Estimating Tail Development Factors: What to do When the Triangle Runs Out

Joseph Boor, FCAS

Abstract:: There are several methods in use today for estimating tail factors. However, most of them are discussed as adjuncts to papers that primarily deal with other subjects. This paper will present a wide variety of method in an understandable format, and includes copious examples

Keywords: Loss reserving, tail factors

1. INTRODUCTION

In many loss reserve analyses, especially those involving long-tail casualty lines, the loss development triangle may end before all the claims are settled and before the final costs of any year are known. For example, it is quite common to analyze U. S. workers compensation loss reserve needs using the ten years of data available in Schedule P of the US NAIC-mandated Annual Statement, while knowing that some of the underlying claims may take as long as fifty years to close. In response to this, actuaries supplement the 'link ratios' they obtain from the available triangle data with a 'tail factor' that estimates the development beyond the last stage of development (last number of months of maturity, usually) for which a link ratio could be calculated.

The tail factor is used just like a link ratio in that it estimates (1.0 + ratio of (final costs after all claims are closed) to (the costs as of the last development stage used)). It is of course included in the product of all the remaining link ratios beyond any given stage of development in calculating a loss development factor to ultimate for that stage of development.

This paper will discuss the methods of computing (really estimating to be precise) tail factors in common usage today. It will also suggest both improvements in existing methods and a new method. It will begin with the simplest class of methods and move forward in increasing complexity.

There are four groups of methods that will be presented:

1. The Bondy (repeat-the-last link-type) methods
2. The Algebraic methods (methods based on algebraic relationships between the paid and incurred triangles)
3. Use of Benchmark Data
4. Curve Fitting Methods

As part of the discussion, commentary on the advantages and disadvantages of each individual method, as well as each class of methods will be included. When the opportunity to discuss an improvement or enhancement that applies to multiple methods presents itself, a brief digression on the enhancement will be included.

2. GROUP 1 - THE BONDY-TYPE METHODS

The Bondy methods all arose from an approach published by Martin Bondy prior to the 1980s. In what was thought to be a period where development decayed rapidly from link ratio to link ratio, he promulgated a practice of simply repeating the last link ratio for use as the tail factor. Since then, several variations of his method that all base the tail factor on the last available link ratio have arisen.

2.1 The Bondy Method

As explained above, the original Bondy method involves simply using the last link ratio that could be estimated from the triangle (the link ratio of the last development stage present in the triangle, or the last stage where the triangle data could be deemed reliable for estimation) as the tail factor. This 'repeat the last link ratio' approach probably seems crude and unreasonable for long-tailed lines, where link ratios decay slowly. However, for fast decaying lines (such as an accident year[1] analysis of automobile extended warranty) this method may work when used as early as thirty-six or forty-eight months of maturity. It must be recognized, though, that in long-tailed lines the criticism is usually justified.

To truly understand this method it also may be best viewed in historical context. The author of the method, Martin Bondy, developed this method well prior to the 1980's. It is commonly believed that during the 1960s and certainly part of the 1970s the courts proceeded at a faster pace and, ignoring the long-tail asbestos, environmental, and mass tort issues that would eventually emerge, general liability was believed to have a much shorter tail than we see today.

It is also of interest to note that there is a theoretical foundation that supports this in certain circumstances. If one assumes that the 'development portion' of the link ratios (the link ratios minus one) are decreasing by one-half at each stage of development, and the last link ratio is fairly low, then the theoretically correct tail factor to follow a link ratio of 1+d is:

$(1+.5d)\times(1+.25d)\times(1+.125d)\times(1+.0625d)\times\ldots\ldots$

Or

$1+(.5+.25+.125+.0625+\ldots)\times d$ + terms involving d^2, d^3, etc.

[1] It should be noted that policy year automobile extended warranty represents an entirely different situation.

Which, per the interest theorem $v+v^2+v^3+....=v/(1-v)$ is equivalent to:

$1+1\times d$ + terms involving d^2, d^3, etc.

Since d is 'small', the other terms will be smaller by an order of magnitude, making the implied tail factor under these assumptions very close to the Bondy tail factor, a repetition of the last link ratio, 1+d. So the Bondy tail factor is 'nearly' equivalent to the tail implied by what will later be called the 'exponential decay' method, with a 50% decay constant.

Of note, this involves two basic assumptions. First that the link ratios decay in proportion to the remaining 'development portion' of the link. Of note, in the absence of any information whatsoever about the decay, that would be as reasonable an assumption as one could reasonably make. Second, that the decay constant is 50%. Again, in the absence of any data whatsoever, one-half would be as reasonable an assumption as one could possibly make. Of course, we do have data in the link ratios before the tail, but it is important to understand this theoretical basis for the Bondy tail factor.

2.2 The Modified Bondy Method

In this method, the last link ratio available from the triangle, call it 1+d, is modified by multiplying the development portion by 2. The result is a development factor like 1+2d. Alternately, the last entire link ratio may be squared, which yields nearly the same value. This has many of the same issues and applications as the basic Bondy method, but it does yield a larger tail than the Bondy method itself. However, for long-tail lines it is still not what would be considered a truly conservative approach, as we will see later. The assumption here is ' The Bondy method seems to underestimate, it should be increased, the easiest thing to do is to multiply the development portion by two.'.

A little algebra and the $v+v^2+v^3+....=v/(1-v)$ theorem show that this is functionally equivalent to 'exponential decay' with a decay coefficient of $^2/_3$.

2.3 Advantages and Disadvantages of the Bondy Methods

The primary advantages of the Bondy methods are that they are extremely simple to execute and easy to understand. Further, they involve relatively straightforward assumptions. However, a major disadvantage is that they tend to greatly underestimate tails of long-tailed, slow-decaying lines.

3. GROUP 2 - THE ALGEBRAIC METHODS

These methods involve initially computing some algebraic quantity that in turn describes a relationship between some aspect of the paid and incurred loss triangles. Then that quantity can be used to generate a tail factor estimate. As with the Bondy method, and almost all tail factor estimation methods, they are based on assumptions. However, in this case each is based on some relatively simple and fairly logical assumption that some numerical relationship known to be true in one circumstance will be true in another.

3.1 Equalizing Paid and Incurred Development Ultimate Losses

This method is the first method discussed with a full theoretical background. It is most useful when incurred loss development essentially stops after a certain stage (i.e., the link ratios are near to unity or unity). Then, due to the absence of continuing development, the current case incurred (sometimes called reported) losses are a good predictor of the ultimate losses for the older or oldest years without a need for additional tail factor development. A tail factor suitable for paid loss development can then be computed as the ratio of the case incurred losses to-date for the oldest (accident[2]) year in the triangle divided by the paid losses to-date for the same (accident) year. That way, the paid and incurred development tests will produce exactly the same ultimate losses for that oldest year.

This method relies on one axiomatic (meaning plainly true rather than an assumption as such) assumption and two true assumptions. The axiomatic assumption is that the paid loss and incurred loss development estimates of incurred loss are estimating the same quantity, therefore the ultimate loss estimates they produce should be equal. The second assumption (the first true assumption) is that the incurred loss estimate of the ultimate losses for the oldest year is accurate. The last assumption is that the other years will show the same development in the tail as the oldest year.

This method may also be generalized to the case where case incurred losses are still showing development near the tail. In that case, the implied paid loss tail factor is

(incurred loss development ultimate loss estimate for the oldest year) / (paid losses to-date for the oldest year).

Of course, in that instance the incurred loss development estimate for the oldest (accident) year is usually the case incurred losses for the oldest year multiplied by an incurred loss tail factor developed using other methods.

This method has a substantial advantage in that it is based solely on the information in the triangle itself and needs no special assumptions. Its weakness is that you must already have a reliable estimate of the ultimate loss for the oldest year before it can be used. An ancillary weakness flows from the assumptions underlying this method. Specifically, if the initial incurred loss development test is driven by a tail factor assumption, this becomes a test that is also based on not only that assumption, but also the assumption that the ratio of the case incurred loss to the paid loss will be the same for the less mature years once they reach the older level of maturity where you are equalizing the paid and incurred loss estimates.

[2] Accident year is used here for illustration. Under similar circumstances, this method would also work in policy year, reinsurance contract year, etc. development.

3.1.1 An example:

Assume that it is just after year-end of 2000. You have pulled the incurred loss triangle from a carrier by subtracting part 4 of Schedule P from part 2 of Schedule P. You have also pulled a paid loss triangle from part 3 of Schedule P. The triangles cover 1991-2000, so 1991 is the oldest year. Say for the sake of argument that the incurred loss link ratios you develop are 2.0 for 12-24 months, 1.5 for 24-36, 1.25 for 36-48, 1.125 for 48-60, 1.063 for 60-72, 1.031 for 72-84, 1.016 for 84-96, 1.008 for 96-108, and 1.004 for 108-120. This conveniently happens to match the exponential decay discussed for the Bondy method, so it makes sense to use 1.004 for the tail factor for development beyond 120 months. Now assume that the latest available (i.e., at 12/31/2000, or 120 months maturity) the case incurred loss[3] for 1991 is $50,000,000 and the corresponding paid loss is $40,000,000. The incurred test ultimate using the 1.004 tail factor is $50,200,000. The paid loss tail factor to equalize the ultimate would be $50,200,000 divided by $40,000,000 or 1.255.

3.1.2 Improvement 1 - using multiple years to develop the tail factor

As stated earlier, the previous method assumes that the current ratio of case incurred loss to paid loss that exists in the oldest year (1991 evaluated at 12/31/2000 in the example above) will apply to the other years when they reach that same level of maturity. For a large high dollar volume triangle with relatively low underlying policy limits that may be a reasonable assumption, but for many reserving applications the 120 month ratio of case incurred to paid loss may depend on whether a few large, complex claims remain open or not. Therefore, it may be wise to supplement the tail factor derived from the oldest available year with that implied by the following year or even the second following year. This method is particularly useful when the later development portion of the triangle has some credibility, but the individual link ratio estimates from the development triangle are not fully credible.

The process of doing so is fairly straightforward. You merely compute the tail factor for each succeeding year by the method above, and divide each by the remaining link ratios in the triangle.

An example using the data above may help clarify matters. Given the data above, assume that 1992 has $50,000,000 of paid loss and $60,000,000 of case incurred loss. Also, assume that your best estimate of the 108-120 paid loss link ratio is 1.01. The incurred loss estimate of the ultimate loss, using the 108-120 link ratio (1.004) and the incurred loss tail factor (also 1.004) is $60,000,000×1.004×1.004, or $60,480,960. The estimated (per incurred loss development) ultimate loss to paid loss ratio at 108 months would then be $60,480,960/$50,000,000, or approximately 1.210. So, 1.210 would then be the tail factor estimate for 108 months. Dividing out the 108-120 paid link ratio (assumed above to be 1.01) gives a tail factor for 120 months of 1.21/1.01 = 1.198. By comparison, the previous analysis using 1991 instead of 1992 gave a 120-month tail factor estimate of 1.255. So it is possible that either 1991 has a high number of claims remaining open, or that 1992 has a low number. Both indicate tail factors in the 120-125 approximate range, though. So averaging

[3] To be technically correct, this would be loss and defense and cost containment under 2003 accounting rules.

the estimates might be prudent. Further, the use of averaging greatly limits the impact of any unusually low or high case reserves that may be present in the oldest year in the triangle.

Note also, that the improvement above involved computing an alternate tail factor using the year with one year less maturity. A similar analysis could also be performed on the next oldest year, 1993, except that two <u>incurred</u> development link ratios plus the tail factor are needed to compute the incurred loss estimate of ultimate. Correspondingly, two <u>paid</u> loss link ratios need to be divided out of the (incurred loss ultimate estimate)/(paid loss to-date ratio for 1993) to estimate the 120 month paid loss tail factor

<u>3.1.2.1 An important note</u>

Further, in this case the improvement involved reviewing the tail factors at various ages from the equalization of paid and incurred loss estimates of the ultimate loss. The core process involves computing tail factors at different maturies, then dividing by the remaining link ratios to place them all at the same maturity. As such, it can also be used in the context of other methods for computing tail factors that will be discussed later in this paper.

<u>3.1.3 A brief digression – the primary activity within each development stage</u>

When using multiple years to estimate a tail factor, it is relatively important that the years reflect the same general type of claims department activity as that which takes place in the tail. For example, in the early 12 to 24 month stage of workers compensation, the primary development activity is the initial reporting of claims and the settlement and closure of small claims. The primary factors influencing development are how quickly the claims are reported and entered into the system, and the average reserves (assuming the claims department initially just sets a 'formula reserve', or a fixed reserve amount for each claim of a given type such as medical or lost time) used when claims are first reported. In the 24 to 36-48 month period, claims department activity is focused on ascertaining the true value of long-term claims and settling medium-sized claims. After 48-60 months most of the activity centers on long-term claims. So, the 12-24 link ratio has relatively little relevance for the tail, as the driver behind the link ratio is reporting and the size of initial formula reserves rather than the handling of long-term cases. Similarly, if the last credible link ratio in the triangle is the 24 to 36 or 36 to 48 link ratio, that triangle may be a poor predictor of the required tail factor.

3.2 The Sherman-Boor[4] Method - Adjusting the Ending Case Using Ratios of Paid Loss to Case Reserve Disposed of

This method, developed by Sherman in Section X of [3] and independently by the author, is the one method that relies solely on the triangle itself and does not require a pre-existing ultimate loss estimate, involve curve-fitting assumptions, or require external data. For data triangles with high statistical reliability as predictors, this can represent the optimum estimation process.

This method involves simply determining the ratio of case reserves to paid loss for the oldest year in the triangle, then adjusting the case reserves by an estimate of the ratio of the unpaid loss to carried case reserves. In essence, the case reserves of the oldest accident year are 'grossed up' to estimate the true unpaid loss using a factor. The estimate of the (true unpaid loss)/(case reserves) factor is based on how many dollars of payments are required to 'eliminate' one dollar of case reserves.

The mathematical formula requires computing a triangle containing <u>incremental</u> rather than cumulative paid losses. In effect, for each point in the paid loss triangle, one need only subtract the previous value in the same row (the first column is of course unchanged). The next step begins with a triangle of case reserves. The incremental case reserve disposed of is calculated as the case reserve in the same row <u>before</u> the data point, less the current case reserve. That represents (as the beginning case reserve – the ending case reserve) the case reserve disposed of. Then the ratios of incremental paid to reserve disposed of at the same points in the triangles are computed. Reviewing these, the adjustment ratio for the ending case reserves is estimated.

3.2.1 An example

Reviewing an example may help the reader follow the calculations discussed earlier. This method requires two triangles, one of paid loss and one of case reserves. Consider the following set of triangles:

Cumulative Paid Loss Triangle						
	12	24	36	48	60	72
1991	1,000	2,000	2,500	2,800	2,950	3,100
1992	1,100	2,400	3,000	3,500	3,900	
1993	1,300	2,500	3,000	3,400		
1994	1,200	2,300	3,100			
1995	1,400	2,800				
1996	1,490					

[4] Of note, this method was first published by Richard Sherman, FCAS in 1984 and developed independently by the author in 1987. Of note, the author used some business materials that contained precursors to this method in 1984-1986 that were developed by a firm of which Mr. Sherman was a principal.

Triangle of Case Reserves Outstanding (Cumulative Case Incurred-Cumulative Paid)						
	12	24	36	48	60	72
1991	1,500	1,300	900	750	600	500
1992	2,000	1,700	1,300	900	600	
1993	1,900	1,700	1,300	1,000		
1994	2,100	2,100	1,500			
1995	2,300	2,000				
1996	2,500					

First, we compute the incremental paid loss triangle. We begin with a given cell in the cumulative paid loss triangle, and then we subtract the previous cell in the same row of the cumulative paid loss triangle. That produces the following triangle.

Incremental Paid Loss Triangle						
	12	24	36	48	60	72
1991	1,000	1,000	500	300	150	150
1992	1,100	1,300	600	500	400	
1993	1,300	1,200	500	400		
1994	1,200	1,100	800			
1995	1,400	1,400				
1996	1,490					

Then we subtract the current cell <u>from</u> the previous cell in the case reserve triangle to obtain the triangle of case reserves disposed of.

Triangle of Incremental Case Reserves Disposed Of						
	12	24	36	48	60	72
1991		200	400	150	150	100
1992		300	400	400	300	
1993		200	400	300		
1994		100	600			
1995		300				
1996						

Then we divide the actual final costs paid (the incremental paid loss), by the assumption-based case reserves eliminated.

Ratio of Paid Loss to Reserves Eliminated						
	12	24	36	48	60	72
1991		500%	125%	200%	100%	150%
1992		433%	150%	125%	133%	
1993		600%	125%	133%		
1994		1100%	133%			
1995		467%				
1996						

Because the early development involves not just elimination of case reserves through payments, but also substantial emergence of IBNR claims, the 12 and 36 columns are presumably distorted. In many lines the 48 month column would still be heavily affected by newly reported large claims, but presumably this is medium-tail business. Looking at the various ratios it would appear that they average around 140%, so we will use that as our adjustment factor for the case reserves.

Pulling the \$500 of case left on the 1991 year at 72 months, and the cumulative paid on the 1991 year of \$3,100, the development portion of the paid loss tail factor would be $(\$500/\$3{,}100)\times 140\% = .161\times 140\% = .226$. So, the paid loss tail factor would be 1.226.

For the incurred loss tail factor, first note that only the 'development portion' of the 140%, or 40%, need be applied (the remaining case is already contained in the incurred). Second, a ratio of the case reserves to incurred loss is technically needed (replacing 1.61 with $\$500/(\$500+\$3{,}100) = .139$). Multiplying the two numbers creates an estimate of the development portion of the tail at $.4\times.139=.056$. So, the incurred loss tail factor estimate would be 1.056.

<u>3.2.2 An Important Note</u>

As is the case with most of the other methods, this method has strengths and weaknesses. Significant strengths of this method are that it requires only the data already in the triangle and that it does not require additional assumptions. The weakness is that it can be distorted if the adequacy of the ending case has changed significantly from the previous year. The reader is advised to also follow Improvement 1 and also evaluate the tail at the next-to-oldest year.

4. GROUP THREE - METHODS THAT USE BENCHMARK DATA

A common solution to the ratemaking problems generated by data with partial statistical reliability (credibility) is to supplement the claims data with a 'complement of credibility'. Of course, tail factor estimation problems stem more from a lack of any data at all after the oldest development stage in the triangle rather than from partially reliable data. But, we can adopt a similar strategy and add outside data in the form of benchmark development factors.

4.1 Directly Using Tail Factors From Benchmark Data

As noted above, many actuaries review benchmark data in selecting tail factors[5]. Benchmark data may come from one of several sources. Perhaps the most common is the use of the data triangles that can be developed from Best's Aggregates and Averages for each of the Schedule P lines. The two larger rating bureaus, the National Council on Compensation Insurance and Insurance Services Office; as well as the Reinsurance Association of America, all publish benchmark loss development data. At its simplest, this method involves copying the derived remaining development factor at the maturity desired for the tail factor.

It is important to note, though, that the quality of the benchmark tail factor as an estimate of the tail depends on how closely the tail development of the benchmark mirrors the tail development of the book of business being analyzed. Considerations such as differences in the way claims are adjusted or reserved, differences in the potential for long-developing high value claims, differences in the initial reporting pattern of claims (claims-made vs. occurrence, whether or there is an innately long discovery period or not, etc.), and differences in the adjudication process of litigated claims can all cause differences in development patterns. It is important to consider those factors along with the statistical reliability of the benchmark triangle when selecting the most appropriate benchmark tail factor.

4.2 Using Benchmark Tail Factors Adjusted to Company Development Levels

One way to address differences between the benchmark development pattern and the development pattern of a given book of business is to try to adjust the benchmark data to more closely mirror the subject book of business. A common practice is to review the relativities of link ratios from the triangle being analyzed to benchmark link ratios. Of course, there is not a tail factor for the triangle being analyzed (we are trying to estimate one). So, instead we can review the quotients (relativities) of subject triangle link ratios to those of the benchmark data at the development stages prior to the tail development stage. The relativities from those stages are used to estimate a adjustment multiplier for the benchmark tail factor. Of note, generally just the development portions ('d' of 1+d) are compared in all the relativities we compute.

[5] It is also common for actuaries to review benchmark data to supplement the portion of the reserve triangle following 72, 60, 48, or even 36 months when the overall triangle has medium credibility and hence has less than medium credibility in the portion that is dominated by activity on a smallish number of claims.

4.2.1 An example

An example will help to illustrate how the process works. Consider the following two patterns:

	Link	
	Ratio	
	Estimated	Benchmark
Months of	By	Link
Maturity	Triangle	Ratio
12	2.000	2.000
24	1.450	1.350
36	1.200	1.150
48	1.150	1.100
60	1.100	1.050
72	1.080	1.030
84	1.050	1.025
96	1.035	1.020
108	1.010	1.010
Tail		1.050

We then simply compute the relativity quotient of the 'development portion' of our triangle-based link ratios to the development portion of the matching benchmark link ratios. Noting that 1÷1 = 100%, .45÷.35 = 129%, .2÷.15 = 133%, etc.

Months of Maturity	Link Ratio Estimated by Triangle	Benchmark Link Ratio	Relativity of Triangle Development to Benchmark
12	2.000	2.000	100%
24	1.450	1.350	129%
36	1.200	1.150	133%
48	1.150	1.100	150%
60	1.100	1.050	200%
72	1.080	1.030	267%
84	1.050	1.025	200%
96	1.035	1.020	175%
108	1.010	1.010	100%
Tail		1.050	
Chosen Ratio			175%
Implied Tail	1.088		

In the case above, we judgmentally select that the triangle development is roughly 175% of benchmark based on the 60 through 108 month relativities. So the .05 development portion of the benchmark tail becomes $.05 \times 1.75 = .0875 \cong .088$. Consequently the entire tail factor, including unity, is 1.088.

4.2.2 Another important note

It is important to consider that adjusting the benchmark tail for actual triangle link ratios is only helpful as long as the link ratios, or at least the broad pattern of link ratios has statistical reliability (predictive accuracy). If not, the uncertainty surrounding the true long-term link ratios of the block of business will cause the adjusted tail factor to lack predictive accuracy.

4.3 *Advantages and Disadvantages of Using Benchmark Data*

When a good benchmark tail factor is available, this is both one of the easiest and also among the most useful methods. However, it is often difficult to find a perfect match in terms of all the factors (claims handling, case reserving, potential for large claims, etc.) that affect loss development. Adjusting the benchmark improves the fit markedly. One could even think of the process of adjusting the benchmark as that of fitting a curve to the link ratios, where the family of curves you are fitting from consists of various relativity-adjusted versions of the benchmark. If the benchmark is remotely related to the book of business being analyzed, that family of curves should be a superior choice to the highly assumption-driven curve families discussed later under curve fitting.

On the other hand, it is often very difficult to obtain the more-mature data needed to create a reliable benchmark tail factor. So, for tail factors beginning at 108 or 120 months, it may be very difficult to find a suitable benchmark.

5. GROUP 4 -THE CURVE FITTING METHODS

As good students of numerical analysis, actuaries long ago realized that they could attempt to extrapolate the tail development by fitting curves to the development before the tail, then using the fitted curve to extrapolate the additional tail development. Some methods have been developed that fit a curve to the paid or incurred loss. Other methods fit to the link ratios. What they all have in common is that they begin with some assumption about the development decay that gives rise to a family of curves, and then select the coefficient(s) that specify the particular member of the family of curves that best fits the data. As with most extrapolations, they are as good as the assumptions that underlie them.

5.1 McClenahan's Method-Exponential Decay of Paid Loss Itself

McClenahan's method (as discussed in [1]) fits a curve to a set of data per an assumption that the incremental paid loss of a single accident year will decay exponentially over increasing maturities of the accident year. In effect, that there was some decay rate 'p' and that the next month's payout on the accidents in a given month would always be 'p' times the current month's payments on that given accident month. He combined that with an assumption that no payments occurr in the first few months of a claim. Putting those pieces together mathematically, he inferred that the payments in a given incremental month of maturity (call it 'm') were

$Ap^{(m-a)}q.$

In this case A is a constant of proportionality and 'p', ($0<p<1$, $q=(1-p)$) represents the decay rate[6] and 'a'[7] represents the average lag time until claims begin to be paid. A theorem from the study of compound interest states that

$$\sum_{m=a}^{\infty} Ap^{(m-a)} q= A\sum_{i=0}^{\infty} p^i q= Aq/(1-p)=Aq/q =A.$$

So A is actually the ultimate loss for the entire year.

Then, under this assumption, the additional payments or incurrals beyond x months are theoretically determined by the basic formula, at least once p and a are estimated. And there are several ways to estimate p and a. For convenience, p is monthly, but p^{12}, the annual decay rate, may be defined as 'r'[8]. Then r may be estimated by reviewing the ratios of incremental paid between m+12 and m+24 months to the incremental paid between m and m+12 months. McClenahan advised that 'a' could be estimated by simply reviewing the average report lag[9] (average date of report-average date of occurrence) for the line of business.. Then, a curve of the form

Ar^y,

where y is the maturity of the accident year in years before each amount of incremental paid can be fit to the incremental dollar amounts of paid loss (or incurred loss, as long as no downward development in incurred loss is present in the development pattern).

Then, McClenahan shows that the percentage remaining unpaid for an entire twelve month accident year at m months of (returning to $p = r^{1/12}$) is

$$(1-p)\times(p^{m+1-a}+ p^{m+1-a-1}+ p^{m+1-a-2}+\ldots+ p^{m+1-a-11})/(12\times(1-p)) = p^{m-a-10} (1- p^{12})/12q$$

The tail factor at m months is of course unity divided by the percentage paid at m months, or

1/(100% - percentage unpaid at m months).

[6] McClenahan's model actually incorporates additional variables for trend, etc that may be collapsed into 'p' for purposes of this analysis.

[7] In Mclenahan's original paper, 'd' is used instead of 'a'. But, since I have used 'd' to denote the development portion of the link ratio or development factor, I am using 'a' to denote the average payment lag.

[8] Please note that the usage of 'r' in this context is different than the usage in McClenahan's original paper. It is used merely because it represents an annual rate.

[9] Note that 'a' applies on a month-by-month basis. So it is technically incorrect to say that the average lag between the beginning of <u>all</u> loss reporting for an accident year is six months (the average lag between inception of the accident year and loss occurrence, at least for a full twelve month accident year) plus 'a' months. To simplify the calculations, the first twelve months can be excluded from the fit

Substituting our formula for the unpaid at 12 months, McClenahan's method produces a tail factor of

$1/\{1 - [p^{m-a-10} (1- p^{12})/12q]\}$

Some algebra reduces that to

$12q/\{12q - p^{m-a-10} (1- p^{12}) \}$,

which provides a nice closed form[10] expression for the tail.

<u>An Example:</u>

Assume that you begin with an 8-year triangle, and generate the following link ratios:

12-24	5.772
24-36	1.529
36-48	1.187
48-60	1.085
60-72	1.042
72-84	1.022
84-96	1.012

The first step is to covert them to a form of dollars paid (remember that there are different paid amounts for different accident years, so we just begin with one hundred dollars for the curve fitting and multiply by the successive link ratios.

Development Stage	Link Ratio	Beginning Maturity	Equivalent Cumulative Paid
12-24	5.772	12	$100.00
24-36	1.529	24	$577.23
36-48	1.187	36	$882.45
48-60	1.085	48	$1,047.38
60-72	1.042	60	$1,136.50
72-84	1.022	72	$1,184.66
84-96	1.012	84	$1,210.68
		96	$1,224.75

[10] It should be noted that while a closed form expression makes the calculations easy, for some audiences, it may be preferable to show the projected link ratios, at least until they are overwhelmingly close to unity.

Then subtract successive cumulative paid amounts to obtain 'normalized to $100 of first year paid' incremental dollars at each stage of development that mirror the actual link ratios.

			Equivalent	Incremental
Development	Link	Beginning	Cumulative	Paid
Stage	Ratio	Maturity	Paid	(Difference)
12-24	5.772	12	$100.00	$100.00
24-36	1.529	24	$577.23	$477.23
36-48	1.187	36	$882.45	$305.22
48-60	1.085	48	$1,047.38	$164.93
60-72	1.042	60	$1,136.50	$89.12
72-84	1.022	72	$1,184.66	$48.16
84-96	1.012	84	$1,210.68	$26.02
		96	$1,224.75	$14.06

Then ratios of the successive 'normalized' incremental paid amounts can be taken.

			Equivalent	Incremental	Year
Development	Link	Beginning	Cumulative	Paid	to Year
Stage	Ratio	Maturity	Paid	(Difference)	Ratio
12-24	5.772	12	$100.00	$100.00	
24-36	1.529	24	$577.23	$477.23	4.7723
36-48	1.187	36	$882.45	$305.22	0.6396
48-60	1.085	48	$1,047.38	$164.93	0.5404
60-72	1.042	60	$1,136.50	$89.12	0.5404
72-84	1.022	72	$1,184.66	$48.16	0.5404
84-96	1.012	84	$1,210.68	$26.02	0.5404
		96	$1,224.75	$14.06	0.5404

As one can see, in this contrived example, the development stage-to-stage ratio is a constant $r = .5404$. It's twelve root p is $p = r^{1/12} = .95$.

That of course only provides p, the average delay must be found as well. Because the answer is contrived to have a=7 months, a= 7 months will work perfectly[11] for this example, but note that McClenahan suggests merely using the report delay for the book of business to determine 'a'.

Using a= 7 months and p = .95, the computed tail factor is

$$12q/\{12q - .95^{m-a-10}(1-.95^{12})\},= .6/\{.6 - .017385(1-.5404)\}= 1.0135.$$

If one reviews the link ratios prior to this, it certainly appears to be reasonable. In fact, extending the payout to additional stages of development will confirm its accuracy.

[11] An interested reader can confirm that a=7 months and p=.95 yields the exact link ratios above.

5.1.1 Advantages and disadvantages of McClenahan's method

At its core, McClenahan's method involves three basic assumptions: First, it assumes that the pattern of paid loss will be a constantly decreasing pattern, at least after all the initial report lags are finished. Second, he assumes that the reduction will always occur in proportion to the size of the most current payout (exponential decay). Third, he assumes that the exponent of decay is constant throughout the entire payout pattern. Logically speaking, if one knew nothing about the individual pattern of the data, but was forced to make some assumptions, those assumptions would seem to be about as minimal and reasonable as possible (excepting perhaps the third). But it is important to remember that they are assumptions and as such will color the predictions the method generates. They do suggest exponential decay of the paid amounts, and exponential decay is a relatively fast decay relative to other forms of asymptotic (far out in the tail) decay. Moreover, it does seem that in practice the decay in paid loss often seems to 'stall out' and show less decay near the tail.

5.1.2 Improvement 2 - exact fitting to the oldest year

A common problem with fitted curves is that the combination of the curve assumptions and the data in the middle of the triangle may create a curve that varies significantly from the development factors at the older stages. McClenahan's method is relatively unique in that the curve is fit to the incremental paid, rather than the link ratios (as will be done in most of the later methods). Nevertheless, we can often improve the quality of the tail prediction by comparing the fitted value to the actual incremental paid loss at the latest stage.

This approach is especially helpful when the curve does not match the shape of the data itself. For example, assume that the assumption of a constant decay rate does not hold. Say the initial year-to-year decay was high at between 36[12] and 48 months, 48 and 60 months, etc., but the decay rate at 84 to 96 months and 96 to 108 months, etc. is much less (i.e., a higher decay factor). Then, the last incremental payments (say between 108 and 120) may be much higher percentagewise than what is implied by the fitted curve.

In that case[13], one need merely multiply the 'development portion' of the tail factor (the tail factor minus one) times the ratio of the actual 108 to 120 increment to the fitted increment. Of course, unity (one) must be added to the final result to produce a proper tail factor.

[12] Note that because of the delay a before payments, etc. begin, the apparent decay between 12 and 24 months and 24 to 36 months is a distortion of the true annual decay.
[13] Assuming that the data has enough volume for the 108 to 120 link ratio to have full credibility.

For example, in the above data, the last incremental data shown is from 96 to 108 months. In that case the fitted value equals the actual 'normalized' value equals $14.06 per a 96 to 108 link ratio of 1.012 and decay rate of .5404. But what if we had the same decay rate overall, but the link ratio from 96 to 108 was 1.018. In that case, the incremental paid would be $21.09, or 150% of the fitted value of $14.06. Then the adjusted tail factor would be:

1+150%(fitted tail factor-1) = 1+150%(1.0135-1)=1+150%×.0135=1.0203.

Note that in the case of McClenahan's method, the ratio used for 'exact fitting' is the ratio of actual to fitted paid loss. In the later methods, where a curve is fit to the 'development portions', a ratio of development portions should be used to produce the exact fit to the last link ratio.

5.1.3 Improvement 1 (using multiple years to estimate the tail) can enhance improvement 2

For McClenahan's method, and all the curve-fitting methods, improvement 1 can only be done in connection with improvement 2. In essence, the concept is to create an exact fit to the next-to-oldest link ratio or 'normalized' paid loss, and perhaps the third-to-last link ratio as well. Then, the implied tail factors can be averaged or otherwise combined into a single tail factor indication. This method is particularly useful when the 'tail' of the triangle has some credibility, but the individual link ratio estimates from the development triangle are not fully credible.

Dev Stage	Link Ratio	Ending Maturity	Equivalent Cumulative Paid	Incremental Paid (Difference)	Year to Year Ratio	Revised Link Ratio	Equivalent Cumulative Paid	Incremental Paid (Difference)
12-24	5.772	12	$100.00	$100.00		5.772	$100.00	$100.00
24-36	1.529	24	$577.23	$477.23	4.7723	1.529	$577.23	$477.23
36-48	1.187	36	$882.45	$305.22	0.6396	1.187	$882.45	$305.22
48-60	1.085	48	$1,047.38	$164.93	0.5404	1.085	$1,047.38	$164.93
60-72	1.042	60	$1,136.50	$89.12	0.5404	1.042	$1,136.50	$89.12
72-84	1.022	72	$1,184.66	$48.16	0.5404	1.044	$1,184.66	$48.16
84-96	1.012	84	$1,210.68	$26.02	0.5404	1.018	$1,236.79	$52.13
		96	$1,224.75	$14.06	0.5404		$1,259.05	$22.26

For example, the table above contains the data cited in the original example of McClenahan's Method (5.1) as the first set of link ratios, equivalent cumulative paid, etc. But, beginning with the 'Revised Link Ratio' column it contains alternate link ratios, etc. for 72 months and later. Using that data, one would still conclude that the fitted annual decline is.5404. But, now the last link is 1.018 (as in 5.1.2 – Improvement 2) instead of 1.012, and that the next-to-last (penultimate) 72-84 link is 1.044 instead of 1.022. In this case, the implied normalized incremental paid between 72 and 84 monthsis now $52.13 instead of the original $26.02. $52.13 is approximately twice $26.02, so the 72-84 activity would imply a tail factor of

1+200%(fitted tail factor −1) = 1+200%(1.0135-1) = 1+200%×.0135 = 1.0270.

The implied tail factor per the 84-96 link ratio is very close to the 1.0203 of the previous example. Note that the normalized paid loss in the 84-96 stage is $22.26 now or roughly 158% of paid loss. That implies a tail factor of

1+158%(1.0135-1) =1+158%×.0135 = 1.0213.

So, averaging the two, a tail factor in the range of 1.024 might be optimal.

5.2 Skurnick's[14] Simplification of McClenahan's Method

Skurnick's approach in [3] is essentially the same as McClenahan's. The difference is that Skurnick does not include the delay constant. Further, Skurnick does not calculate a single decay rate for the entire triangle using selected link ratios. Rather Skurnick fits a curve to each accident year and uses each curve as the sole mechanism of projecting each year's ultimate losses. Mathematically, his tail factor reduces to

$$(1-r)\Big/(1-r-r^{y})$$

where r and y are as before. In this case y denotes the number of years of development at which the tail factor will apply.

An Example

Consider the following incremental loss payouts:

Development	Accident Year	
Stage	1992	1991
12	4000	1000
24	2000	2000
36	1000	1000
48	500	500
60	250	250
72	125	125
84	62.5	62.5
96		31.25

[14] This method is also referred to as the 'Geometric Curve' method.

For illustration of the curve fitting process, the 1992 data produces the following table, when a curve is fit to the natural logarithms of the paid loss in each year (using the identity $\ln(A\times r^y) = \ln(A)+y\times\ln(r)$).

				Fitted Line					
Development	Stage	Amount	Log of	Ln(A) =	8.987	EXP = A =	8000	Fitted	Fit
Stage	in Years	Paid	Amount	Ln(r) =	-.693	EXP = r =	0.5	Curve	Error
12	1	4,000	8.29405					4,000	0
24	2	2,000	7.600902					2,000	0
36	3	1,000	6.907755					1,000	0
48	4	500	6.214608					500	0
60	5	250	5.521461					250	0
72	6	125	4.828314					125	0
84	7	63	4.135167					63	0

The tail factor is then $(1-.5)/(1-.5-.5^7)=.5/(1-.5-.007813) = 1.0159$.

The above is of course a contrived example. But consider the more typical case of the 1991 accident year. In this case, the payments begin low, then decrease after reaching a 'hump' in the 24 month stage. The eventual rate of decrease is still .5, but the curve fit produces:

				Fitted Line					
Development	Stage	Amount	Log of	Ln(A) =	8.294	EXP = A =	4000	Fitted	Fit
Stage	in Years	Paid	Amount	Ln(r) =	-0.578	EXP = r =	0.56123	Curve	Error
12	1	1,000	6.907755					2,245	-1,245
24	2	2,000	7.600902					1,122	878
36	3	1,000	6.907755					561	439
48	4	500	6.214608					281	219
60	5	250	5.521461					140	110
72	6	125	4.828314					70	55
84	7	63	4.135167					35	27
96	8	31	3.442019					18	14

Because of the hump shape 'r' is computed at a higher (i.e., less decay) value, .5613. Hence the tail factor is much larger at

$(1-.5613)/(1-.5613-.5613^7) = .4387/(.4387-.017554) =1.0417$.

5.2.1 Advantages and disadvantages of Skurnick's method

The primary advantage of Skurnick's method, at least relative to McClenahan's method, is that the calculations are much simpler. But correspondingly, this method involves not only all of the assumptions underlying McClenahan's method; a constantly decreasing pattern, exponential decay, and a lack of trend in the decay rate; it adds the assumption of no lag between the accident date and when payments begin. The last assumption is clearly untrue in the vast majority of cases.

As shown above, an additional major disadvantage is that it does not accommodate 'hump shaped' patterns well. The problems with hump-shaped curves serve as an introduction to the next improvement.

5.2.2 Improvement 3 – limit curve fitting to the more mature years

Skurnick's method is a prime candidate for this approach, because it is so common to have a 'hump-shaped' payout curve, whereas by the very nature of the exponential curve, exponential curves are monotonically decreasing. So, it is logical to refocus the tail estimation process, putting primary emphasis on the type of claims activity occurring near the tail.

Going back to the 'Brief Digression' on types of claims activity, the type of claims activity most closely associated with the tail does not begin until after 48 or 60 months. So, it would be logical to just fit the development curve to the paid after 60 months. The result of performing that limited fit on the 1991 data used to illustrate Skurnick's method is shown below.

				Fitted Line					
Development	Stage	Amount	Log of	Ln(A) =	8.987	EXP = A =	8000	Fitted	Fit
Stage	in Years	Paid	Amount	Ln(r) =	-0.69	EXP = r =	0.5	Curve	Error
72	6	125	4.82831					125	0
84	7	63	4.13517					63	0
96	8	31	3.44202					31	0

As expected, this produces the correct decay rate value of 'r' = .5, and the corresponding tail factor of 1.0159.

5.2.1 A note of caution

The above improvement is logical and generally works well with large volume high-credibility data. When the triangle is of 'medium'[15] size and has a fairly high cap on loss size, the triangle will not have full credibility. Therefore, a fit to paid data directly out of the triangle will likely lead to poor tail factor estimates. Of note, Skurnick's method is not the only method where this will yield poor tail estimates. It will happen with all the curve-fitting methods.

5.2.3. Improvements 1 and 2 applied to Skurnick's method

These improvements and their processes have likely been discussed enough earlier in this paper to eliminate a need for examples. Logically, both improvements may be applied while using Skurnick's method.

Method 1, using multiple ending years can be applied by simply fitting the curve to all the payments but the last year, computing the corresponding tail factor for the next-to-last stage of development, and dividing by the last link ratio.

Method 2 can be performed just as it was in McClenahan's method. For example, in the poor curve fit obtained when fitting to all of the 1991 data, the 'development portion' of the fitted tail, 1.0417-1=.0417 could be multiplied by the ratio of the actual incremental paid loss in the 96-108 stage (31, holding the place of the exact value 31.25) to the fitted value (rounded to 18). Note though, that the 'corrected' tail factor is even further off at $1+31\times.417/18 = 1.0718$. This illustration of when improvement 2 does <u>not</u> improve the tail factor prediction is intended to further show what happens when the type of curve fitted is a poor match for the pattern of the data.

5.3 Exponential Decay of the Development Portion of the Link Ratios[16]

This method is the first of several methods that extrapolate the tail factor off the loss development link ratios rather than the paid loss. This method was referred to briefly in the discussion of the Bondy method as a possible source of theoretical underpinnings for the two Bondy methods. The process is very simple. Given a set of link ratios $1+d_1$, $1+d_2$, $1+d_3$,.... $1+d_y$, a curve of the form

$$D\times r^m$$

where D is the fitted development portion of the first link ratio and r is the decay constant, is fit to the d_m's. The easiest way to do so is by using a regression to the natural logarithms of the d_m's. Then, for an ending d_y of small size, the additional development can be estimated by using the previous approach of

[15] It is very difficult to quatify 'medium' in a manner that will work across the different lines of insurance and still be meaningful years in the future. At the time this was written, an example of a 'medium' volume triangle might be a very large workers compensation self-insurance fund.
[16] This method was outlined in Sherman's paper, but likely was already heavily used by actuaries before Sherman's paper was published..

$$\prod_{m=y+1}^{\infty}(1+\ D\times r^{m})\ = \prod_{m=1}^{\infty}(1+\ d_y r^{m})\ \ \cong\ 1 + d_y \sum_{m=y+1}^{\infty} r^m\ = 1 + d_y r/(1\text{-}r).$$

This also automatically introduces Improvement 2 by fitting exactly to the last point. Similar algebra would show that the tail factor is approximated by

$1+\ D\times r^{y+1}/(1\text{-}r)$.

For an ending d_y of larger size, it may be necessary to simply project the link ratios for the next fifteen or so years (until the additional tail is immaterial), then multiply them all together to create a tail factor.

5.3.1 An example

Consider the following sample link ratio data.

Development Stage	Stage in Years	Link Ratio
12	1	1.5
24	2	1.25
36	3	1.125
48	4	1.0625
60	5	1.03125
72	6	1.015625
84	7	1.007813

The astute reader will notice that is a pattern similar to that underlying the Bondy method. In any event, to fit our exponential curve to the development portion, we first subtract unity to obtain the development portion of each link ratio.

Development Stage	Stage in Years	Link Ratio	Development Portion 'd'
12	1	1.5	0.5
24	2	1.25	0.25
36	3	1.125	0.125
48	4	1.0625	0.0625
60	5	1.03125	0.03125
72	6	1.015625	0.015625
84	7	1.007813	0.0078125

Then, as a precursor to curve fitting, we take the natural logarithms of the development portions, or "d's".

Development Stage	Stage in Years	Link Ratio	Development Portion 'd'	Log of d'
12	1	1.5	0.5	-0.69315
24	2	1.25	0.25	-1.38629
36	3	1.125	0.125	-2.07944
48	4	1.0625	0.0625	-2.77259
60	5	1.03125	0.03125	-3.46574
72	6	1.015625	0.015625	-4.15888
84	7	1.007813	0.0078125	-4.85203

Then, we fit a line to those logarithms. Standard commercial spreadsheet software produces:

Development	Stage	Link	Development	Log of	Fitted Curve Values	
Stage	in Years	Ratio	Portion 'd'	d'	Slope	-0.6931
					Intercept	0.0000
12	1	1.5	0.5	-0.69315		
24	2	1.25	0.25	-1.38629		
36	3	1.125	0.125	-2.07944		
48	4	1.0625	0.0625	-2.77259		
60	5	1.03125	0.03125	-3.46574		
72	6	1.015625	0.015625	-4.15888		
84	7	1.007813	0.0078125	-4.85203		

Then, our 'D', or development portion at time zero, is the exponent of the intercept, and the rate of reduction, 'r' is the exponent of the slope. Calculating the exponents and the fitted curve, we get:

Development Stage	Stage in Years	Link Ratio	Development Portion 'd'	Log of d'	Fitted Curve Values			Fitted Curve
					Slope	-0.6931		
					Intercept	0.0000		
12	1	1.5	0.5	-0.69315				1.50000
24	2	1.25	0.25	-1.38629	r = exp(slope)		0.5	1.25000
36	3	1.125	0.125	-2.07944	D = exp(intercept)		1	1.12500
48	4	1.0625	0.0625	-2.77259				1.06250
60	5	1.03125	0.03125	-3.46574				1.03125
72	6	1.015625	0.015625	-4.15888				1.01563
84	7	1.007813	0.0078125	-4.85203				1.00781
	8							1.00391
	9							1.00195
	10							1.00098
	11							1.00049
	12							1.00024
	13							1.00012
	14							1.00006
	15							1.00003
	16							1.00002
	17							1.00001
	18							1.00000
	19							1.00000
	20							1.00000
	21							1.00000
	22							1.00000

Then, for reference we compute the tail factor using both the 'quick' formula usable for small remaining 'development portions', and by multiplying the fifteen fitted link ratios that make up the tail.

Quick Formula Tail	
1+1×(.5^8)/(1-.5) =	1.00781
Product of 8-22 Links	1.00783

As one can see, the difference is negligible.

5.3.2 A more realistic example

The previous example was contrived to make the mathematics clear. Consider the following set of more realistic data.

Development Stage	Stage in Years	Link Ratio
12	1	2.000
24	2	1.250
36	3	1.090
48	4	1.050
60	5	1.040
72	6	1.030
84	7	1.028
96	8	1.020

A curve can be fit to the data using the methodology employed in the previous example.

Development	Stage	Link	Development	Log of	Fitted Curve Values			Fitted	Fit
Stage	in Years	Ratio	Portion 'd'	d'	Slope	-0.4415		Curve	Error
					Intercept	-0.5723			
12	1	2	1	0.0000				1.3628	-0.6372
24	2	1.25	0.25	-1.3863	r = exp(slope)		0.643042	1.2333	-0.0167
36	3	1.09	0.09	-2.4079	D = exp(intercept)		0.56422	1.1500	0.0600
48	4	1.05	0.05	-2.9957				1.0965	0.0465
60	5	1.04	0.04	-3.2189				1.0620	0.0220
72	6	1.03	0.03	-3.5066				1.0399	0.0099
84	7	1.028	0.028	-3.5756				1.0257	-0.0023
96	8	1.02	0.02	-3.9120				1.0165	-0.0035
108	9	1.018	0.018	-4.0174				1.0106	-0.0074

Note that the fit errors exhibit some cyclic behavior, negative as a group at first, then positive from 3-6 years, then negative again at 7-9 year maturities. This suggests that the curve may be a poor fit. That is borne out by the relationship of the tail factor estimates with and without exact fit to the last link ratio:

Quick Formula Tail	
1+D×(r^10)/(1-r) =	1.019108
Product of 8-22 Est. Links	1.019226
After exact fit to last link	
1+.0191×.018/.01061	1.032403

Once again the 'quick approximation' to the tail is almost identical to the precise tail indicated by exponential decay. However, note that because of the poor fit of the curve near the tail, the use of Improvement 2 (exact fitting to the last link ratio) produces a markedly different tail factor. The question of which tail factor is best must now be answered.

To do so, Improvement 3 (fitting the curve solely to the mature years) is in order. In this case, the curve will simply be fit to years 4 (48 months) and beyond. That produces the following fit;

Development	Stage	Link	Development	Log of	Fitted Curve Values			Fitted	Fit
Stage	in Years	Ratio	Portion 'd'	d'	Slope	-0.2073		Curve	Error
					Intercept	-2.1900			
48	4	1.05	0.05	-2.9957				1.0488	-0.0012
60	5	1.04	0.04	-3.2189	r = exp(slope)		0.812748	1.0397	-0.0003
72	6	1.03	0.03	-3.5066	D = exp(intercept)		0.111915	1.0323	0.0023
84	7	1.028	0.028	-3.5756				1.0262	-0.0018
96	8	1.02	0.02	-3.9120				1.0213	0.0013
108	9	1.018	0.018	-4.0174				1.0173	-0.0007

Which produces the following tail estimates:

Quick Formula Tail	
1+D×(r^10)/(1-r) =	1.075166
Product of 10-24 Est. Links	1.075813
After exact fit to last link	
1+.075×.018/.0173	1.078035

Due to the low fit errors, as long as the 48-120 development triangle data that generated the link ratios is credible, this would strongly suggest that a tail factor of around 1.075 is needed. Note also that the 'quick approximation also works well in this instance. In summary, this example illustrates the importance of restricting use of the fitted curve to the portion of the development data that it can reasonably fit.

5.3.3 Advantages and disadvantages of this method

A primary advantage of this method is it's simplicity. The assumption of exponential decay is relatively easy to understand. The calculations have moderate complexity, but an illustration of the fitted values can readily give laypeople comfort that the method is being executed correctly. Of note, this method is 'asymptotically equal' to both McClenahan's and Skurnick's methods, yet is much simpler to execute. That also leads to it's major disadvantage. Because it assumes such a quick decay of the link ratios (exponential decay is faster decay than $1/x$, $1/x^2$, $1/x^3$, etc.), it can easily underestimate the tail.

5.4 Sherman's Method - Fitting an Inverse Power Curve to the Link Ratios

This method, the last[17] of the curve fitting approaches to be discussed, was first articulated by Sherman [2]. Sherman noted[18], while fitting a curve from the McClenahan-Skurnick-Exponential Decay family, that the 'decay ratios' (ratios of successive development portions of link ratios) were not constant as suggested by expoential decay. Rather, as one went further out in the development pattern, the decay ratios rose towards unity (i.e. there was less and less decay as one went further out in the curve). Looking at the data, it appeared that asymptotically, the decay ratios approached unity. Based on this, he posited an 'inverse power' curve of the form $1+a\times t^b$ (t representing the maturity in years) for the link ratios. Sherman then investigated the quality of curve fit to actual industry data for several families of curves, including the inverse power curve. The family that he found generally fit best were the so-called 'inverse power' curves.

The process of fitting an inverse power curve is very similar to that used to fit the exponential curve, excepting that the 'independent variable' used in the curve fit is ln(t). More specifically, the identity

$$\ln(1+d-1) = \ln(d) \cong \ln(1+a\times t^b-1) = \ln(a\times t^b) = \ln(a) + b\times\ln(t)$$

can be used to create an opportunity to base the fitted curve on a simple regression.

[17] Sherman also discussed the fitting of a lognormal curve to the cumulative paid (or implied cumulative paid) and the fit of a logarithmic curve to the link ratios. However, the lognormal fit does not lend itself to easy spreadsheet mathematics, and the logarithmic fit to the link ratios does not produce a unique tail factor. Further, a Sherman discussed, the inverse power curve is a preferable approach.

[18] Mr. Sherman discusses this in Section III of [3].

Unfortunately, this author is not aware of any simple closed form approximation to the tail this curve generates, so the tail factor must be estimated by multiplying together the successive link ratios after the tail begins until the impact of additional link ratios is negligible.

5.4.1 An example

This may best be illustrated by using the initial dataset used for the exponential decay approach:

Development Stage	Stage in Years	Link Ratio
12	1	1.5
24	2	1.25
36	3	1.125
48	4	1.0625
60	5	1.03125
72	6	1.015625
84	7	1.007813

The first step is to calculate the development portion of each link ratio and take natural logarithms of the result.

Development Stage	Stage in Years	Link Ratio	Development Portion 'd'	Log of d'
12	1	1.5	0.5	-0.6931
24	2	1.25	0.25	-1.3863
36	3	1.125	0.125	-2.0794
48	4	1.0625	0.0625	-2.7726
60	5	1.03125	0.03125	-3.4657
72	6	1.015625	0.015625	-4.1589
84	7	1.007813	0.0078125	-4.8520

Those will represent the 'dependent variable' in our regression. Then for the independent variable, we take natural logarithms of the development stage/beginning maturity for the link ratio in years.

Development	Stage	Link	Development	Log of	Log of
Stage	in Years	Ratio	Portion 'd'	d'	Stage in Yrs
				'X'	'Y'
12	1	1.5	0.5	-0.6931	0.0000
24	2	1.25	0.25	-1.3863	0.6931
36	3	1.125	0.125	-2.0794	1.0986
48	4	1.0625	0.0625	-2.7726	1.3863
60	5	1.03125	0.03125	-3.4657	1.6094
72	6	1.015625	0.015625	-4.1589	1.7918
84	7	1.007813	0.0078125	-4.8520	1.9459

Then, we compute the regression parameters.

Development	Stage	Link	Development	Log of	Log of	Fitted Curve Parameters		
Stage	in Years	Ratio	Portion 'd'	d'	Stage in Yrs			
				'X'	'Y'	Slope =	-2.10512	=b
12	1	1.5	0.5	-0.6931	0.0000	Intercept =	-0.20881	
24	2	1.25	0.25	-1.3863	0.6931	a = exp(intercpt)	0.811553	
36	3	1.125	0.125	-2.0794	1.0986			
48	4	1.0625	0.0625	-2.7726	1.3863			
60	5	1.03125	0.03125	-3.4657	1.6094			
72	6	1.015625	0.015625	-4.1589	1.7918			
84	7	1.007813	0.0078125	-4.8520	1.9459			

Following that, we compute the fitted curve values and the fit error.

Development	Stage	Link	Fitted Curve Parameters			Fitted	Fit
Stage	in Years	Ratio				Curve	Error
			Slope =	-2.10512	=b		
12	1	1.5	Intercept =	-0.20881		1.8116	0.3116
24	2	1.25	a = exp(intercept)	0.811553		1.1886	-0.0614
36	3	1.125				1.0803	-0.0447
48	4	1.0625				1.0438	-0.0187
60	5	1.03125				1.0274	-0.0038
72	6	1.015625				1.0187	0.0030
84	7	1.007813				1.0135	0.0057
	8					1.0102	
	9					1.0080	
	10					1.0064	
	11					1.0052	
	12					1.0043	
	13					1.0037	
	14					1.0031	
	15					1.0027	
	16					1.0024	
	17					1.0021	
	18					1.0018	
	19					1.0016	
	20					1.0015	
	21					1.0013	
	22					1.0012	

And, the tail factor estimates are:

Fitted Tail =	1.056977	
Exact Fit to last link		
1+0.056977×0.007813/0.0135		
=	1.032975	.

Even with the utility this adds in the fit, the initial fit produces a tail factor of over 1.05, when the previous exponential decay analysis suggested only 1.00781. The exact fit correction, though, does produce a number that is much closer to the theoretical tail.

Again, one approach is to fit solely to the mature years. That approach produces the following regression calculations:

Development	Stage	Link	Development	Log of	Log of	Fitted Curve Parameters		
Stage	in Years	Ratio	Portion 'd'	d'	Stage in Yrs			
				'X'	'Y'	Slope =	-3.69867	=b
48	4	1.0625	0.0625	-2.7726	1.3863	Intercept =	2.413854	
60	5	1.03125	0.03125	-3.4657	1.6094	a = exp(intercpt)	11.17696	
72	6	1.015625	0.015625	-4.1589	1.7918			
84	7	1.007813	0.0078125	-4.8520	1.9459	.		

And then it produces the following fitted curve:

Development	Stage	Link	Fitted Curve Parameters			Fitted	Fit
Stage	in Years	Ratio				Curve	Error
			Slope =	-3.69867	=b		
48	4	1.0625	Intercept =	2.413854		1.0663	0.0038
60	5	1.03125	a = exp(intercpt)	11.17696		1.0290	-0.0022
72	6	1.015625				1.0148	-0.0008
84	7	1.007813				1.0084	0.0006
	8					1.0051	
	9					1.0033	
	10					1.0022	
	11					1.0016	
	12					1.0011	
	13					1.0008	
	14					1.0006	
	15					1.0005	
	16					1.0004	
	17					1.0003	
	18					1.0003	
	19					1.0002	
	20					1.0002	
	21					1.0001	
	22					1.0001	

And, the tail it produces, although it remains higher than the theoretical tail (at a certain level, the slower decay of the inverse power curve as compared to an exponential curve makes it inevitable that it will produce a higher tail) is much closer to the theoretical tail.

Fitted Tail =	1.017077	
Exact Fit to last link		
1+0.017077×0.007813/0.0084		
=	1.015884	

5.4.2 The more realistic example

Going back to the exponential decay, a tail was fit to the more realistic link ratios shown below:

Development Stage	Stage in Years	Link Ratio
12	1	2
24	2	1.25
36	3	1.09
48	4	1.05
60	5	1.04
72	6	1.03
84	7	1.028
96	8	1.02
108	9	1.018

As in the previous example, we fit an inverse power curve:

Development	Stage	Link	Development	Log of	Log of	Fitted Curve Parameters		
Stage	in Years	Ratio	Portion 'd'	d'	Stage in Yrs			
				'X'	'Y'	Slope =	-1.82492	=b
12	1	2	1	0.0000	0.0000	Intercept =	-0.18424	
24	2	1.25	0.25	-1.3863	0.6931	a = exp(intercpt)	0.83174	
36	3	1.09	0.09	-2.4079	1.0986			
48	4	1.05	0.05	-2.9957	1.3863			
60	5	1.04	0.04	-3.2189	1.6094			
72	6	1.03	0.03	-3.5066	1.7918			
84	7	1.028	0.028	-3.5756	1.9459			
96	8	1.02	0.02	-3.9120	2.0794			
108	9	1.018	0.018	-4.0174	2.1972			

And then we compute the fitted curve values for the link ratios that comprise the tail. Since the link ratios decay so slowly, we project thirty years of additional development instead of fifteen.

Development Stage	Stage in Years	Link Ratio	Fitted Curve Parameters			Fitted Curve	Fit Error
			Slope =	-1.82492	=b		
12	1	2.000	Intercept =	-0.18424		1.8317	-0.1683
24	2	1.250	a = exp(intercpt)	0.83174		1.2348	-0.0152
36	3	1.090				1.1120	0.0220
48	4	1.050				1.0663	0.0163
60	5	1.040				1.0441	0.0041
72	6	1.030				1.0316	0.0016
84	7	1.028				1.0149	1.0239
96	8	1.020				1.0111	1.0187
108	9	1.018				1.0151	-0.0029
	10					1.0124	
	11					1.0105	
	12					1.0089	
	13					1.0077	
	14					1.0067	
	15					1.0059	
	16					1.0053	
	17					1.0047	
	18					1.0043	
	19					1.0039	
	20					1.0035	
	21					1.0032	
	22					1.0030	
	23					1.0027	
	24					1.0025	
	25					1.0023	
	26					1.0022	
	27					1.0020	
	28					1.0019	
	29					1.0018	
	30					1.0017	
	31					1.0016	
	32					1.0015	
	33					1.0014	
	34					1.0013	
	35					1.0013	
	36					1.0012	
	37					1.0011	
	38					1.0011	
	39					1.0010	

That produces the following tail data.

Fitted Tail =	1.114487
Exact Fit to last link	
1+0.11451×0.018/0.0151	
=	1.136502

For comparison, the final 'best estimates' using the exponential decay were in the 1.03-1.05 range. But, those best estimates were based off a fit to just the mature years. So, let us fit the curve solely to the 48+ month data.

Development	Stage	Link	Development	Log of	Log of	Fitted Curve Parameters		
Stage	in Years	Ratio	Portion 'd'	d'	Stage in Yrs			
				'X'	'Y'	Slope =	-1.28108	=b
48	4	1.05	0.05	-2.9957	1.3863	Intercept =	-1.18688	
60	5	1.04	0.04	-3.2189	1.6094	a = exp(intercpt)	0.305171	
72	6	1.03	0.03	-3.5066	1.7918			
84	7	1.028	0.028	-3.5756	1.9459			
96	8	1.02	0.02	-3.9120	2.0794			
108	9	1.018	0.018	-4.0174	2.1972			

However, in this case, the tail is even higher, per the fit

Development	Stage	Link	Fitted Curve Parameters			Fitted	Fit
Stage	in Years	Ratio				Curve	Error
			Slope =	-1.28108	=b		
48	4	1.05	Intercept =	-1.18688		1.0517	0.0017
60	5	1.04	a = exp(intercpt)	0.305171		1.0388	-0.0012
72	6	1.03				1.0307	0.0007
84	7	1.028				1.0252	-0.0028
96	8	1.02				1.0213	0.0013
108	9	1.018				1.0183	0.0003
	10					1.0160	
	11					1.0141	
	12					1.0126	
	13					1.0114	
	Etc.					Etc.	

Multiplying the link ratios that comprise the tail factor together, the estimated tail is:

Fitted Tail =	1.208566
Exact Fit to last link	
1+0.2086×0.018/0.0183	
=	1.20518

So, this illustrates how this method is generally more conservative than the exponential decay method.

5.4.3 Advantages and disadvantages of Sherman's method

Relative to the other curve-fitting methods, this method's primary strengths and weaknesses stem from it's source, although that is mitigated by the fact that in choosing the form of the mathematical curve family that was used (the inverse power curve), Sherman relied heavily on actual data. Specifically, he noted that exponential decay factors flattened heavily (i.e., rose toward unity) at later ages. So, he chose the inverse power curve as his model to reduce the decay at later ages. In a sense, Sherman designed the inverse power curve with an eye toward mathematically correcting an observed deficiency in the exponential decay method. The approach he used to correct exponential decay[19] was merely to find a curve that roughly matched the data he observed. So, since the inverse power approach is based on actual properties of the observed development link ratio curves, and appears to have superior fit to the data, it should arguably be a better predictor of the tail. But on the other hand it also gives no single simple assumption (such as decay proportional to development portion size) that we can test the data against. In other areas, the fit looks a little more mathematically complex to the outsider, but is no more computationally difficult for the practitioner than exponential decay of the link ratios.

5.5 Sherman's Revised Method - Adding Lag to the Inverse Power Curve

In his study of the inverse power curve, Sherman [3] noted that the fit could sometimes be improved by adding a lag parameter to the curve. He used the formula

$1+d \cong 1+a\times(t-c)^b$.

In this case, the mechanics of fitting the curve are somewhat more complex. An example will illustrate the process.

[19] Sherman effectively replaced $1+Dr^t$ from exponential decay with $1+a\times t^b$. Note that a in the inverse power curve plays the same role as D in exponential decay, so really he just replaced r^t, with a constant decay ratio of r by t^b, with a decay rate of $((t+1)\div t)^b$, which is asymptotically one.

5.5.1 Example of fitting an inverse power curve with lag

We first set the lag equal to one (unity) to begin the process, then fit the an inverse power curve reflecting that lag

Development	Stage	Link	Development	Log of	Stage	Log of Rev.	Fitted Curve Parameters		
Stage	in Years	Ratio	Portion 'd'	d'	Minus Lag	Stage in Yrs	Lag =	1	
							Slope =	-1.0273	=b
48	4	1.05	0.05	-2.9957	3.0000	1.0986	Intercept =	-1.8324	
60	5	1.04	0.04	-3.2189	4.0000	1.3863	a = exp(intercpt)	0.1600	
72	6	1.03	0.03	-3.5066	5.0000	1.6094			
84	7	1.028	0.028	-3.5756	6.0000	1.7918			
96	8	1.02	0.02	-3.9120	7.0000	1.9459			
108	9	1.018	0.018	-4.0174	8.0000	2.0794			

Then we compute the link ratios on the fitted curve, and the total squared fit error as well

Development	Stage	Link	Fitted Curve Parameters			Fitted	Fit	Squared
Stage	in Years	Ratio	Lag =	1		Curve	Error	Error
			Slope =	-1.027387872	=b			
48	4	1.05	Intercept =	-1.832444677		1.0385	-0.0115	1.32E-04
60	5	1.04	a = exp(intercpt)	0.160021887		1.0306	-0.0094	8.79E-05
72	6	1.03				1.0254	-0.0046	2.12E-05
84	7	1.028				1.0217	-0.0063	4.00E-05
96	8	1.02				1.0189	-0.0011	1.22E-06
108	9	1.018				1.0167	-0.0013	1.58E-06
								2.84E-04

We note that the total fit error associated with a lag of one is .000284.

Next, in order to estimate the optimum lag, we use a bisection process, following the process above for different potential lags; finding the lowest value of the squared error across a group of values; and progressively narrowing the range. The computations were as follows, and only 7 steps were needed. For reference, at each step of the process the lowest value of the fit error as well as the two adjacent values (the three values generated by the lag points that will be carried to the next step of the process) are in bold.

Stage 1		Stage 2		Stage 3	
	Squared		Squared		Squared
Lag	Error	Lag	Error	Lag	Error
		-1	7.06E-04	-0.5	1.58E-04
-1	**7.06E-04**	-0.5	**1.58E-04**	-0.25	**4.86177E-05**
0	**1.32E-05**	0	**1.32E-05**	0	**1.32454E-05**
1	**2.84E-04**	0.5	**9.22E-05**	0.25	**3.28903E-05**
2	7.50E-04	1	2.84E-04	0.5	9.22146E-05
3	1.08E-03				
Stage 4		Stage 5		Stage 6	
	Squared		Squared		Squared
Lag	Error	Lag	Error	Lag	Error
-0.25	4.86177E-05	-0.125	2.2949E-05	-0.0625	1.62499E-05
-0.125	**2.2949E-05**	-0.0625	**1.625E-05**	-0.03125	1.43034E-05
0	**1.32454E-05**	0	**1.3245E-05**	**0**	**1.32454E-05**
0.125	**1.72333E-05**	0.0625	**1.366E-05**	**0.03125**	**1.30419E-05**
0.25	3.28903E-05	0.125	1.7233E-05	**0.0625**	**1.36599E-05**
Stage 7					
	Squared				
Lag	Error	**Final Selection**		**0.02**	
0	**1.32454E-05**				
0.015625	**1.30389E-05**				
0.03125	**1.30419E-05**				
0.046875	1.32502E-05				
0.0625	1.36599E-05				

Note that as the fit error changes little near the minimum point, a rounded value is acceptable.

Then, that lag value may be used in the final curve fit.

Development	Stage	Link	Fitted Curve Parameters			Fitted
Stage	in Years	Ratio	Lag =	0.02		Curve
			Slope =	-1.253797784	=b	
48	4	1.05	Intercept =	-1.23628766		1.0511
60	5	1.04	a = exp(intercpt)	0.290460507		1.0386
72	6	1.03				1.0307
84	7	1.028				1.0253
96	8	1.02	Fitted Tail =	1.230663894		1.0214
108	9	1.018				1.0185
	10		Exact Fit to last link			1.0162
	11		1+0.2307×0.018/0.0185			1.0144
	12		=	1.224464865		1.0129
	13					1.0117
	14					1.0106
	15					1.0097
	16					1.0090
	17					1.0083
	18					1.0077
	19					1.0072
	20					1.0068
	21					1.0064
	22					1.0060
	23					1.0057
	24					1.0054
	25					1.0051
	26					1.0049
	27					1.0047
	28					1.0045
	29					1.0043
	30					1.0041
	31					1.0039
	32					1.0038
	33					1.0036
	34					1.0035
	35					1.0034
	36					1.0032
	37					1.0031
	38					1.0030
	39					1.0029

Which provides a slightly smaller tail.

Fitted Tail =	1.2306
Exact Fit to last link	
1+0.2307×0.018/0.0185	
=	1.2244

5.5.2 Advantages and disadvantages of introducing lag in the inverse power curve

Summarizing, we can note that while the lag factor may sometimes mitigate the size of the tail, the inverse power in general tends to produce a higher tail than the exponential fit. Although it has not been illustrated herein with actual data, the inverse power curve also generally indicates higher tail factors than McClenahan's and Skurnick's methods, as those methods tend to produce results that are very similar to that of the exponential decay[20]. As before, the inverse power curve's main attraction is that it simply seems to fit the data better. However, in introducing lag it is clear that much computational complexity is added. The practitioner should evaluate whether the additional complexity produces large gains in the accuracy of the estimated tail factor.

6. SUMMARY

Several different methods for assessing tail development were presented, as well as some refinements. Hopefully, this will help the reader in his or her actuarial practice.

[20] That is because they are simply based on exponential decay of the payments rather than the link ratios. A little analysis will show that their decay patterns are about equal for 'large' maturities. If in doubt, simply consider their asymptotic properties.

Appendix 1-Tail Factor Methods Based on Counts

A.1 Introduction

Although they are less commonly used, there are several methods for estimating tail factors that are based on counts. Among these are the Sherman-Diss method, the projected unpaid severity method, and what is really a older year ultimate loss selection method instead of a tail factor method for unexpectedly low open counts that is based on maximum possible costs per claim. All of these methods do have demonstrated limitations, though. So, it is just as important to understand the limitations of each method as it is to understand the methods themselves.

A.2 The Sherman-Diss Method

The Sherman-Diss method described in [4] is a specific example of what could become a class of methods that project the open claim counts at future times, and the cost per claim at each future period. For the first step, this method involves projecting the likelihood that each 'mature' (near the tail maturity) workers compensation claim will still be open next year, the following year, the year following that, etc. using life (mortality) tables and the claimant's current age. Then, the indemnity (wage replacement) benefits each would receive in each future period (if they are still alive to collect benefits as estimated using the life table) is estimated using each worker's current annual benefit, plus an estimate of any inflation in the benefit (should any be allowed under the law of the injured worker's state). The total indemnity tail would then be calculated by extending the probability of each claimant's survival at each future period (the expected open claim counts) times the annual indemnity benefit. For the medical benefits allowed claimants under the workers compensation laws, the probability of survival to each future period is extended by the current medical inflated by an appropriate medical inflation factor. The extension of probability of survival times medical benefits produce the dollars of medical tail.

A.2.1 Pros and cons

Due to the complexity of the calculations and the status of this discussion as an appendix rather than the main paper, an example will not be provided. However, some discussion of this and the other methods in this appendix is certainly in order.

When Sherman and Diss compared their method to other tail factor methods (primarily the curve-fitting methods) on some specific workers compensation data, they found that it produced much higher tail factors than the other methods. However, when they tested their method retrospectively against actual dollar emergence on some Western state fund data, they found that as claimants achieved advanced ages (roughly at thirty to forty-plus years of development) the medical became much higher than that predicted by their method. Per their studies, it appears that as claimants achieve advanced ages, unexpected (at least per life tables and medical) additional development occurs because the main injury may cause related illnesses that are exacerbated by age and because family or spousal care for severely injured claimants must be replaced by nursing home care as the caregivers age and become infirm. So, at least for direct and unlimited workers compensation benefits, it appears that many common methods produce an inadequate tail, but that this method does not fully solve the problem.

Also note that this 'open claim count' method is suitable only for lines where benefits are paid as long as claims remain open. To this author's knowledge, the only lines of insurance that have that feature are workers compensation and disability.

Further, this method was designed for direct and unlimited claim costs, when most insurers purchase some form of specific excess reinsurance that caps the insurer's costs at some 'net retention'. Note however, that method could be revised by accumulating the total projected costs paid to each claimant and eliminating the claim once the net retention is reached[21]. In so doing, each claim would be effectively capped at the retention.

Lastly, this method only directly produces a tail factor for the mature years. If there is a low volume of claims remaining open in the older years (as is often the case), the results of this method will not be a reliable statistic for projecting the tail on the later years (i.e., they will lack credibility).

Qualifications aside, this method does create a powerful tool in the right circumstances. Futher, as time goes by it is possible that other 'remaining open count'-based methods will be developed.

[21] Of note, it is also appropriate to build in any projected costs that exceed the limit of per claim reinsurance purchased.

A.3 The Unclosed Count Method

This method also requires qualifications, but is worth discussion. Just as in workers compensation, the open status of a claim is related to payments. In most other lines the majority of payments occur at the time of claim closing. So, it is reasonable to suppose that there would be a method based on the number of claims yet to close and the average cost of each of those claims. Of course, while it may be relatively easy to estimate the number of claims that will close in the future as long as the actuary is certain that no further IBNR claims will materialize; it is usually very difficult if not impossible to estimate the average costs of closing each claim. However, in some limited circumstances, the average paid loss per closed claim of the oldest accident year may have reliably and permanently plateaued. In those specific circumstances (and only those circumstances), it would be appropriate to multiply the number of unclosed claims by the average paid loss per closed claim from the latest twelve months for the given accident year.

A.3.1 Pros and cons

This method cannot be discussed without discussing the tremendous detraction posed by blithely assuming that the current average paid loss per closed claim will equal the average cost of disposing of the open claim inventory. The author has personally seen general liability data of about 48 months maturity and fairly low volume where the average paid per claim had leveled off at around $5,000 per claim, where only four claims were open, but they were all $20,000+ claims. One major problem was that the maturity was only 48 months. So, the actuary is strongly cautioned to use this only for data of at least 96 months maturity, preferably 120 months, and to carefully review whether the remaining open claims are of the same type, class, average demand, etc. as the claims closed between, say, 96 and 120 months.

The actuary is also cautioned that if the data volume is not overwhelming large, the percentage of claims left open for the older years now may not match the percentage of claims left open at 120 months or so on the more recent years once they reach the 120 month stage. For example, if only four or so claims are left open on the older years, they will lack statistical validity (a form of credibility) in predicting what will be open when the more recent years reach the same development stage. Therefore, they will lack validity in predicting the tail factors for the more current years.

All that being said, under the right circumstances this can be a useful method. One must simply make sure that the set of underlying assumptions hold in whatever circumstance the actuary is using this method.

A.4 The Maximum Possible Loss Method

This method is a variant of the unclosed count method. It, however, does not so much create a tail factor as it does establish a maximum tail for the older years. The core idea of this method is that, given that the maximum net liability of an insurer is some net retention 'R', the liability for all the open claims should not be more than the sum of R-paid to date across all the open claims. So, to use it, given that an accident year is sufficiently mature for no IBNR claims to be reasonably possible, the remaining amounts to reach the retention (R-paid to date) are summed across all remaining open claims in the accident year. The result is not so much an estimate of the tail factor as an upper bound on tail development for that specific year. So, if application of the tail factor to a given year suggests more development than is 'possible' per the remaining amounts to reach the retention in the accident year, the ultimate unpaid loss for that accident year might be capped at the amounts remaining to reach the retention.

In the (fairly unusual) event that there are enough claims left open for this to be a statistically valid predictor of the development of the more recent years, it could be used in estimating the tail factor for all the accident years. But, one would have to be certain that this finding was statistically consistent with the initial tail factor analysis. For example, if the initial tail factor came from a curve fitting, it might be reasonable statistically that the curve fitting was simply using the wrong curve. However, if the initial tail factor came from a 'paid over disposed' method that also used the actual data in the triangle itself, the tail findings would suggest the data is internally inconsistent. In that case, greater care must be taken to understand which method is most accurate for the tail factor to be applied to the more recent years.

A.4.1 Pros and cons

This method improves on the average unpaid loss method by virtue of the fact that the amount to reach the retention need not be estimated. Rather, it is fact. However, it only produces an upper bound, not an actual best estimate.

Like the average unpaid loss method, there are often statistical reliability issues when making inferences about the tail factors of the more recent years. But, one cannot readily dispute the results as an upper bound for the older years on which the method is applied, at least as long as one is certain the prospect of additional IBNR claims is immaterial. So, like the average unpaid loss method, one must be very careful to make sure the proper assumptions hold when using it. But, unlike the average unpaid loss method, it has far more certainty surrounding the loss sizes.

Appendix 2-Developing Case Reserves on the Older Years

This method is also not so much a method for estimating the tail factors to use in incurred or paid loss development as it is a method for estimating ultimate losses in the very mature years. The scenario this addresses is that of a medium-to-low credibility (medium-to-low volume of losses in relation to the net retention) loss triangle. In that scenario it is not unusual for the remaining unpaid loss in the mature to vary significantly depending on whether a large claim, or a few large claims, or no large claims happen to have occurred and still be open in the late development stage. In such circumstances, the standard application of a tail factor may not work simply because there are not enough open claims in the mature years, or even open claims expected in the tail factor, for the law of large numbers to apply. In that case, some recognition of the specific cases remaining open (assuming no further reopenings or IBNR claims) will make the resulting ultimate loss predictions for the older years more accurate.

The process is fairly simple. Given a ratio of what it actually costs to close cases vs. the case reserves held from the 'paid loss to reserve disposed of' method, one simply multiplies that ratio times the case reserve to obtain an estimate of the unpaid loss on each of the very mature years. The ultimate loss estimate for each of those years would simply be the derived unpaid loss estimate plus the paid-to-date for each year.

A word of caution is in order, however. Remember that this method was used to estimate the ultimate loss because the law of large numbers did not work. Therefore, the unpaid losses derived using this method lack credibility in estimating the tail factor for the less mature years. So, if this method is used because the remaining unpaid losses are driven by 'luck of the draw'[22], it is illogical to use the unpaid losses from this method to estimate a tail factor for the less mature years.

Pros and cons

This method's inherent advantage is it's usefulness in low credibility situations. It's disadvantage is that it does not truly produce a tail factor, just some estimates of ultimate loss for the older years. Further, it assumes no reopenings or true IBNR claims. So, it must be used with great caution and respect for it's limitations.

[22] The astute reader will note that the adjusted case reserves are exactly what is used to develop a tail factor in the 'paid loss to reserve disposed of' method. But note that in that instance the tail is presumably based on case reserves that are large enough to have reasonable credibility.

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