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Abstract

This paper is a case study of the quality of clinical judgment in loss reserving for Commercial Auto Liability in the U.S. for accident years 1995 through 2001. Research on clinical vs. statistical prediction in non-insurance fields indicates that relatively simple models frequently produce better results than human experts with access to the same information. To test the quality of clinical judgment vs. statistical prediction in the Commercial Auto Liability loss reserving process, we compared the ultimate loss ratios actually booked by the U.S. insurance industry for these accident years at twelve, twenty-four and thirty-six months of development to comparable loss ratio estimates generated by mechanical application of several basic loss development methods. The booked ultimate loss ratios differed significantly from those indicated by the mechanical application of chain ladder and Bornhuetter-Ferguson methods, implying that the booked ultimate loss ratios were not determined using those methods, at least not without significant adjustment. We then compared all of these booked and estimated loss ratios to the ultimate loss ratios booked as of the end of 2004, which we treated as proxies for the true ultimate loss ratios. In most cases, the mechanically generated ultimate loss ratio estimates were closer to the booked estimates as of the end of 2004 than were the earlier booked loss ratios. The conclusion must be that, either the booked ultimate loss ratios were based on other methods that are inferior to the chain ladder and Bornhuetter-Ferguson or judgmental adjustments were made to the indicated ultimate loss ratios that reduced the quality of the final selections. Further research would be required to determine whether this is a general loss reserving phenomenon or one confined to Commercial Auto Liability during the time period studied.

Keywords: loss reserving, commercial auto liability, chain ladder, Bornhuetter-Ferguson

1. INTRODUCTION

Research on clinical vs. statistical prediction in non-insurance fields indicates that relatively simple quantitative models often produce better results than human experts with access to the same information. "Clinical prediction" refers to the conclusion reached by an expert when presented with a set of facts about a problem of a type with which he or she has experience. "Statistical prediction" refers to the conclusion indicated by a quantitative or statistical formula or model using a set of quantifiable facts about a problem. Clinical prediction does not preclude the use of statistical methods, but where they are employed they are augmented by consideration of other information and the judgment of the expert. For further background on this research see Snijders, Tazelaar and Batenburg [1].

The process of establishing the loss reserve liability to be carried on an insurer's balance sheet generally meets the definition of clinical rather than statistical prediction. Quantitative methods are used to make estimates of ultimate losses, but the estimate of the required loss reserve that is selected for booking on the balance sheet is almost never the unadjusted

output of a formula. Typically, the loss reserve actuary makes adjustments to formula output before making recommendations to executive management. Those recommendations frequently take the form of a range of reasonable estimates. Ultimately, the loss reserve liability selected to be booked on the balance sheet reflects both statistical information and the judgment of the actuary and management.

In this paper we describe the results of a test of the quality of clinical vs. statistical prediction with respect to Commercial Auto Liability ultimate loss ratio estimates for accident years 1995 through 2001 for the U.S. industry in total using Schedule P data reported in Best's Aggregates & Averages. We expected to find insignificant differences in booked ultimate loss ratios from those indicated by the chain ladder and Bornhuetter-Ferguson methods, which we classify as statistical prediction methods¹. To the extent that there were differences, we expected the judgmentally selected loss ratios to be superior. The chain ladder and Bornhuetter-Ferguson methods are relatively crude approaches that do not and cannot incorporate all of the quantitative and qualitative information available about emerging claims. It should be possible to improve upon the estimates that emerge from these methods. Indeed, much of the recent actuarial literature on loss reserving has focused on methods that are statistically, if not qualitatively, superior to the chain ladder and Bornhuetter-Ferguson.

2. SUMMARY OF FINDINGS

In this section we describe the results of our comparison of the industry's booked ultimate loss ratio estimates with statistically predicted ultimate loss ratios. Our purpose was first to determine whether the booked results appear to be based on any of the statistical methods and then to determine whether the booked loss ratios, which were based at least to some extent on clinical judgment, were better or worse than statistically predicted ones.

2.1 Comparison of Clinically and Statistically Predicted Loss Ratios

To test the proposition that the booked ultimate loss ratios for accident years 1995 through 2001 were consistent with estimates indicated by statistical loss development analysis methods, we compared the ultimate loss ratios actually booked by the U.S. insurance industry for these accident years at twelve, twenty-four and thirty-six months of

¹ In fact, our initial purpose in studying Commercial Auto Liability ultimate loss ratios from this period was to determine whether their behavior over time conformed to the model described by Wacek [2], which assumes that selected ultimate loss ratios are largely derived from the loss development models with relatively little injection of judgment.

development to comparable loss ratio estimates generated by mechanical application of the chain ladder and Bornhuetter-Ferguson methods using both paid and case incurred loss data as well as the average of all four of these methods².

Figures A, B and C show comparisons of the clinical and statistical predictions for loss ratio valuations as of twelve, twenty-four and thirty-six months, respectively, after inception of the accident year. In Figure A, which shows the ultimate loss ratio estimates as of twelve months, we see that the booked loss ratio estimates (represented by the dashed line) were almost always the lowest of all of the methods³. If the booked estimates were based on one or more of the statistical methods, we would expect to see the booked loss ratio estimates within the cluster of statistical estimates and not at the edge or outside of it, as they are here. Between 1995 and 1997, the booked estimates seem to track the Bornhuetter-Ferguson case incurred indications, but after that they diverge sharply downward. In Figure B, which compares the ultimate loss ratio estimates as of twenty-four months, we see the same pattern as at twelve months, but it is even clearer. The statistical method estimates were clustered more closely together than at twelve months and this tighter clustering accentuates the divergence of the booked and statistical estimates. For each year from 1999 through 2001 the distance of the booked estimate from the closest statistical estimate was greater than the range of the five statistical predictions! We see the same pattern again in the thirty-sixmonths comparison shown in Figure C, which further reinforces the conclusion that the booked ultimate loss ratios must have been determined by a different process.

Exhibits 1, 2 and 3 make the same comparisons as Figures A, B and C in tabular form. For example, referring to Exhibit 2, we see that the range of statistical ultimate loss ratio estimates for accident year 1999 at twenty-four months was 88.4% to 91.7%, a range of 3.3 loss ratio points. The clinical prediction, represented by the booked loss ratio, was 83.6%, which is nearly five points below the lowest of the statistical estimates. The divergence is even more striking at thirty-six months, where the statistical estimates range from 91.9% to 92.7%, a range of 0.8% points. The booked ultimate loss ratio was 87.7%, again five points below the lowest statistical estimates the size of the range of the statistical estimates! The pattern is similar for accident years 2000 and 2001.

The booked loss ratio estimates were so different from those produced by the chain ladder and Bornhuetter-Ferguson methods and their average that we concluded that the

² For a detailed explanation of the methods and data used to determine these estimates, see Appendix A.

³ The estimate for accident year 1995 is the notable exception. The 1996 and 1997 booked estimates are the lowest (but essentially tied with the B-F case incurred estimates). Each of the 1999-2001 booked estimates is the lowest by a significant amount.

booked loss ratios could not have arisen directly from any of those methods, especially after 1997. To the extent those methods were used, the statistical indications were so heavily adjusted that the final loss ratio estimates selected for booking were effectively independent of those methods.

2.2 Accuracy of Clinically vs. Statistically Predicted Loss Ratios

To test the proposition that the clinically predicted booked ultimate loss ratios were better estimates than the statistical predictions, we compared the clinical and statistical predictions to the ultimate loss ratio estimates booked as of December 2004, which we treated as reasonable proxies for the true ultimate loss ratios⁴.

The clinically predicted loss ratios were *not* better estimates than the purely statistical predictions. In fact, in most cases the booked ultimate loss ratios were far inferior to the mechanically generated ones in predicting the "true" ultimate loss ratios. Figures D, E and F are graphical comparisons of the prediction errors of the various ultimate loss ratio estimation methods for estimates made as of twelve months, twenty-four months and thirty-six months, respectively. A positive error implies a loss ratio projection that is higher than the "true" ultimate loss ratio. A negative error implies a loss ratio projection that is lower than the "true" ultimate loss ratio. A visual inspection of Figures D, E and F makes clear that the clinically predicted loss ratios showed prediction errors of a larger magnitude than the statistical indications for most accident years and all three valuations. Several of the methods showed a negative bias, i.e., a tendency to underestimate the "true" ultimate loss ratio showed the most pronounced negative bias⁵. That negative bias in the booked estimates was not confined to the 1997 through 2000 period and instead was fairly persistent across accident years and at all three valuations.

For a more detailed look, see the tabular summary of the prediction errors provided in Exhibits 4, 5 and 6, which compare the clinical and statistical prediction errors at twelve

⁴ Based on historical development patterns, by December 2004 the expected paid and case incurred losses for the oldest year in our accident year sample, 1995, were both more than 99% of ultimate losses. Even the youngest year, 2001, was substantially developed, with expected paid losses at more than 80% and expected case incurred losses at more than 95% of ultimate losses as of December 2004, leaving little likelihood of development surprises that would materially affect the ultimate loss ratio estimate beyond that date.

⁵ At the twelve months valuation, the mean error of the statistical estimates was -2.2% in 1997; -4.9% in 1998, -6.8% in 1999 and -2.5% in 2000, an average error of -4.1% over the period. Clearly, the statistical methods did not perform well in this time period. However, the errors in the booked estimates at twelve months were much-larger: -5.9% in 1997, -9.2% in 1998, -13.8% in 1999 and -10.4 in 2000, an average of -9.8% for the period. At twenty-four and thirty-six months, respectively, the mean errors of the statistical indications for 1997-2000 were -1.6% and -0.0% compared to -7.1% and -4.1% for the booked estimates.

months, twenty-four months and thirty-six months valuations, respectively. The clinically predicted booked ultimate loss ratio was the most accurate of the estimates in 1995 at all three valuations. However, for *all* other accident years at all three valuations, the clinical prediction proved to be either the least or second least accurate of the six predictive methods. It was the *least* accurate of the six methods in four of the seven accident years as of twelve months, and five of the seven years as of the twenty-four months and thirty-six months valuations. That means that two-thirds of the time *any* of the statistical methods would have been better than the clinical approach that was actually used! The clinical estimates also had by far the highest sum of squared errors of all the methods at all three valuations. Finally, the clinical estimates showed the largest bias (and that bias was negative) at all three valuations.

Statistical prediction outperformed clinical prediction for Commercial Auto Liability ultimate loss ratio estimation by a wide margin in this time period!

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3. CONCLUSIONS

We do not know whether the superiority of statistical loss reserving methods that we saw here with respect to Commercial Auto Liability is confined to the circumstances of that line of business during the time period studied or whether it is a more general phenomenon. That would be an interesting question for further research. All we can say is that the industry would have set more accurate Commercial Auto Liability loss reserves for accident years 1995 through, 2001, if it had simply booked the indications of any one of the five statistical methods we tested (the best of which was the simple average of the two chain ladder and two Bornhuetter-Ferguson estimates).

It is beyond the scope of this paper to explain the poor performance of clinical prediction of Commercial Auto Liability ultimate loss ratios between 1995 and 2001, but let's consider a few possibilities that may also warrant further study.

One possibility is that the negative bias we observed had a purely technical basis arising from the skewness of aggregate loss distributions. In his 1985 paper Stanard [3] made the following observation about chain ladder loss ratio indications in the small samples he studied: "...[T]he median prediction error...was usually negative...but a few large cases of over-prediction made the mean prediction error (the bias) positive." If the industry's Commercial Auto Liability experience comprised individual portfolios that displayed enough skewness to result in the effect that Stanard observed, then perhaps the negative bias we saw resulted merely from chain ladder or other over-projections being judgmentally capped. In

that case the sum of the individual portfolio estimates would be biased low. We don't know whether this effect could be large enough to fully explain the phenomenon we observed.

A second possibility, one suggested by research in other fields, is that the expert judgment exercised by actuaries and management is not always so expert. Perhaps qualitative and even quantitative judgments based on "experience" are risky and even biased. Perhaps what we observed is that even highly trained and experienced insurance professionals can be fooled by "anomalies" in the data that actually are part of the fundamental statistical pattern, the "correction" of which can degrade rather than improve the result.

It is interesting that irrespective of the limitations of the chain ladder and Bornhuetter-Ferguson methods from a theoretical standpoint, they performed better than the method actually used to reserve Commercial Auto Liability from 1995 through 2001. It is a reminder that theoretical advances in loss reserving methodology will have no effect on the accuracy of booked estimates if the indications are ignored or overridden by judgment! We saw that while the chain ladder and Bornhuetter-Ferguson methods underestimated the ultimate loss ratios during the period 1997 through 2000, the addition of clinical judgment *more than doubled* that underestimation. While we must be careful not to over-generalize from this limited study, at very least it suggests that actuaries must be mindful that the exercise of judgment in loss reserving has the potential to compound rather than reduce reserving errors. That is not to say that judgment should never be exercised, but it must be exercised with great care.



Test of Clinical Judgment vs. Statistical Prediction



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FIGURE B



FIGURE C

EXHIBIT 1						
Compa	rison of Cli	inical and S	Statistical U	ltimate Lo	ss Ratio Pre	dictions
	Ad	cident Years	s 1995-2001	as of 12 Mo	nths	
		Stati	stical Predic	tions		Clinical Prediction
	(1) Paid	(2)	(3) Case Inc	(4)	(5)	(6)
Accident Year	Chain Ladder	Paid B-F	Chain Ladder	Case Inc B-F	Average CL & B-F	Actual Booked
1995	89.7%	78.6%	79.9%	77.5%	81.4%	78.3%
1996	92.0%	79.9%	78.7%	76.9%	81.9%	76.7%
1997	88.0%	80.2%	80.0%	78.2%	81.6%	77.9%
1998	85.7%	81.4%	78.6%	78.3%	81.0%	76.7%
1999	90.3%	83.7%	84.7%	82.5%	85.3%	78.3%
2000	89.3%	85.2%	82.9%	82.5%	85.0%	77.1%
2001	80.3%	84.7%	78.9%	81.4%	81.3%	73.5%

Notes.

<u>Column</u>	Comments
(1)	See Appendix Exhibit A-3 (upper portion)
(2)	See Appendix Exhibit A-4 (upper portion)
(3)	See Appendix Exhibit A-6 (upper portion)
(4)	See Appendix Exhibit A-7 (upper portion)
(5)	Simple average of Columns (1) through (4)
(6)	See Appendix Exhibit A-1 ("12 Months" Ratios in upper portion)

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EXHIBIT 2						
Compa	rison of Cli	inical and S	Statistical U	ltimate Lo	ss Ratio Pre	dictions
	Ad	ccident Years	s 1995-2001 s	as of 24 Mo	nths	
		Stati	stical Predict	tions		Clinical Prediction
	(1) Paid	(2)	(3) Case Inc	(4)	(5)	(6)
Accident Year	Chain Ladder	Paid B-F	Chain Ladder	Case Inc B-F	Average CL & B-F	Actual Booked
1995	83.0%	85.9%	77.3%	77.7%	81.0%	78.0%
1996	82.9%	87.0%	78.0%	78.1%	81.5%	77.2%
1997	83.6%	85.4%	79.4%	79.6%	82.0%	78.4%
1998	84.6%	84.9%	80.8%	80.5%	82.7%	78.2%
1999	91.7%	90.8%	89.1%	88.4%	90.0%	83.6%
2000	89.2%	89.4%	87.1%	86.5%	88.0%	80.6%
2001	79.0%	79.8%	79.3%	79.4%	79.4%	73.4%

Notes.

<u>Column</u> <u>Comments</u>

(1) See Appendix Exhibit A-3 (middle portion)

(2) See Appendix Exhibit A-4 (middle portion)

(3) See Appendix Exhibit A-6 (middle portion)

- (4) See Appendix Exhibit A-7 (middle portion)
- (5) Simple average of Columns (1) through (4)
- (6) See Appendix Exhibit A-1 ("24 Months" Ratios in upper portion)

EXHIBIT 3						
Compa	rison of Cl	inical and S	Statistical U	ltimate Lo	ss Ratio Pre	dictions
	A	ccident Year	s 1995-2001	as of 36 Mo	nths	
		Stati	istical Predic	tions		Clinical Prediction
	(1) Daid	(2)	(3)	(4)	(5)	(6)
Accident Year	Chain Ladder	Paid B-F	Case Inc Chain Ladder	Case Inc B-F	Average CL & B-F	Actual Booked
1995	80.0%	80.8%	77.5%	77.4%	78.9%	78.0%
1996	81.5%	81.7%	79.1%	79.0%	80.3%	79.1%
1997	83.5%	83.4%	81.1%	80.9%	82.2%	80.1%
1998	84.7%	84.6%	82.9%	82.8%	83.7%	81.3%
1999	92.7%	92.4%	92.1%	91.9%	92.3%	87.7%
2000	88.7%	88.9%	88.6%	88.5%	88.7%	83.8%
2001	78.3%	78.5%	78.6%	78.6%	78.5%	75.9%

Notes.

<u>Column</u>	Comments
(1)	See Appendix Exhibit A-3 (lower portion)
(2)	See Appendix Exhibit A-4 (lower portion)
(3)	See Appendix Exhibit A-6 (lower portion)
(4)	See Appendix Exhibit A-7 (lower portion)
(5)	Simple average of Columns (1) through (4)
(6)	See Appendix Exhibit A-1 ("36 Months" Ratios in upper portion)



FIGURE D

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in Loss Reserving for Commercial Auto Liability Test of Clinical Judgment vs. Statistical Prediction





in Loss Reserving for Commercial Auto Liability

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Clinical J

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Statistical Prediction

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Accident Year



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Test of Clinical Judgment vs. Statistical Prediction



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EXHIBIT 4							
Con	nparison o	f Clinical a	and Statisti	cal Ultima	ite Loss F	Ratio Predio	ctions
		Accident	Years 1995	2001 as of	12 Months		
			Stati	stical Predic	tions		Clinical Prediction
Accident	(1) Proxy	(2) Paid	(3)	(4) Case Inc	(5)	(6)	(7)
Year	for True Ultimate	Chain Ladder	Paid B-F	Chain Ladder	Case Inc B-F	Average CL & B-F	Actual Booked
1995	78.1%	11.6% *	0.5%	1.8%	-0.6%	3.3%	0.2% +
1996	80.8%	11.2% *	-0.9% +	-2.1%	-3.9%	1.1%	-4.1%
1997	83.8%	4.2%	-3.6%	-3.8%	-5.6%	-2.2% +	-5.9%
1998	85.9%	-0.2% +	-4.5%	-7.2%	-7.6%	-4.9%	-9.2% *
1999	92.1%	-1.8% +	-8.4%	-7.4%	-9.6%	-6.8%	-13.8% *
2000	87.5%	1.8% +	-2.3%	-4.5%	-4.9%	-2.5%	-10.4% *
2001	77.7%	2.7%	7.0% *	1.2% +	3.8%	3.7%	-4.2%
Mean Error 4.2		4.2%	-1.7%	-3.2%	-4.1%	-1.2% +	-6.8% *
Sum of	Errors ²	2.90%	1.59%	1.52%	2.35%	1.07% +	4.52% *
Number o	of Best (+)	3	1	1	0	1	1
Number of Worst (*)		2	1	0	0	0	4

Notes.

Column Comments

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(1)	See Appendix Exhibit A-1 ("December 2004" Ratios in upper portion)
(2)	Exhibit 1, Column (1) minus Exhibit 4, Column (1)
(3)	Exhibit 1, Column (2) minus Exhibit 4, Column (1)
(4)	Exhibit 1, Column (3) minus Exhibit 4, Column (1)
(5)	Exhibit 1, Column (4) minus Exhibit 4, Column (1)
(6)	Exhibit 1, Column (5) minus Exhibit 4, Column (1)
(7)	Exhibit 1, Column (6) minus Exhibit 4, Column (1)

EXHIBIT 5							
Con	nparison o	f Clinical a	and Statisti	ical Ultima	ite Loss F	Ratio Predio	ctions
		Accident	Years 1995-	2001 as of 2	24 Months		
			Statistical I	Predictions			Clinical Prediction
Accident Year	(1) Proxy for True Ultimate	(2) Paid Chain Ladder	(3) Paid B-F	(4) Case Inc Chain Ladder	(5) Case Inc B-F	(6) Average CL & B-F	(7) Actual Booked
1995	78.1%	4.9%	7.8% *	-0.8%	-0.4%	2.9%	-0.1% +
1996	80.8%	2.1%	6.2% *	-2.8%	-2.7%	0.7% +	-3.6%
1997	83.8%	-0.3% +	1.5%	-4.4%	-4.2%	-1.8%	-5.4% *
1998	85.9%	-1.2%	-1.0% +	-5.1%	-5.4%	-3.2%	-7.7% *
1999	92.1%	-0.4% +	-1.3%	-3.0%	-3.7%	-2.1%	-8.5% *
2000	87.5%	1.7%	1.9%	-0.4% +	-1.0%	0.6%	-6.9% *
2001	77.7%	1.3% +	2.1%	1.6%	1.7%	1.7%	-4.2% *
Mean Error 1.2% 2.5% -2.1% -2.2% -0.2% + -		-5.2% *					
Sum of	Errors ²	0.35%	1.13%	0.65%	0.72%	0.30% +	2.39% *
Number o	of Best (+)	3	1	1	0	1	1
Number of Worst (*) 0 2 0 0 0 5			5				

Notes.

Column Comments

(1) See Appendix Exhibit A-1 ("December 2004" Ratios in upper portion)

(2) Exhibit 2, Column (1) minus Exhibit 4, Column (1)

(3) Exhibit 2, Column (2) minus Exhibit 4, Column (1)

(4) Exhibit 2, Column (3) minus Exhibit 4, Column (1)

(5) Exhibit 2, Column (4) minus Exhibit 4, Column (1)

(6) Exhibit 2, Column (5) minus Exhibit 4, Column (1)

(7) Exhibit 2, Column (6) minus Exhibit 4, Column (1)

EXHIBIT 6							
Con	nparison o	f Clinical a	and Statisti	ical Ultima	ate Loss F	Ratio Predic	ctions
		Accident	Years 1995-	-2001 as of :	36 Months		
		· · · ·	Statistical I	Predictions			Clinical Prediction
Accident	(1) Proxy	(2) Paid	(3)	(4) Case Inc	(5)	(6)	(7)
Year	for True Ultimate	Chain Ladder	Paid B-F	Chain Ladder	Case Inc B-F	Average CL & B-F	Actual Booked
1995	78.1%	1.9%	2.7% *	-0.6%	-0.6%	-0.8%	-0.1% +
1996	80.8%	0.7%	0.9%	-1.7%	-1.8% *	-0.5% +	-1.7%
1997	83.8%	-0.3% +	-0.4%	-2.7%	-2.9%	-1.6%	-3.7% *
1998	85.9%	-1.2% +	-1.2%	-2.9%	-3.1%	-2.1%	-4.6% *
1999	92.1%	0.6%	0.3%	0.0% +	-0.2%	0.2%	-4.4% *
2000	87.5%	1.3%	1.5%	1.1%	1.0% +	1.2%	-3.6% *
2001	77.7%	0.7% +	0.8%	0.9%	1.0%	0.8%	-1.8% *
Mean Error 0.5% 0.7% -0.8% -0.9% -0.2% +		-2.8% *					
Sum of Errors ² 0.08% + 0.13% 0.21% 0.23% 0.10%		0.74% *					
Number o	f Best (+)	3	0	1	1	1	1
Number of	Number of Worst (*) 0 1 0 1 0 5					5	

Notes.

(1)	See Appendix Exhibit A-1	("December 2004"	'Ratios in upper portion)
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- (2) Exhibit 3, Column (1) minus Exhibit 4, Column (1)
- (3) Exhibit 3, Column (2) minus Exhibit 4, Column (1)

(4) Exhibit 3, Column (3) minus Exhibit 4, Column (1)

- (5) Exhibit 3, Column (4) minus Exhibit 4, Column (1)
- (6) Exhibit 3, Column (5) minus Exhibit 4, Column (1)
- (7) Exhibit 3, Column (6) minus Exhibit 4, Column (1)

Appendix A

A.1 Sources of Data Used in Analysis

Our analysis of accident years 1995 through 2001 was based on industry aggregate Schedule P data for Commercial Auto Liability as reported in the 1995 through 2005 volumes of Best's Aggregates and Averages⁶. In particular, we used information about net earned premiums, net ultimate losses, net paid losses and net IBNR from Schedule P, Parts 1C, 2C, 3C and 4C, respectively⁷. We determined case incurred losses by subtracting net IBNR from net ultimate losses.

The loss development history for accident years 1995 through 2001 can be found in the 2005 volume of Best's Aggregates and Averages [14], which is a compilation of information from the industry's 2004 Annual Statements. We have tabulated paid, case incurred and booked ultimate loss and loss ratio information from that volume in Appendix Exhibit A-1 for accident years 1995 through 2001 as of twelve, twenty-four and thirty-six months and also as of December 2004. We used the ultimate loss ratio estimates booked as of December 2004 as proxies for the "true" ultimate loss ratios.

We turned to older volumes of Best's Aggregates & Averages for the loss development data needed to make the statistical ultimate loss ratio predictions for accident years 1995 through 2001 at twelve, twenty-four and thirty-six months. For example, we developed the initial expected loss ratio for the Bornhuetter-Ferguson analysis of accident year 1995 as of twelve months using loss development data from the 1994 Schedule P as reported in the 1995 Best's Aggregates and Averages [4]. For the first chain ladder and Bornhuetter-Ferguson analyses of accident year 1995 at twelve months, we augmented the previous loss development factor triangle available from the 1994 Schedule P with 1995 data from the 1996 volume of Best's Aggregate and Averages [5]. We computed the development factors corresponding to development between December 1994 and December 1995 from the data in the 1996 volume and added those development factors to the previous triangle⁸. Similarly, for the analysis of the later accident years and/or later valuation dates we continued to augment the triangle of loss development factors using data from later volumes of Best's Aggregates and Averages. (See [6] through [13].)

⁶ 1995 [4], 1996 [5], 1997 [6], 1998 [7], 1999 [8], 2000 [9], 2001 [10], 2002 [11], 2003 [12], 2004 [13], 2005 [14]

⁷ All references to "net losses" should be understood to include the "defense and cost containment expenses" reported in Parts 2C, 3C and 4C of Schedule P.

⁸ It is more reliable to calculate development factors using data for both numerator and denominator from within a single Schedule P than to take numerator and denominator from Schedules P from successive years, because of slight differences in the companies included in Best's Aggregates and Averages from year to year.

We tabulated the paid loss ratios as of twelve months together with the age-to-age paid development factors in Appendix Exhibit A-2A. The standard format for a triangle of loss development factors shows all development factors for a given accident year in a single row. In that format, the loss development factors associated with the development observed within individual calendar years appear on the positively sloped diagonals.

Appendix Exhibit A-2A departs slightly from the standard format to show all of the development factors observed in a given *calendar year* in a single row rather than on a diagonal. In this format, the development factors associated within individual accident years appear on the negatively sloped diagonals. The advantage of this format is that the five-year average development factors, which are tabulated in the upper section of Appendix Exhibit A-2B, can be computed by reference to five rows of data rather than more complicated references to the five points in a triangular array. This is particularly helpful in this analysis, where we are projecting seven accident years at three different valuations.

The lower section of Appendix Exhibit A-2B shows the cumulative mean development factors to age ten years (which is the outer bound of our development data) and the age ten loss ratios indicated by applying the age twelve months to age ten years development factor to the trailing five-year loss ratio as of twelve months. Those loss ratios, multiplied by a tail factor, are used as initial expected loss ratios in the Bornhuetter-Ferguson analysis as of twelve months.

Appendix Exhibits A-5A and A-5B are the case incurred analogues to Appendix Exhibit A-2A and A-2B. They are tabulations of the case incurred loss ratios as of twelve months and the case incurred development factors based on the Schedule P data contained in the 1995 through 2005 volumes of Best's Aggregates & Averages.

A.2 Clinically and Statistically Predicted Loss Ratios

In this section we describe the source of the booked ultimate loss ratio estimates that we classify as clinical predictions and discuss the details underlying the five judgment-free statistical prediction methods used in our analysis.

A.2.1 Clinically Predicted Ultimate Loss Ratios

The clinical predictions of ultimate loss ratios are available from the 2004 Schedule P compilations that appear in the 2005 Best's Aggregates and Averages. We have tabulated these ultimate loss ratio estimates together with the underlying earned premium and ultimate loss dollars in the upper portion of Appendix Exhibit A-1 in the sections labeled "12

Months", "24 Months" and "36 Months." The earned premium figures are from Schedule P, Part 1C. The ultimate loss figures are from Schedule P, Part 2C.

A.2.2 Statistically Predicted Ultimate Loss Ratios

We made statistical predictions of the ultimate loss ratios for accident years 1995-2001 using the unadjusted results of five loss development methods: 1) the paid chain ladder method, 2) the paid Bornhuetter-Ferguson method, 3) the case incurred chain ladder method, 4) the case incurred Bornhuetter-Ferguson method and 5) the simple average of the results of methods 1-4. We call these statistical predictions because we used the indicated results of each of these methods in every case and injected no subjective judgment by adjusting results that might seem odd or unreasonable.

Paid Chain Ladder Ultimate Loss Ratio Estimates

We determined chain ladder ultimate loss ratio estimates by applying paid loss development factors to paid loss ratios in the standard way. For example, for the 1995 accident year projection at twelve months, we first calculated mean age-to-age factors from historical paid loss data available, using five-year simple means where possible, reflecting the development patterns observed during calendar years 1991 through 1995⁹. These mean age-to-age factors and the cumulative factors they imply out to age ten years are tabulated in Appendix Exhibit A-2B. We used these mean factors as estimates of the appropriate prospective development factors applicable to the 1995 accident year. For the tail factor (age ten years to ultimate) we used the relationship between estimated ultimate losses and paid losses as reported in the 2004 Schedule P, which yielded a factor of 1.009¹⁰. We then multiplied the 1995 paid loss ratio at twelve months by the age twelve months to ultimate loss development factor derived from the age-to-age factors and the tail. We used the same procedure to determine prospective development factors for use with the other accident years and valuations.

Appendix Exhibit A-3 summarizes the calculation of the paid chain ladder ultimate loss ratio estimates for accident years 1995-2001 as of twelve, twenty-four and thirty-six months.

⁹ Appendix Exhibit A-2 shows that for factors corresponding to development from age seven years and beyond, fewer than five loss development; factors were available for the mean calculations.

¹⁰ Based on estimated ultimate losses of \$8,916,383 and paid losses of \$8,835,898 as of December 2004 as reported in Appendix Exhibit A-1.

Paid Bornhuetter-Ferguson Ultimate Loss Ratio Estimates

We determined estimates of the accident year 1995-2001 ultimate loss ratios at twelve, twenty-four and thirty-six months using the version of the Bornhuetter-Ferguson method with paid loss data described below.

First, we determined the initial expected loss ratio for each accident year to be used in the first Bornhuetter-Ferguson analysis at twelve months. Using data available at the beginning of each accident year we calculated the simple mean of the paid loss ratios as of twelve months from the five prior accident years. For example, to calculate the initial expected loss ratio for accident year 1995, we first computed the mean of the 1990-1994 paid loss ratios as of twelve months, (15.0% + 14.5% + 14.4% + 15.5% + 17.6%) / 5 = 15.4%, which we took as the expected paid loss ratio for accident year 1995 at twelve months. We calculated ageto-age development factors in the same way. We then multiplied the 15.4% by the age twelve months to ultimate paid development factor (including the tail factor of 1.009 discussed in the paid chain ladder section) to arrive at 77.7% as the initial Bornhuetter-Ferguson expected loss ratio for accident year 1995. While this procedure is crude, and it is easy to think of ways to improve upon it, in the present circumstances it has the merit of being based only on data available in Schedule P. No additional data or no subjective judgment is required. For 1996 and other accident years we calculated the initial loss ratio for the twelve months valuation in the same way. See Appendix Exhibits A-2A and A-2B for compilations of the historical and average paid loss ratios and age-to-age factors together with implied cumulative development factors out to age ten years on which the initial expected loss ratios were based.

In some versions of the Bornhuetter-Ferguson method the initial expected loss ratio is used not only at the twelve-month valuation but also at all subsequent ones. We believe, however, that it is more common to update the expected loss ratio for the analysis at later valuations, and the version we used for our analysis uses updated expected loss ratios. We again sought to avoid injecting either exogenous information not available from Schedule P or subjective judgment into the analysis, so we adopted the indicated ultimate loss ratio indication from the paid chain ladder method at the previous valuation as the expected loss ratio for all valuation dates beyond twelve months.

The expected loss ratio depends on two quantifiable elements: 1) the expected development in the next twelve months and 2) the expected development beyond the next twelve months. By definition, the first element becomes obsolete in twelve months and is replaced in the estimation process by the actual development that has occurred. In contrast,

in twelve months the second element continues to lie entirely in the future. However, the loss development in the tail observed during the previous twelve months has probably affected our estimate of that future development. In other words, the age-to-ultimate factor has probably been revised to reflect the most recent year of development on the older accident years.

The Bornhuetter-Ferguson ultimate loss ratio estimate typically combines the actual accident year emergence with the updated tail. This can be expressed in formula terms as follows:

BF Loss Ratio = Actual Paid Loss Ratio + ELR ×
$$(1 - \frac{1}{LDF_{sphared}})$$

Because in our formulation the expected loss ratio was explicitly constructed as the product of the expected paid loss ratio and the expected age-to-ultimate development factor (*ELR* = *Expected Paid Loss Ratio* × *LDF*), we concluded that $LDF_{spdated}$ should also be used to update the expected loss ratio as follows:

BF Loss Ratio = Actual Paid Loss Ratio + ELR ×
$$\frac{LDF_{updated}}{LDF} \times (1 - \frac{1}{LDF_{updated}})$$

This adjustment has the effect of updating the expected loss ratio in light of the updated development data to: $ELR = Expected Paid Loss Ratio \times LDF_{updated}$. We recognize that this is not the standard Bornhuetter-Ferguson formulation. However, it is conceptually more consistent with the premise of the expected loss ratio to make this adjustment than not to make it.

The details of the paid Bornhuetter-Ferguson analysis performed for accident years 1995 through 2001 as of twelve, twenty-four and thirty-six months are presented in Appendix Exhibit A-4.

Case Incurred Chain Ladder Ultimate Loss Ratio Estimates

The ultimate loss ratio analysis using the chain ladder method with case incurred loss data paralleled the paid chain ladder analysis. The only differences were that it used case incurred loss data instead of paid loss data and the tail factor (for age ten years to ultimate) was determined from the relationship; between accident year 1995 ultimate losses and case incurred losses (rather than paid losses) reported in the 2004 Schedule P. The case incurred

tail factor determined in this way was 1.002¹¹. See Appendix Exhibit A-5 for compilation of these historical and average case incurred loss ratios and age-to-age factors together with implied cumulative development factors out to age ten years.

The results of the case incurred chain ladder analysis for accident years 1995 through 2001 as of twelve, twenty-four and thirty-six months are summarized in Appendix Exhibit A-6.

Case Incurred B-F Ultimate Loss Ratio Estimates

Similarly, the case incurred Bornhuetter-Ferguson loss ratio analysis paralleled the paid Bornhuetter-Ferguson analysis, except that it used case incurred rather than paid data from Appendix Exhibit A-5. The results from this analysis of accident years 1995 through 2001 as of twelve, twenty-four and thirty-six months are summarized in Appendix Exhibit A-7.

Average of Incurred Chain Ladder and B-F Methods (Paid and Case Incurred)

Ultimate loss ratio selections are rarely determined from only one method. The simple average approach adopted here as a fifth statistical prediction acknowledges in a simple way the practice of combining estimates from different methods.

¹¹ Based on estimated ultimate losses of \$8,916,383 and case incurred losses of \$8,895,998 as of December 2004 as reported in Appendix Exhibit A-1.

APPENDIX EXHIBIT A-1

2004 Annual Statement (U.S. Industry)
Selected Premium and Loss Statistics

				Estimated Ultin	nate Net	Losses and Loss	Expense		
Accident	Net Earned	12 Mont	hs	24 Mont	hs	36 Mont	hs	December	2004
Year	Premiums	Dollars	Ratio	Dollars	Ratio	Dollars	Ratio'	Dollars	Ratio
1995	11,419,308	8,944,478	78.3%	8,909,903	78.0%	8,907,535	78.0%	8,916,383	78.1%
1996	11,945,125	9,164,925	76.7%	9,224,673	77.2%	9,452,826	79.1%	9,660,376	80.9%
1997	12,101,165	9,430,510	77.9%	9,488,547	78.4%	.9,687,547	80.1%	10,141,169	83.8%
1998	12,165,123	9,331,198	76.7%	9,512,292	78.2%	9,885,056	81.3%	10,445,429	85.9%
1999	12,053,631	9,436,430	78.3%	10,073,714	83.6%	10,575,733	87.7%	11,103,268	92.1%
2000	12,929,133	9,966,148	77.1%	10,416,697	80.6%	10,837,941	83.8%	11,037,507	85.4%
2001	14,186,157	10,420,178	73.5%	10,416,359	73.4%	10,761,679	75.9%	11,018,475	77.7%
<u></u>				Case Incur	red Losse	es and Loss Expe	ense		
Accident	Net Earned	12 Mont	hs	24 Mont	hs	36 Mont	hs	December	2004
Year	Premiums	Dollars	Ratio	Dollars	Ratió	Dollars	Ratio	Dollars	Ratio
1995	11,419,308	5,349,752	46.8%	7,155,266	62.7%	8,035,265	· 70.4%	8,895,998	77.9%
1996	11,945,125	5,599,565	46.9%	7,554,912	63.2%	8,590,063	71.9%	9.624.782	80.6%
1997	12,101,165	5,810,562	48.0%	7,761,367	64.1%	8,911,313	73.6%	10,075,215	83.3%
1998	12,165,123	5,725;649	47.1%	7,899,777	64.9%	9,112,603	74.9%	10,357,940	85.1%
1999	12,053,631	6,064,094	50.3%	8,537,262	70.8%	9,923,657	82.3%	10,956,003	90.9%
2000	12,929,133	6,256,104	48.4%	8,793,340	68.0%	10,162,998	78.6%.	10,788,755	83.4%
2001	14,186,157 -	6,350,997	44.8%	8,668,276	61.1%	9,922,085	69.9%	10,503,768	74.0%
			· .	Net Paic	l Losses a	and Loss Expens	e		
Accident	Net Earned	12 Mont	hs	24 Mont	hs	36 Months		December	2004
Year	Premiums	Dollars	Ratio	Dollars	Ratio	Dollars	Ratio	Dollars	Ratio
1995	11,419,308	2,080,653	18.2%	4,400,438	38.5%	6,188,228	54.2%	8,835,898	77.4%
1996	11,945,125	2,298,993	19.2%	4,670,807	39.1%	6,642,691	55.6%	9,532,038	79.8%
1997	12,101,165	2,320,305	19.2%	4,824,751	39.9%	6,916,574	57.2%	9,936,030	82.1%
.1998	12,165,123	2,334,107	19.2%	4,942,814	40.6%	7,062,840	58.1%	10,108,623	83.1%
1999	12,053,631	2,486,813	20.6%	5,329,527	44.2%	7,657,087	63.5%	10,524,675	87.3%
2000	12,929,133	2,652,474	20.5%	5,540,847	42.9%	7,840,880	60.6%	10,279,657	79.5%
2001	14,186,157	2,617,173	18.4%	5,367,450	37.8%	7,607,185	53.6%	9,122,500	64.3%

APPENDIX EXHIBIT A-2A

Accident Year Paid Loss Development Factors By Calendar Year of Observed Development

 Calendar	Age 1	Age	Age	Age	Age	Age	Age	Age	Age	Age	
Year	Loss Ratio	<u>1 - 2</u>	<u>2 - 3</u>	<u>3 - 4</u>	<u>4 - 5</u>	<u>5 - 6</u>	6 - 7	7 - 8	8 - 9	9 - 10	
1990	15.0%	2.291	1.464	1.234	1.120	1.055					
1991	14.5%	2.291	1.454	1.221	1.110	1.059	1.030				
1992	14.4%	2.311	1.465	1.213	1.109	1.057	1.031	1.016			
1993	15.5%	2.242	1.451	1.210	1.105	1.054	1.030	1.015	1.007		
1994	17.6%	2.265	1.456	1.196	1.101	1.050	1.028	1.016	1.008	1.004	
1995	18/2%	2.165	1.449	1.205	1.099	1.048	1.025	1.013	1.008	1.004	
1996	19.2%	2.115	1.422	1.202	1.104	1.047	1.024	1.011	1.005	1.004	
1997	19/225	2.032	1.406	1.209	1.098	1.047	1.024	1.012	1.006	1.004	
1998	19.2%	2.079	1.422	1.197	1.096	1.046	1.020	1.012	1.007	1.003	
1999	20.6%	2118		1.198	1.093	1.049	1.025	1.013	1.006	1.004	
2000	20.5%	2,143	1.429	11.208	1.105	1.047	1.021	1.010	1.004	1.002	
2001	18.4%	2.089	3.437	1.215	1.101		1.023	1.010	1.006	1.002	
2002	14.3%	2.051			1.105	1.049	1.023	1.010	1.007	1.004	
2003	12.8%	2.090	all a think	1196	1.096	1.046	1.020	1.007	1.006	1.004	
	Data Source L	egend									
	包括新用台段指数	1994 Sched	lule P as rep	ported in 19	95 Best's A	ggregates &	Averages				
		1995-2003	Schedules I	as reporte	d in 1995-20	004 Best's A	Aggregates &	& Averages			
	253.W - 2010.5	2004 Schee	lule P as rep	ported in 20	05 Best's A	ggregates &	Averages				
											_

A Test of in Loss Reserving for Commercial Auto Liability Clinical Judgment vs. Statistical Prediction

APPENDIX EXHIBIT A-2B

Accident Year Paid Loss Development Factors

	<u> </u>	Trail	ing Five-Y	lear Aver	age Age t	o Age De	velopmen	it		
	Age 1	Age	Age	Age	Age	Age	Age	Age	Age	Age
<u>Ye</u>	ar <u>Loss Ratio</u>	<u>1 - 2</u>	<u>2 - 3</u>	<u>3 - 4</u>	<u>4 - 5</u>	<u>5 - 6</u>	<u>6 - 7</u>	<u>7 - 8</u>	<u>8 - 9</u>	<u>9 - 10</u>
19	04 15.4%	2.280	1.458	1.214	1.109	1.055	1.030	1.016	1.007	1.004
19	05 16.0%	2.255	1.455	1.209	1.105	1.054	1.029	1.015	1.007	1.004
19	06 17.0%	2.220	1.448	1.205	1.103	1.051	1.028	1.014	1.007	1.004
19	07 17.9%.	2.164	1.437	1.204	1.101	1.049	1.026	1.013	1.007	1.004
19	08 18.7%	2.131	1.431	1.202	1.099	1.048	1.024	1.013	1.007	1.004
19	99 19.3%	2.102	1.427	1.202	1.098	1.048	1.024	1.012	1.006	1.004
20	00 19.8%	2.097	1.423	1.203	1.099	1.047	1.023	1.012	1.006	1.003
20)1 19.6%	2.092	1.426	1.205	1.099	1.047	1.023	1.011	1.006	1.003
20)2 18.6%	2.096	1.427	1.207	1.100	1.048	1.022	1.011	1.006	1.003
20	03 17.4%	2.098	1.426	1.206	1.100	1.047	1.022	1.010	1.006	1.003
		Trailin	a Five Ve	ar Averag	e Develo	ment to	Age 10 V	earc		
10	77 0%	5 000	2 103	ai 71001ag 1 504	1 238	1 117	1 058	1 028	1.012	1.004
10	77 78.0%	4 877	2.175	1.304	1.230	1.117	1.050	1.020	1.012	1.004
10	96 80.5%	4 736	2.105	1.407	1.200	1.115	1.057	1.027	1.012	1.004
19	jo 81.6%	4 548	2 102	1.173	1 215	1 103	1.057	1.025	1.011	1.004
10	08 82.7%	4 4 2 7	2.077	1.165	1.208	1.099	1 049	1.024	1 011	1.004
19	0 83.7%	4 3 3 8	2.064	1 447	1.200	1.097	1.047	1.023	1.010	1.004
20	0 85.2%	4 312	2.001	1 445	1.207	1.094	1.017	1.025	1.009	1.001
20)1 84.5%	4 315	2.062	1 447	1 200	1.092	1.043	1.020	1.009	1.003
20)2 80.8%	4 336	2.069	1 450	1.201	1.092	1.043	1.020	1.009	1.003
20	03 75.2%	4.332	2.065	1.448	1.200	1.091	1.041	1.019	1.009	1.003

Ъ in Loss Reserving for Commercial Auto Liability Test of Clinical Judgment vs. Statistical Prediction

APPENDIX EXHIBIT A-3

Paid Chain Ladder Ultimate Loss Estimates As of Twelve, Twenty-Four and Thirty-Six Months

		As of Twel	ve Month	s	
Accident	Loss Ratio	<u>Loss De</u>	<u>velopmen</u>	<u>t Factors</u>	CL Ult
<u>Year</u>	<u>at 12 Mo.</u>	<u>To 10 Yrs</u>	<u>Tail</u>	<u>To Ult</u>	Loss Ratio
1995	18.2%	4.877	1.009	4.921	89.7%
1996	19.2%	4.736	1.009	4.779	92:0%
1997	19.2%	4.548	1.009	4.590	88.0%
1998	19.2%	4.427	1.009	4.467	85.7%
1999	20.6%	4.338	1.009	4.378	90.3%
2000	20.5%	4.312	1.009	4.352	89.3%
2001	18.4%	4.315	1.009	4.354	80.3%
		<u> </u>	_		
	Δ	a of Twoney	Four Mor		
1	11	is of Twenty-		11115	
Accident	Loss Ratio	Loss De	velopmen	t Factors	CL Ult
<u>Year</u>	<u>at 24 Mo.</u>	<u>To 10 Yrs</u>	Tail	<u>To Ult</u>	Loss Ratio
1995	38.5%	2.134	1.009	2.153	83.0%
1996	39.1%	2.102	1.009	2.121	82.9%
1997	39.9%	2.077	1.009	2.096	83.6%
1998	40.6%	2.064	1.009	2.083	84.6%
1999	44.2%	2.056	1.009	2.075	91.7%
2000	42.9%	2.062	1.009	2.081	89.2%
2001	37.8%	2.069	1.009	2.088	79.0%
		As of Thirty-	Six Mont	hs	
Accident	Loss Ratio	Loss Dev	velopmen	t Factors	CL Ult
<u>Year</u>	<u>at 36 Mo.</u>	<u>To 10 Yrs</u>	<u>Tail</u>	<u>To Ult</u>	<u>Loss Ratio</u>
1995	54.2%	1.463	1.009	1.476	80.0%
1996	55.6%	1.452	1.009	1.465	81.5%
1997	57.2%	1.447	1.009	1.460	83.5%
1998	58.1%	1.445	1.009	1.459	84.7%
1999	63.5%	1.447	1.009	1.460	92.7%
2000	60.6%	1.450	1.009	1.463	88.7%
2001	53.6%	1.448	1.009	1.461	78.3%

APPENDIX EXHIBIT A-4

			As of Twel	ve Months			
	Accident	Loss Ratio	BF	Age to U	Jlt LDF	BF Ult	
	Year	at 12 Mo.	ELR	Current	Prior	Loss Ratio	
	1995'~	18.2%	77.7%	4.921	5.045	78.6%	
	1996	19.2%	78.9%	4.779	4.921	79.9%	
	1997	19.2%	81.2%	4.590	4.779	80.2%	
	1998	19.2%	82.3%	4.467	4.590	81.4%	
	1999	20.6%	83.4%	4.378	4.467	83.7%	
	2000	20.5%	84.5%	4.352	4.378	85.2%	
	2001	18.4%	85.9%	4.354	4.352	84.7%	
						=	
		, A	c of Twenty	Four Month	c		
		Г	is of Twenty	-rour monu	5		
	Accident	Loss Ratio	BF	<u>Age to L</u>	<u>Ilt LDF</u>	BF Ult	
	<u>Year</u>	<u>at 24 Mo.</u>	<u>ELR</u>	Current	<u>Prior</u>	<u>Loss Ratio</u>	
	1995	38.5%	89.7%	2.153	2.182	85.9%	
	1996	39.1%	92.0%	2.121	2.153	87.0%	
	1997	. 39.9%	88.0%	· 2.096	2.121	85.3%	
	1998	40.6%	85.7%	2.083	2.096	84.9%	
	1999	44.2%	90.3%	2.075	2.083	90.8%	
	2000	42.9%	89.3%	2.081	2.075	89.4%	
	2001	. 37.8%	80.3%	2.088	2.081	79.8%	
	· · ·						
			As of Thirty	-Six Months			
			,				
	Accident	Loss Ratio	BF	Age to L	Jlt LDF	BF Ult	
	Year	<u>at 36 Mo.</u>	<u>ELR</u>	<u>Current</u>	<u>Prior</u>	<u>Loss Ratio</u>	
	1995	54.2%	83.0%	1.476	1.486	80.8%	
	1996	55.6%	82.9%	1.465	1.476	81.7%	
	1997	57.2%	83.6%	1.460	1.465	83.4%	
	1998	58.1%	84.6%	1.459	1.460	84.6%	
	1999	63.5%	.91.7%	1.460	1.459	92.4%	
	2000	60.6%	89.2%	1.463	1.460	88.9%	
	2001	53.6%	79.0%	1.461	1.463	78.5%	

Paid Bornhuetter-Ferguson Ultimate Loss Estimates As of Twelve, Twenty-Four and Thirty-Six Months

APPENDIX EXHIBIT A-5A

Accident Year Case Incurred Loss Development Factors By Calendar Year of Observed Development

Calendar	Age 1	Age	Age	Age	Age	Age	Age	Age	Age	Age
Year	Loss Ratio	<u>1 - 2</u>	<u>2 - 3</u>	<u>3 - 4</u>	<u>4 - 5</u>	<u>5 - 6</u>	6 - 7	7 - 8	8 - 9	9 - 10
1990	42.6%	1.392	1.145	1.067	1.034	1.014				
1991	41.7%	1.409	1.128	1.058	1.026	1.011	1.008			
1992	42.0%	1.397	1.124	1.056	1.024	1.014	1.005	1.003		
1993	44.2%	1.345	1.114	1.058	1.025	1.010	1.007	1.003	1.001	1.17
1994	46.7%	1.363	1.123	1.048	1.024	1.010	1.006	1.004	1.002	1.002
1995	46.8%	1.362	1.120	1.051	1.020	1.007	1.002	1.001	1.001	1.001
1996	46.9%	1.337	1.121	1.050	1.024	1.009	1.002	1.002	1.000	1.001
1997	48:0%	1.349	1.1.23	1.060	1.025	1.008	1.007	1.003	1.001	1.000
1998	47.1%	1.336	1.137	1.065	1.024	1.010	1.003	1.002	1.001	1.000
1999	50.3%	1.380	1 148	1.068	1.018	1.010	1.003	1.001	0.999	1.001
2000	48.4%	1.408		1.069	1.028	1.011	1.004	1.000	0.999	0.999
2001	44.8%	1.406		1.682	1.026	1.013	1.006	1.003	1.002	1.002
2002	36.9%	1.365		1.073	1.036	1,017	1.008	1.003	1.003	1.003
2003	24.22/m	1.375	1145		1.022	1.009	1.002	0.998	1.001	1.000
	A									
	Data Source L	egend						1.11		
	1. 学校主任的	1994 Schee	lule P as rej	ported in 19	995 Best's A	Aggregates &	& Averages			- 6 ¹
		1995-2003	Schedules I	P as reporte	ed in 1995-2	2004 Best's	Aggregates	& Average	S	1.1.1
		2004 Schee	lule P as re	ported in 20	005 Best's A	ggregates &	& Averages			
	1.000		17				1925		4	

APPENDIX EXHIBIT A-5B

Accident Year Case Incurred Loss Development Factors

		Trail	ing Five-Y	'ear Aver	age Age t	o Age De	velopmer	nt		
	Age 1	Age	Age	Age	Age	Age	Age	Age	Âge -	Age
Year	Loss Ratio	<u>1 - 2</u>	<u>2 - 3</u> -	·· <u>3</u> 4	4 - 5	5 - 6	6 - 7	7-8	8-9	9 - 10
1994	43.4%	1.381	1:127	1.057	1.027	1.012	1.007	1.003	1.002	1.002
1995	44.3%	1.375	1.122	1.054	1.024	1.011	1.006	1.003	1.001	1.002
1996	· 45.3%	1.361	1.121	1.053	1.023	1.010	1.005	1.002	1.001	1.001
1997	46.5%	1.351	1.120	1.053	1.024	1.009	1.005	1.002	1.001	1.001
1998	47.1%	1.350	1.125	1.055	1.023	1.009	1.004	1.002	1.001	1.001
1999	47.8%	1.353	1.130	1.059	1.022	1.009	1.003	1.002	1.001	1.001
2000	48.1%	1.362	1.137	1.062	1.024	1.010	1.004	1.002	1.000	1.000
2001	47.7%	1.376	1.145	1.069	1.026	1.010	1.004	1.002	1.001	1.000
2002	45.5%	1.379	1.151	1.072	1.028	1.012	1.005	1.002	1.001	1.001
2003	42.9%	1.387	1.153	1.070	1.028	1.012	1.004	1.001	1.001	1.001
		.•••								
		Trailing	g Five-Yea	ir Averag	e Develo	pment to	Age 10 Y	ears		
1994	75.3%	1.732	1.254	1.113	1.053	1.026	1.013	1.007	1.004	1.002
1995	75.4%	1.702	1.237	1.103	1.046	1.022	1.011	1.006	1.003	1.002
1996	75.9%	1.675	1.231	1.098	1.043	1.020	1.009	1.005	1.002	1.001
1997	77.4%	1.663	1.230	1.098	1.043	1.019	1.010	1.005	1.002	1.001
1998	78.5%	1.667	1.235	1.098	1.041	1.017	1.008	1.004	1.002	1.001
1999	80.3%	1.680	1.241	1.099	1.038	1.015	1.006	1.003	1.001	1.001
2000	82.3%	1.710	1.255	1.104	1.040	1.015	1.006	1.002	1.001	1.000
2001	83.9%	1.758	1.278	1.116	1.044	1.018	1.007	1.003	1.001	1.000
2002	81.2%	1.785	1.294	1.124	1.049	1.020	1.008	1.003	1.002	1.001
2003	76.9%	1.792	1.292	1.121	1.047	1.019	1.007	1.003	1.002	1.001

APPENDIX EXHIBIT A-6

Case Incurred Chain Ladder Ultimate Loss Estimates As of Twelve, Twenty-Four and Thirty-Six Months

					-	
		As of Twel	ve Month	s		
Accident	Loss Ratio	Loss De	velopmen	t Factors	CL Ult	
<u>Year</u>	<u>at 12 Mo.</u>	<u>To 10 Yrs</u>	Tail	<u>To Ult</u>	Loss Ratio	
1995	46.8%	1.702	1.002	1.706	79.9%	
1996	46.9%	1.675	1.002	1.679	78.7%	
1997	48.0%	1.663	1.002	1.666	80.0%	
1998	47.1%	1.667	1.002	1.671	78.6%	
1999	50.3%	1.680	1.002	1.683	84.7%	
2000	48.4%	· 1.710	1.002	1.714	82.9%	
2001	44.8%	1.758	1.002	1.762	78.9%	
_					· · · · · · · · · · · · · · · · · · ·	
		(m				
	А	s of I wenty-	Four Mor	iths		
Accident	Loss Ratio	Loss De	velopmen	t Factors	CL Ult	
<u>Year</u>	<u>at 24 Mo.</u>	<u>To 10 Yrs</u>	<u>Tail</u>	<u>To Ult</u>	<u>Loss Ratio</u>	
1995	62.7%	1.231	1.002	1.233	77.3%	
1996	63.2%	1.230	1.002.	1.233	78.0%	
1997	64.1%	1.235	1.002	1.238	79.4%	
1998	64.9%	1.241	1.002	1.244	80.8%	
1999	70.8%	1.255	1.002	1.258	89.1%	
2000	68.0%	1.278	1.002	1.281	87.1%	
2001	61.1%	1.294	1.002	1.297	79.3%	
		As of Thirty-	Six Montl	hs		
Accident	Loss Ratio	Loss De	velopment	t Factors	CL Ult	
<u>Year</u>	<u>at 36 Mo.</u>	<u>To 10 Yrs</u>	<u>Tail</u>	<u>To Ult</u>	<u>Loss Ratio</u>	
1995	70.4%	1.098	1.002	1.101	77.5%	
- 1996	71.9%	1.098	1.002	1.100	79.1%	- 1
1997	73.6%	1.099	1.002	1.101	81.1%	
1998	74.9%	1.104	1.002	1.107	82.9%	
1999	82.3%	1.116	1.002	1.119	92.1%	
· 2000	78.6%	1.124	1.002	1.127	88.6%	
2001	69.9%	1.121	1.002	1.123	78.6%	
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APPENDIX EXHIBIT A-7

		As of Twe	lve Months			
Accident,	Loss Ratio	BF	Age to L	Jlt LDF	BF Ult	
<u>Year</u> .	<u>at 12 Mo.</u>	ELR	Current	Prior	<u>Loss Ratio</u>	
1995	46.8%	75.4%	1.706	1.736	77.5%	
1996	46.9%	75.6%	1.679	1.706	76.9%	
1997 .	48.0%	76.1%	1.666	1.679	78.2%	
1998	47.1%	77.6%	1.671	1.666	78.3%	
1999	50.3%	78.7%	1.683	1.671	82.5%	
2000	48.4%	80.5%	1.714	1.683	82.5%	
2001	44.8%	82.5%	1.762	1.714	81.4%	
 	A	s of Twenty	-Four Month	s		
Accident	Loss Ratio	BF	y-Four Months <u>Age to Ult LDF</u> <u>Current Prior</u>		BF Illt	
Year	at 24 Mo.	ELR	Current	Prior	Loss Ratio	
1995	62.7%	79.9%	1.233	1.240	77.7%	
1996	63.2%	78.7%	1.233	1.233	78.1%	
1997	64.1%	80.0%	1.238	1.233	79.6%	
1998	64.9%	78.6%	1.244	1.238	80.5%	
1999	70.8%	84.7%	1.258	1.244	88.4%	
2000	68.0%	82.9%	1.281	1.258	86.5%	
2001 .*	61.1%	78.9%	1.297	1.281	79.4%	
		As of Thirty	-Six Months			
Accident	Loss Ratio	BF	Arre to U	IIt I DE	BF III+	
Year	at 36 Mo.	ELR	Current	Prior	Loss Ratio	
1995	70.4%	77 3%	1 101	1 101	77 4%	
1996	71.9%	78.0%	1 100	1 101	79.0%	
1997	73.6%	79.4%	1.100	1 100	80.9%	
1998	74.9%	80.8%	1.107	1.100	82.8%	
1999	82.3%	89.1%	1.107	1 107	91.9%	
2000	78.6%	87.1%	1 127	1 1 1 9	88.5%	

Case Incurred Bornhuetter-Ferguson Ultimate Loss Estimates As of Twelve, Twenty-Four and Thirty-Six Months

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Abbreviations and notations

BF, abbreviation for "Bornhuetter-Ferguson"

CL, abbreviation for "chain ladder"

- ELR, expected loss ratio used in Bornhuetter-Ferguson analysis
- LDF, loss development factor

Biography of the Author

Michael Wacek is President of Odyssey America Reinsurance Corporation based in Stamford, CT. Over the course of more than 25 years in the industry, including nine years in the London Market, Mike has seen the business from the vantage point of a primary insurer, reinsurance broker and reinsurer. He has a BA from Macalester College (Math, Economics), is a Fellow of the Casualty Actuarial Society and a Member of the American Academy of Actuaries. He has authored a number of papers.