Financial Pricing Models for Property-Casualty Insurance Products: Modeling the Equity Flows

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This paper and its companion papers present the use of return on capital financial models to price property-casualty insurance products. This paper focuses on the cash flow and equity flow modeling that underlies the financial models. The companion papers complete the description of return on capital pricing models. The first two appendices to this paper – Appendix A on federal income taxes and Appendix B showing the workers' compensation pricing exhibits – apply to all the papers in this series.

FLOW OF FUNDS

Financial models to quantify the expected profitability of a business project consider the net present value of a series of cash flows or the internal rate of return of those cash flows. The models are used to set product prices and to measure business performance.

The cash flows represent the expected flow of funds to and from the suppliers of capital. The suppliers of capital are termed the owners, the investors, the shareholders (or stockholders), or the equityholders; we generally use the term "equityholders" in this paper. Although mutual insurance companies do not have stockholders or investors, and the "ownership" status of their policyholders is not clear, we assume that they face the same capital management constraints as a stock insurance company faces.

For non-regulated industries, the cash flows to and from the company are reasonable proxies for the equity flows to and from the equityholders.

1. A cash flow into the company provides shareholder dividends to the equityholders.
2. A cash flow out of the company necessitates a capital contribution by the equityholders.

The use of company cash flows as a proxy for the implied equity flows assumes that the company cash flows can be paid as dividends to equityholders. For regulated industries, such as insurance and other financial services, this assumption is not correct, because of statutory reserve requirements and risk-based capital requirements.

For pricing property-casualty insurance products, we explicitly examine the expected flow of funds to and from the suppliers of capital – or the implied equity flows – not the flow of funds to and from the company. To calculate the implied equity flows, we project the future cash flows, and we adjust for statutory requirements and federal income taxes.[1]

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[1] When the meaning is clear, we sometimes refer to implied equity flows simply as equity flows.

Cash flow projections are easier for insurance contracts than for many other commercial products. Most insurance underwriting costs are variable, and the overall demand for insurance is relatively stable. In contrast, other commercial products often have large fixed cost components and fluctuating demand from year to year.

Cash flow estimation techniques are not unique to insurance, and actuaries are proficient at these estimation tasks. We assume here that future cash flows to and from the company have been properly estimated and that statutory and tax provisions have been accounted for. Our task is to determine the implied equity flows from which the net present value and the internal rate of return are calculated.

Cash Flows and Equity Flows

In most industries, cash flows to and from the company are reasonable proxies for the cash flows to and from the suppliers of capital. If a company invests $1,000 at time t = 0 and it receives $1,100 at time t = 1, the pricing model assumes that the company's owners provide the $1,000 in capital at time t = 0 and receive the $1,100 at time t = 1.

This is true for unregulated manufacturing or service enterprises and for some utilities. The accounting requirements for these industries are not directly relevant to return on capital pricing models. The models focus on projected cash flows and current income tax liabilities.[2]

Regulated financial institutions – life insurance companies, property-casualty insurance companies, depository institutions, and certain investment firms – are different. For these industries, the cash flows to and from the company do not necessarily reflect the cash flows to and from investors. The potential dividends to equityholders and the capital required from equityholders depend on the statutory funding requirements for loss reserves and on risk-based capital requirements, not just on the cash and similar assets held by the company.

For the major property-casualty insurance transactions – premium collections and loss payments – the two sets of cash flows generally have opposite signs. A premium collection, which is a cash inflow to the company, generally necessitates a capital contribution by the equityholders. A loss payment, which is a cash outflow from the company, generally allows a return of capital to the equityholders.

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[2] Current income tax liabilities are the taxes assessed by the IRS. Accrued tax liabilities are the sum of the current tax liabilities and the deferred tax liabilities. If there are no expected changes in the tax rate, the accrued tax liabilities equals the book income (either GAAP or statutory) times the tax rate. We use a balance sheet orientation – not an income statement orientation – to evaluate the deferred tax assets and liabilities; see SFAS 109. We adjust the statutory deferred tax asset for its admitted portion; see SSAP No. 10.

Solvency monitoring by governmental authorities is the underlying rationale for the difference between the company cash flows and the implied equity flows. The flow of funds to suppliers of capital, the "equityholders," can be inferred from

- the company cash flows,
- capital requirements imposed by regulatory authorities,
- reserve requirements and other accounting regulations.

To distinguish between the company cash flows and the flow of funds to suppliers of capital, we refer to the former as *cash flows* and to the latter as *implied equity flows*. The financial community uses the term "free cash flow" instead of implied equity flow. Atkinson and Dallas [2000], chapter 11, use the term "distributable earnings" instead of implied equity flows.[3]

We summarize below the signs of the company cash flows and the implied equity flows for the four major types of property-casualty insurance transactions. A positive flow means an inflow to the company or to the equityholders. A negative flow means an outflow from the company or the equityholders.

	Company Cash flow	Implied Equity flow
Premiums collected	+	−
Losses paid	−	+
Expenses (including federal income taxes)[4]	−	−
Investment income received	+	+

The actual relationship between the cash flows and the implied equity flows is complex, and the chart above does not do full justice to this topic. The illustration below shows the intuition for the implied equity flows stemming from premium collection. The text of this paper, and the associated flowcharts, graphics, and tables, works through the cash flows and implied equity flows for a more complete illustration. Appendix B shows the full cash flows and implied equity flows for a workers' compensation pricing analysis, using a 50 year (200 quarter) return on capital pricing model.

Illustration: A policy with a premium of $1,000 is written and collected on December 31, 20XX. The agent's commission is 20%, and the capital requirements equal 25% of the written premium. We ignore for the moment the tax liability and the deferred tax asset.

[3] The life insurance pricing model in Atkinson and Dallas [2000] parallels the property-casualty insurance pricing model in this paper. The differences between the models reflect the differences in reserve requirements and federal income tax liabilities between life insurance and property-casualty insurance.

[4] For both life insurance and property-casualty insurance, deferred taxes are included with current tax liabilities. When Atkinson and Dallas wrote their textbook, statutory accounting did not recognize deferred tax assets and liabilities, so they restricted their treatment of deferred taxes to GAAP return on equity models.

- The net cash *inflow* from the policyholder to the company is $1,000 (premium) – $200 (commission) = $800 on December 31.
- The company receives $800 (net of commission), but it must hold a $1,000 unearned premium reserve and it must support the policy with $250 of surplus. The cash *outflow* from the equityholders is ($1,000 – $800) + $250 = $450.

Free Cash Flows and Implied Equity Flows

The distinction between company cash flows and implied equity flows is identical to the distinction between company cash flows and free cash flows. Financial analysts use free cash flows for return on capital pricing models. The use of company cash flows without consideration of changes in net working capital fails to take full account of invested capital.

We summarize below the distinction between non-regulated and regulated industries. For non-regulated manufacturing enterprises, the company cash flows are adjusted for (i) depreciation and amortization and (ii) required investment (or "capital expenditures"). This adjusted income minus the change in net working capital equals the free cash flows. The free cash flows are used to determine the net present value and the internal rate of return.

For regulated insurance enterprises, the statutory income equals the statutory cash flow adjusted for the capitalization and amortization of the unearned premium reserves and the loss reserves. The statutory income *minus the change in required capital* equals the implied equity flow. The implied equity flows determine the net present value and the internal rate of return. The change in net working capital for other industries is equivalent to the change in required capital for insurance enterprises.[5]

Early forms of cash flow pricing models presumed that the cash flows equal the accounting income adjusted for non-cash revenues and expenditures. This presumption is no longer used, because many cash flows do not result in revenues or expenditures. Cash is used to purchase material and supplies to produce goods and services. The purchase of supplies is the exchange of cash for a non-cash asset (inventory). There is no revenue or expenditure on the firm's income statement.

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[5] Some analysts argue that the change in net working capital for manufacturing enterprises is a real cash flow item. It is determined by business and economic constraints, not by regulation, and it reflects the cash expenditures of the firm. In contrast, reserve requirements and risk-based capital requirements for insurance enterprises depend on regulatory mandate, not on business and economic constraints. The perspective in this paper is that a regulatory mandate in a regulated industry is a business constraint.

ILLUSTRATION: AIRCRAFT MANUFACTURING

The following illustration clarifies the difference between cash flows and free cash flows. A non-leveraged (all-equity financed) firm manufactures aircraft. The firm leases the factory, equipment, and work-force, so there are no capital expenditures.

- At time t=0, the firm purchases material and supplies for $10 million to produce an airplane.
- From time t=0 to time t=4, the firm manufactures the airplane, at a cost of $2 million a year in rent and wages.
- The firm sells the airplane one year later, at time t=5, for $25 million, after paying $1 million in storage costs and sales commissions in the last year.

To simplify the computations, we assume that rent and wages are paid at the end of the year.

There are no interest payments, debt payments, amortization, or depreciation of fixed assets for this firm. The earnings for the firm are

- –$2 million at times t=1, 2, 3, and 4 (production expenses), and
- +$14 million at time t=5, calculated as sales revenue of $25 – cost of goods sold of $10 – sales expenses of $1.

There are no non-cash revenues or expenditures in this illustration. At time t=0, when the firm purchased the material and supplies, $10 million in cash is exchanged for $10 million of inventory. The inventory changes in form over the next five years, but its accounting value remains $10 million, with no effect on the balance sheet or the income statement.

If the cost of equity capital is 15% per annum, the apparent NPV of this project is (in millions of dollars)

$$-\$2 / (1.15) - \$2 / (1.15)^2 - \$2 / (1.15)^3 - \$2 / (1.15)^4 + \$14 / (1.15)^5 = \$1.25 .$$

This analysis does not take into account the change in net working capital. At time t=0, there is a cash outflow from the equityholders equal to the increase in the net working capital of $10 million.[6] At time t=5 there is an additional cash inflow to the equityholders equal to the decrease in net working capital of $10 million. The corrected net present value of this project is (in millions of dollars)

$$-\$10 - \$2 / (1.15) - \$2 / (1.15)^2 - \$2 / (1.15)^3 - \$2 / (1.15)^4 + \$24 / (1.15)^5 = -\$3.78.$$

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[6] Net working capital = inventory + accounts receivable + cash on hand – accounts payable.

Proper consideration of the timing of the cash flows turns the $1.25 million indicated profit into a $3.78 million indicated loss.

INSURANCE ANALOGY

The aircraft manufacturing example is elementary. We discussed it because early property-casualty insurance net present value models overstated the returns by mistaken timing of the implied equity flows. A property-casualty insurance enterprise holds no asset called "inventory," but there is an equivalent equity flow stemming from regulatory constraints.[7]

Suppose an insurance policy is written at time t=0, a loss occurs at time t=1, and the loss is settled for $100,000 at time t=5.

- At time t=1, there is a debit on the income statement of $100,000 for the incurred loss. This accounting debit is a non-cash expenditure; it does not affect the cash flows of the firm. $100,000 of policyholders' surplus is transferred to loss reserves on the balance sheet. There is no change in the cash account on the asset side of the balance sheet.
- At time t=5, there is a cash outflow of $100,000 along with an offsetting non-cash reserve reduction of $100,000.

The cash flows noted above are the firm's cash flows, not the implied equity flows (or the free cash flows). They are not the proper base for the IRR or the NPV calculations. The proper perspective is that at time t=1, the insurance company purchases a loss reserve for $100,000. The equityholders no longer have access to these funds.

The loss reserve is like the inventory. It is an inventory of money, instead of an inventory of goods. But this money has changed from free cash that equityholders can use to a stock of funds that is not accessible to the equityholders.

Similarly, when the insurance company collects premium, it uses the funds to purchase an unearned premium reserve, as though it were purchasing a premium inventory. The premium collected is not a free cash flow.

IMPLIED EQUITY FLOWS

The implied equity flows are the implicit flow of funds to and from the suppliers of capital. The implied equity flows may be determined directly from the assets held by the company to support the insurance operations. The required assets comprise three pieces:

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[7] We use the term "implied equity flow" to emphasize that there is not – and there need not be – any actual flow of funds. The term "equityholders" refers to the firm's owners, whether they be common stock investors or other owners of a property-casualty insurance company.

- assets backing the (gross) unearned premium reserves
- assets backing the (full value) loss reserves
- assets backing policyholders' surplus

Because stockholder dividends from insurance enterprises are restricted by statutory accounting rules, statutory accounting determines the implied equity flows.

Illustration: A company begins operations on January 1, 20XX, and writes property-casualty insurance business during the year. The invested capital on December 31, 20XX, equals the sum of three components:

- the gross unearned premium reserves minus the present value of future losses and expenses stemming from unexpired policies
- the full value loss reserves minus the present value of future loss and loss adjustment expenses on claims that have already occurred
- policyholders' surplus to satisfy the NAIC's risk-based capital requirements and similar rating agency capital requirements.

If the company holds only financial assets, the fundamental equation linking implied equity flows and required assets is that

cash flows minus implied equity flows during the accounting period equal the change in required assets from the beginning to the end of the period.

This equation is not correct if the company holds non-financial assets, such as premium receivables and deferred tax assets. Beginning with the codification of statutory accounting in January 1, 2001, almost all companies hold substantial deferred tax assets, stemming from revenue offset and from IRS loss reserve discounting.

Illustration: An insurer writes workers' compensation policies during 20XX. At December 31, 20XX–1, the required assets are zero. During 20XX, it writes $100 million of business. During 20XX, the company collects all the premium, and it pays $25 million in expenses, $18 million in losses, and $5 million in federal income taxes. At year end, it has $50 million in unearned premium reserves and $45 million in full value loss reserves. The capital requirements at year end consist of $12 million in written premium risk charges and $15 million in reserving risk charges.

- The required assets at the end of the year are $50 million + $45 million + $12 million + $15 million = $122 million.
- The cash flows during the year are $100 million – $25 million – $18 million – $5 million = $52 million.

- The implied equity flow is $52 million – ($122 million – $0) = –$70 million.

The negative implied equity flow is a flow from the equityholders to the company. It represents an investment in the insurance operations by the equityholders.

Equity Flows in Practice

Some actuaries object that the equity flows are not real, based on the following reasoning:

> *We speak of an implied equity flow to fund the underwriting loss at policy inception. But there is no actual capital contribution when a policy is written. In contrast, the company cash flows used in other industries are actual transfers of cash.*

This objection is specious. The implied equity flows are real, though they are submerged under a multitude of policies and the other capital structure decisions of the company.

Illustration A: A reinsurer writes a $100 million book of casualty excess-of-loss reinsurance on January 1, 20XX. The risk-based capital requirements are $25 million for the written premium risk charge.

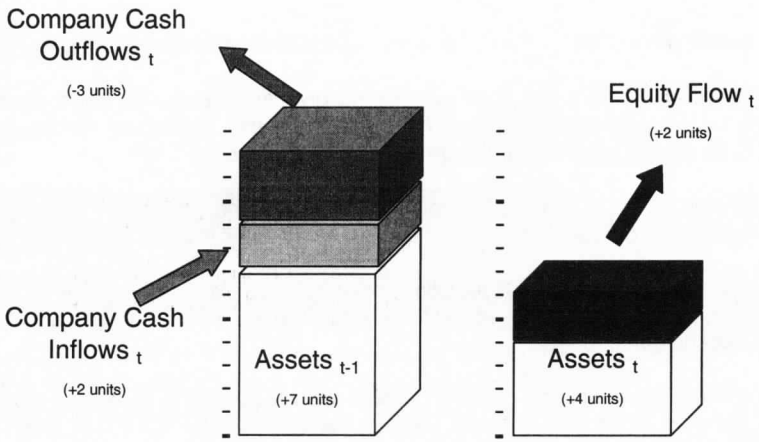
The pricing model uses an implied equity flow of $25 million on January 1. There is, of course, no actual cash flow. But on December 31, 20XX–1, the reinsurer's book of 20XX–1 policies expired, and $25 million of written premium risk capital was freed. The capital is transferred from one block of business to another block of business.

Illustration B: The reinsurer's premium volume increases from $80 million in 20XX–1 to $100 million in 20XX. The written premium risk charge increases from $20 million to $25 million, with a net capital contribution of $5 million. There may be no actual cash flow corresponding to this implied capital contribution. But the reinsurer may have decided not to write other business because of capital constraints, or it may have decided not to pursue other financial activities, such as acquisitions. These are the real world reflections of the implied equity flow.

Illustration C: The reinsurer's premium volume decreases from $120 million in 20XX–1 to $100 million in 20XX. The written premium risk charge decreases from $30 million to $25 million, with a net capital contribution of –$5 million. In this scenario, the reinsurer might pay greater stockholder dividends than it otherwise would have, it might buy back some stock, or it might use the capital to write other business or to engage in other financial activities.

The real world reflections of the implied equity flows may be slow, but eventually they are realized. The pricing model attributes these equity flows to the policies that require the capital.

Determination of the Equity Flow



Cash Flow Definition of Equity Flow:

$$\text{Equity Flow}_t = -\text{Asset Flow}_t + \text{Company Cash Flow}_t$$

$$\text{Company Cash Flow}_t = \text{Co Cash Inflow}_t - \text{Co Cash Outflow}_t$$

$$\text{Asset Flow}_t = \text{Assets}_{(t+1)} - \text{Assets}_t$$

Composition of Assets at time t is a function of business environment constraints:

$$\text{Assets}_t = \text{UEPR}_t + \text{Loss Reserve}_t + \text{Surplus}_t$$

Income Statement Definition of Equity Flow:

$$\text{Equity Flow}_t = \text{Accting Net Income}_t - \Delta\,(\text{Accting Capital})_t$$

For the above Illustration:

$$\text{Asset Flow}_t = (4-7) = -3$$

$$\text{Company Cash Flow}_t = \text{Co Cash Inflow}_t - \text{Co Cash Outflow}_t$$

$$= +2 - 3 = -1$$

$$\therefore\ \text{Equity Flow}_t = -(-3) + (-1) = 2$$

Insurance Transactions

The implied equity flows depend on the cash flows, statutory accounting rules, and capital requirements. The two illustrations below cover the major insurance transactions:

- premium writing
- premium collection
- loss incurral
- loss payment
- expense incurral and payment
- investment income
- federal income tax payments
- deferred tax assets and liabilities

The implied equity flow equals the statutory income minus the change in required capital (see also Robbin [1993; 1998], who uses the same perspective). We use statutory income, not GAAP income, since statutory income and capital requirements determine the funds that are available for distribution to owners.[8] The economic income (the NPV) to the equityholders from issuance of the insurance policy is the present value of the future implied equity flows.

An actual pricing model would use quarterly valuations for the lifetime of the book of business.[9] For this illustration, we use a simplified example with three years of semi-annual valuation periods. We provide item by item documentation, along with graphical depictions of the implied equity flows from premium and loss transactions.

Translating the cash flows, accounting requirements, and capital requirements into implied equity flows is the largest hurdle to proper use of the return on capital pricing models.[10]

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[8] The American Academy of Actuaries *Standard of Practice No. 19, Actuarial Appraisals*, paragraph 5.2.1, emphasizes the centrality of statutory accounting for modeling insurance company cash flows and equity flows: "Distributable Earnings – For insurance companies, statutory earnings form the basis for determining distributable earnings, since the availability of dividends to owners is constrained by the amount of accumulated earnings and minimum capital and surplus requirements, both of which must be determined on a statutory accounting basis. Distributable earnings consist of statutory earnings, adjusted as appropriate to allow for the retention of a portion thereof or the release of a portion of prior accumulated earnings therein, in recognition of minimum capital and surplus levels necessary to support existing business."

[9] Appendix B shows the modeled equity flows for a workers' compensation block of business, using 50 years of quarterly valuation periods, or a total of 200 periods.

[10] Some older papers on property-casualty insurance pricing models have used company cash flows instead of equity flows for the return on capital models. Modeling the cash flows instead of the equity flows provides distorted rate indications. Since the signs of the premium and loss equity flows are the opposite of the signs of the premium and loss cash flows, use of company cash flows instead of implied equity flows transforms an investment project into a borrowing transaction. For an investment project, higher IRR's are

This paper is both a template for the practicing actuary and a teaching text for the actuarial student. We do not simply state the results; we show the calculations step-by-step, so that readers can replicate the procedures.

The illustration here is clear, and the computations are straight-forward. Nevertheless, there are many figures, and it is easy to lose track of the relationships. To keep the intuition clear, we divide the illustration into two pieces. The first piece deals with premiums, expenses, and the associated investment income, federal income taxes, capital requirements, and implied equity flows. The second piece adds losses and the associated equity flows.

The documentation in this paper has three components: (i) textual exposition, (ii) numerical exhibits, and (iii) graphics. Readers may find it helpful to trace the figures in the exhibits and the graphics as they proceed through the text.

ILLUSTRATION A: PREMIUMS

A company writes and collects a $1,000 annual premium on December 31, 20XX. Acquisition expenses of $250 are incurred and paid on that day. Maintenance and general expenses of $150 are incurred and paid evenly over the policy term.

The pre-tax investment yield benchmark is an 8% per annum bond equivalent yield (semi-annual compounding). The marginal tax rate on both underwriting income and investment income is 35%.

No losses are incurred. The capital requirements are based on the NAIC risk-based capital formula. For this scenario, the capital requirements are 25% of annual written premium plus 15% of loss reserves.

We use the following modeling order:

a. underwriting cash flows: premiums, expenses, losses, and taxes on underwriting
b. statutory accounting entries: loss reserves and deferred tax assets
c. required surplus amounts: risk-based capital requirements
d. investment income on the investable assets
e. implied equity flows to and from the equityholders

The same sequence is used for the second half of the illustration as well, which deals with the loss cash flows and the associated components.

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better; for a borrowing transaction, lower IRR's are better.

VALUATION DATES

In theory, we should use continuous accounting changes, cash flows, and implied equity flows. Even if accounting entries are made at specified dates, such as the end of the year, insurance contracts are written throughout the year. An individual policy may have discrete accounting entries, but a policy year has continuous entries, cash flows, and equity flows.

In practice, we use quarterly valuation periods. Many readers find discrete entries simpler than continuous functions. Spreadsheet representations are also easier with discrete entries. The enhanced accuracy from continuous functions is outweighed by the added complexity.[11]

For the illustration here, we use semi-annual valuations, not quarterly valuations. This simplifies the exposition while retaining the structure of the analysis.

Premiums and Premium Receivables

A premium inflow of $1,000 occurs on December 31, 20XX. This illustration assumes full premium collection on the effective date. If the premium is not fully collected on December 31, 20XX, the company shows a premium receivable for the uncollected portion.

- If the premium receivable is an admitted asset, there is no effect on the implied equity flows.
- Any *non-admitted* premium receivable increases the implied equity outflow from the equityholders.
- The premium collection pattern affects the investment income cash flows, even if all premium receivables are admitted. The investment income cash flows affect the implied equity flows in subsequent periods.

For blocks of business with significant deferral of premium collection, such as large account workers' compensation policies, the actuary may estimate the expected non-admitted portion of the premiums receivable asset and increase the required capital contribution.[12]

DISTRIBUTION SYSTEMS AND ACQUISITION EXPENSES

Companies use a variety of distribution systems, such as independent agency, direct writing, salaried sales force, and mass marketing systems. Within each system, there are different

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[11] Pricing models using continuous functions have often been proposed for life insurance contracts, though generally not for property-casualty insurance contracts.

[12] The statutory rules for the non-admitted portion of the premiums receivable are summarized in Feldblum [2002: SchP].

methods of premium billing. The modeling process for premium collection and acquisition expenses depends on the distribution system and the premium billing system.

- For independent agency companies, acquisition costs are primarily agents' commissions. These expenses are incurred on the policy writing date or on the premium collection date.
- For direct writers and for companies with salaried sales forces, acquisition costs include advertising expenses and fixed costs of the agency system. These additional expenses occur before the policy effective date.
- For commercial lines companies writing large accounts, acquisition costs include the costs of developing sales proposals and of soliciting business. These expenses also occur before the policy effective date.

The pricing actuary should use the policy distribution system and the premium billing system consistent with actual company practice.

The illustration in the text uses the independent agency distribution system, with which most readers are familiar. An acquisition expense outflow of $250 occurs on December 31, 20XX. The independent agency distribution system is particularly useful for the exposition in this paper, since it has a large initial underwriting loss, causing a large difference between the company cash flows and the implied equity flows.

General Expenses

A general expense outflow of $150 occurs evenly over the policy term. This can be modeled

- as a single $150 outflow on June 30, 20XX+1
- as three semi-annual outflows of $50 on each half year valuation date from December 31, 20XX, through December 31, 20XX+1.
- as five quarterly outflows of $30 on each quarterly valuation date from December 31, 20XX, through December 31, 20XX+1.

Common actuarial practice is to use quarterly cash flows and equity flows, at least for the first several years. For ease of exposition, this illustration uses a single expense payment at June 30, 20XX+1, not quarterly payments.

Many actuaries model some general expense costs at the policy effective date. For instance, the NAIC defines pre-paid acquisition expenses as commissions, other acquisition expenses, premium taxes, and one half of general expenses (see Feldblum [1997: IEE]). Alternatively, the general expenses incurred on the policy effective date may be included with acquisition costs. This illustration assumes that the $250 of pre-paid acquisition costs includes the general expenses that are incurred on the effective date of the policy.

TAX ON UNDERWRITING INCOME

Taxable underwriting income equals

written premium
- underwriting expenses
- 80% of the change in the gross unearned premium reserve
- paid losses
- the change in the discounted loss and loss adjustment expense reserves.

This may also be written as

statutory earned premium
+ 20% of the change in the statutory unearned premium reserve
- underwriting expenses
- statutory incurred losses
+ the change in the IRS loss and loss adjustment expense reserve discount.

Taxable underwriting income for 20XX is

$200 (income from revenue offset) – $250 (acquisition expenses) = –$50.

The tax outflow is a negative $17.50 (or a tax refund of $17.50). The "tax refund" does not rely on tax carrybacks or carry-forwards. The tax refund stemming from negative taxable income offsets tax liabilities stemming from other insurance contracts and from investment income. We chose an illustration with an acquisition expense greater than 20% of premium to emphasize that expected tax cash flows can be positive or negative.

The taxable premium income of $200 may be evaluated in two ways.

- As written premium minus 80% of the change in the unearned premium reserves = $1000 – 80% × $1000 = $200
- As statutory earned premium income plus 20% of the change in the unearned premium reserves = $0 + 20% × $1000 = $200.

The tax liability is 35% times the taxable income: 35% × ($200 – $250) = –$17.50.

Taxable underwriting income for 20XX+1 equals

$800 of taxable premium income – $150 of general expenses = $650.

The tax liability is \$650 x 35% = \$227.50. Written premium during the year is \$0 and the unearned premium reserve declines from \$1,000 to \$0. We use the same two computation methods:

(i) \$0 – 80% x (–\$1,000) – \$150 = \$650
(ii) \$1,000 + 20% x (–\$1,000) – \$150 = \$650.

We use semi-annual valuations for this illustration, and we assume the tax on underwriting income is incurred evenly between the two halves of the year, or \$227.50 / 2 = \$113.75 in each half year. Alternatively, if all general expenses are assumed to be paid exactly on June 30, 20XX+1, the underwriting income is \$400 – \$150 = \$250 in the first half of the year and \$400 – \$0 = \$400 in the second half of the year.

For the exposition, we model the tax cash flows stemming from premium collection separately from the tax cash flows stemming from incurred losses. We do this to clarify the tax liabilities and the deferred tax assets; there is no qualitative difference in the cash flows.[13]

We have not yet included the investment income cash flows, since these depend on statutory accounting constraints and on the capital requirements.[14]

STATUTORY ACCOUNTING ENTRIES

The following statutory accounting entries are relevant for the implied equity flows:

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[13] In contrast, Myers and Cohn distinguish between the taxes stemming from premium earning and those stemming from loss accruals because they use different capitalization rates to determine the present values of each. See Myers and Cohn [1987] as well as the discussion in Feldblum, [2003: *PCAS* d/d discussion].

[14] In the past, some casualty actuaries differentiated between investment income from policyholder supplied funds and investment income from equityholder supplied funds; cf Bailey [1967]. The investment income on policyholder supplied funds depends on the underwriting cash flows; the investment income on equityholder supplied funds depends also on the accounting entries and the capital requirements. The rationale was that policyholders were entitled to the investment income on their own funds but they were not entitled to the investment income on capital and surplus funds.

Although this distinction is not relevant to return on capital pricing models, it is useful for modeling the source of profits. If premiums are exactly adequate, the profit in the policyholder supplied funds (sometimes called policyholder supplied capital) is needed to fund the difference between the cost of equity capital and the after-tax investment yield on equityholder supplied capital. Myers and Cohn [1978] have a similar perspective, though they fail to take into account the equityholder supplied capital embedded in the statutory loss reserves and the gross unearned premium reserves.

- A unearned premium reserve of $1000 is set up on December 31, 20XX. It is amortized ratably as the insurance protection is provided. For most types of business, the amortization is even over the policy term.[15]

- Surplus of $250 is added to the balance sheet on December 31, 20XX, and it is removed on December 31, 20XX+1. The surplus requirement is *not* amortized over the policy term. This is a consequence of the NAIC risk-based capital formula and the corresponding rating agency formulas. The illustrations evaluate capital requirements by applying the risk-based capital charges to the written premium for the year and to the loss reserves at the valuation date.[16]

- A deferred tax asset of $200 x 35% = $70 stemming from the revenue offset provision is entered on the balance sheet on December 31, 20XX, and it is amortized over the course of the policy term. The full deferred tax asset from revenue offset is recognized on the statutory balance sheet, since it reverses within 12 months of the balance sheet date (for annual policies).[17]

IMPLIED EQUITY FLOW AT POLICY INCEPTION

We discuss the implied equity flows in more detail further below. The implied equity flows affect the investment income, and the investment income affects the implied equity flows. We show the implied equity flow at policy inception before discussing the investment income for the first half of the year to clarify the inter-relationship of these items.

The required assets of the company on December 31, 20XX, equal the $1,000 of unearned premium reserve plus the $250 of required surplus, or $1,250. The company holds statutory assets equal to the following:

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[15] See SFAS 60 and SSAP No. 65. Exceptions occur for certain lines of business. (a) Workers' compensation premiums may be earned when billed for statutory accounting purposes, though not for tax purposes; see SSAP No. 53, "Property-Casualty Contracts – Premiums," paragraph 4 and IRS tax regulation 2001 FED ¶ 26,153, §1.832-4, sections (a)(4) and (a)(5). Companies which use different statutory accounting and tax accounting procedures for recording workers' compensation premium must use two sets of premium writing patterns in the pricing model. We do not show these scenarios in the exhibits. (b) Product warranty unearned premium reserves have more complex computations; see SSAP No. 65, "Property and Casualty Contracts," paragraphs 21-33, and Hayne [1999].

[16] Atkinson and Dallas [2000], chapter 8, use a slightly different procedure. They determine the risk-based capital requirements at year-end dates and they discount at the after-tax investment yield to the beginning of the year.

[17] See SFAS 109 for a general discussion of deferred tax assets and liabilities and SSAP No. 10, "Income Taxes," paragraph 10, for the statutory accounting rules on recognition of deferred tax assets. Appendix A of this paper reviews the post-codification statutory accounting rules for deferred tax assets and liabilities.

+ $1000.00 in cash (written premium collected)
− $ 250.00 in cash (acquisition costs paid)
+ $ 17.50 in cash (tax refund)
+ $ 70.00 (deferred tax asset, which is an admitted non-cash asset)
= $ 837.50.

The remaining capital needed to fund the required assets is provided by equityholders: $1250 − $837.50 = $412.50. We assume that all capital supplied by equityholders is investable.

INVESTMENT INCOME

The invested assets on December 31, 20XX, are $750 (net premium) + $412.50 (capital) + $17.50 (tax refund) = $1,180.00. The deferred tax asset of $70 is not investable.

Equivalently, the total assets required are $1,000 for the unearned premium reserve and $250 in surplus, or $1,250. Subtracting the $70 deferred tax asset, which is not investable, gives $1,180 of investable assets.

- For this illustration, we use an 8% per annum bond equivalent yield with semiannual evaluations. New money investment yields are often given in bond equivalent form.
- The investment income is received on June 30, 20XX+1, and December 31, 20XX+1, with a yield of 4% per half-year.
- The unearned premium reserve declines to $500 on June 30, 20XX+1, and to $0 on December 31, 20XX+1.
- The deferred tax asset declines to $35 on June 30, 20XX+1, and to $0 on December 31, 20XX+1. The change in the deferred tax asset reflects tax payments and refunds in 20XX+1.

The investment income during the period is the product of (i) the assets required at the beginning of the period minus the amount of non-investable (but admitted) assets and (ii) the investment yield during the time period.

Illustration: The investment income earned during the first half of 20XX+1 is 4% × $1,180 = $47.20. The investment income is assumed to be received on June 30, 20XX+1.

The assets required on July 1, 20XX+1, are $500 of unearned premium reserve plus $250 of required capital = $750. The non-investable deferred tax asset is $500 × 20% × 35% = $35, and the investable assets are $750 − $35 = $715. The investment income earned during the second half of 20XX+1 and received on Dec 31, 20XX+1, is $715 × 4% = $28.60.

Investment Income and Underwriting Expenses

For determining federal income taxes on underwriting income, we subtracted underwriting expenses. For determining investment income, we have *not* subtracted the underwriting expenses. At first glance, this appears incongruous, since money paid out as expenses is not available for investment. It might seem that the investable assets should also be reduced for underwriting expenses, just as the underwriting income is reduced for expenses.

The reasoning above is mistaken; it is mentioned here to clarify the equity flow modeling. The investment income is based on the investment yield times the investable assets. The investable assets are based on the statutory reserves and the required capital. Any expenses paid with policyholder supplied funds are replenished with equityholder supplied funds, since the company must hold the gross unearned premium reserve. Expense payments do not affect investable assets in the pricing model.

Rather, an expense outflow causes a federal income tax *inflow* and an implied equity *outflow*.

- The federal income tax *inflow* offsets 35% of the expense paid from the policy premium.
- The implied equity *outflow* offsets the other (1 – 35%) of the expense paid. The implied equity outflow is an investment in the insurance project, so it is modeled as a negative number. The equity outflow is an inflow to the company.[18]

The net change in the company's assets is $0: expense outflow – 35% federal income tax inflow – 65% implied equity flow into the company.

In practice, the amortization of the unearned premium reserves is offset by the accrual of loss reserves. The full pricing model considers premium writing patterns, premium collection patterns, loss accrual patterns, and loss payment patterns. To simplify the exposition, we have separated the premium section from the loss section. This highlights the equity inflows as the premium is earned and the equity outflows as losses are incurred. The numerical exhibits at the end of this paper show the combined premium and loss transactions.

In a companion paper ("Income Recognition and Performance Measurement"), we show net income under different accounting frameworks: statutory, GAAP, tax, fair value, net present value, and internal rate of return. No matter what accounting framework is used to measure income and management performance, the *statutory* accounting framework determines the capital contributions (cf. Atkinson and Dallas [2000], chapter 11).

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[18] The premium is a pre-tax cash flow, so there is an offsetting tax liability or return. An equity flow is an after-tax flow, so there is no offsetting tax liability or return.

TAXES ON INVESTMENT INCOME

We examined earlier the federal income taxes on the net premium income. These are $113.75 each half year, payable on June 30, 20XX+1, and on December 31, 20XX+1; see page ?. Federal income taxes on investment income are (i) $47.20 x 35% = $16.52, paid on June 30, 20XX+1, and (ii) $28.60 x 35% = $10.01, paid on December 31, 20XX+1.

Some analysts presume that taxes are not paid until March or April of the following year. That is not correct. The federal income tax liability is computed for the tax year as whole, but payments to the U.S. Treasury are made quarterly in advance. For simplicity, this illustration shows payments on June 30, 20XX+1, and December 31, 20XX+1. The payment date for the tax liabilities is actually earlier than modeled here, not later.[19]

Although the taxes are paid earlier, the premium is also earned earlier, since it is earned evenly over the policy term. As long as the timing of the premium earning and the federal income taxes on the premium income are consistent, the model is not materially biased.

IMPLIED EQUITY FLOWS

The implied equity flow on any valuation date is (i) a capital distribution if it is positive or (ii) a capital contribution if it is negative. A capital distribution may be a stockholder dividend or a stock repurchase. Since common stocks are cash equivalents, a capital gain – even if unrealized – is also a capital distribution. Shareholders can sell some of their shares to produce a virtual dividend payment.[20]

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[19] The method used in the text is equivalent to using an after-tax investment yield; cf. Atkinson and Dallas [2000], chapter 8. Some pricing models assume that tax liabilities for the current year are paid evenly during the year. The total investment income for the year is $47.20 + $28.60 = $75.80 and the tax is $75.80 x 35% = $26.53. If we assume that taxes are paid evenly between the two halves of the year, the tax payments at each semiannual valuation date are $26.53 ÷ 2 = $13.265.

The IRS allows taxpayers some leeway in the quarterly tax estimates, and companies differ in the timing of their tax payments. Modeling the precise tax payment stream is complex. It does not have a material effect on the rate indications, as long as reasonable assumptions are used.

The procedure in the text of the paper splits the tax on underwriting income evenly between the two halves of the year, but it computes the tax on investment income based on the investment income earned in each half of the year. It is difficult to quantify the amortization of the loss reserve discount between the two halves of the year, so we use an even spread. In contrast, using an after-tax investment yield is not difficult.

[20] The choice between paying dividends and allowing the capital to accumulate in the company depends on the investment opportunities of the company and the tax situation of the equityholders. This choice affects the personal tax liabilities of the investors and cost of holding capital. In practice, it is difficult to model personal income taxes, since they vary with the tax situation of the equityholders.

Illustration: The ABC Insurance Company has 10 million shares outstanding and a market value of $500 million on January 1, 20XX. The stock price is $50 per share. The company earns $50 million in 20XX, and it has a market value of $550 million on December 31, 20XX.

- *Scenario A:* The company pays a stockholder dividend of $5 per share on December 31, 20XX. The market value of the company declines to $500 million after the dividend. The stockholder dividend is the implied equity flow on December 31, 20XX.

- *Scenario B:* The company pays no stockholder dividend. The market value remains $550 million on December 31, 20XX. Since the common stock is a cash equivalent, which can be sold in the open market, the liquid assets of the owners increase by $5 per share. The capital accumulation is the implied equity flow on December 31, 20XX.

We may conceive of the implied equity flow from a balance sheet perspective or from an income statement perspective.

- *Balance sheet perspective:* At valuation dates (that is, between valuation periods), excess assets are distributed to equityholders, and insufficient assets are augmented by equityholder contributions. At the start of each valuation period, the held assets equal the required assets. The required assets are the sum of the liabilities and the required capital.

  During the valuation period, there are two types of changes in the balance sheet entries: (i) cash inflows and cash outflows affect investable assets, and (ii) non-cash increases and non-cash decreases affect non-cash assets and liabilities. The capital requirements may also change from one valuation date to the next valuation date.

  To determine the implied equity flow at any valuation date, we begin with the balance sheet entries at the start of the valuation period and we adjust them for both the cash inflows and outflows and the non-cash increases and decreases, including any change in capital requirements. The adjusted assets at the end of the period minus the required assets at the beginning of the period is the implied equity flow at the end of the period.

- *Income statement perspective:* The implied equity flow at valuation date "*t*" equals the statutory income during the period from "*t*–*1*" to "*t*" minus the change in required capital during this period. A direct charge or credit to surplus at the valuation date is treated as a component of statutory income.

The two definitions are equivalent. We show both methods to determine the implied equity flow on June 30, 20XX+1.

BALANCE SHEET PERSPECTIVE: The assets held at the beginning of the valuation period (on January 1, 20XX+1) are the required assets at that date. This equals $1250, which is the sum

of the liabilities (the unearned premium reserve) and the required capital (the written premium risk charge). The $1250 consists of $1180 of investable assets and a $70 deferred tax asset.

The investable assets on the balance sheet are accumulated for investment income; the non-investable assets are capitalized or amortized.

- Cash inflow for investment income ($47.20 during the valuation period).
- Cash outflows for expenses ($150 in maintenance expenses) and tax accruals ($113.75 tax accrual for underwriting income; $16.52 tax accrual for investment income).[21]
- Amortization of the unearned premium reserve and of the deferred tax asset stemming from revenue offset ($500 amortization of the unearned premium reserve; $35 amortization of the deferred tax asset).

The loss reserves section of this illustration shows the capitalization of the incurred losses during the policy term and the capitalization and amortization of the deferred tax assets through the lifetime of the claims.

The accumulated assets at June 30, 20XX+1 equal the required assets at the beginning of the period adjusted for these cash flows and the non-cash increases and decreases:

Investable assets on December 31, 20XX	=	$1180.00
Investment income (cash inflow)	=	+$ 47.20
General expenses (cash outflow)	=	–$150.00
Federal income taxes on underwriting income (cash outflow)	=	–$113.75
Federal income taxes on investment income (cash outflow)	=	–$ 16.52
Deferred tax asset (non-cash asset)	=	+$ 35.00
Total	=	+$981.93

The required assets on June 30, 20XX+1, equal the statutory reserves plus the required capital. The unearned premium reserves are now $500 and the required capital remains $250, for a total of $750. The implied equity flow to the equityholders on June 30, 20XX+1, is $981.93 – $750.00 = $231.93.

INCOME STATEMENT PERSPECTIVE: The statutory income during the first half of 20XX+1 is shown below. Direct charges or credits to surplus, such as the change in the deferred tax asset, are included with statutory income.

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[21] The tax payments are computed separately for underwriting income and investment income:

- underwriting income: $227.50 for the full year, or $113.75 for each half year, and
- investment income: $16.52 for the first half year and $10.01 for the second half year

Earned premium	=	+$500.00
Investment income	=	+$ 47.20
General expenses	=	–$150.00
Federal income taxes on underwriting income	=	–$113.75
Federal income taxes on investment income	=	–$ 16.52
Change in deferred tax asset	=	–$ 35.00
Total	=	+$231.93

There is no change in the required capital, so the implied equity flow is +$231.93. The two methods are alternative ways of describing the cash flows and balance sheet changes.

We calculate the implied equity flows for the second half of the year in the same two manners. For heuristic purposes, we show a third method below. A change in the implied equity flows between the first half of the year and the second half of the year stems from changes in statutory income or changes in capital requirements. We list these differences below.

a. General expense payments are $150 on June 30, 20XX+1, and $0 in the second half of the year.[22]
b. Investment income is $47.20 in the first half of the year and $28.60 in the second half of the year. The difference in the investment income reflects the difference in the investable assets.
c. The federal income tax on investment income is $16.52 in the first half of the year and $10.01 in the second half of the year.
d. Required capital declines to $0 on December 31, 20XX+1, since we have not yet included losses in the illustration.
e. The tax liability and the amortization of the deferred tax asset are spread evenly over the two halves of the year.

As an alternative to the full calculation, we adjust the implied equity flow from June 30, 20XX+1 with these differences to get the implied equity flow on December 31, 20XX+1:

Equity flow on June 30, 20XX+1	=	+$231.93
Difference in general expenses	=	+$150.00
Difference in investment income	=	+($28.60 – $47.20)
Difference in federal income taxes on investment income	=	–($10.01 – $16.52)
Difference in surplus change	=	+$250.00
Equity flow on December 31, 20XX+1	=	+$619.84

[22] This difference stems from the modeling assumptions. In truth, the maintenance expenses are incurred evenly over the year, but we assume a single payment date to simplify the modeling. For the federal income tax payments on underwriting income, we implicitly assumed that the maintenance expenses are incurred evenly over the course of the year, and we spread the federal income tax on underwriting income evenly over the year.

We show these adjustments to highlight the sources of the implied equity flows. This analysis is helpful for judging the reasonableness of the implied equity flows in each period. The actual pricing model performs separate calculations for each valuation date.[23]

The accompanying graphic shows the cash flow view of the implied equity flows. The following table shows the income statement view of the implied equity flows. These views are various perspectives of the same phenomenon.

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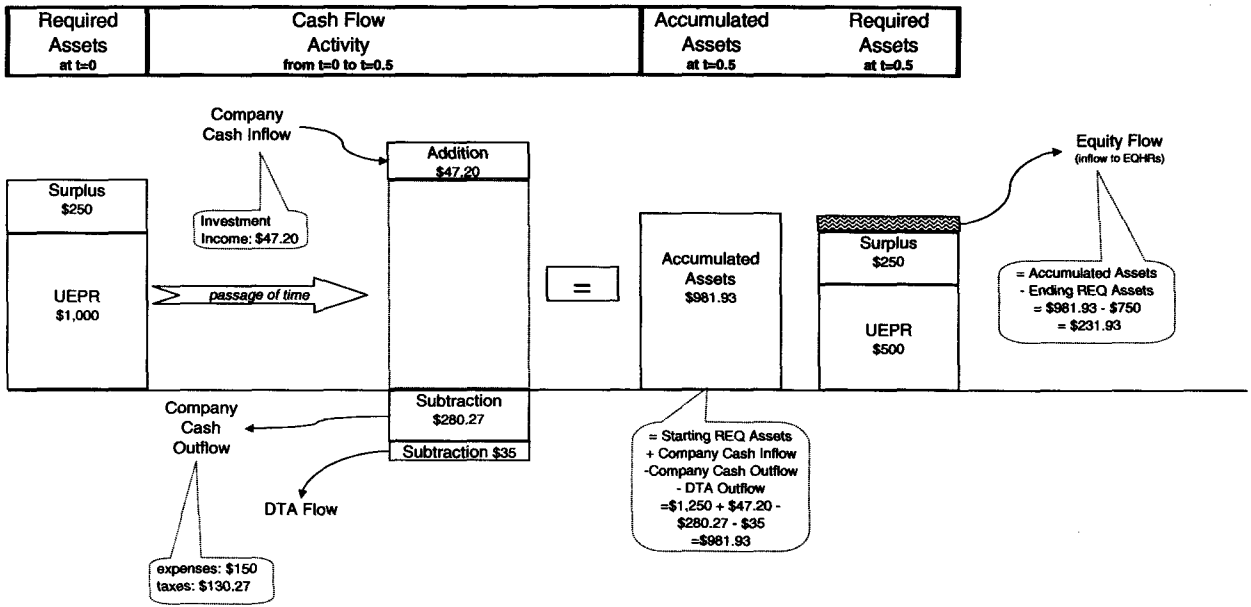
[23] Instead of using values at the end of the period, some actuaries use average values for each entry and implied equity flows in the middle of the valuation period. To do this, we amortize the unearned premium reserve and the deferred tax asset evenly over the policy term. The invested assets decline from $1180 on December 31, 20XX, to $250 on December 31, 20XX+1. The average invested assets are ($1180 + $250) / 2 = $715.00, and the investment income is $715.00 x 8% = $57.20. This is less than the 20XX investment income of $47.20 + $28.60 = $75.80 in the illustration. The lower investment income is offset by the earlier implied equity flows in the alternative model, and there is no material change in the model results.

Cash Flow View of the Equity Flow



Income Statement View of Equity Flows

STATEMENT OF INCOME	0.00	0.50	1.00
Earned Premium		500	500
Undiscounted Incurred Loss		0	0
Paid Taxes		130.27	123.76
Undiscounted Incurred Expenses		150	0
Incurred Costs		280.27	123.76
Investment Income		47.2	28.6
Change in DTA		-35	-35
Net Income		231.93	369.84
CAPITAL and SURPLUS ACCOUNT			
Surplus, end of previous period		250	250
GAINS (and LOSSES) IN SURPLUS			
Net Income		231.93	369.84
Surplus Adjustments (Capital Contributions)		0	-250
Surplus, end of current quarter		250	0
Implied Equityflow		231.93	619.84

CASH FLOWS AND EQUITY FLOWS: PREMIUM AND EXPENSE TRANSACTIONS

The exposition above traces the cash flows and equations by type of transaction. We put the pieces together to show the cash flows and equity flows by valuation date. The four accompanying schematics show the cash flows and equity flows for *Illustration A: Premium and Expense Transactions*. The first schematic shows a summary of the cash flows and equity flows at the three valuation dates relevant to Illustration A. The next three schematics show more detailed information about the cash flows and equity flows at these valuation dates:

- t = 0.0, or December 31, 20XX–1
- t = 0.5, or June 30, 20XX
- t = 1.0, or December 31, 20XX

Each schematic shows transactions among the following seven nodes.

f. *Insurer:* All transactions pass through the insurance company and are taxed at a 35% rate. This causes the multiple layers of taxation of the profit margin in the policy premium.
g. *U/W:* All underwriting flows – premium collections, loss payments, and expense payments – are between the insurance company and the rectangle labeled *U/W*.
h. *IRS:* Tax payments are flows from the insurance company to the IRS; tax refunds are flows from the IRS to the insurance company.
i. *Assets:* To track the flow of funds, we imagine that there is a fiduciary handling the insurer's assets. All cash received by the insurer is sent to the rectangle labeled "assets."
j. *DTA:* The insurance company holds several types of non-cash assets, the most important of which is the deferred tax asset (DTA). There is no cash flow or equity flow underlying the non-cash assets. However, an increase in a non-cash asset causes an equal and offsetting reduction in the implied equity flows. An increase in a non-cash asset, such as a deferred tax asset, is shown as though it were a flow from the *DTA* rectangle to the *Assets* rectangle.
k. *FinMkts:* All cash assets are invested at the benchmark investment yield. The receipt of investment income by the insurance company is shown as a cash flow from the rectangle labeled *FinMkts* to the *Insurer*.
l. *Eqhr:* If the insurer needs more assets than it has from its underwriting and investment operations, the equityholders provide additional capital. If the insurer has excess assets, it distributes the excess capital to its equityholders.

ILLUSTRATION A (Case: No Losses) CASH FLOWS

	Time	0.0	0.5	1.0	1.5	2.0	2.5	3.0
	UW TRANSACTIONS							
(1)	Premium	1,000.00	0.00	0.00	0.00	0.00	0.00	0.00
(2)	Expense - Acquisition	250.00	0.00	0.00	0.00	0.00	0.00	0.00
(3)	Expense - General	0.00	150.00	0.00	0.00	0.00	0.00	0.00
(4)	Loss	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	OTHER							
	DTA							
(5)	Due to Revenue Offset	70.00	35.00	0.00	0.00	0.00	0.00	0.00
(6)	Due to Loss Reserve Discountin	0.00	0.00	0.00	0.00	0.00	0.00	0.00
(7)	TOTAL	70.00	35.00	0.00	0.00	0.00	0.00	0.00
	RESERVES							
(8)	UEPR	1,000.00	500.00	0.00				
(9)	Stat Loss Reserve		0.00	0.00	0.00	0.00	0.00	0.00
(10)	IRS Discount Factors			0.86		0.88		0.90
(11)	Tax Basis Loss Reserve			0.00		0.00		0.00
(12)	Surplus	250.00	250.00	0.00	0.00	0.00	0.00	0.00
(13)	**Total Assets**	1,250.00	750.00	0.00	0.00	0.00	0.00	0.00
(14)	**Investable Assets**	1,180.00	715.00	0.00	0.00	0.00	0.00	0.00
	TAXES							
(15)	Tax Basis UW Revenue	200.00		800.00	0.00	0.00	0.00	0.00
(16)	Tax Basis Expenses	250.00		150.00		0.00		0.00
(17)	Tax Basis Inc Loss	0.00		0.00		0.00		0.00
(18)	Tax Basis Inv Income	0.00	47.20	28.60	0.00	0.00	0.00	0.00
	Valuation of Taxes							
(19)	Tax on UW Income (Annual)	-17.50		227.50		0.00		0.00
(20)	Tax on UW Income (Semi-Annual Pa	-17.50	113.75	113.75	0.00	0.00	0.00	0.00
(21)	Tax on Invest Income	0.00	16.52	10.01	0.00	0.00	0.00	0.00
(22)	Total Semi-Annual Payment	-17.50	130.27	123.76	0.00	0.00	0.00	0.00
	CASH FLOWS							
(23)	Asset Flow	1,250.00	-500.00	-750.00	0.00	0.00	0.00	0.00
(24)	UW Flow	750.00	-150.00	0.00	0.00	0.00	0.00	0.00
(25)	Inv Inc Flow		47.20	28.60	0.00	0.00	0.00	0.00
(26)	Tax Flow	17.50	-130.27	-123.76	0.00	0.00	0.00	0.00
(27)	DTA Flow	70.00	-35.00	-35.00	0.00	0.00	0.00	0.00
(28)	Equityflow	-412.50	231.93	619.84	0.00	0.00	0.00	0.00

U/W Assumptions	Surplus Assumptions	Results
Target Return on Capital = 12.0%	Premium Leverage Ratio = 25%	IRR on Equityflows (annual rate) = 136.8%
Invest Rate of Return = 8.0%	Reserve Leverage Ratio = 15%	
Premium = $1,000		Economic Value Added = $360.08
Dollars of Ultimate Loss = $0		
Combined Ratio = 40%		
Loss Ratio = 0%		

Formulae For Exhibit 1A

CASE: No Losses

$(5)_t = 0.35 * 0.2 * (8)_t$
$(7)_t = (5)_t + (6)_t$

$(12)_t = \quad 0.25^* \text{ WP} \quad \text{for } t = 0,0.5$
$(13)_t = (8)_t + (9)_t + (12)_t$
$(14)_t = (13)_t - (7)_t$

$(15)_t = (1)_t - [\,(8)_t - (8)_{t-1}\,]$
$(16)_t = (2)_t + (2)_{t-0.5} + (3)_t + (3)_{t-0.5}$
$(18)_t = (14)_{t-0.5} * \text{interest rate}$
$(19)_t = [\,(15)_t - (16)_t - (17)_t\,] * 0.35$
$(20)_t = \quad (19)_t \text{ for } t = 1, 2, 3$
$(20)_{t-0.5} = \quad (19)_t \text{ for } t = 0.5, 1.5, 2.5$
$(21)_t = (18)_t * 0.35$

$(23)_t = (13)_t - (13)_{t-0.5}$
$(24)_t = (1)_t - (2)_t - (3)_t - (4)_t$
$(25)_t = (14)_{t-0.5} * \text{interest rate}$
$(26)_t = (22)_t$
$(27)_t = (7)_t - (7)_{t-0.5}$
$(28)_t = -(23)_t + (24)_t + (25)_t + (26)_t$

Exhibit 1C

ILLUSTRATION A (Case: No Losses) Summary of Cash Flows

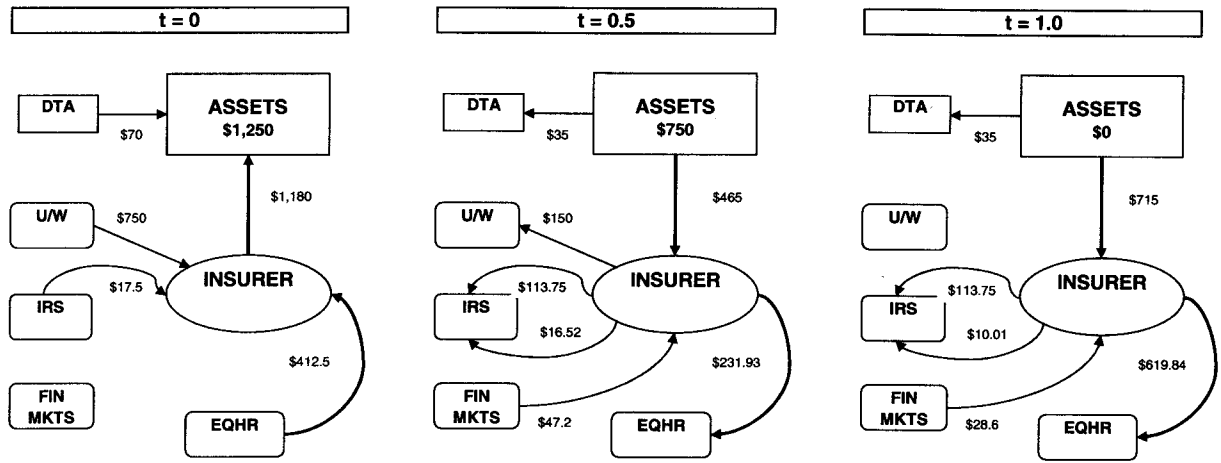


ILLUSTRATION A (No Losses Incurred)

Exhibit 1D

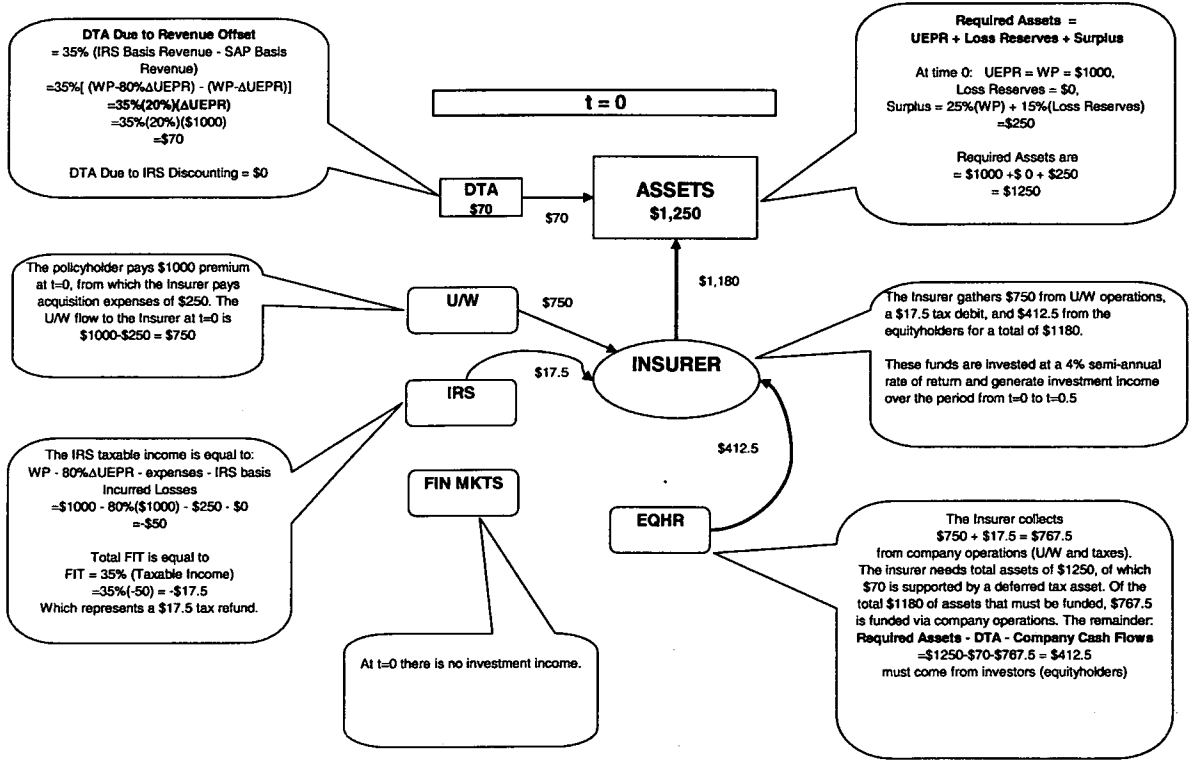


ILLUSTRATION A (No Losses Incurred)

Exhibit 1D

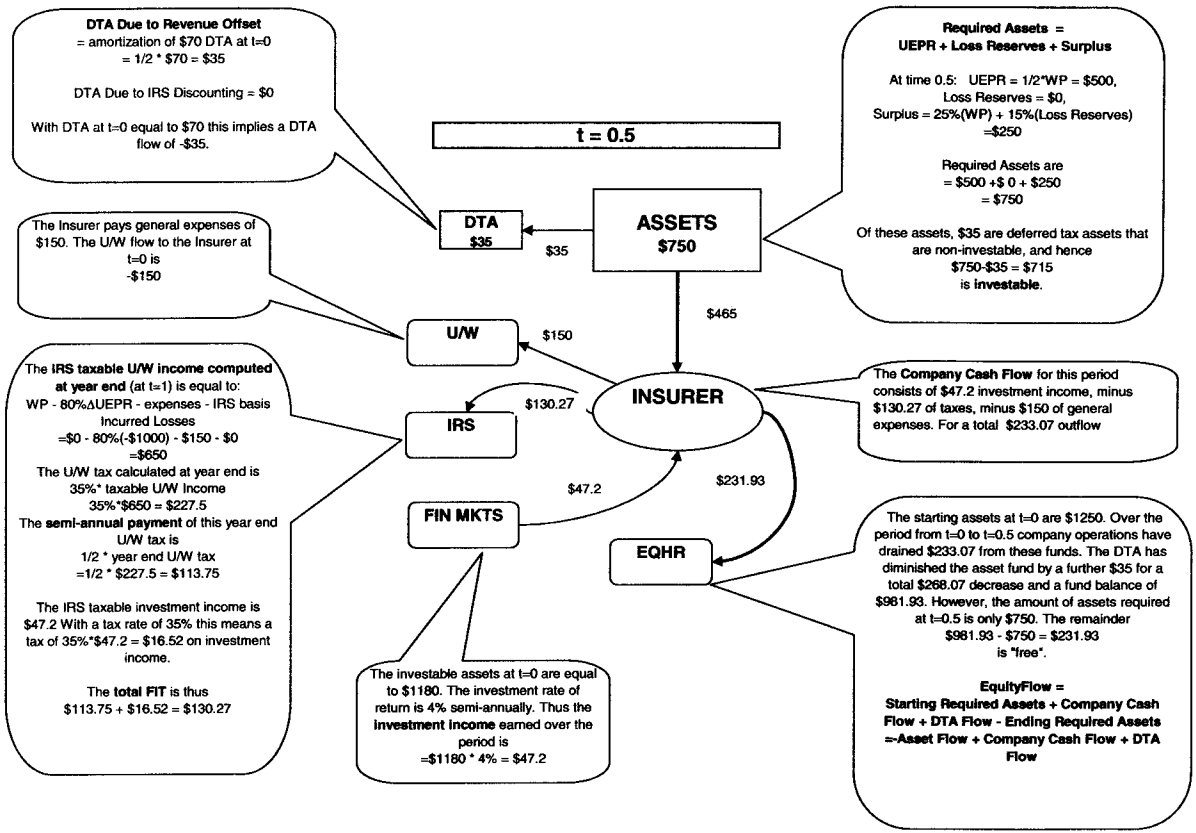


ILLUSTRATION A (No Losses Incurred)

Exhibit 1D

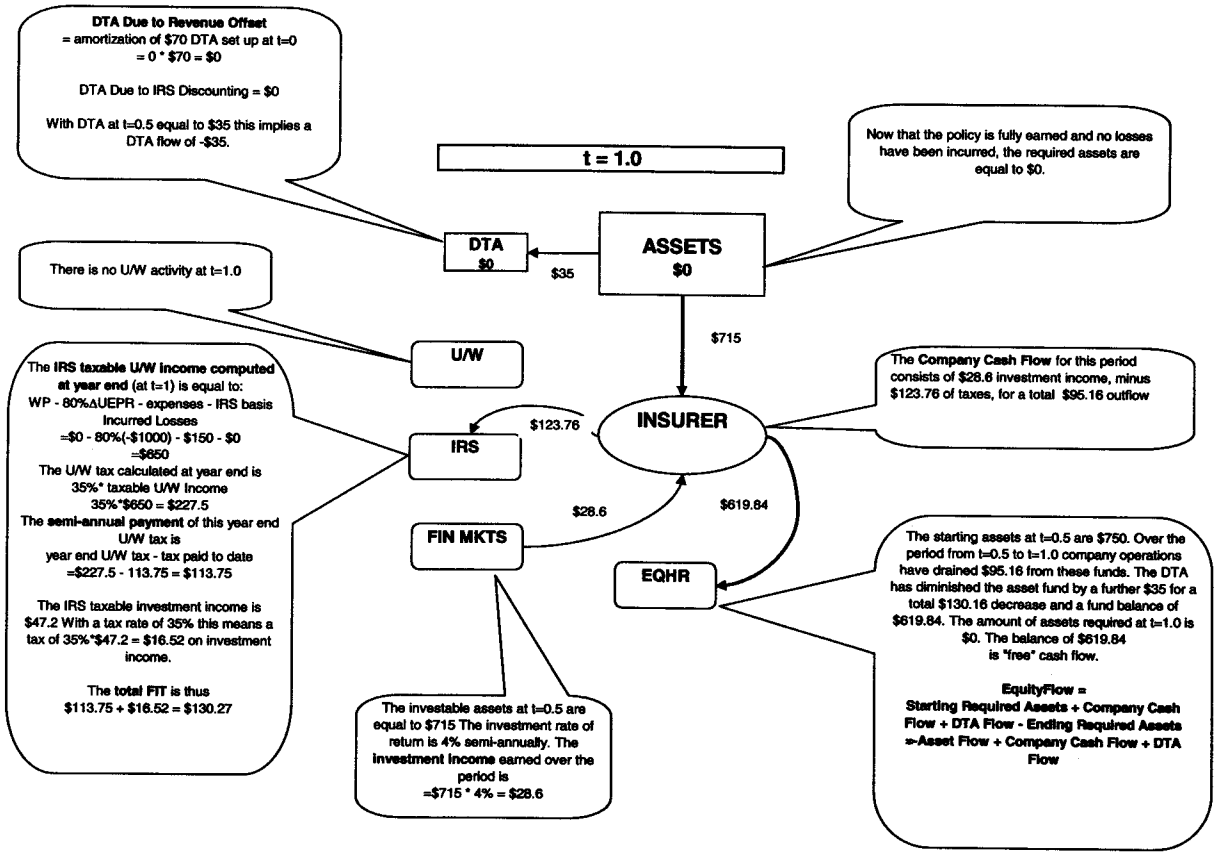


ILLUSTRATION B – LOSS TRANSACTIONS

We retain the scenario in the previous illustration, and we add losses incurred evenly during the policy term and paid over several years. To simplify the illustration, we model the losses as if there were two losses with ultimate values of $400 each occurring on June 30, 20XX+1, and December 31, 20XX+1. Both losses are paid on December 31, 20XX+3. Tax rates and capital requirements are the same as before.

There are several additional cash flows: a loss payment on December 31, 20XX+3 of $800, federal income tax payments or refunds for two and a half years between June 30, 20XX+1, and December 31, 20XX+3, and investment income over the same time period. There are non-cash changes to the balance sheet for loss reserves and for deferred tax assets stemming from IRS loss reserve discounting. There are changes in the capital requirements stemming from the reserving risk charge. The assumption of a single payment date simplifies the computations yet leaves enough detail to highlight the modeling principles.

Federal Income Taxes

We use the IRS loss reserve discount factors for computing taxable income and tax liabilities. The IRS provides loss reserve discount factors for each line of business and accident year.

The text of this paper presumes knowledge of IRS tax calculations and the post-codification statutory accounting rules for deferred tax assets and liabilities. The appendix to this paper contains a more complete exposition of the tax accounting rules.

For the illustration, we assume IRS loss reserve discount factors of 86%, 88%, and 90% for accident year 20XX+1 as of 12 months, 24 months, and 36 months, respectively. Since we are pricing prospectively, the actuary must estimate the future loss reserve discount factors based on federal mid-term rates and either industry or company loss payment patterns.

The offset to taxable income for 20XX+1 equals the tax rate times the change in the discounted loss reserve. The change in the discounted reserve for 20XX+1 is the discounted reserve itself, since losses first occur in 20XX+1. The computation for 20XX+1 is

$$35\% \times 86\% \times \$800 = \$240.80.$$

The offset to taxable income may be viewed either as

(i) 35% of the change in the IRS discounted losses or
(ii) 35% × (statutory incurred losses minus the change in the IRS loss reserve discount).

On December 31, 20XX+1, the IRS discounted reserves are 86% x $800 = $688.00, and the IRS loss reserve discount is (1 – 86%) x $800 = $112.00. Since there are no loss reserves on December 31, 20XX, these figures are also the year-to-year changes in these quantities.

We pro-rate the annual tax liability among the portions of the year to estimate tax payments. For the semi-annual valuation periods in this illustration, we assume tax offsets of $120.40 at each valuation date: June 30, 20XX+1, and December 31, 20XX+1.

The tax rate times the change in discounted reserves during 20XX+2 is

$$35\% \times (88\% \times \$800 - 86\% \times \$800) = \$5.60.$$

The change in discounted reserves is an offset to taxable income, so the tax rate times the change in discounted reserves is the offset to the tax liability for December 31, 20XX+2. We split the offset of $5.60 into equal halves of $2.80 each for the semi-annual valuation periods.

The tax basis incurred loss in 20XX+3 is the paid loss plus the change in discounted reserves:

$$\$800 + (90\% \times \$0 - 88\% \times \$800) = \$96.00.$$

The offset to the tax liability on December 31, 20XX+3, equals

$$35\% \times (\$800 + 90\% \times \$0 - 88\% \times \$800) = \$33.60.$$

We split the $33.60 offset into equal halves of $16.80 each for semi-annual valuation periods.

Unless a policy is written on December 31 or January 1, there are two accident years for tax purposes. This complicates the pricing model, so we have chosen a December 31 effective date for the illustration.[24]

Cash Flow and Equity Flow Patterns

The illustration shows the occurrence of losses and the payment of losses in different years. The company cash flows for incurred losses show the following pattern:

- There are significant cash *inflows* stemming from the offset to taxable income on the dates the losses occur (June 30, 20XX+1, and December 31, 20XX+1). The cash inflow from the offset to taxable income precedes the cash outflow from the payment of losses.
- There is a large cash outflow on the date the loss is paid, or December 31, 20XX+3.

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[24] The exhibits in Appendix B use an effective date of July 1, which is a proxy for the average effective date for policies written evenly through the year. Two accident years are used to evaluate the underwriting tax effects. This is the standard modeling technique for property-casualty insurance policies.

- The offset to taxable income stemming from the unwinding of the IRS loss reserve discount is balanced by the investment income on the assets backing the discounted reserves. The total cash inflow during the period equals the *pre-tax* investment income. The balancing is not perfect, since it depends on the accuracy of the loss reserve discount factors.
- The pricing model treats the federal income taxes on the investment income separately from the tax refund on the amortization of the discount in the IRS loss reserves. The offsetting cash flows are shown separately.
- The investment income on the capital supporting the reserves is not balanced by amortization of the interest discount in the reserves. The supporting capital comprises both the capital embedded in full value reserves and the capital held in surplus for the reserving risk charge.
- Changes in the deferred tax asset are not company cash flows.

The implied equity flows show a different pattern. The losses cause large equity outflows during 20XX+1 and a modest equity inflow on December 31, 20XX+3.

We assume initially that the company posts full value loss reserves on June 30, 20XX+1, and December 31, 20XX+1. If the coverage is priced adequately, the policyholder premium provides for the present value of the expected losses plus an amount to fund the cost of holding capital and the associated federal income taxes.[25] The equityholders fund the loss reserve with assets equal to

- ♦ the difference between the held reserve and the present value of the reserve
- \+ ♦ the capital requirement for reserving risk
- – ♦ the amount of capital provided by the policyholders.[26]

To distinguish the components of the loss reserves and the sources of capital, we conceive of the held reserve as the present value of future loss payments plus the capital embedded in the statutory held reserve.

The *capital requirement* is the explicit capital in the surplus account. For illustration, we assume a capital requirement equal to 15% of held loss reserves. This is consistent with current NAIC risk-based capital requirements.[27]

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[25] On the cost of holding capital, see Feldblum and Thandi, "Target Return on Capital" and "Federal Income Taxes and the Cost of Holding Capital."

[26] Adjustments must be made for the tax refund and the admitted portion of the deferred tax asset stemming from IRS loss reserve discounting.

[27] The risk-based capital requirements for the long-tailed lines of business (except workers' compensation) are higher than 15%, and companies hold surplus about twice the risk-based capital requirements. The covariance adjustment in the risk-based capital formula reduces the effective risk charge by about 50%. The 15% factor understates the capital requirements for the long-tailed casualty lines of

The effects of the reserve valuation rate on the NPV and IRR calculations are examined in a separate paper (Feldblum and Thandi [2002], "Reserve Valuation Rates"). The loss reserve valuation rate is an accounting item; it does not affect the loss cash flows. However, it affects the loss reserve, the capital requirements, the tax payments, and the deferred tax asset, all of which affect the implied equity flows and the internal rate of return. For this paper, we assume that loss reserves are held at full value (undiscounted value).

Computations

On June 30, 20XX+1, the statutory loss reserves are $400 and the required capital is $400 x 15% = $60.00. On December 31, 20XX+1, the statutory loss reserves increase to $800 and the required capital increases to $800 x 15% = $120.00.

The federal income tax liability on December 31, 20XX+1, resulting from the incurred losses is –$240.80, computed as –35% x 86% x $800 . The total deferred tax asset (DTA) is $5.60 + $33.60 = $39.20. This is the deferred tax asset on a GAAP balance sheet, as well as the deferred tax asset in column 1 of the statutory balance sheet (line 15 of page 2 in 2001). Of the $39.20 DTA, only $5.60 reverses within 12 months. This is the admitted portion on the 12/31/XX+1 statutory balance sheet.

We assume that the tax on underwriting income is paid (or the offset to taxable income is received) evenly over the year. For simplicity, we assume that half the deferred tax asset is accrued on June 30, 20XX+1, and the other half is accrued on December 31, 20XX+1.

To fund the incurred losses at June 30, 20XX+1, the policyholders and equityholders must provide assets equal to

the held reserves + the capital requirement – the tax refund – the deferred tax asset, or

$$\$400 + \$60 - \$120.40 - \$2.80 = \$336.80.$$

The policyholder funds are provided by the policy premium. At policy inception, the money is transferred to the unearned premium reserves. Over the course of the year, the money in the

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business and somewhat overstates the capital requirements for workers' compensation. The pricing model in the appendix uses actual factors or best estimates by line of business.

The illustration does not explicitly distinguish between the reserving risk charge applied to held reserves (R_5) and the asset risk charges applied to the assets backing the held reserves (R_1 and R_2). Because the marginal effect of a risk charge varies directly with the magnitude of the charges in its risk category, the marginal effect of the asset risk charges is only about 10% to 20% of the marginal effect of the reserving risk charges for the long-tailed lines of business; see Feldblum [1996: RBC]. One may conceive of the 15% capital requirement as a 13% to 14% reserving risk charge and a 1% to 2% asset risk charge.

unearned premium reserves is transferred to the loss reserves. The equityholder funded capital is a capital infusion at the time of the loss occurrence.

If the policy is adequately priced, the policyholder funds the fair value of the losses plus an amount to cover the cost of holding capital and the associated taxes. The equityholders provide the capital embedded in the reserves and the capital explicitly held in statutory surplus, minus the capital provided by the policyholders.

The assets needed to support the incurred losses at December 31, 20XX+1, are:

$$\$800 + \$120 - \$240.80 - \$5.60 = \$673.60.$$

We have separated the premium transactions from the loss transactions in this illustration to highlight the relationships among the company cash flows and the implied equity flows.

- The premium transactions show the funds supplied at time t=0 by both policyholders and equityholders to support the unearned premium reserve and the initial underwriting loss. At times t=½ and t=1, the funds are distributed to the equityholders.
- The loss transactions show equityholder supplied funds used at times t=½ and t=1 to support the loss reserves. At time t=3, the remaining funds are returned to the equityholders.

In practice, loss reserves and paid losses gradually replace the unearned premium reserves. The policyholder supplied funds collected at time t=0 to support the unearned premium reserve are transferred to support the loss reserves as the premium is earned and the losses are incurred. The profit in the policyholder premium is transferred gradually to the equityholders over the same time period to provide the required return on the invested capital.

In the illustration, there is no change in the undiscounted reserves between December 31, 20XX+1, and December 31, 20XX+3. We use this simplistic scenario to clarify the cash flows and implied equity flows, without having to deal with changing loss reserves and paid losses at each valuation date. In practice, reserves run off gradually.

DEFERRED TAX ASSET: IRS LOSS RESERVE DISCOUNTING

The deferred tax assets are computed at year-end dates. To clarify the exposition, we begin this sub-section with annual valuation periods, not semi-annual periods. We then turn to semi-annual valuation periods to explain the calculation of the deferred tax assets at the mid-year valuation dates.

On December 31, 20XX+1, the offset to *statutory* income stemming from the incurred losses is $800. The federal income tax offset that would result from an offset to taxable income of $800 is 35% × $800 = $280.00.

- The actual offset to taxable income on December 31, 20XX+1, is 86% x $800 = $688.00.
- The offset to the federal income tax liability is 35% x 86% x $800 = $240.80.

The actual federal income tax liability is greater than the federal income tax liability that is implied by the statutory balance sheet. The difference is $280.00 – $240.80 = $39.20.

The difference of $39.20 is recouped by tax refunds in subsequent years. Since statutory accounting recognizes the full value loss reserve on the occurrence date, it should recognize the full tax offset on that date as well. Tax accounting defers the last $39.20 of the offset over the period during which the interest discount unwinds. Both GAAP and statutory accounting treat this as a receivable. It is shown as a deferred tax asset on the balance sheet.

GAAP financial statements recognize the full receivable if the firm expects to collect it. Statutory accounting admits only the portion of the deferred tax asset that is expected to reverse within 12 months of the statement date. To calculate the admitted portion of the deferred tax asset on the December 31, 20XX+1, statutory balance sheet, we estimate the portion of this deferred tax asset that remains on December 31, 20XX+2.

- On December 31, 20XX+2, the offset to taxable income is 88% x $800 = $704.00.
- The offset to the federal income tax liability is 35% x 88% x $800 = $246.40.
- The change in the federal income tax between December 31, 20XX+1, and December 31, 20XX+2, is $246.40 – $240.80 = $5.60.

Date	Statutory	Tax	Difference	Change
12/31/20XX+1	$800	$688	$112	—
12/31/20XX+2	$800	$704	$96	$16

35% x $16 = $5.60.

The expected change in the deferred tax asset from the current valuation date to the valuation date one year hence is the portion of the deferred tax asset that is recognized on the statutory balance sheet.

The appendix explains the calculation of the deferred tax asset when there are multiple loss payments during the year.

INVESTABLE ASSETS

We calculate the investable assets to determine the expected investment income. We show first the procedure for annual valuation periods, and we then extend the computations to semi-annual valuation periods.

Investable Assets, with annual valuation periods: The investable assets equal the total assets minus the admitted portions of any non-investable assets on the statutory balance sheet. The non-investable assets include agents' balances, earned but unbilled premiums, accrued retrospective premiums, and deferred tax assets.

The admitted portion of the DTA is $5.60 at December 31, 20XX+1, and $33.60 at December 31, 20XX+2. The investable assets are

- $920.00 – $5.60 = $914.40 during 20XX+2 and
- $920.00 – $33.60 = $886.40 during 20XX+3.[28]

Investable Assets, with semi-annual valuation periods: The deferred tax asset at December 31 of year X is set up evenly over the course of year X and declines linearly to zero over the course of year X+1. During year X+1, a new deferred tax asset is set up, which declines linearly to zero over the course of year X+2.[29] Each deferred tax asset follows an accrual and amortization pattern shaped like a carot ("/\"). Each carot is two years long.

The rationale for the two year up-down pattern is the assumption that losses are paid evenly during the calendar year. Before a loss is paid, there is a gross (GAAP) deferred tax asset associated with its reserve. The deferred tax asset is admitted on the statutory balance sheet only during the 12 month period immediately prior to its payment.

Illustration: The $5.60 deferred tax asset at December 31, 20XX+1, is accrued evenly over 20XX+1: $2.80 on June 30, 20XX+1, and the remainder on December 31, 20XX+1. It declines to $2.80 at June 30, 20XX+2, and to $0 by December 31, 20XX+2.

A new deferred tax asset of $33.60 is shown at the December 31, 20XX+2 valuation date. It accrues evenly over 20XX+2 ($16.80 on June 30, 20XX+2, and $16.80 on December 31, 20XX+2). It declines to $16.80 at June 30, 20XX+3, and to $0 on December 31, 20XX+3.

The total deferred tax asset on June 30, 20XX+2, is $2.80 + $16.80 = $19.60.

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[28] The implication of this reasoning seems to be that if more of the DTA is not admitted, the investable assets increase. This raises the investment income, which lowers the need for underwriting income, thereby causing a smaller rate indication.

In fact, the investable assets increase only because the non-admitted DTA is replaced by equityholder supplied funds, which are investable. This raises the invested capital, and it more than offsets the higher investment income. The net result is to raise the rate indication, not to lower it.

[29] Since we are using discrete (semi-annual) functions, not continuous functions, "evenly over the course of the year" means one half on June 30 and the other half on December 31.

This modeling procedure interpolates between the deferred tax assets at December 31 of year X and December 31 of year X+1 to derive the deferred tax asset at June 30 of year X+1.

- The deferred tax asset at June 30, 20XX+1, is ($0.00 + $5.60)/2 = $2.80.
- The deferred tax asset at June 30, 20XX+2, is ($5.60 + $33.60)/2 = $19.60.
- The deferred tax asset at June 30, 20XX+3, is ($33.60 + $0)/2 = $16.80.

The change in the deferred tax asset is not itself a cash flow. However, the recognized portion of the deferred tax asset is an admitted statutory asset. A change in the admitted portion of the deferred tax asset causes an implied equity flow. Since the deferred tax asset is not investable but the capital contribution from the equityholders is investable, the change in the deferred tax asset also affects the investment income during the year. A decrease in the deferred tax asset causes an increase in the capital contributed by equityholders. This causes an increase in the expected investment income and an associated increase in the federal income taxes on this investment income.

The total assets held by the company are $920 throughout the two years 20XX+2 and 20XX+3. The investable assets are $914.40 at December 31, 20XX+1, $917.20 at June 30, 20XX+2, $886.40 at December 31, 20XX+2, and $903.20 at June 30, 20XX+2.

Period	Total Assets	Deferred Tax Asset	Investable Assets
1/1– 6/30/XX+2	$920.00	$5.60	$914.40
7/1 – 12/31/XX+2	$920.00	$19.60	$900.40
1/1– 6/30/XX+3	$920.00	$33.60	$886.40
7/1 – 12/31/XX+3	$920.00	$16.80	$903.20

The investment income during the first half of 20XX+2 equals the investable assets times the investment yield, or $914.40 x 4% = $36.58. We use the same computation for each semi-annual valuation period in 20XX+2 and 20XX+3:

Period	Investable Assets	Investment Yield	Investment Income
1/1– 6/30/XX+2	$914.40	4% (half year)	$36.58
7/1 – 12/31/XX+2	$900.40	4% (half year)	$36.02
1/1– 6/30/XX+3	$886.40	4% (half year)	$35.46
7/1 – 12/31/XX+3	$903.20	4% (half year)	$36.13

The implied equity flow at each valuation date equals the statutory income during the preceding period minus the change in capital requirements from the beginning to the end of the period. The components of statutory income during the first half of 20XX+2 are as follows:

- Investment income is $36.58.
- The tax on the investment income is 35% × $36.58 = $12.80.
- The change in the deferred tax asset is $19.60 – $5.60 = $14.00.
- The tax refund for the 20XX+2 amortization of the loss reserve is $5.60. We split the tax refund evenly over the year, giving a tax refund of $2.80 for the first half of 20XX+2.

The total statutory income is $36.58 – $12.80 + $14.00 + 2.80 = $40.58. There is no change in the required capital, so the implied equity flow is +$40.58.

Period	*Investment Income*	*Tax on Inv Income*	*Change in DTA*	*Tax on Reserve Amortization*	*Statutory Income*
1/XX+2	$36.58	$12.80	$14.00	–$2.80	$40.58
2/XX+2	$36.02	$12.61	$14.00	–$2.80	$40.21
1/XX+3	$35.46	$12.41	–$16.80	–$16.80	$23.05
2/XX+3	$36.13	$12.65	–$16.80	–$16.80	$23.48

For the first three half years in the table above, the implied equity flows equal the statutory income. On December 31, 20XX+3, the loss is paid, the loss reserve decreases to $0, and the required capital decreases from $120 to $0. The implied equity flow on December 31, 20XX+3 is

$$\$23.48 - (-\$120.00) = \$143.48.$$

CASH FLOWS AND EQUITY FLOWS: LOSS TRANSACTIONS

The exposition above traces the cash flows and equations by type of transaction. We put the pieces together to show the cash flows and equity flows by valuation date. The five accompanying schematics show the cash flows and equity flows for *Illustration B: Premium, Expense, and Loss Transactions.*

The first schematic shows a summary of the cash flows and equity flows at all seven valuation dates from t=0 through t=3. The subsequent four schematics show more detailed information about the cash flows and equity flows at four of these valuation dates:

- t = 0.5, or June 30, 20XX
- t = 1.0, or December 31, 20XX
- t = 2.5, or June 30, 20XX+2
- t = 3.0, or December 31, 20XX+2

At times t=0.5 and t=1.0, the losses are incurred. The schematics show the relevant cash flows and equity flows for setting up case reserves, added the reserving risk charge to the risk-based capital requirements, obtaining the tax offset for incurred losses, and setting up the deferred tax asset stemming from IRS loss reserve discounting.

At time t=1.5, 2.0, and 2.5, there are no accidents or loss payments. The cash flows and equity flows stem from changes in the IRS discounted reserves, leading to tax payments and changes in the deferred tax asset. In addition, there are investment income flows stemming from the investment return on the assets backing the loss reserves and the reserving risk charge of the risk-based capital requirements.

At time t=3.0, the losses are paid and the assets supporting the reserving risk charge are returned to the equityholders.

The schematics show transactions among the same seven nodes as used for the premium and expense transactions.

ILLUSTRATION B (Case: Losses Incurred) CASH FLOWS

	Time	0.0	0.5	1.0	1.5	2.0	2.5	3.0
	UW TRANSACTIONS							
(1)	Premium	1,000.00	0.00	0.00	0.00	0.00	0.00	0.00
(2)	Expense - Acquisition	250.00	0.00	0.00	0.00	0.00	0.00	0.00
(3)	Expense - General	0.00	150.00	0.00	0.00	0.00	0.00	0.00
(4)	Loss	0.00	0.00	0.00	0.00	0.00	0.00	800.00
	OTHER							
	DTA							
(5)	Due to Revenue Offset	70.00	35.00	0.00	0.00	0.00	0.00	0.00
(6)	Due to Loss Reserve Discountin	0.00	2.80	5.60	19.60	33.60	16.80	0.00
(7)	TOTAL	70.00	37.80	5.60	19.60	33.60	16.80	0.00
	RESERVES							
(8)	UEPR	1,000.00	500.00	0.00				
(9)	Stat Loss Reserve		400.00	800.00	800.00	800.00	800.00	0.00
(10)	IRS Discount Factors			0.86		0.88		0.90
(11)	Tax Basis Loss Reserve			688.00		704.00		0.00
(12)	Surplus	250.00	310.00	120.00	120.00	120.00	120.00	0.00
(13)	**Total Assets**	1,250.00	1,210.00	920.00	920.00	920.00	920.00	0.00
(14)	**Investable Assets**	1,180.00	1,172.20	914.40	900.40	886.40	903.20	0.00
	TAXES							
(15)	Tax Basis UW Revenue	200.00		800.00	0.00	0.00	0.00	0.00
(16)	Tax Basis Expenses	250.00		150.00		0.00		0.00
(17)	Tax Basis Inc Loss	0.00		688.00		16.00		96.00
(18)	Tax Basis Inv Income	0.00	47.20	46.89	36.58	36.02	35.46	36.13
	Valuation of Taxes							
(19)	Tax on UW Income (Annual)	-17.50		-13.30		-5.60		-33.60
(20)	Tax on UW Income (Semi-Annual Pa	-17.50	-6.65	-6.65	-2.80	-2.80	-16.80	-16.80
(21)	Tax on Invest Income	0.00	16.52	16.41	12.80	12.61	12.41	12.64
(22)	Total Semi-Annual Payment	-17.50	9.87	9.76	10.00	9.81	-4.39	-4.16
	CASH FLOWS							
(23)	Asset Flow	1,250.00	-40.00	-290.00	0.00	0.00	0.00	-920.00
(24)	UW Flow	750.00	-150.00	0.00	0.00	0.00	0.00	-800.00
(25)	Inv Inc Flow		47.20	46.89	36.58	36.02	35.46	36.13
(26)	Tax Flow	17.50	-9.87	-9.76	-10.00	-9.81	4.39	4.16
(27)	DTA Flow	70.00	-32.20	-32.20	14.00	14.00	-16.80	-16.80
(28)	Equityflow	-412.50	-104.87	294.93	40.57	40.21	23.05	143.48

U/W Assumptions	Surplus Assumptions	Results
Target Return on Capital = 12.0%	Premium Leverage Ratio = 25%	IRR on Equityflows (annual rate) = 3.0%
Invest Rate of Return = 8.0%	Reserve Leverage Ratio = 15%	
Premium = $1,000		Economic Value Added = -$62.49
Dollars of Ultimate Loss = $800		
Combined Ratio = 120.0%		
Loss Ratio = 80%		

Formulae For Exhibit 1B

CASE: Losses Incurred

$(5)_t = 0.35 * 0.2 * (8)_t$

$(6)_t = 0.35 * \{ [(9)_t - (11)_t] - [(9)_{t+1} - (11)_{t+1}] \}$

$(7)_t = (5)_t + (6)_t$

$(11)_t = (9)_t * (10)_t$

$(12)_t = \begin{cases} 0.25 * WP + 0.15 * (9)t & \text{for } t = 0,0.5 \\ 0.15 * (9)t & \text{for } t >= 1.0 \end{cases}$

$(13)_t = (8)_t + (9)_t + (12)_t$

$(14)_t = (13)_t - (7)_t$

$(15)_t = (1)_t - [(8)_t - (8)_{t-1}]$

$(16)_t = (2)_t + (2)_{t-0.5} + (3)_t + (3)_{t-0.5}$

$(17)_t = (4)_t + (4)_{t-0.5} + [(11)_t - (11)_{t-1}]$

$(18)_t = (14)_{t-0.5} * \text{interest rate}$

$(19)_t = [(15)_t - (16)_t - (17)_t] * 0.35$

$(20)_t = (19)_t \text{ for } t = 1, 2, 3$

$(20)_{t-0.5} = (19)_t \text{ for } t = 0.5, 1.5, 2.5$

$(21)_t = (18)_t * 0.35$

$(23)_t = (13)_t - (13)_{t-0.5}$

$(24)_t = (1)_t - (2)_t - (3)_t - (4)_t$

$(25)_t = (14)_{t-0.5} * \text{interest rate}$

$(26)_t = (22)_t$

$(27)_t = (7)_t - (7)_{t-0.5}$

$(28)_t = -(23)_t + (24)_t + (25)_t + (26)_t$

ILLUSTRATION B (Case: Losses Incurred) Summary of Cash Flows

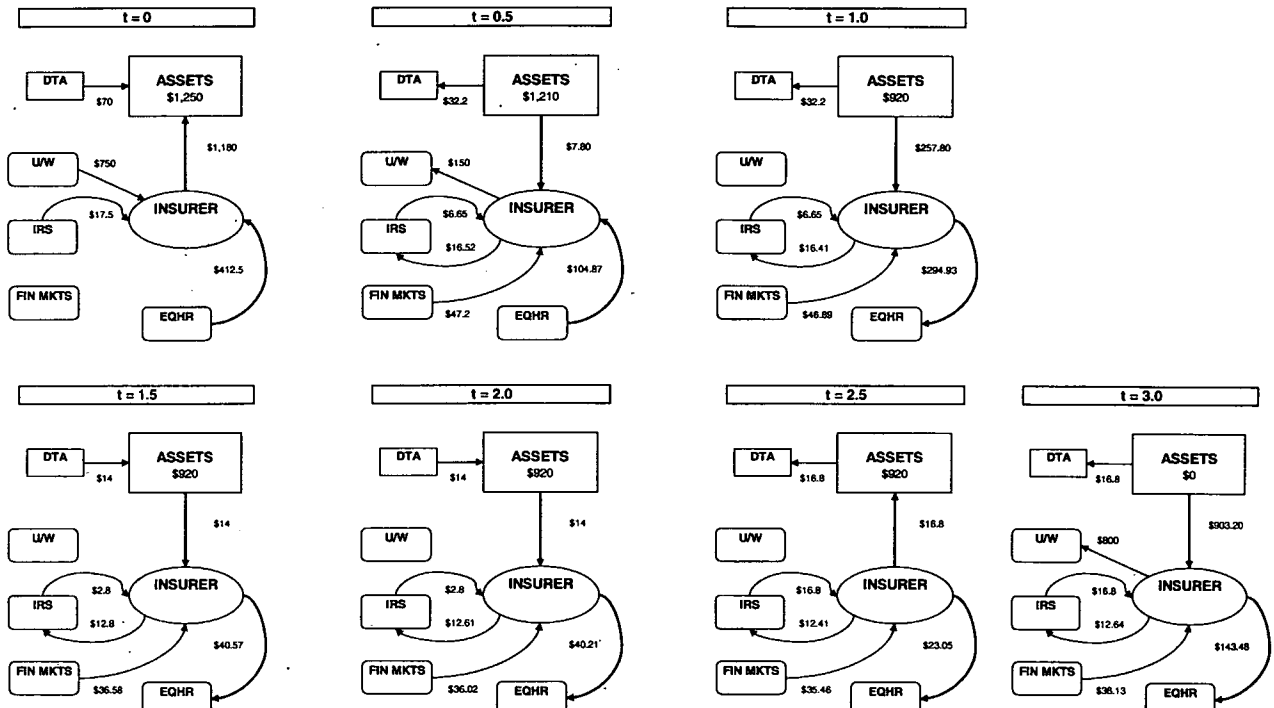


ILLUSTRATION B (Case Losses Incurred)

Exhibit 2D

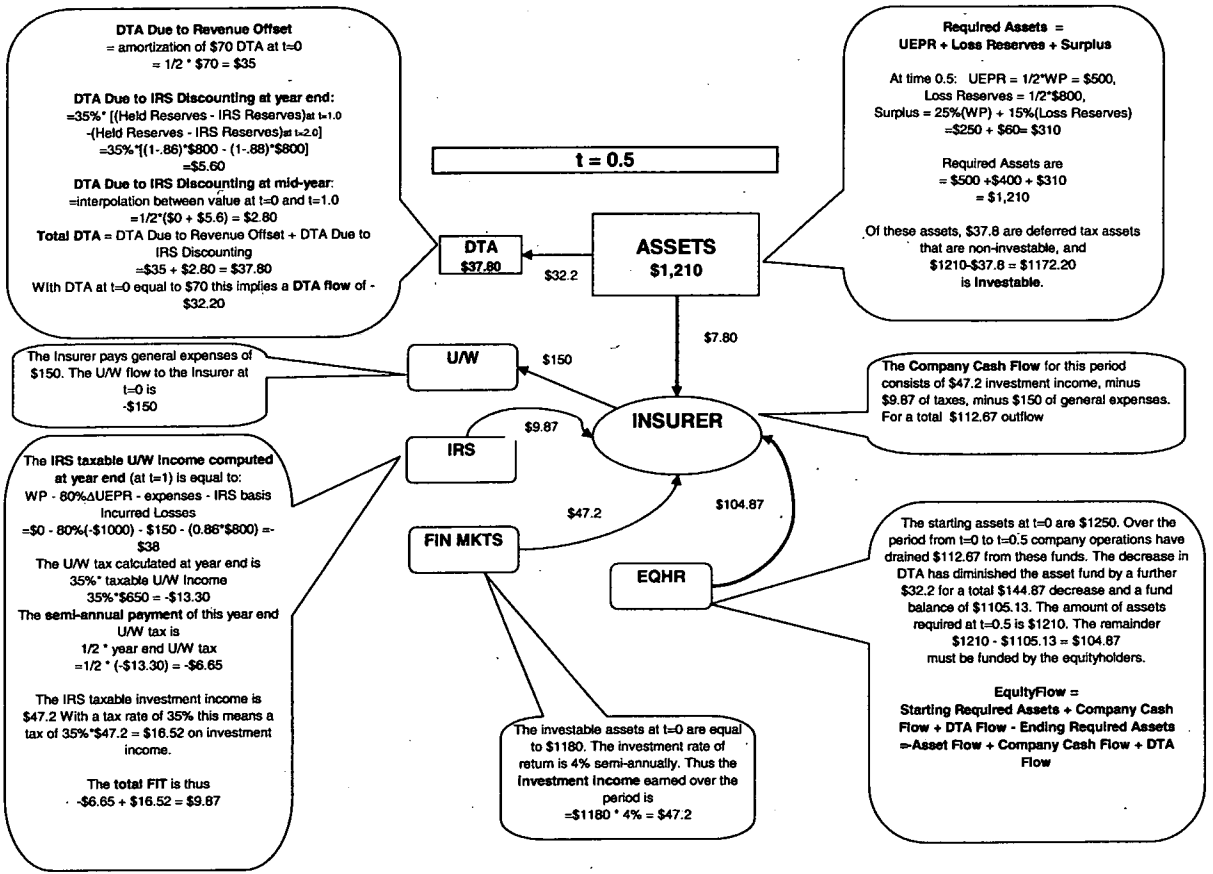


ILLUSTRATION B (Case Losses Incurred)

Exhibit 2D

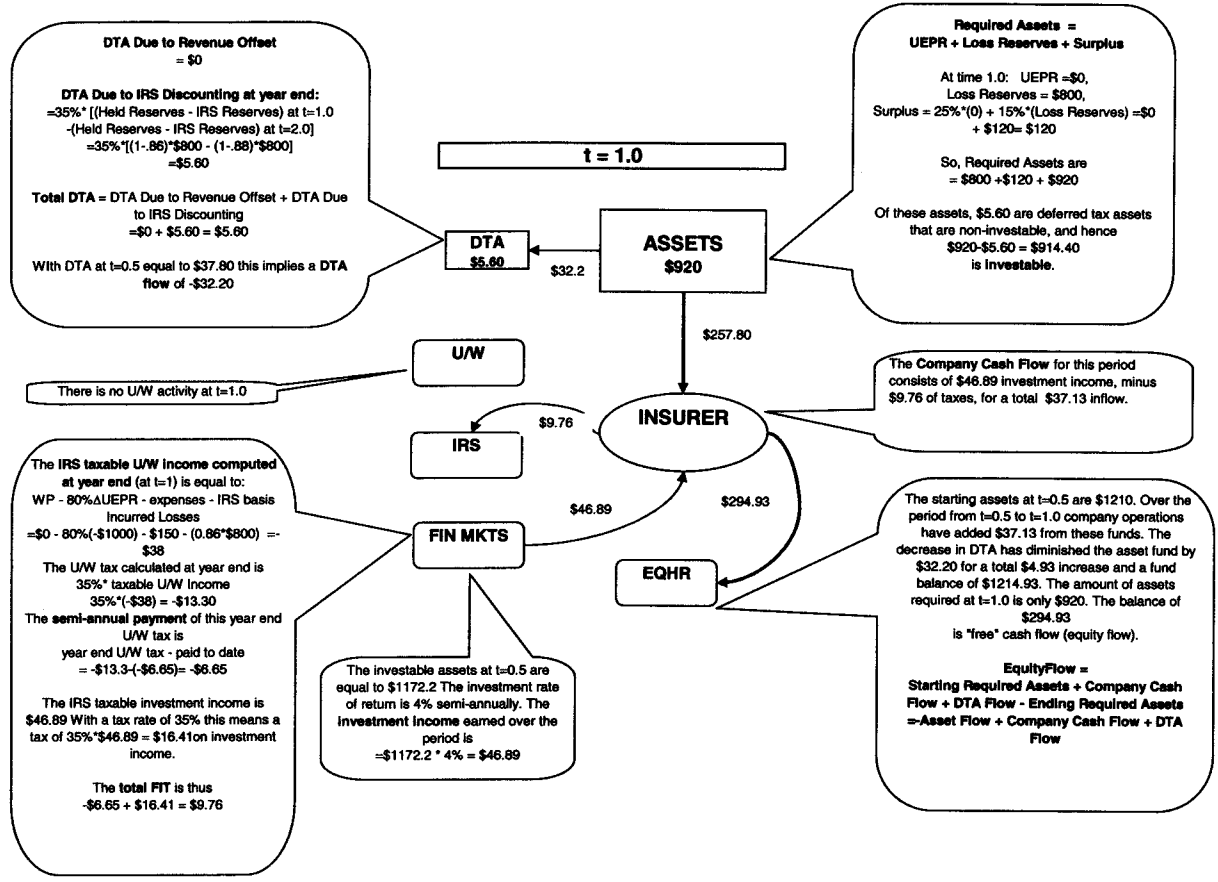


ILLUSTRATION B (Case Losses Incurred)

Exhibit 2D

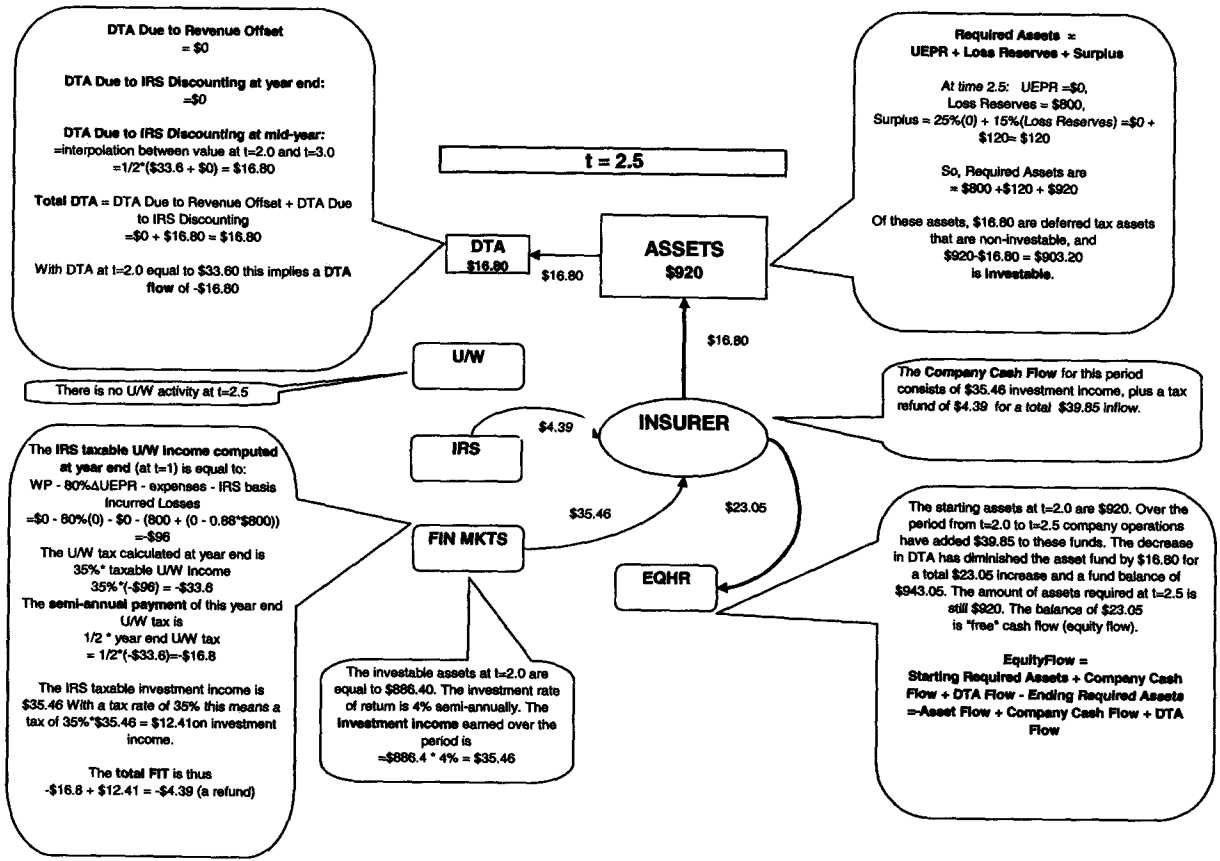
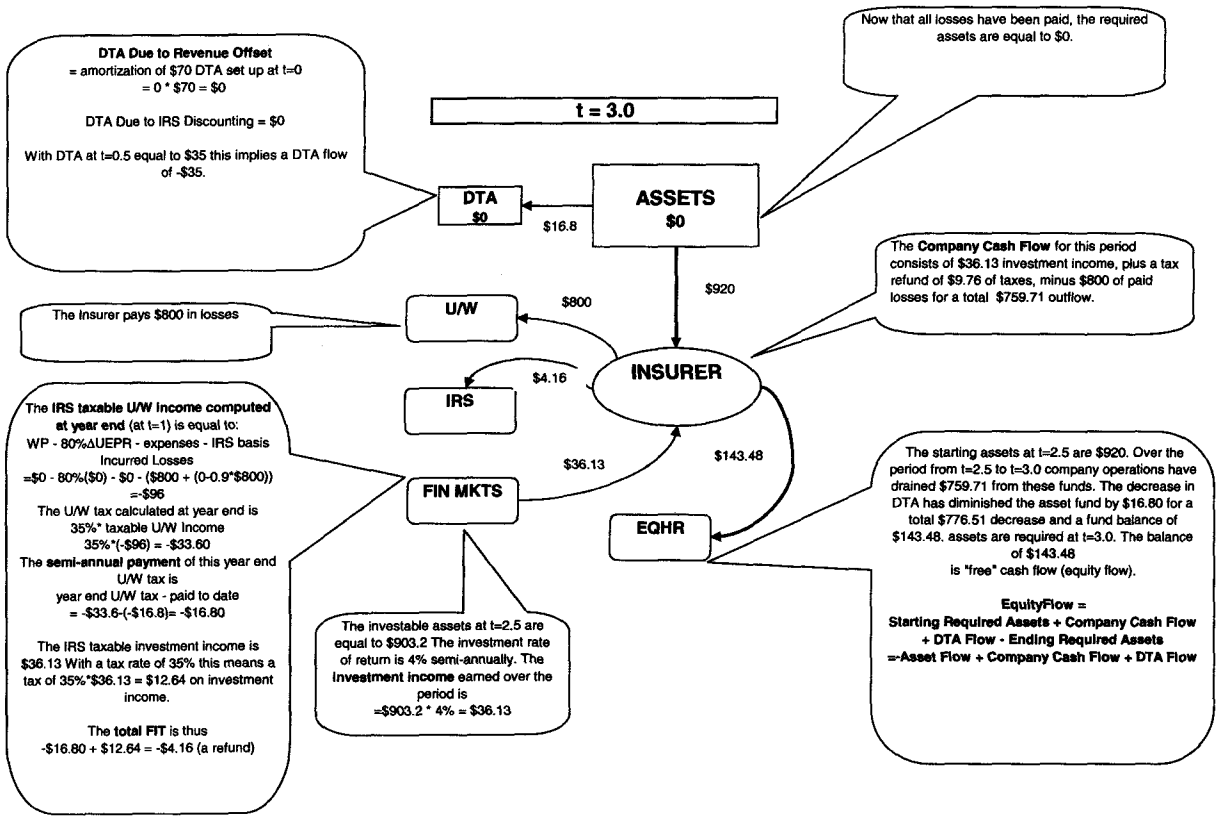


ILLUSTRATION B (Case Losses Incurred)

Exhibit 2D



Premiums and Losses

To complete the modeling of the implied equity flows, we overlay the premium transactions with the loss transactions. On June 30, 20XX+1, and December 31, 20XX+1, there are implied equity flows stemming from both premium and loss transactions.

JUNE 30, 20XX+1: The implied equity flow stemming from the earning of the premiums is +$231.91. This includes the effects of premium earnings, expense payments, federal income taxes, the takedown of one half of the deferred tax asset from revenue offset, and investment income. There is no change during the year in the capital requirements stemming from the written premium RBC charge.

The implied equity flow stemming from the occurrence of the first loss is –$336.80. This includes the effects of loss accrual, federal income taxes, the deferred tax asset stemming from IRS loss reserve discounting, and capital requirements for held loss reserves. The net implied equity flow on June 30, 20XX+1, is +$231.93 – $336.80 = $-104.87.

DECEMBER 31, 20XX+1: The implied equity flow stemming from the earning of the premiums is +$619.84. The implied equity flow stemming from the occurrence of the second loss is –$336.80.

This includes the same items as the implied equity flow on June 30, 20XX+1; it does not include the effect of investment income on the assets held to support the first loss, which was incurred on June 30, 20XX+1. The investable assets supporting the first loss are $400 of loss reserve plus 15% × $400 = $60 of supporting surplus minus $2.80 of deferred tax asset = $457.60. The after-tax investment income on these investable assets is $457.20 × 4% × (1–35%) = $11.89. The net implied equity flow on December 31, 20XX+1, is +$619.84 – $336.80 + $11.89 = $294.93 (premium flows, loss flows, and after-tax investment income on the assets supporting the loss reserves).

Internal Rate of Return and Net Present Value

The internal rate of return is the interest rate that sets the present value of the implied equity flows to zero. The table at the end of this paper shows the cash flows, statutory accounting entries, and implied equity flows for the illustration discussed above. The internal rate of return on the implied equity flows is the solution to the following equation:

$$0 = -412.50 - 104.87/(1+x)^1 + 294.93/(1+x)^2 + 40.57/(1+x)^3 + 40.21/(1+x)^4 + 23.05/(1+x)^5 + 143.48/(1+x)^6.$$

This solution is x = 1.485%, which is a 3.0% effective annual rate ($1.01485^2 = 1.030$).[30]

RATE FILINGS

Since the internal rate of return is less than the cost of capital, the policy generates a loss for the company, not a gain. We have not discussed the cost of capital, but it is at least equal to the investment yield of 8% per annum.

Some regulators presume that a positive internal rate of return implies a profit, even if the profit is not as great as the company desires. The National Council on Compensation Insurance (NCCI) was discomfited by this perception among some state rate regulators in the 1980's, when it filed advisory premium indications for its members.

Net present value models circumvent this misinterpretation. The NPV model shows a dollar gain or loss. An indicated IRR less than the cost of equity capital produces a dollar loss, and an indicated IRR greater than the cost of equity capital produces a dollar gain.

For performance measurement purposes, we use an EVA yardstick in addition to the IRR; see Feldblum and Thandi, "Income Recognition and Performance Measurement." Applying a net present value analysis to the implied equity flows (not to the company cash flows) is similar to an economic value added analysis. Both net present value and economic value added translate the implied return into a dollar amount, so that the gain or loss to the company is more readily understood.

Assumptions and Precision

These illustrations cover the major equity flows that affect financial pricing models for property-casualty insurance. Realistic pricing models have more entries, but they are not conceptually different.

Some readers might dismiss the analysis in the previous sections as needless precision. They presume that the modeling of tax cash flows and deferred tax assets and liabilities imposes excessive costs for little benefit.

This may have been true in past years, when computations were done with pencil and paper or with desk calculators. There is much arithmetic manipulation, but the principles are straightforward. The task of the pricing actuary is to construct the pricing model and to provide reasonable assumptions for the cash flows; the arithmetic is done by computer. Once the model is in place, the computation rules remain the same from year to year.

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[30] By Descartes' rule, the maximum number of real solutions to this polynomial equation is equal to the number of sign reversals. Since there is only 1 sign reversal, the 1.485% solution is unique.

Time demands on casualty actuaries have hindered some companies from developing financially appropriate return on capital pricing models. There is a temptation to use rules of thumb or expedient short-cuts, such as

- traditional combined ratio targets with discounted loss ratios,
- after-tax investment yields instead of explicit modeling of federal income taxes,
- fair values of insurance costs instead of the cost of holding capital.

These short-cuts often lead to severe pricing errors in long-tailed lines of business.

Consumers' Perspective and Suppliers' Costs

We conclude this paper by comparing two perspectives on insurance pricing: the consumers' value perspective and the supplier's cost perspective.

Suppliers: The *equityholder's cost model* described in this paper determines the minimum price that investors demand to fund the insurance product. To optimize shareholders' return on invested capital, an insurer must focus on the amount of invested capital and the return on that capital.

Consumers: The *consumer's value model* focuses on the value received by the consumer from the insurance product. The rational consumer looks at the expected cash flows to and from the insurance company. Pricing an insurance product from the consumer's perspective focuses on expected loss payments for casualty products or expected benefit payments for life insurance products.

The expected loss costs gives the pure premium. By adding underwriting expenses and income taxes on the policy cash flows, and discounting at an appropriate interest rate, one determines the present value of benefits. This is the value of the product to the consumer. The consumer does not include the insurer's cost of capital in the value of the product.

In theory, the consumer's cost of capital should be included in the consumer's value perspective; see the following illustration. In practice, few consumers would set aside the requisite capital to fund the insurance risks.

Illustration: An employer might self-insure its workers' compensation exposures. The self-insured employer faces substantial process risk caused by random loss fluctuations. In theory, it should hold capital to guard against adverse loss fluctuations and to ensure payment of benefits to injured employees. In practice, the employer would pay benefits out of current cash flow, since it has no reserve requirements and no risk-based capital requirements.

The additional capital costs imposed by state regulation cause a discrepancy between the value to the consumer and the cost to the insurer. Even if the rational insurer and the rational

consumer have the same expectations for the insurance cash flows, they calculate different prices for the insurance product. A common solution to this problem in many jurisdictions is an economically inefficient battle of wits among pricing actuaries, regulators, and consumer representatives at state rate hearings.

The magnitude of the discrepancy between the consumer's value perspective and the insurer's cost perspective is not always appreciated by regulatory authorities. This discrepancy was low at the beginning of the 20th century, when property-casualty products covered primarily the short-tailed property lines of business and capital requirements were low. The discrepancy has risen steadily through the 20th century, as casualty lines have increased, payment patterns have lengthened, and more stringent capital requirements have been imposed. The capital to assets ratio for property-casualty insurers is now many times higher than for life insurers, commercial banks, or other financial institutions. The costs of this high capital to asset ratio should be understood and properly weighed by state regulators.

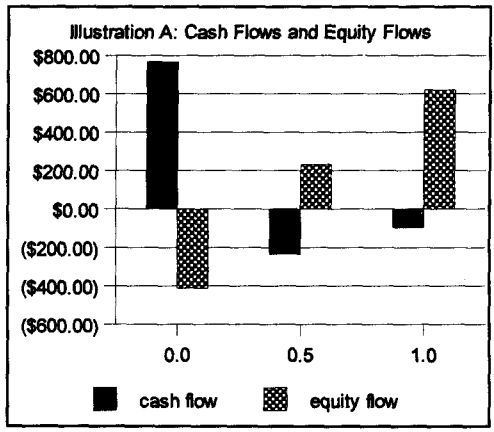
CASH FLOWS AND EQUITY FLOWS

The leitmotif of this paper is that the company cash flows for property-casualty insurance operations are not a suitable proxy for implied equity flows. The accompanying chart shows the cash flows and the associated equity flows for "Illustration A: Premium and Expenses."

Valuation Date	0.0	0.5	1.0
cash flow	$767.50	($233.07)	($95.16)
equity flow	($412.50)	$231.93	$619.94

The premium collection at time t=0 causes a large cash inflow to the company. The unearned premium reserves and risk-based capital requirements cause a large equity outflow at time t=0.

Expenses and tax payments cause cash outflows at times t=½ and t=1. As the premium is earned and the unearned premium reserve is taken down, there are equity inflows.



LOSS TRANSACTIONS

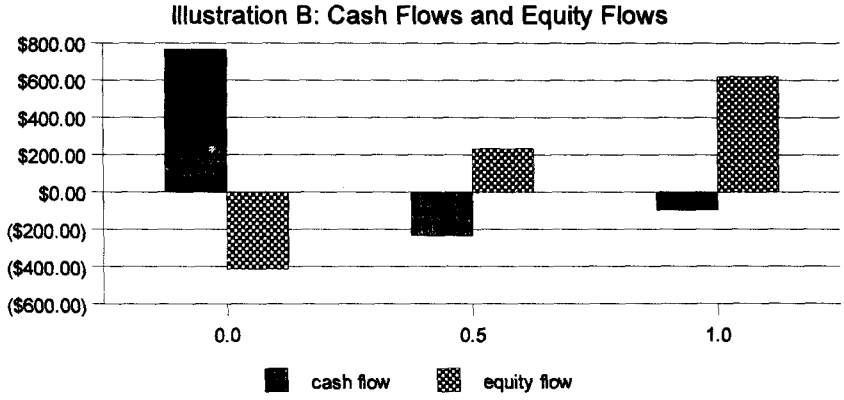
The loss transaction show a similar pattern: company cash inflows are associated with implied equity outflows, and company cash outflows are associated with implied equity inflows. The loss transactions begin at time t=½ and extend through time t=3.

Date	0.5	1.0	1.5	2.0	2.5	3.0
cash flow	$123.20	$135.09	$40.57	$40.21	$23.05	($776.52)
equity flow	($336.80)	($324.91)	$40.75	$40.21	$23.05	$143.48

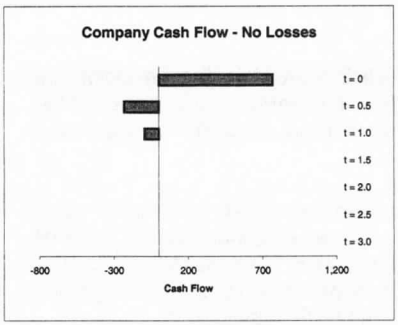
Losses are incurred at times t=½ and t=1. The cash inflows at these dates stem from the federal income tax contra-liabilities, which are 35% of the present value of the incurred losses. The equity outflows stem from the case reserves and the risk-based capital requirements.

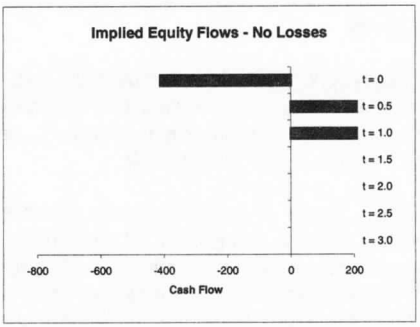
The cash flows and equity flows at times t = 1.5, 2.0, and 2.5, stem from investment income, tax payments, and changes in the deferred tax asset. The equity flows equal the cash flows.

At time t=3, the loss is paid, resulting in a large cash outflow. There is an equity inflow stemming from the takedown of the risk-based capital requirements.

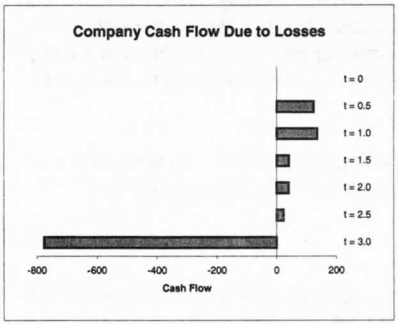


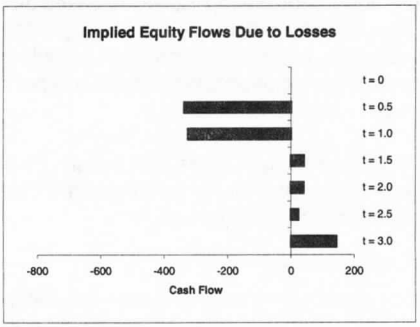
Company Cash Flows and Equity Flows - Case No Losses



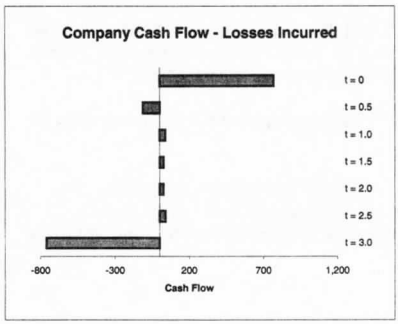


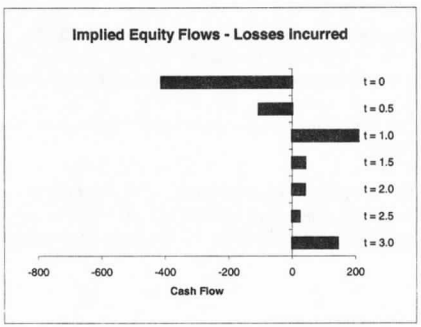
Company Cash Flows and Equity Flows Due to Losses





Company Cash Flows and Equity Flows - Case Losses Incurred





Appendix A: Federal Income Taxes and Deferred Tax Assets

INTRODUCTION

For prospective pricing, the actuary must estimate future IRS loss reserve discount factors and future deferred tax assets stemming from revenue offset and from loss reserve discounting. The post-codification changes to statutory accounting further complicate the consideration of tax effects in actuarial pricing models.

This appendix provide a tutorial through the relevant tax laws and regulations. It provides clear documentation of the IRS provisions concerning loss reserve discounting and the post-codification statutory accounting rules regarding deferred tax assets and liabilities.[31] To avoid repetition, this appendix covers the tax laws and regulations for the series of pricing model papers by S. Feldblum, N. Thandi, E. Schirmacher, and D. Schirmacher.[32]

DATA SOURCES

The equity flows in the pricing model are based on statutory accounting. For non-insurance companies, taxable income depends on general accounting [GAAP] statements. For property-casualty insurance companies, taxable income depends on statutory accounting statements. The exposition in the text of this paper determines taxable income from statutory income with several adjustments.

A. The tax computation begins with statutory pre-tax income from the Underwriting and Investment Exhibit of the Annual Statement: Part 1 for investment income and Parts 2, 2A, and 3 for underwriting income.[33]

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[31] For a general treatment of federal income taxes relating to property-casualty insurance companies, with emphasis on items of particular concern to casualty actuaries, see Sarason, *et al.* [2003].

[32] For more extensive coverage of the tax aspects of insurance pricing, see Feldblum and Thandi, "Federal Income Taxes and the Cost of Holding Capital," Feldblum and D. Schirmacher, "Reinsurance Pricing and Capital Management," and Feldblum, "The Pricing of Commutations."

[33] See the Treasury regulations, 2001FED 26,153, §1.832-4(a)(1): "Gross income means the gross amount of income earned during the taxable year from interest, dividends, rents, and premium income, computed on the basis of the underwriting and investment exhibit of the annual statement."

The Internal Revenue Code lists numerous adjustments, of which the following are the most important for the pricing actuary:

1. The earlier incurral of the tax liability resulting from revenue offset and loss reserve discounting.
2. The effects of anticipated salvage and subrogation and the discounting provisions relating thereto.
3. The reduction of the tax liability resulting from municipal bond income and the dividends received deduction.

B. The additional tax liability resulting from the revenue offset provision is calculated from Part 2 of the Underwriting and Investment Exhibit.[34]
C. Schedule P, Part 1, is used to calculate the additional tax liability resulting from the IRS loss reserve discounting provision. Schedule P, Part 3, may be used to determine the non-admitted portion of the deferred tax asset stemming from the loss reserve discounting.

This appendix focuses on IRS loss reserve discounting and the admitted portion of the resulting statutory deferred tax asset.

Loss Reserve Discounting

For statutory financial statements, calendar year incurred losses equal the losses paid during the year plus the change in the full value loss reserves from the beginning of the year to the end of the year. For federal income tax purposes, the incurred losses during the tax year equal the losses paid during the year plus the change in the *discounted* loss reserves from the beginning of the year to the end of the year.

The determination of discounted loss reserves relies on Schedule P. For a prospective pricing model, the actuary must estimate (i) the discounted loss reserves, (ii) the amount of the discount, and (iii) the deferred tax assets stemming from loss reserve discounting and revenue offset. For retrospective analysis of product profitability, the actuary must consider also the effects of reserve changes on taxable income.[35]

The cost of capital is a major factor for the pricing of insurance contracts, and the double taxation of the investment income on capital funds is a significant component of this cost. The IRS loss reserve discounting provisions and the statutory deferred tax asset affect the cost of holding capital for insurers.[36]

The alternative minimum income tax provisions may also cause earlier incurral of the tax liability. Changes in the incurral dates of the tax liabilities lead to deferred tax assets and liabilities on the statutory balance sheet.

[34] The recognition of taxable revenue from earned but unbilled premiums and accrued retrospective premiums are affected by the January 2000 tax regulations and the statutory accounting codification changes effective on January 1, 2001. The illustration in the text of the paper does not consider earned but unbilled premiums or accrued retrospective premiums. The workers' compensation illustration in Appendix B considers the billing and collection pattern of the policy premiums.

[35] See E. Schirmacher and S. Feldblum, "Retrospective Analysis and Performance Measurement."

[36] See Feldblum and Thandi, [2002], "Federal Income Taxes and the Cost of Holding Capital."

For long-tailed lines of business, the statutory accounting rules cause an underwriting loss during the policy term when losses occur. After policy expiration, the investment income on the assets backing the loss reserves provide steady and positive net income. For tax accounting, the expected investment income on the assets backing the loss reserves offsets the expected amortization of the interest discount in the reserves. The underwriting gain or loss is realized during the policy term, with no expected net gain or loss in subsequent years.[37]

Complete (exact) offsetting depends on the following conditions:

- There are no implicit (undisclosed) discounts in the statutory loss reserves.
- The IRS discount rate equals the investment yield of the company.
- The IRS loss payment pattern equals the actual liquidation pattern for the block of business.
- The company holds fully discounted reserves, with disclosure of the amount of discount.

These conditions are not consistent with current statutory requirements, so complete offsetting is not expected. Nonetheless, they clarify the heuristic illustration below.[38]

Illustration: Offsetting

A one day policy is written on December 31, 20XX, for a net premium of $10,000. One loss occurs on December 31, 20XX, which is paid for $12,100 on December 31, 20XX+2. The term structure of interest rates is flat at 10% per annum. To simplify the illustration, we assume that the IRS loss payment pattern is the same as the actual loss payment pattern here.

In 20XX, statutory accounting shows an underwriting loss of $10,000 – $12,100 = $2,100. The $10,000 net premium is invested at 10% per annum. The investment income is $10,000 × 10% = $1,000 in 20XX+1 and $11,000 × 10% = $1,100 in 20XX+2. There is no underwriting gain or loss in 20XX+1 or 20XX+2, so these are the statutory income amounts.

If we assume a two year IRS loss payment pattern and a discount rate of 10% per annum, the discounted loss reserves are $12,100 / 1.100^2 = $10,000 at December 31, 20XX. Tax accounting shows no underwriting gain or loss in 20XX and a tax liability of $0 for 20XX.

In 20XX+1, investment income is $10,000 × 10% = $1,000. The discounted loss reserve on December 31, 20XX+1, is $12,100 / 1.100 = $11,000. The underwriting loss (or the offset

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[37] Statutory, GAAP, and tax accounting are discussed in detail in Feldblum and Thandi [2002], "Income Recognition and Performance Measurement."

[38] For complete discussion of this subject, see Feldblum and Thandi, [2002], "Reserve Valuation Rates."

to underwriting income) for tax year 20XX+1 equals amortization of the interest discount on the loss reserves, or $11,000 – $10,000 = $1,000. The underwriting loss just offsets the investment income. The net taxable income is $0, and the tax liability is $0.

In 20XX+2, investment income is $11,000 x 10% = $1,100. The incurred loss offset to taxable underwriting income in 20XX+2 is the paid loss plus the change in the discounted loss reserve, or $12,100 (paid on December 31, 20XX+2) + $0 – $11,000 = –$1,100.

This is the amortization of the interest discount on the 12/31/20XX+1 reserve of $11,000. It offsets the investment income in 20XX+2. Taxable income is $0, and the tax liability is $0.[39]

DISCOUNTING PRINCIPLES

The discounted loss reserves are determined from three components:

- The undiscounted loss reserves, as shown in Schedule P, Part 1.
- The loss reserve discount rate, which is promulgated each year by the Treasury.
- The loss payment pattern by line of business, which is determined from Schedule P data.

The illustration below shows the concepts, though the details differ from the IRS computation.

Illustration: The December 31, 20XX, undiscounted loss reserves are $100 million. The loss reserve discount rate is 8% per annum. The $100 million of reserves will be paid in three parts: 50% on December 31, 20XX+1, 30% on December 31, 20XX+2, and 20% on December 31, 20XX+3.[40] The discounted loss reserves equal

$$\$100 \text{ million} \times (50\%/1.08 + 30\%/1.08^2 + 20\%/1.08^3) = \$100 \text{ million} \times 0.879 = \$87.9 \text{ million}.$$

Undiscounted Loss Reserves

The Treasury assumes that the loss reserves in Schedule P, Part 1, are undiscounted values. If discounted values are shown, the losses may be "grossed up" to undiscounted amounts before application of the IRS loss reserve discounting procedure. The "gross-up" is permitted only if the amount of the discount is disclosed in (or with) the Annual Statement.[41]

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[39] Some insurance personnel speak of the post-1986 federal income tax incurral pattern as a "prepayment of taxes by the insurance industry." This is correct from a statutory or GAAP perspective. The IRS would take the opposite view; before 1986 the Treasury helped fund the conservative insurance accounting practices.

[40] This illustration is simplified. The actual tax procedure assumes mid-year payments and a longer loss payment pattern.

[41] See section 846(b)(2) of the Internal Revenue Code: "Adjustment If Losses Discounted on Annual Statement: If the amount of unpaid losses shown in the annual statement is determined on a discounted basis,

Illustration: Schedule P, Part 1, is gross of non-tabular discount and net of tabular discount.

- A company incurs $10,000,000 of accident year 20XX workers' compensation losses, including lifetime pension claim reserves with a tabular discount of $1,000,000.
- The IRS loss reserve discount factor for workers' compensation accident year 20XX reserves is 85%.

If the company does not disclose the tabular discount in the Annual Statement, the offset to taxable income is $10 million × 85% = $8.5 million. If the company does disclose the tabular discount in the Annual Statement, the offset to taxable income is ($10 million + $1 million) × 85% = $9.35 million. The difference in taxable income is $9.35 million – $8.5 million = $0.85 million, and the difference in the tax liability is $0.85 million × 35% = $297,500.

DISCLOSURE AND TIMING COSTS

Reserve discounting is a timing difference; it reverses in subsequent years. The cost to the company is the present value of the expected after-tax investment yield on this money.

Illustration: Suppose the pension reserves are paid (on average) twelve years after policy expiration, and the after-tax investment yield is 6% per annum. The cost to the company is

$$\$297{,}500 \times [\,(1.06^{12} - 1)\,/1.06^{12}] = \$297{,}500 \times 0.503 = \$149{,}651.61.^{42}$$

RESERVE VALUATION RATES

Several of the pricing papers focus on the inter-relationships among the reserve valuation rate, the cost of holding capital, the federal income tax liability, and the indicated premium. We expand the comments above for the application to these papers.

The pricing model separates the undiscounted loss reserves into two pieces:

- the true undiscounted loss reserves, and
- the valuation rate at which the company's reserves are booked.

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and *the extent to which the losses were discounted can be determined on the basis of information disclosed on or with the annual statement,* the amount of the unpaid losses shall be determined without regard to any reduction attributable to such discounting." The required disclosure of *non-tabular* discounts by accident year and by line of business is provided in columns 34 (losses) and 35 (loss adjustment expenses) of Schedule P, Part 1. The required disclosure of *tabular* discounts is shown in note 28 (in the 2001 Annual Statement) to the financial statements, "Discounting of Liabilities for Unpaid Losses or Unpaid Loss Adjustment Expenses."

[42] Because of the statutory deferred tax asset and the capital requirements imposed on insurance companies, the actual cost to equityholders is somewhat different; see Feldblum and Thandi [2002], "Federal Income Taxes and the Cost of Holding Capital," for a full discussion.

If the company holds full value loss reserves, the valuation rate is 0%. A pricing model used by a regulatory agency would use full value loss reserves. Similarly, a pricing model used for a rate filing would use full value loss reserves. A pricing model used for competitive pricing purposes should use the valuation rate implicit in the reserves held by the company.

Illustration: The December 31, 20XX, undiscounted loss reserves are $100 million. The loss reserve discount rate is 8% per annum. The $100 million of reserves will be paid in three parts: 50% on December 31, 20XX+1, 30% on December 31, 20XX+2, and 20% on December 31, 20XX+3. The company values its held reserves at an implicit 5% discount rate.

The reserves on the company's balance sheet are

$$\$100 \text{ million} \times (50\%/1.05 + 30\%/1.05^2 + 20\%/1.05^3) = \$100 \text{ million} \times 0.921 = \$92.1 \text{ million}.$$

If the company's implicit loss reserve discount is not disclosed in its Annual Statement, the IRS treats the held reserves as undiscounted loss reserves. The discounted loss reserves for tax purposes equal

$$\$92.1 \text{ million} \times (50\%/1.08 + 30\%/1.08^2 + 20\%/1.08^3) = \$92.1 \text{ million} \times 0.879 = \$80.96 \text{ million}.$$

The implicit discounting lowers the lowers the tax basis loss reserves. This is a timing effect, not a permanent effect, since the losses actually paid do not depend on the valuation rate used by the company. The company loses the investment income on the early incidence of the federal income tax liability. This loss may be offset by the capital management benefits of having less equityholder supplied funds tied up in full value loss reserves. The capital management benefit depends on the cost of holding capital and on possible rating agency revaluations of the indicated reserves. For a complete treatment of these items, see Feldblum and Thandi, "Reserve Valuation Rates" and Feldblum and Schirmacher, "Reinsurance Pricing and Capital Management."

LIMITATION

The IRS is concerned that a company might claim such a large discount for its statutory loss reserves that the discounted tax-basis loss reserves would be greater than the Annual Statement loss reserves, thereby reducing the tax liability by means of discounting instead of increasing the tax liability. To prevent this, the discounted IRS loss reserves may not be greater than the loss reserves shown in the Annual Statement.[43]

Statutory accounting allows only limited discounting: tabular discounts and exceptional cases of non-tabular discounts. For tabular discounts, most companies use conservative interest rates, such as 3.5% or 4% per annum. For non-tabular discounts, the permissible discount rate for statutory accounting is rarely greater than the discount rate used for IRS loss reserve discounting; see SSAP No. 65 on "Property and Casualty Contracts," paragraph 12.

In most cases, the statutory loss reserves are lower than the IRS discounted loss reserves. The workers' compensation "prior years" row (Part 1D) is an exception. These reserves are primarily indemnity reserves for lifetime pension cases, and many companies use tabular discounts. For this row, the "composite discount factor" used in the IRS discounting calculations assumes (on average) three more years of payment, whereas the pension cases in these reserves may have (on average) a future expected lifetime of 10 to 20 years.

ILLUSTRATION: THE LIMITATION

The workers' compensation prior years row shows unpaid losses and loss adjustment expenses of $30 million. In the Notes to the Financial Statements, the company reports a $10 million tabular discount for these claims. The IRS composite discount factor applicable to these reserves is 90%.

Without the limitation discussed above, the gross loss reserves are $30 million + $10 million = $40 million. The IRS discounted loss reserves are 90% × $40 million = $36 million. Since this exceeds the $30 million of statutory loss reserves, the IRS discounted loss reserves are capped at $30 million.

This rule has a significant effect on the pricing of workers' compensation commutations for permanent total disability cases; see Feldblum [2002], "The Pricing of Commutations," for a full discussion. The rule also affects the pricing of workers' compensation large dollar

[43] See the Internal Revenue Code §846(a)(3): "In no event shall the amount of the discounted unpaid losses with respect to any line of business attributable to any accident year exceed the aggregate amount of unpaid losses with respect to such line of business for such accident year included on the annual statement."

deductible business and workers' compensation excess coverage, since these two types of contract cover primarily long term disability cases.

Discount Rate

The discount rate varies by accident year. For each accident year, the discount rate is the 60 month moving average of the federal mid-term rates ending on the December 1 preceding the accident year. This rate is frozen and applies to that accident year's losses in all future calendar years. In tax parlance, the discount rate is "vintaged." The federal mid-term rate is the average rate on Treasury securities with 3 to 9 years remaining maturity.[44]

The federal mid-term rate is promulgated by the Treasury each month.[45] The 60 month moving average for an accident year can be determined once the last federal mid-term rate has been announced.

Illustration: The loss reserve discounting rate for accident year 20X9 is the 60 month average of the federal mid-term rates from January 1, 20X4, through December 1, 20X8. It can be computed in December 20X8, before the inception of accident year 20X9, so that companies can effectively determine their tax strategies during 20X9.

Yield Projections

The market values of future cash flows are based on the current term structure of interest rates. The date that the liability was incurred is not relevant. In contrast, the IRS bases the discount rate on the incurral year of the liability. The rationale is that the insurance company uses the premium cash flows from the policy to purchase fixed-income securities to fund the future loss payments. The yield on the fixed-income securities is determined at the date of purchase.

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[44] See section 846(c)(2) of the Internal Revenue Code: "Determination of Annual Rate: The annual rate determined by the Secretary under this paragraph for any calendar year shall be a rate equal to the average of the applicable Federal mid-term rates (as defined in section 1274(d) but based on annual compounding) effective as of the beginning of each of the calendar months in the test period. The test period is the most recent 60-calendar-month period ending before the beginning of the calendar year for which the determination is made."

The federal mid-term rates are expressed as bond equivalent yields, since bond coupons are paid semi-annually in the United States. (A bond equivalent yield is a yield with semi-annual compounding.) The IRS loss reserve discounting procedure uses annual compounding, since it assumes that losses are paid in mid-year (i.e., once a year). The bond equivalent yields are converted to effective annual yields before averaging, using the formula $r_a = (1 + r_s/2)^2 - 1$, where r_a is the effective annual yield and r_s is the bond equivalent yield with semi-annual compounding. If the bond equivalent yield is 8% per annum, the equivalent effective annual rate is $(1 + 0.08/2)^2 - 1 = 8.16\%$.

[45] The yield among mid-term securities varies with the remaining maturity, in accordance with the term structure of interest rates. More recently issued securities tend to have slightly lower yields, since they are more marketable. The Secretary of the Treasury selects an appropriate average rate.

If the duration of the assets backing the reserves matches the duration of the loss liabilities, the losses will be paid from the coupon income and the principal repayment from these securities. The yield during the accident year is the relevant investment yield throughout the life of the policies.[46]

For prospective pricing, the actuary must project about two years of future yields. The pricing requirements are most easily seen by illustration.

Illustration – Projected Yields: The pricing actuary is setting rates for policies effective from July 1, 20XX, through June 30, 20XX+1. The losses on these policies extend from July 1, 20XX, through June 30, 20XX+2, since the last policy written under the new rates is effective on June 30, 20XX+1, and remains in effect through June 30, 20XX+2. The losses stemming from policies written under the new rates relate to accident years 20XX, 20XX+1, and 20XX+2.

To project loss reserve discount factors, the pricing actuary must estimate federal mid-term rates through December 1, 20XX+1. If the rate analysis is done in the last quarter of 20XX–1, the actuary must project rates from the last quarter of 20XX–1 through the end of 20XX+1.

The rate projection is generally done in one of two ways:

- The most recent monthly mid-term rate may be repeated for all future months. Alternatively, the average of the most recent three or six monthly mid-term rate may be repeated for all future months.
- The projected rates may be set equal to the current forward rates for the corresponding time period.

The second method is favored by many investment analysts. We explain by means of an illustration.

Illustration: Projecting Treasury Rates: The term structure of interest rates on January 1, 20XX, is upward sloping, as shown in the table below.

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[46] Whether a moving average rate or the current rate is a better predictor for future rates is an open question. Accountants often prefer average rates, on the assumption that the most recent monthly figure may be abnormally high or low. Some financial analysts presume that interest rates revert towards a long-term mean, and a 60 month moving average may be a better reflection of this mean. Other analysts presume that interest rates form a random walk, and the present term structure of interest rates is the best reflection of expected future rates. The dominant view is that the current rate is a better estimator of the rate during the next 12 months than the 60 month moving average is; see Dr Jonathan Benjamini and S. Feldblum, *Dynamic Financial Analysis: a Primer for the Practicing Actuary* [2002].

Table AppA.1: *Term Structure of Interest Rates*

Term	*Spot Rate*	*Term*	*Spot Rate*
1 year	5.00%	*5 years*	6.90%
2 years	6.00%	*6 years*	7.00%
3 years	6.40%	*7 years*	7.10%
4 years	6.70%	*8 years*	7.10%

To project the five year spot rates for 20XX, 20XX+1, 20XX+2, and 20XX+3, we use one of two methods. The first method assumes that the five year spot rate remains unchanged at 6.9%. The second method determines the forward rates for the appropriate periods.

- The five year spot rate on Jan 1, 20XX+1, is estimated as $(1.070^6 / 1.05)^{1/5} - 1 = 7.40\%$.
- The five year spot rate on Jan 1, 20XX+2, is estimated as $(1.071^7 / 1.06^2)^{1/5} - 1 = 7.54\%$.
- The five year spot rate on Jan 1, 20XX+3, is estimated as $(1.071^8 / 1.064^3)^{1/5} - 1 = 7.52\%$.

This method of using forward rates to project future spot rates relies on the pure expectations hypothesis for the term structure of interest rates. Most financial analysts do not subscribe to a pure expectations hypothesis.

Loss Payment Pattern

The IRS determines the expected loss payment pattern by line of business from Schedule P, Part 1. To compute the tax liability, the accountant may use the loss reserve discount factors promulgated by the Treasury. For a financial pricing model, the actuary must estimate the future loss reserve discount factors a year or two in advance. The actuary should understand the relationship of the IRS loss reserve discount factors to actuarially determined loss reserve discount factors to estimate the tax effects on policy pricing; see the discussion below.

Illustration: An actuary is pricing a claim commutation. For a permanent total disability case, the actuarially determined loss reserve discount factors over the next twenty years rise from 70% to 100%, and the IRS loss reserve discount factors are level at about 92%. The IRS discounted reserves are greater than the fair value reserves, leading to a tax credit and a lower commutation price than if actuarial loss reserve discount factors were used; see Feldblum, "The Pricing of Commutations."

Illustration: After a period of falling interest rates, the IRS loss reserve discount factors provide a larger discount than is financially warranted. After a period of rising interest rates, the IRS loss reserve discount factors provide a smaller discount is financially warranted.

Data Grouping

The IRS loss reserve discount factors are determined by Schedule P line of business. The actual loss payment pattern for the business being priced, which is used in the pricing model to determine the implied equity flows, is not relevant for the IRS loss reserve discount factors. The illustrations in this appendix all use Schedule P loss triangles.

Illustration: The average lag between premium collection and loss payment for large dollar deductible workers' compensation business with a $500,000 deductible may be twenty years. The average lag for first dollar workers' compensation business in a state with limited duration permanent total disability cases and no cost of living adjustments may be four years. The IRS loss reserve discount factors are the same for the two sets of business, since both use the Schedule P, Part 1D (workers' compensation) factors.

Deriving Payment Patterns

We determine two sets of loss payment patterns: one for IRS loss reserve discount factors and one for statutory accounting deferred tax assets. Both loss payment patterns are derived from historical loss liquidation patterns. If there were no random fluctuations in the loss payment pattern for any accident year and no systematic changes in the payment pattern over the past ten years, the observed liquidation pattern of the oldest accident year recorded in Schedule P would be sufficient, as illustrated below.

Illustration: We are computing the loss payment pattern for the 20X9 accident year reserves. Suppose that Schedule P, Part 3, shows the following pattern for accident year 20X0:

Exhibit AppA.2: 20X9 Schedule P, Part 3 ($000,000)

Part 3	20X0	20X1	20X2	20X3	20X4	20X5	20X6	20X7	20X8	20X9
20X0	103	226	294	334	363	384	398	412	422	433

In addition, suppose the ultimate incurred losses for accident year 20X0 are $486 million. This estimate may be taken from Schedule P, Part 2, or it may be derived from an actuarial loss reserve projection; see Feldblum [2002: SchP] for estimation procedures.

Schedule P, Part 3, shows cumulative paid losses. The first differences between each adjoining set of figures is the incurred loss paid losses in each 12 month period. The ratio of these incurred loss paid loss figures to the estimated ultimate incurred losses is the percentage of ultimate losses paid in each 12 month period, as shown below.

Exhibit AppA.3: Incremental Loss Payment Pattern from Accident Year 20X0 ($000,000)

Part 3	20X0	20X1	20X2	20X3	20X4	20X5	20X6	20X7	20X8	20X9
1. 20X0	$103	$226	$294	$334	$363	$384	$398	$412	$422	$433
2. percent	0.212	0.465	0.605	0.687	0.747	0.790	0.819	0.848	0.868	0.891
3. incremental	0.212	0.253	0.140	0.082	0.060	0.043	0.029	0.029	0.021	0.023

- Row 1: The row labeled "20X0" shows the cumulative dollars (in millions) of accident year 20X0 losses paid by December 31 of each calendar year from 20X0 through 20X9.
- Row 2: The row labeled "percent" shows the cumulative percentages of accident year 20X0 ultimate losses paid by December 31 of the calendar year in each column.
- Row 3: The row labeled "incremental" shows the incremental percentages of accident year 20X0 ultimate losses paid in each calendar year.

The final row in the table above tells us that 21.2% of an accident year's incurred losses are paid during the accident year, another 25.3% are paid in the 12 months following the accident year, 14.0% are paid in the subsequent 12 months, and so forth. The remaining 10.9% [= 100% − 89.1%] are paid more than 10 years after the inception of the accident year.

The illustration above shows the logic underlying the estimated loss payment pattern. Because of the volatility of loss payments and possible systematic changes over the years, as noted below, we make several adjustments.

- Settlement of large losses may distort the payment pattern in any one accident year.
- The loss payment pattern does not reflect any changes in the intervening nine years.
- This method ignores the information embedded in the observed liquidation of accident years 20X1 through 20X8.

RECENT DATA

To use the most recent data, we examine the dollars paid in calendar year 20X9 divided by the total incurred losses for each accident year. Assume that the paid loss development illustration shows the following associated figures from Schedule P, Parts 2 and 3.[47]

Exhibit AppA.4: Loss Payment Pattern from Successive Accident Years ($000,000) (Data from Schedule P, Parts 2 and 3, from the 20X9 Annual Statement)

Accident Year (1)	*Cum Paid by 20X8 (2)*	*Cum Paid by 20X9 (3)*	*Paid in 20X9 (4)*	*Ultimate Losses (5)*	*Percentage Paid (6)*
20X0	$422	$433	$11	$486	2.26%
20X1	$442	$454	$12	$520	2.31%
20X2	$391	$403	$12	$475	2.53%
20X3	$416	$434	$18	$522	3.45%
20X4	$504	$534	$30	$667	4.50%
20X5	$490	$542	$52	$707	7.36%
20X6	$463	$546	$83	$787	10.55%
20X7	$353	$485	$132	$802	16.46%
20X8	$152	$406	$254	$866	29.33%
20X9		$156	$156	$898	17.37%

The columns show the following figures:

Column (2): Cumulative dollars of loss paid through December 31, 20X8 (from Part 3),
Column (3): Cumulative dollars of loss paid through December 31, 20X9 (from Part 3).
Column (4): Incremental dollars of loss paid in 20X9 (= column (2) minus column (1)).
Column (5): Incurred losses (from Part 2).
Column (6): Incremental dollars of loss paid as a percentage of incurred losses (row 3 / row 4).

Consider the row for accident year 20X4:

Column 2: $504,000 has been paid by 12/31/20X8, or 60 months since inception of the accident year.

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[47] The accident years are shown along the horizontal axis of the table. In the exhibits used for the paid loss chain ladder development method, the accident years are shown along the vertical axis.

Column 3: $534,000 has been paid by 12/31/20X9, or 72 months since inception of the accident year.
Column 4: $30,000 has been paid between 60 months and 72 months.
Column 5: The total accident year 20X4 incurred losses are $667,000.
Column 6: 4.5% (or $30,000 / $667,000) of the incurred losses are paid between 60 months and 72 months since inception of the accident year.

The loss payment pattern in the table above is theoretically sound, though both the IRS and common actuarial practice use slightly different methods.

- This procedure uses figures from Schedule P, Part 3, which shows cumulative paid losses at the current valuation date and the previous valuation date. The IRS used figures from Part 1, perhaps because Part 1 is an audited exhibit whereas Part 3 is not.
- This procedure uses incurred loss losses from a single accident year for each payment period. Common actuarial practice is to use averages from three or more years.

Incremental Percentages and Cumulative Differences

For the lines of business with ten year exhibits, the IRS makes one additional change. The procedure outlined above uses the incremental paid loss percentages in each accident year to estimate the percentage of losses paid in each time interval. The IRS uses the difference in the cumulative paid loss percentages between successive accident years.

Exhibit AppA.5: Loss Payment Pattern Between Accident Years ($000,000)
(Data from Schedule P, Parts 2 and 3, from the 20X9 Annual Statement)

AccYr	20X0	20X1	20X2	20X3	20X4	20X5	20X6	20X7	20X8	20X9
Row 1	$433	$454	$403	$434	$534	$542	$546	$485	$406	$156
Row 2	$486	$520	$475	$522	$667	$707	$787	$802	$866	$898
Row 3	89.1%	87.3%	84.8%	83.1%	80.1%	76.7%	69.4%	60.5%	46.9%	17.4%
Row 4	1.8%	2.5%	1.7%	3.1%	3.4%	7.3%	8.9%	13.6%	29.5%	17.4%

- Row (1) shows the cumulative paid losses at December 31, 20X9, for each accident year.
- Row (2) shows the incurred losses at December 31, 20X9, for each accident year.
- Row (3) shows the ratio of cumulative paid losses to incurred losses.
- Row (4) shows the differences in successive ratios. For accident year 20X9, nothing is paid before calendar year 20X9, so 17.4% of incurred losses are paid in the first 12 months. For losses paid between 12 months and 24 months, we reason as follows.

- ✓ From the 20X8 accident year, we infer that 46.9% of incurred losses are paid by 24 months since inception of the accident year.
- ✓ From the 20X9 accident year, we infer that 17.4% of incurred losses are paid by 12 months since inception of the accident year.
- ✓ This implies that 46.9% – 17.4% = 29.5% of incurred losses are paid between 12 months and 24 months since inception of the accident year.

The figures in row (4) sum to 89.1%. This is the ratio of cumulative paid losses to incurred losses for accident year 20X0.

The illustration above uses figures from Schedule P, Parts 2 and 3. The IRS actually uses figures from Schedule P, Part 1, which include more loss adjustment expenses.

- The Part 1 figures used by the IRS include all loss adjustment expenses.[48]
- The Part 3 figures include only defense and cost containment expenses.

IRS RATIONALE

We summarize the computations as follows:

1. For each accident year in Schedule P, Part 1, we calculate the cumulative paid losses at the current valuation date as a percentage of the incurred losses for that accident year.
2. We take the difference between successive accident years to determine the expected percentage of incurred losses paid in each 12 month interval.
3. We use this procedure for the ten accident years shown in Part 1. If the cumulative paid losses for the oldest year equal 100% of the incurred losses, we stop here. If the cumulative paid losses for the oldest year are less than 100% of the incurred losses, we extend the loss payment pattern for additional years, as described below.

The cumulative paid losses as of the current valuation date are shown in Part 1, column 11, "total net paid." The incurred losses at the current valuation date are shown in column 28, "total losses and loss expense incurred."

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[48] See section 846(f)(2) of the Internal Revenue Code: *The term "unpaid losses" includes any unpaid loss adjustment expenses shown on the annual statement.*

ILLUSTRATION A: NO EXTENSION OF PAYMENTS

Although the concepts are straight-forward, the implementation is complex. We explain the details in this appendix with three illustrations. To *understand* the text of this paper, the reader need not know all the material in this appendix. To *implement* the pricing model described in this paper, the reader must be familiar with the tax rules and regulations.

We proceed incrementally in this appendix. For most casualty lines of business, the IRS loss payment pattern extend up to a maximum of 16 years. We show first the procedure for a line of business with no extension, so the loss payment pattern ceases in the eleventh year. The next illustration shows the extension through the sixteenth year.

The pricing actuary will normally estimate loss reserve discount factors for two or three future accident years. The loss payment pattern can be derived only for the first of these accident years if the company uses its own pattern. If the company uses the industry pattern, the estimation procedure depends on the particular accident years in relation to the redetermination year. For certain items, there is no good way to estimate the required figures. We explain the choices available to the pricing actuary after the illustrations.

The ABC Insurance Company elected to use its own loss payment pattern in the 2007 determination year. This election applies to accident years 2009 through 2013.

It is now July 1, 2010, and the pricing actuary is estimating premium rates for policy year 2011. Losses from policies written in policy year 2011 fall into accident years 2011 and 2012. We estimate IRS loss reserve discount factors for accident year 2011. After completing this illustration, we explain the possible methods of estimating loss reserve discount factors for accident year 2012.

The following figures are taken from ABC's 2009 Annual Statement, Schedule P, Part 1. The procedure shown here is applicable to any ten year line of business. These include all the casualty lines for which financial pricing models are commonly used.

Exhibit AppA.6: Casualty Line of Business Paid and Incurred Losses

Accident Year	Losses + LAE Paid	Losses + LAE Incurred
Prior	250,000	250,000
2000	270,000	275,500
2001	300,000	316,000
2002	320,000	348,000
2003	340,000	386,500
2004	350,000	421,500
2005	370,000	480,500
2006	380,000	550,500
2007	360,000	610,000
2008	330,000	687,500
2009	200,000	571,500

Discount Rate

The discount rate used for the loss reserve discounting procedure is the 60 month rolling average of the federal mid-term rate, from January 2006 through December 2010. Since it is now July 1, 2010, only 54 months are available; the last six months must be estimated.

The actuary has two alternatives for estimating future interest rates, as discussed above.

- Repeat the most recent federal mid-term rate for the remaining months, or use an average of recent federal mid-term rates for the remaining months.
- Determine the federal mid-term rates implied by the current term structure of interest rates.

For this illustration, we assume that the estimated 60 month moving average of the federal mid-term rates is 7% per annum.

Determination Year and Company Election

If the company uses its own data to determine the loss payment pattern, the data are updated each year. If the company uses industry data – that is, if the company uses Treasury factors determined from industry-wide data reported in Best's Aggregates and Averages, the data are updated in determination years (or re-determination years).

Determination years end in a "2" or a "7," and they use aggregate industry data for statement dates ending in a "0" or a "5."

- For determination year 20X2, data as of December 31, 20X0 are used.

- For determination year 20X7, data as of December 31, 20X5 are used.

Once every five years (determination years), the company makes an election to use either the loss reserve discount factors developed by the Treasury, which are based on industry aggregate data or its own loss reserve discount factors, which are based on its own data

The election is made with the company's tax filing for the determination year. It applies to that year and to the succeeding four years. If the company elects to use its own payment patterns, it uses data that are available before the beginning of each tax year. These are the data from two years earlier.

In this illustration, the company made an election with its 2007 tax filing to use its own data. The election applies to the 2007 through 2011 accident years.

PROJECTIONS

If the company uses the industry factors, the loss payment pattern is known until the next determination year. The pricing actuary must estimate only future federal mid-term rates, as discussed earlier.

At determination years, the loss payment pattern changes significantly. There is no simple method of projecting the future Schedule P data. The pricing actuary should use a loss payment pattern based on the actuarial projection, not the IRS projection.

If the company uses its own Schedule P data, the loss reserve discount factors can generally be estimated for one additional accident year only. The factors for the subsequent years should be based on actuarial projection techniques, not the IRS projection technique.

Illustration A: In July 2008, the pricing actuary is estimating loss reserve discount factors for accident years 2009, 2010, and 2011. The industry loss payment patterns were determined in 2007 for accident years 2007 through 2011. No future estimates are needed.

If the company uses its own loss payment patterns, the accuracy of the projection differs by accident year.

- Accident year 2009 uses loss payment patterns based on the 2007 Schedule P, which is available by July 2008. The only projection needed is for federal mid-term rates from July 2008 through December 2008.
- Accident year 2010 uses loss payment patterns based on the 2008 Schedule P entries, which the company's reserving actuary may be able to estimate.
- Accident year 2011 uses loss payment patterns based on the 2009 Schedule P. A projection of Schedule P entries 18 months in advance is unlikely to yield usable figures.

Illustration B: In July 2010, the pricing actuary is estimating loss reserve discount factors for accident years 2011, 2012, and 2013. The industry loss payment patterns are determined in 2012 for accident years 2012 through 2016.

- Accident year 2011 loss payment patterns are based on industry-wide 2005 Schedule P figures, which are available by the summer of 2006.
- Accident year 2012 and 2013 loss payment patterns are based on industry-wide 2010 Schedule P figures, which are not available until the summer of 2011. The IRS loss reserve discount factors are highly sensitive to random loss fluctuations. Attempting to project industry-wide Schedule P figures would not yield accurate discount factors. Instead, the actuary should estimate actuarial loss payment patterns through the tenth year based on the projection techniques discussed below. For years 11 through 16, the actuary should use the IRS procedure for extending the loss payment pattern, using the actuarial estimate for the percentage of losses paid in the tenth year.

One might presume that continuing the industry-wide loss payment patterns for accident years 2007 through 2011 for accident years 2012 and 2013 is a reasonable solution when the loss payment patterns for accident years 2012 and 2013 can not be readily estimated. This is not correct, since the loss payment patterns for accident years 2007 through 2011 are highly sensitive to the random loss fluctuations embedded in the 2005 Schedule P entries. The correct approach is to estimate an actuarial loss payment pattern for the line of business, not to repeat the previous IRS loss payment patterns.[49]

PROSPECTIVE PRICING AND RETROSPECTIVE ANALYSIS

The sensitivity of the IRS loss reserve discount factors to random loss fluctuations means that the actual discount factors may be quite different from the projected discount factors. In these situations, the actual discount factors should be used for retrospective analysis, and the variance should be ascribed to estimation error; see Feldblum [2002: Source of Earnings].

Illustration: In July 2010, the pricing actuary is estimating loss reserve discount factors for accident years 2011, 2012, and 2013. The company uses the industry-wide loss payment patterns. The actuary used actuarial projections for the accident year 2012 and 2013 loss payment patterns, as recommended above.

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[49] The pricing actuary may be tempted to rely on the tax department's projections of future loss reserve discount factors, particularly if the tax department says that they can estimate future loss reserve discount factors. The tax department generally means that they can estimate the factors for accident year 20XX in December 20XX–1 instead of waiting for the official promulgation of the factors by the Secretary of the Treasury in the latter half of 20XX. This is not a "projection"; this is simply an independent computation of the factors. The tax department has no need to project loss reserve discount factors for future years. Many tax accountants would consider the actuary's request for future accident year factors as a misunderstanding of the vintaging provisions in the tax law.

For the retrospective analysis of policy profitability performed in calendar years 2012 and subsequent, the actuary should use the actual factors promulgated by the Treasury, not the original projections. The change from projected to actual is no different for loss reserve discount factors than it is for the loss trend factors in Feldblum [2002: SOE].

VINTAGING

The computed loss reserve discount factors are used for accident year 2011 only. The discount factors for previous accident years at every future valuation date have already been determined and frozen. In tax parlance, they are vintaged. They are not subsequently revised.

Illustration: We determine between 11 and 15 discount factors for accident year 2011. The first ten discount factors are used at valuation dates December 31, 2011, December 31, 2012, December 31, 2013, through December 31, 2020. The final one to five development factors are used at subsequent valuation dates. The development factors are combined into a composite development factor for the prior years row for valuation dates 2021, 2022, and subsequent. The discount factors all use the same discount rate and loss payment pattern. The chart below shows the discount factors and the applicable valuation dates.

Exhibit AppA.7: Valuation Dates for Loss Reserve Discount Factors

Discount Factor	Accident Year	Individual / Composite	Tax Year (Valuation Date)
12 mos	2011	individual	2011
24 mos	2011	individual	2012
. . .	. . .	. . .	. . .
120 mos	2011	individual	2020
132 mos	prior	composite	2021
144 mos	prior	composite	2022

The first ten discount factors apply to accident year 2011 only. They are used at valuation dates between 12 months and 120 months from inception of the accident year, corresponding to tax years 2011 through 2020. For subsequent valuation dates, the discount factor for accident year 2011 is combined with discount factors for other accident years to form a composite discount factor.

Discounting Sequence

The loss reserve discount factor computation can be divided into three steps.

A. Calculate the nominal (undiscounted) amounts for cumulative percentages paid, incremental percentages paid, and percentages unpaid.
B. Calculate the adjustments for long-tailed lines of business showing adjusted incremental percentages paid, long-tail extension of payments, and adjusted percentages unpaid.
C. Apply the appropriate discount rate to obtain the discounted percentages unpaid, and loss reserve discount factors.

UNDISCOUNTED PERCENTAGES

The loss reserve discount factors for this illustration are calculated in Exhibit AppA.1. Column 2 shows the cumulative net paid losses and loss adjustment expenses by accident year at the current statement date. Column 3 shows the incurred net losses and loss adjustment expenses by accident year at the current statement date. These entries include paid losses and loss adjustment expenses, case reserves, and bulk reserves.

Column 4 shows the cumulative percentage paid from inception of the accident year to the current statement date, or column 2 divided by column 3. For accident year 2009, the percentage is $200,000 / $571,500 = 35.00%. For accident year 2008, the percentage is $330,000 / $687,500 = 48.00%.

Assumed Incremental Percentage Paid

Column 5 shows the expected incremental percentage paid in each 12 month period. These entries are the first differences of the series in the previous column:

- For accident year 2009, the cumulative percentage paid at 12 months since inception of the accident year is 35.00%. For the most recent accident year, the incremental percentage paid equals the cumulative percentage paid.
- For accident year 2008, the cumulative percentage paid at 12 months since inception of the accident year is 48.00%. This implies that 48.00% – 35.00% = 13.00% of incurred losses are paid between 12 months and 24 months since inception of the accident year.

Schedule P shows 10 accident years of data, from which we estimate 10 twelve-month intervals of expected loss payments. If any losses remain unpaid at the end of 10 years – that is, if the cumulative paid losses for the oldest accident year does not equal the incurred losses for that accident year – we assume that all these losses are paid in the eleventh year, with the following limitation.

The amount assumed to be paid in the eleventh year is capped by the amount assumed to be paid in the tenth year. The excess amount is assumed to be paid in the twelfth year, but it is also capped at the same limit. The remaining excess is assumed to be paid in the thirteenth year, and so forth. We continue in this fashion through the fifteenth year. The remaining excess is assumed to be paid in the sixteenth year, with no limit. The next illustration (other

liability) shows the computation of an extended loss payment pattern. We defer further explanation of the procedure for that illustration.

The Schedule P entries for the "prior years" row are not used in the computation of the loss reserve discount factors. The reserves and payments in this row relate to various accident years. A "composite" discount factor is used to determine the discounted loss reserves for the prior rows in Schedule P; see the discussion below.

In this illustration, the cumulative percentage paid for the ninth year (2001) is 94.94%, and the cumulative percentage paid for the tenth year (2000) is 98.00%. (The "n^{th}" year here means the "n^{th}" year working backwards from the current valuation date.)[50] The amount assumed to be paid from the end of the ninth year to the end of the tenth year is 98.00% – 94.94% = 3.06%. The amount still unpaid after 10 years is 100.00% – 98.00% = 2.00%. Since 2.00% is less than 3.06%, the full 2.00% is assumed to be paid in the eleventh year. No losses are assumed to be paid after 11 years.

Several of the commercial casualty lines of business have loss payment patterns extending beyond ten years; this is especially true for workers' compensation, other liability, products liability, and medical malpractice. For these lines of business, we don't expect the cumulative paid losses at the end of the tenth year to equal the incurred losses for that year.[51] The next illustration shows the adjustments used for these long-tailed lines of business.

DISCOUNTING COMPUTATIONS

Column 6 shows the percentage of losses unpaid at the end of the accident year, which equals the complement of the cumulative percentage of losses paid. For accident year 2009 in the illustration, the cumulative percentage of losses paid is 35.00%, and the percentage of losses unpaid at the end of the accident year is 100% – 35.00% = 65.00%.

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[50] We estimate the amounts to be paid in future calendar years by looking at old accident years. The difference in the cumulative percentages paid between the n^{th} past accident year and the $(n+1)^{rst}$ past accident year is the percentage assumed to be paid between the end of the n^{th} calendar year from inception of the accident year to the end of the $(n+1)^{rst}$ calendar year from inception of the accident year. The n^{th} accident year working backwards from the most recent accident year corresponds to the n^{th} calendar year working forewards from the current statement date.

[51] The IRS computation of the loss reserve discount factors for all years is heavily influenced by the Schedule P entries for the ninth oldest accident year and the tenth oldest accident year. By random loss fluctuations, any long-tailed line of business may have an 11 year loss payment pattern one year and a 16 year loss payment pattern the next year.

Column 7 shows the discounted percentage of losses unpaid at the end of the accident year. To compute these figures, we assume that all losses are paid at mid-year. We may use either an iterative method, working backwards from the oldest accident year, or a formula method.[52]

Iterative Method

Two percent of the incurred losses are assumed to be paid in the eleventh year, labeled "AY + 10" in the exhibit. We assume that they are paid in mid-year. With a 7.0% discount rate, the discounted value of these losses at the preceding December 31 is $2\%/(1.070)^{0.5} = 1.93\%$.

Going backwards in accident years corresponds to going forwards in calendar years. The "current accident year" in this Schedule P exhibit is 2009, though the computed loss payment pattern is used for accident year 2011, not accident year 2009. The current valuation date for accident year 2011 for which this discount factor applies is December 31, 2011. Accident year AY+1 corresponds to calendar year 2011+1 = 2012. Accident year AY+10 corresponds to calendar year 2011 + 10 = 2021.[53]

To determine the discounted percentage of losses unpaid at the end of the ninth year, we combine two pieces:

i. The percentage of losses assumed to be paid in the tenth year – which are assumed to be paid at mid-year – discounted for half a year to the end of the ninth year.
ii. The discounted percentage of losses unpaid at the end of the tenth year, discounted for an additional year to the end of the ninth year.

In the illustration, the two pieces are as follows.

i. 3.07% of accident year 2011 losses are assumed to be paid in the middle of the tenth year, or July 1, 2020. They are discounted for half a year to December 31, 2019: $3.07\%/1.070^{0.5} = 2.97\%$.
ii. The discounted percentage of accident year 2011 losses unpaid at the end of the tenth year (December 31, 2020) is discounted for a full year: $1.93\%/1.070 = 1.80\%$.

The sum of 2.97% and 1.80% is 4.77%. We continue in this fashion for all accident years. This is the iterative method.

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[52] The assumption that all losses are paid at mid-year is a proxy for an even distribution of paid losses during the year. In truth, losses are paid (on average) earlier than the middle of the year, particularly for losses paid in the 2 or 3 years following the inception of the accident year. The IRS procedure provides a slightly longer discount period than is warranted. This reduces the offset to taxable income and increases the income tax liability. This bias is offset by the shorter payment patterns implicit in the IRS extension past ten years.

[53] For an excellent explanation of this technique, see Salzmann [1984], who uses a similar version to develop a reserving method for allocated loss adjustment expenses.

Formula Method

Alternatively, formulas may be used for each year. The formula for the 2009 accident year in the Schedule P exhibit, which corresponds to accident year 2011 valued at December 31, 2011, is

$$(13.00\% \div 1.07^{0.5}) + (11.02\% \div 1.07^{1.5}) + \ldots + (3.07\% \div 1.07^{8.5}) + (2.00\% \div 1.07^{9.5}) = 52.26\%.$$

LOSS RESERVE DISCOUNT FACTORS

Column 8 shows the loss reserve discount factors used in the tax calculation. These factors are the discounted percentage of unpaid losses at the end of each year divided by the undiscounted percentage of unpaid losses at the end of that year. For accident year 2009, the loss reserve discount factor is 52.26% / 65.00% = 80.3944%. This corresponds to the loss reserve discount factor for accident year 2011 valued at December 31, 2011. If the accident year 2011 undiscounted reserves at December 31, 2011, are $450,000, the corresponding discounted reserves are $450,000 x 80.3944% = $361,775.

The loss reserve discount factor in the preceding row, 81.6659%, is applied to the accident year 2011 reserves on December 31, 2012, not to the reserves of any other accident year. If the 2012 Schedule P reserves for accident year 2011 are $350,000, the 2012 discounted reserves for accident year 2011 are $350,000 x 81.6659% = $281,380.

ILLUSTRATION B – LONG-TAILED EXTENSION OF PAYMENTS

In actual practice, the lines of business for which financial pricing models are appropriate will probably have extended loss payment patterns. The illustration below shows the procedure for extending the loss payment pattern beyond the eleventh year.

The following figures are taken from the 2009 Annual Statement, Schedule P, Part 1H (other liability), of a company that has elected to use its own loss payment pattern for computing discounted reserves for accident year 2011.

Exhibit AppA.8: Paid and Incurred Losses

Accident Year	Losses + LAE Paid	Losses + LAE Incurred
Prior	235,000	250,000
2000	50,000	55,500
2001	55,000	62,000
2002	60,000	70,000
2003	65,000	80,000
2004	70,000	96,000
2005	65,000	103,000
2006	60,000	115,000
2007	50,000	125,000
2008	35,000	140,000
2009	15,000	180,000

The 60 month rolling average of the federal mid-term rate, from January 2006 through December 2010, is 7.0% per annum.

Extension of Payments

The loss reserve discount factors are used for accident year 2011 only. In this illustration, we determine 15 separate loss reserve discount factors. The first ten discount factors are used for valuation dates December 31, 2011, through December 31, 2020. The 11th through the 15th discount factors are used at valuation dates December 31, 2021, through December 31, 2025 as part of the composite discount factor for accident years more than 10 years old.

CAPPING

The amount assumed to be paid in the eleventh year is capped by the amount assumed to be paid in the tenth year. In this illustration, 90.09% – 88.71% = 1.38% of incurred losses are assumed to be paid in the tenth year. The amount remaining unpaid after 10 years is

100.00% – 90.09% = 9.91% of the incurred losses. Only 1.38% is assumed to be paid in the eleventh year. The remaining 9.91% – 1.38% = 8.53% is assumed to be unpaid at the end of the eleventh year.

The 1.38% cap affects the subsequent years as well. The amount assumed to be paid in each of the five years immediately following the tenth year is the lesser of (i) the amount unpaid at the end of the previous year and (ii) the 1.38% cap. We show first an illustration with a loss payment pattern that does not extend through the 16th year before returning to the other liability illustration here.

Illustration: Suppose that the IRS loss reserve discounting procedure indicates that 90.90% is paid within 10 years and 88.10% is paid within nine years. This implies that 90.90% – 88.10% = 2.80% is paid in the tenth year. The amounts assumed to be paid in the 11th, 12th, and 13th years are also 2.80%. Only 9.10% – 3 × 2.8% = 0.70% remains unpaid after thirteen years. This is the amount assumed to be paid in the 14th year.

Whatever remains after 15 years is assumed to be paid in the 16th year, even if it exceeds the 1.38% cap.

Illustration: For illustration B, 9.91% – 5 × 1.38% = 3.01% remains unpaid after 15 years, so 3.01% is assumed to be paid in the sixteenth year.[54]

EXTENDED DEVELOPMENT

We begin the computation of the discounted percentages unpaid at the December 31 preceding the final loss payment. For this (other liability) illustration, the loss payment pattern extends through 16 years, so we begin the computation of the discounted percentage unpaid with the end of the fifteenth year.

3.01% of the accident year 2011 incurred losses are assumed to be paid in the middle of the 16th year, or July 1, 2026. The discounted loss reserve at the end of the 15th year (or December 31, 2025) is $3.01\% / 1.070^{0.5} = 2.91\%$.

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[54] See the Internal Revenue Code §§ 846(d)(3)(C) and (D), "Special rule for certain long-tail lines": In the case of any long-tail line of business, the period taken into account shall be extended (but not by more than 5 years), and the amount of losses which would have been treated as paid in the 10th year after the accident year shall be treated as paid in such 10th year and each subsequent year in an amount equal to the amount of the losses treated as paid in the 9th year after the accident year (or, if lesser, the portion of the unpaid losses not theretofore taken into account). To the extent such unpaid losses have not been treated as paid before the last year of the extension, they shall be treated as paid in such last year. The term "long-tail line of business" means any line of business if the amount of losses which would be treated as paid in the 10th year after the accident year exceeds the losses treated as paid in the 9th year after the accident year.

The discounted percentage unpaid at the end of the 14^{th} year equals the sum of (i) the 2.91% discounted percentage unpaid at the end of the 15^{th} year discounted for an additional full year and (ii) the 1.38% of the incurred losses assumed to be paid on July 1 of the 15^{th} year discounted for half a year. This is $2.91\% / 1.070 + 1.38\% / 1.070^{0.5} = 4.05\%$. (The 0.01 percentage point difference from the figure in the exhibit is a rounding discrepancy.)

Alternatively, we calculate each discounted percentage unpaid by formula. For the 2011 valuation date for the 2011 accident year, the discounted percentage unpaid equals

$$(16.67\% \div 1.07^{0.5}) + (15.00\% \div 1.07^{1.5}) + \ldots + (1.38\% \div 1.07^{13.5}) + (3.01\% \div 1.07^{14.5}) = 70.87\%.$$

Patterns

The pricing model discussed in this paper and its companion papers serves two functions:

- It determines the premium rates needed to provide a target return on capital, and
- It shows the pattern of income recognition under alternative accounting systems.

The unwinding of the interest discount on the loss reserves affects the pattern of income recognition. The income recognition pattern is compared for six accounting systems in Feldblum and Thandi [2002], "Income Recognition and Performance Measurement." The discussion here shows the expected pattern of IRS loss reserve discount factors.

In Illustration B, the loss reserve discount factors are similar for the ten accident years that are separately reported in Schedule P, ranging from 77% to 80%. Some actuaries presume that loss reserve discount factors should be lowest (i.e., furthest below unity) at inception and should increase towards unity as the reserves become more mature. This presumption is that the amount of the discount as a percentage of the remaining reserves is greatest at early maturities and declines to zero at later maturities.

This presumption is correct for the true discount factor for an individual loss. Suppose a loss occurs on July 1, 20X1, and it will be paid on July 1, 20X9. The amount of the discount is greatest on December 31, 20X1, and it declines steadily thereafter.

This presumption is not correct for an accident year. If loss payments follow an exponential decay, as modeled by McClenahan [1975] and Butsic [1981], the loss reserve discount factor remains relatively constant as long as some claims remain unpaid. The expected discount factor depends on the rate of decay and the discount rate, not on the development period. As Butsic [1981] shows, if the loss payments follow an exponential decay, the average remaining time to settlement is constant over the lifetime of the reserves.[55]

The loss reserve discount factors in Illustration B increase steadily in the final six years, from 80% to about 97%. This is caused by the IRS assumption of a constant percentage of incurred losses paid in each development period during the extended part of the loss payment pattern, instead of the declining percentage of incurred losses assumed by an exponential decay pattern.[56] For instance, Illustration B uses a 1.38% figure for each

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[55] For workers' compensation, the decay is slower than exponential. Temporary total claims dominate the early payments; most of these claims are settled within a year or two. Permanent partial disability and permanent total disability claims dominate the reserves for mature years. These claims may remain open for 30 or 40 years. The loss payment pattern is rapid initially but it is very slow by ten years of maturity.

[56] The exponential decay assumes that a constant percentage of the remaining reserves (not of the total incurred losses) is paid in each development period.

development period. The assumption of a final lump sum payment in the last year, whether or not the payment pattern is extended, augments the upward trend in the loss reserve discount factors for mature periods.

Composite Discount Factors

The loss reserve discount factors calculated above are applied to the unpaid losses for the appropriate accident year. Schedule P shows loss reserves by accident year only for the ten most recent years, to which ten separate loss reserve discount factors are applied. The 11th through 15th loss reserve discount factors are applied to the reserves in the Schedule P prior years row, which is not divided into the component accident years.

The IRS loss reserve discounting procedure assumes that all losses are paid no later than the 16th year. The prior years row in Schedule P contain losses that will be paid in the 12th through the 16th year, which use the loss reserve discount factors for years AY+11 through AY+15. A composite discount factor is formed from the five individual discount factors for application to the prior years row.

Each discount factor is the ratio of discounted reserves to undiscounted reserves for a given accident year at a given valuation date. For instance, the "tenth" accident year 2010 discount factor for AY+10 represents the discounted reserves for accident year 2010 at December 31, 2020, divided by the undiscounted reserves for accident year 2101 at December 31, 2020. This discount factor is computed in tax year 2010, not in tax year 2020.

We explain the calculation of the composite discount factor by illustration.

Illustration: Composite Discount Factors

For tax year 2019, Schedule P shows ten individual accident years: 2010 through 2019. Previous accident years – 2009 and prior – are grouped in the prior years row. Since the IRS loss reserve discounting procedure assumes that all losses are paid by the 16th year, we assume that the loss reserves in the prior years row represent losses from accident year 2005 through 2009.

We form a composite discount factor based on the following discount factors:

- Accident year 2005 discount factor for a valuation date 15 years after inception of year.
- Accident year 2006 discount factor for a valuation date 14 years after inception of year.
- Accident year 2007 discount factor for a valuation date 13 years after inception of year.
- Accident year 2008 discount factor for a valuation date 12 years after inception of year.
- Accident year 2009 discount factor for a valuation date 11 years after inception of year.

Some of these loss reserve discount factors use the same loss payment pattern. However, they all use different discount rates, and they are computed in separate years.

Suppose these five loss reserve discount factors are as shown below:

Exhibit AppA.11: Composite Discount Factor

Accident Year (1)	*Valuation Date (2)*	*Undiscounted Reserve (3)*	*Discounted Reserve (4)*	*Discount Factor (5)*
2005	AY + 15	5.0%	4.8%	96.9%
2006	AY + 14	7.2%	6.8%	93.9%
2007	AY + 13	9.1%	8.3%	91.0%
2008	AY + 12	11.7%	10.3%	88.2%
2009	AY + 11	13.3%	11.4%	85.4%
Total	prior years row	46.3%	41.6%	89.8%

The calculation of the individual discount factors is explained earlier. Each discount factor in column 5 is the ratio of the discounted reserves in column 4 to the undiscounted reserves in column 3. The reserve figures in columns 3 and 4 are expressed as percentages of the corresponding year's incurred losses. We compute the total of the five percentages for the discounted reserves and the undiscounted reserves. We divided these totals to obtain the composite discount factor for the prior years row.[57]

PROSPECTIVE PRICING

For the financial pricing model, we must project loss reserve discount factors until all the reserves run off. This time frame is the actual run-off date of the reserves, not the sixteen years assumed by the IRS. We explain the projection process by means of an illustration.

Illustration: The actuary is setting premium rates for policies written in 2005. For this block of business, reserves remaining 20 years after inception of the policy year are not material.

The losses on this block of business fall into accident years 2005 and 2006. Separate loss reserve discount factors are determined for each accident year. We examine here the factors for accident year 2005. The procedure for accident year 2006 is analogous, though it requires more estimation.

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[57] Using a simple average to obtain the "total" row assumes that each year has the same volume of incurred losses. It might seem better to weight the discount factors by the actual percentage of incurred losses by accident year in the prior years row. However, the IRS bases the loss reserve discounting procedure on information contained in the Annual Statement. The distribution of the prior years row reserves by accident year is not found in the Annual Statement.

For valuation dates 12/31, 2005 through 2014, the discount factors are the factors for accident year 2005. For valuation date 12/31/2015, the remaining accident year 2005 reserves appear in the prior years row in the 2015 Schedule P. The discount factor applied to this row is the composite factor determined from the following individual factors:

- accident year 2005 at 11 years
- accident year 2004 at 12 years
- accident year 2003 at 13 years
- accident year 2002 at 14 years
- accident year 2001 at 15 years

The factors for accident years 2001 through 2004 factors are available to the pricing actuary. No additional estimation is required for the composite factor beyond what is required for other accident year 2005 loss reserve discount factors.

For valuation date December 31, 2016, the remaining accident year 2005 reserves appear in the policy years row in the 2016 Schedule P. The loss reserve discount factor applied to this row is the composite factor determined from the following individual factors:

- accident year 2006 at 11 years
- accident year 2005 at 12 years
- accident year 2004 at 13 years
- accident year 2003 at 14 years
- accident year 2002 at 15 years

The pricing actuary has not calculated any accident year 2006 loss reserve discount factors. The random component of the discount factors for the eleventh through the fifteenth years is so great that the projection of these discount factors for future accident years is not feasible.

Instead, the pricing actuary should compile average discount factors for the eleventh through the fifteenth years. Since the random component of these factors is so great, a long-term average should be used. We assume here that the actuary uses ten year averages.

The average loss reserve discount factor for the eleventh year is the average of the eleventh year factors for the ten accident years 1996 through 2005. (If the 2005 factor is only a projection, the actuary may use a ten year average for accident years 1995 through 2004.) This average loss reserve discount factor would be combined with the actual factors for accident years 2002 through 2005 to form the composite factor for accident year 2005 at December 31, 2016.

We continue this process for all subsequent valuation dates. By the December 31, 2020, all five loss reserve discount factors used for the composite discount factor are averages. This same composite factor is used for all subsequent valuation dates.

REVENUE OFFSET

Proper treatment of the tax liability and the deferred tax asset stemming from revenue offset has a material effect on the indicated premiums. This section explains the revenue offset provision in the 1986 Tax Reform Act.

For other industries, sales constitute revenues for income tax purposes. Similarly, premium due is the taxable revenue (as well as the statutory and GAAP revenue) for life insurance companies. For property-casualty insurance companies, earned premium is the revenue for both statutory and taxable income, not written premium or collected premium.

For the statutory income statement, earned premium equals written premium minus the change in the unearned premium reserves. For taxable income, earned premium equals written premium minus 80% of the change in the unearned premium reserves.[58] [59]

- A change in written premium with no change in earned premium does not affect statutory income.
- A change in written premium with no change in earned premium affects the unearned premium reserve and changes the tax liability by means of the revenue offset provision.

Statutory and taxable income also differ in the treatment of accrued retrospective premiums. The statutory vs tax treatment of accrued retrospective premiums is important for the pricing of commercial casualty lines of business, such as workers' compensation and general liability. The pricing model must incorporate the both the statutory and the tax accounting treatment for this asset, as well as the resulting deferred tax asset. This subject is discussed in Feldblum [2002: Schedule P], and it is not repeated here.

The statutory (full expensing) vs GAAP (deferred policy acquisition cost) vs tax (revenue offset) treatment of policy acquisition costs is an important component of the financial pricing model discussed in this paper; see especially Feldblum and Thandi, "Income Recognition and Performance Measurement."

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[58] See the Treasury regulations, 2001 FED 26,153, §1.832-4(a)(3): "The determination of premiums earned on insurance contracts during the taxable year begins with the insurance company's gross premiums written on insurance contracts during the taxable year, reduced by return premiums and premiums paid for reinsurance. This amount is increased by 80 percent of the unearned premiums on insurance contracts at the end of the preceding taxable year, and is decreased by 80 percent of the unearned premiums on insurance contracts at the end of the current taxable year."

[59] Life insurance companies and annuity writers are subject to a DAC-tax that is identical in concept though more complex than the property-casualty tax provision explained here; see Atkinson and Dallas [2000], chapter 9.

ILLUSTRATION: SINGLE POLICY

An insurer writes a policy with a $10,000 written premium on December 31, 20XX, and it pays $2,000 in agents' commissions on that day. Losses of $8,000 are incurred and paid evenly through the policy term. There are no other expenses or losses on this policy. We assume that losses are paid when they are incurred so that we need not deal with IRS loss reserve discounting.

The unearned premium reserve for this policy is $0 on January 1, 20XX, and $10,000 on December 31, 20XX. The change in the unearned premium reserve during the year is $10,000. The earned premium in 20XX is $10,000 of written premium minus the $10,000 change in the unearned premium reserve, or $0. Expenses during 20XX are $2,000, and statutory income during 20XX is –$2,000. Without revenue offset, the federal income tax liability would be 35% × –$2,000 = –$700, or a $700 tax refund.

The unearned premium reserve on December 31, 20XX+1, is $0. The change in the unearned premium reserve during 20XX+1 is –$10,000. The earned premium in 20XX+1 is $0 of written premium minus the –$10,000 change in the unearned premium reserve, or $0 – (–$10,000) = +$10,000. Losses of $8,000 are incurred and paid in 20XX+1. The statutory income is $10,000 – $8,000 = $2,000. The tax liability (ignoring revenue offset) would be 35% × $2,000 = $700.

Statutory accounting recognizes a loss at policy inception and a gradual profit during the remainder of the policy lifetime, thereby preventing companies from recognizing income until it has been fully earned.[60]

Were there no revenue offset provision in the tax code, the U.S. Treasury would fund part of the initial underwriting loss at policy inception. The illustration above shows a tax refund of $700 in 20XX and a tax liability of $700 in 20XX+1. Before 1987, statutory accounting helped the insurance industry defer its tax liabilities. Steady growth (in nominal dollar terms) led to persistent deferral of tax liabilities.

Direct and Indirect Methods

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[60] Some analysts see a conservative bend in statutory accounting's write-off of pre-paid acquisition costs when they are incurred, particularly in comparison with GAAP's capitalization and amortization of the deferred policy acquisition cost asset. This is not quite correct. Statutory accounting is correct from a tangible asset perspective, since the prepaid acquisition costs may be incurred whether or not the company retains the policy. International accounting standards follow statutory accounting on this issue. GAAP capitalizes an "imaginary" asset called DPAC to match revenues and expenses and show a better portrayal of the company's profitability. However, statutory accounting is unduly conservative in its double treatment of underwriting expenses: once when they are incurred and a second time in the gross unearned premium reserves. See Yoheved and Sarason [2003] for further discussion of GAAP and statutory accounting of property-casualty insurance companies.

The Tax Reform Act of 1986 introduced the revenue offset provision of the Internal Revenue Code. The provision may be stated in two equivalent ways. These two perspectives are used in the two fashions of computing taxable income and the federal income tax liability, which are termed here the "direct method" and the "indirect method." The direct method is easier to understand; the indirect method is the method actually used in the Internal Revenue Code for computing taxable income.

1. *Direct method:* The taxable earned premium equals the taxable written premium minus 80% of the change in the unearned premium reserve. This may be stated as "only 80% of the change in the unearned premium reserve is an offset to taxable income."
2. *Indirect method:* Twenty percent of the change in the unearned premium reserve is an *addition* to statutory income for computing taxable income.

We can use either method for the illustration.

Direct method: The taxable earned premium in 20XX equals the taxable written premium minus 80% of the change in the unearned premium reserve, or $10,000 – 80% × ($10,000 – $0) = $2,000 in 20XX. Agents' commissions are $2,000 on December 31, 20XX. Taxable income is $2,000 – $2,000 = $0, and the tax liability is $0.

In 20XX+1, the taxable earned premium equals $0 – 80% × ($0 – $10,000) = $8,000. The losses incurred and paid in 20XX+1 are $8,000. The taxable income is $8,000 – $8,000 = $0, and the tax liability is $0.

Indirect method: Twenty percent of the change in the unearned premium reserve in 20XX is 20% × ($10,000 – $0) = $2,000. The statutory income in 20XX is –$2,000. Taxable income is –$2,000 + $2,000 = $0, and the tax liability is $0.

In 20XX+1, twenty percent of the change in the unearned premium reserve is 20% × ($0 – $10,000) = –$2,000. The statutory income in 20XX+1 is +$2,000. The taxable income is +2,000 – $2,000 = $0, and the tax liability is $0.

ILLUSTRATION B: TWO YEARS

An insurer writes a policy with a $10,000 written premium on July 1, 20XX, and it pays $2,000 in agents' commissions on that day. Losses of $8,000 are incurred evenly over the policy term, and they are paid when they are incurred. On July 1, 20XX+1, the insurer renews the policy for a written premium of $15,000, and it pays $3,000 in agents' commissions on that day. Losses of $12,000 are incurred evenly over the policy term, and they are paid when they are incurred. There are no other expenses on these policies.

Illustration B shows the importance of computing the *change* in the unearned premium reserve during the year. The statutory unearned premium reserve equals $0 on December

31, 20XX–1, $5,000 on December 31, 20XX, $7,500 on December 31, 20XX+1, and $0 on December 31, 20XX+2.

CALENDAR YEAR 20XX

Statutory earned premium is $10,000 written premium minus the ($5,000 – $0) = $5,000 change in the unearned premium reserve; the earned premium is $5,000. Expenses are $2,000, and incurred losses are $4,000. The statutory income in 20XX is $5,000 – $1,000 – $4,000 = –$1,000. There are two methods to calculate the taxable income.

i. *Direct method:* The taxable earned premium is taxable written premium minus 80% of the change in the unearned premium reserve, or $10,000 – 80% × ($5,000 – $0) = $6,000. The taxable income is $6,000 – $2,000 – $4,000 = $0, and the tax liability is $0.
ii. *Indirect method:* Twenty percent of the change in the unearned premium reserve is 20% × ($5,000 – $0) = $1,000. The statutory income in 20XX is –$1,000. The taxable income is –$1,000 + $1,000 = $0, and the tax liability is $0.

CALENDAR YEAR 20XX+1

Statutory earned premium is $15,000 written premium minus the ($7,500 – $5,000) = $2,500 change in the unearned premium reserve; the earned premium is $12,500. Expenses incurred and paid on January 1, 20XX+1, are $3,000, and incurred losses during the year are $4,000 (first six months) + $6,000 (latter six months) = $10,000. The statutory income is $12,500 – $3,000 – $10,000 = –$500. There are two methods to calculate taxable income.

i. *Direct method:* The taxable earned premium is the taxable written premium minus 80% of the change in the unearned premium reserve, or $15,000 – 80% × ($7,500 – $5,000) = $13,000. Expenses and losses are the same as for statutory income. The taxable income is $13,000 – $3,000 – $10,000 = $0, and the tax liability is $0.
ii. *Indirect method:* Twenty percent of the change in the unearned premium reserve is 20% × ($7,500 – $5,000) = $500. The statutory income in 20XX+1 is –$500. The taxable income is –$500 + $500 = $0, and the tax liability is $0.

CALENDAR YEAR 20XX+2

Statutory earned premium is $0 written premium minus the ($0 – $7,500) = –$7,500 change in the unearned premium reserve, or $7,500. Expenses incurred in 20XX+2 are $0, and incurred losses during the year are $6,000. Statutory income is $7,500 – $6,000 = $1,500.

There are two methods to calculate the taxable income.

i. *Direct method:* The taxable earned premium is $0 – 80% x ($0 – $7,500) = $6,000. The taxable income is $6,000 – $6,000 = $0, and the tax liability is $0.

ii. *Indirect method:* Twenty percent of the change in the unearned premium reserve is 20% x ($0 – $7,500) = –$1,500. The statutory income in 20XX+2 is $1,500. The taxable income is $1,500 + –$500 = $0, and the tax liability is $0.

The computation of the admitted portion of the deferred tax asset stemming from IRS loss reserve discounting is based on two items:

- the loss reserve discount factors by accident year and by line of business for the current valuation date and for the valuation date 12 months hence, and
- the company's loss payment pattern by line of business.

The IRS loss payment pattern is used to compute the loss reserve discount factors. The actuary's estimated loss payment pattern is used to compute the admitted portion of the deferred tax asset.

Of all the changes in the NAIC's codification project, the deferred tax asset stemming from IRS loss reserve discounting has the greatest effect on policy pricing and company valuation. The deferred tax assets stemming from revenue offset and loss reserve discounting are used extensively in this paper and in all its companion papers. We present first the requisite background explanations of deferred tax assets and liabilities, and we illustrate the computation of the deferred tax assets relevant for policy pricing.

Current Taxes vs Deferred Taxes

There are two ways of accounting for federal income taxes:

- The incurred tax liability is the tax liability actually incurred by the taxpayer, based on the provisions of the Internal Revenue Code, or
- The accrued tax liability is the tax liability implied by the company's balance sheet, whether GAAP or statutory.

Current taxes are the incurred tax liability. The current year's change to the deferred tax asset or liability is the difference between the incurred tax liability and the accrued tax liability.[61] The change to the deferred tax asset or liability is a direct charge or credit to surplus shown on line 24 of the NAIC Annual Statement. As a direct charge and credit to surplus, it has the same effect on the implied equity flows as though it flowed through the income statement.[62]

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[61] This definition uses a retrospective computation. SFAS 109 requires a prospective computation, which may be different if the tax rate changes or if there are other changes in tax regulations. For simplicity, we use the retrospective viewpoint at first. We explain the prospective viewpoint further below.

[62] Direct charge and credit to surplus are not included in after-tax net income for standard accounting statements, both statutory and GAAP. The implied equity flows depend on the statutory balance sheet entries, not the income statement entries.

Before 2001, insurers could not admit any deferred tax asset or liabilities on the statutory balance sheet. In contrast, GAAP recognizes deferred tax assets and liabilities if they are expected to be realized; see SFAS 109. With the implementation of codification in 2001, statutory accounting recognizes deferred tax liabilities and a portion of deferred tax assets.

Permanent Differences and Timing Differences

Tax accounting differentiates between permanent differences and timing differences, as defined below.

- *Permanent differences* are differences that do not reverse in later accounting periods. The tax exemption for municipal bond interest is a permanent difference.
- *Timing differences* are differences that reverse in later accounting periods. The revenue offset provision creates a timing difference between statutory income and taxable income.

An alternative perspective is to view permanent differences as differences in the tax rates applicable to different sources of income; see Feldblum and Thandi, "Income Recognition and Performance Measurement." For property-casualty insurers, both corporate bond income and municipal bond income are taxable income, but the former has a 35% tax rate and the latter has a 5.25% tax rate; see Feldblum and Thandi, "Investment Yields."

Income Statement vs Balance Sheet

It is tempting to define timing differences as differences in the timing of income between the book income statement (i.e., GAAP or statutory) and the tax income statement. This is not correct.

> *Timing differences are differences between the tax income statement and the income statement implied by the GAAP or statutory balance sheet.*[63]

UNREALIZED CAPITAL GAINS AND LOSSES

For each accounting year, we compute the difference between the book value and the cost of the financial asset. The change in this difference from the previous year to the current year is the unrealized capital gain or loss. For common stocks, the book value is the market value.

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[63] This definition is particularly relevant to the deferred tax liabilities and assets stemming from unrealized capital gains and losses. For the deferred tax assets stemming from revenue offset and loss reserve discounting, we could use the difference between statutory income and taxable income.

Unrealized capital gains and losses are admitted on the statutory (as well as GAAP) balance sheet, though they do not flow through the income statement. They are direct charges and credits to surplus, not a portion of net income.

For tax purposes, capital gains and losses are not part of income until they are realized.

- Unrealized capital gains increase the book value of common stocks on the statutory balance sheet. There is no incurred tax liability in the current tax year. Instead, the reporting company shows a deferred tax liability.
- Similarly, unrealized capital losses decrease the book value of common stocks on the statutory balance sheet. There is no tax refund in the current tax year. Instead, the reporting company shows a deferred tax asset.

Illustration

ABC Insurance Co buys common stock for $50 million on December 31, 20XX.

- On December 31, 20XX+1, the common stock are worth $40 million;
- On December 31, 20XX+2, the common stock are worth $60 million; and
- On December 31, 20XX+3, the common stock are worth $80 million.

The federal income tax rate is 35%. On December 31, 20XX+3, the ABC Insurance Company sells the common stock. We calculate the following accounting entries:

- The unrealized capital gains and losses in years 20XX+1, 20XX+2, and 20XX+3.
- The realized capital gains and losses in years 20XX+1, 20XX+2, and 20XX+3.
- The deferred tax assets and liabilities in years 20XX+1, 20XX+2, and 20XX+3.

Tax year 20XX+1

The market value of the stock has decreased by $10 million. The stock has not been sold yet, so the capital loss is unrealized. There are no realized capital gains and losses.

- On December 31, 20XX, book value – cost = $50 million – $50 million = $0.
- On Dec 31, 20XX+1, book value – cost = $40 million – $50 million = –$10 million.
- The unrealized capital gain or loss = –$10 million – $0 million = –$10 million.

The current balance sheet shows a decline of $10 million. When the stocks are sold, ABC Insurance Company will have an income loss of only $6.5 million, since the capital loss can offset other capital gains, and the company's tax liability will be reduced by $3.5 million. There is a $3.5 million deferred tax asset on the 20XX+1 balance sheet.

Tax year 20XX+2

The stock prices have increased. The unrealized capital gain is the *change* in the difference between book value and cost of the stocks. The unrealized capital gain for 20XX+2 is $20 million. The realized capital gain is again zero, since the stocks have not been sold.

- On December 31, 20XX+1, book value – cost = $40 million – $50 million = –$10 million.
- On December 31, 20XX+2, book value – cost = $60 million – $50 million = +$10 million.
- The unrealized capital gain or loss = +$10 million – (–$10 million) = +$20 million.

The company's balance sheet is $20 million stronger than it was a year ago. However, if the stocks were sold now, the company would realize a gain of only $13 million, since $7 million would go to taxes. The *change* in the deferred tax assets and liabilities is a credit of $7 million. Since we began with a deferred tax asset (a debit) of $3.5 million, we now have a deferred tax liability (a credit) of $3.5 million.

Tax year 20XX+3

The company sells the stock. The difference between market value and cost of the stocks is now $0 (since there are no more stocks on the balance sheet), so the unrealized capital gain is –$10 million.

- On December 31, 20XX+2, book value – cost = $60 million – $50 million = +$10 million.
- On December 31, 20XX+3, book value – cost = $0 million – $0 million = $0 million.
- The unrealized capital gain or loss = $0 million – ($10 million) = –$10 million.

The realized capital gain, which is defined as the sale price minus the purchase price, is +$30 million. The deferred tax assets and liabilities are now zero.[64]

Prospective Pricing

To see the effects of deferred tax assets and liabilities on determining a benchmark investment yield for policy pricing, we consider an illustration of bond and stock returns.

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[64] Unrealized capital gains and losses give rise to deferred tax liabilities and assets, respectively. Realized capital gains and losses affect current taxes; they do not give rise to deferred tax assets and liabilities. An exception stems from the rule that capital losses can offset capital gains but not operating gains.

If capital losses exceed capital gains, the company may carry forward the unused capital losses. The tax rate times the unused capital loss is a deferred tax asset, not a deduction in current tax liabilities.

Capital losses can be carried forward a limited number of years. If during these years the company has not realized sufficient capital gains to offset all the capital losses, the remaining capital losses expire unused, and the deferred tax asset is removed.

Illustration: An insurer invests in bonds yielding 10% per annum and in common stocks that pay no dividends but that are expected to increase in market value by 10% per annum. We examine the effects of each on the benchmark investment yield for a financial pricing model. To keep the illustration clear, we assume that the insurer begins with $100 million of each type of security.

Single Year: During the first year, the bonds yield $10 million. The federal income tax liability is $3.5 million, and the increase in statutory surplus is $6.5 million. The after-tax investment yield for the purposes of the pricing model is 6.5% per annum.

During the first year, the expected change in the common stock value is +$10 million. The expected deferred tax liability is $3.5 million, so the expected change in the statutory balance sheet is a $6.5 million increase in surplus. For the single year scenario, the after-tax investment yield is 6.5% for the common stocks.

For a multi-period scenario, the yields on the bonds and the common stocks are not identical, even when deferred tax assets and liabilities are considered. We show this with a two year scenario. We assume that the investment income in reinvested in the same securities.

Two Years: The bond portfolio yields investment income of $10 million the first year. Of this amount, $3.5 million is paid to the U.S. Treasury, and $6.5 million is reinvested in the bond portfolio. During the second year, the bond portfolio yields investment income of $106.5 million × 10% = $10.65 million. Of this amount, $10.65 million × 35% = $3.7275 million is paid to the U.S. Treasury, and $10.65 million × 65% = $6.9225 million is reinvested in the bond portfolio. The total in the bond portfolio after two years is $106.5 million + $6.9225 = $113.4225. This is a 6.5% annual yield, since $1.065^2 = 1.134225$.

The common stock portfolio appreciates to $110 million the first year. Nothing is paid to the U.S. Treasury, and the company sets up a $3.5 million deferred tax liability on its balance sheet. During the second year, the common stock portfolio appreciates to $121 million. Nothing is yet paid to the U.S. Treasury, and the company increases the deferred tax liability to $21 million × 35% = $7.35 million. The net common stock asset is $121 million – $7.35 million = $113.65 million. This is a 6.607% annual yield, since $1.06607^2 = 1.1365$.

The effect of the tax deferral on the effective investment yield is small for a short holding period. The effect is material for longer holding periods.[65] Formulas for calculating the effective investment yield for different securities are shown in Feldblum and Thandi, "Investment Yields."

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[65] The effective holding period is generally estimated as the reciprocal of the turnover rate. If 5% of the common stocks are sold each year, the effective holding period is 1 ÷ 5% = 20 years. This approximation is not exact, but the difference from the exact result is insignificant.

We sum up the gist of this discussion as follows:

- For calculating after-tax net income, a deferred tax liability is not the same as an actual tax liability.
- For calculating implied equity flows, a deferred tax liability is the same as an actual tax liability.
- For calculating the investment yield, a deferred tax liability is slightly different from an actual tax liability. The magnitude of the difference depends on the effective holding period of the securities.

Statutory Recognition of Deferred Tax Assets

All deferred tax liabilities are recognized on the statutory balance sheet. For most deferred tax assets, the admitted statutory portion equals the entire asset, and statutory accounting is the same as GAAP.[66] In certain instances, only a portion of the deferred tax assets are recognized on the statutory balance sheet. This applies particularly to the deferred tax asset stemming from IRS loss reserve discounting for medium- and long-tailed lines of business.

SSAP No. 10, "Income Taxes," paragraph 10, says:

Gross DTAs shall be admitted in an amount equal to the sum of:

a Federal income taxes paid in prior years that can be recovered through loss carrybacks for existing temporary differences that reverse by the end of the subsequent calendar year;
b The lesser of:

i. The amount of gross DTAs, after the application of paragraph 10 a., expected to be realized within one year of the balance sheet date; or
ii. Ten percent of statutory capital and surplus as required to be shown on the statutory balance sheet of the reporting entity for its most recently filed statement with the domiciliary state commissioner adjusted to exclude any net DTAs, EDP equipment and operating system software and any net positive goodwill; and

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[66] There are two potential differences between GAAP and statutory accounting even when the full deferred tax asset passes the 12 month test:

- Some companies use a valuation allowance on the GAAP balance sheet for deferred tax assets and liabilities that may not reverse.
- Some companies use fair values, or discounted values, for deferred tax assets and liabilities that may not reverse for many years.

c. *The amount of gross DTAs, after application of paragraphs 10 a. and 10 b., that can be offset against existing gross DTLs.*

A gross deferred tax asset is admissible if it will reverse within one year, as required by paragraph (a) and by paragraph (b.i).

The limitation of 10% of surplus in paragraph (b.ii) is applicable for some companies, depending on the circumstances of the company's business and capital. Actuaries estimating the admitted portion of the deferred tax asset for these companies must take this limitation into account.

This is one of the rare instances where the implied equity flows depend not just on the book of business being priced but on all operations of the company. It would be rare for the deferred tax asset stemming from a single policy year to exceed 10% of statutory surplus, but is may occur that the total deferred tax asset of the company exceeds 10% of its surplus.

Illustration: The ABC Insurance Company writes annual policies with effective dates spread evenly through the year. Its premium to surplus ratio is 2 to 1, and its reserves to surplus ratio is 4 to 1. Its average loss reserve discount factor is 80%, and 30% of its deferred tax asset from loss reserve discount will reverse within 12 months. We work out its gross deferred tax asset and the portion admitted on the statutory balance sheet.

To simplify the computations, we use numbers instead of algebraic variables. We assume that statutory surplus is $100 million. Any other figure would work just as well.

The premium to surplus ratio is 2 to 1, so annual premium is $200 million. Since it uses annual policies with effective dates spread evenly through the year, the unearned premium reserves are $200 million × 50% = $100 million. The deferred tax asset stemming from revenue offset is $100 million × 20% × 35% = $7 million.

The reserves to surplus ratio is 4 to 1, so the undiscounted loss reserves are $400 million. The average loss reserve discount factor is 80%, so the discounted reserves are $400 million × 80% = $320 million. The gross deferred tax asset stemming from IRS loss reserve discounting is ($400 million – $320 million) × 35% = $28 million. The portion admitted on the statutory balance sheet is $28 million × 30% = $8.4 million.

The total statutory deferred tax asset is $7 million + $8.4 million = $15.4 million. This is limited to 10% of statutory surplus. Ten percent of statutory surplus is $10 million, so an additional $5.4 million is not admitted.

This illustration is reasonable, but it does not reflect the average company. Various changes in the scenario would reduce or eliminate the effect of the "10% of surplus" restriction.

- A lower premium to surplus ratio would reduce the effect of this restriction. The U.S. insurance industry as a whole has a premium to surplus ratio of about 1 to 1. With a 1 to 1 premium to surplus ratio in the illustration above, the 10% of surplus restriction has no effect on the admitted portion of the deferred tax asset.
- A shorter policy term would reduce the deferred tax asset stemming from revenue offset. With six month policies, the unearned premium reserve would be only half the size and the deferred tax asset stemming from revenue offset would be only half the size.
- The property lines of business do not have slowly liquidating reserves that would generate a significant deferred tax asset from loss reserve discounting.
- Companies with effective dates clustered around January 1 have much lower unearned premium reserves at the end of the year, and therefore have lower deferred tax assets stemming from revenue offset.

Most companies do not have deferred tax assets that will reverse in the coming year and that exceed 10% of policyholders' surplus. The deferred tax asset stemming from revenue offset is usually about 1% to 5% of statutory surplus. The deferred tax asset stemming from IRS loss reserve discounting is larger for companies that predominate in the long-tailed casualty lines of business, but most of this deferred tax asset does not reverse within one year. For companies with low surplus, this restriction is important.

The offsetting against existing gross deferred tax liabilities mentioned in paragraph (c) is relevant for companies with large unrealized capital gains from common stock holdings. The actuary should take this provision into account when quantifying the admitted portion of the deferred tax asset.

Common stock that has suffered an unrealized capital loss may be sold within the next 12 months to realize the tax benefits. A literal reading of the SSAP would permit the recognition of the deferred tax asset only if the company expects to realize the capital loss during the coming calendar year. In practice, most auditors do not require an explicit company expectation to realize the loss in order to admit the deferred tax asset.

Revenue Offset

The deferred tax asset stemming from revenue offset is similar to the deferred tax asset stemming from loss reserve discounting. For annual policies, the entire deferred tax asset will reverse during the coming year, and it is fully admitted on the statutory balance sheet.

BACKGROUND

All acquisition expenses flow through the statutory income statement when they are incurred. No deferred policy acquisition cost (DPAC) asset is entered on the statutory balance sheet.

On GAAP financial statements, acquisition expenses are capitalized on the balance sheet and amortized through the income statement over the term of the policy. The DPAC asset depends on the actual expenses incurred by the company.

For tax purposes, 20% of the written premium is treated as acquisition expenses that are capitalized and amortized over the term of the policy.[67] More precisely, the revenue offset provision defines the taxable earned premium.

- Statutory earned premium equals written premium minus the change in the unearned premium reserves.
- Taxable earned premium equals written premium minus 80% of the change in the unearned premium reserves.

ILLUSTRATION: DPAC OF 20%

An annual policy with a premium of $1,000 and acquisition expenses of $200 is written on December 31, 20XX.

- The statutory balance sheet shows a loss of $200. The written premium of $1,000 is offset by the unearned premium reserve of $1,000, and the incurred acquisition cost of $200 flows through the income statement.
- For tax purposes, the $1,000 written premium is offset by only $800 of unearned premium reserves, leaving a $200 gain. This $200 gain combined with the $200 acquisition cost yields a $0 net gain or loss.

The income implied by the statutory balance sheet – taxable income = –$200 – $0 = –$200.

In 20XX+1, statutory earned premium is $1000, since the entire unearned premium reserve is taken down over the course of the year. The taxable income is $800, since only 80% of the change in the unearned premium reserve is considered. For 20XX+1, the income implied by the statutory balance sheet – taxable income equals $1000 – $800 = $200.

At the end of 20XX+1, the statutory balance sheet equals the implied tax balance sheet. Both show net cash received of $1000 – $200, or the written premium minus the acquisition expense. The temporary balance sheet difference at December 31, 20XX fully reverses by December 31, 20XX+1.

At December 31, 20XX, taxable income is $200 greater than the income implied by the statutory balance sheet. The tax liability for 20XX is 35% x $200 = $70 greater than the tax liability that would be determined from the statutory balance sheet. Since the $70 difference

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[67] Life and health insurers and annuity writers have a similar "DAC-tax."

will reverse over the coming 12 months, it is recognized as a deferred tax asset on the statutory balance sheet.

The deferred tax asset on the statutory balance sheet does not depend on the amount of actual acquisition expenses. In contrast, the deferred tax asset on the GAAP balance sheet depends on the size of the GAAP deferred policy acquisition cost asset relative to the 20% assumption in the revenue offset provision.

ILLUSTRATION: DPAC OTHER THAN 20%

A company writes and collects a $1000 annual premium on December 31, 20XX. Acquisition expenses of $250 are incurred (and paid) on December 31, 20XX. The marginal tax rate on underwriting income is 35%. All acquisition costs are deferrable under GAAP.

Taxable underwriting income for 20XX is $200 (taxable premium income from revenue offset) – $250 (acquisition expenses) = –$50. The tax outflow is a negative $17.50 (or a tax refund of $17.50).[68]

The taxable premium income may be evaluated in either of two ways.

- Taxable earned premium = written premium minus 80% of the change in the unearned premium reserves = $1000 – 80% × $1000 = $200.
- Taxable earned premium = statutory earned premium plus 20% of the change in the unearned premium reserves = $0 + 20% × $1000 = $200.

The tax liability is 35% times the taxable income: 35% × ($200 – $250) = –$17.50.

Taxable underwriting income for 20XX+1 equals $800 of taxable premium income. The tax outflow is $800 × 35% = $280.00. Written premium during the year is $0 and the unearned premium reserve declines from $1000 to $0. We use the same two computation methods: (i) $0 – 80% × (–$1000) = $800, or (ii) $1000 + 20% × (–$1000) = $800.

A deferred tax asset of $70 stemming from the revenue offset provision is entered on the balance sheet on December 31, 20XX, and it is amortized over the course of the policy term. The full deferred tax asset from revenue offset is recognized on the statutory balance sheet, since it reverses within 12 months of the balance sheet date (for annual policies).

On GAAP financial statements, the book income for 20XX is $1000 – $0 = $1000, since all acquisition expenses are capitalized. The taxable income is –$50 (as above), and the tax

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[68] The tax refund stemming from negative taxable income offsets tax liabilities stemming from positive taxable income on other insurance contracts. There is no need to presume tax carrybacks or carryforwards.

liability is –$17.50 (i.e., a refund). GAAP shows a deferred tax liability (not an asset) of $17.50, exactly offsetting the tax refund.

The text of this paper uses an acquisition expense greater than 20% of premium in order to show the differing treatments under GAAP and statutory accounting. The same illustration is carried through to the companion papers.

Loss Reserve Discounting

The deferred tax asset stemming from loss reserve discounting is the most difficult component of the financial pricing model for some readers. The cause of this difficulty lies in the training and experience of many North American actuaries. The actuarial aspects of the pricing model are covered in the examination syllabus, and many actuaries have experience with cash flow analysis at work. Tax accounting is not emphasized on the actuarial syllabus, and few actuaries deal with deferred tax assets in their jobs.

This situation is unfortunate. The concepts involved in tax accounting are not difficult, and they significantly affect policy pricing. There is perhaps no better example than the deferred tax asset stemming from loss reserve discounting. There is no good documentation of the procedures (other than here), and many actuaries never have the opportunity to master the calculations. In truth, the procedure is straight-forward; it takes no more than fifteen or twenty minutes to learn how it is done.

The statutory incurred losses are the paid losses plus the change in the undiscounted loss reserves. The taxable incurred losses are the paid losses plus the change in the *discounted* loss reserves. The difference between statutory and taxable incurred losses is a timing difference. The change in the deferred tax asset is 35% of this difference.

Illustration: A policy is issued on January 1, 20XX, for a premium of $1000 and expenses of $200. Losses of $800 are incurred in 20XX, of which half are paid in 20XX and half are paid in 20XX+1. The IRS loss reserve discount factor at the 12 month valuation is 90%. For simplicity, we assume that the companies earns no investment income.

- The statutory incurred losses in 20XX are $400 of paid losses plus $400 of loss reserve change = $800. Statutory income is $1000 – $200 – $800 = $0. The accrued taxes on income of $0 is $0.
- The taxable incurred losses in 20XX are $400 of paid losses plus $360 of change in discounted loss reserves = $760. Taxable income is $1000 – $200 – $760 = $40. The tax liability on $40 is $14.

The difference between the income implied by the statutory balance sheet and taxable income is $0 – $14 = –$14. The gross deferred tax asset is $14.

Only the portion of the deferred tax asset that reverse within 12 months is admitted on the statutory balance sheet. We examine the statutory income and taxable income for 20XX+1.

- The statutory incurred losses in 20XX+1 are $400 of paid losses plus –$400 of loss reserve change = $0. There is no premium or expense in 20XX+1, so statutory income is $0. The accrued taxes on income of $0 is $0.
- The taxable incurred losses in 20XX+1 are $400 of paid losses plus –$360 of change in discounted loss reserves = $40. There is no premium or expense in 20XX+1, so taxable income is $0 – $40 = –$40. The tax liability is 35% × (–$40) = –$14.

The full difference between statutory and taxable income reverses in 20XX+1, so the full deferred tax asset of $14 is admitted on the statutory balance sheet.

Twelve Month Reversal

We present the formula for computing the admitted portion of the deferred tax asset stemming from loss reserve discounting. The computations are done separately by line of business and by accident year.

Illustration: For accident year 20XX in a given line of business, the loss reserve discount factors are Z_1 at December 31, 20YY, and Z_2 at December 31, 20YY+1. Let "R" be the held loss reserves at December 31, 20YY. Let "P" be the percentage of accident year 20XX reserves that will be paid during calendar year 20XX.

- At December 31, 20YY, the difference between statutory and taxable income for accident year 20XX is $R \times (1 - Z_1)$. The gross deferred tax asset is $35\% \times R \times (1 - Z_1)$.
- At December 31, 20YY+1, the difference between statutory and taxable income for accident year 20XX is $R \times (1 - P) \times (1 - Z_2)$. The gross deferred tax asset is $35\% \times R \times (1 - P) \times (1 - Z_2)$.
- The admitted portion of the deferred tax asset on the statutory balance sheet at December 31, 20YY is $35\% \times R \times [(1 - Z_1) - (1 - P) \times (1 - Z_2)]$.

The value of "P" depends on the actuary's best estimate of the loss payment pattern. It is not the same as the IRS loss payment pattern. To estimate the pattern, we must derive actuarially justified discount factors.

Actuarial Discount Factors

The percentage of losses expected to be paid by each valuation date is the reciprocal of the paid loss development factor.[69] This is a standard actuarial procedure, not peculiar to tax accounting. We show an illustration here for the benefit of readers who have not dealt with reserve liquidation patterns. The illustration uses the same data as the previous illustrations of the IRS loss payment patterns.[70]

The illustration assumes the actuary is determining the deferred tax asset stemming from loss reserve discounting for accident year 20X9 workers' compensation business. Since the amount of the loss reserve discount depends on Schedule P data and the discount is based on Schedule P lines of business, it makes sense to determine the loss liquidation pattern from Schedule P data for Schedule P lines of business.[71]

If the characteristics of the book of business are changing, or if the book of business can be separated into components with different loss liquidation patterns, the pricing actuary should use separate analyses for each component. This is important for accurate policy pricing.

Illustration: An insurer writes two types of workers' compensation coverage:

- First dollar coverage for small and medium size insureds.
- Large dollar deductible policies for large accounts.

Both blocks of business are included in the company's workers' compensation line of business for Schedule P purposes. Both blocks of business have the same loss reserve discount factors for tax purposes. The two blocks of business have different deferred tax assets stemming from the IRS loss reserve discounting, since the actual loss liquidation pattern differs by type of policy. First dollar coverage pays out rapidly. Large dollar deductible (also termed "high deductible") coverage pays out very slowly, since the insurer's payments begin after a large deductible has been pierced. With a deductible of $500,000, which is common for large ("national") accounts, the insurer's payments may begin years after the accident date.

One may be tempted to think that the loss liquidation pattern for calculating the deferred tax asset should be the same as the loss payment pattern for calculating the loss reserve discount factors. This is not correct. The loss reserve discount factors are tax factors. The deferred tax assets are statutory accounting figures. In this illustration, we use the same raw data for the deferred tax assets as we used for the loss reserve discount factors, but the loss

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[69] See Feldblum [2002: SB] for a full discussion of this topic.

[70] A more comprehensive discussion of this illustration may be found in Feldblum [2002: Schedule P].

liquidation pattern for the deferred tax assets is not the same as the loss payment pattern for the loss reserve discount factors.

Exhibit AppA.12 shows the Schedule P, Part 3D entries as they would appear in the 20X9 Schedule P for accident years 20X0 through 20X9 for the loss reserve discount factors estimated above.[72]

Exhibit AppA.12: 20X9 Schedule P, Part 3D ($000)

Part 3	20X0	20X1	20X2	20X3	20X4	20X5	20X6	20X7	20X8	20X9
20X0	103	226	294	334	363	384	398	412	422	433
20X1		111	238	309	356	387	409	428	442	454
20X2			108	221	286	328	354	375	391	403
20X3				111	238	311	357	392	416	434
20X4					135	299	394	458	504	534
20X5						146	314	418	490	542
20X6							159	343	463	546
20X7								146	353	485
20X8									152	406
20X9										156

Paid Loss Link Ratios

We determine the paid loss link ratios from these data. We use these link ratios to calculate the loss liquidation pattern, not to calculate the indicated reserves. Even if the company determines its reserve indications by another reserving method, it would use the procedure described here to determine the loss liquidation pattern.

Paid loss link ratios are the ratios of

i *cumulative paid losses for a specific accident year at a given valuation date to*
ii cumulative paid losses for the same accident year at a valuation date one year earlier.

For instance, the paid loss link ratio from two years to three years of development for accident year 20X6 is $463,000 divided by $343,000, or 1.350. The complete set of link ratios is shown in the table below.

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[72] These data are based on actual Schedule P entries for a large commercial lines insurer that was acquired by a peer company in the mid-1990's. The figures have been disguised, and the accident years have been changed.

Exhibit AppA.13: 20X9 Schedule P, Paid Loss Link Ratios

	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	8 to 9	9 – 10
20X0	2.194	1.301	1.136	1.087	1.058	1.036	1.035	1.024	1.026
20X1	2.144	1.298	1.152	1.087	1.057	1.046	1.033	1.027	
20X2	2.046	1.294	1.147	1.079	1.059	1.043	1.031		
20X3	2.153	1.301	1.148	1.098	1.061	1.043			
20X4	2.215	1.318	1.162	1.100	1.060				
20X5	2.151	1.331	1.172	1.105					
20X6	2.157	1.350	1.179						
20X7	2.418	1.374							
20X8	2.671								

The row labels are accident years; the column captions are development intervals. The caption "2 to 3" means from two years of development to three years of development. We have rotated the triangle, turning the diagonals in Exhibit 3.4 into the columns in Exhibit 3.5.

No link ratio is calculated for the 20X9 accident year, since there is only one valuation. No link ratios are shown for the prior row, since the claims in this row stem from different accident years. For the prior years row, the time since inception of the accident year varies by claim.

We determine averages of the most recent three and the most recent five link ratios, and we select prospective factors from the historical figures and expectations about future conditions. In this illustration, the selected link ratios lie between the three and five year averages.[73]

[73] For the averaging procedures most suitable to reserving analyses, see Feldblum [2002: Schedule P] and Feldblum [2002: Stanard-Bühlmann reserving method].

Exhibit AppA.14: Paid Loss Development (dollars in thousands)

	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	8 to 9	9–10
Averages									
3 year	2.415	1.352	1.171	1.102	1.060	1.044	1.033		
5 year	2.322	1.335	1.162	1.094	1.059				
Select	2.350	1.340	1.170	1.100	1.060	1.040	1.030	1.030	1.020
Cumulative	4.835	2.057	1.535	1.312	1.193	1.125	1.082	1.051	1.020
Pd to Date	$156	$406	$485	$546	$542	$534	$434	$403	$454
Developed	$754	$835	$746	$716	$647	$601	$470	$423	$463
Ultimate	$830	$919	$819	$788	$711	$661	$517	$466	$509
Reserve	$674	$513	$334	$242	$169	$127	$83	$63	$55

PAID LOSS DEVELOPMENT FACTORS

The *cumulative* link ratios, or paid loss development factors, are the cumulative products of the appropriate link ratios (age-to-age factors) in adjacent columns. For instance, the cumulative link ratio from seven to ten years, or 1.082, is the product of 1.030, 1.030, and 1.020, which are the link ratios from seven to eight, eight to nine, and nine to ten years.

We incorporate a paid loss tail factor of +10%, which is not derived from the Schedule P data.

Illustration: The 1 year to 10 years cumulative paid loss development factor is 4.835. The 1 year to ultimate paid loss development factor is 4.835 × 1.100 = 5.319.

Exhibit AppA.15: Paid Loss Development Test of Reserve Adequacy

	1 yr	2 yrs	3 yrs	4 yrs	5 yrs	6 yrs	7 yrs	8 yrs	9 yrs
Pd LDF's	4.835	2.057	1.535	1.312	1.193	1.125	1.082	1.051	1.020
LDF w/ tail	5.319	2.263	1.689	1.443	1.312	1.238	1.190	1.156	1.122
Reciprocal	18.8%	44.2%	59.2%	69.3%	76.2%	80.8%	84.0%	86.5%	89.1%
Incr'tl Pd %	18.8%	25.4%	15.0%	10.1%	6.9%	4.6%	3.2%	2.5%	2.6%

The rows in the table are described below.

- The "Pd LDF's" are the paid loss development factors from each development date to 10 years of maturity, derived from Schedule P, Part 3, data. The paid loss development factor from 1 year to 10 years of maturity is 4.835.
- The "LDF w/ tail" is the paid loss development factors from each development date to ultimate, using a tail factor of +10%. The paid loss development factor from 1 year to ultimate is 5.319.
- The "Reciprocal" of the paid loss development factor to ultimate shows the percentage of losses paid by the development date. The cumulative losses paid by 1 year after the inception of the accident year is 1/5.319 = 18.8% of ultimate paid losses.
- The "Incr'tl Pd %" is the incremental paid losses during each development period as a percentage of ultimate paid losses. The losses paid between 1 year and 2 years after inception of the accident year are 44.2% – 18.8% = 25.4% of ultimate paid losses.

Loss Reserve Discounting

For GAAP financial statements, the deferred tax asset from loss reserve discounting is treated in the same fashion as the deferred tax asset from revenue offset. Both are fully recognized on the balance sheet.

ILLUSTRATION

In the other liability loss reserve discounting illustration in this paper, the accident year 2009 loss reserves for statutory and GAAP balance sheets on December 31, 2009 are $180,000 – $15,000 = $165,000. The corresponding discounted tax basis loss reserves are

$$\$165{,}000 \times 77.8022\% = \$128{,}373.63.$$

The difference between the GAAP loss reserves and the tax basis loss reserves is

$$\$165{,}000.00 - \$128{,}373.63 = \$36{,}626.37.$$

The addition to taxable income stemming from loss reserve discounting for accident year 2009 at December 31, 2009 is $36,626.27 x 35% = $12,819.23. This is the deferred tax asset on the GAAP balance sheet.

The admitted portion of the deferred tax asset on the statutory balance sheet depends on the portion of the loss reserve that will still be unpaid in one year's time. This is an actuarial estimate; it is not the IRS provision used in the loss reserve discounting calculation. We may estimate this amount from Schedule P, Part 3, as discussed earlier.

Suppose the projected paid loss link ratios for other liability are 8.000 at 12 months of development and 5.000 at 24 months of development.

- At 12 months of development, 1/8.000 = 12.5% of incurred losses have been paid and 1 – 1/8.000 = 87.5% of incurred losses are still unpaid.
- At 24 months of development, 1/5.000 = 20.0% of incurred losses have been paid and 1 – 1/5.000 = 80.0% of incurred losses are still unpaid.

We expect 80.0% / 87.5% = 91.428571% of the December 31, 2009, accident year 2009 loss reserves to remain unpaid at December 31, 2010. This amount is $165,000 × 91.4285714% = $150,857.14. The expected IRS discounted reserves at December 31, 2010 equal this amount times the IRS loss reserve discount factor for accident year 2009 at 24 months of development, or 78.7611% in the other liability illustration:

$$\$150,857.14 \times 78.7611\% = \$118,816.75.$$

Implicit Discounting

Some companies implicitly discount reserves for long-tailed lines of business. Implicit discounting means that the company consciously holds less than full value loss reserves (for capital management purposes), not that the company mis-estimates the reserve indication.

One might be tempted to think that the amount of the implicit reserve discount should be taken into consideration when calculating the deferred tax asset. This is not correct. The deferred tax asset must be calculated as if the company held full value loss reserves.[74]

Illustration: An insurer expects to pay a loss for $100,000 in three years. The IRS loss reserve discount factor for this line of business and accident year is 80% for the current valuation date and 85% for the valuation date 12 months hence.

- The (gross) deferred tax asset on the GAAP financial statements is 35% × $100,000 × (1 – 80%) = $7,000.
- The (net admitted) deferred tax asset on the statutory financial statements is 35% × $100,000 × (85% – 80%) = $1,750.

If the insurer implicitly discounts reserves at 5% per annum, its held reserves are $100,000 / 1.05^3 = $86,383.76, and its tax basis reserves are 80% × $100,000 / 1.05^3 = $69,107.01. Its expected held reserves one year hence are $100,000 / 1.05^2 = $90,702.95, and its expected tax basis reserves at that time are 85% × $100,000 / 1.05^2 = $77,097.51.

One might think that the gross (GAAP) and net admitted (statutory) deferred tax assets should be computed as follows:

- Gross (GAAP): 35% × ($100,000 – $69,107.01) = $10,812.55.

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[74] For more complete discussion of this topic, see Feldblum and Thandi, "Reserve Valuation Rates."

- Net admitted (statutory): 35% × ($77,097.51 – $69,107.01) = $2,796.68.

This is not correct. If the company shows a reserve of $86,383.76 on its statutory financial statements, it must treat that reserve at though it were a full value loss reserve for calculating the deferred tax asset. The appropriate calculations are as follows:

- The (gross) deferred tax asset on the GAAP financial statements is 35% × $86,383.76 × (1 – 80%) = $6,046.86.
- The (net admitted) deferred tax asset on the statutory financial statements is 35% × $86,383.76 × (85% – 80%) = $1,511.72.

FEDERAL INCOME TAXES

The previous sections of this appendix explain the federal income tax regulations that are most relevant for property-casualty insurance pricing. This section reviews the tax effects of the accounting transactions in the two illustrations in this paper.

Tables Tx.1 and Tx.2 show the components of the federal income tax liability and the deferred tax assets at each valuation date.

Table Tx.1, "Tax Decomposition," shows the two components of the current tax liability: the tax on underwriting income and the tax on investment income. Each component is divided into two parts: the part stemming from premium revenue and the part stemming from losses and expenses.

Tax Basis

The set of rows under the caption "tax basis" shows the tax basis earned premium, incurred expenses, and incurred losses. The tax basis earned premium is the statutory earned premium adjusted for the revenue offset provision. It is defined either as

written premium – 80% x the change in the unearned premium reserves

or as

statutory earned premium + 20% x the change in the unearned premium reserves

Illustration: For t=0, the written premium is $1000, the statutory earned premium is zero and the unearned premium reserves are $1000. The tax basis earned premium is computed either as $1000 – 80% × ($1000 – $0) = $200 or as $0 + 20% × ($1000 – $0) = $200.

For t=1, the written premium is zero, the statutory earned premium is $1000 and the unearned premium reserves are zero. The tax basis earned premium is computed either as $0 – 80% × ($0 – $1000) = $800 or as $1000 + 20% × ($0 – $1000) = $800.

The tax basis expenses are the same as the statutory expenses. The tax basis incurred loss is the statutory incurred loss adjusted for loss reserve discounting. It is defined either as

paid loss + the change in the discounted reserves

or as

statutory incurred loss – the change in the reserve discount

Illustration: The paid losses are $800 at time t=3.0 and zero before then. The assumed IRS discount factors are 86% at time t=1.0, 88% at time t=2.0, and 90% at time t=3.0. The statutory incurred loss is $800 at time t=1.0 and zero at other valuation dates.

- At time t=1.0, the tax basis incurred loss is computed either as $0 + ($800 × 86% – $0) = $688 or as $800 – [$800 × (1 – 86%) – $0] = $688.
- At time t=2.0, the tax basis incurred loss is computed either as $0 + ($800 × 88% – $800 × 86%) = $16 or as $0 – [$800 × (1 – 88%) – $800 × (1 – 86%)] = $16.
- At time t=3.0, the tax basis incurred loss is computed either as $800 + ($0 × 90% – $800 × 88%) = $96 or as $0 – [$0 × (1 – 90%) – $800 × (1 – 88%)] = $96.

Tax Basis Investment Income

The illustration in the text assumes that all investable assets are fully taxable. If the company holds municipal bonds or common stocks, the yield on the security is adjusted to a pre-tax equivalent yield; see Feldblum and Thandi, "Investment Yields."

The investment income at any valuation date equals investable assets at the previous valuation date times the benchmark investment yield. In the illustration, the benchmark investment yield is an 8% per annum bond equivalent yield, or 4% each half-year.

The investable assets at any valuation date equal the required assets at that date minus the non-cash assets. The only non-cash asset in this illustration is the deferred tax asset.

The tax on underwriting income uses the tax basis earned premium and incurred loss. The tax on investment income stemming from premiums and losses uses the statutory earned premium and incurred loss, with adjustments for the deferred tax asset admitted on the statutory balance sheet.

Illustration: At valuation date t=2.0, or December 31, 20XX+2, the statutory incurred loss is zero and the tax basis incurred loss is $16.00 (see above). The investment income is based on the statutory loss reserves of $800, the statutory capital requirements of $120, and the deferred tax asset of ½ × ($5.60 + $33.60) = $19.60 admitted on the statutory balance sheet. The investment income for the second half of 20XX+2 is

$$8\% \times \tfrac{1}{2} \times (\$800 + \$120 - \$19.60) = \$36.02.$$

Tax Liability

The rows under the caption "Tax on" show the current tax liability by valuation date. The company is in the regular tax environment, with a 35% marginal tax rate. The current tax liability for each cell is 35% of the corresponding figures in the rows under the "Tax Basis" caption.

Illustration: The tax on "underwriting revenue minus expenses" for valuation date t=1.0 is

$$35\% \times (\$800 - \$150) = \$227.50.$$

The rows under the caption "Tax Due to" combine the underwriting income and the investment income for the premium and loss components of the tax. The tax on underwriting income is spread equally across the two halves of each year.

Illustration: The "tax due to writing of policy" on valuation date t=½ is computed as

$$(½ \times \$227.50) + \$16.52 = \$130.27.$$

DEFERRED TAX ASSETS

Table Tx.2 calculates the deferred tax assets on both a statutory and a GAAP basis. The statutory basis deferred tax assets are used for calculating the implied equity flows. The GAAP basis deferred tax assets are used for calculating the GAAP income recognition pattern in Feldblum and Thandi, "Income Recognition and Performance Measurement."

The deferred tax assets for the illustration are computed on December 31 of each year. The deferred tax assets at the June 30 valuation dates in the illustrations are computed by interpolation between the surrounding December 31 dates; see the text of this paper.

The rows under the caption "Tax Basis" show the tax basis premium, expenses, and losses and the federal income tax liability on each. The entries in this table are the same as the entries in Table Tx.1.

The rows under the caption "Statutory Basis" show the corresponding statutory basis premium, expenses, and losses.

- The tax basis premiums are adjusted for revenue offset; the statutory basis premiums are not.
- The expenses are the same for statutory and tax basis entries.
- The statutory incurred losses use full value loss reserves; the tax basis incurred losses use discounted loss reserves.

The rows under the caption "GAAP Basis" show the corresponding GAAP basis premiums, expense, and losses.

- The GAAP premiums equal the statutory premiums.
- The GAAP expenses are adjusted for deferred policy acquisition costs.
- The GAAP losses equal the statutory losses.

The exhibit assumes that all $250 of acquisition expenses are deferrable under GAAP. These expenses are capitalized at time t=0 and amortized over the policy term. Since the exhibit shows only the December 31 valuation dates, it shows a –$250 DPAC expense at

time t=0 and a +$250 DPAC expenses at time t=1. The $250 DPAC expenses is spread evenly over the two halves of the year.

The rows under the caption "Statutory DTA Flow" show the deferred tax assets on the statutory balance sheet. The change in the *gross* deferred tax asset is the tax rate times the difference between taxable income and the income implied by the statutory balance sheet.

Illustration: At time t=0, the taxable income is +$200 from revenue offset and –$250 from incurred expenses, for a total of –$50. The income implied by the statutory balance sheet is $0 from premium and –$250 from incurred expenses. The difference is –$50 – (–$250) = +$200. The deferred tax asset is 35% × $200 = $70.

At time t=2, the taxable income from unwinding of the loss reserve discount is –$16.00. The income implied by the statutory balance sheet is zero. The difference is –$16.00 – $0 = –$16.00. The negative difference implies a reduction in the deferred tax asset of 35% × $16.00 = $5.60, from $39.20 at time t=1 to $33.60 at time t=2.

The net admitted deferred tax asset on the statutory balance sheet is the amount of the gross deferred tax asset that will reverse within 12 months.

Illustration: The entire $70 DTA at time t=0 stemming from revenue offset reverses over the policy term, so all $70 is admitted on the statutory balance sheet.

Illustration: The gross DTA stemming from IRS loss reserve discounting are $39.20 at time t=1 and $33.60 at time t=2. Since only $5.60 of this DTA reverses during year 2, only $5.60 is admitted on the statutory balance sheet at time t=1.

GAAP DEFERRED TAX ASSET

The year to year change in the GAAP deferred tax asset is the tax rate times the difference between taxable income and the income implied by the GAAP balance sheet. There are no admissibility constraints for the GAAP DTA.

Illustration: At time t=0, the taxable income is –$50. Since no premium has been earned or expenses incurred on the GAAP balance sheet at time t=0, the GAAP income is zero. The difference is –$50 – $0 = –$50. The change in the GAAP deferred tax asset is 35% × –$50 = –$17.50. This is shown as a deferred tax liability on the GAAP balance sheet.

TAX DECOMPOSITION

	t = 0	t = 0.5	t = 1.0	t = 1.5	t = 2.0	t = 2.5	t = 3.0
Tax Basis							
(1) UW Revenue	200.00		800.00		0.00		0.00
(2) Expenses	250.00		150.00		0.00		0.00
(3) Inc Loss	0.00		688.00		16.00		96.00
(4) Inv Income							
(4a) on funds due to (Rev - Exp)	0.00	47.20	28.60	0.00	0.00	0.00	0.00
(4b) on funds due to Incurral of L	0.00	0.00	18.29	36.58	36.02	35.46	36.13
(4c) Total Inv Income	0.00	47.20	46.89	36.58	36.02	35.46	36.13
Tax on							
(5) U/W Revenue - Expenses	-17.50		227.50		0.00		0.00
(6) Inc Loss	0.00		-240.80		-5.60		-33.60
(7) Inv Inc							
(7a) on funds due to (Rev - Exp)	0.00	16.52	10.01	0.00	0.00	0.00	0.00
(7b) on funds due to Incurral of L	0.00	0.00	6.40	12.80	12.61	12.41	12.64
Tax Due to							
(8) Writing of Policy (semi-ann paym	-17.50	130.27	123.76	0.00	0.00	0.00	0.00
(9) Incurral of Loss (semi-ann payment)		-120.40	-114.00	10.00	9.81	-4.39	-4.16
(10) Total Tax (semi-ann payment)	-17.50	9.87	9.76	10.00	9.81	-4.39	-4.16

FORMULAS

(5) = 0.35 * [(1) - (2)]
(6) = -0.35 * (3)
(7a) = 0.35 * (4a)
(7b) = 0.35 * (4b)

(8) $t = (7a)_t + 0.5 * [(5)_{t+0.5}]$, for t = 0.5, 1.5, 2.5
(8) $t = (7a)_t + 0.5 * [(5)_t]$, for t = 0.0, 1.0, 2.0, 3.0

(9) $_t = (7b)_t + 0.5 * [(6)_{t+0.5}]$, for t = 0.5, 1.5, 2.5
(9) $_t = (7b)_t + 0.5 * [(6)_t]$, for t = 0.0, 1.0, 2.0, 3.0

(10) $t = (7a)_t + (7b)_t + 0.5 * [(5)_{t+0.5} + (6)_{t+0.5}]$, for t = 0.5, 1.5, 2.5
(10) $t = (7a)_t + (7b)_t + 0.5 * [(5)_t + (6)_t]$, for t = 0.0, 1.0, 2.0, 3.0

Deferred Tax Asset

	t = 0	t = 1.0	t = 2.0	t = 3.0
(1) **Tax Basis**				
a. Revenue	200.00	800.00	0.00	0.00
b. Expense	250.00	150.00	0.00	0.00
c. Incurred Loss	0.00	688.00	16.00	96.00
d. Tax due to Revenue	70.00	280.00	0.00	0.00
e. Tax due to Expense	-87.50	-52.50	0.00	0.00
f. Tax due to Losses	0.00	-240.80	-5.60	-33.60
g. Tax on U/W Total	-17.50	-13.30	-5.60	-33.60
(2) **Statutory Basis**				
a. Revenue	0.00	1,000.00	0.00	0.00
b. Expense	250.00	150.00	0.00	0.00
c. Incurred Loss	0.00	800.00	0.00	0.00
d. Tax due to Revenue	0.00	350.00	0.00	0.00
e. Tax due to Expense	-87.50	-52.50	0.00	0.00
f. Tax due to Losses	0.00	-280.00	0.00	0.00
g. Tax on U/W Total	-87.50	17.50	0.00	0.00
(3) **GAAP Basis**				
a. Revenue	0.00	1,000.00	0.00	0.00
b. Expense	250.00	150.00	0.00	0.00
b' DPAC	250.00	0.00	0.00	0.00
c. Incurred Loss	0.00	800.00	0.00	0.00
d. Tax due to Revenue	0.00	350.00	0.00	0.00
e. Tax due to (Expense - Δ DPA	0.00	-140.00	0.00	0.00
f. Tax due to Losses	0.00	-280.00	0.00	0.00
g. Tax on U/W Total	0.00	-70.00	0.00	0.00
(4) **Statutory DTA Flow**				
a. due to Revenue Offset	70.00	-70.00	0	0
b. due to Loss Reserve Discount	0	39.20	-5.6	-33.6
c. DTA Flow w/out Reversal	70.00	-30.80	-5.60	-33.60
d. DTA w/out Reversal	70.00	39.20	33.60	0.00
e. DTA w/ Reversal	70.00	5.60	33.60	0.00
(5) **GAAP DTA Flow**				
a. due to Revenue Offset				
b. due to Loss Reserve Discounting				
c. DTA Flow	-17.50	56.70	-5.60	-33.60
d. DTA	-17.50	39.20	33.60	0.00

FORMULAE

$(4a)_t = (2d)_t - (1d)_t$

$(4b)_t = (2f)_t - (1f)_t$

$(4c) = (4a) + (4b)$

$(4d)_{t+1} = (4d)_t + (4c)_{t+1}$

$(4e)_0 = (4d)_0$

$(4e)_t = -(4b)_{t+1}$ for t>0

Statutory DTA w/ Reversal is that portion of the DTA that reverses in the year. Hence, it is by definition the negative of the DTA flow w/out reversal that occurs in the subsequent period.

Exhibit AppA.16: illustration A Loss Reserve Discount Factors

Accident Year (1)	Paid Loss + LAE (2)	Incurred Loss + LAE (3)	Cumulative Paid/Incurred Ratio (4)	Incremental Paid/Incurred Ratio (5)	Undiscounted Percentage Unpaid (6)	Discounted Percentage Unpaid (7)	Loss Reserve Discount Factor (8)
AY + 15							
AY + 14							
AY + 13							
AY + 12							
AY + 11							
AY + 10				2.00%	100.00%	0.00%	
2000	$270,000	$275,500	98.00%	3.07%	2.00%	1.93%	96.6735%
2001	$300,000	$316,000	94.94%	2.98%	5.06%	4.77%	94.1800%
2002	$320,000	$348,000	91.95%	3.99%	8.05%	7.34%	91.2271%
2003	$340,000	$386,500	87.97%	4.93%	12.03%	10.71%	89.0399%
2004	$350,000	$421,500	83.04%	6.03%	16.96%	14.78%	87.1281%
2005	$370,000	$480,500	77.00%	7.98%	23.00%	19.65%	85.4281%
2006	$380,000	$550,500	69.03%	10.01%	30.97%	26.07%	84.1740%
2007	$360,000	$610,000	59.02%	11.02%	40.98%	34.04%	83.0660%
2008	$330,000	$687,500	48.00%	13.00%	52.00%	42.47%	81.6659%
2009	$200,000	$571,500	35.00%	35.00%	65.00%	52.26%	80.3944%

Exhibit AppA.17: Illustration B Loss Reserve Discount Factors

Accident Year (1)	Paid Loss + LAE (2)	Incurred Loss + LAE (3)	Cumulative Paid/Incurred Ratio (4)	Incremental Paid/Incurred Ratio (5)	Undiscounted Percentage Unpaid (6)	Discounted Percentage Unpaid (7)	Loss Reserve Discount Factor (8)
AY + 15			100.00%	3.01%	0.00%	0.00%	
AY + 14			96.99%	1.38%	3.01%	2.91%	96.6736%
AY + 13			95.61%	1.38%	4.39%	4.05%	92.3385%
AY + 12			94.23%	1.38%	5.77%	5.12%	88.7803%
AY + 11			92.85%	1.38%	7.15%	6.12%	85.6177%
AY + 10	$235,000	$250,000	91.47%	1.38%	8.53%	7.06%	82.7122%
2000	$50,000	$55,500	90.09%	1.38%	9.91%	7.93%	79.9988%
2001	$55,000	$62,000	88.71%	3.00%	11.29%	8.74%	77.4439%
2002	$60,000	$70,000	85.71%	4.46%	14.29%	11.07%	77.4718%
2003	$65,000	$80,000	81.25%	8.33%	18.75%	14.66%	78.1822%
2004	$70,000	$96,000	72.92%	9.81%	27.08%	21.76%	80.3309%
2005	$65,000	$103,000	63.11%	10.93%	36.89%	29.82%	80.8185%
2006	$60,000	$115,000	52.17%	12.17%	47.83%	38.44%	80.3644%
2007	$50,000	$125,000	40.00%	15.00%	60.00%	47.69%	79.4828%
2008	$35,000	$140,000	25.00%	16.67%	75.00%	59.07%	78.7611%
2009	$15,000	$180,000	8.33%	8.33%	91.67%	71.32%	77.8022%

Appendix B: Workers' Compensation Pricing Exhibits

This appendix describes the implementation of an Excel© based version of the Equity Flow Pricing Model by tracing the steps required to price a fully insured workers' compensation policy (the details of which are described below). The model generates the equity flows associated with the policy being priced. The indicated premium is determined by setting the IRR on the equity flows equal to the target cost of capital. The solution is found by running the goal seek algorithm in Excel©.

To generated the implied equity flows, the model first calculates the company cash flows in the following three categories:

- U/W cash flows
- Investment income cash flows
- Federal income tax flows ("+" denotes a refund; "–" denotes a payment)

The required assets are prescribed by statutory reserve requirements and capital requirements. The asset flow is defined as the change in required assets. The inplied equity flow is calculated by the equation below:

Equity Flow = U/W Flow + Investment Income Flow + Tax Flow – Asset Flow

A positive equity flow denotes a distribution of earnings, or a flow of cash from the insurer to the equityholders, and a negative equity flow denotes a capital contribution, or a payment by the equityholders to the insurer.

Illustration

The illustrative workers' compensation policy is effective on July 1, 20XX. The model assumes effective dates at the inception of a quarter. The July 1 effective date serves as a proxy for a policy year 20XX book of business. The effective date affects the federal income tax calculations and the capital requirements.

The following policy costs serve as inputs to the model:

- The ultimate loss & ALAE,
- The acquisition expenses as a percentage of written premium,
- The general expenses as a percentage of written premium,
- The ULAE as a percentage of ultimate loss & ALAE,
- The policyholder dividends as a percentage of written premium,

The following collection/payment patterns are additional inputs to the model:

- The premium collection pattern,
- The loss & ALAE accident quarter payment pattern,
- The ULAE accident quarter payment pattern,
- The policyholder dividend payment pattern.

The model uses quarterly valuations and assumes all accounting and cash flow activity occur at quarter end.

The model requires the following parameter inputs:

- The benchmark investment yield on invested assets,
- The effective tax rate for both investments and U/W income, which is set at 35% (unless otherwise indicated),
- The surplus leverage ratios (which determine the capital requirements),
- The IRS loss & LAE reserve discount factors.
- The target return on capital.

The effective tax rate and the IRS loss reserve discount factors are either known figures or they are estimated by the pricing actuary. The target return on capital is a discretionary figure that is chosen by company management. The surplus leverage ratios and the benchmark investment yield are a mix of empirical data (such as actual investment yields, risk-based capital requirements, and rating agency capital formulas) and discretionary management choice.

Both the policy characteristics and the parameters are described in more detail in the section on pricing assumptions.

Exhibits

The accompanying exhibits show the quarterly valuation of the various items in the model. Only the first 10 years of valuations are shown. The model which produced these exhibits shows 50 years of quarterly valuations.

Exhibit 1 summarizes the pricing assumptions and shows the pricing results.

Exhibit 2 shows the assumed cash flow patterns (pattern of loss payment, premium collection, policyholder dividend payment, IRS loss reserve discount factors, and tabular discount factors applicable to pension indemnity cases).

Exhibit 3 shows the cash flows and balance sheet items needed to generate the implied equity flow associated with the workers' compensation policy.

Exhibit 4 shows the determination of paid losses and loss adjustment expenses.

Exhibit 5 summarizes the held assets and resulting investment income.

Exhibit 6 shows the Federal Income Tax calculation.

Exhibit 7 summarizes the relevant cash flows and calculates the equity flow.

The exhibits use two conventions for expressing time. The first expresses time in absolute terms: time 0 stands for Jan. 1, 20XX and time 3.50 stands for July 1, 20XX+3. The second convention marks time relative to the policy inception date.

In exhibits 3 through 7 the time column represents absolute time. In exhibit 2 the payment and collection patterns are expressed relative to the age of the policy. The time column for the cash flow patterns is labeled "Age". An age of zero refers to the policy inception date of July 1, 20XX. An age of 1.25 refers to April 1, 20XX+1, which is 1¼ years after policy inception. The exception to this dating convention in exhibit 2 is the IRS discount factors. The discount factors are applicable to specific calendar periods and the time column for these factors represents absolute time (i.e. calendar date). See Appendix A for explanation of the IRS loss reserve discount factors.

Pricing Assumptions

Policy Characteristics

The policy is a fully insured workers' compensation policy effective on July 1.

Premium is collected in the pattern specified in Exhibit 2. The indicated premium, denoted as WP (written premium), is determined by setting the internal rate of return (IRR) of the implied equity flows equal to the target return on capital (TROC) of 12%. The premium of $1,374 shown in all the exhibits is determined by the goal seek algorithm in Excel©. The dollars of premium collected shown in column (4) of Exhibit 3 are calculated as WP × Premium Collection Pattern.

Illustration: From exhibit 2, 18% of the charged premium is collected at policy inception (time 0.5), 17.1% is collected one quarter later (time 0.75), 21.8% is collected two quarters later (time 1.0), 25.3% is collected three quarters later (time 1.25), 7.2% is collected four quarters later (time 1.5), 6.9% is collected five quarters later (time 1.75), etc.

Since the written premium is $1,374, the collection pattern implies that the dollars of premium collected (column (4) exhibit 3) are equal to

1,374 × 18.0% = $247	at time 0.50 (0 years after policy inception)
1,374 × 17.1% = $235	at time 0.75
1,374 × 21.8% = $300	at time 1.00
1,374 × 25.3% = $348	at time 1.25

1,374 × 7.2% = $ 99		at time 1.50
1,374 × 6.9% = $ 95		at time 1.75

The Premium Receivable (column (5) of Exhibit 3) is defined to be
Written Premium – Cumulative Premium Collected.

Illustration: The premium receivable is

$1,374 – $247	= $1,127	at time 0.50
$1,374 – ($247+$235)	= $ 892	at time 0.75
$1,374 – ($247+$235+$300)	= $ 892	at time 1.0

Ultimate Loss & ALAE is $1000, with the Accident Quarter (AQR) loss payment pattern as shown in Exhibit 2. The determination of the dollars of loss payments, given the AQR pattern, as well as the calculation of loss reserves, is described below.

ULAE is 7.2% of Ultimate Loss & ALAE, with a payment pattern as shown in Exhibit 2. The determination of the ULAE payments and reserves is discussed below.

Acquisition expenses are 17.9% of written premium and are paid at policy inception. For the illustration, the acquisition expenses are $1,374 × 17.9% = $246 as shown in column (6) of Exhibit 3.

General expenses are 7.7% of written premium and are assumed to be paid at policy inception. In the illustration, general expenses are $1,374 × 7.7% = $105. Some general expenses actually occur both before and after policy inception. For this illustration we make the simplifying assumption that we can approximate this payment pattern with a single payment at policy inception.

Other Assumptions

We assume an effective annual rate of return on investments of 8% (or $(1+0.08)^{0.25} - 1$ = 1.94277% per quarter). Investment income is earned on all investable assets, which exclude, in particular, the premium receivable asset.

The Federal Income Tax Rate of 35% applies to both underwriting income and investment income.

Surplus is allocated as 43.7% of written premium and is held only for the policy term. Surplus could also be allocated in proportion to loss reserves and held until all losses are paid, if this deemed a more appropriate allocation method.

Loss & ALAE Payments

Because the model uses quarterly cash flows, it requires a quarterly valuation of accident quarter losses. If we needed to determine just the payment of losses on a

single policy we could use the loss payment pattern for the accident year as a whole. This would reproduce the payment pattern for an individual policy. It would not suffice modeling the equity flows in the model, since paid and unpaid losses segregated by accident year are needed to estimate the federal income tax liabilities.

The tax basis discounted loss reserves are the product of the IRS loss reserve discount factor and the held loss & LAE reserves. The discount factor varies by line of business, accident year, and age of the accident year.[1] The appropriate discount factor depends on the year in which the losses occurred. A segregation of losses into the years in which they occur is necessary to determine taxable incurred losses. A description of the segregation procedure follows below.

The policy term spans two accident years, which we term AYR1 and AYR2. For a policy with an effective date during year 20XX, AYR1 extends from January 1, 20XX to December 31, 20XX, and AYR2 extends from January 1, 20XX+1 to December 31, 20XX+1. The portion of the policy term that spans each accident year is given in the table below:

Policy Effective Date	AYR1	AYR2
Jan 1, 20XX	Jan 1, 20XX --> Dec. 31, 20XX	-----
Apr 1, 20XX	Apr 1, 20XX --> Dec. 31, 20XX	Jan 1, 20XX+1 --> May 31, 20XX+1
July 1, 20XX	July 1, 20XX --> Dec. 31, 20XX	Jan 1, 20XX+1 --> June 30, 20XX+1
Oct 1, 20XX	Oct 1, 20XX --> Dec. 31, 20XX	Jan 1, 20XX+1 --> Sept 30, 20XX+1

We assume that losses are incurred evenly over the policy term. We use accident quarter loss and LAE payment patterns to segregate losses into AYR1 and AYR2. The illustration uses an annual workers' compensation policy with an effective date at the beginning of a calendar quarter, so the policy term spans four accident quarters. We assume that one quarter of ultimate losses are incurred each quarter, and that the loss & ALAE payment pattern is the same for each accident quarter.

Given the paid losses by accident quarter, we calculate the accident year losses. The terms accident year 1 and accident year 2 refer to accident years measured from January 1 to December 31. The term AQR1 (or accident quarter 1) refers to the first accident quarter in a given policy.

AYR1 (or AYR2) paid losses consist of the sum of the losses paid in each accident quarter that falls into AYR1 (or AYR2). The classification of accident quarter losses into accident years depends on the policy effective date. The chart below shows the classification of accident quarter losses into accident years:

Policy Effective Date	AYR1	AYR2

[1] The age of the accident year is the valuation date and the inception of the accident year. Accident year 20XX valued at December 31, 20XX+2, is accident year 20XX aged 36 months.

Jan 1, 20XX	(AQR1 + AQR2 + AQR3 + AQR4)	-----
April 1, 20XX	(AQR1 + AQR2 + AQR3)	(AQR4)
July 1, 20XX	(AQR1 + AQR2)	(AQR3 + AQR4)
Oct 1, 20XX	(AQR1)	(AQR2 + AQR3 + AQR4)

Illustration: A portion of the accident quarter loss & ALAE pattern is reproduced below. The pattern is from inception of the policy. For example, accident quarter 3 means the third quarter of the policy term, and accident quarter 5 means the first quarter after expiration of the policy.

Time (from Policy Inception)	AQR Payment Pattern
0.00	0.0000
0.25	0.0480
0.50	0.1210
0.75	0.0819
1.00	0.0622
1.25	0.0543

This pattern and the assumption that $250 (¼ of ultimate losses of $1,000) are incurred in each accident quarter implies the following payment pattern by accident year for a policy inception date of July 1, 20XX:

Policy Effective July 1, 20XX

	AYR1		AYR2		→ AYR1	AYR2
Time	AQR1	AQR2	AQR3	AQR4		
0.00	-	-	-	-		
0.25	-	-	-	-		
0.50	250 * 0.0000 = 0	-	-	-	0 + 0 = 0	
0.75	250* 0.0480 = 12	250 * 0.0000 = 0	-	-	12 + 0 = 12	
1.00	250* 0.1210 = 30	250* 0.0480 = 12	250 * 0.0000 = 0	-	30 + 12 = 42	0 + 0 = 0
1.25	250* 0.0819 = 20	250* 0.1210 = 30	250* 0.0480 = 12	250 * 0.0000 = 0	20 + 30 = 51	12 + 0 = 12
1.50	250* 0.0622 = 16	250* 0.0819 = 20	250* 0.1210 = 30	250* 0.0480 = 12	16 + 20 = 36	30 + 12 = 42
1.75	250* 0.0543 = 13	250* 0.0622 = 16	250* 0.0819 = 20	250* 0.1210 = 30	13 + 16 = 29	20 + 30 = 51

(time 0 refers to Jan 1, 20XX)

Figure 4

A policy with a September 1, 20XX inception date would have the following loss & ALAE payment by accident year:

Policy Effective Sept 1, 20XX

AYR1	AYR 2	→ AYR1	AYR2

Time	AQR1	AQR2	AQR3	AQR4		
0.00	-	-	-	-		
0.25	-	-	-	-		
0.50	-	-	-	-		
0.75	250 * 0.0000 = 0	-	-	-	0	
1.00	250* 0.0480 = 12	250 * 0.0000 = 0	-	-	12	0 + 0 + 0 = 0
1.25	250* 0.1210 = 30	250* 0.0480 = 12	250 * 0.0000 = 0	-	30	12 + 0 + 0 = 12
1.50	250* 0.0819 = 20	250* 0.1210 = 30	250* 0.0480 = 12	250 * 0.0000 = 0	20	30 + 12 + 0 = 42
1.75	250* 0.0622 = 16	250* 0.0819 = 20	250* 0.1210 = 30	250* 0.0480 = 12	16	20 + 30 + 12= 63

(time 0 refers to Jan 1, 20XX) **Figure 5**

Loss & ALAE Reserves (Nominal)

Nominal reserves are statutorily mandated reserves in the absence of discounts. Held reserves and tabular discount are defined in terms of Nominal Reserves.

Once the policy is fully earned the nominal reserves of each accident year are the sum of the unpaid losses of the component accident quarters.

Illustration: For a policy effective at time 0.5 (July 1, 20XX), the AYR1 nominal reserve at time 1.75 (Sept. 1, 20XX+1) equal the unpaid losses on AYR1. The unpaid losses are the AYR1 ultimate losses minus the AYR1 losses paid to date.

- The AYR1 ultimate loss equals ½ × $1,000 = $500.
- The cumulative losses paid by time 1.75 is $(12+42+51+36+29) = $170; see column (5) of exhibit 4 or Figure 4 above.
- The AYR1 unpaid loss at time 1.75 is $500 – $170 = $330; see column (15) of exhibt 4.

- The AYR2 nominal reserve at time 1.75 is the AYR2 ultimate loss minus the AYR2 losses paid to date.
- The AYR2 ultimate loss is ½ × $1,000 = $500.
- The cumulative losses paid by time 1.75 are $(12+42+51) = $105; see column (5) of exhibit 4 or Figure 4 above.
- The AYR2 unpaid loss at time 1.75 is $500 – $105 = $395; see column (15) of exhibt 4.

If a policy is not fully earned at a given valuation date, the recognition of nominal reserves by accident year depends on the policy effective date and the earning of the policy.

Illustration: If the policy effective date is time 0, all losses attributable to AQR1 have occurred by time 0.25. AQR1 losses represent ¼ of all losses attributable to AYR1. The AYR1 nominal reserves at time 0.25 are equal to ¼ of ultimate AYR 1 Losses minus AYR1 Losses Paid to Date.

If the policy effective date is time 0.5, then at time 0.75, all losses attributable to AQR1 have occurred . AQR1 losses represent ½ of all losses attributable to AYR1 → ½ of all losses attributable to AYR1 have occurred. Thus the AYR1 nominal reserves at time 0.75 is equal to ½ of ultimate AYR 1 Losses minus AYR1 Losses Paid to Date).

The AYR1 ultimate losses that are recognized at time T equal the AYR1 ultimate losses times a factor that represents the proportion of the total AYR1 losses that have occurred, less the cumulative amount of losses that have been paid to date. Figure 1 below presents a schematic of the determination of the AYR1 factor. AYR2 is handled in the same fashion. Figure 2 presents the determination of the corresponding AYR2 reserve recognition pattern.

The total amount of losses attributable to AYR1 is equal to the total ultimate losses times the proportion of the policy term that falls in AYR1. The amount of losses attributable to AYR2 is equal to the total ultimate losses times the proportion of the policy terms that falls in AYR2.

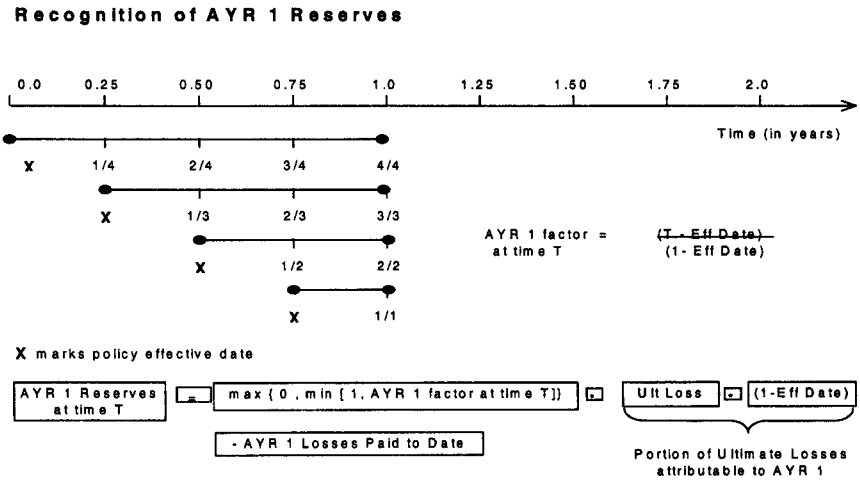

Figure 6

Illustration: We determine the AYR1 nominal reserve valued at time 0.75 for a policy effective at time 0.5.
The AYR1 ultimate loss is ½ × $1,000 = $500.
The AYR1 earnings factor at time 0.75 is the portion of AYR1 losses that have occurred by time 0.75, which equals

$$\frac{0.75-0.5}{1-0.5}=\frac{0.25}{0.5}=0.5$$

The cumulative losses paid by time 0.75 is \$0 + \$12 = \$12; see column (5) of exhibit 4 or Figure 4 above.
The AYR1 nominal reserve at time 0.75 is equal to

AYR1 factor × AYR1 ultimate loss – AYR1 paid to date
= 0.5 × (\$500) – \$12 = \$238

(See column (15) of exhibit 4).

The more complicated formula for the AYR1 reserves at time T, as shown in Figure 6, applies whether or not the policy is fully earned.

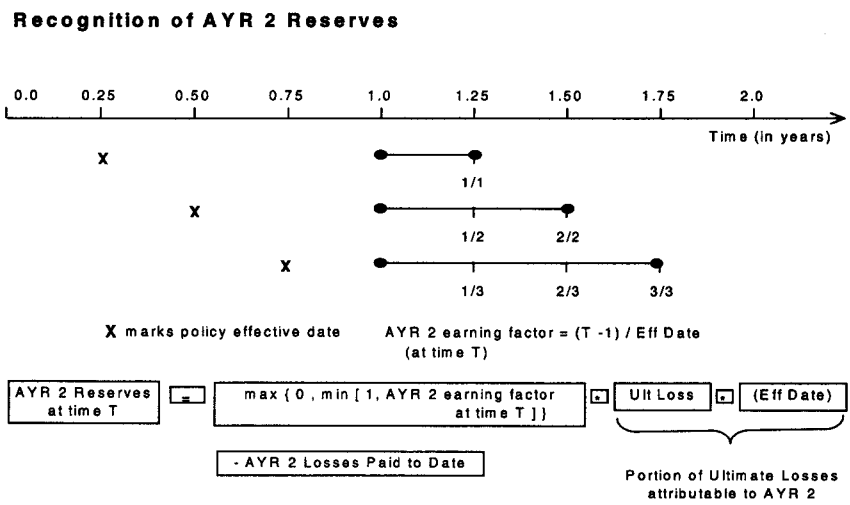

Figure 7

Illustration: We determine the AYR2 nominal reserve valued at time 1.5 for a policy effective at time 0.5.
The AYR2 ultimate loss is ½ × \$1,000 = \$500.
The AYR2 earnings factor at time 1.5 is the portion of AYR2 losses that have occurred by time 1.25, or

$$\frac{1.5-1}{0.5}=\frac{0.5}{0.5}=1.0$$

The cumulative losses paid by time 1.5 are \$0+\$12+\$42 = \$54; see column (6) of exhibit 4 or Figure 4 above.
The AYR2 nominal reserve at time 1.5 is equal to

AYR2 factor x AYR2 ultimate loss – AYR2 paid to date
= 1.0 x (\$500) – \$54 = \$446

(See column (16) of exhibit 4).

Nominal ULAE Reserves

The consideration of nominal ULAE reserves is analogous to nominal loss & ALAE reserves. Instead of using accident quarter loss & ALAE payment patterns we use accident quarter ULAE payment patterns. The ultimate ULAE is estimated as ultimate loss & ALAE x the ULAE ratio. With these modifications, the exposition in the preceding section applies to ULAE as well.

Tabular Discount

Statutory accounting rules allow insurers to discount WC pension indemnity cases. The tabular discounts by line of business and by accident year are disclosed in the notes to the financial statements.[2] To evaluate the tax basis reserves, we need the dollar amount of tabular discount on a given accident year's loss reserves. This dollar amount will vary by age of the accident year.

The dollar amount of tabular discount is calculated by applying the ratio

$$\frac{\text{tabular discount}}{\text{nominal reserves}}$$

to the nominal reserves calculated by the model. This ratio can be based on an analysis of

$$\frac{\text{actual tabular discount}}{\text{actual nominal reserves}}$$

by accident year, using company data.

[2] The tabular discount may also be derived from a comparison of Schedule P, Part 1, which is net of tabular discount, with Schedule P, Part 2, which is gross of tabular discount; see Feldblum [2002: Schedule P].

Held Loss & ALAE Reserves

Held reserves are the sum of nominal loss & ALAE reserves and nominal ULAE reserves, times the level of reserve adequacy, minus the amount of tabular discount:[3]

$$\text{AYR X Held} = \text{Res Adeq} * (\text{AYR X Nominal Loss \& ALAE} + \text{AYR X Nominal ULAE})$$
$$\text{Loss + LAE} \qquad - \text{Tabular Discount}$$

Illustration: For our WC policy we assume that the level of reserve adequacy is 100%. We confirm that the AYR1 held reserve is $246 at time 0.75 (as shown in column (25) of Exhibit 4). From column (15) the nominal loss & ALAE nominal reserve at t=0.75 is $238; the nominal ULAE reserve from column (18) is $18; from column (22) the tabular discount is $10. Thus the AYR1 Held Reserves = 1.0 * ($238 + $18) – $10 = $246.
The AYR1 held reserve at time 1.75 is $337: the nominal loss & ALAE reserve is $330; the nominal ULAE reserve is $26; the tabular discount is $20.
The AYR2 held reserve at time 1.75 is $411: the nominal loss & ALAE reserve is $395; the nominal ULAE reserve is $32; the tabular discount is $16.

Assets

Required Surplus

Surplus is held only for the policy term in our illustration. Surplus is held to cover unforeseen contingencies and to maintain an acceptable level of risk.

Most pricing model use either a premium to surplus leverage ratio or a reserves to surplus leverage ratio or both. The pricing model described here supports not just prospective pricing but also an economic value added performance measurement system. Using a premium to surplus leverage ratio for the surplus assumptions allows a more responsive performance measurement system, which is more likely to be accepted by company personnel. See Kelly [2002] for further discussion.

Illustration: The premium leverage ratio is 43.7%, and he written premium is $1,374. The required surplus for the full policy term (time 0.5 until time 1.5) is 43.7% x $1374 = $601. For time 1.5 and subsequent (once the policy is fully earned) the required surplus is zero.

Total Reserve

The total reserve is the sum of the unearned premium reserve and the held loss & LAE reserves.

[3] The level of reserve adequacy is discussed in Feldblum and Thandi [2002], "Reserve Valuation Rates."

Illustration: At time 0.5 (policy inception) the UEPR is equal to the written premium which is equal to $1,374. Held loss & LAE reserves are equal to 0 → Total Reserves = $1,374
At time 0.75 the UEPR = ¾ * (1,374) = $1,031. The held loss & LAE reserves = $246 → Total Reserves = $1,031 + $246 = $1,277.
At time 1.75 the UEPR = $0. The held loss & LAE reserves = $748 → Total Reserves = $0 + $748 = $748

Required Assets

The assets needed to support the policy (the Required Assets in column (1) of Exhibit 5) equal the total reserves (sum of columns (2) and (3) of Exhibit 5) plus the required surplus (column (4) of Exhibit 5):

$$\text{Required Assets} = \text{Total Reserves} + \text{Required Surplus}$$

Illustration: At time 0.5, Required Assets = $601 + $1,374 = $1,975
At time 0.75, Required Assets = $601 + $1,277 = $1,878
At time 1.5, Required Assets = $0 + $833 = $ 833
At time 1.75, Required Assets = $0 + $748 = $ 748

Income Producing Assets

Not all of the assets held by the company to support the policy generate investment income. The premium receivable (column (5) of Exhibit 3) and the deferred tax asset (column (34) of Exhibit 3) are non-income producing assets:

$$\text{Income Producing Assets} = \text{Required Assets} - \text{Premium Receivable} - \text{DTA}$$

Illustration: At time 0.5 required assets = $1,975, premium receivable = $1,127, and the DTA = $96. Thus Income Producing Assets (column (18) Exhibit 3) = $1,975-$1,127 – $96 = $752.
At time 0.75 the required assets = $1,877, premium receivable = $892, and the DTA = $96. Thus Income Producing Assets = $1,975-$1,127 – $96 = $752.
At time 1.5 the required assets = $833, premium receivable = $145, and the DTA = $9. Thus Income Producing Assets = $833-$145– $9 = $679.
At time 1.75 the required assets = $748, premium receivable = $50, and the DTA = $96. Thus Income Producing Assets (column (18) Exhibit 3) = $1,975-$1,127 – $96 = $752.

Investment Income

The Investment Income earned over a quarter is the product of the quarterly effective investment rate of return times the amount of income producing assets held at the beginning of the quarter:

$$\text{Invest Inc}_{@\text{ time T}} = \text{Qtlry Invest ROR} * \text{Investible Assets}_{@\text{time T-1}}$$

Illustration: Invest Income at time 0.75 = 1.9427% * (Investable Assets at time 0.5) = 1.9427% * $752 = $15 (Investable Assets shown in column (18) of Exhibit 3, Investment Income shown in column (20) of Exhibit 3).
Invest Income at time 1.75 = 1.9427% * (Investable Assets at time 1.5) = 1.9427% * $679 = $13.

Taxes

IRS Discounted Reserves

The tax basis (discounted) reserves are the produce of the held reserves, gross of any tabular discount, and the IRS loss reserve discount factor. The discount factor varies by line of business, by accident year, and by age of the accident year.

The calculation of the loss reserve discount factors is described in Appendix A, along with explanation of the estimation procedures required of the pricing actuary. For this appendix, we take the IRS discount factors as given. We use the accident year 2000 discount factors as the factors for "AYR1" and the accident year 2001 factors for AYR2. The formula for IRS discounted reserves is

IRS Discounted Reserves = IRS Discount Factor * (Held Reserves + Tabular Discount)

Illustration: For the accounting year ending at time 1.0, the AYR1 Held Reserves are $460, the AYR1 tabular discount is $20, and the IRS discount factor is 0.8194 (from columns (25), (22) and (28), respectively, of Exhibit 4). Thus the IRS Discounted Loss & LAE reserves at time 1.0 is equal to 0.8194 * ($460 + $20) = $393.

For the accounting year ending at time 2.0, the AYR1 Held Reserves are $308, the AYR1 tabular discount is $19, and the IRS discount factor is 0.8027 (from columns (25), (22) and (28), respectively, of Exhibit 4). Thus the IRS Discounted Loss & LAE reserves at time 2.0 is equal to 0.8027 * ($308 + $19) = $263.
For AYR2 at time 2.0 the Held reserves are $372, the tabular discount is $16, and the discount factor is 0.8214 (columns (26), (23), and (29) respectively of Exhibit 4). Thus the AYR2 IRS Discounted Reserve is 0.8214*(372+16)=$319.
Thus the total IRS Discounted Reserve at time 2.0 is $263 + $319 = $582.

Taxable U/W Income

The taxable U/W income over an accounting year is

Written Premium - 0.8 * ΔUEPR - Paid Expenses - [Paid Losses + ΔIRS Disc Reserves]

where all activity is over the relevant accounting year.

Illustration: For the accounting year ending at time 1.0 the written premium is $1374 (column (1) Exhibit 3), the change in the UEPR is $687 (column (2) Exhibit 3), the expenses paid are $351 (column (9) Exhibit 3), the paid loss & ALAE is $(42+12)=$64 (column (10) Exhibit 3), the paid ULAE is $1 (column (11) Exhibit 3), and the change in the IRS discounted reserves is $393-$0 =$393 (column (16) Exhibit 3). Thus the Taxable U/W income is 1374-0.8*(687)-351-(42+12+1 +393) = $24.
For the accounting year ending at time 2.0 the written premium is $0, the change in the UEPR is -$687, the expenses paid are $12, the paid loss & ALAE is $(63+78+80+62)=$282, the paid ULAE is $(2+4+5+6)= $16, and the change in the IRS discounted reserves is $582-$393= $289. Thus the Taxable U/W income is 0-0.8*(-687)-12-(282+17 +289) = $49.

The paragraph above determines the annual federal income taxes on U/W income. In practice, taxes are paid quarterly. The taxpayer (the insurance company) projects its annual U/W accounting income and pays one quarter of that amount in each calendar quarter. We estimate the taxes for the two accident years for a given policy, and we spread the tax payments over the quarters in which the policy is effective.

Illustration: U/W Tax at t=0.75 = ½ × (24) × (35%) = $4.20
U/W Tax at t=1.0 = ½ × (24) × (35%) = $4.20
U/W Tax at t=1.25 = ½ × (49) × (35%) = $8.58
U/W Tax at t=1.50 = ½ × (49) × (35%) = $8.58
(See columns (30) and (31) of Exhibit 3.)

Tax on Investment Income

The tax on investment income is paid quarterly as investment income is earned.

Illustration:

Qtrly Tax on Inv Inc at t = 0.75 = Qtrly Inv Inc at t = 0.75 * 35%
= 15 * 35% = $5

Qtrly Tax on Inv Inc at t = 1.75 = Qtrly Inv Inc at t = 1.75 * 35%
= 13 * 35% = $5

Column (36) of Exhibit 3.

Total Tax

The total federal income tax paid each quarter is equal to the sum of the quarterly tax on U/W income and the quarterly tax on investment income.

Illustration: At t=0.75 the total FIT = $4 + $5 = $9
At t=1.75 the total FIT = $4 + $5 = $9

See column (37) of exhibit 3.

Deferred Tax Asset

The calculation of the deferred tax asset is described in Appendix A. We trace the calculation of the deferred tax asset for the workers' compensation policy.

There are two components to the DTA: the portion due to the Revenue Offset; and the portion due to IRS Discounting of Loss&LAE Reserves.

The DTA due to the Revenue Offset is equal to

$$35\% * 20\% * \Delta UEPR$$

Illustration: The DTA due to Revenue Offset at t=0.5 is equal to 35% * 20% * $1,374 = $96.
The DTA due to Revenue Offset at t=0.75 = 35% * 20% * ($1,031) = $72
The DTA due to Revenue Offset at t=1.75 = 35% * 20% * ($0) = $0.

(See column (32) of exhibit 3).

The DTA due to IRS Discounting at the end of Accounting Year X is equal to

35% * [(AYR1 Held Loss Reserve $_{\text{at time X}}$ – AYR1 IRS Loss Reserve $_{\text{at time X}}$) -
(AYR1 Held Loss Reserve $_{\text{at time X+1}}$ – AYR1 IRS Loss Reserve $_{\text{at time X+1}}$)]
+ 35% * [(AYR2 Held Loss Reserve $_{\text{at time X}}$ – AYR2 IRS Loss Reserve $_{\text{at time X}}$) -
(AYR2 Held Loss Reserve $_{\text{at time X+1}}$ – AYR2 IRS Loss Reserve $_{\text{at time X+1}}$)]

The amount in each square bracket is the amount that reverses over the next twelve months, following statutory accounting rules; see Appendix A.

Illustration: At time 2.0 the Held Loss&LAE Reserve for AYR1 is $308, for AYR2 is $372 (columns (25), (26) of Exhibit 4); the IRS Loss Reserves for AYR1 are $263, for AYR2 are $319 (columns (30), (31) of Exhibit 4). At time 3.0 the Held Loss&LAE Reserve for AYR1 is $223, for AYR2 is $262; the IRS Loss Reserves for AYR1 are $191, for AYR2 are $225.Thus the DTA due to IRS Discounting at time 2.0 is

$$35\% * [(308-263) - (223-191)] + 35\% * [(372-319)-(262-225)] = \$10$$

The DTA due to IRS Discounting for any time other than and Accounting Year end is an interpolation year end DTAs (due to IRS discounting).

Illustration: At time 1.0 the DTA due to IRS Discounting is $7, at time 2.0 the DTA due to IRS Discounting is $10, at time 3.0 it is $5. Hence the DTA due to IRS Discounting is $4 at time 0.75 (interpolation between $0 and $7)

$10 at time 1.75 (interpolation between $7 and $10)
(rounded to nearest whole dollar). See column (33) of exhibit 3.

Cash Flows

The relevant cash flows for determining the Equity Flow are described below.

U/W Cash Flow

The underwriting cash flow is defined as

U/W Cash Flow = WP – Paid Expenses – Paid Loss & LAE
Column (38) exhibit 3 = column (1) – column (9) – [column (10) + column (11)]

Illustration: At t=0.5 U/W CF = 1374 – 351 – 0 = $1023
At t=0.75 U/W CF = 0 – 0 – 12 = -$12
At t=1.75 U/W CF = 0 – 12 – 85 = -$97
(See column (38) Exhibit 3).

Investment Income Flow

The investment income cash flow is defined at the quarterly investment income earned each quarter. The calculation is described above

Illustration: At t=0.5 the investment income is $0
At t=0.75 the investment income is $15.
At t=1.75 the investment income is $13.
(See column (39) of Exhibit 3).

Tax Flow

The Tax Cash Flow is defined at the negative (to denote a flow from the company) of the federal income taxes paid that quarter. The calculation of this flow item is described above.

Illustration: At time t=0.5 FIT = 0 → Tax Flow = 0
At time 0.75 FIT = $9 → Tax Flow = -$9
At time 1.75 FIT = $9 → Tax Flow = -$9

(See column (40) Exhibit 3).

DTA Flow

The DTA Flow is defined as the change in the DTA asset over a calendar quarter.

Illustration: At t=0.5, DTA Flow = $DTA_{t=0.5}$ = $96
At t=0.75, DTA Flow = $DTA_{t=0.75} - DTA_{t=0.5}$ = $76 - $96 = -$20
At t=1.75, DTA Flow = $DTA_{t=1.75} - DTA_{t=1.5}$ = $10 - $9 = $1
(See column (41) Exhibit 3)

Asset Flow

The asset flow is defined as the change in the required assets. The composition and calculation of the required assets are described above.

Illustration: At t=0.5 Asset Flow = $Assets_{t=0.5}$ = $1975
At t=0.75 Asset Flow = $Assets_{t=0.75} - Assets_{t=0.5}$ = $1877 - $1975 = -$98
At t=1.75 Asset Flow = $Assets_{t=1.75} - Assets_{t=1.5}$ = $748 - $833 = -$85
(See column (42) of Exhbit 3).

Equity Flow

To compute the Equity Flow at each quarter we use the cash flow definition:

Equity Flow = - *Asset Flow* + *U/W Flow* + *Investment Income Flow* + *FIT Flow*
+ DTA Flow

Recall that we use the convention that a positive equity flow denotes a flow of cash from the insurer to the equityholders, and a negative a payment by the equityholders to the insurer.

Illustration: At t=0.5 Equity Flow = -$1,975 + $1,023 + $0 + $96 = -$856
At t=0.75 Equity Flow = -(-$98) – $12 + $15 – $9 – $20 = $70
At t=1.75 Equity Flow = -(-$85) – $97 + $13 – $9 + $1 = -$7.

SUMMARY OF ASSUMPTIONS AND RESULTS FOR WC Fully Insured Policy

I. UNDERWRITING ASSUMPTIONS

A) Policy Costs	
Expense Ratio (as % WP)	25.6%
Dividend Ratio (as % WP)	5.7%
ULAE Ratio (as % of Loss&ALAE)	7.2%
Ultimate Loss & ALAE	1,000
B) Cash Flow Patterns	
Disc Loss&ALAE to Undisc	73.0%
Duration of Losses (in yrs)	4.3
Disc Premium to Undisc	95.3%
C) Average Effective Date	0.5
D) Level of Reserve Adequacy	
Held to Nominal Reserves	100.0%

II. FINANCE ASSUMPTIONS

A) Investment Rate of Return	
On all Investable Assets	8.0%
B) Federal Income Taxes	
Tax Rate on U/W Income	35.0%
Tax on Investment Income	35.0%
C) Target Return on Capital	
Post-Tax Return	**12.0%**

III. RISK (SURPLUS) ASSUMPTIONS

Reserve Leverage Ratio	0.0%
Premium Leverage Ratio	43.7%

IV. PRICING RESULTS

A) Premium	
Nominal Premium	1,374
Discounted Premium	1,309
B) Summary of Costs	
Disc Loss & LAE	784
Disc Expense (incl PHR Dividends)	416
Disc Taxes	67
	1,267

C) Ratios	Nominal (% of Nominal Prem)	Discounted (% of Disc Prem)
Loss & ALAE Ratio	72.8%	55.8%
ULAE Ratio	5.2%	4.1%
Expense Ratio (incl PHR Dividends	31.3%	31.8%
Combined Ratio	**109.2%**	91.6%

V. PROFITABILITY

	Nominal	Discounted
A) Equity Charge	107.48	42.56
B) IRR on Equity Flows	**12.0%**	

AGE	Accident Quarter Loss & ALAE Payout Pattern	ULAE Payout Pattern	Dividend Payout Pattern	Premium Collection Pattern	Tabular Discount	CYR	IRS Discount Factors	
0.00	0.00%	0.00%	0.00%	18.0%	3.6%	0.00		
0.25	4.80%	1.36%	0.00%	17.1%	4.1%	0.25		
0.50	12.10%	4.89%	0.00%	21.8%	4.5%	0.50		
0.75	8.19%	7.09%	0.00%	25.3%	5.0%	0.75		
1.00	6.22%	8.62%	0.00%	7.2%	5.5%	1.00	0.819398	0.819398
1.25	5.43%	8.46%	15.00%	6.9%	5.9%	1.25		
1.50	5.05%	6.78%	0.00%	0.3%	6.4%	1.50		
1.75	4.40%	5.43%	28.33%	0.3%	6.6%	1.75		
2.00	4.09%	5.02%	0.00%	0.3%	6.8%	2.00	0.802722	0.809786
2.25	3.54%	4.33%	0.00%	0.2%	7.0%	2.25		
2.50	3.21%	4.07%	0.00%	0.2%	7.2%	2.50		
2.75	2.77%	3.65%	28.33%	0.2%	7.9%	2.75		
3.00	2.58%	3.17%	0.00%	0.2%	8.5%	3.00	0.797466	0.804824
3.25	2.18%	2.80%	0.00%	0.1%	9.2%	3.25		
3.50	1.90%	2.56%	0.00%	0.1%	9.8%	3.50		
3.75	1.77%	2.19%	28.33%	0.1%	10.5%	3.75		
4.00	1.58%	2.12%	0.00%	0.1%	11.1%	4.00	0.754828	0.764156
4.25	1.42%	1.77%	0.00%	0.1%	11.7%	4.25		
4.50	1.31%	1.59%	0.00%	0.1%	12.3%	4.50		
4.75	1.15%	1.51%	0.00%	0.1%	12.5%	4.75		
5.00	1.10%	1.28%	0.00%	0.1%	12.7%	5.00	0.733432	0.714034
5.25	0.95%	1.30%	0.00%	0.1%	12.9%	5.25		
5.50	1.01%	1.04%	0.00%	0.1%	13.1%	5.50		
5.75	0.81%	0.99%	0.00%	0.1%	13.7%	5.75		
6.00	0.82%	0.92%	0.00%	0.1%	14.3%	6.00	0.706716	0.678684
6.25	0.65%	0.91%	0.00%	0.0%	14.9%	6.25		
6.50	0.56%	0.84%	0.00%	0.0%	15.5%	6.50		
6.75	0.57%	0.66%	0.00%	0.0%	15.7%	6.75		
7.00	0.54%	0.65%	0.00%	0.0%	15.9%	7.00	0.693485	0.665723
7.25	0.51%	0.60%	0.00%	0.0%	16.1%	7.25		
7.50	0.51%	0.62%	0.00%	0.0%	16.3%	7.50		
7.75	0.45%	0.53%	0.00%	0.0%	16.9%	7.75		
8.00	0.44%	0.51%	0.00%	0.0%	17.6%	8.00	0.666403	0.674000
8.25	0.40%	0.50%	0.00%	0.0%	18.3%	8.25		
8.50	0.35%	0.45%	0.00%	0.0%	19.0%	8.50		
8.75	0.39%	0.40%	0.00%	0.0%	20.0%	8.75		
9.00	0.34%	0.41%	0.00%	0.0%	21.0%	9.00	0.697093	0.706858
9.25	0.33%	0.36%	0.00%	0.0%	22.0%	9.25		
9.50	0.26%	0.36%	0.00%	0.0%	23.0%	9.50		
9.75	0.29%	0.34%	0.00%	0.0%	23.0%	9.75		
10.00	0.26%	0.37%	0.00%	0.0%	22.9%	10.00	0.693861	0.742613

Year Ending	PREMIUM					EXPENSES				LOSS & LOSS ADJUSTMENT EXPENSES						
	WP	UEPR	EP	Collected Premium	Premium Receivable	Acq	Maint.	PHR Dividends	Total	Paid Loss & ALAE	Paid ULAE	Nominal Loss&ALAE (Req Reserve)	Nominal ULAE (Req Reserve)	Tabular Discount	Held Reserve	IRS Disounted Reserve
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
0.00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0.25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0.50	1,374	1,374	0	247	1,127	246	105	0	351	0	0	0	0	0	0	
0.75	0	1,031	344	235	892	0	0	0	0	12	0	238	18	10	246	
1.00	0	687	344	300	592	0	0	0	0	42	1	446	34	20	460	393
1.25	0	344	344	348	244	0	0	0	0	63	2	633	50	20	663	
1.50	0	0	344	99	145	0	0	0	0	78	4	805	64	36	833	
1.75	0	0	0	95	50	0	0	12	12	80	5	725	59	36	748	
2.00	0	0	0	5	45	0	0	0	0	62	6	663	53	36	680	582
2.25	0	0	0	5	40	0	0	22	22	53	5	610	48	35	623	
2.50	0	0	0	5	36	0	0	0	0	47	5	562	43	34	572	
2.75	0	0	0	3	33	0	0	0	0	43	4	520	39	33	526	
3.00	0	0	0	3	30	0	0	0	0	38	3	482	36	33	485	416
3.25	0	0	0	3	27	0	0	22	22	34	3	448	33	32	448	
3.50	0	0	0	3	25	0	0	0	0	30	3	417	30	32	416	
3.75	0	0	0	2	23	0	0	0	0	27	2	391	28	31	387	
4.00	0	0	0	2	21	0	0	0	0	24	2	367	26	31	362	307
4.25	0	0	0	2	19	0	0	22	22	21	2	346	24	31	338	
4.50	0	0	0	2	18	0	0	0	0	19	2	327	22	32	317	
4.75	0	0	0	1	17	0	0	0	0	17	2	311	20	32	299	
5.00	0	0	0	1	16	0	0	0	0	15	1	295	19	33	282	236
5.25	0	0	0	1	14	0	0	0	0	14	1	282	18	32	267	
5.50	0	0	0	1	13	0	0	0	0	12	1	269	17	32	254	
5.75	0	0	0	1	13	0	0	0	0	11	1	258	16	32	242	
6.00	0	0	0	1	12	0	0	0	0	11	1	248	15	31	231	186
6.25	0	0	0	1	11	0	0	0	0	10	1	238	14	31	221	
6.50	0	0	0	1	10	0	0	0	0	9	1	229	13	31	211	
6.75	0	0	0	0	10	0	0	0	0	8	1	221	13	31	203	
7.00	0	0	0	0	9	0	0	0	0	7	1	214	12	30	195	155
7.25	0	0	0	0	9	0	0	0	0	7	1	207	11	30	188	
7.50	0	0	0	0	8	0	0	0	0	6	1	201	11	30	182	
7.75	0	0	0	0	8	0	0	0	0	5	0	196	10	30	176	
8.00	0	0	0	0	8	0	0	0	0	5	0	190	10	30	170	133
8.25	0	0	0	0	7	0	0	0	0	5	0	185	9	30	165	
8.50	0	0	0	0	7	0	0	0	0	5	0	181	9	30	159	
8.75	0	0	0	0	7	0	0	0	0	4	0	176	9	30	155	
9.00	0	0	0	0	6	0	0	0	0	4	0	172	8	30	150	124
9.25	0	0	0	0	6	0	0	0	0	4	0	168	8	31	145	
9.50	0	0	0	0	6	0	0	0	0	4	0	164	8	32	140	
9.75	0	0	0	0	6	0	0	0	0	4	0	161	7	32	136	
10.00	0	0	0	0	6	0	0	0	0	3	0	158	7	33	132	115

	ASSETS				CAPITAL						
Year Ending	Total	Investible	Non-Investible	Pre-Tax Investment Income	Surplus Capital	PHR Funded Capital	EQHR Funded Capital	TOTAL	Contributed Capital	Net Income	Value Added
	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)
0.00	0	0	0	0	0	0	0	0	0	0	0
0.25	0	0	0	0	0	0	0	0	0	0	0
0.50	1,975	752	1,223	0	601	107	856	963	856	0	0
0.75	1,877	910	967	15	601	80	810	890	-46	25	0
1.00	1,748	1,101	647	18	601	56	760	817	-50	23	0
1.25	1,608	1,332	276	21	601	39	721	759	-40	22	0
1.50	833	679	154	26	0	28	60	88	-661	21	0
1.75	748	689	59	13	0	25	68	94	9	2	0
2.00	680	625	55	13	0	24	65	89	-3	2	0
2.25	623	573	49	12	0	23	79	102	13	2	0
2.50	572	528	43	11	0	22	71	93	-8	2	0
2.75	526	487	39	10	0	21	64	85	-7	2	0
3.00	485	450	35	9	0	20	57	77	-7	2	0
3.25	448	417	32	9	0	20	72	92	16	2	0
3.50	416	387	29	8	0	19	67	86	-6	2	0
3.75	387	361	26	8	0	18	61	80	-5	2	0
4.00	362	337	24	7	0	18	56	74	-5	2	0
4.25	338	316	22	7	0	17	73	90	17	2	0
4.50	317	298	20	6	0	16	69	85	-4	2	0
4.75	299	281	18	6	0	16	65	80	-4	2	0
5.00	282	266	16	5	0	15	61	76	-4	2	0
5.25	267	252	15	5	0	14	59	74	-2	2	0
5.50	254	240	14	5	0	14	58	71	-2	2	0
5.75	242	228	14	5	0	13	56	69	-2	2	0
6.00	231	218	13	4	0	13	55	67	-1	2	0
6.25	221	208	13	4	0	12	53	66	-1	2	0
6.50	211	200	12	4	0	12	52	64	-1	2	0
6.75	203	192	11	4	0	11	51	62	-1	1	0
7.00	195	184	11	4	0	11	50	61	-1	1	0
7.25	188	177	11	4	0	10	48	58	-2	1	0
7.50	182	171	11	3	0	10	46	56	-2	1	0
7.75	176	165	11	3	0	10	44	54	-2	1	0
8.00	170	159	11	3	0	9	43	52	-2	1	0
8.25	165	154	11	3	0	9	41	50	-2	1	0
8.50	159	149	11	3	0	9	39	48	-2	1	0
8.75	155	144	10	3	0	8	38	46	-2	1	0
9.00	150	140	10	3	0	8	36	44	-2	1	0
9.25	145	135	10	3	0	8	34	41	-2	1	0
9.50	140	130	10	3	0	7	31	39	-2	1	0
9.75	136	126	10	3	0	7	29	36	-2	1	0
10.00	132	121	10	2	0	7	27	34	-2	1	0

INCOME TAX

CASH FLOW

Year Ending	UW Income				Deferred Tax Asset			Investment Income			UW	Invest Income	FIT	Deferred Assets	Asset	Post-tax Equity
	Statutory UW Inc	Taxable UW Inc	Year End Tax on UW Inc	Qtrly Tax on UW Inc	DTA due to Revenue Offset	DTA due to IRS Disc	DTA	Qtrly Invest Income	Qtrly Tax on Total II	Tax Total						
	(28)	(29)	(30)	(31)	(32)	(33)	(34)	(35)	(36)	(37)	(38)	(39)	(40)	(41)	(42)	(44)
0.00											0	0	0	0	0	0
0.25	0			0	0	0	0	0	0	0	0	0	0	0	0	0
0.50	0			0	96	0	96	0	0	0	1,023.	0	0	96	1,975	-856
0.75	0			4	72	4	76	15	5	9	-12	15	-9	-20	-98	70
1.00	-180	24	9	4	48	7	56	18	6	10	-43	18	-10	-20	-130	73
1.25	0			4	24	8	32	21	7	12	-65	21	-12	-23	-140	61
1.50	0			4	0	9	9	26	9	13	-82	26	-13	-23	-775	682
1.75	0			4	0	10	10	13	5	9	-97	13	-9	1	-85	-7
2.00	155	49	17	4		10	10	13	5	9	-68	13	-9	1	-68	5
2.25	0			-5		9	9	12	4	-1	-80	12	1	-1	-57	-12
2.50	0			-5		8	8	11	4	-1	-52	11	1	-1	-51	10
2.75	0			-5		6	6	10	4	-1	-47	10	1	-1	-46	9
3.00	-25	-55	-19	-5		5	5	9	3	-1	-41	9	1	-1	-41	9
3.25	0			-3		4	4	9	3	0	-59	9	0	0	-37	-14
3.50	0			-3		4	4	8	3	0	-33	8	0	0	-33	8
3.75	0			-3		3	3	8	3	-1	-29	8	1	0	-29	7
4.00	-24	-38	-13	-3		3	3	7	2	-1	-26	7	1	0	-25	7
4.25	0			-3		2	2	7	2	0	-45	7	0	-1	-23	-16
4.50	0			-3		2	2	6	2	0	-20	6	0	-1	-21	6
4.75	0			-3		1	1	6	2	-1	-18	6	1	-1	-19	6
5.00	-21	-29	-10	-3		1	1	5	2	-1	-17	5	1	-1	-17	6
5.25	0			0		1	1	5	2	2	-15	5	-2	0	-15	3
5.50	0			0		1	1	5	2	1	-14	5	-1	0	-13	3
5.75	0			0		1	1	5	2	1	-12	5	-1	0	-12	3
6.00	-1	-3	-1	0		1	1	4	2	1	-11	4	-1	0	-11	3
6.25	0			0		1	1	4	1	1	-10	4	-1	0	-10	3
6.50	0			0		1	1	4	1	1	-10	4	-1	0	-9	3
6.75	0			0		1	1	4	1	1	-9	4	-1	0	-9	3
7.00	-1	-5	-2	0		1	1	4	1	1	-8	4	-1	0	-8	3
7.25	0			0		2	2	4	1	1	-7	4	-1	1	-7	3
7.50	0			0		2	2	3	1	1	-6	3	-1	1	-6	3
7.75	0			0		3	3	3	1	1	-6	3	-1	1	-6	3
8.00	0	-4	-1	0		4	4	3	1	1	-6	3	-1	1	-6	3
8.25	0			-1		4	4	3	1	0	-5	3	0	0	-5	3
8.50	0			-1		4	4	3	1	0	-5	3	0	0	-5	3
8.75	0			-1		4	4	3	1	0	-5	3	0	0	-5	3
9.00	0	-10	-4	-1		4	4	3	1	0	-4	3	0	0	-5	3
9.25	0			-1		4	4	3	1	0	-4	3	0	0	-5	3
9.50	0			-1		4	4	3	1	0	-4	3	0	0	-5	3
9.75	0			-1		4	4	3	1	0	-4	3	0	0	-4	3
10.00	3	-7	-3	-1		5	5	2	1	0	-4	2	0	0	-4	3

LOSS & LAE PAYMENTS

Year Ending	Paid Loss & ALAE							Paid ULAE						
	AQtr 1	AQtr 2	AQtr 3	AQtr 4	AYR 1	AYR 2	TOTAL	AQtr 1	AQtr 2	AQtr 3	AQtr 4	AYR 1	AYR 2	TOTAL
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
0.00	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.25	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.50	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.75	12	0	0	0	12	0	12	0	0	0	0	0	0	0
1.00	30	12	0	0	42	0	42	1	0	0	0	1	0	1
1.25	20	30	12	0	51	12	63	1	1	0	0	2	0	2
1.50	16	20	30	12	36	42	78	2	1	1	0	3	1	4
1.75	14	16	20	30	29	51	80	2	2	1	1	3	2	5
2.00	13	14	16	20	26	36	62	1	2	2	1	3	3	6
2.25	11	13	14	16	24	29	53	1	1	2	2	2	3	5
2.50	10	11	13	14	21	26	47	1	1	1	2	2	3	5
2.75	9	10	11	13	19	24	43	1	1	1	1	2	2	4
3.00	8	9	10	11	17	21	38	1	1	1	1	2	2	3
3.25	7	8	9	10	15	19	34	1	1	1	1	1	2	3
3.50	6	7	8	9	13	17	30	1	1	1	1	1	2	3
3.75	5	6	7	8	12	15	27	1	1	1	1	1	1	2
4.00	5	5	6	7	10	13	24	0	1	1	1	1	1	2
4.25	4	5	5	6	9	12	21	0	0	1	1	1	1	2
4.50	4	4	5	5	8	10	19	0	0	0	1	1	1	2
4.75	4	4	4	5	7	9	17	0	0	0	0	1	1	2
5.00	3	4	4	4	7	8	15	0	0	0	0	1	1	1
5.25	3	3	4	4	6	7	14	0	0	0	0	1	1	1
5.50	3	3	3	4	6	7	12	0	0	0	0	0	1	1
5.75	2	3	3	3	5	6	11	0	0	0	0	0	1	1
6.00	3	2	3	3	5	6	11	0	0	0	0	0	0	1
6.25	2	3	2	3	5	5	10	0	0	0	0	0	0	1
6.50	2	2	3	2	4	5	9	0	0	0	0	0	0	1
6.75	2	2	2	3	4	5	8	0	0	0	0	0	0	1
7.00	1	2	2	2	3	4	7	0	0	0	0	0	0	1
7.25	1	1	2	2	3	4	7	0	0	0	0	0	0	1
7.50	1	1	1	2	3	3	6	0	0	0	0	0	0	1
7.75	1	1	1	1	3	3	5	0	0	0	0	0	0	0
8.00	1	1	1	1	3	3	5	0	0	0	0	0	0	0
8.25	1	1	1	1	2	3	5	0	0	0	0	0	0	0
8.50	1	1	1	1	2	3	5	0	0	0	0	0	0	0
8.75	1	1	1	1	2	2	4	0	0	0	0	0	0	0
9.00	1	1	1	1	2	2	4	0	0	0	0	0	0	0
9.25	1	1	1	1	2	2	4	0	0	0	0	0	0	0
9.50	1	1	1	1	2	2	4	0	0	0	0	0	0	0
9.75	1	1	1	1	2	2	4	0	0	0	0	0	0	0
10.00	1	1	1	1	1	2	3	0	0	0	0	0	0	0

LOSS & LAE RESERVES

Year Ending	Nominal Loss & ALAE Reserves			Nominal ULAE Reserve			Tabular Discount				Held Reserves			IRS Discounted Reserves				
	AYR 1	AYR 2	TOTAL	AYR 1	AYR 2	TOTAL	Discount Factor	AYR 1	AYR 2	Total	AYR 1	AYR 2	TOTAL	IRS Disc Factor AYR 1	IRS Disc Factor AYR 2	AYR 1	AYR 2	TOTAL
	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)	(31)	(32)
0.00	0		0	0		0		0		0	0		0					
0.25	0		0	0		0	0	0		0	0		0					
0.50	0		0	0		0	0.036	0		0	0		0					
0.75	238		238	18		18	0.041	10		10	246		246					
1.00	446		446	34		34	0.045	20		20	460		460	0.8194		393		393
1.25	395	238	633	32	18	50	0.050	20	0	20	408	256	663					
1.50	359	446	805	29	34	64	0.055	20	16	36	369	464	833					
1.75	330	395	725	26	32	59	0.059	20	16	36	337	411	748					
2.00	304	359	663	24	29	53	0.064	19	16	36	308	372	680	0.8027	0.8214	263	319	582
2.25	280	330	610	21	26	48	0.066	18	16	35	283	340	623					
2.50	259	304	562	20	24	43	0.068	18	17	34	261	311	572					
2.75	240	280	520	18	21	39	0.070	17	17	33	241	285	526					
3.00	223	259	482	16	20	36	0.072	16	17	33	223	262	485	0.7975	0.8098	191	225	416
3.25	208	240	448	15	18	33	0.079	16	16	32	207	242	448					
3.50	195	223	417	14	16	30	0.085	17	15	32	192	224	416					
3.75	183	208	391	13	15	28	0.092	17	15	31	179	208	387					
4.00	172	195	367	12	14	26	0.098	17	14	31	167	194	362	0.7548	0.8048	139	168	307
4.25	163	183	346	11	13	24	0.105	17	14	31	157	181	338					
4.50	155	172	327	10	12	22	0.111	17	15	32	148	170	317					
4.75	147	163	311	10	11	20	0.117	17	15	32	140	159	299					
5.00	141	155	295	9	10	19	0.123	17	15	33	132	150	282	0.7334	0.7642	110	126	236
5.25	134	147	282	8	10	18	0.125	17	15	32	126	141	267					
5.50	129	141	269	8	9	17	0.127	16	16	32	120	134	254					
5.75	124	134	258	7	8	16	0.129	16	16	32	115	127	242					
6.00	119	129	248	7	8	15	0.131	16	16	31	110	121	231	0.7067	0.7140	89	98	186
6.25	114	124	238	7	7	14	0.137	16	15	31	105	116	221					
6.50	110	119	229	6	7	13	0.143	16	15	31	101	111	211					
6.75	106	114	221	6	7	13	0.149	16	15	31	97	106	203					
7.00	103	110	214	6	6	12	0.155	16	14	30	93	102	195	0.6935	0.6787	76	79	155
7.25	101	106	207	5	6	11	0.157	16	15	30	90	98	188					
7.50	98	103	201	5	6	11	0.159	16	15	30	87	94	182					
7.75	95	101	196	5	5	10	0.161	15	15	30	85	91	176					
8.00	93	98	190	5	5	10	0.163	15	15	30	82	88	170	0.6664	0.6657	65	69	133
8.25	90	95	185	4	5	9	0.169	15	15	30	79	85	165					
8.50	88	93	181	4	5	9	0.176	16	15	30	77	83	159					
8.75	86	90	176	4	4	9	0.183	16	14	30	74	80	155					
9.00	84	88	172	4	4	8	0.190	16	14	30	72	78	150	0.6971	0.6740	61	62	124
9.25	82	86	168	4	4	8	0.200	16	15	31	70	75	145					
9.50	80	84	164	4	4	8	0.210	17	15	32	67	73	140					
9.75	79	82	161	4	4	7	0.220	17	15	32	65	71	136					
10.00	77	80	158	3	4	7	0.230	18	15	33	63	69	132	0.6939	0.7069	56	59	115

ASSETS

Year Ending	Total Assets	UEPR	Held Loss & LAE Reserves	Required Surplus	Income Producing	Non-Income Producing	II on Total Assets
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.00	0	0	0	0	0	0	0
0.25	0	0	0	0	0	0	0
0.50	1,975	1,374	0	601	752	1,223	0
0.75	1,877	1,031	246	601	910	967	15
1.00	1,748	687	460	601	1,101	647	18
1.25	1,608	344	663	601	1,332	276	21
1.50	833	0	833	0	679	154	26
1.75	748	0	748	0	689	59	13
2.00	680	0	680	0	625	55	13
2.25	623	0	623	0	573	49	12
2.50	572	0	572	0	528	43	11
2.75	526	0	526	0	487	39	10
3.00	485	0	485	0	450	35	9
3.25	448	0	448	0	417	32	9
3.50	416	0	416	0	387	29	8
3.75	387	0	387	0	361	26	8
4.00	362	0	362	0	337	24	7
4.25	338	0	338	0	316	22	7
4.50	317	0	317	0	298	20	6
4.75	299	0	299	0	281	18	6
5.00	282	0	282	0	266	16	5
5.25	267	0	267	0	252	15	5
5.50	254	0	254	0	240	14	5
5.75	242	0	242	0	228	14	5
6.00	231	0	231	0	218	13	4
6.25	221	0	221	0	208	13	4
6.50	211	0	211	0	200	12	4
6.75	203	0	203	0	192	11	4
7.00	195	0	195	0	184	11	4
7.25	188	0	188	0	177	11	4
7.50	182	0	182	0	171	11	3
7.75	176	0	176	0	165	11	3
8.00	170	0	170	0	159	11	3
8.25	165	0	165	0	154	11	3
8.50	159	0	159	0	149	11	3
8.75	155	0	155	0	144	10	3
9.00	150	0	150	0	140	10	3
9.25	145	0	145	0	135	10	3
9.50	140	0	140	0	130	10	3
9.75	136	0	136	0	126	10	3
10.00	132	0	132	0	121	10	2

INCOME TAX

Year Ending	UW Income				Deferred Tax Asset			Investment Income		FIT
	Statutory UW Income	Taxable UW Income	Year End Tax on UW Income	Qtrly Tax on UW Income	DTA: Revenue Offset	DTA: IRS Discounting	DTA: TOTAL	Qtrly Investment Income	Qtrly Tax on Inv Inc	Total
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0.00										
0.25				0	0	0	0	0	0	0
0.50				0	96	0	96	0	0	0
0.75				4	72	4	76	15	5	9
1.00	-180	24	9	4	48	7	56	18	6	10
1.25				4	24	8	32	21	7	12
1.50				4	0	9	9	26	9	13
1.75				4	0	10	10	13	5	9
2.00	155	49	17	4		10	10	13	5	9
2.25				-5		9	9	12	4	-1
2.50				-5		8	8	11	4	-1
2.75				-5		6	6	10	4	-1
3.00	-25	-55	-19	-5		5	5	9	3	-1
3.25				-3		4	4	9	3	0
3.50				-3		4	4	8	3	0
3.75				-3		3	3	8	3	-1
4.00	-24	-38	-13	-3		3	3	7	2	-1
4.25				-3		2	2	7	2	0
4.50				-3		2	2	6	2	0
4.75				-3		1	1	6	2	-1
5.00	-21	-29	-10	-3		1	1	5	2	-1
5.25				0		1	1	5	2	2
5.50				0		1	1	5	2	1
5.75				0		1	1	5	2	1
6.00	-1	-3	-1	0		1	1	4	2	1
6.25				0		1	1	4	1	1
6.50				0		1	1	4	1	1
6.75				0		1	1	4	1	1
7.00	-1	-5	-2	0		1	1	4	1	1
7.25				0		2	2	4	1	1
7.50				0		2	2	3	1	1
7.75				0		3	3	3	1	1
8.00	0	-4	-1	0		4	4	3	1	1
8.25				-1		4	4	3	1	0
8.50				-1		4	4	3	1	0
8.75				-1		4	4	3	1	0
9.00	0	-10	-4	-1		4	4	3	1	0
9.25				-1		4	4	3	1	0
9.50				-1		4	4	3	1	0
9.75				-1		4	4	3	1	0
10.00	3	-7	-3	-1		5	5	2	1	0