A Discussion of "Risk Load for Insurers" by Sholom Feldblum

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RISK LOADS FOR INSURERS

BY SHOLOM FELDBLUM

DISCUSSION BY TRENT R. VAUGHN

Acknowledgment and Caveat Emptor

I would like to acknowledge the comments of the reviewers of this paper. In particular, the reviewers stated that the paper was "unnecessarily argumentative". And ... they may be right about that! I must confess that I do enjoy a good actuarial debate, and, in this case, I strongly believe that Feldblum's risk load methodology represents an unsound application of financial theory to the insurance problem. Even so, let me temper the criticisms below with a couple of caveats. First, an experienced actuary has pointed out to me that Feldblum was not alone in advocating this "modified CAPM" approach to risk loads. At the time that this paper was written, the approach was fairly common. Second, it is important to temper any criticisms of this paper with an acknowledgment of the many contributions that Feldblum has made to the actuarial and insurance literature. Certainly, Feldblum has contributed more to our profession than most actuaries (including myself) could achieve in several lifetimes.

When applying a financial theory to an insurance problem, one should logically follow several rules. First, one should carefully consider the proof of the financial theory, and determine whether the proof makes sense in the insurance setting. Second, one should consider the underlying message of the financial theory, and determine the implications of this message to the insurance problem. Finally, one should be aware of the empirical evidence in support (or contradiction) of the original financial theory. Feldblum's application of the CAPM to the insurance problem fails with regard to the first and second of these rules.

For instance, the actual proof of the CAPM relies on several key assumptions that are incomprehensible in describing the insurance company's choice between writing various lines of business. As an example, the original CAPM proof *assumes that the individual investor can supplement his purchases of marketable securities by borrowing or lending at the risk-free rate of interest, resulting in a linear efficient investment frontier. When an insurance company writes a policy, it invests the policy premium (and supporting surplus) in a variety of financial instruments, including risk-free bonds and/or risky common stocks. From this standpoint, it's unclear how an insurance company faces a choice between writing an insurance policy and borrowing or lending at the risk-free interest rate. As a result, the logic and proof underlying the CAPM cannot rationally be applied to the insurance company's portfolio problem.*

In addition, Feldblum's proposed formula is inconsistent with the very message of the original CAPM. The original CAPM is predicated on the fact that individual investor's can (and do) reduce their risk via individual portfolio diversification. Individual investor diversification is completely outside the scope of Feldblum's formula. Instead, Feldblum's formula measures the "risk" of a given insurance policy only in terms of the insurance company's underwriting portfolio. It does not consider the fact that individual investor's do not hold the insurance company's stock in isolation, but as part of a well-diversified investment portfolio.

In the spirit of full disclosure, let me point out that this paper was soundly rejected for publication in the Proceedings. *As such, I do appreciate the* CAS Forum *as a venue for unique ideas. But, as with all* CAS Forum *articles: let the buyer beware!*

1. INTRODUCTION

Feldblum's paper "Risk Loads for Insurers" discusses various methodologies for estimating the insurance risk load. According to this paper, traditional methods are inadequate. As such, the majority of the paper discusses a proposed methodology for applying modern portfolio theory and the capital asset pricing model (CAPM) to the insurance pricing problem.

Unfortunately, the proposed methodology represents an unsound application of financial theory to an insurance problem. Specifically, the proposed methodology merely borrows the notation of the CAPM, without considering the underlying assumptions and logic of the CAPM paradigm.

Section 2 of this paper will present an actual algebraic proof of the CAPM. In Section 3, we summarize the assumptions underlying the proof and discuss the implications of the result. Section 4 addresses Feldblum's methodology, and points out the unsound nature of that approach. Lastly, Section 5 describes a correct application of the CAPM paradigm to the insurance pricing problem.

2. ALGEBRAIC PROOF OF THE CAPM

Mossin [10] first provided an algebraic formulation of the CAPM proof. In this section, we will briefly outline the key elements of Mossin's proof.[1]

The Investor's Constrained Maximization Problem

The description of the investor's constrained maximization problem is primarily due to Markowitz [5] [6] and Tobin [12]. Markowitz first described the impact of portfolio diversification; Tobin extended the analysis by quantifying the investor's utility of wealth as a function of the mean and variance of total portfolio return.

Assume that there are m individual investors, $i = 1, 2, \ldots m$. Each of these investors possesses an initial wealth amount of w_i, which will be used to purchase securities. At the end of one-period, these securities will be sold, and the proceeds will be used to purchase goods and services for consumption. In other words, we are working with a one-period model of investor behavior. This is also sometimes referred to as a "two-date" model, since the investor purchases securities at time $t=0$ and sells these securities at time $t=1$.

Assume that there are n securities, $j = 1,2, \ldots, n$, each offering a total payment of D_j at the end of one period. Since we are considering a one-period model, D_j can be considered to be a liquidating dividend on the security. In addition, the total payment D_j will be distributed to the various security holders in proportion to the security holder's ownership stake in the firm. For instance, if an individual purchases 25% of the available amount of a security at time 0, then he will be entitled to 25% of the total liquidating dividend on that security at time 1.

Also, assume that for each $j = 1, 2, \ldots, n$, D_j is a normally-distributed random variable. The variance-covariance matrix, Σ, is assumed to be positive-definite; the properties of positive-definite matrices imply that there is no-risk free security (that is, $\mathrm{Var}(D_j)>0$ for all $j = 1, 2, \ldots n$) and no two securities are perfectly negatively correlated (that is, the correlation coefficient for each pair of distinct securities is not equal to negative one). Moreover, in addition to the available

[1] Note: The proof in this section is not exactly identical to Mossin's original proof. We have modified the notation, changed the order, and added several clarifying remarks.

market securities, assume that each investor can borrow or lend at the risk-free rate of interest r_f.

At the beginning to the period, each investor must decide how to allocate his available wealth among the various securities. For instance, let x_{ij} be the proportion of the total issue of security j that is purchased by investor i.. In addition, we will let d_i represent the total dollar amount that the investor lends at the risk-free-rate.[2] At the end of the period, the total payment T_i received by investor i will be given by the following expression:

$$T_i = \sum_{j=1}^{n} x_{ij} D_j + d_i(1+r_f) .$$

Thus, for each investor, the total payment at the end of the period is a normally distributed random variable. The mean and variance of this random variable are given by the following expressions:

$$E(T_i) = \sum_{j=1}^{n} x_{ij} E(D_j) + d_i(1+r_f)$$

$$Var(T_i) = \sum_{j=1}^{n} \sum_{k=1}^{n} x_{ij} x_{ik} Cov(D_j, D_k)$$

Let each investor's utility of end-of-period wealth be given by the function $u_i(w)$, i = 1,2, ... ,m. Moreover, we will assume that each investor is risk-averse (that is, $d^2u_i(w)/dw^2 < 0$) and maximizes the expected value of his utility of end-of-period wealth. Tobin [12] demonstrated that, under these assumptions, each investor's expected utility of end-of-period wealth is a function solely of the mean and variance of the investor's total end-of-period payment. That is, expected utility of end-of-period wealth is given by $E[u_i(T_i)] = f_i[E(T_i),Var(T_i)]$.

Hence, at the beginning of the period, each investor solves a constrained maximization problem. Specifically, each investor will maximize $f_i[E(T_i),Var(T_i)]$ subject to the following wealth constraint:

$$\sum_{j=1}^{n} x_{ij} v_j + d_i = w_i ,$$

where v_j represents the total market value of security j.

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[2] If the investor borrows at the risk-free rate in order to purchase additional securities, then $d_i<0$.

Prior to solving this constrained maximization problem, several specific assumptions should be emphasized. First, we are assuming that the following inputs are all exogenous to the model: (1) the risk preferences of each of the m individuals, as given by their utility of end-of-period wealth functions, (2) the payoff characteristics of the n securities, and (3) the risk-free rate of interest r_f. Second, we are assuming that all assets are marketable and infinitely divisible. Third, we are ignoring taxes and transaction costs. Fourth, we are assuming that investors have homogenous expectations regarding security returns, and that each investor can borrow and lend as much as he wishes at the same risk-free rate of interest. Lastly, we are assuming perfectly competitive security markets; this assumption implies (among other things) that each investor can purchase as much of each security as he wishes at the prevailing market price.

In order to solve the constrained maximization problem, we will utilize the method of Lagrange multipliers:

$$L = f_i[E(T_i), Var(T_i)] + \lambda_i(w_i - \sum_{j=1}^{n} x_{ij}v_j - d_i)$$

Taking partial derivatives with respect to x_{ij} (j=1,2, ... ,n) and d_i and equating them to zero yields:

$$\begin{aligned}\partial L/\partial x_{ij} &= [\partial f_i/\partial E(T_i)][\partial E(T_i)/\partial x_{ij}] + [\partial f_i/\partial Var(T_i)][\partial Var(T_i)/\partial x_{ij}] - \lambda_i v_j \\ &= [\partial fi/\partial E(T_i)]E(D_j) + [\partial f_i/\partial Var(T_i)]\sum_{k=1}^{n} 2x_{ik}Cov(D_j, D_k) - \lambda_i v_j = 0\end{aligned}$$

$$\begin{aligned}\partial L/\partial d_i &= [\partial f_i/E(T_i)][\partial E(T_i)/\partial d_i] - \lambda_i \\ &= [\partial f_i/\partial E(T_i)](1+r_f) - \lambda_i = 0 \\ &\Rightarrow \lambda_i = (1+r_f)[\partial f_i/\partial E(T_i)]\end{aligned}$$

Substituting λ_i into the first set of equations and rearranging yields the following:

$$[\partial f_i/E(T_i)][E(D_j) - (1+r_f)v_j] = -[\partial f_i/Var(T_i)]\sum_{k=1}^{n} 2x_{ik}Cov(D_j, D_k) \qquad (2.1)$$
$$\forall j = 1,2,....,n$$

Thus, each investor (i = 1,2, ... ,m) solves the above system of n equations for the n unknown variables x_{ij}, j=1,2,.....,n.

The Market Clearing Mechanism

Sharpe [11], Lintner [4], and Mossin [10] extended the above analysis to a market equilibrium setting. As each investor solves the above set of equations, security prices (and the resulting total market values, v_j) will adjust to accomodate imbalances between supply and demand. Equilibrium is reached when each investor solves the above equations and the market "clears" for each asset (this market clearing condition will be made more precise later). In this paper, we will ignore the conditions under which equilibrium is attained. Instead, we will assume that equilibrium is reached and examine the properties of the resulting equilibrium.

Arbitrarily select a given investor i and two distinct securities a and b. Taking the ratio of equation (2.1) for these two assets yields the following:

$$\{[\partial f_i / E(T_i)][E(D_a) - (1+r_f)v_a]\} \;/\; \{[\partial f_i / E(T_i)][E(D_b) - (1+r_f)v_b]\}$$
$$= \{-[\partial f_i / Var(T_i)]\sum_{k=1}^{n} 2x_{ik} Cov(D_a, D_k)\} \;/\; \{-[\partial f_i / Var(T_i)]\sum_{k=1}^{n} 2x_{ik} Cov(D_b, D_k)\}$$

After cancelling factors and rearranging terms, we have the following equality:

$$\begin{aligned}&\sum_{k=1}^{n} x_{ik} Cov(D_a, D_k)/[E(D_a) - (1+r_f)v_a]\\ &= \sum_{k=1}^{n} x_{ik} Cov(D_b, D_k)/[E(D_b) - (1+r_f)v_b]\end{aligned} \qquad (2.2)$$

In order for the market to clear, the excess supply for each security must be zero. That is, the sum of the weights for each asset must equal 1. In symbolic terms, the market clearing condition is as follows:

$$\begin{aligned}&\sum_{i=1}^{m} x_{ij} = 1\\ &\forall j = 1,2,\ldots,n\end{aligned} \qquad (2.3)$$

By summing both sides of (2.2) across all investors (i = 1,2,...,m) then applying (2.3) and rearranging terms gives us the following:

$$[E(D_a) - (1+r_f)v_a] / \sum_{k=1}^{n} Cov(D_a, D_k)$$
$$= [E(D_b) - (1+r_f)v_b] / \sum_{k=1}^{n} Cov(D_b, D_k) = \Theta \tag{2.4}$$

Summing equation (2.4) over all assets yields the following:

$$\sum_{j=1}^{n} [E(D_a) - (1+r_f)v_a] / \sum_{j=1}^{n}\sum_{k=1}^{n} Cov(D_j, D_k)$$
$$= [E(D_M) - (1+r_f)v_M] / Var(D_M) = \Theta, \tag{2.5}$$

where D_M is total payment on the market portfolio and v_M is the total value of the market portfolio. That is, $D_M = D_1 + D_2 + \ldots.. + D_n$, and $v_M = v_1 + v_2 + \ldots. + v_n$.

Combining (2.4) and (2.5) yields the following:

$$[E(D_a) - (1+r_f)v_a] / \sum_{k=1}^{n} Cov(D_a, D_k) = [E(D_M) - (1+r_f)v_M] / Var(D_M) \tag{2.6}$$

Using the notation developed above for D_M, note that $\sum_{k=1}^{n} Cov(D_a, D_k)$ can be rewritten as $Cov(D_a,D_M)$. By using this revised notation, equation (2.6) can be solved for the value of an asset under market equilibrium:

$$v_a = \{E(D_a) - [Cov(D_a, D_M) / Var(D_M)][E(D_M) - (1+r_f)v_M]\} / (1+r_f) \tag{2.7}$$

Equation (2.7) can be converted into rates of return by using the following:

$$R_a = (D_a - v_a) / v_a \tag{2.8}$$
$$R_M = (D_M - v_M) / v_M \tag{2.9}$$

Substituting (2.8) and (2.9) into (2.7) yields (after some algebra):

$$E(R_a) = r_f + [Cov(R_a, R_M) / Var(R_M)][E(R_M) - r_f] \tag{2.10}$$

Equation (2.10) is the traditional Sharpe/Lintner/Mossin CAPM.

3. THE CAPM AS A PARADIGM

As demonstrated in the previous section, the proof of the traditional Sharpe/Lintner/Mossin CAPM is predicated on the following key assumptions:

1. Individual investors are risk averse and maximize their expected utility of end-of-period wealth.
2. Investors have homogenous expectations regarding securities with a joint normal distribution of total payments.
3. Investors can borrow and lend as much as they want at the risk-free interest rate.
4. Security markets are perfectly competitive.
5. All assets are marketable and infinitely divisible.
6. There are no taxes, transaction costs or restrictions on short selling.

By rearranging formula (2.10), the CAPM predicts that the equilibrium expected return on an individual asset a will be given by the following formula:

$$E(R_a) - r_f = Cov(R_a, R_M)\{[E(R_M) - r_f]/Var(R_M)\},$$

where R_a is the the return on asset a, R_M is the return on the market portfolio, and r_f is the risk-free rate.

The difference between the expected return on asset a and the risk-free rate, also known as the "risk margin", is thus seen to be the product of two terms: the "risk" of asset a (as given by $Cov(R_a,R_M)$) and the "market price of risk". A common explanation of this definition of "risk" is that investors are only compensated (via the risk margin) for undiversifiable, or "systematic", risk. Investors are not compensated for diversifiable, or "unique", risk. In other words, investors are not concerned about the variance of the asset's return if held in isolation; instead, investors are concerned only with the covariance of that asset's return with the overall market return.

With respect to this interpretation, you sometimes hear the following objection: how can investors ignore the variance of an individual asset's return when that variance contributes to the "risk" of the asset? After all, every asset is included in the market portfolio. Thus, $Var(R_a)$ is actually one of the terms in $Cov(R_a,R_M)$, and thus contributes to the risk premium in asset a's expected return.

This objection, however, is a trifling issue. In real-world security markets, the number of securities n is extremely large. In order to see this, let's re-write the "risk" of the asset, or $Cov(R_a, R_M)$, as the following sum of n terms:

$$Cov(R_a, R_M) = (v_a / v_M)Var(R_a) + \sum_{j \neq a}(v_j / v_M)Cov(R_a, R_j),$$

where, for each security j (j = 1,2, ... ,n), the ratio v_j/v_M is the relative value of asset j as a percentage of the value of the entire market portfolio. In this manner, $Var(R_a)$ is only one of a very large number of terms and v_a/v_M is likely to be very close to zero. As a result, Fama and Miller [2] note that "the variance term in the asset's risk is likely to be trivial relative to the weighted sum of covariances."

Here we can draw an analogy to classic microeconomic price theory. Under the theory of perfect competition, we assume that the individual firm is a price taker; this firm faces a horizontal demand curve and can sell as many units as it wishes at the prevailing market price. If we consider the demand curve for market as a whole, however, price is inversely related to the quantity produced. But isn't each individual firm part of the overall market? How, then, can it sell any given quantity at a fixed price?

The solution to this conundrum lies in the specifications of the economic model; in the model of perfect competition, we require a very large number of producers (and buyers), with no one producer comprising a significant proportion of the overall market. In this case, the actions of any one producer will produce only a negligible impact on the overall market price. Likewise, if apply the CAPM model to a world with a very large number of securities, each security's variance has only a trifling impact on its risk.

4. APPLYING THE CAPM NOTATION TO INSURANCE PRICING

In the past, practicing actuaries have been tempted to borrow the results of the CAPM and apply this notation to insurance pricing. Meyers [7, p.4] describes the rationale as follows: "It would seem desirable to adapt this securities pricing model to the insurance pricing problem. One possible approach would be to let an insurer play the role of the investor and let an insurance policy, or a line of insurance, play the role of the individual security and use the CAPM directly."[3]

[3] As an aside, Meyers adds (in a footnote on the same page), "This is the approach taken by the so called 'Insurance CAPM', which is described in 'Asset Pricing Models for Insurance' by J. David Cummins, ASTIN Bulletin, November 1990, p. 125." It is important to note, however, that Cummins definitely does not use this approach

Feldblum uses this general approach in his paper. Feldblum's methodology essentially applies the CAPM notation to the insurance pricing problem. Feldblum summarizes his approach as follows:

"An insurer chooses lines of insurance (or blocks of business) to maximize its expected return while minimizing its 'risk'. The market return R_m in the CAPM model should be replaced by the return on a fully diversified insurance portfolio. The appropriate equation is $R = R_f + B(R_p - R_f)$, where R_p is the return on the all lines combined insurance portfolio."

In addition, Feldblum derives each line's "beta" by a regression between the operating returns on that line and the operating returns for all property/liability insurance lines combined. Thus, Feldblum's full formula coincides with the Sharpe/Lintner/Mossin CAPM formula, but with an "insurance interpretation" of the variables:

$$E(R_a) - r_f = Cov(R_a, R_P)\{[E(R_P) - r_f]/Var(R_P)\}$$

But is this formula really sound? In a recent PCAS paper, Mildenhall [9] describes the difference between applying a *paradigm* and simply borrowing a *notation*. The approach above simply borrows the CAPM notation while ignoring the major underlying message of the CAPM paradigm. As such, the technique clearly does not represent a logical extension of financial theory to insurance pricing.

Specifically, the CAPM is a paradigm that describes equilibrium in a capital market with risk-averse individuals and a large number of assets. As discussed in the previous section, the main result of the CAPM is that risk-averse investors are only concerned about the systematic, or undiversifiable, risk of individual assets. In this case, corporations, including insurance companies, will not be "risk-averse" in the same sense as individual investors. On the contrary, the CAPM implies that corporations are not concerned about the total variance of results, but only the extent to which these results fluctuate in step with overall economic conditions. Ironically, by simply applying the notation (or framework) of the CAPM to the insurance pricing problem, one is implicitly contradicting the very message of the CAPM paradigm.

The major cause of these problems is that the underlying logic and proof of the CAPM do not apply to the insurance company's choice between individual insurance policies or lines of business. As noted above, the CAPM requires strictly

anywhere in his paper. Instead, Cummins uses a correct application of the CAPM paradigm to an insurance pricing, which will be described in Section 5 of this paper.

risk averse individual investors. In addition, the fundamental CAPM result hinges on the assumption of a large number of assets, as demonstrated in the previous section. Meyers [7] points out a major flaw with replacing the thousands of individual securities in the original CAPM with only a few lines of business in the insurance problem; namely, with only a small number of lines of business, the variance of each line contributes significantly to that line's "beta" (and thus to its risk margin), contradicting a key implication of the CAPM that only undiversifiable risk is relevant.

This problem can be seen clearly by returning to our original analogy from microeconomics. The idea that each individual firm is a price taker hinges on the assumption of a large number of competing firms. Consequently, if we apply the model of perfect competition to a product market with only 15 firms, the underlying logic falls apart.

Furthermore, a closer look at the actual CAPM reveals other assumptions that may need to be modified before applying the proof to insurance markets . In particular, the CAPM assumes that the individual investor incurs no transaction costs in the process of forming a diversified portfolio. This assumption may be reasonable for individual investors, as mutual funds offer extensive diversification in exchange for a relatively low expense charge. Insurance companies, however, incur much more extensive transaction costs in the process of forming a diversified portfolio of insurance policies. Likewise, security markets are generally viewed as perfectly competitive, given the large number of both buyers and sellers, and the widespread availability of information. Insurance markets, on the other hand, may not always be perfectly competitive; for instance, there may only be a handful of insurance companies operating in certain "niche" lines. [4]

Lastly, it is unclear how certain assumptions in the actual CAPM proof even apply to the insurance market. As an example, the CAPM assumes that there are no restrictions on short selling; in symbolic terms, an investor "shorts" a security j by selecting an x_{ij} factor that is less than zero. But how does an insurance company "short" a given line of insurance? Also, the CAPM assumes that the investor has the option of supplementing his purchases and sales in marketable securities by borrowing or lending at the risk-free rate of interest. But what meaning does this have in relation to an insurance company's choice between writing various lines of insurance? When an insurance company writes a policy, it invests the premium in various financial instruments, including risky common stocks and risk-free

[4] The complications of transaction costs in insurance markets and a limited number of lines of business were part of the motivation behind the Competitive Market Equilibrium risk load formula, developed by Meyers [7]. Also, see Meyers [8] for a related discussion of the flaws in Feldblum's methodology.

government bonds. It is unclear how the insurance company faces a “choice” between writing insurance and borrowing or lending money.[5]

5. A CORRECT APPLICATION OF THE CAPM PARADIGM TO THE INSURANCE PRICING PROBLEM

A correct application of the CAPM paradigm to the insurance pricing problem reflects the underlying message of the CAPM: individual investors hold diversified portfolios and only require compensation (in the form of a higher expected return) for undiversifiable risk. In other words, we must recognize that individual investors do not hold the insurance company's common stock in isolation, but only as a small part of a well-diversified portfolio. Hence, the risk margin on the insurer's common stock return is proportional to the "beta" of that common stock, or the extent to which it varies with the overall return on the market portfolio.

The major implication of the CAPM to insurance pricing is that we can't consider the insurer's underwriting results in isolation, because individual investors hold insurance stocks as part of a well-diversified portfolio. Thus, the required return on an insurance company's common stock depends on the correlation between the stock's return and the return on the market portfolio.

As noted in a footnote above, Cummins [1] describes the correct application of the CAPM to the insurance pricing problem. In this formulation, one determines the "fair" premium, or the premium that equates the expected rate of return to the required rate of return. Moreover, the required rate of return is determined in accordance with the CAPM, by examining the correlation between the return on the insurance policy and the return on all securities in the financial marketplace.

Of course, the difficulty in correctly applying the CAPM to the insurance pricing problem involves the necessary parameter estimation. Fortunately, Garven [3] has shown that the option pricing method is consistent with the CAPM approach, while allowing for easier estimation of the necessary parameters.

[5] Actuaries do occasionally attempt to estimate insurance portfolios that lie on the “efficient frontier”. These estimates do not typically include the line tangent to the risk-free rate and the efficient frontier, as is commonly done in the estimation of the efficient frontier of financial securities.

6. SUMMARY

As the financial and insurance sectors continue to consolidate, actuaries are becoming exposed to a myriad of financial theories. As we progress into the 21st century, we will be required to apply these financial theories to problems in insurance. As we complete this endeavor, it is critical to avoid making the same mistakes of the past. A common mistake, as demonstrated in Feldblum's paper, is to simply "borrow" the notation of a financial theory, without considering the assumptions, logic, and implications of the underlying paradigm.

Misapplications of this nature result in more than just bad theory; they also sow widespread confusion. For instance, after reading Feldblum's paper, actuaries will be tempted to partition the total risk of an insurance line of business into two components: the portion that is explained by the variation of operating returns on all insurance lines combined, and the portion that is due specifically to the unique attributes of the line under consideration. The first of these components may be labelled "systematic risk", and the second "unique risk". Using this terminology, systematic risk represents the risk that an insurance company cannot eliminate via diversification across various lines of business.

In the financial world, however, systematic risk represents the underlying risk of a security that an individual investor cannot eliminate via portfolio diversification. In the study and application of finance, the concept of systematic risk pertains to an individual investor's diversification across securities, not to a corporation's attempt to diversify across lines or divisions.

Moreover, the logical framework of the CAPM implies that investors will be rewarded (in a linear manner) only for the risk that cannot be eliminated by individual diversification. There is no sound basis for applying the CAPM to a corporation's choice between various lines (or divisions), and stating that the expected return on a given line will be linearly proportional to the risk that cannot be eliminated via corporate diversification. The two reasons for this are as follows: (1) the assumptions and proof of the CAPM do not apply to a corporation's choice between divisions or lines, and (2) the fundamental message of the CAPM is that individual investor's can diversify on their own; hence corporate diversification is redundant.

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