Consideration of Bias in Chain Ladder Estimates

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Abstract

The chain ladder method may be the most commonly used and well-known approach for estimating ultimate claims. As it is most often employed, the same development pattern is used to project each accident year and its results are generally considered by practitioners to be valid for each accident year. It is the author's contention that, under this application, the chain ladder method will produce biased projections of the ultimate claims for a single accident year. This paper identifies the sources of the bias and provides the actuary with a tool to understand and compensate for a portion of the bias.

Part 1: Notation, Properties and Relationships

This paper utilizes the following notation:

Claims

$Y_{i,j}$ The random variable representing the incremental claims for accident period i and development interval j.

For example, a triangle of incremental claims may be represented by the following:

	Development Interval					
Accident Period	1	2	3	4	5	6
1	$Y_{1,1}$	$Y_{1,2}$	$Y_{1,3}$	$Y_{1,4}$	$Y_{1,5}$	$Y_{1,6}$
2	$Y_{2,1}$	$Y_{2,2}$	$Y_{2,3}$	$Y_{2,4}$	$Y_{2,5}$	
3	$Y_{3,1}$	$Y_{3,2}$	$Y_{3,3}$	$Y_{3,4}$		
4	$Y_{4,1}$	$Y_{4,2}$	$Y_{4,3}$			
5	$Y_{5,1}$	$Y_{5,2}$				
6	$Y_{6,1}$					

$\sum_j Y_{i,j}$ Cumulative claims for accident period i as at the end of development interval j.

$\mu(y)_{i,j}$ The mean of the distribution $Y_{i,j}$.

$\varepsilon(y)_{i,j}$ The random error term for observed claims for accident period i and development interval j.

Incremental Claims Development

$F_{i,j}$ The random variable, typically referred to as the "observed incremental development factor," representing the percentage increase in cumulative claims during interval j for accident period i.

$f_{i,j}$ The quantity that actuaries will typically refer to as the "selected incremental claims development factor." We include the subscript for accident period i; however we recognize that, in practice, the selected development factor rarely differs by accident period. We also assume that this factor is determined based on an examination of $F_{i,j}$ and various arithmetic averages of those observations.

$\mu(f)_{i,j}$ The mean of the distribution $f_{i,j}$.

$\varepsilon(f)_{i,j}$ The random error term for the development factor for accident period *i* and development interval *j*.

Cumulative Claims Development

$C_{i,j}$ The quantity that actuaries will typically refer to as the "cumulative development factor" evaluated at the end of interval *j*. We include the subscript for accident period *i*; however we recognize that, in practice, the selected claims development factor rarely differs by accident period.

Projections of Ultimate Claims

U_i The random variable representing the ultimate claims for accident period *i*.

$D_{i,j}$ The development method projection of ultimate claims for accident period *i* as of the end of interval *j*.

As a result, we have the following properties and relationships:

Claims

(1.1) $E[Y_{i,j}] = \mu(y)_{i,j}$

(2.1) $Y_{i,j} = \mu(y)_{i,j} + \varepsilon(y)_{i,j}$

(3.1) $$\sum_j Y_{i,j} = \sum_j \mu_{i,j} + \sum_j \varepsilon(y)_{i,j}$$

Claims Development

(4.1) $f_{i,j}$ is an estimator of $\mu(f)_{i,}$

(5.1) $F_{i,j} = \mu(f)_{i,j} + \varepsilon(f)_{i,j}$

(6.1) $$C_{i,j} = \prod_{j+1}^{\infty} f_{i,j}$$

Estimated Ultimate Claims

(7.1) $D_{i,j}$ is an estimator of U_i

(8.1) $$D_{i,j} = C_{i,j} \times \sum_j Y_{i,j}$$

(8.2) $$D_{i,j} = C_{i,j} \times \left(\sum_j \mu_{i,j} + \sum_j \varepsilon(y)_{i,j} \right)$$

(8.3) $$D_{i,j} = C_{i,j} \times \sum_j \mu_{i,j} + C_{i,j} \times \sum_j \varepsilon(y)_{i,j}$$

(8.4) $$D_{i,j} = \prod_{j+1}^{\infty} f_{i,j} \times \sum_j \mu_{i,j} + \prod_{j+1}^{\infty} f_{i,j} \times \sum_j \varepsilon(y)_{i,j}$$

Part 2: Bias in the Chain Ladder Method

We should now recognize the following properties of the chain ladder method:

» From 3.1, we recognize that cumulative claims are a function of the expectation of incremental claims for prior periods and the cumulative observed random errors in those prior periods. That is, cumulative claims are a function of all prior observations of incremental claims. From experience, we should recognize that the incremental error terms tend to be correlated. That is, years in which claims are developing adversely or favorably tend to continue to develop in the same manner.

More specifically, through summation of the correlated incremental error terms, there is correlation between the successive observations of cumulative claims. Therefore, we should now recognize that the development factors, $F_{i,j}$, within an accident year, are correlated. As a result, they are highly unlikely to be unbiased with respect to $\mu(f)_{i,j}$ as that would require the sum of the error terms to have an expectation of 0. Although this may be true across multiple years, our experience shows that this is unlikely for a single accident period. This is demonstrated in Part 3 of this paper where we present an example that illustrates what most practitioners observe regularly: that certain accident years have "longer than average" development while others have "shorter than average" development. This occurs because of the correlation of the error terms produces actual development, $F_{i,j}$, that are consistently greater or less than the expectation of development, $\mu(f)_{i,j}$. Finally, since $F_{i,j}$ is typically used to estimate $f_{i,j}$, it is unlikely that $f_{i,j}$ is an unbiased estimate of $\mu(f)_{i,j}$.

» Equation 8.4, provides the mathematical representation of $D_{i,j}$. In order for $D_{i,j}$ to be unbiased, the underlying estimators in 8.4 must also be unbiased. The discussion above provides the rationale for $f_{i,j}$ being considered biased.

Moreover, leaving aside the issue of bias in the development factors, for the chain ladder method to be unbiased, it would require the latest diagonal of observed losses to be "all signal, no noise." This has the following important implications:

> The expectation of the sum of $\varepsilon(y)_{i,j}$, for accident year i would have to equal 0. Even if we relax this requirement and allow the sum of $\varepsilon(y)_{i,j}$ to be "small," we should know from experience and the discussion above that this is often not true.

What we should now recognize is that implementation of the chain ladder method ignores a fundamental truth of the claims emergence process, specifically:

1. the existence of correlations within an accident year, and
2. that the chain ladder method is almost certainly biased.

However, there is a method for consideration (though not elimination) of bias resulting from (1) the presence of error terms and (2) the correlation of error terms within an accident year. This method is the subject of the third part of this paper.

In the discussion above, readers should recognize that we have not yet even explored the impact of environmental factors on both $\mu(f)_{i,j}$ and $\mu(y)_{i,j}$. These factors would include unexpected inflation, changes in limits, changes in case reserving, changes in payment practices and numerous other influences. It is hoped that readers recognize that real-world influences result in the virtual impossibility that development method estimators are unbiased[1].

It is therefore incumbent on practitioners to evaluate whether its convenience is a sufficiently significant benefit to overcome its shortcomings. While this is true of other reserving methods as well, the goal of this paper is to raise the awareness of one particular shortcoming of the chain ladder method.

Part 3: Partial Correction for Bias

Correction for bias in the development factors is beyond the scope of this paper. However, we do have a mechanism for (partially) addressing the bias created by both the presence and the correlation of (cumulative) error terms (the rightmost term of Equation 8.4). These conditions have the result that individual years will experience longer (more) or shorter (less) development than that implied by the selected development pattern. Additionally, also as demonstrated in Equation 8.4, the chain ladder method indiscriminately develops both the signal and noise component of the observed claims value. To address these issues we need to (1) use a tool that separates the "signal" from the "noise" and (2) employ a methodology that tracks the impact of the correlation.

Regression is the typical tool used to isolate the signal from a series of observations of a random variable. We now turn to the question of how to apply principles of regression within the chain ladder method so as to also assess the bias created by the correlation of error terms. To do this, we should recognize that we need not apply the cumulative development factor solely to the last diagonal of the triangle. We can also apply development factors to all prior diagonals as well. We refer to this series of projected ultimate claims as the "retrospective estimates of ultimate claims."

Benefits of Regression

Use of regression in this context has multiple benefits:

(1) Fitting a regression model to the series of projected ultimate claims will (partially) differentiate between the predictive component of $D_{i,j}$ (the first term on the right side of Equation 8.4) and the "noise" (the second term on the right side of Equation 8.4). This will, in effect, reduce the impact of the error terms and therefore partially correct for the bias in the $D_{i,j}$ that results from the noise / error terms.

(2) Testing of the significance of the regression parameters will provide additional insight on the development applicable for any particular year. That is, the regression coefficient will be greater than 0 for years where the ultimate claims estimate is increasing; the regression coefficient will be less than 0 for years where the ultimate claims estimate is decreasing. More specifically, coefficients that are significant and greater than 0 would indicate that the development for a particular year was "longer" than average. Stated differently it would indicate the error terms, $\boldsymbol{\varepsilon(y)}_{i,j}$, were positive. Conversely, coefficients that are significant and less than 0 would indicate that the error terms, $\boldsymbol{\varepsilon(y)}_{i,j}$ were negative. The value of the coefficient would also be an indicator of the strength of the correlation of incremental errors.

[1] To correct for the bias resulting from changes in environmental factors, we would need to incorporate adjustment factors that would offset these biases. The author recognizes that it is likely not possible to calculate adjustment factors for all such changes regardless of the actuarial method selected. However, a frequency-severity method probably best allows for such adjustments as the parameters of that method (i.e. no of claims and value of claims) are specified at the same level of detail that the underlying changes would be expected to influence.

(3) Finally, we could also create a statistic used to measure "net bias" for the development pattern. For example, regression coefficients significant and greater than 0 would contribute +1 to this statistic and coefficients significant and less than 0 would contribute -1 to the statistic. This would allow us to measure whether our development pattern was too long or too short with respect to the claims portfolio under review.

Description of Exhibits
These calculations are demonstrated on the attached exhibits.

Exhibit 1 - The data used in this example is based on the General Liability excluding Mass Torts (combined treaty and facultative) claims data as compiled by the Reinsurance Association of America. This data is presented on Exhibit 1. We also show the selected incremental claims development factors on this exhibit. For simplicity, this presentation assumes that (1) the selected incremental factors are based on volume weighted averages and (2) there is no need for a tail factor.

Exhibit 2 – Exhibit 2 presents the <u>triangle</u> of retrospective estimate of ultimate claims. Each of the entries in the triangle is calculated as the product of the observed claims and appropriate claims development factor.

Exhibit 3 – Exhibit 3 presents the results of a regression model applied to the last five observations of the retrospective ultimate claims triangle. For simplicity we have used a linear regression model in order to conceptually demonstrate the approach. However, reader should recognize that alternative regressions (such as exponential or logarithmic) could also be used as the shape of the curve warranted.

Exhibit 4 – Exhibit 4 presents estimates of ultimate loss as fitted by the regression model.

Exhibit 5 – Exhibit 5 presents a graphical presentation of this model for accident year 1995.

Part 4: Conclusions

Readers should now realize that the chain ladder method is not simply an application of an algorithm to yield a deterministic result. Rather, it is a method that has implicit statistical underpinnings. With this knowledge, we can now turn to an evaluation of the methods from a statistical basis. With this analysis, it becomes apparent that the chain ladder as it is currently applied in practice is not unbiased. Unbiasedness is one of the qualities that we typically desire in statistical estimators – yet practitioners have (implicitly) chosen to ignore this property of the chain ladder method.

The paper then identifies the two primary sources of bias that result from the correlation of error terms in the cumulative observations of claims: (1) bias in the development factor estimators and (2) the bias created by the error terms themselves. The first is beyond the scope of the paper. For the latter, the paper provides a discussion of the use of retrospective estimates of ultimate claims and regression techniques that may be used to address the bias. However, even with these tools, we are not able to completely eliminate its impact.

Part 5: Acknowledgements

The author wishes to thank Bernard Chan, FCAS, MAAA, Katy Siu and the CAS Forum Committee for their reviews of this paper. Any errors that remain herein are the responsibility of the author. As with many research topics, the concepts presented herein are a "work-in-progress." The author would welcome your comments. Please consult the CAS member directory for contact information.

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Analyis of the Chain Ladder Method

RAA Data

GENERAL LIABILITY EXCLUDING MASS TORTS
Combined Treaty and Facultative

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
Cumulative Reported Incurred Claims																
	Evaluations in Years															
Accident Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1989	49,997	139,166	255,241	335,468	402,907	442,665	450,489	458,939	473,975	474,880	480,314	485,644	489,875	493,696	496,089	501,244
1990	70,104	201,662	302,276	416,520	477,142	519,573	556,268	576,183	579,016	592,677	592,901	599,535	600,428	601,523	608,603	
1991	79,614	208,748	335,594	426,513	483,264	543,494	560,866	565,373	575,608	572,829	582,288	590,267	594,035	593,776		
1992	56,265	190,867	322,988	440,379	509,122	566,274	601,800	618,427	628,007	641,911	648,368	653,251	661,002			
1993	68,133	199,866	350,373	447,532	543,208	602,138	632,355	656,249	676,174	684,342	682,951	694,176				
1994	68,530	241,658	436,347	582,913	658,928	732,258	784,302	820,482	829,609	836,716	836,811					
1995	69,055	253,640	423,363	579,181	699,780	776,436	820,855	856,060	896,547	927,569						
1996	102,320	295,607	505,295	711,466	873,714	963,248	1,010,448	1,047,281	1,085,850							
1997	115,360	330,745	620,759	868,440	1,049,837	1,160,867	1,230,741	1,284,842								
1998	138,160	468,526	884,001	1,214,304	1,414,986	1,593,708	1,762,403									
1999	151,311	565,163	982,390	1,385,150	1,739,563	1,971,238										
2000	178,943	562,916	1,279,815	1,739,062	2,226,265											
2001	187,203	671,424	1,324,727	1,916,512												
2002	183,601	692,642	1,377,175													
2003	149,925	494,121														
2004	96,338															
	1 - 2	2 - 3	3 - 4	4 - 5	5 - 6	6 - 7	7 - 8	8 - 9	9 - 10	10 - 11	11 - 12	12 - 13	13 - 14	14 - 15	15 - 16	16 - ult
Sel Incr. Development Factors	3.306	1.872	1.379	1.211	1.115	1.065	1.035	1.026	1.015	1.005	1.012	1.007	1.003	1.009	1.010	1.000
	1 - ult	2 - ult	3 - ult	4 - ult	5 - ult	6 - ult	7 - ult	8 - ult	9 - ult	10 - ult	11 - ult	12 - ult	13 - ult	14 - ult	15 - ult	16 - ult
Cumulative Development Factors	13.862	4.192	2.240	1.624	1.341	1.203	1.130	1.091	1.063	1.047	1.042	1.029	1.022	1.019	1.010	1.000

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(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
Retrospective Estimates of Ultimate Claims																
	Evaluations in Years															
Accident Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1989	693,043	583,443	571,744	544,952	540,379	532,392	508,957	500,749	504,030	497,308	500,331	499,851	500,628	503,141	501,244	501,244
1990	971,762	845,450	677,104	676,617	639,942	624,889	628,464	628,673	615,732	620,669	617,610	617,073	613,607	613,031	614,927	
1991	1,103,587	875,160	751,737	692,851	648,153	653,659	633,659	616,879	612,108	599,884	606,555	607,534	607,073	605,136		
1992	779,932	800,192	723,499	715,376	682,833	681,056	679,905	674,766	667,829	672,228	675,389	672,361	675,510			
1993	944,443	837,919	784,841	726,995	728,550	724,189	714,426	716,034	719,051	716,663	711,414	714,482				
1994	949,935	1,013,130	977,426	946,916	883,753	880,685	886,093	895,229	882,215	876,234	871,686					
1995	957,224	1,063,364	948,341	940,853	938,544	933,817	927,391	934,048	953,398	971,378						
1996	1,418,324	1,239,308	1,131,869	1,155,743	1,171,824	1,158,494	1,141,591	1,142,689	1,154,704							
1997	1,599,078	1,386,620	1,390,511	1,410,741	1,408,040	1,396,171	1,390,475	1,401,892								
1998	1,915,128	1,964,256	1,980,179	1,972,581	1,897,778	1,916,747	1,991,138									
1999	2,097,423	2,369,396	2,200,573	2,250,112	2,333,101	2,370,800										
2000	2,480,457	2,359,976	2,866,809	2,825,026	2,985,865											
2001	2,594,946	2,814,889	2,967,413	3,113,285												
2002	2,545,016	2,903,843	3,084,898													
2003	2,078,207	2,071,560														
2004	1,335,404															

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GENERAL LIABILITY EXCLUDING MASS TORTS
Combined Treaty and Facultative

Regression Analysis

(1) Accident Year	(2) Regression Coefficient	(3) Standard Error	(4) T-Statistics	(5) Intercept	(6) Degrees of Freedom	(7) Critical Value at 5%	(8) Significance	(9) Fitted Ultimate Claims	(10) Chain-Ladder Method	(11) Difference	(12) Percentage Difference	(13) Pattern Bias Statistic
1989	340	398	0.855	496,456	3	3.182	FALSE	496,456	501,244	(4,788)	1.0%	0
1990	(941)	509	1.848	627,480	3	3.182	FALSE	627,480	614,927	12,553	-2.0%	0
1991	1,102	947	1.164	592,010	3	3.182	FALSE	592,010	605,136	(13,125)	2.2%	0
1992	1,549	711	2.179	655,619	3	3.182	FALSE	655,619	675,510	(19,891)	3.0%	0
1993	(1,074)	825	1.301	726,269	3	3.182	FALSE	726,269	714,482	11,787	-1.6%	0
1994	(4,781)	1,848	2.587	925,321	3	3.182	FALSE	925,321	871,686	53,635	-5.8%	0
1995	10,113	3,129	3.232	863,103	3	3.182	TRUE	1,024,909	971,378	53,531	-5.2%	1
1996	(5,005)	3,511	1.425	1,188,893	3	3.182	FALSE	1,188,893	1,154,704	34,189	-2.9%	0
1997	(3,526)	2,265	1.557	1,422,622	3	3.182	FALSE	1,422,622	1,401,892	20,730	-1.5%	0
1998	(3,392)	15,074	0.225	1,968,643	3	3.182	FALSE	1,968,643	1,991,138	(22,496)	1.1%	0
							Total	9,628,223	9,502,098	126,125	-1.3%	1

Note: Regression Model is linear, fit to the latest five retrospective estimates of ultimate clains

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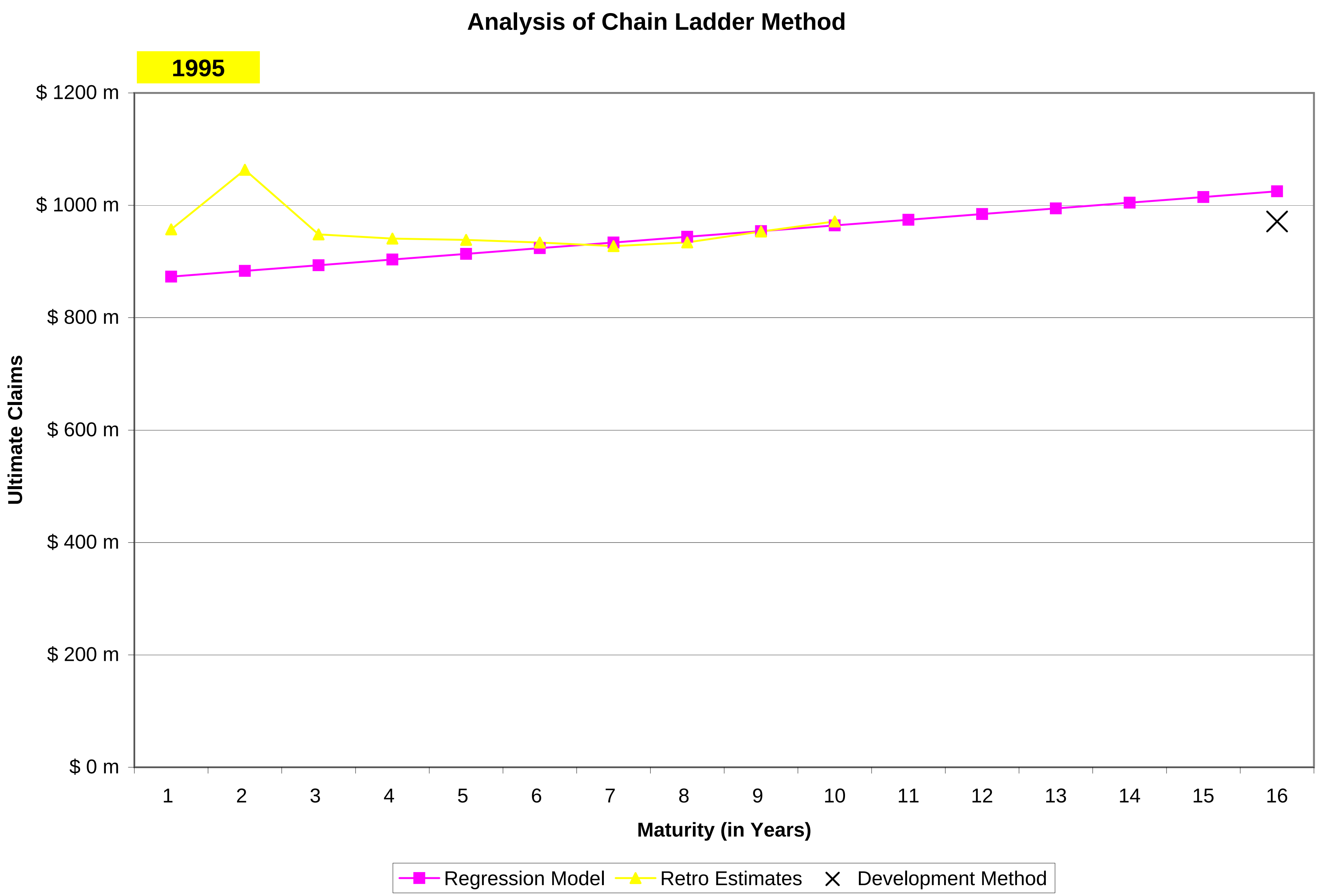
Fitted Estimates of Ultimate Claims

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
	Evaluations in Years															
Accident Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1989	496,797	497,137	497,478	497,818	498,158	498,499	498,839	499,179	499,520	499,860	500,200	500,541	500,881	501,221	501,562	501,902
1990	626,539	625,599	624,658	623,717	622,776	621,835	620,894	619,954	619,013	618,072	617,131	616,190	615,250	614,309	613,368	612,427
1991	593,113	594,215	595,317	596,419	597,521	598,623	599,726	600,828	601,930	603,032	604,134	605,236	606,338	607,441	608,543	609,645
1992	657,169	658,718	660,268	661,817	663,367	664,916	666,466	668,015	669,565	671,114	672,663	674,213	675,762	677,312	678,861	680,411
1993	725,195	724,121	723,047	721,973	720,899	719,825	718,751	717,677	716,603	715,529	714,455	713,381	712,306	711,232	710,158	709,084
1994	920,540	915,759	910,978	906,197	901,415	896,634	891,853	887,072	882,291	877,510	872,729	867,948	863,167	858,386	853,605	848,824
1995	873,216	883,329	893,442	903,555	913,668	923,780	933,893	944,006	954,119	964,232	974,345	984,458	994,570	1,004,683	1,014,796	1,024,909
1996	1,183,888	1,178,884	1,173,879	1,168,874	1,163,870	1,158,865	1,153,861	1,148,856	1,143,851	1,138,847	1,133,842	1,128,838	1,123,833	1,118,828	1,113,824	1,108,819
1997	1,419,096	1,415,569	1,412,043	1,408,517	1,404,990	1,401,464	1,397,937	1,394,411	1,390,885	1,387,358	1,383,832	1,380,305	1,376,779	1,373,253	1,369,726	1,366,200
1998	1,965,251	1,961,859	1,958,468	1,955,076	1,951,685	1,948,293	1,944,902	1,941,510	1,938,118	1,934,727	1,931,335	1,927,944	1,924,552	1,921,161	1,917,769	1,914,377

Exhibit 5



Analysis of Chain Ladder Method
1995
$ 1200 m
$ 1000 m
$ 800 m
$ 600 m
$ 400 m
$ 200 m
$ 0 m
Ultimate Claims
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
Maturity (in Years)
Regression Model
Retro Estimates
Development Method