

*Review of “Capital Allocation for Insurance
Companies” by Stewart C. Myers and
James R. Read Jr.
Practical Considerations for Implementing the
Myers-Read Model*

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Capital Allocation for Insurance Companies **Stewart Myers and James Read**

Practical Considerations for Implementing the Myers-Read Model

A Review by
Kyle Vrieze and Paul Brehm

Introduction.

With their paper, “Capital Allocation for Insurance Companies,” Stewart Myers and James Read have added a great contribution to the finance literature relating to the property and casualty insurance industry. On its face, and taking as given that capital allocation is necessary, the Myers-Read methodology is intuitively appealing and mathematically elegant.

Myers and Read propose a capital allocation methodology based on an options pricing framework. In their model, it is acknowledged that the insurance contract cannot provide a 100% guarantee of indemnification for loss; there is a certain level of default risk. Imagine that the insurance company could purchase an option to put any remaining liabilities to a third party in the event of a default, and that the cost of this put option was related to the volume of risky insurance liabilities as a cost per unit. Myers and Read propose to allocate all of the insurance company’s capital to each of the lines of business of the insurance company, with each line receiving the amount of capital that would equalize the cost of that unit’s default put option per unit of liabilities.

We have experimented with the Myers-Read (hereafter MR) proposal for capital allocation, creating examples or case studies with the characteristics of real-world insurance data. Our review of Myers-Read will accept the mathematical construct of their model for now and instead present some practical considerations actuaries will likely confront in trying to implement the MR capital allocation model. The balance of this paper presents a number of considerations, broadly grouped into the following five sections.

1. What are we trying to allocate, and why?
2. What would a real-world application look like?
3. How do you parameterize the MR model?
4. What about the asset side of MR?
5. How do you expand or contract the lines of business?

What are we trying to allocate, and why?

Let’s step back for a moment and ask ourselves, “What are we trying to achieve with a capital allocation?” Gary Venter [10] said it very well: “Capital allocation is generally not an end in itself, but rather an intermediate step in a decision making process.” The “end” decisions are typically about portfolio mix or business performance measurement

and management. There is a recent body of research that suggests actuaries can assist insurance companies in achieving these ends without the need to allocate capital at all.¹

That said, and for the sake of argument, let's now assume that we have concluded that allocating our company's capital is an admirable task. But we must be clear about *what* we are allocating. Myers and Read state

“...this paper clarifies how option pricing methods can be used to determine how much capital an insurance company should carry and how that capital requirement should be allocated. It is the convention of the insurance industry to refer to capital as surplus.”

“Surplus” is a statutory accounting concept. Indeed, the Myers and Read model could be accused of being an algorithm largely, if only implicitly, written from a regulatory perspective, which might explain the slant. However, beyond regulatory purposes, there is little (no) need to allocate “surplus.”

This is not a criticism of the MR model, but rather a clarification of its application. Rather than “surplus,” we recommend that the practitioner first establish the amount of capital the company needs based on its risk profile.² It is this required level of capital that should be allocated to risk-bearing enterprises in the company. The MR model can do this without loss of generality. Throughout the remainder of this review, we will try to use the term “capital” to reflect the relevant value to be allocated. We will use “surplus” only when quoting or referring to passages in the Myers-Read paper.

What happens when “true” risk-related capital is less than or greater than the recorded level of GAAP or statutory capital? We believe this shortfall or redundancy should be attributed to executive management: to either raise additional capital or put any extra to good use, respectively. It should not be allocated to a risk-bearing activity, as that would tend to distort the very ends we set out to achieve. Allocating too much or too little capital would make it difficult to get an accurate gauge on the true economics of the business for management purposes.

What would a real-world example look like?

The examples used in Myers and Read's original paper, with their evenly sized liabilities, similarly-sized coefficients of variation (cv's), and a single asset class, are not very true-to-life for a typical insurance enterprise. In the discussion that follows, we will work with an example that conforms more to what the actuary might encounter in practice. Our example also has three lines of insurance liabilities, but they are diverse in size and in coefficient of variation. The largest line accounts for the large majority of liabilities, and is intended to represent relatively homogeneous commercial insurance business. The smallest line has a very large cv, and is intended to represent catastrophe business. The

¹ cf Mango [5]. As Don says, “Capital allocation is sufficient but not necessary.”

² There are a variety of ways to do this, cf Lee and Ward [3].

final line contains more highly variable casualty business, and could represent such businesses as assumed reinsurance, excess and surplus lines, or medical malpractice. We also use two asset classes, intended to be representative of stocks and bonds. The following tables summarize the example:

Table 1,
Liabilities:

| | % of liabilities | Coeff. of Variation | Description |
|--------|------------------|---------------------|-----------------------------------------|
| Line A | 86% | 5% | Represents standard commercial business |
| Line B | 4% | 130% | Cat risk |
| Line C | 10% | 30% | Long-tail, high-variance liability |
| Total | | | |

Table 2, Assets:

| | % of assets | Coeff. of Variation | Description |
|---------|-------------|---------------------|-------------|
| Asset 1 | 80% | 3% | Bonds |
| Asset 2 | 20% | 15% | Stocks |

Correlations among lines and between liabilities and assets are shown in Table 3, along with the MR results for this example. Parameterization in general terms will be addressed, but for this particular model, note that the cat risk line is uncorrelated with the other lines, and the general commercial and high-variability casualty businesses are positively correlated. Note, too, the final allocation is shown on the far right in Table 3 as "surplus/liab."

Table 3:

| | % of liabilities | CV | Liability Correlations | | | covariance w/liabilities | covariance w/assets | surplus /liab |
|-----------------------|------------------|------|--------------------------|---------|--------|--------------------------|---------------------|---------------|
| | | | Line A | Line B | Line C | | | |
| Line A | 86% | 5% | 1.0 | 0.0 | 0.3 | 0.003 | -0.001 | 0.14 |
| Line B | 4% | 130% | 0.0 | 1.0 | 0.0 | 0.068 | -0.007 | 2.81 |
| Line C | 10% | 30% | 0.2 | 0.0 | 1.0 | 0.014 | -0.003 | 0.67 |
| Liabilities | 100% | 8.1% | 0.68 | 0.65 | 0.59 | 0.006 | -0.001 | 0.30 |
| Assets | 130% | 4.2% | -0.26 | -0.13 | -0.26 | | 0.002 | |
| surplus | 30% | | | | | | | |
| asset/liab volatility | 10.2% | | | | | | | |
| default value/liab | 0.0184% | | | | | | | |
| Delta | -0.004 | | | | | | | |
| Vega | 0.017 | | | | | | | |
| | | | Asset/Liab. Correlations | | | | | |
| | | | Asset 1 | Asset 2 | | | | |
| | | | 0.2 | 0.1 | 0.2 | | | |
| | | | 0.2 | 0.1 | 0.2 | | | |
| | | | Asset Correlations | | | | | |
| | % of assets | CV | asset 1 | asset 2 | | | | |
| Asset 1 | 80% | 3% | 1.0 | 0.2 | | | | |
| Asset 2 | 20% | 15% | 0.2 | 1.0 | | | | |

This example can be modified somewhat to produce a scenario with a negative capital allocation. If the correlation between Lines A and B is changed to -0.8 , values for Line B's CV between 5% and 66% will produce negative capital ratios for Line B. Here, Line B serves as a valuable hedge that the company is therefore willing to write at a marginal loss. In this case, at a CV in excess of 66%, the hedge value is offset by the capital requirement for Line B's own variability. At a CV less than 5%, Line B won't tend to "wiggle" far enough (in the opposite direction of Line A) to serve as a useful hedge.

Everywhere it appears in the paper, this example will use the MR calculations that arise from the joint lognormal assumption (Appendix 2 to the paper). The joint normal assumption is inappropriate in this case, particularly for the high CV line B, where the normal assumption would imply a reasonable probability for negative liability values. Even the lognormal distribution form may not adequately represent the skewness of a catastrophe risk line such as Line B. Furthermore, there may be segments of the investment portfolio for which the lognormal also may not fairly represent the skewness of returns, such as instruments exposed to significant credit default risk, or those with highly non-linear responses to interest rate changes. We mention this in the context of practical considerations confronting an actuary attempting to implement the MR model because there are a number of real world sources of risk to insurance companies that don't resemble a lognormal curve. Nevertheless, as the authors point out, the appropriate form of probability distribution is an empirical matter. Departing from the lognormal assumption only complicates the calculations; it does not invalidate the basic result.

Using the example in Table 3, we will discuss issues that arise because the allocation assigns all capital to the liability lines, without consideration of the capital consumed by the risky investment portfolio. We will also demonstrate a simple way to subdivide any liability line or asset class in order to produce greater granularity without disturbing the original total capital allocations. But first, we address issues associated with parameterizing the table.

How do you parameterize the MR model?

If the MR model is to be more than of academic interest, we need to be able to parameterize the model. Imagine that you need to fill out the Myers and Read's Table 2 (p. 28) or our Table 3 (above) for your company. The critical parameters that need to be estimated and entered are the standard deviation of the liabilities and the correlations between the classes of liabilities and between the liabilities and the assets.

The standard deviations of the liabilities (unpaid losses) are reasonably approachable. A number of methods exist for calculating these, Mack [4] and Zehnwirth [11] just to name two.

The Holy Grail is to derive the correlation matrix between the unpaid loss liabilities and between the liabilities and the assets. There is currently considerable interest in the search for the Grail. The CAS ERM Committee and the CAS RCM Committee have a

joint call for papers in process now seeking insight into this very issue. The Reinsurance Committee has a similar call. The activity is indicative that no one has a clue³.

One possibility for measuring the MR correlation matrix, at least for the liabilities, would be to start with a measurement of the variance of the entire unpaid loss estimate (σ^2), which is the sum of all of the variance-covariance matrix elements. If we then divided the unpaid loss inventory into two classes, and measured their respective variances (σ_1^2 and σ_2^2), we could calculate the implied covariance (σ_{12}) between the two classes as

$$\sigma_{12} = 0.5[\sigma^2 - (\sigma_1^2 + \sigma_2^2)]$$

By successively splitting aggregate data into classes accordingly, a variance-covariance matrix could be constructed. We're sorry to say that this is still just a thesis on our part; we haven't tried this at home ourselves.

The difficulty in trying to empirically parameterize the MR model is troublesome, but hardly unique. Many popular capital allocation schemes suffer the same fate. Even simple allocation rules such as proportionate standard deviation or variance rules require knowledge about the variance-covariance matrix.

For all the difficulty in expressing the correlation matrix, we have to wonder if it's worth the effort. Recent history, especially events such as September 11, suggest that the nature of the relationship between classes of hazards or between various hazards and other classes of risks may be far too complicated to capture in simple variance-covariance matrix. On the other hand, a conceptual model may still be of value even if it does not capture all the nuances of the real world (think Euclidian geometry, for example).

We alluded to the assumption of normal or lognormal risk distributions in the preceding section. In our experimentation, we relied on the lognormal assumption rather than using any empirical distributions. This was in large part because the math is just so much easier! We did find, however, that the MR model seems to under-allocate capital to highly skewed lines because of this simplifying assumption. Lines that are cat exposed and lines that are credit exposed (financial guarantees, surety) are a couple of examples that we found.

What about the asset side of Myers-Read?

The MR method produces an additive allocation of capital across liability lines. Assigning all costly capital to liability lines implies, correctly, that policyholders bear risk of default not only due to insurance risk, but also due to asset risk.

It is appropriate to expect policyholders to bear the entire cost of capital only if actual investment yields are used in the pricing of coverage. However, in practice, investment

³ For papers on correlations in unpaid losses, see Brehm [1] or Meyers [7].

income is often accounted for in pricing exercises by calculating present values of expected cash flows using a new money Treasury yield curve. This practice is motivated in part by the perception that it would be unfair for insureds to pay a higher premium in order to cover the investment losses of the insurer; therefore it would be inappropriate to use higher, risky yields in pricing exercises. In such a situation, it might be desirable to determine an amount of capital attributable to risky investment activities, and allocate the remainder of capital to liability lines. Then insureds will neither receive the benefit, nor be charged for the capital requirement, of the risk borne by the investments function.

The performance of the investments area could be measured as the asset return in excess of the risk free value of the insurance float compared against the capital consumed by asset risk. Furthermore, the performance of different asset portfolios could be measured against the portfolios' capital consumption. This measurement could have application in asset allocation studies.

While the MR calculation method does not ascribe capital to asset classes, it could be used as the value function in a Shapley value calculation, which produces an additive, order-independent allocation of capital to each source of default risk in the insurance enterprise⁴.

The Shapley value is related to game theory, and is used to calculate the relative power of individuals and coalitions in voting schemes and co-operative games. It is related to the "stand-alone surplus requirement" and "total surplus required for each line, given the other lines" that were discussed in the original MR paper. The idea is to use the MR process to calculate the required capital over all possible combinations of risk sources. In our example from Table 3, we have five sources of risk (three insurance lines and two asset classes). This gives us $2^5 - 1 = 31$ possible combinations of risk sources (this excludes the null set, with no risk sources). We then calculate required capital, using the MR calculation, for each combination of risk sources. Specifically, for each excluded risk source in a given combination, we set the CV for the risk source to zero and adjust the capital ratio until the original default value is returned. This differs somewhat from the method employed by MR when they determined "stand-alone surplus requirement" and "surplus required by each line, given the other two lines". They set the liability value for the excluded line(s) to zero and adjusted the capital ratio until the original default value *as a percent of liabilities* resulted. Their process removed the liability entirely, while ours removes only the risk of the liability or asset.

Once the required capital is calculated for each combination of risk sources, the Shapley value can be computed. Let C represent any one of the 31 combinations of the five risk sources. Let $|C|$ be the number of risk sources that combination C contains. For example, if C contains Line A, Line B and Asset 1 risk, but no Line C or Asset 2 risk, then $|C| = 3$. Let $v(C)$ represent required capital corresponding to combination C and let N represent the set of all 31 combinations C. The Shapley value, $\phi_i(v)$, for risk source i is then given by the following equation:

⁴ Cf Nealon and Yit [9]

$$\phi_i(v) = \sum_{\substack{C \subset N \\ i \in C}} \frac{(|C|-1)!(5-|C|)!}{5!} [v(C) - v(C - \{i\})]$$

The summation is over all the combinations in N that contain risk i. For each combination C, the difference between the capital for C and for the combination created by removing risk i from C is calculated. The weight assigned to this difference is proportional to the (|C|-1)! possible orders in which the |C|-1 risks could be added prior to adding risk i, times the (5-|C|)! orders in which the remaining risks could be added after risk i. All possible orders are contemplated, so the order dependence problem is solved, and the resulting sum of $\phi_i(v)$ for all i equals total required capital.

For our example, the possible combinations of risk sources and their associated MR volatility ratios and capital ratios are shown in Table 4:

Table 4

| | Risk Combination | | | | | MR Volatility Ratio | Surplus Ratio |
|----|------------------|--------|--------|---------|---------|---------------------|---------------|
| 1 | Line A | Line B | Line C | Asset 1 | Asset 2 | 10.2% | 30.0% |
| 2 | Line A | | Line C | Asset 1 | Asset 2 | 8.4% | 23.6% |
| 3 | Line A | Line B | | Asset 1 | Asset 2 | 8.8% | 25.0% |
| 4 | Line A | Line B | Line C | | Asset 2 | 9.3% | 26.5% |
| 5 | Line A | Line B | Line C | Asset 1 | | 9.0% | 25.4% |
| 6 | Line A | | | Asset 1 | Asset 2 | 6.7% | 17.8% |
| 7 | Line A | Line B | | | Asset 2 | 7.9% | 21.8% |
| 8 | Line A | Line B | Line C | | | 8.1% | 22.2% |
| 9 | Line A | | Line C | | Asset 2 | 7.5% | 20.1% |
| 10 | Line A | | Line C | Asset 1 | | 7.1% | 19.0% |
| 11 | Line A | Line B | | Asset 1 | | 7.6% | 20.7% |
| 12 | Line A | Line B | | | | 6.7% | 17.8% |
| 13 | Line A | | Line C | | | 6.1% | 15.8% |
| 14 | Line A | | | Asset 1 | | 5.3% | 13.3% |
| 15 | Line A | | | | Asset 2 | 5.7% | 14.5% |
| 16 | Line A | | | | | 4.3% | 10.2% |
| 17 | | Line B | Line C | Asset 1 | Asset 2 | 8.1% | 22.4% |
| 18 | | | Line C | Asset 1 | Asset 2 | 5.8% | 14.6% |
| 19 | | Line B | | Asset 1 | Asset 2 | 7.1% | 18.9% |
| 20 | | Line B | Line C | | Asset 2 | 7.2% | 19.3% |
| 21 | | Line B | Line C | Asset 1 | | 6.9% | 18.2% |
| 22 | | | | Asset 1 | Asset 2 | 4.2% | 9.9% |
| 23 | | Line B | | | Asset 2 | 6.3% | 16.2% |
| 24 | | Line B | Line C | | | 6.0% | 15.4% |
| 25 | | | Line C | | Asset 2 | 4.6% | 11.2% |
| 26 | | | Line C | Asset 1 | | 4.2% | 9.9% |
| 27 | | Line B | | Asset 1 | | 5.9% | 15.2% |
| 28 | | Line B | | | | 5.2% | 12.9% |
| 29 | | | Line C | | | 3.0% | 6.6% |
| 30 | | | | Asset 1 | | 2.4% | 5.0% |
| 31 | | | | | Asset 2 | 3.0% | 6.6% |

Obviously, this method is far more computationally intensive than the original MR calculation for any significant number n of liability lines or asset classes, since you are essentially performing $2^n - 1$ iterations of a numerical solution for the capital ratio in the MR default value function (unless there is a closed form solution for the capital ratio, but we didn't find one). However, PCs are up to the task (within reason), and an actuary who can translate an algorithm into an Excel macro would make short work of it.

The capital allocation resulting from the Shapley calculation is compared with the original MR allocation in Table 5.

Table 5

| | MR Surplus Allocation | Shapley Surplus Allocation |
|---------|-----------------------------|----------------------------------|
| Line A | 0.12 | 0.08 |
| Line B | 0.11 | 0.09 |
| Line C | 0.07 | 0.05 |
| Asset 1 | | 0.04 |
| Asset 2 | | 0.05 |
| Total | 0.30 | 0.30 |

Differences in capital allocated to liability lines between the MR allocation and the Shapley value follow the marginal diversification characteristics of each liability line. Line A is the line with the highest correlation with liabilities (see Table 3), mainly because it accounts for the large majority of liabilities. At the margin, additional volume in Line A doesn't benefit much from diversification across the other lines, so it receives the largest capital allocation from the MR formula. Line B, the cat risk line, realizes a substantial diversification benefit at the margin because of its very high CV and its independence from the other liabilities (and lower correlation with assets). Its absolute capital allocation under MR is lower than Line A's.

The Shapley allocation looks at all possible combinations of the presence or absence of each risk source, which emphasizes total diversification impact rather than diversification at the margin. This has the effect of crediting Line A for the diversification benefits it imparts to the other lines and to the assets. As an example, consider the effect on the required capital ratio from adding Line B risk to the combination of all the other risk sources. This is the difference between the capital ratio for combination 1 in Table 4 (30.0%) and the capital ratio for combination 2 (23.6%), giving an increase of 6.4%. Now consider the impact of adding Line B risk to all the other risks *except Line A*. This is the difference in capital ratios for combination 17 (22.4%) and combination 18 (14.6%), an increase of 7.8%. The presence of Line A mitigates the capital requirement impact of adding Line B risk.

In further experimentation with the model, we found parameter sets that produced a negative allocation for a segment under MR, but a positive allocation using the Shapley allocation. We also found examples where a negative allocation was produced using both methods.

It could be argued that individual underwriting decisions are made at the margin; therefore the continuous marginal approach of the basic MR is a more appropriate capital allocation for use in pricing. If it is necessary to separate investment risk from the capital cost borne by policyholders, then perhaps the Shapley calculation could be used to determine the portion of capital attributable to investments, and the remainder could be allocated to liability lines in proportion to their original MR allocations. Unfortunately, this mars the elegance of the MR approach, which was a big part of what made it so appealing in the first place.

We promised that you could use this method to measure the performance of the investments function. In this example, the Shapley value assigns almost a third of the capital to the asset portfolio; so, for example, the investment function would realize a 29% pre-tax return on capital under this allocation by beating the risk free return on assets by two points: $1.3 \times .02 / .09 = 29\%$ (assets*spread/capital = investment ROE).

How do you expand the model for greater granularity?

Being in corporate America we take as axiomatic that it is necessary to reorganize the company almost every year. Wouldn't it be nice if the capital allocated to a block of business were impervious to regrouping the liabilities?

One of the attractive features of the M-R calculation is that the resulting capital allocation is additive, meaning that the individual capital requirements add up to the total enterprise capital. In practice, one is likely to frequently encounter the need to go a step beyond additivity to sub-additivity. For the method to work well in practice, there needs to be a way to split the capital allocated to a specified line into component parts of that line, without disturbing the rest of the allocation. For example, the split of line A's capital by region, by producer, or by product type may be of interest. Or it may be desirable to split the catastrophe line into the earthquake and hurricane perils.

This sub-additivity is easy to accomplish with the MR framework. One can expand any liability line into multiple segments, or combine different segments together. The new MR calculation will preserve the total volatility and default value, as well as the original capital allocations to the unaffected segments, under the constraint that the covariances of all the original liability segments with the total company liabilities and with assets are preserved.

Consider the case of expanding Line A into two similarly sized segments, Line A1 and Line A2, with weights, CVs and correlations as shown in Table 6. In this example, values were selected for the individual CVs of Lines A1 and A2, and the correlation coefficient was solved for in order to preserve the total CV for the combined Line A, following the method we describe in the parameterization section.

Table 6:

| | % of liabilities var. | coeff. of | correlations | |
|---------|-----------------------|-----------|--------------|---------|
| | | | Line A1 | Line A2 |
| Line A1 | 40% | 7% | 1.0 | 0.1962 |
| Line A2 | 46% | 6% | 0.1962 | 1.0 |

One can verify that the CV of the sum of lines A1 and A2 is 5%, the same as for the combined Line A. The next problem is determining the correlations of A1 and A2 with Lines B and C, and asset classes 1 and 2. These correlations are constrained by the need to preserve the covariances of the original liability lines with each other and the asset classes. One way to accomplish this is to assume that the correlations with other lines and with asset classes are the same across subdivisions of Line A. Then Line A's original correlation coefficients may be divided by the ratio of the weighted average CV of the subdivisions of Line A (6.4651% in this example) to the original Line A CV of 5%. This adjustment will preserve the necessary covariances, but the resulting correlation matrix is certainly not the only one with this property. Using that adjustment, the new correlations and MR capital allocations are shown in Table 7 (the asset/asset data is unchanged from Table 3).

Table 7:

| | % of liabilities | CV | Liability Correlations | | | | covariance w/liabilities | covariance w/assets | surplus/liab |
|-----------------------|------------------|---------|------------------------|---------|--------|---------|--------------------------|---------------------|--------------|
| | | | Line A1 | Line A2 | Line B | Line C | | | |
| Line A1 | 40% | 7% | 1.0 | 0.1962 | 0 | 0.30935 | 0.003 | -0.001 | 0.15 |
| Line A2 | 46% | 6% | 0.1962 | 1.0 | 0 | 0.30935 | 0.003 | -0.001 | 0.13 |
| Line B | 4% | 130% | 0 | 0 | 1.0 | 0.0 | 0.068 | -0.007 | 2.81 |
| Line C | 10% | 30% | 0.30935 | 0.30935 | 0.0 | 1.0 | 0.014 | -0.003 | 0.67 |
| Liabilities | 100% | 8.1% | 0.53 | 0.53 | 0.65 | 0.59 | 0.006 | -0.001 | 0.30 |
| Assets | 130% | 4.2% | -0.20 | -0.20 | -0.13 | -0.26 | | 0.002 | |
| surplus | | 30% | | | | | | | |
| asset/liab volatility | | 10.2% | | | | | | | |
| default value/liab | | 0.0184% | | | | | | | |
| Delta | | -0.004 | | | | | | | |
| Vega | | 0.017 | | | | | | | |

| Asset/Liab. Correlations | | | | |
|--------------------------|---------|---------|--------|--------|
| | Line A1 | Line A2 | Line B | Line C |
| Asset 1 | 0.16468 | 0.15936 | 0.0 | 0.02 |
| Asset 2 | 0.16468 | 0.15936 | 0.0 | 0.02 |

Note that the global values, such as the asset/liability volatility and default value, are unchanged from Table 3, as is the capital allocation to Lines B and C. The new capital allocation is once again additive. The capital to liability ratios for lines A1 and A2 are very similar to each other and to the original Line A, with a slightly higher capital requirement result for the sub-line with the larger CV.

We caution that this method of subdivision will give erroneous results if the uniform correlation assumption is violated. In other words, the fact that the capital allocation for

the original segments is preserved does not imply that you have found the correct correlations for each of your subdivided lines. As an example, Table 8 shows the same subdivision of Line A into two segments, but this time Line A1 is strongly correlated with Line C, while Line A2 is negatively correlated with Line C. As in Table 7, the global values and the allocations for Lines B and C are unchanged, but the split of the original Line A's allocation between A1 and A2 is quite different.

Table 8:

| | | | Liability Correlations | | | | covariance w/liabilities | covariance w/assets | surplus/liab |
|-----------------------|------------------|---------|------------------------|----------|--------|----------|--------------------------|---------------------|--------------|
| | % of liabilities | CV | Line A1 | Line A2 | Line B | Line C | | | |
| Line A1 | 40% | 7% | 1.0 | 0.1962 | 0 | 0.8 | 0.004 | -0.001 | 0.20 |
| Line A2 | 46% | 6% | 0.1962 | 1.0 | 0 | -0.28958 | 0.001 | -0.001 | 0.09 |
| Line B | 4% | 130% | 0 | 0 | 1.0 | 0.0 | 0.068 | -0.007 | 2.81 |
| Line C | 10% | 30% | 0.9 | -0.28958 | 0.0 | 1.0 | 0.014 | -0.003 | 0.67 |
| Liabilities | 100% | 8.1% | 0.75 | 0.30 | 0.65 | 0.59 | 0.008 | -0.001 | 0.30 |
| Assets | 130% | 4.2% | -0.20 | -0.20 | -0.13 | -0.26 | | 0.002 | |
| surplus | | 30% | | | | | | | |
| asset/liab volatility | | 10.2% | | | | | | | |
| default value/liab | | 0.0184% | | | | | | | |
| Delta | | -0.004 | | | | | | | |
| Vega | | 0.017 | | | | | | | |

| Asset/Liab. Correlations | | | |
|--------------------------|----------|----------|----------|
| | Line A1 | Line A2 | Line B |
| Asset 1 | 0.15468 | 0.15468 | 0.15468 |
| Asset 2 | -0.15468 | -0.15468 | -0.15468 |

Conclusion.

We tried the MR model using fairly realistic insurance company examples. If you *have* to allocate capital, and you are comfortable with the explicit and implicit underlying mathematical assumptions, the MR model is an elegant, intuitively appealing, and tractable method of capital allocation. For this we thank the authors for their contribution.

We stop short of declaring victory in the search for the definitive capital allocation model. The MR model will be hard to parameterize for real-world application. It can yield strange answers when classes of liabilities are not homogenous, when they are highly skewed, or under certain conditions of covariance. Also, the MR model should be used to allocate economic (required) capital not statutory surplus. We further recommend separating the economic capital attributable to the underwriting function from that required for the investment function.

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