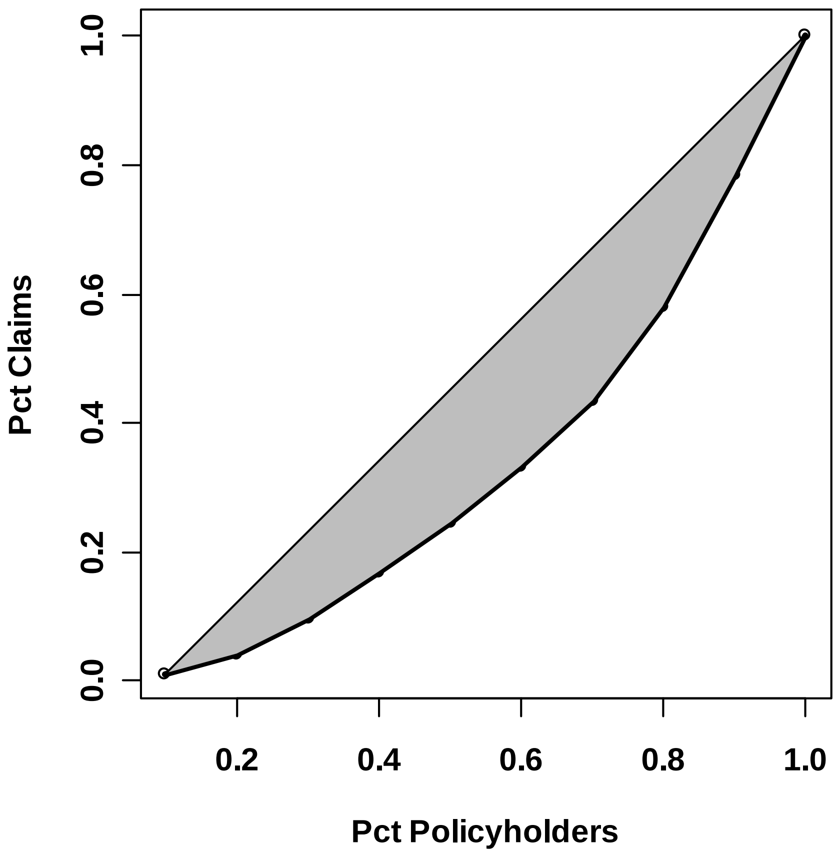
F_i is ith cumulative income

The statistic in (3.5) is also known as the Gini Index. It was introduced by Meyers (2006) as a general procedure for assessing the fit of models. A Lorenz curve can also be constructed from predictive models and the insurance data they are applied to. Figure 3.9 displays an approximation to the Lorenz curve based on the Poisson model. This approximation is based on only 10 groups or deciles of the data,[11] although often more intervals are used.

[11] The test was limited to deciles, as a model with categorical predictors that have only a few levels may have a limited number of possible values.

Figure 3.9

Lorenz Curve for Poisson Predictive Model



The data was also used to compute an approximation to the Gini Index. Table 3.13 presents the approximation Gini Index for the six models.

Table 3.13
Gini Index for Models

Treatment of Variable	Poisson	ZIP
Original Variables	0.1770	0.1830
CHAID, MV Capped	0.1780	0.1800
CHAID, MV Binned	0.1760	0.1800

The out of sample tests in Table 3.13 indicate that the ZIP model fits better than the Poisson. For the Poisson model, it also indicates that preprocessing the variables using CHAID to construct a capped MVR variable improves the fit of the Poisson model but not of the ZIP model.

4. CONCLUSIONS

In this paper, alternatives to the Poisson and negative binomial distributions for count regressions were presented. One alternative makes use of mixed zero-adjusted (zero-inflated and hurdle) distributions. These are mixture models composed of two distinct probability distributions, thus the resulting distribution is not a member of the exponential family of distributions. The alternative provided a significantly better fit to a database of automobile insurance claims than did the Poisson and negative binomial models. Moreover, many other authors (Yip and Yau, 2005; Heilbron, 1994) use zero-inflated and hurdle models to better approximate data than simple Poisson and negative binomial models. In our day-to-day property/casualty insurance modeling, we have found that zero-inflated and hurdle models frequently fit the data better than Poisson and negative binomial models. We have found this to be the case across a number of different lines, including homeowners, personal automobile, and workers compensation. The phenomenon of excess zeros is also commonly encountered in non-insurance applications such as quality control (Lambert, 1992) and biostatistics (Ridout et al., 1998). We tested a small selection of non-insurance databases and zero-adjusted distributions provided a better fit to some of the data. Thus it seems appropriate to test for excess zeros using a test such as Van den Broek's score test. See Appendix C for more information on this test. If excess zeros are indicated, either a zero-inflated or hurdle model is likely to provide a better model than a classical Poisson or negative binomial regression. The testing displayed wide variation between the goodness of fit of the different distributions assessed, suggesting that it is prudent to test several alternative distributions before fitting a model.

A limitation of many GLMs that incorporate categorical variables is over-parameterization. This occurs when more categories are included than are needed. When categories that are statistically equivalent are combined, the over-parameterization is eliminated. In this paper a relative quick and efficient procedure for reducing the cardinality of nominal variables was presented. The procedure in this paper used the CHAID decision tree procedure to statistically determine the appropriate way to combine categories. This paper provided an example where application of the CHAID procedure to group categories of categorical variables improved the fit of the model. Typical predictive modeling applications databases contain a number of categorical variables with many levels. For instance, there may be 100 or more different types of vehicles in a vehicle-type variable, and many are sparsely populated. Since the categorical variables in our data had only a relatively small number of categories, the benefit of preprocessing categorical data was illustrated but could not be fully exploited.

Another limitation of GLMs with numeric predictor variables is that the relationship between the predictor and dependent variable may be nonlinear. The CHAID technique can be used to preprocess numeric variables to approximate the nonlinear relationship.

Supplementary Material

Excel spreadsheets, R and SAS Code will be available on the CAS Web Site.

Appendix A
The Poisson Distribution and the Use of Nonlinear Mixed Modeling Procedures to Fit Parameters

We begin this illustration by using SAS Proc NLMIXED to estimate the parameters of the Poisson. We will derive the log-likelihood of the Poisson from its PDF, to illustrate its use in Proc NLMIXED.

The Poisson PDF is

$$(Y = y \mid \mu) = \frac{e^{-\lambda}\lambda^{y}}{y!} \text{ where } y = 0, 1, 2, 3, \ldots$$

with:

$$mean = E(Y \mid X = x_i) = \lambda$$

$$\text{var} = \sigma^2 = Var(Y \mid X = x) = \lambda$$

It is clear from the above formulas that the mean of the Poisson equals its parameter, lambda.

Differentiating the PDF with respect to our mean parameter, lambda; the log-likelihood:

$$\text{ll=-}\lambda\text{+y*log(}\lambda\text{)-lgamma(y+1)} \, .$$

Below we illustrate the use of this function in a SAS procedure that is used to estimate the parameter of the Poisson. We also illustrate how to directly fit the Poisson, without specifying a likelihood function. Proc NLMIXED is designed to estimate the parameters of nonlinear mixed models. A mixed model arises when some of the independent variables in a model are themselves random realizations from a distribution rather than fixed quantities (see Venables and Ripley, 2002; Faraway, 2006). A discussion of mixed models is beyond the scope of this paper; however, knowledge of how to specify random effects is unnecessary when using Proc NLMIXED to fit common probability distributions such as the Poisson. Below is the SAS code used for the fit:

```
proc sql;
      select max(clm_freq) into :y_max from claims2;
quit;
```

```
%put *** y_max=&y_max.;

%macro estimate;
      %do i = 0 %to &y_max.;
            estimate "p(&i.)" pdf('poisson',&i., lambda);
      %end;
%mend;
proc nlmixed data=claims2;
      parms eta=-0.2;
      lambda = exp(eta);
      y = clm_freq;
      model y ~ poisson(lambda);
/* or /
      loglike = -lambda + y*log(lambda) - lgamma(y + 1);
      model y ~ general(loglike);
/* or /
      pdf = (exp(-lambda)*lambda**y)/fact(y);
      loglike=log(pdf);
      model y ~ general(loglike);
      estimate 'lambda' lambda;
      %estimate;
      predict lambda out=predpoi(keep=clm_freq pred);
      title 'Poisson model via Proc NLMIXED';
run;
title;
```

Poisson model via Proc NLMIXED

The NLMIXED Procedure

Specifications	
Data Set	WORK.CLAIMS2
Dependent Variable	y
Distribution for Dependent Variable	Poisson
Optimization Technique	Dual Quasi-Newton
Integration Method	None

Dimensions	
Observations Used	2812
Observations Not Used	0
Total Observations	2812
Parameters	1

Parameters	
eta	*NegLogLike*
-0.2	3782.77991

Iteration History					
Iter	*Calls*	*NegLogLike*	*Diff*	*MaxGrad*	*Slope*
1	4	3782.75728	0.022624	1.211746	-51.3544
2	5	3782.75696	0.00032	0.002708	-0.00064
3	6	3782.75696	1.728E-9	7.161E-7	-3.2E-9

NOTE: GCONV convergence criterion satisfied.

Fit Statistics	
-2 Log Likelihood	7565.5
AIC (smaller is better)	7567.5
AICC (smaller is better)	7567.5
BIC (smaller is better)	7573.5

Parameter Estimates									
Parameter	*Estimate*	*Standard Error*	*DF*	*t Value*	*Pr > |t|*	*Alpha*	*Lower*	*Upper*	*Gradient*
eta	-0.2045	0.02089	2812	-9.79	<.0001	0.05	-0.2454	-0.1635	7.161E-7

Additional Estimates								
Label	*Estimate*	*Standard Error*	*DF*	*t Value*	*Pr > |t|*	*Alpha*	*Lower*	*Upper*
lambda	0.8151	0.01703	2812	47.87	<.0001	0.05	0.7817	0.8485

SAS Proc NLMIXED also makes it very simple to fit a negative binomial distribution to the sample data. Again, there several ways to specify the distribution. The ability to code with programming statements within Proc NLMIXED is very flexible. One can use the internal specification for negative binomial, specify the negative binomial PDF, take the log and use the model general option, or directly specify the log-likelihood one wishes to solve for it directly. Beside the model fit, we can also ask for some additional statistics such as contrast testing for whether our Negbin dispersion is significantly different from zero (Poisson), as well as the estimated variance and predicted probabilities for each count.

```
%macro estimate;
      %do i = 0 %to &y_max.;
      estimate "p(&i.)"  (gamma(&i. + k)/(gamma(&i. + 1)*gamma(k)))*
                         (((1/k)*mu)**&i.)/(1 + (1/k)*mu)**(&i. + (k));
      %end;
%mend;

proc nlmixed data=claims2;
      parms  b_0=-.2 k=1.4;
      eta = b_0;
      mu = exp(eta);
      y = clm_freq;
/* specify the full log-likelihood */
/*    loglike = (lgamma(y + (1/k)) - lgamma(y + 1) - lgamma(1/k) + */
/*                        y*log(k*mu) - (y + (1/k))*log(1 + k*mu)); */
/*    model y ~ general(loglike);*/
/* or, use the internal negbin(n,p) representation */
      p = exp(-eta)/(1 + exp(-eta));
      model y ~ negbin(1/k,p);
      predict mu out=out2(keep=clm_freq pred);
      contrast 'k = 0' k - 0;
      estimate 'exp(b_0)' exp(b_0);
      estimate 'mean' mu;
      estimate 'k' k;
      estimate 'variance' mu + k*mu**2;
      %estimate;
      title 'Negative Binomial model via Proc NLMIXED';
run;
```

Specifications	
Data Set	WORK.CLAIMS2
Dependent Variable	y
Distribution for Dependent Variable	Poisson
Optimization Technique	Dual Quasi-Newton
Integration Method	None

Dimensions	
Observations Used	2812
Observations Not Used	0
Total Observations	2812
Parameters	1

Parameters	
eta	*NegLogLike*
-0.2	3782.77991

Iteration History					
Iter	*Calls*	*NegLogLike*	*Diff*	*MaxGrad*	*Slope*
1	4	3782.75728	0.022624	1.211746	-51.3544
2	5	3782.75696	0.00032	0.002708	-0.00064
3	6	3782.75696	1.728E-9	7.161E-7	-3.2E-9

NOTE: GCONV convergence criterion satisfied.

Fit Statistics	
-2 Log Likelihood	7565.5
AIC (smaller is better)	7567.5
AICC (smaller is better)	7567.5
BIC (smaller is better)	7573.5

Parameter Estimates									
Parameter	*Estimate*	*Standard Error*	*DF*	*t Value*	*Pr > |t|*	*Alpha*	*Lower*	*Upper*	*Gradient*
eta	-0.2045	0.02089	2812	-9.79	<.0001	0.05	-0.2454	-0.1635	7.161E-7

Additional Estimates								
Label	*Estimate*	*Standard Error*	*DF*	*t Value*	*Pr > |t|*	*Alpha*	*Lower*	*Upper*
lambda	0.8151	0.01703	2812	47.87	<.0001	0.05	0.7817	0.8485

Negative Binomial model via Proc NLMIXED

Specifications	
Data Set	WORK.CLAIMS2
Dependent Variable	y
Distribution for Dependent Variable	Negative Binomial
Optimization Technique	Dual Quasi-Newton
Integration Method	None

Dimensions	
Observations Used	2812
Observations Not Used	0
Total Observations	2812
Parameters	2

Parameters		
b_0	*k*	*NegLogLike*
-0.2	1.4	3560.13868

Iteration History					
Iter	*Calls*	*NegLogLike*	*Diff*	*MaxGrad*	*Slope*
1	3	3502.65393	57.48475	37.59117	-2543.71
2	4	3501.12368	1.53025	11.94101	-2.31042
3	5	3500.97924	0.144435	1.374487	-0.31788
4	6	3500.97779	0.001458	0.050886	-0.003
5	7	3500.97778	1.233E-6	0.00302	-2.59E-6

NOTE: GCONV convergence criterion satisfied.

Fit Statistics	
-2 Log Likelihood	7002.0
AIC (smaller is better)	7006.0
AICC (smaller is better)	7006.0
BIC (smaller is better)	7017.8

Parameter Estimates

Parameter	*Estimate*	*Standard Error*	*DF*	*t Value*	*Pr > |t|*	*Alpha*	*Lower*	*Upper*	*Gradient*
b_0	0.1388	0.07768	2812	1.79	0.0741	0.05	-0.01354	0.2911	0.002728
k	1.4095	0.1006	2812	14.01	<.0001	0.05	1.2122	1.6068	-0.00302

Contrasts

Label	*Num DF*	*Den DF*	*F Value*	*Pr > F*
k = 0	1	2812	196.23	<.0001
k = 1	1	2812	16.56	<.0001

Additional Estimates

Label	*Estimate*	*Standard Error*	*DF*	*t Value*	*Pr > |t|*	*Alpha*	*Lower*	*Upper*
exp(b_0)	1.1489	0.08924	2812	12.87	<.0001	0.05	0.9739	1.3238
mean	1.1489	0.08924	2812	12.87	<.0001	0.05	0.9739	1.3238
k	1.4095	0.1006	2812	14.01	<.0001	0.05	1.2122	1.6068
variance	3.0092	0.5030	2812	5.98	<.0001	0.05	2.0229	3.9956

Appendix B
Count Distribution Parameter Estimation Using SAS Proc GENMOD

Below a Poisson distribution is fit to the data with Proc GENMOD, SAS Generalized Linear Model procedure. The estimate statement applies the inverse of the log link, eponentiating the intercept displaying the estimated mean as 0.82 along with a 95% confidence interval of (0.7824,0.8491). Note that the log likelihood reported in Proc GENMOD is not directly comparable to those reported in Proc NLMIXED or some other software as Proc GENMOD drops the *y*-factorial component of the likelihood as this does not contribute to estimating the mean parameter and can cause numeric instabilities with high counts.

```
proc genmod data=claims2;
      model clm_freq = / link=log dist=Poisson noscale;
      estimate 'mean' intercept 1 / exp;
      title 'Poisson Distribution';
run;
```

SAS Proc GENMOD also makes it very simple to fit a negative binomial distribution to our sample data. Here we simply change the dist= option to our model statement.

```
proc genmod data=claims2;
      model clm_freq = / link=log dist=NegBin;
      estimate 'mean' intercept 1 / exp;
      title 'NegBin model';
run;
```

NegBin Model

The GENMOD Procedure

Model Information		
Data Set	WORK.CLAIMS2	
Distribution	Negative Binomial	
Link Function	Log	
Dependent Variable	CLM_FREQ	#Claims(Past 5 Years)

Number of Observations Read	2812
Number of Observations Used	2812

Parameter Information	
Parameter	*Effect*
Prm1	Intercept

Criteria For Assessing Goodness Of Fit			
Criterion	*DF*	*Value*	*Value/DF*
Deviance	2811	2549.6471	0.9070
Scaled Deviance	2811	2549.6471	0.9070
Pearson Chi-Square	2811	2190.0408	0.7791
Scaled Pearson X2	2811	2190.0408	0.7791
Log Likelihood		-2478.8688	

Analysis Of Parameter Estimates							
Parameter	*DF*	*Estimate*	*Standard Error*	*Wald 95% Confidence Limits*		*Chi-Square*	*Pr > ChiSq*
Intercept	1	-0.2045	0.0306	-0.2645	-0.1445	44.59	<.0001
Dispersion	1	1.4095	0.1006	1.2123	1.6067		

Note: The negative binomial dispersion parameter was estimated by maximum likelihood.

Contrast Estimate Results							
Label	*Estimate*	*Standard Error*	*Alpha*	*Confidence Limits*		*Chi-Square*	*Pr > ChiSq*
mean	-0.2045	0.0306	0.05	-0.2645	-0.1445	44.59	<.0001
Exp(mean)	0.8151	0.0250	0.05	0.7676	0.8655		

Appendix C

SAS Code for SCORE Test

```
/***************************************************************/
/* Van den Broek (1995) score test                            */
/* Van den Broek, Jan,
 A score test for zero inflation in a Poisson distribution,
 Biometrics, 1995, v51, n2, p738-743                          */
/***************************************************************/
proc sql;
      select sum(((clm_freq=0) - exp(-pred))/exp(-pred))**2 as num,
                  sum((1 - exp(-pred))/exp(-pred)) -
count(clm_freq)*mean(clm_freq) as denom,
                        count(clm_freq) as n, mean(clm_freq) as ybar,
```

```
            (sum(((clm_freq=0) - exp(-pred))/exp(-pred))**2) /
                  (sum((1 - exp(-pred))/exp(-pred)) -
count(clm_freq)*mean(clm_freq)) as score,
                        1 - probchi(calculated score, 1) as p format 8.6
                  into :num, :denom, :n, :ybar, :score, :p
                        from out2;
quit;
%put ********************************************************************;
%put *** Van den Broek - Score statistic for extra zeros;
%put *** num=&num, denom=&denom., n=&n., ybar=&ybar., score=&score., p=&p.;
%put ********************************************************************;
```

Van den Brock – Score Statistic for extra zeros

numerator	denom	n	ybar	score	p
1,086,713.00	1,249.30	2812	0.8151	869.9	0.000001

Appendix D

SAS Code for Zero-Inflated Models

This appendix shows how to fit a zero-inflated distribution in SAS Proc NLMIXED. (Also, an experimental procedure under the SAS/ETS product, Proc COUNTREG directly fits ZIP and ZINB models. See http://support.sas.com/kb/26/addl/fusion26161_3_countreg.pdf. Here, we add some options. The parameters statement allows us to specify starting values for our parameters to be estimated. The bounds statement allows us to constrain our zero-inflation factor to the logical range. Again, we utilized the flexibility of programming and the estimate statements to calculate several useful estimates such as the expected number of conditional ZIP mean and variance.

```
%macro estimate;
      %do i = 0 %to &y_max.;
       %if &i.=0 %then %do;
      estimate "p(&i.)" p_0 + (1 - p_0)*pdf('poisson',&i., lambda);
      %end;
            %else %do;
            estimate "p(&i.)" (1 - p_0)*pdf('poisson',&i., lambda);
      %end;
      %end;
%mend;
proc nlmixed data=claims3;
  parameters  p_0=0.57 bll_0=0.5;
  bounds 0<p_0<1;
  eta = bll_0;
  lambda = exp(eta);
  y = clm_freq;
  if y=0 then loglike = log(p_0 + (1 - p_0)*exp(-lambda));
     else loglike = log(1 - p_0) + y*log(lambda) - lambda - lgamma(y + 1);
  model y ~ general(loglike);
  contrast 'p_0 - 0' p_0 - 0;
  estimate "p_0" p_0;
  estimate "Expected zeros=exp(-lambda)" exp(-lambda);
  estimate 'Conditional Poisson Mean (lambda)' lambda;
  estimate 'ZIP Mean (1-p_0)*lambda' (1 - p_0)*lambda;
  estimate 'ZIP Var(1-p_0)*lambda*(1+lambda+(1-p_0)*lambda)'
                     (1 - p_0)*lambda*(1 + lambda + (1 - p_0)*lambda);
/*  estimate "Proportion of 'extra' zeros (theta)" theta; */
  estimate 'theta=p_0/(1-p_0)' p_0/(1 - p_0);
  %estimate;
  predict p_0 out=pred_zi(keep=pred);
  predict lambda out=pred(keep=clm_freq pred);
  predict (1 - p_0)*lambda out=out2(keep=clm_freq pred);

  title 'Zero-Inflated Poisson (ZIP) distribution';
run;
title;
```

Zero-Inflated Poisson (ZIP) Distribution

Specifications	
Data Set	WORK.CLAIMS3
Dependent Variable	y
Distribution for Dependent Variable	General
Optimization Technique	Dual Quasi-Newton
Integration Method	None

Dimensions	
Observations Used	2812
Observations Not Used	0
Total Observations	2812
Parameters	2

Parameters		
p_0	*bll_0*	*NegLogLike*
0.57	0.5	3360.90317

Fit Statistics	
-2 Log Likelihood	6695.2
AIC (smaller is better)	6699.2
AICC (smaller is better)	6699.2
BIC (smaller is better)	6711.1

Parameter Estimates

Parameter	*Estimate*	*Standard Error*	*DF*	*t Value*	*Pr > |t|*	*Alpha*	*Lower*	*Upper*	*Gradient*
p_0	0.5177	0.01231	2812	42.04	<.0001	0.05	0.4935	0.5418	4.247E-7
bll_0	0.5247	0.02658	2812	19.74	<.0001	0.05	0.4726	0.5768	6.041E-6

Contrasts				
Label	Num DF	Den DF	F Value	Pr > F
p_0 - 0	1	2812	1767.27	<.0001

Additional Estimates								
Label	**Estimate**	**Standard Error**	**DF**	**t Value**	**Pr > |t|**	**Alpha**	**Lower**	**Upper**
p_0	0.5177	0.01231	2812	42.04	<.0001	0.05	0.4935	0.5418
Expected zeros=exp(-lambda)	0.1845	0.008289	2812	22.26	<.0001	0.05	0.1683	0.2008
Conditional Poisson Mean (lambda)	1.6899	0.04492	2812	37.62	<.0001	0.05	1.6018	1.7780
ZIP Mean (1-p_0)*lambda	0.8151	0.02331	2812	34.96	<.0001	0.05	0.7694	0.8608
ZIP Var(1p_0)*lambda*(1+lambda+ (1-p_0)*lambda)	2.8568	0.1254	2812	22.79	<.0001	0.05	2.6110	3.1026
theta=p_0/(1-p_0)	1.0733	0.05293	2812	20.28	<.0001	0.05	0.9695	1.1771

Given the additional flexibility introduced with the zero-inflation parameter, The zero-inflated negative binomial (ZINB) distribution fit estimates a very small dispersion parameter, *k*.

```
proc nlmixed data=claims3;
      parms bp_0=.07 bll_0=0.52 k=0.000033;
      bounds k>0;
      eta_zip = bp_0;
      p0_zip = exp(eta_zip)/(1 + exp(eta_zip));
      eta_nb = bll_0;
      mean   = exp(eta_nb);
      y = clm_freq;
      p0 = p0_zip + (1 - p0_zip)*exp(-(y + (1/k))*log(1 + k*mean));
      p_else = (1 - p0_zip)*exp(lgamma(y + (1/k)) - lgamma(y + 1) –
            lgamma(1/k) + y*log(k*mean) - (y + (1/k))*log(1 + k*mean));
      if y=0 then loglike = log(p0);
          else    loglike = log(p_else);
    model y ~ general(loglike);
    estimate "Estimated proportion of 'extra' zeros (theta)" p0_zip;
    estimate 'Estimated Conditional Poisson Mean (Lambda)' mean;
    estimate 'Estimated Unconditional ZIP Mean' (1-p0_zip)*mean;
    estimate 'Estimated Unconditional ZIP Variance'
                                 (1-p0_zip)*mean*(1+p0_zip*mean);
    predict mean out = mean_hat;
    title 'Zero-inflated Negative Binomial ZINB Distribution';
run;
```

Zero-Inflated Negative Binomial ZINB Distribution

Specifications	
Data Set	WORK.CLAIMS3
Dependent Variable	y
Distribution for Dependent Variable	General
Optimization Technique	Dual Quasi-Newton
Integration Method	None

Dimensions	
Observations Used	2812
Observations Not Used	0
Total Observations	2812
Parameters	3

Parameters			
bp_0	*bll_0*	*k*	*NegLogLike*
0.07	0.52	0.000033	3347.62028

Fit Statistics	
-2 Log Likelihood	6695.2
AIC (smaller is better)	6701.2
AICC (smaller is better)	6701.2
BIC (smaller is better)	6719.0

Parameter Estimates

Parameter	*Estimate*	*Standard Error*	*DF*	*t Value*	*Pr > |t|*	*Alpha*	*Lower*	*Upper*	*Gradient*
bp_0	0.07089	0.04949	2812	1.43	0.1521	0.05	-0.02614	0.1679	0.083701
bll_0	0.5246	0.02698	2812	19.44	<.0001	0.05	0.4717	0.5775	-0.11562
k	1.187E-6	0.001040	2812	0.00	0.9991	0.05	-0.00204	0.002041	229.649

Additional Estimates								
Label	*Estimate*	*Standard Error*	*DF*	*t Value*	*Pr > |t|*	*Alpha*	*Lower*	*Upper*
Estimated proportion of 'extra' zeros (theta)	0.5177	0.01236	2812	41.90	<.0001	0.05	0.4935	0.5419
Estimated Conditional ZINB Mean (Lambda)	1.6898	0.04559	2812	37.06	<.0001	0.05	1.6004	1.7793
Estimated Unconditional ZINB Mean	0.8150	0.02340	2812	34.83	<.0001	0.05	0.7691	0.8609
Estimated Unconditional ZINB Variance	1.5280	0.05533	2812	27.61	<.0001	0.05	1.4195	1.6365

Appendix D

SAS Code for Hurdle Model

```
/*****************************************/
/* fit Hurdle - Binomial for 0/1        */
/*****************************************/
data claims4;
      set claims2;
      clm=(clm_freq>0);
run;
proc nlmixed data=claims4;
      parms b_o=-0.52;
      y = clm;
      eta = b_o;
      p1 = exp(eta)/(1 + exp(eta));
      model y ~ binary(p1);
      estimate 'phi' 1-1/(1 + exp(-b_o));
run;
/*****************************************/
/* fit Hurdle - Truncated Poisson       */
/*****************************************/
proc nlmixed data=claims3(where=(clm_freq>0));
   *   parms b_0=0.52;
      eta_lam = b_0;
    lambda = exp(eta_lam);
      y = clm_freq;
      prob = ((exp(-lambda)*(lambda**y))/fact(y))/(1 - exp(-lambda));
      loglike = log(prob);
      model y ~ general(loglike);
      estimate 'lambda' lambda;
      estimate 'conditional mean' lambda/(1 - exp(-lambda));
      estimate 'conditional var' (lambda/(1 - exp(-lambda)))*
                         (1 - (lambda*exp(-lambda))/(1 - exp(-lambda)));
      predict (lambda/(1 - exp(-lambda))) out=tpois_pred;
      title 'Count model for non-Zero Outcomes (Poisson)';
run;
title;
```

Appendix E

Code for Fitting Models

Poisson, negative binomial, ZIP, ZINB, and hurdle GLM regression models are fit with SAS. Below a Poisson regression model is fit to the data with Proc GENMOD. Main effects regressors are added to same setup as above: car use, marital status, area, and sex.

```
/*******************************************************/
/* fit Poisson regression model (including covariates) */
/*******************************************************/
proc genmod data=claims2;
      class car_use mstatus area sex;
      model clm_freq = car_use mstatus area lincome sex
                                  / link=log dist=Poisson;
run;
```

The GENMOD Procedure

Model Information		
Data Set	WORK.CLAIMS2	
Distribution	Poisson	
Link Function	Log	
Dependent Variable	CLM_FREQ	#Claims(Past 5 Years)

Number of Observations Read	2812
Number of Observations Used	2812

Criteria For Assessing Goodness Of Fit			
Criterion	*DF*	*Value*	*Value/DF*
Deviance	2811	4736.2388	1.6849
Scaled Deviance	2811	4736.2388	1.6849
Pearson Chi-Square	2811	4706.1012	1.6742
Scaled Pearson X2	2811	4706.1012	1.6742
Log Likelihood		-2760.6479	

Analysis Of Parameter Estimates							
Parameter	*DF*	*Estimate*	*Standard Error*	*Wald 95% Confidence Limits*		*Chi-Square*	*Pr > ChiSq*
Intercept	1	-0.2045	0.0209	-0.2454	-0.1635	95.82	<.0001
Scale	0	1.0000	0.0000	1.0000	1.0000		

Note: The scale parameter was held fixed.

Model Information		
Data Set	WORK.CLAIMS2	
Distribution	Poisson	
Link Function	Log	
Dependent Variable	CLM_FREQ	#Claims(Past 5 Years)

Number of Observations Read	2812
Number of Observations Used	2812

Class Level Information		
Class	*Levels*	*Values*
CAR_USE	2	Commercial Private
mstatus	2	1. Yes 2. No
area	2	1. Highly Urban/ urban area 2. Highly Rural/ rural area
sex	2	1. M 2. F

Criteria For Assessing Goodness Of Fit			
Criterion	*DF*	*Value*	*Value/DF*
Deviance	2806	4312.6656	1.5369
Scaled Deviance	2806	4312.6656	1.5369
Pearson Chi-Square	2806	4636.5810	1.6524
Scaled Pearson X2	2806	4636.5810	1.6524
Log Likelihood		-2548.8614	

Analysis Of Parameter Estimates								
Parameter		DF	Estimate	Standard Error	Wald 95% Confidence Limits		Chi-Square	Pr > ChiSq
Intercept		1	-1.1801	0.1013	-1.3787	-0.9815	135.66	<.0001
CAR_USE	Commercial	1	0.2831	0.0449	0.1951	0.3711	39.73	<.0001
CAR_USE	Private	0	0.0000	0.0000	0.0000	0.0000	.	.
mstatus	1. Yes	1	-0.0961	0.0425	-0.1794	-0.0127	5.10	0.0239
mstatus	2. No	0	0.0000	0.0000	0.0000	0.0000	.	.
area	1. Highly Urban/ urban area	1	1.3631	0.0835	1.1994	1.5268	266.28	<.0001
area	2. Highly Rural/ rural area	0	0.0000	0.0000	0.0000	0.0000	.	.
lincome		1	-0.0206	0.0061	-0.0327	-0.0086	11.32	0.0008
sex	1. M	1	-0.1206	0.0441	-0.2070	-0.0343	7.49	0.0062
sex	2. F	0	0.0000	0.0000	0.0000	0.0000	.	.
Scale		0	1.0000	0.0000	1.0000	1.0000		

Note: The scale parameter was held fixed.

It is also fairly simple to add regressors to the linear predictor, eta, in Proc NLMIXED. One small complication is that Proc NLMIXED does not offer a class statement, therefore one has to either create desired indicator or dummy variables ahead of time, or as in the example below, use programming statements to create them "on-the-fly," The phrase inside each set of parentheses resolves to either true or false, zero, or one.

```
data claims3;
      set claims2;
      car_usen=0; if car_use='Commercial' then car_usen=1;
      mstatusn=0; if mstatus='1. Yes' then mstatusn=1;
      arean=0;    if area ='1. Highly Urban/ urban area' then arean=1;
      sexn=0;     if sex='1. M' then sexn=1;
run;

proc nlmixed data=claims3;
      eta = b_0 + b_car_use*car_usen + b_mstatus*mstatusn +
                        b_area*arean + b_lincome*lincome + b_sex*sexn;
      lambda = exp(eta);
      loglike = - lambda + clm_freq*log(lambda) - log(fact(clm_freq)) ;
      model clm_freq ~ general(loglike);
      * same results if ll is hardcoded;
/*    model clm_freq ~ poisson(lambda); */
run;
```

or;

```
proc nlmixed data=claims2;
      eta = b_0    + b_car_use*(car_use='Commercial')
                   + b_mstatus*(mstatus='1. Yes')
                   + b_area*(area ='1. Highly Urban/ urban area')
                   + b_lincome*lincome
                   + b_sex*(sex='1. M');
      lambda = exp(eta);
      model clm_freq ~ poisson(lambda);
run;
```

The NLMIXED Procedure

Specifications	
Data Set	WORK.CLAIMS3
Dependent Variable	CLM_FREQ
Distribution for Dependent Variable	General
Optimization Technique	Dual Quasi-Newton
Integration Method	None

Dimensions	
Observations Used	2812
Observations Not Used	0
Total Observations	2812
Parameters	12

Fit Statistics	
-2 Log Likelihood	6404.0
AIC (smaller is better)	6428.0
AICC (smaller is better)	6428.1
BIC (smaller is better)	6499.3

Parameter Estimates									
Parameter	*Estimate*	*Standard Error*	*DF*	*t Value*	*Pr > |t|*	*Alpha*	*Lower*	*Upper*	*Gradient*
bll_0	0.4709	0.1302	2812	3.62	0.0003	0.05	0.2156	0.7262	0.267506
bll_1	0.03851	0.05742	2812	0.67	0.5025	0.05	-0.07408	0.1511	0.084439
bll_2	-0.02516	0.05479	2812	-0.46	0.6461	0.05	-0.1326	0.08227	0.034717
bll_3	0.08448	0.1098	2812	0.77	0.4418	0.05	-0.1309	0.2998	0.089421
bll_4	-0.00321	0.008304	2812	-0.39	0.6994	0.05	-0.01949	0.01308	-0.41592
bll_5	0.01517	0.05627	2812	0.27	0.7874	0.05	-0.09517	0.1255	-0.14471
bp_0	1.2705	0.2256	2812	5.63	<.0001	0.05	0.8282	1.7128	-0.20101
bp_1	-0.5539	0.1235	2812	-4.49	<.0001	0.05	-0.7961	-0.3118	0.240928
bp_2	0.1607	0.1125	2812	1.43	0.1532	0.05	-0.05987	0.3813	0.000136
bp_3	-2.0319	0.1554	2812	-13.07	<.0001	0.05	-2.3367	-1.7272	-0.00329
bp_4	0.04199	0.01865	2812	2.25	0.0245	0.05	0.005415	0.07857	-0.41593
bp_5	0.3006	0.1144	2812	2.63	0.0086	0.05	0.07639	0.5248	-0.07819

Contrasts				
Label	*Num DF*	*Den DF*	*F Value*	*Pr > F*
TEST p_0=0.4468	1	2812	0.02	0.8836

Additional Estimates								
Label	*Estimate*	*Standard Error*	*DF*	*t Value*	*Pr > |t|*	*Alpha*	*Lower*	*Upper*
Estimated proportion of 'extra' zeros (theta)	0.4435	0.02280	2812	19.45	<.0001	0.05	0.3988	0.4882
Estimated Conditional Poisson Mean (lambda)	1.6515	0.07387	2812	22.36	<.0001	0.05	1.5067	1.7963
Estimated Unconditional ZIP Mean ((1-p_0)*lambda)	0.9191	0.04280	2812	21.47	<.0001	0.05	0.8352	1.0031
Estimated Unconditional ZIP Variance ((1-p_0)*lambda*(1+p_0*lambda))	1.5923	0.09299	2812	17.12	<.0001	0.05	1.4099	1.7746

Zero-inflated Poisson regression models can also be easily fitted using Proc NLMIXED. The Zero-inflation parameter can be left as a constant, or a second regression equation can be fitted with the same or different regressors as for the mean parameter, allowing that ZI parameter to vary by group, or even by observation. The mean parameter has a log link, ensuring positivity of the mean, while the ZI parameter has a logit link, ensuring that it remains between zero and one.

```
Parameter Estimates
```

	Parameter	Error	DF	t
bll_0	(0.495)	0.119	2,812	(4.170)
bll_1	0.142	0.052	2,812	2.750
bll_2	(0.010)	0.049	2,812	(2.040)
bll_3	1.211	0.097	2,812	12.460
bll_4	0.015	0.007	2,812	(2.100)
bll_5	0.053	0.051	2,812	(1.040)
bp_0	(0.198)	0.058	2,812	(3.390)

Adding regressors as for the ZI parameter.

```
proc nlmixed data=claims3;
  parameters  bll_0=0 bll_1=0 bll_2=0 bll_3=0 bll_4=0 bll_5=0
                        bp_0=0  bp_1=0 bp_2=0 bp_3=0 bp_4=0 bp_5=0;

  eta_prob = bp_0 + bp_1*car_usen + bp_2*mstatusn + bp_3*arean
                + bp_4*lincome + bp_5*sexn;
  p_0 = exp(eta_prob)/(1 + exp(eta_prob));
 eta_lambda = bll_0 + bll_1*car_usen + bll_2*mstatusn + bll_3*arean
                        + bll_4*lincome + bll_5*sexn;
  lambda = exp(eta_lambda);
 if clm_freq=0 then loglike = log(p_0 + (1-p_0)*exp(-lambda));
               else loglike = log(1-p_0) + clm_freq*log(lambda)
                                    -lambda - lgamma(clm_freq+1);
  model clm_freq ~ general(loglike);
  estimate "Estimated proportion of 'extra' zeros (theta)" p_0;
  estimate 'Estimated Conditional Poisson Mean (lambda)' lambda;
  estimate 'Estimated Unconditional ZIP Mean ((1-p_0)*lambda)'
                                                (1-p_0)*lambda;
  estimate 'Estimated Unconditional ZIP Variance
      ((1-p_0)*lambda*(1+p_0*lambda))' (1-p_0)*lambda*(1+p_0*lambda);

  predict (1-p_0)*lambda out = lambda_hat ;
  title 'ZIP regression model';
run;
```

```
                              ZIP regression model

                               The NLMIXED Procedure
                                  Specifications
                Dependent Variable                          CLM_FREQ
                Distribution for Dependent Variable         General
                Optimization Technique                      Dual Quasi-Newton
                                      Dimensions
                                     Fit Statistics
                      -2 Log Likelihood                       6404.0
```

```
AIC (smaller is better)          6428.0
AICC (smaller is better)         6428.1
BIC (smaller is better)          6499.3
```

Parameter Estimates

Parameter	Estimate	Error	DF	t	Pr	Lower	Upper	Gradient
bll_0	0.469	0.130	2812	3.6	0.0003	0.2136	0.7241	0.0525
bll_1	0.037	0.057	2812	0.65	0.5172	-0.0754	0.1498	-0.0033
bll_2	-0.024	0.055	2812	-0.43	0.6661	-0.1311	0.0838	0.1584
bll_3	0.085	0.110	2812	0.78	0.4371	-0.1300	0.3007	0.0884
bll_4	-0.003	0.008	2812	-0.38	0.7062	-0.0194	0.0132	0.5054
bll_5	0.015	0.056	2812	0.27	0.7863	-0.0951	0.1256	-0.0293
bp_0	1.277	0.226	2812	5.66	<.0001	0.8351	1.7197	0.0822
bp_1	-0.559	0.124	2812	-4.52	<.0001	-0.8018	-0.3169	-0.0772
bp_2	0.165	0.113	2812	1.46	0.1434	-0.0560	0.3855	0.4239
bp_3	-2.035	0.156	2812	-13.08	<.0001	-2.3400	-1.7299	0.0725
bp_4	0.042	0.019	2812	2.24	0.0254	0.0051	0.0782	0.1624
bp_5	0.298	0.114	2812	2.61	0.0092	0.0740	0.5225	-0.3611
Prop Extra 0's	0.445	0.023		19.52	0.0001			

Appendix F

Other Count Datasets

Table F.1 Bailey and Simon Data

ID	Class #	Class	Merit	Exposure	Earned	Claims	Frequency/
1	1	NoYoungMale	A	2,757,520	159,108,000	217,151	0.079
2	5	MarriedYoungMale	A	64,130	5,349,000	6,560	0.102
3	1	NoYoungMale	X	130,706	7,910,000	13,792	0.106
4	2	NonPrincipYoungMale	A	130,535	11,840,000	14,506	0.111
5	1	NoYoungMale	Y	163,544	9,862,000	19,346	0.118
6	5	MarriedYoungMale	x	4,039	345,000	487	0.121
7	5	MarriedYoungMale	Y	4,869	413,000	613	0.126
8	3	Business	A	247,424	25,846,000	31,964	0.129
9	1	NoYoungMale	B	273,944	17,226,000	37,730	0.138
10	2	NonPrincipYoungMale	X	7,233	712,000	1,001	0.138
11	4	YoungMale	A	156,871	18,450,000	22,884	0.146
12	2	NonPrincipYoungMale	Y	9,726	944,000	1,430	0.147
13	5	MarriedYoungMale	B	8,601	761,000	1,291	0.150
14	2	NonPrincipYoungMale	B	21,504	1,992,000	3,421	0.159
15	3	Business	X	15,868	1,783,000	2,695	0.170
16	4	YoungMale	y	21,089	2,523,000	3,618	0.172
17	4	YoungMale	X	17,707	2,130,000	3,054	0.172
18	3	Business	Y	20,369	2,281,000	3,546	0.174
19	4	YoungMale	B	56,730	6,608,000	11,345	0.200
20	3	Business	B	37,666	4,129,000	7,565	0.201

Table F.1.a Chi-Square Test Based on Bailey and Simon Data

Poisson/ZIP	7.8E+16
Negative Binomial	6,672,651
ZINB	6,107,153

Table F.2 Wang, Cockburn and Puterman (1998) Patents data

Obs	Company	Patents	RDS	lgRD
1	ABBOTT LABORATORIES	42	0.0549	4.0869
2	AFFILIATED HOSPITAL PRDS	1	0.0032	-2.0794
3	ALBERTO-CULVER CO	3	0.0078	0.1187
4	ALCON LABORATORIES	2	0.0803	1.8796
5	ALLERGAN PHARMACEUTICALS INC	3	0.0686	1.1033
6	ALZA CORP-CL A	40	3.3319	2.0794
7	AMERICAN HOME PRODUCTS CORP	60	0.0243	4.0953
8	AMERICAN HOSPITAL SUPPLY	30	0.0128	2.8333
9	AMERICAN STERILIZER CO	7	0.0252	1.3915
10	AVON PRODUCTS	3	0.0094	2.6048
11	BARD(C.R.) INC	5	0.0146	0.7957
12	BAXTER TRAVENOL LABORATORIES	59	0.0496	3.5207
13	BECTON, DICKINSON & CO	26	0.0395	3.0001
14	BENTLEY LABORATORIES	3	0.0780	0.5371
15	BOCK DRUG-CL A	0	0.0171	0.7761
16	BRISTOL-MYERS CO	66	0.0347	4.2338
17	CARTER-WALLACE INC	0	0.0569	2.2178
18	CAVITRON CORP	8	0.1095	0.8510
19	CHATTEM INC	2	0.0190	-0.1567
20	CHESEBROUGH-POND'S INC	4	0.0084	1.8358
21	CLINICAL SCIENCES INC	0	0.1003	-1.6045
22	CODE LABORATORIES INC	0	0.0623	0.7071
23	CONCEPT INC	3	0.0707	-0.9916
24	COOPER LABORATORIES	6	0.0359	1.2296
25	DATASCOPE CORP	3	0.0596	-0.5310
26	DEL LABORATORIES INC	0	0.0076	-1.2310
27	DENTSPLY INTERNATIONAL INC	6	0.0185	0.9270
28	DESERET PHARMACEUTICAL	2	0.0080	-1.1332
29	DYNATECH CORP	3	0.0640	-0.0419
30	ELECTRO CATHETER CORP	0	0.0780	-1.8326
31	EVEREST & JENNINGS INTL	1	0.0025	-1.8264

Obs	Company	Patents	RDS	lgRD
32	FABERGE INC	1	0.0040	-0.1985
33	FOREST LABORATORIES INC	0	0.0329	-1.7838
34	GILLETTE CO	25	0.0234	3.5525
35	GUARDIAN CHEMICAL CORP	2	0.0387	-2.5639
36	HELENE CURTIS INDUSTRIES	4	0.0133	0.4523
37	ICN PHARMACEUTICALS INC	1	0.0324	1.0529
38	INSTRUMENTATION LABS INC	1	0.0882	1.4873
39	INTL FLAVORS & FRAGRANCES	51	0.0587	2.7793
40	JOHNSON & JOHNSON	105	0.0446	4.7233
41	JOHNSON PRODUCTS	1	0.0131	-0.6444
42	KEY PHARAMACEUTICALS INC	0	0.0160	-2.9565
43	LA MAUR INC	0	0.0143	-0.9545
44	LILLY (ELI) & CO	166	0.0843	4.7278
45	MALLINCKRODT INC	8	0.0320	2.1831
46	MARION LABORATORIES	6	0.0599	1.5773
47	MERCK & CO	173	0.0821	4.9152
48	????	25	0.0535	3.1807
49	MINE SAFETY APPLIANCES CO	14	0.0226	1.4036
50	NARCO SCIENTIFIC INC	3	0.0397	1.0043
51	NESTLE-LEMUR CO	0	0.0103	-2.3330
52	NEWPORT PHARMACEUTICALS INTL	0	0.7159	-0.1815
53	NOXELL CORP	2	0.0107	0.2670
54	PFIZER INC	93	0.0467	4.4785
55	PURITAN-BENNETT CORP	3	0.0369	0.7105
56	REDKEN LABORATORIES	2	0.0316	0.2979
57	RESEARCH INDUSTRIES CORP	0	0.0355	-2.8647
58	REVLON INC	5	0.0166	2.7622
59	RICHARDSON-MERRELL INC	23	0.0417	3.4383
60	ROBINS (A.H.) CO	11	0.0447	2.5439
61	RORER GROUP	13	0.0401	2.4436
62	SCHERER (R.P.)	0	0.0050	-0.4125
63	SCHERING-PLOUGH	90	0.0618	3.9865

Obs	Company	Patents	RDS	lgRD
64	SEARLE (G.D.) & CO	63	0.0690	3.9620
65	SMITHKLINE CORP	112	0.0813	4.0029
66	SQUIBB CORP	115	0.0409	3.9051
67	STERLING DRUG INC	48	0.0331	3.5909
68	SYBRON CORP	15	0.0323	2.9242
69	SYNTEX CORP	69	0.0859	3.1707
70	TECHNICARE CORP	4	0.0591	1.8089

Table F.2.a Chi-Square Test Based on Patent Data

Distribution	Chi Square
Poisson	7.10E+84
Negative Binomial	584,069
ZIP	6.70E+71
ZINB	249

Table F.3 Deb and Trivedi Hospital Stay Counts

dsn=dt, yvar=hosp, dist=poisson

poisson distribution with mean(xbar)=sample mean= 0.29596

y	Frequency	Percent	Test Percent	Cumulative Frequency	Cumulative Percent
0	3541	80.370	74.380	3541	80.37
1	599	13.600	22.010	4140	93.96
2	176	3.990	3.260	4316	97.96
3	48	1.090	0.320	4364	99.05
4	20	0.450	0.020	4384	99.5
5	12	0.270	0.000	4396	99.77
6	5	0.110	0.000	4401	99.89
7	1	0.020	0.000	4402	99.91
8	4	0.090	0.000	4406	100

Table F.3.a Chi-Square Test Based on Hospital Visit Data

Distribution	Chi Square
Poisson	3.40E+06
Negative Binomial	19,302
ZIP	2.,925
ZINB	25

Table F.4 Deb and Trivedi Office Visit Data

y	Frequency	Percent	Test Percent	Cumulative Frequency	Cumulative Percent
0	683	15.5	0.31	683	15.5
1	481	10.92	1.79	1164	26.42
2	428	9.71	5.18	1592	36.13
3	420	9.53	9.97	2012	45.67
4	383	8.69	14.39	2395	54.36
5	338	7.67	16.62	2733	62.03
6	268	6.08	15.99	3001	68.11
7	217	4.93	13.19	3218	73.04
8	188	4.27	9.52	3406	77.3
9	171	3.88	6.11	3577	81.18
10	128	2.91	3.53	3705	84.09
11	115	2.61	1.85	3820	86.7
12	86	1.95	0.89	3906	88.65
13	73	1.66	0.4	3979	90.31
14	76	1.72	0.16	4055	92.03
15	53	1.2	0.06	4108	93.24
16	47	1.07	0.02	4155	94.3
17	48	1.09	0.01	4203	95.39
18	30	0.68	0	4233	96.07
19	24	0.54	0	4257	96.62
20	16	0.36	0	4273	96.98
21	18	0.41	0	4291	97.39
22	16	0.36	0	4307	97.75
23	10	0.23	0	4317	97.98
24	12	0.27	0	4329	98.25
25	3	0.07	0	4332	98.32
26	9	0.2	0	4341	98.52
27	7	0.16	0	4348	98.68
28	4	0.09	0	4352	98.77
29	3	0.07	0	4355	98.84
30	4	0.09	0	4359	98.93
>30	47	1.04	0	4406	100

Table F.4.a Chi-Square Test Based on Office Visit Data

Distribution	Chi Square
Poisson	2.02E+67
Negative Binomal	2,856
ZIP	2.33E+01
ZINB	4,224

Table F.5 Ridout and Demetrio Apple Shoot Counts

Frequency distributions of the number of roots by 270 shoots of the apple cultivar Trajan									
BAP (muM)	**Photoperiod**								**All**
	8				**16**				
	2	**4**	**9**	**18**	**2**	**4**	**9**	**18**	
No. of roots 0	0	0	0	2	15	16	12	19	64
1	3	0	0	0	0	2	3	2	10
2	2	3	1	0	2	1	2	2	13
3	3	0	2	2	2	1	1	4	15
4	6	1	4	2	1	2	2	3	21
5	3	0	4	5	2	1	2	1	18
6	2	3	4	5	1	2	3	4	24
7	2	7	4	4	0	0	1	3	21
8	3	3	7	8	1	1	0	0	23
9	1	5	5	3	3	0	2	2	21
10	2	3	4	4	1	3	0	0	17
11	1	4	1	4	1	0	1	0	12
12	0	0	2	0	1	1	1	0	5
13	1	1	.	.	.	.	.	.	2
14	.	.	2	1	.	.	.	.	3
17	1	.	.	.	.	.	.	.	1
All	30	30	40	40	30	30	30	40	270

Table F.5.a Chi-Square Test Based on Ridout and Demetrio Apple Shoot Data

Distribution	Chi Square
Poisson	2,694
NB	131
ZIP	76
ZINB	19

Table F.6 Long Biochemists Data
The FREQ Procedure

y	Frequency	Percent	Test Percent	Cumulative Frequency	Cumulative Percent
0	275	30.05	19.28	275	30.05
1	246	26.89	31.74	521	56.94
2	178	19.45	26.12	699	76.39
3	84	9.18	14.33	783	85.57
4	67	7.32	5.90	850	92.90
5	27	2.95	1.94	877	95.85
6	17	1.86	0.53	894	97.70
7	21	2.30	0.13	915	100.00

Table F.6.a Chi-Square Test Based on Biochemists Data

Chi-Square Test for Specified Proportions	
Chi-Square	476.5913
DF	7
Pr > ChiSq	<.0001
WARNING: 25% of the cells have expected counts less than 5. Chi-Square may not be a valid test.	
Sample Size = 915	

Appendix G

A Simple Procedure for Fitting the ZIP Model

A procedure to solve for the ZIP parameters can be set up in Microsoft Excel. The illustration used in this paper optimizes the minimum distance procedure as set forth in Hogg and Klugman (1984), but other statistics can be optimized.

$$\min\left(\sum_k w(k)[F_n(k) - F(k)]^2\right) \tag{G.1}$$

$w(k)$ = weight, $F_n(k)$ = Empirical DF, $F(k)$ = Fitted DF

Hogg and Klugman suggest using a weight of: $w(k) = \dfrac{n_k}{F_n(k)(1 - F_n(k))}$

Table 2.2 displays the spreadsheet setup for the parameter estimation. The parameters phi and lambda have been initialized to those of the Poisson distribution, i.e., no structural zeros, so phi is zero. The sum of column (7) is to be minimized.

Table G.1

Calculation of Zero-Inflated Poisson Parameters: Initialization

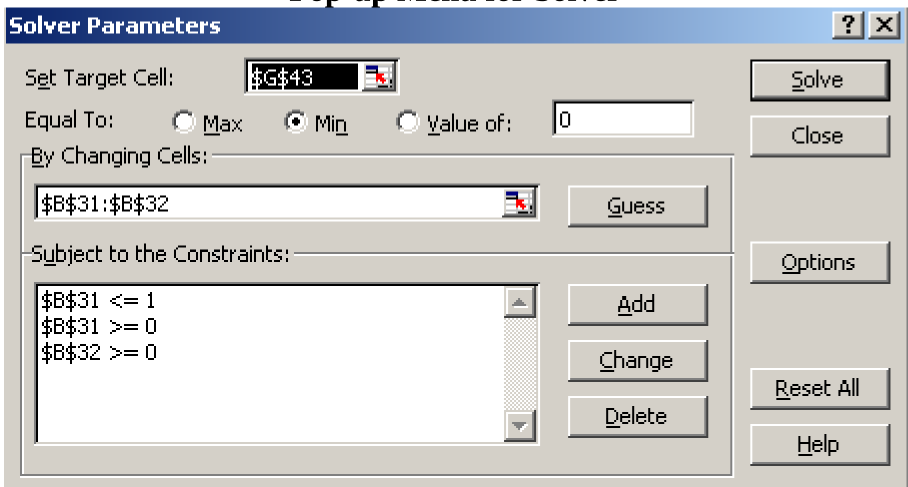
Phi	0					
Lambda	0.82					
No Claims	Actual Count	P(X=x) (2)/SUM(2)	Theoretical P(X=x)	Weight (1)/((2)(1-(2))	Squared Deviation ((3)-(4))^2	Wt Squared Deviation (5)*(6)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0	1,706	0.607	0.44043	7,149	0.02764	197.6
1	351	0.125	0.36115	3,213	0.05585	179.5
2	408	0.145	0.14807	3,289	0.00001	0.0
3	268	0.095	0.04047	3,108	0.00301	9.3
4	74	0.026	0.00830	2,888	0.00032	0.9
5	5	0.002	0.00136	2,817	0.00000	0.0
Sum	2,812				0.08683	387.4

Excel provides the solver function to solve[12] nonlinear optimization problems such as this one. Solver uses a numerical algorithm, such as gradient descent, to solve nonlinear problems. Figure 2.2 displays the pop-up menu that is used with Solver. The menu requires the user to identify a target cell to optimize (here the sum of the weighted squared deviations), the input cells containing the parameters to be estimated and whether the optimization is a minimization or maximization.

[12] Please note you must load the solver add-in to use solver. This can be done from the tools menu, but requires the Microsoft Office disk.

Optionally, the user can specify constraints on the parameters (i.e., for instance phi must be greater than or equal to zero and less than or equal to 1.

Figure G.1
Pop-up Menu for Solver



The Poisson parameters fit with Excel solver are displayed in Table 2.3. The table indicates that on average, 54% of the records have structural zeros. For the remaining policyholders, the mean claim frequency over a five-year period is approximately 1.9. Figure 2.3 compares the negative binomial to the zero-inflated Poisson. The ZIP model appears to provide a better fit to the data.

Table G.2
Fitted Zero-Inflated Poisson

Phi	0.5359					
Lambda	1.9194					Wt
No Claims	Actual Count	P(X=x) (2)/SUM(2)	Theoretical P(X=x)	Weight (1)/((3)(1-(3))	Squared Deviation ((3)-(4))^2	Squared Deviation (5)*(6)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0	1706	0.607	0.60402	7149	0.00001	0.1
1	351	0.125	0.13066	3213	0.00003	0.1
2	408	0.145	0.12540	3289	0.00039	1.3
3	268	0.095	0.08023	3108	0.00023	0.7
4	74	0.026	0.03850	2888	0.00015	0.4
5	5	0.002	0.01478	2817	0.00017	0.5
Sum	2,812				0.00097	3.0

Table G.3
Fitted Zero-Inflated Negative Binomial Model

phi	0					
r	1					
p	0.4561					Wt
No Claims	Actual Count	P(X=x) (2)/SUM(2)	Theoretical P(X=x)	Weight (1)/((3)(1-(3))	Squared Deviation ((3)-(4))^2	Squared Deviation (5)*(6)
(1)	(2)	(3)	(4)	(5)	(3290)	(3291)
0	1706	0.607	0.4561	7149	0.00048	1.5
1	351	0.125	0.2481	3213	0.00018	0.5
2	408	0.145	0.1349	3289	0.00040	1.1
3	268	0.095	0.0734	3108	0.00000	0.0
4	74	0.026	0.0399	2888	0.00000	0.0
5	5	0.002	0.0217	2817	0.00000	0.0
Sum					0.00106	3.1

Table G.4
Fitted Hurdle Poisson Model

phi	0.599					
lambda	1.9286					Wt
No	Actual		Theoretical		Squared	Squared
Claims	Count	P(X=x)	P(X=x)	Weight	Deviation	Deviation
		(2)/SUM(2)		(1)/((3)(1-(3))	((3)-(4))^2	(5)*(6)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0	1706	0.607	0.5990	7149	0.00006	0.4
1	351	0.125	0.1124	3213	0.00015	0.5
2	408	0.145	0.1084	3289	0.00135	4.4
3	268	0.095	0.0697	3108	0.00066	2.0
4	74	0.026	0.0336	2888	0.00005	0.2
5	5	0.002	0.0130	2817	0.00013	0.4
Sum	2,812				0.00239	7.9

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Abbreviations and notations

Collect here in alphabetical order all abbreviations and notations used in the paper

ZIP: Zero-Inflated Poisson	GLM, generalized linear models
ZINB Zero-Inflated negative binomial	CAS Casualty Actuarial Society
CHAID Chi-Squared Automatic Interaction Detection	

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Profit Margins Using Co-Measures of Risk

Mark J. Homan, FCAS, MAAA, CPCU

Abstract:. Insurance policies cover multiple loss components. Lately, there is a move to determining the premium for a policy by combining the components. This has led to the desire to have profit margins that can be combined. This paper demonstrates that profit margins by component are not additive. Those wishing to introduce rating by peril will need to consider how they will determine profit margins as they combine the underlying loss costs. The Excel worksheet used in the examples will be available on the CAS website.

Keywords. Profit Loads; capital allocation; risk loads

1. INTRODUCTION

There is a trend towards rating multi-peril products (i.e., Homeowners and Business Owners) by peril, or splitting rates between catastrophe and noncatastrophe. So the issue of determining the required profit loads naturally arises. The desire is to have profit loads by component so premiums can be determined by component and added together to get the final rate.

For example, the Florida legislature, recognizing the need for an appropriate return on catastrophe risk, required the Office of Insurance Regulation to determine an appropriate profit margin for the catastrophe portion of the Homeowners rate. While it is important for the industry that the legislature recognizes that catastrophe exposed business requires an appropriate return for the risk, they also took the erroneous view that profit margins can be determined by component.

Clearly, the administration of rates, for both companies and state regulators, would be simpler if profit margins could be determined in an additive manner. However, reality once again is not as simple as we would like.

Unfortunately, additive profit margins by component cannot be developed. One may accept the compromise required to treat profit margins as additive. However, this involves significant compromise in some cases, creating significant differences in prices.

Loss costs, which are based on means, are additive. Profit loads are based on risk, reflecting additional moments of the distribution, and higher moments of distributions are not simply additive. Diversification and correlation impact the profit load for the aggregate risk.

The examples in this paper are based on splits between catastrophe and noncatastrophe portions of the risk. The two loss components are treated as independent. This is reasonable in the author's experience, but the methodology can be applied to loss distributions that are correlated. All expenses are treated as variable with no volatility. This is a simplifying assumption to isolate and highlight the interplay of the two loss components. The analysis can be extended to reflect expense variability and the risk that expenses represent.

1.1 Research Context

This paper deals primarily with the required profit margins. It also addresses related issues of capital allocation and ROE.

There are a number of papers in the CAS literature on setting required profit margins, or profit loads. These papers identify that catastrophe exposure is a key consideration in determining the required profit margin. These papers deal with how to determine the profit margin for an aggregate exposure, with all risks combined. No papers or presentations were found that addressed the issue of determining profit margins based on risk component.

1.2 Objective

The paper will evaluate two different approaches for determining profit margins by component. Both approaches will show that profit margins by component are not linear, and as a result, they cannot be added together. The expectation is that the paper can refute the concept of additive component profit margins.

1.3 Outline

The first part of the paper will demonstrate why profit loads cannot be determined by peril or component. Then it will demonstrate how to determine the overall required profit margin using a Risk Coverage Ratio (RCR) approach, and then how to allocate the required profit to component based on risk using an approach algorithm named after the developers Ruhm-Mango-Kreps (RMK). It will also demonstrate the limitations or compromises required in this approach.

2. BACKGROUND AND METHODS

The two methods evaluated are RCR and the RMK algorithm. A brief overview of each method will be provided before using the method to evaluate profit margins for the components. Further information on each method is included in the Appendix..

2.1 Profit Margins using RCR

The initial concept used in this paper to determine a profit margin is the RCR. RCR was introduced to the actuarial community in a paper by David Ruhm [1]. Although RCR does not require surplus, as implied by the title of the paper, it is easy to translate the required price into implied surplus to attribute capital. (More information is provided in the Appendix.)

As a reward-to-risk ratio, RCR balances the required return to the risk. In its basic application, RCR is calculated from the distribution of returns on operating cash flows. In this situation, a common adverse event, or minimum threshold is zero. This means that any scenario where the premium and investment income are insufficient to cover all expenses and losses is considered an adverse event. In other words, any operating loss is bad.

RCR has strong appeal for use in pricing as it includes all adverse events in its determination. The risk metric used in the denominator is related to TVAR (Tail Value at Risk), also known as CTE (Conditional Tail Expectation). The key difference is that TVAR is usually demined at a pre-determined percentile level. For RCR, that percentile is dynamic and will vary based on the shape of the distribution for the line.

Since the RCR is a ratio of reward to risk, each line will have the same cost per unit of risk. In other words, the dollars of return required for each dollar of risk will be uniform across all lines of business.

2.2 Using RMK to Allocate Profit Margin

In Section 3.5 that follows, the RMK (Ruhm-Mango-Kreps) algorithm is used to allocate surplus and thus the profit margin to risk component. RMK is an approach that attributes surplus to risk component in proportion to the component's contribution to aggregate risk. It is solely a methodology to allocate surplus, it does not determine the amount of surplus that is required.

The derivation of this algorithm and its properties are covered in papers available through the CAS. An initial paper by Ruhm and Mango [2] provides the foundation and formulas. Another paper by David Clark [3] provides a practical application of the RMK algorithm. Neither paper will be covered in detail here.

RMK requires setting an initial vector outlining risk appetite. In this paper, all scenarios that generate a net loss are assigned the same weight. Depending on a company's risk appetite, there may be events that cause a more extreme loss that should get higher weight. For example, the weight may be increased in situations where a company is forced to access the capital markets for additional funds. The simpler approach used in this paper works well in practice and adequately outlines the desired concepts. The initial weights then are normalized to average to 1.0, and this becomes the Z-vector discussed in the Mango-Ruhm paper [2].

For the first two examples, since there is only a single loss component, these calculations are uninteresting, but are included to demonstrate that they work in this situation

3. RESULTS AND DISCUSSION

In this section, the required profit load is determined for various combinations of catastrophe and noncatastrophe losses using the Risk Coverage Ratio. Then for the same examples the surplus and profit loads are split to risk component using the RMK algorithm.

3.1 Profit Loads

This section provides a general overview of splitting profit margins into catastrophe loss and noncatastrophe loss components. The examples shown are simplified calculations. Only the volatility in the level of the catastrophe and noncatastrophe loss ratio is reflected. Additional sources of volatility (payment date, interest rate, expense ratio, etc.) are ignored. This allows for illustration of the concepts, without requiring too complex an Excel spreadsheet for the examples. The exhibits show the summary and first 20 scenarios for each simulation. The full Excel spreadsheet is available on the CAS website.

The assumptions used in all examples are shown below:

Expenses	30% (treated as all variable)
Loss Payment	1 year (for both catastrophe and noncatastrophe)
Yield	5.04% before-tax
Tax Rate	35% (ignore tax loss discount)
RCR Target	20

3.2 Separate Profit loads by Component

As a first step, let's look at the profit loads by component for catastrophe separate from noncatastrophe. Exhibit 1A shows the derivation of the required premium for $35 catastrophe loss only with the base assumptions. The catastrophe loss distribution is a sample of 10,000 scenarios from a vendor catastrophe model using a countrywide distribution. The required premium is $90.85. The target combined ratio is 68.5%, or an underwriting profit margin of 31.5%

Exhibit 1B shows the derivation of the required premium for $60 of noncatastrophe loss with the base assumptions. The noncatastrophe loss distribution is based on a lognormal distribution and also uses 10,000 scenarios. The required premium is $97.32. The target combined ratio is 91.7%, or an underwriting profit margin of 8.3%. Adding the noncatastrophe premium to the catastrophe premium yields a total premium of $188.18. (There is an additional cent from rounding in the Excel spreadsheets.)

Comparing the two combined ratios, or profit margins, it is clear that the higher risk represented by catastrophe losses requires a much higher price per dollar of loss. Since expenses are all variable, the required premium is scaleable with losses. So to more directly compare the two premiums, we can scale the noncatastrophe premium to $35 of noncatastrophe loss. That premium would be $56.77, or $34.08 less than what is required for catastrophe losses.

3.3 Profit Load for Combined Components

Now, let's combine the catastrophe and noncatastrophe distributions and create a single loss distribution and an aggregate return distribution. Exhibit 2 shows the resulting required premium ($174.12) and combined ratio (84.6%) for $35 of catastrophe loss and $60 of noncatastrophe loss. Comparing this premium to the total premium of $188.18 from Exhibits 1A and 1B, one can see that the required premium is less on an aggregate basis than the sum of the premiums from each risk separately. The difference in premium of $14.05 is the diversification benefit. The diversification comes from the fact that a bad year on one distribution can be offset, completely or partially, by a lower than expected year on the other distribution. It should be noted that a low catastrophe year will more often offset a bad noncatastrophe result in the same year than the other way around. This is because the catastrophe distribution has a more extreme tail.

3.3.1 Profit Load with a Different Mix

To further illustrate the effect of looking at combined distributions to develop profit margins versus combining the components, let's look at some different splits between catastrophe and noncatastrophe losses.

Exhibit 3A shows the required premium if there is twice as much in catastrophe loss, or $70. The required premium is $264.07. Comparing this to twice the catastrophe premium plus the noncatastrophe premium, which is $279.03, we can see the diversification benefit is $14.96. The diversification benefit is better than Exhibit 2, but only by a small amount. In addition, the target combined ratio is 79.2%, lower than in Exhibit 2.

Exhibit 3B shows the required premium if there is half the amount of catastrophe loss, or $17.50. The required premium is now $130.92. Comparing this to half the catastrophe premium plus the noncatastrophe premium, which is $142.75, we can see the diversification benefit drop to $11.83. Now the target combined ratio is 89.2%, up from Exhibit 2.

I will leave it to the curious reader to download the Excel file from the CAS website and verify the following statements. Clearly, as the catastrophe loss goes to zero, the diversification benefit will go to zero as we will only have the noncatastrophe premium as shown in Exhibit 1A. As the catastrophe loss increases, the diversification increases at a decreasing rate.

3.4 Diversification Benefit

The key difference between separate profit margins and a combined profit margin is reflecting the diversification benefit between the components. From Exhibits 2, 3A and 3B, we can see that the diversification benefit is a nonlinear relationship between the two loss distributions. Since this is a nonlinear relationship, it is clear that one cannot determine separate profit margins for catastrophe and noncatastrophe components and then add them together. The diversification benefit must be considered, and it is not a single factor adjustment in all cases.

3.4.1 Special Case – Complete Correlation

There is a special case where component profit margins would be additive. If the two distributions are completely correlated, there is no diversification benefit from combining them. With no diversification benefit, then the profit margins are the sum of the parts.

3.5 Using RMK to allocate Surplus (and profit margin)

The RMK algorithm is an alternate method for attributing surplus based on contribution to risk. From another perspective, it can be viewed as a method for allocating the diversification benefit.

3.5.1 RMK – Still not a Solution to Component Profit Margins

In Exhibit 2-2, the surplus allocation for the example in Exhibit 2 is derived. This shows that within this example, the surplus is needed predominately for the catastrophe risk. The noncatastrophe risk contributes very little to the operating losses. So the diversification benefit that we discussed above is primarily seen in the noncatastrophe risk. Similar derivations of surplus attribution are shown in Exhibits 3A-2 and 3B-2.

Another way to look at the allocation of diversification benefit is to compare the surplus by component. The surplus indicated for the catastrophe risk only starts at $168.02 (Exhibit 1A), which is reduced only to $160.24 (Exhibit 2-2) in the combined example. In contrast, the required surplus for the noncatastrophe component starts at $61.29 (Exhibit 1B), and this is reduced to $14.53 in the combined example (Exhibit 2-2). This shows that the primary impact of diversification is to reduce the amount of surplus required to support the noncatastrophe risk.

Starting with the allocation of surplus and profit from Example 2, we can try to predict the required profit margins for Examples 3A and 3B. We do this by applying the leverage ratios from Exhibit 2-2 to the liabilities generated in Examples 3A and 3B. This is shown in Exhibit 4. This example shows we would come up short on our estimate of required surplus, and thus profit margin, for both cases.

It is interesting to understand why we are not predicting the correct answer. In both cases, it is because the level of diversification has changed. In Example 3A, we are not getting as much diversification from the noncatastrophe portion of the exposure. Since there is little surplus required for noncatastrophe, the difference in required surplus is moderate. In Example 3B, we have half the catastrophe loss level. Now, the noncatastrophe loss cannot be diversified away as much as in Example 2. In other words, the noncatastrophe risk has more impact on the bottom line, so we need to attribute more surplus to it.

RMK is considered one of the most sophisticated methods of attributing surplus and determining required profit, yet it still cannot provide correct answers for component profit margins that can be used as the mix of risk component changes.

3.5.2 Materiality

Let's shift from the theoretical to the practical. The RCR analysis was sufficient to demonstrate that component profit margins are not additive as risk varies, so why explore the application of RMK? It is because RMK can be used to flex profit margins within a reasonable range of changes in mix by component. The size of the range will depend on one's definition of materiality. Clearly, if there is a theoretical difference that does not translate to a difference in what a policyholder will actually pay (i.e. one that rounds away), then the difference would not be considered material.

For Example 3A, the shortfall is moderate, only $0.78 on the true premium of $264.07. This could very well be determined not to be material. And this is an example where we doubled the catastrophe losses, which is a fairly extreme change. If the increase in catastrophe losses was more moderate, like 10%, the difference would be even smaller and would be more likely to be considered immaterial by many companies.

The examples used in this paper were based on a split between catastrophe and noncatastrophe components. Also, the size of the differences in the split is extreme to more easily demonstrate the points in the paper. While the theoretical conclusions apply equally to more moderate splits, like Homeowners rating by peril, the differences in results are not as great. When the distributions are not as different in shape, then RMK can be used in a broader range without material bias. Or, if the range of changes anticipated in the mix of component is moderate, RMK can be reasonably used.

So, in the end, one may determine that the RMK algorithm creates a practical approach for addressing the component profit margins in certain situations.

4. CONCLUSIONS

Profit margins are based on risk. Risk cannot be evaluated by component in isolation. Risk must be evaluated in the context of the whole, and how the various risks contribute diversification to each other. It is not theoretically possible to create additive component profit margins. However, it is possible using RMK to create profit margins that can be combined within reasonable ranges of mix change.

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Supplementary Material

The Excel workbook containing all the examples in the full 10,000 scenario detail is stored on the CAS website.

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Abbreviations and notations
Collect here in alphabetical order all abbreviations and notations used in the paper
RCR, Rick Coverage Ratio
RMK, Ruhm Mango Kreps
ROE, Return on Equity

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6. APPENDIX

More information on the two approaches used in this paper is provided in this appendix. Recognizing that most readers will not be familiar with RCR or RMK, more information is provided here. This is not intended to replace reading the original papers, but should provide enough information to put this paper in context.

6.1 Risk Coverage Ratio (RCR)

RCR was introduced to the actuarial community in a paper by David Ruhm [1]. As stated in the title of Mr. Ruhm's article, RCR does not require leverage or surplus. The required price, and associated profit margin, is calculated to meet the target RCR. In addition, once the RCR and price are determined, you can use this information to attribute capital.

As a reward-to-risk ratio, RCR balances the required return to the risk. To calculate the RCR, one must first determine an adverse event threshold. This will define both the reward and the risk. The reward is the average return minus the adverse event. The risk is the probability of being below the adverse event threshold times the average amount below the threshold when it is below.

The basic formula is:

$$RCR = (R-m) \,/\, (Pr(x<m)*(m-T)) \tag{6.1}$$

where:

R is average return
m is the adverse event threshold, or minimum return
T is the average of all events below the adverse threshold, or the tail

RCR can be used to attribute capital. After solving for the required price to achieve the target RCR, the expected income from operating flows (O) is known. Given a target return on surplus (ROS) and the yield on surplus (y), it is merely algebra to solve for the surplus.

$$Surplus = O \,/\, (ROS - y) \tag{6.2}$$

The yield that is used in this case should be a risk-free or low-risk yield. There is additional investment risk in the actual investment portfolio for most companies, so the actual portfolio yield is usually higher, but also requires additional surplus to support that risk. The actual portfolio yield can be reflected, but that will require additional modeling to solve for the RCR including portfolio yields and risk. The author recommends the use of LIBOR as a near risk-free yield in determining RCR. LIBOR is the standard rate used in the investment community for modeling, and is available at more durations and time points than Treasury yields.

One of the issues of working with RCR is determining the proper target value. There is no intuitive value that makes sense, nor are there any industry standards that can be used. A recommended approach is to use RCR to attribute surplus for all lines of business and solve for the RCR that attributes all of the company's carried surplus. This becomes the Target RCR value for use in pricing. Using this approach, the total surplus for the company will be attributed, and if all lines are at the target price determined by the RCR, the required return on surplus will be achieved.

6.2 Ruhm-Mango-Kreps (RMK) Algorithm

RMK is a methodology designed to allocate risk charge, and thus capital.

There are some key issues in allocating risk charges, and attributing capital, that the RMK algorithm was created to address. As stated in the paper, "Accounting for aggregate portfolio effects in property-casualty insurance prices has historically created some difficult problems, including:

1) Additivity or sub-additivity of prices;
2) Measuring how much diversification efficiency actually exists;
3) Allocating the diversification benefits back to the individual risks; and
4) Order-dependence."

The authors of the paper go on to state, "The method begins at the aggregate level for evaluating risk, and ends by producing prices for individual risks, effectively allocating the total portfolio risk charge. The result is an internally consistent allocation of diversification benefits, avoiding the difficulties listed above. The method effectively extends any risk-valuation theory used at the aggregate portfolio level to the individual risks comprising the portfolio. The resulting prices are additive, with each risk's price reflecting the degree to which it contributes to total portfolio risk" [2].

RMK starts with an aggregate risk charge, or surplus, determined by some other methodology. RMK is used to distribute the risk charge to component in a consistent manner. Some of the key points from the paper are:

1) The aggregate risk charge is split to the individual risks based on the conditional relationship between the risks' outcomes and the aggregate results for the portfolio.

2) All prices are determined solely by the portfolio-level and the probability structure, so that no other information is required.

3) Correlations between risks (and between each risk and the portfolio) are included in the prices in full detail, via the conditional probabilities. Since diversification is related to these correlations, it is also reflected in the risks through this calculation.

4) Prices produced by this method are additive. The price for each component is made up of its contribution to expected costs and its risk load, or profit margin.

The RMK algorithm requires that you assign a weight to each scenario based on the outcome that reflects the company's risk appetite. In this paper, any loss outcome gets the same weight. The RMK algorithm can handle more complex views on risk, such as assigning higher weights based on the size of the loss. Details on the calculations that are associated with this paper are provided in the Notes to Exhibits section 7.2.

7. NOTES TO EXHIBITS

This section provides more detailed information on the calculations of the various exhibits included in this paper. The Excel file used to develop the exhibits is available from the CAS website.

7.1 RCR Exhibits

The format and formulas in Exhibits 1A, 1B, 2, 3A and 3B are the same. It is just the inputs that vary. So they will be discussed together.

Items

- **Premium (solved)** – This is the premium required to meet the RCR target (below). It is solved for via iteration.
- **Combined (formula)** – Combined ratio determined from average loss and LAE dollars for catastrophe and noncatastrophe in total divided by premium plus the expense ratio.
- **Exp (assumption)** – Expense ratio. All expenses are treated as variable in these examples.
- **Loss (assumption)** – the expected loss and LAE dollars are shown as the average. For each scenario, a lognormal distribution was used to create a loss and LAE figure. The parameters for the lognormal are hypothetical used for these examples. The same distribution is used for all examples, with varying means.
- **Cat Loss (assumption)** – the expected catastrophe loss and LAE dollars are shown as the average. The scenarios for cat loss came from the output of a cat model, manipulated to not reveal any real information. Again, the distribution of cat losses is the same in all examples, just the mean has changed.
- **Loss Lag and Cat Lag (assumption)** – represent the average payment date for the two loss components. A common value of 1.0 years is used for both loss components in these examples.
- **Yield (assumption)** – The yield is the average LIBOR yield for the period of time and duration assumed for investing the flows. A complete discussion of interest rates to use in modeling is beyond the scope of this paper. Suffice it to say that the use of LIBOR to

represent risk-free rates of return is common in the investment and finance community. And that practice has been adopted here. This model can be expanded to look at portfolio yields, but additional capital would be needed to address the increased risk in such a portfolio.

- **Loss Liab and Cat Liab (formula)** – These are the present value of the balance sheet liabilities for noncat loss and LAE and catastrophe loss and LAE, respectively. The formula is shown below:

  $$\text{Liabilities} = \text{Loss} * [1 / (1 + \text{Yield(after-tax)}) \wedge \text{Lag}\,] / (\text{Yield (after-tax)}) \quad (7.1)$$

- **Tax Rate** – Not shown on the exhibits. A 35% tax rate is used in these examples.
- **Net Liab (formula)** – the sum of the loss and cat liabilities. This is the present value of the total balance sheet liabilities. Since both components are on a present value basis, they can be added even if the lags are different.
- **UW Inc (formula)** – The underwriting profit which is the premium minus the sum of expenses, noncatastrophe losses and catastrophe losses, adjusted for taxes.
- **UW Inv Inc (formula)** – this is the investment income on the operating cash flows.

  $$\text{UW Inv Inc} = \text{Net Liab} * \text{Yield (After-tax)} \quad (7.2)$$

- **Tot Inc (Formula)** – Total income, the sum of UW Inc and UW Inv Inc
- **ESD (formula)** – expected surplus drawdown. If the total income is negative, this is the complement of the income. It is zero if the income is positive. In other words, it is the amount of the loss when there is a loss. The average ESD is the risk metric used in calculating RCR. It can also be determined as follows:

  $$\text{Risk} = E(\text{ESD}) = -\Pr(\text{Tot Inc} < 0) * E(\text{Tot Inc} \mid \text{Tot Inc} < 0) \quad (7.3)$$

- **RCR (formula) –** Risk Coverage Ratio. Ratio of Total Income to risk, or:

  $$\text{RCR} = \text{Tot Inc} / E(\text{ESD}) \quad (7.4)$$

- **Target ROS (assumption) –** this is the return on surplus that the company is targeting.
- **Surplus (formula)** – This is the surplus required by the line to translate the operating return (Tot Inc) to the target ROS. It is determined using formula 6.2, restated below using the variable names from these exhibits.

$$\text{Surplus} = \text{Tot Inc} / (\text{Target ROS} - \text{Yield(after-tax)}) \tag{7.5}$$

7.2 RMK Exhibits

The exhibits that demonstrate the application of the RMK algorithm use a common format and set of formulas. They build on information in the corresponding RCR exhibit and are numbered as such. The three exhibits included are 2-2, 3A-2, and 3B-2.

RMK requires a set of weights that can be based on any underlying view of risk. The weights are normalized to sum to 1.0, termed the Z-vector in the paper. There is no requirement as to what sort of risk preference is used to determine the initial weights. In this paper, a simple set of weights is used so all operating losses get the same weight of 1 + (1/RCR), and the positive results are assigned a small weight of (1/RCR).

To start, the premium is apportioned to the components of expense, catastrophe loss and noncatastrophe loss. Then the underwriting gain/loss from each component for each scenario is determined. (Note that since there is no expense volatility in these examples, the expense component drops out and is not shown.) Next the deviation of the investment income for the scenario from the expected is determined. These two pieces are combined to determine the operating gain contribution for the scenario from each component. These figures are used to allocate the risk charge and then the surplus to the components.

<u>**Items**</u>

- **Ave ROE (assumption)** – This is the target ROS from the RCR exhibit.
- **Surplus II (assumption)** – Investment Income (II) on Surplus. This is the yield adjusted to after-tax..
- **Avg Op Rtn (formula)** – Average Operating Return. Viewed as either the Avg ROE minus the Surplus II, or can be calculated from the RCR exhibit as the average Tot Inc divided by the average Net Liab.
- **Surplus (assumption)** – this is the figure determined in the RCR exhibit.
- **Risk Chg (assumption)** – Risk Charge. This is average Tot Inc from the RCR exhibit.

Here it is being viewed as the amount of return required to cover the risk, or as the risk charge.

  - **Total Op Gain (assumption)** – This is the Tot Inc from the RCR exhibit, reproduced here.

- **Crude Weights (formula)** – This is the initial set of weights, before normalization, used in the RMK formula. In these examples, the weights are 1/RCR if the income is positive, and (1 + 1/RCR) if the income is negative. This puts more weight on the loss scenarios. This represents a simple utility function.
- **Z (formula)** – this is the Z-vector referred to in the Ruhm-Mango paper. It is a normalized set of weights calculated as the Crude Weight / E(Crude Weight).
- **Prem Split (formula)** – In order to determine the contribution to the underwriting gain or loss, the premium needs to be split into component. The split here is based on the average cost for each item. Since expenses do not vary, they are not relevant and the portion of premium for expense is not shown. The calculation is very insensitive to the premium split. However, it is easier to interpret the calculations if a reasonable split of the premium is used initially.

  The formula for loss, with cat loss being similar, is:

  $$\text{Prem Split} = \text{Premium} * E(\text{Loss}) \,/\, (E(\text{exp}) + E(\text{loss}) + E(\text{cat loss})) \qquad (7.6)$$

The next items are the six columns. Then the formulas used to calculate across each column will be covered.

- **Loss xCat UW Gain (formula)** – This the contribution of the noncatastrophe loss portion to the UW gain or loss. It is calculated as the difference between the premium split for loss and the loss for the scenario, adjusted for taxes.
- **Cat Loss UW Gain (formula)** – Similar to the above, this is the contribution of the cat loss to the UW gain or loss. It is calculated in the difference between the premium split for loss and the loss for the scenario, adjusted for taxes.
- **Loss xCat Inv Gain/Cat Loss Inv Gain (formula)** – This is the contribution to investment income from the scenario. It is the liability times the yield adjusted for taxes. When there is an underwriting loss, this serves as an offset.
- **Loss xCat Op Gain/Cat Loss Op Gain (formula)** – This is the sum of the UW gain or loss plus the investment income for the component.

The following items are calculated for each column, or component. There are two risk factors, noncatastrophe loss and catastrophe loss, in three different levels – underwriting gain/loss, investment gain and total gain/loss.

- **E(R) (formula)** – Average value for the column, or the risk contribution for the component.
- **E(ZR) (formula)** – Average value of the product of the weight (Z) times the risk contribution, or contribution, for each component.
- **Risk Chg (formula)** – the risk charge for the component, which is E(R) minus E (ZR). This is also the contribution to the average operating return from the component, so the sum across all components will equal the average operating return.
- **Surplus (formula)** – The surplus required for that component. This is calculated using the following formula:

$$\text{Surplus} = \text{Risk Chg} \,/\, \text{Avg Op Return (total)} \tag{7.6}$$

- **Avg Op Rtn (formula)** – Average Operating Return, is calculated for each item as the risk charge, divided by surplus. Given the formula used to get surplus, it will be equal to the average operating return for the total. So, the average operating return for each component should be the same as the overall average operating return, and this acts as a cross check.
- **Surplus II (assumption)** – the investment income on surplus. This is the same as the yield that was used in the total.
- **Avg ROE (formula)** – average ROE is the sum of the Avg Op Rtn and Surplus II.
- **Tot Und/Tot Inv (formula)** – The sum of the surplus for the risk components, noncatastrophe loss and catastrophe loss, at the underwriting and investment level respectively. Note that investment surplus is negative, as it acts to offset the positive surplus for underwriting.
    - **Leverage Ratios (formula)** – The two leverage ratios are shown, which are ratios of liabilities to surplus. The liabilities from the underlying RCR exhibit for the component are divided by the total surplus for that component. These leverage ratios are then used in other models as the expected catastrophe and noncatastrophe losses vary.

The final section in these exhibits is a summary, and shows how the underwriting profit for the component would be derived.

- **Surplus, Yield and Op Income** – are repeats of the items from earlier columns, shown here to see what is used in the following calculations.
- **Op Inv Inc (formula)** – investment income on operating cash flows. This is the liabilities for the component times the yield.

- **UW Income (formula)** – underwriting income, it is the operating income minus the operating investment income. This split shows the composition of the underwriting profit margin.

Sample Calculations

For Row 1 on Exhibit 2

Premium	174.12		
	Dollars	**Ratio to Premium**	
Loss	57.65	33.1%	
Cat Loss	19.62	11.3%	
Expense		30.0%	
Combined Ratio		74.4%	
	Pre-Tax	**Post-Tax**	
Yield	5.04%	3.28%	
Loss Liabilities	55.83		Loss * (1 - 1/(1+ post-tax-yield))/post-tax-yield
Cat Loss Liabilities	19.00		Cat Loss * (1 - 1/(1+ post-tax-yield))/post-tax-yield
Net Liabilities	74.82		Sum of Loss Liabilities and Cat Loss Liabilities
Underwriting Income	29.00		[Premium - (Loss + Cat Loss + Expense)] * (1 - tax rate)
UW Investment Income	2.45		Net Liabilities * post-tax-yield
Total Income	31.45		Sum of UW Income and UW Investment Income

Exhibit 1A

Premium	90.85	Loss Lag	1.000	RCR	20.00
Combined	68.5%	Cat Lag	1.000		
		Yield	5.04%	Target ROS	15.0%
				Surplus	168.02

	Exp	Loss	Cat Loss	Loss Liab	Cat Liab	Net Liab	UW Inc	UW Inv Inc	Tot Inc	ESD
Averages	**0.30**	**0.00**	**35.00**	**0.0**	**33.9**	**33.9**	**18.59**	**1.11**	**19.70**	**0.98**
1	0.30	0.00	19.62	0.0	19.0	19.0	28.59	0.62	29.21	0.00
2	0.30	0.00	90.84	0.0	88.0	88.0	-17.70	2.88	-14.82	14.82
3	0.30	0.00	32.87	0.0	31.8	31.8	19.97	1.04	21.02	0.00
4	0.30	0.00	25.02	0.0	24.2	24.2	25.08	0.79	25.87	0.00
5	0.30	0.00	38.92	0.0	37.7	37.7	16.04	1.23	17.28	0.00
6	0.30	0.00	24.34	0.0	23.6	23.6	25.52	0.77	26.29	0.00
7	0.30	0.00	25.60	0.0	24.8	24.8	24.70	0.81	25.51	0.00
8	0.30	0.00	35.69	0.0	34.6	34.6	18.14	1.13	19.27	0.00
9	0.30	0.00	59.80	0.0	57.9	57.9	2.47	1.90	4.37	0.00
10	0.30	0.00	10.41	0.0	10.1	10.1	34.57	0.33	34.90	0.00
11	0.30	0.00	17.18	0.0	16.6	16.6	30.17	0.54	30.72	0.00
12	0.30	0.00	43.38	0.0	42.0	42.0	13.14	1.38	14.52	0.00
13	0.30	0.00	28.26	0.0	27.4	27.4	22.97	0.90	23.87	0.00
14	0.30	0.00	29.70	0.0	28.8	28.8	22.03	0.94	22.98	0.00
15	0.30	0.00	67.40	0.0	65.3	65.3	-2.47	2.14	-0.34	0.34
16	0.30	0.00	25.96	0.0	25.1	25.1	24.47	0.82	25.29	0.00
17	0.30	0.00	23.56	0.0	22.8	22.8	26.02	0.75	26.77	0.00
18	0.30	0.00	94.33	0.0	91.3	91.3	-19.98	2.99	-16.98	16.98
19	0.30	0.00	15.15	0.0	14.7	14.7	31.49	0.48	31.97	0.00
20	0.30	0.00	55.90	0.0	54.1	54.1	5.01	1.77	6.78	0.00
21	0.30	0.00	20.10	0.0	19.5	19.5	28.27	0.64	28.91	0.00
22	0.30	0.00	46.00	0.0	44.5	44.5	11.44	1.46	12.90	0.00
23	0.30	0.00	36.39	0.0	35.2	35.2	17.69	1.15	18.84	0.00
24	0.30	0.00	34.91	0.0	33.8	33.8	18.65	1.11	19.76	0.00
25	0.30	0.00	19.65	0.0	19.0	19.0	28.56	0.62	29.19	0.00

Exhibit 1B

Premium	97.32	Loss Lag	1.000	RCR	20.00
Combined	91.7%	Cat Lag	1.000		
		Yield	5.04%	Target ROS	15.0%
Cat Premiu	90.85			Surplus	61.29
(From Exh. 1A)					
Total Premium	188.18				

	Exp	Loss	Cat Loss	Loss Liab	Cat Liab	Net Liab	UW Inc	UW Inv Inc	Tot Inc	ESD
Averages	**0.30**	**60.00**	**0.00**	**58.1**	**0.0**	**58.1**	**5.28**	**1.90**	**7.19**	**0.36**
1	0.30	57.65	0.00	55.8	0.0	55.8	6.81	1.83	8.64	0.00
2	0.30	56.99	0.00	55.2	0.0	55.2	7.24	1.81	9.04	0.00
3	0.30	58.41	0.00	56.6	0.0	56.6	6.32	1.85	8.17	0.00
4	0.30	49.37	0.00	47.8	0.0	47.8	12.19	1.57	13.76	0.00
5	0.30	46.47	0.00	45.0	0.0	45.0	14.08	1.47	15.55	0.00
6	0.30	69.27	0.00	67.1	0.0	67.1	-0.74	2.20	1.45	0.00
7	0.30	54.62	0.00	52.9	0.0	52.9	8.78	1.73	10.51	0.00
8	0.30	54.38	0.00	52.7	0.0	52.7	8.94	1.72	10.66	0.00
9	0.30	71.10	0.00	68.8	0.0	68.8	-1.94	2.26	0.32	0.00
10	0.30	57.52	0.00	55.7	0.0	55.7	6.89	1.82	8.72	0.00
11	0.30	75.92	0.00	73.5	0.0	73.5	-5.06	2.41	-2.66	2.66
12	0.30	64.95	0.00	62.9	0.0	62.9	2.07	2.06	4.13	0.00
13	0.30	58.29	0.00	56.4	0.0	56.4	6.40	1.85	8.25	0.00
14	0.30	63.31	0.00	61.3	0.0	61.3	3.13	2.01	5.14	0.00
15	0.30	68.40	0.00	66.2	0.0	66.2	-0.18	2.17	1.99	0.00
16	0.30	64.73	0.00	62.7	0.0	62.7	2.21	2.05	4.26	0.00
17	0.30	63.56	0.00	61.5	0.0	61.5	2.97	2.02	4.99	0.00
18	0.30	50.21	0.00	48.6	0.0	48.6	11.65	1.59	13.24	0.00
19	0.30	46.21	0.00	44.7	0.0	44.7	14.25	1.47	15.71	0.00
20	0.30	68.67	0.00	66.5	0.0	66.5	-0.36	2.18	1.82	0.00
21	0.30	38.50	0.00	37.3	0.0	37.3	19.26	1.22	20.48	0.00
22	0.30	71.73	0.00	69.5	0.0	69.5	-2.34	2.28	-0.06	0.06
23	0.30	56.71	0.00	54.9	0.0	54.9	7.42	1.80	9.22	0.00
24	0.30	57.03	0.00	55.2	0.0	55.2	7.21	1.81	9.02	0.00
25	0.30	79.04	0.00	76.5	0.0	76.5	-7.10	2.51	-4.59	4.59

Exhibit 2

Premium	174.12	Loss Lag	1.000	RCR	20.00
Combined	84.6%	Cat Lag	1.000		
		Yield	5.04%	Target ROS	15.0%
Prior Premium	188.18			Surplus	174.77
Diversification Benefit	14.05				

	Exp	Loss	Cat Loss	Loss Liab	Cat Liab	Net Liab	UW Inc	UW Inv Inc	Tot Inc	ESD
Averages	**0.30**	**60.00**	**35.00**	**58.1**	**33.9**	**92.0**	**17.48**	**3.01**	**20.49**	**1.02**
1	0.30	57.65	19.62	55.8	19.0	74.8	29.00	2.45	31.45	0.00
2	0.30	56.99	90.84	55.2	88.0	143.1	-16.86	4.69	-12.17	12.17
3	0.30	58.41	32.87	56.6	31.8	88.4	19.90	2.90	22.79	0.00
4	0.30	49.37	25.02	47.8	24.2	72.0	30.88	2.36	33.24	0.00
5	0.30	46.47	38.92	45.0	37.7	82.7	23.73	2.71	26.44	0.00
6	0.30	69.27	24.34	67.1	23.6	90.6	18.38	2.97	21.35	0.00
7	0.30	54.62	25.60	52.9	24.8	77.7	27.08	2.54	29.63	0.00
8	0.30	54.38	35.69	52.7	34.6	87.2	20.68	2.86	23.54	0.00
9	0.30	71.10	59.80	68.8	57.9	126.8	-5.86	4.15	-1.71	1.71
10	0.30	57.52	10.41	55.7	10.1	65.8	35.07	2.15	37.23	0.00
11	0.30	75.92	17.18	73.5	16.6	90.1	18.72	2.95	21.67	0.00
12	0.30	64.95	43.38	62.9	42.0	104.9	8.81	3.44	12.25	0.00
13	0.30	58.29	28.26	56.4	27.4	83.8	22.97	2.75	25.72	0.00
14	0.30	63.31	29.70	61.3	28.8	90.1	18.77	2.95	21.72	0.00
15	0.30	68.40	67.40	66.2	65.3	131.5	-9.05	4.31	-4.74	4.74
16	0.30	64.73	25.96	62.7	25.1	87.8	20.28	2.88	23.16	0.00
17	0.30	63.56	23.56	61.5	22.8	84.4	22.60	2.76	25.36	0.00
18	0.30	50.21	94.33	48.6	91.3	140.0	-14.72	4.58	-10.14	10.14
19	0.30	46.21	15.15	44.7	14.7	59.4	39.34	1.95	41.29	0.00
20	0.30	68.67	55.90	66.5	54.1	120.6	-1.74	3.95	2.21	0.00
21	0.30	38.50	20.10	37.3	19.5	56.7	41.14	1.86	42.99	0.00
22	0.30	71.73	46.00	69.5	44.5	114.0	2.71	3.73	6.44	0.00
23	0.30	56.71	36.39	54.9	35.2	90.1	18.72	2.95	21.67	0.00
24	0.30	57.03	34.91	55.2	33.8	89.0	19.47	2.92	22.38	0.00
25	0.30	79.04	19.65	76.5	19.0	95.6	15.07	3.13	18.20	0.00

Exhibit 3A

Premium	264.07	Loss Lag	1.000	RCR	20.00
Combined	79.2%	Cat Lag	1.000		
		Yield	5.04%	Target ROS	15.0%
Prior Premium	279.03			Surplus	339.27
Diversification	14.96				

	Exp	Loss	Cat Loss	Loss Liab	Cat Liab	Net Liab	UW Inc	UW Inv Inc	Tot Inc	ESD
Averages	**0.30**	**60.00**	**70.00**	**58.1**	**67.8**	**125.9**	**35.65**	**4.12**	**39.78**	**1.99**
1	0.30	57.65	39.24	55.8	38.0	93.8	57.17	3.07	60.24	0.00
2	0.30	56.99	181.67	55.2	175.9	231.1	-34.98	7.57	-27.41	27.41
3	0.30	58.41	65.74	56.6	63.7	120.2	39.46	3.94	43.40	0.00
4	0.30	49.37	50.03	47.8	48.4	96.2	55.54	3.15	58.69	0.00
5	0.30	46.47	77.83	45.0	75.4	120.4	39.36	3.94	43.30	0.00
6	0.30	69.27	48.67	67.1	47.1	114.2	43.49	3.74	47.23	0.00
7	0.30	54.62	51.20	52.9	49.6	102.5	51.37	3.36	54.73	0.00
8	0.30	54.38	71.38	52.7	69.1	121.8	38.41	3.99	42.40	0.00
9	0.30	71.10	119.60	68.8	115.8	184.7	-3.81	6.05	2.24	0.00
10	0.30	57.52	20.82	55.7	20.2	75.9	69.23	2.48	71.72	0.00
11	0.30	75.92	34.35	73.5	33.3	106.8	48.48	3.50	51.97	0.00
12	0.30	64.95	86.76	62.9	84.0	146.9	21.54	4.81	26.35	0.00
13	0.30	58.29	56.51	56.4	54.7	111.2	45.53	3.64	49.17	0.00
14	0.30	63.31	59.40	61.3	57.5	118.8	40.39	3.89	44.29	0.00
15	0.30	68.40	134.81	66.2	130.5	196.8	-11.94	6.45	-5.49	5.49
16	0.30	64.73	51.91	62.7	50.3	112.9	44.33	3.70	48.03	0.00
17	0.30	63.56	47.13	61.5	45.6	107.2	48.21	3.51	51.72	0.00
18	0.30	50.21	188.66	48.6	182.7	231.3	-35.11	7.58	-27.54	27.54
19	0.30	46.21	30.30	44.7	29.3	74.1	70.42	2.43	72.85	0.00
20	0.30	68.67	111.79	66.5	108.2	174.7	2.85	5.72	8.57	0.00
21	0.30	38.50	40.20	37.3	38.9	76.2	69.00	2.50	71.49	0.00
22	0.30	71.73	91.99	69.5	89.1	158.5	13.73	5.19	18.93	0.00
23	0.30	56.71	72.78	54.9	70.5	125.4	35.99	4.11	40.10	0.00
24	0.30	57.03	69.81	55.2	67.6	122.8	37.70	4.02	41.73	0.00
25	0.30	79.04	39.31	76.5	38.1	114.6	43.22	3.75	46.98	0.00

Exhibit 3B

Premium	130.92	Loss Lag	1.000	RCR	20.00
Combined	89.2%	Cat Lag	1.000		
		Yield	5.04%	Target ROS	15.0%
Prior Premium	142.75			Surplus	99.39
Diversification	11.83				

	Exp	Loss	Cat Loss	Loss Liab	Cat Liab	Net Liab	UW Inc	UW Inv Inc	Tot Inc	ESD
Averages	**0.30**	**60.00**	**17.50**	**58.1**	**16.9**	**75.0**	**9.19**	**2.46**	**11.65**	**0.58**
1	0.30	57.65	9.81	55.8	9.5	65.3	15.72	2.14	17.86	0.00
2	0.30	56.99	45.42	55.2	44.0	99.2	-7.00	3.25	-3.75	3.75
3	0.30	58.41	16.43	56.6	15.9	72.5	10.92	2.37	13.30	0.00
4	0.30	49.37	12.51	47.8	12.1	59.9	19.35	1.96	21.31	0.00
5	0.30	46.47	19.46	45.0	18.8	63.8	16.72	2.09	18.81	0.00
6	0.30	69.27	12.17	67.1	11.8	78.9	6.63	2.58	9.22	0.00
7	0.30	54.62	12.80	52.9	12.4	65.3	15.74	2.14	17.88	0.00
8	0.30	54.38	17.85	52.7	17.3	69.9	12.62	2.29	14.91	0.00
9	0.30	71.10	29.90	68.8	29.0	97.8	-6.08	3.20	-2.88	2.88
10	0.30	57.52	5.20	55.7	5.0	60.7	18.80	1.99	20.79	0.00
11	0.30	75.92	8.59	73.5	8.3	81.8	4.64	2.68	7.32	0.00
12	0.30	64.95	21.69	62.9	21.0	83.9	3.25	2.75	6.00	0.00
13	0.30	58.29	14.13	56.4	13.7	70.1	12.50	2.30	14.80	0.00
14	0.30	63.31	14.85	61.3	14.4	75.7	8.77	2.48	11.25	0.00
15	0.30	68.40	33.70	66.2	32.6	98.9	-6.80	3.24	-3.56	3.56
16	0.30	64.73	12.98	62.7	12.6	75.2	9.06	2.47	11.52	0.00
17	0.30	63.56	11.78	61.5	11.4	73.0	10.60	2.39	12.99	0.00
18	0.30	50.21	47.17	48.6	45.7	94.3	-3.72	3.09	-0.63	0.63
19	0.30	46.21	7.58	44.7	7.3	52.1	24.61	1.71	26.32	0.00
20	0.30	68.67	27.95	66.5	27.1	93.6	-3.24	3.06	-0.17	0.17
21	0.30	38.50	10.05	37.3	9.7	47.0	28.01	1.54	29.55	0.00
22	0.30	71.73	23.00	69.5	22.3	91.7	-2.00	3.00	1.00	0.00
23	0.30	56.71	18.19	54.9	17.6	72.5	10.88	2.38	13.26	0.00
24	0.30	57.03	17.45	55.2	16.9	72.1	11.15	2.36	13.51	0.00
25	0.30	79.04	9.83	76.5	9.5	86.1	1.80	2.82	4.62	0.00

Exhibit 2-2

Avg ROE	15.00%
Surplus II	3.28%
Avg Op Rtn	11.72%
Surplus	**174.77**
Risk Chg	20.49

Leverage Ratios

	Tot Und	**183.74**	Tot Inv	**(8.97)**	4.00	0.2115
Avg ROE	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%
Surplus II	3.28%	3.28%	3.28%	3.28%	3.28%	3.28%
Avg Op Rtn	11.72%	11.72%	11.72%	11.72%	11.72%	11.72%
Surplus	**15.28**	**168.46**	**(0.75)**	**(8.22)**	**14.53**	**160.24**
Risk Chg	1.791	19.750	-0.087	-0.964	1.70	18.79
E(ZR)	5.331	-15.596	1.991	2.074	7.322	-13.522
E(R)	7.122	4.154	1.903	1.110	9.025	5.265
Prem Split	70.957	41.391				

	Non-Cat	Cat
Surplus	14.53	160.24
Yield	3.28%	3.28%
Op Income	1.70	18.79
Op Inv Inc	1.90	1.11
UW Income	(0.20)	17.68
Pre-tax Margin	(0.31)	27.19

	Total Op Gain	0.12280 Crude Weights	Z	Loss xCat UW Gain	Cat Loss UW Gain	Loss xCat Inv Gain	Cat Loss Inv Gain	Loss xCat Op Gain	Cat Loss Op Gain
1	31.4	0.050	0.407	8.647	14.152	1.829	0.622	10.475	14.774
2	-12.2	1.050	8.550	9.075	-32.139	1.808	2.881	10.883	-29.258
3	22.8	0.050	0.407	8.159	5.540	1.853	1.043	10.011	6.583
4	33.2	0.050	0.407	14.034	10.643	1.566	0.794	15.600	11.437
5	26.4	0.050	0.407	15.917	1.609	1.474	1.234	17.391	2.843
6	21.4	0.050	0.407	1.095	11.085	2.197	0.772	3.292	11.857
7	29.6	0.050	0.407	10.616	10.266	1.733	0.812	12.349	11.078
8	23.5	0.050	0.407	10.776	3.705	1.725	1.132	12.501	4.837
9	-1.7	1.050	8.550	-0.096	-11.966	2.255	1.897	2.160	-10.069
10	37.2	0.050	0.407	8.733	20.139	1.825	0.330	10.558	20.469
11	21.7	0.050	0.407	-3.225	15.740	2.408	0.545	-0.817	16.285
12	12.2	0.050	0.407	3.905	-1.292	2.060	1.376	5.965	0.084
13	25.7	0.050	0.407	8.236	8.537	1.849	0.896	10.085	9.434
14	21.7	0.050	0.407	4.972	7.600	2.008	0.942	6.981	8.542
15	-4.7	1.050	8.550	1.659	-16.908	2.170	2.138	3.829	-14.770
16	23.2	0.050	0.407	4.045	10.033	2.053	0.823	6.099	10.857
17	25.4	0.050	0.407	4.809	11.588	2.016	0.747	6.825	12.335
18	-10.1	1.050	8.550	13.488	-34.412	1.593	2.992	15.080	-31.419
19	41.3	0.050	0.407	16.087	17.056	1.466	0.481	17.553	17.536
20	2.2	0.050	0.407	1.483	-9.427	2.178	1.773	3.662	-7.654
21	43.0	0.050	0.407	21.096	13.839	1.221	0.638	22.317	14.477
22	6.4	0.050	0.407	-0.500	-2.994	2.275	1.459	1.775	-1.535
23	21.7	0.050	0.407	9.263	3.252	1.799	1.154	11.062	4.407

Exhibit 3A-2

Avg ROE	15.00%
Surplus II	3.28%
Avg Op Rtn	11.72%
Surplus	**339.27**
Risk Chg	39.78

	Tot Und	**356.67**	Tot Inv	**(17.41)**	**Leverage Ratios** 9.24	0.2036
Avg ROE	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%
Surplus II	3.28%	3.28%	3.28%	3.28%	3.28%	3.28%
Avg Op Rtn	11.72%	11.72%	11.72%	11.72%	11.72%	11.72%
Surplus	**6.61**	**350.07**	**(0.32)**	**(17.08)**	**6.28**	**332.98**
Risk Chg	0.775	41.042	-0.038	-2.003	0.74	39.04
E(ZR)	9.450	-29.113	1.941	4.223	11.391	-24.890
E(R)	10.224	11.928	1.903	2.220	12.127	14.149
Prem Split	75.729	88.351				

	Non-Cat	Cat
Surplus	6.28	332.98
Yield	3.28%	3.28%
Op Income	0.74	39.04
Op Inv Inc	1.90	2.22
UW Income	(1.17)	36.82
Pre-tax Margin	(1.79)	56.64

0.12190

	Total Op Gain	Crude Weights	Z	LxC UW Gain	Cat Loss UW Gain	Loss xCat Inv Gain	Cat Loss Inv Gain	Loss xCat Op Gain	Cat Loss Op Gain
1	60.2	0.050	0.410	11.749	31.923	1.829	1.245	13.578	33.167
2	-27.4	1.050	8.614	12.178	-60.659	1.808	5.763	13.986	-54.896
3	43.4	0.050	0.410	11.261	14.700	1.853	2.085	13.114	16.785
4	58.7	0.050	0.410	17.136	24.906	1.566	1.587	18.702	26.493
5	43.3	0.050	0.410	19.020	6.837	1.474	2.469	20.494	9.306
6	47.2	0.050	0.410	4.197	25.790	2.197	1.544	6.395	27.334
7	54.7	0.050	0.410	13.719	24.151	1.733	1.624	15.451	25.775
8	42.4	0.050	0.410	13.879	11.029	1.725	2.264	15.603	13.293
9	2.2	0.050	0.410	3.007	-20.312	2.255	3.794	5.262	-16.518
10	71.7	0.050	0.410	11.835	43.897	1.825	0.660	13.660	44.557
11	52.0	0.050	0.410	-0.122	35.100	2.408	1.090	2.286	36.189
12	26.4	0.050	0.410	7.007	1.034	2.060	2.752	9.068	3.787
13	49.2	0.050	0.410	11.339	20.694	1.849	1.793	13.188	22.487
14	44.3	0.050	0.410	8.075	18.820	2.008	1.884	10.083	20.704
15	-5.5	1.050	8.614	4.761	-30.197	2.170	4.276	6.931	-25.920
16	48.0	0.050	0.410	7.148	23.686	2.053	1.647	9.201	25.333
17	51.7	0.050	0.410	7.911	26.795	2.016	1.495	9.928	28.290
18	-27.5	1.050	8.614	16.590	-65.204	1.593	5.985	18.183	-59.219
19	72.8	0.050	0.410	19.190	37.731	1.466	0.961	20.656	38.692
20	8.6	0.050	0.410	4.585	-15.235	2.178	3.546	6.764	-11.689
21	71.5	0.050	0.410	24.198	31.298	1.221	1.275	25.420	32.573
22	18.9	0.050	0.410	2.603	-2.368	2.275	2.918	4.878	0.550
23	40.1	0.050	0.410	12.365	10.124	1.799	2.309	14.164	12.433

Exhibit 3B-2

Avg ROE	15.00%
Surplus II	3.28%
Avg Op Rtn	11.72%
Surplus	**99.39**
Risk Chg	11.65

					Leverage Ratios	
	Tot Und	**104.49**	Tot Inv	**(5.10)**	1.72	0.2586
Avg ROE	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%
Surplus II	3.28%	3.28%	3.28%	3.28%	3.28%	3.28%
Avg Op Rtn	11.72%	11.72%	11.72%	11.72%	11.72%	11.72%
Surplus	**35.61**	**68.87**	**(1.74)**	**(3.36)**	**33.87**	**65.51**
Risk Chg	4.175	8.075	-0.204	-0.394	3.97	7.68
E(ZR)	0.549	-6.697	2.107	0.949	2.656	-5.748
E(R)	4.724	1.378	1.903	0.555	6.627	1.933
Prem Split	67.267	19.620				

	Non-Cat	Cat
Surplus	33.87	65.51
Yield	3.28%	3.28%
Op Income	3.97	7.68
Op Inv Inc	1.90	0.56
UW Income	2.07	7.13
Pre-tax Margin	3.18	10.96

		0.13420							
	Total Op Gain	Crude Weights	Z	LxC UW Gain	Cat Loss UW Gain	Loss xCat Inv Gain	Cat Loss Inv Gain	Loss xCat Op Gain	Cat Loss Op Gain
1	17.9	0.050	0.373	6.249	6.376	1.829	0.311	8.077	6.688
2	-3.8	1.050	7.824	6.677	-16.769	1.808	1.441	8.485	-15.328
3	13.3	0.050	0.373	5.761	2.071	1.853	0.521	7.613	2.592
4	21.3	0.050	0.373	11.636	4.622	1.566	0.397	13.202	5.019
5	18.8	0.050	0.373	13.519	0.105	1.474	0.617	14.993	0.722
6	9.2	0.050	0.373	-1.303	4.843	2.197	0.386	0.894	5.229
7	17.9	0.050	0.373	8.218	4.434	1.733	0.406	9.951	4.840
8	14.9	0.050	0.373	8.378	1.153	1.725	0.566	10.103	1.719
9	-2.9	1.050	7.824	-2.494	-6.682	2.255	0.948	-0.238	-5.734
10	20.8	0.050	0.373	6.335	9.370	1.825	0.165	8.160	9.535
11	7.3	0.050	0.373	-5.623	7.171	2.408	0.272	-3.215	7.443
12	6.0	0.050	0.373	1.507	-1.346	2.060	0.688	3.567	-0.658
13	14.8	0.050	0.373	5.838	3.569	1.849	0.448	7.687	4.017
14	11.2	0.050	0.373	2.574	3.101	2.008	0.471	4.583	3.572
15	-3.6	1.050	7.824	-0.739	-9.153	2.170	1.069	1.431	-8.084
16	11.5	0.050	0.373	1.647	4.317	2.053	0.412	3.701	4.729
17	13.0	0.050	0.373	2.411	5.094	2.016	0.374	4.427	5.468
18	-0.6	1.050	7.824	11.090	-17.905	1.593	1.496	12.682	-16.409
19	26.3	0.050	0.373	13.689	7.828	1.466	0.240	15.155	8.069
20	-0.2	1.050	7.824	-0.915	-5.413	2.178	0.887	1.264	-4.527
21	29.6	0.050	0.373	18.698	6.220	1.221	0.319	19.919	6.539
22	1.0	0.050	0.373	-2.898	-2.196	2.275	0.730	-0.623	-1.467
23	13.3	0.050	0.373	6.865	0.927	1.799	0.577	8.664	1.504

Exhibit 4

	Exh. 2	Exhibit 3A			Exhibit 3B		
	Base	**Estimate**	**Actual**	**Difference**	**Estimate**	**Actual**	**Difference**
Surplus	174.77	335.01	339.27	-1.3%	94.65	99.39	-4.8%
Yield	3.28%	3.28%	3.28%		3.28%	3.28%	
Op Income	20.49	39.28	39.78	-1.3%	11.10	11.65	-4.8%
OP Inv Inc	3.01	4.12	4.12	0.0%	2.46	2.46	0.0%
UW Income	17.48	35.15	35.65	-1.4%	8.64	9.19	-6.0%
Pre-tax Margin	26.89	54.08	54.85		13.29	14.14	
ROE w/estimate		14.85%			14.44%		

Comparison of Minimum Bias and Maximum Likelihood Methods for Claim Severity

Noriszura Ismail, Ph.D. & Abdul Aziz Jemain, Ph.D.

Abstract
The objective of this study is to compare the methods of minimum bias and maximum likelihood by using a weighted equation on claim severity data. The advantage of using the weighted equation is that the fitting procedure provides a faster convergence compared to the classical procedure introduced by Bailey and Simon [1] and Bailey [2]. Furthermore, the fitting procedure may be extended to other models in addition to the multiplicative and additive models, as long as the function of the fitted value is written in a specified linear form. In this study, the minimum bias and maximum likelihood methods will be compared and fitted on three types of claim severity data; the Malaysian data, the U.K. data from McCullagh and Nelder [3] and the Canadian data from Bailey and Simon [1].

Keywords: Minimum bias; maximum likelihood; claim severity; multiplicative; additive.

1. INTRODUCTION

The process of establishing premium rates for insuring uncertain events requires estimates which are made of two important elements; the probabilities or frequencies associated with the occurrence of insured event, and the magnitude or severities of such event. The process of grouping risks of similar risk characteristics for frequencies or severities is known as risk classification where its goal is to group homogeneous risks and charge each group a premium commensurate with the expected average loss. Failure to achieve this goal may lead to adverse selection to insureds and economic losses to insurers. The risks may be categorized according to risk or rating factors; in motor insurance for instance, driver's gender and claim experience, or vehicle's make and capacity, may be considered as rating factors.

In the last forty years, actuarial researchers suggested various statistical procedures for risk classification. For instance, Bailey and Simon [1] suggested the minimum chi-squares, Bailey [2] proposed the zero bias, Jung [4] produced a heuristic method for minimum modified chi-squares, Ajne [5] applied the method of moments also for minimum modified chi-squares, Chamberlain [6] used the weighted least squares, Coutts [7] produced the method of orthogonal weighted least squares with logit transformation, Harrington [8] suggested the maximum likelihood procedure for models with functional form, and Brown [9] proposed the bias and likelihood functions.

In the recent actuarial literature, research on risk classification methods is still continuing and developing. For example, Mildenhall [10] studied the relationship between the minimum bias and the Generalized Linear Models (GLMs), Feldblum and Brosius [11] provided the minimum bias procedures for practicing actuaries, Anderson et al. [12] provided practical insights for the GLMs analysis also for practicing actuaries, Fu and Wu [13] developed and generalized the minimum bias models, Ismail and Jemain [14] bridged the minimum bias and maximum likelihood methods for claim frequency data, and Ismail and Jemain [15] suggested the Negative Binomial and the Generalized Poisson regressions as alternatives to handle over-dispersion in claim frequency or count data.

The objective of this study is to compare the methods of minimum bias and maximum likelihood by using a weighted equation on claim severity data. Although the weighted equation was previously suggested by Ismail and Jemain [14], the application was implemented on claim frequency data. Therefore, this study differs such that the weighted equation will be applied to estimate claim severity or average claim cost which is equivalent to the total claim costs divided by the number of claims. Since the nature of claim frequency and severity is different, the approach taken is also slightly modified. In fact, with a few modifications, the same weighted equation may also be used for loss cost or pure premium which is equal to the total claim costs divided by the exposures, and for loss ratio which is equal to the total claim costs divided by the premiums. However, the weight generally used for fitting loss cost and loss ratio is the exposures.

Several studies have been carried out on claim severity data in the actuarial literature. Since it is well established that the claim cost distributions generally have positive support and are positively skewed, the distributions of Gamma and Lognormal have been used by practitioners for modeling claim severities. As a comparison, several actuarial studies also reported severity results from the Normal distribution. For example, Baxter et al. [16] fit the U.K. own damage costs for privately owned and comprehensively insured vehicles to the weighted linear regression (additive model) by assuming that the variance is constant within classes, McCullagh and Nelder [3] reanalyzed the same data by fitting the Gamma regression model and assuming that the coefficient of variation is constant within classes and the mean is linear on reciprocal scale (inverse model), Brockman and Wright [17] fit the U.K. own damage costs for comprehensive policies also to the Gamma model by using a log-linear regression (multiplicative model), Renshaw [18] fit the U.K. motor insurance claim severity also to the Gamma log-linear regression model, and Fu and Moncher [19] applied several Monte Carlo simulation techniques to examine the unbiasedness and stability of the Gamma, Lognormal and Normal distributions which were fitted on the severity data obtained from Mildenhall [10].

The advantage of using the weighted equation suggested in this study is that the fitting procedure provides a faster convergence compared to the classical procedure introduced by Bailey and Simon [1] and Bailey [2]. Furthermore, the fitting procedure may be extended to other models in addition to the multiplicative and additive models, as long as the function of the fitted value is written in a specified linear form.

In this study, the minimum bias and maximum likelihood methods will be compared and fitted on three types of claim severity data; the Malaysian data, the U.K. data from McCullagh and Nelder [3] and the Canadian data from Bailey and Simon [1].

2. REGRESSION MODEL

In the actuarial literature, various methods have been studied and implemented by actuarial researchers and practitioners for classifying risks. Most of these methods, which also include the minimum bias and maximum likelihood, may be written as a regression model where the explanatory variables are the risk or rating factors. In this study, the regression methods of minimum bias and maximum likelihood will be compared and fitted on claim severity data.

The related data sets for claim severity regression model are (c_i, y_i), where c_i and y_i denotes the average claim cost already adjusted for inflation and the claim count for the ith rating class, $i = 1,2,...,n$, so that the total claim cost is equal to the product of claim count and average claim cost, $y_i c_i$. The response variable and weight for the regression model is the average claim cost, c_i, and the claim count, y_i, respectively.

Consider a regression model with n observations of average claim cost and p explanatory variables inclusive of an intercept and dummy variables. Next, consider a data of average claim costs involving three rating factors, each respectively with three, two, and three rating classes. Thus, the data has a total of $n = 18$ observed average claim costs with $p = 6$ explanatory variables.

Let $\mathbf{c}$ denotes the vector of average claim cost vector, $\mathbf{y}$ the vector of claim count, $\mathbf{X}$ the matrix of explanatory variables where the *i*th row is equivalent to vector $\mathbf{x}_\mathbf{i}^\mathbf{T}$, and $\boldsymbol{\beta}$ the vector of regression parameters. If x_{ij}, $i = 1,2,...,18$, $j = 1,2,...,6$, is the *ij*th element of matrix $\mathbf{X}$, the value for x_{ij} is either one or zero. Table 1 summarizes the regression model for the claim severity data.

Table 1. Data summary

i	c_i	y_i	x_{i1}	x_{i2}	x_{i3}	x_{i4}	x_{i5}	x_{i6}
1	c_1	y_1	1	0	0	0	0	0
2	c_2	y_2	1	0	0	0	1	0
3	$\vdots$	$\vdots$	1	0	0	0	0	1
4			1	0	0	1	0	0
5			1	0	0	1	1	0
6			1	0	0	1	0	1

7			1	1	0	0	0	0
8			1	1	0	0	1	0
9			1	1	0	0	0	1
10			1	1	0	1	0	0
11			1	1	0	1	1	0
12			1	1	0	1	0	1
13			1	0	1	0	0	0
14			1	0	1	0	1	0
15			1	0	1	0	0	1
16			1	0	1	1	0	0
17			1	0	1	1	1	0
18	c_{18}	y_{18}	1	0	1	1	0	1

Moreover, let $\mathbf{f}$, a function of $\mathbf{X}$ and $\boldsymbol{\beta}$, denote the vector of fitted average claim costs. If the function of the fitted average claim cost is log-linear (multiplicative model), the fitted value in the i th rating class is equivalent to

$$f_i = \exp(\mathbf{x}_\mathbf{i}^\mathbf{T}\boldsymbol{\beta}), \tag{1}$$

if the function is linear (additive model), the fitted average claim cost in the i th rating class is equal to

$$f_i = \mathbf{x}_\mathbf{i}^\mathbf{T}\boldsymbol{\beta}, \tag{2}$$

and if the function is inverse (inverse model), the fitted average claim cost in the i th rating class is

$$f_i = \left(\mathbf{x}_\mathbf{i}^\mathbf{T}\boldsymbol{\beta}\right)^{-1}. \tag{3}$$

In fact, a variety of regression models may be created and fitted, as long as the function of the fitted value is written as

$$f_i = \left(\sum_{j=1}^{p} \beta_j x_{ij}\right)^b, \qquad -1 \le b < 0,\ 0 < b \le 1. \tag{4}$$

Thus, the objective of risk classification is to have the fitted average claim cost, f_i, be as close as possible to the observed average claim cost, c_i, for all i.

3. MINIMUM BIAS

Bailey and Simon [1] were among the pioneer researchers that consider bias in risk classification. They introduced the minimum bias method and proposed a famous list of four criteria for an acceptable set of classification rates:

- The rates should reproduce experience for each class and overall, i.e., they should be balanced for each class and overall.
- The rates should reflect the relative credibility of various classes.
- The rates should provide minimum amount of departure from the raw data.
- The rates should produce a rate for each class close enough to the experience so that the differences could reasonably be caused by chance.

3.1 Zero Bias

Bailey and Simon [1] proposed a suitable test for the first criteria by calculating,

$$\frac{\sum_i y_i f_i}{\sum_i y_i c_i}, \tag{5}$$

for each *j* and total. Thus, a set of rates is balanced, i.e., zero bias, if Equation (5) equals 1.00 and automatically, zero bias for each class implies zero bias for all classes.

From this test, Bailey [2] derived a minimum bias model by setting the average difference between the observed and the fitted rates to be equal to zero. In the case of claim severity regression model, the zero bias equation for each *j* can be written in the form of a weighted difference between the observed and the fitted average claim cost,

$$\sum_i w_i (c_i - f_i) = 0, \qquad j = 1,2,\ldots, p, \tag{6}$$

where w_i is equal to $y_i x_{ij}$.

3.2 Minimum Chi-squares

Bailey and Simon [1] also suggested the chi-squares statistics, χ^2, as an appropriate test for the fourth criteria,

$$\chi^2 = K \sum_i \frac{y_i}{f_i}(c_i - f_i)^2,$$

where K is a constant dependent on the data. The same test is also suitable for the second and third criteria as well.

By minimizing the chi-squares, another minimum bias model was derived. For each *j*, the minimum chi-squares equation could be written in the form of a weighted difference between the observed and the fitted average claim cost,

$$\frac{\partial \chi^2}{\partial \beta_j} = \sum_i w_i (c_i - f_i) = 0, \qquad j = 1, 2, ..., p, \tag{7}$$

where w_i is $\frac{y_i(c_i + f_i)}{f_i^2} \frac{\partial f_i}{\partial \beta_j}$.

If the function is log-linear (multiplicative model), the first derivative of the fitted value is equal to

$$\frac{\partial f_i}{\partial \beta_j} = f_i x_{ij}, \tag{8}$$

if the function is linear (additive model), the first derivative is

$$\frac{\partial f_i}{\partial \beta_j} = x_{ij}, \tag{9}$$

and if the function is inverse (inverse model), the first derivative is

$$\frac{\partial f_i}{\partial \beta_j} = -f_i^2 x_{ij}. \tag{10}$$

4. MAXIMUM LIKELIHOOD

Let $T_i = y_i C_i$ be the random variable for total claim costs and assume that the *i*th total claim cost, $y_i c_i$, comes from a distribution whose probability density function is $g(c_i; f_i)$. A maximum likelihood method maximizes the likelihood function,

$$L = \prod_i g(c_i; f_i),$$

or equivalently, the log likelihood function,

$$\ell = \log L = \sum_i \log\left(g(c_i; f_i)\right).$$

Thus, the regression parameters can be obtained by setting $\frac{\partial \ell}{\partial \beta_j} = 0$ for each j, $j = 1, 2, ..., p$.

4.1 Normal

If $T_i = y_i C_i$ is assumed to follow Normal distribution with mean $E(T_i) = y_i f_i$ and variance $Var(T_i) = \sigma^2$, the probability density function is (Brown [9])

$$g(c_i; f_i) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ -\frac{1}{2\sigma^2} \left(y_i c_i - y_i f_i \right)^2 \right\}.$$

The regression parameters may be solved by using the likelihood equation

$$\frac{\partial \ell}{\partial \beta_j} = \sum_i w_i (c_i - f_i) = 0, \qquad j = 1,2,..., p, \tag{11}$$

where w_i is $y_i^2 \frac{\partial f_i}{\partial \beta_j}$. The first derivative of the fitted value is equal to equation (8) for a log-linear function (multiplicative), equation (9) for a linear function (additive), and equation (10) for an inverse function.

4.2 Poisson

If $T_i = y_i C_i$ is Poisson distributed with mean $E(T_i) = y_i f_i$, the probability density function is

$$g(c_i; f_i) = \frac{\exp(-y_i f_i)(y_i f_i)^{y_i c_i}}{(y_i c_i)!}.$$

As a result, the likelihood equation for each j is equal to

$$\frac{\partial \ell}{\partial \beta_j} = \sum_i w_i (c_i - f_i) = 0, \qquad j = 1,2,..., p, \tag{12}$$

but w_i is now equivalent to $\frac{y_i}{f_i} \frac{\partial f_i}{\partial \beta_j}$.

The same weighted equation could also be used to show that the Poisson is actually equivalent to the zero bias if the function of the fitted value is in a log-linear form (multiplicative model). By substituting Equation (8) into Equation (12), the likelihood equation for the Poisson is now equal to

$$\frac{\partial \ell}{\partial \beta_j} = \sum_i w_i (c_i - f_i) = 0, \qquad j = 1,2,..., p,$$

where w_i is $y_i x_{ij}$, and this likelihood equation is equivalent to the zero bias.

4.3 Exponential

Let $T_i = y_i C_i$ be exponential distributed with mean $E(T_i) = y_i f_i$. The probability density function is

$$g(c_i;\, f_i) = \frac{1}{y_i\, f_i} \exp\left(-\frac{c_i}{f_i}\right),$$

and the regression parameters may be solved by using the likelihood equation

$$\frac{\partial \ell}{\partial \beta_j} = \sum_i w_i (c_i - f_i) = 0, \qquad j = 1,2,\ldots, p, \tag{13}$$

where w_i is $\dfrac{1}{f_i^2} \dfrac{\partial f_i}{\partial \beta_j}$.

4.4 Gamma

If $T_i = y_i C_i$ is Gamma distributed with mean $E(T_i) = y_i\, f_i$ and variance $Var(T_i) = v^{-1} y_i^2\, f_i^{\,2}$, the probability density function is

$$g(c_i;\, f_i) = \frac{1}{y_i c_i \Gamma(v)} \left(\frac{v c_i}{f_i}\right)^{v} \exp\left(-\frac{v c_i}{f_i}\right),$$

where v denotes the index parameter. Assuming that v is allowed to vary within classes and written as $v_i = \sigma^{-2} y_i$, the likelihood equation is

$$\frac{\partial \ell}{\partial \beta_j} = \sum_i w_i (c_i - f_i) = 0, \qquad j = 1,2,\ldots, p, \tag{14}$$

where w_i is $\dfrac{y_i}{f_i^2} \dfrac{\partial f_i}{\partial \beta_j}$.

4.5 Inverse Gaussian

The derivation of the weighted equation for an Inverse Gaussian distribution is slightly different. Instead of using the random variable for total claim cost, $T_i = y_i C_i$, the random variable for average claim cost, C_i, is used. Let the random variable for average claim cost, C_i, be distributed as Inverse Gaussian with mean $E(C_i) = f_i$ and variance $Var(C_i) = f_i^{\,3} \tau$. The probability distribution function is (see Mildenhall [10] and Renshaw [18])

$$g(c_i;\, f_i) = \frac{1}{\sqrt{2\pi c_i^3 \tau}} \exp\left\{-\frac{1}{2 c_i\, f_i^{\,2} \tau} (c_i - f_i)^2\right\},$$

where τ denotes the scale parameter. If τ is allowed to vary within classes and written as $\tau_i = \sigma^2 y_i^{-1}$, the likelihood equation is

$$\frac{\partial \ell}{\partial \beta_j} = \sum_i w_i (c_i - f_i) = 0, \qquad j = 1,2,\ldots, p, \tag{15}$$

where w_i is $\dfrac{y_i}{f_i^3}\dfrac{\partial f_i}{\partial \beta_j}$.

4.6 Lognormal

The derivation of the weighted equation for a Lognormal distribution is also slightly different. Let the average claim cost, C_i, be distributed as Lognormal with parameters f_i and $\sigma^2 y_i^{-1}$. Thus, the logarithm of the average claim cost, $\log C_i$, is Normal distributed with mean f_i and variance $\sigma^2 y_i^{-1}$ and the probability density function is now equivalent to

$$g(\log c_i; f_i) = \frac{1}{c_i\sqrt{2\pi\sigma^2 y_i^{-1}}}\exp\left\{-\frac{y_i(\log c_i - f_i)^2}{2\sigma^2}\right\}.$$

The likelihood equation can be written as,

$$\frac{\partial \ell}{\partial \beta_j} = \sum_i w_i(\log c_i - f_i) = 0, \qquad j = 1,2,\ldots p. \tag{16}$$

where w_i is $y_i\dfrac{\partial f_i}{\partial \beta_j}$. Compared to the likelihood equation for other distributions shown by Equations (6), (7), (11), (12), (13), (14) and (15), the Lognormal likelihood equation is slightly different.

5. OTHER MODELS

5.1 Least Squares

The weighted equation may also be extended to other error functions as well. For example, if the sum squares error is defined as (Brown [9])

$$S = \sum_i y_i(c_i - f_i)^2,$$

the regression parameters may be solved by using the least squares equation

$$\frac{\partial S}{\partial \beta_j} = \sum_i w_i(c_i - f_i) = 0, \qquad j = 1,2,\ldots, p, \tag{17}$$

where w_i is $y_i\dfrac{\partial f_i}{\partial \beta_j}$.

The same weighted equation could also be used to show that the least squares is actually equivalent to the zero bias if the function of the fitted value is in a linear form (additive model). By substituting Equation (9) into Equation (17), the likelihood equation for the least squares is

$$\frac{\partial \ell}{\partial \beta_j} = \sum_i w_i (c_i - f_i) = 0, \qquad j = 1,2,..., p,$$

where w_i is equal to $y_i x_{ij}$, and this likelihood equation is equivalent to the zero bias.

5.2 Modified Chi-squares

If the function of error is a modified chi-squares which is defined as

$$\chi^2_{\mathrm{mod}} = \sum_i \frac{y_i}{c_i}(c_i - f_i)^2,$$

the weighted equation is equal to

$$\frac{\partial \chi^2_{\mathrm{mod}}}{\partial \beta_j} = \sum_i w_i (c_i - f_i) = 0, \qquad j = 1,2,..., p, \tag{18}$$

where w_i is $\dfrac{y_i}{c_i}\dfrac{\partial f_i}{\partial \beta_j}$.

Table 2 summarizes the weighted equations for all of the models discussed above.

Table 2: Weighted equations

Models	w_i for weighted equation, $\sum_i w_i(c_i - f_i) = 0$
Zero bias	$y_i x_{ij}$
Minimum χ^2	$\frac{y_i(c_i + f_i)}{f_i^2}\frac{\partial f_i}{\partial \beta_j}$
Normal	$y_i^2 \frac{\partial f_i}{\partial \beta_j}$
Poisson	$\frac{y_i}{f_i}\frac{\partial f_i}{\partial \beta_j}$
Exponential	$\frac{1}{f_i^2}\frac{\partial f_i}{\partial \beta_j}$
Gamma	$\frac{y_i}{f_i^2}\frac{\partial f_i}{\partial \beta_j}$
Inverse Gaussian	$\frac{y_i}{f_i^3}\frac{\partial f_i}{\partial \beta_j}$
Least squares	$y_i \frac{\partial f_i}{\partial \beta_j}$
Minimum modified χ^2	$\frac{y_i}{c_i}\frac{\partial f_i}{\partial \beta_j}$

Models	w_i for weighted equation, $\sum_i w_i(\log c_i - f_i) = 0$
Lognormal	$y_i \frac{\partial f_i}{\partial \beta_j}$

From Table 2, the following conclusions can be drawn:

- If the function of fitted value is in a linear form (additive), the zero bias and the least squares are equivalent.

- If the function of fitted value is in a log-linear form (multiplicative), the zero bias and the Poisson are equivalent.
- The weighted equation, which is in the form of a weighted difference between the observed and the fitted average claim cost, shows that all models are similar and can be distinguished by its weight.
- Since the weighted equation for all models are similar, the regression parameters for all models are expected to be similar. However, the Lognormal regression parameters are expected to be different from the rest of other models because its weighted equation is in the form of a weighted difference between the logarithm of the observed value and the fitted value.

6. FITTING PROCEDURE

The regression fitting procedure suggested in this study provides a faster convergence compared to the classical procedure introduced by Bailey and Simon [1] and Bailey [2]. In the classical procedure, each regression parameter, β_j, $j = 1,2,\ldots,p$, is calculated individually in each iteration whereas in the regression procedure, all of the regression parameters are calculated simultaneously in each iteration.

In the regression fitting procedure, the parameters, β_j, are solved by minimizing,

$$\sum_i w_i (c_i - f_i)^2 , \tag{19}$$

or by equating,

$$\sum_i w_i (c_i - f_i) \frac{\partial f_i}{\partial \beta_j} = 0 , \qquad j = 1,2,\ldots,p . \tag{20}$$

It can be seen that Equation (20) is equivalent to the weighted equation for the minimum bias and maximum likelihood methods shown by Equations (6), (7), (11), (12), (13), (14), (15), (17) and (18). As for Equation (16), the equation is equivalent to the same weighted equation if the value of c_i is replaced by $\log c_i$.

By using Taylor series approximation, it can be shown that the value of vector $\boldsymbol{\beta}$ in the first iteration is

$$\boldsymbol{\beta}_{(1)} = \left(\mathbf{Z}_{(0)}^{\mathbf{T}} \mathbf{W}_{(0)} \mathbf{Z}_{(0)}\right)^{-1} \mathbf{Z}_{(0)}^{\mathbf{T}} \mathbf{W}_{(0)} \left(\mathbf{c} - \mathbf{s}_{(0)}\right), \tag{21}$$

where $\boldsymbol{\beta}_{(0)}$ is the initial value of vector $\boldsymbol{\beta}$, $\mathbf{Z}_{(0)}$ the $n \times p$ matrix whose ij th element is equal to the first derivative of the fitted value evaluated at $\boldsymbol{\beta}_{(0)}$,

$$z_{ij(0)} = \left.\frac{\partial f_i(\boldsymbol{\beta})}{\partial \beta_j}\right|_{\boldsymbol{\beta}=\boldsymbol{\beta}_{(0)}},$$

$\mathbf{W}_{(0)}$ the $n \times n$ diagonal weight matrix evaluated at $\boldsymbol{\beta}_{(0)}$, and $\mathbf{s}_{(0)}$ the $n \times 1$ vector whose i th row is equal to

$$s_i = f_i(\boldsymbol{\beta}_{(0)}) - \sum_{j=1}^{p} \beta_{j(0)} z_{ij(0)}.$$

In the first iteration, the vector of initial values, $\boldsymbol{\beta}_{(0)}$, are required to calculate $\boldsymbol{\beta}_{(1)}$. The process of iteration is then repeated until the solution converges. Since the regression parameters are represented by vector $\boldsymbol{\beta}$, the regression model solves them simultaneously and thus, providing a faster convergence compared to the classical approach.

As an example, the fitting procedure for the least squares additive whereby the weighted equation is equivalent to

$$\sum_i y_i (c_i - f_i) \frac{\partial f_i}{\partial \beta_j} = 0, \quad j = 1, 2, \ldots, p, \tag{22}$$

will be discussed here. By comparing the least squares weighted equation, i.e., Equation (22), with the regression fitting equation, i.e., Equation (20), the i th diagonal element of the weight matrix, $\mathbf{W}_{(0)}$, is equal to y_i and this value is free of $\boldsymbol{\beta}_{(0)}$.

For an additive model, the ij th element of matrix $\mathbf{Z}_{(0)}$ is

$$z_{ij(0)} = \left.\frac{\partial f_i(\boldsymbol{\beta})}{\partial \beta_j}\right|_{\boldsymbol{\beta}=\boldsymbol{\beta}_{(0)}} = x_{ij},$$

and this value is also free of $\boldsymbol{\beta}_{(0)}$.

Therefore,

$$\mathbf{Z}_{(0)} = \mathbf{X},$$

and

$$\mathbf{s}_{(0)} = \mathbf{f}(\boldsymbol{\beta}_{(0)}) - \mathbf{X}\boldsymbol{\beta}_{(0)} = 0,$$

and Equation (21) for the least squares additive may now be simplified into

$$\boldsymbol{\beta}_{(1)} = \boldsymbol{\beta} = (\mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{X})^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{c}. \tag{23}$$

It can be seen that Equation (23) is equivalent to the Normal equation in standard linear regression and the equation also indicates that the regression parameters for the least squares additive may be solved without any iteration.

However, for a multiplicative model, the ij th element of matrix $\mathbf{Z_{(0)}}$ is equivalent to

$$z_{ij(0)} = \left.\frac{\partial f_i(\boldsymbol{\beta})}{\partial \beta_j}\right|_{\boldsymbol{\beta}=\boldsymbol{\beta_{(0)}}} = f_i(\boldsymbol{\beta_{(0)}})x_{ij}.$$

Therefore, matrix $\mathbf{Z_{(0)}}$ may be written as

$$\mathbf{Z_{(0)}} = \mathbf{F_{(0)}X}, \tag{24}$$

where $\mathbf{F_{(0)}}$ is the $n \times n$ diagonal matrix whose i th diagonal element is $f_i(\boldsymbol{\beta_{(0)}})$. Vector $\mathbf{s_{(0)}}$ may now be written as

$$\mathbf{s_{(0)}} = \mathbf{f(\boldsymbol{\beta}_{(0)}) - F_{(0)}X\boldsymbol{\beta}_{(0)}}.$$

Besides multiplicative and additive models, the fitting procedure suggested in this study can also be extended to other regression models and thus, allowing a variety of regression model to be created and applied as long as the function of the fitted value is written as

$$f_i = \left(\sum_{j=1}^{p} \beta_j x_{ij}\right)^b, \qquad -1 \le b < 0, \ \ 0 < b \le 1.$$

As an example, if the fitted average claim cost is assumed to follow an inverse function, i.e., $b = -1$, the ij th element of matrix $\mathbf{Z_{(0)}}$ is equal to

$$z_{ij(0)} = \left.\frac{\partial f_i(\boldsymbol{\beta})}{\partial \beta_j}\right|_{\boldsymbol{\beta}=\boldsymbol{\beta_{(0)}}} = -\left\{f_i(\boldsymbol{\beta_{(0)}})\right\}^2 x_{ij}.$$

Therefore, the equation for matrix $\mathbf{Z_{(0)}}$ may also be written as Equation (24), but the i th diagonal element of matrix $\mathbf{F_{(0)}}$ is equal to $-\{f_i(\boldsymbol{\beta_{(0)}})\}^2$.

An example of S-PLUS programming for the least squares multiplicative is given in Appendix A. Similar programming can also be used for all of the multiplicative, additive and inverse models proposed in this study. Each programming should be differentiated only by the following three elements:

- The vector of fitted average claim cost is equal to $\mathbf{f} = \exp(\mathbf{X}\boldsymbol{\beta})$ for a multiplicative model, $\mathbf{f} = \mathbf{X}\boldsymbol{\beta}$ for an additive model, and $\mathbf{f} = (\mathbf{X}\boldsymbol{\beta})^{-1}$ for an inverse model.

- The equation for $\mathbf{Z}_{(0)}$ is $\mathbf{Z}_{(0)} = \mathbf{X}$ for an additive model, and $\mathbf{Z}_{(0)} = \mathbf{F}_{(0)}\mathbf{X}$ for both multiplicative and inverse models. However, the i th diagonal element of matrix $\mathbf{F}_{(0)}$ is equal to $f_i(\boldsymbol{\beta}_{(0)})$ for a multiplicative model, and $-\{f_i(\boldsymbol{\beta}_{(0)})\}^2$ for an inverse model.
- Each model has its own weight matrix.

7. EXAMPLES

7.1 Malaysian Data

In this study, the methods of minimum bias and maximum likelihood will be compared and fitted on three types of claim severity data; the Malaysian data, the U.K. data from McCullagh and Nelder [3] and the Canadian data from Bailey and Simon [1]. For the Malaysian data, the weighted equation will be applied on a set of private car Third Party Property Damage (TPPD) claim costs obtained from an insurer in Malaysia which covers the legal liability of third party property loss or damage caused by or arising out of the use of an insured motor vehicle. The Malaysian data was based on 170,000 private car policies (1998-2000). The claims, which include both paid and outstanding, were already adjusted for inflation and were provided in Ringgit Malaysia (RM) currency.

The risks for the Malaysian claims were associated with five rating factors namely scope of coverage, vehicle make, vehicle use and gender of driver, vehicle year, and location. Altogether, there were $2 \times 2 \times 3 \times 4 \times 5 = 240$ cross-classified rating classes of claim severities to be estimated. Appendix B shows the rating factors, claim counts and average claim costs for the Malaysian data.

The fitting procedure involves only 108 data points because 132 of the rating classes have zero claim count (or weight). In addition, the models were evaluated using two different tests; the chi-squares and the average absolute difference. The average absolute difference,

$$\frac{\sum_i y_i |c_i - f_i|}{\sum_i y_i c_i},$$

was suggested by Bailey and Simon [1] as a suitable test for the third criteria whereas the chi-squares,

$$\chi^2 = K \sum_i \frac{y_i}{f_i}(c_i - f_i)^2,$$

was proposed by Bailey and Simon [1] as a suitable test for the fourth criteria.

Table 3 and Table 4 give the results of the regression parameters, chi-square values and average absolute difference for the multiplicative and additive models of the Malaysian data. Based on the results, the following conclusions can be made:

- For multiplicative models, the regression parameters for the *Poisson* are equivalent to the *zero bias*.
- For additive models, the regression parameters for the *least squares* are equal to the *zero bias*.
- Except for Lognormal, the regression parameters for multiplicative and additive models are similar. The reason is that the observed average claim costs, c_i, in the Lognormal were replaced by the logarithm of the average claims costs, $\log c_i$.
- Except for Lognormal, the smallest chi-square value is given by the *minimum chi-squares* for both additive and multiplicative models.
- Except for Lognormal, the smallest absolute difference is given by the *least squares* for both additive and multiplicative models.

Table 3: Multiplicative models for Malaysian data

Regression parameters	Zero bias	Minimum χ^2	Normal	Exponential	Poisson	Gamma	Inverse Gaussian	Least squares	Minimum modified χ^2	Lognormal
$\exp(\beta_1)$ Intercept	9242.10	9233.24	9278.99	8938.08	9242.10	9257.87	9281.65	9233.38	9267.88	9.14
$\exp(\beta_2)$ Non-comp	1.16	1.18	1.14	1.17	1.16	1.16	1.16	1.16	1.11	1.01
$\exp(\beta_3)$ Foreign	1.08	1.08	1.07	1.21	1.08	1.09	1.09	1.08	1.08	1.01
$\exp(\beta_4)$ Female	0.90	0.90	0.93	0.80	0.90	0.89	0.88	0.90	0.88	0.99
$\exp(\beta_5)$ Business	0.19	0.19	0.20	0.21	0.19	0.19	0.19	0.19	0.19	0.81
$\exp(\beta_6)$ 2-3 years	0.78	0.78	0.78	0.73	0.78	0.77	0.77	0.78	0.77	0.97
$\exp(\beta_7)$ 4-5 years	0.69	0.69	0.68	0.65	0.69	0.68	0.68	0.69	0.68	0.96
$\exp(\beta_8)$ 6+ years	0.72	0.72	0.71	0.71	0.72	0.72	0.72	0.71	0.72	0.96
$\exp(\beta_9)$ North	0.94	0.94	0.93	0.92	0.94	0.94	0.94	0.94	0.93	0.99
$\exp(\beta_{10})$ East	0.86	0.88	0.84	0.88	0.86	0.87	0.88	0.85	0.83	0.98
$\exp(\beta_{11})$ South	0.94	0.94	0.94	1.04	0.94	0.94	0.93	0.94	0.93	0.99
$\exp(\beta_{12})$ East M'sia	0.94	0.97	0.94	1.06	0.94	0.94	0.93	0.95	0.89	0.99
χ^2	476,081	471,147	492,026	844,318	476,081	477,160	480,147	477,541	517,605	8.16
Absolute difference ($\times 10^3$)	65.62	66.12	66.60	115.76	65.62	66.15	66.63	65.19	66.58	7.83

Table 4: Additive models for Malaysian data

Parameters	Zero bias	Minimum χ^2	Normal	Poisson	Exponential	Gamma	Inverse Gaussian	Least Squares	Minimum modified χ^2	Lognormal
β_1 Intercept	9167	9165	9254	9165	9006	9166	9171	9167	9170	9.13
β_2 Non-comp	1038	1201	931	1034	712	1031	1026	1038	684	0.13
β_3 Foreign	557	597	523	582	1274	606	628	557	555	0.08
β_4 Female	-765	-775	-592	-805	-1537	-838	-863	-765	-884	-0.12
β_5 Business	-4988	-4981	-4908	-4992	-4858	-4997	-5002	-4988	-5024	-1.64
β_6 2-3 years	-1976	-1983	-1968	-1987	-2315	-2000	-2013	-1976	-2009	-0.26
β_7 4-5 years	-2793	-2793	-2896	-2810	-2937	-2832	-2855	-2793	-2850	-0.39
β_8 6+ years	-2505	-2532	-2615	-2508	-2458	-2511	-2515	-2505	-2458	-0.33
β_9 North	-467	-446	-525	-459	-484	-451	-447	-467	-481	-0.07
β_{10} East	-1046	-833	-1084	-946	-782	-869	-815	-1046	-1193	-0.16
β_{11} South	-479	-452	-471	-467	314	-457	-449	-479	-496	-0.07
β_{12} East M'sia	-431	-257	-439	-452	269	-477	-504	-431	817	-0.09
χ^2	468,589	462,541	482,320	467,208	780,662	467,624	469,012	468,589	507,891	8.14
Absolute difference ($\times 10^3$)	64.65	65.42	65.42	64.97	109.19	65.50	66.02	64.65	66.48	7.81

7.2 U.K. Data

The U.K. data provides information on the Own Damage claim counts and average claim costs for privately owned and comprehensively insured vehicles (McCullagh and Nelder [3]). The average claim costs (in Pound Sterling) were already adjusted for inflation and the risks were associated with three rating factors: policyholder's age, car group, and vehicle age. Altogether, there were $8 \times 4 \times 4 = 128$ cross-classified rating classes of claim severities to be estimated. However, the fitting procedure involved only 123 data points because five of the rating classes have zero claim count. In addition to multiplicative and additive models, the severities were also fitted to the inverse models. The results of inverse models were compared to those of McCullagh and Nelder [3], who have applied Gamma regression model on the same severity data by assuming that the regression effects were linear on reciprocal scale.

Table 5, Table 6 and Table 7 give the results of the regression parameters, chi-square values, and average absolute difference for the U.K. data. As expected, except for Lognormal, the regression parameters for each of the multiplicative, additive, and inverse models are similar. In addition, the regression parameters for the Gamma whose fitted value is in the form of an inverse function are equal to the regression parameters produced by the McCullagh and Nelder [3]. The smallest chi-square value for additive, multiplicative and inverse models is provided by the *minimum chi-square*, whereas the smallest absolute difference for additive, multiplicative and inverse models is given by the *Gamma*.

7.3 Canadian Data

The Canadian data was obtained from Bailey and Simon [1] and it provides information on the liability claim counts and average claim costs for private passenger automobile insurance. The data involves two rating factors, namely merit and class, and altogether there were $4 \times 5 = 20$ cross-classified rating classes of claim severities to be estimated. In this study, the claim severities were fitted to the multiplicative and additive models.

Table 8 and Table 9 give the results of the regression parameters, chi-square values, and average absolute difference for the Canadian data. As expected, each of the multiplicative and additive models gives similar estimates for the regression parameters. The smallest chi-square value is provided by the *minimum chi-squares* for both additive and multiplicative models, whereas the smallest absolute difference is given by the *Normal* for both additive and multiplicative models.

Table 5: Multiplicative models for UK data

Parameters	Zero bias	Minimum χ^2	Normal	Poisson	Exponential	Gamma	Inverse Gaussian	Least squares	Minimum modified χ^2	Lognormal
$\exp(\beta_1)$ Intercept	297.57	313.59	279.34	297.57	302.38	286.75	276.52	309.81	257.91	5.61
$\exp(\beta_2)$ 21-24 years	0.98	0.95	1.05	0.98	0.90	1.00	1.02	0.94	1.08	1.01
$\exp(\beta_3)$ 25-29 years	0.91	0.87	0.97	0.91	1.01	0.94	0.97	0.88	1.04	1.00
$\exp(\beta_4)$ 30-34 years	0.88	0.84	0.96	0.88	0.75	0.89	0.90	0.86	1.01	0.99
$\exp(\beta_5)$ 35-39 years	0.70	0.67	0.75	0.70	0.72	0.73	0.76	0.67	0.79	0.95
$\exp(\beta_6)$ 40-49 years	0.77	0.73	0.81	0.77	0.76	0.79	0.80	0.75	0.89	0.97
$\exp(\beta_7)$ 50-59 years	0.78	0.75	0.83	0.78	0.79	0.80	0.82	0.76	0.89	0.97
$\exp(\beta_8)$ 60+ years	0.78	0.74	0.82	0.78	0.75	0.80	0.81	0.77	0.90	0.97
$\exp(\beta_9)$ B	0.99	0.99	0.96	0.99	1.06	1.00	1.01	0.98	0.99	1.00
$\exp(\beta_{10})$ C	1.16	1.16	1.14	1.16	1.17	1.17	1.18	1.15	1.16	1.03
$\exp(\beta_{11})$ D	1.48	1.50	1.53	1.48	1.60	1.49	1.50	1.48	1.45	1.07
$\exp(\beta_{12})$ 4-7 years	0.91	0.91	0.95	0.91	0.89	0.92	0.92	0.90	0.91	0.98
$\exp(\beta_{13})$ 8-9 years	0.70	0.70	0.74	0.70	0.66	0.71	0.72	0.69	0.69	0.94
$\exp(\beta_{14})$ 10+ years	0.49	0.51	0.50	0.49	0.48	0.50	0.50	0.48	0.46	0.87
χ^2	31,410	30,722	32,685	31,410	45,003	31,250	31,948	31,344	34,046	24.03
Absolute difference $(\times 10^3)$	81.30	82.05	83.06	81.30	106.90	80.74	81.24	83.46	82.73	14.56

Table 6: Additive models for UK data

Parameters	Zero bias	Minimum χ^2	Normal	Poisson	Exponential	Gamma	Inverse Gaussian	Least squares	Minimum modified χ^2	Lognormal
β_1 Intercept	298.67	303.94	273.49	288.34	291.89	278.98	270.03	298.67	241.88	5.60
β_2 21-24 years	-5.60	-7.53	17.58	0.31	-10.84	4.96	9.08	-5.60	34.01	0.04
β_3 25-29 years	-24.64	-30.52	-2.01	-16.95	15.31	-9.91	-2.61	-24.64	26.37	-0.01
β_4 30-34 years	-33.22	-43.39	-7.76	-29.34	-47.35	-26.59	-24.37	-33.22	14.17	-0.06
β_5 35-39 years	-87.89	-89.26	-64.78	-75.74	-44.23	-64.82	-53.72	-87.89	-33.45	-0.27
β_6 40-49 years	-66.99	-75.55	-50.51	-60.27	-45.84	-54.15	-47.87	-66.99	-13.68	-0.18
β_7 50-59 years	-63.35	-70.12	-45.49	-55.64	-36.19	-48.60	-41.39	-63.35	-10.87	-0.17
β_8 60+ years	-63.15	-72.15	-47.39	-56.91	-44.32	-51.10	-44.79	-63.15	-10.32	-0.17
β_9 B	-2.46	-0.50	-7.03	-0.21	8.19	2.04	4.11	-2.46	-0.30	0.00
β_{10} C	34.18	35.05	33.89	35.45	25.86	36.41	36.83	34.18	35.84	0.16
β_{11} D	108.66	113.74	123.07	108.76	97.83	108.90	108.62	108.66	96.09	0.39
β_{12} 4-7 years	-24.21	-21.98	-10.57	-21.54	-30.60	-19.62	-18.39	-24.21	-20.39	-0.08
β_{13} 8-9 years	-76.75	-71.63	-59.08	-72.26	-96.51	-69.12	-67.12	-76.75	-74.38	-0.35
β_{14} 10+ years	-126.63	-118.78	-111.15	-121.21	-147.85	-117.94	-116.35	-126.63	-128.54	-0.72
χ^2	34,060	33,200	35,487	33,547	48,796	33,954	35,059	34,060	37,670	24.34
Absolute difference ($\times 10^3$)	87.22	85.47	86.61	85.33	114.54	85.07	86.26	87.22	88.42	14.66

Table 7: Inverse models for UK data

Parameters **(10^4)**	Minimum χ^2	Normal	Poisson	Exponential	Gamma	Inverse Gaussian	Least squares	Minimum modified χ^2	Lognormal
β_1 Intercept	31.23	35.10	32.79	33.24	34.11	35.37	31.30	37.44	1782.06
β_2 21-24 years	3.12	-0.28	2.41	6.04	1.01	-0.11	4.16	-0.74	-7.66
β_3 25-29 years	6.11	2.25	4.75	3.53	3.50	2.30	6.26	0.46	8.40
β_4 30-34 years	6.81	2.50	5.30	11.78	4.62	4.23	6.39	0.83	20.99
β_5 35-39 years	16.11	12.41	14.97	16.36	13.70	12.64	16.61	11.36	93.71
β_6 40-49 years	11.73	8.41	10.28	12.04	9.69	9.25	11.12	5.97	63.00
β_7 50-59 years	11.30	7.78	9.96	11.00	9.16	8.47	10.98	5.89	58.45
β_8 60+ years	11.26	7.88	9.75	12.39	9.20	8.81	10.58	5.32	58.59
β_9 B	0.68	2.06	0.70	-2.82	0.38	-0.08	0.93	0.65	0.34
β_{10} C	-5.60	-5.11	-5.68	-6.51	-6.14	-6.70	-5.29	-5.95	-52.43
β_{11} D	-13.90	-14.27	-13.77	-18.24	-14.21	-14.83	-13.55	-13.60	-123.83
β_{12} 4-7 years	3.99	2.65	3.95	3.39	3.66	3.38	4.21	3.88	28.61
β_{13} 8-9 years	16.33	15.45	16.83	17.42	16.51	16.21	17.14	17.95	123.20
β_{14} 10+ years	38.52	43.50	41.74	33.78	41.54	41.38	41.97	47.09	275.82
χ^2	30,699	32,744	31,032	42,866	31,166	31,731	31,304	33,693	23.80
Absolute difference ($\times 10^3$)	81.29	81.93	80.18	99.61	79.23	79.33	81.46	80.30	14.45

Table 8: Multiplicative models for Canadian data

Parameters	Zero bias	Minimum χ^2	Normal	Poisson	Exponential	Gamma	Inverse Gaussian	Least squares	Minimum modified χ^2	Lognormal
$\exp(\beta_1)$ Intercept	292.00	291.97	291.08	292.00	294.57	291.92	291.84	292.10	292.07	5.68
$\exp(\beta_2)$ Merit X	0.99	0.99	1.00	0.99	0.97	0.99	0.99	0.99	0.98	1.00
$\exp(\beta_3)$ Merit Y	0.99	0.99	0.99	0.99	1.00	0.99	0.99	0.99	0.99	1.00
$\exp(\beta_4)$ Merit B	1.06	1.06	1.07	1.06	1.05	1.06	1.06	1.05	1.06	1.01
$\exp(\beta_5)$ Class 2	1.09	1.09	1.09	1.09	1.12	1.09	1.09	1.08	1.08	1.01
$\exp(\beta_6)$ Class 3	1.02	1.02	1.03	1.02	0.98	1.02	1.02	1.02	1.02	1.00
$\exp(\beta_7)$ Class 4	1.17	1.17	1.18	1.17	1.16	1.17	1.17	1.17	1.17	1.03
$\exp(\beta_8)$ Class 5	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.99
χ^2	49,520	49,470	54,461	49,520	80,313	49,542	49,657	49,609	49,895	27.51
Absolute difference $(\times 10^3)$	10.66	10.59	7.84	10.66	20.38	10.42	10.20	10.94	10.94	1.81

Table 9: Additive models for Canadian data

Parameters	Zero bias	Minimum χ^2	Normal	Poisson	Exponential	Gamma	Inverse Gaussian	least squares	Minimum modified χ^2	Lognormal
β_1 Intercept	291.95	291.83	291.06	291.87	294.77	291.80	291.74	291.95	291.94	5.68
β_2 Merit X	-4.24	-3.38	0.59	-4.05	-10.11	-3.92	-3.82	-4.24	-5.37	-0.02
β_3 Merit Y	-3.45	-3.51	-3.95	-3.58	1.00	-3.68	-3.74	-3.45	-3.71	-0.01
β_4 Merit B	17.11	17.58	20.28	17.53	15.49	17.92	18.28	17.11	17.44	0.06
β_5 Class 2	25.16	25.75	25.13	25.35	35.64	25.54	25.73	25.16	24.63	0.08
β_6 Class 3	4.71	4.80	8.26	4.68	-6.92	4.65	4.62	4.71	4.43	0.02
β_7 Class 4	51.08	51.28	53.30	51.18	47.12	51.30	51.42	51.08	51.01	0.16
β_8 Class 5	-22.92	-22.79	-23.60	-22.99	-25.33	-23.05	-23.11	-22.92	-23.38	-0.08
χ^2	46,776	46,665	51,049	46,713	82,024	46,722	46,790	46,776	47,074	27.22
Absolute difference $(\times 10^3)$	10.08	9.79	7.17	9.86	20.51	9.66	9.49	10.08	10.10	1.79

8. CONCLUSION

This study compares several minimum bias and maximum likelihood methods by using a weighted equation which is written as a weighted difference between the observed and the fitted values. The weighted equation was applied to estimate claim severity or average claim cost which is equivalent to the total claim costs divided by the number of claims.

The weighted equations are summarized in Table 2. Based on the weighted equations, it can be concluded that the equations for *zero bias* and *least squares* are equal if the function of fitted value is linear (additive model) and the equations for *zero bias* and *Poisson* are equal if the function of fitted value is log-linear (multiplicative model). It can also be shown from the weighted equations that all models are similar and can be distinguished by its own weight, except for Lognormal where the observed average claim costs, c_i, were replaced by the logarithm of the average claim costs, $\log c_i$.

The fitting procedure was suggested to be carried out using a regression approach. The advantage of using the regression fitting procedure is that it provides a faster convergence compared to the classical procedure introduced by Bailey and Simon [1] and Bailey [2]. Furthermore, the fitting procedure may also be extended to other models in addition to the multiplicative and additive models, as long as the function of fitted value is written in a specified linear form. A similar programming for the fitting procedure may also be used for all of the multiplicative, additive and inverse models proposed in this study. Each model should be differentiated only by three elements: the vector of fitted average claim cost, $\mathbf{f}$; the equation for matrix $\mathbf{Z}$; and the equation for weight matrix, $\mathbf{W}$.

In this study, the minimum bias and maximum likelihood methods were applied to fit three types of severity data: the Malaysian data, the U.K. data from McCullagh and Nelder [3], and the Canadian data from Bailey and Simon [1]. The models were tested based on the average absolute difference and the chi-square value. Based on the results, except for Lognormal, the smallest chi-square value is given by the *minimum chi-squares*. As for the absolute difference, the smallest value for the Malaysian, U.K., and Canadian data is provided by the *least squares*, *Gamma* and *Normal*, respectively. The U.K. data also showed that the regression parameters for Gamma with an inverse fitted function are equivalent to those produced by the McCullagh and Nelder [3].

When this study was carried out, two main targets were outlined: to provide strong basic statistical justification for the available models, and to search for a match point that is able to merge the available parametric and nonparametric models into a more generalized form. It is hoped that a more friendly and efficient computation approach can be created through both of these targets. As a result, this study managed to not only offer more models which include both parametric and nonparametric approaches, but also a friendlier computation method.

Even though the approach taken in this study was based on statistical parametric theory, the theory can be matched with nonparametric theory as well. For the proposed models, the actuary does not really have to determine the statistical distribution appropriate for the available data; all he needs to do is just determine the weight. Therefore, the main principle which is applied in this approach is the selection of an appropriate weight suitable for the available data. The proposed models may be more flexible and at the same time able to attend both streams of thought in statistics; nonparametric and parametric.

Besides modeling aspects, the suggested regression approach may build a base for efficient computation as well as analysis. The reason is that the regression approach allows the data to be analyzed, interpreted, and predicted with a similar manner to the data analysis, interpretation, and prediction of the regression analysis.

Finally, rewriting the equations of minimum bias and maximum likelihood as a weighted equation has several advantages:

- The mathematical concept of the weighted equation is simpler and hence, providing an easier understanding particularly for insurance practitioners.
- The weighted equation allows the usage of a regression model as an alternative programming algorithm to calculate the regression parameters.
- The weighted equation provides a basic step to further understand the more complex distributions such as Gamma, Inverse Gaussian, and Lognormal.
- The weights of each of the multiplicative, additive and inverse models show that the models have similar regression parameters.

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Appendix A: S-PLUS programming for least squares multiplicative

```
leastsquares.multi <- function(data)
{
#  To identify matrix X, vector cost and vector count from the data
   X <- as.matrix(data[,-(1:2)])
   cost <- as.vector(data[,1])
   count <- as.vector(data[,2])
#  To set initial values for vector beta
   new.beta <- c(10, rep(c(0.01), dim(X)[2]))
#  To start iteration
   for (i in 1:20)
   {
      beta <- new.beta
      fitted <- as.vector(exp(X%*%beta))
      Z <- diag(fitted)%*%X
      W <- diag(count)
      r.s <- cost-fitted+as.vector(Z%*%beta)
      new.beta <- as.vector(solve(t(Z)%*%W%*%Z)%*%t(Z)%*%W%*%r.s)
   }
#  To calculate fitted values, chi-square and absolute difference
   fitted <- as.vector(exp(X%*%new.beta))
   chi.square <- sum((count*(cost-fitted)^2)/fitted)
```

```
   abs.difference <- sum(count*abs(cost-fitted))/sum(count*cost)
#  To list programming output
   list  (expbeta= exp(new.beta), chi.square= chi.square,
         abs.difference= abs.difference)
}
```

Appendix B: Malaysian data

Rating factors					Claim count	Average claim cost (RM)
Scope of coverage	Vehicle make	Vehicle use & gender of driver	Vehicle year	Location		
Comprehensive	Local	Private-male	0-1 year	Central	381	9290
				North	146	8775
				East	44	6447
				South	161	8484
				East Malaysia	8	7785
			2-3 year	Central	422	7220
				North	203	6713
				East	41	6461
				South	164	7298
				East Malaysia	19	4037
			4-5 year	Central	276	6558
				North	145	5220
				East	29	6415
				South	115	5554
				East Malaysia	17	6937
			6+ year	Central	223	6678
				North	150	6230
				East	39	5372
				South	89	5915
				East Malaysia	33	5005
		Private-female	0-1 year	Central	165	9136
				North	55	7876
				East	12	7536
				South	23	6789
				East Malaysia	6	10306
			2-3 year	Central	147	6642
				North	72	5731
				East	12	5038
				South	39	6023
				East Malaysia	8	3977
			4-5 year	Central	56	5545
				North	36	4642
				East	7	4565
				South	23	5038
				East Malaysia	2	3818
			6+ year	Central	51	5709

			North	38	6272
			East	5	2869
			South	23	6243
			East Malaysia	9	3765
	Business	0-1 year	Central	0	0
			North	0	0
			East	0	0
			South	0	0
			East Malaysia	0	0
		2-3 year	Central	0	0
			North	0	0
			East	0	0
			South	0	0
			East Malaysia	0	0
		4-5 year	Central	0	0
			North	0	0
			East	0	0
			South	0	0
			East Malaysia	0	0
		6+ year	Central	0	0
			North	1	1206
			East	0	0
			South	0	0
			East Malaysia	0	0
Foreign	Private-male	0-1 year	Central	94	8986
			North	47	9402
			East	21	7321
			South	38	9170
			East Malaysia	6	11507
		2-3 year	Central	202	8251
			North	85	6772
			East	21	5332
			South	65	5821
			East Malaysia	23	9503
		4-5 year	Central	157	6498
			North	85	8235
			East	15	8758
			South	73	6391
			East Malaysia	24	7047
		6+ year	Central	245	6923
			North	151	6777
			East	44	7563
			South	113	7266
			East Malaysia	64	7047
	Private-female	0-1 year	Central	29	10442
			North	11	7599
			East	2	9492
			South	17	9003

				East Malaysia	6	5867
			2-3 year	Central	46	6460
				North	41	5966
				East	5	3463
				South	13	7329
				East Malaysia	10	5222
			4-5 year	Central	39	4798
				North	15	4921
				East	0	0
				South	16	4384
				East Malaysia	11	6792
			6+ year	Central	47	5197
				North	35	7131
				East	6	6480
				South	9	5152
				East Malaysia	10	7718
		Business	0-1 year	Central	0	0
				North	0	0
				East	0	0
				South	0	0
				East Malaysia	0	0
			2-3 year	Central	0	0
				North	0	0
				East	0	0
				South	0	0
				East Malaysia	0	0
			4-5 year	Central	0	0
				North	0	0
				East	0	0
				South	0	0
				East Malaysia	0	0
			6+ year	Central	0	0
				North	0	0
				East	0	0
				South	0	0
				East Malaysia	0	0
Non-comprehensive	Local	Private-male	0-1 year	Central	0	0
				North	0	0
				East	0	0
				South	0	0
				East Malaysia	0	0
			2-3 year	Central	3	10225
				North	0	0
				East	0	0
				South	1	14265
				East Malaysia	0	0
			4-5 year	Central	1	3619

		North	5	5003
		East	0	0
		South	1	3375
		East Malaysia	0	0
	6+ year	Central	9	8736
		North	5	5142
		East	2	3598
		South	4	8673
		East Malaysia	2	17210
Private-female	0-1 year	Central	0	0
		North	0	0
		East	0	0
		South	0	0
		East Malaysia	0	0
	2-3 year	Central	0	0
		North	1	1563
		East	0	0
		South	0	0
		East Malaysia	0	0
	4-5 year	Central	0	0
		North	1	3619
		East	0	0
		South	0	0
		East Malaysia	0	0
	6+ year	Central	1	2003
		North	0	0
		East	0	0
		South	0	0
		East Malaysia	1	4455
Business	0-1 year	Central	0	0
		North	0	0
		East	0	0
		South	0	0
		East Malaysia	0	0
	2-3 year	Central	0	0
		North	0	0
		East	0	0
		South	0	0
		East Malaysia	0	0
	4-5 year	Central	0	0
		North	0	0
		East	0	0
		South	0	0
		East Malaysia	0	0
	6+ year	Central	0	0
		North	0	0
		East	0	0
		South	0	0

			East Malaysia	0	0
Foreign	Private-male	0-1 year	Central	0	0
			North	0	0
			East	0	0
			South	0	0
			East Malaysia	0	0
		2-3 year	Central	0	0
			North	3	6739
			East	0	0
			South	2	12657
			East Malaysia	0	0
		4-5 year	Central	0	0
			North	3	9796
			East	0	0
			South	0	0
			East Malaysia	3	13812
		6+ year	Central	49	7234
			North	71	7740
			East	6	9383
			South	56	8108
			East Malaysia	22	6207
	Private-female	0-1 year	Central	0	0
			North	0	0
			East	0	0
			South	0	0
			East Malaysia	0	0
		2-3 year	Central	0	0
			North	0	0
			East	0	0
			South	0	0
			East Malaysia	0	0
		4-5 year	Central	0	0
			North	0	0
			East	0	0
			South	0	0
			East Malaysia	0	0
		6+ year	Central	14	6942
			North	15	7462
			East	2	6148
			South	6	10584
			East Malaysia	3	10168
	Business	0-1 year	Central	0	0
			North	0	0
			East	0	0
			South	0	0
			East Malaysia	0	0
		2-3 year	Central	0	0

		North	0	0
		East	0	0
		South	0	0
		East Malaysia	0	0
	4-5 year	Central	0	0
		North	0	0
		East	0	0
		South	0	0
		East Malaysia	0	0
	6+ year	Central	0	0
		North	0	0
		East	0	0
		South	0	0
		East Malaysia	0	0
Total			5,728	-

Disparate Impact and Unfairly Discriminatory Insurance Rates

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Abstract.

Historic actuarial literature, general insurance literature, and legislative histories reveal "unfairly discriminatory rates" to be a cost-based concept. A rate structure is unfairly discriminatory if the insurance premium differences between insureds do not reasonably correspond to differences in expected insurance costs. More recently a new rate concept has arisen in some court cases which is referred to as "disparate impact" (or "adverse impact"). Disparate impact has nothing to do with underlying insurance costs and is solely based on the disproportionate impact of the insurance rate structure on the insurance premiums paid by protected minority classes defined by race, color, religion, sex, or national origin. It would likely be a rare instance where the rate standard of unfairly discriminatory and the concept of disparate impact could be applied simultaneously to a risk classification plan without conflict. It is the author's opinion that if the standard of disparate impact eventually prevails over the historical rate standard of unfairly discriminatory, then accurate risk assessment will be destroyed, adverse selection will be widespread in the insurance marketplace, and coverage availability will suffer.

Keywords. Risk classification plans; risk assessment; credit scoring; insurance law; rate regulation; adverse selection; disparate impact; adverse impact.

1. INTRODUCTION

In today's society, the terms discrimination and disparate impact connote unfairness. Without any historical context as background, it would not be surprising for the average person to mistakenly conclude that the term unfairly discriminatory is redundant, and that the term disparate impact is just another form of unfair rate discrimination. However, a review of insurance literature, legislative histories, and court cases reveal that the terms disparate impact and unfair rate discrimination are fundamentally different. In insurance ratemaking there has always existed a form of rate discrimination which is considered to be fair if the rates are based on underlying insurance costs. On the other hand, disparate impact is defined without any reference to underlying insurance costs.

The origins of the common rate standards applied by actuaries (i.e., reasonable, adequate, not excessive, and not unfairly discriminatory) are discussed in this paper, with special emphasis on the rate standard of unfairly discriminatory. The insurance literature and legislative histories show the four common rate standards to have meanings based entirely on the underlying anticipated insurance costs. It is precisely because these rate standards are cost-based that actuaries have adopted these standards as terms of art, as set forth in Principle 4 of the Casualty

Actuarial Society's Statement of Principles Regarding Property and Casualty Insurance Ratemaking (i.e., CAS Statement of Ratemaking Principles).

More recently, some courts have considered the application of a new standard of disparate impact (or adverse impact) to insurance rate structures. Thus far no court has actually applied the disparate impact standard to insurance rates, but it is only a matter of time before some court does so. The standard of disparate impact has its origins in federal civil rights laws and has been applied by the courts in a range of issues including employment, educational testing, housing, and age discrimination. Unlike unfairly discriminatory rates, disparate impact is not a cost-based concept. If applied to insurance, a risk/rate factor will potentially be said to have a disparate impact if it more adversely impacts a protected minority class than it does the majority class, regardless of its relationship to underlying costs.

It is reasonable to assume a priori that no protected minority class (i.e., race, religion, sex, etc.) will be uniformly distributed throughout any given insurance risk classification plan. This assumption implies that all risk factors used to measure and assess risk are potentially in violation of a disparate impact rate standard, even though each risk factor accurately reflects expected losses and expenses.

If a risk classification plan were changed to eliminate one or more risk factors found to have a disparate impact, the resulting rates would likely be unfairly discriminatory because the rate differences would no longer be based on the underlying insurance costs. Therein lies the inevitable and irreconcilable conflict between the two standards.

This paper concludes with a brief discussion of the potential role of an actuary with the various issues related to disparate impact. Even though disparate impact is not cost-based, and therefore not an actuarial term of art, actuaries do have expertise in measuring the statistical significance of any differences in rate impact between the majority class and a protected minority class. Actuaries could also provide expertise in defining the data needed to measure disparate impact and in establishing the business necessity of any risk factor in question.

2. THE DAWNING OF U.S. RATE REGULATION AND RATE STANDARDS

The origin of property/casualty insurance rate regulation in the U.S. is rooted primarily in the history of fire insurance. It was solvency concerns and destructive price competition in the fire insurance business in the 1800's that spurred the need for cost-based actuarial ratemaking procedures and the need for rate regulation by the states.

In the early to mid-1800's local boards (i.e., voluntary associations of insurers) were organized to provide a means of sharing loss data and to enforce uniform rates among the insurers. Uniform rates were desired so that rates were adequate to protect against insolvencies and were not unfairly discriminatory. The primary concern with unfairly discriminatory rates, often stated at the time, was that rich and powerful insureds could unfairly negotiate lower rates than were being charged to less influential insureds, even though their degree of risk and underlying insurance costs did not warrant a lower rate.

In 1866 a national association of insurers, the National Board of Fire Underwriters (i.e., NBFU), was formed to gather industrywide data and to develop a uniform rate schedule. The NBFU decreased the need for local boards. During the ensuing profitable years the insurers regularly violated their NBFU membership agreements by engaging in destructive rate-cutting. On the verge of disbanding just prior to the 1871 Chicago fire, the insurer insolvencies which followed the Chicago fire gave new life to the need for rate discipline and new life to the NBFU. But profitability soon returned to fire insurers and destructive rate-cutting returned to the market. Rampant rate-cutting caused the NBFU to finally disband in 1887, thereby shifting "control" of fire insurance rates back to local boards and associations.

Federal legislation in the 1880's, which outlawed combinations of insurers in restraint of trade, led about half the states to adopt anti-compact laws between 1885 and 1907. The anti-compact laws sharply reduced the ability of local boards to maintain uniform, adequate, and fairly discriminatory rates. The pressing need for insurers to associate so as to create a combined, credible fire insurance database and the existing lack of discipline in fire insurance rating practices in the late 1800's led to many proposals for state regulation of rates.

3. UNFAIRLY DISCRIMINATORY RATES

3.1 Early Rate Regulatory Laws

The first modern-style rate regulation statute was enacted in Kansas in 1909. The Kansas law required fire insurance rates to be filed with the Insurance Commissioner and required the rates to be reasonable, not excessive, adequate to the safety and soundness of the insurer, and not unjustly discriminatory. Unjust discrimination was defined as charging different rates to persons with "risks of a like kind and hazard".

Soon after enactment of the Kansas law, although largely as the result of the insolvencies and the subsequent sharp fire insurance rate increases ensuing from the fires following the great San Francisco Earthquake of 1906, the New York legislature appointed the Merritt Committee and launched an investigation of fire insurance rates. The Merritt Committee Report led to New

York's first rate regulatory law in 1911. This law permitted insurers to gather data and act in concert to set rates through rate bureaus. The New York law also required fire insurance rates to be filed with the Superintendent of Insurance and prohibited unfairly discriminatory rates. The law and the Merritt Committee Report made it clear that rates were considered to be unfairly discriminatory if different rates were charged to risks in the same class or of essentially the same hazard. Class rate differentials based on differences in risk and loss experience were expressly permitted by the New York legislation.

New York, working through the National Convention of Insurance Commissioners (i.e., NCIC), offered its new fire insurance rate law as a prototype for other states. Many states (e.g., New Jersey in 1913) did adopt similar rate regulatory laws which permitted collusive rate setting through rate bureaus and prohibited unfairly discriminatory rates. Consistently, the clear purposes of these early laws were to permit collusion in regard to data gathering and rate setting, and to ensure that rates were established commensurate with the degree of risk and hazard being insured. In a speech before the NCIC in 1915, the New Jersey Insurance Commissioner spoke about the need to base insurance rates on the degree of risk being insured and the unfair discrimination that arose when "some people were getting insurance for less than it was worth and others were paying for it."

3.2 McCarran-Ferguson and Modern Rate Regulation

The enactment of Public Law No. 15 (i.e., McCarran-Ferguson) on March 9, 1945 reaffirmed the right of the states to regulate insurance by providing an antitrust exemption for insurance to the extent that insurance was regulated by state laws. McCarran-Ferguson spurred a new and modern round of state rate regulatory laws throughout the United States. As a result of McCarran-Ferguson, the National Association of Insurance Commissioners (i.e., NAIC) immediately turned its attention to drafting model rate regulatory laws that could be considered for adoption by the majority of state legislatures which were scheduled to begin to meet next in 1947. The 1945 NAIC proceedings indicate that the model laws and the rate standards were based largely on existing state rate regulatory statutes, as witnessed by the following quote from the May 12, 1945 Report of the Subcommittee on Federal Legislation:

> "On the subject of rate regulation the Committee felt that there were well-defined patterns available based upon the actual experience of a number of states which could be used as a foundation for the drafting of rate regulatory statutes at this time. This fact was recognized by certain segments of the insurance industry which prepared so-called model rating bills based largely upon existing statutes

> and which were used as guides for the enactment of rate regulatory laws recently in several states."

The NAIC's model fire/marine and casualty/surety rate regulatory bills of 1946 utilized the rate standards of not excessive, inadequate or unfairly discriminatory and required that rates be based on consideration of past and prospective loss and expense experience. These model bills specifically allowed for the grouping of risks by classifications for the establishment of rates. Classification rates could be modified for individual risks if, and only if, the modification was based on "variations in hazards or expense provisions, or both."

The NAIC model bills were a pervasive influence on individual state legislatures. It is not at all surprising that the rate regulatory laws throughout the U.S. today contain similar, if not the same, language as the 1946 NAIC model bills. As an example, the influence of the 1946 NAIC model bills on individual state rate regulatory laws can be found in the California McBride-Grunsky Act of 1947 (S.B.1572). This California statute prohibited rates that were unfairly discriminatory and specifically allowed for differences in rates between risk classifications, if the rate differences were based on the differences in the underlying hazard or expenses.

A new rate regulatory statute was established in California in 1988 with the passage of Proposition 103. Proposition 103 reestablished the unfairly discriminatory rate standard, as well as placed certain restrictions on some rate factors used in rating personal auto insurance. Subsequent to the passage of Proposition 103 new rate regulations were adopted and some lower courts addressed the definition of unfairly discriminatory rates in California. In this author's opinion thus far there have been no changes in California to the traditional concept that rates should be based on expected costs and not be arbitrary.

4. DISPARATE IMPACT ON INSURANCE RATES

4.1 History

The concept of disparate impact[1] has its roots in certain federal civil rights laws, including the Civil Rights Acts of 1866, 1964, and 1991 and the Fair Housing Act (42 U.S.C. Sec. 3604) (i.e., FHA). Broadly speaking, this category of federal laws prohibits discrimination based on race,

[1] Note: As in this paper, the terms disparate impact and adverse impact are generally used interchangeably to mean that a protected minority class is being adversely and disproportionately impacted as compared to the impact on the majority class. Disparate impact and adverse impact are both distinguished from disparate treatment, which involves intent to discriminate in a way that is prohibited by federal civil rights law.
In this paper the terms disparate impact and adverse impact are used with the recognition that the impact may occur in neutral processes without the specific intent to violate any civil rights prohibitions. Disparate treatment, based on the intent to violate discrimination prohibitions, is not related to actuarial considerations, is a mutually exclusive theory from disparate impact, and is not addressed in this paper.

color, religion, sex, or national origin. The seminal disparate impact case was decided by the U.S. Supreme Court in Griggs v. Duke Power 401 U.S. 424, 430-32; 1971. The Civil Rights Act of 1991 codified the disparate impact findings in Griggs.[2]

Disparate impact has been defined by various courts as an unintentional discrimination against the protected minority class and its existence is not necessarily illegal. If a plaintiff is able to establish that a specific practice leads to a significantly higher adverse impact on the protected minority class than on the majority class, the defendant then has the burden and opportunity to prove that the practice in question has "legitimate business reasons" or "business necessity" (see Watson v. Forth Worth Bank & Trust, 487 U.S. 977, 978; 1988). Even if the defendant is successful in showing the practice in question is of a business necessity, the plaintiff still has the opportunity to show that other practices would serve the defendant's business purposes without disparate impact against the protected minority class (see Albermarle Paper Co. v. Moody, 422 U.S. 405, 425; 1975).

In summary, past court decisions seem to suggest that a business practice with disparate impact on a protected minority class will be considered illegal by the courts if:

a. there is a significantly higher adverse impact on a protected minority class than on the majority class, and

b. either the practice in question cannot be shown to have a legitimate business necessity, or an alternate practice is shown to achieve the business purpose without the disproportionate adverse impact on the protected minority class.

4.2 Measurement of Significance

The Uniform Guidelines on Employee Selection Procedures (adopted in 1978 by the EEOC, U.S. Civil Service Commission, Department of Labor, and the Department of Justice) provided the so-called "4/5's Rule" as a guideline for employment selection practices. This guideline allows for some disproportionate adverse impact against the protected minority class as long as the impact is not considered to be significant by the court. The adverse impact is considered to be significant only when the "4/5's Rule" is failed. For example, if 60% of the job applicants in the majority class are hired and only 50% of the job applicants in the minority class are hired, the difference in impact is considered not significant and not discriminatory. This is because the hiring rate of the minority class is more than 80% of that of the majority class.

[2] Employment Discrimination Law, American Bar Association, Barbara Lindemann and Paul Grossman, Volume I; Chapter 3, 2007.

The "4/5's Rule" to determine significance is not the only test of significance that has been used by the courts. In some cases, statistical tests of significance or a showing of a disparity of two or more standard deviations have also been applied to determine if the adverse impact is significant enough to be a problem. To guard against a relatively small difference being considered statistically significant because of a large sample size, some courts have required that a statistically significant disparity also have a practical significance.

Although anything is possible in terms of future lawsuits, it is the author's opinion that the "4/5's Rule" may not be accepted as a test of significance for insurance ratemaking. It is more likely that the determination of significance of any disparate impact of insurance rates will be based on statistical tests of significance.

4.3 Application to Insurance Practices

There is a strong legal argument that federal civil rights laws, including the FHA, should not be applied to the pricing and underwriting of insurance because of the McCarran-Ferguson exemption. Thus far the courts have rejected this McCarran-Ferguson argument. However, most of the insurance cases in which the courts have rejected the McCarran-Ferguson defense have involved claims of either fraud or intentional discrimination (i.e., disparate treatment).

One such "disparate treatment" case was NAACP v. American Family Mutual Insurance Company (978 F.2d 287, Seventh Circuit, 1992). The complaint in the American Family case involved an alleged violation of the FHA due to charging higher rates for residential property insurance in racial minority neighborhoods. The Court observed that there was an important distinction between disparate treatment and disparate impact because the nature of insurance inherently requires risk classification and discrimination by degree of risk. As the Court said in the American Family case, "risk discrimination is not race discrimination."

In a more recent insurance case (DeHoyos, et al. v. Allstate, et al., 345 F.3d 290, Fifth Circuit, 2003), it was charged that Allstate's residential property insurance rates had a racially disparate impact because of the use of credit-based insurance scores in its rate structure. This was a true disparate impact case, rather than a disparate treatment case, because intent to racially discriminate was not at issue. The Fifth Circuit ruled that McCarran-Ferguson did not preempt the application of the FHA in this case. Allstate appealed the preemption decision to the U.S. Supreme Court, which refused to take the case. After the Supreme Court declined to review the preemption decision of the Fifth Circuit, Allstate settled the case. Even though the Court never had the opportunity to address the issues of disparate impact (i.e., the existence and significance of the difference, the business purpose, or the potential substitutes for credit data) in the Allstate case, it is only a matter of time before some court does.

5. DEFINITIONS

5.1 Unfairly Discriminatory

As previously discussed, the definition of unfairly discriminatory insurance rates has historically and consistently been related to the underlying costs of providing insurance. Prior to the first rate regulatory law in Kansas, insurance literature consistently refers to the unfairness of charging different rates to risks with similar risks of loss and similar hazards. The literature surrounding the adoption of the first rate regulatory laws in Kansas, New York, New Jersey and the 1946 NAIC model rate regulatory bills are consistent on this point.

Professor C. Arthur Williams, Jr.[3] has put forward what is probably the most commonly used, and the most succinct, definition of unfairly discriminatory insurance rates as follows:

> "An insurance rate structure will be considered to be unfairly discriminatory. . . ., if allowing for practical limitations, there are premium differences that do not correspond to expected losses and average expenses or if there are expected average cost differences that are not reflected in premium differences"

5.2 Actuarial Term of Art

It is precisely because the concept of unfairly discriminatory insurance rates has historically been a cost-based concept, that actuaries adopted that rate standard as a term of art. Although this term of art was embodied in much of the early actuarial literature, it was not until 1988 that the CAS Statement of Ratemaking Principles was formally adopted, which declared in Principle 4:

> "A rate is reasonable and not excessive, inadequate, or unfairly discriminatory if it is an actuarially sound estimate of the expected value of all future costs associated with an individual risk transfer."

5.3 Disparate Impact

Court cases reveal that the term disparate impact is not a cost-based concept and, therefore, it is not currently considered to be an actuarial term of art. Disparate impact is strictly a standard based on a significantly disproportionate and adverse impact on a protected minority class defined by race, color, religion, sex, or national origin. In an insurance context disparate impact has nothing to do with the underlying costs of providing insurance.

5.4 Conflict in Definitions

[3] *Insurance, Government, and Social Policy*, The S.S. Huebner Foundation for Insurance Education, C. Arthur Williams, Jr., Chapter 11, Price Discrimination in Property and Liability Insurance, 209-242.

It is likely that the rate standard of unfairly discriminatory will be in direct conflict with the application of a disparate impact standard to insurance rates. This conflict will potentially exist for nearly every risk factor used to develop property/casualty insurance rates because protected classes, most if not all of the time, will not be evenly distributed throughout the various risk classifications. If a court or legislature were to order that all disparate impacts be eliminated from insurance premiums, it is likely that accurate risk assessment would be destroyed, resulting in unfairly discriminatory rates. Paraphrasing a 1915 NAIC speech by the New Jersey Insurance Commissioner, unfairly discriminatory rates mean that some people would pay less than the insurance was worth, at the expense of other people who would be required to pay more than the insurance is worth in order to subsidize the under-payers. It is possible that the only rate structure which could survive a strict disparate impact standard is "one-rate-for-all." If such a scenario materializes, adverse selection would be rampant in the insurance market and coverage availability would suffer.

6. ROLE OF THE ACTUARY

6.1 Determination of Unfair Discrimination

The role of the actuary in determining underlying insurance costs and verifying that the rate structure is not unfairly discriminatory is well-established and uniquely actuarial in nature. The costs which an actuary considers in a review of any rate structure are prospective losses, prospective expenses, and an appropriate provision for risk commensurate with the cost of capital necessary to support the insurance mechanism.

6.2 Determination of Disparate Impact

The role of the actuary with disparate impact issues has not yet been fully established. Certainly actuaries are not trained to opine on social policies or to determine which minority classes deserve the protection of the law. Society's definition of overall fairness needs to be left to the legislatures and courts.

However, actuaries do possess the unique expertise to measure the impact on insurance rates of any risk factor and to determine the degree and statistical significance of any apparent disparate impact on any protected minority group defined by law. If a court were to find that a particular risk factor had a disparate impact on the insurance premiums of a protected minority group and the disparate impact was statistically significant enough to be of concern to the court, actuaries would be uniquely qualified to opine on the predictive power and business necessity of the risk factor in question, as well as opine on any risk factors that might replace the risk factor in question.

6.2.1 An example

When disparate impact arises in the context of insurance rates, it will likely be an issue with personal auto or residential dwelling insurance. The risk factor in question could be territory rate factors, because racial groups likely differ in their geographical distributions. Or in the case of auto insurance, the risk factor in question could be age of driver, gender of driver, credit-based insurance scores, or etc. In the case of homeowners insurance, rates based on the age of the home have already been challenged as having a disparate impact. Since the distribution across the various rate classes of racial groups is likely to vary somewhat for every risk factor, there is a potential for "disparate impact" with every risk factor.

For purposes of this example, assume the risk factor in question is credit-based insurance scores as applied to personal auto insurance. In its July 2007 report in the U.S. Congress, the Federal Trade Commission (i.e., "FTC Study") found that credit-based insurance scores "are effective predictors of risk" for auto insurance. The FTC also found that credit-based insurance scores "are distributed differently among racial and ethnic groups, and this difference is likely to have an effect on the insurance premiums that these groups pay, on average". While the FTC did not attempt to actually measure the effect on auto insurance premiums, or opine on the statistical significance of any premium impact, the mere suggestion of a "likely" unequal impact on average premiums raises the spectre of disparate impact for this risk factor.

The following sections discuss the role of an actuary in a hypothetical lawsuit where the charge is that credit-based insurance scores have a disparate impact on the auto insurance premiums for a protected racial minority.

6.2.2 Data to determine disparate impact

However a court or legislature might define disparate impact as applied to insurance practices, it is highly likely that the determination of its existence will involve sophisticated analyses of data. Unlike employment/hiring cases, it will be difficult, if not impossible, to accurately apply any racial disparate impact definition to insurance rates in an objective, statistical way because the racial data needed are simply not available.

The FTC Study was based on racial information for each policyholder obtained from the Social Security Administration. Due to limitations in this data prior to 1981, the FTC also relied on a Hispanic surname match and Census tract data to identify some Hispanics, Asians, and Native Americans. The reliance on a surname match and Census tract data to identify Hispanics, Asians, and Native Americans for policyholder records prior to 1981 raises concerns about the accuracy of those racial identifications. Plaintiffs in disparate impact cases will likely have access to databases that are even less perfect than the database available to the FTC.

In our hypothetical lawsuit, there will be no racial information in the insurer's policyholder records that can be produced through the discovery process. Neither the plaintiff nor the defendant will have access to the Social Security Administration's database, as did the FTC. In order to carry the burden of showing disparate impact on any racial group, the plaintiff will necessarily be restricted to a conjecture and inference of each policyholder's race based on surname matches, Census tract data, or other potentially inaccurate indicia of race.

Since actuaries routinely use data to analyze insurance rates, actuaries will be able to offer this hypothetical court a great deal of expertise with regard to the reliability and credibility of any demographic data used to measure the extent of disparate impact on insurance rates.

6.2.3 Statistical significance of disparate impact

Assume the plaintiff is able to convince the court that its data are of sufficient accuracy and that some adverse disparate impact actually exists on the average premiums paid by a protected minority group. The next question before this hypothetical court is whether the disparate impact is significant enough to be of concern.

Since historically the "4/5's Rule" relied on by some courts in employment/hiring cases has been applied to binary decisions, (i.e., the decision to hire or not hire), it is not obvious how it would be applied to insurance rates. Perhaps as long as the impact on the average insurance premiums for a protected minority group is no greater than 20% of the impact on the premiums for the majority, then the disparate impact is deemed acceptable. However, this is only one of many possible tests that might be applied in disparate impact litigation. It is likely the plaintiff in this hypothetical lawsuit will argue for a narrower range of acceptability.

Actuaries are well-qualified to opine on the statistical and practical significance of any disparate impact found by the court, whether the degree of significance is based on some variation of the "4/5's Rule" or on the application of other common statistical tests of significance.

6.2.4 Business necessity and potential replacements

If our hypothetical court finds that credit-based insurance scores disparately, and significantly, impact the insurance premiums of a protected minority group, and if this hypothetical case then proceeds in the same way that similar employment/hiring cases have proceeded, the burden would then shift to the defendant insurer to show the business necessity of credit-based insurance scores.

Actuaries are uniquely qualified to conduct a multi-variate analysis of the defendant insurer's loss data to statistically prove the degree to which credit-based insurance scores add value and precision to the risk assessment process. The FTC's finding that credit-based insurance scores are effective predictors of auto insurance risk would likely be corroborating evidence.

It is important to note that the actuarial analysis supporting the business necessity of credit-based insurance scores (i.e., predictive power) will rely on obtainable, objective claim loss data, just as did the FTC Study. The analysis of predictive power does not rely on any inaccurate racial data, thereby avoiding the data problems associated with determining the existence of a disparate impact on any protected minority group.

Finally, an actuary would be uniquely qualified to opine on the effectiveness of any proposed alternative rate factors; how the elimination of the risk factor in question would create a rate structure that is unfairly discriminatory in violation of the state's rate regulatory standards; and be able to explain how the resulting adverse selection would lead to coverage availability problems in the market.

7. CONCLUSION

The concept of unfairly discriminatory rates has traditionally been cost-based, meaning that rates reflect the underlying risk and hazard. The concept of disparate impact has no relationship to the underlying insurance costs and refers solely to the adverse, significant disproportionate impact of one or more rate factors on a protected minority class.

The standards of unfair discrimination and disparate impact will potentially be in conflict because of the likelihood that protected minority classes will not be proportionately distributed throughout the various risk classifications. If the standard of disparate impact prevails over the standard of unfairly discriminatory rates, important risk factors will likely be banned from insurance rating plans. The elimination of even one proven risk factor will result in a rate structure that is unfairly discriminatory. Accurate risk assessment will be destroyed; adverse selection will be rampant; and coverage availability problems will likely arise.

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Biography of the Author

Michael J. Miller is a consulting actuary with EPIC Consulting, LLC. He is a FCAS and MAAA and has primarily focused his practice on ratemaking related issues. He has twice been elected to the CAS Board of Directors, has served the CAS as its Vice President–Research, has chaired the Risk Classification Committee and was the first chair of the Ratemaking Committee.

Monitoring Renewal Rate Change on Cat-Exposed Excess Property Business

Ira Robbin, Ph.D.

Abstract

This paper explains why a commonly-used metric of pricing performance, the Renewal Rate Change statistic, might not give true indications of the real rate change on Catastrophe (Cat)-Exposed Excess Property business. At the account level, false readings may arise when the renewal and expiring policies cover different layers or different sets of locations. When that happens, premium changes stemming from such differences are confounded with real changes in rate level. The paper presents a proposal to appropriately reflect coverage layer and location schedule differences. The proposal involves use of Cat Loss Simulation models to estimate the percentage by which the premium for an account should change in response to such differences. Once individual policy rate changes are correctly calculated, there is a potential problem in aggregating the individual results to a correct portfolio total. Concrete examples are presented to demonstrate that weighting with Renewal Premiums is incorrect and will lead to an overly optimistic answer. The paper then proposes alternative weights that lead to an unbiased result.

Keywords Rate Monitor, Renewal Rate Change, Excess Property, Catastrophe Models

1. INTRODUCTION

The Renewal Rate Change is a popular pricing metric, but it can give misleading indications when used to measure the rate change on Catastrophe-Exposed Excess Property business. There are potential problems both at the account level, in defining rate change for an account, and at the portfolio level, in weighting together the individual account rate changes to get a portfolio total.

For an individual account, the nominal change in rates between the renewal and expiring policies will not necessarily provide a valid rate change comparison when those policies cover different excess layers or cover different sets of locations. We will estimate the relative effect such differences should have on rate level by calculating how they impact Technical Premium[1] rates. This will allow us to offset the Nominal Renewal Rate Change for the effect of these differences and thus arrive at a better measure of the real rate change. The necessary Technical Premiums will be computed using results from a Catastrophe (Cat) Loss Simulation model.[2]

At the portfolio level, an optimistic bias is introduced if individual account rate changes are aggregated into a portfolio total using Renewal Premium weights. We will show what is wrong with

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[1] Technical Premium is here used to denote an indicated risk-loaded premium that is calculated directly from a set pricing algorithm without influence of schedule rating or other judgment modification.

[2] A cat loss simulation model runs thousands of modeled events against input locations and insurance coverages to arrive the estimated distribution of insurance losses from specified catastrophe perils. See Burger, Fitzgerald, White, and Woods [2] for a description of this type of model.

Renewal Premium weights and then propose better ones.

1.1 The Lack of Actuarial Literature on the Subject

There are many articles on rates and rate changes, but we have not found any that directly address the concerns we have identified regarding coverage layer changes, location schedule changes, and aggregation averaging. The one paper that is focused on rate monitors notes that "renewal rate change reports often provide misleading indications of rate changes due to changes in the underlying mix of business on each policy,"[3] but does not present a solution to that problem. Price monitors have also been the subject of discussion at several actuarial professional meetings,[4] but those discussions have not yet found their way into the actuarial literature. There also may be internal corporate memos documenting computations similar to those we will present. Unfortunately, those articles are not part of the public domain. Our belief is that this will be the first paper in the literature to directly address the key questions we have identified about renewal rate change computations for excess property business.

1.2 Industry Reported Rate Increases

In the aftermath of the Katrina, Rita, and Wilma hurricanes of 2005, many industry pricing surveys reported significant rate increases on hurricane-exposed property business.[5] Since we do not know how insurance companies computed renewal rate change figures by account or how they aggregated them into portfolio totals, we do not know if there were any distortions in those reported rate change figures. However, if we succeed in our objective, so that our methods, or improved variations, are adopted as industry standard practices, it may help to dispel questions about the computational validity or comparability of reported rate increases following the next major cat event.

1.3 Uses of the Renewal Rate Change Statistic

The Renewal Rate Change statistic is one of the most popular pricing metrics. It is used in two major ways. First, it is used by actuaries as one of several inputs needed to compute on-level premiums and to project rate adequacy. Second, it is used by managers as a gauge of pricing performance.

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[3] Vaughn [8], p. 506.
[4] See the presentations by Kundrot [4], Nyce [5], and Palisi [6].
[5] For example, the May, 2006 Expert Commentary on the First Quarter Market Survey from The Council of Insurance Agents and Brokers (CIAB) [3] had the quote "Overall, rates are down, *except* for property cat exposures (which are) up more than 25 percent…" (Italics in original).

1.3.1 On-Level Factors, New and Nonrenewed Business

In this paper, we will focus on measuring the Renewal Rate Change and only the Renewal Rate Change. So our discussion will only briefly touch on two other important pieces in the measurement of the overall rate: the contribution due to newly written business and the impact resulting from nonrenewed business.

Since the calculation of Renewal Rate Change omits consideration of new business, it provides at best an incomplete picture of the overall rate level.[6] We hypothesize that relative price adequacy between new and renewal business varies with the insurance cycle. When the market is tight and nonrenewed accounts go searching for a new carrier, they are in a weak bargaining position and may pay a large premium. Then a new business account may be more adequately priced than a renewal.[7] On the flip side, when the market softens and companies are flush with capacity, underwriters will agree to cut price to gain new accounts and meet volume targets. Then a new account may be relatively underpriced.

The Renewal Rate Change statistic also neglects any consideration of nonrenewed business. In general, the nonrenewed book is a mix of adequately priced accounts which departed for better deals elsewhere and poorly priced accounts which were cancelled or nonrenewed by company underwriters. The mix between the two likely varies over the insurance cycle and should change when the insurance company revises its pricing and underwriting policies.

A detailed computation of on-level factors would require going beyond looking at rate change on renewal accounts to also consider rate adequacy levels on new and nonrenewed accounts. However, in practice this is seldom done. Most on-level factors are derived solely from rate change statistics on renewal business. This is technically an incomplete treatment that might lead to inaccuracy in computed on-level factors.

1.3.2 Management Uses of Rate Monitors

The Renewal Rate Change statistic is one of several rate change statistics that may be used by company executives to monitor pricing. It is conceptually easy to understand, treats each underwriter fairly, and fosters clear lines of responsibility. These are some of the reasons the

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[6] Vaughn [8] p. 506 states "there are several drawbacks associated with renewal rate change reports. For instance, the renewal rate change report does not monitor the price level changes associated with new-business policies."

[7] Boor [1] describes a scenario that supports our hypothesis. He states "accounts may be 'orphaned' and unable to find coverage. This produces a situation where accounts are willing to accept higher prices for the benefit of having insurance coverage when it is hard to obtain." p. 3.

Renewal Rate Change statistic is generally accepted by both business line managers and underwriters.

Rate monitors are used in different ways by executives in managing insurance companies. Some executives set monitor targets and explicitly look at price monitor performance when evaluating underwriters and business units. These executives believe a connection between monitor performance and compensation will act as a powerful incentive for underwriters to push the line on price as far as it can go. Other executives disagree. They feel that too much emphasis on a statistic may distract underwriters from sound underwriting and may even spur them to find ways to "game" the system. We will not argue one way or the other about how rate change statistics should be used in managing a company. Our intent is to make actuaries aware that there may be widely divergent philosophies in different companies. Further, our discussion of the Renewal Rate Change statistic should not be construed as an endorsement or criticism for using that statistic, or any pricing statistic, as a measure of underwriter performance.

1.4 Loss Ratios Instead of Rate Monitors?

Since what really matters are results, one might play devil's advocate and question whether rate monitors have any use at all in managing the business. Why not just look at loss ratios? Isn't the proof of price performance ultimately in the results? Why supplement the loss ratio perspective with anything else? There are several general answers to this line of questioning. First, there is the matter of timing. Rate monitors are a leading indicator that can help us predict what loss ratio to expect on the business we are writing.[8] Loss ratios are a backward look at results on what has been written. So rate monitors give management a more timely indication of possible problems ahead. Second, there is the point that loss development introduces a lag before loss ratios results can be estimated with reasonable accuracy. When the lag runs into years, there may be some legitimate question about who is responsible for current results. Consider any long-tailed casualty line, where an underwriter can switch companies every few years and, with any luck, will stay one step ahead of the loss tail. Why should current underwriters be held to account for the shipwreck caused by a prior crew? In contrast, a price monitor would typically show the pricing achieved by current underwriters over the most recent prior month or quarter, with perhaps a lag of a month or two.[9]

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[8] Wang and Faber [9] support this view stating, "What the insurance industry needs is leading indicators…" and that "…companies need to diligently track rate level changes" (p. 59).

[9] Price monitor values can also age over time due to late bookings, premium audits, endorsements, and retro adjustments. In most cases, price monitor development is essentially complete after a few months.

The final problem with focusing solely on the loss ratio is that it can be too volatile. On this point, Cat-Exposed Excess Property is a good example. The loss ratio is usually quite low for every insurance company and underwriter, except when a catastrophe occurs. Then everyone's loss ratio is in the stratosphere. With so many underwriters doing poorly that year, it is hard to legitimately single out a specific underwriter for having had a poor showing. Of course, this is an oversimplification: a good underwriter will carefully control exposure aggregation, will write a geographically diversified mix of business, and utilize other underwriting practices which should translate into better results in any particular year and over the cycle. Nonetheless, the point still stands: a good cat underwriter will often lose money in a bad year and a poor cat underwriter will often make money in a good year.

1.5 Organization of the Paper

In Chapter 2, we will define the Renewal Rate Change statistic at the account level and show how to make the necessary adjustments. Then, in Chapter 3 we will examine how to aggregate the individual account rate changes into a portfolio average. All key formulas are documented in Appendix A, while Appendix B contains a discussion of data issues.

2. RENEWAL RATE CHANGE BY ACCOUNT

In this chapter we will first explain why it is necessary to go beyond the Nominal Renewal Rate Change on Cat-Exposed Excess Property business. Then we will walk through a hypothetical Excess Property example and show how to make account level adjustments for coverage layer and location schedule differences between the renewal and expiring policies. To avoid complications, we will assume all accounts have Specified Peril catastrophe coverage.[10]

2.1 Coverage Layer and Location Schedule Changes

For Cat-Exposed Excess Property business, it is not at all unusual for accounts to have renewal and expiring policies that provide different layers of coverage or that have different schedules of insured locations. For many other types of insurance business, the vast majority of accounts renew policies with coverage layers that are the same as the expiring ones and that have minimal or

[10] Accounts often have policies that cover Flood, Fire, Terrorism and other perils. Some polices may provide All-Risk Coverage or they may provide Difference in Conditions (DIC) Coverage.

predictable changes in exposure. Why is the Cat-Exposed Excess Property business different? Our answer is that Property Cat market has inherent characteristics which push the insured and the insurance company to initiate changes more frequently than for many other lines. In addition, much of this business is placed through Surplus Lines brokers. It is possible that accounts in the Nonadmitted market are subject to more frequent program changes than Admitted market accounts.

2.1.1 Changes Initiated by the Insured

The insured typically decides to revise its insurance program[11] in response to market price swings or capacity fluctuations. These are endemic to the Cat-Exposed Property market. In the aftermath of a significant cat, capacity gets tight, and the rate on Cat-Exposed Property skyrockets. The insured, who may be strapped for cash following a cat, is in no position to pay the huge premium increase needed to renew its expiring program. The insured may thus feel forced to retain more risk, either by changing the layer of insurance it purchases or, less frequently, by removing some locations from its insurance program. After a few years, prices will drop as naive capacity floods the market, and the process will reverse itself.

2.1.2 Changes Initiated By the Insurance Company

There are several situations in which an insurance company might initiate changes. A company faced with a downgrade by a rating agency, for instance, may need to reduce its exposed limits. To do this, it might reduce its percentage shares of layers with large limits or write different layers with lower limits. In another scenario, changes on individual accounts flow from a deliberate change made in the company's underwriting strategy. For example, the company might have been writing large shares of high excess layers that kept it above the "working layer." When the account is up for renewal, the company may have adopted a "ventilation" strategy under which it aims to write modest percentages of several disconnected layers, including some working layers.

2.1.3 Changes Initiated By the Surplus Lines Broker

A broker may sometimes be the driving force behind changes in an account's insurance program. It is possible that splitting a large layer in two or combining two small layers into one can result in an overall price reduction to the insured. As well, a broker may want to introduce new carriers to help

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[11] Beyond changing coverage layers and location schedules, changes are also made with respect to per location deductibles, per occurrence deductibles, occurrence hour range definitions (contiguous 96 hours versus 128 hours, etc.), location or peril sublimits, Business Interruption (BI) waiting periods and limits, and so forth.

spur competition for the account. This might necessitate cutting back on the shares of the incumbent writers or splitting layers to make room for the new carriers. A broker also may feel the need to make such changes in order to stave off competition from another Surplus Lines broker. Note that some program structure revisions can be made without altering the customer's overall layer of insurance coverage, even though they will alter the specific layers written by particular insurance companies.

2.2 Defining Real Rate Change

Before making adjustments to a nominal rate change figure in order to obtain the real rate change, we should first discuss the concept of a "real" rate change. When the expiring and renewal policies have identical coverages and location schedules, the real rate change is the same as the nominal rate change: both equal the change in the charged rate.[12] When they are not identical, we need to eliminate the portion of the nominal rate change which stems from differences in the layer of coverage or differences in the schedule of locations. What remains is the "real" rate change.

To introduce some theoretical precision about what portion of a rate change should be attributed to changes in the coverage layer and the location schedule, suppose we had a single manual showing perfectly adequate rates for both the expiring and renewal coverage layers and location schedules. By the phrase, "perfectly adequate rates," we mean a set of actuarially sound indicated rates that satisfy all rules of mathematical consistency and which within those rules faithfully reflect the risk-return preferences of the company.[13] In principle, the company should be equally willing to write any coverage layer and location schedule combination at the perfectly adequate rates listed in the theoretical manual. In this sense, all the rates in the manual are at the same rate level.

Our proposed conceptual approach is to use rate relativities from this hypothetical manual of perfectly adequate rates in order to ascertain the effect of coverage and location schedule differences between the renewal and expiring policies. For instance, if the manual says there is a 10% rate difference between the renewal and expiring layers, then we will back out that 10% rate difference when computing the real rate change.

We should note that perfectly adequate manual pricing does <u>not</u> imply the manual rates are

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[12] A rate in this context is the premium per unit of exposure.

[13] In defining perfect adequacy, we make reasonable assumptions as needed to avoid complications due to minimum premium rules, commission differences, and other similar factors.

calibrated to produce the same expected loss ratio for all coverages. Depending on how we price risk, we might reasonably have lower expected loss ratios for high excess layers than for ground-up limited layers.

Also, it should be emphasized we are using only one manual, the current one, in evaluating perfectly adequate premiums and in deriving the impact of coverage layer and location schedule changes. To be specific, we will assume the current manual is perfectly adequate as of the effective date of the renewal policy, even though it contains rates for layers and locations from the expiring policy. Note the manual of adequate rates could change from year to year -- for instance, due to the effect of trend. An account with no real rate change might thus end up with a renewal policy less adequately priced than the expiring policy.

To implement this framework, we have to clear a major hurdle: the lack of a manual of perfectly adequate rates for a Cat-Exposed Excess Property account. The proposal we will later present entails using Cat Loss Simulation models to generate Technical Premium rates that will serve as the best available estimates of perfectly adequate rates.

2.3 A Hypothetical Example Account

We will now look at an example in which there are both coverage layer and location schedule differences. To begin our example, suppose Wayne's Widgets is a major widget manufacturer and vendor. Assume it has a factory, a warehouse, and several retail stores in two hurricane-exposed states. Suppose our insurance company has written cat coverage on Wayne's Widgets for two years running. Table 2.3.1 summarizes the premiums, coverages, and exposures for the expiring and renewal policies.

Table 2.3.1

Premium, Coverage, and Exposure Summary
Wayne's Widgets

	Expiring	Renewal
Premium	\$50,000	\$40,000
Coverage	\$5m p/o \$25m x \$5m	\$2.5m p/o \$10m x \$15m
Company Limit	\$5,000,000	\$2,500,000
Layer 100% Limit	\$25,000,000	\$10,000,000
Attachment	\$5,000,000	\$15,000,000
Exposure (TIV)	\$30,000,000	\$25,000,000

The coverage for the expiring policy is $5 million part of $25 million excess of $5m and is written as "$5m p/o $25m x $5m," This means the insurance company has a 20% share of the $25m x $5m layer. The exposure is the Total Insured Value (TIV) of locations covered under the policy. This value is gross of deductibles.

Based on this data, we calculate the Nominal Rate, where the Nominal Rate equals the 100% Layer Premium Per $100 of TIV. The 100% Layer Premium is the premium for the full layer. To calculate it, we take the company premium and divide by the share. The reason the rate change is calculated with rates on a 100% basis is that this prevents changes in share from being incorrectly counted as rate changes. The Nominal Rate Change for our sample account is shown in Table 2.3.2.

Table 2.3.2

Nominal Rate Change
Wayne's Widgets

	Expiring	Renewal
Premium	$50,000	$40,000
Coverage	$5m p/o $25m x $5m	$2.5m p/o $10m x $15m
Company Limit	$5,000,000	$2,500,000
Layer 100% Limit	$25,000,000	$10,000,000
Attachment	$5,000,000	$15,000,000
Exposure (TIV)	$30,000,000	$25,000,000
Company Share	20%	25%
Layer 100% Premium	$250,000	$160,000
Rate per $100 TIV	$0.8333	$0.6400
Nominal Rate Change		-23%

2.3.1 Coverage Layer Differences

Our initial conclusion, based on the negative Nominal Rate Change, is that pricing has slipped. Yet this initial conclusion does not seem right. While the rate has dropped 23%, the 100% layer limit has dropped by 60%, from $25 million to $10 million. This suggests the charged rate could also fall by 60% without reducing rate level.

The beneficial effect of limit reduction is directly incorporated in another pricing statistic, the Rate on Line (ROL).[14] For the expiring policy, the 100% premium is $250,000 and the 100% limit is

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[14] ROL is defined as the 100% Layer Premium divided by 100% of the limit of the layer to be insured.

$25,000,000. So, the ROL is $10,000 per million. Similarly we can derive an ROL of $16,000 per million for the renewal policy. The ROL statistic thus indicates an improvement of 60%.

Raising the attachment reduces loss in many scenarios and this also points towards an improvement in the real rate level. Taken together, the reduction in the limit and the increase in the attachment would lead us to conclude the real rate change is positive, not negative.

2.3.2 Location Schedule Differences

A location schedule shows the location and Total Insured Value (TIV) of covered structures. Separate TIVs are shown for Structures, Contents, Business Interruption (BI), Extra Expense, and possibly other coverages. The location schedule should also have age of structure, construction type, occupancy class, protection rating, and other information about each location. The location

Table 2.3.2.1

Location Schedules

Wayne's Widgets

Location Schedule			**Expiring Policy**				
Loc Number	Description	Year Built	Street Address	City	State	ZIP	TIV
1	Co HQ	2005	12 Shady Lane	Pleasantville	AA	12345	$3,000,000
2	Warehouse	1977	Industrial Park Center	East Town	AA	12222	$8,000,000
3	Factory	1995	22 Fast Lane	Grime	AA	12288	$7,000,000
4	Retail Store A	2001	Harbor St Marina	ShoreHarbor	AA	10225	$3,000,000
5	Retail Store B	1998	Pier 7	Lighthouse	AA	10245	$3,000,000
6	Retail Store C	1982	Beach Lane Landing	Cape Shark	AA	10255	$3,000,000
7	Retail Store D	1999	Dock 15	Fishtown	BB	31288	$3,000,000
7	Total						$30,000,000

Location Schedule			**Renewal Policy**				
Loc Number	Description	Year Built	Street Address	City	State	ZIP	TIV
1	Co HQ	2005	12 Shady Lane	Pleasantville	AA	12345	$3,000,000
3	Factory	1995	22 Fast Lane	Grime	AA	12288	$7,000,000
4	Retail Store C	1982	Beach Lane Landing	Cape Shark	AA	10255	$3,000,000
5	Retail Store D	1999	Dock 15	Fishtown	BB	31288	$3,000,000
8	Retail Store E	1993	13 Canal Street	SandyShore	AA	12255	$3,000,000
9	Retail Store F	2003	81 Peninsula Drive	WaveCrest	BB	31224	$3,000,000
10	Retail Store G	1992	Seashore Mall	Seashore	BB	31288	$3,000,000
7	Total						$25,000,000

schedules for our fictitious account are shown in Table 2.3.2.1.

For convenience and simplicity, we have summarized the TIVs to a single number and omitted the construction code, occupancy class, and other such information. For any real application, such information could have a material effect on the answer and should not be omitted. In comparing renewal and expiring schedules in our example, we note the insured removed the warehouse and two of its retail stores from the program, and replaced them with three other stores. We also observe that more of the TIV is concentrated in state BB for the renewal than it was for the expiring policy. The impact of these mix changes is not knowable in advance. However, the Nominal Rate Change will misconstrue as real rate change any rate movement that might arise from such location schedule differences.

2.4 Technical Premium Based on Cat Loss Simulation Model Statistics

As previously outlined, our proposed solution entails quantifying the impact that coverage layer and location schedule differences have on Technical Premium rates. In our application, a Technical Premium denotes an indicated premium computed by machine algorithm without schedule rating or other judgment modification. We will assume the Technical Premium includes provision for risk-loaded loss. We will also suppose that it does not reflect any minimum premium or minimum rate on line constraints.

Our proposal is to compute Technical Premiums using results from a cat loss simulation model. The pure loss provision in the premium will be equal to the Average Annual Loss (AAL) from the model and the risk load could be based on any of a variety of risk metrics generated by the model. These risk metrics include the Variance, Standard Deviation, Probable Maximum Loss, and Tail Conditional Expectation. We will neither define all these metrics nor discuss their particular advantages or disadvantages here.[15] Instead, in order to demonstrate our proposal, we will focus on one widely accepted risk metric, the Probable Maximum Loss (PML),[16] and compute our risk load as 5% of the PML. Including a loading factor of 1.50 for expenses, we arrive at the following illustrative Technical Premium formula:

$$\text{Technical Premium} = 1.5 \bullet \left(AAL + .05 \bullet PML \right) \tag{2.4.1}$$

[15] See Robbin and DeCouto [8] for one discussion of various risk metrics.
[16] In the sense that we are using it, the PML is associated with a return period. For example, the 100-year PML is the size of cat loss that on average occurs no more frequently than 100 years.

2.5 Cat Loss Simulation Model Runs

As previously outlined, our proposed solution entails quantifying the impact that coverage layer and location schedule differences have on Technical Premium rates. To avoid confusing a change in share with a change in the 100% coverage layer, we will refer to a difference in the 100% layer of coverage as a difference in Coverage Structure. In our example, there is a Coverage Structure change from the 25m x 5m layer to the 10m x 15m layer.

We will also decompose location schedule differences into Exposure Magnitude and Location Mix components. In our example, there is an Exposure Magnitude movement in the TIV reduction from $30m to $25m. There are also Location Mix changes in the types of structures covered and in the distribution of the structures between states and within states.[17]

We will separately quantify the effects of Coverage Structure and Location Mix differences. One could estimate the combined impact of changes in Coverage Structure and Location Mix with one overall factor. However, it is useful to have a breakdown of their separate effects. Especially when these adjustments point in different directions, it may be somewhat unconvincing to provide a single number summary without showing the offsetting contributions made by Coverage Structure and Location Mix differences.

To implement this "separate effects" approach, we will make a series of runs with our Cat Loss Simulation model. We will start with the Expiring Location Schedule and Expiring Coverage Structure, then change the location schedule and finally change the 100% coverage layer. Thus, the sequence of cat runs we are proposing is as shown in Table 2.5.1.

It is important to emphasize we are using only one version of a cat loss simulation model in evaluating the impact of these changes. Typically we would employ the most recent version of the model as it incorporates the latest knowledge and advances in methodology. To switch models would be analogous to switching measuring sticks between two measurements: we would be unsure how much of any difference was due to a real difference and how much was due to the switch in our measuring stick.

[17] For a limited excess layer, a uniform change in the values of all locations does not necessarily lead to the same proportionate change in the 100% layer loss.

Table 2.5.1

Cat Runs for Renewal Rate Change Monitoring

Number	Location Schedule	100% Coverage Layer	Cat Model
1.	Expiring	Expiring	Current
2.	Renewal	Expiring	Current
3.	Renewal	Renewal	Current

2.6 Adjustment Factor Formulas

To translate the 100% Layer Technical Premiums from the three cat runs listed in Table 2.5.1 into Technical Rates, we divide by the appropriate TIVs. We will then define Location Mix and Coverage Structure Adjustment Factors (MXAF and CSAF respectively) by taking the following ratios of Technical Rates:

$$MXAF = \frac{\text{Rate based on Renewal Location Schedule and Expiring 100\% Coverage Layer}}{\text{Rate based on Expiring Location Schedule and Expiring 100\% Coverage Layer}} \qquad (2.6.1)$$

$$CSAF = \frac{\text{Rate based on Renewal Location Schedule and 100\% Renewal Coverage Layer}}{\text{Rate based on Renewal Location Schedule and 100\% Expiring Coverage Layer}} \qquad (2.6.2)$$

The derivation of adjustment factors for our sample account is shown in Table 2.6.1. Note under our definitions the Location Mix Adjustment Factor will be unity if there is no change in the location schedule and the Coverage Structure Adjustment Factor will be unity if there is no change in the 100% layer of coverage. An adjustment factor value above unity means that some of the Nominal Renewal Rate increase is absorbed in covering the impact of the associated location schedule or coverage layer change. See Appendix A for more compact versions of these formulas.

Table 2.6.1

Technical Premiums, Technical Rates and Adjustment Factors

Wayne's Widgets

	Expiring Exposure Expiring Layer	Renewal Exposure Expiring Layer	Renewal Exposure Renewal Layer
Coverage	\$5m p/o \$25m x \$5m	\$5m p/o \$25m x \$5m	\$2.5m p/o \$10m x \$15m
Exposure (TIV)	\$30,000,000	\$25,000,000	\$25,000,000
Company Share	20%	20%	25%
100% AAL	\$50,000	\$40,000	\$15,000
100% PML	\$15,000,000	\$11,000,000	\$5,000,000
Technical Premium	\$300,000	\$225,000	\$97,500
Technical Rate	\$10.00	\$9.00	\$3.90
Adjustment Factor		0.900	0.433

2.7 Renewal Rate Change Formula

We are now finally ready to calculate the Renewal Rate Change (ΔR) by netting the impacts of the Location Mix and Coverage Structure changes from the Nominal Renewal Rate Change (ΔN).[18] The formula is:

$$\Delta \mathrm{R} = \frac{(1+\Delta N)}{MXAF \cdot CSAF} - 1 \tag{2.7.1}$$

Here ΔN is the Nominal Rate Change, MXAF is the Location Mix Adjustment Factor, and CSAF is the Coverage Structure Adjustment Factor.

In Table 2.7.1, we apply this formula to Wayne's Widgets. Table 2.7.1 shows how the Nominal Renewal Rate decrease in our example has been transformed into a sizeable (adjusted) Renewal Rate increase. We think this more accurately represents the real rate change.

[18] An alternative terminology is to call ΔR the Effective Renewal Rate Change and refer to ΔN as the Renewal Rate Change. We choose to call ΔR the Renewal Rate Change and refer to ΔN as the Nominal Renewal Rate Change.

Table 2.7.1

Renewal Rate Change

Wayne's Widgets

	Expiring	Renewal
Premium	\$50,000	\$40,000
Coverage	\$5m p/o \$25m x \$5m	\$2.5m p/o \$10m x \$15m
Exposure (TIV)	\$30,000,000	\$25,000,000
Company Share	20%	25%
Layer 100% Premium	\$250,000	\$160,000
Rate per \$1000 TIV	\$8.33	\$6.40
Nominal Rate Change		-23%
Location Mix Adjustment Factor (MXAF)		0.9000
Coverage Structure Adjustment Factor (CSAF)		0.4333
Renewal Rate Change		97%

2.8 Premium Reconciliation

It is useful to reconcile the Expiring and Renewal Premiums. The reconciliation proceeds mathematically as follows:

$$P_{REN} = P_{EXP} \cdot \frac{s_{REN}}{s_{EXP}} \cdot \frac{TIV_{REN}}{TIV_{EXP}} \cdot MXAF \cdot CSAF \cdot (1 + \Delta R) \tag{2.8.1}$$

In words, to go from the Expiring Premium to the Renewal Premium, we need to adjust for changes in Share, Exposure Magnitude, Exposure (location) Mix, Coverage Structure (100% layer), and then reflect the true Renewal Rate Change.

For our example account we get:

$$40,000 = 50,000 \cdot \frac{.25}{.20} \cdot \frac{25}{30} \cdot .90 \cdot .433 \cdot (1.97) \tag{2.8.2}$$

From this perspective, we see the overall premium change has been split between a real rate level change component and other components that are not counted as real rate change contributors. These non-rate change factors include volume scaling factors such as change in Share and change in Exposure Magnitude. They also include rate movement factors due to Location Mix and Coverage Structure changes.

2.9 Notional Expiring Premium

We can also infer what the Expiring Premium would have been if it were based on the same location schedule and layer of coverage as the Renewal policy. We will call this the Notional Expiring Premium and denote it as $P_{NXP.}$[19] Following the definition, it is derived by starting with the Expiring Premium and adjusting for Share, Exposure Magnitude, Location Mix, and Coverage Structure changes. It is not hard to see this is equivalent to the Renewal Premium net of the Renewal Rate Change.

$$P_{NXP} = P_{EXP} \cdot \frac{TIV_{REN}}{TIV_{EXP}} \cdot \frac{s_{REN}}{s_{EXP}} \cdot MXAF \cdot CSAF = \frac{P_{REN}}{(1+\Delta R)} \tag{2.9.1}$$

For our Wayne's Widgets example, we find that P_{NXP} =40,000/ (1.97) = \$20,312. This is far less than the actual Expiring Premium of \$50,000.

2.10 Account Renewal Rate Change Summary

To summarize our derivation of Renewal Rate Change for an account, we first put the rates on a 100% basis to eliminate any distortion due to change in share. Then we adjusted the Nominal Rate Change for Location Mix and Coverage Structure changes. The adjustments were derived by using a Cat Loss simulation model to compute, in sequence, the impact such changes would have on Technical Premium rates. We assumed the relationship between Technical Premiums rates should apply to the charged premium rates. So, for example, if changing the 100% coverage layer moved the Technical Premium Rate by 10%, then we assumed 10 points of any Nominal Rate Increase would be attributable to the difference in Coverage Structure. The final Renewal Rate Change is the Nominal Rate Change net of these Location Mix and Coverage Structure adjustment factors. We have also seen how the Renewal Rate Change thus defined can be reconciled with the absolute change in premium.

2.11 Alternative Renewal Rate Change Estimates

Once we go beyond the simple calculation of Nominal Renewal Rate Change and attempt to reflect the impact of differences between the Renewal and Expiring policies, we introduce questions about how to evaluate the rate impact of such differences. So, while all actuaries should arrive at the

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[19] Another name for what we call the Notional Expiring Premium is the Renewal Premium at Expiring Rates.

same Nominal Rate Change for an account, they might arrive at alternative estimates of the Renewal Rate Change. Such divergent estimates could arise from the use of alternative Cat Simulation Models and Technical Premium formulas.[20]

On working excess layers, we have found that most models and formulas produce percentage adjustments often within a few points of one another, and seldom more than five to ten points apart. In contrast, on very high excess layers, we may see significantly different results when different models and formulas are used. As Cat Simulation Models improve over time, we would expect the range of differences to become narrower and we would also expect major differences to be found on smaller sets of accounts.[21] A key point is that all actuarially sound models and formulas should produce consistent results, even for high excess layers. For example, if the 100% layer limit is reduced and all else remains unchanged, the Coverage Structure Adjustment Factor will always be less than unity since the reduced coverage and reduced risk imply a reduced rate.

While it is disheartening not to have an indisputable exact answer, the use of estimates is common in many aspects of current actuarial practice. Part of the art inherent in actuarial science is in selecting appropriate parameters, formulas, and models in order to derive a reasonable range of estimates and then in selecting a final pick within that range. Such is the case in the procedure we are proposing for estimating the real rate change.

3. PORTFOLIO AVERAGE RATE CHANGE

Once we have the Renewal Rate Change for each policy, the question then is how to aggregate results to get the portfolio average Renewal Rate Change. It is a common practice to simply take a weighted average of the rate changes, where Renewal Premiums are used as weights. However, as we will demonstrate, this practice leads to an overly optimistic estimate of rate change. The bias goes one way: rate level improvements are not as substantial as indicated and rate level decreases are worse than indicated. Weights based on Expiring Premiums are better, but they give an out-of date and potentially distorted picture. We will present an alternative in which the weights are based on the Notional Expiring Premiums defined in Chapter 2. We will walk through a series of hypothetical

[20] This problem is not unique to Cat-Exposed Property insurance. Using increased limits factors, one could derive coverage structure adjustments for casualty accounts and different sets of factors would lead to different rate change estimates.

[21] Other implementation and data issues are discussed in Appendix B.

scenarios comparing the weighted average rate changes that result from these weights.

3.1 Renewal Premium Weighting Bias

Suppose we have a portfolio of four risks. Assume for the base case that the Exposure

Table 3.1.1

Portfolio Weighted Average Rate Change
Base Case

	Expiring			**Renewal**					
Risk	Prem (000)	Share	TIV mill	Prem (000)	Share	TIV mill	Nominal Rate Change	Rate Change	NXP Prem (000)
A	$200	25%	100	$280	25%	100	40%	40%	$200
B	$200	25%	100	$120	25%	100	-40%	-40%	$200
C	$50	25%	200	$70	25%	200	40%	40%	$50
D	$50	25%	50	$30	25%	50	-40%	-40%	$50
Total	$500	25%		$500	25%				$500
Weights									
Renewal Prem							16%	16%	
Expiring Prem							0%	0%	
NXP Prem							0%	0%	

Magnitude stays constant, as does the Share, Location Mix, and Coverage Structure. Only the rates change as shown in Table 3.1.1.

In the Base Case we have rate changes by account that end up generating the same overall premium for the expiring and renewal portfolios. Despite the fact that we are getting the same exact money to cover the same exact exposures overall, the weighting of rate changes on Renewal Premiums incorrectly indicates there is a sizeable overall rate increase. This happens because the Renewal Premium weighting algorithm inherently gives undue emphasis to those policies that had rate increases while deemphasizing those that had decreases. In other words, it overcounts rate increases and undercounts rate decreases. As a consequence, the result from weighting with Renewal Premium is biased. It paints an overly optimistic picture of any actual portfolio level increase and masks the true extent of any portfolio level decrease.

3.2 Expiring Premium Weighting Inaccuracy

In the Base Case, there was no difference between the actual Expiring Premium and the Notional Expiring Premium. Now we will modify our example by increasing our share on the accounts that had rate increases. These share increases boost Renewal Premiums, but they do not impact the individual account rate increases. This "Shares Up on Accounts with Rate Increases" scenario is shown in Table 3.2.1.

Table 3.2.1

Portfolio Weighted Average Rate Change
Shares Up on Accounts with Rate Increases

	Expiring			**Renewal**					
Risk	Prem (000)	Share	TIV mill	Prem (000)	Share	TIV mill	Nominal Rate Change	Rate Change	NXP Prem (000)
A	$200	25%	100	$560	50%	100	40%	40%	$400
B	$200	25%	100	$120	25%	100	-40%	-40%	$200
C	$50	25%	200	$140	50%	200	40%	40%	$100
D	$50	25%	50	$30	25%	50	-40%	-40%	$50
Total	$500	25%		$850	43%				$750
Weights									
Renewal Prem							26%	26%	
Expiring Prem							0%	0%	
NXP Prem							13%	13%	

Since more business is from accounts that had increases, we should expect to see a rate increase for the overall portfolio. However, because changing shares has no impact on the Expiring Premium, a weighting on Expiring Premium is the same as in the Base Case and yields no portfolio rate increase.

Next, in Table 3.2.2, we look at an example in which the TIVs increase only for the accounts that had rate increases.

Table 3.2.2

Portfolio Weighted Average Rate Change
TIVs Up on Accounts with Rate Increases

	Expiring			**Renewal**					
Risk	Prem (000)	Share	TIV mill	Prem (000)	Share	TIV mill	Nominal Rate Change	Rate Change	NXP Prem (000)
A	$200	25%	100	$504	25%	180	40%	40%	$360
B	$200	25%	100	$120	25%	100	-40%	-40%	$200
C	$50	25%	200	$126	25%	360	40%	40%	$90
D	$50	25%	50	$30	25%	50	-40%	-40%	$50
Total	$500	25%		$780	25%				$700
Weights									
Renewal Prem							25%	25%	
Expiring Prem							0%	0%	
NXP Prem							11%	11%	

In Table 3.2.2, we assume the TIV changes only impact the Exposure Magnitudes. Therefore these exposure differences do not alter the rate change for any account, yet they do change the relative weight of an account within the portfolio. With more weight now given to accounts that had rate increases, we should now expect to see an overall rate increase. This overall rate increase does appear when Notional Expiring Premium weights are used. In contrast, the weighted average as computed with Expiring Premium weights incorrectly shows no overall rate increase. This is not surprising. By definition, the Expiring Premium weights cannot respond to this TIV-driven change in portfolio mix.

Finally, in Table 3.2.3 we look at a scenario where accounts have Renewal Rate Changes that offset one another in such a way that the total Notional Expiring Premium equals the total Renewal Premium. This means there has been no overall rate change.

Table 3.2.3

Portfolio Weighted Average Rate Change
Offsetting Coverage Strucuture Changes

	Expiring			**Renewal**					
Risk	Prem (000)	Share	TIV mill	Prem (000)	Share	TIV mill	Nominal Rate Change	Rate Change	NXP Prem (000)
A	$200	25%	100	$280	25%	100	40%	75%	$160
B	$200	25%	100	$120	25%	100	-40%	-50%	$240
C	$50	25%	200	$70	25%	200	40%	17%	$60
D	$50	25%	50	$30	25%	50	-40%	-25%	$40
Total	$500	25%		$500	25%				$500
Weights									
Renewal Prem							16%	31%	
Expiring Prem							0%	9%	
NXP Prem							-5%	0%	

When we weight on Notional Expiring Premiums, we correctly end up with that 0% overall portfolio rate change. Weighting with either Expiring or Renewal Premium incorrectly overstates the rate increase. Further, this scenario demonstrates that a weighted average taken with Notional Expiring Premiums does not have to fall between the corresponding weighted averages computed with Renewal Premiums and Expiring Premiums respectively.

3.3 Notional Expiring Premium Weighting

We have seen through concrete examples that Notional Expiring Premium weights are demonstrably superior to either Renewal Premium or actual Expiring Premium weights. They do not overcount rate increases and undercount rate decreases like Renewal Premiums do. They do not ignore changes in the weight of an account within the portfolio as actual Expiring Premiums do. There is an intuitive appeal to derive weights by taking the Renewal Premiums and backing out the Renewal Rate Changes. The resulting weights will be appropriately sensitive to relative importance of an account within the current portfolio. As noted in Equation (2.9.1), the resulting premiums are algebraically equivalent to the Notional Expiring Premiums. To summarize, our proposal is to compute the overall portfolio rate change ($\Delta R(TOT)$) using Equation (3.3.1).

$$\Delta R(TOT) = \frac{\sum P_{NXP}(i) \cdot \Delta R(i)}{\sum P_{NXP}(i)} = \frac{\sum \frac{P_{REN}}{1+\Delta R(i)} \cdot \Delta R(i)}{\sum \frac{P_{REN}(i)}{1+\Delta R(i)}} \tag{3.3.1}$$

3.4 Weighted Harmonic Average Interpretation

We have felt it intuitively appealing to present our procedure as the computation of an arithmetic weighted average of rate changes and to focus our attention on arriving at the proper weights. This allowed us to directly demonstrate the error in using Renewal Premium weights when computing the arithmetic weighted average rate change and to further argue that Notional Expiring Weights should be used instead. Another approach is to retain the Renewal Premium Weights, but to use a different type of average, called the harmonic average. This interpretation is mathematically presented in the derivation shown in Equation (3.4.1).

$$\Delta R(TOT) = \frac{\sum \frac{P_{REN}}{1+\Delta R(i)} \cdot \Delta R(i)}{\sum \frac{P_{REN}(i)}{1+\Delta R(i)}} \tag{3.4.1}$$

$$= \frac{\sum \frac{P_{REN}}{1+\Delta R(i)} \cdot (1+\Delta R(i))}{\sum \frac{P_{REN}(i)}{1+\Delta R(i)}} - 1 = \frac{\sum P_{REN}(i)}{\sum \frac{P_{REN}(i)}{1+\Delta R(i)}} - 1$$

The latter ratio of sums in Equation (3.4.1) is the Weighted Harmonic Average of the Renewal Rate Change Factors with weights based on Renewal Premiums. Some actuaries may be more comfortable with this interpretation.

We may also use a mathematical argument based on harmonic averages to buttresses our intuitive reasoning about why the Renewal Premium Weighted Average of Rate Changes is biased upward. It can be easily shown that the Weighted Harmonic Average is always less than or equal to the

Weighted Arithmetic Average. The inequality in Equation (3.4.2) holds.

$$\Delta R(TOT) = \frac{\sum P_{REN}(i)}{\sum \frac{P_{REN}(i)}{1+\Delta R(i)}} - 1 \tag{3.4.2}$$

$$\leq \frac{\sum P_{REN}(i)\cdot(1+\Delta R(i))}{\sum P_{REN}(i)} - 1 = \frac{\sum P_{REN}(i)\cdot \Delta R(i)}{\sum P_{REN}(i)}$$

It follows from Equation (3.4.2) that weighting Renewal Rate Changes with Renewal Premium weights produces an overly optimistic result.

4. CONCLUSION

We have presented solutions to the two major actuarial problems that can cause the Renewal Rate Change metric to be misleading on Cat-Exposed Excess Property business. Our first problem was to figure out how to adjust the nominal rate changes so as to properly account for differences in the layer structure and location mix of the renewal and expiring policies. Our solution was to derive Technical Premiums with the latest cat loss simulation model, first changing the locations and then changing the coverage layer. From these we computed Technical Premium rates and then derived adjustment factors by taking appropriate ratios between these rates.

Our second major problem was to find which weights to use when computing the overall portfolio rate change as a weighted average of the individual account rate changes. We found there is an optimistic bias in the results when Renewal Premium weights are used. We also demonstrated that weighting on Expiring Premiums is flawed, as it is insensitive to changes in Share and to portfolio level TIV mix changes. Our solution was to weight on premiums adjusted to be at the expiring rate level, but based on the renewal location schedule, share, and layer structure. We called these the Notional Expiring Premiums. We have seen these are equivalent to Renewal Premiums Net of Rate Change. We have also shown our approach is equivalent to taking the Weighted Harmonic Average of Rate Changes while using the Renewal Premiums as weights.

We hope some practical benefit will come from our efforts to resolve actuarial questions about how to calculate Renewal Rate change statistics on Cat-Exposed Excess Property business. Accounts in this business have been subject to large rate swings, the magnitude of which may have been obscured by changes in location schedules and coverage layers and by actuarially incorrect ways of aggregating individual account rate changes. It is our intent to foster development of a uniform and actuarially valid approach to computing the Renewal Rate change on this business so as to increase the accuracy and public credibility of reported rate change statistics. While the use of different Cat Simulation models may lead to different Renewal Rate Change estimates, a reasonable range of estimated effects due to changes in Coverage Structure and Location Mix is better than ignoring the effects of such changes altogether. We would encourage others to write on these and other actuarial issues inherent in rate monitoring statistics so that a more extensive literature on price monitors develops over time.

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Disclaimers

The opinions expressed are solely those of the author and are not presented as a statement of the position of Endurance American Insurance Company or any parent or affiliated company. The reader is warned that the author assumes no liability for any damages that may result directly or indirectly from use of the methods presented in this paper.

Appendix A – Notation and Formulas

In Table A.1, we define our basic notation:

Table A.1

Basic Notation

P	Premium at Company Share
P100	Premium at 100%
K	Company Limit
K100	100% Layer Limit
T	Total Insured Value
S	Company Share
N	Nominal Rate

For the i[th] policy, Equations (A.1),)A.2), (A.3), and (A.4) hold:

$$s(i) = K(i)/K100(i) \tag{A.1}$$

$$P(i) = s(i) \cdot P100(i) \tag{A.2}$$

$$N(i) = P100(i)/T(i) \tag{A.3}$$

$$P(i) = N(i) \cdot s(i) \cdot T(i) \tag{A.4}$$

Use subscripts EXP and REN to denote whether a variable is for the expiration or renewal policy respectively. Define the Nominal Renewal Rate Change for the i[th] policy as:

$$\Delta N(i) = \frac{N_{REN}(i) - N_{EXP(i)}}{N_{EXP}(i)} \tag{A.5}$$

We will suppress the policy "(i)" notation to simplify the formulas and derivations that follow unless it is needed for clarity.

To make mix and coverage change adjustments, we define the following 100% Technical Premium Rates:

Table A.2

Technical Premium Rate	100% Layer	Exposures
$Q_{REN,REN}$	Renewal	Renewal
$Q_{REN,EXP}$	Renewal	Expiring
$Q_{EXP,EXP}$	Expiring	Expiring

The Location Mix and Coverage Structure adjustment factors are defined as follows:

$$MXAF = \frac{Q_{REN,EXP}}{Q_{EXP,EXP}} \tag{A.6}$$

$$CSAF = \frac{Q_{REN,REN}}{Q_{REN,EXP}} \tag{A.7}$$

The Renewal Rate Change, ΔR, is given as:

$$\Delta R = \frac{(1+\Delta N)}{MXAF \bullet CSAF} - 1 \tag{A.8}$$

In words, to get the Renewal Rate Change we start with the Nominal Rate Change and net out the Mix Change and Coverage Structure Change adjustments. Next we define the Notional Expiring Premium by taking the Expiring Premium and adjusting for changes in Exposure Magnitude, Share, Location Mix, and Coverage Structure.

$$P_{NXP} = P_{EXP} \cdot \frac{TIV_{REN}}{TIV_{EXP}} \cdot \frac{s_{REN}}{s_{EXP}} \cdot MXAF \cdot CSAF \tag{A.9}$$

It follows from these definitions that:

$$P_{NXP} = \frac{P_{REN}}{(1+\Delta R)} \tag{A.10}$$

Weighting with the Notional Expiring Premium, we define the overall portfolio Renewal Rate Change as:

$$\Delta R(TOT) = \frac{\sum P_{NXP}(i) \cdot \Delta R(i)}{\sum P_{NXP}(i)} \tag{A.11}$$

The overall portfolio Renewal Rate Change can be equivalently express as the Weighted Harmonic Average of the Renewal Rate Change Factors using Renewal Premium weights:

$$\Delta R(TOT) = \frac{\sum P_{REN}(i)}{\sum \frac{P_{REN}(i)}{1+\Delta R(i)}} - 1 \tag{A.12}$$

To see this consider that

$$\frac{\sum P_{NXP}(i) \cdot \Delta R(i)}{\sum P_{NXP}(i)} = \frac{\sum \frac{P_{REN}(i)}{1+\Delta R(i)} \cdot \Delta R(i)}{\sum \frac{P_{REN}(i)}{1+\Delta R(i)}} \tag{A.13}$$

$$= \frac{\sum \frac{P_{REN}(i)}{1+\Delta R(i)} \cdot ((1+\Delta R(i)) - 1)}{\sum \frac{P_{REN}(i)}{1+\Delta R(i)}} = \frac{\sum P_{REN}(i)}{\sum \frac{P_{REN}(i)}{1+\Delta R(i)}} - 1$$

APPENDIX B - DATA AND MODELING ISSUES

The reliability of the Renewal Rate Change statistic depends heavily on having accurate data. The database fields for the calculations we have outlined include premium, coverage layer (attachment, company limit, and layer limit) parameters, and location schedule information for "matched" renewal and expiring policies. Most experienced actuaries will have their own ways of checking the data for reasonability and flagging questionable data items. Our purpose here is to point out a few data issues beyond the usual ones. We also want to mention a few of issues that might arise is using Cat Loss simulation models in the rate monitoring process.

B.1 Matching

The matching is often more difficult than it might first appear. Sometime an account will have a check mark in a renewal status field, yet its expiring policy may be missing. The account name and number may have changed, and there may be no easy automated way to pair up wayward accounts. Coverages must also be matched, as a renewal and expiring account may sometimes have different coverages. For example, Business Interruption (BI) may be covered on the expiring policy but not the renewal. In that case, the premium allocated for BI on the expiring policy should be removed.

B.2 Annualized Premium

Another issue is that the expiring and renewal policy may have different durations. One way to handle this is to compute rates with annualized premiums and exposures. Though it is an unnecessary duplication for many accounts, it is worth having a separate annualized premium field. This field can also be used to adjust for midterm endorsements. While annualized premiums are recommended for use in computing rates and rate changes, the actual and not the annualized premiums should be used when weighting rate changes to obtain the portfolio average rate change.

B.3 Aging of Statistics

The actuary may also find it instructive to look at how the monitor statistics "age" over time. In a rush to get the latest pricing statistics, management may insist on getting monitor statistics produced as quickly as possible after a period closes. Yet this may miss many pricing changes, new business bookings, and nonrenewals that come in later. There may also be a spate of endorsements, extensions, cancellations, or back-outs of endorsements. Producing refreshed versions of monitor statistics may reveal a characteristic pattern where, for example, a 10% increase on first look declines to 2% over the next two months as booking is "trued up" in the system. Studying a series of refreshed monitors may also turn up specific offices, lines of business, or underwriters with stellar initial statistics that evaporate over time.

B.4 Quality of Location Schedule Data

The data on location schedules is usually checked by cat modelers during the cat modeling process. However, as with any data from whatever source, the actuary would be well advised to check it. The cat modeler will typically do statistical data checks to tell, for example, that 90% of the locations are geo-coded to the zip code level or better. With large location schedules, having another set of human eyes review the data can often turn up potential anomalies the machine may miss. For example, duplicate entry errors may be masked by street addresses with names or numbers that differ slightly. The actuary should also check the TIV data. Locations with small TIVs should not be ignored. The TIVs for some of those locations may have been incorrectly entered in units of thousands. A comparison should also be done of the new schedule against the prior one. If both schedules have identical location addresses and values, the actuary should request an update. If some locations in the new schedule are missing street addresses, the actuary may find them in the prior schedule. Data fields on Construction, Occupancy, and other characteristics are also shown on the location schedule. Having accurate entries will usually have a material impact on the quality of answers coming out of any cat loss simulation model. It is thus worthwhile to ask the underwriter to contact the broker for more definitive information if for example half the construction types are listed as "unknown." Overall, the key to achieving better data quality is to work cooperatively with underwriters and cat modelers.

B.5 Implementation Issues

Before attempting to implement the procedures suggested in this paper, the actuary would be well advised to run sensitivity tests and sample computations for a large set of renewal accounts. When evaluated on actual accounts, a Technical Premium formula that is superior to another in theory may turn out to be inferior in practice. A robust formula that always yields sensible results that move in the right direction may be preferable to one that for unknown reasons produces bizarre results on a small percentage of accounts even while performing in stellar fashion on the others.

Part of the problem is that current cat loss simulation models are not quite up to the task we have set for them. They were designed for portfolio analysis and they do a reasonably credible job costing reinsurance treaties. However, anomalous results can sometimes crop up for individual insureds, especially for those with a small number of locations. With currently available models, portfolio impact statistics[22] on individual accounts also tend to misbehave more than comparable stand-alone statistics. Advances in cat loss simulation modeling should mitigate problems of this sort over time.

[22] The portfolio impact version of a statistic is the difference between its value on the portfolio after adding the additional insured and its value on the initial portfolio.

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Abbreviations and Notations

Cat, Catastrophe
ROL, Rate on Line
PML, Probable Maximum Loss
TIV, Total Insured Value
AAL, Average Annual Loss
NXP, Notional Expiring Premium

Biography of the Author

Ira Robbin is currently Senior Vice-President and Chief Pricing Actuary for the US and London at Endurance Worldwide Insurance. Ira received a Bachelor's Degree in Math from Michigan State University and a PhD in Math from Rutgers University. Then he took an actuarial research position with the Insurance Company of North America (INA) and has been working at insurance and reinsurance companies ever since. He has headed large risk property and casualty pricing units, developed pricing algorithms, and produced a number of price monitors. He has authored several Proceedings, Forum, and Study Note papers on a range of subjects, taught exam preparation classes, and made numerous presentations at actuarial meetings.

Statistical Trend Estimation with Application to Workers Compensation Ratemaking

Frank Schmid

Abstract

Motivation. Estimating the trend of the severity, frequency, and loss ratio rates of growth is an integral part of NCCI ratemaking. The time series from which such trend estimation has to be derived are typically short and volatile, comprising only 19 observations or, equivalently, 18 rates of growth. Thus, separating signal (i.e., trend) from (white) noise is particularly challenging.

Method. NCCI has developed a Bayesian Statistical Trend model that is geared toward extracting the trend in short and high-volatility time series. This model has been optimized by minimizing the root mean squared prediction error across NCCI states using three-year hold-out periods (as the applicable forecasting horizon is typically around three years).

Results. We present trend estimates for severity, frequency, and loss ratio rates of growth for an unidentified state. The model is robust to outliers and delivers stable, yet time-varying trend estimates.

Conclusions. The statistical properties of the model are conducive to rate stability and, at the same time, allow the practicing actuary to recognize changes in trend.

Availability. The model runs in WinBUGS 1.4.3 (www.mrc-bsu.cam.ac.uk/bugs) within the R (www.r-project.org) package R2WinBUGS (http://cran.r-project.org). WinBUGS is administered by the MRC Biostatistics Unit, University of Cambridge, UK; R is administered by the Technical University of Vienna, Austria. WinBUGS and R are GNU projects of the Free Software Foundation and hence available free of charge.

Keywords. Trend and loss development; Bayesian methods; time series; Workers Compensation.

1. INTRODUCTION

Estimating the trend of the frequency, severity, and loss ratio rates of growth is an integral part of NCCI ratemaking. The time series on which such trend estimation rests are typically short, comprising only 19 observations or, equivalently, 18 rates of growth. Further, these time series may display high degrees of volatility. Thus, separating signal (i.e., trend) from (white) noise is critical for discerning the trend. To achieve this objective, NCCI has developed and implemented a Bayesian state-space model that is designed to elicit the trend in short and volatile time series. This model has been optimized by minimizing the root mean squared prediction error (RMSPE) across NCCI states using three-year hold-out periods (as the applicable forecasting horizon typically amounts to about three years).

The Statistical Trend model runs in WinBUGS 1.4.3 (www.mrc-bsu.cam.ac.uk/bugs) within the R (www.r-project.org) package R2WinBUGS (http://cran.r-project.org). WinBUGS is administered by the MRC Biostatistics Unit, University of Cambridge, UK; R is administered by the Technical University of Vienna, Austria. WinBUGS and R are GNU projects of the Free Software Foundation and hence available free of charge.

1.1 Research Context

Forecasting is a signal extraction and signal extrapolation problem. Measurement errors cause the quantities of interest (such as the rates of growth of indemnity severity, medical severity, and frequency) to be observed with (presumably Gaussian) noise, thus obscuring the signal. In forecasting, the signal is the quantity of interest, because it is the signal on which future observations center. Specifically, it is the objective of a forecasting model to educe from historical observations the process that generates the unobservable signal. Because a forecasting model replicates the data-generating process of the signal (as opposed to replicating the observations themselves), its quality cannot be judged by the (in-sample) fit to the observed data, as gauged, for instance, by the R^2 or similar measures of goodness of fit. In fact, good fit to heretofore observed data harbors the risk of overfitting. Such overfitting may imply that the forecasts do not center on the signal, thus giving rise to potentially large forecasting errors. The risk of overfitting affords parsimony a critical role in time series modeling.

As an example, consider a game of dice. In any roll of a pair of dice, the expected value of the outcome is 7. This expected value is the signal, which manifests itself as the mean outcome as the number of rolls goes to infinity. The signal offers an unbiased forecast for any future toss; the difference between the observations and the signal is noise. Thus, among all possible forecasting models, the one that simply produces the time-invariant signal of 7 as its forecast has the lowest expected root mean squared prediction error. Yet, this forecasting algorithm offers the worst in-sample fit possible, as the model has no explanatory power with regards to the variation of the outcome around the expected value. Not surprisingly, a least-squares regression of the 36 possible outcomes on the time-invariant signal reveals an R^2 equal to zero.

Two common properties in time series are nonstationarity and mean reversion. In the example above, nonstationary is equivalent to a time-varying mean; instead of invariably equaling 7, this mean would change in time. As will be argued below, in workers compensation, the frequency rate of growth (and, as a result, the loss ratio rate of growth) should be treated as nonstationary.

Mean reversion, on the other hand, implies that the outcomes of the mentioned rolls of dice are not independent draws, thus causing serial correlation. In games of chance, such mean reversion is associated with the gambler's fallacy, which rests on the (erroneous) belief that below-average outcomes of past rolls of dice are to be corrected by above-average outcomes in the future (instead of simply being diluted). Although the business cycle may introduce mean reversion in the severities and frequency rates of growth, such mean reversion is likely to be weak and, more importantly, cannot be expected to improve the quality of the forecasts in short non-stationary time series due to lack of precision in estimating such mean reversion.

Traditionally, NCCI estimates trends using the exponential trend approach, which is a linear regression of logarithmic levels on a sequence of integers that indicate time.

1.2 Objective

Recent advances in statistical modeling offer ways of dealing with the problem of estimating trend rates of growth from times series that are short, volatile, and potentially nonstationary. The state-space modeling framework, along with the Metropolis-Hastings algorithm for estimating Bayesian models by means of Markov-Chain Monte Carlo (MCMC) simulation, makes such sophisticated statistical modeling available to the practicing actuary.

1.3 Outline

What follows is the presentation of a multiequation model for forecasting the trend in the rates of growth of the indemnity and medical severities, frequency, and the respective loss ratios. This model is then applied to a "paid" data set of an unidentified U.S. state for the time period 1987–2005. The last section offers conclusions and guidance for implementation of this model in actuarial practice.

2. BACKGROUND AND METHODS

In the context of NCCI ratemaking, frequency is defined as the ratio of the developed (to the 5^{th} report) number of claims to the developed (to the 5^{th} report), on-leveled (to the current loss-cost or rate level, depending on the case), and wage-adjusted premium. Severity is defined as the ratio of the developed, on-leveled, and wage-adjusted loss to the developed (to the 5^{th} report) number of claims. When defined in such way, the product of indemnity (medical) severity and frequency equals the indemnity (medical) loss ratio. In consequence, the logarithmic rate of growth of the loss ratio equals the sum of the logarithmic rates of growth of frequency and the applicable severity; in what follows, this property is referred to as the add-up constraint.

The model estimates the five rates of growth (the two severities, frequency, and the two loss ratios) simultaneously. Covariances among these variables account for common shocks. For instance, the severities and frequency are subject to common shocks because they share the wage adjustment; further, the severities and frequency share components of the on-leveling for changes in benefits levels. The joint estimation of the growth rates also allows for implementing the mentioned add-up constraint. This constraint ensures that, at any point in time, the estimated rates of growth of the indemnity (medical) severity and frequency are consistent with the estimated rate of growth of the indemnity (medical) loss ratio.

The model uses logarithmic rates of growth, because conventional rates of growth have a lower bound at minus 1 and, hence, violate the assumption of normality. These logarithmic rates of growth are then transformed into conventional rates of growth to obtain the forecast rates of growth and, after adding 1, the NCCI trend factors. Further, the (conventional) rates of growth are compounded over the multiyear forecasting horizon or, equivalently, the NCCI trend period; the number of years of this trend period is typically not an integer. Adding 1 to the compounded rates of growth delivers the NCCI adjustment factors. The purpose of the adjustment factor is to scale up the levels of the variables of interest (the severities, frequency, and the loss ratios) from the end of the experience period (i.e., the time period for which there are observations available) to the end of the trend period (i.e., the end of the forecasting horizon).

Note that transforming logarithmic rates of growth into conventional rates of growth necessitates a bias-adjustment when such transformation is done on the expected value; this is because, for a normally distributed variable x, $\mathrm{E}[e^x] = e^{\mathrm{E}[x]+\sigma^2/2}$. Because the model is estimated by means of Monte Carlo simulation, such transformation happens "draw by draw" (instead of on the expected value) and, thus, no bias-adjustment is necessary.

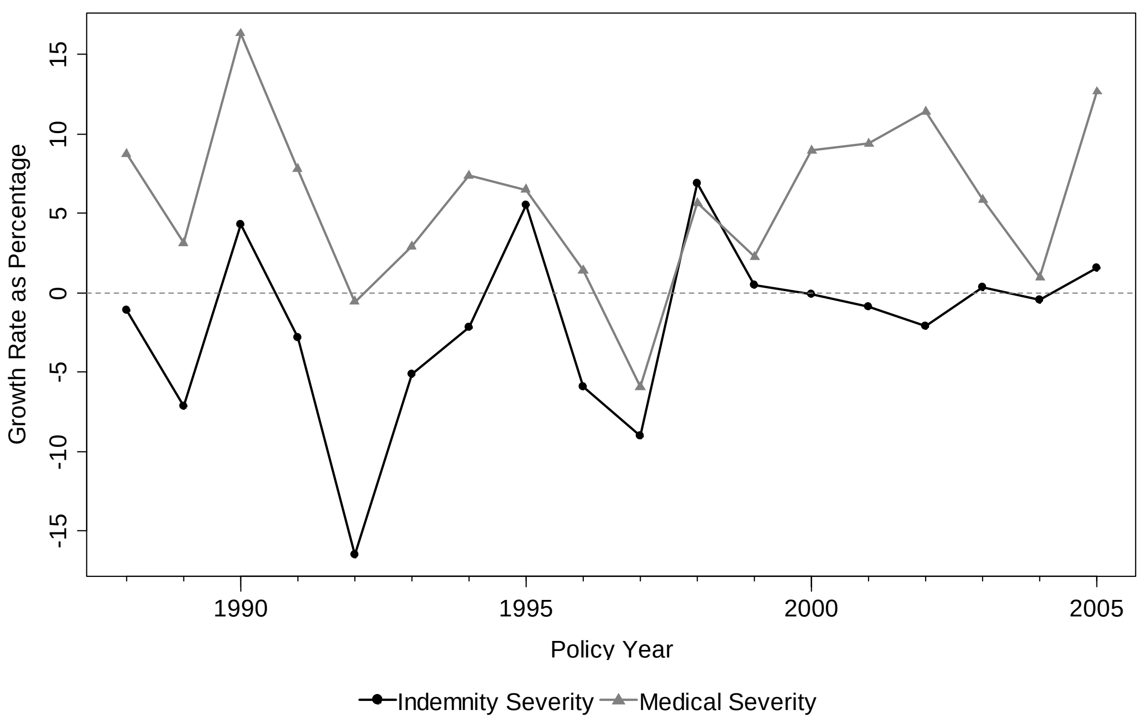
For the time period 1988-2005, Chart 1 shows for an unidentified state the (conventional) rates of growth of the indemnity and medical severities. Chart 2 displays the growth rate for frequency. Finally, Chart 3 presents the growth rates of the corresponding loss ratios.

Although Charts 1 through 3 are not necessarily representative of NCCI states, they are typical in that they are indicative of nonstationarity (i.e., time-variation in the mean) in the growth rate of frequency, but less so in the severities. Note that because the sum of two time series is nonstationary if at least one of the individual series is nonstationary, a time-varying mean in the growth rate of frequency implies time-varying means in the growth rates of both loss ratios. For instance, as Chart

2 shows, the growth rate of frequency was higher at around the year 1990 than it was ten years later; and because the variation in the mean growth rate of frequency was not offset by a change (in opposite direction) of the growth rates of the severities, such variation is mirrored in the means of the growth rates of the loss ratios (see Chart 3).

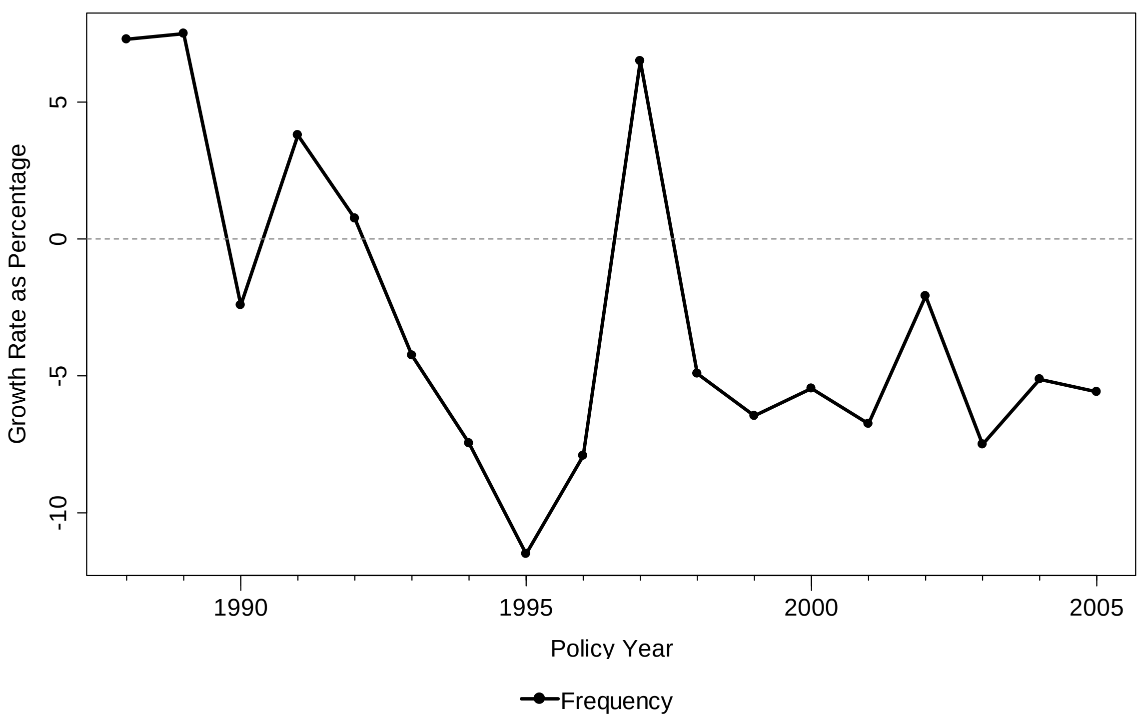
Time series can be checked for nonstationarity, but such unit root tests have little power for short time series; as a consequence, these tests favor the null hypothesis of a (pure) random walk (see, for instance, Hamilton [4]). As will be argued below, stationarity and nonstationarity are limiting cases of a smoothed random walk. Frequently, neither stationarity (a time-invariant mean) nor a (pure) random walk is an appropriate assumption for forecasting models that rely on short and volatile time series.

Chart 1: Growth Rates of Indemnity and Medical Severities, Policy Years 1988–2005



Another property frequently present in time series is serial correlation. Such serial correlation may originate in mean reversion, as caused by the business cycle. First, the rate of frequency growth may be hypothesized to vary with economic activity as the least productive workers are the last to be hired in an economic upturn and the first to be laid off in a downturn. Second, wage growth is a (lagging) indicator of economic activity; hence, the wage-adjusting of losses (severities) and premium (frequency) may introduce mean reversion into the severities and frequency series. On the other hand, the business cycle in the United States has been fairly shallow over the past 20 years; there were only two official recessions (1990/91 and 2001) according to the NBER Business Cycle Dating Committee (http://www.nber.org/cycles/cyclesmain.html) and, according to the Federal Reserve Bank of Saint Louis Fred2 database (https://research.stlouisfed.org/fred2), only one-quarter of negative GDP growth. In conclusion, discerning a shallow mean-reverting process in time series as short and volatile as those depicted in Charts 1 through 3 harbors the risk of overfitting and is likely to add little predictive power to the forecasts.

Chart 2: Growth Rate of Frequency, Policy Years 1988–2005



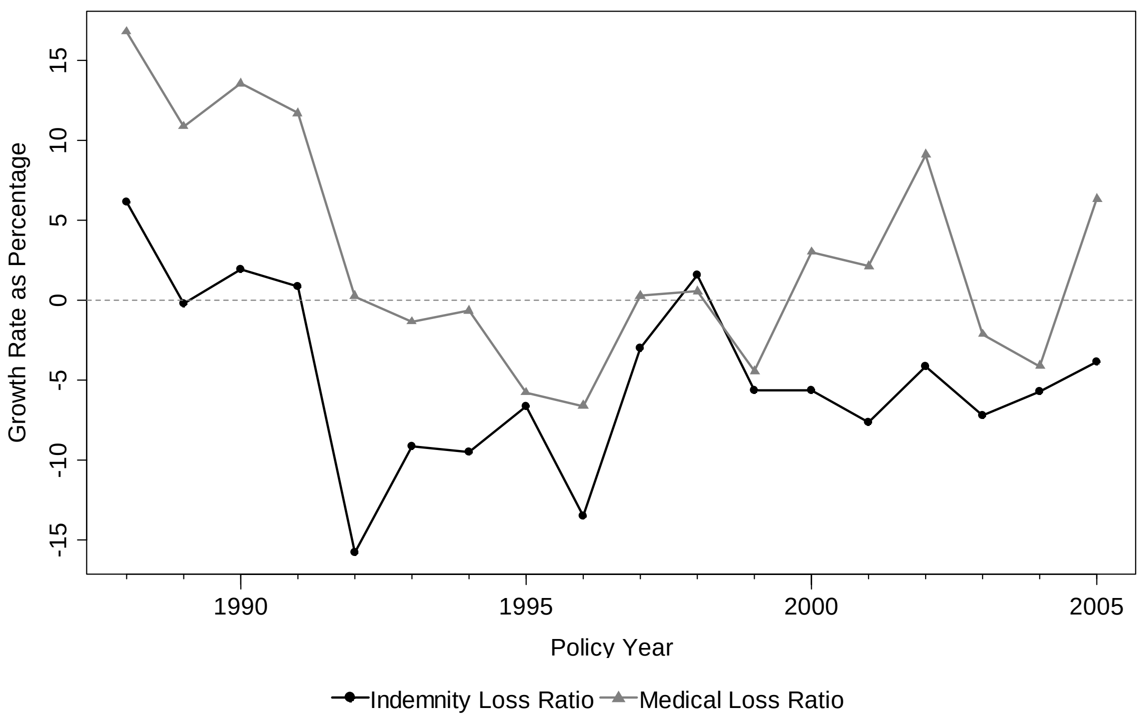
As mentioned, the forecasting model makes use of the concept of the smoothed random walk. For illustration, a simple model of a smoothed random walk may be written as follows:

$$y_t \sim \mathrm{N}(x_t, \sigma_y^2)\ ,\ \ t = 1, \ldots, T \tag{1.1}$$

$$x_t \sim \mathrm{N}(x_{t-1}, \sigma_x^2)\ ,\ \ t = 2, \ldots, T \tag{1.2}$$

where $\mathrm{N}(\mu, \sigma^2)$ indicates a normal distribution with mean μ and finite variance σ^2. Equation (1.1) states that the variable y_t is observed with measurement noise σ_y^2 around the unobservable mean x_t; in state-space format, this equation is called the measurement equation. Equation (1.2) states that the mean x_t is time-varying as described by a random walk with innovation variance σ_x^2; in state-space format, this equation is called the transition equation.

Chart 3: Growth Rates of Indemnity and Medical Loss Ratios, Policy Years 1988–2005



There are two limiting cases to model (1.1–1.2), one of which is the case of stationarity, and the other one is the (pure) random walk. We obtain stationarity by setting the innovation variance σ_x^2 in the transition equation to zero:

$$y_t \sim \mathrm{N}(x_t, \sigma_y^2)\ ,\ \ t = 1,...,T \tag{2.1}$$

$$x_t = x_{t-1}\ ,\ \ t = 2,...,T \tag{2.2}$$

Alternatively, we obtain the limiting case of a pure random walk by setting the measurement noise σ_y^2 to zero:

$$y_t = x_t\ ,\ \ t = 1,...,T \tag{3.1}$$

$$x_t \sim \mathrm{N}(x_{t-1}, \sigma_x^2)\ ,\ \ t = 2,...,T \tag{3.2}$$

In the general case where neither time-variation in the mean (nonstationarity) nor measurement noise can be excluded, model (1.1-1.2) applies. In such a general model, it is necessary to determine how much of the time-variation in the dependent variable y_t should be assigned to noise (σ_y^2); the remainder represents innovation (σ_x^2). This allocation decision, which determines the degree of smoothing of the dependent variable, may be assigned to an algorithm such as the Kalman filter (as discussed in Evans and Schmid [2]; for a general discussion of the Kalman filter, see, for instance, Hamilton [4]). Note that for any given set of observations y_t, $t = 1,...,T$, there is only one degree of freedom in determining the optimal degree of smoothing, as choosing σ_y^2 determines σ_x^2, and vice versa.

Unfortunately, the Kalman filter does not necessarily deliver the optimal degree of smoothing; in short and volatile time series in particular, the Kalman filter assigns more time variation to innovations in the mean than is conducive to minimizing the forecasting error.

2.1 The Model

The model to be introduced in this section is Bayesian. Such Bayesian models have a number of advantages over frequentist approaches, among which is the ease at which even very complex models can be estimated. For instance, if there were missing values in the severity, frequency, or loss ratio series, the model shown below would interpolate of its own accord, based on the estimated random walk properties.

The model is estimated using the Metropolis–Hastings algorithm, which computes the (posterior) distributions of the model parameters by means of Markov–Chain Monte Carlo simulation. For an introduction to Bayesian modeling see Gelman et al. [3].

Equation (4) below represents a system of transition equations for the rates of severity and frequency growth, which describe (smoothed) random walks; the innovations to these variables (i.e., the changes to their means) follow a multivariate normal distribution. Equation (5) states that the initial values for the three mentioned growth rates are also draws from a multivariate normal; the expected values of this normal are zero, but the covariance matrix imposes little restrictions on the means of their posterior distributions. Equation (6) describes the measurement equations, inclusive of the add-up constraint; in the measurement equations, the model fits to the observed values the process that is stated in Equation (4). Equations (7) through (10) describe the prior distributions for the covariance matrices of the initial states, the innovations, and the measurement noise; these covariance matrices are modeled on Wishart distributions.

$$\begin{pmatrix} z_{s_{ind},t} \\ z_{s_{med},t} \\ z_{fr,t} \end{pmatrix} \sim \mathrm{N}\left(\begin{pmatrix} z_{s_{ind},t-1} \\ z_{s_{med},t-1} \\ z_{fr,t-1} \end{pmatrix}, \mathbf{\Omega}_1 \right), \ t = 2,\ldots,T \tag{4}$$

$$\begin{pmatrix} z_{s_{ind},1} \\ z_{s_{med},1} \\ z_{fr,1} \end{pmatrix} \sim \mathrm{N}\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \mathbf{\Omega}_2 \right) \tag{5}$$

$$\begin{pmatrix} s_{ind,t} \\ s_{med,t} \\ fr_t \\ lr_{ind,t} \\ lr_{med,t} \end{pmatrix} \sim \mathrm{N}\left(\begin{pmatrix} z_{s_{ind},t} \\ z_{s_{med},t} \\ z_{fr,t} \\ z_{s_{ind},t} + z_{fr,t} \\ z_{s_{med},t} + z_{fr,t} \end{pmatrix}, \mathbf{\Omega}_3 \right), \ t = 1,\ldots,T \tag{6}$$

$$\mathbf{\Omega}_i \sim \mathrm{W}(\mathbf{R}_i, 1000), \ i = 1,3 \tag{7}$$

$$\mathbf{\Omega}_i \sim \mathrm{W}(\mathbf{R}_i, 10)\ ,\ i = 2 \tag{8}$$

$$\mathbf{R}_1 = \mathbf{R}_2 = 0.01 \times I_{3\times 3} \tag{9}$$

$$\mathbf{R}_3 = 0.2 \times I_{5\times 5} \tag{10}$$

where N and W indicate normal and Wishart distributions, respectively. The variables $s_{ind,t}$ and $s_{med,t}$ are the logarithmic rates of growth of indemnity and medical severities, respectively. The variable fr_t is the logarithmic rate of growth of frequency, and the variables $lr_{ind,t}$ and $lr_{med,t}$ are the logarithmic rates of growth of the indemnity and medical loss ratios, respectively. $I_{3\times 3}$ and $I_{5\times 5}$ symbolize identity matrices. The larger the diagonal elements of $\mathbf{R}_3$ are, the greater the degree of smoothing is. The matrix $\mathbf{R}_i (i = 1,...,3)$ is a scale matrix that "represents an assessment of the order of magnitude of the covariance matrix" $\mathbf{\Omega}_i^{-1} (i = 1,...,3)$ (WinBUGS [5]). (Note that the WinBUGS notation for the normal distribution makes use of the precision matrix, which is the inverse of the covariance matrix.)

If (and only if) there is a predictable upswing in future economic activity, the model employs a covariate (explanatory variable). In this case then, the rate of frequency growth is modeled as the sum of a (smoothed) random walk and a standard regression component; this standard regression component hosts the covariate. The covariate of choice is the change in the rate of unemployment. As argued, in an economic upswing, the growth rate of frequency can be expected to rise as currently employed labor is utilized more intensively and currently unemployed labor is put back to work. Predictable upswings in economic activity typically happen in the wake of natural disasters; an example of such an event is Hurricane Katrina in 2005.

When including a covariate, equations

$$\begin{pmatrix} z_{s_{ind},t} \\ z_{s_{med},t} \\ \bar{z}_{fr,t} \end{pmatrix} \sim \mathrm{N}\left(\begin{pmatrix} z_{s_{ind},t-1} \\ z_{s_{med},t-1} \\ \bar{z}_{fr,t-1} \end{pmatrix}, \mathbf{\Omega}_1 \right),\ t = 2,...,T \tag{11}$$

$$\begin{pmatrix} z_{s_{ind},1} \\ z_{s_{med},1} \\ \overline{z}_{fr,1} \end{pmatrix} \sim \mathrm{N}\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \ \mathbf{\Omega}_2 \right) \tag{12}$$

substitute for Equations (4) and (5), and the following two equations are added to the model:

$$\lambda \sim \mathrm{N}(0, \tau_\lambda) \tag{13}$$

$$z_{fr,t} = \overline{z}_{fr,t} - \lambda \cdot \delta_t, \ t = 1, \ldots, T \tag{14}$$

where the variable δ_t is the t to $t-1$ (accident year; policy year: $t+1$ to $t-1$) difference in the rate of unemployment. The prior for the parameter λ is a normal distribution with an expected value of zero and a variance (τ_λ^{-1}) that must be chosen sufficiently small to prevent λ from picking up noise.

The model generates forecasts by moving the (logarithmic) rates of growth of frequency and the severities forward according to the innovation variances of the random walks described in Equation (4) (or Equation (11), if applicable), based on the estimated innovation covariance matrix $\hat{\Omega}_1^{-1}$. As is the case with all estimated parameters of the models, the forecasts come as posterior distributions, the means of which serve as the point estimates. The posterior distributions of the forecast trend and adjustment factors offer credible intervals, based on the chosen probability level (e.g., 95 percent). These credible intervals differ in important ways from traditional, frequentist confidence intervals. Whereas in frequentist statistics the true value either lies within a given confidence interval or not, the (Bayesian) credible interval is indeed a probabilistic statement about its location; see Carlin and Louis [1]. Note that the credible intervals are statements about the *trend* rates of growth, rather than the realized rates of growth (which are the sum of trend and noise).

3. RESULTS AND DISCUSSION

In what follows we apply the model (without a covariate, that is, Equations 4 through 10) to an unidentified U.S. state. The observations for the severities, frequency, and the loss ratios run from policy years 1987 through 2005, which renders 18 rates of growth (1988–2005).

As mentioned, the objective of the model is to generate trend factors, which are estimates of the trend rates of growth. By means of scaling up these trend factors to the trend period (i.e., the forecasting horizon), we obtain the adjustment factors.

NCCI typically computes adjustment factors not just for the final year, but also for the penultimate and antepenultimate years of the experience period. For instance, if the experience period ends with policy year 2005, then these alternative adjustment factors attach to the policy years 2004 and 2003, respectively; the corresponding alternative trend periods end on the same point on the calendar year axis as does the trend period that attaches to policy year 2005. (Note that the model is estimated only once; in the example above, this means that the trend factors that attach to policy years 2004 and 2003 are based on the same estimation as the trend factor that attaches to policy year 2005, thus utilizing all observations of the experience period.)

For a given policy year, the trend period runs from the midpoint of the policy year to the midpoint of what is known at NCCI as the effective period. The effective period is defined as the period in which the filed rate or loss cost (depending on the case) is going to be in effect. The midpoint of a policy year or an effective period is based on the assumed monthly premium distribution; such premium distribution may vary across states. As mentioned, the trend period attaches to the final year of the experience period, with alternative trend periods attaching to the penultimate and antepenultimate policy years. For the case at hand, this final year of the experience period is policy year 2005, and the trend period equals 3.001 years, rounded to the third decimal. Correspondingly, the trend period that attaches to the penultimate (antepenultimate) policy year of the experience period is 4.001 (5.001) years of length.

When the change in the rate of unemployment is used as a covariate for frequency growth, then this variable is measured by the two-year difference of the rate of unemployment for policy years and by the first (i.e., one-year) difference of the rate of unemployment for accident years. For instance, for policy year 2005, the pertinent two-year difference is the change in level between the end-of-calendar-year 2006 and the end-of-calendar-year 2004 values. For accident year 2005, the first difference is the change in level between the end-of-calendar-year 2005 and the end-of-calendar-year 2004 values. These end-of-calendar-year numbers of the unemployment rate refer to the final quarter of the year and are averaged over the three months in the quarter. (We average the rate of unemployment because only quarterly forecasts for the rate of unemployment are available.) In determining the trend estimates for the unidentified state discussed below, no covariate was employed.

As mentioned, the model presented above must be calibrated such that it minimizes the prediction error. This calibration is done by choosing the optimal degree of smoothing, as it manifests itself in the diagonal elements of the scale matrix $\mathbf{R}_3$; the prediction error is gauged by the RMSPE. To determine the optimal degree of smoothing, we ran the model with a holdout period of 3 years for all NCCI states with an array of smoothing parameters; the diagonal elements of $\mathbf{R}_3$ (which determine the degree of smoothing) were varied equidistantly on a logarithmic scale. The 3-year holdout period corresponds (approximately) to the applicable trend periods (of typically little more, but sometimes little less than 3 years). As shown in Chart 4, the RMSPE, aggregated across all NCCI states varies systematically with the degree of smoothing (which is represented by an index, not the actual magnitude of the diagonal elements of $\mathbf{R}_3$); the prediction error is large for little smoothing (low index values), because little smoothing entails a great deal of fitting to noise; also, the prediction error is large for extensive smoothing (high index values), because a high degree of smoothing insufficiently accommodates the nonstationarity of the underlying growth series.

Based on data from an unidentified state, the model is estimated using WinBUGS 1.4.3 within the R package R2WinBUGS. We sample 50,000 times, following a burn-in of 50,000 iterations.

The results for the severities, the frequency and the loss ratios are displayed in Charts 5 through 7. The dashed vertical lines in these charts indicate the beginning of the trend period, which attaches to the final year of the experience period (policy year 2005).

Chart 5 displays the actual, fitted, and forecast trend growth rates for indemnity and medical severities. The mean rates of severity growth show little time variation, although the indemnity growth rate is slightly trending up. The chart demonstrates that, for both series, the forecasts are not sensitive to the respective final observed value, which is desirable as any observed value contains potentially a great deal of noise.

Chart 6 depicts the actual, fitted, and forecast trend growth rates for frequency. Here, there is clearly evident a downtrend in the mean rate of growth. Also, note the insensitivity of the model to the outlier in the year 1997.

Chart 7 exhibits the actual, fitted, and forecast trend growth rates for the indemnity and medical loss ratios. The medical loss ratio trend rate of growth clearly follows the trend in frequency growth, while the indemnity loss ratio trend rate of growth is also influenced by the uptrend in the trend growth rate of indemnity severity.

Charts 8 and 9 present the posterior distributions for the estimated trend factors and adjustment factors for frequency, the indemnity and medical severities, and the indemnity and medical loss ratios.

Chart 4: Root Mean Squared Prediction Error (RMSPE) as a Function of the Degree of Smoothing

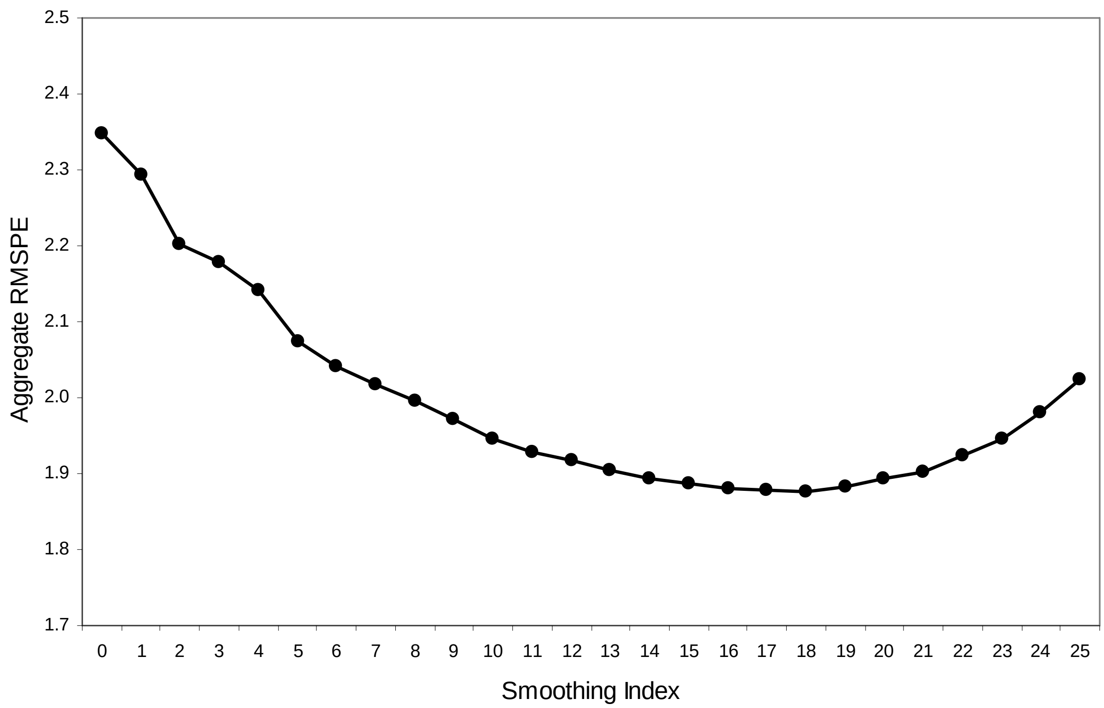


Chart 5: Growth Rates of Indemnity and Medical Severities (Actual, Fitted, and Forecast), Policy Years 1988–2005 (Forecasts: 2006–2009)

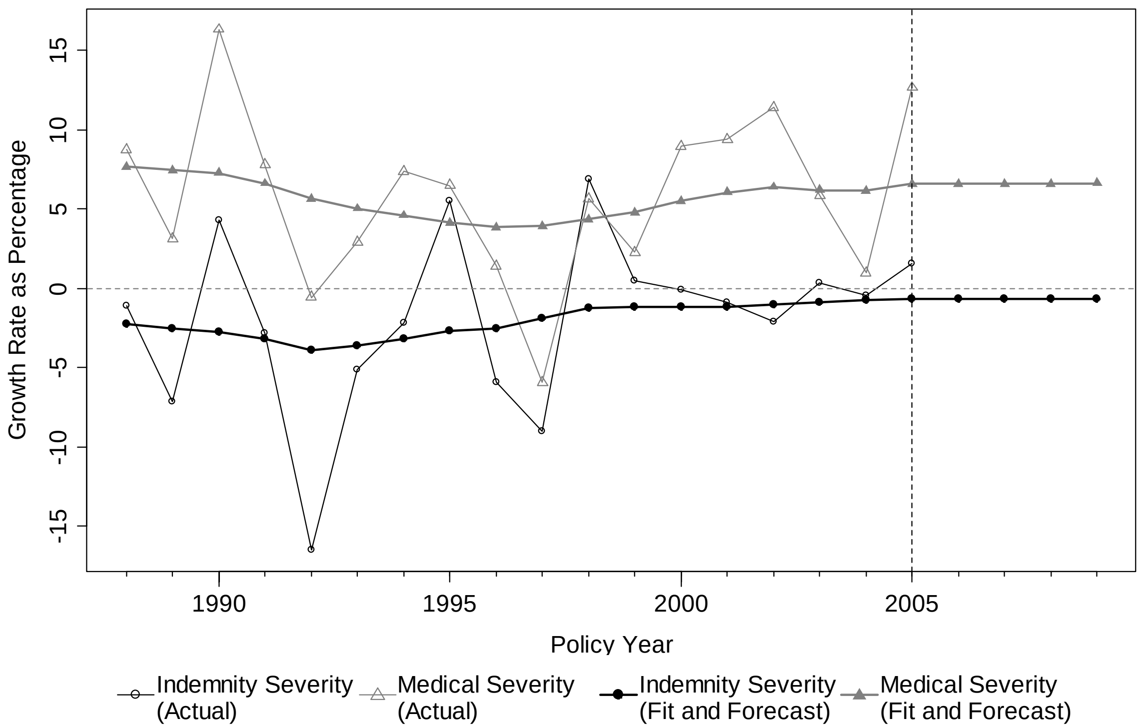


Chart 6: Growth Rate of Frequency (Actual, Fitted, and Forecast), Policy Years 1988–2005 (Forecasts: 2006–2009)

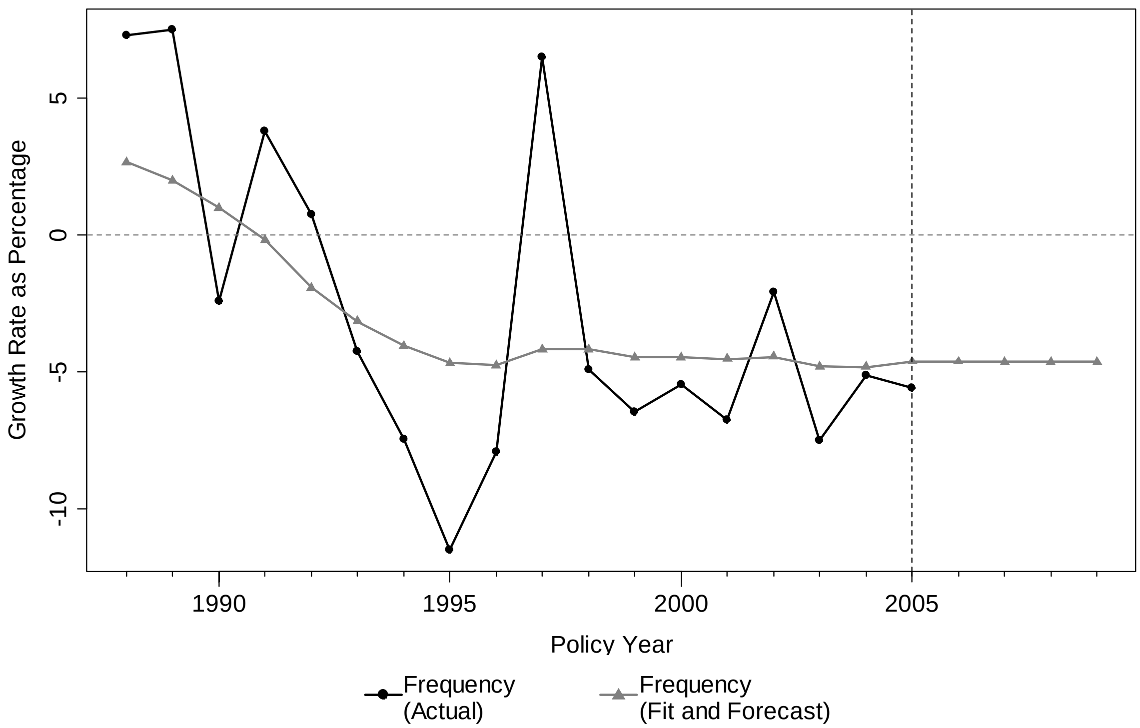


Chart 7: Growth Rates of Indemnity and Medical Loss Ratios (Actual, Fitted, and Forecast) Policy Years 1988–2005 (Forecasts: 2006–2009)

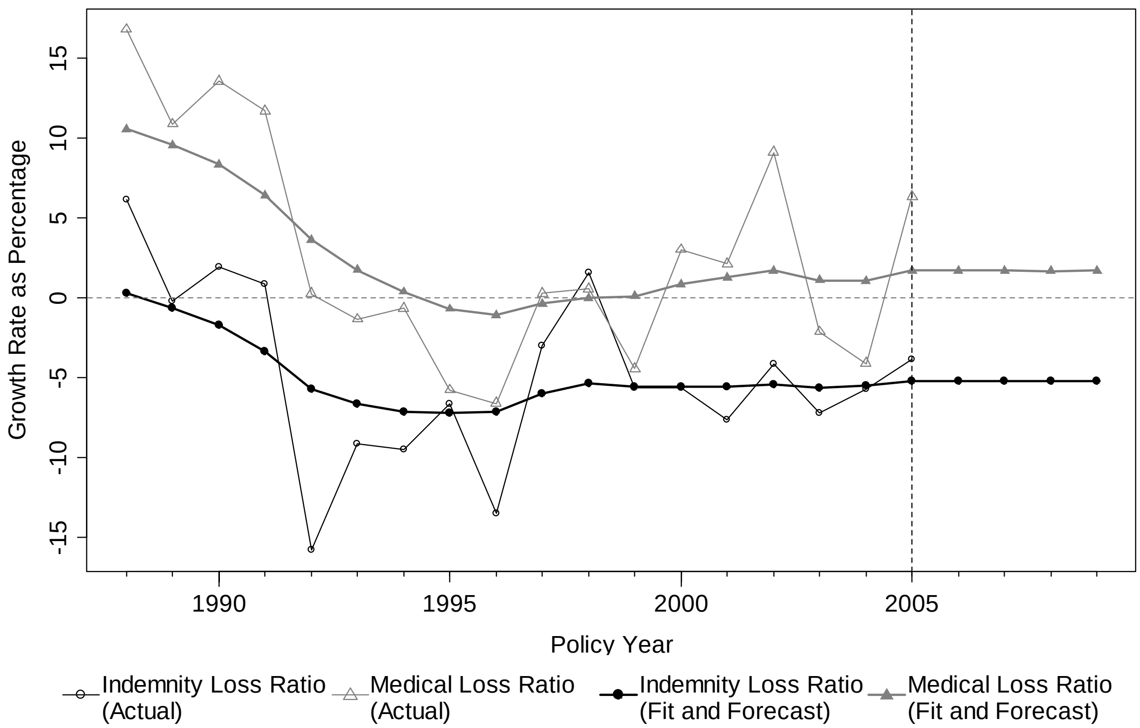
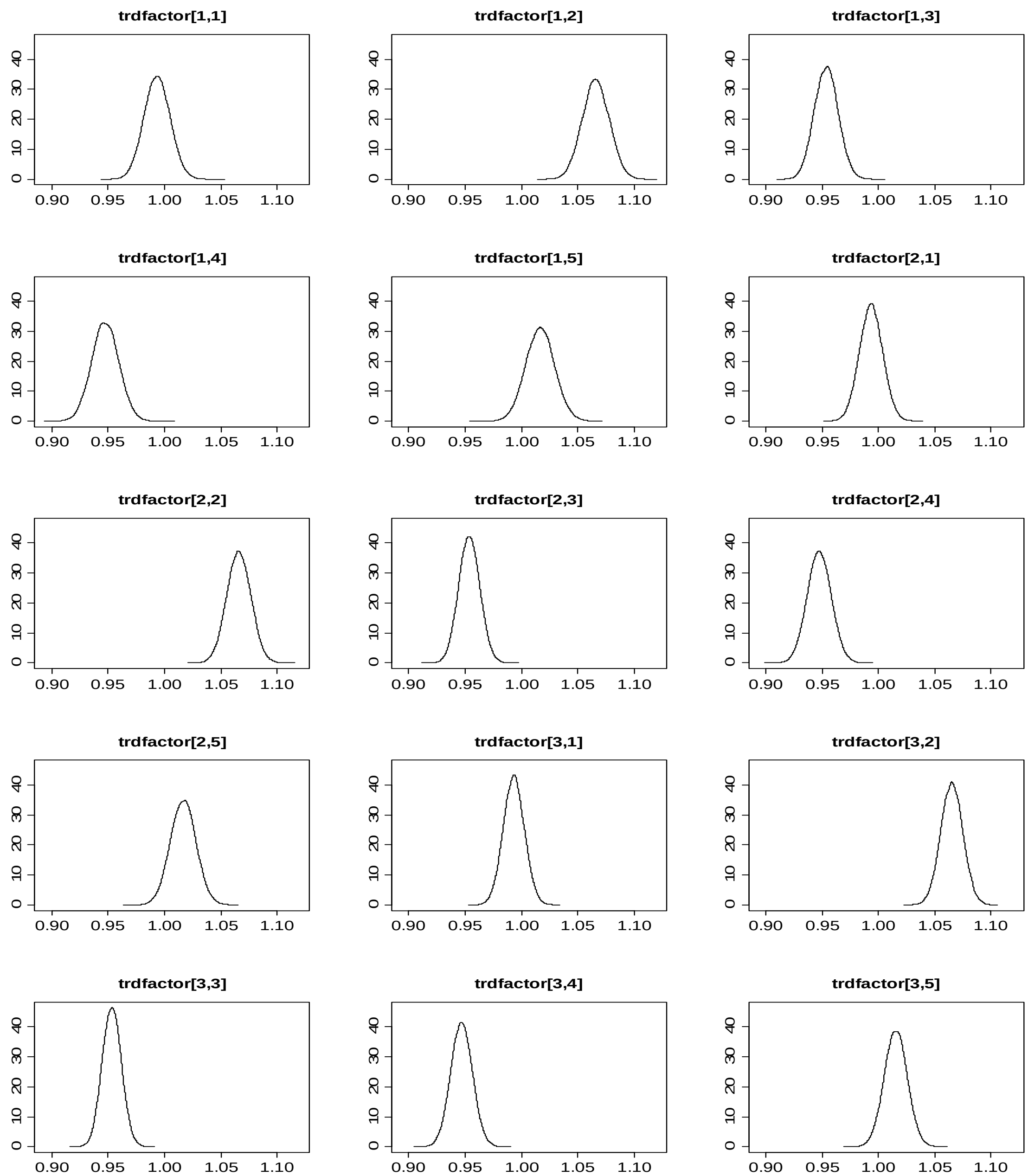
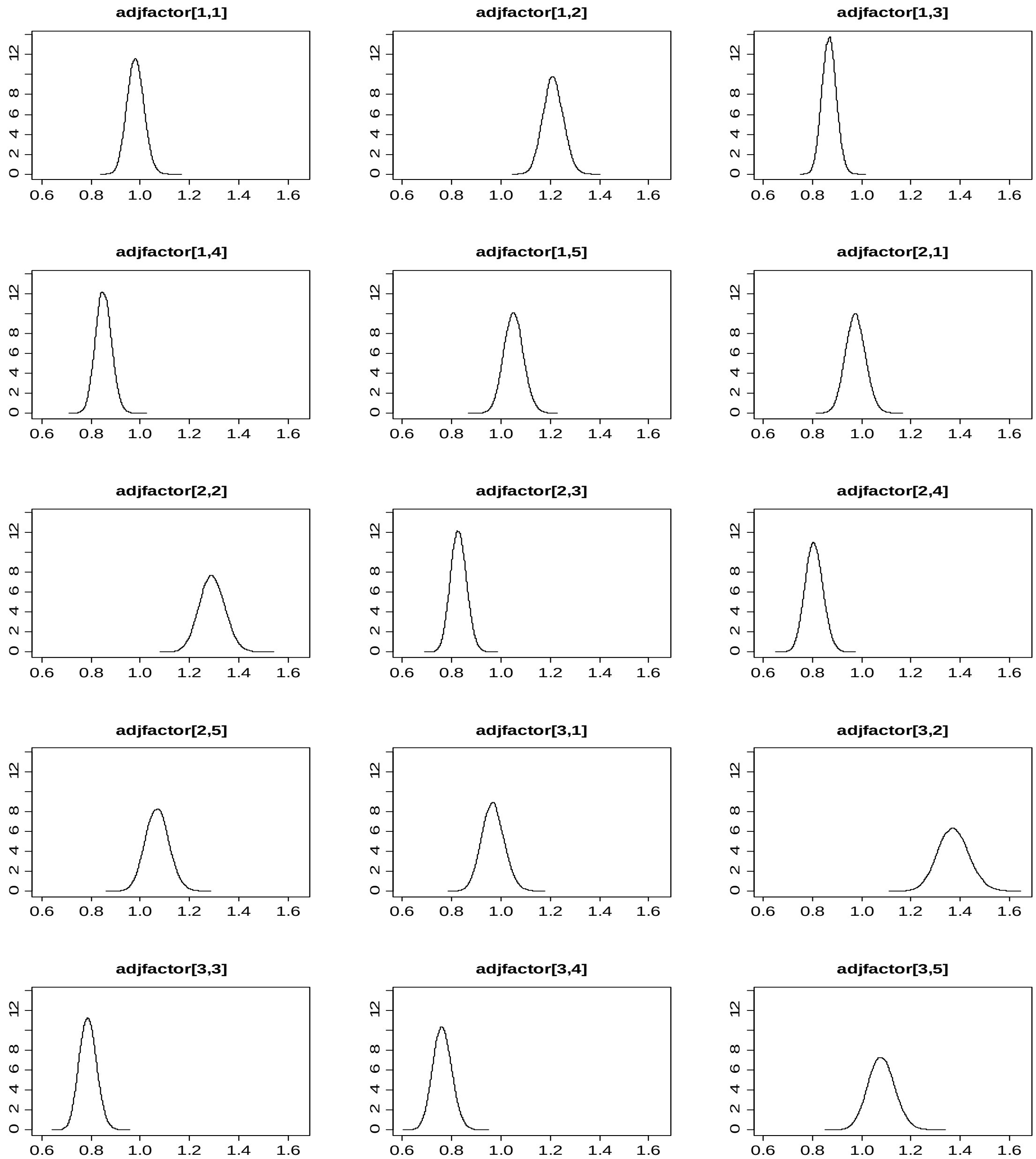


Chart 8: Posterior Densities for Trend Factors



Note: The first index in brackets refers to the policy year of the experience period at which the trend factor attaches (1: final; 2: penultimate; 3: antepenultimate). The second index represents the series (1: indemnity severity; 2: medical severity; 3: frequency; 4: indemnity loss ratio; 5: medical loss ratio).

Chart 9: Posterior Densities for Adjustment Factors



Note: The first index in brackets refers to the policy year of the experience period at which the adjustment factor attaches (1: final; 2: penultimate; 3: antepenultimate). The second index represents the series (1: indemnity severity; 2: medical severity; 3: frequency; 4: indemnity loss ratio; 5: medical loss ratio).

Table 1 exhibits the trend and adjustment factors, along with 95 percent credible intervals. Note that these intervals are not necessarily symmetric around the forecast values.

Table 1: Trend Factors and Adjustment Factors

Policy Year Paid							
Frequency							
Year	Trend Period	TF Lower Bound	**Mean Trend Factor(TF)**	TF Upper Bound	AF Lower Bound	**Mean Adjustment Factor(AF)**	AF Upper Bound
2003	5.001	0.9370	0.9533	0.9698	0.7223	0.7878	0.8579
2004	4.001	0.9357	0.9537	0.9721	0.7664	0.8276	0.8929
2005	3.001	0.9334	0.9537	0.9745	0.8131	0.8677	0.9253
Indemnity Severity							
Year	Trend Period	TF Lower Bound	**Mean Trend Factor(TF)**	TF Upper Bound	AF Lower Bound	**Mean Adjustment Factor(AF)**	AF Upper Bound
2003	5.001	0.9751	0.9932	1.0120	0.8817	0.9674	1.0600
2004	4.001	0.9735	0.9934	1.0140	0.8980	0.9746	1.0560
2005	3.001	0.9713	0.9935	1.0160	0.9164	0.9809	1.0490
Medical Severity							
Year	Trend Period	TF Lower Bound	**Mean Trend Factor(TF)**	TF Upper Bound	AF Lower Bound	**Mean Adjustment Factor(AF)**	AF Upper Bound
2003	5.001	1.0460	1.0651	1.0840	1.2530	1.3718	1.4970
2004	4.001	1.0450	1.0660	1.0870	1.1930	1.2921	1.3960
2005	3.001	1.0430	1.0660	1.0890	1.1340	1.2118	1.2930
Indemnity Loss Ratio							
Year	Trend Period	TF Lower Bound	**Mean Trend Factor(TF)**	TF Upper Bound	AF Lower Bound	**Mean Adjustment Factor(AF)**	AF Upper Bound
2003	5.001	0.9282	0.9468	0.9656	0.6888	0.7617	0.8395
2004	4.001	0.9268	0.9474	0.9683	0.7378	0.8063	0.8792
2005	3.001	0.9243	0.9475	0.9709	0.7897	0.8509	0.9151
Medical Loss Ratio							
Year	Trend Period	TF Lower Bound	**Mean Trend Factor(TF)**	TF Upper Bound	AF Lower Bound	**Mean Adjustment Factor(AF)**	AF Upper Bound
2003	5.001	0.9954	1.0153	1.0350	0.9772	1.0800	1.1900
2004	4.001	0.9945	1.0166	1.0390	0.9783	1.0689	1.1650
2005	3.001	0.9919	1.0166	1.0420	0.9759	1.0512	1.1310

Note: The trend period is measured in years. The interval between upper and lower bounds covers 95 percent of the probability mass of the distribution of the forecast.

4. CONCLUSIONS

NCCI has developed a Bayesian statistical model for estimating the trend rates of growth of the indemnity and medical severities, frequency, and the indemnity and medical loss ratios in the context of ratemaking. The model is purpose-built for short, volatile and potentially nonstationary time series and calibrated to minimize the prediction error. Further, the model accounts for common shocks, is robust to outliers, and is capable of interpolating where observations for the mentioned rates of growth are missing. Finally, by means of incorporating an add-up constraint, the model ensures consistent forecasts for the five time series in question.

Acknowledgment

Thanks to Harry Shuford for comments and to Chris Laws, Jose Ramos, and Manuel de la Guardia for research assistance.

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Abbreviations and notations

MCMC, Markov-Chain Monte Carlo
NCCI, National Council on Compensation Insurance
RMSPE, Root Mean Squared Prediction Error

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Cost of Capital Estimation with Application to Workers Compensation Ratemaking

Frank Schmid and Martin Wolf

Motivation. This paper discusses how NCCI estimates the cost of capital in its ratemaking framework. The implementation of this actuarial concept in ratemaking is challenging because financial economics offers more than one model for estimating the cost of capital. Even where there is agreement on the model, there may be questions about how to arrive at its input components.
Method. NCCI computes the cost of equity capital using the Discounted Cash Flow (DCF) and the Capital Asset Pricing Model (CAPM) approaches. The DCF method employs forecasts for the rate of dividend growth from Value Line Publishing, Inc. The CAPM model utilizes *betas* from Value Line Publishing, Inc., and historical returns on T-bills and the stock market from Morningstar, Inc.
Results. The two approaches to estimating the cost of capital are conceptually different and their estimates are similar, yet not identical.
Conclusions. In ratemaking, NCCI relies on two concepts of estimating the cost of equity capital in workers compensation. Important inputs to these approaches rest on long-term averages, thus making these methods robust to short-term economic fluctuations.
Availability. Historical returns on T-bills and the stock market are available from Morningstar, Inc. Dividend growth rates and CAPM *betas* are available from Value Line Publishing, Inc.
Keywords. Dividend Growth Model, Equity Valuation, Workers Compensation

1. INTRODUCTION

Central to developing the underwriting profit provision in actuarial ratemaking is the *total financial needs model*, which states that "the sum of underwriting profit, miscellaneous (non-investment) income, investment income from insurance operations, and investment income on capital, after income taxes, will equal the cost of capital" (see Actuarial Standards Board [1], p. 8). From this perspective, the cost of capital is an integral part of ratemaking at NCCI. What follows is a discussion of how NCCI estimates the cost of capital in its ratemaking framework.

Estimating the cost of capital is challenging, and the academic discussion surrounding this concept shows little signs of abating (see, for instance, Dimson, Marsh, and Phillips [6], Goetzmann and Ibbotson [8], and McGrattan and Prescott [17]). In actuarial practice, it is critical to follow a parsimonious, transparent, and robust approach for arriving at cost of capital estimates. At the same time, the approach should periodically be scrutinized and possibly updated in the light of new academic research findings.

1.1 Research Context

NCCI employs two concepts for estimating the cost of capital in the context of ratemaking in workers compensation; these two approaches and their implementation are periodically reviewed. What follows is a discussion of the two methods for estimating the cost of equity capital, along with a detailed description of how NCCI implements these approaches.

1.2 Objective

NCCI estimates of the cost of equity capital rest on both the discounted cash flow (DCF) and the capital asset pricing model (CAPM) concepts. The DCF approach employs estimates of the current dividend yield and forecasts for the rate of dividend growth from Value Line Publishing, Inc. The CAPM model utilizes *betas* from Value Line Publishing, Inc., and historical returns on T-bills and the stock market from Morningstar, Inc. Important inputs to these models rest on long-term averages, thus affording these methods robustness to short-term economic fluctuations.

The computed cost of capital is used within NCCI's internal rate of return (IRR) model to calculate an underwriting contingency provision (UCP, which is a profit factor) in those states where NCCI files rates (as opposed to loss costs).

NCCI's IRR model calculates the internal rate of return (based on changes in shareholder equity) of a $1 million workers compensation insurance policy written at proposed rates. The model incorporates all cash flows related to the policy, including factors such as premium inflows, losses, underwriting expenses, policyholder dividends, federal income taxes, and investment income earned on reserves and surplus. The model incorporates quarterly cash flows for the first five years and annual flows thereafter (through year 24 or 35, depending on the version of the model).

Once the IRR model has been estimated, the model is then backsolved to the cost to capital to calculate the UCP. The calculated UCP is used by state actuaries as an advisory input in their ultimate rate filing. The actual value of the UCP included in the filing (if any) depends on state regulatory practices as well as actuarial judgment. (The UCP is included as part of the expense provision that underlies the rate filing.)

1.3 Outline

In what follows we describe the two most widely used approaches in the estimation of the cost of capital and demonstrate how NCCI implements these methods to compute the cost of capital in workers compensation for ratemaking purposes. Further, we provide cost of capital estimates based on data available as of May 25, 2007.

2. BACKGROUND AND METHODS

The implementation of the actuarial concept of the *total financial needs model* in ratemaking is challenging because financial economics offers more than one approach to estimating the cost of capital. And even where there is agreement on the choice of the model, there often is no consensus on how to estimate the values of the parameters to be fed to this model.

Myers and Borucki [19] describe the DCF model as the "most widely used approach to estimate the cost of equity capital to regulated firms in the United States." The other commonly applied approach is the CAPM, which may be implemented as a one-factor model (which is our preferred approach) or a three-factor model (which is an approach that we do not pursue). NCCI employs both concepts in estimating the cost of equity capital for the property and casualty (P&C) insurance industry with application to ratemaking in workers compensation.

The cost of equity capital in the insurance industry has been studied by Fama and French [8] and, more recently, by Cummins and Phillips [5]—both studies implement the CAPM as one-factor and three-factor models. We will compare our methodology and results to theirs in Section 2.2.3.

2.1 The Discounted Cash Flow Model

Generally, the price of a financial asset is the present value of its (future) cash flow. When it comes to a share of stock, the cash flow consists of dividend payments. On a more aggregate level, such as the entire stock of a corporation, an industry or a country, cash returned to shareholders also includes cash dispensed in share repurchase programs, so long as these shares are retired (instead of being handed out to executives and employees, in which case the shares change hands but no cash is paid out on net).

A widely used model for the valuation of corporate stock, which is described in many corporate finance textbooks, is the Gordon dividend growth model (see Gordon [10]).

The Gordon model is suitable only for mature industries (such as the P&C industry) because this model assumes that (1) the industry in question returns cash to shareholders and that (2) the amount of cash paid to shareholders grows at a steady rate. As mentioned, cash returned to shareholders includes cash dispensed by means of repurchasing shares for the purpose of retiring them.

The Gordon model requires only two inputs for determining the cost of capital of an industry, which are the *effective* dividend yield of the stock (i.e., cash paid out by means of dividends and share repurchases, divided by the stock market valuation) and the rate at which the amount of so dispensed cash grows. Prospective dividend growth rates are available on company-level and industry-level bases from disinterested third parties, which is a key factor in a regulatory proceeding.

The following sections provide a summary of the NCCI methodology for estimating the effective dividend yield, the prospective rate of growth of dividends, and how these estimates are combined to generate a measure for the cost of capital within a DCF context.

2.1.1 The Present Value of Future Dividend Payments

Conceptually, a share of stock has an infinite lifetime. This assumption is not invalidated by the fact that stock may be repurchased and retired, because the value of the share when repurchased is again the present value of all future dividends, which are paid out lump sum to those that sell shares back to the corporation.

In a generalized, two-stage Gordon dividend growth model, there are n periods during which the dividend payments grow at a forecast rate g^{fct}, followed by an infinite number of periods during which the dividend grows at its long-term sustainable rate g^{avg}.

In general, the present value of a stock, V_0, that pays a dividend for n periods, equals

$$V_0^n = D_0 \cdot \sum_{t=1}^{n} \frac{(1+g)^t}{(1+k)^t}, \tag{2.1}$$

where D_0 is the dividend paid at the present time (the end of period 0), g is the rate of growth of this dividend, and k is the cost of capital. This formula can be simplified to

$$V_0^n = D_0 \cdot \frac{1+g}{k-g} \cdot \left(1 - \frac{(1+g)^n}{(1+k)^n}\right). \tag{2.2}$$

For an infinite number of periods, the present value reads

$$V_0^\infty = D_0 \cdot \frac{1+g}{k-g}. \tag{2.3}$$

Hence, the present value of a stock the dividend payment of which grows at the rate g^{fct} for n periods, followed by an infinite number of periods at which the dividend grows at the rate g^{avg}, equals

$$V_0 = D_0 \cdot \left[\frac{1+g^{fct}}{k-g^{fct}} \cdot \left(1 - \frac{\left(1+g^{fct}\right)^n}{(1+k)^n} \right) + \frac{\left(1+g^{fct}\right)^n \cdot \left(1+g^{avg}\right)}{k-g^{avg}} \cdot \frac{1}{(1+k)^n} \right]. \qquad (2.4)$$

As a consequence, we obtain the marginal cost of capital by means of solving the following equation for k:

$$\frac{1}{\frac{D_0}{V_0}} = \left[\frac{1+g^{fct}}{k-g^{fct}} \cdot \left(1 - \frac{\left(1+g^{fct}\right)^n}{(1+k)^n} \right) + \frac{\left(1+g^{fct}\right)^n \cdot \left(1+g^{avg}\right)}{k-g^{avg}} \cdot \frac{1}{(1+k)^n} \right], \qquad (2.5)$$

where D_0/V_0 is the current (end of period 0) dividend yield. If it is assumed that the dividend payments occur in the middle of the fiscal year, then the equation above still offers the correct answer for the cost of capital, assuming that the dividend yield D_0/V_0 has been observed in the middle of fiscal year 0.

The solver module in Microsoft Excel provides a means of iteratively solving for k, given inputs for the dividend yield and the forecast and long-term average rates of dividend growth.

2.1.2 The Effective Dividend Yield

The NCCI estimate of the dividend yield of the P&C industry for NCCI ratemaking rests on dividend yield data of individual insurers as published by Value Line Publishing, Inc. [23]. Value Line Publishing, Inc. data are developed in a consistent manner and are generally viewed as reliable. Analyses based on Value Line Publishing, Inc. information are used in many regulatory settings, especially in the insurance industry and the utilities sector (see, for instance, Cummins and Phillips [5], and Morin [18]).

NCCI defines the domain of P&C companies pertinent to determining the cost of capital in workers compensation as consisting of 32 corporations, 29 of which are selected from the Value Line P&C Industry Grouping (PMI was omitted due to its specialization on residential mortgages) and another three (AIG, Hartford Financial Services, and Unitrin) are taken from the Diversified Financial Services Grouping. (AIG, Hartford Financial Services, and Unitrin have significant P&C business as a percent of total; moreover, AIG and Hartford are major writers of workers compensation insurance.) Of these 32 corporations, which are detailed in Table 1, 29 companies currently pay dividends.

The dividend yield is estimated by the (unweighted) average of the dividend yields of these 29 dividend-paying corporations; this average amounts to 1.84 percent. The use of an unweighted average (as opposed to an average weighted by the corporations' stock market valuations) rests on the premise that any company is as a good a representative of the industry as any other. Further, such an unweighted industry average is robust to possible discontinuities in the dividend payments over time of any single corporation; otherwise, the industry average may be affected by the idiosyncrasies of a few large companies. For the record, weighting individual company dividend yields by market capitalization reduces the average dividend yield (prior to adjustment for share repurchases) to 1.56 percent from 1.84 percent. The reduction largely reflects the 33 percent weight on the 0.98 percent dividend yield of AIG. (The market capitalization of AIG at the time this study was prepared was about one-third of the total market cap of all dividend-paying companies in the Value Line sample of companies used in this study.)

The list of companies in Table 1 includes insurers that pursue more than the P&C line of business (but may also have some life insurance business, for instance)—these companies may be labeled as diversified. Similar to calculating a so-called pure play *beta*, a pure play approach to calculating the dividend yield would necessitate exclusion of diversified companies. But whereas the CAPM *beta* is systematically affected by diversification (as diversification moves the *beta* closer to the unit value), there is no similar well-established theory regarding the relation between diversification and the dividend yield.

Table 1: Current Dividend Yield, Percent

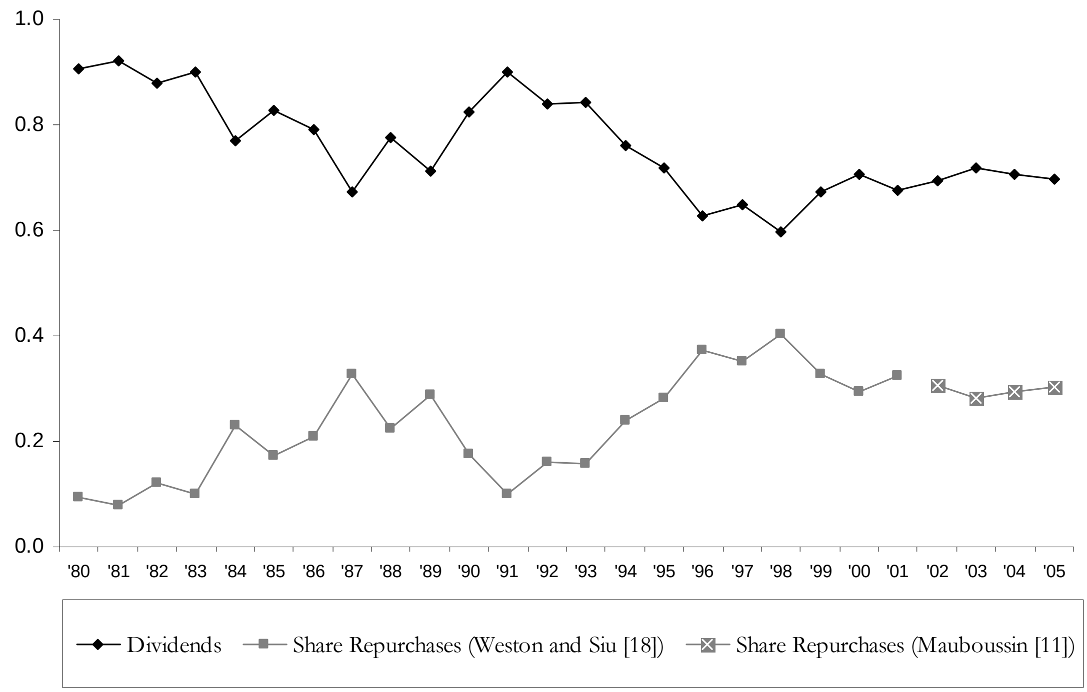
Company	Current Div. Yield
21st Century Insurance Group	2.90
ACE Limited	1.82
Alleghany Corporation	n/a
The Allstate Corporation	2.52
American Financial Group, Inc.	1.15
American International Group, Inc.	0.98
Assured Guaranty Ltd.	0.55
W.R. Berkley Corporation	0.58
Berkshire Hathaway Inc.	n/a
The Chubb Corporation	2.18
Cincinnati Financial Corporation	3.22
CNA Financial Corporation	0.83
Erie Indemnity Co.	3.08
Everest Re Group, Ltd.	1.89
The Hanover Insurance Group, Inc.	0.65
The Hartford Financial Services Group, Inc.	2.05
HCC Insurance Holdings, Inc.	1.28
Markel Corporation	n/a
Max Capital Group Ltd.	1.00
Mercury General Corporation	3.85
Ohio Casualty Corporation	1.68
Old Republic International Corporation	2.89
PartnerRe Ltd.	2.44
The Progressive Corporation	0.20
RenaissanceRe Holdings Ltd.	1.60
RLI Corp.	1.44
Safeco	1.91
Selective Insurance Group, Inc.	1.89
Transatlantic Holdings, Inc.	0.89
The Travelers Companies, Inc.	2.00
Unitrin, Inc.	3.88
XL Capital Ltd	2.10
Average	**1.84**

Source: Value Line Publishing, Inc. [23]. Note: "n/a" signifies non-dividend paying companies.

2.1.2.1 Adjustment for Share Repurchases

Dividends are not the only means by which companies return cash to shareholders. Chart 1 shows that share repurchases have become an important factor since the mid-1980s, and currently account for about 30 percent of all cash returned to shareholders. As argued above, to the extent that such share buybacks return cash to shareholders, the repurchases have to be added to the dividend payments when computing the *effective* dividend yield.

Chart 1: Fractions of Dividends and Share Repurchases in Total Cash Paid out to Shareholders



Source: Weston and Siu [26]; Mauboussin [16].

Quantifying the amount of cash that is returned to shareholders in stock repurchases is not straightforward. Compared to dividends, where management tends to maintain a steady payment pattern over time, share buybacks are "lumpy" and vary with overall stock market conditions. For instance, as shown in Chart 1, stock repurchases increased greatly during the stock market run-up of the mid-to-late 1990s. Another complicating factor is that such buybacks tend to be concentrated in a relatively small number of companies. Finally, not all shares that are repurchased are also retired; instead, repurchased shares may be handed out to employees or executives as part of their compensation. When repurchased shares are passed on to employees or executives, then there is no cash returned to shareholders in the aggregate; see Liang and Sharpe [14].

Table 2 provides data on share repurchases and cash dividends for the chosen set of 32 companies. The data, which were obtained from 10K reports filed with the U.S. Securities and

Exchange Commission (SEC; http://www.sec.gov), cover the period 2004-2006. As shown in the table, share buy-backs equaled $5.53 billion in the period 2004-2006, thus exceeding the cash dividends of $5.05 billion for the same period by 9.6 percent. Based on these numbers, one could argue that the *effective* dividend yield is about two percentage points above the reported 1.84 percent dividend yield. However, in light of the variable and skewed nature of share buy-backs in the P&C industry, and the possibility that some of the repurchased shares may fund share-based compensation and stock option grants, NCCI takes a conservative stance and estimates that the effective dividend yield exceeds the reported dividend yield by half a percentage point. (22 of 32 companies in the NCCI P&C company data set engaged in share repurchases in the period 2004-2006; of those 22 companies that repurchased shares, only one reported a reissuance of shares in its 10K statement; this reissuance amounted to about $250 million or, equivalently, 4 percent of the average annual share repurchases of all 22 companies taken together.)

2.1.3 The Prospective Rate of Growth in Dividends

Forecasting dividend growth is subject to the same principles as economic forecasting in general. Whereas in the short run, the path of future economic activity may be discernable in a fairly accurate way, in the long run, the growth of economic activity mean-reverts to a long-term average.

Table 2: Dividends and Common Stock Repurchases, 2004-2006

Company	Dividends			Common Stock Repurchases			Dividends	Repurchases
	2006	**2005**	**2004**	**2006**	**2005**	**2004**	**2004-2006 Average**	**2004-2006 Average**
21st Century Insurance Group	27.6	13.7	8.5	-	-	-	16.6	-
ACE Limited	312.0	253.0	226.0	-	-	-	263.7	-
Alleghany Corporation	-	-	-	39.2	-	-	-	13.1
The Allstate Corporation	873.0	830.0	756.0	1,770.0	2,484.0	1,373.0	819.7	1,875.7
American Financial Group, Inc.	38.2	33.1	35.1	-	-	-	35.5	-
American International Group, Inc.	1,638.0	1,421.0	730.0	20.0	176.0	1,083.0	1,263.0	426.3
Assured Guaranty Ltd.	10.5	9.0	4.6	171.1	19.0	6.0	8.0	65.4
W.R. Berkley Corporation	29.4	19.1	23.5	45.1	0.6	0.3	24.0	15.3
Berkshire Hathaway Inc.	-	-	-	-	-	-	-	-
The Chubb Corporation	403.0	330.0	291.0	1,228.0	135.0	-	341.3	454.3
Cincinnati Financial Corporation	228.0	204.0	177.0	120.0	61.0	59.0	203.0	80.0
CNA Financial Corporation	-	-	-	-	-	-	-	-
Erie Indemnity Co.	86.1	83.9	55.1	217.4	99.0	54.1	75.0	123.5
Everest Re Group, Ltd.	39.0	25.4	22.4	-	-	-	28.9	-
The Hanover Insurance Group, Inc.	15.4	13.4	-	200.2	-	-	9.6	66.7
The Hartford Financial Services Group, Inc.	460.0	345.0	325.0	-	-	-	376.7	-
HCC Insurance Holdings, Inc.	38.9	27.6	20.0	-	-	-	28.9	-
Markel Corporation	-	-	-	45.9	15.9	3.4	-	21.7
Max Capital Group Ltd.	14.3	9.0	5.5	0.0	7.4	4.9	9.6	4.1
Mercury General Corporation	105.0	93.9	80.6	-	-	-	93.2	-
Ohio Casualty Corporation	22.3	11.5	-	98.7	38.9	-	11.3	45.9
Old Republic International Corporation	135.8	300.7	91.6	-	-	-	176.0	-
PartnerRe Ltd.	125.4	118.9	92.3	(17.2.0)	(102.4)	152.5	112.2	10.9
The Progressive Corporation	25.0	23.7	23.3	1,214.5	482.8	1,628.5	24.0	1,108.6
RenaissanceRe Holdings Ltd.	60.4	57.0	53.8	-	0.7	38.8	57.1	13.2
RLI Corp.	19.0	15.9	12.6	37.6	-	0.0	15.9	12.5
Safeco	130.2	118.9	104.8	1,165.2	255.9	663.0	118.0	694.7
Selective Insurance Group, Inc.	22.8	19.9	17.3	116.4	22.9	8.7	20.0	49.3
Transatlantic Holdings, Inc.	33.6	29.0	24.7	-	6.3	1.2	29.1	2.5
The Travelers Companies, Inc .	702.0	628.0	642.0	1,120.0	33.0	23.0	657.3	392.0
Unitrin, Inc.	119.8	117.4	113.5	89.9	48.9	-	116.9	46.3
XL Capital Ltd	277.7	276.7	270.5	5.6	5.5	4.6	274.9	5.3
Total	**5,992.4**	**5,428.7**	**4,206.8**	**7,687.4**	**3,790.4**	**5,103.9**	**4,934.3**	**5,522.0**

Source: 10K reports, U.S. Securities and Exchange Commission, http://www.sec.gov. Note: Numbers are stated in millions of U.S. dollars. The Travelers Companies, Inc. were formerly known as The St. Paul Travelers Companies.

In a similar vein, the NCCI discounted cash flow approach draws on forecasts for the immediate future before reverting to a long-term average. Specifically, NCCI uses Value Line Publishing, Inc. forecasts for the dividend growth of the P&C industry for a five-year horizon before transitioning to a long-term average rate of growth of the industry; the long-term average rate of industry growth is gauged by the long-term average rate of growth of total financial assets of property-casualty insurance companies as stated in the Flow of Funds accounts of the Federal Reserve (http://www.federalreserve.gov/RELEASES/z1). This approach, which relies on professional forecasts for the near term and on a long-term average for the time thereafter, recognizes the benefit of near-term growth rates in accounting for short-term cyclical factors but, at the same time, restrains any potential optimism bias on the part of the analyst (see Easterwood and Nutt [7]).

To be specific, for the first five years of the forecasting period, we calculate the rate of dividend growth of the industry as an (unweighted) average across 28 (of the currently 29 dividend-paying) corporations based on company-level Value Line Publishing, Inc. [23] "five-year-ahead" forecasts, which are displayed in Table 3 below. (Value Line Publishing, Inc. did not offer a forecast for Ohio Casualty, which declared dividends in the years 2005 and 2006 but, prior to that, had not declared dividends four years running.) As of May 25, 2007, the estimated rate of dividend growth equals 11.19 percent.

The rate of growth that applies in perpetuity after the initial five-year period is calculated in two steps. First, we estimate for the period 1952-2005 the long-term real (that is, inflation-adjusted) rate of growth of the P&C industry based on its total financial assets; to this end, we deflate (that is, inflation-adjust) this measure of industry size, using the implicit price deflator of the Gross Domestic Product. (The Gross Domestic Product Deflator is published by the Department of Commerce, Bureau of Economic Analysis, as part of the National Income and Product Accounts; http://www.bea.gov.) Second, we multiply this long-term real rate of growth of the industry by the rate of expected inflation, which we gauge by the spread between the yields on 10-Year Treasury notes and 10-year Treasury Inflation-Indexed Securities. The resultant long-term rate of dividend growth equals 7.68 percent, based on data available as of May 25, 2007.

Table 3: Dividend Growth

Company	Forecast 2011
21st Century Insurance Group	30.0
ACE Limited	7.5
Alleghany Corporation	n/a
The Allstate Corporation	9.0
American Financial Group, Inc.	2.5
American International Group, Inc.	21.0
Assured Guaranty Ltd.	15.0
W.R. Berkley Corporation	13.0
Berkshire Hathaway Inc.	n/a
The Chubb Corporation	7.0
Cincinnati Financial Corporation	7.0
CNA Financial Corporation	n/a
Erie Indemnity Co.	10.0
Everest Re Group, Ltd.	28.0
The Hanover Insurance Group, Inc.	23.0
The Hartford Financial Services Group, Inc.	14.5
HCC Insurance Holdings, Inc.	15.5
Markel Corporation	n/a
Max Capital Group Ltd.	12.0
Mercury General Corporation	5.5
Ohio Casualty Corporation	-- *
Old Republic International Corporation	12.0
PartnerRe Ltd.	6.0
The Progressive Corporation	20.5
RenaissanceRe Holdings Ltd.	3.0
RLI Corp.	12.0
Safeco	8.0
Selective Insurance Group, Inc.	6.0
Transatlantic Holdings, Inc.	12.0
The Travelers Companies, Inc.	3.0
Unitrin, Inc.	1.0
XL Capital Ltd	-2.0
Average	**11.19**

Source: Value Line Publishing, Inc. [23]. Note: * indicates value labeled "not meaningful" by Value Line Publishing, Inc.

2.1.3.1 The TIIS Spread

The spread between the yields of conventional and Treasury Inflation-Indexed Securities (TIIS) offers an objective, market-based estimate of future inflation. The advantage of such an inflation gauge over opinion surveys is that it reflects actions taken by investors in the market place (see Kwan [13]).

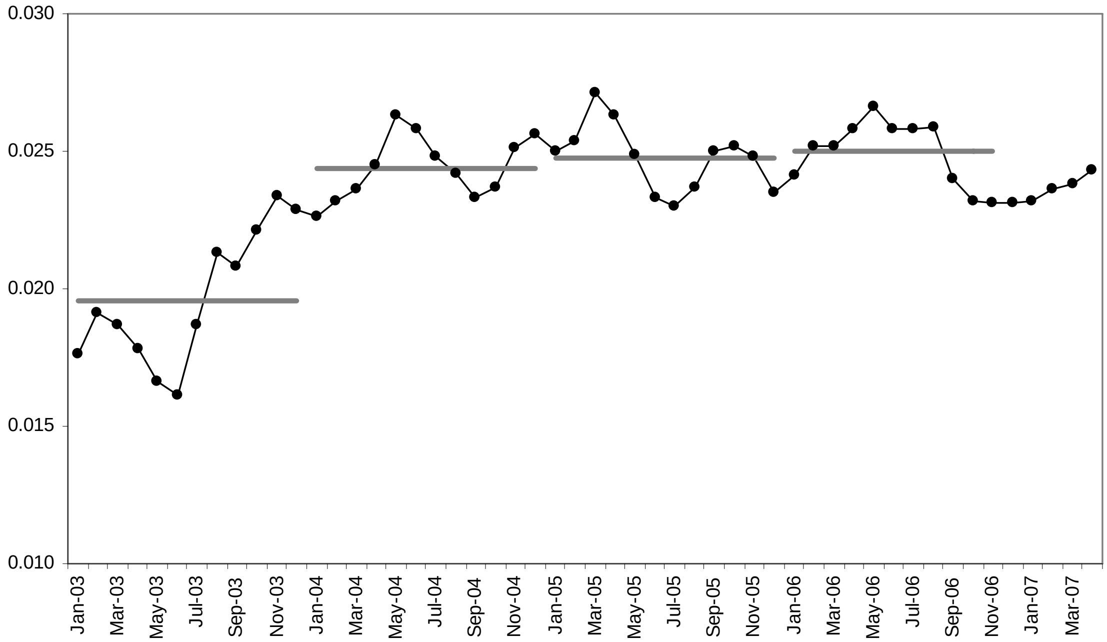
TIIS, which are also known as TIPS (Treasury Inflation-Protected Securities), were introduced by the Treasury Department in 1997 as a new class of government debt. Although the coupon yield (ratio of interest payment to principal) of TIIS is fixed for the time to maturity, the actual coupon payments rise according to the rate of inflation as the principal adjusts to the CPI (Consumer Price Index). The average future rate of inflation for which an investor is indifferent between holding a conventional Treasury note and holding a TIIS, is known as the break-even rate of inflation. This break-even rate of inflation may serve as a gauge of the rate of inflation that investors expect on average for the time to maturity (see Kwan [13]). To smooth out noise, we measure such inflation expectations by the average of the past 12 monthly observed TIIS spreads.

For the time period ending in April 2007, the trailing average of 12 monthly TIIS spreads equals 2.44 percent, with relatively small variations on a month-to-month basis. As Chart 2 shows, inflation expectations as gauged by a rolling trailing 12-month TIIS spread have changed only little since 2004.

2.1.4 The DCF Estimate

The DCF approach delivers an estimate for the cost of capital of 10.62 percent, based on data available as of May 25, 2007. When using a dividend yield average weighted by market capitalization (of 1.56 percent, instead of an equally weighted dividend yield average of 1.84 percent), the cost of capital amounts to 10.26 percent.

Chart 2: Spread between Rate on Conventional and TIIS 10-Year Government Securities



Source: U.S. Department of the Treasury, Office of Debt Management; daily observations, not seasonally adjusted. Note: Observations are charted on a monthly basis; horizontal bars indicate calendar year averages.

2.2 The CAPM

The Sharpe-Lintner (one-factor) CAPM rests on the fundamental insight that the total risk of an equity investment can be broken down into a component that can be eliminated by means of diversification, and a residual component known as systematic risk; see Sharpe [22] and Lintner [15]. Because diversification is brought about by holding (a representative slice of) the entire market, the risk that cannot be eliminated is the one that correlates with the market. The degree of such correlation with the market of an individual stock is known as *beta* (β). Further, only systematic risk generates a risk premium in the marketplace, because this is the risk component that has to be born by the investor. From this it follows that the return an investor demands for holding a given stock equals

$$k = r_f + \beta \cdot (r_m - r_f), \tag{2.6}$$

where r_f is the return on the risk-free asset (commonly referred to as the risk-free rate of return) and r_m is the expected return on the market portfolio. The difference between the expected return on the equity market and the return on the risk-free asset, $r_m - r_f$, is known as the equity risk premium. The return k defines the marginal cost of equity capital.

According to the cost of capital equation stated above, differences in the cost of capital across companies are due to differences in the degree to which their returns co-vary with the return on the stock market as a whole. A *beta* equal to one indicates that the company in question offers the same expected return as the market as a whole ($k = r_m$). For companies with *betas* greater than one, the cost of capital exceeds the expected return on the market portfolio.

2.2.1 Inputs to the CAPM

Employing the CAPM for the purpose of estimating the cost of capital of the P&C industry necessitates estimating the risk-free rate of return, r_f, the *beta* of the industry, β, and the expected market return, r_m. Below follows a description of the NCCI estimates of these three variables.

2.2.1.1 Risk-Free Yield

Only short-term rates are free from both default *and* inflation risks. Thus, we gauge the risk-free rate of return, r_f, by a short-term Treasury yield; see Bodie and Merton[4]. Because short-term interest rates mean-revert as they follow the monetary tightening and easing cycle of the Federal Reserve, NCCI uses a long-term average as measured by the arithmetic return on U.S. Treasury bills with about 30 days to maturity for the period 1926-2006; this value equals 3.8 percent (see Morningstar, Inc. [20]).

Note that the choice of the risk-free rate has only a minor impact on the NCCI CAPM cost of capital estimate because the *beta* of the P&C industry is close to unity, as shown below; for a *beta* of unity, the risk-free rate of return drops out of the cost-of-capital equation.

2.2.1.2 The *Beta*

NCCI obtains company-level estimates for the *betas* of the mentioned set of 32 P&C companies from the Value Line Publishing, Inc. [23]. These *betas*, which are displayed in Table 4, have been adjusted (by Value Line) for their tendency to mean-revert to unity, as suggested by Blume [3]. Specifically, a Blume-adjusted *beta* is the sum of a constant (0.35) and the weighted original estimated CAPM *beta* (weight: 0.67).

We use two alternative ways of aggregating the company-level *betas* to an industry *beta*. The first approach is to calculate the industry *beta* as an unweighted average of the Blume-adjusted (one-factor) CAPM *betas* displayed in Table 4; this is our favored approach. The reason for using an unweighted average in computing the pertinent industry statistic was stated above in connection with the industry dividend yield. Again, using an unweighted average views any corporation operating in the P&C industry as good a representative of the industry as any other. Further, an unweighted average is robust to mean reversion, although the Blume-adjustment already diminishes this problem. Using the unweighted average of Blume-adjusted company-level *betas*, we arrive at a P&C industry *beta* of 0.95, based on data available as of May 25, 2007.

The second approach that we pursue in aggregating company-level *betas* into an industry *beta* is the (stock market capitalization-weighted) full-information *beta* concept detailed in Kaplan and Peterson [12]. For this purpose, we collect the stock market capitalization of the P&C companies listed in Table 4, the (one-factor) CAPM *betas* and the stock market capitalizations of the Value Line Life Insurance companies (see Table 5), as well as the business volume of the these Value Line P&C and Life companies, broken by line of insurance. We distinguish only between P&C and life business, thus ignoring any residual (of which there was only one, which originated in the two-percent banking business of Aegon); the weights were scaled such that P&C and life add up to 100 percent. Business volume is measured (in lexicographic order) either by earned premium, net earned premium, revenue or gross written premium, whichever allowed us to break down the business by line of insurance. (When calculating the full-information industry *beta*, we exclude Berkshire Hathaway from the list of P&C companies for the purpose of estimating the industry *beta*, as this company is a conglomerate with many lines of business outside the insurance industry. Further, the *betas* of the life companies and the values for the stock market capitalization of the P&C companies are of more recent vintage than the *betas* of the P&C companies.) Using the full-information industry *beta* approach, we arrive at a P&C industry *beta* of 1.097, based on data available as of May 25, 2007.

2.2.1.3 The Prospective Return on the Market Portfolio

The expected rate of return on the equity market is a heavily debated issue in financial economics; mostly, this debate is stated in terms of the equity risk premium. Remember that the equity risk premium is the *expected* return on the market in excess of the risk-free rate.

A measure of the equity risk premium that suggests itself is the *realized* return on the stock market in excess of the return on short-term Treasury bills. Based on such realized returns, the equity risk premium for the period 1926-2006 equals 8.6 percent; this calculation rests on the difference between the arithmetic mean returns on the S&P 500 and on T-bills, as published by Morningstar, Inc. [21]. However, it has been argued that gauging the equity risk premium by past returns overstates its value; this is because the 20th century stock market return came in part at the expense of a secular decline in the dividend yield, which may not repeat itself in the future (see Arnott and Bernstein [2]).

In a comprehensive study of market expectations and realized equity returns over more than 200 years, Goetzmann and Ibbotson [9] find that the realized equity risk premium of the 20th century, although much higher than what finance scholars had expected at the time, was not out of line with historical experience. At the same time, these authors confirm that, indeed, there has been a secular decline in the dividend yield in the history of the U.S. stock market.

Table 4: CAPM *Betas* P&C

Company	***Beta*** (Blume-adjusted)	Market Capitalization (Billons of U.S. dollars)
21st Century Insurance Group	0.90	1.9
ACE Limited	1.35	19.7
Alleghany Corporation	0.60	3.4
The Allstate Corporation	0.90	30.0
American Financial Group, Inc.	1.00	3.4
American International Group, Inc.	1.25	118.0
Assured Guaranty Ltd.	0.60	1.2
W.R. Berkley Corporation	0.90	5.7
Berkshire Hathaway Inc.	0.65	168.0
The Chubb Corporation	1.05	20.6
Cincinnati Financial Corporation	0.90	6.5
CNA Financial Corporation	1.00	9.5
Erie Indemnity Co.	0.70	2.7
Everest Re Group, Ltd.	1.05	6.0
The Hanover Insurance Group, Inc.	1.65	2.4
The Hartford Financial Services Group, Inc.	1.30	22.7
HCC Insurance Holdings, Inc.	0.90	3.4
Markel Corporation	0.80	4.6
Max Capital Group Ltd.	0.90	1.7
Mercury General Corporation	0.85	2.8
Ohio Casualty Corporation	0.95	2.6
Old Republic International Corporation	1.05	3.6
PartnerRe Ltd.	0.95	4.3
The Progressive Corporation	0.90	13
RenaissanceRe Holdings Ltd.	0.70	3.9
RLI Corp.	0.80	1.4
Safeco	0.85	5.3
Selective Insurance Group, Inc.	0.85	1.2
Transatlantic Holdings, Inc.	0.80	4.2
The Travelers Companies, Inc.	1.25	34.0
Unitrin, Inc.	1.05	2.4
XL Capital Ltd	1.05	10.3
Unweighted Average	**0.95**	**---**

Source: Value Line Publishing, Inc. [23] (*betas*) and [24] (market capitalization).

Table 5: CAPM *Betas* Life

Company	***Beta*** (Blume-adjusted)	Market Capitalization (Billons of U.S. dollars)
Aflac, Inc.	0.80	31.5
Aegon	1.55	26.6
Delphi Financial Group	0.95	1.4
Genworth Financial	1.15	10.3
Lincoln National	1.30	14.3
Manulife Financial	0.95	58.0
Metlife, Inc.	1.05	44.0
Nationwide Financial	1.10	6.6
The Phoenix Companies	1.35	1.4
Protective Life Corp	0.95	2.9
Prudential Financial	1.15	36.0
Reinsurance Group	0.95	3.4
Torchmark Corp	0.90	5.7
Unum Group	1.50	8.0
Unweighted Average	**1.12**	**---**

Source: Value Line Publishing, Inc. [25].

Specifically, Goetzmann and Ibbotson [9] show that over the period 1792-1925, the difference between the arithmetic mean return on stocks and the arithmetic mean rate of inflation was 7.08 percentage points (7.93 percent minus 0.85 percent); this compares to a 8.63 percentage point difference between the arithmetic mean returns on the U.S. stock market (12.39 percent) and on Treasury bills (3.76 percent) for the period 1926-2004 (see Goetzmann and Ibbotson). Assuming a stable difference between the risk-free rate of return and the rate of inflation, the findings by Goetzmann and Ibbotson suggest that the realized equity risk premium over the period 1792-1925 was 2.19 percentage points ([12.39 minus 3.12] minus [7.93 minus 0.85] percent) lower than over the period 1926-2004 (see Goetzmann and Ibbotson).

We now detail how to calculate the arithmetic mean return on the S&P 500 stock market index for the period 1926 through 2006, adjusted for the secular decline in the dividend yield. First, the pertinent 2006 total return index value, TRI^{2006}, is adjusted such that the implied dividend yield equals the dividend yield observed in the year 1926 (as advocated in section 2.2.1.3):

$$\frac{TRI^{2006} \times \text{dividend}\, yield^{2006}}{adjusted\ TRI^{2006}} = dividend\ yield^{1926}. \qquad (2.7)$$

Second, we calculate the geometric mean annual return as the arithmetic mean annual return in logarithmic space:

$$return^{geometric} = \frac{\log(adj.\ TRI^{2006}) - \log(TRI^{1926})}{80}. \tag{2.8}$$

Third, we obtain the desired arithmetic mean return by exponentiating and bias-adjusting:

$$return^{arithmetic} = \exp(return^{geometric} + \hat{\sigma}^2/2), \tag{2.9}$$

where $\hat{\sigma}^2$ is an estimate of the variance of the actual annual logarithmic returns (which equal the first differences in the annual logarithmic *TRI* values).

The numerical values associated with these calculations are displayed in Table 5. As this table shows, the historical arithmetic average annual return on the S&P 500 index equals 11.39 percent when adjusted for the secular decline in the dividend yield, down from the actually observed 12.3 percent (per calculation of Morningstar, Inc. [21]). The implied equity risk premium equals 7.59 percent (11.39 percent minus 3.8 percent), which compares to an unadjusted risk premium of 8.6 percent (using data before rounding; see Morningstar, Inc. [21]).

2.2.2 Bottom-Line CAPM Estimate

Taken together, we arrive at a CAPM-based cost of capital estimate of 11.02 percent, based on data available as of May 25, 2007; the implied estimate of the equity risk premium equals 7.59 percent.

Table 6: Total Return Calculation

Total Return Index in 1926 (TRI^{1926}) (1)	1,334.79
Dividend Yield in 1926 (2)	5.41%
Total Return Index in 2006 (TRI^{2006}) (3)	3,679,817.89
Dividend Yield in 2006 (4)	2.01%
Adjustment for Share Repurchases	0.50%
Estimated Effective Dividend Yield	2.51%
Adjusted Total Return Index ($Adj.\ TRI^{2006}$)	1,707,272.26
Arithmetic Mean Using Adjusted Total Return Index	11.39%
Arithmetic Mean Using Actual Total Return Index	12.3%

Source: Own calculations; Morningstar, Inc. [20]: (1) p. 204, (2) p. 228, (3) p. 205, and (4) p. 229.

According to Equation (2.7), $Adj.\ TRI^{2006}$ equals the product of (1) TRI^{1926} (3,679,817.89) and (2) the ratio of the 2006 and 1926 dividend yields (inclusive of share repurchases) (2.51/5.41), thus resulting in *Adj. TRI* ^{2006} of 1,707,272.26. The computation of the arithmetic and geometric mean returns is based on Equations (2.8) and (2.9).

2.2.3 Discussion

As mentioned, our *betas* were obtained from the (one-factor) CAPM; we aggregate these *betas* in two alternative ways to a *beta* for the P&C industry. In one aggregation approach, we Blume-adjust the company-level *betas* and then calculate an unweighted average. In the other approach, we estimate a full-information industry *beta* as suggested by Kaplan and Peterson [12] and implemented for the insurance industry by Cummins and Phillips [5]; here, the (one-factor) CAPM company-level *betas* are not Blume-adjusted.

Alternative to using (one-factor) CAPM *betas*, three-factor *betas* may be used—the Fama French three-factor model may be viewed as a generalization of the (one-factor) CAPM. In their 1997 study of the (one-factor) CAPM and three-factor *betas* of major U.S. industries, Fama and French [8] find little difference between these two *betas* (as mentioned above) and little difference between the implied risk premiums—the risk premium obtained with the three-factor model exceeds the 5.14 percent risk premium of the (one-factor) CAPM by only 0.58 percentage points. On the other hand, Cummins and Phillips [5] in their 2005 paper, find that the three-factor *betas* greatly exceed the (one-factor) CAPM *betas*, thus resulting in large differences in the estimated costs of equity capital between these two approaches. Based on market-value weighted estimates for the P&C industry, these authors come up with a (one-factor) CAPM *beta* of 0.843 (their Table 4, Panel B) and the Three-Factor *beta* of 1.099 (their Table 5, Panel B); the corresponding implied cost of equity capital equal 12.0 percent (one-factor CAPM; Table 4, Panel D) and 19.1 percent (three-factor model; Table 5, Panel D).

We are skeptical of the three-factor model because of its lacking theoretical foundation and, with regard to the Cummins and Phillips [5] study, we are unconvinced of (1) the seven percentage-point difference in the implied cost of equity capital between the two approaches (remember that Fama and French [8] found only a small difference in risk premiums between the two models) and (2) of the high cost of equity capital of 19.1 percent. Note that the arithmetic mean stock market return of large (small) capitalization stocks from 1926 to 2000 ran at only 12.3 (17.4) percent, which casts doubt on the proposed 19.1 percent cost of equity capital for the P&C industry, whose *beta* was estimated in the neighborhood of unity by Fama and French [8]. In fact, the Cummins and Phillips estimate of the cost of equity capital of 12.0 percent (as obtained with the one-factor CAPM, market-value weighted; their Table 4, Panel D) agrees more with the U.S. long-term average large cap stock return and with our estimate of the cost of capital.

It is worthy of note that our implementation of the CAPM generates a risk premium that is fairly stable over time; this is because our model rests on long-term averages. An alternative approach to estimating the cost of capital is to choose a level for the equity risk premium and then deduce the cost of capital by adding this risk premium to the short-term or long-term Treasury yield (depending on whether the risk premium is measured over short-term or long-term Treasuries). A major drawback of such an approach is the high degree of cross-sectional variation in opinions regarding the appropriate level of risk premium, as well as high degree of time-variation of such opinions, as documented by Graham and Harvey [11]. For a discussion of this subject matter in relation to NCCI ratemaking, see Wolf [27].

3 THE OVERALL COST OF CAPITAL

The DCF and CAPM are market-based approaches to estimating the cost of capital. That is, they both use financial market data to develop estimates for the expected return demanded by the marginal investor. The DCF method uses current stock prices and dividend yields as key inputs, but is sensitive to the projected growth in dividends. In contrast, the CAPM is sensitive to the choice of the industry *beta* and, perhaps more critically, to the projected return on the market portfolio and the implied equity risk premium.

Given the uncertainty surrounding the projected dividend growth and the ongoing debate among financial economists regarding the equity risk premium, it seems appropriate not to rely on one method alone. For this reason, the NCCI estimate of the cost of capital of the P&C industry is computed as an average of the estimates delivered by the DCF and CAPM approaches. Such averaging is likely to lead to less variation in the cost of capital estimate over time, which is a desirable feature in a regulatory setting.

Table 6 summarizes the DCF and CAPM results based on the NCCI cost of capital methodology; this table displays an overall cost of capital of 10.82 percent.

Note that NCCI updates its cost of capital estimates throughout the year as (1) Value Line Publishing, Inc., releases quarterly updates for the DCF inputs and the CAPM *beta* coefficients and (2) Morningstar, Inc. publishes annual updates of the historical returns on T-bills and the S&P 500 stock price index, which enter the CAPM.

Table 7: Cost of Equity (CE) Estimates Using Data as of May 25, 2007

CE Discounted Cash Flow (DCF) Model	10.62 %
Current Dividend Yield	2.34%
Forecast Avg. Ann. Growth in Dividends	11.19%
Long-term Avg. Ann. Growth in Dividends	7.68%
CE Capital Asset Pricing Model (CAPM)	11.02 (12.12)%
Risk-Free Rate	3.8%
beta	0.95 (1.097)%
Market Return	11.39%
Equity Risk Premium	7.59%
Overall Cost of Capital	10.82 (11.37)%

Note: CE indicates the cost of equity capital; in parentheses are the results using full-information *betas*. Some of the data used in the full-information *beta* approach is of more recent vintage (December 2007 and February 2008), as documented in the footnotes to Tables 4 and 5.

4. CONCLUSIONS

The cost of capital is an integral part of ratemaking at NCCI, as its value is a key element used in the specification of the profit factor. NCCI uses both the DCF and CAPM approaches in estimating of the cost of capital of the P&C industry. The data that feed into these estimates are from publicly available sources that are frequently cited in similar analyses prepared for regulatory proceedings; these sources include governmental institutions (U.S. Treasury Department and the Board of Governors of the Federal Reserve System) and private organizations (Value Line Publishing, Inc. and Morningstar, Inc.). In addition, NCCI uses long-term averages where appropriate, for instance when estimating the prospective dividend growth in the DCF analysis, and the risk-free rate and market return in the CAPM. The use of such long-term averages makes the cost of capital estimates robust to short-term economic fluctuations. The practice of averaging the DCF and CAPM estimates in determining the ultimate cost of capital acknowledges model uncertainty and uncertainty in the employed data inputs. Further, such averaging reduces the time variation of the cost of capital estimates, which is a desirable attribute from a regulatory perspective.

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Abbreviations and notations

CAPM, Capital Asset Pricing Model
CE, Cost of Equity Capital
DCF, Discounted Cash Flow
P&C, Property and Casualty
NCCI, National Council on Compensation Insurance

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Applications of the Offset in Property-Casualty Predictive Modeling

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Abstract: Generalized Linear Model [GLM] theory is a commonly accepted framework for building insurance pricing and scoring models. A helpful feature of the GLM framework is the "offset" option. An offset is a model variable with a known or pre-specified coefficient. This paper presents several sample applications of offsets in property-casualty modeling applications. In addition, we will connect the offset option with more traditional actuarial techniques such as exposure and premium adjustments. A recurring theme of the discussion is that actuarial modelers have at their disposal several conceptually related techniques that can be used to eliminate the impact of variables that (for whatever reason) are not intended for inclusion in a model, despite the fact that they might be correlated with both the target variable and other predictive variables. Examples discussed in this paper include a class plan analysis as well as a tier scoring application. Sample SAS code for fitting GLMs will be provided in the body of the paper.

Key Words: Offset, Residual, Generalized Linear Models, GLM, Predictive Modeling, Ratemaking, SAS

Introduction

In recent years, property-casualty insurance companies have widely embraced predictive modeling as a strategic tool for competing in the insurance marketplace. Predictive modeling – and in particular the use of Generalized Linear Models – was originally introduced as a method for improving the precision of personal auto insurance pricing. The use of predictive modeling was subsequently extended to homeowners and commercial lines as well. Today, predictive modeling is a core strategic capability of many top insurers and is applied in such key operations as marketing, underwriting, pricing, and claims management.

Property-casualty insurance is a complex and dynamic business. As is often observed, it is unique in that the ultimate cost of its basic product is unknown at the time of sale. A plethora of risk factors affects the cost of providing insurance. Many of these are well understood and are reflected in the price of insurance. For example, a typical automobile insurance rating plan contains more than 20 variables, including a wide range of driver, vehicle, and territorial characteristics [1]. However, the cost of providing insurance is also greatly influenced by such dynamic and exogenous factors as the underwriting cycle, medical inflation, variations in the size of jury awards, and poorly understood exposures such as asbestos and mold.

It is therefore practically impossible for actuarial models to be "comprehensive" in the sense of including all relevant variables that affect the number and size of claims. The non-ideal nature of actuarial models is compounded by the real-world fact that insurance data is often incomplete, inconsistently coded, and generally "dirty". In addition, many relevant variables (such as vehicle symbol, rating territory, or Workers Comp industry classifications) are "massively categorical", leaving individual insurers with insufficiently credible data to estimate their own rating factors as part of a rating plan optimization exercise.

For these and other reasons, actuarial modelers face a generic problem: in many, if not most, modeling situations, they are forced to exclude variables that are relevant to predicting frequency and size of loss. If these "omitted variables" are correlated with both the target variables and one or more of the other modeling variables, they will bias the estimates of the corresponding model parameters [2]. This phenomenon is commonly known as "omitted variable bias" [OVB].

In short, it will never be possible to build a single actuarial "super model" that accounts for every single determinant of loss. To avoid the peril of OVB, actuaries therefore must often "adjust for" or otherwise accommodate the effects of omitted variables as part of their model design and model construction process. Commonly known factors which potentially bias property-casualty predictive modeling results include the underwriting cycle and external environmental changes (i.e., time), variation in loss maturity, distribution channel, variation in rate adequacy across states and through time, and a changing competitive landscape, to name a few.

A traditional actuarial response to the problem of OVB is to adjust the model's target variable (more precisely, the exposure or premium component of the target variable). A conceptually similar technique that has long been in the arsenal of actuarial modelers is running a "preliminary" regression model on the variables to be omitted (such as policy year or state) and then using the *residuals* of this model as the target variable going forward. More recently, actuaries have embraced the offset option from Generalized Linear Model theory [3-7]. Each of these techniques offers a way of avoiding OVB. That is, each technique offers a way of accounting for the effect of omitted variables in a way that avoids biasing the model's parameters.

This paper will review the basics of GLM theory and the GLM offset option, provide various sample applications of the offset, and draw connections between the offset option and traditional actuarial techniques.

Background: GLM Theory and the Offset

Recall that a Generalized Linear Model [GLM] relates the expected value of the target variable ($\mu \equiv E[Y]$) to a linear combination of predictive variables ($\beta \cdot X$) via a "link function" $g(\cdot)$:

$$g(\mu) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_p X_p \equiv \beta \cdot X$$

In addition to the linearity assumption implicit in the above equation, GLM theory assumes that the target variable is distributed by the 2-parameter family of distributions known as the *exponential family*. The exponential family encompasses a wide range of distributional forms including Normal, Gamma, Binomial, Poisson, Negative Binomial, and many others. The exponential family density function is expressed as:

$$f_Y(y;\theta,\varphi) = \exp\{(y\theta - b(\theta))/a(\varphi) + c(y,\varphi)\}$$

The two parameters in this family, θ and φ, are known as the *canonical parameter* and *dispersion parameter*, respectively. As we will see, these are related to the mean and variance, respectively, of Y.

Two mathematical facts are helpful in interpreting this seemingly complicated expression:

$$E[Y] = \mu = b'(\theta)$$

and:

$$Var(Y) = b''(\theta)a(\varphi)$$

It is common to denote $b''(\theta)$ as $V(\mu)$ and call it the "variance function". (N.B.: the "variance function", $V(\mu)$, is not the same thing as the variance of Y.) Furthermore, the function $a(\varphi)$ is often specified to be φ/ω, where ω is a prior weight (such as exposure or premium). Therefore, we have the following expression that relates the variance of Y to the mean of Y:

$$Var(Y) = \frac{\varphi}{\omega} V(\mu)$$

In the special case of un-weighted, ordinary least squares [OLS] regression, we have: $g(\mu)=1$ (identity link), $\omega\equiv 1$ (each observation is given equal weight), $a(\varphi)=\sigma^2$ (merely a different naming convention), $b(\theta)=\theta^2/2$, and $c(y,\varphi) = -\frac{1}{2}\{y^2/\sigma^2 + \log(2\pi\sigma)\}$. The reader can verify that these substitutions result in the familiar expressions for the Normal distribution $N(\mu,\sigma^2)$ and homoskedasticity (constant variance):
:

$$f_Y(y;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-(y-\mu)^2/2\sigma^2\right\}$$

$$Var(Y) = \sigma^2$$

In OLS regression, the modeler selects the target variable, the appropriate set of predictive variables, as well as the prior weights ω, and must verify that the assumptions of normality (in particular homoskedasticity) and the linearity on the additive scale (i.e., identity link) are satisfied. In the broader GLM framework, the normality and linearity assumptions are each relaxed. The normality assumption is replaced with the much weaker assumption that the distribution of Y is from the exponential family; and linearity is replaced with linearity on the scale determined by the link function. Commonly used distribution/link function combinations are displayed below:

Distribution	$V(\mu)$	Link	Sample Application
Normal	1	identity	General applications
Poisson	μ	log	Frequency modeling
Binomial	$\mu(1-\mu)$	logit	Retention, cross-sell
Gamma	μ^2	log	Severity modeling
Tweedie	μ^p, $p\in(1,2)$	log	Pure Premium modeling

Note that what is often called "choosing a distribution" for a GLM is tantamount to choosing the variance function $V(\mu)$ that relates the variance of Y to the mean.

With the basic GLM framework in hand, we can turn to the offset feature. An *offset* is simply an additional model variable, ξ, whose coefficient is constrained to be 1:

$$g(\mu) = \beta \cdot X + \xi$$

In the case of OLS regression, this amounts to subtracting ξ from the target variable prior to running the regression. Therefore, offsets are not typically discussed in the context of OLS regression. Suppose that ξ is the predicted value of Y from a "preliminary" regression model. Then, specifying ξ as an offset is equivalent to using the residual from the preliminary regression as the target variable of the regression of interest. As mentioned above, this is a well known method of removing the effects of a group of nuisance variables from the target variable prior to running the model in order to avoid omitted variable bias.

In the remainder of this paper, we will discuss offsets in the context of multiplicative models, i.e., models constructed using the log link function.

As an aside, it is interesting to note that the offset was originally an afterthought in the development of Generalized Linear Models theory by Nelder and Wedderburn in 1972 [3]. Quoting from the book by Hilbe [8, page 130]:

> *"Offsets were first conceived by John Nelder as an afterthought to the [Iteratively Reweighed Least Squares] algorithm he and Wedderburn designed in 1972. The idea began as a method to put a constant term directly into the linear predictor without that term being estimated. It affects the algorithm only directly before and after regression estimation. Nelder only later discovered that the notion of an offset could be useful for modeling rate data."*

Offsets as a Measure of Exposure

As the above quote suggests, the offset is most commonly discussed as a measure of exposure in the context of Poisson regression. For example, it shows up in essentially the same way in both actuarial and epidemiological work. In both cases, offsets are often interpreted as a measure of exposure. In the latter setting, the exposure might be the number of people exposed to a pathogen; and the response would be the number of people who contract the disease. In the former setting, the exposure might be the number of car-years insured; and the response would be the number of claims incurred. In both settings, the value of the response is assumed to be roughly proportional to the value of the exposure.

As the final equation in the previous suggestion indicates, the offset must be on the same scale as the linear predictor $\beta \cdot X$. Therefore, in the auto example above, log(exposure) would be used as an offset. That is: $\xi = \log(u)$ where u ("units") denotes exposure:

$$\log(E[C]) = \beta \cdot X + \log(u)$$

In the Poisson case, this is mathematically equivalent to replacing claim count with claim *frequency* (claims divided by exposures: $F=C/u$) as the target variable; using exposure as the weight; and dispensing with the offset:

$$\log(E[F]) = \beta \cdot X$$

These two models' specifications are summarized in the table below:

	Option 1	Option 2
GLM family:	Poisson	Poisson
Target:	C	F
Weight:	(none)	u
Offset:	$\log(u)$	(none)

Option 2 is the more commonly adopted model specification. The equivalence of these two specifications is demonstrated in Appendix A.

Exposure Adjustments and the Offset

To avoid omitted variable bias, actuaries commonly perform as a preliminary step various exposure or premium adjustments to remove the effects of variables not included in the model. Such adjustments are commonly used in pricing plan analyses for reasons including pricing structure complexity, data availability, data credibility, business or regulatory considerations, competitive considerations, and the desire to mitigate policyholder impacts.

For example, suppose we wish to model claim frequency in terms of the following variables:

- Multi-car indicator
- Driver age
- Vehicle use
- Symbol
- Territory

Because of the large number of Territory and Symbol categories, the analyst might wish to estimate Symbol and Territory factors in a separate analysis. Merely dropping these variables from the model with no further action would raise the problem of OVB.

Suppose, for example, that a certain territory has a disproportionately large number of young drivers. If Territory were simply excluded from the model with no further adjustment, the Driver Age variable would act partly as a proxy for territory. The final rating plan, including both Territory and Driver Age, would overcharge young drivers in this hypothetical territory.

This problem is sometimes dealt with by adjusting the exposure field. Assuming a completely multiplicative rating plan, adjusting the exposures means simply multiplying exposures either by the existing territory and symbol relativities, or by a set of relativities that have been estimated in a separate modeling exercise. As above, let u denote the exposure measure and let $\tau_i\sigma_j$ denote the product of the Territory and Symbol relativities for Territory i and Symbol j. We compute f_{adj} =

$c/(u^*\tau_i\sigma_j)$. We use this adjusted frequency (claims divided by adjusted exposure) quantity rather than unadjusted frequency ($f=c/u$) as the target variable.

Given the above discussion and the result of Appendix A, it should be clear that one could equivalently use the *un*-adjusted frequency field f as the target variable and also include $\log(\tau_i\sigma_j)$ as an offset term in the model:

$$\log(E[F]) = \beta_{multi} + \beta_{driverAge} + \beta_{vehicleUse} + \log(\tau\sigma)$$

In other words, the traditional actuarial response to the OVB problem is equivalent to using a strategically selected offset term, that is, adjusting exposures is equivalent to including the pre-specified rating factors as an offset in the model and allowing the remaining factors to conform to this offset.

Loss Ratio Modeling and the Offset

The discussion in the previous section is analogous to the distinction between Loss Ratio and Pure Premium models. Suppose we wish to construct a credit scoring model, for eventual use in target marketing, company placement, and pricing refinement. Suppose also that the current rating plan is up-to-date, with no base rate or rating relativity changes needed. Examples of the variables used to construct the credit scoring model might be number of late payments in the past x days, balance-to-limit ratio, and number of derogatory public records in the past y years.

Using Pure Premium as the target variable in such a model would obviously introduce the possibility of omitted variable bias. It is possible that some of the parameters in the resulting credit scoring model would "double count" a penalty or credit given in one of the existing rating factors [9]. The traditional actuarial response to this problem is to use Loss Ratio rather than Pure Premium as the target variable [10]. This is analogous to the above discussion of adjusted exposures in Pure Premium modeling: we replace $loss/u$ (Pure Premium) with $loss/(u^*\tau\sigma\cdots\upsilon)=loss/prem$ (Loss Ratio) as the target variable.

This is conceptually equivalent to using dollars of loss as the target variable, and including log(prem) as the offset term in the model:

$$\log(E[loss]) = \beta_{latePay} + \beta_{balToLim} + \beta_{derog} + \ldots + \log(prem)$$

In this way, Loss Ratio modeling as an alternative to Pure Premium modeling can be viewed as yet another instance of strategically using the offset feature to avoid the problem of omitted variable bias.

Please note that our point is not to recommend that actuaries abandon the use of Loss Ratio as a target variable in favor of using Pure Premium or dollars of loss with an offset. We only wish to make the point that modeling Loss Ratio rather than Pure Premium is conceptually yet another instance of using an offset to integrate prior constraint into one's model. In loss ratio modeling, the goal is to build a scoring model to be layered on top of the existing rating plan. The prior constraint is therefore the current rating plan in its entirety, properly adjusted and on-leveled.

Using the Offset to Constrain Selected Rating Factors

Another useful application of the offset is constraining certain rating factors to take on pre-specified values. Constraints such as these are often motivated by regulatory and marketing considerations [7,11]. For example:

- The insurance marketplace might demand that that the discount for multi-car or home-auto package policies be no greater than 15%, regardless of the indication of a statistical analysis.
- California's Proposition 103 requires that a good driver discount be at least 20% below the rate the insured would otherwise be charged.

In both cases, we must constrain the values of certain rating factors in advance, and allow the remaining rating factors to optimally conform to these constraints. The offset allows one to easily integrate constraints such as these into one's model. This is only a short step from the exposure adjustment example discussed above. We will give an example with the added complexity that we wish to constrain some, but not all, of the levels of a certain rating variable.

In passing, we should note that offsets should not be applied blindly or in a mechanical fashion. Werner and Guven [12] provide a helpful example of a case in which one would *not* want the other factors in a rating plan to help "make up for" a prior constraint to a rating factor. In general, one should be mindful of the caveat that no modeling decisions (modeling technique, target variable design, choice of predictive variables and offsets, modeling dataset design, and so on) should be made without due regard for the business context of one's work.

Suppose we wish to optimize two factors of a multiplicative rating plan: driver age group (with values {1,2,3,4}) and multi-car indicator. We have already multiplied the exposures by all other rating variables as described in the exposure adjustment section above. Our target variable is adjusted frequency: claim count divided by adjusted exposure. Details of the dataset used in this and the following examples can be found in Appendix B.

Let us further assume that (either for competitive or regulatory reasons) the relativities for DRIVER_AGE_GROUP 3 and 4 must be constrained to take on the values 1.05 and 1.25, respectively. The following SAS code shows how to build a model that incorporates this constraint.

Model 1

```
data freq_data; set input;
     FREQ_ADJ = CLAIM_COUNT / EXPOSURE_ADJ;
     offset_factor = 1;
      if DRIVER_AGE =3  then offset_factor=1.05;
      if DRIVER_AGE =4  then offset_factor=1.25;
     logoffset=log(offset_factor);

     if DRIVER_AGE_NEW in (1,2)
            then DRIVER_AGE_NEW = DRIVER_AGE;
            else DRIVER_AGE_NEW = 99;
run;

proc genmod data=freq_data;
```

```
            class DRIVER_AGE_NEW;
            weight EXPOSURE_ADJ;
            model FREQ_ADJ = DRIVER_AGE_NEW MULTICAR
                 / dist=poisson
                   link=log
                   offset= logoffset;
      run;
```

Table 1 – Model 1 Output

Variable	variable value	beta	e^{beta}
DRIVER_AGE_NEW	1	0.75	2.11
DRIVER_AGE_NEW	2	0.65	1.91
DRIVER_AGE_NEW	99	0.00	1.00
MULTICAR	1	-0.27	0.77
MULTICAR	0	0.00	1.00

In this example, we constrain DRIVER_AGE by letting the offset take on the constrained values for age groups 3 and 4; and the 1.0 for the other age groups. At the same time, we re-code the age group values 3 and 4 to the value 99 to ensure that the model parameters for these levels will be 0. (This is a SAS trick: SAS treats the highest value of a categorical value as the base category.) Therefore, the model estimates "beta" parameters for age groups 1 and 2, as well as the multi-car indicator, subject to the constraint that age groups 3 and 4 must have relativities of 1.05 and 1.25, respectively. The final relativity for each level of DRVER_AGE and MULTICAR will by exp(beta + log_offset) = e^{beta}*offset. The final rating relativities are displayed below.

Table 2 – Combining Model 1 Parameters with Offset Values

Variable	variable value	model beta	e^{beta}	offset	final relativity
DRIVER_AGE	1	0.75	2.11	1	2.11
DRIVER_AGE	2	0.65	1.91	1	1.91
DRIVER_AGE	3	0.00	1.00	1.05	1.05
DRIVER_AGE	4	0.00	1.00	1.25	1.25
MULTICAR	1	-0.27	0.77	1	0.77
MULTICAR	0	0.00	1.00	1	1.00

Construction of a Cross-coverage Tier Score

In many insurance rating plans, a "tier" structure is a rating component that is layered on top of a class plan. In most cases, tier pricing is applied on a *policy* level across coverages. The purpose of rating tiers is to include in the pricing process further variables – such as personal credit score or not-at-fault accidents – which are not part of standard class plans. Rating tiers can also be used to capture interaction effects between the class plan variables, such as the interaction between driving record and driver age, which are not fully reflected due to the limitation of pricing structures. In the next example we illustrate how an offset technique can be used to create a cross-coverage tier structure.

Suppose we wish to add a tier structure to an existing standard personal auto class plan with two coverages: property damage liability (PD) and comprehensive (Comp). The tiers are to be comprised of two factors: number of policy-level not-at-fault accidents in the past 3 years (NAF) and credit score (CREDIT). The tier structure and the tier score are required to be the same across the two coverages.

Suppose we start with two separate data files, one for PD liability and one for Comp. Table 3 shows some sample records of the two data files on the exposure/vehicle level. Note that the PD and Comp adjusted exposures were calculated using the logic described in the section of "Exposure Adjustments and Offset". Specifically, the PD liability adjusted exposure is the unadjusted exposure multiplied by the corresponding territory factors; and the Comp adjusted exposure is the unadjusted exposure multiplied by the corresponding factors for territory, vehicle symbol and deductible.

Table 3

Sample Records from PD Dataset

Policy Number	Vehicle Number	Credit Score Group	Policy Level NAF Count	Current Plan Rating Factor	Adjusted Exposure	Incurred Loss	Adjusted Pure Premium
00003	1	2	1	0.41	0.73	0	0
00004	1	0	0	1.63	1.46	1664	1143
00005	1	0	0	0.58	1.25	0	0
00006	1	2	0	0.61	0.52	0	0
00007	1	1	0	1.12	1.25	1344	1077

Sample Records from Comp Dataset

Policy Number	Vehicle Number	Credit Score Group	Policy Level NAF Count	Current Plan Rating Factor	Adjusted Exposure	Incurred Loss	Adjusted Pure Premium
00003	1	2	1	1.38	1.07	0	0
00004	1	0	0	0.79	1.60	495	309
00005	1	0	0	1.05	1.51	566	375

Our first step is to simply "stack" these two datasets together, adding a coverage indicator to identify whether the record is PD vs. Comp.

Table 4

PD and Comp Combined Dataset

Policy Number	Vehicle Number	PD_IND	Credit Score Group	Policy Level NAF Count	Current Plan Rating Factor	Adjusted Exposure	Incurred Loss	Adjusted Pure Premium
00003	1	1	2	1	0.41	0.73	0	0
00004	1	1	0	0	1.63	1.46	1664	1143
00005	1	1	0	0	0.58	1.25	0	0
00006	1	1	2	0	0.61	0.52	0	0
00007	1	1	1	0	1.12	1.25	1344	1077
00003	1	0	2	1	1.38	1.07	0	0
00004	1	0	0	0	0.79	1.60	495	309
00005	1	0	0	0	1.05	1.51	566	375

The following GLM can be used to estimate the parameters for NAF and Credit:

Model 2

Input Dataset:	Stacked dataset
Target Variable:	Pure Premium (loss / exposure_adj);
Predictive Variables:	Credit, NAF
Distribution:	Tweedie
Link:	Log
Offset:	PD_Relativity*$\beta_1\beta_2\cdots\beta_p$ (product of existing rating plan factors)
Weight:	exposure_adj;

In the above model specification, we are using the offset to reflect the existing rating plan factors. We must also account for the variation in Pure Premium between the two coverages: clearly we expect a higher Pure Premium for PD records than Comp records. Not including a PD indicator in the model design would lead to a particularly egregious example of omitted variable bias.

In the above model design, we choose to include the PD relativity as an offset factor along with the rating plan factors other than credit score and not-at-fault accident count. Note that other model designs are possible. For example, it would also be possible to include the PD relativity as part of the exposure adjustment step. Either way, we must perform a preliminary analysis to estimate the Pure Premium relativity for PD vs. Comp, and include this relativity either as part of the offset or the exposure adjustment step.

Because our target variable in this example is Pure Premium, the Tweedie is an appropriate choice of distributions. This has been discussed extensively in the actuarial literature [4,6], so we will review this topic only briefly. For claim count (or frequency) modeling, it is customary to assume that the variance of the target variable is proportional to the mean: $V(\mu)=\varphi\mu$. This is the "Poisson" model design used in the previous examples. For severity modeling, it is customary to assume that the variance is proportional to the square of the mean: $V(\mu)=\varphi\mu^2$. This is known as a "Gamma" model design. Pure Premium is the sum of a (Poission distributed) random number of (Gamma distributed) sizes of loss. It is a convenient mathematical fact that the variance of this target variable is proportional to the mean raised to a power between 1 and 2, $p\epsilon(1,2)$: $V(\mu)=\varphi\mu^p$. This model design is also exponential family, and is known as the "Tweedie".

Unfortunately, the commonly used SAS statistical package does not automatically support the Tweedie model in the GENMOD GLM modeling procedure. One alternative to GENMOD is to fit Tweedie models using the NLMIXED procedure. Details of this are given in Appendix C.

Table 5 shows the parameter estimates from the above Tweedie model. The PD relativity used in the offset is 3.44.

Table 5 – GLM Output and Pure Premium Relativities

Parameter	Estimate	Pure Premium Relativity
credit_grp_0	1.09	2.96
credit_grp_1	1.23	3.44
credit_grp_2	0.74	2.10
credit_grp_3	-0.14	1.15
credit_grp_4	0.00	1.00
naf_pol_0	-0.15	0.86
naf_pol_1	-0.03	0.97
naf_pol_2	0.00	1.00

Thus the tier score for a policy with NAF=1 and Credit=2, for example, is exp(0.74 - 0.03)=2.03. Please note that the PD indicator is not used to calculate the tier factor.

Sequential Modeling

The previous two examples, building a credit score and a tiering structure "on top of" an existing rating plan, may be thought of as exercises in "sequential modeling". By "sequential modeling" we mean building a model to account for variation not already explained by a pre-specified model. The pre-specified model (the existing rating plan in the above examples) in other words serves an as "offset" when building the second model.

Sequential modeling techniques have a wide range of applications. As noted above, the first two examples – estimating rating plan factors after Territory and Symbol factors have been determined in a separate analysis; and building a credit scoring model on top of an existing rating plan – are examples of sequential modeling. Sequential modeling can also be useful for regulatory compliance. For example, California's Proposition 103 requires that safety/driving record and mileage be the greatest determinants of auto premiums. Insurers typically use sequential methods when developing their rates in California.

We will give one final example of sequential modeling before closing the paper. In this example, we will estimate first the main effects of a rating plan and then an interaction term in sequential fashion. There can be many motivations for sequential modeling strategies such as the one exemplified here. For example, perhaps the interaction factors will be used only in certain states; but the main effect factors are desired to be common across all states. Sequential modeling using an offset would be a practical way to approach such a situation. Another motivation might be that one wishes to keep the main effects model simple, without the complication of estimating an interaction term in the same step.

In this final example, we suppose we are modeling PD pure premium using the three rating variables: driver age group, multicar indicator, and pleasure use indicator. We will build an initial GLM model for these three main effects. We will next build a second model – using the first model score as an offset – to estimate the factors for a driver age/pleasure use interaction term. As discussed above, the "main effects" rating plan might be used nationally; the additional interaction factors might be implemented in selected states.

Model 3
Target Variable: PD Pure Premium (pd loss / pd exposure_adj);
Predictive Variables: DRIVER_AGE, MULTICAR, PLEASURE_USAGE
Distribution: Tweedie
Link: Log
Offset: (none)
Weight: exposure_adj;

The rating factors resulting from this model are displayed in the table below.

Table 6 – Model 3 Parameter Estimates and Pure Premium Relativities

Variable	Value	Beta	e^{beta}
MULTICAR	1	-0.26	0.77
MULTICAR	0	0.00	1.00
DRIVER_AGE	1	0.37	1.45
DRIVER_AGE	2	0.04	1.04
DRIVER_AGE	3	-0.83	0.44
DRIVER_AGE	4	0.00	1.00
PLEASURE	1	-0.36	0.70
PLEASURE	0	0.00	1.00

Let η denote the linear component of the scoring formula corresponding to the table above: $\eta = \beta_{DRIVER\_AGE} + \beta_{MULTICAR} + \beta_{PLEASURE}$. We will use η as the offset in the model for Step II of the sequence.

Model 4
Target Variable: PD Pure Premium (pd loss / pd exposure_adj);
Predictive Variables: DRIVER_AGE * PLEASURE
Distribution: Tweedie
Link: Log
Offset: η
Weight: exposure_adj;

Model 4 differs from Model 3 only in the choice of predictive variables; and the fact that we're using the linear component of the Model 3 scoring formula (ETA) as the offset. Note although exp(η) is Model 3's estimate of PD Pure Premium, we are using η, not exp(η), as the offset in Model 4 (below). This is because we are building a multiplicative model (using the log link function). Therefore the offset must be on the log scale.

Table 7 displays the rating factors resulting from Model 4.

Table 7 – Model 4 Parameter Estimates and Pure Premium Relativities

DRIVER_AGE	PLEASURE	Model 3 Estimates	Pure Premium Relativity
1	1	0.54	1.72
1	0	0.63	1.88
2	1	0.45	1.57
2	0	0.78	2.18
3	1	0.65	1.92
3	0	0.75	2.12
4	1	-0.05	0.95
4	0	0.00	1.00

In states for which Model 4's interaction factors are not used, the factors in Table 6 constitute the rating plan. In states for which the interaction is intended to be used, we must integrate the results of tables 6 and 7. This is done in tables 8 and 9:

Table 8 – Pure Premium Relativities for Type

Variable	Value	Relativity
MULTICAR	1	0.77
MULTICAR	0	1.00

Table 9 – Pure Premium Relativities for DRIVER_AGE and PLEASURE

	PLEASURE	
DRIVER AGE	PLEASURE=1	PLEASURE=0
1	1.74	2.71
2	1.63	2.26
3	0.71	0.97
4	0.90	1.00

Conclusion

The GLM offset feature is a practical and versatile tool for dealing with a variety of issues such as: data constraints, credibility issues (as in Symbol factor development), regulatory considerations (e.g. California's Proposition 103), the desire to layer a further rating, scoring, or tier model on top of an existing rating plan (credit scoring, tier factor development), and the need to add state-specific variations to a basic countrywide rating plan (sequential modeling).

Generally speaking, the offset option is helpful when omitted variable bias [OVB] threatens to distort one or more model parameters. The classic use of an offset is to incorporate a measure of exposure when modeling *rates*. For example if some records in a personal auto dataset correspond to 6-month policies while other records correspond to 12-month policies, then it is appropriate to use (log of) months of exposure as an offset. Failure to do so would raise the specter of OVB: model variables correlated with months of exposure might possibly pick up some of the variation that should be explained by months of exposure. This would result in biased parameter estimates.

Beyond this classical use, the offset option is helpful in a number of actuarial applications. For example, we have described how the offset option can be used to build GLM models subject to certain rating factor constraints; to optimize the rating factors of some, but not all, of the variables in a rating plan; and to build predictive or rating models in sequential fashion. We discussed credit scoring, tier variable creation, and state-exception sub-models as examples of actuarial pricing models built in sequential fashion.

The offset option provides actuaries with a unifying framework – encompassing such traditional techniques as exposure adjustments and loss ratio modeling as an alternative to pure premium modeling – for avoiding omitted variable bias. It is therefore appropriate to consider using an offset when performing a multivariate analysis subject to variable exclusions or other a priori constraints.

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Appendix A: Two Equivalent Ways of Modeling Frequency with Poisson Regression

Suppose we wish to model claim frequency F as a generalized linear function of several covariates $\{X_1, X_2, \ldots, X_N\}$. Let C denote the number of claims for a given policy, and u (for "units") denote number of exposures. Then: $F = C/u$.

We will demonstrate that the following two ways of modeling F are equivalent:

	Option 1	Option 2
GLM family:	Poisson	Poisson
Target:	C	F
Weight:	(none)	u
Offset:	$\log(u)$	(none)

Let us start with Option 1 and demonstrate that it is equivalent to Option 2. Let i denote the observation number. The Poisson regression assumption is that:

$$P(C_i = c_i) = \frac{e^{-\lambda_i}\lambda_i^{c_i}}{c_i!}$$

Where

$$\lambda_i = \exp(\log(u) + \alpha + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_N X_N)$$

Note that if we let:

$$\mu_i = \exp(\alpha + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_N X_N)$$

Then we have the relationship: $\lambda_i = u * \mu_i$.

The log-likelihood function for the Option 1 Poisson regression model is:

$$l(c \mid \alpha, \beta) = \sum_i \log\left(\frac{e^{-\lambda_i}\lambda_i^{c_i}}{c_i!}\right) = \sum_i -\lambda_i + c_i \log(\lambda_i) + \kappa$$

(For simplicity we are assuming that over-dispersion does not exist in the data. That is, $\varphi=1$.) We can recast the above expression in terms of F and μ:

$$\begin{aligned} l(c \mid \alpha, \beta) &= \sum_i -u_i\mu_i + c_i \log(u_i\mu_i) + \kappa = \sum_i -u_i\mu_i + c_i \log(\mu_i) + \kappa' \\ &= \sum_i u_i(-\mu_i + c_i / u_i \log(\mu_i) + \kappa') = \sum_i u_i(-\mu_i + f_i \log(\mu_i) + \kappa') \\ &= \sum_i u_i \log\left(\frac{e^{-\mu_i}\mu_i^{f_i}}{\kappa''}\right) \end{aligned}$$

In the above expressions, $\{\kappa, \kappa', \kappa''\}$ denote constants that do not depend on the model parameters. This last expression is the log-likelihood function for the Poisson regression, cast in the terms Option 2.

Appendix B: Details of the Dataset Used in Examples

The data used in this paper was simulated by Deloitte Consulting using a typical private passenger auto (PPA) rating structure. The data consists of 50,000 vehicle-level records corresponding to 24,993 single-car policies and 11,038 multi-car policies. Two coverages, Property Damage liability (PD) and Comprehensive (Comp), were simulated for each vehicle. By construction, 50% of the vehicles have exposures in both coverages, while the other 50% of the vehicles have PD exposure.

The following rating variables were simulated for each vehicle record:

Multicar indicator	{0,1}	"0" – single car "1" – multi Car
Policy age	{0,1,2,…,15}	
Driver age group	{1,2,3,4}	
Pleasure use indicator	{0,1}	"1" – Pleasure Use "0" – Not Pleasure Use
Credit score group	{0,1,2,3,4}	
Territory	{T1, T2, T3, T4}	
Vehicle symbol	{1,2,3,4,5}	
Policy-level at fault accidents	{0,1,2+}	
Policy-level not at fault accidents	{0,1,2+}	

All of these variables are treated as categorical variables in the examples described in the body of this paper.

The following target fields were also simulated for each vehicle record: PD incurred loss, PD claim count (7,414 claims, or 14%), PD exposure, Comp incurred loss, Comp claim count (6,143 claims, or 12.2%) and Comp exposure.

Appendix C: The Tweedie Compound Poisson Model and Corresponding SAS Code

Matthew Flynn, Ph.D.

Following Smyth & Jorgenson [14], section 4.1, page 11, the Tweedie Compound Poisson joint likelihood function as:

$$f(n, y; \varphi/w, \rho) = a(n, y; \varphi/w, p)\exp\left\{\frac{\omega}{\varphi} t(y, \mu, p)\right\}$$

with

$$a(n, y; \varphi/w, p) = \left\{\frac{(w/\varphi)^{\alpha+1} y^{\alpha}}{(p-1)^{\alpha}(2-p)}\right\}^{n} \frac{1}{n!\Gamma(n\alpha)y}$$

where α = (2-*p*)/(*p*-1) and .

$$t(y, \mu, p) = y\frac{\mu^{1-p}}{1-p} - \frac{\mu^{2-p}}{2-p} \quad t(y, \mu, p) = y\frac{\mu^{1-p}}{1-p} - \frac{\mu^{2-p}}{2-p}.$$

The SAS codes using "Proc NLMIXED" to fit the above likelihood function for the cross coverage tiering score example in the paper is given as follows:

```
proc nlmixed data=the_appended_dataset;
      parms p=1.5;
      bounds 1<p<2;
      eta_mu                    =                    b0                 +
      c1*(credit_grp=1)+c2*(credit_grp=2)+c3*(credit_grp=3)+c4*(credit_grp=4)
      + naf1*(naf_pol=1)+naf2*(naf_pol=2)
      + coverage_COMP*(coverage='COMP');
      mu = exp(eta_mu + current_factor);
      eta_phi = phi0 +
            phi_c1*(credit_grp=1)+                      phi_c2*(credit_grp=2)+
      phi_c3*(credit_grp=3)+    phi_c4*(credit_grp=4)+    phi_naf1*(naf_pol=1)+
      phi_naf2*(naf_pol=2)
      + phi_coverage_COMP*(coverage='COMP');
      phi = exp(eta_phi);
      n = claims;
      w = insured;
      y = pp;
    t = ((y*mu**(1 - p))/(1 - p)) - ((mu**(2 - p))/(2 - p));
    a = (2 - p)/(p - 1);
    if (n = 0) then
      loglike = (w/phi)*t;
    else
      loglike = n*((a + 1)*log(w/phi) + a*log(y) - a*log(p - 1) - log(2 - p))
                        - lgamma(n + 1) - lgamma(n*a) - log(y) + (w/phi)*t;
      model y ~ general(loglike);
      replicate adjexp;
      estimate 'p' p;
run;
```

The above codes can be broken down into the following major sections:

- First we call the Proc NlMIXED, addressing the desired input dataset:

  ```
  proc nlmixed data=the_appended_dataset;
  ```

  The PARMS statement provides a starting value for the algorithm's parameter search. Multiple starting values are allowed, as well as input from datasets (from prior model runs, for example). With some domain knowledge we anticipate this parameter to be in the neighborhood of 1.5.

  ```
  parms p=1.5;
  ```

  Parameters can also be easily restricted to ranges, such as to be positive, and here we require the estimated Tweedie power parameter to fall between one and two.

  ```
  bounds 1<p<2;
  ```

- Next we specify the linear model/predictor for the mean response. Proc NLMIXED does not have the convenient CLASS statement of some of the other regression routines, like Proc GENMOD or Proc LOGISTIC. However, the design matrix can be created "on-the-fly", so to speak, by effectively including programming statements in the Proc NLMIXED code. Here, we create dummy variables by coding the linear model with logical statements. For example, the phrase, (credit_grp=1) resolves to either true (1) or false (0) at runtime, creating our desired indicator variables to test discrete levels of right-hand side variables. As a reminder, for a GLM, the linear predictor is required to be *linear in the estimated parameters*, so non-linear effects such as high powers of covariates or splines can be accommodated.

  ```
  eta_mu                    =                    b0                    +
  c1*(credit_grp=1)+c2*(credit_grp=2)+c3*(credit_grp=3)+c4*(credit_grp=4)
  + naf1*(naf_pol=1)+naf2*(naf_pol=2)
  + coverage_COMP*(coverage='COMP');
  ```

- Next we create a log link that maps the linear predictor to the mean response. That log link on the left hand side, becomes an exponential as the inverse link (on the right-hand side).

  ```
  mu = exp(eta_mu + current_factor);
  ```

- A great feature of using Proc NLMIXED is its flexibility. Here we are specifying what Smyth & Jorgenson [13] refer to as a double GLM. Instead of a single constant dispersion constant, we can fit an entire second linear model with log link for the dispersion factor.

  ```
  eta_phi = phi0 +
  phi_c1*(credit_grp=1)+   phi_c2*(credit_grp=2)+   phi_c3*(credit_grp=3)+
  phi_c4*(credit_grp=4)+ phi_naf1*(naf_pol=1)+ phi_naf2*(naf_pol=2)
  + phi_coverage_COMP*(coverage='COMP');

  phi = exp(eta_phi);
  ```

- Proc NLMIXED allows a number of datastep style programming statements. Here we are assigning input dataset variables claims, insured, and pp as new variables (n, w and y) to be used subsequently in building out our likelihood equation. That way, one can easily adapt pre-existing code to a particular input dataset, without requiring modifications to the "guts" of the log-likelihood equation (it is complicated enough already).

```
n = claims;
w = insured;
y = pp;
```

- Now one can begin to specify the loglikelihood. Here, for clarity, we build it out in several steps. Simply refer to the Tweedie Compound Poisson likelihood described above from Smyth & Jorgensen [13], and lay it out.

```
t = ((y*mu**(1 - p))/(1 - p)) - ((mu**(2 - p))/(2 - p));

a = (2 - p)/(p - 1);

if (n = 0) then
      loglike = (w/phi)*t;
else
      loglike = n*((a + 1)*log(w/phi) + a*log(y) - a*log(p - 1) - log(2
      - p)) - lgamma(n + 1) - lgamma(n*a) - log(y) + (w/phi)*t;
```

Proc NLMIXED includes several pre-specified likelihoods, for example, Poisson and Gamma, the GENERAL specification allows the great flexibility to specify one's desired model specification.

```
model y ~ general(loglike);
```

Weights can either be included directly in the loglikelihood above, or with the handy REPLICATE statement. Each input record in the dataset represents an amount represented by the input variable "adjexp".

```
replicate adjexp;
```

The ESTIMATE statement can easily calculate and report a variety of desired statistics from one's model estimation. Here, we are interested in the Tweedie Power parameter.

```
estimate 'p' p;
```

Without using any of the additional "Mixed" modeling power, Proc NLMIXED performs as a great Maximum Likelihood Estimator using a variety of numeric integration techniques.

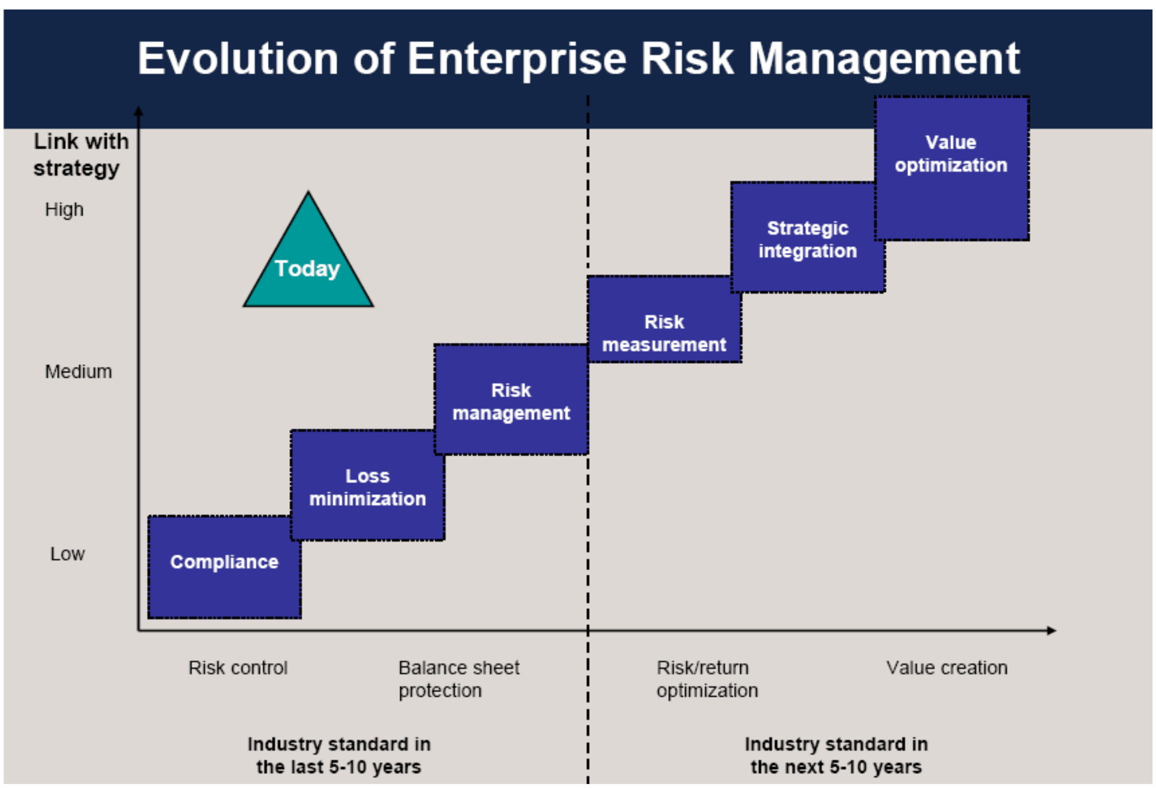
ERM and Actuaries

David Ingram

Enterprise Risk Management is both new and old. Actuaries have always been highly involved in risk management and they have much to learn about the new field of Enterprise Risk Management. These seemingly contradictory statements are the subject of this paper. This paper is an attempt to create a context for the understanding of ERM that can identify both the historic and new roles for actuaries and other risk professionals in managing risks for any type of firm.

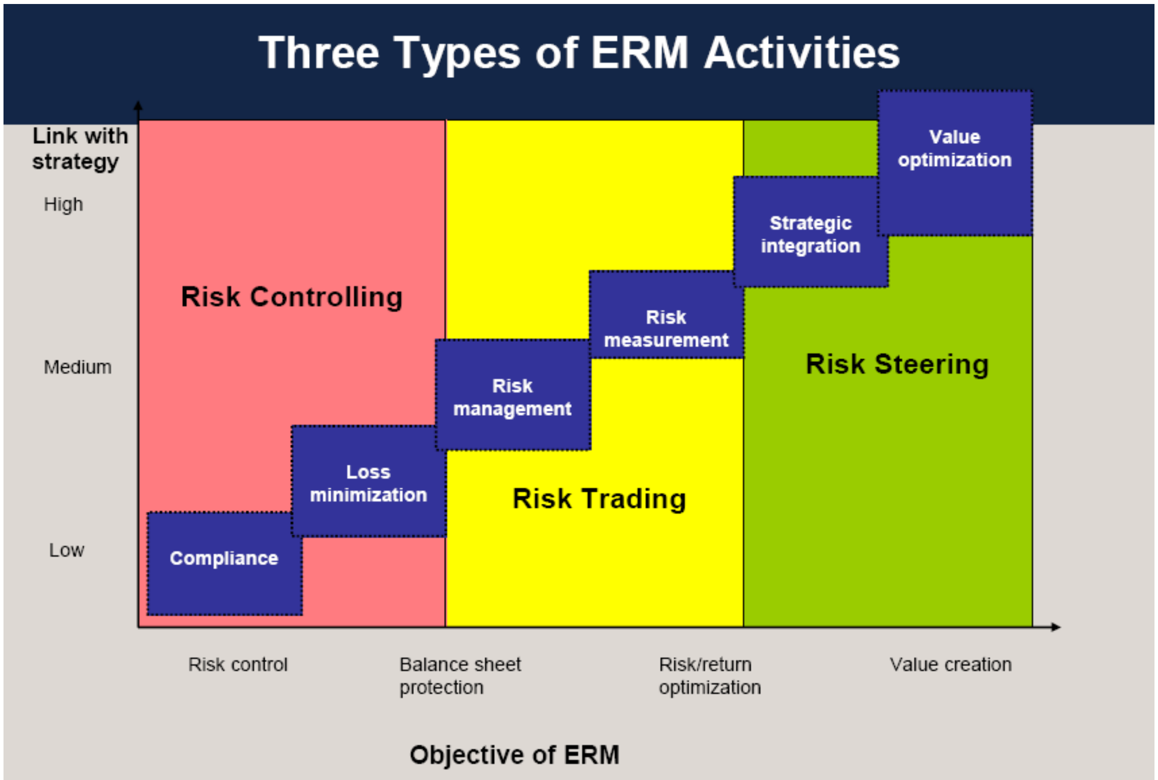
THREE TYPES OF ERM

The activities of Enterprise Risk Management can be grouped into three broad areas. Those are Risk Controlling, Risk Trading and Risk Steering. Some discussions of these three types of risk management present them as a progression for less advanced to more advanced activity.



This chart (adapted from Mark Puccia Standard & Poor's presentation) shows a progression from risk management programs that are risk control oriented through programs that feature risk management and measurement to ultimately evolve to value creation and optimization activities.

The three areas of risk management that are mentioned above can be seen as groupings of the six boxes on this stairway.



These three types of ERM activities have different objectives and different firms have applied these types of ERM with different degrees of emphasis.

Risk Controlling is a fundamental activity that seeks to restrict exposure to potential losses or risks. Almost all business activities include some amount of risk control activity. In insurance companies, the major risk controlling activities included underwriting of insurance risks, underwriting of credit risks, authority limits and exposure limits for each of those areas. It also included internal audit and other functions for controlling operational risks. Eventually, some firms added in controls around other risks such as interest rate and equity risks using ALM and hedging as a risk control processes. In banks, the same sorts of credit and operational risk controlling activities existed. In non-financial firms, there was often a large added physical component to loss controlling. Safety and industrial engineering programs worked on physical risks. In addition, many non-financial firms have large exposure to physical property risks that are insurable. So management of an insurance program became a major risk control process. In addition, there are supply chain and raw materials risks. These are managed by a variety of techniques, including but not limited to hedging. And in all firms, managing foreign exchange and liquidity risks were practiced to varying degrees.

Most commonly, these risks were management completely in isolation by specialists in each particular risk element. This is the most traditional picture of risk management. The new part of risk controlling that ERM brings is the possibility of bringing all of these risks to the same table, looking at them on some comparable basis and determining the degree to which a firms wants to retain or reduce exposure to risks on a consistent basis from a top down point of view.

Actuarial involvement in risk controlling has traditionally been limited. Actuaries have worked as underwriters and as investment managers in insurers but that has mostly been stepping outside of the actuarial role into another specialty. Actuaries have taken more prominent roles in development and monitoring of limits in areas such as cat risk, where the measurement depended on complex models. Those models also require collaboration with scientists who are specialists in the specific catastrophes. Actuaries play a major role in ALM and hedging of insurance exposures there also usually in collaboration with specialists in the investments that make up half of the equation.

The new role for actuaries in risk controlling is the development, maintenance and interpretation of comprehensive risk models that can be used to bring all risks onto the same basis for top level discussion and ultimately the determination of overall risk tolerance and decisions on which risks to retain to use up that tolerance. The first awakening from this process is the first time glance at the actual risk profile of the firm and the realization that some risks are managed very tightly while others are quite the opposite. Once that discussion has been held, actuaries than also have a role to help to translate that risk tolerance back into measures that are more familiar to managers in each risk area.

Risk controlling is the area of ERM that is addressed by COSO and AS/NZ Risk Management Standard. It is often dismissed by actuaries as being of relatively little importance or not even a part of ERM. However, the risk management activities of most non-financial firms fall largely into the area of risk controlling.

When actuaries look at potential roles outside of financial services for ERM activities, there needs to be recognition of the large amount of physical risk that is present in many non-financial firms that actuaries are totally unprepared to help manage. These risks are most often managed by engineers or people with similar backgrounds.

Many firms will never go beyond risk controlling in their risk management programs. And that might well be the best use of management time and resources. That will depend on the major limiting factor of the firm. Financial firms are usually faced with capital as a major limiting factor and retained risk as a major driver of capital needs. So financial firms must have risk and risk management at the heart of many management discussions. For non-financial firms, risk will influence need for cash or access to cash and availability of cash and capital, but there may be other much more important limiting factors. In the recent times, more and more firms have leveraged themselves more and more creating a situation where risk and the resulting volatility in cashflow needs are now of very high importance. But there are other firms, such as the large well established technology firms that are awash in cash and have plenty of capital but have very different key limiting factors that push risk management to a lower level on the priority list which means that risk controlling is the only type of risk management that will be undertaken.

As the actuarial profession seeks to promote an "all industry" approach to professional involvement in risk management, these factors need to be taken into account. And in addition, the most likely area of actuarial involvement, for creation of comprehensive risk models needs to be developed into a practice area where an actuary could be able to learn from established methods for modeling of all risks. These methods have not been established, which makes it extremely difficult to sustain a standard of professionalism that is equivalent to traditional areas of actuarial practice.

RISK TRADING

Risk Trading is the second major type of ERM activities. Modern ERM can be traced to the trading businesses of banks. Hard lessons from uncontrolled risk trading led to the development of processes and standards for controlling the traded risks. A major element in these systems is the function of valuing, or in other words, pricing of risks. For this paper, all activities that include the deliberate acquisition of risks for the purpose of making a profit by management of a pool of risks to be risk trading. With that definition, insurance and reinsurance companies can be seen to be pure risk trading firms. And actuaries are at the heart of this activity as major players in the pricing and valuation of insurance risks. With this way of organizing risk management activities, it is clear that most actuarial is and has always been risk management. In fact, as usually boasted, actuaries probably have over 100 years more experience in risk management than any others active in this part of the field of risk management.

ERM changes the risk pricing by introducing a consistent view of pricing of risk margins across all risks. For actuaries and insurance products this has taken the form of economic capital and cost of capital pricing.

But the activity in this area that has developed in banking and that has, until very recently, been driving regulatory thinking has gone in different directions than actuarial risk trading. Actuaries have entered into parallel activities, but most often with totally different objectives.

At a fundamental level, actuarial practice has been organized along the basic insurance principle of diversification. Risk pricing for insurers has been a compromise between the cost of risk to the customer with a highly specific risk and the cost of risk to the insurer with a diversified pool of risks.

The banking approach to risk trading has been that of replication. It is an extension of the fundamental activity of trading away market inefficiencies of prices between different markets. Risks are taken and offset. Banks keep the difference between one version of a risk and a replicating position based on other securities. Different risk trading strategies employ different degrees of replication. The closer the strategy is to exact replication, usually the lower the margin per trade. For example, Long Term Capital Management used a trading strategy firms developed at Solomon Brothers that offset positions in 20 year treasuries with positions in 19 year treasuries. Other risk trading employed by banks is the so called "carry trade" where the banks borrow funds in a currency with very low interest rates (most often Japan in recent years) and lend those same funds in a market with higher interest rates, usually for a much longer term.

One of the underlying principles of bank trading risk management is liquid markets for risk. Another major principle is the atomization of risk and the dispersal of risk. This is seen as a solution to a classic bank problem of risk concentration. Historically, banks operated locally and had very highly concentrated loan portfolios that focused on their local businesses. Atomization of risk, usually referred to as securitization, allows for drastic reduction in the risk concentration of banks.

Actuaries use very similar techniques to bank risk trading when they do ALM and hedging. However, similar seeming activities are classified as risk controlling for insurers and trading for banks. This is because in most cases, banks do risk trading to achieve profits while insurers do ALM and hedging to reduce or eliminate risk. This is fairly arbitrary and the reader can decide that form of activity, rather than objective of activity is the more important classification criteria.

It is important note that bankers and bank regulators have been very vocally promoting the risk management practices that have been devised for their risk trading as THE platform for all risk management. In fact, they have struggled mightily to force the credit and operational risks of banks into this framework. Meanwhile, insurers and insurance regulators have also been drawn into this path as well.

The idea is that trading of all risks will enhance the risk management of all risks. And even if there is no trading of a risk, that the best course of action for risks that are not traded is to pretend that they are traded.

This approach has caused the management of many, many non-financial firms to conclude that risk management does not apply to them. They would contend that their risks do not even slightly fit into the risk trading model. This discussion is intended to show how they fit into ERM and this author heartily agrees that most risks will not ever fit into a risk trading model.

Recent events perhaps call into question the entire risk trading model of banks. The idea that inexact replication can be a reliable activity can only be supported if it is believed that the degree of residual risk can be determined reliably. However, there have been repeated market events that show that periodically, the small and moderate amounts of expected residual risk can become mammoth when economic activity falls into a transition period between predictable regimes.

Another major difference between the actuarial approach to risk pricing and the banking approach is the reference basis. Actuarial pricing and valuation tends to reference the cashflows in a fundamental analysis approach to risk. Banking pricing and valuation tend to reference other prices.

In recent years, actuaries have been heavily criticized and self critical because of the failure to sufficiently reference market prices. One problem for actuarial valuations has been the treatment of market risk margins. When actuaries cannot find a cahsflow basis for risk margins, those margins are treated as any other cashflows and are then a positive contribution to value. This leads to the illogical result that an investment with a large risk margin that was purchased for $100 would be considered to be worth $120 immediately by the actuarial valuation.

In addition, and even more important to a discussion of risk management, actuaries were valuing risky options that were implicitly granted inside of an insurance contract at values much, much lower than the price of replicating options purchased in the financial markets. In some cases, actuaries were underestimating both the expected losses from the options and the risk margins.

However, the recent credit crisis shows that market evaluations of risk margins are not perfect either. In the period preceding the sub prime meltdown, market based margins for risk for many financial instruments were at or near historical lows. A fundamental evaluation of the potential risk would have suggested that the market was not paying appropriately for taking risks.

Risk trading is also done by some of the non-financial firms that have built trading platforms to support hedging of their raw materials costs. These firms seek to get profits from the insights into the movements in market prices and their knowledge of the actual activities related to the underlying goods. Enron was the largest proponent of this activity. Many other firms have continued in these practices after the demise of Enron, but in a much more controlled fashion.

RISK STEERING

Management has always looked to choose strategies that enhance firm value. ERM provides an entire new and more quantitative approach to this high level activity.

At the macro level, management will leverage the risk and reward information that comes from the ERM systems to optimize the risk reward mix of the entire portfolio of insurance and investment risks that they hold. Proposals to grow or shrink parts of the business and choices to offset or transfer different major portions of the total risk positions can be viewed in terms of risk adjusted return. This can be done as part of a capital budgeting / strategic resource allocation exercize and can be incorporated into regular decision making. Some firms bring this approach into consideration only for major ad hoc decisions on acquisitions or divestitures and some use it all of the time.

There are several common activities that may support the macro level risk exploitation:

Economic Capital. Realistic risk capital for the actual risks of the company is calculated for all risks and adjustments are made for the imperfect correlation of the risks. Identification of the highest concentration of risk as well as the risks with lower correlation to those higher concentration risks is the risk information that can be exploited. Insurers will find that they have a competitive advantage in adding risks to those areas with lower correlation to their largest risks. Insurers should be careful to charge something above their "average" risk margin for risks that are highly correlated to their largest risks. In fact, at the macro level as with the micro level, much of the exploitation results from moving away from averages to specific values for sub classes.

Risk Adjusted Product Pricing. Product pricing reflects the cost of capital associated with the economic capital of the product as well as volatility of expected income. Product profit projections show the pure profit as well as the return for risk of the product. Risk adjusted value added is another way of approaching this that has the advantage that it does not favor shrinkage of the business as a rate driven risk adjusted rate of return does.

Capital Budgeting. The capital needed to fulfill proposed business plans is projected based on the economic capital associated with the plans. Acceptance of strategic plans includes consideration of these capital needs and the returns associated with the capital that will be used. Risk exploitation as described above is one of the ways to optimize the use of capital over the planning period.

Risk Adjusted Performance Measurement (RAPM). Financial results of business plans are measured on a risk-adjusted basis. This includes recognition of the economic capital that is necessary to support each business as well as the risk premiums and loss reserves for multi-period risks such as credit losses or casualty coverages.

Risk Adjusted Compensation. An incentive system that is tied to the risk exploitation principles is usually needed to focus attention away from other non-risk adjusted performance targets such as sales or profits. In some cases, the strategic choice with the best risk adjusted value might have lower expected profits with lower volatility. That will be opposed strongly by managers with purely profit related incentives. Those with purely sales based incentives might find that it is much easier to sell the products with the worst risk adjusted returns. A risk adjusted compensation situation creates the incentives to sell the products with the best risk adjusted returns.

A fully operational risk steering program will position a firm in a broad sense similarly to an auto insurance provider with respect to competitors. There, the history of the business for the past 10 years has been an arms race to create finer and finer pricing/underwriting classes. As an example, think of the underwriting/pricing class of drivers with brown eyes. In a commodity situation where everyone uses brown eyes to define the same pricing/underwriting class, the claims cost will be seen by all to be the same at $200. However, if the Izquierdo Insurance Company notices that the claims costs for left-handed, brown-eyed drivers are 25% lower than for left handed drivers, and then they can divide the pricing/underwriting into two groups. They can charge a lower rate for that class and a higher rate for the right handed drivers. Their competitors will generally lose all of their left handed customers to Izquierdo, and keep the right handed customers. Izquierdo will had a group of insureds with adequate rates, while their competitors might end up with inadequate rates because they expected some of the left-handed people in their group and got few. Their average claims costs go up and their rates may be inadequate. So Izquierdo has exploited their knowledge of risk to bifurcate the class, get good business and put their competitors in a tough spot.

Risk Steering can be seen as a process for finding and choosing the businesses with the better risk adjusted returns to emphasize in firm strategic plans. Their competitors will find that their path of least resistance will be the businesses with lower returns or higher risks.

JP Morgan in the current environment is showing the extreme advantage of macro risk exploitation. In the subprime driven severe market situation, JP Morgan has experienced lower losses than other institutions and in fact has emerged so strong on a relative basis that they have been able to purchase several other major financial institutions when their value was severely distressed. And by the way, JP Morgan was the firm that first popularized VaR in the early 1990's, leading the way to the development of modern ERM. However, very few banks have taken this approach. Most banks have chosen to keep their risk information and risk management local within their risk silos.

Actuaries play a key role in providing the information for risk steering with economic capital and return/value modeling.

This is very much an emerging field for non-financial firms and may prove to be of lower value to them because of the very real possibility that risk and capital is not the almost sole constraint on their operations that it is within financial firms as discussed above.

Implications for Actuaries

This framework shows how much actuarial work can be seen as risk management.

It also shows very briefly summarizes the differences between traditional actuarial work and

ERM.

The framework shows the ERM work that is not now performed by actuaries, the risk management work that is not performed by actuaries and the work that arises from a shift to ERM that could be undertaken by actuaries.

Actuaries need to consider how the profession should relate to the areas of risk management where actuaries are not primary players but have related roles. These areas exist within insurance and include some of the largest risks such as equity risk, credit risk, underwriting and operational risk.

Actuaries need to consider how the profession should relate to areas of risk management where actuaries currently have no current connection.

	Banks	Insurers	Non Financial	Main Idea
Risk Controlling	– Basel II – has risk controlling focus, but thinking is trading risk based. – Credit Risk management has traditionally fallen here, but with advent of credit trading (CDS) has been moving it into Risk Trading. – ORM should be in this box, trading risk focus of banks forces ORM into a trading risk approach.	– Solvency 2– has risk controlling focus, but thinking is copy of banking risk trading approach. – RBC – Underwriting Standards – Investment Policies – ORM is developing – ALM & Hedging – Reinsurance	COSO Cost benefit approach to risk management. Insurance is primarily a risk controlling tool.	To keep risk within tolerance. To limit potential losses. Main tools are underwriting/risk selection and loss control activities to reduce frequency & severity of unavoidable losses. Works with gross positions. Starts as silos. Eventually in aggregate.
Risk Trading	– Market Risk of trading books – Valuation models – Focus is solely on getting market price and volatility of market price. – Usually no fundamental analysis of risks	– Most insurers have buy and hold approach to risk – Some do trading of market risk for profit – Pricing of many insurance products – Valuation of insurance risks – Initially based solely on fundamental analysis of risks – Shifting to market price of risk	– Applies only to hedging of raw materials and prices of products. – Non-Financials not familiar with controlling of trading risks leading to frequent mismatches between risk appetite and hedging positions.	To get the prices right on risks to make trade-offs. Starts in Silos. Eventually consistent across risks. To limit net positions. Tools are risk structuring & risk trading. Counterparty risk becomes key.
Risk Steering	Not usually A few banks use RAROC With risk trading view Risk Steering doesn't make sense, since positions change constantly. All risk management is seen to be tactical.	Some trying to use EC & RAROC or EC & cost of capital in Embedded value	Not usually considered by non-financials Rarely is there any clearly articulated risk reward trade-off standards.	To balance aggregate risks & understand aggregations and diversifications in the businesses in order to improve the spread for risk and the return for net firm-wide risk.
Comments	Banks started with Risk Trading and they are trying to force all of their ERM activity into a trading framework	Insurers try to copy bank ERM, but it doesn't translate well because they are predominantly buy & hold risk takers. May cause insurers to shift to more risk trading.	Non-Financials protest the application of trading centered bank ERM ideas to their businesses where very few of the risks are traded.	

Large Scale Analysis of Persistency and Renewal Discounts for Property and Casualty Insurance

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Abstract

In this paper, we study the issue of whether a price discount for renewal business is warranted for property and casualty insurance. The discount is motivated by the fact that new business with insurance coverage lapse, or new business in general, may perform worse than renewal business. The study is based on a total of 25 books of insurance business with a total amount of almost $29 billion of premium. The data cover all the primary property and casualty lines of business, including personal Auto and Homeowners as well as commercial Business Owners Policies, Auto, WC, GL and Property. The data do indicate that new business universally has a higher loss ratio and a lower retention rate than renewal business across all the 25 books of business. We will attempt to offer reasons as to why such difference exists between new and renewal business for insurance.

Keywords: Persistency discount, renewal discount, loss ratio, retention.

1. INTRODUCTION

It may be known to the property and casualty insurance industry that new business possesses higher risk than renewal business. Stable and persisting insureds are generally bringing in more profits to insurers, while insureds who frequently switch from one carrier to another are usually poor risks [1]. For example, the research report by Conning [2] indicates that new business loss ratios can vary from 10% higher to more than 30% higher than renewal business, depending on the line of business and underwriting cycle. As a result, the industry may want to surcharge new business or award discounts to their renewal business.

One primary principle for insurance pricing is that "*A rate is an estimate of the expected value of future costs*" [3]. In other words, two risks with same characteristics should be charged the same rate. Therefore, such price differentiation between new and renewal business has caused some debate in the past because some people believed that insurance rates should not be unfairly discriminatory due to the length of an insured staying with a carrier. For example, in California, over the last decade, the new business surcharge or persistent discount debate has been one of the insurance regulation focuses. California regulators once barred automobile insurance companies from levying surcharges against new customers who drove without coverage [4]. After this bar was lifted later, consumer advocate groups also filed separate lawsuits against companies who use a customer's lack of prior insurance as a factor in determining premiums [5]. On the other hand, the insurance industry did propose to allow drivers who renew coverage with their current insurer to receive discounts on basis of the argument that actuarial evidence shows drivers who maintain insurance for an extended period of time with an insurer have lower loss costs [6].

Different states may have different regulations on such a new business surcharge or renewal business discount [7]. We conducted a survey with the department of insurance in various states. Regulations for some states are silent on the topic, while other states do not prohibit price differentiation as long as insurers can provide support for the discount. The survey result suggests that most of the states appear to allow the price differentiation. Table 1 in appendix summarizes the highlight of the responses from the departments of insurance we contacted.

There is a difference between persistency discount and renewal discount. The persistency discount rewards a lower rate to new business without prior insurance lapse. Hence, the discount essentially implies a surcharge to new business with insurance lapse. On the other hand, the renewal discount results in a lower rate for renewal business. Therefore, the renewal discount implies a surcharge to new business as a whole. Since insurance companies in general do not capture data well that can allow us to differentiation new business with or without prior insurance, our study focuses on the total new business.

Setting aside public policy and regulation considerations, the key actuarial and rating questions for the issue are:

- Is it true that new business in general performs worse than renewal business?
- If yes, what are the reasons for such a difference?

Several published studies before have noted that renewal business in general exhibits continuing improvement in loss ratio as the business has stayed with the same insurer for multiple terms [1,2, 8-10]. One study further attributes such improvement to the fact that as an insured stays longer with the same insurer, the insurer is able to obtain more information about the insured, including a verified loss history, the condition of the insured property and the degree of cooperation by the insured in settling claims [8]. This enhanced information about the insured enables the insurer to select desirable risks and thus improve the performance of its book. A persisting insured could also provide income over multiple terms and spread the acquisition cost and other underwriting costs over a long period of time to achieve lower average expenses per year, which provides savings to the insurer in addition to the improvement in loss ratio.

While the issue has a long history and several studies were published before on the issue, we believe that additional research, especially a study that utilizes the real industry data, can be done to help the industry gain a better understanding of it. Through our work on data mining and predictive modeling in past several years, we have studied a fairly large amount of data from a wide range of insurers. The data enables us to review the performance difference between new and renewal business in detail. In this paper, we will share our findings and knowledge on this issue based on our experience with the industry data in the past. In addition, we will bring in macroeconomic data for insurance exposures on drivers, vehicles, property, and business [11-13] as well as insurance industry data from AM Best [14] to compare with our finding. We believe by putting all the information and data together, we can offer in-depth insights on why new business and renewal business perform differently for the property and casualty insurance.

2. DATA

We have studied a total of 25 books of business with a total amount of premium of $29 billion. The 25 different books are from a wide range of carriers, including national, multi-line carriers as well as regional, mono-line carriers, and they cover all the major primary lines of business for property and casualty insurance, including personal Auto and Homeowners, as well as commercial BOP, Auto, Property, GL, Package, and Workers' Compensation. The data as a whole spans across the last underwriting cycle from late 1990 to mid 2000. Tables 2 in the appendix shows some details of the data used in this paper. Tables 3-5 shows the performance difference in several characteristics between new business and renewal business for these 25 books.

In addition, Tables 6-9 show the historical macroeconomic data for the drivers, vehicles, homes and businesses [11-13]. The data indicates the underlying exposure information for the U.S. property and casualty insurance industry. Finally, Table 10 shows the historical industry premium data for different lines of business from AM Best [14].

3. RESULTS AND DISCUSSION

Table 3 indicates that new business show a higher loss than renewal business. The data further indicates that all of the 25 books of business under study show such result. On average, the new business loss ratio is 13 points worse than the renewal business. The fact that new business has a higher loss ratio than renewal business is the primary reason why insurance companies are interested in offering a price discount for their renewal business.

Our experience further indicates that as the renewal business continues to age, the loss ratio will continue to improve. The renewal business' loss ratio will be close to the overall average loss ratio around 3 to 5 years after the business is on the book. In other words, insurance carries need to invest a couple years on a new business before the business turns into profit. It also suggests that long-time, loyal customers bring in the highest share of profit for the carriers. Such loss ratio-policy age pattern we have seen in our data is consistent with the study result by D'Arcy [8].

Another result given in Table 3 is that new business appears to have a higher turnover rate than renewal business. Similar to the loss ratio result, all of the 25 books are showing a lower retention rate for the new business. On average, renewal business has a 6 point higher retention rate than new business.

In general, there are three reasons why an insured is not retained by a carrier. First, the insured's exposure stops to exist, for example, termination of business operation or discontinued ownership of a car or a property. The second reason is because the insured voluntarily switches insurance from one carrier to another. Multiple factors may trigger an insured to switch its carrier, and they may include price shopping, dissatisfaction of the service, agent's action etc., to name a few. The third reason is because the carrier terminates the policy due to its own action. For example, insurer carriers always take underwriting action to manage the poor risks on their book, and the action may includes terminating the insurance contract, raising the price, limiting the coverage, restricting the selection of payment plan, etc. Such underwriting action inevitably will result in some risks leaving the carrier to seek another carrier. We can expect that the latter two reasons, insured's voluntary switch from one carrier to another and the action by insurance carriers, are the primary reasons for the fact that new business has a lower retention rate than renewal business. Later, we will bring in additional macroeconomic data and other insurance statistics to further explain the retention difference between new and renewal business.

While Table 3 clearly indicates that loss ratio for new business is worse than renewal business, it may not support the fact that new business has more risk or higher pure premium than renewal business. This is because insurance companies may need to offer low, competitive price in the market place in order to compete for new business. However, for the data used in this study, it is not possible to compare pure premium between new and renewal business. Therefore, we have come up with another analysis to address this issue and question, and the result is given in Table 4.

For personal insurance, the rate is less flexible, so it is hard to manipulate price to compete for new business. On the other hand, the price for commercial insurance is fairly flexible because typical commercial line pricing contains several subjective and flexible price components. Commercial carriers can apply these flexible components compete for new business price. One commonly used flexible price component is scheduled credit/debit or individual risk modification factor, IRPM. Analyzing how commercial insurance carriers apply scheduled credits and debits will allow us to understand their pricing strategy in the market place for new business. In Table 4, we show, by the major commercial lines of business, the average percentage of policies receiving credits vs. debits between new and renewal business. Table 4 indicate that, while the result is somewhat mixed for policies receiving credits, the new business appears to receive less debits than renewal business. The result does suggest that insurance companies may charge less for new business than renewal in order to compete for new business. Such pricing strategy for competing new business may partially contribute to the fact that new business has a higher loss ratio than renewal business. However, the magnitude of credit and debit difference in Table 4 does not seem to be large enough to account for the loss ratio difference in Table 3.

The next analysis we have performed is that we selected 3 books from Table 2, all of them commercial, and for each book, and we split the data into 2 groups. One group contains the risks that were retained for the next term with the same insurer, while the other group contains the risks which were not retained. Then, between the two groups, we measure and compare two characteristics: loss ratio and business financial credit score. The result is given in Table 5.

The first characteristic for comparison is loss ratio. We find that the group which was retained for the next term has a lower loss ratio than the group which was not retained. This suggests that insurance companies appear to retain more of their "profitable business" than their "unprofitable business".

In addition to loss ratio, we also compare a financial credit score between the 2 groups. The financial credit score data we use is developed by Dunn and Bradstreet. The score is a measurement of the likelihood for a business to fulfill its future financial obligation, such as payment on time. The score we use for comparison has a scale of 1 to 100, and the higher the score, the better the financial condition. Table 5 shows, again, a better average credit score for business retained than business not retained.

From the loss ratio and credit score comparison, we can see that the quality of the retained business is better than the quality of non-retained business. This is consistent with the fact that insurance companies do take underwriting action to manage poorer risks on their books. It also suggests that as the non-retained business becomes new business for another carrier, the quality of the new business is worse the renewal business for the carrier.

Another result given in Table 3 is that on average, the new business accounts for 20% of the total business for the 25 books under study. We can expect that an insurer's new business should compose two different portions of risks. The first portion is the first-time insurance buyers, for example, first time drivers with a new drivers' license, a new vehicles, a first-time home owner or property owner, or a newly established business or property that need insurance coverage, etc. In other words, from the perspective of the insurance industry as whole, this portion of risks is the "true" new business. The second portion is the risks which did not renew their insurance with prior insurance carriers. In other words, while they are "new business" for the insurer, the business is from other carrier's renewal book. .

In order to research the two compositions of the new business, we bring in additional macroeconomic and insurance data. Tables 6 to 9 show the 20 years of statistics, from 1986 to 2006, for drivers, vehicles, homes, property, and business in the U.S. The statistics indicates the underlying exposure information for the overall US property and casualty insurance industry, and it shows that the growth rate in the overall exposure is fairly minor, much less than the average of "20%" new business for the 25 books under study.

In addition, Table 10 shows 10-year history of premium dollars for the personal lines, commercial property lines and commercial casualty lines combined from AM Best. Again, the total industry premium growth rate over the last 10 years has been very mild and is less than the average new business percentage for the insurance data used in this study.

Another fact about insurance carriers in accepting and underwriting their new business is that typically they are tougher on the "truly new exposures", such as newly established businesses or drivers who just obtained their driver licenses. For example, to our knowledge, many commercial line carriers will not accept a commercial risk with less than 3 years of history, or if they accept, they will apply their higher priced company or restrict their schedule credits. Therefore, many commercial line carriers have very few first time established businesses on their books. Similar experience can be applied to personal auto carriers, whose books typically have very few first-time youthful drivers.

From the macroeconomic statistics for the overall industry exposure data, the total industry premium data, and the standard insurance industry practice on accepting new business; we can conclude that the majority of an insurance company's new business comes from other insurance company's renewal business, and not from the truly new business as a first time insurance buyer.

Let us put together the performance comparison results and the industry exposure information from Table 3 to Table 10, and we can then begin to describe the dynamic process of new and renewal business for insurance companies. Such a dynamic cycle can make us understand why there is a difference in performance between new and renewal businesses.

Insurance companies constantly trade and swap risks between themselves. Most of the new business for an insurance company comes from other insurance companies' renewal book. Since every insurance company underwrites its book and takes action against the poorly performing risks, one reason for insureds to leave their carriers and seek insurance for another company is due to the result of the underwriting action by the existing company, such as non-renewal or increase in renewal price. Of course, they may also voluntarily change insurance carriers due to a wide range of other reasons, such as shopping for cheaper rates or not being satisfied with their carriers for service. No matter what the reasons are for insureds to leave their insurance carriers, our study shows that overall, they possess worse characteristics, such as higher loss ratios or worse credit scores, than the insureds who stay and renew their policies with their existing insurance carriers. After leaving the existing insurance carriers, they most likely become another company's new business, unless their exposure stops to exist. Since the new business in general possesses poorer risk characteristics, our study shows that for all the 25 different books of data under study, the new business' performance for loss ratio and retention is universally worse than the renewal business. Sometimes, insurance carriers' business strategy of using flexible pricing components to compete for new business will worsen new business' loss ratio even more. Such a dynamic cycle suggests that renewal business is subsidizing new business for the property and casualty insurance. It is due to such differences in loss ratio, retention, and risk quality that the insurance industry is interested in deploying a price difference in their rating between new and renewal business.

4. CONCLUSIONS

We believe that the data underlying this research is very credible and can represent the general result for the property and casualty insurance industry. Our study clearly shows that for property and casualty insurance carriers, the new business performance is worse than the renewal business. The new business appears to have higher loss ratios and worse retention than renewal business. Our experience further indicates that as renewal business ages, its performance will become even better.

We believe that the reasons why new business performance is worse than the renewal business is two fold: (1) The first time insurance buyers are less experienced in dealing with managing their insurance risks, and (2) Those who are not the first time insurance buyers but seek new insurance carriers, typically have worse risk characteristics and may be price shoppers. Actuarially, new business surcharges or renewal business discounts appear to be justifiable by the data in this study.

While we believe that new business surcharge or renewal business discount can be justified, there is still an issue: if a new risk and a renewal risk are the same in their characteristics, why can they be charged differently just because one risk is a new business and the other one is a renewal business? One key reason is because insurance carriers have more knowledge of their renewal business than their new business.

When a risk has been with a carrier for several years, the carrier will know the risk's loss experience with the carrier. The carrier also knows many other details about the risk, such as its premium paying history, its coverage change and endorsement records, etc. When the risk leaves the carrier and become a new business to another company, some of the important information may not be known to the new company because such information is not captured during the new business writing and binding process. Even if the new company does collect some of the information, it is in a way that is not verifiable or can be manipulated by the insured. Also, for writing new business, there is a balance of gathering more information verse "ease to do business". Gathering too much information when writing new business may cause undue burden on agents or brokers.

For example, for commercial insurance, while many insurance companies will ask for prior loss runs for new business and will use the loss run to underwrite the new business, the data on the loss run typically is not passed to the data system and therefore is not captured in the pricing database. Therefore, prior loss history of a new business is subsequently lost after the new business is written. Unless the insurance industry enhances its information gathering practice and collect much more information for new business underwriting and pricing, the industry probably will continue to experience worse performance for their new business than their renewal business.

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Appendices

Table 1: Summary of the Survey Responses on Price Differentiation between New and Renewal Business

State	Response from the Department of Insurance
AZ	There should be no difference in the premium that is charged between new business and renewal business if all the risk characteristics are the same
CA	Persistency is a permitted rating factor for personal auto in California
FL	It would be very unusual for companies to file a different price for new versus renewal.
IL	We do not have a rating law in Illinois. A lot of personal lines insurers give renewal discounts. Commercial rates are not filed at all.
LA	Louisiana law does not prohibit insurance companies from offering discounts for renewal policies
MO	The rating laws do not delineate between new and renewal business, rather they speak to rates in general.
NC	Does not prohibit difference for new and renewal business.
ND	We do allow companies to file renewal discounts
NJ	The NJ regulations do not prohibit companies from charging higher premium for new business versus renewal business or offer discounts for renewal business.
NM	We do allow carriers to charge more for new policies.
NY	We do allow renewal discounts and they are heavily used. These are often tied to "claim free" discounts.
OH	If a company provides support that there is a cost difference between new and renewal business then they can reflect the difference in their rates.
OR	An insurer can charge more for new business, or offer a persistency discount, provided the difference is supported statistically.
PA	If a company has reasonable, actuarial support that demonstrates the appropriateness of "lower" rates for renewal business than for new business (i.e., lower expenses and/or lower losses), rates based upon this support would be acceptable.
TX	There isn't anything that speaks directly to new business vs. renewal business for property and casualty insurance but any price difference between the two would be subject to the rate standards in the statutes.
WA	Renewal discounts are permitted in Washington, as there is no statute or regulation prohibiting them

Table 2: Summary of the Data

Line of Business	Number of Books	Total Premium, in Billions	Data Period
BOP	4	$4.9	1995 to 2004
Commercial Package	3	$4.7	1996 to 2004
Commercial Auto	4	$3.6	1998 to 2005
General Liability	2	$1.1	1995 to 2004
Commercial Property	3	$1.7	1995 to 2002
WC	4	$3.9	1996 to 2004
Personal Auto	3	$2.0	1997 to 2005
Personal Home	2	$6.8	1997 to 2003
Total	25	$28.7	1995 to 2005

Table 3: Comparison of Loss Ratio and Retention between New Business and Renewal Business

Line of Business	Number of Books	Average % of New Business	Average of Loss Ratio Difference, New – Renewal*	Average of Retention Difference, New – Renewal*
BOP	4	19%	18%	-5%
Commercial Package	3	19%	9%	-7%
Commercial Auto	4	19%	15%	-5%
General Liability	2	22%	7%	-8%
Commercial Property	3	17%	17%	-8%
WC	4	27%	11%	-3%
Personal Auto	3	16%	12%	-3%
Personal Home	2	23%	15%	-19%
Total	25	20%	13%	-6%

* For all the 25 books under study, the loss ratio is higher and the retention is lower for the new business than the renewal business.

Table 4: Comparison of Percentage of Policies Receiving Schedule Credits or Debits between New and Renewal Business for Commercial Lines

Line of Business	Number of Books	Average Percentage of Policies Receiving Credit		Average Percentage of Policies Receiving Debit	
		New	Renewal	New	Renewal
BOP	4	15%	16%	3%	8%
Commercial Package	3	16%	18%	5%	11%
Commercial Auto	4	20%	14%	2%	9%
General Liability	2	30%	29%	12%	23%
Commercial Property	3	29%	30%	5%	12%
WC	4	7%	7%	1%	1%

Table 5: Comparison of Loss Ratio and Financial Credit Score between Retained Business and Non-Retained Business for 3 Selected Commercial Books:

Line of Business	Total Premium	Loss Ratio Difference, Non Retained - Retained	Difference in Business Financial Score, Non Retained – Retained *
BOP	$690 Millions	+4 points	-5
General Liability	$533 Millions	+4 points	-2
Commercial Property	$345 Millions	+7 points	-3

* The business financial credit score used is published by Dunn and Bradstreet. The score is on 1-100 scale, and the higher the score the better the financial credit.

Table 6: US Licensed Drivers Statistics

Year	Total Licensed Drivers	Annual Growth
1986	159,487,000	1.7%
1987	161,818,461	1.5%
1988	162,853,255	0.6%
1989	165,555,295	1.7%
1990	167,015,250	0.9%
1991	168,995,076	1.2%
1992	173,125,396	2.4%
1993	169,968,825	-1.9%
1994	175,409,447	3.2%
1995	176,634,467	0.7%
1996	179,539,340	1.6%
1997	182,709,204	1.8%
1998	184,980,177	1.2%
1999	187,170,420	1.2%
2000	190,625,023	1.9%
2001	191,275,719	0.3%
2002	194,295,633	1.6%
2003	196,165,667	1.0%
2004	198,888,912	1.4%
2005	200,548,972	0.8%
2006	202,810,438	1.1%

Source: Office of Highway Policy Information, Highway Statistics Publications

Table 7: US Motor Vehicles Statistics

Year	Private & Commercial Vehicles	Annual Growth	Publicly Owned Vehicles	Annual Growth	Total	Annual Growth
1986	172,763,183	2.4%	2,937,156	0.9%	175,700,339	2.3%
1987	175,998,790	1.9%	2,997,857	2.1%	178,909,773	1.8%
1988	181,322,995	3.0%	3,069,679	2.4%	184,392,674	3.1%
1989	184,197,489	1.6%	3,158,617	2.9%	187,356,106	1.6%
1990	185,540,912	0.7%	3,257,002	3.1%	188,797,914	0.8%
1991	184,829,525	-0.4%	3,306,944	1.5%	188,136,469	-0.4%
1992	186,960,290	1.2%	3,401,938	2.9%	190,362,228	1.2%
1993	190,642,869	2.0%	3,420,613	0.6%	194,063,482	1.9%
1994	194,531,748	2.0%	3,513,617	2.7%	198,045,365	2.1%
1995	197,941,202	1.8%	3,588,819	2.1%	201,530,021	1.8%
1996	202,713,708	2.4%	3,651,448	1.8%	206,365,156	2.4%
1997	204,079,162	0.7%	3,674,498	0.6%	207,753,660	0.7%
1998	207,840,942	1.8%	3,775,611	2.8%	211,616,553	1.9%
1999	212,474,300	2.2%	3,834,323	1.6%	216,308,623	2.2%
2000	217,566,789	2.4%	3,908,384	1.9%	221,475,173	2.4%
2001	226,646,079	4.2%	3,782,247	-3.2%	230,428,326	4.0%
2002	225,772,196	-0.4%	3,847,783	1.7%	229,619,979	-0.4%
2003	227,475,999	0.8%	3,913,999	1.7%	231,389,998	0.8%
2004	233,266,291	2.6%	3,976,325	1.6%	237,242,616	2.5%
2005	237,139,650	1.7%	4,054,324	2.0%	241,193,974	1.7%
2006	240,059,464	1.2%	4,106,222	1.3%	244,165,686	1.2%

Source: Office of Highway Policy Information, Highway Statistics Publications

Table 8: US Total Housing Inventory Statistics

Year	Estimated Total Housing (000s)	Annual Growth
1986	99,318	2.0%
1987	101,811	2.5%
1988	103,653	1.8%
1989	105,729	2.0%
1990	106,283	0.5%
1991	107,276	0.9%
1992	108,316	1.0%
1993	109,611	1.2%
1994	110,952	1.2%
1995	112,655	1.5%
1996	114,139	1.3%
1997	115,621	1.3%
1998	117,282	1.4%
1999	119,044	1.5%
2000	119,628	0.5%
2001	121,480	1.6%
2002	119,297	-1.8%
2003	120,834	1.3%
2004	122,187	1.1%
2005	123,925	1.4%
2006	126,012	1.7%

Source: US Census Bureau, Housing Vacancies and Homeownership

Table 9: US Business Statistics

Time Period	Initial Year Establishments	Percent of Net Growth
1995-1996	5,878,957	1.6%
1996-1997	5,970,420	2.5%
1997-1998	6,120,714	1.1%
1998-1999	6,187,599	1.0%
1999-2000	6,248,411	0.8%
2000-2001	6,297,423	0.8%
2001-2002	6,345,890	0.6%
2002-2003	6,386,609	1.1%
2003-2004	6,455,018	1.4%

Source: US Census Bureau, Statistics of U.S. Businesses

Table 10: AM Best Statistics for US Property& Casualty Insurance Industry

LOB	Year	Premiums earned (in $1,000)	Growth Rate
Total US Personal Lines	1996	123,722,772	
	1997	129,529,545	4.7%
	1998	134,910,240	4.2%
	1999	139,053,922	3.1%
	2000	146,442,174	5.3%
	2001	155,377,660	6.1%
	2002	171,055,476	10.1%
	2003	189,414,491	10.7%
	2004	204,074,773	7.7%
	2005	212,766,944	4.3%
	2006	217,629,797	2.3%
Total US Commercial Property	1996	5,639,304	
	1997	5,893,398	4.5%
	1998	5,937,140	0.7%
	1999	6,205,553	4.5%
	2000	6,459,054	4.1%
	2001	7,617,844	17.9%
	2002	7,528,501	-1.2%
	2003	10,110,915	34.3%
	2004	10,430,080	3.2%
	2005	11,138,340	6.8%
	2006	11,976,705	7.5%
Total US Commercial Casualty Lines	1996	104,742,557	
	1997	105,914,101	1.1%
	1998	105,305,898	-0.6%
	1999	103,930,114	-1.3%
	2000	110,111,876	6.0%
	2001	120,055,783	9.0%
	2002	141,695,628	18.0%
	2003	159,335,190	12.5%
	2004	174,887,038	9.8%
	2005	176,755,172	1.1%
	2006	181,148,749	2.5%

Source: AM Best