A NONLINEAR REGRESSION MODEL OF INCURRED BUT NOT REPORTED LOSSES by Scott Stelljes

Discussion by Jeffrey H. Adams, FCAS

The paper by Stelljes [1] the subject of this discussion is a welcome addition to the Casualty Actuarial Society literature on nonlinear regression for loss reserving. This discussion will predominantly concern a key assumption made in [1]. In particular, on page 361:

"Based on the assumption that the incremental pure premiums for different development intervals are independent, the variance of IBNR pure premium is the sum of the variances of the incremental pure premiums for the remaining development intervals."

It may be true that the *historical* incremental pure premiums can be considered independent, but it does not follow that the future *fitted* incremental pure premiums are independent. An analogous situation exists for ordinary linear regression, where the *hat* matrix provides for the covariance of the fitted values. Since the variance of the sum of random variables depends on covariance between the random variables, the variance of the reserve will depend on the covariance of the incremental IBNRs.

After providing a brief review on traditional nonlinear regression in section 2, the bulk of this discussion is concerned with two issues. First, modifying the methods of [1] to reflect covariance among the fitted values and is described in section 3. Second, there are times when a reliable insurance trend factor is not available. In such circumstances the actuary needs to derive the trend as part of the model, as in the model on page 359 of [1]. [1] succinctly describes the problems with such an approach. Section 4 discusses this latter model and shows simulation is not required to calculate confidence intervals. The last section, section 5 will discuss some miscellaneous issues.

2. BRIEF REVIEW OF NON LINEAR REGRESSION BASED ON THE BOOK BY MYERS, MONTGOMERY, VINING [4].

Let y be the dependent variable. Let x be a vector of explanatory variables, and **B** a vector of parameters. We then assume the following function:

(2.1) $y = f(x,\mathbf{B}) + \varepsilon$

ε are the errors and are assumed to be independent normal, with the means zero and constant variance σ^2.

(When fitting the data, this assumption should be checked to see if the error assumption is tenable since insurance claim data is often skewed or the errors may be heteroscedastic. [1] notes the heteroscedasticity and thus modifies the error term).

(2.2) $E(y) = f(x,\mathbf{B})$, denotes the expectation of y.

For example let y = $x_1*B_1/(B_3 + x_2*B_2)$ + error. The expectation of y is $f(x,\mathbf{B})$ and is $x_1*B_1/(B_3 + x_2*B_2)$.

Typically, **B** is unknown and replaced with parameter estimates. Based on significance tests (see (2.7) below), it is possible fewer parameters are necessary. Insignificant parameters can be discarded and the function refit.

The parameters may be estimated through nonlinear least squares using the iterative Gauss-Newton method (or other methods).

The (asymptotic) variance covariance matrix of parameter estimators **b** is

(2.3) $\text{var}(\mathbf{b}) \cong \hat{\sigma}^2 (\mathbf{D}^T \mathbf{D})^{-1}$

(2.4) $D_{ij} = \partial f(x_i, \mathbf{B})/\partial B_j$) is evaluated at final parameter estimates.

In (2.4) i refers to the vector of explanatory variables for observation i, and the j refers to the j'th parameter.

An estimate of the error variance is

(2.5) $\hat{\sigma}^2 = \hat{\varepsilon}^T \hat{\varepsilon}/(n-p)$, n is the number of observations fit, and p the number of parameters in **B**.

(2.6) $\hat{\varepsilon} = y - f(x,\mathbf{b})$

(2.7) A parameter significance test is (**b** ÷ (standard error of the parameter)), which is asymptotically the normal distribution. The denominator is the square root of the appropriate element from the diagonal of the asymptotic variance covariance matrix of the parameters (2.3), or for weighted regression (2.11).

Let g(**b**) be a function of the parameter estimators and observations. Then

(2.8) $E(g(\mathbf{b})) \cong g(\mathbf{B})$

The approximate (asymptotic) variance covariance matrix of g(**b**) is

(2.9) $\text{var}(g(\mathbf{b})) \cong \mathbf{d}^T \text{var}(\mathbf{b})\, \mathbf{d}$, where

(2.10) $\mathbf{d}^T = [\, \partial g(\mathbf{B})/\partial B_1, \ldots, \partial g(\mathbf{B})/\partial B_p]$ is evaluated at the estimated parameters.

Equations (2.9) emphasizes the discussion in section 1 regarding the non-independence of fitted values. (Take g(**b**) as the predicted values, then (2.9) can be used to derive the covariance of the predicted values).

If weighted non linear regression is used with a diagonal matrix $\mathbf{V} = \mathrm{var}(y_i) = \mathrm{diag}\{\sigma_1^2,.....,\sigma_n^2\}$; $\sigma_i^2 = \sigma^2 / w_i$, and w_i are the weights then

(2.11) $\mathrm{var}(\mathbf{b}) \cong (\mathbf{D}^T\mathbf{V}^{-1}\mathbf{D})^{-1}$

Weighted non linear regression may be used in the presence of heteroscedasticity.

Let $\mathbf{W} = \mathrm{diag}\{w_1,...,w_n\}$, then

(2.12) $\hat{\sigma}^2 = \hat{\varepsilon}^T(\mathbf{W})\hat{\varepsilon}/(n-p)$ is the mean square error, and

(2.13) $\hat{\sigma}_i^2 = \hat{\sigma}^2 / w_i$, provides an estimate for $\mathbf{V}$.

After the fit, the model assumptions must be checked. Checks include the usual regression error plots.

For loss reserving, errors should also be checked by accident quarter. The accident quarter fitted values by age, should be plotted against the dependent variable pure premium values. This will appraise the fit and the homogeneity of the accident quarters.

3. THE EQUATIONS APPLIED TO LOSS RESERVING WHEN EXTERNAL TREND IS USED

Let c_i represent the accident quarter exposures for observation i. In [1], the exposures are not inflation sensitive and external inflation factors were utilized to trend the incremental pure premiums. If the exposures are inflation sensitive, no additional inflation adjustment is generally required. (However, you may statistically test whether an additional trend factor is required by fitting (4.1) and (4.2). This will be discussed in section 4). If no additional inflation adjustment is required, the methods in section 4 may be applied, and no simulation is required for confidence intervals.

Start with the basic equation given in [1] for future observation(s) y, the future incremental pure premium(s). There is only one explicit explanatory variable x, the valuation age.

(3.0) $f(x,\mathbf{B}) = B_1\exp(xB_2) + B_3\exp(xB_4)$

(3.1) $y = f(x,\mathbf{b}) + \varepsilon / (w^{1/2})$

Multiply (3.1) by exposure c gives

(3.2) $cy = cf(x,\mathbf{b}) + c\,\varepsilon / (w^{1/2})$

Taking the variance of (3.2) gives

(3.3) $\mathrm{variance}(cy) = \mathrm{variance}(cf(x,\mathbf{b})) + \mathrm{variance}(c\,\varepsilon / w^{1/2})$

Now take g(**b**) = cf(x,**b**), and then apply (2.9), (2.10), and (2.11) giving,

(3.4) variance(c y) $\cong$ $\mathbf{d}^T \text{var}(\mathbf{b})\, \mathbf{d} + (c^2)\hat{\sigma}^2 / w$

For equation (3.4) use equation (2.12) to valuate $\hat{\sigma}^2$.

The second term on right hand side of (3.4) is a diagonal matrix, diag $=\{c_i^2 \hat{\sigma}_i^2\}$.

The expectation of (3.2) is

(3.5) E(cy) $\cong$ c f(x,**b**) = g(**b**)

(3.5) provides the vector of means, and (3.4) provides the variance covariance matrix, for a multinormal distribution. It is that distribution that must be sampled to provide an IBNR array. Then, each IBNR value is multiplied by the simulated trend factor, as explained in [1]. Doray [6] page 648 explains a method for simulating the multinormal. The simulations in this discussion were performed in R version 2.4.1 (2006-12-18) (C) 2006 The R Foundation for Statistical Computing.

Exhibit 1 displays a summary and the key results of this discussion. The first four columns are reproduced from Table 3.2.1 of [1]. Columns (7) and (8) are calculated assuming all off diagonal elements of the matrix of (3.4) are set to zero, and then doing 1000 simulations of the multinormal distribution, after which simulated trend factors (using the [1] trending approach) are applied. That is essentially the method in [1]. Columns (5) and (6) are also based on 1000 simulations but incorporate covariance terms of the full matrix (3.4). Although the expected total IBNR are essentially the same in columns (3), (5), (7), and the standard deviations of the total IBNR of (4) and (8) are essentially the same, the standard deviations of the total IBNR in column (6) is significantly higher. Column (6) is the appropriate standard deviation.

Exhibit 2 column (5) and (10) provides a partial listing of the vector of 780 means (3.5) used to simulate the pre- trended IBNRs (these are at calendar quarter 40 level). Exhibit 3 provides a portion of the 780 by 780 variance covariance matrix (3.4).

Accident quarter variances are estimated as a by-product of simulating the entire southeast portion of the loss “triangle”, and should not add up to the variance of total IBNR.

4. THE EQUATIONS APPLIED TO LOSS RESERVING WHEN NO EXTERNAL TREND IS USED

Let y be the incremental losses divided by an inflation or non inflation sensitive exposure base. We use the rejected trend model on page 359 of [1] shown as (4.1) below. (See section 5 paragraph g regarding the extrapolation issue briefly discussed in [1]).

Let B_5 be the trend, u the calendar quarter, and age be the accident quarter valuation age. If an inflation sensitive exposure base is used, B_5 provides for excess trend. (I have assumed the same weights as in [1]. Normally the appropriate weights need to be individually selected for each model).

After the fit, significance levels of the parameters can be checked. If B_5 is not significant then there is no trend other than what is contemplated by the exposure base and age, then $\exp(uB_5)$ may be dropped from equation (4.1) and the model refit.

(4.1) $f(\text{age},u,\mathbf{B}) = (B_1 \exp(B_2\text{age}) + B_3\exp(B_4\text{age}))\exp(uB_5)$

(Denote u and age by the explanatory variable vector x.)

(4.2) $y = f(x,\mathbf{B}) + \varepsilon / (w^{1/2})$

Assume (4.1), (4.2) have been fit to the historical incremental pure premiums. The focus will now be on the future incremental pure premiums.

Using the estimated parameters **b** in (4.2), multiply (4.2) by c to get the future incremental losses:

(4.3) $c\,y = cf(x,\mathbf{b}) + c\,\varepsilon / (w^{1/2})$

Taking the variance of (4.3) gives

(4.4) $\text{variance}(cy) = \text{variance}(cf(x,\mathbf{b})) + \text{variance}(c\,\varepsilon/w^{1/2})$

Now take $g(\mathbf{b}) = cf(x,\mathbf{b})$ and apply (2.9), (2.10), and (2.11) giving

(4.5) $\text{variance}(c\,y) \cong \mathbf{d}^T \text{var}(\mathbf{b})\,\mathbf{d} + (c^2)\hat{\sigma}^2/ w$

For equation (4.5), use equation (2.12) to evaluate $\hat{\sigma}^2$. The second term on the right hand side of (4.5) is a diagonal matrix, $\text{diag} = \{c_i^{\,2}\,\hat{\sigma}_i^{\,2}\}$.

The expectation of (4.3) are the expected future incremental losses

(4.6) $E(cy) \cong g(x,\mathbf{b})$

Now form the sum of the future incremental losses denoted by R for reserve giving

(4.7) $R = \Sigma\, cy$, the sum taken over the southeast portion of the loss “triangle”.

The expectation of R is the mean total reserve and is given by

(4.8) $E(R) \cong \Sigma\, g(x,\mathbf{b})$, the sum taken over the southeast portion of the loss “triangle”.

The variance of R denoted by var(R) is

(4.9) $\text{var}(R) = \Sigma\ \Sigma\ \text{cov}(c_i y_j,\, c_j, y_j)$

In (4.9), the sum is taken over all future observations (i,j) in the southeast portion of the loss triangle. The covariance terms in (4.9) are from (4.5).

Using the normality assumption, the confidence interval for the reserve becomes

(4.10) $E[R] \pm z \cdot var(R)^{1/2}$, z is the appropriate standard normal value.

Applying section 4 equations to Exhibit A data from [1] provides the following:

The estimated parameters for b_1, b_2, b_3 ,b_4, b_5 are 2.364885501 -0.077678377 21.611842502 -0.566532596 0.009735732. The MSE is 2759171.

The parameter variance covariance matrix derived from equation (2.11) is

	b_1	b_2	b_3	b_4	b_5
b_1	0.308171765	-3.248550e-03	2.13645749	-2.257342e-02	-2.046082e-03
b_2	-0.003248550	8.684792e-05	-0.01130499	4.756396e-04	-5.492841e-06
b_3	2.136457489	-1.130499e-02	31.07109676	-2.940411e-01	-1.983273e-02
b_4	-0.022573418	4.756396e-04	-0.29404108	6.488308e-03	-1.960661e-05
b_5	-0.002046082	-5.492841e-06	-0.01983273	-1.960661e-05	3.016662e-05

The parameter standard deviations are the square roots of the diagonal:

0.555132205, 0.009319223, 5.574145384, 0.080550033, 0.005492415 .

The 95% confidence intervals using t(.025,590-5) are

	b_1	b_2	b_3	b_4	b_5
Lower	1.274593134	-0.095981606	10.664076447	-0.724735018	-0.001051543
Upper	3.45518287	-0.05937519	32.55962508	-0.40833007	0.02052296

The trend parameter b_5 is just shy of significance at the 95% level, but will be used.

Exhibit 1, column (9) displays the estimated IBNRs and corresponds to equation (4.6) summed over the accident quarter's IBNRs. The IBNR, by accident quarter and in total, compare favorably with columns (3), (5), and (7), although a bit higher probably due to the higher trend (.0097 versus .005 used by the author). The total IBNR standard deviation calculated using the square root of (4.9) is 3782848, and using (4.10) with z =1.96 provides a 95% reserve confidence interval of : (25254267 , 40083031).

Simulation may also be used to determine confidence intervals. (4.6) provides the vector of means, and (4.5) provides the variance covariance matrix for a multinormal distribution. Exhibit 2 columns (4) and (9) provides a partial listing of the vector of 780 means that may be used to simulate the IBNRs. Exhibit 2 columns (4) and (5) are not comparable, since column (4) already includes trend, while column (5) is still at calendar quarter 40 level. The same applies for columns (9) and (10).

If confidence intervals are desired by accident quarter, the multinormal distribution can be simulated. Accident quarter variances are estimated as a by-product of simulating the entire southeast portion of the triangle, and of course will not add up to the variance of total IBNR. Alternatively, equation (4.9) may be used limiting the summation to the appropriate accident quarter ages. For example, consider accident quarter 4. The portion of the variance covariance matrix (4.5) corresponding to the fourth accident quarter's three IBNR elements is

age	38	39	40
38	605880842	3367957	3291128
39	3367957	582719205	3225207
40	3291128	3225207	560981739

Adding up these nine figures provide the variance for the fourth accident quarter IBNR, which is 1769350371, and a standard deviation of 42064. The diagonal elements are the individual IBNR variances. For example, the variance of the incremental IBNR for accident quarter 4 age 39 is 582719205. Exhibit 1, column (10) displays the standard deviations for the accident quarter IBNRs calculated in such a fashion.

Exhibit 4 displays a partial portion of the variance covariance matrix as calculated in (4.5).

5. MISCELLANEOUS ISSUES

a) On page 354 of [1] "Furthermore, Narayan...remarks that dollar based regression models do not take into account changing levels of exposure. This is a serious flaw because the amount of loss in an accident period is highly correlated to the number of earned exposures." I would concur with this assessment and would suggest incorporating exposure as an explanatory variable in GLM or regression methods, or perhaps an offset in GLM. England and Verrall [2] discuss incorporating exposure in stochastic loss reserving. Incorporating exposure should act to reduce the number of parameters in a GLM or regression type model.

b) Page 231 of [1] formula (2.3.1) should have included the weight function in the minimization since weighted least squares is being performed i.e minimize

$$\sum_{i=1}^{n} w_i(y_i - f(x_i,\mathbf{B}))^2$$

This must have been a typo, and conversations with Stelljes has confirmed this.

c) Page 371 of [1] "Some of the models could be applied to cumulative instead of incremental data." (Page 370 in [1] does note that if autocorrelation occurs other models exist). In my limited experience fitting a single curve to an array of cumulative accident year or report year data results in autocorrelation which violates linear and nonlinear regression assumptions. In addition, heteroscedasticity tends to occur. A plot of the cumulative data for each incurred year versus the fitted curve will help detect autocorrlation as well as detect non-homogeneity of the accident years. A further problem with fitting cumulative data occurs when the estimated ultimate pure premium for a particular incurred year is below the actual emerged pure premium for that year. One way around these problems may be to fit a separate curve to each accident year as in Clark [3] and Kazenski[5]. Kazenski asserts he has detected no autocorrelation using such an approach.

d) Traditional nonlinear regression assumes the error terms are normal which is a symmetric distribution with a range $-\infty$ to $+\infty$. Incremental pure premium data may actually be skewed and can hardly ever be highly negative, therefore, using the normal distribution is approximation at best.

e) Page 358 of [1] formula (2.2.2) should use the square root of the weight, not just the weight. This appears to have been a typo, and conversations with Stelljes has confirmed this. See equation (3.1) above.

f) A note regarding the parameter estimates and the data used for fitting.

[1] excluded the first evaluation of an accident quarter and all evaluations prior to the twenty first calendar quarter when fitting the equation. The same was done in this discussion, both in section 3 and section 4 and section 5 paragraph g. Also, Stelljes [1] has informed me the raw incremental pure premiums (Exhibit A in [1]) are first trended to calendar quarter 40 using a constant trend factor of exp(.005) per calender quarter prior to fitting them. The same was done for the section 3 calculations. Using Exhibit A data (kindly supplied by Stelljes as a computer file), I was able to replicate the following from [1]: parameters on page 362, matrix inversion of $(F'WF)^{-1}$ on page 363, the confidence interval of (-40259,56186) for accident quarter 2 on page 364, and finally, the mean square error of 2987236 on page 364. The parameters in [1] on page 362: 3.1994, -.0754, 29.4446, -.5480 correspond to estimates of B_1, B_2, B_3, B_4 in equation (3.0) of this discussion and are used in section 3.

Keeping within the limited scope of this paper, various diagnostics for the section 4 or section 5 paragraph g fittings have not been performed. Those diagnostic procedures are widely discussed in nonlinear regression texts and should be applied in practice. No claim is made that the fitted parameters are actually the best. Nonlinear regression requires initial starting values, and there is no guarantee the solution will converge, let alone converge to the global minimum mean square error.

g) Extrapolating

In section 4, if B_5 is significant, formula (4.5) extrapolates beyond the fitting space, (in the example for calendar quarters past 40). Discussions with Stelljes, and page 359 in [1] cautions against extrapolating. Pages 86-88 in [4] provides for a confidence interval of a “future observed response”, and seems silent on the issue of extrapolating. Using the approaches in section 4, an alternative model is:

(5.1) $f(age,aqtr,\mathbf{B}) = (B_1 \exp(B_2 age) + B_3 \exp(B_4 age))\exp(B_5\, aqtr)$

where aqtr the accident quarter. Using the same data as in section 4, results from (5.1) were very close to those of (4.1), but even (5.1) will also extrapolate beyond the fitting space when B_5 is significant.

If the variances as calculated by (4.5) appear unreasonable in the extrapolated region, perhaps a ceiling or floor may be required after some point. This seems to be an area requiring further research.

h) On the one hand, the approach in [1] (and section 3), assume the availability of an external trend and that the estimates of the parameter in the model are independent of the trend. On the other hand, it's nonlinear regression model is not extrapolated, only the trend needs to be extrapolated. The section 4 model allows for estimation of internal trend and allows for covariance among all the parameters (including trend), but does require extrapolation when B_5 is significant. Neither method is perfect.

REFERENCES

[1] Scott Stelljes, "A Nonlinear Regression Model of Incurred But Not Reported Losses", Casualty Actuarial Society Forum, Fall 2006 Featuring Reserves Call Papers, pp. 353-377.

[2] Peter D. England and Richard J. Verrall, "A flexible Framework for Stochastic Claims Reserving", Proceedings of the Casualty Actuarial Society 2001 Volume LXXXVIII, pp. 1-38.

[3] Harold E. Clarke , "Recent Developments in Reserving for Losses in the London Reinsurance Market", Proceedings of the Casualty Actuarial Society 1988 Volume LXXV, pp. 1-48.

[4] Raymond H. Myers, Douglas C. Montgomery, G. Geoffrey Vining, Generalized Linear Models With Applications in Engineering and the Sciences, 2002 John Wiley and Sons, Inc., pp. 63-92 discuss nonlinear regression. This book also provides accessible explanations of linear regression, GLM, GEE and GAM.

[5] Paul M. Kazenski, "A Nonlinear Modeling Approach to Assessing the Accuracy of Property-Liability Insurer Loss Reserves", University of Hawaii - Manoa, February 1994.

[6] Louis Doray, "IBNR Reserve Under a Loglinear Location-Scale Regression Model", Casualty Actuarial Society Forum Spring 1994, Volume Two, pp. 607-652.

EXHIBIT I

(1) Accident Quarter	(2) Exposure	(3) [1] Expected Value	(4) [1] Standard Deviation	(5) Discussion Paper section 3 Expected Value	(6) Discussion Paper section 3 Standard Deviation	(7) Check [1] Expected Value	(8) Check [1] Standard Deviation	(9) Discussion Paper section 4 Expected Value	(10) Discussion Paper section 4 Standard Deviation
2	50,801	8,190	24,518	7,489	24,719	7,616	24,912	8,010	23,601
3	51,187	16,643	35,835	16,767	36,204	16,816	33,944	16,872	33,922
4	51,146	26,310	44,192	28,985	45,415	24,058	44,909	26,443	42,064
5	51,527	36,541	51,941	33,429	51,328	37,975	52,022	37,157	49,402
6	52,348	49,099	58,839	49,399	59,053	48,470	60,416	49,380	56,446
7	52,480	61,528	65,232	60,100	69,592	60,716	65,327	62,191	62,790
8	53,148	75,340	71,800	75,401	72,824	76,815	72,159	76,954	69,266
9	53,924	91,671	78,552	93,025	79,352	90,003	80,072	93,486	75,738
10	54,403	109,065	85,433	112,127	88,895	108,506	87,839	111,208	81,966
11	54,557	124,874	91,436	126,736	94,084	125,926	91,494	129,920	87,919
12	55,083	144,622	96,258	149,578	100,674	141,166	94,407	151,342	94,221
13	55,292	168,450	103,341	175,839	107,340	166,273	101,628	173,891	100,296
14	55,899	192,189	108,233	189,828	117,084	183,868	108,754	199,906	106,864
15	56,067	215,948	115,108	218,495	119,945	218,185	113,886	226,736	113,100
16	57,025	247,643	123,187	245,486	126,152	249,288	119,610	259,542	120,393
17	57,071	279,736	129,481	277,633	136,171	279,801	129,502	291,148	126,815
18	57,317	311,248	134,933	305,717	133,675	311,388	134,122	326,584	133,667
19	57,907	346,819	143,714	346,509	143,603	336,674	140,549	367,375	141,225
20	58,285	388,878	149,405	383,582	151,327	387,150	152,150	410,598	148,789
21	59,096	433,974	157,772	435,640	164,002	427,185	163,959	461,162	157,349
22	59,193	479,592	165,473	474,623	173,326	478,486	161,765	510,590	165,192
23	59,524	530,342	173,337	524,379	177,440	528,747	169,566	566,470	173,823
24	59,745	583,879	177,894	585,037	183,270	573,480	175,996	626,235	182,747
25	60,427	645,944	188,083	652,774	204,720	639,599	194,014	696,579	193,112
26	60,155	705,701	195,557	709,139	199,170	706,895	193,614	761,641	202,285
27	60,568	776,239	207,953	776,419	222,299	788,439	203,526	841,356	213,588
28	60,708	852,632	215,059	863,905	225,281	844,677	209,276	924,383	225,219
29	60,262	925,896	222,578	921,837	235,006	924,073	229,328	1,005,182	236,460
30	60,606	1,012,197	233,755	1,015,105	247,787	1,016,063	247,362	1,107,100	250,821
31	60,580	1,109,304	251,368	1,099,773	268,201	1,094,682	247,988	1,212,155	265,684
32	60,648	1,213,637	258,802	1,227,733	267,445	1,221,054	254,047	1,330,473	282,513
33	61,159	1,344,114	277,079	1,325,154	281,107	1,348,687	269,254	1,473,989	302,862
34	61,462	1,492,000	292,032	1,470,864	296,064	1,509,526	298,480	1,633,463	325,285
35	61,934	1,660,873	312,021	1,664,619	328,967	1,665,426	304,419	1,826,677	351,853
36	61,716	1,858,275	333,112	1,867,446	348,580	1,863,920	337,684	2,040,965	380,446
37	61,837	2,123,409	361,113	2,128,841	352,122	2,140,963	343,229	2,330,037	417,181
38	62,285	2,514,004	394,000	2,499,739	392,466	2,521,633	404,654	2,738,893	466,097
39	62,728	3,055,695	450,062	3,069,822	465,666	3,061,935	443,104	3,329,815	532,473
40	63,180	3,892,584	522,958	3,892,268	515,975	3,878,801	501,528	4,232,741	633,498
Totals		30,105,085	1,350,093	30,101,242	2,210,162	30,104,966	1,348,733	32,668,649	3,782,848

Exhibit 2

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
			Section 4	Section 3				Section 4	Section 3
			Incremental	Incremental				Incremental	Incremental
aqtr	age	expos	IBNR	IBNR	aqtr	age	expos	IBNR	IBNR
2	40	50801	8010	7964	40	2	63180	846121	795568
3	39	51187	8723	8653	40	3	63180	553745	520639
3	40	51187	8150	8024	40	4	63180	381677	357292
4	38	51146	9420	9323	40	5	63180	278847	258771
4	39	51146	8801	8646	40	6	63180	215972	198022
4	40	51146	8223	8018	40	7	63180	176251	159387
5	37	51527	10256	10128	40	8	63180	150042	133789
5	38	51527	9583	9392	40	9	63180	131801	115967
5	39	51527	8953	8710	40	10	63180	118339	102858
5	40	51527	8365	8077	40	11	63180	107812	92679
6	36	52348	11261	11095	40	12	63180	99153	84382
6	37	52348	10522	10289	40	13	63180	91736	77348
6	38	52348	9831	9542	40	14	63180	85192	71207
6	39	52348	9185	8849	40	15	63180	79299	65733
6	40	52348	8582	8206	40	16	63180	73920	60784
7	35	52480	12202	11994	40	17	63180	68967	56268
7	36	52480	11400	11123	40	18	63180	64381	52123
7	37	52480	10651	10315	40	19	63180	60120	48304
7	38	52480	9952	9566	40	20	63180	56153	44776
7	39	52480	9298	8871	40	21	63180	52454	41513
7	40	52480	8687	8227	40	22	63180	49002	38491
8	34	53148	13355	13098	40	23	63180	45780	35692
8	35	53148	12478	12147	40	24	63180	42771	33098
8	36	53148	11658	11264	40	25	63180	39960	30693
8	37	53148	10893	10446	40	26	63180	37335	28463
8	38	53148	10177	9688	40	27	63180	34882	26395
8	39	53148	9509	8984	40	28	63180	32591	24478
8	40	53148	8884	8332	40	29	63180	30450	22700
9	33	53924	14645	14330	40	30	63180	28450	21051
9	34	53924	13683	13289	40	31	63180	26581	19522
9	35	53924	12784	12324	40	32	63180	24835	18104
9	36	53924	11944	11429	40	33	63180	23203	16789
9	37	53924	11160	10599	40	34	63180	21679	15570
9	38	53924	10427	9829	40	35	63180	20255	14439
9	39	53924	9742	9115	40	36	63180	18925	13391
9	40	53924	9102	8453	40	37	63180	17682	12418
10	32	54403	15968	15589	40	38	63180	16520	11516
10	33	54403	14919	14457	40	39	63180	15435	10680
10	34	54403	13939	13407	40	40	63180	14421	9904
10	35	54403	13024	12433					
10	36	54403	12168	11530					
10	37	54403	11369	10693					
10	38	54403	10622	9916					
10	39	54403	9924	9196					
10	40	54403	9273	8528					

Exhibit 3

aqtr		2	3	3	4	4	4
aqtr	age	40	39	40	38	39	40
2	40	602,854,869	3,112,619	3,014,973	3,204,500	3,110,126	3,012,558
3	39	3,112,619	631,054,179	3,136,270	3,334,387	3,235,699	3,133,757
3	40	3,014,973	3,136,270	607,458,435	3,228,848	3,133,757	3,035,448
4	38	3,204,500	3,334,387	3,228,848	655,671,475	3,331,716	3,226,262
4	39	3,110,126	3,235,699	3,133,757	3,331,716	630,546,122	3,131,247
4	40	3,012,558	3,133,757	3,035,448	3,226,262	3,131,247	606,969,439
5	37	3,319,123	3,454,230	3,344,343	3,557,965	3,451,463	3,341,664
5	38	3,228,371	3,359,226	3,252,901	3,459,481	3,356,535	3,250,295
5	39	3,133,294	3,259,802	3,157,102	3,356,535	3,257,191	3,154,573
5	40	3,034,999	3,157,102	3,058,060	3,250,295	3,154,573	3,055,610
6	36	3,458,533	3,599,973	3,484,812	3,708,826	3,597,089	3,482,020
6	37	3,372,008	3,509,268	3,397,630	3,614,656	3,506,457	3,394,908
6	38	3,279,810	3,412,750	3,304,731	3,514,602	3,410,016	3,302,084
6	39	3,183,218	3,311,742	3,207,405	3,410,016	3,309,089	3,204,836
6	40	3,083,357	3,207,405	3,106,785	3,302,084	3,204,836	3,104,296
7	35	3,546,852	3,692,652	3,573,802	3,805,149	3,689,694	3,570,939
7	36	3,467,254	3,609,050	3,493,599	3,718,178	3,606,160	3,490,801
7	37	3,380,511	3,518,116	3,406,197	3,623,770	3,515,299	3,403,469
7	38	3,288,080	3,421,355	3,313,064	3,523,465	3,418,615	3,310,410
7	39	3,191,245	3,320,093	3,215,493	3,418,615	3,317,433	3,212,917
7	40	3,091,132	3,215,493	3,114,619	3,310,410	3,212,917	3,112,124
8	34	3,663,703	3,815,168	3,691,541	3,932,367	3,812,112	3,688,584
8	35	3,591,998	3,739,655	3,619,291	3,853,584	3,736,659	3,616,392
8	36	3,511,387	3,654,989	3,538,068	3,765,505	3,652,061	3,535,234
8	37	3,423,540	3,562,897	3,449,553	3,669,896	3,560,044	3,446,790
8	38	3,329,933	3,464,905	3,355,235	3,568,314	3,462,129	3,352,547
8	39	3,231,865	3,362,353	3,256,422	3,462,129	3,359,660	3,253,813
8	40	3,130,478	3,256,422	3,154,264	3,352,547	3,253,813	3,151,737
9	33	3,778,996	3,936,224	3,807,710	4,058,262	3,933,071	3,804,660
9	34	3,717,196	3,870,872	3,745,440	3,989,782	3,867,772	3,742,440
9	35	3,644,444	3,794,256	3,672,136	3,909,849	3,791,217	3,669,194
9	36	3,562,656	3,708,354	3,589,726	3,820,484	3,705,384	3,586,851
9	37	3,473,527	3,614,918	3,499,919	3,723,479	3,612,023	3,497,116
9	38	3,378,553	3,515,495	3,404,224	3,620,414	3,512,679	3,401,497
9	39	3,279,053	3,411,446	3,303,968	3,512,679	3,408,713	3,301,321
9	40	3,176,185	3,303,968	3,200,318	3,401,497	3,301,321	3,197,755
10	32	3,861,688	4,023,507	3,891,031	4,149,543	4,020,284	3,887,914
10	33	3,812,564	3,971,189	3,841,533	4,094,311	3,968,008	3,838,456

Exhibit 4

aqtr		2	3	3	4	4	4
aqtr	age	40	39	40	38	39	40
2	40	557,010,999	3,076,909	3,007,116	3,181,310	3,115,232	3,044,109
3	39	3,076,909	583,099,003	3,141,809	3,324,615	3,255,594	3,181,296
3	40	3,007,116	3,141,809	561,346,864	3,249,349	3,183,066	3,111,471
4	38	3,181,310	3,324,615	3,249,349	605,880,842	3,367,957	3,291,128
4	39	3,115,232	3,255,594	3,183,066	3,367,957	582,719,205	3,225,207
4	40	3,044,109	3,181,296	3,111,471	3,291,128	3,225,207	560,981,739
5	37	3,309,854	3,459,950	3,381,707	3,580,453	3,506,202	3,426,258
5	38	3,248,025	3,395,306	3,319,801	3,513,545	3,442,061	3,364,808
5	39	3,180,038	3,324,227	3,251,437	3,439,980	3,371,218	3,296,648
5	40	3,106,968	3,247,836	3,177,730	3,360,919	3,294,831	3,222,924
6	36	3,465,072	3,623,357	3,541,518	3,750,844	3,673,110	3,589,404
6	37	3,408,292	3,563,915	3,484,814	3,689,234	3,614,287	3,533,269
6	38	3,344,022	3,496,650	3,420,281	3,619,536	3,547,345	3,469,022
6	39	3,273,491	3,422,845	3,349,190	3,543,076	3,473,604	3,397,971
6	40	3,197,796	3,343,649	3,272,681	3,461,044	3,394,244	3,321,287
7	35	3,571,152	3,735,590	3,651,328	3,868,496	3,788,379	3,702,096
7	36	3,521,668	3,683,685	3,602,101	3,814,585	3,737,218	3,653,557
7	37	3,463,277	3,622,482	3,543,587	3,751,066	3,676,435	3,595,423
7	38	3,397,363	3,553,427	3,477,220	3,679,434	3,607,512	3,529,163
7	39	3,325,166	3,477,814	3,404,282	3,601,029	3,531,782	3,456,095
7	40	3,247,795	3,396,803	3,325,924	3,517,048	3,450,436	3,377,400
8	34	3,708,164	3,880,415	3,793,016	4,020,164	3,936,970	3,847,360
8	35	3,667,175	3,837,284	3,752,499	3,975,213	3,894,734	3,807,677
8	36	3,615,573	3,783,079	3,700,941	3,918,824	3,841,063	3,756,609
8	37	3,554,928	3,719,439	3,639,966	3,852,693	3,777,639	3,695,822
8	38	3,486,652	3,647,838	3,571,034	3,778,341	3,705,975	3,626,814
8	39	3,412,007	3,569,596	3,495,454	3,697,135	3,627,427	3,550,925
8	40	3,332,125	3,485,894	3,414,395	3,610,295	3,543,208	3,469,358
9	33	3,846,268	4,026,672	3,936,130	4,173,643	4,087,347	3,994,380
9	34	3,815,764	3,994,377	3,906,352	4,139,764	4,056,121	3,965,605
9	35	3,772,676	3,948,954	3,863,507	4,092,331	4,011,365	3,923,380
9	36	3,718,786	3,892,266	3,809,438	4,033,270	3,954,991	3,869,594
9	37	3,655,700	3,825,989	3,745,804	3,964,313	3,888,716	3,805,947
9	38	3,584,857	3,751,625	3,674,094	3,887,013	3,814,085	3,733,966
9	39	3,507,548	3,670,523	3,595,643	3,802,764	3,732,479	3,655,021
9	40	3,424,928	3,583,889	3,511,646	3,712,812	3,645,138	3,570,336