# A DATABASE IN 3-D

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*Abstract*

*Three-dimensional geometry and calculus are useful conceptual and analytical tools for working with valuations of insurance statistics. Geometry can be used to provide a pictorial representation of a database and illustrate differences between calendar, exposure/accident and policy year concepts. Calculus can be used to estimate on-level, trended and developed statistics used in ratemaking and reserving. Also, three dimensions lead to several practical two-dimensional methods when the generality of three dimensions is not required.*

## 1. INTRODUCTION

An insurer keeps track of various information over the course of doing business. Databases are maintained recording the result of exposure base, premium and claims transactions for financial reporting, statistical reporting, ratemaking and reserving purposes, for example. The goal of this paper is to develop a pictorial representation of a database as three dimensions (3-D) with values attached to points. The result is a simple, conceptual and analytical tool useful for working with valuations of insurance statistics. Our approach is summarized as follows:

- Section 2 develops 3-D as a pictorial representation of a database.
- Section 3 includes conceptual applications of Section 2 using an example from ratemaking.
- Section 4 includes closing comments.
- The Appendix provides further discussion of the 3-D approach and analytical applications of Section 2 using methods from basic calculus.

## 2. A DATABASE IN 3-D

A variety of exposure base, premium and claims transactions make up a database and are collected to produce reports. In this section we: 1) represent a database as points in 3-D with values attached; and 2) represent a report as a collection of points in 3-D with a collective value attached.

### *2.1 A Database in 3-D*

For the transaction: "On a policy issued 1-1-04 covering a claim incurred 8-1-04, pay $5,000 on 2-1-05."; we identify three dates and a value: 1) "1-1-04", the policy date or date the policy was written or issued; 2) "8-1-04", the exposure date or date the policy was in force and exposed to loss; 3) "2-1-05", the valuation date or date the transaction was made; and 4) "$5,000 paid losses", the amount and type of statistic. By defining xyz-space coordinates x, y and z as **policy date**, **exposure lag** and **valuation lag**, respectively – so that x+y and x+y+z are defined as exposure date and valuation date, respectively – we can represent the transaction as the "valued" point in 3-D, ((1-1-04,7,6),5000), length 1 on an axis equal to 1 month.

TABLE 1

TRANSACTION, ((X,Y,Z),S), 3-D COORDINATES

| Combinations | Interpretations* |
| --- | --- |
| • x, y, z | • policy date, exposure lag, valuation lag |
| • x+y, x+y+z | • exposure date, valuation date |
| • y+z, z | • policy age, exposure age |

*Policy, exposure and valuation date are synonymous with issue/written, in-force and transaction date, respectively. Exposure date and lag become claim (or accident) date and lag, respectively, when dealing with claim statistics. Note the two interpretations for the z-coordinate as a type of lag and age.

2.1.a. *In 3-D represent the transaction: "To a policy written, exposed and valuated at times $x_o$, $x_o+y_o$ and $x_o+y_o+z_o$, respectively, assign* $s_o$ *statistic units."; as valued point $((x_o,y_o,z_o),s_o)$.*

Table 1 lists the various definitions and interpretations of xyz-coordinate combinations used throughout the paper. We plot several transactions on a policy issued 1-1-04 next, length 1 on an axis equal to 1 month [1].

2.1.a.1. The initial transaction to record $4,320 in written premium is made at issue so that the valuation date equals the policy date; ((1-1-04,0,0),4320).

2.1.a.2. The transaction to record $12 in earned premium on 3/1/04 is made at the time the policy is in-force on 3/1/04; ((1-1-04,2,0),12) [2].

2.1.a.3. On 6/1/04 corrective transactions are made since the policy was actually written for $8,640 and recorded in error. Transaction ((1-1-04,0,5),4320) corrects for written premium transaction 2.1.a.1 and transaction ((1-1-04,2,3),12) corrects for earned premium transaction 2.1.a.2.

2.1.a.4. An endorsement for additional coverage for $400 written premium is processed mid-term on 9/1/04; ((1-1-04,8,0),400).

2.1.a.5. The policy is cancelled without penalties on 12/1/04 with a full refund of a month's premiums unearned on the base policy for $720(=$8,640/12) and on the endorsement for $100(=$400/4); ((1-1-04,0,11),-720) & ((1-1-04,8,3),-100).

2.1.a.6. The transaction to record a claim incurred 8/1/04 is made on 8/15/04; ((1-1-04,7,0.5),1).

A transaction's policy and valuation dates are recorded in practice. Exposure date – unless associated with a date of loss – is not recorded and is an abstract concept we introduce for purposes of the presentation. Imagine that an annual policy is made up of 365 separate daily policies, 8,760 separate hourly policies, etc., until we view the policy as a **post**[3] continuum. We introduced exposure date so that transactions tracked by policy in practice could be tracked by post in theory.

[1] We invite the reader to express each transaction in the form provided by 2.1.a by identifying the three dates and value, and plot all transactions in the same 3-D diagram.

[2] This transaction is "implicit" as no actual record is made to record earnings.

[3] Mnemonic for "a Policy expOSed to loss at a point in Time".

We extend 2.1.a to apply to a database of transactions. For simplicity we assume all the transactions in the database are for the same statistic.

2.1.b. *In 3-D represent a database of transactions for a single statistic by D=(P,f).*

In 2.1.b, P is the collection of points, (x,y,z), resulting from plotting transactions using 2.1.a and *f* is the rule assigning value to points of P, *f*(x,y,z), derived as the sum of the statistic over all transactions assigned to point (x,y,z). Thus, D=(P,*f*) is a collection of points of the form, ((x,y,z),*f*(x,y,z)). Figure 1(a) is a generic database D with the *f*-values assumed color-coded to simplify the illustration. Note:

2.1.b.1 D ranges from the collectible (e.g., written premium transactions in company records) to the hypothetical (e.g., written premium projections).

2.1.b.2 D (or more precisely P in D=(P,*f*)) appears differently by type of statistic. Under some basic conventions D is: 1) confined to the x-axis for written statistics; 2) confined to the xy-plane for earned statistics; and 3) unrestricted in space for claim statistics. [4]

2.1.b.3 D appears differently by coverage for the same statistic. Private Passenger Auto Physical Damage paid losses close rather quickly when compared to Workers Compensation paid losses. Transaction activity that occurs long after the accident date is reflected in larger z-coordinate values for points in D. Thus, we would expect $D^*$ for auto paid losses to be generally closer to the xy-plane than $D^{**}$ for Workers Compensation paid losses.

### *2.2 A Report in 3-D*

A report involves collecting statistical information from specific database transactions. In 3-D this amounts to identifying a subset of space along with the total statistic value associated with that subset. The report: "Accident Year 2001 as of 12/31/2002 totaled $31.3 Million Paid Losses."; is a very basic type of report we call a **valuation** characterized by: 1) a **data organization** (i.e., Accident Year 2001); 2) a **status** (i.e., "as of 12/31/2002"); and 3) a statistic **level** (i.e., $31.3 Million Paid Loss).

2.2.a. *In 3-D represent a valuation by O.V.s.*

In 2.2.a, O and V (O containing V) are subsets of space determined by the valuation's data organization and status, respectively, and level s assigned to V is the total statistic for points $P \cap V$. [5] We often write V for O.V.s, O and s understood.

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[4] The conventions arise under the assumptions written, earned and claim statistic transactions ((x,y,z),s) are made *only* at policy inception (i.e., x+y+z equals x), moment in force (i.e., x+y+z equals x+y) and after the date of loss (i.e., x+y+z exceeds x+y), respectively, and so reduce to forms ((x,0,0),s), ((x,y,0),s) and ((x,y,z>0),s), respectively - assuming non-negative coordinates x, y and z. 2.1.a.3 includes counterexamples for the written and earned statistic conventions. As for a counterexample to the claim statistic convention, confine hypothetical transactions to record ultimate claim counts to the xy-plane.

[5] If $P \cap V = \varnothing$, then s is undefined. We consider two valuations distinct if they differ at either of O or V but *equivalent* if their V's have the same intersection with P. We can talk about the level or equivalence (to another subset) of an arbitrary subset A by considering the valuation where A=O=V.

TABLE 2
VALUATION, O.V.S, ILLUSTRATED

| Valuation Data Organizations, <u>O, as Subsets of Space</u> | Valuation Statuses, <u>V, as Subsets of O</u> |
|---|---|
| • Policy issued at time t with term k: All points with policy date x=t and exposure lag y at most k, (t, y≤k,z). | • As of date *t*: V is all points of O with valuation date x+y+z at most *t*. |
| • Post/Accident at exposure lag e on a policy issued at time t: All points with policy date x=t and exposure/accident lag y=e, (t,e,z). | • At policy age *a*: V is all points of O with policy age y+z at most *a*. |
| • Exposure/Accidents at time t: All points with exposure/accident date x+y=t, (x,t-x,z). | • At exposure/accident age *e*: V is all points of O with exposure/accident age z at most *e*. |
| • Time t: All points with valuation date x+y+z=t, (x,y,t-x-y). | • Over calendar period t1 to t2: V is all points of O with valuation date x+y+z from t1 to t2, inclusive. |
| • Policy, Exposure/Accident or Calendar Period t1 to t2: All points with policy, exposure/accident or valuation date from t1 to t2, inclusive. | • At ultimate: V equals O. This is equivalent to *t*, *a* and *e* becoming infinite in the as-of-date-t, at-policy-age-*a* and at-exposure-age-*e* statuses, respectively. |

Note: Level s is the collective value of P∩V. See Table 1 for various date, lag and age definitions.

Thus, using 2.2.a our valuation example is represented by drawing all points, (x,y,z), with accident date x+y in the year 2001 and assigning $31.3 Million Paid Losses to the subset with valuation date x+y+z at most 12/31/2002.[6] We close this section with comments on the primary valuation data organizations and statuses, which we define with drawing instructions in Table 2 and illustrate in Figure 1.

2.2.a.1. A **policy** provides coverage during its term, coverage at any point mid-term referred to as a **post**[7]. Figure 1(b) shows the annual policy written 12/31/04, which by Table 2 is drawn as all points with policy date x equal to 12/31/04 and exposure lag y at most 1 year. The post (and the **claim** it covers) on 6/30/05 is drawn as all points in the policy with exposure/claim date x+y equal to 6/30/05.

2.2.a.2. An **exposure**[8] is coverage from all policies in force at a point in time and so concurrent claims are covered by an exposure. Figure 1(b) shows the exposure (and concurrent claims) on 1/1/03, drawn as all points with exposure date x+y equal to 1/1/03.

2.2.a.3. A point in **time**, itself, is drawn as all points with a given valuation date. Figure 1(b) shows time 12/31/02 as all points with valuation date x+y+z equal to 12/31/02.

2.2.a.4. Finally, policies, exposures and times combine to form **periods** of the same. Figure 1(b) shows Calendar Year (CY) 2002, Exposure/Accident Year (E/AY) 2003 and Policy Year (PY) 2004.

2.2.a.5. Valuation **statuses** for data organizations in 2.2.a.1-2.2.a.4 indicate which points to collect for a total statistic. **As-of-date, at-policy-age** and **at-exposure-age** statuses collect transactions for a data organization thru a certain valuation date, age of underlying policies and age of underlying

[6] The valuation is drawn in Figure 2.
[7] Introduced in Section 2.1 and equal to the intersection of a policy and an exposure as point sets.
[8] We use the term "exposure" as a coverage concept. Other uses (e.g., type of insured, insurance coverage limit) are found in the literature.

exposures, respectively. For example, in Figure 1(c) the status: 1) as-of-date-w=1/1/04 collects transactions with valuation date x+y+z ≤ 1/1/04; 2) at-policy-age-w=18 months collects transactions with policy age y+z ≤ 18; and 3) at-exposure-age-w=18 months collects transactions with exposure age z ≤ 18. **Over-calendar-period** status identifies transactions made on or between two dates, and **at-ultimate** status takes into account all transactions, past, present and future.

## 3. APPLICATIONS

In this section we apply D=(P,*f*) conceptually using the geometry of P to illustrate ideas from ratemaking. Analytical applications, where we consider how *f* behaves on P using calculus in 1, 2 or 3 dimensions as required, are reserved for the Appendix.

XYZ Company proposes a rate change for PY 2004 to be effective 1-1-04. They estimated ultimate losses and ultimate earned premiums (written using the current manual) for PY 2004 to be $45.0 million and $67.3 million, respectively. With a permissible loss ratio target of 65%, a rate level change of 2.9%(=45.0/67.3/0.65-1) was indicated. We sketch their approach as follows, using Figure 2 as a guide.

3.a. LOSS PROJECTION (Figure 2(a)): PY 2004 ultimate losses are estimated from AY(=EY) 2001 losses. As of the latest valuation, 12-31-02, AY 2001 paid losses are $31.3 million. Development to ultimate would add another $9.6 million. Finally, $4.1 million trends or conforms the AY 2001 experience to the PY 2004 basis. The result is $45.0 (=31.3+9.6+4.1) million in estimated ultimate losses for PY 2004.

3.b. PREMIUM PROJECTION (Figure 2(b)): PY 2004 ultimate earned premium is estimated from CY 2001 earned premium. CY 2001 earned premium was $60.3 million. Of that amount, $45.9 million was earned on policies written under the current manual that became effective 1-1-01. The remaining $14.4 million would be increased $0.6 million if underlying policies had been written using the current manual. Finallly, $6.3 million trends the CY 2001 experience to the PY 2004 basis. The result is $67.3 (=60.3+0.6+6.3) million in estimated ultimate earned premium for PY 2004.

In practice we might use several AYs and CYs in pricing PY 2004, applying some weighted average of the results to derive our final estimate. Moreover, instead of using AY 2001 losses and CY 2001 premium, we could use losses and premiums from the same data organization.

## 4. CONCLUSIONS

We represented a database pictorially by assuming three dates and a statistic value could be associated with each transaction. Note:

4.a. In constructing a database in 3-D we first draw all policies then populate those policies with transactions. A policy, exposure and post/claim are infinite in height. We generally assume D consists of points with non-negative xyz-coordinates. By allowing z to be less than zero we can represent transactions made prior to policy issuance. For example, transaction ((1-1-04,0,-1),$1,000) is a $1,000 premium renewal received 1 month in advance of the 1-1-04 renewal date.

4.b. In 3-D the origin should, in principle, correspond to a date before or on the effective date of the very first policy written. Policy term may be unlimited (e.g., a title insurance policy) and the picture for a policy is independent of the claim "trigger" (e.g., occurrence or claims-made triggers).

4.c. The list of valuation data organizations and statuses in Table 2 is not exhaustive, but representative of the valuations that often arise in practice. A variety of valuations can be found in Schedule P of the NAIC Annual Statement. CY, AY, EY and PY valuations can be found in Schedule P, Parts 1, 2, 6 and 7, respectively. Moreover, we consider PY, EY and CY the fundamental data organizations, with AY a special case of EY when we are dealing with claim statistics.

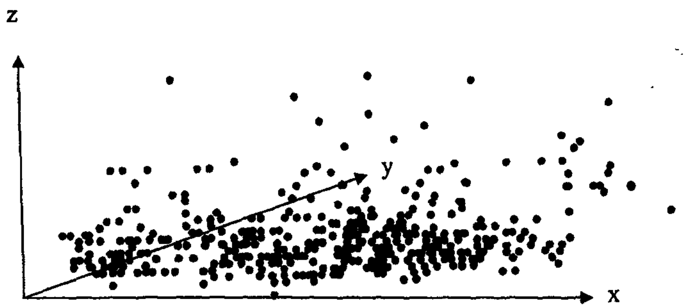
4.d. Development triangles in Schedule P for AY, EY and PY are on an as-of-date-t basis. We could also set up development triangles on an at-policy-age or at-exposure/accident-age basis. All three approaches partition a data organization using planes at ever increasing height in the z or "development" direction.

## REFERENCES

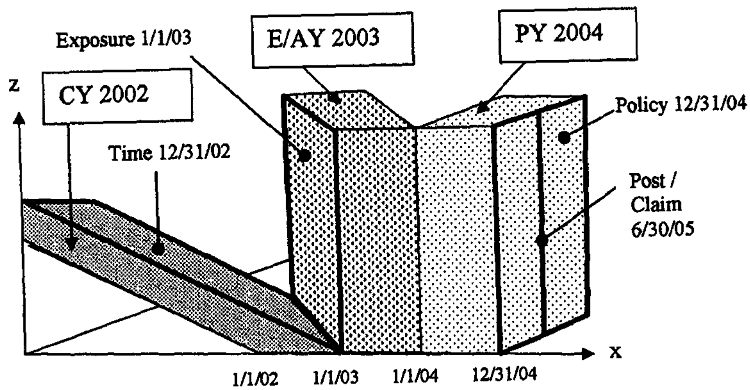
[1] Miller, D. L. and Davis, G. E., " A Refined Model for Premium Adjustment," PCAS LXIII, 1976, pp. 117-124.

[2] Cook, C. F., "Trend and Loss Development Factors," PCAS LVII, 1970, pp. 1-14.

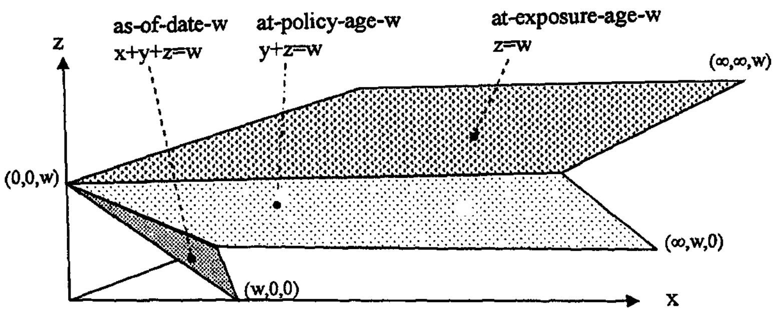
FIGURE 1
A DATABASE IN 3-D




(a) A database in 3-D, D=(P,*f*)

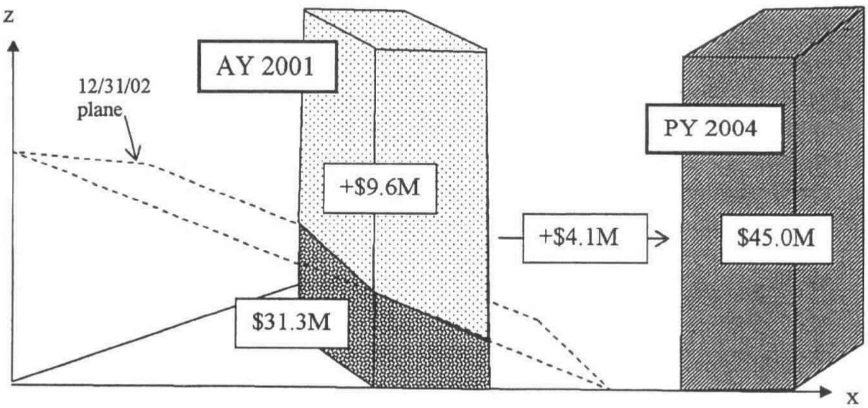



(b) Basic data organizations

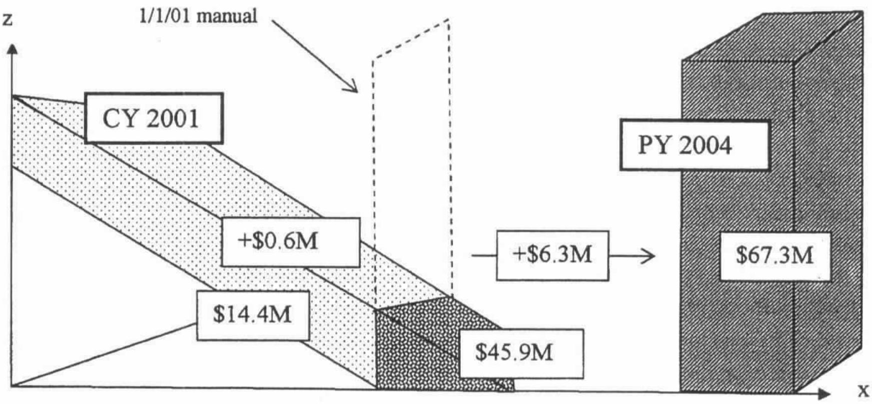



(c) Basic status planes

FIGURE 2
RATEMAKING ILLUSTRATED




(a) Loss Projection: Accident Year 2001 to Policy Year 2004




(b) Premium Projection: Calendar Year 2001 to Policy Year 2004

## APPENDIX

### ANALYTICAL APPLICATIONS

Let D=(P,*f*) be a database in 3-D [9] and O.V.s. a valuation. We assume set V and density *f* sufficiently defined so that we may calculate s as the integral of *f* over V. We apply densities for various statistics for D in 1-, 2- and 3-dimensions, dimensions limiting valuation variety.[10] Five basic analytical applications are described below:

A.1. *Developed, On-Level & Trended Statistics*: Developing involves determining the change in level for two valuations of the same data organization. On-leveling involves determining the change in level for some V under different rules $f_1$ & $f_2$. Finally, trending involves determining the change in level for $V_1$ & $V_2$ under the same rule *f*.

A.2. *Average Value and Average Point*: Define the *average value* of V as s/|V| and the *average point* or *center* of V as that with average coordinates, v*=(x*,y*,z*) [11]. The center provides the average policy, exposure/claim and valuation date, for example. The center for *f*(x,y) constant or uniform over certain V in the xy-plane is readily determined as follows:

   a. For V a rectangle or parallelogram, x* is the midpoint of the x range and y* the midpoint of the y-range for V.
   b. For V an isosceles right triangle, x* is 1/3 into the x-range and y* 1/3 into the y-range of V, starting from the vertex at the right angle.

**For the examples that follow, in xyz-space length 1 on an axis equals 1 year and x=0 and x=1 correspond to dates 1/1/00 and 1/1/01, respectively.**

EXAMPLE 1. As an example of an on-level calculation, we estimate the change in earned premium for CY 2001 at actual and current rate levels, given: i) policies are annual term; ii) the manual effective 1/1/01 represents the current rate level; and iii) $D_{EP}$ for earned premium is confined to the xy-plane with density $f_{EP}$(x,y)=(4000x)($100) for policies written prior to 1/1/01 and $f_{EP}$(x,y)=(4000x)($100)(1.2), thereafter.[12] In Figure 3(a) we show CY 2001 split by rate level. The desired factor is On-level EP ÷ Actual EP, calculated as follows:

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[9] With **R** the set of real numbers, $f$:P⊆ $\mathbf{R}^3 \rightarrow \mathbf{R}^1$ implies D is the graph of *f*, a subset of $\mathbf{R}^3 \times \mathbf{R}^1$.

[10] Two subsets A and B of space are said to be equivalent if they have the same intersection with P, written "A≡B". For example, D confined to the x-axis, xy-plane and xz-plane implies CY≡E/AY≡PY, CY≡E/AY and E/AY≡PY, respectively.

[11] |V|>0 is the content (i.e., length, area or volume) of V and v* is defined only when f>0 on V with x*, y* and z* calculated as the integral of x*f*/s, y*f*/s and z*f*/s over V, respectively.

[12] Here: i) 4000x is the density for written exposure base units (e.g., car years, payroll, stadium seats, etc.) earned uniformly with respect to lag y; ii) $100 is the average written premium; and iii) factor 1.2 represents a 20% increase in rate level on 1/1/01.

$$OEP\ /\ AEP\ =\int_0^1\int_{1-y}^{2-y}480{,}000\,xdxdy\ /[\int_0^1\int_{1-x}^{1}400{,}000\,xdydx\ +\int_1^2\int_0^{2-x}480{,}000\,xdydx\,]$$

1.059= \$480,000 / [\$400,000(1/3)+\$480,000(2/3)].

If exposure writings were uniform instead of increasing (e.g., replace 4,000x with constant 4,000 in the two densities) then the on-level factor becomes:

1.091= \$480,000 / [\$400,000(1/2)+\$480,000(1/2)].

Example 1 is taken from [1]. In that paper the authors perform the same on-level factor calculations, however using a geometric orientation based on the traditional parallelogram method. *For D confined to the xy-plane, the transformation sending point (x,y) to (x+y,y) results in illustrations under the traditional parallelogram method.*[13]

EXAMPLE 2. As an example of a trend calculation, we estimate the change in ultimate loss ratios between PY 2001 @12/31/01 and PY 2004, given: i) ½-year policy terms; and ii) 3% accident year loss cost trend; and iii) 1% on-level policy year premium trend. We assume:

- ultimate loss and ultimate earned premium densities take the forms $f_{UL}(x,y)=wl\times(1.03)^{x+y}$ and $f_{EP}(x,y)=wp\times(1.01)^{x}$, respectively, w, l and p exposure base unit, loss cost and premium constants, respectively[14]; and
- loss and premium levels for V are estimated by $s\cong f(v^*)|V|$ where v* is the uniform center of V, so that $l(103)^{x^*+y^*}/p(1.01)^{x^*}$ is the loss ratio estimate for V.

Applying (A.2.a) & (A.2.b) to Figure 3(b), the centers under uniformity for PY 2001 @12/31/01 and PY 2004 are (1.389,0.222)[15] and (4.5,0.25), respectively. The desired trend factor estimate is therefore 1.06378 (= $(1.03^{(4.75-1.611)}\ /\ 1.01^{(4.5-1.389)})$.

Example 2 supports a common calculation made in practice. Using actual in place of approximate integrations results in a trend factor of 1.06380. Integration has the advantages of following directly from the density assumptions and differentiating between V's with the same uniform center.

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[13] For example, apply the transformation to Figure 3. Several 2-D plotting methods also arise from "collapsing" 3-D. In particular, we note mappings of (x,y,z) to 2-D planes (x,y), (x+y,y), (x,y+z), (x+y,z) and (x+y+z,y).

[14] $l(1.03)^t$ is the result of an exponential fit of a series of accident year average loss costs. With the on-leveling adjustment treated separately, $p(1.01)^t$ is the result of an exponential fit of a series of policy year average earned premiums at current rate level.

[15] The center for EY 2001 is the weighted average of centers for its components from PY 2000 and PY 2001. Thus, solving for $(x_o,y_o)$ in: (1.25,.25)= .25(1-1/6,.50-1/6)+.75$(x_o,y_o)$; yields $(x_o,y_o)$=(1.389,.222).

EXAMPLE 3. We provide the calculations behind the XYZ Company loss and premium values presented in 3.a and 3.b from Section 3. In Figure 3(b) we show the "footprint" of C/E/AY 2001 and PY 2004 in the xy-plane. Assume ½ year policy terms. Let $D_{PL}$ for paid losses be 3-dimensional with density, $f_{PL}(x,y,z)=196{,}000(\$400)e^{.03x+.015y-z}$. Let $D_{EP}$ for earned premium be 2-dimensional with density $f_{EPa}(x,y) = 196{,}000(\$575)e^{.04x}$ for x prior to 1/1/01 and $f_{EPb}(x,y) = 196{,}000(\$600)e^{.04x}$ thereafter. The three loss and three premium integrations required are as follows:

$$AY2001\_PL@12/31/02 = \int_0^{0.5}\int_{1-y}^{2-y}\int_0^{3-x-y} f_{PL}(x,y,z)dzdxdy = \$31{,}329{,}071$$

$$AY2001\_PL@ultimate = \int_0^{0.5}\int_{1-y}^{2-y}\int_0^{\infty} f_{PL}(x,y,z)dzdxdy = \$40{,}852{,}442$$

$$PY2004\_PL@ultimate = \int_0^{0.5}\int_4^{5}\int_0^{\infty} f_{PL}(x,y,z)dzdxdy = \$45{,}036{,}196$$

$$CY2001\_EP@actual = \int_0^{.5}\int_{1-y}^{1} f_{EPa}(x,y)dxdy + \int_0^{.5}\int_1^{2-y} f_{EPb}(x,y)dxdy$$
$$= \$14{,}444{,}165 + \$45{,}977{,}558 = \$60{,}421{,}723$$

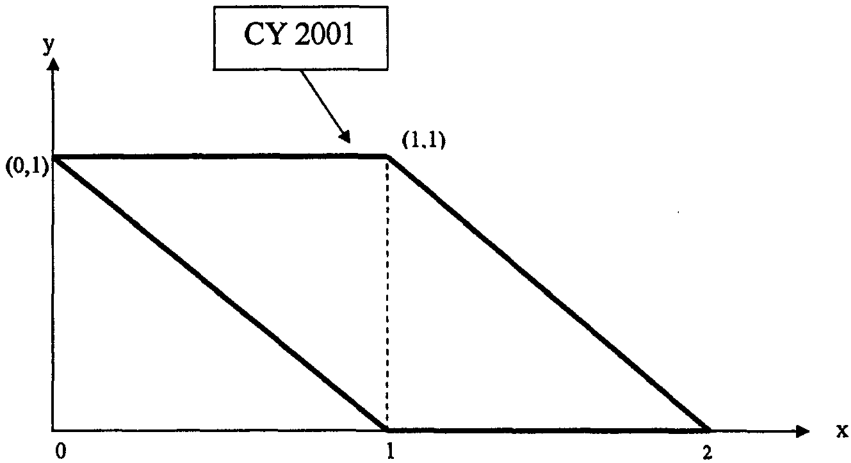
$$CY2001\_EP@on-level = \int_0^{.5}\int_{1-y}^{2-y} f_{EPb}(x,y)dxdy = \$61{,}049{,}730$$

$$PY2004\_EP@on-level = \int_4^{5}\int_0^{0.5} f_{EPb}(x,y)dydx = \$67{,}301{,}286.$$

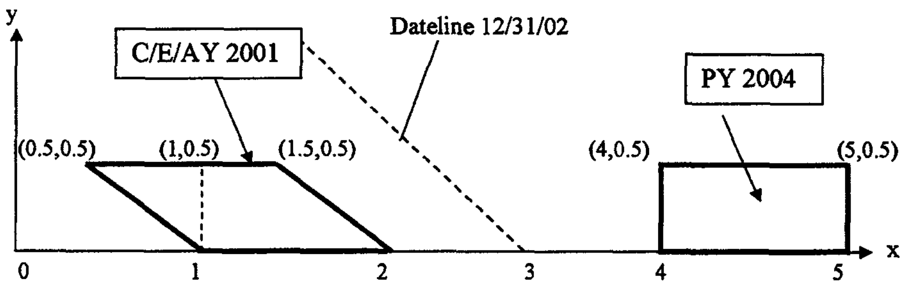
Example 3 densities were developed from assumptions on the rate at which units are written and earned, and the relationship between premiums and losses for a given risk. The densities took the forms $f_{EP}(x,y) = w(x)e(x,y)p(x,y)$ and $f_{PL}(x,y,z) = w(x)e(x,y)l(x,y)q(z)$, where:

- $w(x)=10^5e^{.01x}$ is the rate at which units are written at time x;
- $e(x,y)=1.96=2(.98)$ is the rate at which units written at time x are earned at lag y. The integral of $e(x,y)dy$ over term $0\le y\le 1/2$ equals 98% due to a 2% cancellation rate on average;
- $p(x,y)$ is the post premium: the product of a base rate ($575 for x prior to 1/1/01 and $600 thereafter) and premium relativity $e^{.03x}$.
- $l(x,y)$ is the post loss cost: the product of base loss cost $400, loss cost relativity $e^{.005x}$ and inflation factor thru the date of loss $e^{.015(x+y)}$.
- $q(z)= e^{-z}$ is the portion of loss $l(x,y)$ paid at lag z. The integral of $q(z)$ over $0\le z\le\infty$ equals 100%.

**FIGURE 3**
EXAMPLES 1, 2 & 3 DIAGRAMS




(a) Example 1 diagram




(b) Examples 2 & 3 diagram