Loss Development in Workers Compensation in the Presence of Legislative Reform

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Motivation. Legislative reforms affect loss development patterns in various ways. Some legislative innovations may affect new policy (or accident) years only, while others have diagonal effects as they affect both new and existing claims. Modeling these effects is critical for adequacy in ratemaking and reserving.
Method. Using a Bayesian state-space model, workers compensation triangles are developed subject to the applicable legislative stipulations. Most importantly, this model is capable of accommodating the legislative environment as it evolves over time.
Results. The model is applied to an unidentified state, which experienced a reform cluster in the period 1990/92. The model shows how this reform cluster affects the ultimate loss and the 19th-to-ultimate tail factors.
Conclusions. Ultimate losses are not only dependent on the legislative environment at time of loss, but are also affected by how this legislative environment evolves over time. The statistical model is capable of quantifying the effects of such legislative changes on the loss development pattern.
Availability. The model runs in OpenBUGS 2.2.0 (http://mathstat.helsinki.fi/openbugs/) within the R (www.r-project.org) package BRugs 0.3-3 (http://cran.r-project.org). OpenBUGS is administered by the Department of Mathematics and Statistics of the University of Helsinki, Finland; R is administered by the Technical University of Vienna, Austria. OpenBUGS and R are GNU projects of the Free Software Foundation and, hence, available free of charge.

Keywords. Workers Compensation, Trend and Loss Development, Reserving Methods.

1. INTRODUCTION

Workers compensation is a line of insurance that operates in a legal environment that is subject to frequent and (sometimes) sweeping changes. Such legislative changes affect the loss development patterns in ratemaking and reserving in powerful and complex ways. Traditional loss development models do not acknowledge the specific legal environment in which the losses have been observed, nor are these models capable of incorporating changes in the legal setting into the loss development pattern; as a consequence, these models are not capable of quantifying the impacts of changes to legal stipulations on the ultimate loss or tail factor.

What follows is a Bayesian state-space model of loss development that explicitly accounts for the legal environment in which the losses of a given (indemnity or medical) triangle were observed. Accounting for the legal environment means translating legal stipulations into data inputs, which are then fed into the model. The model is set up to accommodate a wide array of legal changes, among which are changes to the stipulated rates of escalation (for indemnity) and (any) factors that bear on the rate at which incremental payments decay in development net of the calendar-year effect.

1.1 Research Context

A wide array of loss development models have been suggested, some of which are extensions of traditional actuarial methods (mostly related to the chain ladder; see, for instance, Mack [5]), while other models cast loss development into a time series framework (see, for instance, de Jong and Zehnwirth [2], de Jong [1], and Verrall [9]). For overviews on loss development models, see England and Verrall [3] and Taylor [8]. Bayesian modeling of loss development using the software platform BUGS (Bayesian inference Using the Gibbs Sampler) has been pioneered by Scollnik [6,7]. The model presented here draws on Scollnik [7].

1.2 Objective

The objective of the loss development model introduced in this paper is to give the practicing actuary a framework for developing losses in a changing legal environment. By acknowledging changes in pertinent legal stipulations, the model is capable of delivering values for the ultimate loss (and, hence, the tail factor) that are adequate for ratemaking and reserving. Specifically, the model allows for quantification of reform impacts on the ultimate loss and the tail factor.

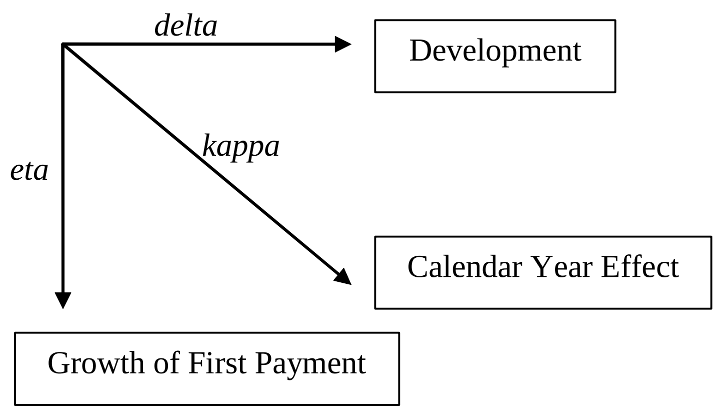
1.3 Outline

The following section first outlines the basic structure of the Bayesian state-space model of loss development and then, in a sub-section, applies this model to an unidentified state. This application details how regulatory information is fed into the model and how the model quantifies the reform impact. Section 3 presents the results of this empirical analysis. Section 4 offers conclusions.

2. BACKGROUND AND METHODS

The Bayesian state-space model of loss development employed in the analysis of legislative reform treats incremental payments as a three-dimensional time series problem. Specifically, the incremental payments are driven by three time processes, which are growth of the first payment, development, and the calendar-year effect; these processes are illustrated in Exhibit 1.

Exhibit 1: Time Processes in Loss Development



The model fits to (the logarithms of) incremental payments and, at the same time, employs a stochastic cumulative sum (cusum) constraint to ensure that, for any development year, the sum of the estimated incremental payments for a given policy (or accident) year add up (approximately) to the observed cumulative payment for that policy (or accident) year.

As an example, consider the stylized triangle displayed in Exhibit 2. Let $y[i,j]$ be the (natural) logarithm of the incremental payment of policy (or accident) year i in development year j, which materializes as a draw from a normal distribution with expected value $b[i,j]$. Then, the expected value of the logarithm of the first payment in the first policy (or accident) year, $b[1,1]$, develops into $b[1,2] = b[1,1] + delta[2] + kappa[1,2]$, where $b[1,2]$ is the expected value of the logarithm of the second incremental payment in the first policy (or accident) year. The parameter $delta[2]$ is the rate of decay (which is expressed as a logarithmic rate of growth) of the calendar-year effect-adjusted incremental payments from development year 1 to development year 2, whereas the term $kappa[1,2]$ is the calendar-year effect (which, again, is expressed as a logarithmic rate of growth) from calendar year 1 to calendar year 2. Note that the calendar-year effect is not restricted to be uniform along a given diagonal—for instance, $kappa[2,3]$ is allowed to differ from $kappa[3,2]$; this is because different types of indemnity claims (which consist of Temporary Total [TT], Permanent Partials [PP], Permanent Totals [PT], and Fatals) may escalate at different rates and the fraction of the various types in the total may change across development years. Finally, for the expected value of the logarithm of the first payment in the second policy (or accident) year, $b[2,1]$, we can write

$b[2,1] = b[1,1] + eta[2]$, where $eta[2]$ (which is again expressed as a logarithmic rate of growth) equals the change in expected values.

Exhibit 2: Stylized Triangle

Policy (or Accident) Year

	Development Year		
	1	2	3
1	y[1,1]	y[1,2]	y[1,3]
2	y[2,1]	y[2,2]	
3	y[3,1]		

The run-off rate (*delta*) is estimated using a smoothed random walk specification; the smoothing is obtained by scaling the innovation variance with a Gompertz function. The rate of growth of the expected value of the first incremental payment (*eta*) is also estimated using a smoothed random walk; unlike the innovation variance of the run-off rate (which decreases as development progresses), the innovation variance of *eta* is constant. (The smaller the innovation variance, the smoother is the estimated trajectory of growth rates.)

The model draws on expert information in determining the prior for the calendar-year effect, which manifests itself in the growth rate *kappa*. For indemnity benefits inflation, these expert priors are the rates of escalation as stipulated in the law; these stipulated rates of escalation may vary by type of claim. Additionally, the expert priors for the rates of escalation may vary by policy (or accident) years and development years. The expert prior for medical benefits inflation is the rate of growth of the Medical Care component of the CPI (Consumer Price Index; www.bls.gov), M-CPI for short.

The model develops future losses subject to the assumption that the expert priors for the (non-constant) rates of inflation follow random walks, starting at the final observed rates. The purpose of these random walks is to incorporate uncertainty about the future rates of inflation. The innovation variances of these random walks have to be determined by an expert based on the actual behavior of the applicable inflation series. Due to the skewed, lognormal distribution of the incremental payments, greater uncertainty about future rates of inflation (that is, greater innovation variances in the random walks) implies higher expected values of incremental payments and, all else being equal, a larger tail factor.

The model assumes that beyond the final observed development year, the projected run-off rate is the minimum of the final estimated run-off rate (that is, the run-off rate that applies in the final observed development year) and a mortality-based run-off rate. Starting with the final estimated logarithmic run-off rate, this logarithmic mortality-based run-off rate decreases linearly in every development year such that in development year 60, this rate equals the current official (logarithmic) mortality rate for age 80. Beyond age 80 (development years 61 through 70), the (logarithmic) mortality-based run-off rate equals the (logarithm of the) official mortality rate for the applicable age. The mortality information originates from the Social Security Administration (Periodic Life Table, www.ssa.gov). Where indemnity benefits are not granted for life (due to an age limit or an otherwise stipulated restriction in the duration of benefits), the number of payments is reduced accordingly, as detailed in the following section.

For details on the model, see Appendixes 2 and 3; Appendix 4 offers a list of variables. The model was estimated using Markov chain Monte Carlo simulation; for introduction to this estimation technique see, for instance, Gilks, Richardson, and Spiegelhalter [4]. The equations were coded in BUGS and run in R (using BRugs [Version 0.3-3, which utilizes OpenBUGS 2.2.0 beta from February 2006]) with a burn-in of 40,000 iterations, followed by a sample of 40,000 iterations, of which every fourth draw entered the posterior distribution (to mitigate autocorrelation in the Markov chains).

2.1 The Reform Impact of an Unidentified State

This section presents an application of the loss development model for the purpose of studying the impact of legislative reform on the loss development pattern, and the tail factor in particular. The model is applied to a loss triangle of policy year data; the first report of payments of any given policy year comprises 24 months of experience. The policy years in the loss triangle range from 1980 through 2005. The triangle, which is displayed in Exhibit A-1 in Appendix 1, is incomplete due to a missing upper left-hand side triangle, a missing upper right-hand side triangle, and a missing lower left-hand side (single-observation) triangle.

The purpose of the analysis is to study the reform impact in an unidentified state; this state experienced major reforms in workers compensation in the years 1982, 1986, 1990 (effective

September 1), and 1992 (indemnity-related reforms effective May 18, and medical-related benefits reform effective November 1). The 1982 and 1986 reforms are not broken out because the first diagonal in the triangle refers to the year 1988. The reform impact of interest is the one of the 1990/1992 reform cluster; for this purpose, we define the time window 1988-1989 as the pre-reform period, and the window 1993-2005 as the post-reform period. Four of the most significant impacts of the 1990/1992 reforms were (1) the introduction of escalation of indemnity benefits at the rate of the CPI (regardless of the date of the injury) for PT disability claims and PP disability claims in May 1991 (beyond 312 weeks of benefits; indemnity benefits for fatal claims had been escalating at a fixed rate of 4 percent since June 1986); (2) a limitation of the duration of TT disability claims to 52 weeks; (3) closer scrutiny regarding continued eligibility of indemnity benefits; and (4) an indemnity retirement offset that is immediate for accidents past age 55 or, otherwise, sets in five years prior to the official retirement age. Whereas the introduction of a cost-of-living adjustment is captured in the model as a calendar-year effect (as such adjustment started applying to claims of any maturity), the time limitation on TT claims, the increased scrutiny regarding continued eligibility, and the social security offset can be expected to bear on the run-off rate (*delta*). The run-off rate (*delta*) picks up the effect of a social security offset to the extent that such offset kicks in for (older) claimants within the first 20 development years (as these are the development years covered by the data). Yet, because the social security offset may not be fully captured by the run-off rate (due to there being [younger] claimants for whom the offset does not kick in within the 20 observed development years), the model assumes (as an approximation) a 50 percent reduction of the incremental indemnity payments past development year 40. Note that the increased scrutiny regarding continued eligibility of indemnity benefits may spill over into medical benefits, thus causing medical claims to close faster. Hence, we expect the 1990/1992 reform cluster to lead to a faster run-off not only in indemnity but also in medical incremental payments. (Note that although the most significant impacts of the 1990/1992 reform cluster were the indemnity reforms mentioned above, the 1990/1992 reform cluster also included a medical reform in November 1992, as mentioned above.)

Exhibit A-2 in Appendix 1 details the shapes of the pre-reform and post-reform triangles. The area of the pre-reform triangle for which there is data is shaded gray; this area comprises all observations between (and inclusive of) the 1988 diagonal and the 1989 policy year. The post-reform triangle is bordered by a solid line and consists solely of post-1992 diagonals. Note that the model does not fit to (the six) observations between (and inclusive of) the 1990 policy year and the 1992 diagonal, although these observations are assigned to the pre-reform period for the purpose of the post-reform estimation. The pre-reform and post-reform loss development processes are estimated simultaneously. The missing upper left-hand side triangle (diagonals 1980 through 1987) is given its own trajectory of run-off rates, which is the same for both the pre-reform and the post-reform estimation. Finally, for the post-reform estimation, the run-off rates that apply to the diagonals from 1988 through 1992 are allowed to differ from the estimated pre-reform run-off rates.

Although the pre-reform triangle consists only of policy-year data prior to 1990, this triangle includes elements through the 2005 diagonal. To the degree that the 1990/1992 reform cluster affected existing (instead of only new) claims (for instance, by accelerating their closure), the model may underestimate the impact of the reform cluster on the ultimate loss; however, the post-reform ultimate losses (and tail factors) would still be accurate, as argued below. For the data set at hand, the pertinent (future) policy year for ratemaking is 2008.

Unlike the pre-reform triangle, the post-reform triangle consists only of diagonals observed in the pertinent legislative environment. Yet, only in the first column of the post-reform triangle do all observations fall into the post-reform regime. As development time increases, the post-reform triangle phases in observations from the previous legislative setting, as indicated by the step function that defines the post-reform triangle in Exhibit A-2. For instance, in the first development year, all 13 incremental payments (of which the one for policy year 2005 is missing) are from the post-reform period. In the second development year, there are again 13 incremental payments (of which none are missing), but only 12 originate in the post-reform regime; and so on. The progressive phasing in of observations from the prior legislative regime rests on the premise that the run-off rates (but not necessarily the level of payments) of the post-reform regime approach the pre-reform run-off rates as development time advances; this is because the rates of decline of calendar-year effect-adjusted incremental payments deep in development may predominantly be driven by factors immune to the reforms, such as mortality. (It is because the reform may affect the level of payments deep in development [due to its effect on the run-off rates early in development] that the pre-reform run-off rates in the post-reform estimation are allowed to differ from the pre-reform run-off rates in the pre-reform estimation.) If the run-off rates (of the pre-reform policy years) deep in development are indeed immune to the reform, then the model estimates accurately both the pre-reform and post-reform ultimate losses. If, on the other hand, the run-off rates (of the pre-reform policy years) deep in development are affected by the reform, then the model underestimates the reform impact (but still estimates the post-reform ultimate loss accurately because it is the post-reform development pattern that materializes in the post-reform diagonals). But then there is a third situation where the model is not able to quantify the post-reform ultimate loss (as well as the impact of the reform). Such situation arises when the reform affects the run-off deep in development of new claims only, as is the case when a second-injury fund is eliminated. Because the reform takes many years to play out in the data (that is, manifest itself in incremental payments of new claims deep in development), the model is incapable of quantifying such reform impact immediately.

When estimating the loss development model, the pre-reform and post-reform triangles are estimated simultaneously, subject to the constraint that the two triangles have identical calendar-year effects, identical rates of growth of the expected value of the logarithm of the first payment, and identical variances in the measurement equations of the incremental payments. For details on the model, see Appendices 2 and 3.

3. RESULTS

Odd-numbered charts exhibit the indemnity results, whereas even-numbered charts display the results for medical.

Charts 1 and 2 show the indemnity and medical benefits estimated run-off rates (*delta*) along the development year axis—remember that the run-off rates are the rates of growth of the incremental payments, adjusted for the calendar-year effect. As mentioned, the run-off rates beyond the final observed year of development incorporate mortality information. Whereas the displayed run-off rates for medical benefits (Chart 2) describe the trajectory of the run-off rate as employed in the computation of the ultimate loss (and, hence, the tail factor), the run-off trajectory of the incremental indemnity payments (Chart 1) needs adjustment before inputting it into the computation of the ultimate loss or the tail factor; this is because indemnity benefits may not be granted for life, or there may be a social security offset. (If there is an immediate social security offset that applies regardless of the age of the claimant, then such offset is captured by the trajectory of the run-off rate *delta*.) In the unidentified state in question, effective May 1992, a social security offset applies to accidents that happen past age 55 or within five years of the legal retirement age. As a result of this legislative change, the incremental payments for development years 41 through 70 (70 being the final development year) were reduced by 50 percent of what would be projected otherwise.

Charts 3 and 4 present the expert priors (lines with full circles) and the posteriors for the calendar-year effect in the second development year (which is the first year of escalation). Due to this being a policy year triangle, the prior in the displayed second development year comprises 18 months of inflation (which is the time difference between the mid-points of the first 24 months of experience and the subsequent 12 months of experience). Note that, in general, a systematic difference between the expert prior for the calendar-year effect and the (unknown) workers compensation-specific rate of inflation factors into the run-off rate *delta*. Specifically, if for all incremental payments the actual (logarithmic) rate of benefits inflation exceeds the expert prior by a constant *c* (which may be positive or negative), then such constant will be absorbed by the rate of decay (*delta*) of the calendar-year effect-adjusted incremental payments—in statistical terms, the parameter *c* is unidentified.

Whereas the prior for medical inflation (Chart 4) is the M-CPI for all policy years, the prior for indemnity escalation (Chart 3) is a weighted average of the legally stipulated rates of escalation (of which the model accommodates two non-zero rates of escalation in addition to the zero rate [no escalation]). For instance, for the second development year, the rate of escalation that applies to a given type of claim (for a given policy or accident year) is weighted by the fraction of (incremental) losses associated with the given type of claim in the first development year. (Note that the fraction of incremental losses that applies to a given type of claim for a given development year is held constant for every policy (or accident) year, as such information is not available for every single policy or accident year.) Before policy year 1984, there was no escalation of indemnity benefits. Then, in policy year 1984, the escalation of fatal claims (at 4 percent), as introduced in June 1986, shows up in the prior (to the extent that this policy year was affected by the legislative change). The weight of such escalation increased in policy year 1985 before reaching (in policy year 1986) the level that corresponds to the fraction of Fatal (incremental) losses in the first development year. This level of escalation then rose again in policy year 1989 when in May 1991 PT claims started escalating at the CPI rate of inflation. This escalation of PT claims reached its full weight (at the fraction of PT incremental losses in the first development year) in policy year 1991. Note that because CPI inflation varies over time, the expert prior for the escalation of indemnity claims shows time variation even after 1991 (as indicated by the slight bumps in the applicable line in Chart 3).

Charts 5 and 6 displays the priors (lines with full circles) and the posteriors for the calendar-year effect of the latest observed diagonal; remember that there are no observations available for the final six values of the latest diagonal, which is why for these values the posterior equals the prior. Again, note that the first value on the diagonal comprises 18 months of inflation. For medical benefits, the expert prior for the calendar-year effect (which is the M-CPI; Chart 6) is uniform along the diagonal, except for the first value, which comprises inflation of a longer time period. For indemnity benefits, the expert prior for the rate of escalation (as determined by the pertinent legal stipulations) varies along the diagonal (beyond the initial change caused by switching from 18 months of inflation to 12 months of inflation); this is because diagonals span several development years. As a given set of indemnity claims develops, the proportions of incremental payments going to the various claim types (TT, PP, PT, and fatal) change; if these claim types escalate at different rates, then the expert prior for the escalation of the total of incremental payments within a given calendar year (diagonal) varies by development year. As mentioned, fatal claims escalate at four percent and PT claims escalate at the rate of CPI inflation; because the fraction of these claims is small in the first development year, the expert prior for the rate of escalation embedded in the total incremental payments in the second development year is close to zero. As development progresses, the fractions of incremental payments that apply to these two types of claim increases, as indicated by the rising line (full circles) in Chart 5 for development years 2 through 6. After 312 weeks of benefits, PP claims start escalating at the rate of CPI inflation. With TT claims having expired (or technically behaving like PT or PP claims), all claims escalate from development year 6 onward. (Fatal claims keep escalating at the stipulated four percent, whereas all other claims escalate at the CPI rate of inflation.)

Charts 7 and 8 show for $1 of initial (that is, first report) payment, kernel density estimates for the impact of the reform-induced change in the run-off rate (*delta*) on the ultimate loss for (the future) policy year 2008; remember that the first year comprises 24 months of development. Note that the payments are adjusted for the calendar-year effect; otherwise, studying the reform-induced difference in the ultimate loss would require choosing a specific pre-reform reference year (because of the time variation of the rate of inflation). Breaking out the reform impact on medical benefits is straightforward as for medical benefits, legislative reforms generally feed into the run-off rate *delta*. (Remember that any systematic difference between the workers compensation-specific medical inflation and M-CPI inflation are captured by the run-off rate *delta*; hence, any changes to the difference between these two inflation rates will be reflected in changes to *delta*.) Breaking out the reform impact on the ultimate loss of indemnity is more demanding than isolating such impact on the ultimate loss of medical; this is because legislative changes may not only change the run-off rate but also affect the stipulated rate of escalation, age limit for benefits, duration of benefits, or social security offset. The reform impact on the ultimate loss in indemnity, as depicted in Chart 7, is adjusted for the calendar-year effect, which means that the legislative changes to the applicable rates of escalation are not captured. Of course, the impact of the change in escalation can be broken out as well, but this requires choosing a specific reference year, as the CPI rate of inflation varies over time. (Alternatively, the ultimate losses of the various policy years [per $1 of initial payment] could be presented in a chart similar to Charts 9 and 10, which display the tail factors by policy year, while fully accounting for reform impacts.) As mentioned, to the extent that the 1990/92 reform cluster led to faster closing of existing claims and this way affected the run-off rates of post-1992 diagonals for pre-1990 policy years, the reform impact displayed in Charts 7 and 8 may be understated; this is because, even though the post-reform losses are accurately estimated, the "as-if-pre-reform" post-reform ultimate losses may be understated. Most interestingly, Chart 8 shows that the 1990/92 reform cluster indeed reduced the ultimate loss for medical (per $1 of initial payment), thus pointing to a faster run-off of medical payments due to increased scrutiny regarding continued eligibility for indemnity payments. As mentioned, the 1990/1992 reform cluster pertained mainly to indemnity benefits, but there was also a medical benefits reform, which occurred in November 1992.

Charts 9 and 10 exhibit the 19th-to-ultimate tail factors, differentiated by pre-reform and post-reform period; the post-reform period includes the future policy year (2008) of interest to ratemaking. The displayed tail factors rest on two alternative concepts. The first concept ("Tail Factors Based on b") computes the tail factors based on the estimated data-generating process. The second concept ("Tail Factors Based on *y.hat*") computes the tail factor based on the estimated incremental payments. Generally, for future policy (or accident) years, depending on the case, the two concepts generate the same number. The tail factor trajectory that stretches over the entire analyzed time period offers actual pre-reform tail factors (to the left of the left-most vertical separator) and "as-if-pre-reform" post-reform tail factors (to the right of the right-most vertical separator). The vertical differences between the "as-if-pre-reform" post-reform tail factors and the actual post-reform tail factors gauge the (full) reform impact. As argued above, to the extent that the reform cluster affected post-1992 diagonals for pre-1990 policy years, the "as-if-pre-reform" post reform tail factor may be understated.

Charts 11 and 12 offer a demonstration of how sensitive tail factors are to the rate of inflation that applies to the pertinent future policy year 2008. For indemnity, this rate of inflation is the rate of growth of the CPI, which is the (post-reform) stipulated rate of escalation for PP claims (after 312 weeks of benefits) and PT claims; the rate of escalation of fatal claims is kept at four percent. For medical, the rate of inflation is the M-CPI. Note that, due to the convexity of the tail factor in the rate of inflation, greater variability in the rate of inflation entails larger tail factors when averaged across policy years.

Charts 13 through 18 are diagnostic tools. These charts gauge how well the model has been calibrated; they display by policy year (Charts 13 and 14), development year (Charts 15 and 16) and calendar year (Charts 17 and 18) the difference between the log incremental payments predicted by the data-generating process (*b*) and the actual log incremental payments (*y*); the solid line indicates the median difference. Early in development, the solid lines in Charts 15 and 16 must be close to zero; late in development, these lines may turn jagged as outliers (in the percentage difference between observed and predicted payments) become more likely. The diagnostic Charts 13 through 18 signify that the model is well calibrated (as the median differences [solid lines] show no persistent departure from the zero line); in particular, the calendar-year effect (Charts 17 and 18) is properly captured.

Charts 19 and 20 are another set of diagnostic tools. These charts inform about data outliers and may serve as data quality indicators. The charts display by policy year the difference between the actual log cumulative payments (*z*) and the fitted log cumulative payments (*z.hat*) along the development year time axis. Based on experience, values within the interval (-0.005; 0.005) indicate that the model is able to replicate the underlying data. Values outside this interval but within the interval (-0.01; 0.01) have to be considered outliers. Values outside the interval (-0.01; 0.01) must be considered data points of poor quality.

Chart 1: Indemnity: Trajectory for delta (Run-off Rate, Calendar-Year Effect-Adjusted); "9": Pre-Reform; "8": Post-Reform

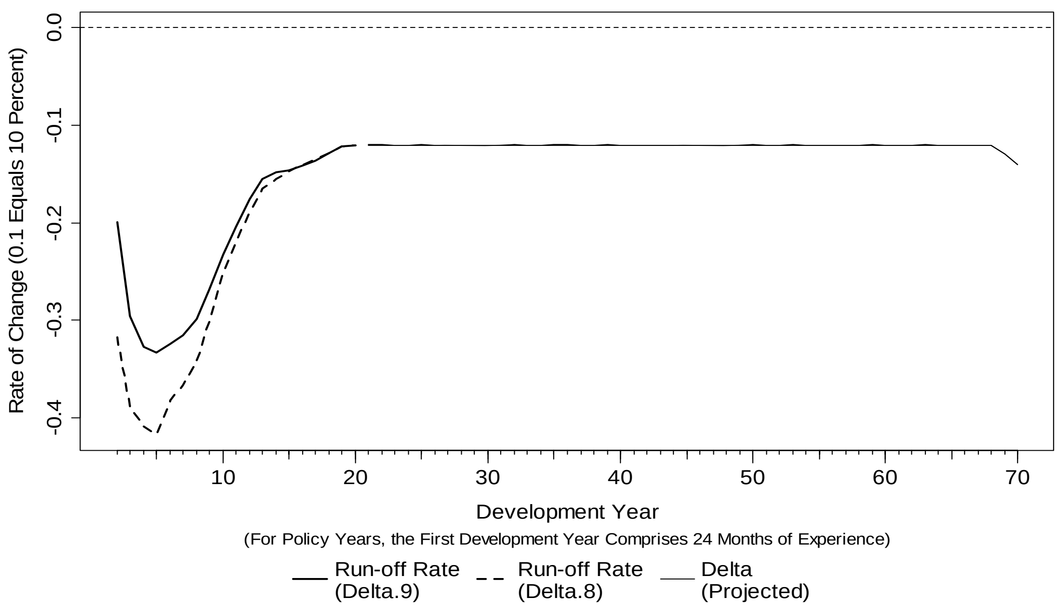


Chart 2: Medical: Trajectory for delta (Run-off Rate, Calendar-Year Effect-Adjusted); "9": Pre-Reform; "8": Post-Reform

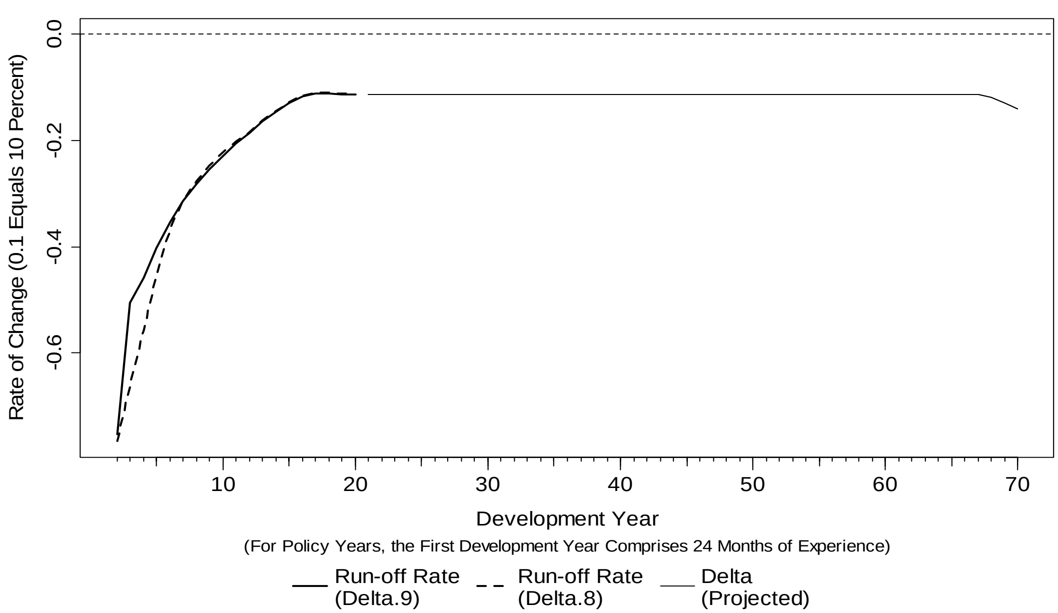


Chart 3: Indemnity: Calendar-Year Effect, Second Development Year

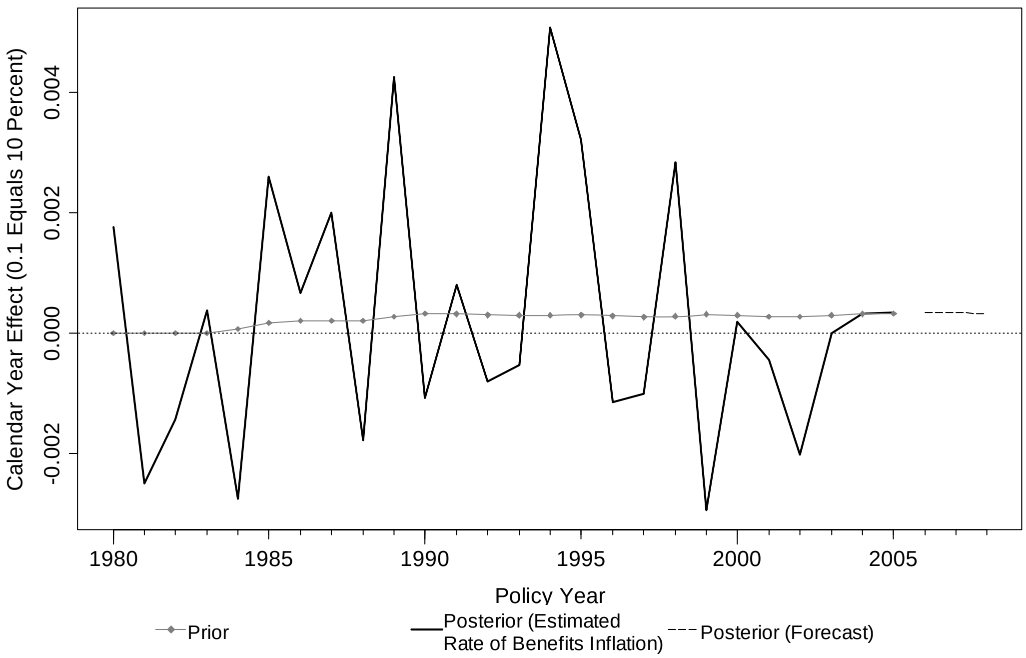


Chart 4: Medical: Calendar-Year Effect, Second Development Year

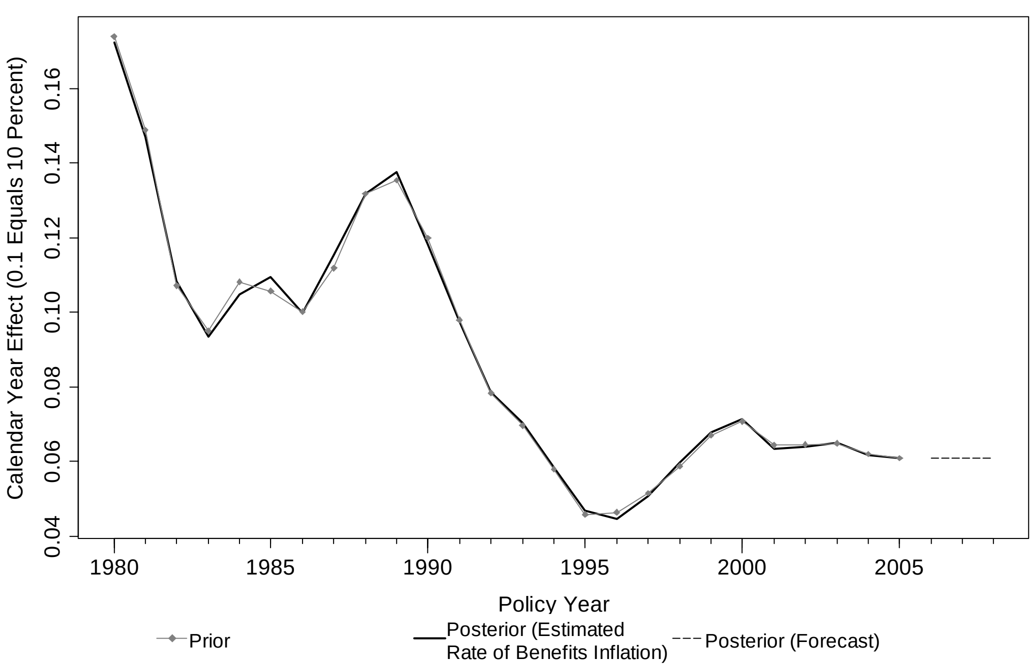


Chart 5: Indemnity: Calendar-Year Effect, Final Diagonal

Chart 6: Medical: Calendar-Year Effect, Final Diagonal

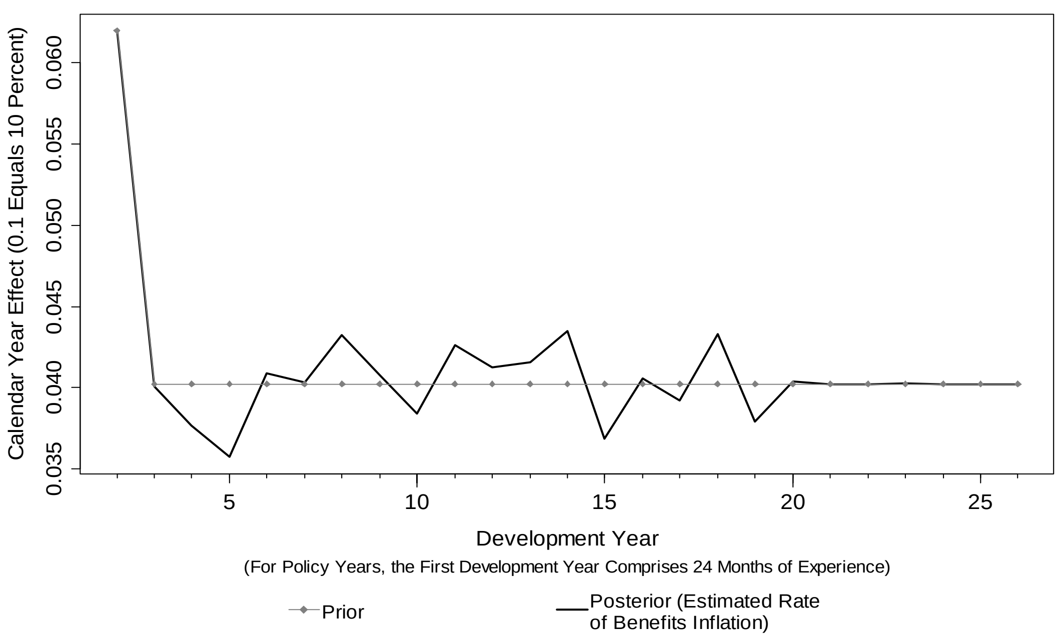


Chart 7: Indemnity: Reform Impact on the Ultimate Loss per $1 of First Report Payment (Adjusted for Calendar-Year Effect); Kernel Density Estimation

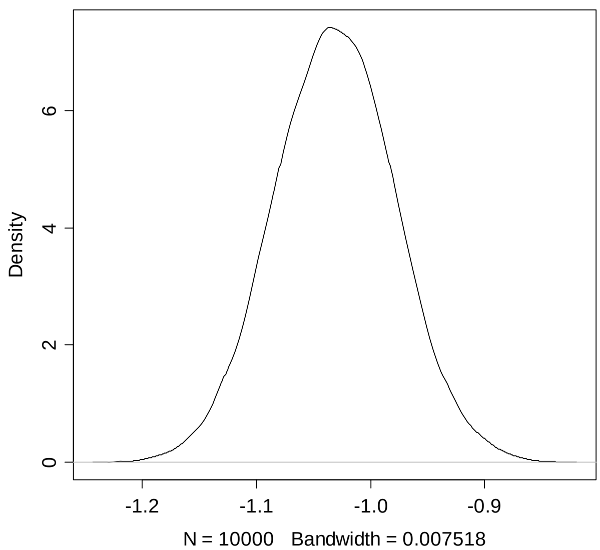


Chart 8: Medical: Reform Impact on the Ultimate Loss per $1 of First Report Payment (Adjusted for Calendar-Year Effect); Kernel Density Estimation

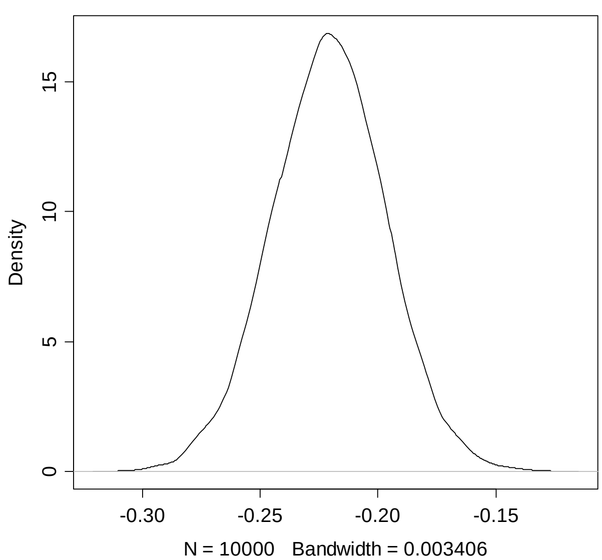


Chart 9: Indemnity: Tail Factor (Vertical Separators Border Reform Cluster)

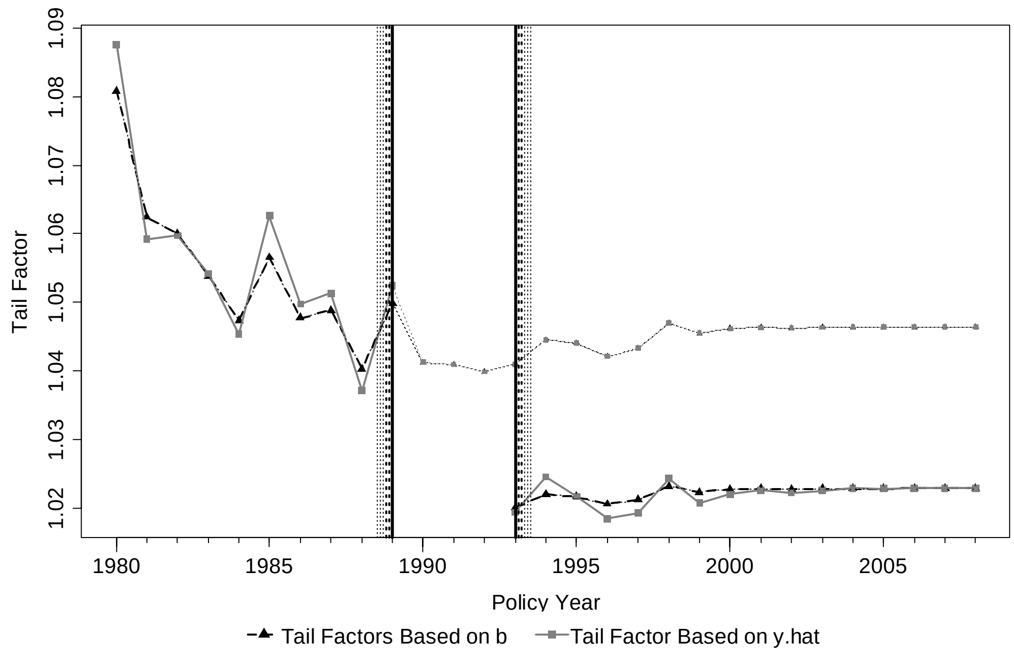


Chart 10: Medical: Tail Factor (Vertical Separators Border Reform Cluster)

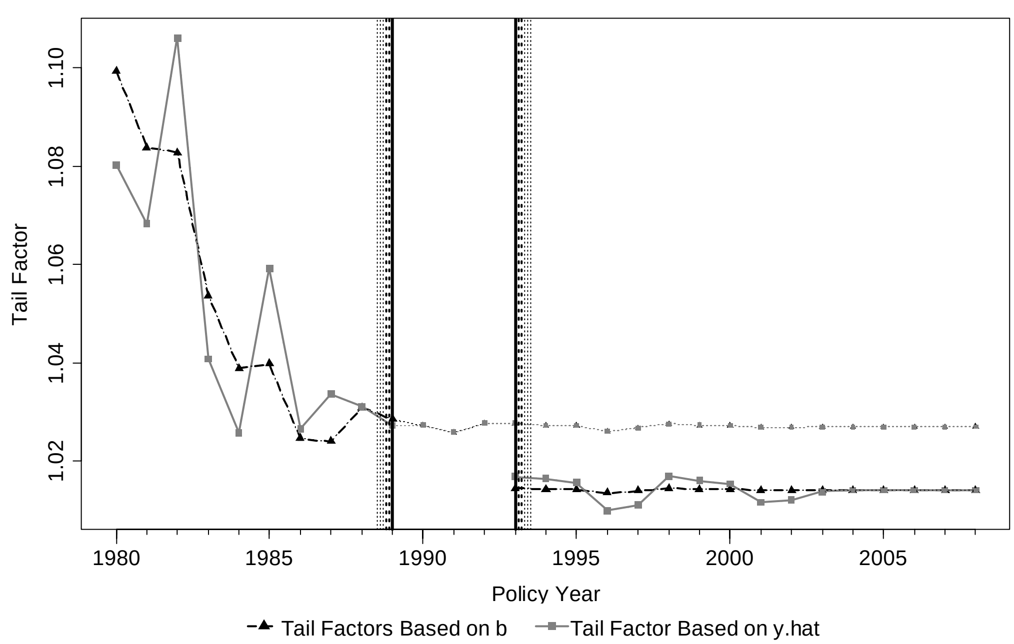


Chart 11: Indemnity: Sensitivity of Tail Factor to Official Rate of Inflation (CPI) for Policy Year 2008

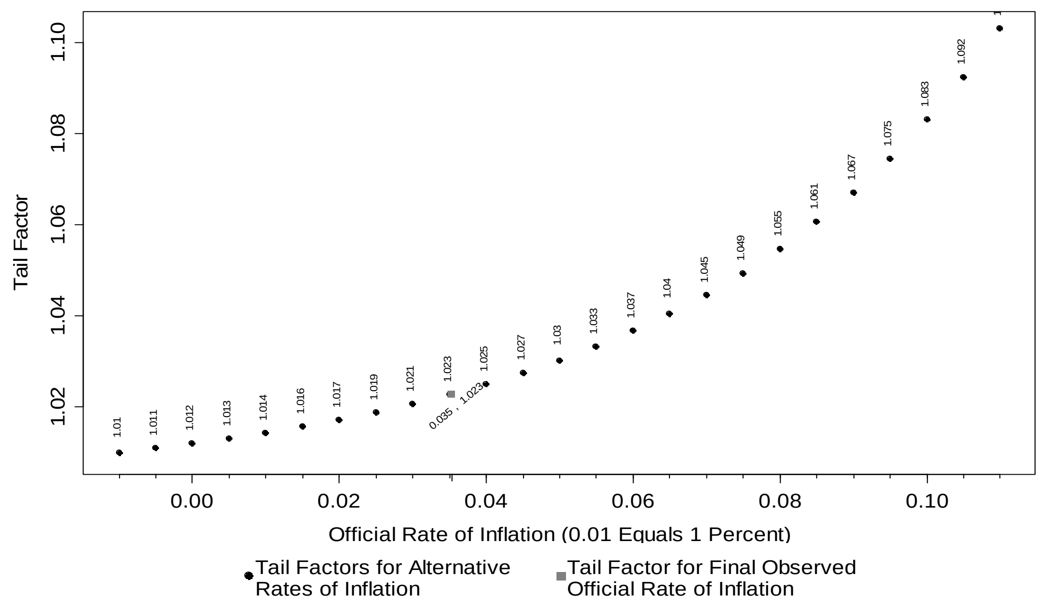


Chart 12: Medical: Sensitivity of Tail Factor to Official Rate of Inflation (M-CPI) for Policy Year 2008

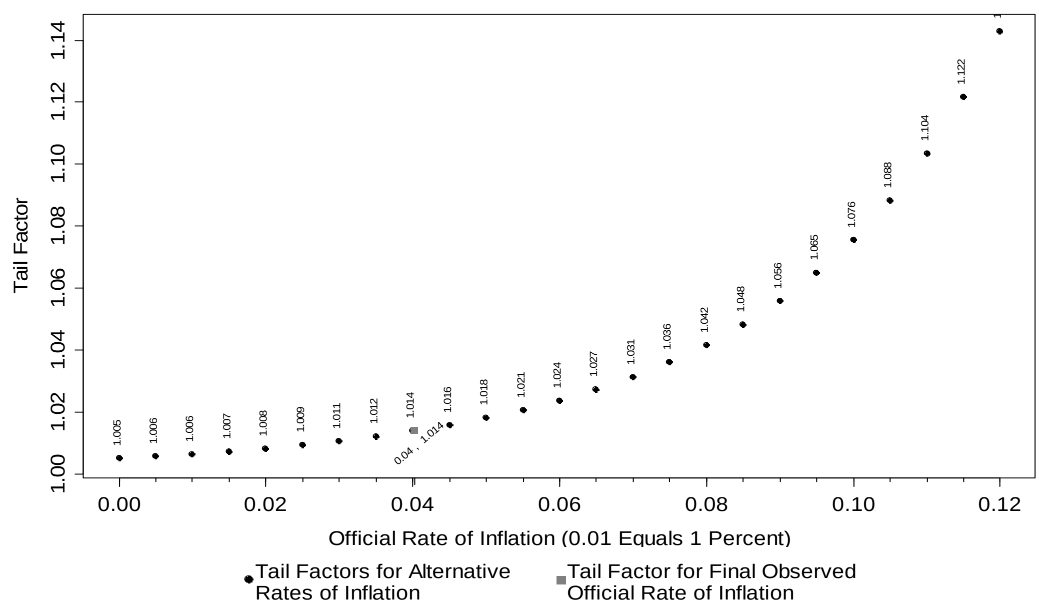


Chart 13: Indemnity: Difference between Actual Observations (*y*) and Estimated Process (*b*) by Policy Year, Post-Reform

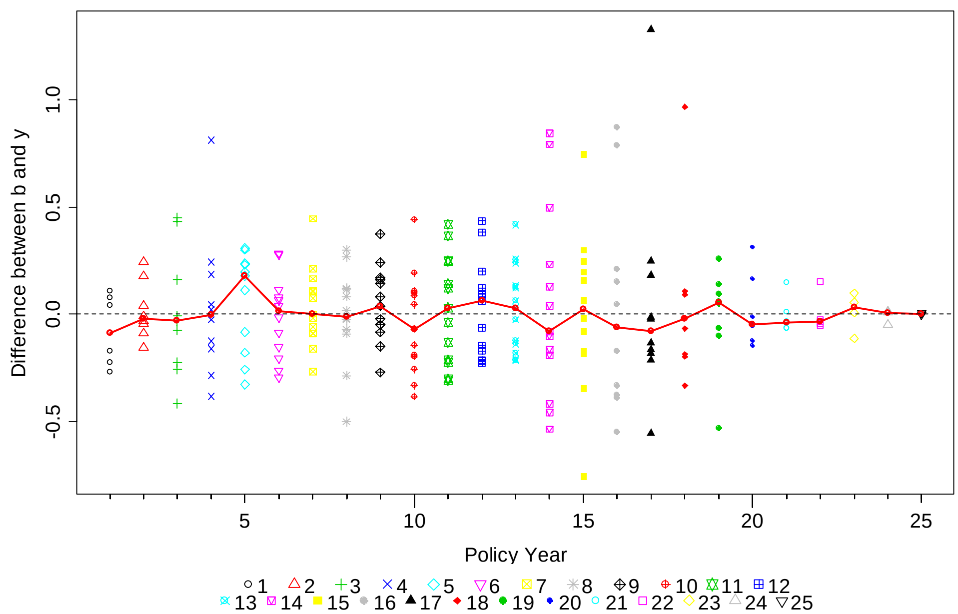


Chart 14: Medical: Difference between Actual Observations (*y*) and Estimated Process (*b*) by Policy Year, Post-Reform

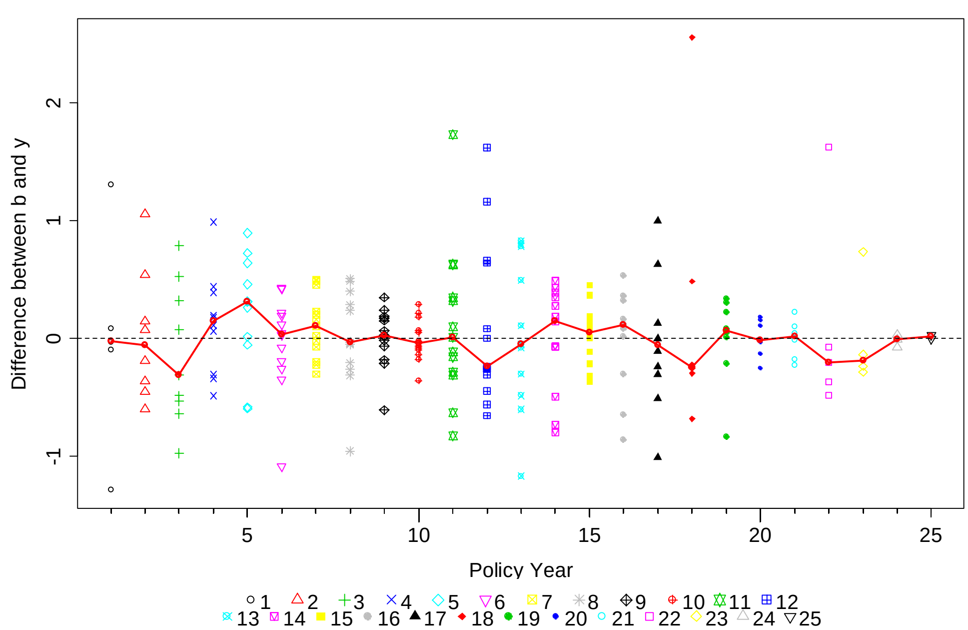


Chart 15: Indemnity: Difference between Actual Observations (*y*) and Estimated Process (*b*) by Development Year, Post-Reform

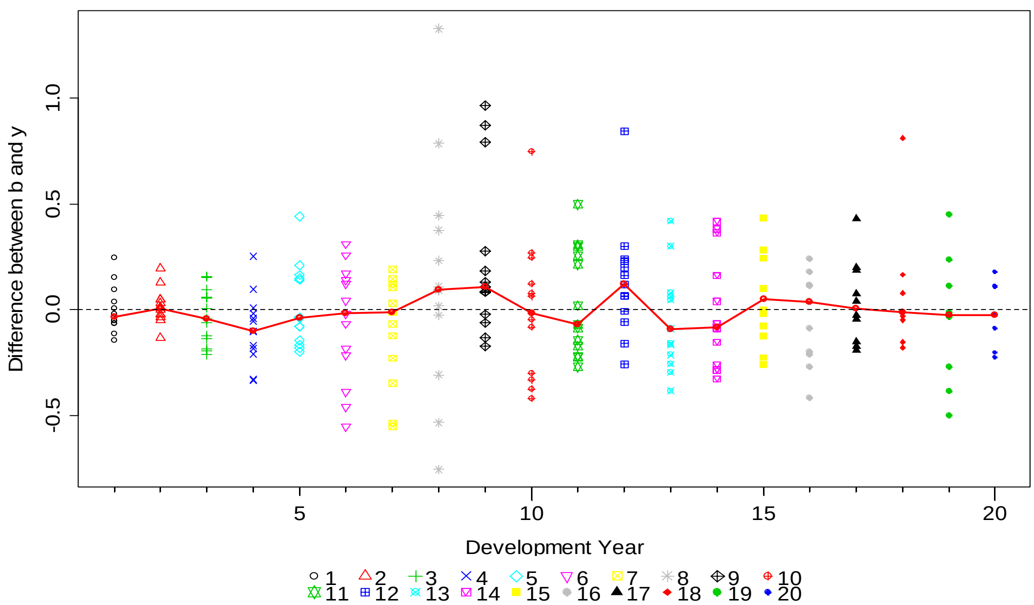


Chart 16: Medical: Difference between Actual Observations (*y*) and Estimated Process (*b*) by Development Year, Post-Reform

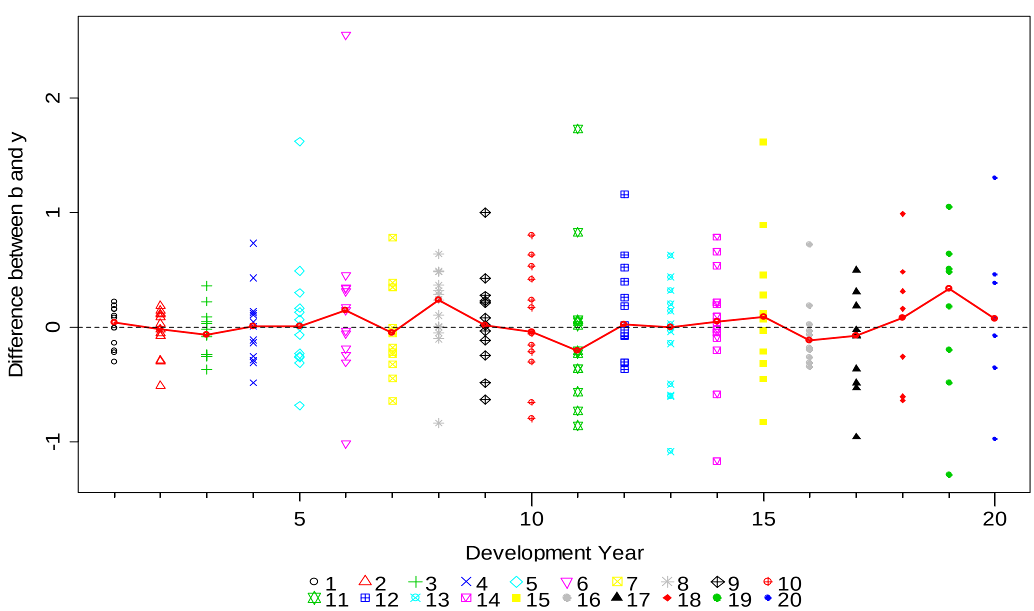


Chart 17: Indemnity: Difference between Actual Observations (*y*) and Estimated Process (*b*) by Diagonal (Calendar Year), Post-Reform

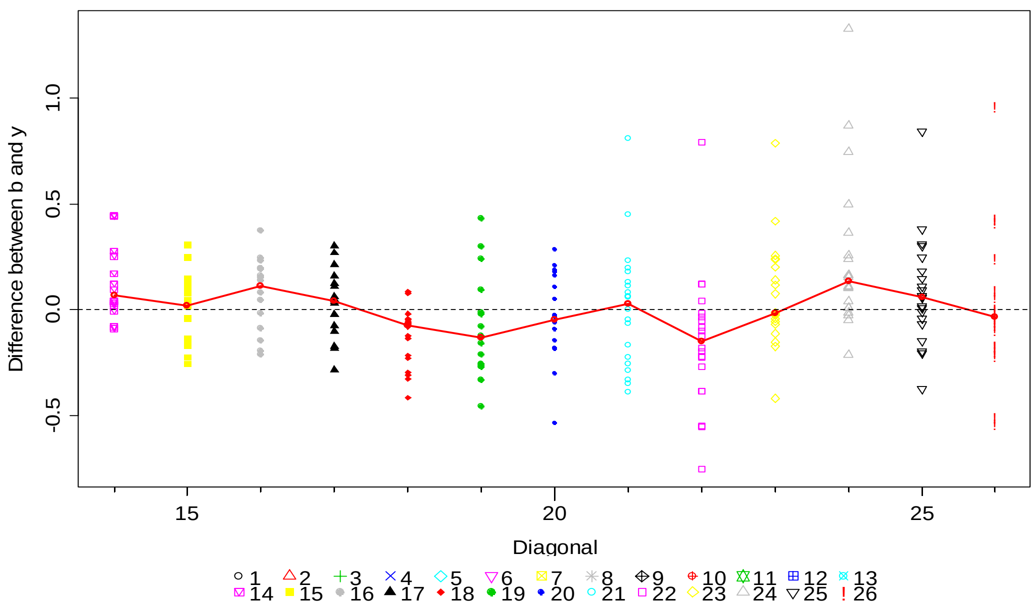


Chart 18: Medical: Difference between Actual Observations (*y*) and Estimated Process (*b*) by Diagonal (Calendar Year), Post-Reform

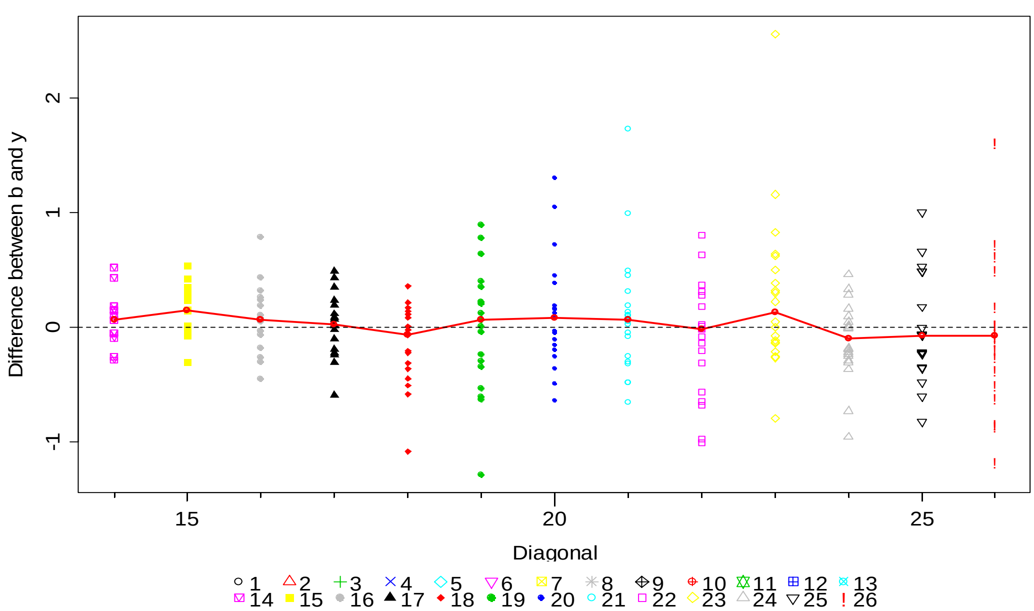


Chart 19: Indemnity: Actual Log Cumulative minus Predicted Log Cumulative Payments, Post-Reform

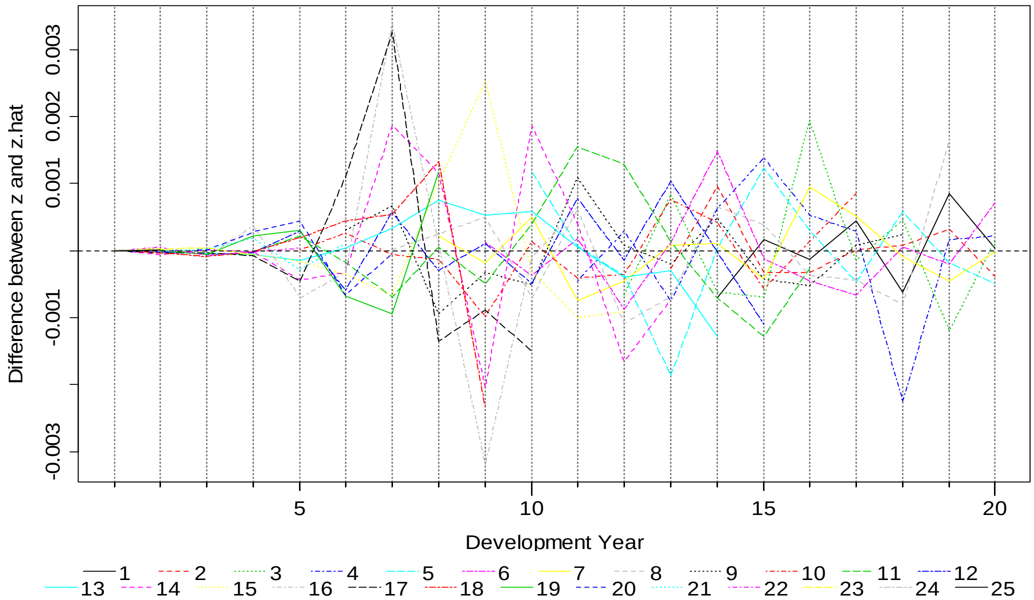
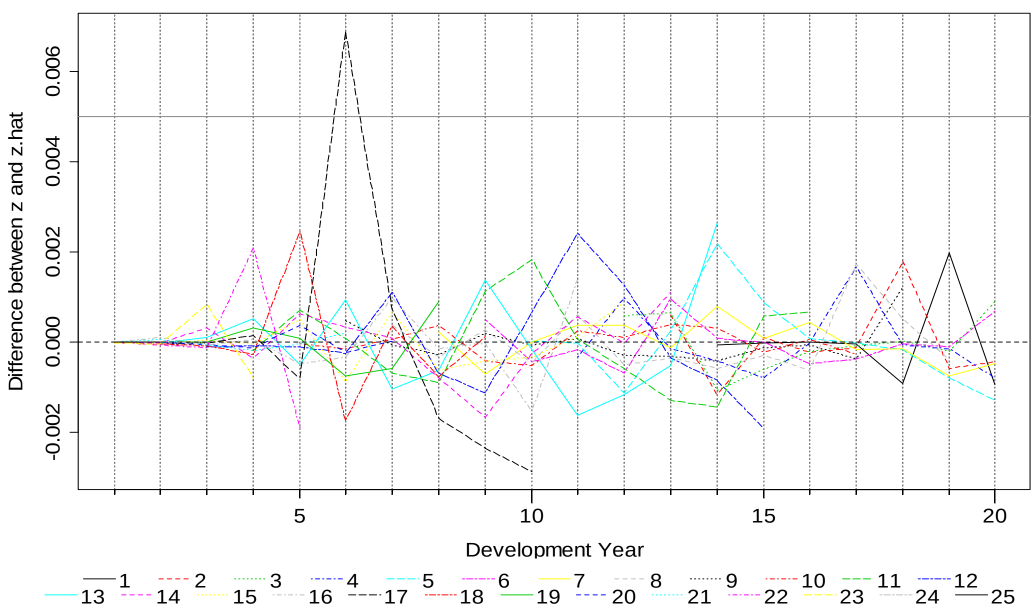


Chart 20: Medical: Actual Log Cumulative minus Predicted Log Cumulative Payments, Post-Reform



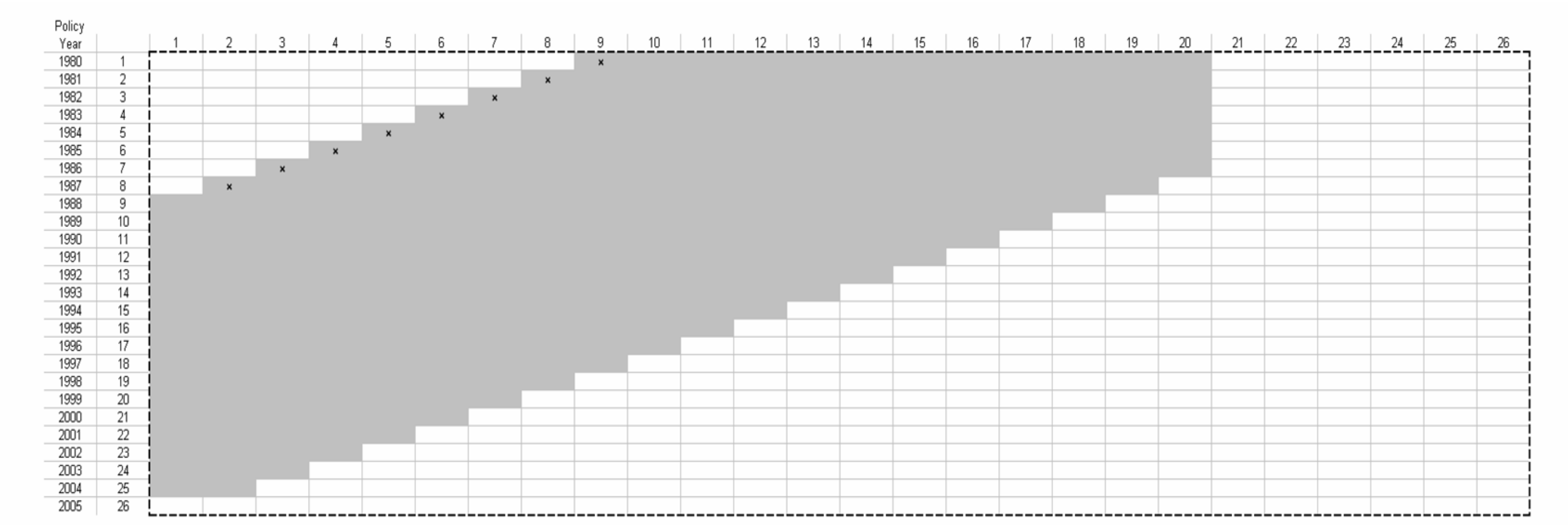
4. CONCLUSIONS

A loss development model has been presented that explicitly accounts for the legislative environment that applies to the time period during which the losses have been observed. Most importantly, the model accommodates changes in the legislative environment, which may be multi-faceted, having either diagonal (calendar year) or horizontal (policy year) effects (or both). The application of the model to an unidentified state demonstrates how, due to its high degree of flexibility, the model is capable of accommodating complex changes to loss development patterns. Further, the model is able to break out and quantify individual aspects of the legislative reform, such as calendar-year effects versus changes to the (calendar-year effect-adjusted) run-off.

Most interesting to the practicing actuary is the ability of the model to incorporate expert information as Bayesian priors in the estimation process. As shown, such expert priors may be legally stipulated rates of escalation (for indemnity) or information on medical price inflation at large (where more detailed information on the inflation embedded in medical benefits is unavailable).

Appendix 1

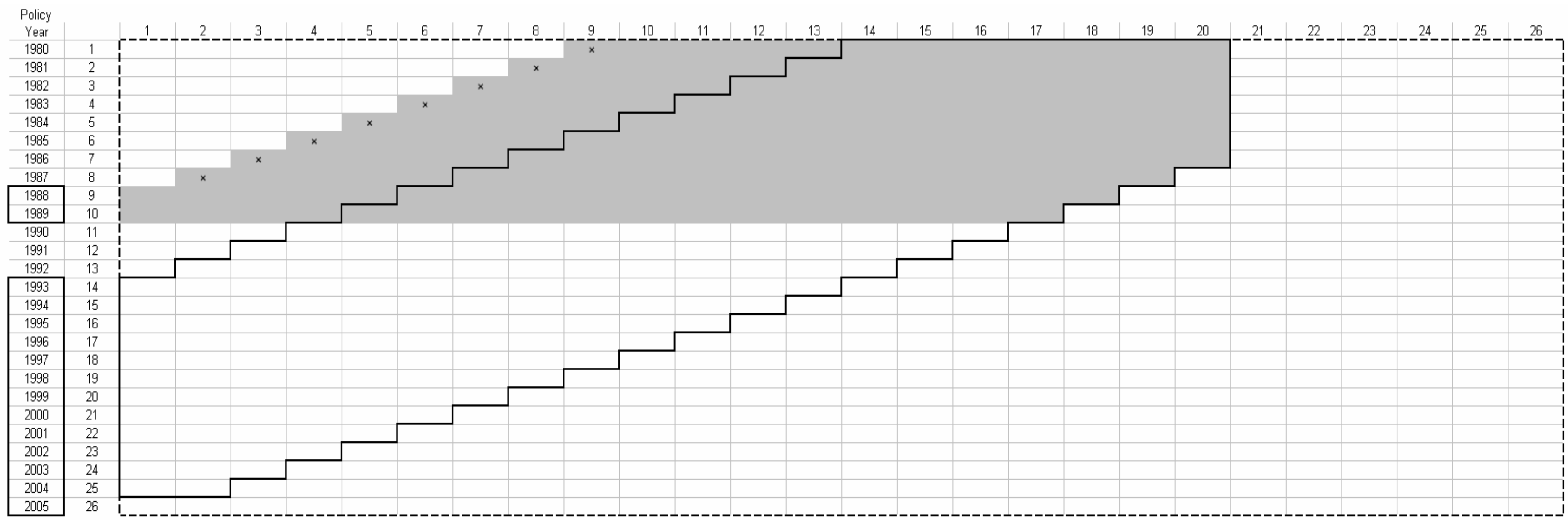
Exhibit A-1: Loss Triangle Template, Indemnity and Medical



Note: Available payments are shaded gray. For the cells marked by the symbol ×, only cumulative (but no incremental) payments are available.

Appendix 1, *cont.'d*

Exhibit A-2: Loss Triangle Template, Pre-Reform and Post-Reform



Note: The payments constituting the pre-reform triangle are shaded gray; the payments forming the post-reform triangle are framed by a solid line. For the cells marked by the symbol ×, only cumulative (but no incremental) payments are available.

Appendix 2: Pre-Reform Model (Model Type 9)

$$y_{i,j} \sim \mathrm{N}(b_{i,j,9}, \sigma_y^2) \begin{cases} i = 1,\ldots,rg-1,\ j = cg-i+2,\ldots,cf-i+1 \\ i = 2,\ldots,rf,\ j = cf-i+2,\ldots,cf \\ i = rf+1,\ldots,rh_9,\ j = cf-i+2,\ldots,c-i+1 \\ i = cf+2,\ldots,rh_9,\ j = 1,\ldots,c-i+1 \\ i = rg,\ldots,rh_9,\ j = 1,\ldots,cf-i+1 \end{cases} \tag{A2-1}$$

$$cs.mean_{i,j,9} \begin{cases} = \hat{z}_{i,j,9} - z_{i,j} \quad \text{for} \begin{cases} i = 1,\ldots,rg-1,\ j = cg-i+1,\ldots,cf-i+1 \\ i = 2,\ldots,rf,\ j = cf-i+2,\ldots,cf \\ i = rf+1,\ldots,rh_9,\ \ j = cf-i+2,\ldots,c-i+1 \\ i = cf+2,\ldots,rh_9,\ \ j = 1,\ldots,c-i+1 \\ i = rg,\ldots,rh_9,\ \ j = 1,\ldots,cf-i+1 \end{cases} \\ = mv_{\mu,i,j,9} \quad \text{for} \begin{cases} i = 1,\ldots,rg-1,\ j = 1,\ldots,cg-i \\ i = 1,\ldots,rf-1,\ j = cf+1,\ldots,c-i+1 \\ i = rg_9+1,\ldots,r,\ j = 1,\ldots,c-i+1 \\ i = 2,\ldots,r,\ j = c-i+2,\ldots,c \end{cases} \end{cases} \tag{A2-2}$$

$$\hat{y}_{i,j,9} = \mathrm{N}(b_{i,j,9}, \sigma_y^2)\ ,\ i, j = 1,\ldots,c \tag{A2-3}$$

$$\hat{z}_{i,j,9} = \log\left(\sum_{k=1}^{j} \exp\left(\hat{y}_{i,k,9}\right)\right)\ ,\ i, j = 1,\ldots,c \tag{A2-4}$$

$$\mathbf{mv}_9 \sim \mathrm{N}(\mathbf{mv}_{\mu,9}, \mathbf{\Omega}_1) \tag{A2-5}$$

$$\mathbf{mv}_{\mu,9} \sim \mathrm{N}(\mathbf{mv}_9, \mathbf{\Omega}_2) \tag{A2-6}$$

$$cs_{i,j,9} = 0\ ,\ i, j = 1,\ldots,c \tag{A2-7}$$

$$\mathbf{cs}_{i,9}{}' \sim \mathrm{N}(\mathbf{cs.mean}_9{}', \mathbf{\Omega}_1)\ ,\ i = 1,\ldots,r \tag{A2-8}$$

$$b_{r_h,1,9} \sim \mathrm{N}(y_{r_h,1,9}, \sigma_{b.init}^2) \tag{A2-9}$$

$$b_{i,1,9} = b_{i+1,1,9} - \Lambda_{i+1,1,9} \;,\; i = 1,\ldots,r_{h,9} - 1 \qquad \text{(A2-10)}$$

$$\Lambda_{i+1,1,9} = \eta_{i+1} \;,\; i = 1,\ldots,r_{h,9} - 1 \qquad \text{(A2-11)}$$

$$\eta_i \sim \mathrm{N}(\eta_{i-1}, \sigma_\eta^2) \;,\; i = 3,\ldots,r \qquad \text{(A2-12)}$$

$$\eta_2 \sim \mathrm{N}(0, \sigma_\eta^2) \qquad \text{(A2-13)}$$

$$b_{i,1,9} = b_{i-1,1,9} + \Lambda_{i,1,9} \;,\; i = r_{h,9} + 1,\ldots,r \qquad \text{(A2-14)}$$

$$\Lambda_{i,1,9} = \eta_{i,1} \;,\; i = r_{h,9} + 1,\ldots,r \qquad \text{(A2-15)}$$

$$b_{i,j,9} = b_{i,j-1,9} + \Lambda_{i,j,9} \;,\; i = 1,\ldots,r-1,\; j = 2,\ldots,c-i+1 \qquad \text{(A2-16)}$$

$$\Lambda_{i,j,9} = \delta_{j,9.pre} + \kappa_{i,j} \;,\; i = 1,\ldots,r_{g,9} - 1,\; j = 2,\ldots,c_{g,9} - i + 1 \qquad \text{(A2-17)}$$

$$\Lambda_{i,j,9} = \delta_{j,9} + \kappa_{i,j} \;,\; i = 1,\ldots,r_{g,9} - 1,\; j = c_{g,9} - i + 2,\ldots,c - i + 1 \qquad \text{(A2-18)}$$

$$\Lambda_{i,j,9} = \delta_{j,9} + \kappa_{i,j} \;,\; i = r_{g,9},\ldots,r,\; j = 2,\ldots,c-i+1 \qquad \text{(A2-19)}$$

$$\delta_{j,9} \sim \mathrm{N}(\delta.prior_j, \sigma_{\delta,2}^2) \;,\; j = 2,3 \qquad \text{(A2-20)}$$

$$\delta_{j,9} \sim \mathrm{N}(\delta_{j-1,9}, \sigma_{\delta,j}^2) \;,\; j = 4,\ldots,cf \qquad \text{(A2-21)}$$

$$\delta_{j,9} \sim \mathrm{N}(\delta_{cf}, \sigma_{\delta,1}^2) \;,\; j = cf + 1,\ldots,c \qquad \text{(A2-22)}$$

$$\sigma^2_{\delta,j} = \sigma^2_{\delta,1} \cdot 10^{-\alpha+\alpha \cdot e^{-\beta \cdot e^{-\gamma \cdot (j-1)}}} \text{ , } j = 4,...,cf;\ \alpha, \beta, \gamma > 0 \tag{A2-23}$$

$$\sigma^2_{\delta,2} \text{ , } \sigma^2_{b.init} \text{ large} \tag{A2-24}$$

$$\sigma^2_{\delta,1} \text{ small} \tag{A2-25}$$

$$\kappa_{i,j} \sim \mathrm{N}(\mu_{i,j}, \sigma^2_{\kappa}) \text{ , } i = 1,...,r;\ j = 2,...,c \tag{A2-26}$$

$$\mu_{i,j} = \lambda_{1,j} \cdot \pi_{1,i+j} + \lambda_{2,j} \cdot \pi_{2,i+j} \text{ , } i = 1,...,r;\ j = 2,...,c,\ \lambda_{1,j} + \lambda_{2,j} \le 1 \tag{A2-27}$$

where y, and $\hat{y}$ are the observed and estimated logarithmic incremental payments, respectively. For negative incremental payments, the corresponding values of y are coded as missing values. The indexes i and j indicate policy (or accident) and development years, respectively; $r = c$ signifies the number of years in the loss triangle. The parameter c_f signifies the column with the final value for the cumulative (and incremental) payment in the first $r_f - 1$ rows, where the first $r_f - 1$ rows are those affected by the cut-off in reported development. The parameter c_g signifies the first column that has a value for the cumulative payment in the first row; note that the first incremental payment in this row is located in column $c_g + 1$. The parameter $r_g (= c_g)$ indicates the first row that has a value for the cumulative (and thus incremental) payment in the first column.

The parameter $r_{g,9} (= c_{g,9})$ indicates the row (column) with the first pre-reform incremental payment in the first column (row). If there was no structural break prior to the reform of interest, then $r_{g,9} (= c_{g,9}) = r_g (= c_g)$. Conversely, if there was such a possible structural break, then the parameter $r_{g,9} (= c_{g,9})$ indicates the first row (column) with an incremental payment in the first column (row) that belongs to the post-structural-break pre-reform period.

Equation (A2-1) fits the observations of the logarithmic incremental payments to a normal distribution. Equation (A2-2) defines the deviation of the estimated logarithm of the cumulative payment ($\hat{z}_{i,j,9}$, where the index 9 indicates pre-reform) in policy (or accident) year j and development year i from and the observed logarithm of the cumulative payment ($z_{i,j}$); this deviation is denoted $cs.mean_{i,j}$, where cs stands for cumulative sum. Equation (A2-3) simulates the predicted values of the logarithmic incremental payments; these predicted values feed into the estimated logarithmic cumulative payments in Equation (A2-4). Where such cumulative sum does not exist (to the right of the final diagonal, up to the final observed development year), $cs.mean_{i,j}$ is replaced by a draw from a multivariate distribution, $mv_{\mu,i,j,9}$, as shown in Equation (A2-6). Specifically, the row vector $\mathbf{cs.mean}_i$ comprises the differences between the predicted and observed logarithmic cumulative payments of row i for those columns for which observed logarithmic cumulative payments are available; for all other columns, the elements of $\mathbf{cs.mean}_i$ are taken from a vector of (expected) values that generates a multivariate normal distribution of the same variance as the one that $\mathbf{cs.mean}_i$ is fitted to. The covariance matrices $\mathbf{\Omega}_{1,2}^{-1}$ are modeled on Wishart distributions. Equations (A2-5) and (A2-6) generate a distribution the $mv_{\mu,i,j,9}$ can be drawn from; the distributions of the observed and the generated values of $\mathbf{cs.mean}_i$ share the same covariance matrix, $\mathbf{\Omega}_1^{-1}$. Equation (A2-7) stipulates that the observed differences between the logarithms of the observed and estimated cumulative payments be zero, on average. Equation (A2-8) represents the cumulative sum (cusum) constraint. This stochastic constraint ensures that, for every cell of the loss triangle, the sum of estimated incremental payments lines up (approximately) with the observed cumulative payment. The cusum constraint also serves as a means of interpolating between incremental payments when there is a missing value (due to a negative incremental payment).

Equation (A2-9) initializes for the upper-left hand side region (where no observations are available for the first incremental payment) the first logarithmic increment payment on the first logarithmic incremental payment of the first row for which such a payment is available (denoted as row r_h).

Equations (A2-10-), (A2-11), and (A2-14) through (A2-19) describe the process displayed in Exhibit 1. Equation (A2-12) describes the random walk of *eta*, and Equation (A2-13) its starting value. Equation (A2-21) describes the random walk of *delta*, and Equation (A2-20) describes how the first two values of delta are estimated before the random walk sets in, whereas Equation (A2-22) details how delta is extrapolated into the future after the random walk ends with the final observed development year. Equation (A2-23) describes a Gompertz function for the innovation variance of the random walk of *delta*; this innovation variance approaches the variance displayed in Equation (A-25). The variance for estimating the first two values of *delta* (that is, before the random walk sets in) is shown in Equation (A2-24). Finally, Equations (A2-26 and A2-27) detail how the calendar-year effect is estimated using an expert prior on the rate of escalation (indemnity) and inflation (medical).

The model has two layers of noise, which implies that there are two predicted values (for each observed value of incremental payment). First, there is the variable b, which aggregates the three processes (run-off in development, growth of expected value of first payment, and calendar-year effect). Second, there is the variable $y.hat$, which is a draw from a normal distribution, the expected value of which is b. Where there are no observations (the run-off triangle is squared, the tail is estimated, and future policy or accident years are forecast), the variable $y.hat$ corresponds to the expected value, b. The variable $y.hat$ gauges the ability of the model to replicate the observed incremental payments.

The variables $\pi_{i,j}$ $(i=1,2)$ are expert priors for (logarithmic) rates of inflation, which may vary by policy (or accident) and development years. (For policy years, the first prior in any given policy (or accident) year comprises inflation for a period of 18 months, this being the time difference between the mid-point of the initial 24 months of experience and the subsequent 12-month period.) The model accommodates two non-zero rates of inflation, differentiated by type of claim; this is important for indemnity claims (but irrelevant for medical claims). Thus, the prior for the calendar-year effect in any given development year, j, is a weighted average of three (one zero and two non-zero) expert rates of inflation, the weights being the fractions of dollars in incremental payments that apply to up to two differently inflating claim types in development year $j-1$, $\lambda_{k,j}$, $k=1,2$ (while a third claim type may inflate at a zero rate). If there is only one claim type (as is the case for medical claims) or all claim types escalate at the same rate, then $\pi_{2,j}$ and $\lambda_{2,j}$ equal zero for all j, and $\lambda_{1,j}$ equals 1 for all j.

Specifically, for indemnity, the expert prior for the (logarithmic) calendar-year effect equals the official (logarithmic) rate of inflation relevant to the cost-of-living adjustment, weighted by the fractions of incremental dollars that have been paid on escalating claims in the development year $j-1$, λ_j. The official rate of inflation pertinent to cost-of-living adjustment may be the rate of growth of the state-level average weekly wage (as measured by the Quarterly Census of Employment and Wages, QCEW, http://www.bls.gov) or the U.S. CPI (Consumer Price Index, http://www.bls.gov), depending on the applicable legislative provision; we apply an observation and implementation lag of 14 months. The expert inflation prior for medical benefits is the (contemporaneous logarithmic) rate of growth of the Medical Care component of the U.S. CPI.

The QCEW average weekly wage is calculated as the ratio of the total wage bill for the calendar year, summed up over four quarterly values, and then divided by the average employment for the calendar year; this average employment for the calendar year is calculated from 12 monthly numbers. The Medical Care component of the CPI is the published annual calendar year number.

It is important to note that the rate of growth of the expected value of the first incremental payment (η) is specified in nominal terms, which means that the rate of inflation is not broken out. As a consequence, the mentioned inflation modeling applies solely to the way the incremental payments inflate in development but has no bearing on the how the first incremental payment inflates from one policy (or accident) year to the next.

The chosen set of hyper-parameters of the prior distributions has been calibrated to incremental payments, the logarithm of which fall into the range of 7 to 11; the incremental (and cumulative) payments of the loss triangle that is to be analyzed have to be normalized accordingly. With such normalization, the chosen set of hyper-parameters accommodates any sufficiently well-behaved triangle. As a consequence, the final calibration of the model when applied to a loss triangle is done solely by choosing the three parameters of the Gompertz function, with one exception; this exception concerns the variance of the rate of growth of the expected value of the first payment, as exhibited in Equations (A2-12, 13). For triangles with a high degree of variation in the rate of growth of the first incremental payment (such as percentage point differences in the higher double digits), a larger variance is needed. Further, the parameters of the Gompertz function need to be chosen. This Gompertz function serves the purpose of smoothing the run-off rate δ by means of controlling the innovation variance of the random walk. The Gompertz function accommodates convex, concave, and "S"-shaped trajectories of this variance. The first parameter of the Gompertz function, α, determines the upper asymptote; the parameter β is (roughly) a horizontal shift parameter, and the parameter γ determines the rate of the growth (that is, the steepness and curvature). The choice of the parameters β and γ is ultimately a matter of judgment, especially for small triangles. Several diagnostic charts have been developed (as discussed in the body of the text) that assist in this choice.

Note that the pre-reform and post-reform models have all variances in common; further, the two models have a common calendar-year effect and common rates of growth of the expected value of the first payment. For all scalar variances in the model, there are gamma distributions used as priors.

Appendix 3: Post-Reform Model (Model Type 8)

$$y_{i,j} \sim \mathrm{N}(b_{i,j,8}, \sigma_y^2) \begin{cases} i = 1,\ldots,rg_8 - 1,\ j = cg_8 - i + 1,\ldots,cf - i + 1 \\ i = 2,\ldots,rf,\ j = cf - i + 2,\ldots,cf \\ i = rf + 1,\ldots,cf + 1,\ j = cf - i + 2,\ldots,c - i + 1 \\ i = cf + 2,\ldots,rh_8,\ j = 1,\ldots,c - i + 1 \\ i = rg_8,\ldots,cf,\ j = 1,\ldots,cf - i + 1 \end{cases} \tag{A3-1}$$

$$cs_{i,j,8} \begin{cases} = \hat{z}_{i,j,8} - z_{i,j} & \text{for} \begin{cases} i = 1,\ldots,rg_8 - 1,\ j = cg_8 - i + 1,\ldots,cf - i + 1 \\ i = 2,\ldots,rf,\ j = cf - i + 2,\ldots,cf \\ i = rf + 1,\ldots,cf + 1,\ \ j = cf - i + 2,\ldots,c - i + 1 \\ i = cf + 2,\ldots,rh_8,\ \ j = 1,\ldots,c - i + 1 \\ i = rg_8,\ldots,cf,\ \ j = 1,\ldots,cf - i + 1 \end{cases} \\ = mv_{\mu,i,j,8} & \text{for} \begin{cases} i = 1,\ldots,rg_8 - 1,\ j = 1,\ldots,cg_8 - i \\ i = 1,\ldots,rf - 1,\ j = cf + 1,\ldots,c - i + 1 \\ i = rh_8 + 1,\ldots,r,\ j = 1,\ldots,c - i + 1 \\ i = 2,\ldots,r,\ j = c - i + 2,\ldots,c \end{cases} \end{cases} \tag{A3-2}$$

$$\hat{y}_{i,j,8} = \mathrm{N}(b_{i,j,8}, \sigma_y^2)\ ,\ i, j = 1,\ldots,c \tag{A3-3}$$

$$\hat{z}_{i,j,8} = \log\left(\sum_{k=1}^{j} \exp\left(\hat{y}_{i,k,8}\right)\right)\ ,\ i, j = 1,\ldots,c \tag{A3-4}$$

$$\mathbf{mv}_8 \sim \mathrm{N}(\mathbf{mv}_{\mu,8}, \mathbf{\Omega}_1) \tag{A3-5}$$

$$\mathbf{mv}_{\mu,8} \sim \mathrm{N}(\mathbf{mv}_8, \mathbf{\Omega}_2) \tag{A3-6}$$

$$cs_{i,j,8} = 0\ ,\ i, j = 1,\ldots,c \tag{A3-7}$$

$$\mathbf{cs}_{i,8}{}' \sim \mathrm{N}(\mathbf{cs.mean}_8{}', \mathbf{\Omega}_1)\ ,\ i = 1,\ldots,c \tag{A3-8}$$

$$b_{r_h,1,8} \sim \mathrm{N}(y_{r_h,1,8}, \sigma^2_{b.init}) \qquad \text{(A3-9)}$$

$$b_{i,1,8} = b_{i+1,1,8} - \Lambda_{i+1,1,8} \ , \ i = 1,\ldots, r_{h,8} - 1 \qquad \text{(A3-10)}$$

$$\Lambda_{i+1,1,8} = \eta_{i+1} \ , \ i = 1,\ldots, r_{h,8} - 1 \qquad \text{(A3-11)}$$

$$b_{i,1,8} = b_{i-1,1,8} + \Lambda_{i,1,8} \ , \ i = r_{h,8} + 1,\ldots, r \qquad \text{(A3-12)}$$

$$\Lambda_{i,1,8} = \eta_i \ , \ i = r_{h,8} + 1,\ldots, r \qquad \text{(A3-13)}$$

$$b_{i,j,8} = b_{i,j-1,8} + \Lambda_{i,j,8} \ , \ i = 1,\ldots, r-1, \ j = 2,\ldots, c-i+1 \qquad \text{(A3-14)}$$

$$\Lambda_{i,j,8} = \delta_{j,9.pre} + \kappa_{i,j} \ , \ i = 1,\ldots, r_{g,9} - 1, \ j = 2,\ldots, c_{g,9} - i + 1 \qquad \text{(A3-15)}$$

$$\Lambda_{i,j,8} = \delta_{j,89} + \kappa_{i,j} \ , \ i = 1,\ldots, r_{g,9} - 1, \ j = c_{g,9} - i + 2,\ldots, c_{g,8} - i + 1 \qquad \text{(A3-16)}$$

$$\Lambda_{i,j,8} = \delta_{j,89} + \kappa_{i,j} \ , \ i = r_{g,9},\ldots, r_{g,8} - 1, \ j = 2,\ldots, c_{g,8} - i + 1 \qquad \text{(A3-17)}$$

$$\Lambda_{i,j,8} = \delta_{j,8} + \kappa_{i,j} \ , \ i = 1,\ldots, r_{g,8} - 1, \ j = c_{g,8} - i + 2,\ldots, c - i + 1 \qquad \text{(A3-18)}$$

$$\Lambda_{i,j,8} = \delta_{j,8} + \kappa_{i,j} \ , \ i = r_{g,8},\ldots, r, \ j = 2,\ldots, c - i + 1 \qquad \text{(A3-19)}$$

$$\delta_{j,8} \sim \mathrm{N}(\delta.prior_j, \sigma^2_{\delta,2}) \ , \ j = 2,3 \qquad \text{(A3-20)}$$

$$\delta_{j,8} \sim \mathrm{N}(\delta_{j-1,8}, \sigma^2_{\delta,j}) \ , \ j = 4,\ldots, cf \qquad \text{(A3-21)}$$

$$\delta_{j,8} \sim \mathrm{N}(\delta_{cf}, \sigma^2_{\delta,1}) \ , \ j = cf + 1,\ldots,c \tag{A3-22}$$

$$\delta.diff_j = \delta_{j,8f} - \delta_{j,9} \ , \ j = 4,\ldots,cf \tag{A3-23}$$

$$\delta.diff.z_j \sim \mathrm{N}(\delta.diff_j, \sigma^2_{\delta,j}) \ , \ j = 4,\ldots,cf \tag{A3-24}$$

$$\delta.diff.z_j = 0 \ , \ j = 4,\ldots,cf \tag{A3-25}$$

$$\kappa_{i,j} \sim \mathrm{N}(\mu_{i,j}, \sigma^2_{\kappa}) \ , \ i = 1,\ldots,r; \ j = 2,\ldots,c \tag{A3-26}$$

$$\mu_{i,j} = \lambda_{1,j} \cdot \pi_{1,i+j} + \lambda_{2,j} \cdot \pi_{2,i+j} \ , \ i = 1,\ldots,r; \ j = 2,\ldots,c, \ \lambda_{1,j} + \lambda_{2,j} \le 1 \tag{A3-27}$$

The parameter $r_{g,8} (= c_{g,8})$ indicates the row (column) of the first post-reform incremental payment in the first column (row). Equations (A3-23, 24, and 25) define the convergence constraint for the run-off rates of the pre-and post-reform triangles; this constraint becomes tighter as development progresses. Note that the pre-reform run-off rates of the post-reform triangle are allowed to differ from the run-off rates of the pre-reform triangle (except for the $\delta_{j,9.pre}$ area). For the definitions of the variables parameters, see Appendix 2. Further, see Appendix 4 for a complete list of variables.

Acknowledgment

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5. REFERENCES

[1] De Jong, Piet, "Forecasting Runoff Triangles," *North American Actuarial Journal* 10, 28-38, 2006.
[2] De Jong, Piet, and Ben Zehnwirth, "Claims Reserving, State-Space Models and the Kalman Filter," *Journal of the Institute of Actuaries* 110, 157-181, 1983.
[3] England, Peter, and Richard Verrall, "Stochastic Claims Reserving in General Insurance," *Journal of the Institute of Actuaries* 129, 1-76, 2002.
[4] Gilks, Wally R., Sylvia Richardson, and David J. Spiegelhalter, *Markov Chain Monte Carlo in Practice,* (Boca Raton, FL: Chapman & Hall/CRC, 1996).
[5] Mack, Thomas, "Which Stochastic Model is Underlying the Chain Ladder Method?" *Insurance, Mathematics and Economics* 15(2/3), 133-138, 1994.
[6] Scollnik, David, "Bayesian Reserving Models Inspired by Chain Ladder Methods and Implemented Using WinBUGS," http://math.ucalgary.ca/~scollnik/balducci/Papers/resnaaj.pdf, April 2004.
[7] Scollnik, David, "Implementation of Four Models for Outstanding Liabilities in WinBUGS: A discussion of a paper by Ntzoufras and Dellaportas," *North American Actuarial Journal* 6(1):128-136, 2002.
[8] Taylor, Gregory, *Loss Reserving: An Actuarial Perspective*, (Boston: Kluwer, 2000).
[9] Verrall, Richard, "Modeling Claims Run-Off Triangles with Two-Dimensional Time Series," *Scandinavian Actuarial Journal* 129-138, 1989.

Abbreviations and notations

BUGS, Bayesian inference Using the Gibbs Sampler
CPI, Consumer Price Index
MCMC, Markov Chain Monte Carlo (Simulation)
M-CPI, Medical Care Component of the CPI
NCCI, National Council on Compensation Insurance
QCEW, Quarterly Census of Employment and Wages
PP, Permanent Partial (Claims)
PT, Permanent Total (Claims)
TT, Temporary Total (Claims)

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