A Survival Model Approach to Non-Life Run-off Triangle Estimation

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Abstract

Motivation. Most standard loss reserving techniques do not explicitly consider the rate at which claims will close, or the expected amount of time that a claim will remain open. Consideration of the time until closure allows one to calculate the amount of time until a block of claims will run-off. Further, it allows one to take explicit assumptions with regard to interest and inflation into account.

Method. By observing the closure rates for claims by age, a survival function is produced. This function can be used to determine the future lifetime of a claim at any age and the number of claims remaining open at any time.

Results. The method applied to a set of sample data generates a complete picture of the future pattern of claim disposal.

Conclusions. The method presented here grounds the projection of future claim run-off in theory common to life actuaries and opens up the life toolset to the analysis of non-life data.

Keywords. Reserving; Survival models.

1. INTRODUCTION

This paper will present a method to estimate the length of time that claims for a book of non-life insurance will remain open. This method is based largely on theory common to life actuaries but rarely used in the non-life field. This technique requires data presented in a manner slightly different than that with which non-life actuaries are accustomed to working. Nevertheless, the concepts are straightforward and intuitive and the data storage and computation requirements are not onerous. Coupled with a standard technique to forecast the emergence of unreported claims, an estimate for the time required for the complete run-off of a portfolio is produced.

1.1 Objective

Loss development techniques have traditionally sought to produce an estimate of the total quantum of losses remaining to be paid, e.g., the total reserve position. More recently, attention has been directed to consideration of the variance around both that estimate and the actual realized value of payments. Timing of payments to be made or a statement about the total amount of time to run-off a reserve is not always considered. When calculating the transfer price of a block of non-life (re)insurance liabilities, or calculating the amount of capital required to support the run-off, this is highly relevant. In those cases where a discounted value of reserves is needed, the standard approach is to take the results of an analysis of a paid loss triangle. Doing so doesn't allow one to directly observe or consider the manner in which a typical claim settles.

1.2 Outline

The paper proceeds as follows: First, the survival model is briefly reviewed. Next, the data required for the technique is described and the estimation of the survival model parameters is outlined. Results for a set of test data are shown and discussed. The method is then compared with several well-known methods. We conclude by addressing several unresolved issues and also by discussing some of the applications of this technique.

2. BACKGROUND AND METHODS

2.1 Review of Survival Model Mathematics

2.1.1 The Survival Function

A survival function, denoted $S(x)$, measures the probability that a value will be greater than or equal to some threshold x. When so stated, the function is easily seen to be equal to 1 minus the cumulative probability function or

$$S(x) = 1 - F(x) \tag{2.1}$$

This function follows the normal rules associated with probability distribution functions, with the additional requirement that x be non-negative. So, $S(0) = 1$ and $S(\infty) = 0$, or "the probability that a life will survive past age 0 is 1 and the probability that a life will survive to age infinity is zero." The terms "life" and "age" need not refer to an actual life, be it human or otherwise, but may refer to anything that has a well-defined temporal start and end point.

Although $S(x)$ can be defined continuously, the function is often given in a discrete form, using integral values for the age x. For convenience, the notation ${}_a p_x$, where x represents a starting age and a represents some future time period is often used. This can be read as "the probability that a life aged x survives for an additional time period a." Note that this probability is conditional on having attained age x. Mathematically, this is stated as follows:

$${}_a p_x = \frac{S(a+x)}{S(x)} = \frac{1-F(a+x)}{1-F(x)} = \frac{\int_{a+x}^{\infty} f(y)dy}{\int_{x}^{\infty} f(y)dy} \tag{2.2}$$

The notation ${}_aq_x$ represents the probability that a life aged x will terminate within time period a. This is the logical complement of the probability implied by ${}_ap_x$ (a life must either survive or terminate within a stated period of time) and is therefore equal to

$${}_aq_x = 1 - {}_ap_x. \tag{2.3}$$

When a is omitted a time period of one year is assumed. Given a set of factors for ages x through $x+a$-1, one can calculate the probability of survival for any duration a by multiplying successive factors as follows:

$${}_a p_x = p_x p_{x+1} \cdots p_{x+a-1} = \prod_{i=0}^{a-1} p_{x+i} \tag{2.4}$$

We use the random variable $K(x)$ to describe the future lifetime for a life aged x. Its expectation and variance for discrete probabilities are as follows:

$$E\left[K(x)\right] = \sum_{i=0}^{\infty} {}_{i+1}p_x \tag{2.5}$$

$$E\left[K^2(x)\right] = \sum_{i=0}^{\infty} (2i+1)_{i+1}p_x \tag{2.6}$$

$$Var(K(x)) = E\left[K^2(x)\right] - E\left[K(x)\right]^2 = \sum_{i=0}^{\infty} (2i+1)_{i+1}p_x - \left(\sum_{i=0}^{\infty} {}_{i+1}p_x\right)^2 \tag{2.7}$$

The derivation of the above formulae can be found in London [8] or Bowers [4].

2.1.2 Estimation of the Survival Function

London describes two different types of studies that may be performed to estimate a survival function. A longitudinal study examines a cohort of lives from age zero until the time of death. A cross-sectional study examines a group of lives of various ages for a fixed period of time.

For reasons that will be made clear below, the focus of this paper is a cross-sectional study. Here, a set of lives are observed between two points in time. For each life, the quantities y_i and z_i represent the age at which the observation period begins and ends, respectively. Note that the life may not survive until age z_i. For an age interval $(x, x+1]$, the quantities $x+r_i$ and $x+s_i$ are defined as the ages at which life i is scheduled to enter and exit that age interval. For example, if one observed a group of lives between 1 January 2004 and 31 December 2005, a life with birth date 16 February 1972 would have the following values:

$y_i = 31.87$
$z_i = 33.87$

For age interval (31, 32)	For age interval (32, 33)	For age interval (33, 34)
$x+r_i = 31.87$	$x+r_i = 32.00$	$x+r_i = 33.00$
$x+s_i = 32.00$	$x+s_i = 33.00$	$x+s_i = 33.87$

For each age interval, the probability of death within one year, q_x is estimated using the following estimator:

$$\hat{q}_x = \frac{d_x}{\sum_{i=1}^{n}(s_i - r_i)} \tag{2.6}$$

where n represents the number of observed lives and d_x represents the number of observed deaths. London shows that this estimator can be derived using the method of moments or maximum likelihood.

In cases where the exact age is not known, s_i and r_i are taken to be 1 and 0, respectively. In this case, the estimate of q_x is simply equal to:

$$\hat{q}_x = {}^{d_x}\!/_{n_x} \tag{2.7}$$

The estimated survival function is constructed by combining equations 2.3 and 2.4 as follows:

$$\hat{S}(x) = \prod_{i=0}^{x-1} \hat{p}_x = \hat{p}_0 \hat{p}_1 \ldots \hat{p}_{x-1} = (1-\hat{q}_0)(1-\hat{q}_1)\ldots(1-\hat{q}_{x-1}) \tag{2.8}$$

London makes two assumptions.

(1) Each $\hat{p}_x$ is binomially distributed with mean p_x and variance $\frac{p_x q_x}{n_x}$.

(2) The $\hat{p}_x$ s are independent.

These two assumptions allow us to state the following about the sample estimate of the expected future lifetime:

$$E\left[\hat{K}(x)\right] = E\left[\sum_0^\infty {}_{i+1}\hat{p}_x\right] = \sum_0^\infty E\left[{}_{i+1}\hat{p}_x\right] = \sum_0^\infty {}_{i+1}p_x = E\left[K(x)\right], \tag{2.9}$$

$$E\left[\hat{K}^2(x)\right] = E\left[\sum_0^\infty (2i+1)_{i+1}\hat{p}_x\right] = \sum_0^\infty (2i+1)E\left[{}_{i+1}\hat{p}_x\right] = \sum_0^\infty (2i+1)_{i+1}p_x = E\left[K^2(x)\right], \tag{2.10}$$

$$Var(\hat{K}(x)) = E\left[\hat{K}^2(x)\right] - \left(E\left[\hat{K}(x)\right]\right)^2 = E\left[K^2(x)\right] - \left(E\left[K(x)\right]\right)^2 = Var(K(x)). \tag{2.11}$$

In other words, the sample estimate of expected future lifetime is an unbiased estimate whose variance is independent of sample size.

A number of other techniques exist to estimate the survival function. Their implementation and appropriateness will not be explored in this paper.

2.2 Survival Model Methods as Applied to Non-Life Run-Off

A claim may be regarded as analogous to a life. It begins and ends at a fixed point in time. Its future remaining lifetime at any point is a random variable. A group of homogenous claims will likely exhibit similar survival patterns in the same way that humans with common characteristics will exhibit similar mortality. In the same way that human lifespans change over time, due to any number of factors such as nutrition, environment, changes in lifestyle, or advances in medicine, claim survival patterns may also change over time. A number of factors may influence non-life survival characteristics: claim department practice, legislative changes, behavior of insureds or cedants, to name but a few.

In general, a claim cannot be observed from time zero, the date of accident. There is generally a lag between when a claim occurs and when the claim is reported to a (re)insurer. This lag will vary depending on the characteristics of the claim and the type of coverage. First-party primary claims will be reported more quickly than third-party excess claims. This means that a claim may already be several years old when it can first be observed. For this reason, claim survival functions can only be estimated using the cross-sectional study described above.

2.2.1 The Data

The data was taken from a transactional database, which showed a history of claim payments made in each year, the date the claim occurred, and the status of the claim. Here, we define age as the calendar year minus the accident (or underwriting or reporting) year, plus one.[1] Note that when the age is so defined, the values for r_i and s_i in equation (2.6) are 0 and 1, respectively.

If no payment is made, a record is still kept to indicate that the claim remains open. This allows one to determine whether or not the claim will remain open in the following year.[2] So, for each payment year, for each age, one can calculate the total number of claims open as well as the number of claims that will terminate in the following year. The figures were summed for all payment years. The results are shown in Appendix A.

Note that this data may also be presented in a triangle format. The resultant triangles would be the number of claims open and the incremental number of claims closed during the period. These are shown as Appendices C and D.

2.2.2 The Method

With the data so arranged, the calculations proceed simply. Refer to n_x as the number of claims of age x and d_x as the number of claims of age x which will close. An estimator for the probability of claim closure for each age is given by formula 2.7.

Note that if the data is given as a triangle, summing across payment years is equivalent to taking the sum of the accident year rows.

At this point, a model has been developed for the expected value of the future life of all claims that have been reported. To forecast the emergence of new claims, a standard chain-ladder technique can be used. This will yield projections of the number of claims with respective ages for all future time periods. The same survival function can be applied to this set of IBNR claims.

The future lifetime for the book is equal to the maximum of the future lifetimes for all claims. As will be seen below, this is not necessarily the same as the expected future lifetime for the youngest claim present in the sample. The expected future lifetime is a quantity which depends on attained age. For non-life claims, it is often true that the longer a claim has been open, the longer it can be expected to remain open.

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[1] Note that some authors refer to this quantity as "lag."

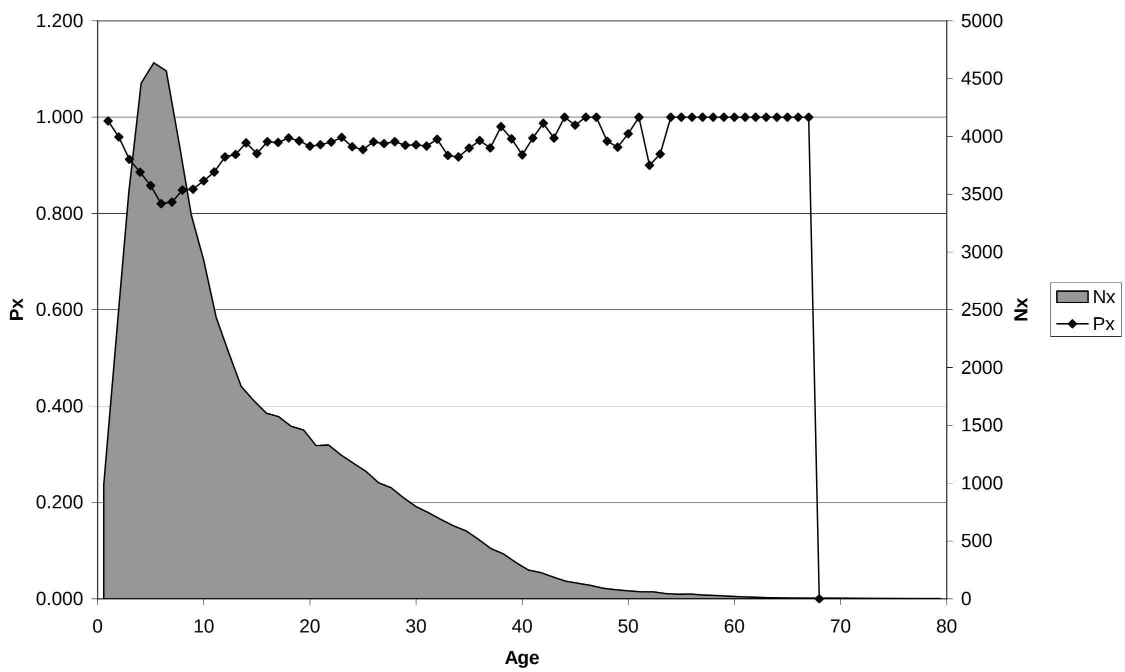
[2] Note that for the most recent year, it is impossible to determine whether or not a claim will terminate.

3. RESULTS AND DISCUSSION

3.1 Results

The forecast method was applied to a set of data that includes, among other things, excess bodily injury claims. The data has been randomly altered to conceal its identity, however the broad conclusions remain. Claim payment records for over 40 years were available. The earliest payment year includes information on currently open claims, so the oldest potential age can be, and indeed is, greater than 40 years. The oldest age in the sample was 68 years.

The chart below plots p_x against the age of a claim. The shaded surface shows n_x.



The likelihood of a claim persisting for an additional year drops for claims of low age, but then raises to a relatively high and constant survival probability beyond 14 years. Claims older than 14 years are very likely permanent bodily injury claims that will last as long as the claimant remains alive.

The fluctuation in probabilities beginning around age 38 is due to a reduction in sample data. Specifically, the number of observed claim closures drops below 10 at this age and is zero for some ages. This is a worrisome result. In effect, what it means is that one cannot truly know what's happening to claims that have been open for a very long time. That is to say that the likelihood that a very old claim will close in any given period of time is not easily estimable via statistical methods given this particular sample data.

This is a situation with which casualty actuaries are familiar. When reserving, one usually has the problem of how to estimate a tail factor. There are a number of techniques discussed in the non-life literature as to how to go about this. When revising a mortality table, one not only adjusts and extrapolates the estimates for high ages—the "tail" of the table—one smoothes the estimates for all ages. This revision of sample estimates is referred to as "graduation." London [9] gives a useful introduction to several graduation methods used by life actuaries. Contrast this with the typical non-life approach where age-to-age and tail factors are each calculated and judgmentally adjusted individually.

In this case, the sample estimate was adjusted by using the Whitaker-Henderson method of graduation. This technique can be considered ad hoc. The intent is to produce a revised set of estimates that represents a blend between smoothness and reproduction of the sample estimates. To do this, one minimizes the sum of the differences between the estimates and the squared difference between the sample estimates and revised values. A parameter ε controls the relative weight one places on smoothness and reproduction of the sample.

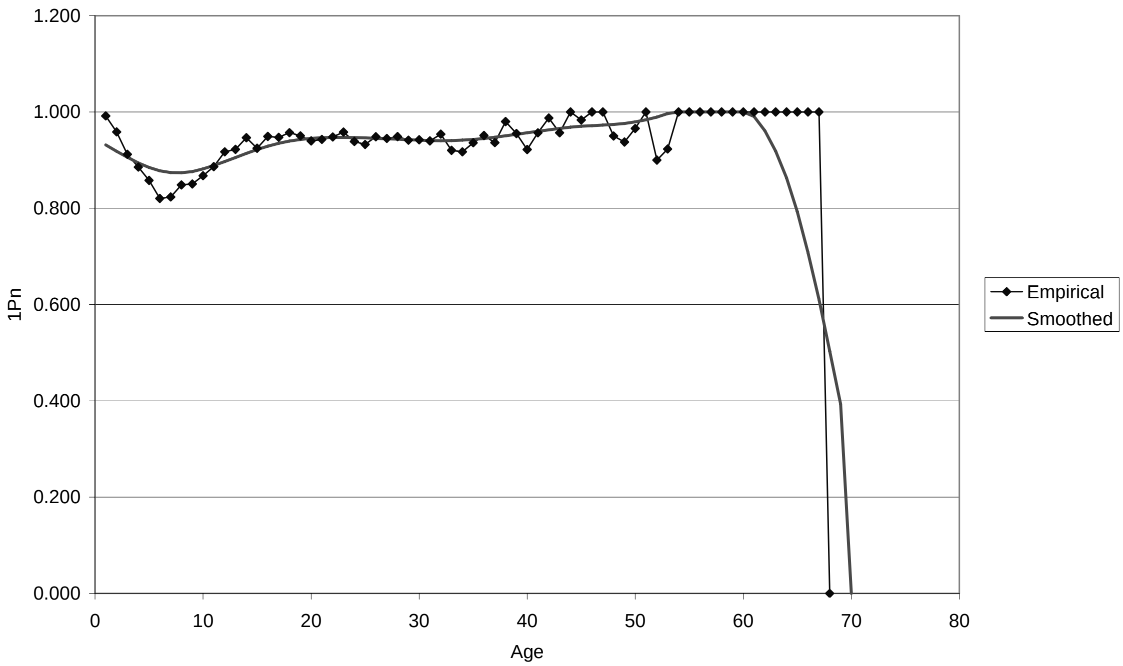
This quantity is given as follows:

$$\sum_{x=0}^{\max-3}(\Delta^3 p'_x)^2 + \varepsilon\sum_{x=0}^{\max}\left(p'_x - \hat{p}_x\right)^2 \tag{3.1}$$

where p' is the revised estimate.

In addition to smoothing the results, the technique can also be used to extrapolate beyond the maximum age in the sample. In this case, 70 was selected as the maximum feasible age of a claim. A claim age of 70 would imply a claimant age of 70 plus the age of the claimant at the time of injury. This is well within a reasonable maximum for a human life for claimants of a very young age, but not claimants who make a claim later. However, it is possible that beneficiaries or a claimant's estate may also receive claim payments. Further, the claimant may be a corporation or some other nonhuman entity. In either case, the age of a claim could be greater than the feasible length of one human life.

The chart below shows the results of the smoothing.



This is but one option. One could also extrapolate the survival model or construct a parametric survival model. An alternative would be to rely on a human mortality table. This could be appropriate for lifetime bodily injury cases. Diss and Sherman [5] use this method to estimate a tail factor for workers compensation business. (Note that there has been some research into differences between the mortality for bodily injury claimants and the general population. In particular, see Barnett [2] and Gillam [7].)

Yet another alternative would be to replace the sample estimates with judgmentally derived survival probabilities, possibly determined in conjunction with the claims department. Note that when speaking with non-actuaries it is likely far easier to pose the question "What is the likelihood that a claim that has been open for 40 years will stay open for another year?" than to ask "Is a tail factor of 1.025 at development year 40 reasonable?"

The smoothed results are given in Appendix B, along with an estimate of the expected future lifetime for each age. For this sample, the expected future lifetime is 20.3; it should take at least another 20 years for the business to completely run-off. However, there is a possibility that it will take quite a while longer. Assuming a normal approximation and using the standard deviation as given in formula 2.7, there is a 5% chance that this book could take 46.1 years to completely close—a difference of over 25 years.

3.2 Comparison with Other Techniques

Observation of the rate at which non-life claims close is not a new idea. Although claim count information is used less often, several well-known papers present techniques for handling this data. Following are comments on how this method compares with others.

Both Adler and Kline [1] and Berquist and Sherman [3] discuss a claims closure ratio. This ratio is defined as the number of claims closed in a given development period divided by the number of ultimate claims. Thus, the claims closure ratio depends on an estimate of the ultimate number of claims having already been made. In both cases, they presume that the future ratio of closed claims to ultimate for any development period will be the same as the most recent calendar year. Adler and Kline presume that the rate of claim closure is a stable figure that depends on the amount of claims remaining to be closed for a particular accident year. Note that this is different than what is presumed here. Here it is assumed that the survival of a particular claim is determined by the characteristics of the claim itself. The implication of both Adler and Kline and Berquist and Sherman is that a claim has an expected lifetime, which is more or less fixed, and that its time of settlement can change only because of the workflow characteristics of the claim department.

Fisher and Lange [6] describe a claims disposal ratio. Here too, they calculate this as the ratio of claims disposed of in any particular year to the total number of claims. Because they are working with report year data, the number of claims is known for each year and need not be estimated.

Teng and Sherman [10] present a reserving technique that utilizes an estimate of claims closure ratios similar to what is presented here. The closure ratio is calculated as the number of claims closed in any particular period to the number of claims reported up to the beginning of that period. Because closed claims will always remain in the population of claims reported to date, this quantity is not the same as the probability that a claim will terminate given that it survives to a particular development period. In fact, what Teng and Sherman are estimating is $1 - S(x)$. Because $S(x)$ depends on the individual p_xs, one could argue that the method presented here may be more appropriate given that it develops a specific estimate of the survival probability at each age.

3.3 Enhancements

There are a number of ways that this technique could be enhanced. At present, no distinction is made as to the way in which a claim is closed. If one were aware of certain effects, such as an active commutation strategy, or the influence of particular cedants or insureds, those claims could be removed from the sample.

The Whitaker-Henderson method is but one option for graduation of an empirical survival model. One could also apply the standard battery of smoothing and trending methods. As noted earlier, the use of a survival model does not obviate the need to select a tail factor. However, unlike some techniques applied to triangle data, a survival model requires the actuary to posit an upper bound for the length of time that a claim will remain open and state the likelihood of attaining that age.

It is commonly accepted that claim closure patterns change over time due to any number of influences. Less common is an objective method to forecast those changes. Life actuaries do attempt to project mortality trends into the future. Adoption of those techniques may help shed light on the dynamics of non-life claim behavior.

In order to convert this method into one for which an estimate of reserves could be calculated, as estimate of the size of prospective payments would have to be incorporated. Among the advantages of constructing the reserving model in this way are that one can integrate estimates for future inflation explicitly. This is effectively what Teng and Sherman have done.

4. CONCLUSIONS

The use of survival models, though understood in principle, is not common to non-life actuaries. The ability to examine data in this way opens up a number of interesting possibilities, including the use of techniques developed in the fields of population growth and demography. In the view of this author, equally important is a philosophical shift away from triangulated data towards a more fundamental consideration of the dynamics of the claim. Every actuary appreciates that the dynamics of claim generation and settlement are complex and change over time, but the methods currently available for the analysis of aggregate claim triangles do not easily lend themselves to taking these forces into account. It is hoped that this approach will serve as a step towards changing that.

Acknowledgment

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Appendix A

Age	*N*	*D*	*qx*	*px*	*S*(*x*)
1	985	8	0.008	0.992	0.992
2	2270	94	0.041	0.959	0.951
3	3522	309	0.088	0.912	0.867
4	4461	510	0.114	0.886	0.768
5	4636	659	0.142	0.858	0.659
6	4566	820	0.180	0.820	0.541
7	3958	699	0.177	0.823	0.445
8	3322	503	0.151	0.849	0.378
9	2927	438	0.150	0.850	0.321
10	2433	322	0.132	0.868	0.279
11	2126	242	0.114	0.886	0.247
12	1838	152	0.083	0.917	0.227
13	1715	133	0.078	0.922	0.209
14	1607	86	0.054	0.946	0.198
15	1575	119	0.076	0.924	0.183
16	1492	76	0.051	0.949	0.174
17	1459	77	0.053	0.947	0.164
18	1325	57	0.043	0.957	0.157
19	1331	66	0.050	0.950	0.150
20	1243	75	0.060	0.940	0.141
21	1172	67	0.057	0.943	0.132
22	1101	57	0.052	0.948	0.126
23	1003	42	0.042	0.958	0.120
24	961	59	0.061	0.939	0.113
25	875	59	0.067	0.933	0.105
26	796	41	0.052	0.948	0.100
27	744	41	0.055	0.945	0.094
28	686	35	0.051	0.949	0.090
29	631	37	0.059	0.941	0.084
30	588	34	0.058	0.942	0.079
31	515	31	0.060	0.940	0.075
32	435	20	0.046	0.954	0.071
33	389	31	0.080	0.920	0.066
34	314	26	0.083	0.917	0.060
35	249	16	0.064	0.936	0.056
36	226	11	0.049	0.951	0.054
37	188	12	0.064	0.936	0.050
38	151	3	0.020	0.980	0.049
39	133	6	0.045	0.955	0.047
40	115	9	0.078	0.922	0.043
41	92	4	0.043	0.957	0.041
42	79	1	0.013	0.987	0.041
43	69	3	0.043	0.957	0.039
44	60	0	0.000	1.000	0.039
45	60	1	0.017	0.983	0.038
46	46	0	0.000	1.000	0.038

Age	*N*	*D*	*qx*	*px*	*S*(*x*)
47	39	0	0.000	1.000	0.038
48	40	2	0.050	0.950	0.036
49	32	2	0.063	0.938	0.034
50	29	1	0.034	0.966	0.033
51	23	0	0.000	1.000	0.033
52	20	2	0.100	0.900	0.030
53	13	1	0.077	0.923	0.027
54	11	0	0.000	1.000	0.027
55	7	0	0.000	1.000	0.027
56	6	0	0.000	1.000	0.027
57	8	0	0.000	1.000	0.027
58	6	0	0.000	1.000	0.027
59	7	0	0.000	1.000	0.027
60	6	0	0.000	1.000	0.027
61	3	0	0.000	1.000	0.027
62	3	0	0.000	1.000	0.027
63	3	0	0.000	1.000	0.027
64	3	0	0.000	1.000	0.027
65	2	0	0.000	1.000	0.027
66	2	0	0.000	1.000	0.027
67	2	0	0.000	1.000	0.027
68	1	1	1.000	0.000	0.000

Appendix B

Age	Empirical p_x	Smoothed p_x	Capped p_x	$S(x)$	$\hat{K}(x)$	Std.Dev.$(\hat{K}(x))$	95th
1	0.992	0.931	0.931	0.931	11.203	13.527	33.454
2	0.959	0.918	0.918	0.855	11.110	13.754	33.734
3	0.912	0.906	0.906	0.775	11.162	14.040	34.256
4	0.886	0.894	0.894	0.693	11.363	14.380	35.015
5	0.858	0.885	0.885	0.613	11.713	14.761	35.993
6	0.820	0.878	0.878	0.538	12.203	15.166	37.150
7	0.823	0.874	0.874	0.470	12.816	15.571	38.428
8	0.849	0.874	0.874	0.411	13.523	15.950	39.759
9	0.850	0.876	0.876	0.360	14.290	16.281	41.069
10	0.868	0.881	0.881	0.318	15.077	16.546	42.292
11	0.886	0.889	0.889	0.282	15.842	16.737	43.371
12	0.917	0.897	0.897	0.253	16.549	16.855	44.273
13	0.922	0.906	0.906	0.229	17.170	16.908	44.981
14	0.946	0.914	0.914	0.209	17.690	16.907	45.500
15	0.924	0.922	0.922	0.193	18.103	16.867	45.846
16	0.949	0.929	0.929	0.179	18.410	16.798	46.040
17	0.947	0.935	0.935	0.168	18.625	16.710	46.110
18	0.957	0.939	0.939	0.158	18.761	16.610	46.082
19	0.950	0.943	0.943	0.149	18.837	16.503	45.982
20	0.940	0.945	0.945	0.140	18.871	16.390	45.830
21	0.943	0.947	0.947	0.133	18.879	16.271	45.642
22	0.948	0.947	0.947	0.126	18.872	16.147	45.431
23	0.958	0.947	0.947	0.119	18.864	16.015	45.206
24	0.939	0.947	0.947	0.113	18.865	15.873	44.974
25	0.933	0.946	0.946	0.107	18.880	15.720	44.737
26	0.948	0.945	0.945	0.101	18.914	15.551	44.493
27	0.945	0.944	0.944	0.095	18.970	15.364	44.242
28	0.949	0.943	0.943	0.090	19.051	15.155	43.979
29	0.941	0.942	0.942	0.085	19.158	14.918	43.696
30	0.942	0.941	0.941	0.080	19.290	14.649	43.385
31	0.940	0.941	0.941	0.075	19.444	14.342	43.034
32	0.954	0.940	0.940	0.071	19.614	13.992	42.629
33	0.920	0.940	0.940	0.066	19.792	13.597	42.156
34	0.917	0.941	0.941	0.063	19.964	13.153	41.598
35	0.936	0.943	0.943	0.059	20.115	12.663	40.943
36	0.951	0.945	0.945	0.056	20.229	12.131	40.182
37	0.936	0.947	0.947	0.053	20.294	11.562	39.313
38	0.980	0.950	0.950	0.050	20.302	10.964	38.336
39	0.955	0.953	0.953	0.048	20.247	10.342	37.259
40	0.922	0.956	0.956	0.046	20.123	9.706	36.088
41	0.957	0.960	0.960	0.044	19.925	9.065	34.836
42	0.987	0.963	0.963	0.042	19.654	8.427	33.515
43	0.957	0.966	0.966	0.041	19.315	7.795	32.137
44	1.000	0.968	0.968	0.040	18.916	7.169	30.707
45	0.983	0.970	0.970	0.038	18.467	6.542	29.227
46	1.000	0.972	0.972	0.037	17.977	5.905	27.690

Age	Empirical p_x	Smoothed p_x	Capped p_x	$S(x)$	$\hat{K}(x)$	Std.Dev.($\hat{K}(x)$)	95th
47	1.000	0.973	0.973	0.036	17.452	5.250	26.087
48	0.950	0.974	0.974	0.035	16.889	4.573	24.411
49	0.938	0.976	0.976	0.034	16.277	3.887	22.670
50	0.966	0.979	0.979	0.034	15.600	3.223	20.901
51	1.000	0.984	0.984	0.033	14.842	2.640	19.184
52	0.900	0.989	0.989	0.033	13.990	2.229	17.656
53	0.923	0.997	0.997	0.033	13.036	2.096	16.484
54	1.000	1.005	1.000	0.033	12.036	2.096	15.484
55	1.000	1.013	1.000	0.033	11.036	2.096	14.484
56	1.000	1.019	1.000	0.033	10.036	2.096	13.484
57	1.000	1.024	1.000	0.033	9.036	2.096	12.484
58	1.000	1.025	1.000	0.033	8.036	2.096	11.484
59	1.000	1.021	1.000	0.033	7.036	2.096	10.484
60	1.000	1.010	1.000	0.033	6.036	2.096	9.484
61	1.000	0.991	0.991	0.032	5.083	2.048	8.451
62	1.000	0.961	0.961	0.031	4.250	1.912	7.395
63	1.000	0.919	0.919	0.029	3.536	1.722	6.369
64	1.000	0.863	0.863	0.025	2.938	1.502	5.408
65	1.000	0.793	0.793	0.020	2.444	1.268	4.529
66	1.000	0.708	0.708	0.014	2.038	1.028	3.730
67	1.000	0.610	0.610	0.008	1.701	0.778	2.981
68	0.000	0.503	0.503	0.004	1.393	0.488	2.197
69	0.000	0.393	0.393	0.002	1.000	0.000	0.000
70	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Appendix C

Number of open claims

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1990	22	39	54	87	79	92	63	48	42	30	27	25	25	21	22	24
1991	13	24	44	57	61	71	40	35	31	27	26	20	18	17	16	
1992	10	30	54	66	63	69	53	49	45	44	39	35	27	22		
1993	9	41	56	69	67	60	63	49	48	36	37	34	31			
1994	14	58	86	107	146	139	118	123	111	93	82	79				
1995	30	99	146	192	208	217	251	202	176	171	102					
1996	40	94	152	189	236	221	230	221	160	130						
1997	40	149	215	300	329	415	297	342	248							
1998	28	78	122	195	198	236	236	197								
1999	29	78	200	391	414	424	375									
2000	36	120	291	391	352	366										
2001	54	143	257	270	309											
2002	27	74	116	130												
2003	12	23	41													
2004	8	15														
2005	4															

Appendix D

Incremental number of closed claims

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1990	0	1	1	8	13	24	15	10	12	5	1	2	4	1	1	4
1991	0	0	0	5	9	20	16	10	8	6	2	6	0	4	6	
1992	0	0	7	8	9	14	9	9	7	3	7	4	6	4		
1993	0	0	4	7	14	8	9	13	7	1	7	9	12			
1994	0	2	5	13	20	31	18	19	22	14	19	24				
1995	0	3	16	18	26	34	48	34	30	59	26					
1996	1	4	16	20	47	33	38	42	48	36						
1997	0	2	5	19	30	88	48	85	67							
1998	0	0	8	14	27	42	69	62								
1999	0	3	9	60	72	106	93									
2000	0	1	22	43	84	125										
2001	0	7	12	44	87											
2002	0	4	16	30												
2003	0	2	12													
2004	0	4														
2005	0															

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Abbreviations and notations

$S(x)$, survival function

${}_aq_x$, probability that a life aged x will die within the next a years

r_i, difference between the age when observation begins and the most recent integral age

d_x number of deaths observed during a period of observation

${}_ap_x$, probability that a life aged x will survive for a years

$K(X)$, the future lifetime of a life aged x

s_i, difference between the next integral age and the age at which the life exits observation

n_x, number of lives at age x during the observation period

Biography of the Author

Brian Fannin is an actuary at the Munich Re Group, responsible for the pricing and underwriting of retrospective reinsurance contracts for the Customized Portfolio Solutions unit. Prior to his current role, he spent three years in Munich Re's Integrated Risk Management unit where he participated on a variety of ERM projects. Prior work experience includes treaty and facultative reinsurance and primary commercial lines pricing. Brian may be contacted at BFannin@MunichRe.com