Incorporating Systematic Risk Into The RMK Framework

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Abstract
The RMK pricing algorithm provides a method for pricing insurance contracts or reinsurance deals. This paper discusses the incorporation of systematic, or non-diversifiable, risk into the RMK framework.

1. A SIMPLE EXAMPLE OF THE RMK METHOD

Ruhm/Mango (2003) present a simple illustration of the RMK pricing algorithm. Specifically, this simple example assumes that the insurance (or reinsurance) company writes two risks, each with the following state-dependent loss vector:

State	Risk 1 Loss	Risk 2 Loss	Portfolio Loss	Probability
1	$100	$100	$200	35%
2	$100	$200	$300	15%
3	$200	$100	$300	25%
4	$200	$200	$400	25%

The RMK algorithm incorporates an adjustment for risk by means of a set of *outcome-specific weights*. For this example, Ruhm/Mango utilize the following set of risk-averse outcome weights:

Portfolio Outcome	Risk-Averse Outcome Weight
$200	0.500
$300	1.000
$400	1.250

These risk-averse outcome weights are similar to Mango's (2003) concept of a *cost function*. Mango points out that such a function can be interpreted as a corporate utility function; that is, in some sense, management has determined that a $300 aggregate loss is "twice as bad" as a $200 aggregate loss.[1]

These risk-averse weights are then normalized (scaled so that their expected value is one) to produce the following vector of normalized weights:

[1] Fama and Miller (1972) point out the many theoretical difficulties involved in interpreting and determining a "corporate" utility-of-wealth function. However, for purposes of this paper, we will assume that such a function has been determined by some means.

Portfolio Outcome	Normalized Weight
\$200	0.563
\$300	1.127
\$400	1.408

The RMK method then determines the "risk load" for each of the two risks according to the following formula:

Risk Load for Risk i = Cov(Ri,Z),

> where Ri is the loss amount for each of the two risks (i = 1, 2),
> and Z is the vector of normalized weights.

Thus, the resulting risk load is \$13.38 for Risk 1 and \$12.11 for Risk 2. The final premium is then determined by discounting the expected loss for each risk (at the risk-free rate of interest), then adding the risk load. Assuming that losses are payable at the end of one year, and a risk-free interest rate of 2%, the final RMK premiums are as follows:

Premium for Risk 1 = \$150/1.02 + \$13.38 = \$160.44
Premium for Risk 2 = \$140/1.02 + \$12.11 = \$149.36

2. THE FINANCIAL PRICING METHOD

By comparison, let's utilize a financial pricing method to price each of the risks in the previous example. If we ignore default costs, then the financial premium formula reduces to the following equation:

Premium = Present Value of Expected Loss (at risk-adjusted rate) + Capital * Cost of Capital

According to the Capital Asset Pricing Model (CAPM), the risk-adjusted discount rate for the loss amount depends on the relationship between the loss random variable and the return on the market portfolio. Let's assume the following state-specific returns (Rm) for the market portfolio:

State	Return on Market Portfolio (Rm)
1	+25%
2	+10%
3	+4%
4	-5%

For each of the two risks, the present value of the expected loss at the risk-adjusted rate, or PV(Ri), is determined according to the certainty-equivalent version of the CAPM:[2]

PV(Ri) = E(Ri) / (1 + Rf) – [Lambda * Cov(Ri,Rm)] / (1+Rf),

> Where E(Ri) is the expected loss amount for each of the two risks,
> Rf is the risk-free rate of interest,
> And Lamda is the "market price of risk" given by:
>> Lambda = [E(Rm) – Rf] / Var(Rm)

According to our assumptions regarding the return on the market portfolio, we calculate the following values:

Cov(R1,Rm) = -5.25
Cov(R2,Rm) = -3.75
Lambda = 5.56
PV(R1) = $175.65
PV(R2) = $157.68

In the financial formula, the "cost of capital" is primarily due to double taxation and agency costs. Let's arbitrarily assume that the cost of capital is 10% of the required capital. The required capital for each policy is generally determined by allocating the total capital down to the risk, or policy, level. In the financial method, this allocation method is generally based on some form of Option Pricing Theory (OPT). However, for simplicity, let's assume that a total capital amount is $200, and that it will be allocated in proportion to the expected loss amount for each risk. The premium for each risk is then given as follows:

Premium for Risk 1 = $175.65 + 10% of $103.45 = $186.00
Premium for Risk 2 = $157.68 + 10% of $96.55 = $167.33

3. EXPLAINING THE DIFFERENCES BETWEEN THE RMK AND FINANCIAL PREMIUMS

In the previous two sections, the Financial method resulted in a much higher required premium for each policy than the RMK method. There are two major reasons for this discrepancy.

First, the RMK method requires a "calibration" to ensure that the resulting combined ratio and return on equity are in accordance with the overall corporate objectives. Mango (2003) discusses the issue of calibration in detail, but the procedure is outside the scope of this paper. Presumably, the overall return implied by management's risk-averse outcome weights would determined; if this overall return falls short of corporate targets, there would need to be a feedback loop back to management to adjust the weights. The

[2] See the Appendix to Chapter 9 of Brealey and Myers (2000) for a derivation of this formula.

procedure would continue until a set of weights had been identified that resulted in an acceptable corporate return.

Second, the financial method incorporates additional data and assumptions regarding the state-return on the market portfolio. In other words, the financial method incorporates the "systematic risk" of the loss variables, whereas the RMK method did not. Since these loss variables possessed a negative covariance with the market return, the incorporation of systematic risk resulted in an increase in the required premium.

This begs the question: can we adjust the RMK method from Section 1 to reflect the market return data – and the "systematic risk" of the loss variables? This question will be explored in the following section.

4. A METHOD FOR REFLECTING SYSTEMATIC RISK IN THE RMK ALGORITHM

Mango (2004) presents a simplified flow-chart method for incorporating systematic risk into the RMK framework. Essentially, the method combines the results of the insurer's underwriting portfolio and the insurer's asset portfolio to produce a state-specific *net income* distribution. This net income distribution then serves as the *reference portfolio* for the RMK application.

In order to determine this net income distribution, we need to develop some assumptions regarding the insurer's investment (or asset) portfolio. For this example, let's assume that 80% of the insurer's assets are invested in risk-free bonds, earning the risk-free rate of 2%; the remaining 20% of the insurer's assets are invested in the market portfolio, earning the state-specific returns provided in Section 2.

Since we are now dealing with net income, management's risk preferences must be stated in terms of various net income amounts (as opposed to aggregate loss amounts). Let's assume that management has developed the risk-averse outcome weights as a function of various net income amounts. Again, there is an intuitive interpretation of this risk aversion function.[3] For instance, let's say that the outcome-specific weight is 1.25 for net income of $50 and 0.25 for net income of $150; in this sense, management views a net income result of only $50 as being "five times as bad" as a higher net income result of $150.

In this case, the RMK method requires an iterative approach, since the resulting premium amount impacts both the underwriting income and the investment income.[4] With the asset allocation assumptions above – together with some assumed values for the risk-averse outcome weights -- the resulting premium is $171.77 for Risk 1 and $160.47 for Risk 2. The following chart and formulas provide the details of the calculation:

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[3] And, again, we will ignore the theoretical and practical difficulties involved in determining this function.
[4] Investment income is impacted since total assets are equal to total premium plus total surplus.

State	Probability	Aggregate Loss	Market Return	Net Income	Mgt. Risk Weight	Normalized Weight (Z)
1	0.35	$200	25%	$167.37	0.25	0.230
2	0.15	$300	10%	$51.41	1.25	1.149
3	0.25	$300	4%	$45.02	1.25	1.149
4	0.25	$400	-5%	-$64.56	2.00	1.839

Risk Load for Risk 1 = Cov(R1,Z) = $24.71
Risk Load for Risk 2 = Cov(R2,Z) = $23.22
Premium for Risk 1 = $150/1.02 + $24.71 = $171.77
Premium for Risk 2 = $140/1.02 + $23.22 = $160.47

Also, it may be helpful to illustrate the calculation of the net income amount for state 1. In this state, the income variables are as follows:

Underwriting Income = Total Premium – Aggregate Loss = $332.24 - $200 = $132.24
Total Assets = Total Premium + Surplus = $332.24 + $200 = $532.24
Assets Invested in Market Portfolio: 20% of $532.24 = $106.45
Assets Invested in Bond Portfolio: = 80% of $532.24 = $425.79
Investment Income from Market Portfolio = 25% return on $106.45 = $26.62
Investment Income from Bond Portfolio = 2% return on $425.79 = 8.52
Total Income[5] = $132.24 + $26.62 + $8.52 = $167.38

Net income for the other states is determined in a similar manner.

5. POTENTIAL PROBLEMS WITH THE MANGO ADJUSTMENT FOR SYSTEMATIC RISK

In some sense, the method in Section 4 does provide an adjustment for systematic risk, since the insurer's net income depends (to a certain extent) on the return on the market portfolio. However, the sensitivity of the insurer's net income to the market return will depend on the insurer's asset allocation. For example, if the insurer is invested entirely in risk-free bonds, then net income will be unaffected by market return.

Moreover, in a practical situation, the insurance company invests in many more asset types than simply a "market portfolio" and risk-free bonds. Insurers may invest in corporate bonds, some sampling of common and preferred stocks, real estate, etc. In addition, the insurer's common stock portfolio may not be fully diversified, but invested in only a handful of individual stock holdings. In this case, the net income approach will reflect the risk characteristics of the insurer's asset portfolio, but it would be incorrect to say that it has "incorporated systematic risk" into the analysis.

[5] This is actually total income prior to federal income taxes. We are ignoring federal income taxes in this example.

As an alternative, we may wish to calculate the insurer's net income distribution by fully utilizing the return on the representative market portfolio -- that is, make the assumption that the insurer is 100% invested in the market portfolio. This approach is still subject to the following drawbacks:

1. Instead of using the assumption of 100% in the market portfolio, we could have used some other hypothetical mixture, such 75% in the market portfolio and 25% in risk-free bonds. It isn't clear which representative mixture best incorporates "systematic risk" into the net income distribution. And, in general, the resulting risk loads (and premiums) will vary on the basis of the assumed allocation.
2. It becomes much harder to provide any intuitive meaning to the *risk-averse outcome weight*. The subject of these outcome weights is now a complicated intermingling of the market return volatility and the insurance portfolio volatility – and may bear little resemblance to the actual net income result for the insurance company in any particular state.
3. There are already a variety of financial approaches for reflecting the systematic risk of a cash flow (e.g. CAPM, APT, Fama-French Three Factor Models). These models are not based on judgmental assessments of management's risk preferences, but financial theories regarding equilibrium in capital markets. By combining the adjustments for systematic risk and insurance risk into one step, we are not able to utilize these financial theories regarding systematic risk.

It is possible, in theory, to determine a set of risk-averse outcome weights for the RMK procedure that will duplicate the premiums from the financial model.[6] This, however, provides little guidance to the actuary who is pricing a reinsurance deal "from scratch". That is, assuming that the answer is not known in advance, the pricing actuary must determine a set of risk-averse outcome weights from a reference portfolio that has little (if any) intuitive or practical meaning.

6. AN ALTERNATIVE METHOD FOR INCORPORATING SYSTEMATIC RISK INTO RMK

As an alternative to the method in Section 4, we can accommodate systematic risk within an RMK framework simply by discounting the expected losses at a risk-adjusted discount rate. In other words, simply utilize the RMK risk loads from Section 1, but adjust the discount rate for the losses in accordance with financial theory.

For instance, according to the certainty-equivalent version of the CAPM, the present value of expected losses for each of the two risks was given as follows (per Section 2):

PV(R1) = \$175.65
PV(R2) = \$157.68

[6] Assuming that the surplus allocation in the financial model is additive, which it is in this case.

According to the RMK method of Section 2, the risk loads (excluding systematic risk) were \$13.38 for Risk 1 and \$12.11 for Risk 2. By adding these risk loads to the present value (at the risk-adjusted rate) of losses, we get the following premiums:

Premium for Risk 1 = \$175.65 + \$13.38 = \$189.03
Premium for Risk 2 = \$157.68 + \$12.11 = \$169.79

By using this method, we can also "bridge the gap" between the financial method and the RMK method. Specifically, in the financial method in Section 2, we allocated capital in proportion to expected losses. As an alternative, let's allocate capital in accordance with the risk-averse outcome weights assigned to the various aggregate loss amounts in Section 1. In other words, allocate capital to each risk in proportion to that risk's relative contribution to the covariance between the aggregate loss outcome and the Z-vector. For instance, the percentage of capital allocated to Risk 1 is Cov(R1,Z) / Cov (Aggregate Portfolio Outcome,Z) = \$13.38 / \$25.49 = 52.5%.[7] According to the Z-vector from Section 1, the \$200 capital would then be allocated at \$104.99 for Risk 1 and \$95.01 for Risk 2. The financial premiums then become:

Risk 1 Premium = \$175.65 + 10% of \$104.99 = \$186.15
Risk 2 Premium = \$157.68 + 10% of \$95.01 = \$167.18

Lastly, the final reconciliation issue is simply a problem of "calibration" (per the terminology in Mango). That is, there is no reason to expect that the total capital (\$200) and cost of capital (10%) in the financial model will produce the same ROE as the RMK method. But, on an individual policy level, the ratio between the premiums for each risk is the same. Thus, we can complete the reconciliation by changing either the total capital or the cost of capital in the financial model. Let's change the cost of capital to 12.75%, which will complete the reconciliation:

Risk 1 Premium = \$175.65 + 12.75% of \$104.99 = \$189.03
Risk 2 Premium = \$157.68 + 12.75% of \$95.01 = \$169.79

7. SUMMARY

This paper has presented two proposed methods for incorporating systematic risk into the RMK pricing algorithm. The Mango (2004) method is plagued by an assortment of theoretical and practical problems. In short, the best method for incorporating systematic risk into the RMK framework is simply to discount the expected losses at a risk-adjusted hurdle rate. This risk-adjusted rate can be determined by any one of the common financial pricing models, including the CAPM, the APT, or the Fama-French Three Factor Model.

[7] Also, note that this is just the ratio of each of the individual risk loads to the total risk load.

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