Pricing for Systematic Risk

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Abstract

In recent years, financial methods have emerged as the dominant approach for establishing insurance profit loadings. Financial theory suggests that prices should reflect systematic risk only, with no reward for diversifiable risk. This principle is applied to the pricing of insurance exposures actively traded in a secondary market. The resulting Systematic Risk Pricing Model differs from the Capital Asset Pricing Model in that it determines the price rather than the rate of return for each exposure. In order to reconcile the two pricing models, the amount of capital invested in a security in the Capital Asset Pricing Model is reinterpreted as the price for the exposure. Under the Systematic Risk Pricing Model, the price for the exposure is determined without regard for the insurer's cost of capital. In this method, an exposure's rate of return represents the profit margin, that is, the expected profit for an exposure in relation to its price. Due to the inconsistency of the CAPM with this result, the interpretation of CAPM rate of return as the market capitalization rate used to discount future income to present value is abandoned. An in-depth examination of the CAPM identifies a number of conceptual errors with the model, the most serious of these being that the CAPM substitutes the variability of the price of the exposure over time for the true risk of the exposure. A mathematical derivation of the CAPM from the Systematic Risk Pricing Model is presented to identify the faulty assumptions underlying the model.

Pricing for Systematic Risk

Introduction

One of the ongoing challenges in insurance risk pricing is to determine an appropriate profit margin to include in an insurer's rates. Several distinct approaches to this problem have been developed. In recent years, the actuarial literature has focused primarily on the use of financial analysis methods to evaluate the insurer's rate of return on capital. In comparison, the economic literature emphasizes the use of risk pricing methods such as the expected utility theory model to determine the certainty equivalent price for a transfer of risk. A discussion of these and other methods can be found in Feldblum (1990).

At first glance, the financial and economic approaches to insurance risk pricing appear to be irreconcilable. Financial methods such as the discounted cash flow models described in Bingham (1993) and Feldblum (1992) stress the role of capital, risk-adjusted rates of return, and returns on alternative investment opportunities. In comparison, the expected utility theory (EUT) model as described in Borch (1990) gives little or no consideration to the insurer's capital or to the exposure's rate of return. While financial methods take into account the timing of future cash flows in the analysis and operate under the assumption that time and risk are essentially inseparable, the EUT model avoids any reliance on the time value of money by assuming that the indemnities are paid immediately after the premium is collected. The two methods also differ in their treatment of risk diversification. D'Arcy and Dyer (1997) note that the Capital Asset Pricing Model (CAPM) rewards an investor only for systematic risk, that portion of the risk that cannot be eliminated by diversification over the market. In comparison, the expected utility theory approach evaluates the price for each risk transfer in isolation from all other transactions. In essence, the EUT model determines the price for each exposure based on its own risk and gives no consideration to the effect of risk diversification on price.

Rather than attempt to address all of the differences between the financial and economic approaches to insurance risk pricing, this paper will focus exclusively on the issue of risk diversification and its effect on price. Diversification is a strategy for reducing the risk of an insurer or investor. For insurance, risk diversification relies on the law of large numbers. By insuring a large number of independent identically distributed exposures, an insurer is able to reduce the variance of the average insured damages so that its results become more stable and predictable. The effect of risk diversification in securities markets is similar except that the returns on securities tend to be correlated with one another, thereby limiting an investor's ability to reduce risk. Markowitz (1991) provides a discussion of optimizing portfolio selection in security markets based on the objective of minimizing the investor's variance while simultaneously achieving a selected expected return.

In addition to reducing risk, diversification may also have an effect on price. Based on the principle that risk determines return, the reduction in risk that an insurer can achieve through diversification should be expected to lead to a reduction in the price it requires for a transfer of risk. For securities markets, an explicit measurement of the impact of risk diversification on price is provided by the CAPM pricing formula, which determines the expected return for a security based on its systematic risk and the expected return for the market as a whole. However, since insurance exposures are not actively traded in securities markets, the CAPM approach is not directly applicable to insurance pricing. Despite this, a variety of insurance pricing methods that rely directly or indirectly on the CAPM have been developed. One example is the use of the CAPM for determining underwriting betas, as discussed in Feldblum (1990). Feldblum discounts this particular approach since it "quantifies the risk faced by the investor in insurance stocks, not the risk of the insurer." As an alternative, he proposes an insurance pricing formula (p. 187) analogous to the CAPM in which the market return is replaced by the rate of return on a fully diversified insurance portfolio. One aspect of this formula is that it quantifies the effect of risk diversification on the price for an insurance exposure. On the other hand, Feldblum (p. 189) observes that his proposed approach leaves unanswered the question of whether the insurer should be rewarded solely for its insurance risk or also for unrelated risks such as asset valuation fluctuations. A review of the actuarial literature shows that this issue is also relevant to other insurance risk pricing methods, particularly those based on a total rate of return approach.

Since the CAPM, either directly or indirectly, has been the basis for any number of insurance pricing models, it seems appropriate to consider whether the interpretations and conclusions drawn from the CAPM are valid. In order to provide a fresh perspective to this issue, this paper will address the fundamental principle underlying the CAPM, the concept that risk diversification has an effect on price. The following section considers the effect of risk diversification on the insurer's price for the portfolio as a whole. Pricing for the individual exposures within a portfolio is considered in the remainder of this paper.

Risk Diversification and the Price for the Portfolio

In order to examine the effect of risk diversification on price, consider an insurer that provides coverage to a large number of identically distributed exposures from a single market segment. The notation $X_1, \ldots, X_n$ will be used to represent the insurer's damages from a set of n exposures selected from the market segment. The insurer is assumed to have a systematic, consistent, and non-judgmental procedure for determining a unique price $P(X)$ for each exposure X. This price is required to depend entirely on the risk of each individual exposure, without consideration of the effect of risk diversification on price. For instance, the insurer might evaluate its price by means of an expected utility theory model. Since the insurer needs to be rewarded for risk, the price for each exposure is required to be no less than the expected damages and no greater than the maximum damages so that $E(X) <= P(X) <= max(X)$. This inequality indicates that the insurer expects to earn a profit but that it has the potential to lose money on each transaction.

From the insurer's perspective, the worst possible portfolio is one in which the exposures are perfectly correlated with one another. In this situation, the insurer obtains no benefit from risk diversification since the variance of the average damages per exposure is identical to the variance of any individual exposure, i.e., $V(\Sigma X_i/n) = V(X_1)$. Since each exposure is priced for its own risk, the insurer's premium for the portfolio is $\Sigma P(X_i)$ or $nP(X_1)$. In comparison, if the insurer's portfolio consists of n independent exposures, the variance of the average damages per exposure is significantly reduced since $V(\Sigma X_i/n)$ is now equal to $V(X_1)/n$. Due to the greater risk of the first portfolio, the insurer should be willing to insure the second portfolio at a lower price. This effect of risk diversification on the price for the portfolio would also reduce the insurer's price for the individual exposures within the portfolio. As a result, the final price charged for each exposure would depend not only on the risk of the exposure but also on the amount of risk diversification the insurer achieves.

Secondary Market Pricing

The discussion of risk diversification in the previous section raises two issues. The first issue, determining the effect of risk diversification on the price for the portfolio, is beyond the scope of this paper. The second issue, how to determine the price for the individual exposures given the price for the portfolio, is considered below.

The basis for this analysis will be the assumption that all insurance exposures are actively traded in a secondary market (i.e., an insurance exchange) that functions as an intermediary between insurers and the capital markets. Even though this is not a realistic model of insurance markets, the benefit of this approach is that it provides a basis for analyzing the relationship between insurance prices and the capital markets. The secondary market is considered to consist of a large number of buyers and sellers with no market participant being large enough to have an influence on price. The market is required to clear, with each exposure selected by some market participant. The secondary market premium for each exposure is determined by competition within the market subject to certain restrictions to be discussed. Due to the effect of risk diversification on price, the premium for the secondary market as a whole may be less than the sum of the prices that would be charged if each exposure were priced for its own risk. Also, since insurers require a return for accepting risk, the premium for the secondary market as a whole needs to exceed the total expected damages. In order to eliminate opportunities for insurers to earn risk-free returns, the price established in the secondary market can be assumed to determine the price charged in the primary market. At the outset of this analysis, the time value of money will be disregarded by requiring that all premiums and indemnities be paid instantaneously. Transaction and insurer overhead expenses will be disregarded.

Let the exposures $X_1, \ldots, X_n$ represent the entire collection of exposures transferred into the secondary market. Each X_i is a random variable whose outcome x_i represents the actual damages incurred. To improve marketability, each exposure X_i is permitted to be divided into smaller units aX_i, where $0 < a <= 1$. The aggregate exposure for the entire market will be designated as $W = \Sigma X_i$, with P_W being the premium for the secondary market as a whole. P_W may differ from $P(W)$, the insurer's price for W when considered as a single exposure. The premium P_W is required to be no greater than the total of the individual risk premiums $\Sigma P(X_i)$ in recognition of the effect of risk diversification. Also, since insurers need to be rewarded for accepting risk, the premium P_W is required to be not less than $E(W)$ so that $E(W) <= P_W <= \Sigma P(X_i)$. The secondary market premium for an individual exposure X_i will be designated as $P(X_i\,;\,W)$. This premium is assumed to depend exclusively on X_i and the portfolio W. Subjective elements are not permitted to influence the price.

Since the exposures are actively traded, the secondary market prices needs to be consistent with the ability of market participants to take advantage of opportunities for risk-free returns. For example, consider two exposures X_1 and X_2 with premiums of $P(X_1\,;\,W)$ and $P(X_2\,;\,W)$, respectively. If market prices are such that the premium $P(X_1 + X_2\,;\,W)$ for the combined exposure $X_1 + X_2$ differs from the sum $P(X_1\,;\,W) + P(X_2\,;\,W)$ of the premiums for the individual exposures, a market participant may be able to acquire and restructure the exposures in order to earn a risk-free return. Arbitrage opportunities such as this can be eliminated by requiring that prices in the secondary market be additive:

$$(1) \qquad P(X_1 + X_2 ; W) = P(X_1 ; W) + P(X_2 ; W)$$

For the market as a whole, the sum of the premiums for the individual exposures in the portfolio must be equal to the premium for the market portfolio W. Since $\Sigma X_i = W$:

$$(2) \qquad P_W = \Sigma P(X_i ; W) = P(W ; W)$$

The risk margin for each exposure will be defined as the amount of premium in excess of the expected damages. Using the notation M_i to represent the risk margin for X_i, M_i is defined as $P(X_i ; W) - E(X_i)$. Similarly, the risk margin for W is defined as $M_W = P_W - E(W)$. Given these definitions, equation (2) can be restated as:

$$(3) \qquad M_W = \Sigma M_i$$

Based on the price additivity rule in (1), the price for a pro-rata portion of an exposure should be the pro-rata price. This can be demonstrated for any positive integer m since the price additivity rule requires that $P(X_i ; W) = mP(X_i/m ; W)$. Similarly, for any rational number k/m, where k is a positive integer such that $k <= m$, $P((k/m)X_i ; W) = (k/m)P(X_i ; W)$. While this suggests that $P(aX_i ; W) = aP(X_i ; W)$ for all a in the range $0 <= a <= 1$, this paper will adopt a more limited pricing rule that applies only to W. For all multipliers a with $0 <= a <= 1$, it will be assumed that:

$$(4) \qquad P(aW ; W) = aP(W ; W) = aP_W$$

Since $aW + (-a)W = 0$, equation (4) can be extended to all constants a in the range from -1 to 1, and subsequently to any positive or negative value.

In addition, the market price for a certain outcome c is required to be equal to that outcome:

$$(5) \qquad P(c ; W) = c$$

These pricing rules provide the basis for the development of the Systematic Risk Pricing Model.

The Systematic Risk Pricing Model

In order to determine market prices, the first requirement is to consider how each exposure contributes to the risk of the portfolio W. Each exposure X_i can be decomposed into two components $\beta_i W$ and U_i, where:

$$(6) \qquad X_i = \beta_i W + U_i$$

The value for β_i can be selected to ensure that $\beta_i W$ and U_i are uncorrelated with one another:

(7) $\beta_i = Cov(X_i, W) / V(W)$

Mathematically, $\beta_i W$ is the projection of the vector X_i on the vector W, where the covariance function is used as the inner product operator. Since $\Sigma\beta_i = 1$, equation (6) implies that $\Sigma U_i = 0$. This result indicates that the uncertainty arising from the uncorrelated components U_i for each exposure is completely eliminated by diversification over the portfolio. Since this implies that $\Sigma E(U_i) = 0$ and $\Sigma P(U_i\,;\,W) = 0$, this implies the diversifiable risk adds nothing to the risk margin for the portfolio:

(8) $\Sigma\,[P(U_i\,;\,W) - E(U_i)] = 0$

In essence, the U_i represent a zero sum game. Equation (8) indicates that any surcharge for diversifiable risk included in the premium for exposure X_i would be offset by a credit on the premium for another exposure. Since exposures having positive risk margins on their diversifiable risk components will be more attractive than those having negative risk margins, competition among market participants should be expected to eliminate both the positive and negative risk margins for diversifiable risk. More generally, it will be assumed that the market does not reward diversifiable risk. Consequently, for any exposure U included in but uncorrelated with the portfolio W:

(9) $P(U\,;\,W) = E(U)$

This result can now be used to determine the market price for exposure X_i. Based on equations (6) and (9), the price for X_i is:

(10) $P(X_i\,;\,W) = E(X_i) + \beta_i M_W$

The Systematic Risk Pricing Model in equation (10) determines the price for X_i entirely in terms of its contribution $\beta_i W$ to the systematic (i.e., non-diversifiable) risk of the portfolio, while the diversifiable component of risk U_i makes no contribution to the risk margin for X_i.

This result can also be stated in terms of the standard deviation of the exposure and correlation between the exposure and the market portfolio. As a first step, express β_i as $\rho_i\sigma_i / \sigma_W$, where the correlation coefficient ρ_i is defined as $Cov(X_i, W) / (\sigma_i\ \sigma_W)$. Next, define λ as M_W / σ_W, the risk margin of the portfolio per unit of standard deviation. Substituting these into equation (10) yields an alternate form of the Systematic Risk Pricing Model:

(11) $P(X_i\,;\,W) = E(X_i) + \lambda\rho_i\sigma_i$

Based on this result, the premium for an exposure can be less than the expected damages, that is, $P(X_i\,;\,W) < E(X_i)$, whenever the correlation coefficient with the portfolio is negative. Even though this is contrary to expectations, it arises because the Systematic Risk Pricing Model determines price based on the systematic risk rather than on the total risk of the exposure.

Equation (10) can also be expressed in terms of the relationship between the risk margins for X_i and the portfolio W:

$$(12) \qquad M_i = \beta_i M_W$$

Using the results developed above, it can now be shown that the pro-rata pricing rule in (4) applies to every exposure and not simply to the portfolio W. Let X_i be an exposure in W and define Y as aX_i for some a. Equation (10) states that the price for Y is $P(Y\,;\,W) = E(Y) + \beta_Y M_W$. Substituting $E(Y) = aE(X)$ and $\beta_Y = Cov(Y, W) / V(W) = a\beta_i$ into this formula demonstrates that $P(aX_i\,;\,W) = aP(X_i\,;\,W)$.

At this point, it may be worthwhile to briefly review these results. The Systematic Risk Pricing Model in (10) has several elements in common with the Capital Asset Pricing Model. Both models are developed for markets in which exposures are actively traded, and both reward only systematic risk. The CAPM develops its results for an identifiable set of exposures in a real secondary market, while the Systematic Risk Pricing Model presumes the existence of an imaginary insurance exchange. The precise nature of this exchange has not been defined – it may include exposures from only a single market segment or from all market segments combined.

One important difference between the two models is that the Systematic Risk Pricing Model determines the price for an exposure while the CAPM determines the expected rate of return on capital for an investment. As indicated by equations (3) and (12), the Systematic Risk Pricing Model is simply a method for allocating the risk margin for the portfolio to the individual exposures with the portfolio. This is not the only method that can be used to accomplish this result. For example, the standard actuarial expense loading formula allocates a portfolio's risk margin to individual exposures through the use of a loss cost multiplier applied to the expected damages. Other allocation bases, such as standard deviation or variance, could also be used for this purpose.

One of the most interesting aspects of the Systematic Risk Pricing Model is that the insurer's prices are determined without the need to allocate the insurer's capital to market segments. Rather than being a flaw in the model, this demonstrates that insurance exposures can be priced for systematic risk without reference to the capital markets. According to equation (10), the risk margin for an insurance exposure depends on the relationship between the exposure and the portfolio W and on the risk margin M_W for the portfolio, and not on the insurer's cost of capital. The portfolio W may represent a single market segment for one insurer or it may be the complete book of business across the entire insurance industry. The model also indicates that the price for an exposure should be based exclusively on its systematic risk and not on unrelated risks, such as asset valuation fluctuations, to which the insurer is exposed.

As a final observation, it should be noted that the Systematic Risk Pricing Model places only very minimal restrictions on the market premium P_W. The only requirement is that P_W falls within the range from $E(W)$ to $\Sigma P(X_i)$. If P_W is at the lower end of the range, the price $P(X_i\,;\,W)$ for each exposure would be equal to its expected value $E(X_i)$. If P_W is at the upper end of the range, then the secondary market simply redistributes the individual risk premiums $P(X_i)$ among the exposures. In this situation, some premiums could increase while others would decrease in relation to the individual risk prices $P(X_i)$ offered in the primary market. However, whenever the secondary market price $P(X_i\,;\,W)$ for an exposure exceeds its individual risk price $P(X_i)$, the policyholder's willingness to participate in the secondary market may be affected.

The Variance Pricing Formula for Independent Exposures

For independent exposures, the Systematic Risk Pricing Model implies that the insurer's risk margin for each exposure should be proportional to the variance of each exposure. To demonstrate this result, evaluate β_i under the assumption that the exposures are independent:

$$\beta_i = Cov(X_i, W) \,/\, V(W) = V(X_i) \,/\, V(W) = \sigma_i^2 \,/\, \sigma_W^2 \tag{13}$$

Next, define k as $M_W \,/\, \sigma_W^2$, the portfolio's risk margin per unit of variance. This can be substituted into (10) to obtain the variance pricing formula:

$$P(X_i\,;\,W) = E(X_i) + k\sigma_i^2 \tag{14}$$

Miccolis (1977) provides an application of the variance pricing formula to the pricing of liability increased limits factors. The difference between this result and the approach described by Miccolis is that the Systematic Risk Pricing Model determines the value for k based on the risk margin M_W for the portfolio, whereas Miccolis determines the value for k based on a judgmentally selected risk margin for the basic limits policy.

As an application of this result, suppose that an insurance advisory organization (i.e., ISO) provides both $E(X_i)$ and σ_i^2 at each policy limit. Given this information, the insurer can select an arbitrary value for the parameter k in order to determine $P(X_i\,;\,W)$ at each policy limit. Based on these results, the insurer's increased limits factor at a selected policy limit can be defined as the value of $P(X_i\,;\,W)$ at the higher limit divided by the corresponding value at the basic limit. This approach enables an insurer to revise its increased limits factors without the need to develop revised estimates of $E(X_i)$ and σ_i^2.

The Insurance Analogue to the Capital Asset Pricing Model

The next topic to be considered is the relationship of the Systematic Risk Pricing Model to the Capital Asset Pricing Model. According to Brealey and Myers (1996), the CAPM states that the expected rate of return for a security is determined by the security's beta:

$$E(r_s) - r_f = \beta_s\,(E(r_M) - r_f) \tag{15}$$

where:

$$\beta_s = Cov(r_s, r_M) \,/\, V(r_M) \tag{16}$$

and r_s and r_M represent the rate of return on the security and the market, respectively. The value r_s is generally described as the risk adjusted rate of return or the cost of capital.

In order make a comparison between the two models, the Systematic Risk Pricing Model needs to be restated in terms of its implied rate of return. For any particular outcome x_i of X_i, define R_i as the observed return $P(X_i\,;\,W) - x_i$ for that outcome. Since $E(R_i) = M_i$, the expected return is simply the risk margin in the premium. Similarly, let R_W represent the observed return $P_W - w$ for the market segment given the outcome w so that the expected return is $E(R_W) = M_W$. With these definitions, equation (12) can be restated as:

$$(17) \qquad E(R_i) = \beta_i\, E(R_W)$$

Next, express the expected returns R_i and R_W as rates of return in relation to the price for the exposure. Define $r_i = R_i \,/\, P(X_i\,;\,W)$ and $r_W = R_W \,/\, P(W)$ and substitute in the equation above:

$$(18) \qquad E(r_i)P(X_i\,;\,W) = \beta_i\, E(r_W)P(W)$$

To be consistent with the CAPM, define β'_i as:

$$(19) \qquad \beta'_i = Cov(r_i\,,\, r_W) \,/\, V(r_W)$$

Since β'_i is equal to $\beta_i P(W) \,/\, P(X_i\,;\,W)$, equation (18) can be restated as:

$$(20) \qquad E(r_i) = \beta'_i\, E(r_W)$$

This result represents the insurance analogue to the Capital Asset Pricing Model. Since the damages for each exposure are paid at time 0, the adjustment for the time value of money in equation (15) is unnecessary. However, in order to analyze the two models on a consistent basis, assume that the premium and the damages for each exposure X_i in market segment W are paid at time 1 rather than at time 0. Let $P(X_i\,;\,W)$ be the market price for X_i at time 1 as determined by equation (10). Since the premium is a constant, it can be discounted to present value at the risk-free rate. Denote the discounted premium for X_i as $PV_i = vP(X_i\,;\,W)$. Similarly, let $PV_W = vP_W$ be the discounted premium for the portfolio. Next, reverse the sign on the damages so that cash outflows are treated as negative values. This adjustment is needed for consistency with the CAPM treatment of investment gains as positive values. Finally, define the rate of return for an exposure as the return earned at the end of the period divided by the price for the exposure at the start of the period. On this basis, the rate of return on X_i is defined as $r_i = (x_i - PV_i)/PV_i$ while the rate of return r_W on the portfolio is $(w - PV_W)/PV_W$. Given these definitions, and with β'_i defined as in (19), the Systematic Risk Pricing Model can be used to show that:

$$(21) \qquad E(r_i) - r_f = \beta'_i\,(E(r_W) - r_f)$$

This structure of this result is identical to that of the Capital Asset Pricing Model formula in equation (15). The primary difference between this formula and the CAPM is that the rate of return in (21) is defined in relation to the price for the exposure rather than in relation to the amount of capital invested. This result demonstrates that the exposure's expected rate of return has no relationship to its cost of capital. Instead, the rate of return in (21) is simply the insurer's profit margin, that is, the expected profit divided by the premium for the exposure. It should also be noted that equation (21) was obtained by discounting the premium rather than the uncertain damages for the exposure. For this reason, the rate of return in equation (21) does not represent the risk-adjusted rate at which uncertain future cash flows for an insurance exposure can be discounted to present value.

To complete this analysis, the relationship between the insurance pricing formula in (21) and the CAPM formula in (15) needs to be addressed. Since the rationale used to develop the Systematic Risk Pricing Model can also be applied to security pricing, the two models should be consistent with one another. The primary difference between the models can be immediately reconciled by recognizing that the price for a security in a secondary market is equivalent to the amount of capital invested. In other words, the return on capital for a security is also its return on price. One issue this raises is that the conclusion from the previous paragraph can also be applied to security pricing. This result shows that the interpretation of the CAPM rate of return as the risk-adjusted rate at which uncertain future cash flows can be discounted to present value is inconsistent with the Systematic Risk Pricing Model.

Discounting Future Cash Flows to Present Value

In the previous section, the different interpretations of the Capital Asset Pricing Model and the Systematic Risk Pricing Model were reconciled by recognizing that the amount of capital invested in a security is equivalent to its price. A second difference that needs to be addressed is the relationship between time and risk in the two models. For the Capital Asset Pricing Model, time is treated as an essential element of risk. Specifically, the CAPM rate of return represents the rate at which uncertain future cash flows are discounted to present value. In comparison, the Systematic Risk Pricing Model considers time and risk to be independent of one another. The difference between the two models is illustrated by equation (20), in which the Systematic Risk Pricing Model has been used to determine the rate of return for an exposure whose outcomes are paid at time 0. Since the exposure has no time element, the CAPM cannot be used to determine its rate of return.

One issue that arises from the independence of time and risk in the Systematic Risk Pricing Model is that the model provides no information on how to discount uncertain future cash flows to present value. In order to investigate this issue, recall that equation (10) determines the relationship between the price for each exposure and the portfolio risk margin $M_W = P_W - E(W)$. Provided that the portfolio price P_W meets certain reasonability conditions, it may be possible to determine how uncertain future damages should be discounted to present value. Let W_0 and W_1 be two portfolios having identical damage distributions except that the damages for W_0 are paid at time 0 while the damages for W_1 are paid at time 1. The first assumption is that the price for the portfolio is independent of time. If P_0 and P_1 represent the insurer's pricing functions at times 0 and 1 respectively, this requires that the price $P_0(W_0)$ at time 0 be identical to the price $P_1(W_1)$ at time 1. Second, for positive values k close to 1, the price for a portfolio kW is assumed to be k times the price for the original portfolio, $P(kW) = kP(W)$. By substituting v for k, where v is the discount factor corresponding to the risk-free rate r_f, this implies that $vP_0(W_0) = P_1(vW_1)$.

To apply these assumptions to the pricing of an individual exposure within the portfolio, let X_1 be an exposure with damages paid at time 1 and let X_0 be the identical set of damages paid at time 0. Since the premium at time 1 for exposure X_1 is $P_1(X_1 ; W_1)$, the premium payable at time 0 is $vP_1(X_1 ; W_1)$. This premium can be compared to the premium based on discounting the uncertain damages to present value at the risk-free rate. The discounted payments for X_1 at time 0 are represented by vX_0 while the discounted payments for W_1 are vW_0. In accordance with equation (10), the premium at time 0 for the discounted damages is:

$$(22) \qquad P_0(vX_0 ; vW_0) = E(vX_0) + \beta_0 (P_0(vW_0) - E(vW_0))$$

By applying the portfolio pricing assumptions, this can be expressed as:

$$(23) \qquad P_0(vX_0 ; vW_0) = v[E(X_0) + \beta_0 (P_0(W_0) - E(W_0))]$$

The right hand side of this formula is equal to $vP_0(X_0 ; W_0)$. Since the values for β at time 0 and time 1 are identical, equation (10) ensures that $P_0(X_0 ; W_0)$ is identical to $P_1(X_1 ; W_1)$. Based on this result, the premium $vP_1(X_1 ; W_1)$ for the exposure X_1 at time 0 is equivalent to the premium $P_0(vX_0 ; vW_0)$ obtained by discounting the uncertain future damages X_1 and W_1 to time 0 at the risk-free rate.

Given the two reasonability assumptions regarding the portfolio price, the preceding analysis has shown that the price for the uncertain future cash flows can be obtained by discounting each outcome to present value at the risk-free rate. This result reaffirms that the rate of return shown in equation (21) does not represent the rate at which uncertain future cash flows should be discounted to present value. While this conclusion has been developed from the context of Systematic Risk Pricing Model, it also applies to the CAPM. Accordingly, the assumption that the CAPM rate of return can be used to discount uncertain future cash flows to present value needs to be abandoned.

The Capital Asset Pricing Model

In equation (21), the insurance analogue to the Capital Asset Pricing Model, r_i has been defined as $(x_i - PV_i)/PV_i$. This expresses the rate of return for an exposure X in terms of the uncertain future outcome x_i and the price PV_i at time 0 for the exposure. The systematic risk of the exposure, represented by $\beta_i W$, is based on the correlation between the outcomes for the exposure and the outcomes for the secondary market W. In comparison, the CAPM defines the rate of return for a security in terms of its price at two points in time. More specifically, the CAPM rate of return is defined as $r_X = (P_{X1} - P_{X0})/P_{X0}$, where P_{X0} and P_{X1} represent the price for a security X at times 0 and 1 respectively. The systematic risk of the security is based on the correlation between the rate of return r_X for the security and the rate of return $r_W = (P_{W1} - P_{W0})/P_{W0}$ for the market W as a whole. At first glance, the two methods for pricing for systematic risk appear to be reasonably consistent with one another. However, a more careful examination of the both methods leads to the identification of a number of significant conceptual problems with the CAPM. These problems are severe enough to undermine the validity of the CAPM as risk pricing model.

The most basic problem with the CAPM is that the expected rate of return for a security is determined without any consideration being given to the performance of the business that underlies the security. Since a security represents ownership of the income generated by a business, a relationship between the performance of the business and the price for its security is essential. For insurance exposures, the Systematic Risk Pricing Model determines the price for an exposure based on the uncertainty of its insured damages rather than on the variability of its price in the secondary market. Similarly, the risk for a security should be based on the uncertain future cash flows (e.g., stockholder dividends) for the business underlying the security rather than on the variability of the price for the security in the secondary market. In essence, the CAPM mistakenly substitutes the price variability of an exposure in the secondary market for the uncertainty of the future cash flows from the business. Price variability over time is not the proper measure of the risk of an exposure.

The CAPM suffers from a second problem that arises out of its relationship between present and future prices. In the CAPM approach, the price P_{X0} for a security at time 0 adjusts to ensure that the security achieves its cost of capital. Since the cost of capital expresses the relationship between the current and future price, this implies that the current price P_{X0} for a security is a function of the distribution of its future price P_{X1}. However, if similar reasoning is applied to any specific future price P_{X1}, the price P_{X1} for the security at time 1 depends on the distribution of its price P_{X2} even further in the future. This establishes an iterative and indeterminate process for determining the price for a security. The pricing procedure fails because price is treated as both an input and an output of the analysis. The Systematic Risk Pricing Model avoids this problem by evaluating price in terms of the true risk of an exposure rather than in terms of the variability of the price of the exposure over time.

An additional problem with the Capital Asset Pricing Model is that the CAPM approach is based on a fundamental misinterpretation of a security as a risk exposure. For an insurance policy, the risk of an insured exposure is essentially static throughout the policy term. That is, the insurer expects the risk characteristics of the exposure throughout the policy period to be consistent with the insurer's expectations at the time the policy is issued. If the risk characteristics for a policy change mid-term, the insurer often has a contractual right to cancel coverage. In comparison, a security represents ownership of the future income of a business. Unlike an insurance policy, a business is dynamic in the sense that the company management can take actions that affect its future income. Actions that might influence the risk profile of a business include product pricing changes and decisions to enter or exit individual market segments. Since a business has a measure of control over its risk profile, a business cannot be interpreted as a static risk exposure. More properly, a business represents a risk exposure X_0 at one point in time and a different risk exposure X_1 at another point in time. Since the risk profile of a business can change over time, there is no reason to think that the CAPM β value for its security will remain stationary over time. However, without a stable value for β, the CAPM formula is not useful for determining the expected rate of return for a security.

Another problem with the CAPM becomes apparent based on a direct comparison between the Systematic Risk Pricing Model and the CAPM. In order to determine the CAPM β for a security, the correlation between the rate of return for the security and the rate of return for the market as a whole needs to be evaluated. However, the Systematic Risk Pricing Model in equation (10) considers the price $P_X = P(X\,;\,W)$ for the exposure X and P_W for the market W to be constants rather than random variables. If the price for a security is not a random variable, neither is the security's rate of return in the secondary market. Accordingly, the correlation between the two rates of return, and consequently the CAPM β for the security, is not a meaningful concept.

Since the Capital Asset Pricing Model is based on portfolio selection theory, the observation that the CAPM is invalid also raises a challenge to portfolio selection theory. Portfolio selection theory describes how to construct a portfolio with the least risk for a selected expected rate of return. In order to apply this procedure, the investor needs to know the mean and variance of the rate of return for each security in the secondary market. The problem with this statement is that the types of information available to the investor are more likely to be related to the performance of the business underlying the security than they are to the rate of return for the security in the secondary market. For instance, an investor may have information on prospective economic conditions that can be anticipated to affect the income for a business in the current year. Whether or not this information has already been incorporated into the market price for the security may not be evident. If the information is consistent with the prior assumptions that were used to set the current market price, it may have no effect on the future rate of return for the security. On the other hand, the information may alter the perceived risk profile of the business so that the initial risk exposure X_0 is transformed into a new risk exposure X_1. In this situation, the information may have an effect on the security's future rate of return.

The problems discussed above can also be demonstrated mathematically in the derivation of the CAPM from the Systematic Risk Pricing Model. Consider a security X that has an unknown selling price of P_{X1} versus an original purchase price of P_{X0}, and let the rate of return for X be defined as $r_X = (P_{X1} - P_{X0})/P_{X0}$. Since the future prospects for the business underlying the security can change over time, the investor can be considered to own a share of the original exposure X_0 at time 0 and a different exposure X_1 at time 1. The price for X at each point in time is determined in a secondary market W, or more accurately, a secondary market W_0 at time 0 and a different secondary market W_1 at time 1. The price at each point in time, $P_{X0} = P_0(X_0\,;\,W_0)$ and $P_{X1} = P_1(X_1\,;\,W_1)$, can be developed from the Systematic Risk Pricing Model for times $t = 0$ and $t = 1$ such that $P_t(X_t\,;\,W_t) - E(X_t) = \beta_t(P_t(W_t) - E(W_t))$ for each value of t. In this formulation, X_t and W_t are random variables that represent the underlying primary exposures. However, in order to develop the CAPM, the random variables X_t and W_t need to be treated as constants while the corresponding prices $P_t(X_t\,;\,W_t)$ and $P_t(W_t)$ are treated as variables. Following this reasoning, define $y(t) = P_t(X_t\,;\,W_t)$ and $z(t) = P_t(W_t)$. Since the risk of the exposure and the risk of the secondary market both change over time, the value of β can also change over time. For this analysis, make the assumption that β_t can be considered to be constant over brief periods of time so that $d\beta_t/dt = 0$. From this, it follows immediately that $dy/dt = \beta_t\, dz/dt$. This indicates that the instantaneous rate of change of price $y(t)$ for the exposure X in the secondary market is proportional to the instantaneous rate of change of $z(t)$, the price for the market as a whole, where the proportionality constant β_t is evaluated in terms of the underlying exposures X_t and W_t, consistent with the Systematic Risk Pricing Model.

A different perspective on this result can be obtained by treating the prices $P_t(X_t\,;\,W_t)$ and $P_t(W_t)$ for $t = 0$ and 1 as random variables rather than as real valued functions of time. For this analysis, let Y_t be defined as $P_t(X_t\,;\,W_t)$ and Z_t as $P_t(W_t)$. An immediate problem with this analysis is that equation (10) requires Y_t and Z_t to be perfectly correlated with one another. To circumvent this issue, introduce independent error terms ε_t with an expected value of 0 into the Systematic Risk Pricing Model so that the price for each exposure at the two points in time can be expressed as:

$$(24) \qquad Y_0 - E(X_0) = \beta_0(Z_0 - E(W_0)) + \varepsilon_0$$

and:

$$(25) \qquad Y_1 - E(X_1) = \beta_1(Z_1 - E(W_1)) + \varepsilon_1$$

Under the assumption that the expected cash flows for W_0 and X_0 both increase at the risk-free rate so that $E(X_1) = (1 + r_f)E(X_0)$ and $E(W_1) = (1 + r_f)E(W_0)$, and assuming that the proportionality term is constant over time so that $\beta_0 = \beta_1, = \beta$, it follows immediately that:

$$(26) \qquad (Y_1 - (1 + r_f)Y_0) = \beta(Z_1 - (1 + r_f)Z_0) + \varepsilon_1 - \varepsilon_0$$

where $\beta = cov(X_1, W_1)/V(W_1)$. Using equation (25), β can be expressed in terms of the price random variables Y_1 and Z_1, so that $\beta = cov(Y_1, Z_1)/V(Z_1)$. After applying the expectation operator (and removing unnecessary parentheses), equation (26) can be restated as:

$$(27) \qquad EY_1 - (1 + r_f)EY_0 = \beta(EZ_1 - (1 + r_f)EZ_0)$$

Next, recall that the CAPM formula defines beta as $\beta' = cov(r_Y, r_Z)/V(r_Z)$ where $r_Y = (Y_1 - Y_0)/Y_0$ and $r_Z = (Z_1 - Z_0)/Z_0$. If Y_0 and Z_0 can be considered to be constants, then β' can be evaluated as $\beta Z_0/Y_0$. Substituting this into equation (27) gives $(EY_1 - Y_0)/Y_0 - r_f = \beta'((EZ_1 - Z_0)/Z_0 - r_f)$, which can be expressed more succinctly as:

$$(28) \qquad E(r_Y) - r_f = \beta'(E(r_Z) - r_f) \quad \text{where } \beta' = cov(r_Y, r_Z)/V(r_Z)$$

This completes the proof of the CAPM formula. It should be noted that the proof relies on a number of questionable assumptions. The most questionable of these is that the variability of the price for the security over time, rather than the uncertainty of the underlying exposure itself, is the proper measure of the risk of an exposure. In addition, the proof requires β to be constant over time so that $\beta_0 = \beta_1$. It also assumes that $E(X_1) = (1 + r_f)E(X_0)$, and $E(W_1) = (1 + r_f)E(W_0)$. While these conditions might be reasonable approximations over brief periods of time, they are not likely to be valid over longer periods. For example, β (and hence β') will immediately change each time a new exposure enters the secondary market. If β is not constant, the CAPM is not a useful method for security market pricing. The CAPM proof also requires that the prices Y_0 and Z_0 at time 0 to be known values. If the current prices are known, this negates the value of the CAPM as a means for evaluating the current price for a security based on its future cash flows.

Conclusion

Financial theory suggests that prices should reflect systematic risk only, with no compensation given to the diversifiable risk of each exposure. By applying this principle to the pricing of insurance exposures actively traded in a secondary market, the Systematic Risk Pricing Model in equation (10) has been derived. This model determines the price for an exposure based on its contribution to the risk of the portfolio in which it resides. Given the price for the portfolio, the price for each exposure within the portfolio can be determined without reference to the insurer's or the exposure's cost of capital. The Systematic Risk Pricing Model can be interpreted as being simply a method for allocating the risk margin for the portfolio to the individual exposures within the portfolio. In the special case where the exposures are independent, the risk margin for each exposure is proportional to its variance. Other methods, such as the standard actuarial loss cost multiplier approach, can also be used for this purpose.

The Systematic Risk Pricing Model differs from the Capital Asset Pricing Model in that it determines the price rather than the rate of return for each exposure. A careful examination of the CAPM leads to the identification of a number of significant conceptual problems within the model, the most serious of which is that the model substitutes the variability of the price of an exposure over time for the true risk of the exposure. It also relies on the questionable assumption that the value of the CAPM β is stable over time. Due to these problems, the CAPM cannot be considered to be a valid risk pricing model.

The principle that exposures should be priced solely on the basis of their systematic risk, as described in this paper, is also open to interpretation. Based on its construction, the Systematic Risk Pricing Model is relevant for exposures that are actively traded in a secondary market. Since this is not a realistic assumption for insurance markets, the Systematic Risk Pricing Model may not be the most realistic method for determining prices for insurance exposures. Other methods for allocating the risk margin of the portfolio to the individual exposures within the portfolio, such as the use of loss cost multipliers, may be more suitable.

A related issue with regard to pricing for systematic risk is the size of the portfolio over which the insurer's systematic risk is evaluated. Due to the insurer's ability to reduce its risk by insuring a large number of independent exposures, the insurer's price should decrease in response to its success in diversifying risk within each market segment. What may not be as evident is that the insurer's ability to diversify its risk across market segments need not have an effect on the insurer's price. This issue is addressed in the companion piece to this paper, "The Cost of Conditional Risk Financing." As described in that paper, the risk pricing function for a well-diversified insurer that retains the benefits of risk diversification across market segments can be completely determined provided that the insurer operates under a capital preservation objective. After the insurer uses its risk pricing function to determine its premium for each market segment, the Systematic Risk Pricing Model or another model can be applied to determine the premium for the individual exposures within each market segment. In combination, the two pricing models are capable of completely determining the price an insurer should charge for each exposure.

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