Comparison of Risk Allocation Methods – Bohra-Weist DFAIC Distributions

By Trent R. Vaughn, FCAS

Abstract This study compare the results of several risk allocation methods for a realistic insurance company example.

INTRODUCTION

This purpose of this study is to compare the results of several risk allocation methods for a realistic insurance company example. The basis for the study is the fitted loss distributions of Bohra and Weist (2001), which were derived from the hypothetical data for DFA Insurance Company (DFAIC). This hypothetical data was distributed by the Casualty Actuarial Society's Committee on Dynamic Financial Analysis, as part of its 2001 call for papers.

In addition, Ruhm and Mango (2003) utilized these fitted distributions to produce 2000 simulated loss scenarios for DFAIC. This detailed simulation data was included in a spreadsheet that accompanied their paper. The Ruhm-Mango simulation data is also an important source of input for this study.

All of the data, analysis and results for the study are shown on the accompanying Excel Workbook ("bohra-weist data.xls"). The actual Bohra-Weist fitted distributions are shown on the "Data for Study" sheet. This sheet also provides some explanatory notes regarding the Ruhm-Mango simulation data. The actual Ruhm-Mango simulation data, sorted in ascending order, is shown on several different sheets, including the "RMK Capital Consumption" sheet.

In general, the calculations for each of the individual methods are displayed on a separate sheet of the Workbook. There is also a "summary" sheet that summarizes the resulting allocation and pricing for each method. In order to focus on differences in the allocation results for the various methods, each of the methods has been "calibrated" to the same overall corporate premium level. This overall premium amount is $1,242,777, which represents a total risk loading of $100,000. This total corporate risk load could be based on a financial pricing model (such as the Fama-French 3-Factor Model), or it could simply be based on a judgmental ROE or combined ratio goal that has been set by the Board of Directors.

In this study, we have also ignored complications caused by existing reserves, loss discounting, and long-tailed payouts.[1] In other words, we are assuming that these loss distributions apply to a start-up insurance company with no existing reserves. We are also implicitly ignoring differences in the duration of payments for the various lines.

Some of the methods in this study also require a specific value for the policyholders' surplus of DFAIC. We have assumed that surplus at the time of writing is $900,000. This implies a premium-to-surplus ratio of $1,242,777 / $900,000 = 1.38. The expected return-on-equity (ROE) for the company, ignoring investment income, is $100,000 / $900,000 = 11.1%.

Many of the methods in this study[2] directly determine the capital cost allocation for each of the subject lines of business. For these methods, the resulting premium by line is then determined by the following formula:

Premium = Expected Loss + Pro-Rata Allocation of Risk Load x $100,000

Note that the expected loss in this formula is undiscounted. Also, there is no provision made for underwriting and loss adjustment expenses.

The remaining methods in this study[3] directly determine the premium for each of the subject lines of business. For these methods, the total premium is "calibrated" at $1,242,777. The corresponding capital cost allocation by line is then determined according to the following formula:

Capital Cost Allocation = (Premium – Expected Loss) / $100,000

The remainder of this paper will provide explanatory notes for each of the methods, followed by short summary of the observations and results of the study.

MYERS-READ METHOD

Myers-Read (2001) proposed a capital allocation method that is based on Option Pricing Theory (OPT). Myers/Read provided a separate version of their formula for both lognormal and normal underlying loss distributions. The calculations on the "Myers-Read"

[1] See Venter (2002) for a discussion of these issues.
[2] Namely, the following methods: Myers-Read, RMK with Capital Consumption, RMK with Variance, Covariance, XTVaR99, and XTVaR with Expected Loss Cutoff.
[3] The following methods: Variance Load, Standard Deviation Load, RCR, Wang Transform, PH Transform

sheet of this workbook are based on their lognormal model, which in turn utilizes the lognormal version of Margrabe's formula.[4]

Technically, this formula requires that the distribution of aggregate (i.e. all lines combined) losses and asset values is joint lognormal. Myers-Read point out that "if each line's future loss is lognormal, then the overall loss cannot be lognormal." However, they also state: "The following derivations of default values and surplus allocations assume that total losses (the sum of all lines' losses) and asset values are joint lognormal. The authors believe that this is a reasonable approximation even when individual lines' losses are also lognormal."

Thus, the Myers-Read lognormal model is well-suited to the lognormal fitted distributions of Bohra-Weist. However, there is an important caveat. On the top of p. 556, Myers-Read provide a formula for approximating the variance of the "lognormal" aggregate losses from the individual line data.[5] But the authors point out that this formula only provides a close approximation when "the line-by-line loss volatilities are not large." For this reason, we can't utilize the lognormal line data of Bohra-Weist directly in the Myers-Read formula – because the volatility of the HO-xCat line is extremely large relative to the other lines, and the approximating formula on p. 556 will not work. Thus, in the Myers-Read sheet, the HO-xCat and HO-Cat lines of Bohra-Weist have been combined into a single "Homeowners" line. The mean and standard deviation for this combined Homeowners line is based on the sample mean and sample standard deviation for Homeowners in the Ruhm/Mango simulations.[6]

The actual Myers-Read sheet in this study is set up exactly like the tables in the Myers-Read paper. In fact, all of the headings and labels are exactly the same.[7] The "standard deviation" in column D is expressed as a percentage of the mean, as in the Myers-Read tables. Thus, in more precise terms, it is actually the "coefficient of variation" of the lognormal line distributions.

The Myers-Read formula also requires additional assumptions regarding the standard deviation of asset returns and correlations between asset values and the individual line losses. These assumptions are shown in cells D15 through I15.

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[4] See Margrabe (1978) for details.
[5] This formula is cell J14 on the "Myers-Read" sheet.
[6] For details, see the explanatory notes on the "Data for Study" sheet.
[7] We have, however, omitted the normal distribution results shown on the Myers-Read tables, since the Bohra-Weist data assumes lognormality.

RMK METHODS

In the previous section, the actual Bohra-Weist fitted lognormal distributions were utilized to determine the Myers-Read allocation. RMK procedures generally do not utilize fitted loss distributions. Instead, these fitted distributions are used to generate a large number of simulated scenarios; the actual RMK calculations are then performed on the simulated scenarios.

Ruhm/Mango (2003) generated a set of 2000 simulated scenarios from the Bohra-Weist fitted lognormal distributions. Additional notes on the simulation data are contained on the "Data for Study" sheet. The remaining methods in this study use the simulation data, as opposed to the actual fitted distributions.

Clark (2005) discusses various practical applications of the RMK methodology, including the allocation of risk load by component. In general, the RMK procedure requires a "riskiness leverage ratio" as a benchmark for the measurement of risk. Clark provides RMK risk allocation examples that are based on both a capital consumption and a variance risk measure.

The "RMK Capital Consumption" sheet determines the DFAIC allocation corresponding to Clark's Exhibit 3a. The raw Ruhm-Mango simulation data, sorted in ascending order, is shown in columns A through H. The riskiness leverage ratio (column R) is based on capital consumed (column O). In this procedure, the maximum amount of capital consumed is equal to the available surplus of $900,000. This capping reflects Kreps' (2005) statement that "once you are buried it doesn't matter how much dirt is on top."

The "RMK Variance" sheet determines the allocation corresponding to Clark's Exhibit 3b. In this case, the riskiness leverage ratio is based on variance, instead of capital consumption. Clark points out that RMK with variance is equivalent to an allocation by the covariance method. In order to verify this assertion, I have also provided an explicit covariance-based allocation on the "covariance" sheet of the workbook.

VARIANCE AND STANDARD DEVIATION LOAD

Feldblum (1990) describes the variance and standard deviation load methods. Bault (1995) clarifies the underlying assumptions behind each of these two methods. The standard deviation load method sets the premium for each line according to the following formula:

Premium = Expected Loss + T x Standard Deviation

The formula for the variance load method is similar in form:

Premium = Expected Loss + T x Variance

The "variance and stdev" sheet displays the DFAIC premium calculation by line for each of these two methods. The calculations are based on the Ruhm-Mango simulation data, as opposed to the Bohra-Weist fitted distributions.[8] For each method, the "T" value (cells C2014 and C2015) is calibrated to the desired overall premium goal, and this T value is then used to determine the premium for each of the lines. The corresponding capital cost allocation is then determined according to the formula shown in the first section of this paper (see cells I9 through J14 on the "Summary" sheet for details).

EXCESS TAIL VALUE AT RISK (XTVaR)

Venter, Major, and Kreps (2005) discuss risk allocations that are based on the XTVaR risk measure. In order to utilize this method, one must select a "cutoff point" for the tail. Venter, Major, and Kreps provide the following guidance for this selection:

> *One possibility for establishing a cutoff probability for tail risk measures would be to use the probability of having any loss of capital at all. Then XTVaR would be the average loss of capital when there is a loss of capital. Another possible choice is the probability that capital is exhausted. The former is arguably more relevant to capital allocation, in that it charges for any use of capital rather than focusing on the shortfalls upon its depletion.*
>
> *On the other hand, policyholders tend to be sensitive to default. Studies suggest that they demand premium reductions one or two orders of magnitude greater than the expected value of the default cost in order to accept less than certain recovery. This is in part due to undiversified purchases of insurance. Thus the value of default has meaningful pricing effects, and policyholder concerns become quite relevant to shareholders as well.*

In this study, we have included an XTVaR calculation that is based on average loss of capital, as well as a version that is based roughly on default or insolvency. The "XTVaR Expected Loss" sheet utilizes a cutoff point that is equal to aggregate expected losses of $1,140,291.[9] As such, this version focuses on average loss of capital. The "XTVaR99" sheet utilizes a cutoff point that is equal to the 99th percentile of the aggregate loss distribution; hence, this version focuses on default outcomes.

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[8] The calculations for these methods could also be easily performed on the fitted distributions. Provided that we have obtained a sufficiently large number of simulations, the premium and allocation results should be very similar.

[9] As in the previous section, the XTVaR calculations are based on the simulation data, as opposed to the Bohra-Weist distributions. In theory, the XTVaR formulas could also be applied directly to the fitted distributions.

Both the XTVaR with Expected Loss Cutoff and the RMK with Capital Consumption methods focus on the measurement of average capital consumed. Not surprisingly, the two methods also produce very similar capital cost allocations, as shown on the "summary" sheet. In fact, the only difference between these two methods lies in the RMK's capping procedure; that is, capital consumption in the XTVaR calculation is allowed to exceed the available surplus of $900,000. In other words, if we were to eliminate the capping procedure on the "RMK Capital Consumption" sheet, the two methods would produce an identical result.[10]

WANG TRANSFORM

Wang (2002) provides a practical discussion and description of the Wang transform method.[11] On the "Wang Transform" sheet, we have utilized this method to determine the DFAIC premiums and capital allocation.

The calculations are based on the Ruhm/Mango simulation data, shown in columns A through H. Column J assigns objective probabilities f(x) = 1/2000 to each scenario. Column K adds up the objective probabilities f(x) to get the cumulative probabilities F(x). Column L applies the Wang transform to F(x) to get the adjusted F*(x); the Sharpe ratio for this Wang transform is shown in cell 2007. Column M determines the risk-adjusted probability weights f*(x) from F*(x).

The Sharpe ratio is calibrated to produce a risk-adjusted mean of $1,242,777 for the total aggregate loss distribution. This Sharpe ratio is then used to determine the risk-adjusted means for each of the individual lines.

THE PROPORTIONAL HAZARDS (PH) TRANSFORM

Wang (1998) describes the Proportional Hazards (PH) transform. The "PH Transform" sheet applies this methodology to the DFAIC data. As with the other methods, the PH Transform has been calibrated to produce an overall risk load of $100,000; this results in a value for "r" of roughly 5/8 (see cell C2007). This r-value is then utilized to determine the PH-mean for the individual lines.

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[10] This can be verfied by changing cell L2013 on the "RMK Capital Consumption" sheet to some very high value, say $10,000,000.
[11] For a more rigorous discussion and derivation, see Wang (2000).

RISK COVERAGE RATIO (RCR) METHOD

Ruhm (2001) describes the application of the risk coverage ratio (RCR) to insurance ratemaking and capital allocation. The "RCR" sheet applies the RCR method to the DFAIC data. Columns J through O display the ROE's by line and total for each of the 2000 simulated scenarios. Note that these ROE's are allowed to fall below -100% for very unfavorable scenarios.

The surplus allocation is required as an input to this method, as shown in cells C2011 to H2011. Premiums are then set (in cells C2012 through G2012) so that each line has the same risk coverage ratio. Provided that each of the lines is assigned a surplus amount greater than $0, the resulting RCR premiums by line are independent of the selected surplus allocation.[12] This can be easily verified by changing the surplus allocation in cells C2011 through F2011.

The Risk-to-Reward (or "R2R") Method is conceptually very similar to the RCR method. In fact, the two methods have been shown to produce identical results. This is verified in the "R2R" sheet of the Excel workbook.

MANGO CAPITAL CONSUMPTION

Mango (2003) introduced the capital consumption approach to capital loads. The "Mango Capital Consumption" sheet in this workbook was actually supplied by Don Mango. The methodology requires a key exogenous parameter, or utility "exponent" in cell R5. Mango provided the following explanatory remarks:[13]

> *[This method] uses the .... utility-type function to actually calculate scenario-level capital costs based on total UW loss. Costs are then spread back to LOB based on LOB UW loss only. This is different from [Clark's] RMK Capital Consumption approach, where "winners" are rewarded -- that is, LOB UW gains as well as UW losses @ scenario level are factored in, with the gains actually serving to reduce allocated capital (or cost). The approach I have done here mirrors the appendix of my Capital Consumption paper.*

Obviously, the exponent is cell R5 is a key input. Mango notes that his original choice for that exponent produced an allocation result that was very close to the Myers/Read allocation. The exponent was then adjusted to approximate the Myers/Read result as closely as possible.

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[12] See Ruhm (2001) for an algebraic proof of this statement. Although not discussed it Ruhm, it is worth noting that this statement is only true if we allow for ROE values that are less than -100%.
[13] Source: Correspondence in private listserv group.

However, note that changes in this exponent parameter can have a big impact on the resulting allocation. Specifically, as the exponent parameter is increased (starting from a baseline of zero), more capital is allocated to Homeowners, and capital for all other lines is reduced (monotonically). In the actual "Mango Capital Consumption" sheet in the workbook, I have "calibrated" this exogenous parameter to produce the desired total risk load of $100,000 (as shown in cell S2009).[14]

SUMMARY AND CONCLUSIONS

The "summary" sheet displays the allocation results and resulting premiums for each of the methods in this study. The most significant difference between the various allocation results involves the relative allocation between the Homeowners line (which is highly cat prone) and the remaining lines. The standard deviation method allocates 59.4% of surplus to Homeowners, and the variance method allocates 88.9%. Many of the other methods fall within this s.d.-to-variance "band"; in fact, the allocations produced by the PH Transform, Covariance, Myers/Read, RMK with Variance, Mango Capital Consumption, and XTVaR99 are all remarkably similar. However, both the Wang Transform and RCR/R2R methods resulted in an Homeowners allocation that was lower than the s.d.-to-variance "band". And both the XTVaR with Expected Loss Cutoff and D. Clark's RMK with Capital Consumption resulted in an extremely low Homeowners allocation, relative to the other methods.

In general, the conceptual dichotomy between "cat prone" and "non cat prone" lines represents an important business management issue in our industry. This is especially true in the wake of Hurricanes Katrina and Rita, as most of the popular catastrophe models are now producing much higher PML's for the industry. As such, the decision regarding the percentage of capital and capital costs that should be allocated to cat-prone lines has critical implications on pricing, marketing and reinsurance purchases. Moreover, depending on the capital allocation model that you use, you'll get a much different answer to this problem.

[14] This is consistent with the approach taken in this paper for other methods that require an exogenous parameter (for example, the Sharpe ratio of the Wang Transform or the "r" of the PH-Transform). That is, the exogenous parameter has been "calibrated" to the same overall target risk load or premium.

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