USING A CLAIM SIMULATION MODEL FOR RESERVING & LOSS FORECASTING FOR MEDICAL PROFESSIONAL LIABILITY

By Rajesh V. Sahasrabuddhe, FCAS, MAAA

Abstract
Various recent papers have included criticisms related to the use of link-ratio techniques for estimating ultimate losses. While this paper does not review these criticisms, it does outline development characteristics of medical professional liability1 losses that would lead the actuary to believe that link-ratio techniques may not always be the best available option for projecting ultimate losses. The paper then proceeds to provide a model that addresses these weaknesses and then extends this model for loss forecasting applications.

More specifically, this paper provides a framework for evaluating medical professional liability[1] loss exposures. The concepts used in this model are more fully discussed and described in other statistical textbooks or refereed actuarial journals. This paper is intended to provide a synthesis of existing distinct processes. Rather than repeat those discussions, a bibliographical reference is provided. The bibliography included, therefore, should be considered a critical section of this paper.

Specifics of the modeling within the framework presented are the responsibility of the actuary implementing the model. While the paper does include alternatives that may be considered within the framework, it is not intended to be a comprehensive listing of these alternatives.

This model has been developed in recognition of data availability issues for self-insured healthcare facilities. However, this model may easily be expanded for use in an insurance company context or for evaluating other medical professional liability exposures.

I. Motivation and Rationale

A recent paper published in the CAS Forum included the following statement:

---

[1] This model may be extended to other general liability or professional liability exposures. This model is not appropriate for coverages subject to partial payments such as workers compensation.

"...for most, if not all, cumulative arrays the assumptions made by the standard link ratio techniques are not satisfied by the data, ..."[2] [1]

Without providing a statistical analysis to prove this statement, there are a number of intuitive reasons why we would expect this statement to be true for medical professional liability losses.

(1) Claims-made medical professional liability is generally considered a "short-tailed" coverage in comparison to other liability coverages. Occurrence coverage is generally considered "long-tailed." This leads to the natural conclusion that on an occurrence basis, the majority of loss development may be attributed to claims that are incurred but not reported ("IBNR"). Consistent with common actuarial usage, this type of development is referred to as "pure IBNR" emergence. This development should be distinguished from the development on known claims, which will be referred to as bulk development. Link ratio techniques assume future development is a function of prior cumulative experience. This is inconsistent with the understanding developed that future development is actually due to newly reported claims. These newly reported claims do not necessarily have any relationship to past claims.

This relationship between pure IBNR and bulk development may be driven by the fact that healthcare institutions, in general, are conservative by their very nature. In the aggregate, case reserves established by these conservative institutions tend to be reasonably adequate.

(2) A model is defined as "a simplified mathematical description" [3] of a more complicated process. Loss development approaches would not appear to satisfy this definition since future development is not entirely a function of cumulative losses. Therefore the "mathematical description" is not consistent with the process being modeled.

(3) Link ratio techniques are generally based on the analysis and review of loss development triangles. Given the long-tail nature of occurrence coverage the predictive ability of loss development triangles is severely compromised by inflation. Emergence in the 10th calendar period for the

[2] That same paper was later published in the Proceedings with softer language: "Most loss arrays don't satisfy the assumptions of standard link ratio techniques."

10th prior accident period is likely to differ from the for the current accident period due to inflationary pressures.

(4) Loss development data for self-insureds may be subject to various limits and deductibles. This further compounds the inflation problem. Even if we obtain triangles at constant limits and deductibles on a nominal basis, the development for each period will be different on a real, or inflation-adjusted, basis.

In addition, if actuaries choose (as they often do) to select a single development pattern and apply it to every exposure period[3], they are making the implicit assumption that trend acts in one direction – across exposure periods. Any other "direction" of trend, i.e. across settlement period, report period or maturity, would be inconsistent with the development patterns as they are used in general practice.

These factors would compromise estimates using traditional Bornhuetter-Ferguson ("B-F") or additive techniques as well as link-ratio methods.

(5) When information is aggregated, information is lost. Fundamentally, by aggregating loss information into somewhat arbitrary accident year groupings, information provided by individual claim detail is lost. It is also critical to recognize that loss development is a statistical model. In this model, parameters (loss development factors) are estimated using data (loss triangles).

The framework described herein is based on multiple underlying stochastic models and is likely to be more robust. This is because we are estimating fewer parameters with more information. However, there may be residual uncertainties that cannot be eliminated.

From a practical perspective, these methods also suffer from the following problems:

(1) To many users of actuarial information, risk (deviation from the point-estimate) is just as important as, if not more important than, the point-estimate itself. We may be able to develop statistical measures of the uncertainty involved in the selection of loss development factors - also

---

[3] "exposure period" is intended as a generalized term for accident period, policy, period, report period, etc.

known as parameter risk. However, we have not been able to determine the model specification risk. This risk can be quite large since the assumption of link ratio models may not be consistent with the underlying cause of loss development.

(2) Oftentimes loss triangles are simply not available. This is particularly the case for self-insureds. Self-insured entities often keep a current loss database and generally do not track aggregate loss development.

(3) For many self-insureds, excess insurance is only available on a claims-made basis. However, accruals need to be made on an occurrence basis. This would require that actuarial analyses recognize differences in limits and retentions that are dependant on the report date of the claim. Link-ratio methods do not easily recognize these differences.

The goal of this paper is to present a model that overcomes these limitations. Specifically, we present a model that is adaptable, accounts for inflation, estimates risk, and is easily extendable for loss forecasting applications.

This model has the following benefits relative to link-ratio models because:

(1) This model reduces model specification error. This is due to that fact that this model that attempts to replicate claims process. The model includes the following phases of the actual claim life cycle: an accident occurs, the claim is reported, and the claim is settled for some amount. It would be naïve to believe that each and every driver of the claim process is (or can be) included in the model presented herein. However, the model better satisfies the definition of a "model" as stated above.

(2) The model is "unified" and easily adaptable to provide consistent estimation of pure IBNR, bulk reserves and prospective loss forecasts.

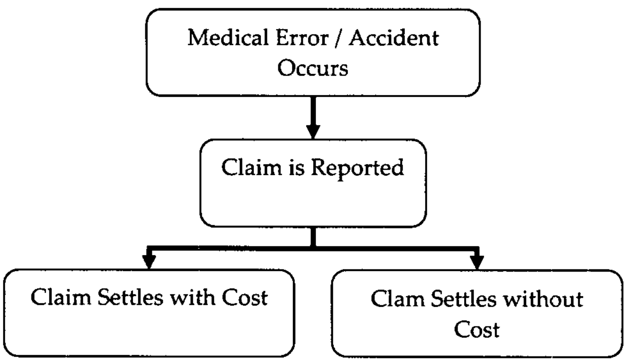
(3) The model is specifically designed to be used in a simulation environment. Given that insurance involves bearing risk – a reserve or loss forecast model should measure that risk. Use of simulation techniques is necessary in analyzing these exposures to provide an estimate of variability. Insureds retaining risk require this information as they are quite concerned with the variability in the point estimate. A model that yields only a point estimate does not accomplish this goal.

A claim level simulation model allows for the evaluation various per-occurrence and aggregate coverage alternatives in a prospective loss forecast. This information is particularly useful for insureds considering changes in their insurance program.

II. Model Overview

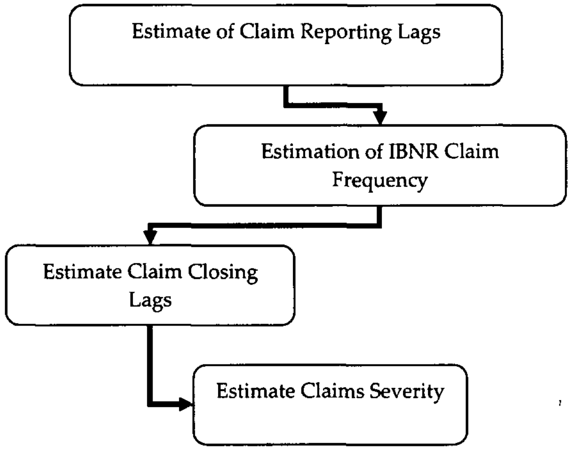
The loss-reserving model estimates indemnity and expense reserves for pure IBNR separately from bulk reserves. Each reserve component is estimated using a frequency × severity methodology. Model specification error is reduced with a frequency × severity model as it attempts to replicate the claims process. This is a benefit relative to link-ratio, additive or B-F methods, which only provide proxy models for loss movements in aggregate. The claim process replicated by the frequency × severity may be illustrated as follows:

FIGURE 1



Based on the diagram above, the model is specified in the following order:

FIGURE 2



The model used to estimate required bulk reserves or prospective loss forecasts are simply "special cases" of the model used to estimate pure IBNR. For this reason, the model for pure IBNR is presented first and the special cases follow.

III. The General Model – Evaluation of Pure IBNR

Claims Reporting Lag

Recognizing usual self-insured data limitations, the model employs an approach that does not require claim triangles. In typical self-insured loss runs, observed report lag for each claim will be available. These lags are calculated as the difference between the report date and the accident date for each claim. A statistical distribution may then be fit to these observed report lags.

In doing so, it must be recognized that the observed report lags have a problem similar to that found with deductible claim data[4]. With deductible claim data, our observations will not include losses below the deductible – i.e. claims are said to be truncated from below. With observed report lag data, our observations will not include claims that have not yet been reported – i.e. the observations are truncated from above. The observations, *W*, will follow a conditional distribution with the following density function:

[4] The parallel to the deductible loss data may be understood by reviewing Hogg & Klugman [6] (p. 129 –130).

$$f_w(x) = f_X(x) / F_X(M) \text{ where M = (valuation date - accident date)}$$
$$= 0 \text{ for } x > (\text{valuation date} - \text{accident date})$$

Using this density function, we can solve for the parameters of the lag distribution using maximum likelihood techniques and spreadsheet optimization tools. The likelihood function for individual claim data may be written as:

$$L(\theta) = L(\theta; w_1, w_2, w_3, w_4, \ldots, w_n)$$
$$= \prod_j f_w(x_j; \theta)$$

Taking logarithms, we have:

$$\ln(L(\theta)) = \sum_j \ln(f_w(x_j; \theta))$$

where θ represents the parameter(s) of the selected distribution[5]

Using spreadsheet optimization tools, we can solve for the parameter(s), θ, which maximize the likelihood function. This analysis is presented on Exhibit 1. Column (4) of this exhibit shows the calculated observed lag (in days). The maximum report lag is calculated as shown in Column (5). The conditional likelihood and conditional log likelihood are calculated in Columns (6) and (7), respectively. The mathematics of this lag model are described in Weissner [2].

The cumulative distribution function provides our claims reporting pattern. The stochastic distribution also is easily used in a simulation analyses.

Determining Pure IBNR Claim Frequency

The report lag model is then used to estimate pure IBNR claims frequency. IBNR claims are estimated using a B-F approach. This estimation is illustrated on Exhibit 2.

For the B-F calculation, the claims reporting pattern is provided by the cumulative distribution function of the report lag model and the a priori ultimate claim estimate is determined using the average of development method estimates of ultimate claims of the mature claims periods (Column (9)).

[5] In general, we tend to use Rayleigh or Weibull distributions for lags.

The "Percent Reported" (Column (8)) is calculated using the cumulative distribution function of the lag model as determined on Exhibit 1. The function is evaluated at the difference between the data evaluation date and the midpoint of the accident period.

This is not the only approach available for estimating IBNR claims; however, based on the understanding that future claims are unrelated from prior claims, approaches, such as B-F, where IBNR claims are estimated (largely) independently of reported claim count are desirable.

Determination of Claims Closing Lag Model
The development of a conditional closing lag distribution is identical to the development of the claim reporting lag with one fundamental difference. Since the closing lag in this model represents the difference between the report date and the date of closing, the observed closing lags are truncated (from above) at the difference between the valuation date and the report date. As the reader will recall, the observed reporting lags are truncated (from above) at the difference between valuation date and the accident date. As this process is identical to that for reporting lags, the calculations underlying a closing lag model are not included in this paper.

Claims Settlement Model
Professional liability claims will settle with one of the following outcomes: (1) no payment, (2) indemnity and expense, (3) indemnity only, or (4) expense only. Depending on the quantity of available data, it may be necessary to collapse settlement outcomes into: (1) "no payment" and (2) "with cost" outcomes. The first step in determining our claim settlement model is to estimate the probability of each of these possible claim settlement outcomes.

It is recommended that the distribution of claim settlements be reviewed based on both closing year and accident year bases. Closing years are preferred as they better capture changes in claims settlement practices. Changes in claims settlement practices tend to apply to claims closed after a given date regardless of the accident date of the claim. When reviewing accident year distributions of settlements, only accident years that are completely or nearly completely closed should be considered. Consideration of immature periods may bias results towards the more quickly closed "no payment" or "expense only" settlement types. This type of review is presented on Exhibit 3.

Claims Cost Models

There are numerous previously published papers and texts describing methods to estimate stochastic claims cost models. The development of claims cost models is beyond the scope of this paper. The following is a partial listing of relevant papers and texts on this topic that the interested reader should review.

- » *Klugmanm Panjer, & Wilmot – Loss Models* [3]
- » *Keatinge – Modeling Losses with the Mixed Exponential Distribution* [4]
- » *Philbrick – A Practical Guide to Single Parameter Pareto Distribution* [5]

In practice, mixed distribution models appear to best describe claims severity and are easily adaptable to the simulation process. Each component of the mixture represents a "type of claim." For example, a mixture of two lognormals, a point mass and a uniform distribution many be used to describe "small normal claims" (first lognormal), "large normal claims" (second lognormal), "losses clustered at the limit of insurance" (point mass) and "shock claims" (uniform distribution), respectively. An example of this model is presented in Exhibit 4 and is used later in this paper.

Credibility

Credibility of the loss data used in the estimation of the model parameters is an issue with every model. However, relative to development or B-F models, credibility should be less of an issue for the model presented herein. Credibility becomes an issue as actuaries attempt to estimate more parameters with fewer data. Relative to other models, this model estimates fewer parameters from more data.

Actuaries should recognize that, in estimating development patterns, each selected link-ratio is a parameter that is estimated from the observation in a column of observed link ratios. In addition to these link ratios, development methods or B-F methods may add other estimated parameters such as *a priori* loss estimates. Even with a 10-year development triangle, this may require the actuary to estimate more than 10 parameters. The model presented herein should generally require the estimation of fewer than 10 parameters. In addition, because this model relies on claim level detail, we have significantly more data - or information - relative to models in which claim level detail is collapsed into accident years – thereby destroying information.

Oftentimes, when credibility is an issue with B-F or development models, actuaries will simply rely on "industry data" or some other external source. This of course has a (generally unstated) credibility problem caused by lack of homogeneity.

An additional advantage of the model is that parameter uncertainty can be explicitly considered in this model. The following is a listing of relevant papers that the author has reviewed that provide uncertainty models that may be incorporated into the model framework:

» *Heckman, Philip E.; Meyers, Glenn G.- The Calculation of Aggregate Loss Distributions from Claim Severity and Claim Count Distributions* [7]

» *Kreps, Rodney - Parameter Uncertainty in (Log) Normal Distributions* [8]

Since we tend to employ lognormal claim cost models, we tend to use the derivation of parameter uncertainty as described by Kreps [8].

Claims Simulation

We now have all the elements necessary to simulate the pure IBNR reserve. Our simulation proceeds according to the claims process illustrated in Figure 1.

Step 1. Simulation of the number of claims – the number of claims is simulated using a stochastic model with a mean equal to the estimated IBNR claims frequency discussed previously. A Poisson model is often used to simulate the number of claims; however there is no requirement to do so. (We can also use a prior distribution for the Poisson parameter to incorporate parameter risk – which, as is well known, results in a negative binomial model.)

Step 2. Simulation of accident year – For each of the simulated claims, the accident year is simulated. The distribution of claims by accident year is based on the discrete distribution of IBNR claims by accident year derived in our estimation of IBNR claims frequency.

Step 3. Simulation of accident date – the accident date is simulated using a uniform distribution between the inception and expiration of the accident year simulated in Step 2. It is recognized that accident dates are not uniformly distributed throughout the accident year. That is, it is expected that there are more IBNR claims resulting from accidents occurring later within the simulated accident year. However, this is not considered to be a material weakness in the model and consideration of this nuance is a further area of development left to the interested actuary.

Step 4. Simulation of report date – the report date is simulated by adding a simulated report lag to the simulated accident date from Step 3. In this simulation, it must be recognized that the domain of possible report lags has a minimum value of the difference between the data evaluation date and simulated accident date. (For pure IBNR claims, the report date of the claim must, by definition, be greater than the valuation date of the data.) Therefore the report lag should be simulated using a truncated distribution.

Step 5. Simulation of claim closing date – the claims closing date is simulated by adding a simulated closing lag to the simulated report date from Step 4.

Step 6. Determination of present value factor – the present value factor applicable to each claim is calculated using the claim closing date. In general, for professional liability coverage, all indemnity is paid at claims closing and partial payments are not an issue. The calculated present value factor should consider that expenses are paid in advance of the claim closing date. It would not be overly difficult to develop a pattern for the payment of expenses; however, this consideration is beyond the scope of this paper.

Step 7. Simulation of claims outcomes – using the discrete distribution of claims settlement probabilities, we simulate the outcome of each claim.

Step 8. Simulation of claim value – the final step is estimation of claims settlement values. Using a mixed distribution model, the simulation of claim cost is a two-step process. The first step is the determination of which component of mixture generates the loss. This is simulated using a discrete distribution and the weight of each component in the mixture. The second step is to determine the claim value. This is simulated directly using the parameters and model-form of the component of the mixture that generates the loss.

Using the parameter uncertainty model described in Kreps [8], the severity parameters may also be simulated in each iteration.

The parameters of stochastic distribution of claim values should be adjusted for trend between the simulated accident date and the date at which the claims severity model is evaluated. The discussion of the impact of trend on loss distributions (and their parameters) is contained in Hogg & Klugman [6]. Essentially, this allows for each and every claim to be individually adjusted for trend. Furthermore, the model allows for trend adjustment based on accident date, report date or date of closing. The ability of the model to respond to trend in this manner is a benefit relative to link-ratio methods.

This simulation is shown on Exhibit 5. While spreadsheet tools can be used to simulate all these values, simulation software packages (often spreadsheet add-ins) will facilitate this process.

Through this process we have now have a comprehensive listing of all IBNR claims and all necessary information on those claims to:

- assign claims to report period,
- assign limits and retentions given claims made excess coverages, and
- calculate claims at various indemnity and / or expense retentions.

IV. A Specific Case – Bulk Reserves

The estimation of bulk reserves is simply a special case of the IBNR simulation model. Bulk reserves are simply the difference between case reserves and ultimate claims values. The process described above may also be applied to known claims with the following exceptions:

- The number of reported open claims is known and therefore does not need to be simulated.

- The accident dates and report dates of report claims are known and do not need to be simulated.

- In this simulation of closing lags it must be recognized that the domain of possible closing lags has a minimum value of the difference between the data valuation date and the actual report date. Therefore the closing lag should be simulated using a truncated distribution.

» Consideration may be given to the reserved value of the claim. If no consideration is given, the actuary (implicitly) assumes that the case reserve provides no predictive information. This may be a valid assumption for immature claims. For more mature claims, the reported value of the case should be considered in the simulation. Generally, this consideration results in "shifting" of the weights of the components of the mixed severity model or truncation of the severity distribution. Further research in this area is left to the interested reader.

» The simulated severity model should be truncated from below as the paid-to-date value of the claim. This may be a conservative adjustment as it will not allow claims to settle at their current value. An alternative would be to not consider paid amounts and allow individual claims to simulate at less then the paid value (most optimistic) or censor the resulting claim values to the paid value.

The reader should notice that these exceptions simply change the parameters of the simulation of ultimate values on known claims. The basic framework is identical to that used for the simulation of pure IBNR claims.

V. A Specific Case – Prospective Loss Forecast

The estimation of loss forecasts is also simply a special case of the IBNR simulation model. Loss forecasting and pure IBNR estimation are almost identical since no information is known about either claim type. The model is adjusted as follows to simulate prospective losses:

» The *a priori* estimate used in the B-F calculation is used as the mean estimate of prospective claims.[6]

» All claims occur within a single accident period. Therefore, the accident year need not be simulated.

[6] Essentially, this procedure is identical to that employed for pure IBNR frequency. The percentage of claims reported for a prospective period is by definition 0% and the estimated ultimate number of claims using a Bornhuetter –Ferguson model would be identical to the *a priori* frequency.

VI. Simulating From Truncated Distributions

Many aspects of this model require simulation from truncated distributions. Many simulation software packages allow for truncated distributions. If the actuary is either using spreadsheet software to perform the simulations or using simulation software that can not accommodate truncated distributions, the actuary can use inverted distributions to sample from a truncated distribution. Specifically,

» Calculate the CDF at the truncation points. For example, for distribution truncated from below at a, calculate F(a).

» Sample from a uniform distribution between the truncation points. In this example, the sampling would be between F(a) and 1.00 (= F(infinity)). We will designate the sampled value as U.

» Calculate the value at which the CDF is equal U. In this example, the value would be equal to $F^{-1}(U)$ and provides our sampled value from a truncated distribution.

VII. Conclusions and Areas for Further Research

The framework of the model presented herein provides a model that is adaptable, accounts for inflation, estimates risk (both process and parameter), and is easily extendable for loss forecasting applications. Finally, this model allows for consistency in the estimation of loss forecasts and loss reserves. The model attempts to replicate the claims process rather than representing a proxy model for future emergence.

The goal of this paper is to present a model framework. However, as with all actuarial models, this model remains a "work-in-progress." Several areas for model enhancement are listed below and left to the practioner:

» Relationships between lags and claim costs: In the current model, claims severity is independent of report lag and closing lags are independent of claims severity. The prevailing theory is that larger claims are reported later and take longer to settle.

» Additional methods to incorporate claim information on known claims: The model provides one method by which known claim information (specifically, case reserve values) can be considered in the calculation of bulk reserves. In the evaluation of a large number of known open claims, no consideration may be necessary as all claim types would be assumed to by represented in the sample. However, for situations involving the evaluation of a smaller number of open claims, it would be desirable to develop models under which the simulated severity considered as much of the information regarding these claims as possible.

Essentially because the model is simulating the measurable aspects of the claims process, it allows the actuary an almost limitless opportunity to study various relationships.

BIBLIOGRAPHY

[1] Barnett, Glen; and Zehnwirth, Ben, *"Best Estimates for Reserves,"* Casualty Actuarial Society Forum, 1998 Vol: Fall, Proceedings of the Casualty Actuarial Society 1978 Vol: LXV Page(s): 1-9, Casualty Actuarial Society: Arlington, Virginia.

[2] Weissner, Edward W., *"Estimation of the Distribution of Report Lags by the Method of Maximum Likelihood,"* Proceedings of the Casualty Actuarial Society 1978 Vol: LXVPage(s): 1-9, Casualty Actuarial Society: Arlington, Virginia.

[3] Klugman, S.A.; Panjer, H.H.; and Willmot, G.E., *Loss Models: From Data to Decisions*, 1998, John Wiley and Sons: New York.

[4] Keatinge, Clive L., *"Modeling Losses with the Mixed Exponential Distribution,"* Proceedings of the Casualty Actuarial Society 1999 Vol: LXXXVI Page(s): 654-698, Casualty Actuarial Society: Arlington, Virginia.

[5] Philbrick, Stephen W., *"A Practical Guide to Single Parameter Pareto Distribution,"* Proceedings of the Casualty Actuarial Society 1985 Vol: LXXII Page(s): 44-84, Casualty Actuarial Society: Arlington, Virginia.

[6] Hogg, Robert V. and Klugman, Stuart A., *Loss Distributions*, 1984, John Wiley and Sons, New York

[7] Heckman, Philip E.; Meyers, Glenn G., *"The Calculation of Aggregate Loss Distributions from Claim Severity and Claim Count Distributions;"* Proceedings of the Casualty Actuarial Society 1983 Vol: LXX Page(s): 22-61, Casualty Actuarial Society: Arlington, Virginia.

[8] Kreps, Rodney E., *"Parameter Uncertainty in (Log) Normal Distributions,"* Proceedings of the Casualty Actuarial Society 1997 Vol: LXXXIV Page(s): 553-580, Casualty Actuarial Society: Arlington, Virginia.

Hospital for Injured Actuarial Students

Estimation of Losses and Expense as of 9/30/2002

Estimation of Report Lag

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Observations			(2) - (1)	(3) - (1)	f (x) / F (M)	ln (6)
Incident Date	Report Date	Valuation Date	Report Lag (Days) (x)	Maximum Report Lag for Claim (M)	Conditional Likelihood (L)	Conditional Log-Likelihood ln(L)
01/06/97	01/31/97	09/30/02	25	2,093	1.305E-04	-8.9439
01/09/97	01/31/97	09/30/02	22	2,090	1.149E-04	-9.0714
01/06/97	01/31/97	09/30/02	25	2,093	1.305E-04	-8.9439
01/06/97	01/31/97	09/30/02	25	2,093	1.305E-04	-8.9439
01/07/97	01/31/97	09/30/02	24	2,092	1.253E-04	-8.9846
04/01/97	04/23/97	09/30/02	22	2,008	1.149E-04	-9.0714
01/04/97	06/02/97	09/30/02	149	2,095	7.353E-04	-7.2153
01/04/97	06/02/97	09/30/02	149	2,095	7.353E-04	-7.2153
02/28/97	06/22/97	09/30/02	114	2,040	5.763E-04	-7.4590
02/28/97	06/22/97	09/30/02	114	2,040	5.763E-04	-7.4590
02/28/97	06/22/97	09/30/02	114	2,040	5.763E-04	-7.4590
04/11/97	07/14/97	09/30/02	94	1,998	4.804E-04	-7.6410
04/13/97	07/14/97	09/30/02	92	1,996	4 706E-04	-7 6615
01/05/97	07/24/97	09/30/02	200	2,094	9 420E-04	-6 9675
04/06/97	07/28/97	09/30/02	113	2,003	5 715E-04	-7.4672
07/08/97	07/31/97	09/30/02	23	1,910	1 201E-04	-9 0270
07/09/97	07/31/97	09/30/02	22	1,909	1 149E-04	-9.0713
03/22/97	08/18/97	09/30/02	149	2,018	7.353E-04	-7.2153
03/22/97	08/18/97	09/30/02	149	2,018	7.353E-04	-7.2153
05/19/97	09/05/97	09/30/02	109	1,960	5.526E-04	-7.5009
06/30/97	09/26/97	09/30/02	88	1,918	4 510E-04	-7.7040
06/27/97	09/26/97	09/30/02	91	1,921	4.657E-04	-7.6719
02/20/97	10/09/97	09/30/02	231	2,048	1.051E-03	-6 8583
02/15/97	11/03/97	09/30/02	261	2,053	1.142E-03	-6.7748
09/14/97	11/05/97	09/30/02	52	1,842	2.701E-04	-8.2169
07/14/97	11/12/97	09/30/02	121	1,904	6 090E-04	-7.4036
07/14/97	11/12/97	09/30/02	121	1,904	6.090E-04	-7.4036
04/01/97	12/10/97	09/30/02	253	2,008	1 119E-03	-6.7951
11/01/01	08/29/02	09/30/02	301	333	4.935E-03	-5.3115
11/01/01	08/29/02	09/30/02	301	333	4.935E-03	-5.3115
11/20/01	09/10/02	09/30/02	294	314	5.397E-03	-5.2219
11/20/01	09/10/02	09/30/02	294	314	5.397E-03	-5.2219
11/20/01	09/10/02	09/30/02	294	314	5.397E-03	-5.2219
07/12/01	09/11/02	09/30/02	426	445	3.430E-03	-5.6753
01/19/00	09/12/02	09/30/02	967	985	4.762E-04	-7 6496
09/20/01	09/23/02	09/30/02	368	375	4.390E-03	-5 4285
09/20/01	09/23/02	09/30/02	368	375	4.390E-03	-5 4285
⋮	⋮	⋮	⋮	⋮	⋮	⋮
01/13/01	09/25/02	09/30/02	620	625	1 854E-03	-6 2901
09/10/01	09/25/02	09/30/02	380	385	4.240E-03	-5.4632
08/11/02	09/25/02	09/30/02	45	50	3.593E-02	-3.3263
09/10/01	09/25/02	09/30/02	380	385	4 240E-03	-5.4632
02/03/00	09/25/02	09/30/02	965	970	4.834E-04	-7.6347
01/09/00	09/27/02	09/30/02	992	995	4.280E-04	-7.7565
10/01/01	10/02/02	09/30/02	366	364	4 605E-03	-5.3806
08/08/01	10/04/02	09/30/02	422	418	3.777E-03	-5.5787
04/07/01	10/10/02	09/30/02	551	541	2.436E-03	-6 0174
10/03/01	10/15/02	09/30/02	377	362	4.686E-03	-5.3631
10/03/01	10/15/02	09/30/02	377	362	4.686E-03	-5.3631
10/07/01	10/15/02	09/30/02	373	358	4.761E-03	-5.3472
08/24/00	10/21/02	09/30/02	788	767	1.035E-03	-6.8735
10/17/01	10/21/02	09/30/02	369	348	4.980E-03	-5 3023
04/13/00	10/23/01	09/30/02	558	900	1 470E-03	-6.5228
04/13/00	10/23/01	03/31/02	558	717	1 749E-03	-6 3489
			Conditional Log- Likelihood ln(L)			-9150 5841

Report Lag Model	Model Parameters	Rayleigh
	b =	437 290
	Estimation	MLE
	Σln (*L*)	-9,150.58

Exhibit 1

Hospital for Injured Actuarial Students

Estimation of Losses and Expense as of 09/30/2002

Estimation of IBNR Claims

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Calendar Period		Avg of (1) & (2)		(4) - (3)			CDF ((5))	(7) / (8)	(9) / ((6))	A.*(6)*(1-(8))
Inception	Expiry	Average Accident Date	Valuation Date	Time Available to Report (X)	Exposure	Claims Reported to Date	Expected Percent Reported	Estimated Ultimate Claims	Frequency	Estimated IBNR Claims at 09/30/2002
1/1/1997	12/31/1997	07/02/97	09/30/2002	1,916	2,741	294	100%	294.02	0.107	0.02
1/1/1998	12/31/1998	07/02/98	09/30/2002	1,551	2,838	324	100%	324.60	0.114	0.56
1/1/1999	12/31/1999	07/02/99	09/30/2002	1,186	2,843	322	97%	330.35	0.116	7.65
1/1/2000	12/31/2000	07/01/00	09/30/2002	821	2,929	278	83%	335.75	0.115	53.63
1/1/2001	12/31/2001	07/02/01	09/30/2002	455	3,144	105	42%	251.18	0.080	194.83
1/1/2002	12/31/2002	07/02/02	09/30/2002	90	3,322	8	2%	381.74	0.115	346.26
				Total	17,816	1,331				602.95

A. Selected *a priori* frequency 0.106

Exhibit 2

Hospital for Injured Actuarial Students

Estimation of Losses and Expense as of 09/30/2002

Claims Settlement Model

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
					(4) / ((3)+(4)+(5))	(3) / ((3)+(4)+(5))	(5) / ((3)+(4)+(5))
Period		Closed w/ Indemnity	Expense Only Claim	Closed with no payment	% of Expense Only	% of Closed with Indemnity	% of Closed with no payment
Total by Occurrence Year							
Inception	Expiry						
1/1/1997	12/31/1997	77	103	71	41.0%	30.7%	28.3%
1/1/1998	12/31/1998	63	98	72	42.1%	27.0%	30.9%
1/1/1999	12/31/1999	44	83	75	41.1%	21.8%	37.1%
1/1/2000	12/31/2000	13	42	71	33.3%	10.3%	56.3%
1/1/2001	12/31/2001	1	4	23	14.3%	3.6%	82.1%
1/1/2002	12/31/2002	0	0	2	0.0%	0.0%	100.0%
Total		198	330	314	39.2%	23.5%	37.3%
Total by Closing Year							
Inception	Expiry						
1/1/1997	12/31/1997	0	0	0			
1/1/1998	12/31/1998	2	5	4	45.5%	18.2%	36.4%
1/1/1999	12/31/1999	12	37	19	54.4%	17.6%	27.9%
1/1/2000	12/31/2000	43	58	23	46.8%	34.7%	18.5%
1/1/2001	12/31/2001	68	105	60	45.1%	29.2%	25.8%
1/1/2002	12/31/2002	72	114	65	45.4%	28.7%	25.9%
not coded		1	11	143	7.1%	0.6%	92.3%
Total		198	330	314	39.2%	23.5%	37.3%
		Selected Distribution			40.0%	24.0%	36.0%

Exhibit 3

Hospital for Injured Actuarial Students

Estimation of Losses and Expense as of 09/30/2002

Severity Model

Model:	Mixed	Claim Type -->		Normal Small	Normal Large		Limit Loss		Shock
		Component Model -->		Lognormal	Lognormal		Point Mass		Uniform
			μ_i =	10.204	11.542	Mean	1,000,000	Minimum	1,000,000
			σ_i =	0.932	1.187	Std Dev.	-	Maximum	6,000,000
Truncation Point - Maximum Possible Claim				1,000,000	1,000,000		None		None
		w_i =		0.566	0.378		0.047		0.009

Exhibit 4

Hospital for Injured Actuarial Students

Estimation of Losses and Expense as of 09/30/2002

Simulation of True IBNR Claims
Sample Iteration

(1) Claim No.	(2) Accident Year	(3) Date of Occurrence	(4) Report Date	(5) Type of Claim	(8) Indemnity & Expense
1	2001	01/19/01	12/22/03	Normal Large	68,854
2	2001	04/30/01	01/08/04	Normal Small	63,381
3	2002	03/19/02	07/15/04	Normal Small	8,323
4	2000	09/23/00	01/17/03	Normal Small	14,616
5	2002	07/06/02	05/07/04	Shock Loss	2,004,453
6	2002	09/16/02	11/02/03	Normal Small	44,159
7	2002	07/12/02	09/13/03	Normal Large	32,163
8	2001	11/15/01	12/31/03	Normal Small	41,512
9	2001	12/26/01	03/19/05	Normal Small	93,695
10	2002	08/16/02	10/07/03	Normal Small	79,486
11	2002	01/08/02	10/28/02	Normal Small	41,606
12	2002	03/04/02	09/22/03	Limit Loss	1,000,000
13	2001	03/18/01	11/06/02	Normal Small	37,381
14	2002	06/01/02	04/03/04	Normal Large	294,091
⋮	⋮	⋮	⋮	⋮	⋮

Exhibit 5