IRR, ROE, and PVI/PVE

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Abstract:

This paper presents three related measures of the return on a Property-Casualty insurance policy. These measures are based on a hypothetical Single Policy Company model. Accounting rules are applied to project the Income and Equity of the company and the flows of money between the company and its equity investors. These are called Equity Flows. The three measures are: i) the Internal Rate of Return (IRR) on Equity Flows, ii) the Return on Equity (ROE), and iii) the Present Value of Income over the Present Value of Equity (PVI/PVE). The IRR is the yield achieved by an equity investor in the Single Policy Company. The ROE is the Growth Model Calendar Year ROE computed on a book of steadily growing Single Policy business. The PVI/PVE is computed by taking present values of the projected Income and Equity of the Single Policy Company. The paper includes new results relating the PVI/PVE and ROE to the IRR. Beyond developing the foundation and theory of these return measures, the other main goal of the paper is to demonstrate how to use the measures to obtain risk-sensitive prices. To do this, Surplus during each calendar period is set to a theoretically required amount based on the risk of the venture. The main source of risk arises from uncertainty about the amount and timing of subsequent loss payments. With the IRR and PVI/PVE, the indicated prices are those needed to achieve a fixed target return. The indicated price using the Growth Model is that needed to hit the target return at a specified growth rate. With the Growth Model, one can also compute the premium-to-surplus leverage ratio for the Book of Business when it achieves equilibrium. The ability to relate indicated pricing to a leverage ratio, growth rate, and return is an advantage of Growth Model and could lead to greater acceptance of its results. The paper includes sensitivity analysis on the returns and on the indicated profit provisions. In the presentation, the analysis of return is initially done for a single loss scenario. Later, there is discussion on how to model the return when losses are a random variable instead of a single point estimate. Finally, there is a comparison of the approach in this paper versus that of the Discounted Cash Flow model.

Keywords: ROE, IRR, PVI/PVE

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1. INTRODUCTION

In this paper, we will present three related ways to measure the return on an insurance policy. The three measures are:

- The Internal Rate of Return on Equity Flows (IRR)

- The Growth Model Calendar Year Return on Equity (ROE)
- The Present Value of Income Over Present Value of Equity (PVI/PVE)

Then we will demonstrate how to use these measures to price Property-Casualty insurance products. We will do this from the perspective of a pricing actuary conducting analysis for a stock insurance company. Whether any of these methods is appropriate in another context is a subject outside the scope of our discussion.

There is nothing novel about using measures of return to price products. The idea is simple enough: any venture with return above a given target hurdle rate is presumably profitable enough to be undertaken. The indicated price for a product can then be defined as the one at which its expected return hits the target. Within the context of internal corporate pricing analysis, corporate management usually sets the target return and a common target is generally used for all insurance ventures.

A significant problem in Property and Casualty insurance pricing applications is that there is no one universally accepted measure of return. The sale of an insurance policy leads to cash flows, underwriting income, investment income, income taxes, and equity commitments that may span several years. How do we distill all this into one number, the return on the policy?

Our three measures are based on two related, but distinct, notions of return on a policy. The first idea is to define return from the perspective of an equity investor who supplies all the capital required to support the policy and who in return receives all the profits it generates. The other idea is to generalize the return achieved by a corporation so that it can be applied to a policy. GAAP ROE (Return on Equity) is a commonly accepted measure of corporate calendar year return. We have two ways to adapt this to a single policy. One is to extend GAAP ROE beyond a single calendar year so that it can handle multi-year ventures. The other is to generate a hypothetical book of business and then measure its ROE. Thus we will end up with three measures of return.

To ensure necessary precision in our analysis, we will define our measures of return by modeling a hypothetical company, the Single Policy Company, which writes a particular policy, the Single Policy. The Single Policy Company writes no other business and is liquidated when the last loss and expense payment is made. Suppose we consider a particular loss scenario and have a model for its anticipated premium, loss, and expense cash flows. We can then apply accounting rules to derive the underwriting income for the Single Policy Company. With other assumptions about investment returns, Statutory Surplus requirements, and taxes, we can derive the company's Investment Income, Income Tax, GAAP After-Tax-Income and GAAP Equity for each accounting period. We will also model a related hypothetical company, the Book of Business Company. This company has a portfolio consisting entirely of Single Policy business. Each period it writes a policy that is a scaled version of the Single Policy. The Book of Business Company begins operations when it writes its first policy and is liquidated after the last loss and expense payment is made on the last policy. We can project the Income Statement and Balance Sheet for the Book of Business Company. Our three profitability measures are defined from the Single Policy and Book of Business Company constructs.

The IRR on Equity Flows is the return that would be achieved by an equity investor in the Single Policy Company. It is a total return measure that reflects the equity requirements, underwriting income, investment income, and taxes associated with the policy by accounting period over time.

PVI/PVE is another measure of profitability based on the Single Policy Company model. It is a generalization of GAAP ROE defined as the ratio of the present value of income valued as of the end of year 1 over the present value of equity. We will show that PVI/PVE will also equal IRR if the present values are computed using a rate equal to the IRR.

Growth Model Return on Equity (ROE) is defined as the Calendar Year ROE that will eventually be achieved by the Book of Business Company if it grows at a constant rate. Under the constant growth assumption, the company will attain an equilibrium in which its Calendar

Year ROE stays constant. We will show that Growth Model ROE equals IRR if the growth rate is also the IRR.

We will derive indicated prices from our return measures. We want these indicated prices to be consistent and sensitive to risk. We also want them to reasonably reflect management's risk-return preferences. To achieve this, we will set Surplus in our model based on a theoretical requirement, and not on an allocation of actual Surplus. Since each of our return measures is sensitive to the effects of leverage, the resulting prices will vary with risk. There are several ways to derive theoretical Surplus requirements and we will not advocate any particular method. We will assume that one has been chosen and that it incorporates any necessary portfolio correlation and order adjustments.

We have said Surplus in our model is a theoretically required amount based on the risk of the venture. But what risk are we talking about? While there is some risk related to the investment of assets, the principal risk in Property and Casualty insurance ventures stems from uncertainty about the timing and amount of loss payments[1]. That is the sole risk we will consider in setting Surplus for our model.

Our initial Surplus is based on the distribution of the present value of ultimate losses. This seemingly innocuous statement has major implications in pricing analysis. For if we vary the premium, we do not change the losses and therefore do not change the amount of surplus. The conclusion is that variations in pricing should lead to variations in the premium-to-surplus

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[1] Robbin and DeCouto[15] argue that the risk measure should act on the present value of underwriting cash outflow, where underwriting cash outflow is loss plus expense less premium. This allows consistent treatment of swing rating plans and contingent commissions, where the premium or expense may be functions of the loss. We will simplify matters in this discussion and assume premium and expense are not adjusted retrospectively.

ratio. In order to see this, consider an example in which the required surplus is derived from the loss distribution and is equal to $50. Suppose the initial premium is $100, so that the initial premium-to-surplus ratio is 2.00. Now consider the situation when the premium is changed to $110. Since the loss distribution is unchanged while the premium has been increased, the required initial surplus should still suffice[2]. Let us suppose it stays at $50. Even though the required surplus has not changed, the leverage ratio is now 2.20 (2.20= 110/50).

The situation is even more complicated when we consider the duration of surplus commitments. Following our logic one step further, we should set surplus at each point in time based on the risk associated with unpaid losses. Since it may take many years for all loss to be paid on a policy, the surplus will evolve over several years. This underscores the conclusion that when pricing analysis is being conducted the proper way to set surplus is not with a fixed premium-to-surplus ratio. This does not mean that in a different context, such as in solvency regulation or rating agency analysis, that comparisons against fixed premium-to-surplus ratios would not be appropriate.

As a caution we should note that our discussion has not addressed the question of comparability between insurance ventures and alternative non-insurance ventures. Since delving into this larger question would take us too far afield from our main topic, we will not consider it further. Also, we should note that in the modeling examples in this paper, Surplus is set simply as a fixed percentage of the expectation of the present value of unpaid losses. This is done in order to clarify the presentation. In any actual application, this loading percentage should vary with the risk by policy and development age.

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[2] Robbin and DeCouto [15] discuss two sorts of capital requirements. One is called Level Sensitive and it declines as the premium rate is increased. The other is called Deviation Sensitive and it stays invariant when the premium rate changes. The approach in this paper is equivalent to the Deviation Sensitive approach.

An equivalent, but different, approach to pricing can likely be obtained by using a fixed and common Surplus requirement for all insurance ventures in conjunction with target returns that vary with risk. In order to avoid debate on which approach is better, we will allow that our preference for using a fixed target return on risk-sensitive capital may be largely aesthetic.

The IRR on Equity Flows has already been presented in the Robbin [13] and Feldblum [8] Study Notes. It has also been used in NCCI rate filings. Appel and Butler [1] have previously addressed some criticisms of the IRR approach. The PVI/PVE has also been presented by Robbin [13] and it appears to be equivalent to the NVP Return developed by Bingham [2].

The Growth Model ROE has some connection to previous work done by Roth [16]. In it, he showed how to convert calendar year figures into a true measure of current year return. He also advocated a target return that includes provision for growth as well as the current return needed for shareholders. The Growth Model ROE provides a way to implement these ideas in a pricing context. With it, the actuary can relate indicated pricing with a calendar year ROE, growth rate, and leverage ratio. These are metrics of interest to insurance company executives and could lead to greater acceptance of the results.

Our analysis will also touch on some of the differences between alternative approaches. First it is important to clarify differences between different IRR models. Some authors have discussed an IRR that is an IRR on underwriting cash flows (paid premium less paid loss and paid expense). There has rightly been criticism that this IRR may not even be defined when the flows switch sign more than once. This may not happen frequently in such models, but the counterexamples given by critics are not unduly atypical.[3] However, as we shall later see, it would be very unusual for the Equity Flows we define to change sign more than once. So this criticism generally does not apply to our IRR on Equity Flows.

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[3] See D'Arcy [5] p525.

Discounted Cash Flow models have many features in common with our three models, but there are important differences. Perhaps most notable is the tautological point that they are focused on underwriting cash flows. As a consequence, they either omit or need to graft on factors such as the accounting treatment of expenses and Surplus requirements. Consider that these methods have no direct way to reflect the conservative treatment of expenses under Statutory Accounting or, equivalently, no direct way to reflect the Deferred Acquisition Balance under GAAP. While some DCF methods do account for taxes on investment income related to Surplus, their results are relatively insensitive to the leverage effects of Surplus. As well, there is no way to study the impact on return from holding discounted loss reserves.

In Section 2, we will present the Single Policy Model. We will use it to define the IRR on Equity Flows in Section 3 and the PVI/PVE Measure in Section 4. In Section 5 we will construct the Book of Business Growth model and define the Growth Model Equilibrium Calendar Year ROE. In Section 6, we will consider modeling returns when the loss can be a random variable instead of a single point estimate. In Section 7, we will study the sensitivity of our return measures to the premium, Surplus level, the interest rate, and the loss payout pattern. We will do this with reserves held at full value or discounted. Then, in Section 8, we will show how to use these measures to derive profit provisions. We will examine the sensitivity of these profit provisions to the Surplus level, the interest rate, and the loss payout pattern. In Section 9 we will compare our approach against the Risk-Adjusted Discounted Cash Flow procedure.

2. THE SINGLE POLICY COMPANY MODEL

Our objective here is to show how to model the accounts of the Single Policy Company based on assumptions about the underwriting results and cash flows of the Single Policy. Our specific goal is to derive the Income and Equity of the Single Policy Company. We will often make simplifying assumptions as this will make it easier to understand the procedure[4]. When modeling actual policies for business analysis, sufficient detail should be incorporated.

An initial assumption we will make is that results are exactly as anticipated. Thus, we will derive a return that is really a return "if all goes just as planned". Later, we will discuss modeling when there is a distribution of possible outcomes.

Before modeling the various income statement, cash flow, and balance sheet accounts, we need to carefully state our indexing conventions. We will use a subscript, j, to denote the value of an income item or cash flow occurring at the end of the j^{th} accounting period. Similarly, a balance sheet account with a subscript, j, denotes its value as of the end of the j^{th} accounting period. We use the subscript, j=0, for a cash flow to indicate the flow takes place at policy inception. As well we use the j=0 subscript for a balance sheet account to denote its initial value. However, we will assume that income can only be declared at the end of an accounting period so that any income item with a j=0 subscript is automatically zero. This is an important assumption. If we were working with an accounting system with some income or loss declared at inception, we would adopt a modified accounting system that would defer that income to the end of the first period and post the appropriate deferred balance as a debit or credit to surplus. To simplify the analysis, we will also assume that no cash flows take place at intermediate times and that the value of a balance sheet account stays constant during a period. This implies the average value of a balance sheet account <u>during</u> the $(j+1)^{st}$ period is equal to its value as of the end of the $(j)^{th}$ period. We will use annual accounting in presenting our model. We will later add a few comments on refining the accounting to a quarterly or monthly basis. Finally, we will assume that the last loss payment is made exactly "n" periods after policy inception and that the Single Policy Company is then liquidated.

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[4] See Feldblum [8] for a more extensive discussion of modeling details.

As regards accounting conventions, our general approach will be to use Statutory Accounting and make some of the adjustments needed to derive GAAP Income and GAAP Equity. Our Income and Equity will also reflect some simplifications. Nonetheless, unless there is a need to make a distinction, we will refer to our Income and Equity as "GAAP".

With these conventions we define booked underwriting income for the j^{th} accounting period:

(2.1

$$U_j = EP_j - IL_j - IX_j$$

$$\text{for } j = 1, 2, \ldots, n$$

Here U is underwriting gain, EP is earned premium, IL is incurred loss, and IX is incurred underwriting and general expense. The loss includes loss adjustment expense. The incurred loss is calculated on a calendar period accounting basis so that it reflects posted IBNR adjustments as well as case incurred losses. However, the loss reserve is not necessarily held at full value, but could be discounted. In the examples in the Exhibits, we compute expense as the sum of a fixed amount plus a component that varies with premium. We assume the Statutory Incurred Expenses are incurred according to a fixed pattern, while the GAAP Expenses are incurred as premium is earned. The difference between Statutory and GAAP Incurred Expense to date is called the Deferred Acquisition Cost Balance (DAC). To keep matters simple, we ignore policyholder dividends.

Next we turn to the very critical question of how Equity is handled in our model. Our assumption is that Equity will be derived from Statutory Surplus and that the Statutory Surplus will adhere to pre-set requirements. We define S_j as the Required Surplus as of the end of the j^{th} period. In later examples, we will always set Required Surplus as a fixed percentage of the expected discounted unpaid loss. However, for our initial purposes, it is not so important how it is set, as the fact that it is set in advance. We can then derive Q_j, the required GAAP Equity.

We make the simplifying assumption that the only difference between GAAP and Statutory Accounting is in the treatment of initial expenses. Thus, we only need to adjust Q_0 for Deferred Acquisition Costs (DAC). Under this hypothesis we have:

(2.2

$$Q_0 = S_0 + DAC$$
$$Q_j = S_j \text{ for } j = 1,2,\ldots,n$$

Note that $Q_n = 0$ since that is the time when the last loss is paid.

Next we define assets as the sum of Statutory Reserves and Statutory Surplus:

(2.3

$$A_j = UEPR_j + XRSV_j + LRSV_j + S_j$$
$$\text{for } j = 0, 1, 2, \ldots n$$

This equation embodies the fundamental accounting principle that the balance sheet must balance. Here UEPR is the Unearned Premium, XRSV is the Statutory Expense Reserve, LRSV is the Loss Reserve, and S is the Surplus. The Loss Reserve is the calendar period loss reserve, inclusive of IBNR as well as case reserves. We could write a similar equation under GAAP. While the Equity would differ from Statutory Surplus and the expense reserves would be different, the resulting assets would be the same under the simplifying assumptions we have made[5]. Note the basic balance sheet formula is used here to define the assets. In contrast, when evaluating real companies, the assets are given and it is the surplus that is then derived by subtracting the liabilities.

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[5] As long as there are no GAAP assets such as Goodwill that do not exist in Statutory Accounting, we will have equality between GAAP and Statutory Assets even though the liabilities may differ.

Next we derive invested assets:

(2.4

$$IA_j = A_j - RECV_j$$

$$\text{for } j = 0, 1, 2, \ldots, n$$

In this formula, we use RECV to denote receivables and amounts recoverable.

With invested assets we can compute investment income for each accounting period. Letting "i" denote the risk-free return on invested assets, we have:

(2.5

$$II_j = i \cdot IA_{j-1}$$

$$\text{for } j = 1, 2, \ldots, n$$

We define pre-tax income as the sum of investment income and underwriting income:

(2.6

$$INCPTX_j = U_j + II_j$$

$$\text{for } j = 1, 2, \ldots, n$$

To handle taxes, we define taxable underwriting income, UITX, and taxable investment income IITX by period. We let t_U denote the tax rate on underwriting income and t_I the tax rate on the taxable investment income. We then compute the tax each period via:

(2.7

$$TAX_j = t_U UITX_j + t_I IITX_j$$
$$\text{for } j = 1, 2, \ldots, n$$

Note we are allowing income taxes to be negative. Also note that taxes in our simplified model are paid when the income is declared. A more realistic approach might utilize carry-forwards and carry-backs in the tax calculation. We would also apply the reserve discounting, unearned premium disallowance, and other provisions of the current US tax code. As well, we would model GAAP in more detail by setting up a deferred tax balance to reflect differences between tax basis and accounting basis income. While the model could be made more elaborate and realistic along these lines, we will avoid complications by using our simplified approach in this paper. In any real-world application, the actual tax code should be modeled in detail. A final note on taxes is that in our examples we will simplify matters by using a common tax rate for underwriting and investment income.

Finally, we define after-tax income:

(2.8

$$I_j = INCPTX_j - ITAX_j$$
$$\text{for } j = 1, 2, \ldots, n$$

Now that we have the Income and Equity accounts of the Single Policy Company, we are ready to define the return on the Single Policy.

3. THE IRR ON EQUITY FLOWS

We now define equity flows as the flows of money between an equity investor and a company. The flows of money could be due to the purchase of stock, the payment of dividends, or the repurchase of stock. We suppose the equity flows are given by the reconciliation formula: equity flow equals income less the change in the equity balance[6]. This presumes any capital shortfall will by corrected by using equity capital[7]. Under this definition, flows of investor capital into the company carry a negative sign, while payments from the company to the investors carry a positive sign.

To compute the Equity Flow, F, we add the Income and subtract the increase in the Single Policy Company's Equity:

For j= 0, we set

(3.1

$$F_0 = I_0 - Q_0 = -Q_0$$

For j= 1,2, ..., n, we set:

(3.2

$$F_j = I_j - (Q_j - Q_{j-1}) = I_j - \Delta Q_{j-1}$$

Figure 1 depicts this general construction.

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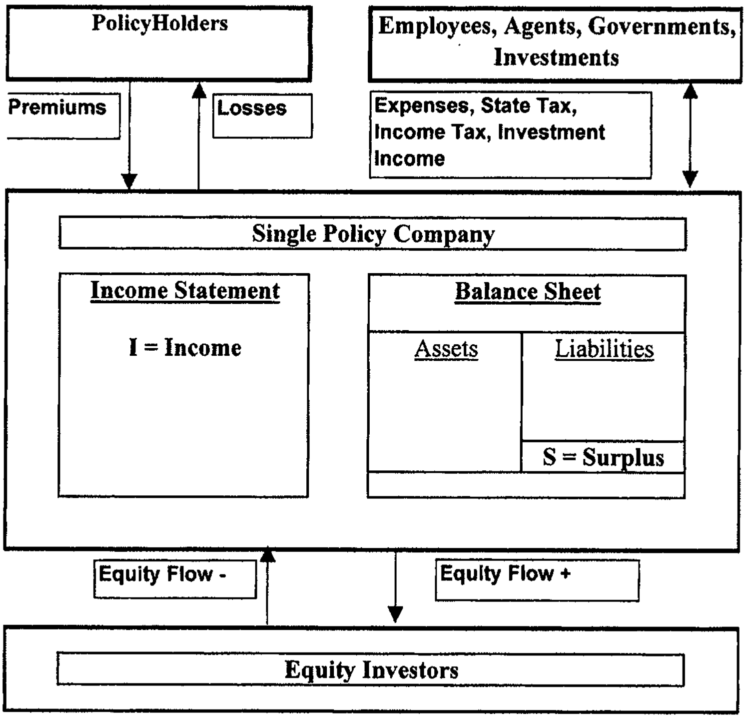
[6] This is a simplified version of the formula in Roth[15].

[7] In other words, we will not consider the use of debt and other non-equity capital in meeting the Surplus requirements.

Figure 1



For the insurance applications we are considering, the initial equity flow, F_0, will always be negative. There are two reasons for this. First, the initial commitment of equity needed to fund the Surplus, S_0, contributes the amount $-S_0$, to the initial equity flow. Second, there is a commitment of equity associated with the Deferred Acquisition Cost balance. This is also called the "Equity in the Unearned Premium Reserve". It arises from the conservative treatment of expenses in Statutory Accounting under which acquisition expenses are incurred up-front rather than as the premium is earned.

The IRR on Equity Flows, y, solves the IRR equation:

(3.3

$$\sum_{j=0} F_j \cdot (1+y)^{-j} = \sum_{j=0} F_j \cdot w^j = 0 \quad \text{where } w = (1+y)^{-1}$$

The IRR, if it exists and is unique, is comparable to the interest rate on a loan or the yield rate on a bond. However, since IRR is in general the solution to a nth degree polynomial, there might be multiple real roots. In that case, for each real root, the equity flows can be decomposed into a series of lending and borrowing transactions at the rate of interest equal to that root. For example, if the flows are (–200, +420, -220), the roots are 0% and 10%. With 0%, a loan of 200 is made from A to B and paid back after one year, and then a loan of 220 is made from B to A and it is paid back a year later. The decomposition is: (-200, 420, -220) = (-200, 200, 0) + (0, 220, -220). For the 10% interest rate, the decomposition is (-200,420, -220) = (-200, +220, 0) +(0, 200, -220). This is shown in the following chart.

Figure 2

	Combined Flow from A to B	**Loan From A to B**		**Loan From B to A**	
Time	**FV Flows**	**FV Flows**	**PV @ 10%**	**FV Flows**	**PV @ 10%**
0	-200	-200	-200	0	0
1	420	220	200	200	181.82
2	-220	0	0	-220	-181.82

While multiple roots are a general problem for IRR analysis[8], they do not arise, except in pathological cases, when computing the IRR on the anticipated Equity Flows for a Single Policy. This is because the Equity Flows in our model only switch signs once. As previously noted, the initial Equity Flow is negative due to the up-front commitment of Surplus and the posting of the Deferred Acquisition Cost balance. After that, during the period the premium is earned, the Equity Flows could be negative or positive depending on the amount of underwriting loss and expense in relation to premium and on whether reserves are held at full value or are discounted. Thereafter, the Equity Flows are all positive. This is due to the earning of investment income and the takedown of Surplus[9]. Also, note that anticipated deferred premium payments or salvage and subrogation loss recoveries and other factors that could lead to reversals in the sign of the net underwriting cash flows do not lead to reversals in the sign of the Equity Flows we have defined. This is true because such payments do not impact the booked underwriting gain. With only one sign change in the Equity Flows, there will be only one root to the polynomial equation and the IRR on the Equity Flows will thus be unique.

On the first sheets of Exhibits 2 and 3 are examples showing the accounts of the Single Policy Company for a hypothetical policy. In each case, the resulting equity flows switch signs once and as a result the IRR is unique. Exhibit 2 is the base case. In Exhibit 3 we show results when loss reserves are discounted. Our ability to do this stems from having an underlying corporate structure with balance sheets and income statements. With Discounted Cash Flow models, there is no natural way to model the distinction.

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[8] Sign reversals are a problem for single policy cash flow analysis as shown in D'Arcy [5].

[9] There is also an implicit assumption that reserves, if discounted, will be discounted at a consistent rate that is less than the anticipated risk-free immunized investment rate. Pathological examples can be constructed by abruptly altering the reserve discount rate from one period to the next. This could lead to reversals in the sign of the Equity Flows.

Two objections that have been raised against IRR are, first, it may not exist due to multiple roots to the IRR equation, and, second, it has an implicit reinvestment assumption at a rate different from the market rate. Appel and Butler [1] have already answered these on general grounds. To eliminate the sign changes that lead to multiple roots, they introduced preferential borrowing and lending rules between a firm and a project under the assumption that "...a transfer of a loan to a future date must be accomplished at the market rate of interest". While we agree with Appel and Butler on general grounds, we do not need such a sweeping argument. We may grant there are general problems with IRR analysis when the flows change sign more than once, but the Equity Flows we are analyzing only experience one sign change. So, for our particular application, that is not an issue.

4. THE PVI/PVE MEASURE

While the IRR on the Equity Flows is an intuitive measure comparable to the interest rate on a loan, we would also like to define a single policy ROE, a measure expressed as the ratio of income over equity. In calendar year accounting it makes perfect sense to take the ratio of income for the year over the initial (or average) equity for the year. However, the Single Policy generates Income over many years and it has Equity requirements that may span more than one year. To summarize the multi-year Income and Equity associated with the Single Policy, we will take present values. The result is a measure of return, PVI/PVE, the ratio of the present value of income over the present value of equity.

Let r_I be the interest rate we will use to discount Income and let r_Q be the interest rate we will use to discount Equity. We set $w_I = (1+r_I)^{-1}$ and $w_Q = (1+r_Q)^{-1}$. Assume the last loss payment for the Single Policy is made at the end of "n" years. Then PVI/PVE is given as:

(4.1

$$\text{PVI} / \text{PVE} = \frac{(1 + r_I) \cdot \sum_{j=1}^{n} I_j \cdot w_I^{-j}}{\sum_{j=0}^{n-1} Q_j \cdot w_Q^{-j}}$$

Note the formula is effectively discounting income to the end of the first year. This is done to make the definition of return consistent with the usual definitions of ROE and interest rate. In those definitions, income is taken at the end of the year and is not discounted. Note that under our definition a one-year venture has PVI/PVE equal to the interest rate and is independent of the rates used for discounting[10].

We have allowed for possibly different rates to be used for discounting numerator and denominator. However, our favored approach is to discount both at the same rate and we will henceforth assume a common rate is used unless otherwise stated. Also, we believe that in the PVI/PVE context, the appropriate rate for discounting is the cost of capital. We favor the cost of capital over the risk-free rate because the Single Policy Company can borrow at the cost of capital. The thought is that the Single Policy Company could use borrowed money to give its equity investors the PVI/PVE return each year. The income generated by the Single Policy Company in subsequent years would be used to repay the loans. We have previously mentioned a criticism against IRR: that it uses implicit rates of reinvestment at non-market rates of interest. It is hard to raise a similar criticism against PVI/PVE when the discounting is done using the cost of capital. The rate is explicit and it is the market rate for the company.

For a numerical example, suppose the Single Policy has a two-year payout pattern and assume the Single Policy Company will have Equity of 40.0 for year one, and 22.0 for year two.

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[10] If \$100 is put in a bank account at the start of the year and earns \$10 of interest paid at the end of the year, the return is 10%. The \$10 is not discounted.

Using our indexing notation, we would have $Q_0 = 40.0$, $Q_1 = 22.0$, and $Q_3 = 0$. Now assume income of 5.0 for year one and 4.4 for year two. With our notation, this would translate to $I_0 = 0.0$, $I_1 = 5.0$ and $I_2 = 4.4$. Using a 10.0% rate for discounting, the present value of the income at the end of year one would be 9.0 (5.0 + 4.4/1.1). The present value of the equity would be 60.0 (40.0 + 22.0/1.1). Thus the resulting PVI/PVE would be 15.0% (9.0/60.0).

Next we will show that PVI/PVE is equal to the IRR if the rates for discounting are set equal to the IRR.

(4.2

Result Relating PVI/PVE and IRR: If $r_I = r_Q$ = IRR, then PVI/PVE = IRR.

Proof: Let y = IRR and $w = (1+y)^{-1}$. Then from the IRR Equation we have

(4.3

$$0 = \sum_{j=1}^{n} I_j \cdot w^{-j} - Q_0 - \sum_{j=1}^{n-1} (Q_j - Q_{j-1}) \cdot w^{-j} + Q_{n-1} w^{n}$$

It follows that:

(4.4

$$\sum_{j=1}^{n} I_j \cdot w^{-j} = Q_0 + (Q_1 - Q_0)w + (Q_2 - Q_1)w^2 + \ldots + (Q_{n-1} - Q_{n-2})w^{n-1} - Q_{n-1}w^n$$

$$= (1-w) \cdot \sum_{j=0}^{n-1} Q_j \cdot w^{-j}$$

Dividing both sides by the present value of the equity, we obtain:

(4.5

$$1 - w = \frac{\sum_{j=1}^{n} I_j \cdot w^{-j}}{\sum_{j=1}^{n} Q_j \cdot w^{-j}}$$

and multiplying by $(1+y)$ leads to the desired result.

This result can be viewed as a way to interpret IRR. Under this interpretation, IRR is a PVI/PVE measure in which the rates for discounting change with the profitability of the policy. Note the idea that these rates should change is antithetical to the PVI/PVE approach. Under the PVI/PVE approach, these rates are, in principle, fixed before modeling the particular result for a policy. In Exhibits 2 and 3 we show the two PVI/PVE that result from use of two different discount rates. The first is based on a common rate of 12.0% and the second is based on a rate equal to the IRR.

Now, suppose we set the target IRR, target PVI/PVE, and the PVI/PVE discounting rates equal to the cost of capital and derive the resulting profit provisions. According to our theory, the two measures will generate identical profit provisions. So in the end, as far as indicated profit provisions are concerned, we arrive at the same answer whether we use IRR or PVI/PVE. In that situation, PVI/PVE does not provide an alternative to IRR, but rather another justification for the validity of an indicated IRR-derived profit provision.

5. BOOK OF BUSINESS GROWTH MODEL

We will construct a book of Single Policy business by writing a policy at the start of each accounting period. Each policy is a scaled version of the Single Policy. By summing contributions from all prior policies we can derive the income statement items, cash flows, and balances for the Book of Business Growth Company. If the scaling factors are generated from a uniform growth rate, we can express the accounts for the Book of Business Company as polynomial functions of the growth rate. We will see that the company goes through a start-up phase during which its reserves, assets, surplus and investment income all increase at a rate higher than the generating growth rate. Eventually, the company reaches an equilibrium growth phase at which point all accounts increase at the generating growth rate. We will measure the calendar period return for the Book of Business Growth Company.

Before we can properly analyze the Book of Business Company, we need to convert our indexing notation from one that refers to timing to one that refers to accounting period. We do this by introducing beginning of period (BOP) and end of period (EOP) suffix notation. The conversion is straightforward. Balance sheet accounts having a subscript, "0", get converted to accounts with a suffix BOP and a subscript "1". In other words, the balance at time t=0 is viewed as the balance for the beginning of period 1. For a balance sheet account, B_t, with time value index, t, strictly larger than zero, we define the ending balance at the end of period "t", $BEOP_t$, to be equal to B_t. Under our assumptions, this is the starting value for the next period, so that we have: $BBOP_{t+1} = B_t$. Also, since we have assumed that income is only declared at the end of time periods, the translation is very easy for income accounts: an account with a timing subscript t becomes an end of period account for period t. Figure 3 provides a simple numerical example of the conversion to accounting period notation.

Figure 3

Single Policy - Timing Notation		
	Equity	Income
t	Q	I
0	40.0	0.0
1	22.0	5.0
2	0.0	4.4

Single Policy - Accounting Notation			
	Equity		Income
Period	QBOP	QEOP	IEOP
1	40.0	22.0	5.0
2	22.0	0.0	4.4

Next, we will extend this notation to the Book of Business Growth Company, by adding a prefix G in front of a Single Policy Company variable. We assume the business premium volume is growing at a fixed rate of growth, g, and that a new scaled version of the Single Policy is added to the Growth Company at the start of each period. We let "n" denote the number of periods till all loss is paid for the Single Policy. We can then translate a Single Policy Balance Sheet account, B, to the corresponding beginning of period and end of period balances for the Book of Business Growth Company using the following formulas:

(5.1

$$GBBOP_k = \sum_{i=0}^{k-1} B_i \cdot (1+g)^{k-1-i}$$

(5.2

$$GBEOP_k = \sum_{i=1}^{k} B_i \cdot (1+g)^{k-i}$$

For example, the Equity at the beginning of year two would be $GQBOY_2 = Q_0(1+g)+Q_1$ and the Equity at the end of year two would be $GQEOY_2 = Q_1(1+g)+Q_2$.

The summations in formulas 5.1 and 5.2 can be readily understood with a policy contribution diagram:

Figure 4

Book of Business with n=2
Balance Sheet Account Growth - Policy Contribution Diagram

Policy	Year 1 BOY	Year 1 EOY	Year 2 BOY	Year 2 EOY	Year 3 BOY	Year 3 EOY	Year 4 BOY	Year 4 EOY
1	B_0	B_1	B_1					
2			$(1+g)*B_0$	$(1+g)*B_1$	$(1+g)*B_1$			
3					$(1+g)^2*B_0$	$(1+g)^2*B_1$	$(1+g)^2*B_1$	
4							$(1+g)^3*B_0$	$(1+g)^3*B_1$

To provide a numerical example, suppose the Single Policy had Equity balances: $Q_0 = 40.0$ and $Q_1 = 22.0$, and $Q_2 = 0$.. Assume the Growth Company writes the Single Policy at the beginning of year one and writes a 10% larger version of the Single Policy at the start of year two. Using 5.1 and 5.2, the total Equity for the two policies at the beginning of year two would be 66.0 (40.0*1.1+ 22.0). The total Equity would then drop to 24.2 (22.0*1.1) at the end of year two. Using our growth model notation, we would write $GQBOY_1 = 40.0$, $GQEOY_1 = 22.0$, $GQBOY_2 = 66.0$, and $GQEOY_2 = 24.2$.

It is important to note that, even though we have assumed end of period balances for one period are identical to the starting balances for the next period for the Single Policy, the same is not true for the Growth Company. This is true because a new policy is added to the Growth Company portfolio at the start of the next period. The balances from the new policy show up in beginning of period balances for that next period.[11].

[11] For example, since a new policy is written on 1/1/(y+1), the unearned premium balance on 12/31/y is different from the unearned premium balance on 1/1/(y+1).

We will next write a formula for Growth Company income statement accounts. However, under our assumptions, the beginning of period income will always be zero. So we only need supply a formula for "end of period" income items:

(5.3

$$GIEOP_k = \sum_{j=1}^{k} I_j \cdot (1+g)^{k-j}$$

Again, a policy contribution diagram can be useful in understanding the summation:

Figure 5

Book of Business with n=2
Income Account Growth - Policy Contribution Diagram

	Year 1		Year 2		Year 3		Year 4	
Policy	**BOY**	**EOY**	**BOY**	**EOY**	**BOY**	**EOY**	**BOY**	**EOY**
1		I_1		I_2				
2				$(1+g)*I_1$		$(1+g)*I_2$		
3						$(1+g)^2*I_1$		$(1+g)^2*I_2$
4								$(1+g)^3*I_1$

To continue with our numerical example, suppose the Single Policy had income of 5.0 at time t=1 and income of 4.4 at time t=2. Under the Growth Model, this would translate to income of 5.0 at the end of year one and 4.4 at the end of year two. Again supposing a 10% larger version of the policy was written at the start of year two, the total income for the Book of Business Company would be 5.0 at the end of year one and 9.9 (9.9 = 5.0*1.1 + 4.4) at the end of year two. The ROE for year two would be 15.0% (.15 = 9.9/66.0).

Now we consider what happens when the Growth Company has been growing for "n" periods. After that, all income statement and balance sheet accounts will be increasing at the growth rate and we say the business is in the Equilibrium Growth Phase. When this equilibrium has been reached, the formulas can be written as:

(5.4

$$GBBOP_{n+k} = (1+g)^k \sum_{j=0}^{n-1} B_j \cdot (1+g)^{n-1-j} = (1+g)^{k+n-1} \sum_{j=0}^{n-1} B_j \cdot (1+g)^{-j}$$

(5.5

$$GBEOP_{n+k} = (1+g)^k \sum_{j=1}^{n-1} B_j \cdot (1+g)^{n-j} = (1+g)^{k+n-1} \sum_{j=1}^{n-1} B_j \cdot (1+g)^{-(j-1)}$$

So, for example, if n=2, the Equity at the beginning of the fourth year would be given as:

(5.6

$$GQBOY_4 = (1+g)^3 (Q_0 + Q_1 (1+g)^{-1})$$

The Equity at the end of the fourth year would be:

(5.7

$$GQEOY_4 = (1+g)^3 (Q_1)$$

The general formula for income in the kth year of equilibrium is:

(5.8

$$GIEOP_{n+k} = (1+g)^k \sum_{j=1}^{n} I_j \cdot (1+g)^{n-j} = (1+g)^{k+n-1} \sum_{j=1}^{n} I_j \cdot (1+g)^{-(j-1)}$$

We can now compute ROE when the Book of Business Growth Company is in the Equilibrium Growth Phase. Our ROE will be defined as the ratio of end of period Income over beginning of period Equity. For any year in the Equilibrium Growth Phase, the ratio will be:

(5.9

$$\mathrm{ROE} = \frac{\sum_{j=1}^{n} I_j \cdot (1+g)^{-(j-1)}}{\sum_{j=0}^{n-1} Q_j \cdot (1+g)^{-j}}$$

A key observation is that Equilibrium Growth ROE is a function of the growth rate. We are now ready to show that if the growth rate is equal to the IRR on Equity Flows, then the ROE will also equal that IRR.

(5.10

Result Relating IRR and CY Growth ROE: Calendar Year ROE in the Equilibrium Growth phase will equal IRR if the Book of Business is growing at a uniform growth rate equal to the IRR..

Proof. Let g= IRR and set $w = (1+g)^{-1}$. We rewrite the IRR defining equation 2.11 as follows

(5.11

$$\sum_{j=1}^{n} I_j \cdot w^j = Q_0 + \sum_{j=1}^{n-1} (Q_j - Q_{j-1}) \cdot w^j - Q_{n-1} w^n$$

Expanding the right hand side and regrouping, we have

(5.12

$$\sum_{j=1}^{n} I_j \cdot w^j = (1-w) \cdot Q_0 + (1-w) Q_1 w^1 + \ldots$$
$$= \frac{g}{1+g} \sum_{j=0}^{n-1} Q_j \cdot w^j$$

Therefore it follows that:

(5.13

$$\frac{\sum_{j=1}^{n} I_j \cdot w^j}{\sum_{j=0}^{n-1} Q_j \cdot w^j} = \frac{g}{1+g}$$

From that we derive:

(5.14

$$g = \frac{\sum_{j=1}^{n} I_j \cdot (1+g)^{-(j-1)}}{\sum_{j=0}^{n-1} Q_j \cdot (1+g)^{-j}} = \text{ROE}$$

Thus we have proved our desired result.

The reader may note that this proof is essentially the same as the proof for the PVI/PVE result, with the growth rate playing the role of the rate used for discounting. The Growth Model ROE also provides another interpretation of IRR. Consider that once in the Equilibrium Growth Phase the Equity increases from one year to the next by the factor, (1+g). When g equals the IRR, our result says that ROE is equal to the growth rate g. The conclusion is that all the Income is being used to support growth and that the Income generated is all that is needed to support growth at that rate. In other words the end of period Income from one period equals the increase in beginning of period Equity for the next period. So, when we find IRR we are finding the maximal self-sustaining growth rate. It is self-sustaining in the sense that equity investors need supply no more capital once the Equilibrium Growth Phase is reached.

In Exhibits 2 and 3 we show Growth Model accounts for our example. We do this in two stages. First in Sheet 2 of these exhibits, we restate the Single Policy Model accounts using our Beginning of Year (BOY) and End of Year (EOY) accounting conventions. Then, we show growth results in Sheet 3, all at a common growth rate of 5.0%. We compute ROE for each year in the Growth Model. A summary table displays IRR and ROE results. The ROE summary results are for the Equilibrium Growth Phase. In Exhibits 2 and 3, we also have a Sheet 4 that displays accounts where the calculations have been done using a growth rate equal to the IRR. For those scenarios, the ROE equals the IRR, thus demonstrating our theoretical result. For the Sheet 3 scenarios, the two measures are not equal.

If we compare Sheet 3 ROE results by year in Exhibit 2, which is based on full value reserves, versus the comparable ones in Exhibit 3, which is based on discounted reserves, we find that they are nearly identical in equilibrium. However, during the start-up years, the ROE based on discounted reserves is quite a bit higher. This is true even though leverage ratios are unrealistically high in the initial years in both models. Were the leverage ratios reduced in those initial years, the ROEs would decline in both cases. So, in the case when reserves are held full value, the pattern of low ROEs in the initial years rising up to the equilibrium value would be even more pronounced. This leads us by example to a general observation: rapid growth tends to depress ROE, but this can be countered by discounting reserves. Thus, our theory tends to make us more apt to scrutinize the adequacy of reserves and capital in a rapidly growing firm that posts a high ROE and has a heavy concentration in long-tailed lines of business.

We have presented models constructed on an annual basis. It is straightforward to build comparable models on a quarterly or monthly basis, because the accounting rules allow us to do so. Quarterly equity flows can thus be computed and a quarterly effective IRR can be derived from them. PVI/PVE presents a little bit of a problem. Because we have four equity values each year instead of one, our PVE denominator will be roughly four times as large as the PVE from the annual model. On the other hand, the PVI numerator does not necessarily increase or decrease in moving from an annual model to a quarterly one. Two alternatives that have been proposed to deal with this are: i) view the return as a quarterly effective return or ii) annualize the return by dividing the Equity roughly by 4.[12] For ROE we have comparable choices. We could take income for a quarter and divide it by the equity for that quarter. The result would be a quarterly return. The alternative is to take a full year's income and divide it by the average equity for the four quarters. We will not do that in our demonstration. Our point is simply that it is not terribly difficult to extend our models to a quarterly basis. That would allow us to achieve greater accuracy.

6. RETURNS WHEN LOSS IS A RANDOM VARIABLE

We have derived our return measures by modeling results of hypothetical corporations under the assumption all goes as planned. In particular, we have modeled loss as a single point estimate. We now explore how to compute the returns when loss is a random variable. Assume we have a loss distribution consisting of a finite number of loss scenarios and associated probabilities. To be complete, we could also have a more complicated set of scenarios, each consisting of a loss amount and a loss payout pattern. But, for our current work, we will assume it is only the loss amount that varies.

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[12] See Robbin [13] for a more in-depth discussion of annualization.

Our plan is to model the Income, Surplus, and Equity Flows of each scenario. At first this would seem to be easy. We could just plug the loss amount for each scenario into our model and let it run. However, the problem is a bit harder than that. We can identify at least three major related issues that need to be resolved. The first is whether to let our Single Policy Company go bankrupt in adverse scenarios. The second is the related issue of how to set Surplus. The third is how to model the timing of when the actual ultimate loss is recognized.

We could let our Single Policy go bankrupt in very unprofitable scenarios. The opposite approach is to keep it afloat by implicitly assuming the equity investors will pump in as much money as is needed. This is over and above the initial or planned commitment of Capital. A compromise position is to assume the equity investors post some fixed amount of extra money that could be tapped if needed. The rental of this extra capital should carry a charge. In a setup suggestive of the shared assets paradigm for insurance developed by Mango [10], we could model a Holding Company that would back a portfolio of different Single Policy Company subsidiaries. The Holding Company would assess a "use of extra equity" charge against each Single Policy Company and would be an intermediary between the equity investors and these subsidiary companies. The required segregated Holding Company capital would then depend on the amount of capital in each Single Policy Company subsidiary, the odds each subsidiary would need to draw on Holding Company funds, and the covariance between results of the subsidiaries. While this is conceptually attractive as well as more realistic, it is complicated. We will leave implementation of this approach as a topic for future research. Instead, we will model a company that does not go bankrupt. While this approach has some conceptually debatable underpinnings, it is the easiest to implement. Further, as we will later argue, it provides a conservative estimate of what would result from a more complete model.

In regard to what Surplus requirement should be used, we believe, on theoretical grounds, that all scenarios should start with the same initial Surplus. The reason is simple: at the outset there is no way to know what scenario will ensue. Under our procedure, the initial Surplus would thus be set as a percentage of the expected present value of unpaid losses. The expectation would be taken with respect to all scenarios. After that the situation gets more complicated. As results are posted for the first accounting period, company management may have a better idea than at the start which scenarios are more likely than others. In theory it would then set the Surplus based on its revised estimate of present value of unpaid losses. While this is in some sense realistic as well as conceptually appealing, it is complicated. For our current purposes, we will opt again for the simplest approach and assume a common amount of Surplus at each point in time for all scenarios. The common amount of Surplus would be set at a given point in time as a percentage of the expected present value of unpaid losses. In concept, the percentage would be based on a risk measure operating on the distribution of the present value of unpaid losses. In the examples we use the same percentage for all evaluations.

Now we turn to the question of when to recognize the ultimate loss in a given scenario. Initially, we know only the expected loss over all scenarios. Within any particular scenario, the discrepancy must eventually be recognized on the books of the Single Policy Company. The timing of this recognition will impact underwriting income, loss reserves, investment income, income taxes, and equity flows. Our approach is to recognize the difference at the end of the first accounting period.[13] An alternative is to set reserves equal to the expected ultimate loss times the percent of loss unpaid. The expectation is over all scenarios. Under this approach, the difference between the ultimate loss in the particular scenario and the expected ultimate over all scenarios would be recognized piecemeal as the losses are paid. Various intermediate recognition algorithms could also be used and all the methods could be adjusted to handle reserve discounting. While it is somewhat unrealistic to assume complete recognition of the ultimate loss at the first evaluation, this leads to the simplest algorithm. As well, we will argue that it is the most conservative approach.

Use of our simplest solutions to each of these problems leads to a very convenient modeling result: the average income, average equity, and average equity flow over all scenarios

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[13] In a quarterly model, we would recognize one fourth of the difference at the end of each of the first four quarters.

are the same as those resulting when the model is run on the average scenario. In Exhibit 5, we illustrate this with a three-point loss distribution. What this means is that we do not need to separately model all the scenarios to find the returns. Our results for the average scenario will suffice.

An important caveat is that this observation only applies when the premium and expenses are fixed and do not vary with the loss. With Retrospective Rating plans, for example, the premium varies with the loss, and is further subject to Maximum and Minimum Retro Premium restrictions. The average underwriting loss for such a plan does not in general equal the underwriting loss that results from the average loss scenario. So we would need to model the full distribution when dealing with a Retro Plan. However, when complications of that sort are not present, we have found that our simplifying assumptions will allow us to legitimately reduce the distribution of losses to a single scenario.

What have we lost by adopting these simplifications? The answer is that the major factor we are missing is consideration of the default scenarios in which the Single Policy Company fails to meet its obligations to policyholders. We have incorrectly assumed the equity investors would keep the company afloat rather than letting it become insolvent. In effect, we have neglected to put a cap on the downside risk to the equity investors. Because we have not done so, the amounts lost by the investors in adverse scenarios are greater in our model than those that would be indicated in a more sophisticated model. The conclusion is that our model leads to a more conservative average result. In other words, our returns are lower than what they would be if we had modeled the default option. Though our simplified approach would thus be inappropriate for some applications, such as modeling Guarantee Fund assessments, its conservative answers are arguably the answers that are most useful in internal corporate pricing analysis. In that context, the more complete models can exhibit inadequate sensitivity to the tail of the loss distribution. While increasing the relative weight of the tail does increase the risk measure and thus the required Surplus, this is partly offset by the assumption that the equity investors can walk away from the big events. With our simplified approach, there is no walking away and, therefore, no offset. Thus the returns we derive are sensitive to tail events. We feel this is more appropriate in the pricing context of our discussion.

7. SENSITIVITY OF RETURNS

Before going further, it is useful to study how our three measures of return respond to changes in premium, Surplus, interest rate, and payout pattern. We will do this with a simple example. Base case assumptions are shown in Exhibit 1.

The sensitivity of return with respect to premium is of interest when pricing a particular product or policy. Perhaps the return on a product is initially below target at the premium suggested by an agent or broker. Knowing the sensitivity to premium will provide us an intuition about much more premium it will it take to get to the target. Summary premium sensitivity results for our example are shown in Exhibit 4 on Sheets 1 and 2. Reserves are held at full value for Sheet 1 and are discounted in Sheet 2. All Growth Model results assume a 5.0% growth rate and all PVI/PVE results assume discounting at 12.0%. These selections would be appropriate if we suppose that corporate management has targeted a 5.0% growth rate and a 12.0% calendar year ROE. As might be expected, due to the fact that all three models share a common foundation, there is not much difference in the results. Only when returns are negative in the low premium scenario do we see any real difference and even that is fairly modest. In that scenario, the IRR is not quite as negative as the PVI/PVE.

As premiums increase by a constant increment, the returns increase, but in a slightly nonlinear fashion. The IRR goes up at a slightly increasing rate, while the PVI/PVE and ROE rise at a slightly decreasing rate. While a full explanation of the nonlinearities would require detailed analysis, we can at least indicate that our assumptions regarding Deferred Acquisition are part of the explanation as regards PVI/PVE and Growth Model ROE. According to these assumptions, an increase in premium leads to an increase in DAC and thus to an increase in PVE and GAAP Equity in the respective models. The increase in the DAC component of Equity slightly moderates the increase in returns caused by the premium increase. Another consequence of our modeling assumptions is that, counterintuitively, an increase in premium can lead to a reduction in investment income in the second year of the policy. This happens since we have supposed some premium is not paid till the second year. The assets in that year are equal to Reserves plus Surplus and do not change when premium is increased. However the rise in premium boosts the Receivables and thus decreases the investible assets.

Note that the different premium scenarios have different premium-to-surplus leverage ratios. This is in accord with our assumption that the Surplus requirement is driven by the loss distribution. Since all the premium sensitivity scenarios thus have the same amount of Surplus and differing amounts of premium, they end up with different leverage ratios. Another observation is that the change in Equilibrium Growth Model ROE as the result of a change in premium is the same whether reserves are held at full value or are discounted. This makes intuitive sense since the amount of Equity in our model is independent of whether actual reserves are held at full value or are discounted.

Now we examine the sensitivity of our returns to changes in the level of Surplus. This might be of interest when comparing products with different levels of risk. The different levels of risk would translate into different Surplus loading factors for the products. The results for our example are shown in Exhibit 4, Sheet 3. There is nothing surprising: more Surplus produces returns closer to the after-tax yield on investment, no matter which of our return measures is used. However, the sensitivity is perhaps lower than might be guessed in advance. As we increase our loading factor for Surplus so that the Growth Model premium-to-surplus ratio drops from around 3.0 to around 2.0, the returns drop by a bit less than 2 points. The major reason for this is that the after-tax return on investment of the Surplus is fixed and immune to the effects of leverage. So, of the roughly 11.7% returns we get in our low Surplus scenario, nearly 4.0% is achieved on the Surplus itself and only the increment of 7.7% is due to the insurance venture. To get a rough estimate of the Surplus sensitivity in moving from leverage of 3.0 to leverage of 2.0, we would multiply the 7.7% by 2/3 to get 5.1%. The difference of 2.6% is higher than our observed difference of nearly 2.0%, but it suggests that the observed sensitivity is plausible.

We next look at the sensitivity of our returns to changes in the interest rate. As is to be expected, the higher interest rates yield higher returns. They are even a bit higher than one might initially have guessed. This is due to our method of setting Surplus values as a percentage of the present value of unpaid loss. As the interest rate increases, these present values decline. This reduces the amount of Surplus, and so the Growth Model leverage ratios increase.

Finally we turn to examine sensitivity due to changes in the payout pattern. To make the analysis cleaner, we changed our Surplus-loading factor between scenarios so that all scenarios would have the same Growth Model leverage ratio. Implicitly we are assuming that the longer tailed scenarios have lower risk that just offsets the larger commitment of Surplus due to their longer duration. The results are just as expected: longer payout patterns lead to higher returns. The effects are significant. We see that a change in duration of half a year can change the return by over 2 points. This result is sensitive to the interest rate assumption of 6.0% used in our analysis. With a higher rate, we would see even greater sensitivity.

To summarize, the returns exhibit appropriate sensitivities that we can intuitively explain after the fact, even if we did not entirely foresee them beforehand. We should caution that the particular results we have presented are critically dependent on our modeling assumptions. The results would differ if the required Surplus or the Deferred Acquisition balance were computed differently.

8. INDICATED PROFIT PROVISIONS

We define the Indicated Profit Provisions and Indicated Premiums for each of our measures by solving for the profit provision and resulting premium that yields a return equal to the selected target return. Results are shown in Exhibit 5 assuming a target of 12.0%. All results assume reserves are held at full value. Recall that for PVI/PVE we also need to choose a rate for discounting income and equity. We again chose 12.0% under our logic that the cost of capital is a natural target and the natural rate to use for such discounting. However, according to our result relating IRR and PVI/PVE, when the same rate is used for the target and for discounting, we will end up with a PVI/PVE equal to the IRR. Thus our indicated profit provisions for IRR and PVI/PVE are identical. With the Growth Model ROE, we used a growth rate target of 5.0%. If we had used a growth target of 12.0%, results for ROE would have also been the same as for IRR. However, we have no logic that compels such a choice. Rather, we have assumed that management has specified a long-term growth target of 5.0% and a target calendar year return of 12.0%.

In Sheet 1, we examine sensitivity of the Indicated Profit Provisions to changes in the level of Surplus. We change the level of Surplus by varying the Surplus-loading factor. As we would anticipate, higher Surplus loading factors give rise to higher profit provisions. However, the leverage ratios do not follow a direct inverse relation with the loading factors. The divergence arises because the premium is also changing between scenarios. As shown in Exhibit 5, the ROE profit provision moves from –1.97% to –0.13% in response to a change in Surplus loading factors that reduces the Growth Model leverage ratio from 3.09 to 2.15.

Next we examine sensitivity of indicated premiums to a changes interest rates while keeping the target return fixed. Results are shown in Sheet 2. Raising the interest rate leads to a reduction in the profit provision. This is in accord with our intuition. With more investment income we need less underwriting income to achieve the target. The IRR and ROE results are similar, but not identical. With our loss payout pattern duration of only 2.0 years, moving the interest rate up one point reduces the indicated profit provision by a bit less than 2.0 points. The result also depends on our Surplus-loading factor. With a higher loading factor, we could drive sensitivity down. The results can also be explained by noting that interest rates impact the leverage ratio in our model. On the one hand, increasing the interest rate reduces the present value of unpaid loss. That reduces the Surplus. On the other hand, higher interest rates reduce the indicated premium, assuming the target return stays fixed. This happens because they reduce the difference between that target return and the after-tax investment return as well as increase the investment income on our full value reserves. The net tradeoff between the reduction in Surplus and the reduction in Premium as seen in our results is that the leverage ratios decrease modestly with an increase in the interest rate.

Finally, we turn to sensitivity analysis of the indicated profit provisions with respect to changes in the loss payout pattern. Results are shown in Sheet 3. To facilitate comparisons, we adjust our loading factors for Surplus in order to achieve a constant Equilibrium Growth Model leverage ratio in all scenarios. We see, as expected, that the results show significant response to the duration of the payout pattern. Increasing the duration by half a year moves the profit provision down by just over 2.0 points when the interest rate is 6.0%.

To summarize, despite a few subtleties, the models produce Indicated Premiums that are appropriately responsive to changes in key inputs. Next, we will compare our corporate structure approach with the Risk-Adjusted Cash Flow Model.

9. COMPARISON TO THE RISK-ADJUSTED DISCOUNTED CASH FLOW MODEL

The Risk-Adjusted Discounted Cash Flow Model (RA DCF) has often been used in pricing. However, it takes a different approach to pricing than the one we have taken. Instead of finding the Indicated Premium needed to hit a fixed target return on Risk-sensitive Surplus, the RA DCF approach is to find the Fair Premium directly. The Fair Premium is defined as the sum of loss, expense, and income tax cost components. Each component is discounted. However, since losses are a risky cash flow, they are discounted at a risk-adjusted rate.

In words, the formula is

(9.1

Fair Premium =
PV of Loss at the Risk - Adjusted Rate + PV of Expense
+ PV of Tax on Investment Income on Surplus and Premium net of Expense
+ PV of Tax on Underwriting Income from Premium less Expense
- PV of Tax Reduction for Losses at the Risk - Adjusted Rate

For a single period example, we can write the formula in mathematical symbols as follows:

(9.2

$$P = \frac{L}{(1+r_A)} + \frac{X}{(1+r_f)} + \frac{T_I \cdot r_f \cdot (P - X + S)}{(1+r_f)} + \frac{T_U \cdot (P - X)}{(1+r_f)} - \frac{T_U \cdot L}{(1+r_A)}$$

Here P stands for premium, L is loss, and X is expense. The losses are discounted at a risk adjusted rate, r_A, which is less than or equal to the risk –free rate, r_f. The tax rate on investment income is T_I and the tax rate on underwriting income is T_U. Here S stands for Surplus. Note that the Fair Premium includes a provision for the tax on the investment income from both the Surplus and the balance of underwriting cash flows.

The risk-adjusted rate is a key parameter in the RA DCF model. As D'Arcy and Dyer [6] note, determination of this rate is a "thorny issue"[14]. They describe two approaches. One is to view the adjustment "as a form of compensation to the insurer for placing its capital at risk in the insurance contract"[15]. The second is to derive the risk-adjustment from principles of the Capital Asset Pricing Model (CAPM). This is the approach used by Myers and Cohn [12] in

[14] D'Arcy and Dyer [6], p.342.

[15] D'Arcy and Dyer [6], p.342.

their original paper introducing the model. Under CAPM, there should be no charge for process risk, only for systematic risk related to the covariance of insurance losses with returns on the stock market. This covariance is known as "beta. The determination of beta has been the subject of some disagreement. Some believe beta is close to zero. For example, Vaughn [17] notes:: " For many P/L lines, indemnity losses possess very little systematic risk. As such, the risk-free rate is often used as an acceptable approximation …"[16]. However, Derrig [7] and others have used a non-zero, CAPM-based beta in rate filings.

This short introduction to the RA DCF model is necessarily incomplete, but it will suffice to allow us to reasonably compare that model against the procedure we have presented. The most obvious distinction is that the RA DCF is a method to determine premium without need to assume a target return. In our models, the Indicated Premium is that needed to achieve a given target return (or target return at a given target growth rate for the Growth Model).

The next major distinction is that the RA DCF model has no underlying corporate or accounting structure, while such a framework is the basis for defining our returns. Because of this, the RA DCF has no natural way to reflect the conservative treatment of expenses under Statutory Accounting. In our corporate model, this was handled by making an adjustment to GAAP Equity for Deferred Acquisition Costs. As well, there is no natural way in the RA DCF framework to reflect reserve discounting. While reserve discounting does not impact underwriting cash flows, it does impact the flow of funds to equity investors. Our corporate model of Equity Flows takes this into account.[17]

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[16] Vaughn [17], p. 406

[17] Another anomaly caused by lack of an accounting substructure is that the balance of investible assets does not automatically decay to zero. However, since it usually decays to a positive or negative balance close to zero and the RADCF provision is for the present value of taxes on the investment income on the balance, the practical impact of the non-disappearing balance is usually negligible.

The next point of distinction concerns the role of Surplus. In the RA DCF, it plays no direct major role. There is a provision in the Fair Premium for the present value of the tax on the investment of the Surplus, but this is usually small. Consider a one-year example assuming a 3.0 leverage ratio, 6.0% interest rate, and a 35% tax rate. The full value tax in that case would come to around 0.69%. Not only is the effect small, the sensitivity to changes in Surplus is even smaller. Reducing the leverage ratio to 2.0 in our example produces a full value tax of 1.05%. The difference of 0.36% is significantly smaller than the 1.84% difference (-0.13% -(-1.97%)) seen in our Growth Model ROE results. Further, if the tax rate were zero, the Fair Premium would be independent of Surplus. In contrast, in our models the leverage effect of Surplus has a critical impact on the results. It is revealing that in some RA DCF models[18], Surplus is assumed to be larger than the amount needed to ensure that there is essentially no chance of insolvency. This view of Surplus is effectively tantamount to regarding it as a "free" good; there is more than enough of it to go around. However, in the corporate context of our models, Surplus is in scarce supply.

Another major difference between the models concerns their sensitivity to risk. As we previously noted, risk sensitivity in the RA DCF model depends on how beta is selected. Yet, that selection is problematic. If we follow Vaughn and use no risk-adjustment, RA DCF pricing would have no sensitivity to risk. Since we believe pricing ought to be risk-sensitive, we would disagree with this implementation of the RADCF: it is an RADCF without the "RA". If we follow others who use CAPM to derive a non-zero beta, we would have some risk sensitivity. However, those methods have typically been applied at a line of business level for the industry. It is not obvious how to extend them to pricing different products within a line for a single company.

Finally, we could follow those who set the beta so as to provide an adequate return on risk-sensitive capital. In that case, we would look to our approach to arrive at the Indicated

[18] See Vaughn [17].

Premium and solve for the beta that leads to the same answer. While the presentation of that result as a RA DCF calculation might be useful in some situations, it forces us to think about risk sensitivity in terms of changes in beta. Within our framework, risk sensitivity depends on the Surplus requirement formula and the spread between the target return and the after-tax yield on investment. We believe actuaries and insurance company management find it more intuitive to think in those terms. Further, though there are disagreements about how to set theoretical Surplus, they are not as severe as the disagreements over beta.

Ultimately we feel the methods arise in different contexts and reflect different perspectives in pricing. Others have noted these differences[19]. Management, we believe, will be far less interested in knowing the Fair Premium for a product than it will be in knowing the Indicated Premium needed to attain its risk-return objectives. One the other hand, as the title of the Myers and Cohn paper [12], " A Discounted Cash Flow Approach to Property-Liability Insurance Rate Regulation" makes clear, that model was originally developed to handle pricing in a regulatory arena. From a policyholder or regulatory perspective, there may be much greater concern with finding the Fair Premium than knowing whether the premium is adequate for shareholders to achieve the expected return they desire. While the Fair Premium may contain some compensation for the equity investors of the insurance company, those investors may or may not find that compensation acceptable.

One other issue that must be clarified is that there are discounting methods, such as the one developed by Butsic [4], in which the losses are discounted at a risk-adjusted rate, yet which are closer to our method than to the RA DCF approach. In Butsic's model, the rate adjustment depends explicitly on the equity requirement and a given target return. Butsic sets the equity requirement as a percentage of the discounted loss reserve. He also computes an IRR that is conceptually the same as our IRR on Equity Flows. He finds the premium needed to hit a given target return. What Butsic shows is that if reserves are discounted at just the right rate,

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[19] See Bingham [2].

then the ROE for each year is equal to the IRR and the target return. His rate for discounting losses is given as:

(9.3

$$r_A = i - e(R - i)$$

Here i is the risk-free rate, R is the target return, and e is the equity loading factor relative to the discounted reserve.

What Butsic has done is to show how to modify the accounting system to bring it into accord with economic reality so the anticipated calendar year returns each year would be the same as the IRR. If we were to discount reserves in our model according to Butsic's formula, we would obtain the same results.

10. CONCLUSION

We have covered many topics and now it is time to summarize what has been accomplished. The first step in our journey was to define our three measures based on a hypothetical corporate structure. Looking back we can see that this structure enforced a certain discipline in our analysis. We had to be precise about the amount of Surplus being held and about the flows of money to and from equity investors. The structure allowed us to reflect the impact of the DAC adjustment in GAAP and the effect of reserve discounting. Having a corporate structure that incorporates accounting rules is a critical aspect of our approach. Further we can conclude that models without sufficient corporate structure cannot fully capture key aspects of the return on an insurance venture, at least not the return to an equity investor or to the insurance company.

We proved results relating PVI/PVE and ROE to IRR and used these to provide new interpretations of IRR. We found that, with some simplifying assumptions, we could conveniently use a single average loss scenario to obtain the average return when the loss is a random variable. We then argued that these simplifying assumptions led to a conservative answer that was appropriate in the internal corporate context of our pricing analysis. With examples, we explored the sensitivity of our returns to changes in premium adequacy, Surplus level, interest rate, and payout pattern.

Our examination of the sensitivity of indicated profit provisions showed that these models should lead to reasonably responsive risk-sensitive prices for insurance products. The risk-sensitive pricing was obtained by using risk-sensitive Surplus requirements in conjunction with a fixed target return.

We have seen the Growth Model ROE emerge as a very strong contender to the IRR on Equity Flows. While there was not much of a difference in the results, the Growth Model allows us to directly relate product pricing to long-term calendar year ROE and growth rate targets. It also produces a calendar year premium-to-surplus leverage ratio for the Book of Business in equilibrium. This could be compared against industry benchmarks.

We have discussed why results from our models would differ from those of others such as the Risk-Adjusted Discounted Cash Flow model. This was done in an attempt to increase understanding. While some of our comments could be taken as critical, we have not gone so far as to say there is anything inappropriate about using other approaches in other contexts. In some regulatory situations, it may well be better to use the RA DCF model than any of the three we have presented.

There already is a significant body of literature on other ways of pricing in general[20] and on other ways of pricing insurance products in particular[21]. However, we feel we have demonstrated a methodology for deriving indicated prices that should be appropriate for internal corporate pricing analysis. We believe each of our three measures of return could reasonably be used in that context. Methods similar to ours are in common use and we hope our work furthers their acceptance. In conclusion, while we have left some theoretical questions unresolved and frequently adopted simplifying assumptions, we believe we have nonetheless demonstrated three variants of an approach to pricing that is both sound and practical.

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[20] For example, the Black-Scholes formula for pricing options does not use a target return.

[21] See D'Arcy and Dyer [6], Derrig [7], and Robbin [13] for various alternative approaches to pricing property and casualty insurance products.

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Exhibit 1

Profit Measure Examples
Assumptions

Rates	
Investment Return	6.00%
Tax Rate	35.00%
PVI/PVE Discount Rate Selection	12.00%
Growth Rate Target	5.00%

Surplus Requirements	
Ratio to PV Unpaid Loss	31.5%
Rate for Discounting Unpaid Loss	6.00%

Patterns

Earning and Incurral

Year	Earned Premium	Full Value Incurred Loss	Stat Incurred Expense	GAAP Incurred Expense
0	0.0%	0.0%	60.0%	0.0%
1	100.0%	100.0%	40.0%	100.0%
2	0.0%	0.0%	0.0%	0.0%
3	0.0%	0.0%	0.0%	0.0%
4	0.0%	0.0%	0.0%	0.0%

Payment Patterns

Year	Paid Premium	Paid Loss	Paid Expense
0	75.0%	0.0%	30.0%
1	20.0%	25.0%	45.0%
2	5.0%	50.0%	20.0%
3	0.0%	25.0%	5.0%
4	0.0%	0.0%	0.0%
Total	100.0%	100.0%	100.0%
PV Factor (t=0)	0.9832	0.8908	0.9445

Underwriting

		Loss	Expense
Fixed		72.00	10.00
Variable		0.0%	20.0%

Exhibit 2
Sheet 1

Single Policy Company

UW Assumptions			Financial Assumptions		IRR and PVI/PVE Results		
	Amount	Ratio	Interest Rate	6.00%	IRR	10.74%	10.74%
Premium	100.0	100.0%	Tax Rate	35.00%	PVI/PVE Discount Rate	12.00%	10.74%
Loss	72.0	72.0%	Rsv Discount Rate	0.00%	PVI	6.05	6.10
Expense	30.0	30.0%	S as % of PV Unpaid Loss	31.50%	PVE	56.52	56.78
Combined	102.0	102.0%	PV Loss Discount for S Calc	6.00%	PVI/PVE	10.71%	10.74%

Year	Earned Premium	Incurred Loss	Stat Incurred Expense	Stat UW Income	Paid Premium	Paid Loss	Paid Expense	UW Cash Flow
0	0.0	0.0	18.0	-18.0	75.0	0.0	9.0	66.0
1	100.0	72.0	12.0	16.0	20.0	18.0	13.5	-11.5
2	0.0	0.0	0.0	0.0	5.0	36.0	6.0	-37.0
3	0.0	0.0	0.0	0.0	0.0	18.0	1.5	-19.5
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total	100.0	72.0	30.0	-2.0	100.0	72.0	30.0	-2.0

Year	Unearned Premium	Loss Reserve	PV Unpaid Loss	Stat Expense Reserve	Total Stat Reserves	Surplus	Assets	Receivables	Invested Assets	Inv Income
0	100.0	0.0	64.1	9.0	109.0	20.2	129.2	25.0	104.2	
1	0.0	54.0	50.0	7.5	61.5	15.7	77.2	5.0	72.2	6.3
2	0.0	18.0	17.0	1.5	19.5	5.3	24.8	0.0	24.8	4.3
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.5
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Year	DAC	GAAP Equity	GAAP Incurred Expense	GAAP UW Income	GAAP Pre-tax Income	Income Tax	Income	Change in Equity	Equity Flow
0	18.0	38.2	0.0	0.0	0.0	0.0	0.0	38.2	-38.2
1	0.0	15.7	30.0	-2.0	4.3	1.5	2.8	-22.5	25.2
2	0.0	5.3	0.0	0.0	4.3	1.5	2.8	-10.4	13.2
3	0.0	0.0	0.0	0.0	1.5	0.5	1.0	-5.3	6.3
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total			30.0	-2.0	10.1	3.5	6.6	0.0	6.6

Exhibit 2
Sheet 2

Single Policy Company- BOY and EOY Accounting

UW Assumptions			Financial Assumptions		IRR and PVI/PVE Results		
	Amount	Ratio	Interest Rate	6.00%	IRR	10.74%	10.74%
Premium	100.0	100.0%	Tax Rate	35.00%	PVI/PVE Discount Rate	12.00%	10.74%
Loss	72.0	72.0%	Rsv Discount Rate	0.00%	PVI	6.05	6.10
Expense	30.0	30.0%	S as % of PV Unpaid Loss	31.50%	PVE	56.52	56.78
Combined	102.0	102.0%	PV Loss Discount for S Calc	6.00%	PVI/PVE	10.71%	10.74%

Year	Earned Premium EOY	Incurred Loss EOY	GAAP Incurred Expense EOY	GAAP UW Income EOY	Paid Premium BOY	Paid Premium EOY	Paid Loss EOY	UW Cash Flow BOY	UW Cash Flow EOY
1	100.0	72.0	30.0	-2.0	75.0	20.0	18.0	66.0	-11.5
2	0.0	0.0	0.0	0.0	0.0	5.0	36.0	0.0	-37.0
3	0.0	0.0	0.0	0.0	0.0	0.0	18.0	0.0	-19.5
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Year	Unearned Premium BOY	Unearned Premium EOY	Loss Reserve BOY	Loss Reserve EOY	Stat Expense Reserve BOY	Stat Expense Reserve EOY	Total Stat Reserves BOY	Total Stat Reserves EOY
1	100.0	0.0	0.0	54.0	9.0	7.5	109.0	61.5
2	0.0	0.0	54.0	18.0	7.5	1.5	61.5	19.5
3	0.0	0.0	18.0	0.0	1.5	0.0	19.5	0.0
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Year	Surplus BOY	Surplus EOY	Assets BOY	Assets EOY	Receivables BOY	Receivables EOY	Invested Assets BOY	Investment Income EOY
1	20.2	15.7	129.2	77.2	25.0	5.0	104.2	6.3
2	15.7	5.3	77.2	24.8	5.0	0.0	72.2	4.3
3	5.3	0.0	24.8	0.0	0.0	0.0	24.8	1.5
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Year	DAC BOY	DAC EOY	GAAP Equity BOY	GAAP Equity EOY	GAAP Pre-tax Income EOY	Income Tax EOY	GAAP Income EOY
1	18.0	0.0	38.2	15.7	4.3	1.5	2.8
2	0.0	0.0	15.7	5.3	4.3	1.5	2.8
3	0.0	0.0	5.3	0.0	1.5	0.5	1.0
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Exhibit 2
Sheet 3

Book of Business Growth Company

UW Assumptions	Amount	Ratio	Financial Assumptions		IRR and ROE Results	
Premium	100.0	100.0%	Interest Rate	6.00%	IRR	10.74%
Loss	72.0	72.0%	Tax Rate	35.00%	EQ Growth ROE	10.90%
Expense	30.0	30.0%	Rsv Discount Rate	0.00%	EQ Growth P/S	2.50
Combined	102.0	102.0%	S as % of PV Unpaid Loss	31.50%	Growth Rate	5.00%
			PV Loss Discount for S Calc	6.00%		

Year	Earned Premium EOY	Incurred Loss EOY	GAAP Incurred Expense EOY	GAAP UW Income EOY	Paid Premium BOY	Paid Premium EOY	Paid Loss EOY	UW Cash Flow BOY	UW Cash Flow EOY
1	100.0	72.0	30.0	-2.0	75.0	20.0	18.0	66.0	-11.5
2	105.0	75.6	31.5	-2.1	78.8	26.0	54.9	69.3	-49.1
3	110.3	79.4	33.1	-2.2	82.7	27.3	75.6	72.8	-71.0
4	115.8	83.3	34.7	-2.3	86.8	28.7	79.4	76.4	-74.6

Year	Unearned Premium BOY	Unearned Premium EOY	Loss Reserve BOY	Loss Reserve EOY	Stat Expense Reserve BOY	Stat Expense Reserve EOY	Total Stat Reserves BOY	Total Stat Reserves EOY
1	100.0	0.0	0.0	54.0	9.0	7.5	109.0	61.5
2	105.0	0.0	54.0	74.7	17.0	9.4	176.0	84.1
3	110.3	0.0	74.7	78.4	19.3	9.8	204.2	88.3
4	115.8	0.0	78.4	82.4	20.3	10.3	214.5	92.7

Year	Surplus BOY	Surplus EOY	Assets BOY	Assets EOY	Receivables BOY	Receivables EOY	Invested Assets BOY	Investment Income EOY	P/S
1	20.2	15.7	129.2	77.2	25.0	5.0	104.2	6.3	4.95
2	37.0	21.9	212.9	106.0	31.3	5.3	181.7	10.9	2.84
3	44.2	23.0	248.4	111.3	32.8	5.5	215.6	12.9	2.50
4	46.4	24.1	260.8	116.8	34.5	5.8	226.4	13.6	2.50

Year	DAC BOY	DAC EOY	GAAP Equity BOY	GAAP Equity EOY	GAAP Pre-tax Income EOY	Income Tax EOY	GAAP Income EOY	GAAP ROE
1	18.0	0.0	38.2	15.7	4.3	1.5	2.8	7.23%
2	18.9	0.0	55.9	21.9	8.8	3.1	5.7	10.24%
3	19.8	0.0	64.0	23.0	10.7	3.8	7.0	10.90%
4	20.8	0.0	67.2	24.1	11.3	3.9	7.3	10.90%

Exhibit 2
Sheet 4

Book of Business Growth Company

UW Assumptions	Amount	Ratio	Financial Assumptions		IRR and ROE Results	
Premium	100.0	100.0%	Interest Rate	6.00%	IRR	10.74%
Loss	72.0	72.0%	Tax Rate	35.00%	EQ Growth ROE	10.74%
Expense	30.0	30.0%	Rsv Discount Rate	0.00%	EQ Growth P/S	2.58
Combined	102.0	102.0%	S as % of PV Unpaid Loss	31.50%	Growth Rate	10.74%
			PV Loss Discount for S Calc	6.00%		

Year	Earned Premium EOY	Incurred Loss EOY	GAAP Incurred Expense EOY	GAAP UW Income EOY	Paid Premium BOY	Paid Premium EOY	Paid Loss EOY	UW Cash Flow BOY	UW Cash Flow EOY
1	100.0	72.0	30.0	-2.0	75.0	20.0	18.0	66.0	-11.5
2	110.7	79.7	33.2	-2.2	83.1	27.1	55.9	73.1	-49.7
3	122.6	88.3	36.8	-2.5	92.0	30.1	79.9	80.9	-74.6
4	135.8	97.8	40.7	-2.7	101.9	33.3	88.5	89.6	-82.6

Year	Unearned Premium BOY	Unearned Premium EOY	Loss Reserve BOY	Loss Reserve EOY	Stat Expense Reserve BOY	Stat Expense Reserve EOY	Total Stat Reserves BOY	Total Stat Reserves EOY
1	100.0	0.0	0.0	54.0	9.0	7.5	109.0	61.5
2	110.7	0.0	54.0	77.8	17.5	9.8	182.2	87.6
3	122.6	0.0	77.8	86.2	20.8	10.9	221.3	97.0
4	135.8	0.0	86.2	95.4	23.1	12.0	245.0	107.4

Year	Surplus BOY	Surplus EOY	Assets BOY	Assets EOY	Receivables BOY	Receivables EOY	Invested Assets BOY	Investment Income EOY	P/S
1	20.2	15.7	129.2	77.2	25.0	5.0	104.2	6.3	4.95
2	38.1	22.8	220.3	110.4	32.7	5.5	187.6	11.3	2.91
3	47.6	25.2	268.8	122.2	36.2	6.1	232.6	14.0	2.58
4	52.7	27.9	297.7	135.4	40.1	6.8	257.6	15.5	2.58

Year	DAC BOY	DAC EOY	GAAP Equity BOY	GAAP Equity EOY	GAAP Pre-tax Income EOY	Income Tax EOY	GAAP Income EOY	GAAP ROE
1	18.0	0.0	38.2	15.7	4.3	1.5	2.8	7.23%
2	19.9	0.0	58.0	22.8	9.0	3.2	5.9	10.13%
3	22.1	0.0	69.6	25.2	11.5	4.0	7.5	10.74%
4	24.4	0.0	77.1	27.9	12.7	4.5	8.3	10.74%

Exhibit 3
Sheet 1

Single Policy Company

UW Assumptions			Financial Assumptions		IRR and PVI/PVE Results		
	Amount	Ratio	Interest Rate	6.00%	IRR	10.99%	10.99%
Premium	100.0	100.0%	Tax Rate	35.00%	PVI/PVE Discount Rate	12.00%	10.99%
Loss	72.0	72.0%	Rsv Discount Rate	6.00%	PVI	6.22	6.23
Expense	30.0	30.0%	S as % of PV Unpaid Loss	31.50%	PVE	56.52	56.73
Combined	102.0	102.0%	PV Loss Discount for S Calc	6.00%	PVI/PVE	11.01%	10.99%

Year	Earned Premium	Incurred Loss	Stat Incurred Expense	Stat UW Income	Paid Premium	Paid Loss	Paid Expense	UW Cash Flow
0	0.0	0.0	18.0	-18.0	75.0	0.0	9.0	66.0
1	100.0	68.0	12.0	20.0	20.0	18.0	13.5	-11.5
2	0.0	3.0	0.0	-3.0	5.0	36.0	6.0	-37.0
3	0.0	1.0	0.0	-1.0	0.0	18.0	1.5	-19.5
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total	100.0	72.0	30.0	-2.0	100.0	72.0	30.0	-2.0

Year	Unearned Premium	Loss Reserve	PV Unpaid Loss	Stat Expense Reserve	Total Stat Reserves	Surplus	Assets	Receivables	Invested Assets	Inv Income
0	100.0	0.0	64.1	9.0	109.0	20.2	129.2	25.0	104.2	
1	0.0	50.0	50.0	7.5	57.5	15.7	73.2	5.0	68.2	6.3
2	0.0	17.0	17.0	1.5	18.5	5.3	23.8	0.0	23.8	4.1
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.4
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Year	DAC	GAAP Equity	GAAP Incurred Expense	GAAP UW Income	GAAP Pre-tax Income	Income Tax	Income	Change in Equity	Equity Flow
0	18.0	38.2	0.0	0.0	0.0	0.0	0.0	38.2	-38.2
1	0.0	15.7	30.0	-2.0	8.3	2.9	5.4	-22.5	27.8
2	0.0	5.3	0.0	-3.0	1.1	0.4	0.7	-10.4	11.1
3	0.0	0.0	0.0	-1.0	0.4	0.1	0.3	-5.3	5.6
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total			30.0	-2.0	9.8	3.4	6.4	0.0	6.4

Exhibit 3
Sheet 2

Single Policy Company- BOY and EOY Accounting

UW Assumptions	Amount	Ratio	Financial Assumptions		IRR and PVI/PVE Results		
Premium	100.0	100.0%	Interest Rate	6.00%	IRR	10.99%	10.99%
Loss	72.0	72.0%	Tax Rate	35.00%	PVI/PVE Discount Rate	12.00%	10.99%
Expense	30.0	30.0%	Rsv Discount Rate	6.00%	PVI	6.22	6.23
Combined	102.0	102.0%	S as % of PV Unpaid Loss	31.50%	PVE	56.52	56.73
			PV Loss Discount for S Calc	6.00%	PVI/PVE	11.01%	10.99%

Year	Earned Premium EOY	Incurred Loss EOY	GAAP Incurred Expense EOY	GAAP UW Income EOY	Paid Premium BOY	Paid Premium EOY	Paid Loss EOY	UW Cash Flow BOY	UW Cash Flow EOY
1	100.0	68.0	30.0	2.0	75.0	20.0	18.0	66.0	-11.5
2	0.0	3.0	0.0	-3.0	0.0	5.0	36.0	0.0	-37.0
3	0.0	1.0	0.0	-1.0	0.0	0.0	18.0	0.0	-19.5
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Year	Unearned Premium BOY	Unearned Premium EOY	Loss Reserve BOY	Loss Reserve EOY	Stat Expense Reserve BOY	Stat Expense Reserve EOY	Total Stat Reserves BOY	Total Stat Reserves EOY
1	100.0	0.0	0.0	50.0	9.0	7.5	109.0	57.5
2	0.0	0.0	50.0	17.0	7.5	1.5	57.5	18.5
3	0.0	0.0	17.0	0.0	1.5	0.0	18.5	0.0
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Year	Surplus BOY	Surplus EOY	Assets BOY	Assets EOY	Receivables BOY	Receivables EOY	Invested Assets BOY	Investment Income EOY
1	20.2	15.7	129.2	73.2	25.0	5.0	104.2	6.3
2	15.7	5.3	73.2	23.8	5.0	0.0	68.2	4.1
3	5.3	0.0	23.8	0.0	0.0	0.0	23.8	1.4
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Year	DAC BOY	DAC EOY	GAAP Equity BOY	GAAP Equity EOY	GAAP Pre-tax Income EOY	Income Tax EOY	GAAP Income EOY
1	18.0	0.0	38.2	15.7	8.3	2.9	5.4
2	0.0	0.0	15.7	5.3	1.1	0.4	0.7
3	0.0	0.0	5.3	0.0	0.4	0.1	0.3
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Exhibit 3
Sheet 3

Book of Business Growth Company

UW Assumptions			Financial Assumptions		IRR and ROE Results	
	Amount	Ratio	Interest Rate	6.00%	IRR	10.99%
Premium	100.0	100.0%	Tax Rate	35.00%	EQ Growth ROE	10.85%
Loss	72.0	72.0%	Rsv Discount Rate	6.00%	EQ Growth P/S	2.50
Expense	30.0	30.0%	S as % of PV Unpaid Loss	31.50%	Growth Rate	5.00%
Combined	102.0	102.0%	PV Loss Discount for S Calc	6.00%		

Year	Earned Premium EOY	Incurred Loss EOY	GAAP Incurred Expense EOY	GAAP UW Income EOY	Paid Premium BOY	Paid Premium EOY	Paid Loss EOY	UW Cash Flow BOY	UW Cash Flow EOY
1	100.0	68.0	30.0	2.0	75.0	20.0	18.0	66.0	-11.5
2	105.0	74.4	31.5	-0.9	78.8	26.0	54.9	69.3	-49.1
3	110.3	79.1	33.1	-1.9	82.7	27.3	75.6	72.8	-71.0
4	115.8	83.1	34.7	-2.0	86.8	28.7	79.4	76.4	-74.6

Year	Unearned Premium BOY	Unearned Premium EOY	Loss Reserve BOY	Loss Reserve EOY	Stat Expense Reserve BOY	Stat Expense Reserve EOY	Total Stat Reserves BOY	Total Stat Reserves EOY
1	100.0	0.0	0.0	50.0	9.0	7.5	109.0	57.5
2	105.0	0.0	50.0	69.5	17.0	9.4	171.9	78.8
3	110.3	0.0	69.5	72.9	19.3	9.8	199.0	82.8
4	115.8	0.0	72.9	76.6	20.3	10.3	209.0	86.9

Year	Surplus BOY	Surplus EOY	Assets BOY	Assets EOY	Receivables BOY	Receivables EOY	Invested Assets BOY	Investment Income EOY	P/S
1	20.2	15.7	129.2	73.2	25.0	5.0	104.2	6.3	4.95
2	37.0	21.9	208.9	100.7	31.3	5.3	177.6	10.7	2.84
3	44.2	23.0	243.2	105.8	32.8	5.5	210.4	12.6	2.50
4	46.4	24.1	255.3	111.0	34.5	5.8	220.9	13.3	2.50

Year	DAC BOY	DAC EOY	GAAP Equity BOY	GAAP Equity EOY	GAAP Pre-tax Income EOY	Income Tax EOY	GAAP Income EOY	GAAP ROE
1	18.0	0.0	38.2	15.7	8.3	2.9	5.4	14.07%
2	18.9	0.0	55.9	21.9	9.8	3.4	6.4	11.38%
3	19.8	0.0	64.0	23.0	10.7	3.7	6.9	10.85%
4	20.8	0.0	67.2	24.1	11.2	3.9	7.3	10.85%

Exhibit 3
Sheet 4

Book of Business Growth Company

UW Assumptions	Amount	Ratio	Financial Assumptions		IRR and ROE Results	
			Interest Rate	6.00%	IRR	10.99%
Premium	100.0	100.0%	Tax Rate	35.00%	EQ Growth ROE	10.99%
Loss	72.0	72.0%	Rsv Discount Rate	6.00%	EQ Growth P/S	2.58
Expense	30.0	30.0%	S as % of PV Unpaid Loss	31.50%	Growth Rate	10.99%
Combined	102.0	102.0%	PV Loss Discount for S Calc	6.00%		

Year	Earned Premium EOY	Incurred Loss EOY	GAAP Incurred Expense EOY	GAAP UW Income EOY	Paid Premium BOY	Paid Premium EOY	Paid Loss EOY	UW Cash Flow BOY	UW Cash Flow EOY
1	100.0	68.0	30.0	2.0	75.0	20.0	18.0	66.0	-11.5
2	111.0	78.5	33.3	-0.8	83.2	27.2	56.0	73.3	-49.8
3	123.2	88.1	37.0	-1.9	92.4	30.2	80.1	81.3	-74.7
4	136.7	97.8	41.0	-2.1	102.5	33.5	88.9	90.2	-82.9

Year	Unearned Premium BOY	Unearned Premium EOY	Loss Reserve BOY	Loss Reserve EOY	Stat Expense Reserve BOY	Stat Expense Reserve EOY	Total Stat Reserves BOY	Total Stat Reserves EOY
1	100.0	0.0	0.0	50.0	9.0	7.5	109.0	57.5
2	111.0	0.0	50.0	72.5	17.5	9.8	178.5	82.3
3	123.2	0.0	72.5	80.4	20.9	10.9	216.5	91.3
4	136.7	0.0	80.4	89.3	23.2	12.1	240.3	101.4

Year	Surplus BOY	Surplus EOY	Assets BOY	Assets EOY	Receivables BOY	Receivables EOY	Invested Assets BOY	Investment Income EOY	P/S
1	20.2	15.7	129.2	73.2	25.0	5.0	104.2	6.3	4.95
2	38.2	22.8	216.6	105.1	32.7	5.5	183.9	11.0	2.91
3	47.7	25.3	264.3	116.7	36.3	6.2	227.9	13.7	2.58
4	53.0	28.1	293.3	129.5	40.3	6.8	253.0	15.2	2.58

Year	DAC BOY	DAC EOY	GAAP Equity BOY	GAAP Equity EOY	GAAP Pre-tax Income EOY	Income Tax EOY	GAAP Income EOY	GAAP ROE
1	18.0	0.0	38.2	15.7	8.3	2.9	5.4	14.07%
2	20.0	0.0	58.1	22.8	10.3	3.6	6.7	11.48%
3	22.2	0.0	69.9	25.3	11.8	4.1	7.7	10.99%
4	24.6	0.0	77.6	28.1	13.1	4.6	8.5	10.99%

Exhibit 4
Sheet 1

Return Measures
Sensitivity to Premium **Full Value Reserve**

Scenario	1	2	3	4	5	6	7
Premium	80.00	85.00	90.00	95.00	100.00	105.00	110.00
Combined Ratio	122.50%	116.47%	111.11%	106.32%	102.00%	98.10%	94.55%
Resulting Growth Model P/S	2.00	2.12	2.25	2.37	2.50	2.62	2.75
Returns							
IRR	-7.00%	-2.74%	1.65%	6.15%	10.74%	15.40%	20.10%
PVI/PVE	-9.21%	-4.07%	0.96%	5.89%	10.71%	15.43%	20.05%
ROE	-8.47%	-3.47%	1.42%	6.21%	10.90%	15.49%	19.99%
Change in Returns							
IRR		4.27%	4.39%	4.50%	4.59%	4.66%	4.70%
PVI/PVE		5.14%	5.03%	4.92%	4.82%	4.72%	4.62%
ROE		5.00%	4.89%	4.79%	4.69%	4.59%	4.50%

Assumptions for All Scenarios

Financial

Interest Rate	6.00%
Tax Rate	35.00%
Reserve Discount Rate	0.00%
Surplus as % of PV Unpaid Loss	31.50%
Rate for PV Calculation	6.00%
Rate for PVI/PVE Discounting	12.00%
ROE Growth Rate	5.00%

Assumptions for All Scenarios

Underwriting

Premium		Loss		Expense	
	varies	Fixed	72.00	Fixed	10.00
				Variable	20.0%
Premium Payment		Loss Payout		Expense Payout	
Year	%	Year	%	Year	%
0	75.0%	0	0.0%	0	30.0%
1	20.0%	1	25.0%	1	50.0%
2	5.0%	2	50.0%	2	25.0%
3	0.0%	3	25.0%	3	0.0%
4	0.0%	4	0.0%	4	0.0%

Exhibit 4
Sheet 2

Return Measures
Sensitivity to Premium **Discounted Reserve**

Scenario	1	2	3	4	5	6	7
Premium	80.00	85.00	90.00	95.00	100.00	105.00	110.00
Combined Ratio	122.50%	116.47%	111.11%	106.32%	102.00%	98.10%	94.55%
Resulting Growth Model P/S	2.00	2.12	2.25	2.37	2.50	2.62	2.75
Returns							
IRR	-7.74%	-3.23%	1.42%	6.16%	10.99%	15.87%	20.79%
PVI/PVE	-8.89%	-3.75%	1.27%	6.19%	11.01%	15.73%	20.34%
ROE	-8.52%	-3.53%	1.36%	6.15%	10.85%	15.44%	19.94%
Change in Returns							
IRR		4.52%	4.64%	4.75%	4.83%	4.88%	4.92%
PVI/PVE		5.14%	5.03%	4.92%	4.82%	4.72%	4.62%
ROE		5.00%	4.89%	4.79%	4.69%	4.59%	4.50%

Assumptions for All Scenarios

Financial

Interest Rate	6.00%
Tax Rate	35.00%
Reserve Discount Rate	6.00%
Surplus as % of PV Unpaid Loss	31.50%
Rate for PV Calculation	6.00%
Rate for PVI/PVE Discounting	12.00%
ROE Growth Rate	5.00%

Assumptions for All Scenarios

Underwriting

Premium		Loss		Expense	
	varies	Fixed	72.00	Fixed	10.00
				Variable	20.0%
Premium Payment		Loss Payout		Expense Payout	
Year	%	Year	%	Year	%
0	75.0%	0	0.0%	0	30.0%
1	20.0%	1	25.0%	1	50.0%
2	5.0%	2	50.0%	2	25.0%
3	0.0%	3	25.0%	3	0.0%
4	0.0%	4	0.0%	4	0.0%

Exhibit 4
Sheet 3

Return Measures
Sensitivity to Surplus

Scenario	1	2	3	4	5	6	7
Surplus as % of PV Unpaid Loss	25.50%	27.50%	29.50%	31.50%	33.50%	35.50%	37.50%
Resulting Growth Model P/S	3.08	2.86	2.67	2.50	2.35	2.22	2.10
Returns							
IRR	11.73%	11.37%	11.04%	10.74%	10.46%	10.21%	9.97%
PVI/PVE	11.72%	11.35%	11.02%	10.71%	10.42%	10.16%	9.92%
ROE	11.96%	11.57%	11.22%	10.90%	10.60%	10.33%	10.09%

Assumptions for All Scenarios

Financial

Interest Rate	6.00%
Tax Rate	35.00%
Reserve Discount Rate	0.00%
Surplus as % of PV Unpaid Loss	varies
Rate for PV Calculation	6.00%
Rate for PVI/PVE Discounting	12.00%
ROE Growth Rate	5.00%

Assumptions for All Scenarios

Underwriting

Premium		Loss		Expense	
Fixed	100.00	Fixed	72.00	Fixed	10.00
				Variable	20.0%
Premium Payment		Loss Payout		Expense Payout	
Year	%	Year	%	Year	%
0	75.0%	0	0.0%	0	30.0%
1	20.0%	1	25.0%	1	50.0%
2	5.0%	2	50.0%	2	25.0%
3	0.0%	3	25.0%	3	0.0%
4	0.0%	4	0.0%	4	0.0%

Exhibit 4
Sheet 4

Return Measures
Sensitivity to Interest Rate

Scenario	1	2	3	4	5	6	7
Interest Rate	4.50%	5.00%	5.50%	6.00%	6.50%	7.00%	7.50%
Resulting Growth Model P/S	2.44	2.46	2.48	2.50	2.52	2.53	2.55
Returns							
IRR	7.48%	8.56%	9.65%	10.74%	11.84%	12.93%	14.04%
PVI/PVE	7.38%	8.48%	9.59%	10.71%	11.83%	12.96%	14.10%
ROE	7.54%	8.65%	9.77%	10.90%	12.03%	13.18%	14.33%

Assumptions for All Scenarios

Financial

Interest Rate	varies
Tax Rate	35.00%
Reserve Discount Rate	0.00%
Surplus as % of PV Unpaid Loss	31.50%
Rate for PV Calculation	varies
Rate for PVI/PVE Discounting	12.00%
ROE Growth Rate	5.00%

Assumptions for All Scenarios

Underwriting

Premium		Loss		Expense	
Fixed	100.00	Fixed	72.00	Fixed	10.00
				Variable	20.0%
Premium Payment		Loss Payout		Expense Payout	
Year	%	Year	%	Year	%
0	75.0%	0	0.0%	0	30.0%
1	20.0%	1	25.0%	1	50.0%
2	5.0%	2	50.0%	2	25.0%
3	0.0%	3	25.0%	3	0.0%
4	0.0%	4	0.0%	4	0.0%

Exhibit 4
Sheet 5

Return Measures
Sensitivity to Payout Pattern

Scenario		1=Base	2	3	4	5	6	7
Loss Pattern	Year							
	0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	1	25.00%	100.00%	50.00%	0.00%	0.00%	0.00%	0.00%
	2	50.00%	0.00%	50.00%	100.00%	50.00%	0.00%	0.00%
	3	25.00%	0.00%	0.00%	0.00%	50.00%	100.00%	50.00%
	4	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	50.00%
Surplus % of PV Unpaid Loss		31.50%	58.96%	40.72%	31.10%	25.68%	21.87%	19.32%
Resulting Growth Model P/S		2.50	2.50	2.50	2.50	2.50	2.50	2.50
Indicated Profit Margins								
IRR Method		10.74%	6.34%	8.60%	10.82%	12.85%	14.83%	16.61%
PVI/PVE Method		10.71%	6.33%	8.55%	10.79%	12.88%	14.97%	16.92%
ROE Method		10.90%	6.35%	8.65%	10.95%	13.15%	15.34%	17.43%

Assumptions for All Scenarios

Financial

Interest Rate	6.00%
Tax Rate	35.00%
Reserve Discount Rate	0.00%
Surplus as % of PV Unpaid Loss	varies
Rate for PV Calculation	12.00%
Rate for PVI/PVE Discounting	12.00%
Growth Rate	5.00%

Assumptions for All Scenarios

Underwriting

Premium		Loss		Expense	
		Fixed	72.00	Fixed	10.00
				Variable	20.0%
Premium Payment		Loss Payout		Expense Payout	
Year	%	Year	%	Year	%
0	75.0%	0	varies	0	30.0%
1	20.0%	1	varies	1	45.0%
2	5.0%	2	varies	2	20.0%
3	0.0%	3	varies	3	5.0%
4	0.0%	4	varies	4	0.0%

Exhibit 5
Sheet 1

Indicated Profit
Sensitivity to Surplus

Scenario	1	2	3	4	5	6	7
Surplus as % of PV Unpaid Loss	25.50%	27.50%	29.50%	31.50%	33.50%	35.50%	37.50%
Resulting Growth Model P/S	3.09	2.87	2.69	2.53	2.38	2.26	2.15
Indicated Profit Margins							
IRR Method	-1.79%	-1.49%	-1.20%	-0.90%	-0.61%	-0.32%	-0.03%
PVI/PVE Method	-1.79%	-1.49%	-1.20%	-0.90%	-0.61%	-0.32%	-0.03%
ROE Method	-1.97%	-1.65%	-1.34%	-1.04%	-0.73%	-0.43%	-0.13%

Assumptions for All Scenarios

Financial

Interest Rate	6.00%
Tax Rate	35.00%
Reserve Discount Rate	0.00%
Surplus as % of PV Unpaid Loss	varies
Rate for PV Calculation	6.00%
IRR Target Return	12.00%
PVI/PVE Target Return	12.00%
Rate for PVI/PVE Discounting	12.00%
ROE Target Return	12.00%
ROE Target Growth Rate	5.00%

Assumptions for All Scenarios

Underwriting

Premium		Loss		Expense	
		Fixed	72.00	Fixed	10.00
				Variable	20.0%
Premium Payment		Loss Payout		Expense Payout	
Year	%	Year	%	Year	%
0	75.0%	0	0.0%	0	30.0%
1	20.0%	1	25.0%	1	25.0%
2	5.0%	2	50.0%	2	50.0%
3	0.0%	3	25.0%	3	25.0%
4	0.0%	4	0.0%	4	0.0%

Exhibit 5
Sheet 2

Indicated Profit
Sensitivity to Interest Rate

Scenario	1	2	3	4	5	6	7
Interest Rate	4.50%	5.00%	5.50%	6.00%	6.50%	7.00%	7.50%
Resulting Growth Model P/S	2.56	2.55	2.54	2.53	2.52	2.50	2.49
Indicated Profit Margins							
IRR Method	1.91%	0.98%	0.05%	-0.90%	-1.86%	-2.82%	-3.80%
PVI/PVE Method	1.91%	0.98%	0.05%	-0.90%	-1.86%	-2.82%	-3.80%
ROE Method	1.88%	0.92%	-0.05%	-1.04%	-2.03%	-3.03%	-4.05%

Assumptions for All Scenarios

Financial

Interest Rate	varies
Tax Rate	35.00%
Reserve Discount Rate	0.00%
Surplus as % of PV Unpaid Loss	31.50%
Rate for PV Calculation	varies
IRR Target Return	12.00%
PVI/PVE Target Return	12.00%
Rate for PVI/PVE Discounting	12.00%
ROE Target Return	12.00%
ROE Target Growth Rate	5.00%

Assumptions for All Scenarios

Underwriting

Premium		Loss		Expense	
		Fixed	72.00	Fixed	10.00
				Variable	20.0%
Premium Payment		Loss Payout		Expense Payout	
Year	%	Year	%	Year	%
0	75.0%	0	0.0%	0	30.0%
1	20.0%	1	25.0%	1	25.0%
2	5.0%	2	50.0%	2	50.0%
3	0.0%	3	25.0%	3	25.0%
4	0.0%	4	0.0%	4	0.0%

Exhibit 5
Sheet 3

Indicated Profit
Sensitivity to Payout Pattern

Scenario		1=Base	2	3	4	5	6	7
Loss Pattern	Year							
	0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	1	25.00%	100.00%	50.00%	0.00%	0.00%	0.00%	0.00%
	2	50.00%	0.00%	50.00%	100.00%	50.00%	0.00%	0.00%
	3	25.00%	0.00%	0.00%	0.00%	50.00%	100.00%	50.00%
	4	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	50.00%
Surplus % of PV Unpaid Loss		31.50%	62.00%	41.76%	31.08%	25.03%	20.77%	17.90%
Resulting Growth Model P/S		2.53	2.53	2.53	2.53	2.53	2.53	2.53
Indicated Profit Margins								
IRR Method		-0.90%	2.96%	1.02%	-0.97%	-2.88%	-4.85%	-6.72%
PVI/PVE Method		-0.90%	2.96%	1.02%	-0.97%	-2.88%	-4.85%	-6.72%
ROE Method		-1.04%	2.94%	0.98%	-1.09%	-3.16%	-5.34%	-7.52%

Assumptions for All Scenarios

Financial

Interest Rate	6.00%
Tax Rate	35.00%
Reserve Discount Rate	0.00%
Surplus as % of PV Unpaid Loss	varies
Rate for PV Calculation	6.00%
IRR Target Return	12.00%
PVI/PVE Target Return	12.00%
Rate for PVI/PVE Discounting	12.00%
ROE Target Return	12.00%
ROE Target Growth Rate	5.00%

Assumptions for All Scenarios

Underwriting

Premium		Loss		Expense	
		Fixed	72.00	Fixed	10.00
				Variable	20.0%
Premium Payment		Loss Payout		Expense Payout	
Year	%	Year	%	Year	%
0	75.0%	0	varies	0	30.0%
1	20.0%	1	varies	1	45.0%
2	5.0%	2	varies	2	20.0%
3	0.0%	3	varies	3	5.0%
4	0.0%	4	varies	4	0.0%

Exhibit 6
Sheet 1

Results for Three Point Loss Distribution
Sensitivity to Premium   Full Value Reserve

Scenario	1			2			3			Average over All Scenarios		
Probability	40.00%			40.00%			20.00%					
Premium	100.00			100.00			100.00			100.00		
Loss	60.00			72.00			96.00			72.00		
Combined Ratio	90.00%			102.00%			126.00%			102.00%		
Returns												
IRR	24.11%			10.74%			-11.63%			10.74%		
PVI/PVE	23.79%			10.71%			-15.45%			10.71%		
Results by Year	Equity	Income	Equity Flow	Equity	Income	Equity Flow	Equity	Income	Equity Flow	Equity	Income	Equity Flow
Year												
0	38.20	0.00	-38.20	38.20	0.00	-38.20	38.20	0.00	-38.20	38.20	0.00	-38.20
1	15.74	10.56	33.02	15.74	2.76	25.22	15.74	-12.84	9.62	15.74	2.76	25.22
2	5.35	2.47	12.86	5.35	2.82	13.21	5.35	3.52	13.91	5.35	2.82	13.21
3	0.00	0.85	6.20	0.00	0.97	6.32	0.00	1.20	6.55	0.00	0.97	6.32

Exhibit 6
Sheet 2

Single Policy Company **Loss Scenario 1**

E[Loss]	72.0

UW Assumptions			Financial Assumptions		IRR and PVI/PVE Results	
	Amount	Ratio	Interest Rate	6.00%	IRR	24.11%
Premium	100.0	100.0%	Tax Rate	35.00%	PVI/PVE Discount Rate	12.00%
Loss	60.0	60.0%	Rsv Discount Rate	0.00%	PVI	13.45
Expense	30.0	30.0%	S as % of E[PV Unpaid Loss]	31.50%	PVE	56.52
Combined	90.0	90.0%	PV Loss Discount for S Calc	6.00%	PVI/PVE	23.79%

Year	Earned Premium	Incurred Loss	Stat Incurred Expense	Stat UW Income	Paid Premium	Paid Loss	Paid Expense	UW Cash Flow
0	0.0	0.0	18.0	-18.0	75.0	0.0	9.0	66.0
1	100.0	60.0	12.0	28.0	20.0	15.0	13.5	-8.5
2	0.0	0.0	0.0	0.0	5.0	30.0	6.0	-31.0
3	0.0	0.0	0.0	0.0	0.0	15.0	1.5	-16.5
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total	100.0	60.0	30.0	10.0	100.0	60.0	30.0	10.0

Year	Unearned Premium	Loss Reserve	Expected PV Unpaid Loss	Stat Expense Reserve	Total Stat Reserves	Surplus	Assets	Receivables	Invested Assets	Inv Income
0	100.0	0.0	64.1	9.0	109.0	20.2	129.2	25.0	104.2	
1	0.0	45.0	50.0	7.5	52.5	15.7	68.2	5.0	63.2	6.3
2	0.0	15.0	17.0	1.5	16.5	5.3	21.8	0.0	21.8	3.8
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.3
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Year	DAC	GAAP Equity	GAAP Incurred Expense	GAAP UW Income	GAAP Pre-tax Income	Income Tax	Income	Change in Equity	Equity Flow
0	18.0	38.2	0.0	0.0	0.0	0.0	0.0	38.2	-38.2
1	0.0	15.7	30.0	10.0	16.3	5.7	10.6	-22.5	33.0
2	0.0	5.3	0.0	0.0	3.8	1.3	2.5	-10.4	12.9
3	0.0	0.0	0.0	0.0	1.3	0.5	0.9	-5.3	6.2
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total			30.0	10.0	21.4	7.5	13.9	0.0	13.9

Exhibit 6
Sheet 3

Single Policy Company **Loss Scenario 2**

E[Loss]	72.0

UW Assumptions			Financial Assumptions		IRR and PVI/PVE Results	
	Amount	Ratio	Interest Rate	6.00%	IRR	10.74%
Premium	100.0	100.0%	Tax Rate	35.00%	PVI/PVE Discount Rate	12.00%
Loss	72.0	72.0%	Rsv Discount Rate	0.00%	PVI	6.05
Expense	30.0	30.0%	S as % of E[PV Unpaid Loss]	31.50%	PVE	56.52
Combined	102.0	102.0%	PV Loss Discount for S Calc	6.00%	PVI/PVE	10.71%

Year	Earned Premium	Incurred Loss	Stat Incurred Expense	Stat UW Income	Paid Premium	Paid Loss	Paid Expense	UW Cash Flow
0	0.0	0.0	18.0	-18.0	75.0	0.0	9.0	66.0
1	100.0	72.0	12.0	16.0	20.0	18.0	13.5	-11.5
2	0.0	0.0	0.0	0.0	5.0	36.0	6.0	-37.0
3	0.0	0.0	0.0	0.0	0.0	18.0	1.5	-19.5
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total	100.0	72.0	30.0	-2.0	100.0	72.0	30.0	-2.0

Year	Unearned Premium	Loss Reserve	Expected PV Unpaid Loss	Stat Expense Reserve	Total Stat Reserves	Surplus	Assets	Receivables	Invested Assets	Inv Income
0	100.0	0.0	64.1	9.0	109.0	20.2	129.2	25.0	104.2	
1	0.0	54.0	50.0	7.5	61.5	15.7	77.2	5.0	72.2	6.3
2	0.0	18.0	17.0	1.5	19.5	5.3	24.8	0.0	24.8	4.3
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.5
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Year	DAC	GAAP Equity	GAAP Incurred Expense	GAAP UW Income	GAAP Pre-tax Income	Income Tax	Income	Change in Equity	Equity Flow
0	18.0	38.2	0.0	0.0	0.0	0.0	0.0	38.2	-38.2
1	0.0	15.7	30.0	-2.0	4.3	1.5	2.8	-22.5	25.2
2	0.0	5.3	0.0	0.0	4.3	1.5	2.8	-10.4	13.2
3	0.0	0.0	0.0	0.0	1.5	0.5	1.0	-5.3	6.3
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total			30.0	-2.0	10.1	3.5	6.6	0.0	6.6

Exhibit 6
Sheet 4

Single Policy Company **Loss Scenario 3**

E[Loss]	72.0

UW Assumptions			Financial Assumptions		IRR and PVI/PVE Results	
	Amount	Ratio	Interest Rate	6.00%	IRR	-11.63%
Premium	100.0	100.0%	Tax Rate	35.00%	PVI/PVE Discount Rate	12.00%
Loss	96.0	96.0%	Rsv Discount Rate	0.00%	PVI	-8.73
Expense	30.0	30.0%	S as % of E[PV Unpaid Loss]	31.50%	PVE	56.52
Combined	126.0	126.0%	PV Loss Discount for S Calc	6.00%	PVI/PVE	-15.45%

Year	Earned Premium	Incurred Loss	Stat Incurred Expense	Stat UW Income	Paid Premium	Paid Loss	Paid Expense	UW Cash Flow
0	0.0	0.0	18.0	-18.0	75.0	0.0	9.0	66.0
1	100.0	96.0	12.0	-8.0	20.0	24.0	13.5	-17.5
2	0.0	0.0	0.0	0.0	5.0	48.0	6.0	-49.0
3	0.0	0.0	0.0	0.0	0.0	24.0	1.5	-25.5
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total	100.0	96.0	30.0	-26.0	100.0	96.0	30.0	-26.0

Year	Unearned Premium	Loss Reserve	Expected PV Unpaid Loss	Stat Expense Reserve	Total Stat Reserves	Surplus	Assets	Receivables	Invested Assets	Inv Income
0	100.0	0.0	64.1	9.0	109.0	20.2	129.2	25.0	104.2	
1	0.0	72.0	50.0	7.5	79.5	15.7	95.2	5.0	90.2	6.3
2	0.0	24.0	17.0	1.5	25.5	5.3	30.8	0.0	30.8	5.4
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.9
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Year	DAC	GAAP Equity	GAAP Incurred Expense	GAAP UW Income	GAAP Pre-tax Income	Income Tax	Income	Change in Equity	Equity Flow
0	18.0	38.2	0.0	0.0	0.0	0.0	0.0	38.2	-38.2
1	0.0	15.7	30.0	-26.0	-19.7	-6.9	-12.8	-22.5	9.6
2	0.0	5.3	0.0	0.0	5.4	1.9	3.5	-10.4	13.9
3	0.0	0.0	0.0	0.0	1.9	0.6	1.2	-5.3	6.6
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total			30.0	-26.0	-12.5	-4.4	-8.1	0.0	-8.1