Estimation and Application of Ranges of Reasonable Estimates

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INTRODUCTION

Until about 30 years ago, the term "range of reasonable estimates" was not generally applied to the loss[1] reserving process. While reserving actuaries were often asked, usually by management, to assess the range around the reserve values, more often than not the actuary could get away with "plus or minus five percent" as a range. The question "five percent of what?" went largely unasked. The result was a general agreement that the carried reserves were within five percent of those needed so long as the five percent could be applied, as required, to unpaid losses or ultimate losses or company assets or industry assets or GDP.

In his 1973 review of David Skurnick's paper *A Survey of Loss Reserving Methods*, Robert Anker describes three ranges: the "absolute range," which is the range from the lowest indication of any method to the highest indication of any method; the "likely range," representing the range from the lowest selected value of any method to the highest selected value of any method; and the "best estimate range."[2] I believe the development of the concept that would become the "range of reasonable estimates" started with the Anker review.

[1] The term "loss" is used herein for simplicity and should be interpreted as "loss and/or loss adjustment expense"
[2] *Proceedings* of the Casualty Actuarial Society Vol. LX, p. 59

In 1988 the CAS Board adopted the *Statement of Principles Regarding Property and Casualty Loss and Loss Adjustment Expense Reserves* (Statement of Principles) which included the following two principles:

3. *The uncertainty inherent in the estimation of required provisions for unpaid losses or loss adjustment expenses implies that a range of reserves can be actuarially sound. The true value of the liability for losses or loss adjustment expenses at any accounting date can be known only when all attendant claims have been settled.*
4. *The most appropriate reserve within a range of actuarially sound estimates depends on both the relative likelihood of estimates within the range and the financial reporting context in which the reserve will be presented.*

With the adoption of the statutory *Statement of Actuarial Opinion* on loss reserves, the Committee on Property and Liability Financial Reporting of the American Academy of Actuaries promulgated the interpretation *that a reserve makes a "reasonable provision" if it is within the range of reasonable estimates of the actual outstanding loss and loss adjustment expense obligations*, where the *range of reasonable estimates is a range of estimates that would be produced by alternative sets of assumptions that the actuary judges to be reasonable, considering all information reviewed by the actuary*.[3]

In 2000, the Actuarial Standards Board adopted ASOP No. 36 – *Statements of Actuarial Opinion Regarding Property/Casualty Loss and Loss Adjustment Expense Reserves*

[3] Property and Casualty Practice Note 1994-2, p.28.

wherein “range of reasonable estimates” is described *as a range of estimates that could be produced by appropriate actuarial methods or alternative sets of assumptions that the actuary judges to be reasonable*.

This paper will discuss the concept of a range of reasonable estimates, will describe some methods for determining ranges, will demonstrate a sound basis for the aggregation of ranges from individual line of business (or other subdivision) ranges, and will recommend a basis for the application of the range to individual loss reserving decisions.

RANGE OF REASONABLE ESTIMATES

It is unfortunate that the language of actuarial loss reserving has produced “reasonable” as the primary modifier of “estimate.” Not only does it inexorably lead to the implication that all estimates outside the range of reasonable estimates are unreasonable, it also lends itself to circular definition as in ASOP No. 36, Section 3.6.4 as quoted above. The Statement of Principles language, combining *reasonable* assumptions with *appropriate* methodology to produce *actuarially sound* estimates would have been preferable.

Whatever the language, it is clear that the range arises from the uncertainty associated with the problem of estimating future loss payments and that the purpose of the range is to reflect not only the process variance but the parameter variance as well. This is clear from both the *Statement of Principles*, which seems to deal primarily with the process variance, and from the ASOP No. 36 language focusing on methods and assumptions.

While likelihood is a consideration cited in Principle 4, a sound range will not necessarily contain the most likely result. As an example, suppose that as of a reserving date an actuary estimates that there is a .01 probability of a $1 million IBNR loss on a policy, with the probability of a $0 IBNR being .99. The actuary might reasonably reserve to the expectation of $10,000 or might add some risk margin and reserve to $20,000 or $50,000. But it would not be reasonable to reserve at $0, even though it represents both the mode and the median of the loss distribution. The concept of actuarial soundness demands that we discard the answer we expect to be precisely accurate 99 times out of a hundred and adopt instead a reserve which we expect will always be wrong!

Carrying on with our example we note that the range of reasonable estimates might include neither of the only two possible outcomes. This is an important distinction. The range of reasonable estimates is not intended to include all, or perhaps even a majority of the *possible* values.

FINANCIAL CONDITION AND THE RANGE OF REASONABLE ESTIMATES

Although the Statement of Principles clearly indicates that the selected value within the range of reasonable estimates may depend upon the financial condition of the company[4] the range itself may depend upon such condition as well. Again considering our example above, in the context of a billion dollar surplus, the range might be from $0 to $20,000 with the midpoint representing the expectation and the range encompassing 99% of the

[4] Principle 4 "The most appropriate reserve within a range of actuarially sound estimates depends on both the relative likelihood of estimates within the range and the financial reporting context in which the reserve will be presented."

probability. But in the context of a million dollar surplus, that is, where the million dollar loss could render the company insolvent, $0 would probably not be a reasonable estimate for the liability. In that instance, it would be hard to contend that anything less than the expectation of $10,000 could be considered reasonable.

The fact that the bottom of the range will tend to increase with the materiality of the estimate is easily understood if we recall that the provision for uncertainty in the carried reserves should reflect not only the uncertainty of the individual reserve value, but the impact of that uncertainty upon financial condition as well. Where surplus is low, the provision for uncertainty will tend to increase and, since the range is intended to include only those values which the actuary believes would represent reasonable reserves, the range will increase as well.

METHODS FOR ESTIMATING RANGES

Assumed Allowable Deviations

As long as there have been actuarial estimates of loss reserves, there have been CEOs asking for some quantification of the accuracy of those estimates. As mentioned above, for years, actuaries were able to respond to such requests with the assurance that the reserves were "within plus or minus five percent." Regulators and the IRS also used percentage benchmarks, typically five percent of carried reserves. With this history, it is not surprising that the earliest quantifications of ranges of reasonable estimates tended to be percentages of reserves.

Unfortunately, the method does not work very well. The inherent differences between lines, for example commercial property (with high but reasonably ascertainable losses) and excess workers' compensation, require different assumed allowable percentage deviations. Calculation of the appropriate deviations by line is tantamount to calculation of the range of reasonable estimates. In addition, the requirements of the actuarial standards of practice are such that there must be a demonstrable and documented basis for a material assumption.

Alternative Methods

One common approach to the establishment of a range of reasonable estimates is for the actuary to apply multiple methods to the same line of business and to use the results to estimate the range. In applying this method, the actuary must be careful to discard any results which are inconsistent with the other indications. If the paid loss development method and the incurred loss development method are producing indications which are materially different, the difference may not constitute a range but an unexplained difference. It is also important that each method and related assumptions be individually reasonable.

The actuary using this method should also compare the results line-to-line. If the incurred loss development method always produces the low indication, it is likely that there has been a change in the underlying development – perhaps a decrease in case reserve adequacy.

Finally, this method benefits from the application of multiple and independent methods. The addition of a frequency-severity method to a set of loss development and Bornhuetter-Ferguson indications adds additional information to the process and produces a more representative range.

Alternative Assumptions

Occasionally I have seen situations in which an actuary varies the assumptions, as opposed to the methods, to establish the range. For example, the actuary will pick the highest and lowest reasonable incremental development factors at each age and use the highest to generate the high end of the range and the lowest to generate the low end. This approach produces ranges which are too wide. The probability that each age-to-age development for each accident year will be at the low end of the observed history is too low to make the resultant indication reasonable. There is, however, a method which the actuary can use to vary the assumptions and produce information which is useful in the establishment of a range of reasonable estimates. This method is the *method of convolutions*.

Method of Convolutions

The general availability of powerful computing resources has made it possible to apply techniques which would have been impossibly time-consuming in earlier times. The method of convolutions as applied to the loss development methodology is simply the application of each combination of the observed age-to-age factors to the current data and then the combination of the resultant individual year indications to produce a large number of indications arising out of the observed history.[5]

As an example, consider the hypothetical incurred loss development data in Table 1:

Table 1

Accident	Case Incurred by Age				
Year	12	24	36	48	60
1998	$1,503,839	$2,490,404	$4,266,948	$6,144,355	$6,266,584
1999	1,535,773	3,028,897	4,874,340	7,348,570	
2000	1,989,915	3,574,304	5,790,811		
2001	1,660,687	3,031,952			
2002	2,224,336				

Which give rise to the development factors in Table 2:

Table 2

Accident	Incremental Development Factors			
Year	12-24	24-36	36-48	48-60
1998	1.656	1.713	1.440	1.020
1999	1.972	1.609	1.508	
2000	1.796	1.620		
2001	1.826			

For purposes of illustration, we assume that all claims are settled by age 60. Our observed history then gives rise to 4! or 24 different combinations of development factors for the 2001 year, 3! or 6 combinations for the 2000 year, 2 for the 1999 year and 1 for the 1998 year. Combining these indications produces 24 x 6 x 2 x 1 = 288 convolutions which can be sorted into a surrogate cumulative aggregate IBNR distribution.

[5] To the best of my knowledge the first documented construction of a convolution distribution of reserve outcomes was carried out by C. K. Stan Khury *circa* 1992.

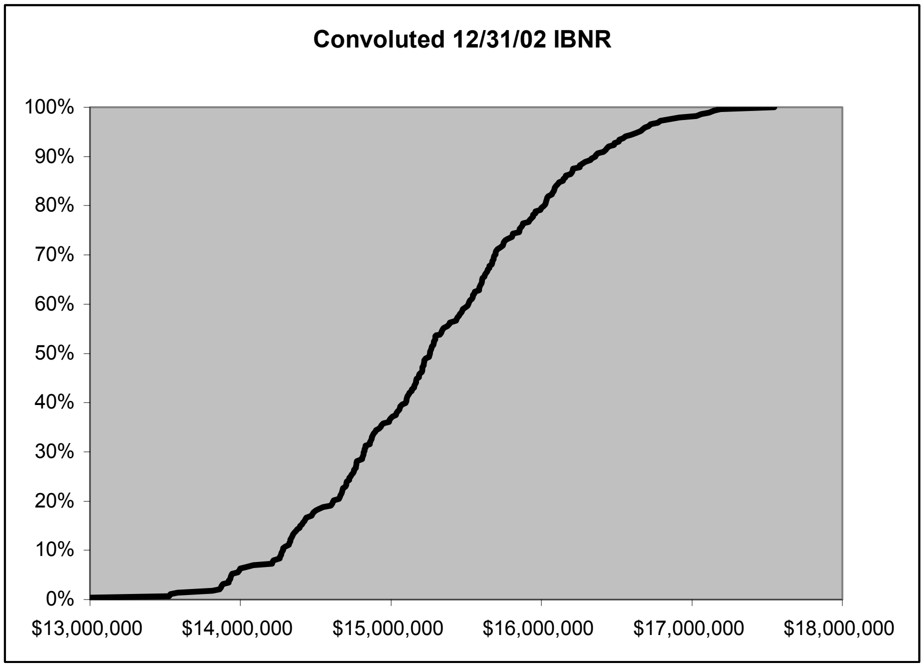

Figure 1

Continuing with our example, let's assume that we have established our "best estimate" IBNR using the unweighted average of the observed factors as shown in Table 3:

Table 3

Accident Year	Incurred Losses	Average Factor	Ultimate Factor	Indicated IBNR
1998	$6,266,584	1.000	1.000	$0
1999	7,348,570	1.020	1.020	146,971
2000	5,790,811	1.474	1.503	2,912,778
2001	3,031,952	1.648	2.477	4,478,193
2002	2,224,336	1.813	4.491	7,765,157
Total				$15,303,099

Plotting this estimate against our convolutions we see that it falls at approximately the 54th percentile, about what we would expect for a lognormal distribution.[6]

[6] The distribution of the product of independent normally-distributed random variables is lognormal.

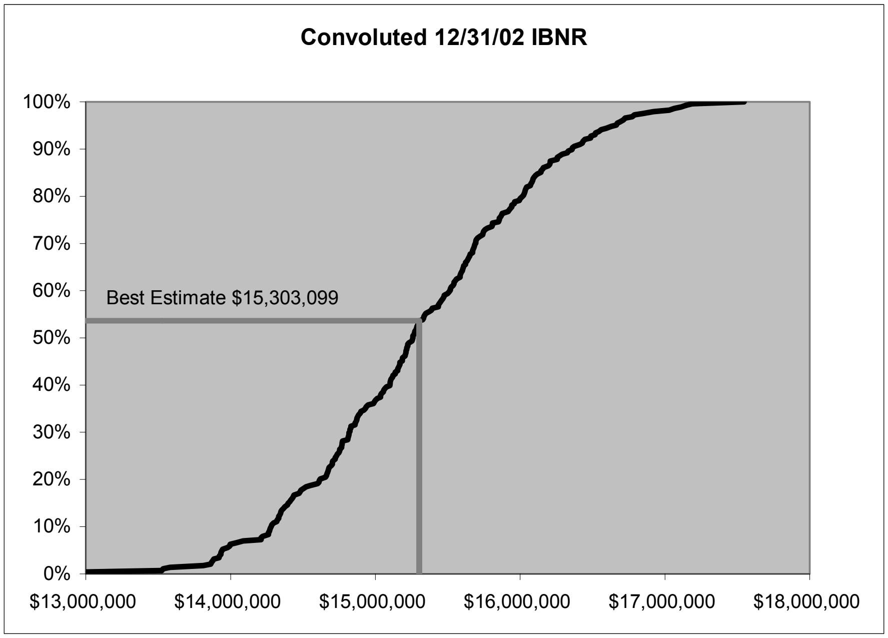

Figure 2

In a similar manner we can plot the extent of whatever we may consider a reasonable range. In this case, with the small triangle, we might select an 80% range from 10% to 90% as follows:

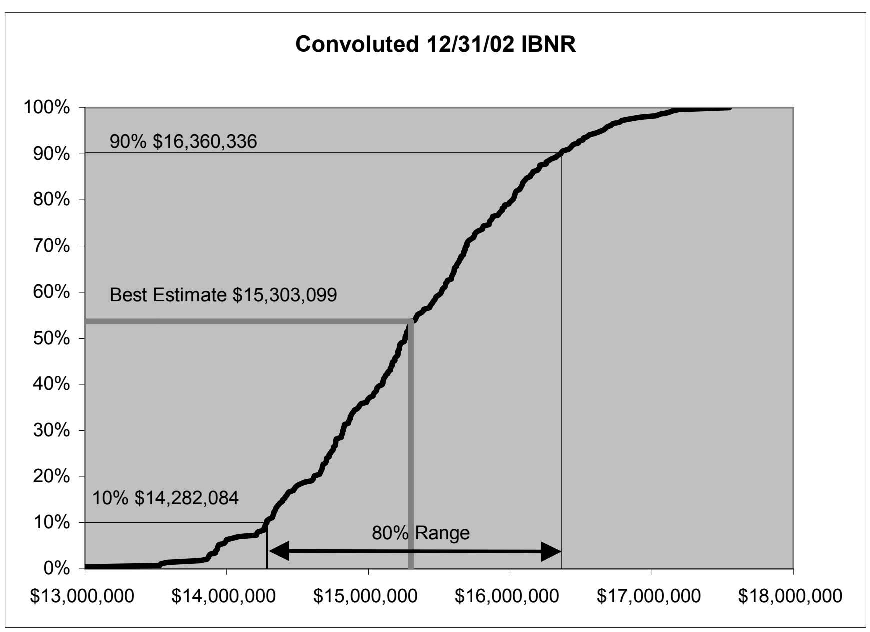

Figure 3

It is preferable that the method of convolutions be applied to several methods, not just a single method as in the above example. The convolutions from multiple methods can be combined into a single distribution of estimates.

The number of convolutions tends to get out of hand quickly. The number of individual estimates for a k-by-k development factor triangle is $\prod_{1}^{k} k!$ which is a manageable 288 for the 4-by-4 triangle of our example, but becomes 5,056,584,744,960,000 for an 8-by-8 triangle. Even the fastest of personal computers can take a while to calculate 5 quadrillion values. In such cases, it sacrifices little to limit each convolution to the youngest four by four triangle, with the 5^{th} value to ultimate being assumed as the product of the average observed 5^{th} and subsequent incremental factors.

<table>
<tr><th rowspan="2">Year</th><th colspan="8">Age</th></tr>
<tr><th>1-2</th><th>2-3</th><th>3-4</th><th>4-5</th><th>5-6</th><th>6-7</th><th>7-8</th><th>8-Ult</th></tr>
<tr><td>1</td><td></td><td></td><td></td><td></td><td>Factor</td><td>Factor</td><td>Factor</td><td>Factor</td></tr>
<tr><td>2</td><td></td><td></td><td></td><td>Factor</td><td>Factor</td><td>Factor</td><td>Factor</td><td>CNV 1x1</td></tr>
<tr><td>3</td><td></td><td></td><td>Factor</td><td>Factor</td><td>Factor</td><td>Factor</td><td colspan="2">CONVOLUTED 2x2</td></tr>
<tr><td>4</td><td></td><td>Factor</td><td>Factor</td><td>Factor</td><td>Factor</td><td colspan="3">CONVOLUTED 3x3</td></tr>
<tr><td>5</td><td>Factor</td><td>Factor</td><td>Factor</td><td>Factor</td><td colspan="4">CONVOLUTED 4x4</td></tr>
<tr><td>6</td><td>Factor</td><td>Factor</td><td>Factor</td><td colspan="4">CONVOLUTED 4x4</td><td>Avg. 8-Ult</td></tr>
<tr><td>7</td><td>Factor</td><td>Factor</td><td colspan="4">CONVOLUTED 4x4</td><td colspan="2">Avg. 7-Ult</td></tr>
<tr><td>8</td><td>Factor</td><td colspan="4">CONVOLUTED 4x4</td><td colspan="3">Avg. 6-Ult.</td></tr>
<tr><td>9</td><td colspan="4">CONVOLUTED 4x4</td><td colspan="4">Avg. 5-Ult</td></tr>
</table>

This cuts the number of convolutions for the 8-by-8 triangle to $1!\times2!\times3!\times4!\times4!\times4!\times4!\times4! = 95{,}551{,}488$.

AGGREGATION OF RANGES

The combination of individual line of business or line and year ranges into an actuarially sound aggregate range of reasonable estimates requires some consideration. Recall that the range is of estimates, not possibilities, and only *reasonable* estimates are included within the range, the lows and highs of the individual years cannot be added to generate the range for the line and the lows and highs of the ranges for the individual lines cannot be added to generate the range for the aggregate reserve. The individual lows and highs represent the extremes of the actuary's reasonable estimates and while the low or high might be reasonable for a single year within a single line, it would not be reasonable to reserve to the sum of the lows or the sum of the highs.

If we posit a situation where we have four lines being reserved and four open accident years within each line and we assume that for each year within each line the proper reserve is either the low or the high, each with 50% probability, the chance that either the sum of the lows or the sum of the highs will be the proper reserve is $.5^{16}$ or 0.001526%. The actual distributions of estimates are such that the probabilities of the lows or highs being the proper reserves are well below 50%.

Viewed differently, suppose we were asked to estimate the range of reasonable estimates for the number of heads in the toss of ten true coins. We might select from 3 to 7 heads as our range knowing (or at least being able to determine) that we would expect this result about 89% of the time. But in a toss of 100 true coins the range from 30 to 70 heads would not be the range of reasonable estimates, comprising as it does 99.99678%

of the expected results. The range most consistent with the individual ranges of 3 to 7 would be from 42 to 58 heads out of the toss of 100 true coins which results in an expectation of about 91%. Note that adding the individual highs and lows for the years and or lines of business is the equivalent of adopting the 30 to 70 range.

How then can we combine the individual line and/or year ranges into a reasonable aggregate range? If we make the assumption that individual estimates are independent the solution is straightforward. Knowing that if x and y are independent random variables with variances $V(x)$ and $V(y)$ that $V(x+y) = V(x) + V(y)$[7] and assuming that our individual ranges represent k standard deviations (of the individual distributions of estimates) in width, then the width of the aggregate range is the square root of the sum of the squares of the widths of the individual estimates. The placement of the aggregate best estimate, the sum of the individual best estimates, is then determined by weighting the position within the range of each best estimate by the ratio of that best estimate to the total of the best estimates. An example of this process is shown in Table 4:

[7] See, for example Brunk, H.D. *An Introduction to Mathematical Statistics*, Blaisdall, 1965, p.91

Table 4

Line [1]	Accident Year [2]	Total Needed Reserves ($000) Low [3]	Best Estimate [4]	High [5]	Range Width [5]-[3] [6]	Square of Width $[6]^2$ [7]	Calculated Width √[7] [8]
Auto BI	1999	$450	$500	$600	$150	22,500	
	2000	2,700	3,000	3,500	800	640,000	
	2001	6,000	7,000	7,500	1,500	2,250,000	
	2002	9,000	11,000	14,000	5,000	25,000,000	
	Total		$21,500			27,912,500	$5,283
Auto PD	1999	$90	$100	$115	$25	625	
	2000	1,400	1,500	1,650	250	62,500	
	2001	2,800	3,000	3,300	500	250,000	
	2002	6,800	7,500	8,400	1,600	2,560,000	
	Total		$12,100			2,873,125	$1,695
Total	Total		$33,600			30,785,625	$5,548

Line [1]	Accident Year [2]	Best Estimate Weight [4]/Sum[4] [9]	Best Est. Position in Range {[4]-[3]}/{[5]-[3]} [10]	Weighted Position in Range [9]×[10] [11]	Calculated Position in Range [11]/[9] [12]	Calculated Low [4]-{[12]×[8]} [13]	Calculated High [8]+[13] [14]
Auto BI	1999	1.488%	0.3333	0.004960			
	2000	8.929%	0.3750	0.033482			
	2001	20.833%	0.6667	0.138889			
	2002	32.738%	0.4000	0.130952			
	Total	63.988%		0.308284	0.481783	$18,955	$24,238
Auto PD	1999	0.298%	0.4000	0.001190			
	2000	4.464%	0.4000	0.017857			
	2001	8.929%	0.4000	0.035714			
	2002	22.321%	0.4375	0.097656			
	Total	36.012%		0.152418	0.423244	$11,383	$13,078
Total	Total	100.000%		0.460702	0.460702	$31,044	$36,592

In our simple example in Table 4, the total range of reasonable estimates around the aggregate best estimate of $33,600 is from $31,044 to $36,592.

We know that the individual estimates are not strictly independent. Where traditional loss development methodology is used, incremental development assumptions affect multiple years producing some correlation between estimated ultimate losses for years. Court decisions, regulatory climate and economic conditions impact ultimate losses for multiple lines. For the most part, however, these are outweighed by the independent stochastic nature of the observed frequencies and severities which form the basis for the projections and the fact that it is the unpaid losses which are the subject of the range. Given the computational difficulties introduced in any attempt to measure and reflect the covariance matrix in the aggregation of the ranges, the assumption of independence seems a reasonable approach.

APPLICATION OF RANGES

Having established a basis for the determination of actuarially sound ranges of reasonable estimates, it is natural to turn to the application of those estimates. In order to examine the question of how to apply the concept of a range of reasonable estimates it is important to understand that while each reserve value within the range is presumed to be reasonable, that is meets the ASOP requirement that it *could be produced by appropriate actuarial methods or alternative sets of assumptions that the actuary judges to be reasonable*, not all values within the range are qualitatively equal. The low and high values represent the demarcation between presumably sound and presumably unsound reserves, and the actuary establishing a reserve should adhere to the requirements of the *Statement of Principles* and should consider *both the relative likelihood of estimates within the range and the financial reporting context in which the reserve will be presented*.

The proper application of the range will depend to a great extent upon the “ownership” of the estimate. If the actuary is opining upon the reasonableness of a carried reserve which the company has already established without knowing the results of the opining actuary’s analysis, and that reserve is within the opining actuary's range of reasonable estimates it is deemed reasonable. We refer to such a reserve as being "untutored."

If, however, the company knows the results of the opining actuary's analysis before establishing the reserve and then selects a reserve at the low end of the opining actuary's range, the company no longer "owns" the estimate. In such a case, if the reserve is not one which the opining actuary would have established in accordance with the *Statement of Principles*, it does not represent a reasonable reserve. To allow the low end of the range to serve as a target reserve is to subjugate the opining actuary's best estimate to his or her view of what reserve might be established by a hypothetical actuary using different methods and applying different assumptions.