Risk Transfer Testing of Reinsurance Contracts: Analysis and Recommendations

CAS Research Working Party on Risk Transfer Testing

Abstract

This paper was prepared in response to a call from the American Academy of Actuaries Committee on Property and Liability Financial Reporting (COPLFR). The call requested ideas about how to define and test for risk transfer in short duration reinsurance contracts as required by FAS 113 and SSAP 62. These accounting standards require that a reinsurance contract must satisfy one of two conditions in order to qualify for reinsurance accounting treatment: 1) the contract must transfer "substantially all" of the underlying insurance risk, or failing that, 2) it must at least transfer "significant" insurance risk. The paper presents methods to test for both conditions, but the main focus is on testing for "significant" risk transfer. The shortcomings of the commonly used "10-10" test are discussed and two alternative testing frameworks are presented as significant improvements over "10-10". The first of these, which is presented in detail, is based on the expected reinsurer deficit (*ERD*). Conceptually, that approach is a refinement and generalization of "10-10" that addresses its major shortcomings. The second framework, based on the right tail deviation (*RTD*), is presented more briefly. It has certain desirable properties but at the cost of greater complexity.

Keywords: risk transfer testing, FAS 113, "10-10" test, downside risk, expected reinsurer deficit (*ERD*), right tail deviation (*RTD*), tail value at risk (*TVaR*), parameter uncertainty

1. INTRODUCTION

The purpose of this paper is to propose an improved framework for testing short-duration reinsurance contracts for risk transfer compliance with FAS 113. Under that accounting statement, reinsurance accounting is allowed only for those indemnity contracts that transfer insurance risk. The aim of the paper is to present a theoretically sound but practical approach to determining whether a contract meets the risk transfer requirements of FAS 113.

1.1 Context

The working party that prepared this paper was formed by the CAS to respond to a call by the American Academy of Actuaries Committee on Property and Liability Financial Reporting (COPLFR) for the submission of actuarially sound ideas about how to define and test for risk transfer in reinsurance transactions. The American Academy call arose out of the need for a constructive response from the actuarial profession following some widely publicized cases of alleged abuse of finite reinsurance and related accounting principles. Those cases have led to renewed scrutiny of reinsurance contracts to ascertain whether they comply with the existing accounting requirements and to a broader inquiry as to whether

FAS 113 goes far enough in specifying the manner in which contracts will be accounted for either as reinsurance or otherwise.

In a letter dated June 13, 2005, and addressed to members of the CAS, the chair of COPLFR framed the request as follows:

"Property/casualty actuaries interested in contributing suggestions...are asked to submit responses to one or more of the following questions:

1. What is an effective test for risk transfer? (Respondents are asked to focus on actuarial methodology and provide examples as appropriate.)
2. What criteria should be used to determine whether a reinsurance contract transfers significant risk to the reinsurer? (Respondents are asked to focus on decision criteria used to evaluate the results of the test described in question #1.)
3. What safe harbors, if any, should be established so that a full risk transfer analysis does not have to be completed for each and every reinsurance contract (i.e., in what instances is risk transfer "reasonably self-evident" and therefore cash flow testing is not necessary to demonstrate risk transfer)?
4. What are the advantages and disadvantages of the suggested approach versus other approaches commonly used?"

There is very little published actuarial literature on the subject. The only significant paper appears to be the one prepared in 2002 by the CAS Valuation, Finance, and Investments Committee entitled, "Accounting Rule Guidance Statement of Financial Accounting Standards No. 113—Considerations in Risk Transfer Testing"[1]. That paper provided an excellent summary of FAS 113 and the risk transfer testing methods that emerged in response (including the "10-10" test) as well as a discussion of a number of alternative methods. However, the paper was fairly muted in its criticism of "10-10", and it did not strongly advocate replacing it with an alternative.

In this paper we seek to respond to all four of the questions posed by COPLFR. The members of the working party believe the time has come to be explicit about the shortcomings of the "10-10" test that has come into common use and to advocate its replacement with a better framework. Accordingly, in this paper we include an extensive critique of the "10-10" test and describe two frameworks, one in detail and the other in summary, that would be significant improvements over "10-10". We also identify methods for determining whether individual reinsurance contracts should be subject to detailed testing.

The frameworks described in the paper primarily address the issue of developing a more consistent and rigorous quantitative approach for the evaluation of risk transfer. As a result, the approaches described might reduce the potential for accounting mistakes simply by virtue of the higher level of clarity and consistency that result from their application. But the working party wants to make it very clear that no quantitative methodology will ever be fully successful in detecting intentional attempts at fraud or accounting abuse. Regulators and auditors face a difficult but necessary task in ferreting out the motives and intent of the producers of accounting statements. Actuaries are important partners and advisors in the area, especially in areas such as risk transfer. But it would be a mistake to think that actuaries or any other quantitative expert can provide a formula that reduces the analysis of intent, good or bad, to a simple (or even complex) calculation. This is important, because many of the alleged acts that have topped recent headlines are in fact much more about bad intent than risk transfer. No matter how good this working party's work, the methodologies developed here would not likely have prevented many of the alleged abuses, at least not without other efforts to discern the intent of the transactions.

At the same time, it is important to remember that in most reinsurance transactions the parties are acting in good faith and their intentions are good. Just as a mathematical test cannot identify bad intent, it cannot by itself discern the likely good intent of the parties. Therefore, the failure of a contract to meet a quantitative risk transfer test should not result in denial of reinsurance accounting treatment to a transaction without a thorough review of the all aspects of the deal, including the question of intent.

1.2 Disclaimers

While this paper is the product of a CAS working party, its findings do not necessarily represent the official view of the Casualty Actuarial Society. Moreover, while we believe the approaches we describe are very good examples of how to address the issue of risk transfer, we do not claim they are the only acceptable ones.

In the course of the paper, in order to make our ideas as clear as possible, we present a number of numerical examples that require assumptions about the distribution of losses and appropriate threshold values for the risk transfer tests we describe. We recognize that any loss model we choose is an approximation to reality at best and might even be a poor one, and that with respect to the decision about appropriate risk transfer threshold values, other constituencies, including regulators, accountants and outside auditors have a key role to play. In making such assumptions for purposes of illustration, we are not necessarily endorsing any particular loss model or threshold value.

In many of our examples we display the results of calculations to two decimal places, which suggests an unreasonably high level of precision. We do so only in order to highlight the differences in what are frequently very small numbers. We are not suggesting that use of two decimal places is appropriate in the practical application of the methods we describe.

Throughout the paper we use the FAS 113 definition of the reinsurer's loss, which ignores brokerage and the reinsurer's internal expenses. Our use of that definition should not be construed to mean that we endorse that definition for any purpose other than testing reinsurance contracts for compliance with FAS 113.

1.3 Organization of Paper

The paper is structured in nine sections.

Section 1 describes the impetus for and context of the paper as well as a summary of the risk transfer requirements of FAS 113, which we treat as a reasonable framework for evaluating risk transfer, subject to a fair interpretation of the critical elements of "reasonably possible" and "substantially all". To meet the FAS 113 risk transfer requirements, a contract must satisfy one of two conditions: 1) the reinsurer must assume "substantially all" of the underlying insurance risk, or 2) the reinsurer must assume "significant" insurance risk and it must be "reasonably possible" that the reinsurer may realize a "significant" loss.

In Section 2 we present a systematic approach for determining whether "substantially all" of the underwriting risk has been transferred under a reinsurance contract. If "substantially all" the risk has been transferred, then the contract meets the risk transfer requirement of FAS 113 without it being necessary to show that the risk transfer is "significant". This section partially addresses the third question.

In Section 3 we present a detailed critique of the "10-10" test itself and how it has been applied in practice. We first describe the emergence of the "10-10" approach as a method of testing contracts for "significant" risk. Then we illustrate the application of the "10-10" benchmark to three reinsurance contracts that clearly contain risk, including a property catastrophe contract and two quota shares of primary portfolios. All the tested contracts "fail" the "10-10" test, implying that the test is flawed. In the context of one of the examples we also emphasize the importance of taking parameter uncertainty into account in the risk assessment. Finally, we point out some unintended consequences of "10-10", namely that it implicitly imposes price controls on reinsurance contracts. We conclude that "10-10" is inadequate as a measure of risk and therefore unsuitable as a universal test for determining the "significance" of risk transfer. At best, one may argue that "10-10" is a sufficient test for risk transfer. It is not, however, a necessary condition.

Section 4 discusses two specific shortcomings of "10-10" and describes a different approach that addresses those shortcomings, thus addressing the first, second and fourth questions to varying degrees. The improved test we present here is based on the *expected reinsurer deficit* (*ERD*), which incorporates present value underwriting loss frequency and severity into a single measure. The loss severity embedded in the *ERD* is the tail value at risk (*TVaR*) measured at the economic breakeven loss ratio. We show that the *ERD* test is effectively a variable *TVaR* standard. We point out that a "significance" threshold of $ERD \geq 1\%$ has the merit of a certain amount of continuity with the "10-10" but without that test's major shortcomings. In order to address concerns that "10-10" might not be a strict enough standard, we also suggest the possibility of a supplemental minimum downside requirement. However, we do not advocate retesting of contracts already on the books that have already been found to pass "10-10".

Section 5 shows the application of the *ERD* test to the same contracts tested in Section 3 as well as to additional quota share contracts with loss ratio corridors or loss ratio caps, as well as to excess swing-rated contracts and individual risks. Using an illustrative standard of $ERD \geq 1\%$, we show that contracts that most people would consider risky receive a "passing" score, with one exception. This further addresses the first two questions.

Section 6 discusses the identification of contracts subject to the "significant" risk requirement, but which do not require individual testing, and thus addresses the third question. The NAIC is considering a requirement that the CEO and CFO attest that a risk transfer analysis has been completed for all reinsurance contracts, except those for which it is "reasonably self-evident" that significant risk has been transferred. We seek to put some definition to "reasonably self-evident". In this section we illustrate the application of the

$ERD \geq 1\%$ test to several classes of reinsurance contracts with certain structural features. We show, using conservative assumptions, that 1) standard catastrophe excess of loss treaties, 2) contracts covering individual risks and 3) certain other excess of loss reinsurance structures, could all be "pre-qualified" as meeting the "significant" risk requirement (unless there is reason to believe they include other features that might affect the amount of risk transferred). We also describe an additional approach that could potentially be used to further expand the set of such contracts.

Section 7 discusses the possible evolution of risk measurement beyond the application to risk transfer testing that is the focus of this paper. This section offers an alternative way to address the first two questions. It briefly presents a framework proposed based on *right tail deviation* (*RTD*) that tightly links risk transfer testing and risk loading. We present two examples. While the *RTD*-based approach has theoretical appeal, it has the drawback of being more complex and thus less understandable to a non-actuarial audience than the *ERD* approach.

Section 8 is a summary of the key points of the paper.

Section 9 provides suggested priorities for areas of further research.

Appendix A gives the mathematics underlying the *ERD* test. Appendix B explains the comparison between S&P 500 equity risk and quota share reinsurance risk (which is used in examples in Sections 3 and 5). References are listed in Section 10, which follows the appendices.

1.4 Background

FAS 113 ("Accounting and Reporting for Reinsurance of Short-Duration and Long-Duration Contracts") was implemented in 1993[1] to prevent, among other things, abuses in GAAP accounting for contracts that have the formal appearance of reinsurance but do not transfer significant insurance risk and thus should not be eligible for reinsurance accounting. FAS 113 amplified the earlier requirement of FAS 60 that reinsurance accounting only applies to contracts that transfer insurance risk. SSAP 62, which largely incorporates the same language as FAS 113, was implemented shortly thereafter to address the same issues

[1] It was issued in December 1992 for implementation with respect to financial statements for fiscal years commencing after December 15, 1992. Since insurance companies generally have fiscal years that coincide with calendar years, in effect it was implemented for the 1993 fiscal year.

with respect to statutory accounting. Our references to FAS 113 should be understood to refer collectively to FAS 113 and SSAP 62.

In order for a contract to qualify for reinsurance accounting treatment in accordance with FAS 113, it must transfer insurance risk from an insurer to a reinsurer. To meet the risk transfer requirement, a reinsurance contract must satisfy one of two conditions:

1. It must be evident that "the reinsurer has assumed substantially all of the insurance risk relating to the reinsured portion of the underlying insurance contracts" (paragraph 11), or
2. The reinsurer must "assume significant insurance risk under the reinsured portions of the underlying insurance contracts"(paragraph 9a) and it must be "reasonably possible that the reinsurer may realize a significant loss from the transaction" (paragraph 9b).

We are aware that our presentation of the two FAS 113 conditions in this order (i.e., first the paragraph 11 condition and then the paragraph 9 condition) is unusual. In practice, the "significant" risk requirement has often been considered first, and only if the contract "fails" is paragraph 11 considered. However, because part of our aim is to determine how to avoid testing every contract, we find it useful to start with the consideration of whether the contract meets the risk transfer requirement by virtue of "substantially all" the underlying risk having been transferred. If it does, then the "significant" risk question does not need to be considered at all. Accordingly, throughout the paper we will present and work with the FAS 113 risk transfer conditions in that conceptual order.

This paper is not intended to be a critique of FAS 113. We treat FAS 113 as it is currently constructed as a reasonable framework for evaluating risk transfer, subject to a fair interpretation of the critical elements of "reasonably possible" and "substantially all", despite some reservations about its focus on the financial effects (excluding brokerage and internal expenses) of a transaction on the reinsurer alone.

While all reinsurance contracts must satisfy the requirements of FAS 113, it is up to each company to determine which contracts should be subjected to detailed testing and which contracts clearly satisfy the requirements of FAS 113 based upon inspection. In this paper we describe an approach that can help guide both ceding companies and reinsurers through that decision process.

2. DETERMINING WHETHER THE CONTRACT TRANSFERS "SUBSTANTIALLY ALL" UNDERLYING INSURANCE RISK

We suggest it makes sense to begin by determining whether the contract meets the FAS 113 condition of transferring "substantially all" the insurance risk. If it does, then the contract meets the risk transfer requirement. If it does not, then the contract is subject to the other condition that the risk transfer must be "significant".

What is the "insurance risk relating to the...underlying insurance contracts?" We see it as the ***downside risk*** associated with the cedent's portfolio of insurance, i.e., the exposure faced by the underwriter to incurring a loss. If the downside risk assumed by the reinsurer is essentially the same as that faced by the cedent with respect to the original unreinsured portfolio, then the contract transfers "substantially all" the insurance risk.

The trivial case is a quota share or other proportional contract with a flat ceding commission equal to the ceding company's expense ratio, where there are no features such a sliding scale commission, profit commission, loss ratio corridor or aggregate loss ratio limit. In such a case, the comparison between the ceding company's position and that of the reinsurer is obvious. The contract clearly transfers not only "substantially all" the risk to the reinsurer but literally all of it. Facultative reinsurance is often written on this basis, but more often than not, quota share treaties include one or more of the features identified above.

Sliding scale and/or profit commission features are often used by reinsurers as incentives to reinforce the ceding company's motivation to underwrite its business in a disciplined way. Their use can promote a win-win situation for the ceding company and the reinsurer. These and other features such as loss ratio corridors or caps appear frequently in traditional reinsurance contracts as a means of making otherwise unattractive treaties acceptable to the reinsurance market. Usually the context for incorporation of caps or corridors is poor historical underwriting experience in the portfolio for which reinsurance is being sought. The ceding company believes it has taken the necessary corrective actions to turn the portfolio around, but the reinsurance market is skeptical. The inclusion of caps and corridors in a reinsurance contract can often make it possible for a ceding company that has confidence in its own business plan to obtain the reinsurance capacity it requires to execute that plan. Sometimes, but not always, such features have the effect of taking "too much" risk out of a reinsurance deal to allow the "substantially all" requirement to be met. We need to be able to compare the downside risk in the ceding company's unreinsured policies with the downside risk of the reinsurer.

We describe two ways of making this comparison – there may be other good methods as well – and illustrate them with an example. The first method is easier to understand but is not always conclusive, while the second method is somewhat more complicated but can always be applied.

Method 1 – Comparison of All Underwriting Downside Scenarios

Compare the cedent's underwriting margin over a range of loss ratios on the original unreinsured portfolio to the reinsurer's underwriting margin over the same range of loss ratios. The cedent's underwriting margin is defined as 100% less its unreinsured loss ratio less its actual expense ratio on the unreinsured portfolio[2]. The reinsurer's underwriting margin is defined as 100% less its assumed loss ratio less the ceding commission[3]. If the cedent's margin equals or exceeds the reinsurer's margin for the loss ratios that imply an underwriting loss, then clearly the reinsurer has assumed "substantially all" of the insurer's downside risk. Even if the cedent's margin is less than the reinsurer's margin, if that difference is small (as it is in Example 2.1), then the "substantially all" test may be met. Note that unless there are significant cash flow differences between the ceding company and the reinsurer, it is not necessary to conduct a full analysis of cash flows, since they will affect both parties in the same way.

Method 2 – Comparison of Cedent and Reinsurer Expected Underwriting Deficits

Compare the expected underwriting deficits (*EUD*) of the cedent and the reinsurer. The *EUD* can be calculated either directly as the pure premium of an aggregate excess of loss

[2] Expenses before reinsurance divided by premiums before reinsurance. Whether expenses should be marginal or average is a matter of debate.

[3] This definition of the reinsurer's underwriting margin does not reflect other expenses of the reinsurer, including brokerage and internal expenses. While this approach to measuring the reinsurer's profitability is consistent with the FAS 113 definition, it does not reflect economic reality.

cover attaching at the breakeven loss ratio or as the product of the frequency and severity of underwriting loss, (*Freq*(*UL*) and *Sev*(*UL*), respectively)[4].

If the *EUD* faced by the reinsurer is greater than or equal to the *EUD* of the cedent, then the "substantially all" test is clearly met. Because "substantially all" is less than "all", if the *EUD* faced by the reinsurer is within a small tolerance of the expected underwriting deficit faced by the cedent, say, within 0.1%, then we would also say the "substantially all" test is met.

Let's consider an example to illustrate these two methods.

Example 2.1: Non-Standard Auto Share with Sliding Scale Commission

Suppose a quota share of a non-standard auto portfolio is under consideration. The ceding commission is on a sliding scale. A minimum commission of 19.5% is payable if the loss ratio is 73% or higher. The commission slides up at a rate of one point for every one point of reduction in the loss ratio ("1:1 slide") below 73%, up to 30% at a loss ratio of 62.5%. The commission increases above 30% at a rate of 0.75% for every one point of loss ratio reduction ("0.75:1 slide") below 62.5%, up to a maximum commission of 39%, which is achieved at a loss ratio of 50.5% or lower. The ceding company's direct expense ratio on the subject business is 20%, so at the minimum ceding commission of 19.5%, it recoups virtually all of its direct costs. Its underwriting breakeven loss ratio is 80%. The reinsurer's FAS 113 underwriting breakeven loss ratio (i.e., ignoring brokerage and reinsurer internal expenses) is 80.5%.

The results of Method 1 are given in Table 1 and the accompanying Chart 1. The table compares the ceding company's expense ratio and underwriting margin on the unreinsured portfolio over a wide range of loss ratios to the reinsurer's ceding commission expense and underwriting margin at the same loss ratios. The accompanying chart compares the ceding company's margin and the reinsurer's margin graphically. From Table 1 and Chart 1 we see that above an 80% loss ratio (the ceding company's breakeven on the unreinsured portfolio), the ceding company's margin and reinsurer's margin are virtually undistinguishable, which

[4] If x represents the loss ratio and B is the underwriting breakeven loss ratio, then

$EUD = \int_B^\infty (x-B)\, f(x)\, dx = Freq(UL) \cdot Sev(UL)$, where $Freq(UL) = \int_B^\infty f(x)\, dx$ and $Sev(UL)$ is the

"tail value at risk" (*TVaR*) at the underwriting breakeven: $Sev(UL) = \int_B^\infty (x-B)\, f(x)\, dx \,/\, \int_B^\infty f(x)\, dx$

indicates the reinsurer has assumed "substantially all of the insurance risk" of the reinsured policies.

TABLE 1

"Substantially All" Risk Transfer Analysis - Method 1
Comparison of Reinsurer vs. Cedent Margins
Example 2.1

Subject Loss Ratio	Cedent Expense Ratio	Cedent Margin	Reinsurance Ceding Commission	Reinsurer Margin
30.0%	20.0%	50.0%	39.0%	31.0%
50.5%	20.0%	29.5%	39.0%	10.5%
62.5%	20.0%	17.5%	30.0%	7.5%
73.0%	20.0%	7.0%	19.5%	7.5%
80.0%	20.0%	0.0%	19.5%	0.5%
80.5%	20.0%	-0.5%	19.5%	0.0%
100.0%	20.0%	-20.0%	19.5%	-19.5%

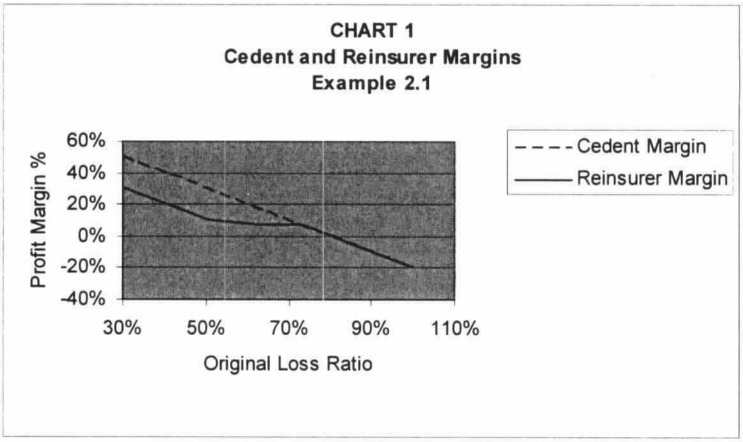

Table 2 summarizes the Method 2 comparison of expected underwriting deficits. It shows the insurer's and reinsurer's comparative underwriting downside risk by examining their respective *Freq(UL)*, *Sev(UL)* and *EUD*. In this example, the ceding company's frequency of underwriting loss is 11.28% vs. 10.45% for the reinsurer. The ceding company's underwriting loss severity is 8.33% vs. the reinsurer's 8.48%. The ceding company's *EUD* is 0.94% vs. the reinsurer's *EUD* of 0.89%[5]. While these measures vary slightly between the ceding company and the reinsurer, they are clearly very close. Thus, we would say that Method 2 also indicates that the reinsurer has assumed "substantially all" of

TABLE 2

"Substantially All" Risk Transfer Analysis - Method 2
Reinsurer vs. Cedent Margins in Downside Scenarios
Example 2.1

	Breakeven Loss Ratio	*Freq(UL)*	*Sev(UL)*	*EUD*
Cedent	80.0%	11.3%	8.3%	0.940%
Reinsurer	80.5%	10.5%	8.5%	0.886%
Difference	-0.5%	0.8%	-0.2%	0.054%

the ceding company's downside risk and the contract therefore meets the risk transfer requirements of FAS 113.

We conclude that in this example either Method 1 or Method 2 indicates the contract transfers "substantially all" the underlying insurance risk to the reinsurer.

While this approach works most naturally for quota share contracts, it can potentially be applied to excess of loss treaties as well. In that case, the reinsurer's *EUD*, calculated in the same way as above in the quota share case as a ratio to the ceded premium, should be compared to the cedent's *EUD* on the portion of the original subject portfolio which is exposed to the same risks as the excess of loss reinsurance contract. If the reinsurer's *EUD*

[5] Losses have been modeled using a lognormal distribution modified for parameter uncertainty, the details of which are not important for this example.

is close to or greater than the cedent's, then the reinsurer can be judged to have assumed "substantially all" the cedent's insurance risk in this context. For example, suppose the portion of original insurance risk assumed by a catastrophe reinsurance contract covering a portfolio of business has a 1% probability of a claim of a certain size. In that case the reinsurance of that portion of the risk also requires no more than a 1% probability of loss of the same size, because the *EUD*s of the ceding company and the reinsurer are the same with respect to the original catastrophe exposure.

If our argument about the applicability of the comparative *EUD* approach to excess of loss contracts and contracts with loss ratio caps is not found to be compelling, note that in section 6 we will also demonstrate that catastrophe reinsurance and some other contracts with aggregate loss limitations can meet the "significant" risk requirement under many circumstances.

Finally, there is a case to be made that, to the extent that a ceding insurance company is limited in its ability to meet net losses by its surplus, it is reasonable to allow a similar limitation of the reinsurer's aggregate liability. If this is accepted, then it is possible to calculate the minimum loss ratio cap that can be imposed by the reinsurer without violating the condition that "substantially all" of the underlying risk has been transferred. This potentially represents a third way of determining whether the "substantially all" risk transfer condition has been met.

For example, suppose a ceding company enters into a whole account quota share reinsurance arrangement that results in a net premium to surplus ratio of 200%. If the quota share has a ceding commission of 25% (approximating the ceding company expenses), then a loss ratio cap as low as 125% would be consistent with the transfer of "substantially all" of the risk, because at a combined ratio of 150% the ceding company has lost all of its surplus. Naturally such an interpretation would have to be made after due consideration of all other relevant features of the reinsurance contract in question.

If a contract does not meet the "substantially all" test, then it is subject to the second FAS 113 condition that "significant risk" must be transferred in order for the contract to qualify for reinsurance accounting. We now turn our attention to the question of what constitutes "significant" risk.

3. "SIGNIFICANT" RISK TRANSFER AND THE "10-10" TEST

3.1 "10-10" and its Shortcomings

A contract that does not meet the FAS 113 requirement for risk transfer by transferring "substantially all" the underlying insurance risk is subject to the second condition that "significant" risk be transferred. The so-called "10-10" test emerged in the years following the implementation of FAS 113 as a common benchmark for determining whether a reinsurance contract satisfies the requirement of a reasonable chance of "significant" loss to the reinsurer, which the test defines as "at least a 10% chance of a 10% loss". "10-10" is usually referred to as a "risk transfer" test, which implies an understanding of "risk" as a measure of exposure to loss rather than as exposure to volatility of results. "10% chance of a 10% loss" is usually interpreted to mean that the underwriting loss at the 90th percentile (of the probability distribution of underwriting results[6]) must be at least 10% of the ceded reinsurance premiums, where both underwriting loss and premiums are understood to be present values. Another term for "the underwriting loss at the 90th percentile" is "the value at risk" at the 90th percentile" or "$VaR_{90\%}$" with respect to the underwriting result. Accordingly, the "10-10" test can also be succinctly described as requiring $VaR_{90\%} \geq 10\%$.

The "10-10" benchmark arose as an informal method for testing whether purported reinsurance contracts contained sufficient risk transfer to meet the requirements of FAS 113 under the reasonable chance of significant loss criterion. It was not intended to be a universally applicable risk transfer test. Indeed, it has long been recognized that many reinsurance contracts having the characteristics of low underwriting loss frequency but high severity (such as property catastrophe excess of loss reinsurance) fail "10-10" on the basis that the probability of a 10% loss is less than 10%. In addition, if they do not meet FAS 113 risk transfer requirements by virtue of transferring "substantially all" risk, ordinary quota share reinsurance of many primary insurance portfolios (e.g., low limits private passenger auto), which have the characteristics of high frequency of underwriting loss but relatively low severity, may also fail. Until recently that was not seen as a problem because experienced practitioners understood the target of FAS 113 to be highly structured contracts that limited the transfer of insurance risk. As a consequence, traditional reinsurance contracts were typically not even tested.

[6] Low percentiles represent better results; high percentiles represent poorer results. Underwriting losses are represented as positive numbers. References to "underwriting results" and "underwriting losses" should be understood to refer to present values.

In the wake of the recent revelations of new accounting abuses related to "reinsurance contracts" apparently involving little or no risk transfer, the situation has changed. There is greater sentiment now that (a) more contracts should be routinely tested for significant risk transfer and (b) "10-10" is not a stringent enough standard. The view that "10-10" may not be stringent enough arises in part from the fact that some highly structured contracts have been carefully engineered to allow for exactly a 10% probability of a 10% loss and little or no possibility of a loss greater than 10%.

It is clear from the failure of the "10-10" benchmark to correctly identify both catastrophe excess of loss and some quota share reinsurance as risky and its failure to flag certain highly structured contracts as not significantly risky that "10-10" is insufficiently discriminating to serve as a universal measure of risk transfer in reinsurance contracts. We need a better test for measuring significant risk transfer in contracts that are subject to that requirement.

The interpretation of FAS 113's paragraph 9b is a critical issue. Paragraph 64 states that "an outcome is reasonably possible if its probability is more than remote." Despite this definition, the expectation appears to have developed that "reasonably possible" means a probability substantially greater than "remote". While the accounting literature gives no specific guidance on these probabilities, a 10% chance has come to be widely accepted as the smallest probability that should be categorized as "reasonably possible." It is our position that a different interpretation of "reasonably possible" is more appropriate, one that depends on the context of the risk and recognizes that some weight should be given to loss scenarios that, while rare, are not remote.

In particular, we propose that, in establishing the threshold probability for "reasonably possible", consideration must be given to the probability of loss (and indeed the size of that loss) arising from the reinsured portions of the underlying insurance contracts. For example, in the context of catastrophe reinsurance, "reasonably possible" should be associated with a probability that reflects the inherently low probability of the covered event. For other reinsured portfolios, where the inherent probability of loss is greater, "reasonably possible" is appropriately associated with a higher probability value.

This interpretation goes a long way toward eliminating the apparent inconsistency of according reinsurance accounting to some contracts that do not satisfy an invariant probability threshold of 10%. That property catastrophe contracts are typically accorded reinsurance accounting treatment even though they often do not meet a "reasonable possibility" requirement, defined as 10%, implicitly reflects this kind of interpretation.

In section 4 we will present a framework for capturing the interaction between the "reasonably possible" and "significant loss" components of paragraph 9b in a way that automatically makes the appropriate contextual adjustment without having to resort to situation-based arguments.

First, let us continue our critique of "10-10".

3.2 Illustration of the Shortcomings of "10-10"

Through a series of examples we will show why "10-10" is an unsatisfactory test for establishing whether or not a reinsurance contract transfers significant risk. Example 3.1 illustrates the application of the test to a property catastrophe contract and shows that it "fails" to transfer significant risk. Example 3.2 illustrates the application (and misapplication) of "10-10" to a low volatility primary quota share, given a set of historical loss ratio experience. We also use that example to warn of the pitfalls of simply fitting a loss distribution to on-level loss ratio experience and using that for risk transfer analysis. Example 3.3 shows that a quota share of an insurance portfolio having the volatility characteristics of the S&P 500 would frequently fail the "10-10" test.

We begin with the property catastrophe example.

Example 3.1: Property Catastrophe Excess of Loss Reinsurance

A property catastrophe reinsurance contract paying a premium equal to 10% of the limit[7] is typically priced to a loss ratio of around 50%. That implies an expected loss of 5% of the limit. Catastrophe reinsurance contracts, especially for higher layers, run loss free or have small losses in most years but occasionally have a total limit loss. This pattern is illustrated by the simplified catastrophe loss distribution shown in Table 3 below.

[7] This is frequently referred to as a "10% rate on line".

TABLE 3 Catastrophe Loss Distribution for Example 3.1		
Loss as % of Limit	Loss as % of Premiums	Probability of Given Loss
0%	0%	67%
5%	50%	20%
10%	100%	10%
100%	1000%	3%
5%	50%	100%

The loss at the 90th percentile of the catastrophe loss distribution is 100% of premiums. Assuming standard reinstatement premium provisions, the 90th percentile of the underwriting result distribution is an underwriting profit of 10% of premiums (100% original premiums plus 10% reinstatement premiums minus 100% loss). This contract fails the "10-10" test.

There is universal agreement among accountants, regulators, insurers, reinsurers and rating agencies that contracts like this one are risky. Clearly, the failure of "10-10" to identify the contract in this example as risky is an indication of a problem with "10-10" and not the contract.

Example 3.2: Primary Quota Share Reinsurance

Assume a cedent and reinsurer have negotiated a quota share treaty on a primary insurance portfolio. The treaty has a ceding commission of 25%. Does the treaty contain "significant" risk transfer[8]?

[8] Let's assume the treaty does not meet the condition of transferring "substantially all" of the underlying risk, perhaps because the cedent's expenses are substantially greater than the ceding commission. As a result the treaty is subject to the "significant" risk transfer requirement.

To measure the risk transferred we need to model the prospective underwriting result. Because the underwriting result is the breakeven loss ratio minus the actual loss ratio, the key to modeling the underwriting result is the probability distribution of the prospective loss ratio x. There are a number of reasonable actuarial methods for modeling prospective loss ratios[9]. In actuarial pricing applications the principal focus is on the mean of the prospective loss ratio distribution. Not much attention is paid to the full distribution. In contrast, risk transfer analysis requires the full distribution. This means there are pitfalls associated with using the output from the pricing analysis for the risk transfer analysis without full consideration of the issues affecting the full loss ratio distribution.

Let's review the underwriting experience analysis of the insurance portfolio that is the subject matter of the quota share. Five years of loss ratio experience is available together with information of varying quality about historical loss development and claim trends as well as the rate level history and the cedent's expectation of rate actions during the treaty period. This is summarized in Table 4, which shows the reported, estimated ultimate and estimated ultimate "on-level" loss ratios[10] together with the loss development, premium on-level and loss on-level factors used in the analysis. The means, variances and standard deviations of the on-level loss ratios x_i and their natural logs $\ln x_i$ are tabulated using the assumption that exposure has been constant over the experience period.

The historical experience has been poor. Given the ceding commission of 25% and ignoring brokerage and internal expenses (as per FAS 113), the reinsurer's present value breakeven loss ratio is 75%[11]. Three of the five years have estimated ultimate loss ratios significantly greater than 75% and in two of the years the loss ratio is over 75% even on a reported basis. The good news is that the ceding company has taken action to increase rates significantly, which results in estimated on-level loss ratios that are much lower than the actual historical loss ratios. The on-level mean of 70.67% compares very favorably with the

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[9] The models we use for the purposes of illustrating the issues related to risk transfer testing are not intended to be prescriptive and are independent of the risk measurements we describe.

[10] This means the loss ratios have been adjusted to reflect the projected premium rate and claim cost levels expected to apply during the treaty term.

[11] Note that given typical brokerage of 1.5% and internal expenses of 3% to 5%, reinsurers would regard their real breakeven loss ratio as 68.5% to 70.5%, depending on expenses. As we shall see, this treaty is a breakeven or slightly worse than breakeven proposition and would not be attractive to most reinsurers.

historical mean of about 80%. Moreover, the on-level loss ratios are not very variable as indicated by the standard deviations of 7.45% with respect to x and 10.88% with respect to $\ln x$.

TABLE 4

On-Level Loss Ratio Experience

For Quota Share in Example 3.2

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Accident Year	Reported L/R	Age to Ult Factors	Est Ult L/R	Prem On-Level Factors	Loss On-Level Factors	On-Level L/R x_i	$\ln x_i$
1	92.8%	1.039	96.4%	1.963	1.364	67.0%	-0.401
2	75.6%	1.048	79.3%	1.737	1.307	59.7%	-0.516
3	77.0%	1.095	84.3%	1.376	1.246	76.4%	-0.269
4	61.2%	1.141	69.9%	1.139	1.181	72.5%	-0.321
5	52.5%	1.415	74.3%	1.061	1.111	77.8%	-0.251
				Mean	$\bar{x}$	70.7%	-0.352
				*Var**	s^2	0.554%	1.18%
*Unbiased				*St. Dev.**	s	7.45%	10.88%

We are first going to illustrate ***how not to apply*** the "10-10" benchmark in this scenario. We do this in order to point out the problems associated with this approach, which we believe may be in relatively common use.

Let's assume the underlying random process governing the prospective loss ratio is lognormal. Then the "best fit" distribution, given the on-level loss ratio experience, is defined by parameters $\mu = \bar{x}$ and $\sigma = s$. From this it is easy to determine whether the present value underwriting loss corresponding to $VaR_{90\%}$ exceeds 10%. If B is the present value FAS 113 breakeven loss ratio and FV and PV represent "future value" and "present value" operators, respectively, then from the characteristics of the lognormal distribution we know that

$$N^{-1}(90\%) = \frac{ln[FV(B + VaR_{90\%})] - \mu}{\sigma} \tag{3.1}$$

which implies

$$VaR_{90\%} = PV(e^{\mu + N^{-1}(90\%) \cdot \sigma}) - B \tag{3.2}$$

If ceded loss payments lag ceded premium payments by one year on average, the risk free interest rate is 5%, $\mu = \bar{x} = -0.3518$ and $\sigma = s = 10.88\%$, then formula (3.2) implies

$$\begin{aligned} VaR_{90\%} &= e^{(-0.3518)+(1.2815)(0.1088)} \cdot (1.05^{-1}) - .75 \\ &= 2.02\% \end{aligned}$$

Since "10-10" requires $VaR_{90\%} \geq 10\%$, according to this analysis the quota share treaty in this example does not transfer "significant" risk. In fact, the $VaR_{90\%}$ of 2.02% suggests that the treaty contains hardly any risk at all. Yet when we look back at the historical experience, we see that the reinsurer would have lost more than 10% in one year and would have lost money over the entire period. The conclusion that the reinsurer does not face a "reasonable possibility of significant loss" seems strange.

Why did we get this result? There are two reasons. The first, as we hinted at the beginning, has to do with inadequacies in the loss model we selected. The second has to do with shortcomings in the "10-10" test itself.

Let's discuss the problem with the approach we described for identifying a loss ratio model. Fundamentally, the problem is that we fitted a single distribution to the on-level loss ratios and then used that distribution as though we knew with certainty that it is the correct one. In that case the only source of risk being modeled is process risk, because we have assumed we have the correct model. In fact, there are multiple sources of parameter uncertainty, some of which we enumerate below:

- The ultimate loss estimates might be wrong;
- The rate level history might be inaccurate;
- The prospective rate changes assumptions might be wrong;
- The historical claim trend estimates might be inaccurate;
- The prospective claim trend assumptions might be wrong;
- The experience period might be too short to include rare but very large losses;
- The prospective loss ratios might not be lognormally distributed;
- The lognormal assumption is right, but the "best fit" distribution is not the actual;
- Cash flow timing assumptions, particularly regarding claims, might be wrong;
- The prospective exposure mix might be different from expected;
- For multi-year reinsurance contracts, the level of parameter uncertainty from all sources increases as the length of the coverage period increases.

In any actuarial application where the knowledge of the loss distribution itself and not just its mean is important, it is very important that the modeling be based on loss models that incorporate parameter uncertainty, which is an important and frequently underestimated source of risk[12]. Risk transfer testing, given its dependence on the right tail of the loss ratio distribution is one of those applications.

Accordingly, actuaries should be cautious about placing too much confidence in a single distribution fitted to estimated loss ratios. Where the estimates are the result of applying large development and/or on-level factors, the likelihood of parameter error is especially large, and appropriately large adjustments must be made to the distribution to account for it.

While it is beyond the scope of this paper to discuss specific methods for estimating the impact of parameter uncertainty, for the sake of illustration, suppose the effect of reflecting parameter uncertainty in the current example is to increase σ in the lognormal model to 15%. If we constrain μ such that $E(x)$ remains unchanged, then $\mu = -0.3571$ and formula (3.1) yields $VaR_{90\%} = 5.76\%$, which still fails to meet the "10-10" threshold for

[12] Kreps[2] and Van Kampen [3] provide examples of large effects in loss reserve estimates and aggregate excess pure premiums, respectively, due to the recognition of parameter uncertainty.

"significant" risk transfer. In this case, an adjustment to try to take account of parameter uncertainty is not sufficient to show "significant" risk transfer in the contract, at least if we use "10-10" to measure it.

The next example brings into question the appropriateness of the "10-10" criterion of $VaR_{90\%} \geq 10\%$ by examining its implications for how we think about stock market risk.

Example 3.3: Primary Quota Share Reinsurance (Volatility of S&P 500)

Assume we are considering a quota share treaty on a second primary insurance portfolio. As in Example 3.2 the treaty ceding commission is 25%, which implies a FAS 113 breakeven present value loss ratio of 75%. Suppose this portfolio has the distributional and volatility characteristics commonly attributed to the S&P 500 equity index and an on-level loss ratio of 70%. This implies an assumption that the prospective loss ratio is lognormally distributed[13] with a mean of 70%. Let's also assume the claim payments lag premiums by one year. In order to pass the "10-10" test, which requires a present value loss ratio of at least 85% at the 90th percentile, if the risk free interest rate is 5%, the minimum value of the lognormal σ parameter is about 21%[14].

Actual annualized volatility in the price of the S&P 500 index exchange traded fund (symbol SPY) between early May 2004 and early May 2005 was 10.64%.[15] On May 4, 2005, the broadly based CBOE Volatility Index (VIX), a measure of the expected annualized volatility in the S&P 500 stock index implied by the market pricing of index options, closed at 13.85%. The market was using a higher estimate of future volatility for pricing purposes than that observed in the recent past, which might reflect an adjustment for parameter uncertainty or simply the opinion that volatility would increase. Both estimates of σ fall

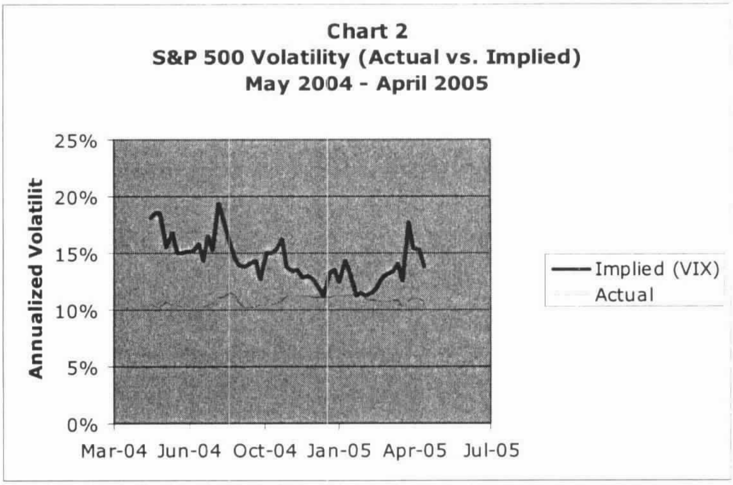
[13] For a discussion of the basis for this assumption, see Appendix B.

[14] $\sigma = \dfrac{ln[(.85)(1.05)] - \mu}{N^{-1}(.9)}$ and $\mu = ln(.70) - .5\sigma^2$ imply $\sigma = 20.6\%$ or 236%, the former being the only reasonable solution in this context. This threshold assumes a ceding commission of 25%, a risk free interest rate of 5% and lognormal stock prices. The threshold will vary depending on the parameters.

[15] Calculated as the annualized standard deviation of weekly log returns $\ln(P_w / P_{w-1})$ between May 2004 and May 2005.

below the threshold of 21% required to pass "10-10", implying that a "quota share" of the S&P 500 index[16] would fail to meet the FAS 113 requirement for significant risk transfer!

This is not merely a temporary aberration. During the period from early May 2004 through early May 2005 the actual volatility observed on a one-year look-back basis averaged 10.77%. Over the same time period, VIX averaged 14.39%. Chart 2 shows this graphically. The persistent pattern of VIX greater than actual historical volatility suggests that VIX reflects an adjustment for parameter uncertainty rather than a forecast that volatility will increase.

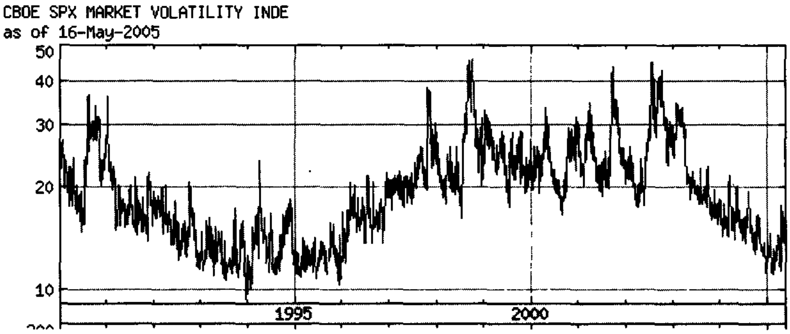


Source: Yahoo! Finance

Over a longer period of time the market opinion of the prospective volatility of the S&P 500 has varied considerably, ranging from a high of about 50% in 2002 to a low of about 9% in 1993[17]. Chart 3 shows this graphically.

[16] We put "quota share" in quotation marks because the S&P 500 index transaction comparable to a quota share of an insurance portfolio involves a short sale. Since a short sale is usually considered to be even riskier than a long position, the failure to "pass" a risk transfer test is all the more surprising. See Appendix B for details.

CHART 3



Source: Yahoo! Finance

Chart 4 shows the probability of a present value loss of 10% or more on the quota share of this example, given σ = VIX values as of the last trading day of each year from 1990 through 2004 plus May 4, 2005. It shows that the probability exceeds 10%, given the VIX values at the end of 1990 and those for every December from 1996 through 2002. However, the probability is less than 10%, given the VIX values from every December 1991 through 1995 and those for December 2002 and 2003 as well as that for May 2005[18]. Almost no one would argue that an investment in equities, even in a diversified portfolio such as the S&P 500, is not risky. Yet the implication of the "10-10" benchmark is that a quota share reinsurance that has the same volatility characteristics ascribed to the S&P 500 by the options market over the period since 1990 would have been considered risky only about half the time! Unless the intention is to set the bar for "significant" risk at a level higher than the typical volatility of the S&P 500, we must conclude that the "10-10" criterion is an inadequate measure of significant risk.

[17] For more information about VIX and its calculation, see the white paper published by the CBOE, which is available at its website: http://www.cboe.com/micro/vix/vixwhite.pdf. The paper included the history between 1990 and August 2003.

[18] The data underlying Chart 4 can be found in Appendix B.

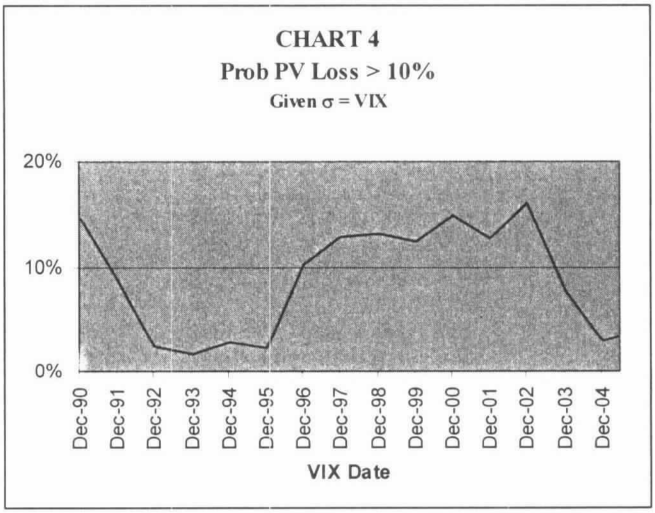


Table 5 illustrates the "10-10" analysis for a quota share of a portfolio whose loss ratio has the volatility characteristics of the S&P 500, for two volatility scenarios: 9% (representing the low end of the VIX range since 1990) and 13.85% (representing the VIX value on May 4, 2005). The ceding commission is 25%. The table shows (a) the loss at the 90th percentile of the present value underwriting result distribution, and (b) the probability of a present value loss of 10% or more, for σ = 9% and 13.85%. Both of these volatility scenarios fail to meet the "10-10" threshold for significant risk transfer.

If σ = 9%, which represents the low end of the range of S&P 500 implied volatility since 1990, the quota share actually has a negative loss (i.e., small profit) at the 90th percentile ("10% chance of a (0.49%) or greater loss") and a miniscule 0.30% probability of a 10% loss or more. This scenario fails the "10-10" test badly!

For σ = 13.85% Table 5 shows a 10% chance of a 3.85% or greater loss and a 3.41% chance of a 10% loss or more. This contract scenario also fails "10-10" by a long way[19].

[19] Note that even at an expected loss ratio of 75%, which is the treaty breakeven point, there is a 10% chance of only a 9.49% or greater loss. See Appendix B (Table B-2) for details about the sensitivity of the analysis to changes in the expected loss ratio assumption.

TABLE 5 "10-10" Risk Transfer Analysis for Quota Share in Example 3.3 Given Portfolio with Volatility of S&P 500			
VIX	σ	(a) 90th Percentile P.V. Underwriting Loss	(b) Probability of ≥ 10% P.V. Underwriting Loss
Low	9.00%	(0.49%)	0.30%
May 2005	13.85%	3.85%	3.41%

For further discussion of the comparability of quota share reinsurance with the S&P 500, see Appendix B.

3.3 Unintended Consequences: The Impact of "10-10" on Reinsurance Pricing

There is a further troubling implication of "10-10". It implicitly imposes price controls on reinsurance contracts at such a low level that, if that benchmark were to be enforced as a rule, reinsurance capacity for certain types of business is likely to be reduced, if not eliminated entirely.

To illustrate this we will assume the prospective loss ratio is lognormally distributed[20]. The mean of a lognormal distribution is given by

$$E(x) = e^{\mu + 0.5\sigma^2} \tag{3.3}$$

If we solve for μ in formula (3.1) and substitute the result for the μ in formula (3.3) we obtain the formula for *E(x)* constrained by $VaR_{90\%} = 10\%$:

[20] We choose the lognormal merely for purposes of illustration. A different distribution might be more appropriate.

$$E(x) = Exp\{\ln[FV(B + VaR_{90\%})] + N^{-1}(90\%) \cdot \sigma + 0.5\sigma^2\} \tag{3.4}$$

For example, in the treaty scenario with no ceding commission, $B + VaR_{90\%} = 110\%$, and the minimum permissible loss ratio is:

$$E(x) = Exp\{\ln[FV(110\%)] + 1.2815 \cdot \sigma + 0.5\sigma^2\} \tag{3.5}$$

Table 6 is a tabulation of the minimum permissible loss ratios allowed by "10-10" for a range of values of σ and average net claim payment lags of zero, one year, two years and three years. Chart 5 is a graphical representation of the data in Table 6. We see that for small values of σ and claim lags of a year or more, the minimum permissible loss ratios are greater than 100%, implying the reinsurer is required to price its business at an underwriting loss even before taking into account brokerage and its own internal expenses. Even at somewhat higher values of σ that might correspond to certain excess of loss business, the reinsurers's net underwriting margins (after typical brokerage of 10% and comparable internal expenses) are quite low.

For example, given $\sigma = 9\%$ and assuming no claim payment lag (and hence no investment income), the reinsurer's minimum permissible loss ratio is 98.4%. That implies a maximum allowable margin before brokerage and internal expenses of 1.6%. The maximum permissible loss ratio rises as the claim payment lag increases. The effect of the $VaR_{90\%} = 10\%$ constraint is that all the investment income earned as a result of the claim payment lag is credited to the cedent, and the present value of the reinsurer's margin remains at 1.6%. For example, given a three-year payment lag and a 5% interest rate, the breakeven loss ratio is 115.8% and the minimum permissible loss ratio is 113.9%, which leaves a future value margin for the reinsurer of 1.9%. The present value of that 1.9% is 1.6%. Clearly, given brokerage costs and internal expenses, no reinsurer could afford to write business at such a meager margin.

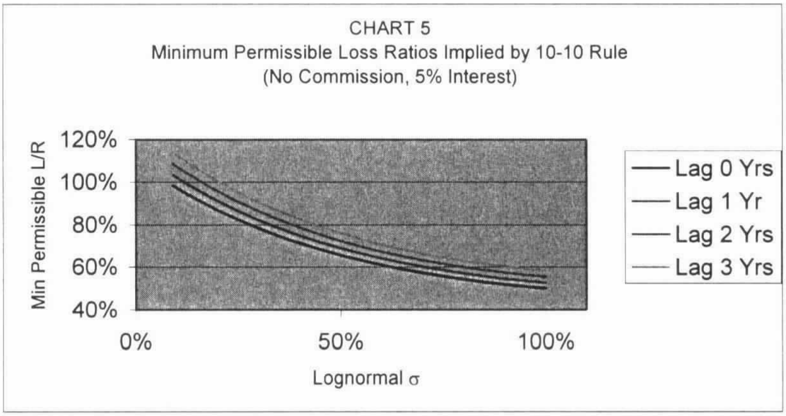
TABLE 6

Minimum Permissible Loss Ratio
Implied by "10-10"

Contracts with No Ceding Commission
Interest at 5% per annum

By σ and Claim Lag

σ	No Lag	1 Yr Lag	2 Yr Lag	3 Yr Lag
9.0%	98.4%	103.3%	108.5%	113.9%
10.0%	97.3%	102.1%	107.2%	112.6%
11.0%	96.1%	100.9%	106.0%	111.3%
12.0%	95.0%	99.8%	104.7%	110.0%
13.0%	93.9%	98.6%	103.5%	108.7%
14.0%	92.8%	97.5%	102.4%	107.5%
15.0%	91.8%	96.4%	101.2%	106.3%
20.0%	86.8%	91.2%	95.8%	100.5%
25.0%	82.4%	86.5%	90.8%	95.4%
30.0%	78.3%	82.3%	86.4%	90.7%
40.0%	71.4%	74.9%	78.7%	82.6%
50.0%	65.7%	69.0%	72.4%	76.0%
60.0%	61.0%	64.1%	67.3%	70.7%
75.0%	55.7%	58.5%	61.4%	64.5%
100.0%	50.3%	52.9%	55.5%	58.3%



In light of our earlier discussion of parameter uncertainty, it may well be that σ values as low as 9% will never be used in practice. However, the problem remains to some extent at higher values of σ. For example, for $\sigma = 30\%$ the maximum gross reinsurer's margin is 21.7% (100% less the minimum loss ratio with no claims lag). If the reinsurance is on an excess of loss basis, brokerage is likely to be 10% and internal expenses are likely to be a similar amount. That leaves only 1.7% as a net present value margin for the reinsurer, which is not likely to be attractive.

3.4 Section Summary

The discussion in this section should make it clear that the "10-10" benchmark is a flawed measure of "significant" risk transfer. The test used to measure risk transfer should accurately distinguish between contracts that clearly contain significant risk from those that don't. That "10-10" fails to identify both catastrophe reinsurance treaties and contracts with the characteristics of equity investments as risky tells us that it is a poor test. "10-10" also implies very restrictive caps on reinsurance pricing that can never have been intended. At the same time it has received criticism from the other direction that it does not do an adequate job of screening out contracts that meet its minimum requirements but in such a contrived way that the intent of FAS 113 is thwarted. For all of these reasons it makes sense to identify a better test than "10-10", which we seek to do in the next section.

4. TOWARD A BETTER TEST

There are at least two major shortcomings of the "10-10" test. First, the focus on the present value loss only at the 90th percentile ($VaR_{90\%}$) ignores the information in the remainder of the tail represented by the percentiles beyond the 90th. A better test would take account of the loss potential in the right tail of the distribution, which sometimes can be extreme (as in the case of catastrophe reinsurance). Second, both the 10% probability and 10% loss thresholds are arbitrary. The risk transfer test should be generalized to allow for both low frequency-high severity (e.g., 5%-20%) and high frequency-low severity (e.g., 20%-5%) combinations.

The first shortcoming could be remedied by replacing $VaR_{90\%}$ with the mean severity of present value underwriting losses at and beyond the 90th percentile, a measure known as the "tail value at risk" or $TVaR_{90\%}$ [21]. This measure of severity incorporates the information about the loss potential in the right tail that the "10-10" test misses. Indeed, the 2002 VFIC paper suggested replacing $VaR_{90\%}$ in the "10-10" test with $TVaR_{90\%}$. However, simply replacing $VaR_{90\%}$ with $TVaR_{90\%}$ is not by itself a full solution to the problems associated with "10-10", because it leaves unaddressed that test's second shortcoming that the 10% thresholds wrongly screen out low frequency-high severity and high frequency-low severity contracts.

That second shortcoming can be corrected by relaxing the requirement that the probability of loss and the severity of loss must both exceed 10%. We can do this by making use of the fact that the *expected reinsurer deficit* (*ERD*)[22] is equal to the probability (or *frequency*) of the present value underwriting loss times its average *severity*, where the latter is *TVaR* measured at the economic breakeven point. Since *ERD* incorporates information about both the frequency and severity of the reinsurer's downside risk into a single measure, it makes sense to use that measure to define a threshold for measurement of significant risk transfer rather than to define it in terms of frequency and severity separately:

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[21] Also known as the "tail conditional expectation" or "*TCE*", *TVaR* has been praised by VFIC[1] , Meyers [4], and others as a coherent measure of risk as well as for its incorporation of the information contained in the right tail of the distribution.

[22] The *ERD* is the expected cost of all present value underwriting loss scenarios. It is also the expected value of Mango's [5] contingent capital calls. Conceptually, it is related to the *EUD* defined in Section 2, but the *EUD* is defined in nominal terms and the *ERD* is defined in present value terms.

$$ERD = Freq \times Sev \geq A \tag{4.1}$$

where A represents the threshold above which a contract is considered to have provisionally "passed" the "significant" risk transfer test and below which is it is considered to have "failed". ***Freq*** and ***Sev*** refer to the frequency of present value loss and the average severity of such loss, respectively. See Appendix A for the mathematical definitions of all the elements of formula (4.1).

This approach, which we will refer to as the "*ERD* Test", addresses both shortcomings of the "10-10" test by (a) reflecting the full right tail risk in the definition of severity and (b) replacing separate frequency and severity requirements with a single integrated measure that treats low frequency-high severity, high frequency-low severity and moderate frequency-moderate severity contracts in the same way.

We will illustrate the application of the ***ERD*** test with a threshold A of 1%, because it has the merit of a certain amount of continuity with the "10-10" test[23]. The way to think about that is that first we have changed the $VaR_{90\%} \geq 10\%$ embodied in the "10-10" test to $TVaR_{90\%} \geq 10\%$. Then we have generalized the ***TVaR*** standard to allow contracts having a wide variety of frequency-severity combinations, including 5%-20%, 10%-10% and 20%-5%, to meet the requirement for "significant" risk transfer. $ERD \geq 1\%$ is effectively a variable ***TVaR*** standard that defines "significant" as $TVaR_{1-Freq} \geq \dfrac{1\%}{Freq}$. One implication of this is that any contract that passes "10-10" will also pass a standard of $ERD \geq 1\%$.

Chart 6 shows the "significant" risk transfer frontiers for $ERD \geq 1\%$ and three ***TVaR*** standards ("10-10" as well as "5-20" and "20-5") plotted in terms of frequency and severity. Frequency-severity combinations above and to the right of the frontiers represent "significant" risk. We see that a fixed ***TVaR*** "10-10" standard would exclude contracts with loss frequencies less than 10% and severities less than 10% that the ***ERD*** standard would accept as "significant". As a generalized ***TVaR*** standard, a $ERD \geq 1\%$ standard would accept $TVaR_{95\%} \geq 20\%$ or $TVaR_{90\%} \geq 10\%$ or $TVaR_{80\%} \geq 5\%$, etc.

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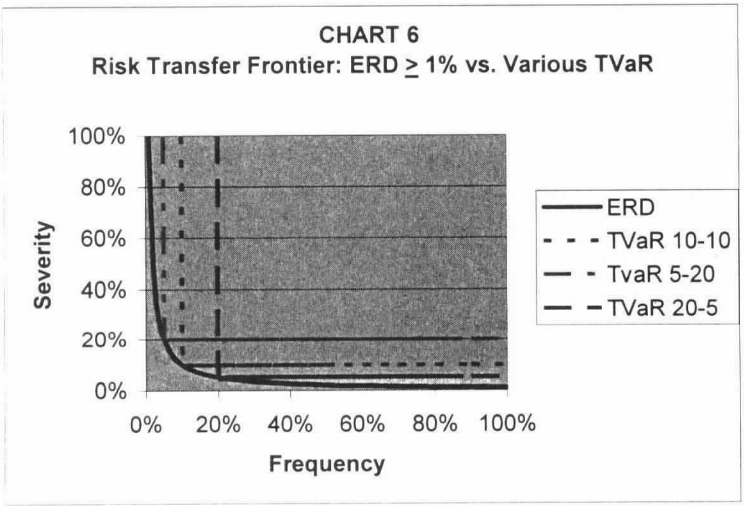
[23] Whether that is the proper threshold warrants further research.



To address the issue of contracts that have been engineered to remove most or all of the potential for a loss greater than 10% in the right tail, which some criticize as too small, we suggest consideration of a supplemental requirement that there be the potential for a reinsurer loss of some minimum threshold, say, 15% or 20% of premiums. That would eliminate very low loss ratio caps.

We are not advocating that every reinsurance contract be tested for significant risk transfer. It should be possible to conclude that some contracts have adequate risk transfer without formally testing them. In section 6 we will suggest some ways to do that. However, we *are* suggesting that the *ERD* test (possibly together with the supplemental test) could be applied to all contracts that are subject to the "significant" risk transfer requirement with the confidence that it would produce consistently reasonable results.

We believe the *ERD* test (with or without the supplemental component), if adopted, should only be applied prospectively and not to contracts already on the books.

5. ILLUSTRATION OF THE *ERD* TEST

In this section we apply the proposed test to the contracts used in the examples of Section 3 as well as several additional examples.

Example 5.1: Property Catastrophe Excess of Loss Reinsurance

If we apply the *ERD* test to the catastrophe reinsurance contract described in Example 3.1, that contract now easily passes muster for risk transfer. Again assuming normal reinstatement premium provisions, which call for an additional premium equal to the original premium times the proportion of the limit that has been exhausted, *Freq*=3%, *Sev*= $TVaR_{97\%}$=800% and *ERD* =24%. Because of the large contribution from *Sev* to *ERD*, this contract now easily surpasses the standard of $ERD \geq 1\%$.

TABLE 7

ERD / Max Downside

For Standard Cat XL Contracts

By Rate on Line

Rate on Line	Poisson λ	*ERD* *	Reinsurer Max Downside*
1.0%	0.5%	49.0%	19545%
2.0%	1.0%	48.0%	9678%
3.0%	1.5%	47.0%	6364%
4.0%	2.1%	46.0%	4651%
5.0%	2.6%	45.1%	3726%
7.5%	3.9%	42.6%	2373%
10.0%	5.3%	40.2%	1711%
12.5%	6.7%	37.9%	1315%
15.0%	8.1%	35.6%	1051%
20.0%	11.1%	31.0%	723%
25.0%	14.2%	26.6%	530%
30.0%	17.5%	22.3%	402%
40.0%	24.6%	14.2%	246%
50.0%	32.4%	6.6%	157%

* Ratio to expected premium

Assumptions.
- One reinstatement of limit for 100% A.P.
- Investment income effects ignored
- Poisson model with parameter λ
- Expected loss ratio 50%

In fact, using conservative assumptions, contracts having the same structure as the standard property catastrophe treaty[24] can be shown to exceed the $ERD \geq 1\%$ threshold (as well as a supplemental minimum potential downside threshold) if the upfront rate on line $ROL \leq 50\%$. Table 7 summarizes the *ERD* and potential downside values (ignoring investment income) for contracts having rates on line ranging from 1% to 50%, based on the simplifying assumptions that the expected loss ratio is 50%, all claims are total limit losses and that claims are Poisson distributed. On the basis that every rate on line in Table 7 easily passes the *ERD* test even without the supplemental downside requirement, we suggest that any reinsurance contract having this structure be deemed to meet the requirements for "significant" risk transfer. Clearly, such contracts are subject to the "significant" risk transfer requirement, but because we have, in effect, pre-qualified them as a class, the requirement to demonstrate significant risk transfer can be waived.

Example 5.2: Primary Quota Share Reinsurance

We applied the *ERD* test to the primary quota share contract described in Example 3.2. Again assuming a one-year net claim payment lag[25], a 5% interest rate and a lognormal σ of 15%, we calculated the frequency and severity, respectively, of present value underwriting loss to be 21.53% and 6.91%, which corresponds to an *ERD* of 1.49%[26]. This *ERD* value surpasses the $ERD \geq 1\%$ standard. Moreover, because there is no limit on the reinsurer downside potential, it would meet the suggested supplemental requirement. Therefore, this contract meets the "significant" risk transfer requirement.

Example 5.3: Primary Quota Share Reinsurance (Volatility of S&P 500)

In this example we test the same quota share that was the subject of Example 3.3. That quota share covered an insurance portfolio with the same loss ratio volatility as an S&P 500 index investment. The ceding commission is 25%. The frequency, severity and *ERD*

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[24] The standard property catastrophe treaty provides two loss limits, the second one paid for with a contingent "reinstatement" premium at the same rate on line as the first one.

[25] Using this simplifying assumption, we can focus on the present value of the losses only, measured at the time the premium is received, because the present value factor applicable to premiums and losses for the period up to the premium receipt date is the same. The ratio of discounted *ERD* to discounted premium using the full claim and premium payment lags is equal to the ratio of discounted *ERD*, using the net claim lag, to undiscounted premium.

[26] If the prospective loss ratio is lognormally distributed, $ERD = PV[E(x) \cdot N(d1) - FV(B) \cdot N(d2)]$, where N is the normal cdf, $d1 = [\ln(E(x)/FV(B)) + 0.5\sigma^2]/\sigma$ and $d2 = d1 - \sigma$.

characteristics of such a portfolio are summarized in Table 8 for the two volatility scenarios modeled in Example 3.3. For volatility of 13.85% the $ERD \geq 1\%$ standard is met. However, at the historically low volatility of 9%, a portfolio with S&P 500 volatility characteristics has an *ERD* of only 0.28% and thus fails the $ERD \geq 1\%$ standard by a wide margin. That creates a conundrum – is it ever reasonable to consider the S&P 500 to be without risk? If not, a 1% threshold for *ERD* is too high.

TABLE 8

ERD Risk Transfer Analysis for Quota Share in Example 5.3

Given Portfolio with Volatility of S&P 500

σ	*Freq*	*Sev*	*ERD*
9.00%	8.8%	3.2%	0.28%
13.85%	17.9%	6.0%	1.07%

Next, we will use the *ERD* test to assess quota share contracts with features such as loss ratio caps and corridors that reduce the loss exposure of the reinsurer. These features appear frequently in traditional reinsurance contracts as a means of making otherwise unattractive treaties acceptable to the reinsurance market.

Example 5.4: Reinsurance with 25% Ceding Commission and 5-Point Loss Ratio Corridor

Table 9 shows the downside risk measures *Freq*, *Sev* and *ERD* for a quota share or excess contract that provides a 25% ceding commission and requires the ceding company to retain any losses that fall within a five point loss ratio corridor from 75% to 80%. We assume the prospective loss ratio is lognormally distributed, with a mean of 70% and a range of values for σ. Claim payments are assumed to lag premium payments by one year.

Table 9 shows that for lower volatility business, represented here by lognormal σ values of 10% and 15%, a treaty with the 5 point loss ratio corridor removes enough risk from the deal that the *ERD* falls below 1%, indicating that the risk transfer is not significant. For the σ values of 25% and higher, the *ERD* significantly exceeds the 1% threshold. Clearly, the effect of a loss ratio corridor depends on the characteristics of the reinsured business, and in some circumstances such treaty feature is entirely appropriate.

TABLE 9

ERD Risk Transfer Analysis for Contract With 25% Ceding Commission and Loss Ratio Corridor from 75% to 80%

σ	*Freq*	*Sev*	*ERD*
10%	3.1%	3.2%	0.10%
15%	9.1%	6.0%	0.59%
20%	15.6%	9.2%	1.43%
25%	19.7%	12.6%	2.47%
30%	22.4%	16.2%	3.63%
40%	25.6%	23.9%	6.13%
50%	26.9%	32.4%	8.74%

Example 5.5: Reinsurance with 25% Ceding Commission and 95% Loss Ratio Cap

We now consider the effect of an aggregate loss ratio cap of 95% (instead of a loss ratio corridor) on the same subject matter business discussed in Example 5.4. Table 10 shows frequency, severity and *ERD* for σ values ranging from 10% to 50%. Except for the case of $\sigma = 10\%$ (where *ERD* =0.41%) the aggregate loss ratio cap is at a high enough level that the 1% threshold is exceeded, and for the higher values of σ by a wide margin.

Note that in the case of $\sigma = 10\%$, the *ERD* associated with a contract with no loss ratio cap is also 0.41%, indicating that the cap at 95% has no significant effect on the risk transferred to the reinsurer. On that basis, the contract with a 95% cap transfers "substantially all" the risk in the underlying portfolio, and even though it does not transfer "significant" risk, it meets the risk transfer requirements of FAS 113.

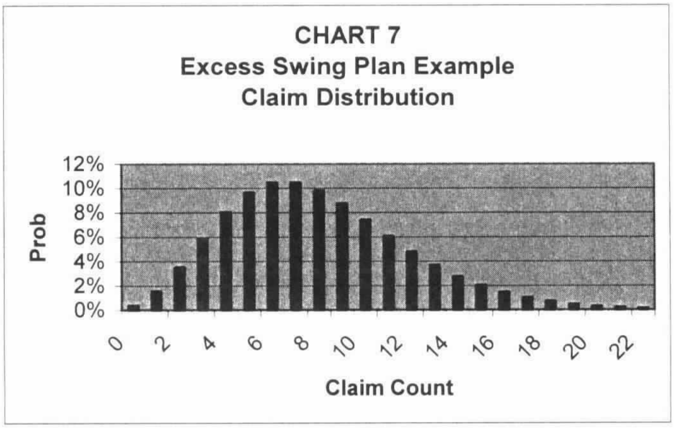
TABLE 10
ERD Risk Transfer Analysis for Contract With 25% Ceding Commission and Loss Ratio Cap of 95%

σ	*Freq*	*Sev*	*ERD*
10%	11.0%	3.8%	0.41%
15%	19.5%	6.5%	1.27%
20%	24.5%	8.9%	2.18%
25%	27.6%	10.7%	2.94%
30%	29.4%	12.0%	3.53%
40%	31.1%	13.8%	4.29%
50%	31.4%	14.9%	4.69%

Example 5.6: Excess Swing-Rated Reinsurance

It is common for "working layer" excess of loss reinsurance to be structured on a "swing-rated" basis, which means the premium is based in part on the losses ceded to the treaty. Typically, the premium formula calls for ceded claims to be multiplied by a loading factor to reflect a margin for the reinsurer, subject to a minimum and maximum. In primary insurance this structure is known as a "retrospective experience rating plan". The purpose of such plans is to allow the ceding company to fund its own excess claims up to the point beyond which it would become too painful and to cede the excess claims beyond that point to the reinsurer. To the extent that the excess claims experience is good, the ceding company benefits from a lower rate. Reinsurers often like these plans because they provide strong incentives, both positive and negative, to the ceding company to minimize excess claims. Ceding companies often find these plans attractive because they believe their realized rate will be significantly less than under a flat-rated plan.

While minimizing risk transfer is not usually the driving force behind the structuring of a swing plan, such a structure typically does transfer less risk than a flat-rated excess of loss treaty covering the same business. To illustrate this, suppose the expected excess losses are $4 million. If the total premiums on the subject portfolio are $50 million, this can be expressed as a loss cost of 8%. For the sake of discussion let's assume the excess claim count can be modeled using a negative binomial distribution with an mean of 8 claims[27] and that only total limit claims are possible. The claim distribution is shown graphically in Chart 7.



Suppose the swing plan calls for an excess reinsurance premium equal to excess claims times 100/80, subject to a minimum of 4% of subject premiums and a maximum of 16%. That results in the excess rate distribution shown in Chart 8. The expected value of the premium rate under this plan is 9.71%. The alternative is a contract with a flat rate of 11.43%.

[27] Specifically, using the Microsoft Excel function for the negative binomial probability, Prob(COUNT)= NEGBINOMDIST(COUNT, 8, 0.5)

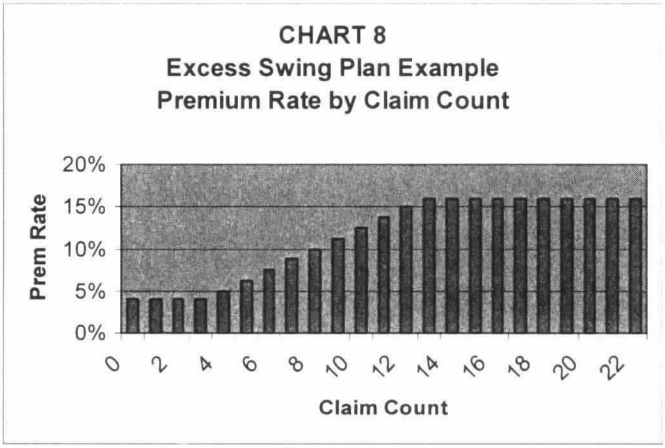


Table 11 summarizes the *ERD* analysis for both the flat-rated and swing-rated plans, assuming that there is a negligible claim payment lag. It shows that the swing plan has an *ERD* of 0.97%, just under the 1% threshold for significant risk. With some minor restructuring this contract would be able to pass the *ERD* test. In contrast, the flat-rated plan has an *ERD* of 4.70%, which is well above the threshold. Note that the mean severity of loss faced by the reinsurer is greater in the case of the swing plan than in the flat-rated plan, but because the probability of loss is much lower, the swing plan *ERD* falls below the threshold for "significant" risk. This is a good illustration of why severity (*TVaR*) by itself is an unreliable indicator of risk.

TABLE 11

ERD Risk Transfer Analysis

Swing-Rated vs. Flat-Rated Excess

Plan	Rate	*Freq*	*Sev*	*ERD*
Swing	9.71%	3.2%	30.4%	0.97%
Flat	11.43%	18.0%	26.2%	4.70%

Example 5.7: Individual Risks

One of the well known drawbacks of the "10-10" test is that if it were applied to individual insurance contracts or facultative reinsurance contracts, it would in almost all cases indicate that they do not contain "significant" risk, which strikes virtually everyone as unreasonable. In this example, using simplifying but not unreasonable assumptions we will show that the *ERD* test correctly identifies individual risk contracts as containing significant risk.

We assume that a portion of the premium for every individual risk contract is attributable to the potential for a limit loss. Since it is very large losses rather than partial losses that are most likely to put the insurer or reinsurer into deficit, we will ignore the potential for small losses and focus on limit losses. Let's assume that the pure premium for total limit losses is 10% of the total premium. Since a limit loss can occur only once in a policy period, let's assume the probability of such a loss is Bernoulli distributed with a probability equal to this 10% times the total premium rate on line (i.e., the total premium divided by the limit). From that we can calculate the *ERD* and the maximum downside potential.

The results are shown in Table 12 for rates on line ranging from 0.5% up to 83.33%. We see that any individual risk paying a rate on line of less than 83.33% would exceed a $ERD \geq 1\%$ standard for "significant" risk. We display such a wide range of rates on line, because we want to show that virtually all individual risks, ranging from personal lines policies to large commercial policies with a high level of premium funding, can be shown to meet the "significant" risk requirement using the *ERD* test.

Above a rate on line of 83.33%, the maximum downside falls below 20% of premium, which is a potential threshold for our proposed minimum downside requirement. Thus, individual risks with rates on line above 83.33% would fail to show "significant" risk. While this is a highly idealized example and further research would be appropriate to refine the methodology, we believe it is sufficiently realistic to "pre-qualify" virtually all individual risk contracts as containing significant risk and thus make it unnecessary to test them individually.

TABLE 12

ERD / Max Downside
For Individual Risk Contracts

By Rate on Line

Rate on Line	Limit Loss Prob	*ERD*	Reinsurer Max Downside
0.5%	0.05%	9.95%	19900%
1.0%	0.10%	9.90%	9900%
2.5%	0.25%	9.75%	3900%
5.0%	0.50%	9.50%	1900%
10.0%	1.00%	9.00%	900%
25.0%	2.50%	7.50%	300%
50.0%	5.00%	5.00%	100%
75.0%	7.50%	2.50%	33%
83.3%	8.33%	1.67%	20%

Assumptions.
- Investment income effects ignored
- Bernoulli probability of limit loss
- Total limit loss ratio 10%

5.1 Section Summary

In this section we have shown that the *ERD* test produces mostly reasonable results when applied to a variety of reinsurance structures covering insurance portfolios having a wide range of risk characteristics. Using the $ERD \geq 1\%$ standard together with reasonable contract assumptions we have demonstrated that catastrophe excess of loss reinsurance and individual risk contracts generally contain significant risk, which is a common sense result that eludes the "10-10" test. We also showed that loss ratio corridors and loss ratio caps are acceptable under some circumstances but not under others, and similarly that swing-rated excess reinsurance must be structured with care to ensure that it transfers significant risk while still meeting the reinsurer's and ceding company's other goals. The only unreasonable result we produced was that a quota share contract with a ceding commission of 25% and the prospective volatility characteristics of the S&P 500 (as measured by VIX) does not always meet the "significant" risk requirement. VIX has ranged as low as 9% in the period since 1990. Volatility parameters below about 13% produce *ERD* results (in the quota share we tested) that suggest insignificant levels of risk. This is an anomalous result because it suggests that under some circumstances an investment related to the S&P 500 index should not be considered risky, a conclusion that does not seem reasonable.

In summary, given these results and the findings in Section 4, we conclude that:

1. The *ERD* methodology described here, with a 1% threshold for significant risk transfer, is numerically comparable to the "10-10" benchmark;
2. The *ERD* methodology is qualitatively superior to that benchmark; and
3. If the 1% *ERD* method were adopted as a de facto standard replacing the "10-10", we would consider that a significant improvement.

6. IDENTIFICATION OF CONTRACTS SUBJECT TO "SIGNIFICANT" RISK REQUIREMENT THAT DO NOT REQUIRE INDIVIDUAL TESTING

Apart from those contracts for which it can be demonstrated that they transfer "substantially all" the risk inherent in the underlying insurance policies, all purported reinsurance contracts are subject to the requirement that they transfer "significant" risk. Unless a contract is tested, it is impossible to know whether or not it meets the requirement. However, the implication that it is necessary to test every single reinsurance contract is daunting. For many ceding companies buying excess of loss reinsurance, it might even be impossible. Ceding companies often buy excess coverage not only to transfer risk but also to obtain pricing for excess exposure they themselves do not fully understand, which they can factor into their own insurance rates. Under such circumstances, to ask ceding companies to model such exposure to demonstrate compliance with FAS 113 seems unreasonable.

Ideally, we would like to find a way to partition the set of all reinsurance contracts subject to the "significant" risk requirement into the subset containing those that we can reasonably expect will pass if they were tested and the subset comprising all other contracts. The former subset would be exempt from individual testing, while the latter subset would have to be tested individually. The purpose of this section is to begin to identify elements of the first subset of contracts that do not require individual testing.

Example 6.1: Individual Risk and Catastrophe Excess of Loss Contracts

In Section 5 we showed that 1) standard catastrophe excess of loss contracts and 2) individual risk contracts, generally possess *ERD* characteristics that indicate these two classes of contracts meet the "significant" risk requirement, and that it is therefore unnecessary to test contracts within those classes individually.

Example 6.2: Other Excess of Loss Contracts

By virtue of analysis similar to that for individual risk and catastrophe excess of loss contracts, it is possible to add a further large subset of excess of loss contracts (treaty and facultative) to the category of contracts that do not require individual testing. Table 13 summarizes the *ERD* analysis for excess of loss contracts with no ceding commission and rates on line ranging from 1% to 500% and aggregate limits no less than one full limit or 200% of premiums, whichever is greater. The term "rate on line" is most frequently used in connection with catastrophe excess of loss treaties and other excess contracts where the ratio of premium to limit[28] is far less than 100%, so a rate on line of 500% might be surprising. However, it is common for "working layer" excess of loss contracts to be priced with the expectation that there will be between several and many claims during the coverage period. Under typical pricing assumptions, a 500% rate on line implies the expectation that excess claims will be equivalent to about three total limits losses.

Our analysis assumes a Poisson distribution for claim frequency and that all claims are limit losses. Theoretically, we should use a negative binomial, but because that makes the tail fatter and thus easier to pass the *ERD* test, the Poisson assumption is conservative. We assume an expected loss ratio of 70%, another conservative assumption. In a competitive market the expected loss ratio can be expected to be higher, especially for the higher rate on line business. We assume an interest rate of 5% and a 5-year claim payment lag (which makes this analysis suitable for reasonably long tail as well as short tail business).

On the basis that every rate on line in Table 13 from 1% to 500% passes the *ERD* test even without the supplemental downside requirement coming into play, we suggest that any excess of loss contract having this structure (and no loss sensitive or other features that might call the contract's status into question) be deemed to meet the requirements for

[28] Note that the limit used in the denominator is the risk or occurrence limit, depending on the coverage, not the aggregate limit except in the case of aggregate excess coverage.

TABLE 13

Expected Reinsurer Deficit / Max Downside

For Long/Short Tail XL Contracts with
Aggregate Limit ≥ One Limit or 200% Loss Ratio

By Rate on Line

Rate on Line	Poisson λ	Expected Reinsurer Deficit*	Reinsurer Max P.V. Downside*
1.0%	0.7%	54.0%	7735%
2.5%	1.8%	52.6%	3034%
5.0%	3.5%	50.5%	1467%
10.0%	7.0%	46.2%	684%
15.0%	10.5%	42.1%	422%
25.0%	17.5%	34.3%	213%
50.0%	35.0%	16.7%	57%
75.0%	52.5%	6.9%	57%
100.0%	70.0%	8.8%	57%
200.0%	140.0%	5.0%	57%
300.0%	210.0%	2.9%	57%
400.0%	280.0%	1.8%	57%
500.0%	350.0%	1.3%	57%

* Ratio to premium

Assumptions.
- Loss cap of greater of one limit or 200% L/R
- No ceding commission
- Poisson model with parameter λ
- Claim payment lag 5 years
- Interest rate 5% per annum
- Expected loss ratio 70%

"significant" risk transfer. Excess of loss contracts with no aggregate limit clearly fall into this category as well. All such contracts are subject to the "significant" risk transfer requirement. However, because we have, in effect, pre-qualified them as a class, the requirement to demonstrate significant risk transfer can be waived.

Example 6.3: Contracts with Expected Loss Ratios Above a Minimum Permissible Loss Ratio Threshold

There is a further general approach to expanding the set of contracts subject to "significant" risk testing that do not need to be tested individually. In Section 3 we noted that one unreasonable implication of the "10-10" test is a cap on reinsurance pricing at such a low level that, if it were enforced, would likely lead to a reduction of reinsurance capacity. The $ERD \geq 1\%$ standard we have proposed also implies a cap on reinsurer margins. Fortunately, the *ERD* standard we have illustrated implies a significantly higher maximum permissible present value margin for the reinsurer than the "10-10" test does.

Table 14 shows maximum permissible present value margins and corresponding minimum permissible loss ratios implied by $ERD \geq 1\%$ for claim lags of zero, one year, two years and three years with respect to contracts for which the prospective loss ratio can be modeled using a lognormal distribution[29]. The results are shown for σ values ranging from 9% to 100%. Note that for each value of σ, the permissible loss ratios increase in nominal terms with the claim lag, but the present values are all the same. The allowable margins for the σ values at the low end of the range might make reinsurance of such low risk portfolios impossible unless the reinsurance is structured to meet the "substantially all" risk transfer test. For example, the maximum permissible present value margin for $\sigma = 9\%$ of only 7.1%, while much higher than the 1.6% permitted under "10-10"[30], does not allow a reinsurer much, if any, upside potential, after deducting brokerage and internal expenses. That is one reason to consider the possibility that an *ERD* threshold of 1% might be too high. On the other hand, in light of our discussion in Section 3 about parameter uncertainty, it might turn out to be the case that realistic prospective estimates of σ will, in practice, generally exceed the low end of the range, making this concern irrelevant.

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[29] Where the lognormal assumption is not appropriate, similar tables could be constructed for other loss ratio models.

[30] See Table 6. It is worth noting that the *ERD* $\geq$3% mentioned in the 2002 VFIC paper as a possible threshold would result in an even lower maximum permissible present value margin of 1.2%! A threshold of 3% is clearly too high.

TABLE 14

Maximum Margins / Minimum Permissible Loss Ratios
Implied by $ERD \geq 1\%$

Contracts with No Ceding Commission
Interest at 5% per annum

Tabulated by σ and Claim Lag

	Max	Minimum Permissible Loss Ratio			
	P.V.	Lag 0	Lag 1	Lag 2	Lag 3
σ	Margin	Yrs	Yr	Yrs	Yrs
9.0%	7.1%	92.9%	97.5%	102.4%	107.5%
10.0%	8.4%	91.6%	96.2%	101.0%	106.0%
11.0%	9.7%	90.3%	94.8%	99.6%	104.6%
12.0%	11.0%	89.0%	93.5%	98.2%	103.1%
13.0%	12.3%	87.7%	92.1%	96.7%	101.6%
14.0%	13.6%	86.4%	90.8%	95.3%	100.1%
15.0%	14.9%	85.1%	89.4%	93.9%	98.6%
20.0%	21.3%	78.7%	82.7%	86.8%	91.1%
25.0%	27.4%	72.6%	76.2%	80.0%	84.0%
30.0%	33.2%	66.8%	70.1%	73.6%	77.3%
40.0%	43.7%	56.3%	59.1%	62.1%	65.2%
50.0%	52.6%	47.4%	49.8%	52.2%	54.9%
60.0%	60.1%	39.9%	41.9%	44.0%	46.2%
75.0%	69.1%	30.9%	32.5%	34.1%	35.8%
100.0%	79.5%	20.5%	21.6%	22.6%	23.8%

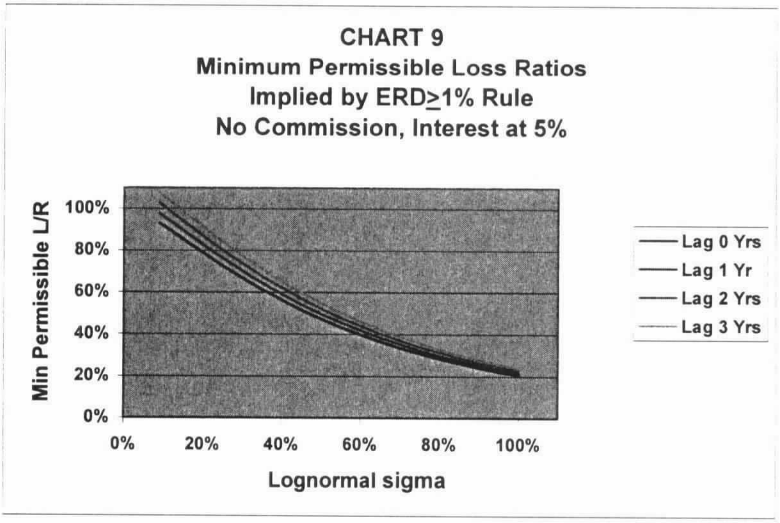
The maximum margins implied by $ERD \geq 1\%$ for larger values of σ seem more reasonable. For example, for $\sigma = 30\%$, the allowable present value margin is 33.2%, which is a more reasonable ceiling[31].

The implication of this for our present discussion is that if a contract with no ceding commission is priced to an expected loss ratio that is greater than the minimum permissible loss ratio shown for the relevant σ and claim lag (and the other assumptions are reasonable), then the contract will meet the $ERD \geq 1\%$ standard that indicates significant risk transfer. We present this as an illustration of how the subset of contracts that do not

[31] In contrast, a threshold of $ERD \geq 3\%$ implies a maximum permissible present value margin of 22.0%, which is about the same as that implied by "10-10".

require detailed testing for significant risk transfer could be expanded beyond the catastrophe excess of loss, individual risk and other excess of loss contracts we identified earlier. Any contract that is priced to an expected loss ratio that exceeds the minimum permissible loss ratio would be exempt from individual testing. Additional research is necessary to fully realize this approach.

Chart 9 shows the minimum permissible loss ratios in Table 14 graphically.



Example 6.4: Contracts with Immaterial Premiums

Contracts or programs that involve the cession of small amounts of premium should be exempt from individual testing, unless there is reason to suspect that they might materially distort either the ceding company's or reinsurer's financial statements. A reasonable definition of small might be the smaller of $1 million and 1% of total gross premiums. The rationale for this exception is that small premium cessions by definition have a very limited impact on either party's financial statements. Any distortion resulting from minimal risk transfer below the significance threshold would be immaterial.

7. POSSIBLE EVOLUTION OF RISK TRANSFER MEASUREMENT

The context of the paper is risk transfer testing. However, the notion of risk transfer is also integral to the pricing of insurance and reinsurance products. Risk transfer is what gives rise to risk premiums and the potential for profit. Many methods already exist for explicitly or implicitly adding a profit load to a reinsurance contract. It seems reasonable that a risk loading method used to determine needed profits could be turned into a risk transfer test as well. Although this paper does not address the issue directly, the *ERD* risk transfer test described in earlier sections of this paper measures tail value at risk (*TVaR*), which is a valid method for producing risk and profit loads. In fact, given the coherent nature of *TVaR*, it is considered a superior method for risk loading by many practitioners.

At least one major insurance company has used the *ERD* framework in pricing and enterprise risk management for several years, in the form of the *risk coverage ratio* (*RCR*) described by Ruhm [6]. In practice, that risk measure has produced results for the company that are reasonable and consistent across a broad variety of actual risks, due in large part to its good technical properties and its relative transparency.

As noted before, this working party is not endorsing any single specific method for risk transfer testing. Thus, rather than doing more work on our *ERD* example to show its full implications for risk loading, we will show another (much briefer) example here where risk loading and risk transfer testing are tightly linked.

The approach we examine here is based on the *right tail deviation* (*RTD*), a framework proposed by Wang and developed from concepts he has written about extensively [7] [8].

For a given aggregate distribution function $F(x)$ (derived from some convolution of frequency and severity distributions), we transform the distribution using the following formula:

$$F^{*}(x) = 1 - \sqrt{1 - F(x)} \tag{7.1}$$

Because $0 < F(x) < 1$ for all x, it is fairly easy to see that $F^{*}(x) < F(x)$ for all x, which implies the following expected value relationship:

$$E^{*}(x) \geq E(x) \tag{7.2}$$

The interpretation is that the transform has "loaded" the original distribution for risk. The difference between E^{*} and E is the risk load, for any layer of the distribution. Thus, we can use E^{*} instead of E to represent a fully risk loaded pure premium. The reason this approach is appealing is that the transformed distribution is itself another loss distribution, meaning that all the ordinary mathematics of loss distributions carry over. Relating this to financial mathematics, it is generally assumed that assets like equities are themselves transformed distributions, although this is not usually explicitly stated. The transform in the financial economic model is the so-called state price, which enforces no-arbitrage pricing [9].

If one wants to think about the risk load independently, it is easily captured as:

$$RTD(x) = E^{*}(x) - E(x) \tag{7.3}$$

Under this approach, the risk load *RTD* might be adjusted (i.e. multiplied) by some constant factor α to produce the final profit load. Note that Wang has generalized this model to consider other exponents of transformation (i.e. instead of just the power of 0.5, any power between 0 and 1 exclusive).

There are a couple of ways in which the *RTD* could be used to devise a risk transfer test. One way would be to treat αRTD as the maximum permissible reinsurer's margin consistent with "significant" risk transfer. That is essentially the same approach that was described in Example 6.3. The difference is that in that example, we derived the risk load consistent with a "significant" risk transfer threshold of $ERD \geq 1\%$, whereas here we would determine the risk load component αRTD first and then effectively determine the risk transfer threshold that is consistent with it.

A second way would be to devise a risk transfer test that compares the full premium (not just the margin) with a multiple of αRTD using the following procedure, which is similar to one outlined by Wang:

1. Compute expected loss of the contract under the untransformed distribution $F(x)$;
2. Note the premium for the deal (however computed—allows for market pricing);
3. Compute *RTD* for the deal using the transformed distribution and formula (7.3);

4. Define the ***maximum qualified premium*** as some multiple of *RTD* (Wang suggests 3-5x[32]);

5. The "significant" risk transfer threshold is defined as "***maximum qualified premium*** $\geq$ ***premium***"[33].

We will look at two examples of this approach. The first is the catastrophe excess of loss contract described in Examples 3.1 and 5.1. The second example addresses a questionable scheme for creating a reinsurance structure that apparently meets the "significant" risk transfer requirement by combining two unrelated coverages to produce just enough risk transfer to pass. This is an important example, because this method separates the reinsurance premium into higher risk and lower risk components and thus has potential to identify highly structured reinsurance contracts that satisfy other quantitative tests but do not meet the spirit of FAS 113[34].

Example 7.1: Property Catastrophe Excess of Loss Reinsurance

If we apply the *RTD* qualified premium approach to the property catastrophe excess of loss example discussed in Examples 3.1 and 5.1, we see that the contract easily meets this *RTD*-based risk transfer requirement. Table 15 shows the catastrophe loss distribution originally shown in Table 3 with an additional column for the "transformed" probability based on the $F^*(x)$ determined from formula 7.1. $E^*(x)$, expressed both in terms of premiums and limit, is shown at the bottom of the table as 203% and 20%, respectively.

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[32] The issue of the appropriate multiplier of *RTD* warrants further research. A multiple of 4 appears to imply that traditional quota shares like those discussed in Examples 3.2 and 3.3 do not contain significant risk transfer, which suggests the effective threshold may be set too low.

[33] Wang has a suggested giving partial credit in cases where the maximum qualified premium is less than the actual reinsurance premium. However, we prefer to focus on the risk characteristics of the contract as a whole.

[34] This comes at the cost of some complexity. The subdivision into risky and less risky components depends on the values chosen for α, the multiplier for αRTD, and the exponent in formula (7.1), choices that are made more difficult by the fact that it is difficult to ascribe an intuitive meaning to these parameters.

TABLE 15 Catastrophe Loss Distribution Example 7.1			
Loss as % of Limit	Loss as % of Premiums	Actual Probability of Given Loss	Transformed Probability* of Given Loss
0%	0%	67%	43%
5%	50%	20%	21%
10%	100%	10%	19%
100%	1000%	3%	17%
5%	50%	100%	100%
20%*	203%*		

In terms of premium, *RTD*=203%-50%=153%. Using a multiplier of 4x, the "qualified" premium proportion is 612%, which is well in excess of the threshold of 100% required for significant risk transfer.

Example 7.2: "Highly Structured" Mix of Low Risk and High Risk Portfolios

We now move on to the example of potential manipulation. In this case, the deal structure consists of a base portfolio with very little risk mixed with a highly risky catastrophe layer. The overall structure is designed to barely pass risk transfer using the "10-10" criterion.

The low risk portfolio has expected losses of $8 million with lognormal σ value of only 1%. To maximize the low risk nature of this portfolio, its premium is $8 million—no load for expense or profit at all.

The catastrophic portfolio we add to this deal is a $1.6 million layer with a 12.5% chance of loss. For simplicity, if a loss occurs, it is a total loss. Thus, the expected loss for this piece is $200,000. Let's assume the premium is $500,000, for a 40% expected loss ratio.

First, let us consider the two pieces separately. The low risk portfolio has an untransformed expected loss of $8 million and a transformed expected loss of $8.1 million. The maximum qualified premium is only $0.4 million, leaving $7.6 million unqualified. This piece falls far short of the "significant" risk standard.

The catastrophic portfolio has an untransformed expected loss of $200,000 and a transformed expected loss of $666,000. The maximum qualified premium is well in excess of the actual premium of $500,000, thus easily meeting the *RTD*-based "significant" risk standard.

Now consider the combined distribution. The combined contract has a premium of $8.5 million. A 10% loss over this would be an attachment of $9.35 million, and the probability of this occurring is 12.5% (very close to the cat loss alone, of course). Thus, this contract passes the "10-10" test. But Wang's method gets closer to the truth. The transformed expected losses are only $8.65 million vs. $8.2 million untransformed, producing maximum qualified premiums of only $1.8 million, leaving $6.7 million unqualified, well short of the 100% required for "significant" risk transfer.

Note that this method penalizes the combination even more than the sum of the components (the *RTD* of the combined deal is $450,000, whereas the sum of the *RTD*s of the two deals is about $570,000)[35]. It is not clear whether this phenomenon, i.e., the *RTD*-based approach of the highly contrived structure being less than sum of the *RTD* of the separate components, represents the general case. However, it does suggest the intriguing possibility that this approach could perhaps be developed into a quantitative test to detect reinsurance structures that appear to pass certain quantitative threshold, but which do not meet the spirit of FAS 113.

This is as far as we will pursue the *RTD* ideas here. The *RTD* approaches have some appeal and added properties that the *ERD* method does not, at the cost of increased complexity. As noted previously, the working party is not specifically advocating any particular method. This example shows that other methods could be used instead of the *ERD* example that we have examined in some detail. Ultimately, a combination of market and regulatory factors will determine what methods are actually deployed.

[35] This is due to the diversification of the combined deal, which is of course the correct treatment.

8. SUMMARY

The purpose of this paper has been to contribute constructively to the thinking about what should be understood by the term "risk transfer" in the context of FAS 113 by framing a comprehensive response to the four questions posed by COPLFR.

In particular, we have responded to the first two questions by describing two approaches for assessing the significance of risk transfer that are superior to the "10-10" test that is in common use. The first approach, which we have described and illustrated in detail, is based on the expected reinsurer deficit (*ERD*). The second approach, which we outline more briefly, is based on the concept of right tail deviation (*RTD*). We have responded to the third "safe harbor" question in two parts. First, we have described a framework for determining whether a purported reinsurance contract meets the FAS 113 risk transfer requirement by virtue of the cession of "substantially all" of the underlying insurance risk to the reinsurer. Second, we have begun to identify groups of contracts that are subject to the "significant" risk requirement of FAS 113, but which can be exempted from detailed individual testing, because we have established that contracts falling within the group can reasonably be expected to pass the "significance" test, if they were actually tested.

In particular, the following classes of contracts fall into the category of transferring "substantially all" of the original insurance risk, unless they include features that reduce the reinsurer's *expected underwriting deficit* (*EUD*) below that which the cedent would face on its unreinsured portfolio:

- Proportional facultative reinsurance with effective ceding commissions no less than cedent expenses;
- Proportional treaties with effective minimum ceding commissions no less than cedent expenses;
- Proportional facultative or treaty reinsurance for which it can be shown that the reinsurer's *EUD* is essentially the same as the cedent's *EUD* on the unreinsured subject portfolio, irrespective of whether the contract includes a loss ratio corridor, loss ratio cap or other risk mitigating feature;
- Excess of loss facultative or treaty reinsurance for which it can be shown that the reinsurer's *EUD* is essentially the same as the cedent's *EUD* on the portion of the

original subject portfolio that is exposed to the same risks as the excess of loss contract;

- Whole account quota share contracts with loss ratio caps no lower than the point at which the ceding company would exhaust its surplus.

To address the question of how to measure "significant" risk transfer, we have proposed an *ERD* test as an improvement over the "10-10" test, which arose in the 1990s as a way to test "finite risk" reinsurance contracts for compliance with FAS 113. The "10-10" test was not originally intended to be applied to traditional reinsurance contracts, and usually it was not. In the wake of recent real and alleged reinsurance accounting abuses, there is an increasing sentiment that a wider class of reinsurance contracts beyond those classified as "finite" need to be tested for significant risk transfer. Because it has come into widespread use, the "10-10" test has become the de facto standard for reinsurance risk transfer testing, despite the fact that it has never been endorsed by any professional body nor subjected to serious critical scrutiny.

We have also addressed COPLFR's fourth question. Throughout the paper we have discussed the advantages of our described approaches over the "10-10" test that is commonly used today. We have demonstrated that "10-10" is inadequate for use as a universal risk transfer test, because it cannot correctly identify contracts that are clearly risky. We have proposed an improved alternative test based on the concept of the *expected reinsurer deficit*, or *ERD*, which incorporates both frequency and severity of underwriting loss into a single measure. The embedded severity measure is the *TVaR* at the economic breakeven point. *TVaR* has the advantages over *VaR* of reflecting all the information in the right tail of the underwriting result distribution as well as being a coherent measure of risk.

We have shown that the proposed $ERD \geq 1\%$ threshold correctly classifies as "risky[36] a quota share treaty that has the loss ratio volatility characteristics of the S&P 500 stock index. This is important because the standard for assessing reinsurance risk should be consistent with those in other financial markets.

We have also shown that low frequency-high severity reinsurance contracts (such as catastrophe excess of loss treaties) and high frequency-low severity contracts (such as traditional primary quota share treaties) pass the *ERD* test, provided loss mitigating features

[36] Provided the risk characteristics of the treaty are not too distorted by a large ceding commission.

such as loss ratio caps and/or corridors do not remove too much risk from the contracts (in which case a "failure" is entirely appropriate).

In summary, while we are not explicitly endorsing any single model or framework, because the *ERD* methodology described here (with a 1% risk transfer threshold) is numerically comparable to the current "10-10" benchmark and is superior in almost every way to that benchmark, if the 1% *ERD* method were adopted as a de facto standard replacing the "10-10", we would consider that a good outcome.

To address the concern in some quarters that the *ERD* test is not always stringent enough with respect to the potential for a large loss by the reinsurer, we have suggested consideration of a supplemental requirement that the reinsurer face a minimum downside potential of 15% or 20% of premiums.

Among contracts that are subject to the "significant" risk transfer requirement, under the "significance" standard embodied in $ERD \geq 1\%$ the classes of contracts listed below would not be subject to individual testing, because they have already been found to meet the requirement under very general conditions. It is therefore possible to say about contracts falling into the categories on the list below that the significance of their risk transfer is "reasonably self-evident". This is a preliminary list. We believe it may be possible to expand it considerably.

- Individual risk contracts;
- Short tail excess of loss treaties in the standard catastrophe excess structure, i.e., one reinstatement of the limit for 100% additional premium, with rates on line of up to 50%;
- Other excess of loss contracts with aggregate limits of no less than the greater of one occurrence (or risk) limit and 200% of premiums, no ceding commissions, and rates on line of up to 500%;
- Proportional and excess contracts having an expected loss ratio above the minimum permissible loss ratio implied by the $ERD \geq 1\%$ standard (or other standard as may be agreed);
- Contracts involving immaterial premiums.

Other contracts should be considered for significance testing, even if they appear to fall into one of the safe harbor categories, for the simple reason that they have greater potential to attract attention, and it is better to be prepared. This group includes, for example, 1) contracts involving large premium cessions, 2) those which, when accounted for as reinsurance, would substantially alter surplus or the ratio of premiums to surplus, and 3) contracts involving unusual structures, especially those that look contrived (e.g., a primary quota share combined with catastrophe protection on a different portfolio). Contracts in category 3 may be structured to narrowly meet the quantitative requirements for "significant" risk transfer, but they might still reasonably be disqualified on other grounds. Thus, a quantitative risk transfer test such as the *ERD* will not be adequate in all cases. However, we believe the *ERD* would do a good job of discriminating between contracts with significant risk and those without significant risk in all but cases involving contrived structures.

We have also pointed out that other risk transfer tests besides *ERD* can and should be considered, particularly in the context of reconciling risk transfer testing to the process of determining risk and profit loads. One such example, based on the ***right tail deviation***, has certain desirable properties but comes at the cost of greater complexity. Other approaches could surely be used and should be the subject of future research.

It is important to remember that any risk transfer test requires a model of the prospective underwriting results and the related cash flows. In cases where there is relevant and credible loss experience, identifying a model is often straightforward, though it is always important to appropriately adjust the historical loss experience to prospective levels and to be conscious of the uncertainty in the model parameters. Where there is little or no relevant historical experience, the model must be chosen on the basis of the similarity of the subject portfolio to other ones with the same general characteristics. In such cases there will be greater uncertainty about the parameters, which should be reflected in the structure of the model.

9. SUGGESTED PRIORITIES FOR FURTHER RESEARCH

The *ERD* test proposed in this paper should be seen as an example of a reasonable framework for assessing the significance of risk transfer in reinsurance contracts. We have demonstrated that it is a clear improvement over "10-10", but we do not claim that it is the only reasonable approach. Indeed, we briefly described another promising, albeit more complicated, method, namely, Wang's *RTD* framework. There may be others. We urge the CAS to encourage further research on this subject, perhaps through a call for papers.

We recommend the following research priorities in order to quickly arrive at a more effective assessment of risk transfer according to FAS 113 as well as to provide for continuing research in relation to future improvements.

9.1 Immediate "Level 1" Research – Consensus on Thresholds

1. Determination of an appropriate pass threshold for the comparison methodologies presented in Section 2 to determine whether or not "substantially all" of the insurance risk has been transferred. This may include determining a single applicable testing methodology (i.e., limiting the test to just one of the two methods presented);

2. Determination of an appropriate "pass" threshold framework for the *ERD* test presented in Section 4. In particular, is the 1% threshold illustrated in this paper appropriate, or would some other threshold be more appropriate? In addition, should there be a supplemental requirement that the reinsurer's potential loss be greater than or equal to some minimum amount? (We considered a minimum underwriting loss of 20% in some of our examples.);

3. Determination of the contract categories and financial characteristics of contracts that will not be required to be individually tested for "significant" risk transfer (because they have previously been analyzed and found generally to pass the significance test). This depends on item 2. Given a standard of $ERD \geq 1\%$, we demonstrated that individual risks, short tail excess of loss contracts in the standard catastrophe excess of loss structure within a certain rate on line range, other excess treaties within a certain rate on line range that have aggregate limits that are not too large, and other contracts with expected loss ratios above a minimum permissible loss ratio threshold, should not be required to be individually tested because we have determined they will pass if they were tested. It may be possible to expand that set of contracts "pre-qualified" for "significant" risk in that same way. If an *ERD* threshold different from 1% is adopted, the set of contracts that can be pre-qualified for "significant" risk may change.

9.2 On-Going "Level 2" Research – Other Methods

1. Continued research on methodologies and thresholds for determining whether or not "substantially all" of the insurance risk has been transferred;

2. Continued research for methodologies that assess risk transfer within the "reasonably possible" chance of a "significant" loss. As stated earlier, the Wang transformation could be one example of such a method;
3. Continued research into appropriate methods for incorporation of parameter uncertainty into models used for risk transfer testing.

Appendix A

Definition of Downside Risk Measures

Suppose B represents the amount of (present value) claims corresponding to the reinsurer's economic "breakeven" point, before taking into account brokerage and internal expenses (the FAS 113 definition):

$$B = P - C \tag{A.1}$$

where P represents the ceded premiums and C represents the ceding commissions payable on ceded premiums, if any. If $C = 0$, then the breakeven loss amount is equal to the premiums.

Let x denote the random variable for the prospective losses. (It may be more convenient in practice to work with loss ratios, but here we are using loss dollars.) Then the expected cost of FAS-113-defined present value loss scenarios PV($Loss > 0$) (which ignore all reinsurer expenses other than ceding commissions), also known as the present value expected reinsurer deficit or *ERD*, expressed as a dollar amount, is:

$$ERD = E[(PV(Loss) > 0)] = PV \int_{FV(B)}^{\infty} (x - FV(B)) \cdot f_x(x)dx \tag{A.2}$$

As the pure premium cost of underwriting loss scenarios, *ERD* is a measure of the reinsurer's underwriting downside risk[37].

[37] Note that the *ERD* is the expected present value of the contingent capital calls described by Mango [5].

The probability or frequency of the insurer incurring a present value loss $PV(Loss) > 0)$ is:

$$Freq = \Pr ob[PV(Loss) > 0] = \int_{FV(B)}^{\infty} f_x(x)dx \tag{A.3}$$

The expected severity of underwriting loss, given $PV(Loss) > 0)$, is

$$Sev = E[(PV(Loss) \mid PV(Loss > 0)]$$

$$= \frac{\int_{FV(B)}^{\infty} (x - FV(B)) f_x(x)dx}{\int_{FV(B)}^{\infty} f_x(x)dx}$$

$$= \frac{ERD}{\Pr ob[PV(Loss > 0)]} \tag{A.4}$$

Note that Sev is the Tail Value at Risk (for present value underwriting loss) described by Meyers [4] as a coherent measure of risk and by the CAS Valuation, Finance, and Investments Committee [1] for potential use in risk transfer testing of finite reinsurance contracts. Meyers (p. 239) gives the following formula for $TVaR_\alpha$:

$$TVaR_\alpha = VaR_\alpha + \frac{EPD(VaR_\alpha)}{1-\alpha} \tag{A.5}$$

At the present value breakeven loss point *B*, $\alpha = F_x(B) = \int_0^{FV(B)} f_x(x)dx$. The present value loss at the breakeven loss is zero, implying $VaR_\alpha = 0$. That leaves only the second term. Because $EPD(VaR_{F_x(B)}) = ERD$ and $1-\alpha = 1 - F_x(B) = \Pr ob[PV(loss > 0)]$, when the variable of interest is present value underwriting loss, (A.5) equates to formula (A.4).

For a quota share with no loss ratio caps or corridors, the reinsurer's loss ratio is identical to the ceding company's loss ratio on the subject portfolio and their distributions are identical[38]:

$$f_x(x) = f_y(y)$$

If there are no loss ratio caps or corridors, it is often still convenient to express the random variable x for the reinsurer's loss ratio in terms of the subject portfolio's loss ratio random variable y. For example, given a 5-point loss ratio corridor between 75% and 80% with respect to the subject portfolio, the reinsurer's loss ratio $x(y)$ is:

$$x(y) = \begin{array}{ll} y & \text{if } y \le 75\% \\ 75\% & \text{if } 75\% < y < 80\% \\ y - 5\% & \text{if } y \ge 80\% \end{array}$$

In this case, given $B = 75\%$, formula (A.2) for *ERD* would be expressed in terms of y as follows:

$$ERD = PV \int_{FV(B_y)}^{\infty} (y - FV(B_y)) \cdot f_y(y)dy$$

$$ERD = PV \int_{FV(80\%)}^{\infty} (y - FV(80\%)) \cdot f_y(y)dy$$

where $B_y = B + 5\%$. Similarly, Formulas (A.3) for frequency and (A.4) and severity can be expressed in terms of y.

[38] Because it is easier to compare the cedent and reinsurer positions if we use loss ratios rather than loss dollars, this part of the discussion is in terms ratios to premiums.

Appendix B

Discussion of Analogy to Stock Market Risk

In this appendix we compare S&P 500 equity risk[39] to the risk in a quota share reinsurance treaty. We begin by discussing the basis of the lognormal assumption. Then, in Example B.1, we show how the cash flows and economics of the quota share described in Example 3.3 can be replicated by an S&P 500 index transaction. That transaction takes the form of a short sale. In that scenario, the short seller loses money if the S&P 500 index closes higher than its level at the time of the short sale, just as the reinsurer loses money if the actual loss ratio exceeds the breakeven loss ratio. The appendix also includes Table B-1, which shows the data underlying Chart 4 and Table B-2, which shows the sensitivity of "10-10" test results for the quota share in Example 3.3 to the expected loss ratio.

Basis of Lognormal Assumption

It is possible, perhaps even likely, that stock prices are not lognormally distributed. However, stock price movements are commonly assumed by financial economists to follow Brownian motion through continuous time, which implies that stock returns over infinitesimal time intervals are normally distributed and stock prices are lognormally distributed after any finite time interval. For example, see Hull [10] Chapter 11 (p. 228) and Baxter-Rennie [11] Chapter 3 (p. 51). The latter says, "It is not the only model for stocks...but it is simple and not that bad." The Black-Scholes call option pricing formula was originally derived using a Brownian motion assumption. It has subsequently been shown that it can also be derived from the assumption that "asset prices are lognormally distributed under the martingale measure Q."[Ibid, p. 181].

At the same time there is some disagreement with the Brownian motion/lognormal assumption. See for example Peters [12], Chapter 3 (p. 27), who presented evidence that the distribution of actual stock market *returns* has a higher peak and fatter tails than predicted by a normal distribution and found, "The stock market's probability of a three-sigma event is roughly twice that of the Gaussian random numbers." [Ibid, p. 29]. He argues that because "capital market theory is, in general, dependent on normally distributed

[39] In order to simplify the discussion we ignore dividends, which could easily be incorporated in the example, but at the cost of complicating the comparison.

returns"[Ibid, p. 25], the Efficient Market Hypothesis, Capital Asset Pricing Model and Modern Portfolio Theory all rest on a shaky foundation. We don't take a position in that debate. However, we do wish to point out that our use of a lognormal distribution is consistent with the mainstream view.

The fact is that doubling the probability at the three-sigma level does not have a significant practical effect. We can adjust for Peter's finding of a fatter tail in the stock return distribution. A Student's t distribution with 30 degrees of freedom has twice the probability of a three-sigma event as the corresponding normal. It has a higher peak and fatter tails.

If we replace the lognormal stock price model with a "log t" model, "10-10" test values for the Example 3.3 quota share with $\sigma = 9\%$ and $\sigma = 13.85\%$ still fall far short of the significance threshold. For $\sigma = 9\%$, the 90th percentile result is still a small profit of 0.29% and the probability of a 10% loss rises to just 0.51%. For $\sigma = 13.85\%$, we find a 90th percentile loss of 4.17% and a probability of a 10% loss of 3.91%. These values are only slightly higher than those arising from the lognormal model. There is no practical effect of the non-normality observed by Peters.

Example B.1: Replicating a Quota Share with 25% Ceding Commission

Suppose the quota share in Example 3.3 involves ceded premiums of $10 million. Given a ceding commission of 25%, the net proceeds to the reinsurer total $7.5 million. Similarly, if S&P 500 "spiders" (symbol SPY) are trading at $117 a share (as they were in early May 2005), a short sale of 64,103 shares also yields net proceeds to the seller of $7.5 million. The expected loss ratio on the quota share is 70%, implying expected losses of $7 million. Claim payments are expected to lag premiums by one year. This is equivalent to the short seller estimating the expected value of SPY in one year's time as $109.20, or $7 million in total for the short position. (A short seller would generally not short the stock if he did not expect it to decline.) In order for the reinsurer to suffer a $1 million present value loss (10% of the ceded premiums), given a risk free interest rate of 5%, the loss ratio would need to reach 85% times 1.05, or 89.25%. In order for the short seller to incur a $1 million present value loss, the stock price would have to reach $139.23[40]. These are the threshold levels for "passing" the "10-10" test.

As discussed in Example 3.3, in order for either the loss ratio to exceed 89.25% or the stock price to exceed $139.23 with a probability of 10% (these being fundamentally identical scenarios), the lognormal σ parameter must be at least 20.6%.

If we remove the 25% ceding commission from the quota share terms and instead provide for a premium cession net of a 25% expense allowance, then the "10-10" threshold for a 10% / $750,000 present value loss to the reinsurer is 82.5% times 1.05, or 86.63%. The comparable "10-10" threshold for the short seller is a stock price of $135.14. Exceeding these thresholds requires a σ value of at least 17.9%.

Data Underlying Chart 4

Table B-1 shows the data underlying Chart 4, which plots the probability of a 10% present value loss on the quota share defined in Example 3.2, given a 70% expected loss ratio, 25% ceding commission and σ values equal to VIX as of the last trading day of each year from 1990 through 2004 plus May 4, 2005.

[40] $1 million loss amounts to $15.60 per share, implying a present value share price of $132.60 and a future value share price of $139.23.

TABLE B-1 "10-10" Risk Transfer Analysis for Quota Share in Example 2.3 Given Portfolio with Volatility of S&P 500 VIX Data Underlying Chart 4			
VIX Date	VIX	(a) 90th Percentile P.V. Underwriting Loss	(b) Probability of ≥10% P.V. Underwriting Loss
Dec 1990	26.4%	15.3%	14.6%
Dec 1991	19.3%	8.8%	8.8%
Dec 1992	12.6%	2.7%	2.3%
Dec 1993	11.7%	1.9%	1.6%
Dec 1994	13.2%	3.3%	2.8%
Dec 1995	12.5%	2.7%	2.3%
Dec 1996	20.9%	10.3%	10.3%
Dec 1997	24.0%	13.1%	12.9%
Dec 1998	24.4%	13.5%	13.2%
Dec 1999	23.4%	12.6%	12.4%
Dec 2000	26.9%	15.7%	14.9%
Dec 2001	23.8%	12.9%	12.7%
Dec 2002	28.6%	17.3%	16.1%
Dec 2003	18.3%	7.9%	7.8%
Dec 2004	13.3%	3.4%	2.9%
May 2005	13.9%	3.9%	3.4%

Sensitivity of "10-10" Test Values to Expected Loss Ratio Assumption

Table B-2 shows the sensitivity of the values shown in Table 5 to changes in the expected loss ratio. It shows that our conclusions with respect to the "10-10" test apply even with high assumed levels for the expected loss ratio. For example, even in the case of no expected profit and the higher May 2005 implied volatility levels, the "10-10" rule is not met.

TABLE 5

"10-10" Risk Transfer Analysis

for Quota Share in Example 2.3

Given Portfolio with Volatility of S&P 500

Sensitivity to Expected Loss Ratio

VIX	σ	Expected Loss Ratio	(a) 90^{th} Percentile P.V. Underwriting Loss/(Profit)	(b) Prob of $\geq 10\%$ P.V. Underwriting Loss/(Profit)
Low	9.00%	65%	(5.81%)	0.02%
Low	9.00%	67.5%	(3.15%)	0.08%
Low	9.00%	70%	(0.49%)	0.30%
Low	9.00%	62.5%	2.18%	0.93%
Low	9.00%	75%	4.84%	2.40%
May 2005	13.85%	65%	(1.78%)	0.92%
May 2005	13.85%	67.5%	1.04%	1.85%
May 2005	13.85%	70%	3.85%	3.41%
May 2005	13.85%	62.5%	6.67%	5.82%
May 2005	13.85%	75%	9.49%	9.25%

10. References

[1] CAS Valuation, Finance, and Investments Committee "Accounting Rule Guidance Statement of Financial Accounting Standards No. 113--Considerations in Risk Transfer Testing", *Casualty Actuarial Society Forum*, Volume: Fall, 2002, pp. 305-338.

[2] Kreps, Rodney E., "Parameter Uncertainty in (Log)Normal Distributions", PCAS LXXXIV, 1997, p. 553-580, http://www.casact.org/pubs/proceed/proceed97/97553.pdf

[3] Van Kampen, Charles E., "Estimating the Parameter Risk of a Loss Ratio Distribution", Casualty Actuarial Society Forum, Volume: Spring, 2003, p. 177-213, http://www.casact.org/pubs/forum/03spforum/03spf177.pdf

[4] Meyers, Glenn G., "The Cost of Financing Insurance", *Casualty Actuarial Society Forum*, Volume: Spring, 2001, pp. 221-264, http://www.casact.org/pubs/forum/01spforum/01spf221.pdf.

[5] Mango, Donald F., "Capital Consumption: An Alternative Methodology for Pricing Reinsurance", *Casualty Actuarial Society Forum*, Volume: Winter, 2003, pp. 351-379, http://www.casact.org/pubs/forum/03wforum/03wf351.pdf

[6] Ruhm, David L., "Risk Coverage Ratio: A Leverage-Independent Method of Pricing based on Distribution of Return," ASTIN Colloquium, July 2001, http://www.actuaries.org/ASTIN/Colloquia/Washington/Ruhm.pdf

[7] Wang, Shaun S., "Premium Calculation by Transforming the Layer Premium Density", *ASTIN Bulletin*, Volume: 26:1 (1996), pp. 71-92, http://www.casact.org/library/astin/vol26no1/71.pdf

[8] Wang, Shaun S., "A Universal Framework for Pricing Financial and Insurance Risks", *ASTIN Bulletin*, Volume: 32:2 (2002), pp. 213-234, http://www.casact.org/library/astin/vol32no2/213.pdf

[9] Panjer, Harry H., et al, Financial Economics, Schaumburg, IL, The Actuarial Foundation, 1998.

[10] Hull, John, *Options, Futures and Other Derivatives*, Third (International) Edition, London, Prentice Hall International, Inc., 1997.

[11] Baxter, Martin and Andrew Rennie, *Financial Calculus*,, London, Cambridge University Press, 1996.

[12] Peters, Edgar E., *Chaos and Order in the Capital Markets*, New York, John Wiley & Sons, Inc., 1991

Abbreviations and notations

10-10, 10% chance of 10% loss benchmark
CAS, Casualty Actuarial Society
COPLFR, Committee on Property and Liability Financial Reporting
E(x), expected value of *x*
E(x)*, expected value of transformed *x*
ERD, expected reinsurer deficit
EUD, expected underwriting deficit
F(x), aggregate distribution function
F(x)*, transformed aggregate distribution function
FAS 113, Financial Accounting Standard No. 113
Freq, probability of present value loss
Freq(UL), probability of underwriting loss
FV, future value operator
N(z), standard normal distribution function
N^{-1}(prob), standard normal inverse dist function
PV, present value operator
RTD, right tail deviation
S&P 500, Standard & Poor's 500 stock index
Sev, severity of present value loss
Sev(UL), severity of underwriting loss
SSAP, Statement of Statutory Accounting Principles
TVaR, tail value at risk
$TVaR_\alpha$, tail value at risk at α probability level
VaR, value at risk
VaR_α,, value at risk at α probability level

Biographies of Working Party Members

Michael Wacek (Working Party Chair) is President of Odyssey America Reinsurance Corporation based in Stamford, CT. Over the course of 20 years in the industry, including nine years in the London Market, Mike has seen the business from the vantage point of a primary insurer, reinsurance broker and reinsurer. He has a BA from Macalester College (Math, Economics), is a Fellow of the Casualty Actuarial Society and a Member of the American Academy of Actuaries. He has authored several papers.

John Aquino is Executive Vice President of Benfield, Inc. in Chicago. He has over twenty years of experience in the reinsurance industry and actuarial consulting. John has also been a frequent speaker at industry meetings. He has served as President of the Midwest Actuarial Forum and also on various committees of the CAS. He holds a bachelor's degree in Math and a MBA in Finance from the University of Chicago and is a Fellow member of the CAS and a Member of the American Academy of Actuaries.

Todd Bault is Senior Research Analyst covering non-life insurance equities for Sanford C. Bernstein & Co., LLC, a New York-based research firm. In 2004, Todd was ranked the #1 non-life insurance analyst by Institutional Investor's annual poll. Risk measurement and quantification is a favorite research topic. Todd is a Fellow of the CAS and a Member of the American Academy of Actuaries.

Paul Brehm is Senior Vice President and Instrat Manager for Guy Carpenter. Paul spent 22 years at St Paul Travelers, most recently as the Chief Actuary. He holds a BS degree in Economics from the University of Minnesota. He is a Fellow of the CAS and a Member of the American Academy of Actuaries. Paul is a former chair of the CAS Valuation, Finance, and Investment Committee and has authored several papers.

Elizabeth Hansen is a Managing Director at Guy Carpenter, based in Minneapolis, MN. She is responsible for all quantitative resources in the Mid-America region as a regional manager of Instrat. Elizabeth holds a bachelor's degree in Mathematics from Luther College in Decorah, Iowa. She is a Fellow of the Casualty Actuarial Society, a member of the American Academy of Actuaries and a frequent presenter at industry conferences.

Pierre Laurin is Senior Vice President and Director of Reinsurance with Zurich in North America. He is responsible for the corporate reinsurance treaties and has underwriting oversight over all business unit related treaties. He has a degree in actuarial science from the University of Laval in addition to a Master of Science from Western University. He is a Fellow of the CAS, of the CIA and a Member of the American Academy of Actuaries. He has participated on the CAS examination committee, and is a frequent presenter at industry symposia on reinsurance topics.

Mark Littmann is a principal with PricewaterhouseCoopers LLP, based in the firm's Hartford, CT, office. His practice areas have included reserving, financial modeling, valuations, benchmarking claims and actuarial practices, financial reporting, and actuarial software systems. He has been the actuarial group's thought-leader for evaluating the internal control implications for actuaries arising from Section 404 of the Sarbanes-Oxley Act. He co-authored the firm's paper on the practical implications of implementing fair value accounting for property/casualty loss reserves. He earned a BA degree in Mathematics and Economics from Valparaiso University. He is a Fellow of the CAS and a Member of the American Academy of Actuaries.

Karen Pachyn is Senior Vice President and Chief Pricing Actuary for the North American Broker segment of GE Insurance Solutions. In this role, she leads a team of actuaries pricing treaty reinsurance in the U.S. and Canada. She is an FCAS, MAAA and CPCU, and is currently involved in the CAS Committee on Special Interest Seminars. She has previously participated on a number of other CAS Committees and is a past President of the Midwestern Actuarial Forum. She has a BA from Illinois Wesleyan University.

Deborah Rosenberg is the Deputy Chief Casualty Actuary for the New York State Insurance Department. She is a Fellow of the CAS and a Member of the American Academy of Actuaries. Deborah is the current Vice President of Administration for the CAS and also a member of the Task Force on Publications and the Task Force on Reserving Principles.

David Ruhm is Assistant Vice President at Hartford Investment Management Company in Hartford, CT. His areas of responsibility include portfolio risk management, financial modeling and enterprise risk management. He has a bachelor's degree in Mathematics from the University of California, San Diego and is a Fellow of the CAS. David has published several papers on risk theory and capital management. He participates on the CAS Theory of Risk Committee, and is a frequent presenter at industry conferences.

Mark van Zanden is a structured risk underwriter with Catlin Insurance Company Ltd. in Bermuda. He is responsible for originating, structuring, analyzing and underwriting highly tailored (re)insurance transactions. He has a degree in Mathematics and Statistics from the University of Western Ontario in London, Canada. He is a Fellow of the CAS and of the Canadian Institute of Actuaries. He is a CFA charterholder. He has over ten years of experience designing and analyzing alternative risk transfer (re)insurance transactions.