Consistent Measurement of Property-Casualty Risk-Based Capital Adequacy

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Abstract

This paper is a review and case study of Butsic's expected policyholder deficit (EPD) framework for measurement and maintenance of risk-based capital adequacy for property-casualty insurance companies, the promise of which is that long term solvency protection can be achieved by periodic assessment and adjustment of risk-based capital using a consistent and short time horizon, e.g., one year, for risks on both sides of the balance sheet. Using a common one-year EPD risk measure to assess all risks, the case study examines the exposure to capital exhaustion during the period 1999 through 2004 arising from 1) U.S. Commercial Auto Liability accident year 1999 underwriting and reserving and 2) investment in the stocks comprising the S&P 500. The case study results indicate that NAIC and rating agency risk-based capital requirements for Commercial Auto Liability are significantly higher than necessary to meet stated solvency objectives and much higher than those demanded for common stock investments. That disparity probably exists for other lines of business as well. The consistent measurement of all time-dependent risks described in the paper is relevant not only to risk-based capital applications but to enterprise risk assessment and management as well.

Keywords: risk-based capital, expected policyholder deficit, stochastic loss models, Commercial Auto Liability, enterprise risk management, transfer value

1. INTRODUCTION

The thesis of this paper is that U.S. regulatory and rating agency[1] risk-based capital factors used to allocate capital for at least some non-catastrophe underwriting and reserve risks are significantly higher than necessary to meet stated solvency objectives. These factors are too high both in absolute terms and relative to those that are applicable to insurance company assets such as common stocks. The reason for this disparity is that the risks related to insurance underwriting and reserving and those associated with investing in common stocks have been measured inconsistently. When the risks are measured consistently, less risk-based capital is required to support underwriting activity and loss reserves or more capital is required to support the holding of assets such as common stocks, or possibly both.

Our thesis is based on the application of Robert Butsic's approach to measurement of risk-based capital adequacy, which makes use of a clearly defined and consistent time horizon for assessing and managing underwriting, asset and other risks. In his Michelbacher-Prize-winning 1992 paper [5] on risk-based capital and solvency issues for the

[1] A.M. Best and Standard and Poor's.

property-casualty industry, Butsic showed that long term solvency protection can be achieved by periodically rebalancing risk-based capital to maintain a constant (and low) target exposure to capital exhaustion over a short prospective time horizon[2]. He used the *expected policyholder deficit* (EPD) with a time horizon of one year as the measure of exposure to capital exhaustion[3]. Butsic's framework incorporates a consistent time horizon for measuring and allocating capital for risks on both sides of the balance sheet. The risk-based capital requirements for asset risks as well as underwriting and reserve risks are all calibrated to the same target one-year EPD. The capital allocated to support a block of assets or underwriting risks is adjusted up or down at the end of each year to produce a prospective one-year EPD that matches the target value[4].

The idea that an insurer needs to allocate capital to minimize the risk of underwriting and reserve related capital exhaustion occurring only within the next year seems to run counter to the imperative that allocated capital be sufficient to deal with the risk of insolvency over the indefinite time horizon encompassing ultimate claim settlement. It apparently leaves the insolvency risk beyond the next year "unfunded." In fact, that is not the case. Butsic's breakthrough insight was that such longer term risk can be addressed effectively, one year at a time, as it comes into the one-year time horizon in future periods (in the same way that common stock risk has historically been handled). Because capital is recalibrated each year to the target EPD, any capital inadequacy short of total exhaustion that has emerged during a year can be corrected at the end of that year. In that way the small prospective exposure to capital exhaustion at the start of each successive one-year time horizon is maintained at the target level. If an insurer cannot recapitalize at the level consistent with the target EPD, in all circumstances except those in which the capital has been exhausted there still will be sufficient assets to facilitate liquidation of the risk portfolio. If the regulators do not immediately intervene, rating agencies can calculate the EPD associated with the reduced

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2 We refer, in particular, to his discussion on pages 327-335. A slightly amended version of Butsic's paper was later published in the *Journal of Risk and Insurance* under the same title [6]. In that version the discussion appears on pages 668-675.

3 The expected policyholder deficit with a time horizon of one year is defined as the expected value of the amount by which available assets, including allocated capital, will be inadequate to satisfy all claims one year in the future. A policyholder deficit with respect to asset risk arises when a fluctuating asset value falls below the value of unpaid losses (which is assumed to be fixed). A policyholder deficit with respect to underwriting-related risks arises when the fluctuating transfer value of unpaid losses exceeds the value of the available assets (which is assumed to be fixed). The EPD expressed as a ratio to the expected unpaid losses as of the beginning of the year can be viewed as the expected value of the proportion of the outstanding policyholder claims that will be unrecoverable because of insurer insolvency. Butsic used a one-year time horizon to illustrate his framework. It could, of course, be more or less than one year.

4 Note that this framework can easily be adapted to use a risk measure other than the EPD. The principle that the chosen risk measure be used consistently to assess all risks consistently over successive short time horizons is more important than the risk measure itself (provided the risk measure is a sound one).

capital level and adjust the insurer's financial strength rating to reflect the increased insolvency risk[5].

A major advantage of Butsic's approach is that its consistent measurement of all risks over a common time horizon allows us to compare and ultimately combine risks that have different natural time horizons. That enables us to make clear and meaningful statements about an insurer's exposure to capital exhaustion in the next year. In contrast, it is less clear what a statement about an insurer's exposure to capital exhaustion means when it reflects a mixture of time horizons. Should capital exposed over different time horizons be calibrated to different target EPDs, perhaps 1% for one-year horizons and 4% for four-year horizons, or should the target be fixed irrespective of time horizon? How do risks having different time horizons interact? For example, if we measure risks only at their ultimate time horizons, if we add a four-year horizon risk to a portfolio with a one-year horizon, how do we measure the one-year capital exhaustion risk of the new portfolio? Butsic's framework allows us to avoid these questions by focusing on a common time horizon from the start.

A significant obstacle to the full implementation of Butsic's approach has been that, while it is relatively easy to calculate for asset risks, it is more difficult to determine the one-year EPD for underwriting and reserve risks. Butsic explained the concept and illustrated the calculation, but he did not describe a model or method for doing the calculation in practice. The issue is that the value of the one-year EPD is a function of a time-dependent loss distribution for which actuaries historically have had no use. However, a recent paper by Wacek [12] on the path of the ultimate loss ratio estimate describes a framework that can be used to model that distribution. We will use the approach outlined in that paper together with actual industry loss experience to illustrate the application of Butsic's framework for measuring underwriting and loss reserve risk.

One of our aims is to revive interest in Butsic's approach to the assessment of risk-based capital requirements, and, in particular, the use of a clear and consistent time horizon for measuring all of the solvency risks faced by an insurance company. In this paper we present a review and illustration of the key concepts of his framework using insurance industry and stock market experience from the period 1999 through 2004. By using actual experience to parameterize stochastic stock price and loss ratio models, we show that Butsic's framework is not only of theoretical interest but can be practically applied in the real world. While we

[5] That is more or less what happens today (though both A.M. Best and Standard and Poor's base their capital factors for underwriting and reserve risks on an *ultimate time horizon* EPD methodology [1][11]). Best, for example, reports that an EPD ratio of greater than 1% indicates a BCAR score of less than 100 and a rating of less than B+, and makes clear that capital adequacy is a key element of its rating analysis. See [1], page 5.

focus on the analysis of non-catastrophe risks, Butsic's framework can also be applied to the analysis of the threat to solvency posed by property catastrophe loss events.

The late 1990s were challenging years for the U.S. insurance industry. For our case study of underwriting and reserve risk within the Butsic framework, we used industry data for Commercial Auto Liability, a line that experienced particularly poor accident year underwriting results in the late 1990s. We focused on accident year 1999, which had not only the highest estimated ultimate loss ratio of any accident year for that line in the period 1995 through 2004 but also one that proved difficult for the industry to estimate accurately. The ultimate loss ratio estimate of 78.3% booked as of December 1999 had to be increased repeatedly, reaching 92.1% as of December 2004[6]. While there is evidence that the industry could have made better estimates at early valuations using information available at the time, even those better estimates underestimated the ultimate loss ratio significantly[7]. The magnitude and unpredictability of the 1999 ultimate loss ratio makes that accident year a good choice for stress-testing the Butsic framework.

The period 1999 through 2004 was also a turbulent one for the U.S. stock market. The S&P 500 rose by more than 20% in two of the six years, declined for three consecutive years (including one year by more than 20%) and ended 2004 about 8% above its level at the beginning of 1999[8]. That volatility makes the S&P 500 in this period a good candidate for a case study of risk-based capital analysis of diversified common stock investments.

Our case study reveals that during this period, if the risk in both portfolios is measured consistently, the insolvency risk embedded in the industry Commercial Auto Liability underwriting and reserve portfolios was a small fraction of that inherent in the diversified common stock portfolio represented by the S&P 500. Our modeling of the increased risk associated with individual insurers (compared to the industry as a whole) also indicated much lower insolvency risk than investment in the S&P 500. Moreover, we found that the amount of risk-based capital required to achieve a target one-year EPD for underwriting and reserve risks consistent with the 1% target sometimes cited for common stocks [1][8] to be

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6 According to the industry 2004 Schedule P data reported in the 2005 edition of Best's Aggregates & Averages [4]. These loss ratios were calculated from "incurred net losses and cost containment expenses" reported in Part 2C and "net premiums earned" reported in Part 1C.

7 See Wacek [13], which was a case study of the relative quality of clinical judgment and statistical prediction in Commercial Auto Liability loss reserving for accident years 1995 through 2001. The paper concluded that the statistical prediction methods performed far better than the clinical methods actually used to set the reserves, but noted that they also did not perform well. For example, the accident year 1999 loss ratio actually booked at twelve months underestimated the ultimate loss ratio by 13.8 loss ratio points, while the mean of the statistical estimates underestimated it by 6.9 loss ratio points.

8 Including dividends, the annual S&P 500 total returns during the period 1999 through 2004 were as follows: +21.0%, -9.1%, -11.9%, -22.1%, +28.7% and +10.9%. Source: Berkshire Hathaway 2005 Annual Report [3].

much lower than NAIC and rating agency requirements as of December 2006, while we found the capital required for common stocks, at least during the period covered by our study, to be higher. Our study was too narrow in scope to support general conclusions, but the strikingly lower risk level we found for Commercial Auto Liability suggests that consistently calibrated risk-based capital factors would probably also be lower than those promulgated by the NAIC and the rating agencies for the underwriting and reserve risk associated with other insurance lines.

1.1 Organization of the Paper

The paper comprises three main sections including this introduction, plus four appendices containing more technical and detailed material. The heart of the paper is Section 2, where we first define the one-year policyholder deficit and its expected value with respect to 1) common stock asset risk and 2) underwriting and reserve risk, and then illustrate these definitions by applying them to the actual performance of the S&P 500 and the U.S. industry Commercial Auto Liability 1999 accident year between January 1999 and December 2004. In addition, we extend the industry analysis to model underwriting and reserve risks at the insurer level. Section 3 comprises a brief summary and our conclusions. Appendix A describes the source and use of the loss development data used in the paper. It also includes exhibits that summarize the calculation of statistical ultimate loss ratio estimates for accident year 1999 at successive annual valuations using unadjusted historical loss development patterns. Appendix B shows the derivation and illustration of a formula for Butsic's "transfer value of unpaid losses," a key element in the calculation of the one-year actual and expected policyholder deficits with respect to underwriting activity. Appendix C describes the stochastic modeling used to estimate the loss distributions underlying the calculation of the policyholder deficit with respect to underwriting and reserve risks. It discusses the sources of variation in future loss ratio estimates, describing in detail how this is manifested in the ultimate loss ratio estimates produced by the loss development methods used in the paper. It also discusses our modeling of the estimated ultimate loss ratio and the policyholder deficit distributions, explaining our application of Monte Carlo simulation and how we reflected parameter uncertainty in the modeling. Appendix D discusses the policyholder deficit and intermediate calculations pertaining to the U.S. industry Commercial Auto Liability 1999 accident year experience at annual valuations between December 1999 and December 2004.

2. THE POLICYHOLDER DEFICIT AND ITS EXPECTED VALUE

The distinctive element of Butsic's risk-based capital framework is its use of the EPD, calculated over a short time horizon, to calibrate risk-based capital consistently for both asset risks and underwriting and reserve risks. In order to determine the EPD for a specified time horizon, we first need to define the policyholder deficit for that time horizon and then to estimate the mean of its distribution.

For common stocks, modeling the distribution is relatively easy, because the idea that stock prices follow a time-dependent stochastic process, and one that can be modeled, is well-established. On the other hand, modeling the one-year EPD for underwriting and reserve risk is more difficult, because it requires a time-dependent perspective of ultimate loss ratio estimates, a perspective that is not required for most actuarial applications. For that reason we begin our discussion with common stocks.

2.1 Actual and Expected Policyholder Deficits - Stocks

If the required capital to asset ratio for common stocks is c, then an initial stock investment of A_0 made from assets matching expected unpaid losses L_0 requires a concurrent risk-based capital allocation of $C_0^R = c \cdot A_0$.

If the allocated capital C_0^R earns interest at the risk-free rate r and the value of the stock investment after one year is A_1, then the value of the capital at the end of the year is equal to the change in value of the stock investment plus the initial capital value with interest[9]:

$$C_1 = A_1 - A_0 + C_0^R(1+r) \tag{2.1}$$

If the value of the available assets at the end of the year, $A_1 + C_0^R(1+r)$, falls below the expected unpaid losses $L_0 = A_0$, then there is, by definition, a funding deficit with respect to the unpaid policyholder claims. Setting $S_1 = A_0 - C_0^R(1+r)$, we can express that policyholder deficit as:

$$PD_1 = S_1 - A_1 \tag{2.2}$$

To determine the *expected* policyholder deficit from the vantage point of investment inception, we need to model the prospective year-end policyholder deficit. We cannot use

[9] This formulation assumes no change in the estimate of the total claims value. It also assumes that any claims paid during the period are settled at the end of the year, allowing the stock investment to be held for the full year, and that any gain (or loss) $A_1 - A_0$ on the stock investment is transferred to (or from) the "capital account" at the end of the year, leaving assets in the "investment account" equal to the initial A_0 required to match the initial expected unpaid losses L_0.

A_1 and PD_1 for this purpose, because until a year after investment inception, they are unknown and uncertain. However, A_1 and PD_1 are prefigured by random variables a_1 and pd_1, defined as of investment inception, which represent the respective values, one year out, of the stock investment and the policyholder deficit. We can express the one-year expected policyholder deficit $E_0(pd_1)$ as of investment inception as the following function of a_1:

$$E_0(pd_1) = \int_0^{S_1} (S_1 - a_1) f(a_1) da_1, \tag{2.3}$$

which is recognizable as the expected expiry value of a one-year European put option on the stock investment with a strike price of S_1.

$E_0(pd_1)$ is the expected value, as of investment inception (hence the E_0), of the policyholder deficit after one year of investment results. Because it is a measure of the capital exhaustion risk associated solely with prospective investment performance, the value of $E_0(pd_1)$ given by Formula (2.3) can be described as the one-year EPD with respect to common stock asset risk.

The classical stock price model assumes that price changes can be explained by geometric Brownian motion, which implies that future stock prices after any finite time interval are lognormally distributed. Accordingly, we will assume that a_1 is lognormal. This allows us to restate Formula (2.3) as:

$$E_0(pd_1) = E(a_1) \cdot (N(d_1) - 1) - S_1 \cdot (N(d_2) - 1), \tag{2.4}$$

where $d_1 = \dfrac{\ln(E(a_1)/S_1) + 0.5\sigma^2}{\sigma}$ and $d_2 = d_1 - \sigma$. $N(d_1)$ and $N(d_2)$ are values of the standard normal cumulative distribution function evaluated at d_1 and d_2, respectively[10].

If we assume that the initial investment A_0 is funded by assets corresponding to the unpaid claims liability L_0 (i.e., $A_0 = L_0$), then $E_0(pd_1) > 0$ implies that policyholders can expect to recover less than 100% of the value of their unpaid claims. Butsic advocated that the risk-based capital factor *c* be chosen to target a selected EPD ratio that identifies this shortfall, namely, $\dfrac{E_0(pd_1)}{L_0}$.

[10] For some purposes it may be desirable to know the *present value* of the one-year EPD, which is given by the Black-Scholes formula for the value of a one-year European put option:

$PV(E_0(pd_1)) = A_0 \cdot (N(d_1) - 1) - S_1 e^{-r} \cdot (N(d_2) - 1)$, where $d_1 = \dfrac{\ln(A_0/S_1) + r + 0.5\sigma^2}{\sigma}$ and $d_2 = d_1 - \sigma$.

Feldblum [8] reported that Butsic, as part of his work for the American Academy of Actuaries Property/Casualty Risk-Based Capital Task Force, had "calibrated the common stock charge using a 1% 'expected policyholder deficit'" and on that basis argued that a 15% risk-based capital charge was more appropriate than 30% (which was also under consideration)[11]. While we do not know what parameter assumptions Butsic used for his calibration, his conclusion seems about right for the time period in which he did his work. The long term standard deviation of U.S. stock market returns is about 20%[12], and the value of the CBOE VIX index, which measures the prospective annualized volatility (σ) of the S&P 500 index implied by the market prices of short term options on that index, hovered around 20% during the early 1990s[13]. If we assume $\sigma = 20\%$, together with a prospective expected annual stock return of 10%, risk-free rate $r = 5\%$ (both simple rates of return), $A_0 = 1$ and $c = 15\%$, then $S_1 = 1 - 0.15 \times 1.05 = 0.8425$ and Formula (2.4) produces an EPD ratio one year out of 0.81%:

$$\frac{E_0(pd_1)}{L_0} = 1.1 \cdot (0.9241 - 1) - 0.8425 \cdot (0.8916 - 1) = 0.81\%$$

Suppose the expected unpaid claim liability after one year is L_1 and, after transferring assets of $A_1 - A_0$ back to the investment account[14], the insurer makes a matching stock investment of A_1[15]. Because the capital $C_0^R = c \cdot A_0$ was intended to minimize the risk of capital exhaustion arising from the stock investment A_0 over the one-year time horizon just ended, the allocated risk-based capital needs to be adjusted to maintain the target EPD ratio with respect to the updated investment value A_1 for the year ahead. In particular, risk-based capital of $C_1^R = c \cdot A_1$ is required to hold the stock investment A_1. After the transfer of $A_1 - A_0$ back to the investment account, the capital account balance is $C_0^R(1+r)$, which means that a calibrating capital adjustment of $C_1^R - C_0^R(1+r)$ is necessary. If $C_1^R - C_0^R(1+r) > 0$, then the capital provider must contribute additional capital. $C_1^R - C_0^R(1+r) < 0$ implies that capital can be released to the capital provider.

If we reset $A_0 = A_1$ and $C_0^R = C_1^R$ at the beginning of each year, we can use Formulas (2.1) through (2.3) to determine C_1, PD_1 and $E_0(pd_1)$ for any one-year period.

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11 Best has also reported that its capital factor of 15% for common stocks "is consistent with A.M. Best's goal of calibrating the baseline capital factors to a 1% expected policyholder deficit." See [1], page 6.

12 Dimson, Marsh and Staunton [7] put it at 20.2% for the period 1900 through 2000 (page 55).

13 This can be seen in the chart for symbol ^VIX at the Yahoo! Finance website displayed for the maximum time range: (http://finance.yahoo.com/q/bc?s=%5EVIX&t=my).

14 See footnote 9.

15 It would be sheer coincidence, of course, for L_1 to match A_1. However, we want to illustrate the capital consequences of a buy-and-hold stock investment policy.

2.1.1 Case Study – S&P 500: 1999-2004

In this section we illustrate the calculation of one-year expected and actual policyholder deficits for each year from 1999 through 2004 with respect to $100 of assets invested on January 1, 1999 in the stocks comprising the S&P 500 index.

We begin by summarizing the performance of the S&P 500 during the period 1999 through 2004 against the backdrop of the estimated probability distributions from which it arose. We used the classical stock price model to estimate successive distributions of the value one year out of an investment in the stocks comprising the S&P 500 for each year from 1999 through 2004. For each of these distributions, we assumed a prospective expected annual return of 10% (9.53% continuously compounded) and annualized volatility (σ) equal to the closing value of the CBOE Volatility Index (VIX) on the last trading day before the beginning of each year[16].

Figure A is a plot of the actual performance during this period of a $100 investment made on January 1, 1999 against the backdrop of 95% confidence intervals from these six successive distributions of stock investment values one year out[17]. The connected square dots reflect the actual S&P 500 total return performance including dividends from the beginning of 1999 through the end of 2004. The triangles highlight the successive one-year confidence intervals. The vertical side of each triangle marks the confidence interval range. Because each confidence interval is a function of the state of knowledge as of the prior year valuation, in order to stress that temporal connection we connected the endpoints of each confidence interval to the investment value one year earlier.

For example, during 1999 an investment in the S&P 500 returned 21.0% and the $100 initial investment grew to $121.00 at the end of December. Of course, on January 1, 1999, when the $100 investment was made, that result was far from certain. At that time the 95% confidence interval for the value of the investment on December 31, 1999 indicated a range of $66.16 to $172.31.

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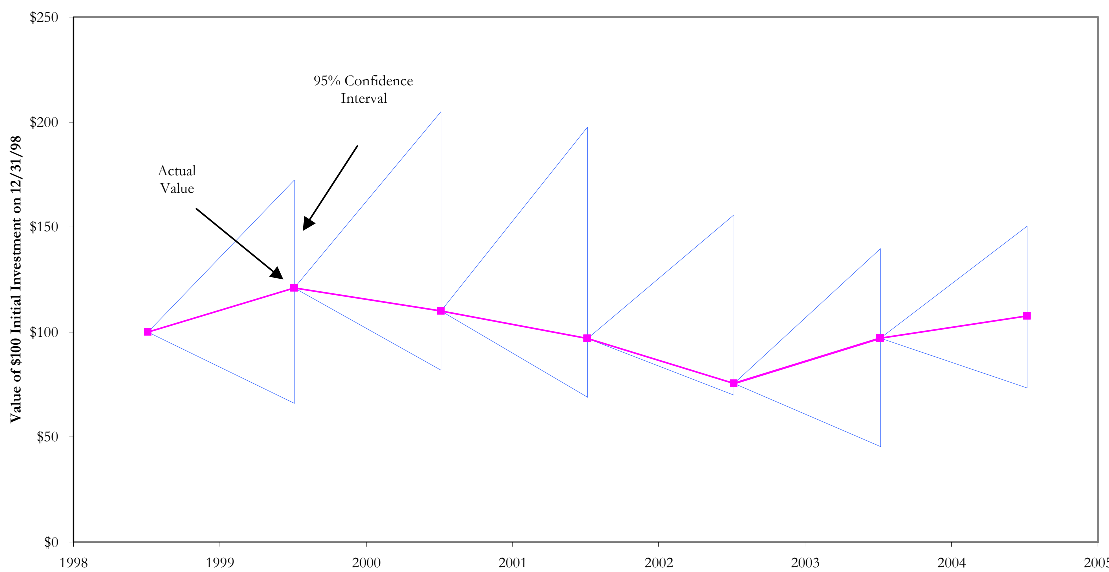
[16] These prospective σ estimates were: 24.42% (1999), 23.4% (2000), 26.85% (2001), 20.45% (2002), 28.62% (2003) and 18.31% (2004).

[17] See Table 1 for the actual investment values plotted here. According to the general formula for a 95% lognormal confidence interval, $E(x)\cdot\exp(\pm1.96\sigma-0.5\sigma^2)$, the endpoints of the confidence intervals are as follows: $66.16-$172.31 (1999), $81.87-$204.87 (2000), $68.95-$197.53 (2001), $69.91-$155.85 (2002), $45.48-$139.66 (2003) and $73.40-$150.46 (2004).

FIGURE A
Actual S&P 500 Performance: 1999-2004
Within 95% Confidence Intervals for One Year Horizon
Total Return Including Dividends



In 2000 the actual S&P 500 return was a loss of 9.1%, which reduced the value of the investment from $121.00 in January to $109.99 at the end of December. At the beginning of 2000 the 95% confidence interval for that ending value was a range of $81.87 to $204.87. The actual value of the investment at the end of each year during the 1999 through 2004 period fell within each year's 95% confidence interval. However, in 2002, when the total return on the S&P 500 was a loss of 22.1%, the ending investment value fell close to the bottom of the confidence interval.

Table 1 shows the expected and actual one-year policyholder deficit ratios for the period 1999 through 2004. The actual policyholder deficits were calculated using Formula (2.2), assuming capital ratio $c = 15\%$ [18] and risk-free rate $r = 5\%$, which imply a capital exhaustion threshold S_1 equal to 84.25% of the beginning market value each year. The EPDs were calculated from Formula (2.4) using the same assumptions together with an assumed prospective expected annual stock return of 10% and the previously described VIX-based σ estimates.

[18] We assumed the same 15% capital factor for common stocks used by the NAIC, A.M. Best and S&P in their capital models as of December 2006.

During most of this period the one-year EPD ratio, given $c = 15\%$, was significantly greater than 1%, averaging about 1.5% over the period. It reached 2.59% in 2003 when σ peaked at 28.6%. In 2002 the S&P 500 declined enough that an investment in it would have resulted in an actual policyholder deficit equal to 6.35% of the January 2002 investment value[19]. Given the high average EPD ratio during the period, it is not surprising that an actual policyholder deficit would have emerged in at least one year. A one-year EPD ratio of 1.5% corresponds roughly to an annualized 15% chance of a 10% deficit. Over six years the probability of a deficit in one or more years is over sixty percent[20]!

TABLE 1

Expected and Actual Policyholder Deficits 1999-2004
S&P 500 Investment (1)

		Beginning	Ending	Capital	Policyholder Deficit	
		Market	Market	Exhaustion		
Calendar		Value	Value	Threshold	Expected	Actual
Year	σ	A_0	A_1	S_1	$E_0(pd_1)$	PD_1
1999	24.4%	$100.00	$121.00	$84.25	1.63%	0.00%
2000	23.4%	121.00	109.99	101.94	1.42%	0.00%
2001	26.9%	109.99	96.90	92.67	2.17%	0.00%
2002	20.5%	96.90	75.49	81.64	0.89%	6.35%
2003	28.6%	75.49	97.15	63.60	2.59%	0.00%
2004	18.3%	97.15	107.74	81.85	0.57%	0.00%

(1) 15% capital, 5% risk-free rate, 10% expected stock return

Figure B shows how the risk-based capital allocated using a 15% factor at the beginning of each year was affected by the investment performance during the year. It also shows how, at the end of each year, the risk-based capital was rebalanced to match the prospective 15% requirement. In 2002 the capital was totally depleted and a policyholder deficit

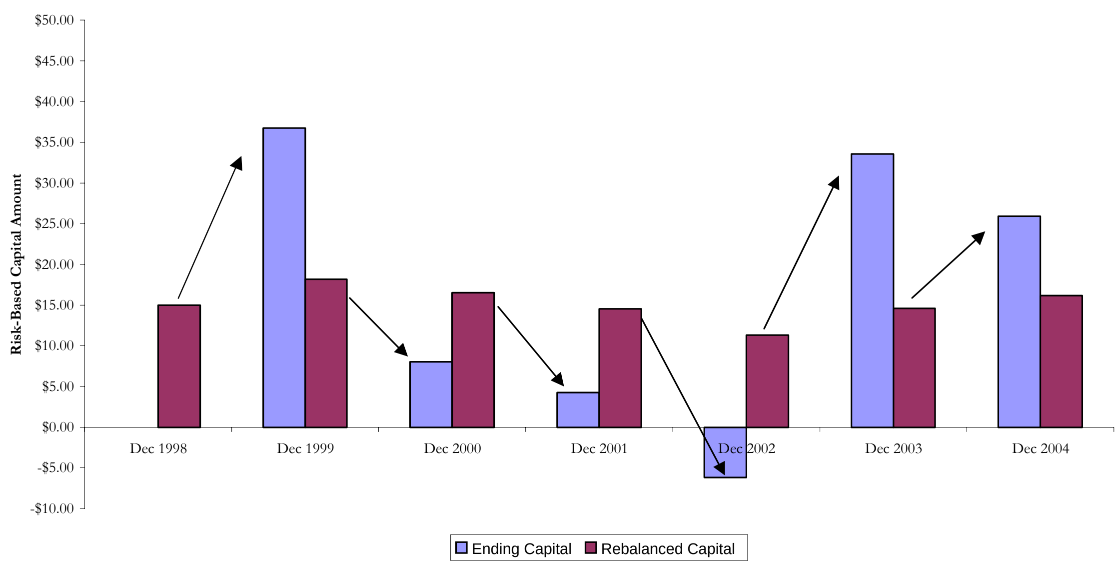
[19] $(\$81.64 - \$75.49) / \$96.90 = 6.35\%$.

[20] The binomial probability of no deficit in six years, given a 15% annual chance of deficit, is about 38%.

emerged. In order to continue trading forward into 2003, the capital provider would have needed both to fund that deficit and recapitalize to the 15% level.

This recap of historical policyholder deficit experience assumed a constant capital ratio of 15%. Given the widely varying EPD ratios indicated by the VIX estimates of σ, if the objective is to maintain a constant one-year EPD ratio, then it is necessary to adjust the capital ratio c to reflect expected S&P 500 volatility in the year ahead. If the c had been recalibrated at the beginning of each year to correspond to a prospective one-year EPD ratio of 1%, then the capital ratios would have been as follows: 19% (1999), 18% (2000), 22% (2001), 14% (2002), 24% (2003) and 11% (2004).

FIGURE B
Risk-Based Capital for S&P 500 Investment: 1999-2004
Investment of $100 on January 1, 1999
Required Capital = 15% of Investment Market Value



2.2 Actual and Expected Policyholder Deficits – Underwriting[21]

For our analysis of underwriting and reserve risks, we will focus initially on a single line of business for a single accident year and later discuss the implications of the more realistic scenario that involves reserves from multiple accident years. We will begin with

[21] To avoid a proliferation of variable names, we will reuse the capital and policyholder deficit related notation from Section 2.1. In particular, we will redefine C_0^R, C_1, C_1^R, $E_0(pd_1)$, PD_1, pd_1 and S_1 to reflect the underwriting related context of this section.

underwriting risk. The total available assets one year after accident year inception comprise the allocated risk-based capital and the underwriting assets derived from premiums[22], plus interest earned on these assets during the year[23]. If we assume that the underwriting assets $T(L_0)$ and allocated capital C_0^R earn interest at the risk-free rate *r*, then the value of the assets S_1 available at the end of the year to fund the claim obligation that was assumed at accident year inception is:

$$S_1 = (C_0^R + T(L_0)) \cdot (1 + \tfrac{3}{4} r)$$

In nominal terms, the end of year value of the estimated claim obligation L_0 that was assumed at inception is the sum of the estimated unpaid losses L_1 at year-end and the claims paid during the year P_1. The *transfer value* of that liability $T(L_1 + P_1)$ is the price at which $L_1 + P_1$ can be removed from the insurer's balance sheet one year after accident year inception.

In order to quantify $T(L_1 + P_1)$, let us look at the transfer values of the paid and unpaid loss components separately. $T(L_1)$ is the price a third party would charge to assume the liability for unpaid losses L_1 at one year of development, which Butsic defined as the present value of L_1 plus a risk charge to reflect the uncertainty in the unpaid losses[24]. $T(P_1)$ is the price claimants would demand to defer payment of their claims until year-end. Assuming that the claims comprising P_1 are paid, on average, halfway through the year, then their year-end transfer value is $T(P_1) = P_1 \cdot (1 + \tfrac{1}{2} r)$. The total year-end transfer value of $L_1 + P_1$ is $T(L_1 + P_1) = T(L_1) + T(P_1)$.

The capital position one year after accident year inception is equal to the value of the underwriting and capital assets less the transfer value of the loss liability[25]:

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[22] These are premiums net of expenses only. Claims paid during the year are treated as part of the loss liability.

[23] We assume that half of the accident year earned premiums is written and collected before the beginning of the accident year and thus earns interest for the full year. We assume the other half of the earned premiums is collected, on average, halfway through the year and earns (simple) interest for six months. Capital is assumed to be allocated as premiums are collected and to earn interest accordingly.

[24] Conceptually, this is identical to the theoretical market price of a stock, which also reflects the present value of the future realizable cash flows of the company and an appropriate risk premium. In an efficient market, the theoretical and actual market prices should be the same. In his discussion of the value of insurance claims, Butsic used the terms "market value" and "transfer value" interchangeably. Because there is not an active market after policy inception for the buying and selling of loss reserves, we prefer the term "transfer value," which has a more theoretical connotation. We derive a formula for $T(L_n)$ for $n \geq 1$ in Appendix B, which reflects the recapture of the cost of the allocated risk-based capital. That approach has also been discussed in connection with the EU's Solvency II initiative. See the UK FSA's Solvency II discussion paper [10], page 25.

[25] This formulation assumes that any shortfall (or surplus) in the assets available to fund losses is transferred from (or to) the "capital account," leaving the correct amount in the "underwriting account" to fund losses exactly.

$$C_1 = S_1 - T(L_1 + P_1), \tag{2.5}$$

If $T(L_1 + P_1) > S_1$, then the ending capital $C_1 < 0$, which implies a policyholder deficit of:

$$PD_1 = T(L_1 + P_1) - S_1 \tag{2.6}$$

While at age twelve months $T(L_1 + P_1)$ and PD_1 take on specific values, at accident year inception their values are unknown and uncertain. Let t_1 and pd_1 be random variables, defined at accident year inception, that correspond to $T(L_1 + P_1)$ and PD_1, respectively. t_1 represents the transfer value, one year out, of the unpaid losses embedded in the premiums at accident year inception and pd_1 represents the policyholder deficit, one year out, viewed from the vantage point of accident year inception.

We can express the one-year expected policyholder deficit as of accident year inception as the following function of t_1:

$$E_0(pd_1) = \int_{S_1}^{\infty} (t_1 - S_1) f(t_1) dt_1, \tag{2.7}$$

which is recognizable as the expected expiry value of a one-year European call option, with a strike price of S_1, on the transfer value, one year out, of the estimated unpaid losses at inception[26].

$E_0(pd_1)$ is the expected value as of inception of the policyholder deficit after one year of development. It is calculated at accident year inception before any actual claims have been reported. Because it is a measure of the capital exhaustion risk associated solely with prospective underwriting activity, the value of $E_0(pd_1)$ given by Formula (2.7) can be described as the one-year EPD with respect to underwriting risk.

Butsic advocated calibration of risk-based capital ratios to produce consistent EPD ratios for all asset, underwriting and reserve risks. Suppose the required underwriting risk-based capital C_0^R at accident year inception is defined as a certain percentage c_0 of the premiums net of expenses $T(L_0)$, which implies $C_0^R = c_0 \cdot T(L_0)$. If the target one-year EPD ratio for common stocks is set at 1% of expected unpaid losses, then the underwriting risk-based capital factor c_0 should likewise be chosen to produce a value of $E_0(pd_1)$ equal to 1% of L_0. However, because L_0 is not observable, a practical alternative is to calibrate the EPD to 1% of the loss provision implied by premiums net of expenses $T(L_0)$.

[26] Note that while the classical theory of stock prices implies that a_1 (the stock price random variable) is lognormal, t_1 is lognormal only under very narrow circumstances, which means we cannot simply use the stock price model to value the underwriting risk EPD. We also cannot use the Black-Scholes call formula to determine the present value of the one-year EPD.

At twelve months after accident year inception, our interest turns from the adequacy of the loss provision in the premiums to the adequacy of the loss reserves. The loss reserve L_1 must be funded by assets equal to $T(L_1)$. In addition, capital of $C_1^R = c_1 \cdot L_1$ is required to support the loss reserves for the next twelve months, bringing the total required accident year underwriting and capital assets to $C_1^R + T(L_1)$. Because the ending capital C_1 will rarely match the prospective required capital C_1^R, a calibrating capital adjustment of $C_1^R - C_1$ is necessary. $C_1^R - C_1 > 0$ implies that the capital provider must contribute additional capital in that amount. $C_1^R - C_1 < 0$ implies that capital can be released to the capital provider in that amount.

From one year of accident year development and beyond, the successive one-year EPDs measure the capital exhaustion risk associated only with the prospective uncertainty in the loss reserve estimates. We *could* also refer to the risk arising from loss reserve uncertainty as underwriting risk, since it arises only as a result of past underwriting activity. However, because the risk-based capital convention is to refer to the risk arising from loss reserves as reserve risk, we follow that convention of separating the total risk in the accident year into its underwriting and reserve components.

2.3 Actual and Expected Policyholder Deficits – Loss Reserves

Following the capital rebalancing at the end of the first year of development, the combined risk-based capital and underwriting assets total $C_1^R + T(L_1)$. By the end of the second year the value of available assets S_2 (including interest earned for the full year) is:

$$S_2 = (C_1^R + T(L_1)) \cdot (1 + r)$$

During the second year of development, the loss reserve L_1 is reduced by paid claims P_2. At the end of the year $L_1 - P_2$ is replaced by a revised loss reserve L_2, which is based on the loss development observed during the year. $L_2 + P_2$ is the one-year hindsight estimate of L_1, with a transfer value of $T(L_2 + P_2) = T(L_2) + P_2 \cdot (1 + \frac{1}{2} r)$. The economic value of the allocated capital at the end of the second year of development is given by:

$$C_2 = S_2 - T(L_2 + P_2) \tag{2.8}$$

$T(L_2 + P_2) > S_2$ implies a policyholder deficit of:

$$PD_2 = T(L_2 + P_2) - S_2 \tag{2.9}$$

Let t_2 and pd_2 represent the random variables, defined at age one year, corresponding to $T(L_2 + P_2)$ and PD_2, respectively. We can then express the one-year EPD at age one year as the following function of t_2:

$$E_1(pd_2) = \int_{S_2}^{\infty} (t_2 - S_2) f(t_2) dt_2, \tag{2.10}$$

which is the same as Formula (2.7) except that the subscripts are different. The one-year EPD with respect to loss reserve risk at twelve months is expressible as the expected expiry value of a one-year European call option on the transfer value, one year out, of the unpaid losses as of twelve months.

The risk-based capital required to support the unpaid loss liability L_2 (which itself is funded by assets of $T(L_2)$) in the period from two to three years of development is $C_2^R = c_2 \cdot L_2$. Because the ending capital at two years is C_2 a calibrating capital adjustment of $C_2^R - C_2$ is required, resulting in an additional capital contribution or a capital release.

The same process is repeated at each successive valuation at one-year intervals until all accident year claims have been paid. The key loss-related formulas applicable to the valuation *n* years after accident year inception (for $n \geq 1$) are as follows:

$$C_n^R = c_n \cdot L_n \tag{2.11}$$

$$S_{n+1} = (C_n^R + T(L_n)) \cdot (1 + r) \tag{2.12}$$

$$C_{n+1} = S_{n+1} - T(L_{n+1} + P_{n+1}) \tag{2.13}$$

$$PD_{n+1} = T(L_{n+1} + P_{n+1}) - S_{n+1}, \text{ if } T(L_{n+1} + P_{n+1}) > S_{n+1} \tag{2.14}$$

$$E_n(pd_{n+1}) = \int_{S_{n+1}}^{\infty} (t_{n+1} - S_{n+1}) f(t_{n+1}) dt_{n+1} \tag{2.15}$$

In practice, an insurer's unpaid losses almost never pertain to a single accident year. This is important, because the objective of minimizing exposure to capital exhaustion due to reserve risk does not require minimizing that exposure with respect to each accident year individually, but rather for all accident years collectively. As we will see in our case study discussed in Section 2.4.1, this makes a big difference in the amount of required capital. The key formulas for working with unpaid losses arising from multiple accident years are given in Section C.7 of Appendix C.

2.4.1 Case Study – Commercial Auto Liability Accident Year 1999: 1999-2004

In this section we illustrate the calculation of one-year expected and actual policyholder deficits at the industry level for each year from 1999 through 2004 with respect to accident year 1999 claims arising from $100 of Commercial Auto Liability earned premiums. We also discuss the impact of loss reserves from multiple accident years as well as our modeling of the policyholder deficit calculations at the insurer level.

We begin by summarizing the performance of statistical ultimate loss ratio estimates during the period 1999 through 2004 against the backdrop of the estimated probability distributions from which they arose.

The statistical ultimate loss ratio estimates – so called because they reflected no actuarial or other clinical judgment – are simple averages of the unadjusted estimates produced by four traditional loss development methods: 1) paid chain ladder, 2) paid Bornhuetter-Ferguson, 3) case incurred chain ladder, and 4) case incurred Bornhuetter-Ferguson. We chose the four-method average because Wacek [13], in a comparison of the accuracy of various loss projection methods for accident years 1995 through 2001, found the four-method average to be the most accurate estimator of the ultimate loss ratio over that period for Commercial Auto Liability at twelve, twenty-four and thirty-six months of development.

TABLE 2

Accident Year 1999 Ultimate Loss Ratio Estimates
Commercial Auto Liability
As of December 31 each Year

Calendar Year	CL Paid	CL Case Incurred	B-F Paid	B-F Case Incurred	Mean of Methods
1999	90.3%	84.7%	83.7%	82.5%	85.3%
2000	91.7%	89.1%	90.8%	88.4%	90.0%
2001	92.7%	92.1%	92.4%	91.9%	92.3%
2002	93.5%	92.9%	93.4%	92.9%	93.2%
2003	93.1%	92.2%	93.1%	92.2%	92.6%
2004	91.6%	91.8%	91.7%	91.8%	91.7%

Table 2 shows the estimates of the ultimate loss ratio from the four methods and their mean at the end of each year from 1999 through 2004[27]. The mean of methods estimate as of the end of 1999 was 85.3%. Unanticipated loss development, i.e., development beyond that implied by the historical development patterns, during the next three years led to

[27] See Appendix Exhibit A-4 for the details of the calculation of each of these ultimate loss ratio estimates. For a full description of the four methods, see Appendix A, Section A.2.2, of [13].

increases in the ultimate loss ratio estimate in each of 2000, 2001 and 2002 to 90.0%, 92.3% and 93.2%, respectively[28]. Then the pattern of unfavorable development deviation reversed itself, and in 2003 and 2004 unexpectedly favorable development led to slight reductions in the ultimate loss ratio estimate to 92.6% and 91.7% at the end of 2003 and 2004, respectively.

Each of these actual ultimate loss ratio estimates can be viewed as one outcome from the distribution of potential loss ratio estimates arising from a stochastic loss development process. For example, from the vantage point of accident year inception, the ultimate loss ratio of 85.3% estimated as of the end of 1999 was one of many potential estimates. If the observed loss development during 1999 had been different, then the ultimate loss ratio estimate would also have been different. We estimated the distribution of these different ultimate loss ratio estimates using a Monte Carlo simulation process that stochastically modeled the loss development observed during 1999, combined it with what was already known at accident year inception, and then applied the four loss development methods described above to the simulated experience. We used the set of ultimate loss ratio estimates produced from 10,000 Monte Carlo trials as a discrete approximation of the distribution of the ultimate loss ratio estimate one year out (at the end of 1999) from the vantage point of accident year inception. We followed the same procedure to model the distribution of the ultimate loss ratio one year out at each successive annual valuation from accident year inception (which we have just described) through December 2003[29].

Figure C is a plot of the path of the 1999 accident year ultimate loss ratio estimate (the four-method average) for Commercial Auto Liability against the backdrop of 95% confidence intervals from the distributions of the estimated ultimate loss ratio one year out[30]. The connected square dots reflect the actual statistical ultimate loss ratio estimates for successive annual valuations ranging from the beginning of 1999 through the end of 2004. As in Figure A, the triangles highlight the successive one year confidence intervals, where the vertical side of each triangle marks the confidence interval range.

[28] Because the estimated ultimate loss ratios were purely statistical estimates calculated from the unadjusted indications of the four loss development methods, the main source of subsequent upward or downward revisions in the estimates was the deviation of observed loss development from that predicted by historical patterns. An additional source of minor deviations was the behavior of the five-year moving averages of historical development used to estimate prospective development. See footnote 32.

[29] See Appendix C for a detailed description of how these distributions were estimated.

[30] The endpoints of the confidence intervals are as follows: 76.1%-85.1% (1999), 83.6%-91.6% (2000), 87.9%-93.0% (2001), 90.4%-94.3% (2002), 91.5%-95.0% (2003) and 92.0%-93.4% (2004).

FIGURE C
Path of Accident Year 1999 Ultimate Loss Ratio Estimate: 1999-2004
Within 95% Confidence Intervals for One Year Horizon
Commercial Auto Liability

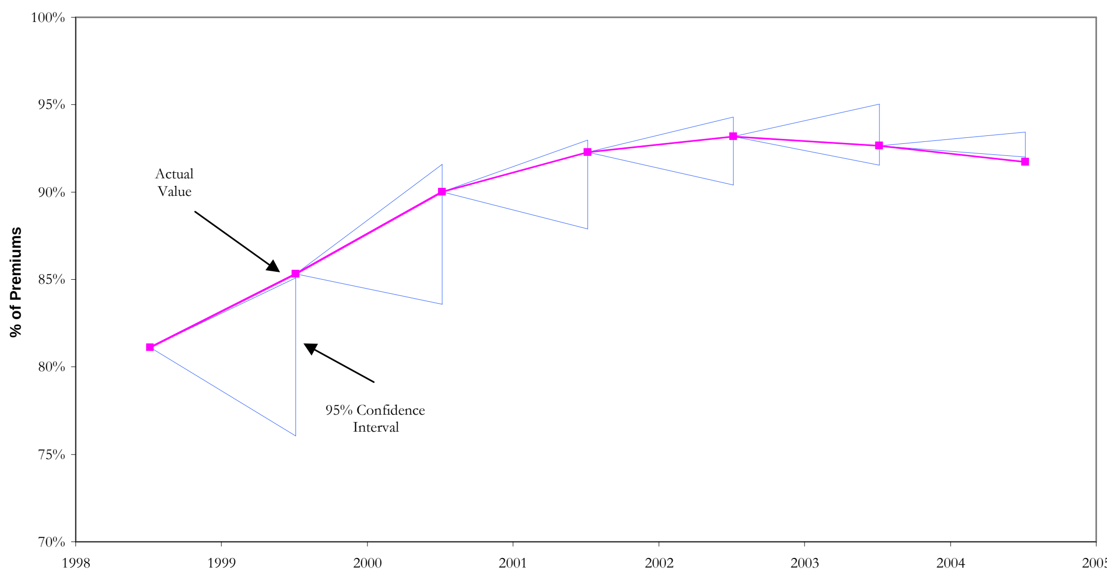


Figure C shows that at the beginning of the accident year on January 1, 1999, the initial ultimate loss ratio estimate was 81.1%[31]. At that time the 95% confidence interval for the ultimate loss ratio estimate one year out (i.e., at the December 31, 1999 valuation) was bounded by 76.1% on the low end and 85.1% on the upper end[32]. Based on the actual loss emergence during calendar year 1999 the four-method average ultimate loss ratio estimate as of December 31, 1999 was 85.3%. Unusual paid and case incurred loss development during calendar year 1999 led to an upward revision in the ultimate loss ratio estimate to just above the upper end of the 95% confidence interval.

At the December 31, 1999 valuation, the 95% confidence interval for the ultimate loss ratio estimate one year out (i.e., as of December 31, 2000) was a range of 83.6% to 91.6%. One year later, at the December 31, 2000 valuation, the ultimate loss ratio estimate was

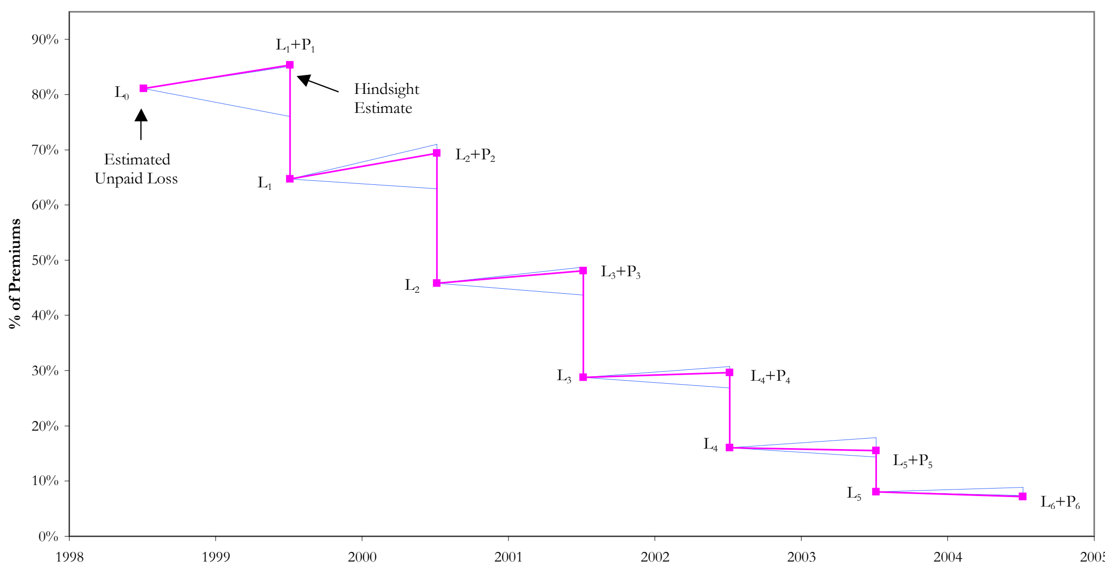
[31] This the mean of the paid and case incurred Bornhuetter-Ferguson initial expected loss ratios, 83.4% and 78.7%, respectively, which were based purely on 1998 and prior accident year experience. See [13] for more information about how that was done. This purely statistical estimate ignored other objective information, which, if available, might have improved this estimate.

[32] The mean of the ultimate loss ratio estimate one year out was 80.3%, which is different from the 81.1% estimate as of January 1, 1999 because it reflects a slight difference in the five data points comprising the development factor means. The estimate one year out drops the calendar year 1994 development observation from each development factor calculation and replaces it with the mean of the 1994-1998 observations.

revised from 85.3% to 90.0%, based on loss development observed during calendar year 2000.

The ultimate loss ratio estimates continued to increase at the December 31, 2001 and 2002 valuations, before declining slightly at the December 31, 2003 and 2004 valuations[33]. Note that the width of the confidence intervals became smaller in successive years, which reflects the declining proportion of unpaid claims (the only source of uncertainty) within the loss ratio.

FIGURE D
Accident Year 1999 Actual & Hindsight Unpaid Loss Ratio Estimates: 1999-2004
Within 95% Confidence Intervals for One Year Horizon
Commercial Auto Liability



Because the threat to solvency arises from the potential for adverse deviation inherent in the unpaid portion of the ultimate loss ratio estimate, let's look at the behavior of the accident year 1999 unpaid loss ratio over the same period. Figure D is a plot showing the succession of unpaid loss estimates (L_n) and their hindsight re-estimates one year later $(L_{n+1}+P_{n+1})$ against the backdrop of 95% confidence intervals for the latter. The connected square dots represent the actual and one-year hindsight estimates of the unpaid loss ratio at

[33] The development of both paid and case incurred losses during 2004 was extremely light compared to the historical pattern. Referring to the columns labeled "Age 5-6" in Appendix Exhibits A-1A and A-1B, we see that the paid age-to-age factor of 1.032 was the lowest by far of eleven factors and the case incurred age-to-age factor of 1.007 was tied for lowest of eleven.

successive annual valuation between accident year inception and 2004. The triangles show the 95% confidence intervals for the hindsight estimates one year out, and as such provide a visual representation of the potential exposure of the unpaid loss estimate to upward or downward revision in the next year.

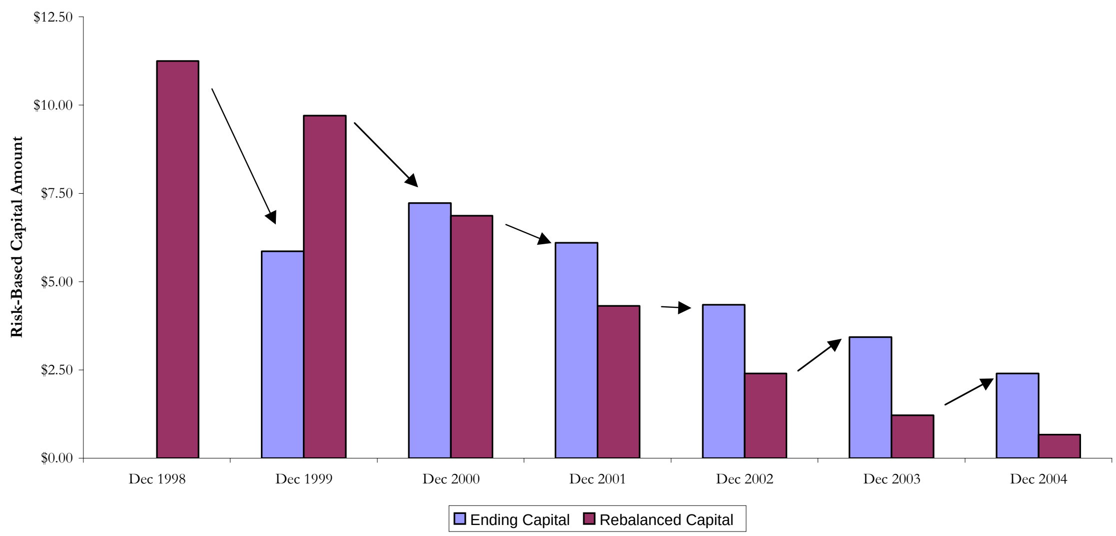
The initial unpaid loss ratio estimate at accident year inception was $L_0 = 81.1\%$. One year later the hindsight estimate of L_0 as of December 1999 was $L_1 + P_1 = 85.3\%$, representing the sum of the paid loss ratio P_1 of 20.6% and the unpaid loss ratio L_1 of 64.7%. The upward sloping line connecting L_0 and $L_1 + P_1$ indicates adverse loss development, which is quantified by the difference $L_1 + P_1 - L_0 = 4.2\%$ of premiums (5% of L_0).

The hindsight estimate of L_1 as of December 2000 was $L_2 + P_2 = 69.4\%$, the sum of the period paid loss ratio $P_2 = 23.6\%$ and the unpaid loss ratio $L_2 = 45.8\%$. The upward sloping line between L_1 and $L_2 + P_2$ indicates further adverse loss development of 4.7% of premiums (7% of L_1).

The next two years, 2001 and 2002 saw a continuation of the pattern of the hindsight estimates of unpaid losses exceeding the beginning of year estimates. The adverse development was 2.3% of premiums (5% of L_2) in 2001 and 0.8% of premiums (3% of L_3) in 2002. In 2003 and 2004 the unexpected loss development turned favorable.

The cumulative adverse loss development from accident year inception through December 2002 totaled 12% of premiums. That seems to support the argument for a large capital requirement for Commercial Auto Liability. However, the fact that this shortfall emerged over four years rather than a single year is extremely important. Because the required amount of risk-based capital is determined annually, any erosion of allocated capital caused by adverse loss development is replenished at the end of the year. If capital exhaustion can be avoided for each of the four years in succession, then clearly capital exhaustion is also avoided for the four years as a block. In that context the adverse loss development seen in Commercial Auto Liability between 1999 and 2002, which averaged about 5% of the unpaid loss estimate at the beginning of each year, was much more manageable than the volatility seen in the S&P 500, which lost 22% of its value in a single year (2002) and 38% over three years (2000-2002).

FIGURE E
Risk-Based Capital for Accident Year 1999 Unpaid Losses: 1999-2004
Commercial Auto Liability
1999 Premiums of $100
Required Capital = 15% of Unpaid Losses



This can be seen in Figure E, which shows the effect on capital of the accident year 1999 loss development observed between accident year inception and December 2004, assuming an expense ratio of 25%, risk free rate of 5% and a required capital ratio of 15%[34]. It shows, for example, that because the year-end 1999 ultimate loss ratio estimate of 85.3% exceeded the funding capacity of the premiums, a portion of the initial capital of $11.25 (15% of $75) had to be diverted to fund losses and capital ended the year at $5.65. That implied a loss to insurers, but capital was far from exhausted. At the end of 1999 capital had to be topped up to $9.70 (15% of unpaid losses of $64.69). During 2000 further adverse development resulted in a reduction in capital to $7.23. However, the required capital going forward (15% of unpaid losses of $45.80) was only $6.87, which meant that $0.36 of capital could be withdrawn. Subsequent increases in the ultimate loss ratio estimate in 2000 and 2001

[34] We chose the same 15% capital factor used for common stocks in order to facilitate the comparison of the relative riskiness of Commercial Auto Liability insurance and an investment in the S&P 500. As of December 2006, the NAIC and S&P both used capital factors for underwriting risk that equate to about 22% of premiums net of 25% expenses, and capital factors for reserve risk that equate to 16% and 10%, respectively, of undiscounted loss reserves. Note that, unlike the NAIC and Best, S&P does not use covariance or diversification adjustments, so its effective factors on a comparable basis are at least 50% higher than those given here. Best has not published its underwriting and reserve factors, but we have observed Best capital factors for Commercial Auto Liability greater than 15%.

resulted in further capital drawdowns. However, in none of the years was capital close to being exhausted[35].

Table 3 shows the expected and actual one-year policyholder deficit ratios for the period 1999 through 2004 in tabular form. The actual policyholder deficits were calculated using Formula (2.6) for the December 1999 valuation and Formula (2.9) for the 2000 through 2004 valuations. For further details of these calculations, see Appendix Exhibit D.

TABLE 3

Expected and Actual Policyholder Deficits 1999-2004

Industry Commercial Auto Liability - Accident Year 1999 (1)

Calendar Year	n (2)	Beginning Unpaid Loss Provision L_n (3)	Hindsight Unpaid Loss $L_{n+1}+P_{n+1}$	Transfer Value Hindsight Unpaid Loss $T(L_{n+1}+P_{n+1})$	Capital Exhaustion Threshold S_{n+1}	Policyholder Deficit Expected	Policyholder Deficit Actual
1999	0	$75.00	$85.32	$83.84	$89.48	0.01%	0.00%
2000	1	$64.69	$69.39	$68.78	$76.01	0.01%	0.00%
2001	2	$45.80	$48.07	$47.89	$54.05	0.00%	0.00%
2002	3	$28.76	$29.65	$29.62	$34.03	0.00%	0.00%
2003	4	$16.01	$15.48	$15.48	$18.95	0.04%	0.00%
2004	5	$8.07	$7.15	$7.10	$9.54	0.02%	0.00%

(1) 15% capital, 5% risk-free rate
(2) Lag from inception (in years) as of beginning of year
(3) $T(L_0)$ for 1999

[35] See Appendix Exhibit D for the details underlying the calculation of capital and policyholder deficits with respect to the underwriting and reserve risks associated with the Commercial Auto Liability accident year 1999 between 1999 and 2004.

The expected policyholder deficit $E_n(pd_{n+1})$ was calculated using Formula (2.7) for the 1999 valuation $(n=0)$ and Formula (2.10) for the 2000 and subsequent valuations $(1 \le n \le 5)$[36]. A capital requirement of $15\% \cdot T(L_n)$ and risk-free rate $r = 5\%$ implied capital exhaustion thresholds $S_1 = 1.15 \cdot T(L_0) \cdot (1 + \frac{3}{4} \cdot 0.05)$ for $n=0$ and $S_{n+1} = 1.15 \cdot L_n \cdot 1.05$ for $1 \le n \le 5$. The policyholder deficit was determined by comparing the transfer value $T(L_{n+1} + P_{n+1})$ of the hindsight estimate $L_{n+1} + P_{n+1}$ to the capital exhaustion threshold S_{n+1}. We have expressed the policyholder deficits in Table 3 as ratios to L_n[37]. Throughout the 1999-2004 period the one-year EPD was barely greater than zero. The largest EPD value was 0.04% (4 basis points) in 2003 and its average was less than 1.5 basis points, just 1% of the EPD calculated for the investment in the S&P 500! Moreover, despite the persistent pattern of upward adjustment in the hindsight reserve estimates, the actual policyholder deficit remained zero throughout the 1999-2004 period.

These expected and actual policyholder deficit calculations assumed that accident year 1999 was the sole source of Commercial Auto Liability loss reserves. If, instead, we assume that there were also loss reserves from a number of other accident years, then the one-year EPD with respect to total reserve risk approaches zero. To illustrate this simply, let us pretend that the loss development statistics tabulated in Table 3 with respect to accident year 1999 over several calendar years instead pertained to loss development observed during calendar year 2000 with respect to accident years 1995 through 1999 as shown in Table 4. To create Table 4 we mapped the accident year 1999 loss reserves at each development age (shown in Table 3) to the accident year that would be the same age in calendar year 2000[38]. In effect, we assumed that the accident loss exposure was constant from 1995 through 1999 and the development patterns observed in that period were similar to those we saw for accident year 1999 as it developed.

At the beginning of 2000, the total unpaid loss provision with respect to these hypothetical accident years 1995-1999 was \$163.33. At the end of 2000 the hindsight loss estimate for this block of reserves increased to \$169.74 and the transfer value of that hindsight estimate was \$168.27. The capital exhaustion threshold, which reflects the beginning of year total underwriting and risk-based capital assets plus interest, was \$191.78.

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36 Strictly speaking we calculated the EPDs from discrete approximations of the underlying distributions achieved through Monte Carlo simulation, rather than by integrating the actual continuous density functions as implied by the references to Formulas (2.7) and (2.10). In particular, we approximated the application of Formulas (2.7) and (2.10) by using Formulas (2.6) and (2.9) for each Monte Carlo trial and then computing the mean policyholder deficit over all trials.

37 $T(L_0)$ for $n=0$.

38 This is a purely illustrative assumption for the purpose of showing the effect that holding multiple accident years' reserves has on the policyholder deficit calculations.

That implied an actual policyholder deficit of zero. We also found the one-year EPD to be negligible. We assumed the unpaid accident year losses were independent, but if they were anything less than totally correlated, the expected and actual policyholder deficits for the five accident years' unpaid losses would always be lower than for the accident years individually.

TABLE 4

Expected and Actual Policyholder Deficits

Illustration of Multiple Accident Years' Reserves Effect in 2000

Industry Commercial Auto Liability – Hypothetical Accident Years 1995-1999

Accident Year	Beginning Unpaid Loss Provision	Hindsight Unpaid Loss	Transfer Value Hindsight Unpaid Loss	Capital Exhaustion Threshold	Policyholder Deficit	
					Expected	Actual
1995	$8.07	$7.15	$7.10	$9.54	0.02%	0.00%
1996	$16.01	$15.48	$15.48	$18.95	0.04%	0.00%
1997	$28.76	$29.65	$29.62	$34.03	0.00%	0.00%
1998	$45.80	$48.07	$47.89	$54.05	0.00%	0.00%
1999	$64.69	$69.39	$68.78	$76.01	0.01%	0.00%
1995-99	$163.33	$169.74	$168.87	$191.78	0.00%+	0.00%

15% capital, 5% risk-free rate

Clearly, compared to the risk in the diversified common stock portfolio exemplified by the S&P 500, at the industry level the exposure to capital exhaustion posed by the accident year 1999 Commercial Auto Liability underwriting and reserve risks, given the same 15% capital ratio used with the S&P 500, was negligible. However, because solvency concerns are focused on the exposure that individual insurers have to capital exhaustion, and not on the exposure of the industry as a whole, we need to address the question of insolvency risk at the insurer level.

While we did not have access to the individual insurer data comprising the industry experience and thus could not model capital exhaustion exposure at the insurer level directly, we were able to model it indirectly by making use of the relationship between insurer "total risk" and "industry risk" reported by the American Academy of Actuaries Property/Casualty Risk-Based Capital Task Force [2]. The Task Force reported "total risk" coefficients of variation for ultimate time horizon underwriting and reserve risks of 23.3% and 17.2%, respectively, and corresponding "industry risk" coefficients of 12.2% and 6.4%, equating to ratios of 2.7 and 1.9[39]. To approximate the risk faced by individual insurers, we multiplied the standard deviations of the age-to-age development factor natural logarithms tabulated in Appendix Exhibits A-2A and A-2B by factor of 3 (rounding up from 2.7 and 1.9). We then repeated the same one-year EPD analysis that we had performed using the industry data[40].

This procedure produced a one-year EPD with respect to underwriting risk of 0.50% and a one-year EPD with respect to the reserve risk arising from the five accident years 1995-1999 of 0.03%, both of which are much lower than the average one-year EPD of 1.5% for the S&P 500 using the same risk-based capital factors.

Calibrated to a 1% target one-year EPD, the indicated Commercial Auto Liability capital factors for individual insurers would have been 5% for underwriting risk and 4% for reserve risk. These indicated factors are much lower than those promulgated by the NAIC and the rating agencies[41].

3. SUMMARY AND CONCLUSIONS

In this paper we have provided a detailed roadmap for the application of Butsic's framework for insurance company solvency protection and illustrated it with a case study using historical data. The results of the case study support Butsic's contention that insurance company solvency can be ensured by the periodic assessment and rebalancing of

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[39] See [2], Exhibit 3, Sheets 1 and 2, pages 155-156.

[40] A factor of 3 increased the coefficient of variation of a lognormal random variable by slightly more than 3. See Appendix C for details of the loss development model used for the industry and company analyses.

[41] See footnote 34 for a recap of those factors. Another simple way to measure the relative variability of an investment in the S&P 500 and Commercial Auto Liability insurance is to compare the coefficients of variation of the random variables a_{n+1} and t_{n+1}. The c.v. of a_{n+1} ranged from 18% to 29%. In contrast the coefficients of variation for t_{n+1} (in particular, for t_1, which corresponds to underwriting risk, and $\sum t_i$, which corresponds to total reserve risk) were much lower at 9% for underwriting and 3% for total reserves. S&P states that its 15% capital factor for common stocks is equal to the standard deviation of S&P 500 annual returns since 1945 [11] (page 35). Ignoring the fact that our research indicated a higher standard deviation for the S&P 500, if S&P had been consistent in its approach, it would have set its capital factors at 9% for underwriting risk and 3% for reserve risk instead of at 22% and 10%.

capital to maintain a constant target EPD ratio over a short time horizon. A striking finding from the case study is that the amount of capital needed to support the problematical Commercial Auto Liability line in the worst accident year of the "soft market" of the late 1990s was significantly less than that required by the NAIC, Best and S&P. That Butsic's solvency framework with a capital ratio of 15% or less would work so well in the face of the severe deterioration in the accident year 1999 ultimate loss ratio estimate to 91.7% at the end of 2004 is a testament to its robustness. A second striking finding is that the riskiness of investment in a diversified common stock portfolio appears to be underappreciated by the NAIC and the rating agencies, both in absolute terms and relative to Commercial Auto Liability insurance. The risk-based capital they required to support such an investment was consistent with a one-year EPD ratio averaging 1.5% over the 1999 through 2004 period (fifty times the Commercial Auto Liability EPD ratio at the same capital level!) and was insufficient to prevent actual exhaustion of that allocated capital during 2002.

It is important that any solvency framework measure the risk of capital exhaustion *consistently* across lines of insurance and both sides of the balance sheet. While our case study is too limited in scope to permit sweeping conclusions, the results with respect to Commercial Auto Liability are startling enough to suggest that the capital requirements of other insurance lines should be studied as well. We have a strong suspicion that the capital requirements of the NAIC, Best and S&P with respect to other lines of insurance are also overstated, both in absolute terms and relative to their requirements for common stock investment[42].

We believe the bias in favor of common stock investment embedded in the current (December 2006) capital factors is unintentional and has resulted from the unconscious use of inconsistent methods of measuring risk. It appears that the risk associated with common stock investment has been measured using a time horizon of about one year, while underwriting and reserve risks have been measured over a much longer time horizon. Butsic argued the importance of using a consistent time horizon in the early 1990s at the time when the NAIC began implementing its risk-based capital framework. However, either because there was no practical way to incorporate his insights or because they were not properly understood, his ideas have languished, and for far too long[43].

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[42] We have in mind the largest U.S. primary lines of business, which lend themselves well to Schedule P analysis. Unfortunately, because of data quality and heterogeneity issues, Schedule P does not shed much light on this question for International, Special Liability and the Nonproportional Reinsurance lines.

[43] Butsic was a member of the American Academy of Actuaries Property/Casualty Risk-Based Capital Task Force, but that did not prevent the Task Force from employing an EPD methodology for underwriting and

Butsic's concept of a short time horizon for risk measurement that is used consistently for all types of risk also has obvious application to enterprise risk management and other updated approaches to solvency risk management. While we have focused on the measurement and calibration of asset and underwriting-related risks separately, clearly the ultimate objective of solvency management is to minimize the likelihood and cost of insolvency from all of the risks, alone or in combination, inherent in an insurance enterprise. The EPD measured over a consistent short time horizon is a good measure of that enterprise-wide risk. We advocate calibrating the enterprise-wide capital requirement to a target EPD that measures all risks simultaneously over the same time horizon. We have illustrated the EPD measure using a time horizon of one year, but it is easy to see the potential merits of shorter time horizons, such as quarterly or even monthly. While there are obvious practical obstacles to implementing such a framework in the near term, conceptually we can imagine a solvency framework in which capital is recalibrated on a daily basis!

Meanwhile, it is important that the issue of the capital required by the existing risk-based capital models to support property-casualty insurance operations be taken up again and with some urgency. This is important, because in recent years rating agencies have shown an inclination to increase underwriting-related capital requirements by increasing capital factors directly and/or indirectly by increasing the capital adequacy ratios that correspond to their various ratings. While no responsible insurance professional can be opposed to a strong solvency regime, requiring more capital than is actually required to meet stated solvency objectives increases the cost of insurance and unnecessarily impedes the ability of insurers to compete with alternative methods of managing risk. Our aim in preparing this paper has been to stimulate thoughtful discussion of this important issue, which we hope will ultimately lead to actions by regulators and rating agencies to adapt their risk-based capital models to reflect more accurately the real risks embedded in insurance company underwriting and loss reserving activities.

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reserve risks that assumed a time horizon that encompassed ultimate claim settlement [2]. Best and S&P both state that their risk-based capital models use that same ultimate time horizon EPD methodology [1] [11].

4. APPENDICES

APPENDIX A

Historical Loss Development and Accident Year 1999 Estimates

The main source of the loss development data used in this paper was the Best's Aggregates & Averages compilation of industry Schedule P information for Commercial Auto Liability that was tabulated in Wacek [13] as sets of paid and case incurred loss development factors (in that paper's Appendix Exhibits A-2A and A-5A, respectively)[44].

The upper half of Appendix Exhibit A-1A of this paper shows the 1) paid loss ratio through one year of development, and 2) age-to-age paid development factors (from age 1-to-2 through age 9-to-10), that were observed during calendar years 1994 through 2004 with respect to accident years 1999 and prior. The calendar year 1994 through 2003 data is from Best as tabulated by Wacek [13]. The calendar year 2004 information was derived directly from the industry Schedule P compilation contained within 2005 edition of Best's Aggregates & Averages [4]. The age 10-to-ultimate paid development factor implied by the relationship between the accident year 1995 reported ultimate and age 10 paid losses (1.009) is also tabulated here. The lower half of Appendix Exhibit A-1A displays the natural logarithms of the loss ratios and development factors shown in the upper half of the exhibit.

Appendix Exhibit A-1B is the case incurred loss analogue to Appendix Exhibit A-1A. The upper half of the exhibit displays the case incurred loss ratios through one year of development and age-to-age case incurred loss development factors. The source of that data is largely Appendix Exhibit A-5A of Wacek [13], supplemented by calendar year 2004 data from the 2005 edition of Best's Aggregates & Averages [4]. The age 10-to-ultimate case incurred development factor implied by the relationship between the accident year 1995 reported ultimate and age 10 case incurred losses (1.002) is also tabulated here. The lower half of the exhibit gives the corresponding natural logarithms.

Appendix Exhibits A-2A and A-2B display trailing five-year simple means and standard deviations of the loss ratio and development factor natural logarithms shown in the lower halves of Appendix Exhibits A-1A and A-1B, respectively. The means and standard deviations tabulated in Appendix Exhibit A-2A were used as estimates of the parameters μ and σ, respectively, of lognormal random variables representing 1) the paid loss ratio through one year of development, and 2) age-to-age paid loss development factors. For the

[44] Best's Aggregates & Averages, 1995-2005 editions. See [13] for full details.

insurer level analysis we multiplied these estimates σ by a factor of three. In order to preserve the same expected lognormal development factors as those found in the industry analysis, we adjusted the corresponding estimates of μ by adding the term $0.5\sigma^2 \cdot (1 - f^2)$, where $f = 3$.

The means and standard deviations tabulated in Appendix Exhibit A-2B were used to parameterize lognormal random variables representing 1) the case incurred loss ratio through one year of development, and 2) age-to-age case incurred loss development factors. For the insurer analysis, we made the same adjustment to the σ estimates that we described in the previous paragraph with respect to paid development factors.

Appendix Exhibits A-3A and A-3B display the expected values of the lognormal random variables parameterized using the means and standard deviations displayed in Appendix Exhibits A-2A and A-2B, respectively. To a very close degree of approximation, the implied mean age-to-age and age-to-ultimate development factors match those computed directly from the development factor data.

Appendix Exhibit A-4 summarizes the use of the historical loss development data to estimate accident year 1999 Commercial Auto Liability ultimate loss ratios using paid and case incurred chain ladder and Bornhuetter-Ferguson loss development methods at annual valuations from December 1999 through December 2004. We applied these four loss development methods as described in Appendix A of [13] using age-to-ultimate development factors from Appendix Exhibits A-3A and A-3B.

APPENDIX B

Estimating the Transfer Value of Unpaid Losses

According to Butsic, the transfer value of the unpaid loss liability should equal the present value of the expected future loss payments plus a risk charge for the potential for adverse deviation[45]. That definition was echoed in the UK FSA's February 2006 discussion paper on the EU's Solvency II initiative: "An unbiased valuation of insurance liabilities would reflect the best estimate plus a margin determined by the cost of capital required by the market to bear the risk of holding the liability[46]." In this appendix we derive a formula for this transfer value based on the capital required to support the unpaid loss liability and the required return on that allocated risk-based capital.

The transfer value $T(L_n)$ of unpaid losses L_n at development age $n \geq 1$ years is the sum:

[45] See [5], page 330, footnote 15.
[46] See [10], page 25.

$$T(L_n) = PV(L_n) + R'_n, \tag{B.1}$$

where $PV(L_n)$ is the present value sum of the future loss payments at the risk-free rate r and R'_n is the present value sum, at the same rate r, of the future risk charges associated with unpaid losses.

The calculation of the first term $PV(L_n)$ of $T(L_n)$ requires knowledge of the amounts and timing of the expected future loss payments $P_{n+1}, P_{n+2}, P_{n+3}, \ldots, P_{n+k}$, where k represents the number of future loss payments. If we assume that all loss payments are made at the midpoint of each payment year, then the value of $PV(L_n)$ is given by the formula:

$$PV(L_n) = (1 + \tfrac{1}{2} r) \cdot A_n, \tag{B.2}$$

where $A_n = P_{n+1} \cdot v + P_{n+2} \cdot v^2 + P_{n+3} \cdot v^3 + \ldots + P_{n+k} \cdot v^k$ and $v = \dfrac{1}{1+r}$. A_n is the present value sum of the loss payments under the assumption that they are made at the *end* of each year. $1 + \tfrac{1}{2} r$ is the adjustment factor required to reflect our assumption that loss payments are made at the *midpoint* of each year.

If the annual risk charge related to unpaid losses is expressed as a percentage return on the allocated risk-based capital $C_n^R = c_n \cdot L_n$, then the second term R'_n in Formula (B.1) can be expressed as:

$$R'_n = r'_n \cdot L_n \cdot v + r'_{n+1} \cdot L_{n+1} \cdot v^2 + r'_{n+2} \cdot L_{n+2} \cdot v^3 + \ldots + r'_{n+k-1} \cdot L_{n+k-1} \cdot v^k, \tag{B.3}$$

where $r'_n, r'_{n+1}, r'_{n+2}, \ldots, r'_{n+k-1}$ are the required annual returns expressed in terms of unpaid losses. To determine these required returns we assume that the capital provider demands an annualized after-tax return on equity of *roe* commensurate with the risk it is assuming for each year the capital is exposed. Given a tax rate of *tax*, the annual pre-tax return requirement on the allocated risk-based capital is $\dfrac{roe}{1-tax}$, of which r will be provided by interest earned on the capital itself. If the allocated capital is $c_n \cdot L_n$, then the required risk charge for each development period $n \geq 1$ is $c_n \cdot L_n \cdot (\dfrac{roe}{1-tax} - r)$. This risk charge can be expressed as an annual rate of return on L_n of:

$$r'_n = c_n \cdot (\frac{roe}{1-tax} - r) \tag{B.4}$$

If $c_1 = c_2 = c_3 = \cdots = c_n$ for all $n \geq 1$, i.e., the risk-based capital charges applicable to loss reserves are identical irrespective of the development age of the reserves, then we can drop the subscript from r'_n and restate Formula (B.3) as:

$$R'_n = r' \cdot (L_n \cdot v + L_{n+1} \cdot v^2 + L_{n+2} \cdot v^3 + \ldots + L_{n+k-1} \cdot v^k)$$

$$= r' \cdot (v \sum_{n+1}^{n+k} P_i + v^2 \sum_{n+2}^{n+k} P_i + v^3 \sum_{n+3}^{n+k} P_i + \ldots + v^k P_{n+k})$$

$$= r' \cdot (P_{n+1} \cdot v + P_{n+2} \cdot (v + v^2) + P_{n+3} \cdot (v + v^2 + v^3) + \ldots + P_{n+k} \cdot (v + v^2 + v^3 + \cdots + v^k))$$

$$= r' \cdot (P_{n+1} \cdot \frac{1-v}{r} + P_{n+2} \cdot \frac{1-v^2}{r} + P_{n+3} \cdot \frac{1-v^3}{r} + \ldots + P_{n+k} \cdot \frac{1-v^k}{r})$$

$$= \frac{r'}{r} \cdot (\sum_{n+1}^{n+k} P_i - (P_{n+1} \cdot v + P_{n+2} \cdot v^2 + P_{n+3} \cdot v^3 + \ldots + P_{n+k} \cdot v^k))$$

and finally as:

$$R'_n = \frac{r'}{r} \cdot (L_n - A_n) \tag{B.5}$$

Then the transfer value $T(L_n) = PV(L_n) + R'_n$ can be expressed in terms of A_n as:

$$T(L_n) = (1 + \tfrac{1}{2} r) \cdot A_n + \frac{r'}{r} \cdot (L_n - A_n) \tag{B.6}$$

or in terms of $PV(L_n)$ as:

$$T(L_n) = PV(L_n) + \frac{r'}{r} \cdot (L_n - PV(L_n) + \frac{r \cdot PV(L_n)}{2 + r}) \tag{B.7}$$

Appendix Exhibit B-1 illustrates the risk charge and transfer value calculations using the unpaid losses as of December 1999 associated with \$100 of premiums from accident year 1999. The expected payment pattern was derived from the simple average age-to-age (annual) paid development factors observed during the five calendar years 1995 through 1999[47]. The illustration assumes a capital factor of 15% of unpaid losses, a required after-tax return on allocated capital of 15%, tax rate of 35% and risk-free return of 5%.

The left side of the exhibit summarizes the transfer value calculation. The unpaid losses of \$64.69 at the end of 1999 (and beginning of 2000) had a present value of \$58.62. The present value sum of the future annual risk charges was \$4.06. The sum of these two components, \$62.69, represents the transfer value of the \$64.69 of loss reserves at the beginning of 2000. One year later at the end of 2000 (and beginning of 2001), the unpaid losses were expected to decline to \$43.59. On that basis and the expected future payment pattern, the present values of the unpaid losses and future risk charges, were \$39.93 and \$2.51, respectively, yielding a transfer value of \$42.44. Observe that a risk charge must be

[47] See the "1999" row in the "Trailing Five-Year Average Development to Ultimate" section of Appendix Exhibit A-3A.

added to the present value of the unpaid losses at each valuation date at which there remain unpaid losses, which in this illustration is out through the end of 2009.

The right side of the exhibit is a reconciliation of the transfer value calculations. It shows that invested cash equal to the transfer value of $62.69 at the beginning of 2000 would earn $2.61 during 2000. That principal and interest would be sufficient to pay expected claims of $21.10 plus a cash risk charge of $1.75 to the capital provider, leaving a balance of $42.44 at the end of the year. That ending balance matched the expected transfer value of unpaid losses at that time. The reconciliation shows that the transfer values calculated on the left side of the exhibit are such that all losses and risk charges can be paid as due (assuming the size and timing of loss payments are as expected.)

Appendix Exhibit B-2 summarizes the present value of the remaining loss reserves and the related risk charge (from Formula (B.5)), based on trailing five-year paid loss development experience, as of each calendar year-end from 1999 through 2004. The sum of these two present values is the transfer value of the remaining reserves (expressed as a percentage of remaining reserves).

APPENDIX C

C.1 Stochastic Modeling of Losses

The premise underlying the stochastic loss models used in this paper is that age-to-age loss development can be represented using a lognormal model. Our approach is closely related to the one described by Wacek [12], which was an elaboration of an idea first presented by Hayne [9]. We assumed that both paid and case incurred loss development patterns are lognormal.

Sources of Variation in Future Ultimate Loss Ratio Estimates

In general, a future estimate of the ultimate loss ratio with respect to a particular accident year depends mainly on the loss development that occurs between now and the time the future estimate is made[48]. That loss development affects the future ultimate loss estimate in two ways. The first and most direct effect arises from the loss development observed with respect to the subject accident year itself. More development generally implies a larger

[48] There can also be a minor effect that arises from the use of moving averages of historical development measures. For example, if prospective development in the tail is estimated using the five-year mean of historical development factors, then one year later when the tail is re-estimated, the earliest development factor will have dropped out of the calculation and a factor reflecting more recent development will have entered. The difference between the dropped factor and the added factor can have a small effect on the revised tail.

future ultimate loss ratio estimate than less development. The second effect arises from the loss development observed with respect to earlier accident years. That loss development affects the estimation of development in the tail of the subject accident year beyond the future valuation date. Again, more development generally implies a larger future ultimate loss ratio estimate than less development.

Our interest is in estimates of the ultimate loss ratio for accident year 1999 one year out from the vantage point of a succession of annual valuation dates from accident year inception ($n = 0$) through December 2003 ($n = 5$), where n refers to years of development.

From the vantage point of accident year 1999 inception ($n = 0$), the ultimate loss ratio estimate one year out will depend on loss development observed during calendar year 1999 with respect to: 1) accident year 1999 (from inception to age one year) and 2) 1998 and prior accident years.

In general, at each annual valuation through December 2003 corresponding to $0 \le n \le 5$ years of development, the ultimate loss ratio estimate one year out is a function of: 1) accident year 1999 development during calendar year $1998 + n + 1$ (from age n to $n + 1$), and 2) development on 1998 and prior accident years observed during calendar year $1998 + n + 1$.

To estimate the parameters of the random variables representing these loss development effects, we used industry Commercial Auto Liability loss development experience from Best's Aggregates & Averages, which is tabulated in Appendix Exhibits A-1A and A-1B mainly in the form of paid and case incurred age-to-age development factors (and their natural logarithms), respectively. See Appendix A for a full description of this data and its source.

C.2 Paid Chain Ladder

At n years of development the accident year 1999 paid chain ladder ultimate loss ratio estimate one year out is the product of the accident year 1999 cumulative paid loss ratio one year out (at age $n + 1$) and the paid age $n + 1$-to-ultimate tail factor one year out (at age $n + 1$). The first factor of this product reflects accident year 1999 development in the next year. The second factor reflects the effect of development of accident years 1998 and prior on the calculation of the tail factor one year out.

Modeling the First Source of Variation – Accident Year Development

To model accident year development over the course of the next year from the perspective of accident year 1999 inception ($n = 0$), we calculated the mean $\bar{y} = -1.678$ and standard deviation $s = 0.041$ of the natural logarithms of the paid loss ratios through one year of development observed over the five most recent calendar years (1994 through 1998)[49]. We used $\bar{y}$ and s as estimates of the parameters μ and σ of the lognormal random variable p_1 representing the paid loss ratio that will be observed one year out at the end of 1999. The *expected* value of the paid loss ratio at the end of 1999 $E(p_1)$ implied by these parameters was 18.7%. The *actual* paid loss ratio P_1 observed at the end of 1999 was 20.6%.

At the end of 1999 ($n = 1$) we went through a similar procedure. Let $\bar{y}$ and s represent the mean and standard deviation, respectively, of the natural logarithms of the age 1-to-2 development factors observed over the five most recent calendar years (1995 through 1999). We used $\ln P_1 + \bar{y} = \ln(20.6\%) + 0.743 = -0.836$ and $s = 0.024$ to estimate the parameters μ and σ of the lognormal random variable representing the cumulative paid loss ratio $P_1 + p_2$ that will be observed at the end of 2000[50]. These parameters implied an *expected* cumulative paid loss ratio as of the end of 2000 $E(P_1 + p_2)$ of 43.4%. The *actual* cumulative paid loss ratio $P_1 + P_2$ observed at the end of 2000 was 44.2%.

Generally, at $1 \le n \le 5$ years of development, to model the accident year 1999 paid loss ratio one year out at the end of $1999 + n$, we used $\ln(\sum_{i=1}^{n} P_i) + \bar{y}$ and s as estimates of the parameters μ and σ of the lognormal random variable $\sum_{i=1}^{n} P_i + p_{n+1}$, where P_i is the actual partial loss ratio paid during period *i*, and $\bar{y}$ and s are the mean and standard deviation, respectively, of the natural logarithms of the age n to $n+1$ development factors observed over the five most recent calendar years ($1999 + n - 5$ through $1999 + n - 1$). The *expected* cumulative paid loss ratio out year out, i.e., as of the end of $1999 + n$, is $E(\sum_{i=1}^{n} P_i + p_{n+1})$ and the *actual* cumulative paid loss ratio is $\sum_{i=1}^{n} P_i + P_{n+1}$.

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[49] Appendix Exhibit A-2A summarizes these calculations, which are based on data in Appendix Exhibit A-1A. For the insurer level analysis we used $3s$ in place of s and $\bar{y} + 0.5s^2 \cdot (1 - 3^2)$ to model the greater variability of an individual insurer's development factors, while preserving the original lognormal expected value development factors.

[50] The random variable for the cumulative paid loss ratio can also be defined multiplicatively as $P_1 \cdot d_{1,1-2}$, where $d_{1,1-2}$ is the lognormal random variable at age $n = 1$ representing the age 1-to-2 development factor that will manifest itself over the next year, with parameters estimated by $\bar{y} = 0.743$ and $s = 0.024$. We prefer the additive formulation, because it preserves the annual components of the cumulative paid loss ratio.

We calculated these parameter estimates for $1 \le n \le 5$ and tabulated them, together with the expected and actual paid loss ratios one year out, in Appendix Exhibit C-1A in the column labeled "Paid L/R."

Modeling the Second Source of Variation – Tail Factor Revision

The revised tail factor one year out is the product of the mean age-to-age factors one year out. If five-year means are used, four of the five development factors to be used in the mean age-to-age factor calculations are already known. The fifth development factor is unknown, because it represents the development to be observed during the next year, but we can model it as a random variable. Because it involves four constants and a random variable, the mean age-to-age factor one year out is a random variable.

We modeled the revised tail factor one year out in three steps. First, we estimated the parameters of each age-to-age development factor random variable. These random variables modeled the age-to-age development to be observed during the next year. Next, we estimated the parameters of the *mean* age-to-age factors one year out. These mean age-to-age factor random variables combined the four known development factors and the random variable determined in step one. Finally, the mean age-to-age random variables were multiplied together to obtain the random variable for the revised tail factor out year out. Because the final step is difficult to carry out analytically, we used Monte Carlo simulation to model the revised tail factor random variables.

We illustrate the first two steps of this process for the age 1-to-2 development factor at accident year inception ($n = 0$). Referring to Appendix Exhibit A-1A, the age 1-to-2 development factors observed in calendar years 1994 through 1998 (with respect to accident years 1993 through 1997) and their natural logarithms were 2.265, 2.165, 2.115, 2.032, 2.079 and 0.817, 0.772, 0.749, 0.709, 0.732, respectively. That implied $\bar{y} = 0.756$ and $s = 0.041$, which we took as estimates of the μ and σ parameters of the random variable $d_{0,1-2}$ for the age 1-to-2 development factor to be observed in 1999 (with respect to accident year 1998). That is step one.

Because the historical development factors can be thought of as lognormal random variables with a σ parameter of zero, the random variable for the mean age 1-to-2 development factor one year out $\bar{d}_{1,1-2}$ has estimated parameters $\hat{y}$ (for μ) and $\hat{s}$ (for σ) of

$\frac{1}{5}(0.772+0.749+0.709+0.732+0.756)=0.744$ and $\frac{1}{5}\cdot 0.041=0.008$[51]. This random variable has an expected value of 2.104[52].

The same process was repeated for each age-to-age factor out to the factor for development from age nine to ten years. The age ten years to ultimate factor was treated as a constant. The product of all of the mean age-to-age factors one year out is the tail factor. The results from the perspective of accident year inception ($n=0$) are tabulated in Appendix Exhibit C-1A in the rows corresponding to Valuation Date "12/98." At the far right we also show the effect of multiplying the paid loss ratio one year out by the product of the revised age-to-age factors one year out to produce the estimated ultimate loss ratio estimate one year out. Here we see that at accident year inception the expected paid chain ladder ultimate loss ratio estimate one year out was 82.0%[53]. However, after observing the actual development during 1999, the paid chain ladder ultimate loss ratio estimate was revised to 90.3%.

In general, to model the tail factor one year out at $0 \le n \le 5$ years of development, we followed the same procedure that we described for $n=0$. In the first step we calculated the mean $\bar{y}$ and standard deviation s of the natural logarithms of the paid loss age-to-age development factors separately for each development period beyond age $n+1$ years (from age $n+1$-to-$n+2$ out through age 9-to-10) observed over the five calendar years $1999+n-5$ through $1999+n-1$. We took these as estimates of the parameters of the distributions of age-to-age factors that would be observed during $1998+n+1$. In the second step we combined the four age $n+1$-to-$n+2$ development factor observations from calendar years $1999+n-4$ through $1999+n-1$ with the random variable for $1999+n$ development whose parameters we estimated in step one. The results from the perspective of all annual valuation dates from December 1998 ($n=0$) through December 2003 ($n=5$) are tabulated in Appendix Exhibit C-1A. The combined effects of accident year development and tail factor revision are embodied in the actual and expected ultimate loss ratio estimates one year out shown at the far right.

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51 For the insurer level analysis we did not adjust the four known data points (0.772, 0.749, 0.709 and 0.732) to offset the effect of multiplying s by a factor of three. This resulted in a slight upward bias in the distributions of the mean development factors making up the tail.

52 Note that this matches the simple average comprising the 1995 through 1998 development factors and the 1994 through 1998 development factor mean. If we were interested only in the development factor itself and not also its variability, it would be easier to work directly with the development factor data.

53 If the reader is puzzled about why this is different from the 83.5% shown in Appendix Exhibit A-3A as the implied paid chain ladder estimate at inception of the ultimate loss ratio, note that 82.0% is the estimate at inception of the paid chain ladder ultimate loss ratio *one year out*, which reflects the dropping of the 1994 development factors and addition of the estimate of 1999 development.

C.3 Case Incurred Chain Ladder

The modeling of the random variables for case incurred loss development was the same in every respect as that for paid loss development, except that case incurred loss data was used to estimate the parameters rather than paid loss data.

The parameter estimates for the case incurred random variables one year out, together with the expected and actual case incurred loss ratios one year out, are tabulated in Appendix Exhibit C-1B in the column labeled "Case Inc L/R" for annual valuation from accident year inception through December 2003. The parameter estimates for the case incurred age-to-age factor random variables one year out, together with the expected and actual case incurred age-to-age factors one year out, are tabulated in the body of the same Appendix Exhibit C-1B. The far right column shows the values one year out of the expected and actual ultimate loss ratio estimates.

C.4 Bornhuetter-Ferguson – Paid and Case Incurred

We applied the paid and case incurred Bornhuetter-Ferguson methods described in Appendix A, Section A.2.2, of Wacek [13] to loss development experience simulated using the random variables described in sections C.2 and C.3. At accident year inception ($n = 0$) we used the same initial expected loss ratios that were used in that paper: 83.4% for the paid method and 78.7% for the case incurred method. Separately for the paid and case incurred versions, we set the expected loss ratio for subsequent valuations equal to the chain ladder ultimate loss ratio estimate from the prior valuation, which was the convention used in [13].

C.5 Incorporation of Parameter Uncertainty

If we could have been certain about our lognormal parameter estimates, we would have simulated loss development experience using the lognormal random variables described in the foregoing sections of this appendix. Given a uniform random number R, the corresponding lognormal random number $LN^{-1}(\mu, \sigma, R)$ is:

$$LN^{-1}(\mu, \sigma, R) = \exp(\mu + N^{-1}(R) \cdot \sigma), \qquad \text{(C.1)}$$

where μ and σ are the usual lognormal parameters and N^{-1} is the standard normal inverse distribution function.

However, we did not (and could not) know the true values of μ and σ. We had only parameter estimates $\hat{y}$ and $\hat{s}$. Because of that parameter uncertainty, we used a log t (rather than lognormal) random variable to simulate random values representing loss development experience:

$$LT^{-1}(\hat{y}, \hat{s}, R, k) = \exp(\hat{y} + T_{k-1}^{-1}(R) \cdot \hat{s}\sqrt{k + 1/k}) \qquad \text{(C.2)}$$

where $\hat{y}$ and $\hat{s}$ are the estimates of the lognormal parameters μ and σ, R is a uniform random number and T_{k-1}^{-1} is the inverse distribution function for the Student's *t* distribution with $k-1=4$ degrees of freedom (k representing the number of data points used to estimate the parameter). The $\sqrt{k+1/k}$ factor reflects the fact that both parameters are uncertain. In much statistical analysis involving the Student's *t* distribution it is assumed that μ is known and only σ is uncertain. We know here that both are uncertain. See Wacek [14] for a detailed discussion of parameter uncertainty in lognormal models.

C.6 Monte Carlo Simulation

Because the loss development random variables are hard to work with analytically (especially because we incorporated parameter uncertainty), we used Monte Carlo simulation to model chain ladder and Bornhuetter-Ferguson ultimate loss ratio estimates one year out as of each annual valuation date from inception through December 2003 using both paid and case incurred methods. For each of 10,000 Monte Carlo trials, we determined ultimate loss ratio estimates from each of the four loss development methods, and selected their unadjusted simple mean U_{n+1} as the best estimate of the ultimate loss ratio one year out. Appendix C-2A illustrates, for one Monte Carlo trial, the simulation of the paid chain ladder and Bornhuetter-Ferguson ultimate loss ratio estimates one year out from the vantage point of accident year inception ($n=0$). Appendix C-2B illustrates the simulation of the case incurred chain ladder and Bornhuetter-Ferguson ultimate loss ratio estimates one year out from the same vantage point[54]. We used the same uniform random numbers for the paid and case incurred simulations, reflecting our assumption that paid and case incurred loss development are not independent.

The ultimate loss ratio estimate has two stochastic elements corresponding to paid and unpaid losses, which we needed to separate in order to determine the transfer value of the ultimate loss ratio estimate one year out: $T(L_{n+1} + P_{n+1}) = T(L_{n+1}) + T(P_{n+1})$.

Therefore, in addition to U_{n+1}, for each trial we also tabulated the simulated values one year out of the period paid loss ratio P_{n+1} and the unpaid portion L_{n+1} of U_{n+1} (given by $L_{n+1} = U_{n+1} - \sum_{i=1}^{n+1} P_i$), as well as the transfer values $T(L_{n+1} + P_{n+1})$ and $T(L_{n+1})$. The transfer values were determined using the approach described in Appendix B.

[54] Note that Appendix Exhibits C-2A and C-2B use the same principles and format as Exhibit 11 in [12].

For each Monte Carlo trial we calculated the value of ending capital C_{n+1} and the ending policyholder deficit PD_{n+1} using the formulas in Section 2. The expected policyholder deficit was computed as the mean over 10,000 random trials:

$$E_n(pd_{n+1}) = \tfrac{1}{10,000} \sum_{i=1}^{10,000} PD_{n+1,i} \tag{C.3}$$

Formula (C.3) is a discrete approximation of Formula (2.15), which uses the continuous random variable t_{n+1} corresponding to $T(L_{n+1} + P_{n+1})$.

C.7 Reserves from Multiple Accident Years

If we let AY refer to the most recent of the accident years $AY - i$ ($i \geq 0$) with unpaid losses, the key loss-related formulas applicable to the valuation *n* years after AY's inception (for $n \geq 1$) of all accident years together are as follows:

$$_{AY-i}C^{R}_{n+i} = c_{n+i} \cdot {}_{AY-i}L_{n+i} \tag{C.4}$$

$$_{AY-i}S_{n+1+i} = ({}_{AY-i}C^{R}_{n+i} + {}_{AY-i}T(L_{n+i})) \cdot (1+r) \tag{C.5}$$

$$_{All}S_{n+1} = \sum_{i\geq 0} ({}_{AY-i}C^{R}_{n+i} + {}_{AY-i}T(L_{n+i})) \cdot (1+r) \tag{C.6}$$

$$_{AY-i}C_{n+1+i} = {}_{AY-i}S_{n+1+i} - T({}_{AY-i}L_{n+1+i} + {}_{AY-i}P_{n+1+i}) \tag{C.7}$$

$$_{All}C_{n+1} = {}_{All}S_{n+1} - \sum_{i\geq 0} T({}_{AY-i}L_{n+1+i} + {}_{AY-i}P_{n+1+i}) \tag{C.8}$$

$$_{All}PD_{n+1} = \sum_{i\geq 0} T({}_{AY-i}L_{n+1+i} + {}_{AY-i}P_{n+1+i}) - {}_{All}S_{n+1}, \text{ if } {}_{All}C_{n+1} < 0 \tag{C.9}$$

$$E_n({}_{All}pd_{n+1}) = \int_{_{All}S_{n+1}}^{\infty} (t_{n+1} - {}_{All}S_{n+1}) f(t_{n+1}) dt_{n+1}, \tag{C.10}$$

where t_{n+1} is abbreviated notation for $_{All}t_{n+1} = \sum_{i\geq 0} {}_{AY-i}t_{n+1+i}$.

APPENDIX D

Accident Year 1999 Actual Policyholder Deficits: 1999-2004

This appendix gives details of the capital and policyholder deficit calculations arising from the *actual* Commercial Auto Liability accident year 1999 industry experience evaluated at successive annual intervals from December 1999 through December 2004. We used an analogous process to determine capital and policyholder deficits in the Monte Carlo analysis described in Appendix C. The centerpiece of our discussion is Appendix Exhibit D. Key results from that exhibit are summarized in Table 3 and Figure D in Section 2.

At the beginning of 1999, risk-based capital was established at 15% of premiums net of expenses and subsequently recalibrated to maintain funding equal to 15% of loss reserves at the end of each calendar year. A negative capital balance at the end of a year implied a policyholder deficit. If the "capital account" was under-funded at the end of any year, either because of a policyholder deficit or the recalibration requirement, the capital provider had to deposit additional cash. If this account was over-funded, the capital provider could withdraw the excess cash.

At accident year inception we assumed \$100 of premiums and an underwriting expense ratio of 25%, which implied initial underwriting assets $T(L_0)$ of \$75. The initial required capital was \$11.25 (15% of \$75). For purposes of calculating interest earned on the underwriting and capital assets, we assumed that half of the premium cash was available on January 1, 1999 in the form of an unearned premium portfolio and that the other half was available, on average, on July 1, 1999. Assuming the capital was allocated as the premiums were received and a risk free rate of 5%, interest of \$3.23 was earned during 1999. The December 1999 value of the underwriting and capital assets, including interest, was S_1=\$75.00+\$11.25+\$3.23=\$89.48.

Meanwhile, the hindsight re-estimate $L_1 + P_1$ as of December 1999 of the initial loss estimate L_0 was \$85.32, comprising paid losses P_1 of \$20.63 and an unpaid loss liability L_1 of \$64.69. The total transfer value of the hindsight losses $T(L_1 + P_1)$ was \$83.84, reflecting a paid loss transfer value $T(P_1)$ of \$21.15 and an unpaid loss transfer value $T(L_1)$ of \$62.69[55].

Now let's look at the "capital account". The ending capital as of December 1999 was \$5.65, which was the difference between the available assets S_1 and the transfer value of the hindsight losses $T(L_1 + P_1)$. Because the capital balance remained positive, the policyholder deficit was zero.

However, based on the loss reserve of \$64.69 as of December 1999, the prospective capital requirement was \$64.89$\times$15%, or \$9.70, which meant that the capital account was under-funded by \$4.06. In order to meet the ongoing capital requirement, the capital provider had to contribute \$4.06 of additional capital at the end of 1999[56]. (This is summarized graphically in Figure D in Section 2 of the paper. The initial capital of \$11.25

[55] The paid loss transfer value assumes that claims were settled, on average, on July 1, 1999. The risk charge embedded in the unpaid loss transfer value is consistent with a 15% target after-tax return on the capital supporting the loss reserves, tax rate of 35%, risk-free interest rate of 5% and capital/reserve ratio of 15%. The loss payout pattern was derived from paid loss development experience through the end of calendar year 1999. See Appendix B for the theoretical basis and numerical illustration of the calculation.

[56] If the capital provider had failed to recapitalize, it would have been possible for the regulator to arrange an immediate transfer of the unpaid loss liability at the transfer value.

was reduced to $5.65 by the end of 1999, but was replenished to $9.70 to meet the prospective capital requirement based on year-end 1999 loss reserves.)

The same process was repeated for calendar year 2000. At the end of the year the sum of capital and underwriting assets ($9.70+$62.69) plus interest of $3.62 resulted in total available assets of $76.01, which was more than enough to cover the $68.78 transfer value of hindsight losses, despite an increase in the ultimate loss ratio estimate from 85.3% to 90.0%. Capital was reduced from $9.70 at the beginning of the year to $7.23 at the end, but it was far from exhausted, so again the actual policyholder deficit was zero.

At the beginning of 2001 capital again had to be reestablished at 15% of unpaid losses, or $6.87. Because the ending capital balance in December 2000 was $7.23, the capital provider could withdraw $0.36. 2001 saw further deterioration in the ultimate loss ratio estimate to 92.3%, and capital declined to $6.17 by year-end, but the policyholder deficit was zero.

We will leave it to the reader to review the details of the development of accident year 1999 from the end of 2001 through the end of 2004 as tabulated in Appendix Exhibit D, pointing out only that no policyholder deficit emerged at any valuation.

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Abbreviations and notations

A_n, value of S&P 500 investment at time *n*

a_{n+1}, random variable, at time *n*, for value of S&P 500 investment one year out (*n*+1)

AY, accident year

C_n^R, required risk-based capital at time *n*

C_{n+1}, ending capital at time *n*+1

c, risk-based capital factor for common stocks

c_0, risk-based capital factor for underwriting

c_{n+1}, risk-based capital factor for loss reserves at time *n*+1

$d_{n,age-age+1}$, random variable for age-to-age+1 loss development factor

$\bar{d}_{n+1,age-age+1}$, random variable, at time *n*, for mean age-to-age+1 loss development factor one year out (*n*+1)

$E_n(pd_{n+1})$, expected value, at time n, of policyholder deficit one year out (*n*+1)

EPD, expected policyholder deficit

L_n, unpaid losses at time *n*

$L_{n+1}+P_{n+1}$, one-year hindsight estimate of L_n

LN^{-1} (μ,σ,prob), inverse lognormal distribution function

LT^{-1} ($\hat{y}$,$\hat{s}$,prob,*k*), inverse log *t* distribution function based on k-point sample

N^{-1}(prob), inverse standard normal distribution function

n, lag (years) from inception at beginning of year

$n+1$, lag (years) from inception at end of year

P_{n+1}, paid losses between time *n* and *n*+1

PD_{n+1}, policyholder deficit at time *n*+1

p_{n+1}, random variable, as of time *n*, for paid losses between time *n* and *n*+1

pd_{n+1}, random variable, as of time *n*, for policyholder deficit one year out (*n*+1)

PV, present value operator

R, random number from unit uniform distribution

R_n', present value risk charge at time *n*

r, risk-free interest rate, per annum

r_n', risk charge, per annum, as a rate on L_n

roe, after-tax target return on equity capital

S_{n+1}, strike price, at time *n*, of insolvency option one year out (*n*+1)

s, five-year standard deviation of LDF logs

$\hat{s}$, estimate of σ used in log *t* simulations

$T(L_n)$, transfer value of unpaid losses at time *n*

$T(P_{n+1})$, transfer value of paid losses between time *n* and *n*+1

T_{k-1}^{-1} (prob), Student's *t* inverse distribution function with *k*-1 degrees of freedom

t_{n+1}, random variable, as of time *n*, transfer value of L_n one year out (*n*+1)

tax, corporate income tax rate

$\bar{y}$, five-year mean of LDF logs

$\hat{y}$, estimate of μ used in log *t* simulations

U_n, estimated ultimate loss at time *n*

v, one-year PV factor: $=1/(1+r)$

μ, first parameter of lognormal

σ, second parameter of lognormal

Biography of the Author

Michael Wacek is President of Odyssey America Reinsurance Corporation based in Stamford, CT. Over the course of more than 25 years in the industry, including nine years in the London Market, Mike has seen the business from the vantage point of a primary insurer, reinsurance broker and reinsurer. He has a BA from Macalester College (Math, Economics), is a Fellow of the Casualty Actuarial Society and a Member of the American Academy of Actuaries. He has authored a number of papers.

APPENDIX EXHIBIT A-1A

Commercial Auto Liability Accident Year Paid LDFs and their Natural Logarithms
By Calendar Year of Observed Development

Calendar Year	Age 1 Loss Ratio	Age 1 - 2	Age 2 - 3	Age 3 - 4	Age 4 - 5	Age 5 - 6	Age 6 - 7	Age 7 - 8	Age 8 - 9	Age 9 - 10	Age 10-Ult
1994	17.6%	2.265	1.456	1.196	1.101	1.050	1.028	1.016	1.008	1.004	
1995	18.2%	2.165	1.449	1.205	1.099	1.048	1.025	1.013	1.008	1.004	
1996	19.2%	2.115	1.422	1.202	1.104	1.047	1.024	1.011	1.005	1.004	
1997	19.2%	2.032	1.406	1.209	1.098	1.047	1.024	1.012	1.006	1.004	
1998	19.2%	2.079	1.422	1.197	1.096	1.046	1.020	1.012	1.007	1.003	
1999	20.6%	2.118	1.434	1.198	1.093	1.049	1.025	1.013	1.006	1.004	
2000		2.143	1.429	1.208	1.105	1.047	1.021	1.010	1.004	1.002	
2001			1.437	1.215	1.101	1.046	1.023	1.010	1.006	1.002	
2002				1.215	1.105	1.049	1.023	1.010	1.007	1.004	
2003					1.096	1.046	1.020	1.007	1.006	1.004	
2004						1.032	1.019	1.009	1.006	1.003	1.009
Natural Logarithms of Age 1 Loss Ratio and Age-to-Age Factors Shown Above											
1994	-1.739	0.817	0.375	0.179	0.096	0.049	0.028	0.016	0.008	0.004	
1995	-1.703	0.772	0.371	0.187	0.094	0.047	0.025	0.013	0.008	0.004	
1996	-1.648	0.749	0.352	0.184	0.099	0.046	0.024	0.011	0.005	0.004	
1997	-1.652	0.709	0.341	0.189	0.093	0.046	0.024	0.012	0.006	0.004	
1998	-1.651	0.732	0.352	0.180	0.091	0.045	0.020	0.012	0.007	0.003	
1999	-1.578	0.750	0.360	0.181	0.089	0.048	0.025	0.013	0.006	0.004	
2000		0.762	0.357	0.189	0.100	0.046	0.020	0.009	0.004	0.002	
2001			0.362	0.195	0.097	0.045	0.023	0.010	0.006	0.002	
2002				0.194	0.099	0.048	0.023	0.010	0.007	0.004	
2003					0.092	0.045	0.020	0.007	0.006	0.004	
2004						0.032	0.019	0.009	0.006	0.003	0.009

APPENDIX EXHIBIT A-1B

Commercial Auto Liability Accident Year Case Incurred LDFs and their Natural Logarithms
By Calendar Year of Observed Development

Calendar Year	Age 1 Loss Ratio	Age 1 - 2	Age 2 - 3	Age 3 - 4	Age 4 - 5	Age 5 - 6	Age 6 - 7	Age 7 - 8	Age 8 - 9	Age 9 - 10	Age 10-Ult
1994	46.7%	1.363	1.123	1.048	1.024	1.010	1.006	1.004	1.002	1.002	
1995	46.8%	1.362	1.120	1.051	1.020	1.007	1.002	1.001	1.001	1.001	
1996	46.9%	1.337	1.121	1.050	1.024	1.009	1.002	1.002	1.000	1.001	
1997	48.0%	1.349	1.123	1.060	1.025	1.008	1.007	1.003	1.001	1.000	
1998	47.1%	1.336	1.137	1.065	1.024	1.010	1.003	1.002	1.001	1.000	
1999	50.3%	1.380	1.148	1.068	1.018	1.010	1.003	1.001	0.999	1.001	
2000		1.408	1.154	1.069	1.028	1.011	1.004	1.000	0.999	0.999	
2001			1.162	1.082	1.036	1.013	1.006	1.003	1.002	1.002	
2002				1.073	1.036	1.017	1.008	1.003	1.003	1.003	
2003					1.022	1.009	1.002	0.998	1.001	1.000	
2004						1.007	1.005	1.002	1.002	1.001	1.002

Natural Logarithms of Age 1 Loss Ratio and Age-to-Age Factors Shown Above

Calendar Year	Age 1 Loss Ratio	Age 1 - 2	Age 2 - 3	Age 3 - 4	Age 4 - 5	Age 5 - 6	Age 6 - 7	Age 7 - 8	Age 8 - 9	Age 9 - 10	Age 10-Ult
1994	-0.761	0.310	0.116	0.047	0.023	0.010	0.006	0.004	0.002	0.002	
1995	-0.758	0.309	0.114	0.049	0.020	0.007	0.002	0.001	0.001	0.001	
1996	-0.758	0.291	0.114	0.049	0.024	0.009	0.002	0.002	0.000	0.001	
1997	-0.734	0.300	0.116	0.058	0.025	0.008	0.007	0.003	0.001	0.000	
1998	-0.754	0.289	0.128	0.063	0.023	0.010	0.003	0.002	0.001	0.000	
1999	-0.687	0.322	0.138	0.065	0.018	0.010	0.003	0.001	-0.001	0.001	
2000		0.342	0.143	0.067	0.028	0.010	0.004	0.000	-0.001	-0.001	
2001			0.150	0.079	0.035	0.013	0.006	0.003	0.002	0.002	
2002				0.070	0.035	0.017	0.008	0.003	0.003	0.003	
2003					0.022	0.009	0.002	-0.002	0.001	0.000	
2004						0.007	0.005	0.002	0.002	0.001	0.002

APPENDIX EXHIBIT A-2A

Commercial Auto Liability Accident Year Paid Loss Development
Mean and Standard Deviations of Natural Logarithms of LDFs

Trailing Five-Year Mean Age-to-Age Development Factor Natural Logarithms

Cal Year	Age 1 Loss Ratio	Age 1 - 2	Age 2 - 3	Age 3 - 4	Age 4 - 5	Age 5 - 6	Age 6 - 7	Age 7 - 8	Age 8 - 9	Age 9 - 10	Age 10-Ult *
1998	-1.678	0.756	0.358	0.184	0.095	0.047	0.024	0.013	0.007	0.004	0.009
1999		0.743	0.355	0.184	0.093	0.047	0.023	0.012	0.006	0.004	0.009
2000			0.352	0.185	0.094	0.046	0.023	0.011	0.006	0.003	0.009
2001				0.187	0.094	0.046	0.022	0.011	0.006	0.003	0.009
2002					0.095	0.046	0.022	0.011	0.006	0.003	0.009
2003						0.046	0.022	0.010	0.006	0.003	0.009
2004							0.021	0.009	0.006	0.003	0.009

Trailing Five-Year Standard Deviation of Age-to-Age Development Factor Natural Logarithms

Cal Year	Age 1 Loss Ratio	Age 1 - 2	Age 2 - 3	Age 3 - 4	Age 4 - 5	Age 5 - 6	Age 6 - 7	Age 7 - 8	Age 8 - 9	Age 9 - 10	Age 10-Ult *
1998	0.041	0.041	0.014	0.005	0.003	0.001	0.003	0.002	0.001	0.000	0.000
1999		0.024	0.011	0.004	0.004	0.001	0.002	0.001	0.001	0.000	0.000
2000			0.007	0.005	0.005	0.001	0.002	0.001	0.001	0.001	0.000
2001				0.006	0.004	0.001	0.002	0.001	0.001	0.001	0.000
2002					0.005	0.002	0.002	0.001	0.001	0.001	0.000
2003						0.002	0.002	0.002	0.001	0.001	0.000
2004							0.002	0.001	0.001	0.001	0.000

* Age 10 to Ultimate development implied in 2004 Annual Statement for accident year 1995

APPENDIX EXHIBIT A-2B

Accident Year Case Incurred Loss Development
Mean and Standard Deviations of Natural Logarithms of LDFs

Trailing Five-Year Mean Age-to-Age Development Factor Natural Logarithms

Cal Year	Age 1 Loss Ratio	Age 1 - 2	Age 2 - 3	Age 3 - 4	Age 4 - 5	Age 5 - 6	Age 6 - 7	Age 7 - 8	Age 8 - 9	Age 9 - 10	Age 10-Ult *
1998	-0.753	0.300	0.118	0.053	0.023	0.009	0.004	0.002	0.001	0.001	0.002
1999		0.302	0.122	0.057	0.022	0.009	0.003	0.002	0.001	0.001	0.002
2000			0.128	0.061	0.024	0.009	0.004	0.002	0.000	0.000	0.002
2001				0.067	0.026	0.010	0.004	0.002	0.001	0.000	0.002
2002					0.028	0.012	0.005	0.002	0.001	0.001	0.002
2003						0.012	0.004	0.001	0.001	0.001	0.002
2004							0.005	0.001	0.001	0.001	0.002

Trailing Five-Year Standard Deviation of Age-to-Age Development Factor Natural Logarithms

Cal Year	Age 1 Loss Ratio	Age 1 - 2	Age 2 - 3	Age 3 - 4	Age 4 - 5	Age 5 - 6	Age 6 - 7	Age 7 - 8	Age 8 - 9	Age 9 - 10	Age 10-Ult *
1998	0.011	0.010	0.006	0.007	0.002	0.001	0.002	0.001	0.001	0.001	0.000
1999		0.014	0.011	0.008	0.003	0.001	0.002	0.001	0.001	0.001	0.000
2000			0.013	0.007	0.004	0.001	0.002	0.001	0.001	0.001	0.000
2001				0.008	0.006	0.002	0.002	0.001	0.001	0.001	0.000
2002					0.008	0.003	0.002	0.001	0.002	0.001	0.000
2003						0.003	0.003	0.002	0.002	0.001	0.000
2004							0.002	0.002	0.001	0.001	0.000

* Age 10 to Ultimate development implied in 2004 Annual Statement for accident year 1995

APPENDIX EXHIBIT A-3A

Implied Lognormal Mean Accident Year Paid Loss Development Factors
Based on Mean and Standard Deviations of Natural Logarithms of LDFs
Commercial Auto Liability

Trailing Five-Year Average Age-to-Age Development

Cal Year	Age 1 Loss Ratio	Age 1 - 2	Age 2 - 3	Age 3 - 4	Age 4 - 5	Age 5 - 6	Age 6 - 7	Age 7 - 8	Age 8 - 9	Age 9 - 10	Age 10-Ult *
1998	18.7%	2.131	1.431	1.202	1.099	1.048	1.024	1.013	1.007	1.004	1.009
1999		2.102	1.427	1.202	1.098	1.048	1.024	1.012	1.006	1.004	1.009
2000			1.423	1.203	1.099	1.047	1.023	1.012	1.006	1.003	1.009
2001				1.205	1.099	1.047	1.023	1.011	1.006	1.003	1.009
2002					1.100	1.048	1.022	1.011	1.006	1.003	1.009
2003						1.047	1.022	1.010	1.006	1.003	1.009
2004							1.021	1.009	1.006	1.003	1.009

Trailing Five-Year Average Development to Ultimate

Cal Year	Age 1 Loss Ratio	Age 1 - 2	Age 2 - 3	Age 3 - 4	Age 4 - 5	Age 5 - 6	Age 6 - 7	Age 7 - 8	Age 8 - 9	Age 9 - 10	Age 10-Ult *
1998	83.5%	4.468	2.096	1.465	1.219	1.109	1.058	1.033	1.020	1.013	1.009
1999		4.378	2.083	1.460	1.215	1.107	1.056	1.032	1.019	1.013	1.009
2000			2.075	1.459	1.213	1.103	1.053	1.030	1.018	1.013	1.009
2001				1.460	1.211	1.102	1.053	1.030	1.018	1.012	1.009
2002					1.212	1.102	1.052	1.029	1.018	1.012	1.009
2003						1.101	1.051	1.028	1.018	1.012	1.009
2004							1.049	1.027	1.018	1.012	1.009

* Age 10 to Ultimate development implied in 2004 Annual Statement for accident year 1995

APPENDIX EXHIBIT A-3B

Implied Lognormal Mean Accident Year Case Incurred Loss Development Factors
Based on Mean and Standard Deviations of Natural Logarithms of LDFs
Commercial Auto Liability

Trailing Five-Year Average Age-to-Age Development

Cal Year	Age 1 Loss Ratio	Age 1 - 2	Age 2 - 3	Age 3 - 4	Age 4 - 5	Age 5 - 6	Age 6 - 7	Age 7 - 8	Age 8 - 9	Age 9 - 10	Age 10-Ult *
1998	47.1%	1.350	1.125	1.055	1.023	1.009	1.004	1.002	1.001	1.001	1.002
1999		1.353	1.130	1.059	1.022	1.009	1.003	1.002	1.001	1.001	1.002
2000			1.137	1.063	1.024	1.010	1.004	1.002	1.000	1.000	1.002
2001				1.069	1.026	1.010	1.004	1.002	1.001	1.000	1.002
2002					1.028	1.012	1.005	1.002	1.001	1.001	1.002
2003						1.012	1.004	1.001	1.001	1.001	1.002
2004							1.005	1.001	1.001	1.001	1.002

Trailing Five-Year Average Development to Ultimate

Cal Year	Age 1 Loss Ratio	Age 1 - 2	Age 2 - 3	Age 3 - 4	Age 4 - 5	Age 5 - 6	Age 6 - 7	Age 7 - 8	Age 8 - 9	Age 9 - 10	Age 10-Ult *
1998	78.7%	1.671	1.238	1.100	1.043	1.020	1.011	1.007	1.004	1.003	1.002
1999		1.684	1.244	1.101	1.040	1.018	1.009	1.005	1.004	1.003	1.002
2000			1.258	1.107	1.042	1.018	1.008	1.004	1.003	1.003	1.002
2001				1.119	1.046	1.020	1.009	1.005	1.003	1.003	1.002
2002					1.052	1.023	1.010	1.006	1.004	1.003	1.002
2003						1.021	1.009	1.005	1.004	1.003	1.002
2004							1.010	1.005	1.004	1.003	1.002

* Age 10 to Ultimate development implied in 2004 Annual Statement for accident year 1995

APPENDIX EXHIBIT A-4

Accident Year 1999 Ultimate Loss Ratio Estimates
Commercial Auto Liability

Chain Ladder Methods

Calendar Year	Dec Paid L/R	Dec Case L/R	Paid LDF	Case LDF	CL Paid Ult L/R	CL Case Ult L/R
1999	20.6%	50.3%	4.378	1.684	90.3%	84.7%
2000	44.2%	70.8%	2.075	1.258	91.7%	89.1%
2001	63.5%	82.3%	1.460	1.119	92.7%	92.1%
2002	77.2%	88.3%	1.212	1.052	93.5%	92.9%
2003	84.6%	90.2%	1.101	1.021	93.1%	92.2%
2004	87.3%	90.9%	1.049	1.010	91.6%	91.8%

Paid Bornhuetter-Ferguson Method

Calendar	Dec	BF Paid	Age to Ult LDF		BF Paid
Year	Paid L/R	ELR	Current	Prior	Ult L/R
1999	20.6%	83.4%	4.378	4.468	83.7%
2000	44.2%	90.3%	2.075	2.083	90.8%
2001	63.5%	91.7%	1.460	1.459	92.4%
2002	77.2%	92.7%	1.212	1.211	93.4%
2003	84.6%	93.5%	1.101	1.102	93.1%
2004	87.3%	93.1%	1.049	1.051	91.7%

Case Incurred Bornhuetter-Ferguson Method

Calendar	Dec	BF Case	Age to Ult LDF		BF Case
Year	Case L/R	ELR	Current	Prior	Ult L/R
1999	50.3%	78.7%	1.684	1.671	82.5%
2000	70.8%	84.7%	1.258	1.244	88.4%
2001	82.3%	89.1%	1.119	1.107	91.9%
2002	88.3%	92.1%	1.052	1.046	92.9%
2003	90.2%	92.9%	1.021	1.023	92.2%
2004	90.9%	92.2%	1.010	1.009	91.8%

Summary All Methods

Calendar Year	CL Paid Ult L/R	CL Case Ult L/R	BF Paid Ult L/R	BF Case Ult L/R	Mean Ult L/R
1999	90.3%	84.7%	83.7%	82.5%	85.3%
2000	91.7%	89.1%	90.8%	88.4%	90.0%
2001	92.7%	92.1%	92.4%	91.9%	92.3%
2002	93.5%	92.9%	93.4%	92.9%	93.2%
2003	93.1%	92.2%	93.1%	92.2%	92.6%
2004	91.6%	91.8%	91.7%	91.8%	91.7%

APPENDIX EXHIBIT B-1

Calculation of Expected Transfer Values of Accident Year 1999 Unpaid Losses
Loss Payment Patterns Based on Paid Development Through 1999
Per $100 of Commercial Auto Liability Premiums

Calculation

Cal Year	Beginning Unpaid Losses	Paid Losses Expected in Period	PV Beginning Unpaid Losses	Risk Charge in Period	PV Total Risk Charge	Beginning Transfer Value of Unpaid Losses
2000	$64.69	$21.10	$58.62	$1.75	$4.06	$62.69
2001	43.59	17.17	39.93	1.18	2.51	42.44
2002	26.42	11.60	24.33	0.72	1.46	25.78
2003	14.82	6.74	13.65	0.40	0.81	14.47
2004	8.08	3.61	7.42	0.22	0.45	7.88
2005	4.47	1.87	4.09	0.12	0.26	4.35
2006	2.59	0.99	2.38	0.07	0.15	2.53
2007	1.60	0.53	1.48	0.04	0.09	1.56
2008	1.07	0.32	1.01	0.03	0.05	1.06
2009	0.76	0.76	0.74	0.02	0.02	0.76

Reconciliation

Beginning Cash	Interest Earned	Paid Losses Expected in Period	Risk Charge Paid in Period	Ending Cash
$62.69	$2.61	-$21.10	-$1.75	$42.44
42.44	1.69	-17.17	-1.18	25.78
25.78	1.00	-11.60	-0.72	14.47
14.47	0.55	-6.74	-0.40	7.88
7.88	0.30	-3.61	-0.22	4.35
4.35	0.17	-1.87	-0.12	2.53
2.53	0.10	-0.99	-0.07	1.56
1.56	0.07	-0.53	-0.04	1.06
1.06	0.05	-0.32	-0.03	0.76
0.76	0.02	-0.76	-0.02	0.00

Interest Rate	5.0%	Eff Risk Chg Rate	2.71%
Capital Ratio	15.0%		
Target ROE	15.0%		
Tax Rate	35.0%		

APPENDIX EXHIBIT B-2

Projected Loss Payout Patterns for Use with Accident Year 1999 Commercial Auto Liability
Based on Trailing Five-Year Paid Development Experience

5-Year Period Ending	Remaining Reserves PV Factor	Remaining Reserves PV Risk Chg	Period Paid Development as % of Ultimate Losses Age 1 - 2	Age 2 - 3	Age 3 - 4	Age 4 - 5	Age 5 - 6	Age 6 - 7	Age 7 - 8	Age 8 - 9	Age 9 - 10	Age 10-Ult
1999	90.62%	6.28%	25.2%	20.5%	13.8%	8.0%	4.3%	2.2%	1.2%	0.6%	0.4%	0.9%
2000	91.66%	5.73%		20.4%	13.9%	8.2%	4.3%	2.2%	1.1%	0.6%	0.3%	0.9%
2001	92.25%	5.42%			14.1%	8.1%	4.3%	2.1%	1.1%	0.6%	0.3%	0.9%
2002	92.37%	5.36%				8.2%	4.3%	2.1%	1.1%	0.6%	0.3%	0.9%
2003	92.12%	5.49%					4.3%	2.1%	1.0%	0.6%	0.3%	0.9%
2004	91.44%	5.85%						2.0%	0.9%	0.6%	0.3%	0.9%

5-Year Period Ending	Cumulative Paid Development as % of Ultimate Losses Age 1	Age 2	Age 3	Age 4	Age 5	Age 6	Age 7	Age 8	Age 9	Age 10
1999	22.8%	48.0%	68.5%	82.3%	90.4%	94.7%	96.9%	98.1%	98.7%	99.1%
2000		48.2%	68.6%	82.5%	90.6%	94.9%	97.1%	98.2%	98.8%	99.1%
2001			68.5%	82.6%	90.7%	95.0%	97.1%	98.2%	98.8%	99.1%
2002				82.5%	90.7%	95.0%	97.2%	98.2%	98.8%	99.1%
2003					90.8%	95.1%	97.3%	98.2%	98.8%	99.1%
2004						95.3%	97.3%	98.2%	98.8%	99.1%

Present values reflect 5% risk free rate
Capital allocation 15% of reserves, tax rate 35%, target return on equity 15%.

APPENDIX EXHIBIT C-1A

Commercial Auto Liability Accident Year Paid Loss Development
Applicable to Stochastic Modeling of Accident Year 1999 Losses One Year Out

Lognormal Parameters and Expected vs. Actual Values of Random Variables One Year Out

Val Date		Paid L/R	Age 1 - 2	Age 2 - 3	Age 3 - 4	Age 4 - 5	Age 5 - 6	Age 6 - 7	Age 7 - 8	Age 8 - 9	Age 9 - Ult	Est Ult L/R
12/98	ŷ	-1.678	0.744	0.355	0.185	0.094	0.046	0.023	0.012	0.007	0.013	
	ŝ	0.041	0.008	0.003	0.001	0.001	0.000	0.001	0.000	0.000	0.000	
	Expected	18.7%	2.104	1.426	1.203	1.099	1.047	1.023	1.012	1.007	1.013	82.0%
	Actual	20.6%	2.102	1.427	1.202	1.098	1.048	1.024	1.012	1.006	1.013	90.3%
12/99	ŷ	-0.836		0.352	0.183	0.093	0.047	0.023	0.012	0.006	0.013	
	ŝ	0.024		0.002	0.001	0.001	0.000	0.000	0.000	0.000	0.000	
	Expected	43.4%		1.422	1.201	1.098	1.048	1.023	1.012	1.006	1.013	89.9%
	Actual	44.2%		1.423	1.203	1.099	1.047	1.023	1.012	1.006	1.013	91.7%
12/00	ŷ	-0.464			0.185	0.094	0.046	0.022	0.012	0.006	0.012	
	ŝ	0.007			0.001	0.001	0.000	0.000	0.000	0.000	0.000	
	Expected	62.9%			1.203	1.098	1.048	1.023	1.012	1.006	1.012	91.7%
	Actual	63.5%			1.205	1.099	1.047	1.023	1.011	1.006	1.012	92.7%
12/01	ŷ	-0.267				0.094	0.046	0.022	0.011	0.006	0.012	
	ŝ	0.006				0.001	0.000	0.000	0.000	0.000	0.000	
	Expected	76.6%				1.099	1.047	1.022	1.011	1.006	1.012	92.7%
	Actual	77.2%				1.100	1.048	1.022	1.011	1.006	1.012	93.5%
12/02	ŷ	-0.164					0.047	0.023	0.011	0.006	0.012	
	ŝ	0.005					0.000	0.000	0.000	0.000	0.000	
	Expected	84.9%					1.048	1.023	1.011	1.006	1.012	93.6%
	Actual	84.6%					1.047	1.022	1.010	1.006	1.012	93.1%
12/03	ŷ	-0.121						0.022	0.010	0.006	0.012	
	ŝ	0.002						0.000	0.000	0.000	0.000	
	Expected	88.6%						1.022	1.010	1.006	1.012	93.1%
	Actual	87.3%						1.021	1.009	1.006	1.012	91.6%

APPENDIX EXHIBIT C-1B

Commercial Auto Liability Accident Year Case Incurred Loss Development
Applicable to Stochastic Modeling of Accident Year 1999 Losses One Year Out

Lognormal Parameters and Expected vs. Actual Values of Random Variables One Year Out

Val Date		Rptd L/R	Age 1 - 2	Age 2 - 3	Age 3 - 4	Age 4 - 5	Age 5 - 6	Age 6 - 7	Age 7 - 8	Age 8 - 9	Age 9 - Ult	Est Ult L/R
12/98	ŷ	-0.753	0.298	0.118	0.055	0.023	0.008	0.004	0.002	0.001	0.003	
	ŝ	0.011	0.002	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	
	Expected	47.1%	1.347	1.125	1.056	1.023	1.009	1.004	1.002	1.001	1.003	78.6%
	Actual	50.3%	1.353	1.130	1.059	1.022	1.009	1.003	1.002	1.001	1.003	84.7%
12/99	ŷ	-0.385		0.124	0.059	0.022	0.009	0.004	0.002	0.001	0.003	
	ŝ	0.014		0.002	0.002	0.001	0.000	0.000	0.000	0.000	0.000	
	Expected	68.1%		1.132	1.060	1.023	1.009	1.004	1.002	1.001	1.003	85.0%
	Actual	70.8%		1.137	1.062	1.024	1.010	1.004	1.002	1.000	1.003	89.1%
12/00	ŷ	-0.217			0.063	0.023	0.010	0.004	0.001	0.000	0.002	
	ŝ	0.013			0.001	0.001	0.000	0.000	0.000	0.000	0.000	
	Expected	80.5%			1.065	1.024	1.010	1.004	1.001	1.000	1.002	89.3%
	Actual	82.3%			1.069	1.026	1.010	1.004	1.002	1.001	1.003	92.1%
12/01	ŷ	-0.128				0.026	0.011	0.004	0.001	0.000	0.003	
	ŝ	0.008				0.001	0.000	0.000	0.000	0.000	0.000	
	Expected	88.0%				1.026	1.011	1.004	1.001	1.000	1.003	92.1%
	Actual	88.3%				1.028	1.012	1.005	1.002	1.001	1.003	92.9%
12/02	ŷ	-0.096					0.013	0.005	0.002	0.001	0.003	
	ŝ	0.008					0.001	0.000	0.000	0.000	0.000	
	Expected	90.8%					1.013	1.005	1.002	1.001	1.003	93.0%
	Actual	90.2%					1.012	1.004	1.001	1.001	1.003	92.2%
12/03	ŷ	-0.091						0.005	0.002	0.001	0.003	
	ŝ	0.003						0.001	0.000	0.000	0.000	
	Expected	91.3%						1.005	1.002	1.001	1.003	92.4%
	Actual	90.9%						1.005	1.001	1.001	1.003	91.8%

APPENDIX EXHIBIT C-2A

Monte Carlo Simulation of Estimated Ultimate Loss Ratio One Year Out - Paid Development Methods
Accident Year 1999 at Inception

Illustration of One Random Trial - Reflecting Parameter Uncertainty
Commercial Auto Liability

Devt Period	Expected LDF	Sample Size k	Degrees of Freedom k-1	Uniform Random Number R	$T_4^{-1}(R)$	$\hat{y}$	$\hat{s}$	$\sqrt{\frac{k+1}{k}}$	Random Paid LR: Accident Yr Devt *	Random LDF: Revised Tail *
9-Ult	1.013	5	4	0.034	-2.481	0.013	0.000	1.095		1.013
8-9	1.007	5	4	0.665	0.460	0.007	0.000	1.095		1.007
7-8	1.012	5	4	0.879	1.373	0.012	0.000	1.095		1.013
6-7	1.023	5	4	0.954	2.202	0.023	0.001	1.095		1.025
5-6	1.047	5	4	0.056	-2.032	0.046	0.000	1.095		1.047
4-5	1.099	5	4	0.110	-1.456	0.094	0.001	1.095		1.098
3-4	1.203	5	4	0.729	0.664	0.185	0.001	1.095		1.204
2-3	1.426	5	4	0.205	-0.918	0.355	0.003	1.095		1.422
1-2	2.104	5	4	0.025	-2.779	0.744	0.008	1.095		2.051
0-1	18.7%	5	4	0.333	-0.467	-1.678	0.041	1.095	18.3%	
	4.387								18.3%	4.270

* $= \exp(\hat{y} + T_{k-1}^{-1}(R) \cdot \hat{s}\sqrt{(k+1)/k})$

Revised **Paid Chain Ladder** Loss Ratio Estimate One Year Out
= 18.3% x 4.27 = **78.0%**

Revised **Paid Bornhuetter-Ferguson** Loss Ratio Estimate One Year Out
= 18.3% - 18.7% + 18.7% x 4.27 = **79.4%**

APPENDIX EXHIBIT C-2B

Monte Carlo Simulation of Estimated Ultimate Loss Ratio One Year Out - Case Incurred Development Methods
Accident Year 1999 at Inception

Illustration of One Random Trial - Reflecting Parameter Uncertainty
Commercial Auto Liability

Devt Period	Expected LDF	Sample Size k	Degrees of Freedom k-1	Uniform Random Number R	$T_4^{-1}(R)$	$\hat{y}$	$\hat{s}$	$\sqrt{\frac{k+1}{k}}$	Random Case LR: Accident Yr Devt *	Random LDF: Revised Tail *
9-Ult	1.003	5	4	0.034	-2.481	0.003	0.000	1.095		1.003
8-9	1.001	5	4	0.665	0.460	0.001	0.000	1.095		1.001
7-8	1.002	5	4	0.879	1.373	0.002	0.000	1.095		1.002
6-7	1.004	5	4	0.954	2.202	0.004	0.000	1.095		1.005
5-6	1.009	5	4	0.056	-2.032	0.008	0.000	1.095		1.008
4-5	1.023	5	4	0.110	-1.456	0.023	0.000	1.095		1.023
3-4	1.056	5	4	0.729	0.664	0.055	0.001	1.095		1.057
2-3	1.125	5	4	0.205	-0.918	0.118	0.001	1.095		1.124
1-2	1.347	5	4	0.025	-2.779	0.298	0.002	1.095		1.339
0-1	47.1%	5	4	0.333	-0.467	-0.753	0.011	1.095	46.8%	
	1.668								46.8%	1.658

$* \;=\; \exp(\hat{y} + T_{k-1}^{-1}(R) \cdot \hat{s}\sqrt{(k+1)/k})$

Revised Case Incurred **Chain Ladder** Loss Ratio Estimate One Year Out
= 46.8% x 1.658 = **77.6%**

Revised **Case Incurred Bornhuetter-Ferguson** Loss Ratio Estimate One Year Out
= 46.8% - 47.1% + 47.1% x 1.658 = **77.8%**

APPENDIX EXHIBIT D

Calculation of Actual Policyholder Deficits 1999-2004

Commercial Auto Liability – Accident Year 1999

Premiums of $100

25% Expenses / Required Capital 15% of Unpaid Losses / 5% Interest

Calendar Year	1999	2000	2001	2002	2003	2004
n [1]	0	1	2	3	4	5
$T(L_n)$	$75.00	$62.69	$44.61	$28.09	$15.65	$7.88
C_n^R	11.25	9.70	6.87	4.31	2.40	1.21
$r \cdot (C_n^R + T(L_n))$ [2]	3.23	3.62	2.57	1.62	0.90	0.45
S_n	$89.48	$76.01	$54.05	$34.02	$18.95	$9.54
U_{n+1} [3]	85.32	90.02	92.29	93.17	92.65	91.73
P_{n+1}	20.63	23.58	19.31	13.64	7.41	2.74
L_{n+1}	64.69	45.80	28.76	16.01	8.07	4.41
$L_{n+1} + P_{n+1}$	85.32	69.38	48.07	29.65	15.48	7.15
$T(P_{n+1})$	21.15	24.17	19.79	13.98	7.60	2.81
$T(L_{n+1})$	62.69	44.61	28.09	15.65	7.88	4.29
$T(L_{n+1} + P_{n+1})$	$83.84	$68.78	$47.89	$29.62	$15.48	$7.10
C_{n+1}	$5.86	$7.23	$6.17	$4.40	$3.47	$2.44
PD_{n+1}	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
C_{n+1}^R	9.70	6.87	4.31	2.40	1.21	0.66
$C_{n+1}^R - C_{n+1}$	4.06	(0.36)	(1.85)	(2.00	(2.26)	(1.78)

1 n is the lag in years from accident year inception at beginning of year

2 $\frac{3}{4} r \cdot (C_0^R + T(L_0))$ for n=0

3 U_{n+1} is the estimated ultimate loss amount at age n+1