Parameter Estimation for Bornhuetter/Ferguson

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Abstract: The Bornhuetter/Ferguson loss reserving method consists of selecting a development pattern and, for each accident year, an initial ultimate loss ratio. From these, the reserve estimate is derived. In this paper, the usual way to obtain the development pattern from the chain ladder link ratios is criticized because it assumes a multiplicative connection between past and future loss amounts whereas the Bornhuetter/Ferguson method establishes an additive connection (i.e. an independence). Therefore, an alternative approach to derive and select a development pattern is proposed.

Furthermore, the raw data usually contain some implicit information about the underwriting cycle. This paper shows how this information can be extracted from the data and used in the selection of the initial ultimate loss ratios.

Altogether the proposed approach is believed to align with the concepts of Bornhuetter and Ferguson better than the conventional approach does. The result is a standalone reserving method which does not rely upon the use of chain ladder elements.

Keywords. Loss reserving, Bornhuetter/Ferguson, Development pattern, Initial ultimate loss ratio

1. Introduction

Let $C_{i,k}$ denote the cumulative loss amount (either paid or incurred) of accident year i after k years of development, $1 \le i, k \le n$, and v_i be the premium volume of accident year i. Then $C_{i,n+1-i}$ denotes the current loss amount of accident year i. Let further $S_{i,k} = C_{i,k} - C_{i,k-1}$ denote the incremental loss amount (with $C_{i,0} = 0$) and U_i the (unknown) ultimate loss amount of accident year i. Then $R_i = U_i - C_{i,n+1-i}$ is the (unknown true) loss reserve for accident year i. For an easier exposition of the ideas, we assume in the beginning that n is large enough such that there is no significant loss development beyond development year n. We will eliminate this assumption at the end of section 3.

Bornhuetter/Ferguson (BF) introduced their method to estimate R_i in 1972 in order to cope with a major weakness of the chain ladder (CL) method. Therefore, we will first examine this weakness: The CL uses link ratios $\hat{f}_k$ in order to project the current loss amount $C_{i,n+1-i}$ to ultimate, i.e. it estimates $\hat{U}_i^{CL} = C_{i,n+1-i}\hat{f}_{n+2-i} \cdot \ldots \cdot \hat{f}_{n-1}\hat{f}_n$. Therefore, the CL reserve is

$$\hat{R}_i^{CL} = \hat{U}_i^{CL} - C_{i,n+1-i} = C_{i,n+1-i}\left(\hat{f}_{n+2-i} \cdot \ldots \cdot \hat{f}_n - 1\right).$$

This means that the reserve is heavily dependent upon the current loss amount $C_{i,n+1-i}$. This can lead to a nonsensical reserve $\hat{R}_i^{CL}$ = 0 for accident years where currently no claims are paid or reported which is not unusual in excess-of-loss reinsurance for the most recent accident year(s).

The BF method avoids this dependency upon the current loss amount $C_{i,n+1-i}$. The indicated BF reserve is defined as

$$\hat{R}_i^{BF} := \left(1-\hat{b}_{n+1-i}\right)\hat{U}_i$$

where

> $\hat{U}_i = v_i\hat{q}_i$ with an *a priori* estimate $\hat{q}_i$ of the ultimate loss ratio (ULR) $q_i := U_i/v_i$ for accident year i,
>
> $b_k \in [0, 1]$ is the percentage of ultimate losses expected to be known after development year k.

Note that $\hat{q}_i$ is called the *a priori* (or *initial*) estimate of the ULR, in contrast to the posterior estimate $(C_{i,n+1-i} + \hat{R}_i^{BF})/v_i$ of the ULR. This a priori estimate is different from the posterior estimate if and only if $C_{i,n+1-i} \neq \hat{b}_{n+1-i}v_i\hat{q}_i$. The percentages $(b_1, b_2, ..., b_n)$ constitute the expected cumulative development pattern (with $b_n = 1$ due to our preliminary assumption regarding n) and $1-\hat{b}_{n+1-i}$ is therefore the expected outstanding loss percentage of accident year i.

Thus, in order to apply the BF method, the actuary has to estimate the parameters q_i and b_k for all i and k. In practice, the b_k are derived from the CL link ratios in the following way:

$$b_n = 1,\ \hat{b}_{n-1} = \hat{f}_n^{-1},\ \hat{b}_{n-2} = \left(\hat{f}_{n-1}\hat{f}_n\right)^{-1}, \ldots,\ \hat{b}_1 = \left(\hat{f}_2 \cdot \ldots \cdot \hat{f}_n\right)^{-1}.$$

The method itself does not provide an objective approach for the determination of the a priori estimate $\hat{q}_i$. In practice, the q_i are estimated in a variety of ways, often based upon last year's estimate and/or pricing and market information. At worst, this practice can make the estimate $\hat{q}_i$ appear manipulated in order to achieve a reserve of a desired size. At best, the use of the CL pattern makes it difficult to view the BF method as a standalone reserving method.

Moreover, the use of the CL link ratios assumes that the unknown losses are a direct multiple of the already known losses at each point of the development. This contradicts the basic idea of the independence between $C_{i,n+1-i}$ and $\hat{R}_i^{BF}$ which was fundamental to the origin of the BF method.

Therefore, this paper develops an alternative approach to estimating the BF parameters q_i and b_k without the use of CL concepts along with rather clear guidance on how to arrive at an a priori estimate for the ultimate loss ratio q_i. Through this approach, the BF method becomes a true alternative to the CL method.

2. Estimation of the Development Pattern

If we already have an a priori estimate for U_i (e.g. from the traditional approach as outlined above), we are able to estimate the appropriate development pattern. From the BF reserve formula $\hat{R}_i^{BF} = \left(1-\hat{b}_{n+1-i}\right)\hat{U}_i$ we deduce

$$\hat{b}_{n+1-i} = 1-\frac{\hat{R}_i}{\hat{U}_i} = \frac{\hat{U}_i-\hat{R}_i}{\hat{U}_i} \approx \frac{C_{i,n+1-i}}{\hat{U}_i}.$$

As previously stated, the ≈-sign is a strict equality only if the a priori estimate $\hat{U}_i$ equals the posterior $C_{i,n+1-i} + \hat{R}_i$, i.e. if $C_{i,n+1-i} = \hat{b}_{n+1-i}\hat{U}_i$. This will not be the case for every i but should be true on average, at least approximately, otherwise the pattern $\hat{b}_1, \hat{b}_2, \ldots$ would not fit to the data. Therefore, the previous approximate equation suggests the estimator

$$\hat{b}_k := \sum_{i=1}^{n+1-k} C_{ik} \Big/ \sum_{i=1}^{n+1-k} \hat{U}_i$$

as weighted average of the ratios $C_{i,k}/\hat{U}_i$. This direct way of estimating the cumulative pattern b_1, b_2, ... may lead to inversions, i.e. $\hat{b}_k > \hat{b}_{k+1}$, because each $\hat{b}_k$ is based on a different number of accident years. In order to avoid such inversions, we use the corresponding increments

$$\hat{\beta}_k := \sum_{i=1}^{n+1-k} S_{ik} \Big/ \sum_{i=1}^{n+1-k} \hat{U}_i$$

and obtain $\hat{b}_k$ by adding up the $\hat{\beta}_k$, i.e. take

$$\hat{b}_k = \hat{\beta}_1 + \ldots + \hat{\beta}_k$$

and supplement it with $\hat{b}_{n+1} = 1$.

This is the development pattern as suggested by the BF reserve formula itself. This pattern is different from the CL pattern as can be seen e.g. from the numerical example below. Of course, the $\hat{\beta}_k$ should be smoothed and decreasing towards 0. This can be achieved by smoothing selections much as one would do when selecting CL link ratios. We will apply such a procedure together with the estimation of the ultimate loss ratio in the next section. But the actuary who wants to stay with the traditional BF way to arrive at an estimate for U_i can stop reading here and just use the specific BF pattern derived above.

3. Estimation of the Initial Ultimate Loss Ratios

As said in the introduction, the BF method aims at developing an estimate for q_i which does not directly depend on the losses $C_{i,n+1-i}$ known to-date and can be similarly obtained by another actuary. The procedure proposed here employs a three-steps approach. The first step considers the average incremental loss ratio (ILR)

$$\hat{m}_k := \sum_{i=1}^{n+1-k} S_{ik} \Big/ \sum_{i=1}^{n+1-k} v_i$$

of development year k observed to-date. The sum $\hat{m}_1 + \ldots + \hat{m}_n$ of all average ILRs is an a priori estimate of the ultimate loss ratio of an average accident year (if the development is assumed to be finished after n years). Note that in determining this a priori estimate, the known loss experience $C_{i,n+1-i}$ of any fixed accident year i is taken into account only marginally (as opposed to the CL estimate for U_i).

In the second step, we leverage the fact that the ultimate loss ratio q_i of accident year i is highly influenced by the level of the rate adequacy of that particular year. The rate adequacy is determined by two factors: the rate level and the loss level, which together yield the level of the loss ratio. But whereas in rate making we have to determine a sufficient absolute rate level – sufficient to pay all costs of the business -, for reserving purposes it is sufficient to judge the relative level of rate adequacy of an accident year as compared to the other accident years. With this information we can translate the (almost) known loss ratio of the oldest accident year(s) into predictions for the more recent accident years. Thus, we have to estimate the rate level change and the loss cost trend only. This is much easier because, at the time of reserving, we know the degree to which any rate changes have been realized and we know already some part of the losses of each accident year. This information should therefore be used for the assessment of the rate adequacy in addition to the information from the time of rate making.

Thus, we analyze what the run-off data tell us about the rate adequacy. If an accident year i has a below average rate adequacy (as compared to the other accident years considered), then the premium volume v_i is smaller than it should be for an average accident year. Therefore, most of its observed individual incremental loss ratios

$$\frac{S_{i,1}}{v_i}, \frac{S_{i,2}}{v_i}, \ldots, \frac{S_{i,n+1-i}}{v_i}$$

will be higher than the corresponding averages

$$\hat{m}_1, \hat{m}_2, \ldots, \hat{m}_{n+1-i},$$

at least after we have eliminated any unusually large individual losses as is normally done with any loss reserving method. In order to arrive at a single figure indicating the emerged relative rate adequacy level of accident year i (as compared to the average level of all accident years considered) we use the weighted average

$$r_i := \sum_{k=1}^{n+1-i} \frac{\hat{m}_k}{\sum \hat{m}_j} \cdot \frac{S_{ik}/v_i}{\hat{m}_k} = \sum_{k=1}^{n+1-i} S_{ik} \Big/ \sum_{k=1}^{n+1-i} (v_i \hat{m}_k) = \frac{C_{i,n+1-i}/v_i}{\sum_{k=1}^{n+1-i} \hat{m}_k}$$

of the ratios of $S_{i,k}/v_i$ and $\hat{m}_k$. Thus, r_i is the ratio of the current individual loss ratio $C_{i,n+1-i}/v_i$ of accident year i divided by the corresponding a priori average loss ratio. Therefore, r_i can be called a *loss ratio index*.

As seen from the premium perspective, r_i indicates the factor by which the premium v_i has to be multiplied in order to adjust it to the average rate adequacy level of the accident years $i = 1, \ldots, n$ considered. From this perspective, r_i can be called an *on-level premium factor*. Again, the factor r_i does not necessarily bring the premium v_i to the sufficient absolute size; it only achieves that – in relation to $v_i r_i$ instead of v_i - all accident years have approximately the same ultimate loss ratio $U_i/(v_i r_i) \approx \hat{m}_1 + \ldots + \hat{m}_n$, may the latter be profitable or not. At this stage we can already state that, if the r_i's and the $\hat{m}_k$'s are plausible, then

$$(\hat{m}_1 + \ldots + \hat{m}_n) r_i$$

is a reasonable a priori estimate of the ultimate loss ratio $q_i = U_i/v_i$ (if the development is assumed to be finished after n years).

As a third step, we have to check the plausibility of r_i. Initially we realize that the paid data and the incurred data will yield different values for r_i. But of course, these should be identical because they relate to the same premium v_i and losses U_i for either set of data. Without additional knowledge, we would therefore use the straight average $\left(r_i^{paid} + r_i^{inc}\right)/2$ or – as we deal with factors - rather the geometric mean

$$\overline{r}_i = \sqrt{r_i^{paid} \cdot r_i^{inc}} \ .$$

The calculation of the r_i's should be based on the data of a rather large portfolio in order to have the factors r_i be as reliable as possible. This large portfolio could be comprised of several run-off triangles for which the reserving is done separately, but which are assumed to have undergone similar changes in rate adequacy level.

Normally, we also have some information from pricing available, i.e. the rate changes effected and an estimate of the loss trend. The ratio r_i/r_{i-1} of any two consecutive years should be checked against the ratio of the loss trend and the effective rate change imbedded in v_i (in combination these represent the indicated change of the rate adequacy level). For instance, if from year i-1 to year i a loss increase of +10% is expected but a rate change of only +5% has been achieved, the ratio r_i/r_{i-1} should be close to 1.10/1.05 indicating a deterioration of the loss ratio by 4.8% (= 1.10/1.05 - 1). If not, we have to make a decision between these two ratios, e.g. form a credibility-weighted average of both values.

For the most recent accident years $i=n$ and $i=n$-1 we probably will trust the pricing information more than the r_i-estimate from the data, as the latter only relies on one or two entries in the triangle. At an extreme, r_i could be 0, which would be nonsensical and must obviously be adjusted. The size of r_i for the first accident year can in principle be chosen arbitrarily, because its rate adequacy level (loss ratio level) will be taken into account in a subsequent adjustment of $\hat{m}_k$, see below. Therefore it can be left as it comes out of the formula in order to keep the $\hat{m}_k$ at the intuitive incremental loss ratio level.

What really matters are the relativities r_i/r_{i-1}. Therefore, we first select the values for these relativities based on all information available and then, starting with a selection for r_1^*, derive from these the resulting selections r_i^* for each accident year i. With these selected r_i^*, all adjusted premium volume figures $v_ir_i^*$, $1 \le i \le n$, should ultimately lead to (approximately) the same rate adequacy level, i.e. yield similar values of $U_i/(v_ir_i^*)$.

At next year's reserve calculation, the data triangle will contain an additional diagonal which will result in changes to all r_i. But the ratios r_i/r_{i-1} have the same interpretation as before. Therefore, due to the arbitrariness of r_1^*, we can keep the "old" r_1^* and – as long as no changes in the ratios r_i^*/r_{i-1}^* are indicated – also keep the other r_i^* and just add a new r_{n+1}^* based on a plausible ratio r_{n+1}^*/r_n^*.

Before using r_i^* for the estimation of q_i we have to adjust the average incremental loss ratios $\hat{m}_k$ because these were based on the unadjusted premium volume figures v_i. Therefore we replace $\hat{m}_k$ with

$$\hat{\hat{m}}_k := \sum_{i=1}^{n+1-k} S_{ik} \Big/ \sum_{i=1}^{n+1-k} (v_i r_i^*).$$

Often this will result in minor changes only. Major changes may happen for the last two or three development years or generally with data where the sizes of v_i or r_i^* vary significantly.

The adjusted ILRs $\hat{\hat{m}}_k$ of the last few development years could still produce unintuitive results, again due to the limited number of data points. Of course, these incremental values should be smooth and decreasing towards 0. Therefore, a smoothing approach is reasonable, and we denote the ILRs finally selected with $\hat{m}_k^*$.

At this point we will abandon the unrealistic assumption of not having any development beyond development year n. This is simply achieved by selecting an average tail ratio $\hat{m}_{n+1}^*$ (which may be 0 or even negative, like any other $\hat{m}_k^*$), to supplement the ILRs $\hat{m}_k^*$, $1 \le k \le n$, already selected.

Using these selected ILRs, we now have

$$\hat{m}^* := \hat{m}_1^* + ... + \hat{m}_n^* + \hat{m}_{n+1}^*$$

as an adjusted estimate for the ULR at average rate adequacy level. Of course, the paid data should have the same estimated ULR $\hat{m}^*$ as the incurred data. If that is not the case, we must adjust some $\hat{m}_k^*$, especially $\hat{m}_{n+1}^*$, to achieve the equality $\hat{m}_{paid}^* = \hat{m}_{inc}^*$. This finally yields the a priori estimate $\hat{q}_i := r_i^* \hat{m}^*$ for the ULR of accident year i and the corresponding amount $\hat{U}_i := v_i r_i^* \hat{m}^*$.

In contrast to the traditional BF procedure, this procedure gives the actuary the possibility to consolidate the general pricing and market information available with the trends and relativities contained in the paid and incurred data triangle. Moreover, this procedure uses a detailed decomposition of the initial ultimate loss ratio $\hat{q}_i = r_i^* (\hat{m}_1^* + ... + \hat{m}_{n+1}^*)$ into its components rate adequacy and development pattern. This makes the procedure easier to be followed or peer-reviewed by any other actuary.

4. Estimation of the Development Pattern (continued)

Now, we insert the result $\hat{U}_i = v_i r_i^* \hat{m}^*$ of the previous section into the formula derived for $\hat{\beta}_k$ in section 2 and obtain

$$\hat{\beta}_k = \frac{\sum_{i=1}^{n+1-k} S_{ik}}{\sum_{i=1}^{n+1-k} \hat{U}_i} = \frac{\sum_{i=1}^{n+1-k} S_{ik}}{\sum_{i=1}^{n+1-k} v_i r_i^* \hat{m}^*} = \frac{\hat{\hat{m}}_k}{\hat{m}^*}.$$

Here we see that the numerator $\hat{\hat{m}}_k$ may differ from the finally selected $\hat{m}_k^*$, as the denominator reflects the selected ILRs. Therefore it is logical to select

$$\hat{\beta}_k^* := \frac{\hat{m}_k^*}{\hat{m}^*}.$$

This finally leads to

$$\hat{b}_k^* := \hat{\beta}_1^* + \ldots + \hat{\beta}_k^* = \frac{\hat{m}_1^* + \ldots + \hat{m}_k^*}{\hat{m}_1^* + \ldots + \hat{m}_{n+1}^*}.$$

This is the genuine BF development pattern which is different from the CL pattern (see the numerical example below).

5. Putting it all Together

Altogether, we have the following steps of calculation:

$\hat{m}_k = \sum_{i=1}^{n+1-k} S_{ik} \Big/ \sum_{i=1}^{n+1-k} v_i$ raw incremental loss ratio (ILR) at development year k

$r_i = \sum_{k=1}^{n+1-i} S_{ik} \Big/ \sum_{k=1}^{n+1-i} \left(v_i \hat{m}_k\right)$ raw on-level premium factor for accident year i

r_i^* = selected on-level premium factor for accident year i (same for paid and incurred)

$\hat{m}_k^*$ = selected average ILR at development year k

(smoothed version of $\hat{\hat{m}}_k = \sum_{i=1}^{n+1-k} S_{ik} \Big/ \sum_{i=1}^{n+1-k} \left(v_i r_i^*\right)$)

$\hat{q}_i = r_i^* \left(\hat{m}_1^* + \ldots + \hat{m}_n^* + \hat{m}_{n+1}^*\right)$ *a priori* ULR for accident year i, including tail ratio $\hat{m}_{n+1}^*$

$\hat{U}_i \quad = v_i\hat{q}_i = v_i r_i^* \left(\hat{m}_1^* + ... + \hat{m}_{n+1}^*\right)$ *a priori* estimate of ultimate losses for accident year i

$\hat{b}_k^* \quad = \dfrac{\hat{m}_1^* + ... + \hat{m}_k^*}{\hat{m}_1^* + ... + \hat{m}_{n+1}^*}$ avg. cumulative percentage paid (incurred) at development year k

$\hat{R}_i \quad = \left(1 - \hat{b}_{n+1-i}^*\right)\hat{U}_i = v_i r_i^* \left(\hat{m}_{n+2-i}^* + ... + \hat{m}_{n+1}^*\right)$ loss reserve for accident year i

With this way of estimating its parameters q_i and b_k, the BF method is truly a standalone reserving method which is completely independent of the CL method. As shown in section 2, this way of calculating the pattern b_1, b_2, ... can also be used if the a priori estimates $\hat{q}_i$ and $\hat{U}_i = v_i\hat{q}_i$ are arrived at in a different (e.g. traditional) way. Thus, even if one does not like to work with m_k and r_i, one should at least adopt the estimation of the pattern as outlined above and avoid using the CL pattern.

6. Numerical Example

Data from General Liability Excess business are used to demonstrate the method. Exhibit A contains the premiums v_i and the incremental amounts $S_{i,k}$ of the incurred and the paid losses for the accident years 1992 – 2004 and development years 1 to 13. Some negative amounts have been kept in order to demonstrate that this does not lead to distortions. Exhibits B and C show the detailed results of the calculations for the incurred and the paid data respectively. These two exhibits are subdivided into three column blocks and two row blocks indicating the order of calculation: Columns (A) through (C) and rows (1) through (2) are the given data in aggregated form. From these the various components are calculated in the following order:

Rows (3) through (4),

Columns (D) through (G),

Rows (5) through (9),

Columns (H) through (M).

In the headings of column (H) and row (9), (8#) stands for the last number in row (8), i.e. $\hat{m}^*$. The suffix $_{+k}$ in rows (2), (3) and (5) stands for summation over i, i.e. $\sum_{i=1}^{n+1-k}$, The term "post." in columns (L) and (M) stands for "posterior". The bold headings **r_i***, **m_k*** and **Tail-ILR** indicate those positions where selections were required. These selections have been made in the following way:

Before selecting $\mathbf{r}_i^*$ we looked at Exhibit D where the raw r_i from column (E) are plotted for both paid and incurred data. The graph shows that the two sets of data are reasonably consistent, except for accident year 2004. Therefore, for i = 1992, …, 2002, we selected $\mathbf{r}_i^*$ as the geometric mean between the paid r_i and the incurred r_i. For i = 2003 and 2004, we have set $\mathbf{r}_i^* = 0.50$ for both, incurred and paid. The latter choice is not based on any further information. It is just an example. As mentioned earlier, information from pricing should also be used when making the selection. But even without this, the resulting $\mathbf{r}_i^*$ seem to give a realistic picture of the rather extreme rate adequacy level changes over the years considered. These $\mathbf{r}_i^*$ correspond to the following adequacy changes:

i-1→i	92→93	93→94	94→95	95→96	96→97	97→98	98→99	99→00	00→01	01→02	02→03	03→04
$\mathbf{r}_i^*/\mathbf{r}_{i-1}^*$	0.89	0.95	0.94	1.52	1.49	1.26	1.54	0.72	0.66	0.79	0.67	1.00

If we interpret r_i a loss ratio index, the above figures imply that we assume a decrease of the loss ratio index r_i from 1992 to 1993 of 11% (= 0.89 - 1) and an increase of 52% from 1995 to 1996.

$\mathbf{m}_k^*$ has been taken from row (6) ($\tilde{m}_k$) for development years k = 1, …, 7. All the other $\mathbf{m}_k^*$ have been selected in order to make the development smoothly decreasing. Of course, other selections would have been possible. The **Tail-ILR** for incurred has been selected to be 0 and the **Tail-ILR** for paid has been selected such that the sum $\hat{m}^*$ of all paid ILRs equals that of the incurred-ILRs which is 137.9%. Note that the traditional way to apply BF will yield exactly the same reserve R_i as obtained in column (K) if we use $1.379 \cdot \mathbf{r}_i^*$ as initial loss ratio and the pattern from row (9).

Finally, Exhibit E shows a comparison between the raw development pattern as proposed here and the pattern derived from the raw CL factors. More precisely, the BF pattern is a plot of $\hat{b}_k^{raw} = \dfrac{\hat{m}_1 + \ldots + \hat{m}_k}{\hat{m}_1 + \ldots + \hat{m}_n}$ using the raw ILR's m_k of row (4), whereas the CL pattern is a plot of $\hat{b}_k^{CL} = (\hat{f}_{k+1} \cdot \ldots \cdot \hat{f}_n)^{-1}$ with $\hat{f}_k = \sum_{i=1}^{n-k} C_{i,k+1} \Big/ \sum_{i=1}^{n-k} C_{i,k}$. We see that the raw BF pattern is clearly different from the raw CL pattern for either data set.

7. Final Remarks

As with any reserving method, this approach to estimating the parameters (i.e. the reserve) relies on implicit assumptions. One main assumption has already been addressed in the beginning: the data observed to-date and the amounts still outstanding are independent. This assumption is a cornerstone of the BF method. As the assumption should hold at any point in time, it essentially means that all incremental amounts $S_{i,1}$, ..., $S_{i,n}$ of each accident year are assumed to be independent. This would be violated if claim payments or bookings of case reserves were not done in the same way each year, especially if high payments in one calendar year would be followed by rather delayed payments in the following year(s). Similarly, the independence of the accident years is implicitly assumed in the estimation of m_k. This independence assumption is normally less problematic but could also be violated by calendar year effects. A more critical assumption is that the development pattern is consistent across all accident years. Of course, this assumption is not unique to this approach, as it is also implicit in the traditional BF method, as well as in the CL. This assumption should be especially borne in mind when selecting the accident years upon which the parameter estimates are to be based.

The way in which the parameters r_i and m_k are estimated consists of starting with an estimate for m_k which then is used to estimate r_i. The latter is adjusted and then used to arrive at an improved estimate for m_k. Thus, it may be tempting to again use this improved estimate of m_k to improve the estimate for r_i. But one must be cautious here. External judgment has already been applied in developing these parameters, and therefore any further changes based on the run-off data would only serve to dilute the (presumably desired) impacts of those judgments. Similarly, a purist might be tempted to iterate the estimations without any adjustments in between, i.e. to start with $\hat{m}_k$ and r_i as given in section 4, and with $\hat{\hat{m}}_k$ as in section 3, but then to use the latter for calculating $\tilde{r}_i = \sum_{k=1}^{n+1-i} S_{ik} \Big/ \sum_{k=1}^{n+1-i} \left(v_i \hat{\hat{m}}_k\right)$. This would then be iterated by calculating new estimates, first for m_k then for r_i by using the corresponding estimates obtained immediately before. Indeed, this procedure will quickly converge upon and yield exactly the same reserves as the CL does (for a full triangle only). This is not surprising, since proceeding in this way implies that we fully believe all the information contained in the data, without any input of external information. Thus we see that the input of external information is vital for the BF method.

For the CL, a methodology of assessing the variability of the reserves has been established in recent years. See e.g. the papers by Murphy or Mack in the 1994 CAS Spring Forum. Therefore, one would like to have this for BF, as well. For this purpose, we refer to the fact that our way of modeling the BF method can be seen as a cross-classified model, as in automobile rating, based upon the assumption $E(S_{i,k}/v_i) = r_i m_k$. Thus it can be treated using Generalized Linear Models.

However, this would use the "wrong" volume v_i instead of $v_i r_i$. Moreover, an appropriate assumption for the variance is necessary, too. Therefore, it may seem easier to use the alternative approach of embedding this BF model into the classical credibility IBNR model (see the author's paper "Improved Estimation of IBNR Claims by Credibility Theory" in the journal *Insurance: Mathematics & Economics* of 1990). In this way, the rate level r_i would be treated as a random variable. In any case, the issue of reserve variability deserves a separate paper.

8. References

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Abbreviations

BF: Bornhuetter/Ferguson

CL: chain ladder

ILR: incremental loss ratio

ULR: ultimate loss ratio

About the author

The author is retired (since 2006) chief actuary non-life of Munich Re. He studied Mathematics at the universities of Munich and Mannheim (Germany) and then worked more than 30 years for Munich Re. He is a member of the German Actuarial Association and a honorary member of the Institute of Actuaries (UK) and of the SAV (Switzerland). He has written several papers, mainly published in the ASTIN Bulletin. He has been awarded the 2nd-3rd prize at the CAS prize paper competition on variability of loss reserves in 1992 and the Charles A. Hachemeister prize in 1994.

Exhibit A

Incremental Incurred Loss Amounts

Acc.Year	Premium	Dev.Yr. 1	2	3	4	5	6	7	8	9	10	11	12	13
1992	41020	7362	3981	4881	5080	3806	2523	792	731	-1	241	-347	3	-115
1993	57547	5400	7208	7252	4946	4394	3198	3039	-771	988	-495	-182	1251	
1994	60940	2215	12914	6494	5585	2211	3363	2126	445	421	118	849		
1995	63034	1109	6581	5833	4827	5672	8638	12	146	4054	-625			
1996	61256	6220	10065	10343	11259	9032	1207	26	4221	378				
1997	57231	1324	6579	16428	17453	20457	3209	7103	101					
1998	91137	5772	12714	22918	33920	20709	33941	28483						
1999	96925	8563	47206	59695	60043	50458	5129							
2000	167021	11771	48696	84750	77361	39404								
2001	148494	11259	27000	38648	51890									
2002	165410	11855	27183	25927										
2003	228239	6236	18214											
2004	226454	7818												

Incremental Paid Loss Amounts

Acc.Year	Premium	Dev.Yr. 1	2	3	4	5	6	7	8	9	10	11	12	13
1992	41020	234	4643	6249	3530	6539	2737	2546	1815	335	110	18	26	-1
1993	57547	1994	4936	4825	6180	7659	1951	5110	611	776	409	48	1327	
1994	60940	-75	3208	7853	7127	5360	3876	3426	1440	1283	67	1616		
1995	63034	236	2202	4125	5003	4189	9064	2202	2064	3244	1179			
1996	61256	976	4719	9397	13253	6106	4975	3049	4719	2715				
1997	57231	-730	3353	12904	10642	16491	8886	7228	8512					
1998	91137	539	5238	14901	24865	20274	17769	32934						
1999	96925	725	14900	34676	43595	52621	27480							
2000	167021	312	6442	43596	88702	38812								
2001	148494	2988	9921	20357	34585									
2002	165410	260	7181	22202										
2003	228239	994	3049											
2004	226454	2411												

Exhibit B

Reserve Calculation for Incurred Data

(A) Acc. Year i	(B) v_i	(C) $C_{i,n+1-i}$	(D) Σm_k from (4)	(E) r_i (C)/(B)/(D)	(F) r_i^* selected	(G) $v_i r_i^*$ (B)*(F)	(H) q_i (F)*(8#)	(I) U_i (B)*(H)	(J) $1-b_{n+1-i}$ from (9)	(K) R_i (I)*(J)	(L) post. U_i (C)+(K)	(M) post. ULR (L)/(B)
1992	41,020	28,937	132.6%	0.53	0.57	23,421.1	78.7%	32,299.9	0.0%	0.0	28,937.0	70.5%
1993	57,547	36,228	132.9%	0.47	0.51	29,206.9	70.0%	40,279.1	0.1%	29.2	36,257.2	63.0%
1994	60,940	36,741	131.6%	0.46	0.48	29,464.7	66.7%	40,634.6	0.2%	88.4	36,829.4	60.4%
1995	63,034	36,247	131.4%	0.44	0.46	28,717.6	62.8%	39,604.3	0.6%	229.7	36,476.7	57.9%
1996	61,256	52,751	131.8%	0.65	0.69	42,376.1	95.4%	58,440.6	1.3%	762.8	53,513.8	87.4%
1997	57,231	72,654	129.7%	0.98	1.03	58,985.8	142.1%	81,346.9	2.8%	2,241.5	74,895.5	130.9%
1998	91,137	158,457	128.3%	1.36	1.30	118,381.1	179.1%	163,258.7	6.4%	10,417.5	168,874.5	185.3%
1999	96,925	231,094	118.7%	2.01	2.01	194,439.7	276.7%	268,150.6	15.5%	41,569.7	272,663.7	281.3%
2000	167,021	261,982	107.1%	1.46	1.44	240,660.2	198.7%	331,893.1	24.0%	79,509.4	341,491.4	204.5%
2001	148,494	128,797	84.7%	1.02	0.94	140,323.8	130.3%	193,519.8	38.7%	74,977.1	203,774.1	137.2%
2002	165,410	64,965	52.4%	0.75	0.74	122,950.0	102.5%	169,559.7	60.5%	102,656.5	167,621.5	101.3%
2003	228,239	24,450	24.4%	0.44	0.50	114,119.5	69.0%	157,381.6	80.5%	126,690.1	151,140.1	66.2%
2004	226,454	7,818	5.9%	0.58	0.50	113,227.0	69.0%	156,150.7	95.0%	148,318.1	156,136.1	68.9%

(1) Dev.Yr. k		1	2	3	4	5	6	7	8	9	10	11	12	13	
(2) S_{+k}		86,904	228,341	283,169	272,364	156,143	61,208	41,581	4,873	5,840	-761	320	1,254	-115	
(3) v_{+k}	from (B)	1,464,708	1,238,254	1,010,015	844,605	696,111	529,090	432,165	341,028	283,797	222,541	159,507	98,567	41,020	
(4) m_k	(2)/(3)	5.9%	18.4%	28.0%	32.2%	22.4%	11.6%	9.6%	1.4%	2.1%	-0.3%	0.2%	1.3%	-0.3%	
(5) $(vr^*)_{+k}$	from (G)	1,256,273.4	1,143,046.4	1,028,926.9	905,976.9	765,653.0	524,992.9	330,553.2	212,172.2	153,186.4	110,810.3	82,092.7	52,628.0	23,421.1	
(6) $\tilde{m}_k$	(2)/(5)	6.9%	20.0%	27.5%	30.1%	20.4%	11.7%	12.6%	2.3%	3.8%	-0.7%	0.4%	2.4%	-0.5%	**Tail-ILR**
(7) m_k^*	selected	6.9%	20.0%	27.5%	30.1%	20.4%	11.7%	12.6%	5.0%	2.0%	1.0%	0.5%	0.2%	0.1%	0.0%
(8) Σ(7)		6.9%	26.9%	54.4%	84.5%	104.9%	116.5%	129.1%	134.1%	136.1%	137.1%	137.6%	137.8%	137.9%	137.9%
(9) b_k	(8)/(8#)	5.0%	19.5%	39.5%	61.3%	76.0%	84.5%	93.6%	97.2%	98.7%	99.4%	99.8%	99.9%	100.0%	100.0%

Exhibit C

Reserve Calculation for Paid Data

(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)	(J)	(K)	(L)	(M)
Acc. Year i	v_i	$C_{i,n+1-i}$	Σm_k	r_i	r_i^*	$v_i r_i^*$	q_i	U_i	$1-b_{n+1-i}$	R_i	post. U_i	post. ULR
			from (4)	(C)/(B)/(D)	selected	(B)*(F)	(F)*(8#)	(B)*(H)	from (9)	(I)*(J)	(C)+(K)	(L)/(B)
1992	41,020	28,781	114.5%	0.61	0.57	23,421.1	78.7%	32,299.9	3.5%	1,118.6	29,899.6	72.9%
1993	57,547	35,826	114.5%	0.54	0.51	29,206.9	70.0%	40,279.1	4.9%	1,979.1	37,805.1	65.7%
1994	60,940	35,181	113.1%	0.51	0.48	29,464.7	66.7%	40,634.6	6.4%	2,585.8	37,766.8	62.0%
1995	63,034	33,508	112.1%	0.47	0.46	28,717.6	62.8%	39,604.3	8.5%	3,381.8	36,889.8	58.5%
1996	61,256	49,909	111.3%	0.73	0.69	42,376.1	95.4%	58,440.6	12.2%	7,109.0	57,018.0	93.1%
1997	57,231	67,286	108.3%	1.09	1.03	58,985.8	142.1%	81,346.9	17.2%	14,024.5	81,310.5	142.1%
1998	91,137	116,520	102.7%	1.24	1.30	118,381.1	179.1%	163,258.7	25.2%	41,168.2	157,688.2	173.0%
1999	96,925	173,997	89.6%	2.00	2.01	194,439.7	276.7%	268,150.6	37.6%	100,850.1	274,847.1	283.6%
2000	167,021	177,864	75.1%	1.42	1.44	240,660.2	198.7%	331,893.1	48.2%	160,000.6	337,864.6	202.3%
2001	148,494	67,851	52.4%	0.87	0.94	140,323.8	130.3%	193,519.8	63.2%	122,259.5	190,110.5	128.0%
2002	165,410	29,643	24.3%	0.74	0.74	122,950.0	102.5%	169,559.7	82.2%	139,350.9	168,993.9	102.2%
2003	228,239	4,043	6.4%	0.28	0.50	114,119.5	69.0%	157,381.6	94.9%	149,426.8	153,469.8	67.2%
2004	226,454	2,411	0.7%	1.44	0.50	113,227.0	69.0%	156,150.7	99.4%	155,171.6	157,582.6	69.6%

(1) Dev.Yr. k		1	2	3	4	5	6	7	8	9	10	11	12	13	
(2) S_{+k}		10,864	69,792	181,085	237,482	158,051	76,738	56,495	19,161	8,353	1,765	1,682	1,353	-1	
(3) v_{+k}	from (B)	1,464,708	1,238,254	1,010,015	844,605	696,111	529,090	432,165	341,028	283,797	222,541	159,507	98,567	41,020	
(4) m_k	(2)/(3)	0.7%	5.6%	17.9%	28.1%	22.7%	14.5%	13.1%	5.6%	2.9%	0.8%	1.1%	1.4%	0.0%	
(5) $(vr^*)_{+k}$	from (G)	1,256,273.4	1,143,046.4	1,028,926.9	905,976.9	765,653.0	524,992.9	330,553.2	212,172.2	153,186.4	110,810.3	82,092.7	52,628.0	23,421.1	
(6) $\tilde{m}_k$	(2)/(5)	0.9%	6.1%	17.6%	26.2%	20.6%	14.6%	17.1%	9.0%	5.5%	1.6%	2.0%	2.6%	0.0%	**Tail-ILR**
(7) m_k^*	selected	0.9%	6.1%	17.6%	26.2%	20.6%	14.6%	17.1%	11.0%	7.0%	5.0%	3.0%	2.0%	2.0%	4.8%
(8) Σ(7)		0.9%	7.0%	24.6%	50.8%	71.4%	86.0%	103.1%	114.1%	121.1%	126.1%	129.1%	131.1%	133.1%	137.9%
(9) b_k	(8)/(8#)	0.6%	5.1%	17.8%	36.8%	51.8%	62.4%	74.8%	82.8%	87.8%	91.5%	93.6%	95.1%	96.5%	100.0%

Exhibit D

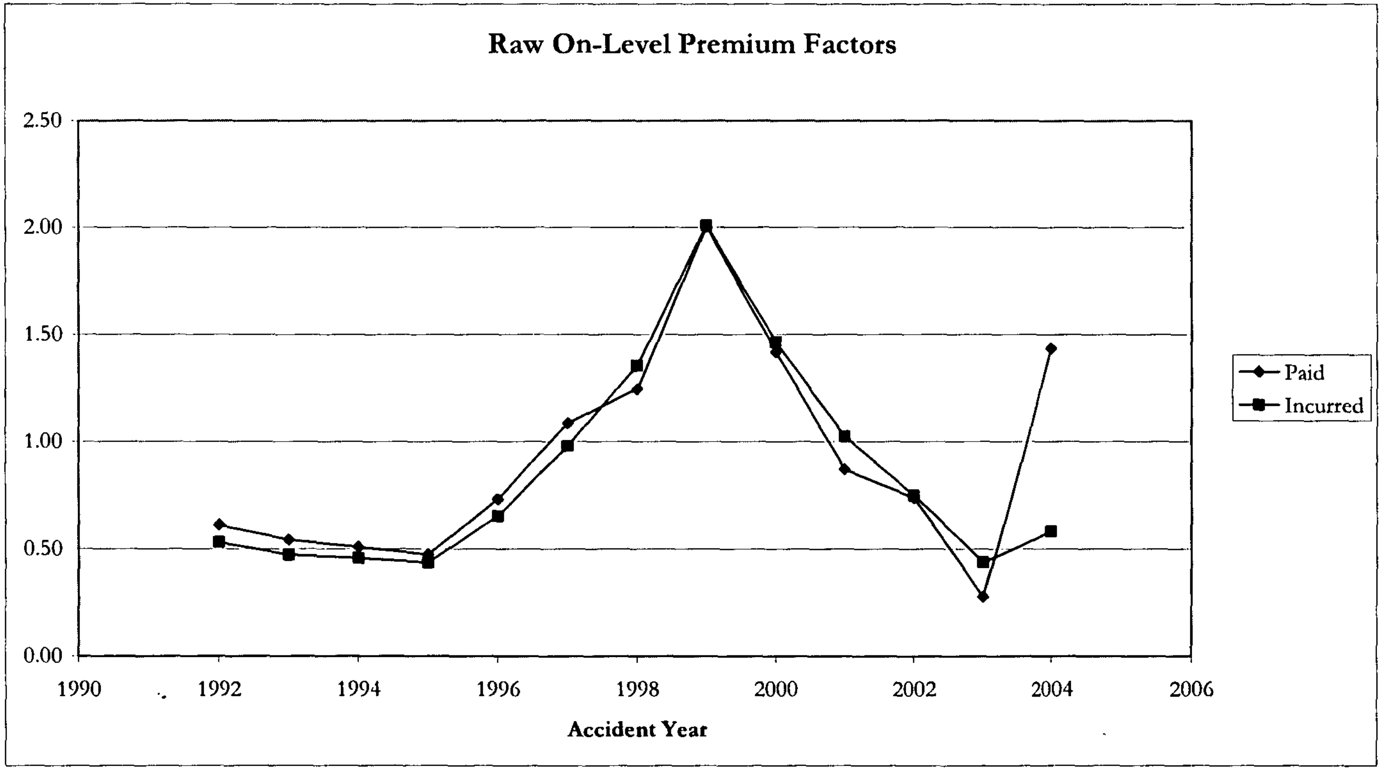
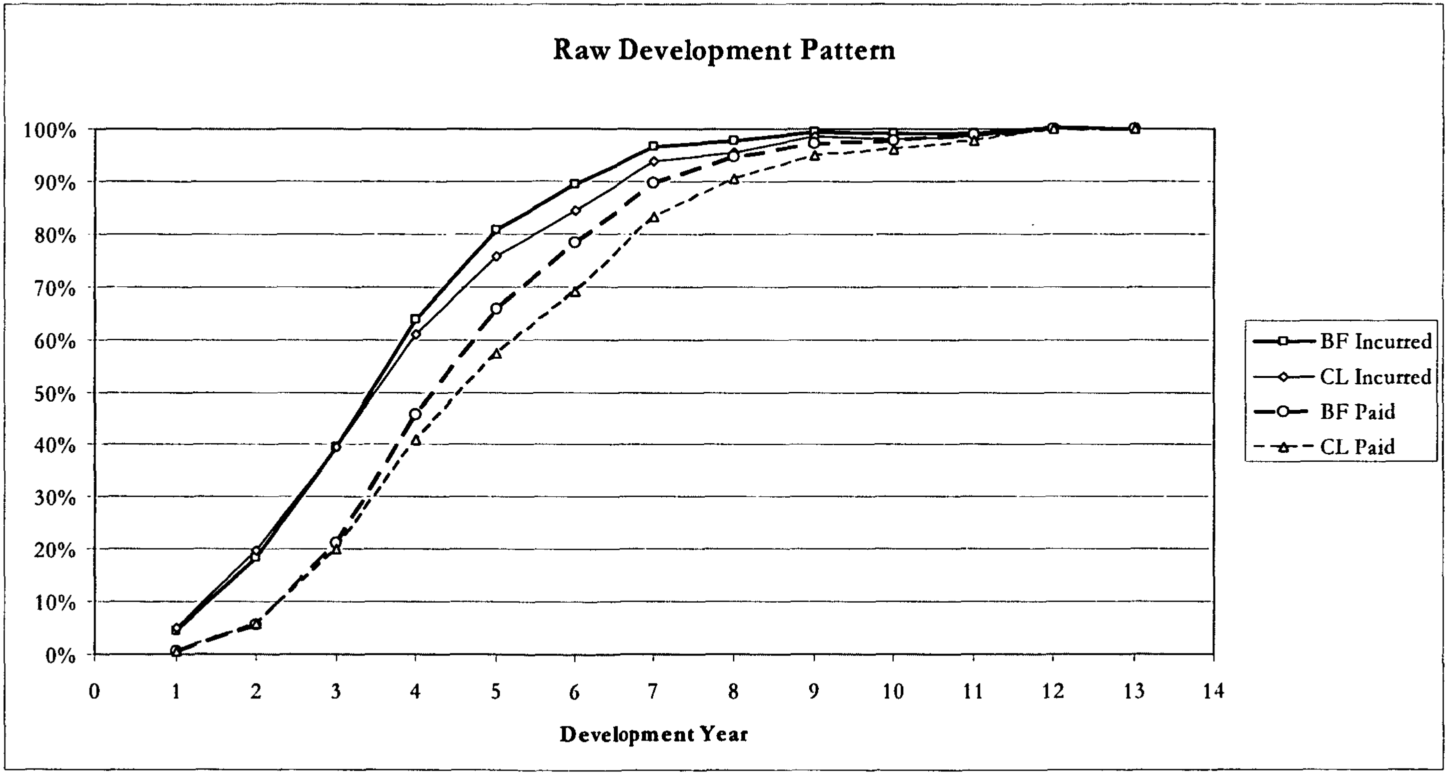

Raw On-Level Premium Factors
2.50
2.00
1.50
1.00
0.50
0.00
1990
1992
1994
1996
1998
2000
2002
2004
2006
Accident Year
Paid
Incurred

Exhibit E


Raw Development Pattern
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80%
70%
60%
50%
40%
30%
20%
10%
0%
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
Development Year
BF Incurred
CL Incurred
BF Paid
CL Paid