Probabilistic Framework for Evaluating Materiality and Variability in Loss Reserve Estimates

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I. Abstract

A reserve[1] point estimate is usually presented without explicit quantitative reference to the variability associated with it. The literature provides little guidance on how to go about providing such quantitative representations. In this paper the authors present a new function, called the *coefficient of estimation*, as a measure of the placement of a reserve point estimate on the continuum of reserve estimates defined by the underlying aggregate loss distribution. The authors further use this coefficient to discuss six commonly used reserving terms to illustrate how the variability inherent in a point reserve estimate may be quantified. The authors also illustrate these ideas with six different demonstrations for each of two lines of business, including tables and charts depicting underlying aggregate loss distributions. The authors conclude the paper with a series of observations to amplify some of the salient issues as well as set some boundaries for the usefulness of the proposed coefficient of estimation.

1 In this paper "reserve" refers only to a loss or loss adjustment expense reserve.

II. Background

Currently available guidance for the actuary who is analyzing loss and loss adjustment expense reserves provides references to various elements of reserves that suggest a stochastic approach to reserving. Yet a number of these terms are left undefined or are not defined in a manner that suggests probabilistic quantification. For example, Principle 3 of the Statement of Principles Regarding Property and Casualty Loss and Loss Adjustment Expense Reserves (Reserving Principles) states:

> *The uncertainty inherent in the estimation of required provisions for unpaid losses or loss adjustment expenses implies that a range of reserves can be actuarially sound.*

Principle 4 states:

> *The most appropriate reserve within a range of actuarially sound estimates depends on both the relative likelihood of estimates within the range and the financial reporting context in which the reserve will be presented.*

Other references can be found in Actuarial Standard of Practice No. 36 (ASOP 36) *Statements of Actuarial Opinion Regarding Property/Casualty Loss and Loss Adjustment Expense Reserves*. Section 3.6.5 states:

> *The potential variation in the actual amount that will be needed to pay unpaid claims gives rise to uncertainty in the reserve estimates. An adverse deviation occurs when such a variation results in paid amounts higher than provided for in the reserves.*

Section 2.6 of ASOP 36 also provides a definition of *expected value estimate*:

> *An estimate of the mean value of an unknown quantity where the mean value represents a probability-weighted average of the quantity over the range of all possible values.*

The December 2001 American Academy of Actuaries Property and Casualty Practice Note discusses *materiality* as follows:

Requiring the use of professional judgment and placing importance on intended purpose both emphasize the role of qualitative considerations in evaluating materiality. Actuaries will naturally also focus on quantitative considerations related to judgments on materiality. No formula can be developed that will substitute for professional judgment by providing a materiality level for each situation.

The above citations are but a few that illustrate the fact that important elements of reserves that need to be addressed by the actuary are presented in a manner that suggests their stochastic nature, yet fall short of providing the tools to define the elements of the reserves reflecting their probabilistic nature.

It is clear that commonly used actuarial terms such as *best estimate, range of reasonableness, confidence interval, provision for uncertainty (risk margin), reasonableness,* and *materiality* can be found throughout the actuarial principles, standards, and other literature. And it is also clear that these terms have a stochastic element to them although none of the above referenced documents includes any suggestion as to how the stochastic element may be indicated or quantified.

The lack of rigorous definition of these terms has, in the authors' opinion, led to the use of numerous caveats in actuarial work products. These caveats offer the reader little insight into the actual degree of confidence the actuary places in the estimate. For example, a typical caveat in an actuarial report that contains reserve estimates states: "The ultimate value of the liability for future development, when all losses are reported and settled, may vary, perhaps significantly, from the estimates in this report." The reader of this caveat, although duly warned that there will be variation in the actual results from that which was estimated by the actuary, has little understanding of the amount of variability present in the estimates. Such is the state of the art today.

In this paper, the authors propose to discuss some of the more commonly used terms in a way that associates a probability statement with each. It is our hope that these concepts, if used by actuaries in estimating and communicating reserve estimates, will lead to a greater understanding of the variability associated with loss reserve estimates.

III. Foundational Framework: Aggregate Loss Distributions

When a reserve point estimate is put forth, there is always the implicit understanding that a specific estimate is but one of a number of plausible alternative reserve estimates, each of which is actuarially sound.[2] Taken a step further, it is also implicit that there is an underlying distribution of reserve estimates that contains the set of all such reserve estimates along with their associated probabilities. A major premise of this paper is that until and unless the actuary identifies and makes use of such distributions, it is not possible to communicate meaningfully and completely about reserve estimates, their expected degree of adequacy, and their inherent variability.

Although the construction of such distributions is beyond the scope of this paper, we will discuss the subject very briefly in order to complete the foundational work for this paper. Generally speaking, such distributions exist in one of two ways: as *assumed* distributions (expressed in closed form, using parameters suggested by the raw data utilized in deriving a reserve estimate) and as *empirical* (or deterministic) distributions.

Assumed distributions require the actuary to select the type of distribution function (i.e., the "shape" of the distribution) as well as its parameters. There is currently substantial literature in this area and thus the subject requires little discussion beyond acknowledging the availability of such distributions.[3]

Empirical distributions, on the other hand, are those that arise naturally from considering all the available data and compiling all possible outcomes contemplated by various actuarial methodologies. One example is the set of all possible outcomes produced by

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[2] "The uncertainty inherent in the estimation of required provisions for unpaid losses or loss adjustment expenses implies that a range of reserves can be actuarially sound." Lines 121-122 of *Statement of Principles Regarding Property And Casualty Loss And Loss Adjustment Expense Reserves* as adopted by the Casualty Actuarial Society in May 1988.

[3] See, for example: Heckman & Myers, PCAS Vol. LXX (1983); Hewitt & Lefkowitz, PCAS Vol. LXVI (1979); Klugman & Rahykha, PCAS Vol. LXXX (1993); and Hayne, PCAS Vol. LXXVI (1989).

every possible combination of link ratios calculated from an incurred loss development array. This calculation, when carried out completely, delivers a specific distribution of outcomes of ultimate losses, along with associated frequencies of occurrence, from which specific probabilities of adequacy can be derived and associated with various reserve estimates.[4]

For our purposes, we assume that at time **t** the actuary has constructed—for a well-defined cohort of claims—a relevant distribution of outcomes, either assumed or empirical, from which a cumulative frequency distribution of outcomes is constructed. In turn, the cumulative frequency distribution of outcomes may be used to associate a probability that the final value, when it becomes known, when all claims are finally settled, will be less than or equal to the proposed reserve estimate.

This distribution, however constructed at time **t**, in turn yields a mean reserve value, $\mathfrak{R}$, a median, **M**, along with a standard deviation, σ. And since $\mathfrak{R}$, **M**, and σ are all identified at time **t**, we will designate them as $\mathfrak{R}_t$, $\mathbf{M}_t$, and σ_t, respectively. The entire discussion of commonly used actuarial terms builds on these values. Also, we will make use of some familiar probability notations as follows:

$\mathbf{P_t(X)}$: The probability at time **t** that the ultimate value will fall in the interval $(-\infty, \mathbf{X})$.

$\mathbf{P_t(A,B)}$: The probability at time **t** that the ultimate value will fall in the interval **(A,B)**. Note that $\mathbf{P_t(A,B) = P_t(B) - P_t(A)}$.

IV. The Basic Idea: The Coefficient of Estimation

The basic idea advanced in this paper is that a reserve estimate, in order to be meaningfully and completely presented, needs to be associated with a statement that

[4] The time needed to complete such constructions may render such distributions impossible to produce. However, various approximation algorithms can be useful in compressing the problem to the point where the construction of such distributions is perfectly possible.

gives the user an idea of the probability of adequacy of the proposed estimate. We call that proposed probability statement the *coefficient of estimation* (denoted by **CE**) that is associated with the reserve estimate.

The coefficient of estimation, **CE** at time **t**, denoted by $\mathbf{CE_t}$, of a point estimate of reserves, also calculated at time **t**, denoted by $\mathbf{X_t}$, is defined by:

$$\mathbf{CE_t(X_t) = 100(P_t(X_t) / P_t(\mathfrak{R}_t))}$$

Some immediately obvious properties of **CE**:

1. The domain of **CE** is $(-\infty, +\infty)$.

2. The range of **CE** is $[0, 100/ P_t(\mathfrak{R}_t)]$.

3. $\mathbf{CE_t(\mathfrak{R}_t) = 100}$.

4. **Limit** $\mathbf{CE_{\mathfrak{R}\to M,\, X\to\infty}(X) = 200}$. Note that the limit of **200** may be approached from above or below.

5. $\mathbf{CE_t(X_t) = \alpha(P_t(X_t))}$, where α is a scalar given by $\mathbf{100/P_t(\mathfrak{R}_t)}$. Thus the shape of the graph of **CE** is identical to the shape of the graph of the underlying cumulative frequency distribution of outcomes. This phenomenon is illustrated in the charts described in Section VI.

6. For the great majority of distributions of aggregate insurance losses, because they are usually skewed to the left, one can expect $\mathfrak{R}$ to be greater than **M**. One can expect this relationship to hold in all but some very special situations where one is dealing with a great predominance of small claims with only occasional claims of significant values.

7. The **CE** function has an associated inverse function, which is denoted by $\mathbf{CE}^{-1}$. In other words, given a coefficient of estimation **a** one can identify a unique estimate **e** such that $\mathbf{CE}^{-1}(\mathbf{e}) = \mathbf{a}$.

We now propose that a point estimate of reserves, **R**, derived at time **t**, denoted by $\mathbf{R_t}$, be presented along with its associated coefficient of estimation, $\mathbf{CE_t(R_t)}$. Thus the full reserve estimate statement would appear as $\mathbf{(R_t, CE_t(R_t))}$.

Because reserve estimates are subject to uncertainty, it follows that one would want to allow leeway with respect to values of $\mathbf{CE_t}$ that are near $\mathbf{CE_t(R_t)}$. In other words one would like to find a way to accord values that are in the neighborhood of $\mathbf{R_t}$ substantially the same meaning as $\mathbf{R_t}$. This idea in turn gives rise to a number of related concepts, including but not limited to: *range of reasonableness, confidence intervals, best estimate, materiality, reserve strengthening, and risk margin*. Given the basic framework suggested by $\mathbf{(R_t,CE_t(R_t))}$, we can proceed to make the extensions such that each term is framed probabilistically, and more specifically, in terms of $\mathbf{CE_t}$.

V. Discussion of Selected Reserving Terminology

In the remainder of this paper we discuss six commonly used terms placing each of them in a probabilistic context using the foundational framework set forth above. In Section VI all of these terms are illustrated with live examples.

A. **Best Estimate**. This term, essentially undefined in the literature, is often used to label the actuary's final selection of a point estimate of reserves. A little reflection suggests a number of possible meanings:

   1. It is the actuary's selected point estimate, in a literal sense, given all the quantitative and qualitative information as well as relying on his education, experience, and judgment.

2. It is the actuary's selected point estimate, among several options of plausible outcomes.

3. It is the mean value of reasonable estimates derived by actuary, each of which is equally plausible.

4. It is the weighted mean value of reasonable estimates derived by the actuary, thus recognizing the respective likelihood of each of a number of plausible outcomes.

5. It is the mean of an underlying distribution of outcomes.

And there are others. While we do not propose to suggest than any of these meanings is the proper one, because different actuaries are likely to use the term to mean different things, we do propose that the actuary, in addition to using the term *best estimate*, attach to it the associated coefficient of estimation. Thus a point estimate, when described as a best estimate will have two dimensions to it: one is the proposed "standard" meaning implied by its coefficient of estimation (and therefore having to identify and use the underlying and implied distribution of outcomes) and the other is the colloquial meaning that the actuary intends.

This convention is a special case of the general proposition advanced in this paper, that a reserve estimate be set forth as a pair of values, $(\mathbf{R_t}, \mathbf{CE_t}(\mathbf{R_t}))$. Note that, if the actuary chose not to use the term *best estimate*, the general convention would also have relevance—as the point estimate is simply the actuary's selection given the data and the circumstances at time $\mathbf{t}$.

A subtle implication of the $(\mathbf{R_t}, \mathbf{CE_t}(\mathbf{R_t}))$ convention is the implicit requirement that is placed on the actuary in the event the selected reserve has a coefficient of estimation that is significantly different from **100**. The actuary would have to make the case, much more directly than heretofore required, as to why his estimate should be so different from the indicated reserve $\mathfrak{R}$. This duty applies regardless of whether the coefficient of estimation is much higher or much lower than **100**. One collateral issue in this discussion is the level at which the difference between the coefficient of estimation and **100** is significant. It is not possible to set hard and fast rules for such standards. However, the use of some proportion of the standard deviation might be useful. For example, one may use the following coefficients of estimation to establish the standard of significance for further investigation: $\mathbf{CE_t(\mathfrak{R}_t - 0.5\,\sigma_t)}$ and $\mathbf{CE_t(\mathfrak{R}_t + 0.5\,\sigma_t)}$. And in any event, whatever the actuary uses as the standard for significance, it should be disclosed so that there is no mystery as to what is operating.

Similarly, we need to point out that the mere fact of a reserve estimate having a coefficient of estimation equal to **100** is not, by itself, dispositive. The actuary still has the professional duty to identify for himself, and certainly include such demonstration in his work papers, the rationale for the selection of the underlying frequency distribution of outcomes.

B. **Range of Reasonableness**. A use of this concept is generally on the order of a *range of reasonableness* being defined to be within ± 5% (or some other increment) of the selected reserve estimate". The immediate problem with such statements is that the invoked degree of tolerance means different things depending on the shape of the distribution of outcomes. This concept, in reality, does little more than introduce a bit of speculation in the communication process as no real information is imparted to the user as to the underlying variability of the reserve estimate.

A way to eliminate the problem associated with the use of this terminology is to actually identify the coefficient of estimation associated with the endpoints of the specified degree of tolerance. In symbols, suppose that one is advancing an estimate, $\mathbf{R_t}$, then, given a tolerance amount δ_t, an amount that may be defined absolutely or as a proportion of $\mathbf{R_t}$, one can calculate the following coefficients of estimation: $\mathbf{CE_t(R_t - \delta_t)}$ and $\mathbf{CE_t(R_t + \delta_t)}$.

When one states that the ultimate value is within δ_t of the reserve estimate $\mathbf{R_t}$, one can immediately append a coefficient of estimation to this statement, using the underlying distribution. In this manner, the user can have a sense of the significance of the indicated tolerance δ_t. For example, for a personal line of business, such as private passenger automobile liability, where the loss distribution can be expected to be compact and to exhibit a high degree of central tendency, a tolerance of δ_t, yielding a change in the coefficient of estimation, say $\mathbf{\Delta CE}$, would almost certainly yield a smaller change in the coefficient of estimation if the line of business is commercial auto liability. This is because the commercial auto liability loss distribution can be expected to be much flatter (*i.e.*, inherently more disperse, more skewed) than the private passenger automobile liability distribution.[5] The user can now view the meaning of a reserve coupled with a statement of tolerance through the prism of the associated coefficients of estimation.

We should also note that although this discussion is couched in terms of the interval $\mathbf{(R_t\text{-}\delta_t,R_t\text{+}\delta_t)}$, in reality the real concern is with the degree of adequacy of the two endpoints – and therefore the coefficients of estimation for the two endpoints. For example, the regulatory authorities can be expected to be more concerned with $\mathbf{CE_t(R_t\text{-}\delta_t)}$. On the other hand, the IRS can be expected to be more concerned with $\mathbf{CE_t(R_t\text{+}\delta_t)}$.

This discussion would not be complete if the possibility of the converse of this proposition were not considered. If the degree of tolerance is stated as a probabilistic tolerance – that is a tolerance in the coefficient of estimation, π_t, around $\mathbf{CE_t(R_t)}$, then, once again using the underlying distribution, one can calculate the absolute amount of the range that corresponds to the suggested tolerance in the coefficient of estimation, π_t. In other words, one can identify an amount δ_t' such that:

[5] This phenomenon is illustrated in the Section VI.

$$CE_t(R_t+\delta_t') - CE_t(R_t-\delta_t') = 2\pi_t$$

The idea of some voluntarily[6] identified boundaries, or, as more commonly known, a range of reasonableness, whether set in absolute terms or in probabilistic terms, is immediately placed in context such that a user may be able to appreciate in concrete terms the significance of the suggested range of reasonableness.

In concluding this section, it should be noted that no quantitative definition of the *range of reasonableness* is provided. After the preceding discussion, it is obvious that no such definition is possible. What we created is the framework in which such language may be used meaningfully. In other words, given a numerical tolerance in the reserve estimate, one is able to produce a corresponding probabilistic statement. All three items (the reserve estimate, the numerical tolerance, plus the associated coefficient of estimation associated with the numerical tolerance) are required elements that need to be present in order to be fully credible in the use of the *range of reasonableness.*

Until such time as the Actuarial Standards Board (ASB) may adopt a uniform benchmark for what constitutes a *range of reasonableness*, the concept is destined to remain a function of the training, experience, and judgment of the individual actuary as well as the facts and circumstances of the case under consideration. In other words no two actuaries need adopt the same standard in order for this concept to operate. However, what we have done in this paper is to identify the three elements of the statement that need to be present in order to be able to view consistently various statements about the *range of reasonableness*.

[6] The term "voluntarily" is used to indicate that it is a choice of the presenting actuary to employ such boundaries. It is not required *per se* by any principle or standard. However, what we are suggesting in this paper is that if the actuary chooses to go down this voluntary path, then he has the obligation to follow through with a complete presentation of these boundaries and their probabilistic significance.

C. **Confidence Interval**. The idea of a *confidence interval* is an extension (or a generalization) of the concept of *range of reasonableness*. In the preceding section we identified the three elements necessary in order to be able to use the language of a *range of reasonableness*. Thus in the affirmative case where a reserve estimate, $\mathbf{R_t}$, is advanced, a numerical tolerance, δ_t, is selected, one can identify the coefficient of estimation of the resultant endpoints, given by $\mathbf{CE_t(R_t\text{-}\delta_t)}$ and $\mathbf{CE_t(R_t+\delta_t)}$. The confidence interval concept is identical in all respects except that the connection to $\mathbf{R_t}$ is eliminated. In other words, the confidence interval can refer to *any* interval. Thus given any two numerical values, at a time **t**, such as $\mathbf{A_t}$ and $\mathbf{B_t}$, one is able to calculate the coefficient of estimation for each of the endpoints of the interval $\mathbf{(A_t,B_t)}$, based on the underlying distribution, yielding $\mathbf{CE_t(A_t)}$ and $\mathbf{CE_t(B_t)}$. In other words, the range of outcomes implied by $\mathbf{A_t}$ and $\mathbf{B_t}$ is now associated with the respective coefficients of estimation and thus yielding valuable insight as to the significance of the interval $\mathbf{(A_t,B_t)}$.

Note that, as in the case of the range of reasonableness, the converse of this proposition is also possible. Given two coefficients of estimation, one can calculate the corresponding interval with endpoints having the given coefficients of estimation.

D. **Materiality**. As noted earlier, the December 2001 American Academy of Actuaries Property and Casualty Practice Note discusses *materiality* as follows:

> *Requiring the use of professional judgment and placing importance on intended purpose both emphasize the role of qualitative considerations in evaluating materiality. Actuaries will naturally also focus on quantitative considerations related* to judgments on materiality. *No formula can be developed that will substitute for professional judgment by providing a materiality level for each situation.*

While this statement is reasonable in that it leaves the determination of materiality to the actuary, there is no guidance as to the elements that need to be present in order to make a coherent statement about materiality. We propose to fill this gap in the following paragraphs.

First, the idea of materiality is a comparative concept. That is, the difference between *two* quantities is the object of materiality discussions. For example, given a reserve estimate, $\mathbf{R}_t$, then an alternate reserve estimate, $\mathbf{R}'_t$, is materially different from $\mathbf{R}_t$ if and only if the difference between the two estimates is more than a specified benchmark.

Second, materiality has to be set against some benchmark. The practice note does not provide guidance on this point. While this is fine, as the selection of the benchmark is left to the judgment of the actuary, the suggestion advanced in this paper is that such a benchmark needs to be disclosed as part of the actuary's statement on materiality – along with the rationale for such selection. The practice note affords the actuary great latitude, both qualitatively and quantitatively, in selecting such a standard. Alternatively, a benchmark may be stated in terms of probabilistic increments – pertaining to the coefficient of estimation. In other words, a benchmark may be stated as the maximum difference in the coefficients of estimation of the two amounts being compared.

One way to illustrate how these concepts can be pulled together is to recognize that we have two immediate *a priori* amounts to be compared: $\mathbf{R}_t$ and $\mathbf{R}'_t$, and then we also note that we have σ_t to form the foundation of an objective benchmark. For example, the actuary can set his benchmark as the difference between $\mathbf{CE}_t(\mathfrak{R}_t + 0.5\sigma_t)$ and $\mathbf{CE}_t(\mathfrak{R}_t)$. Thus, within this framework, the difference between two estimates would be material if and only if:

$$|\mathbf{CE}_t(\mathbf{R}_t)-\mathbf{CE}_t(\mathbf{R}'_t)| > (\mathbf{CE}_t(\mathfrak{R}_t + 0.5\sigma_t) - \mathbf{CE}_t(\mathfrak{R}_t)).$$

We also need to point out that one need not go to such lengths as to calculate complicated standards such as $(\mathbf{CE}_t(\mathfrak{R}_t + 0.5\sigma_t) - \mathbf{CE}_t(\mathfrak{R}_t))$. Any other standard that is appropriate, in the judgment of the actuary, may be used provided the actuary identifies the rationale for such selection.

Another interesting possibility for identifying the standard of materiality is to set it as a function of the company's surplus – say some fraction, β, of surplus, **S**, denoted by β**S**. In that case the corresponding probabilistic condition for materiality would be set as: $|\mathbf{CE_t(R_t)\text{-}CE_t(R'_t)}| > (\mathbf{CE_t(\mathfrak{R}_t + \beta S) - CE_t(\mathfrak{R}_t)})$.

Yet another way that the materiality standard may be set is in terms of solvency standards. That is, selecting an increment that maintains the company's quantitative elements of solvency as may be set in the IRIS tests (such as maintaining a maximum premium-to-surplus ratio). Note that in the examples advanced here the full latitude afforded the actuary by the practice note is fully preserved. What these ideas advance is the manner in which the actuary may state his judgment as to materiality using the underlying loss distribution.

E. **Provision for uncertainty (risk margin).** ASOP 36 defines *risk margin* as: *An amount that recognizes uncertainty; also known as a provision for uncertainty.* Note that this definition provides a very wide berth for the actuary to set any risk margin he deems appropriate. Once again, while this is fine as far as it goes, in this paper we break down this statement such that the actuary is still free to set his own standard for the appropriateness of a specific risk margin, yet is able to produce a coherent statement of the meaning and basis for his selection.

First, given that the risk margin is an *amount*, we begin by searching for the types of bases that may be used to arrive at such an amount – which we may designate as the risk margin. The most obvious and natural benchmark to examine is a measure of dispersion of the underlying loss distribution. One measure of dispersion we have identified in this discussion is σ_t. Another element of establishing a basis for a *risk margin* is the size of the surplus of the company – in that any risk margin that is built into $\mathbf{R_t}$ is an amount that serves to directly reduce the otherwise available surplus. And this observation notes the obvious linkage between the size of the surplus (either on a pre- or post-risk margin basis) and the adequacy of reserves (including any risk margins that may be used). These are complicated relationships and any light one can shed on the issue in communicating them to the user has to be helpful.

The concept of a risk margin is similar to the idea of converting *materiality* into a probabilistic statement. Thus, when an actuary adds a risk margin, in fact he is increasing the probability of adequacy of his otherwise applicable estimate. Using our adopted notation, if an indicated reserve $\underline{\mathbf{R}}_t$ (set before any risk margin is added) is increased by some risk margin, $\Delta\underline{\mathbf{R}}_t$, then we can identify a change in the coefficients of estimation of the two alternative estimates: $|\mathbf{CE}_t(\underline{\mathbf{R}}_t+\Delta\underline{\mathbf{R}}_t)-\mathbf{CE}_t(\underline{\mathbf{R}}\mathbf{t})|$. The risk margin is now stated in probabilistic terms. Once this amount is given, one can see the extent to which the risk margin is significant – by making use of the characteristics of the underlying loss distribution. For example, if adding a risk margin causes the coefficient of estimation to increase from **88** to **90**, one can question whether the addition of the risk margin to the otherwise applicable estimate is significant. On the other hand, if the increase is from **88** to **98**, one may view $\Delta\underline{\mathbf{R}}_t$ as a legitimate candidate to be designated as the risk margin. We should note that at this point the linkage between *materiality* and *risk margin* is clear. In other words, a *risk margin* is material if it exceeds some benchmark that is selected and motivated by the actuary.

The discussion is concluded by noting an implicit condition that should be observed whenever an actuary makes use of the terminology "*risk margin*". Saying that a *risk margin* is added to an otherwise indicated reserve estimate that merely brings $\mathbf{R}_t$ closer to $\mathfrak{R}_t$ may be inadvertently misleading. In this case the coefficient of estimation of the final reserve, $\mathbf{R}_t(=\underline{\mathbf{R}}_t+\Delta\underline{\mathbf{R}}_t)$ inclusive of a risk margin, is simply raised closer to **100**, the condition under which the proposed estimate is simply approaching the mean of the underlying loss distribution. In this case it is clear that a true risk margin is not provided – in spite of using the terminology of risk margins. At least it is not obvious how such a statement can be meaningful. Using our notation: if an indicated reserve $\underline{\mathbf{R}}_t$ is increased by some risk margin, $\Delta\underline{\mathbf{R}}_t$, then, absent some very unusual conditions, which should be fully explained, one should be able to expect that $\mathbf{CE_t(R_t) > 100}$. If this is not part of the outcome of adding a risk margin, additional explanation and rationale needs to be provided by the actuary.

F. **Reserve Strengthening**. This language is often used in actuarial reports. Its meaning has never been established in the actuarial literature. One common usage occurs in connection with strengthening case loss reserves. That is generally understood to mean that the case loss reserves are now established to be closer to the ultimate settlement values than is historically indicated. This is often used to justify a lower-than-indicated aggregate reserve. In this paper when we refer to reserve strengthening, we are talking about strengthening of the total reserve (the sum of case reserves and IBNR reserves) in relation to what might have been done normally.

The basic idea of (the total) reserve strengthening simply suggests that the total carried reserve is materially closer to the ultimate value than would be the case had the otherwise indicated reserve been carried. Note here that there is no concept of the passage of time anywhere in the reserve strengthening idea. It is an instantaneous concept.

Thus for our purposes, we begin by identifying some indicated reserve, denoted by $\underline{\mathbf{R}}_t$. This reserve is arrived at by using a particular set of assumptions and methods (denoted by $\underline{\mathbf{A\&Ms}}$), that are consistent or identical to the assumptions and methods used in the past. The actuary, then, for good and sufficient reason, determines that a different set of assumptions and methods is more appropriate (denoted by **A&Ms**) is more appropriate. And in so doing, if applying **A&Ms** yields a higher reserve than the reserve produced using $\underline{\mathbf{A\&Ms}}$, we can now say that the reserves are strengthened. We can set this condition probabilistically by noting that the reserve are strengthened if and only if:

$$CE_t(R_t|A\&Ms) > CE_t(\underline{R_t}|\underline{A\&Ms})$$

We should note here that **<u>A&Ms</u>** are those used in the prior period. In other words, if the actuary continues with the same **<u>A&Ms</u>** as in the past, then the idea of reserve strengthening cannot be meaningful. Also note this is not introducing an element of time in our construction. Time here is used to simply identify and anchor the assumptions and methods that form the baseline.

With just these six illustrations, it is now possible to appreciate that practically any of the "soft" language that may be used to represent reserve estimates may be converted to a probabilistic basis. While that is not an end unto itself, the use of probabilistic representations makes it possible to harden the representations that actuaries make in connection with the presentation of loss reserve estimates.

VI. A Demonstration

This section contains a number of simple demonstrations of the concepts advanced in this paper. For our purposes, we are given two sets of raw data, one set is for line of business A (commercial automobile liability) and one for line of business B (private passenger automobile liability), as of a specific time **t**, from which we are able to construct two loss distributions, one for each line of business.[7] The following tables and charts are included at the end of this paper:

1. Tables A1 and B1 contain a compressed form of the cumulative frequency distributions for lines of business A and B, respectively.[8]

2. Tables A2 and B2, contain a compressed form of the coefficients of estimation associated with each of the significant outcomes in the underlying loss distributions for lines of business A and B, respectively.

[7] From this point forward, we will omit the reference to **t**, as all valuations and associated statements are as of time **t**.
[8] The full distribution using the intervals shown in Table A requires 15 pages to set forth completely.

3. Charts A1 and B1 show graphs of the cumulative frequency distributions set forth in Tables A1 and B1, for lines of business A and B, respectively.

4. Charts A2 and B2 show the graphs of the frequency distributions that underlie the graphs shown in Charts A1 and B1, for lines of business A and B, respectively.

5. Charts A3 and B3 show the graphs of the coefficients of estimation shown in Tables A2 and B2 for lines of business A and B, respectively.

The key parameters of the underlying loss distributions are calculated to be:

$$\mathfrak{R}(A) = \$3{,}486{,}577 \qquad \sigma(A) = \$1{,}754{,}637$$

$$\mathfrak{R}(B) = \$7{,}148{,}286 \qquad \sigma(B) = \$899{,}038$$

For the rest of this section, we will erect a number of scenarios and discuss the application of the concepts advanced in this paper to each scenario as appropriate, in turn illustrating the application of the particular facts to one of the terms discussed above.

Scenario 1. Best Estimate.

Line A. In this scenario suppose the selected point estimate of reserves for line of business A is **$3,000,000**. The reporting actuary calls this his best estimate. Our first observation, drawing on the values in Table A2, page 1, is that **CE($3,000,000) = 79.8**. Note that the coefficient of estimation of the mean of the distribution is **100**. That is **CE($3,486,577) = 100**. Thus even though the **$3,000,000** estimate is **$486,577** away from the mean of the underlying distribution (giving a preliminary and unconfirmed indication of a reserve deficiency), this amount represents a significant deviation from the mean of the distribution. The actuary then would endeavor to provide the rationale for departing from the mean of the distribution to the extent that he has. We should also note that the final representation of the reserve estimate is **($3,000,000 ; 79.8)**

Line B. In this scenario suppose the selected point estimate of reserves for line of business B is **$7,100,000**. The actuary calls this his best estimate. According to Table B2, **CE($7,100,000) = 97.4**. Note that the coefficient of estimation for the mean of the distribution is **100**. That is **CE($7,148,286) = 100**. The estimate of **$7,100,000** is **$48,286** from the mean of the underlying distribution (giving a preliminary and unconfirmed indication of an appropriate reserve selection – not redundant and not inadequate). Since the proximity of the point estimate to the mean of the loss distribution is not necessarily dispositive of the condition of the loss reserves, the actuary has the obligation to review the contemporaneous facts on operations to satisfy himself that there is nothing in the environment that would serve as a counter-indicator to the **$7,100,000** estimate. Assuming that the search turns up no significant counter indicators that would discredit the indicated estimate, the actuary would represent the statement of the reserve estimate as **($7,100,000 ; 97.4)**.

Scenario 2. Range of Reasonableness.

Suppose the reserving actuary has provided a voluntary range of reasonableness that each of his estimates has a range of reasonableness of **10%**. Now we review the significance of this statement as discussed above:

Line A. For this line of business the range of reasonableness represents **10%** of **$3,000,000**, or **$300,000**. Thus the range of reasonableness is **($2,700,000 ; $3,300,000)**. We note that the coefficients of estimation of the endpoints are as follows: **CE(2,700,000) = 68.0** and **CE(%3,300,000) = 92.0**. The interesting outcome here is that the distribution is substantially symmetrical about the **$3,000,000** estimate, in that the **CE** values at the boundaries are also symmetrical about the **CE** of the estimate (i.e., **79.8** is almost exactly halfway between **68** and **92**). These **CE**'s also indicate that the **10%** range of reasonableness is a fairly narrow range given the spread of the distribution of the **CE**'s. In other words the bulk of the expected outcomes remains outside the indicated range of reasonableness.

Line B. For this line of business the range of reasonableness represents **10%** of **$7,100,000**, or **$710,000**. Thus the range of reasonableness is **($6,390,000 ; $7,810,000)**. We note that the coefficients of estimation of the endpoints are as follows: **CE($6,390,000) = 39.5** and **CE($7,810,000) = 147.3**. Thus, the **CE** of the original estimate, at **97.4**, extends to cover the interval of **CE**'s consisting of **(39.5 ; 147.3)**. The indication is that the distribution is somewhat symmetrical about the selected estimate. More specifically, the **CE** of the point estimate, at **97.4,** is **57.8** points greater than the **CE** of the lower bound of the range of reasonableness and **49.9** points less than the **CE** of the upper bound of the range of reasonableness. Finally, the range of **10%** appears to cover the vast bulk of the distribution of possible outcomes.

It is noteworthy that the **10%** range of reasonableness covers a band of **CE**'s that spans **24.0** points **(= 92.0 - 68.0)** for line A while the same **10%** range of reasonableness spans **107.8** points **(= 147.3 – 39.5)** for line B. The reason for this difference is that the distribution for line A is much flatter than the distribution for line B. In evaluating these observations, it is useful to recall that the range of outcomes for the **CE** function is **200**.

Scenario 3. Confidence Interval.

Line A. For this scenario, suppose the actuary has calculated an interval of possible outcomes but did not select a point estimate.[9] The interval in the instant case is given as **($2,800,000 ; $3,800,000)**. We calculate the **CE**'s for these values: **CE($2,800,000) = 71.4** and **CE(3,800,000) = 110.8**. The spread of **CE**'s that corresponds to this confidence interval is **39.4** points **(= 110.8 – 71.4)**. The **$3,000,000** estimate is within the interval – but is near the low end. The final reserve statement by the reviewing actuary may well contain a remark to point out the flatness of the distribution and that

[9] This situation arises often in the case of one actuary reviewing the work of another, such as the actuary for an audit firm. Here the actuary calculates a range and tests the estimate of the audit client against the interval he has derived.

the bulk of the possible outcomes remain outside the indicated confidence interval. Even though it is obvious that the selected point estimate is within the confidence interval, the value of the **CE**'s in this case is to assist the actuary in finding out just how much of the distribution is actually covered by interval of coefficients of estimation in relation to the coefficient of estimation of the point estimate of the reserve being tested.

Line B. For this scenario, we are told that an actuary has calculated an interval of possible outcomes but did not select a point estimate. The interval in the instant case is given as **($7,000,000 ; $8,000,000)**. We calculate the **CE**'s for these values: **CE($7,000,000) = 91.3** and **CE($8,000,000) = 157.1**. The spread of **CE**'s that corresponds to this confidence interval is **65.8** points **(= 157.1 – 91.3)**. The **$7,100,000** estimate is within the interval – but is near the low end. However, the **CE**, even for the lower boundary of the confidence interval is in the neighborhood of the mean of the distribution so that the reviewing actuary could easily accept this value without reservation. The opining actuary may well include a comment in his opinion to express the high degree of comfort that is indicated by the selected point estimate of the reserves under review. Once again, even though it is obvious that the selected point estimate is within the confidence interval, the value of the **CE**'s in this case is to assist the actuary in finding out just how much of the distribution is actually covered by the interval of coefficients of estimation in relation to the coefficient of estimation of the point estimate of the reserve being tested.

Scenario 4. Materiality.

Line A. For this scenario, suppose the actuary has estimated the reserve at **$4,000,000**. The question arises as to the materiality of the difference between this estimate and the carried reserve at **$3,000,000**. The respective **CE**'s are: **CE($3,000,000) = 79.8** and **CE($4,000,000) = 117.3.** The reviewing actuary decides to use the materiality threshold as half the standard deviation. In this case that amount is **$877,319**. Following the construction from earlier in this paper, the **CE** spread that is implied by this standard is **|CE($3,486,577) - CE($4,363,896)| = |100.0 – 128.4| = 28.4** points.[10] On the other hand, the absolute value of the difference between **CE($3,000,000)** of **79.8** and **CE($4,000,000)** of **117.3,** is **37.5** points. Accordingly, since **37.5 > 28.4**, one is able to conclude that the difference is material.

Line B. For this scenario, suppose the actuary has estimated the reserve at **$7,500,000**. The question arises as to the materiality of the difference between this estimate and the carried reserve at **$7,100,000**. The respective **CE**'s are: **CE($7,100,000) = 97.4** and **CE($7,500,000) = 128.9**. The reviewing actuary again decides to use the materiality threshold as half the standard deviation. In this case that amount is **$449,519**. Following the construction from earlier in this paper, the **CE** spread that is implied by this standard is **|CE($7,148,286)-CE($7,597,805)|=|100.0–133.1| = 33.1** points. On the other hand, the absolute value of the difference between **CE($7,100,000)** of **97.4** and **CE($7,500,000)** of **128.9,** is **31.5** points. Accordingly, since **33.1 > 31.5**, one is able to conclude that the difference is not material.

Even if a different standard for materiality is used, such as a percentage of surplus, the mechanics illustrated above are applicable.

Scenario 5. Risk Margin.

Line A. In this scenario the actuary would like to consider adding a risk margin to his reserve estimate. The standard the actuary selects that the risk margin must meet in order to be considered material is **25%** of σ. The question is what is the amount that corresponds to this additional potential risk margin. **25%** of σ for this line of business is **$438,659**. Next we calculate the spread in **CE**'s that is represented by the difference between the **CE** of the mean of the distribution and the **CE** of the proposed higher value (mean of the distribution plus the proposed risk margin of **25%** of σ). Thus the spread is given by **|CE($3,486,577) – CE($3,925,236)| = |100.0 – 115.0| = 15.0** points. We already know that the **CE** of the original estimate is given by **CE($3,000,000) = 79.8**. Thus we are looking for that amount which, when added to **$3,000,000** will yield a **CE** of

[10] $4,363,896 = the mean + one half the standard deviation = $3,486,577 + $877,319.

94.8 (=79.8 + 15.0). Consulting Table A2, and locating the cell with the coefficient of estimation that is closest to **94.8**, yields a total reserve of **\$3,382,405**[11], which in turn yields a risk margin of **\$382,405 (= \$3,382,405 – \$3,000,000)**.

<u>Line B</u>. In this scenario the actuary again would like to consider adding a risk margin to his reserve estimate. Once again the standard the actuary selects that the risk margin must meet in order to be considered material is **25%** of σ. The question then is what is the amount that corresponds to this additional potential risk margin. **25%** of σ for this line of business is **\$224,260**. Next we calculate the spread in **CE**'s that is represented by the difference between the **CE** of the mean of the distribution and the **CE** of the proposed higher value (mean of the distribution plus the proposed risk margin of **25%** of σ). Thus the spread is given by **|CE(\$7,148,286) – CE(\$7,372,546)|=|100.0-119.2| = 19.2** points. We already know that the **CE** of the original estimate is given by **CE(\$7,100,000) = 97.4**. Thus we are looking for that amount which, when added to \$**7,100,000** will yield a **CE** of **116.6 (=97.4+19.2)**. Consulting Table B2 yield a total reserve of **\$7,317,595**, which in turn yields a risk margin of **\$217,595 (= \$7,317,595 – \$7,100,000)**.

We should note that the difference in the spread of the distributions is showing up rather remarkably in these examples. For example, using the same standard of materiality of **25%** of σ, the amount of risk margin for line A, **\$382,405**, is equal to **13%** of the otherwise selected point estimate, while the amount of risk margin for line B, **\$217,595**, is equal to **3%** of the otherwise selected estimate. Clearly the shape of the distribution is a significant variable in interpreting the reserve estimates as well as collateral issues related to them, such as risk margins.

Scenario 6. Reserve Strengthening.

For this scenario, suppose the actuary, having arrived at the estimates in Scenario 1, using assumptions and methods that were used the last time reserves were set, **<u>A&Ms</u>,**

[11] Once the appropriate cell is located, we simply use the midpoint of the corresponding interval.

is considering an alternative set of assumptions and methods, **A&Ms**. He has done the work and the new estimates are given as **$3,100,000** for line A and **$7,500,000** for line B. While it is clear that the new reserve estimates are higher than the original estimates, it is not clear that either one represents a reserve strengthening. Let us now consider if these new reserve estimates represent a strengthening.

Line A. We begin by noting that **CE($3,100,000|A&Ms) = 84.1**. Note that for the original reserve **CE($3,000,000|A&Ms) = 79.8**. The **4.3** point increase in **CE** does not suggest that this is a true strengthening. We can also invoke a standard of materiality which could be used to identify the increase in reserves as a strengthening or not. For our illustrative purposes we shall use the standard of **25%** of σ. This standard implies that a change in **CE** of less than **15** points is not material (See Scenario 5 for the derivation). Thus, using that standard we can conclude that the increase in reserves in this case is not material.

Line B. Once again we begin by noting that **CE($7,500,000|A&Ms) = 128.9**. And again note that for the original reserve **CE($7,100,000|A&Ms) = 97.4**. The increase in **CE** due to the revision in assumptions and methods is **31.5** points. Following the same standard of materiality of **0.25**σ yields a spread in **CE** of **19.2** points as the requirement to meet before we can pronounce a change to be material. In the instant case, the proposed change in reserves due to the revised methods and assumptions is **31.5** points, which is greater than the threshold standard of **19.2** points, and hence we are able to conclude that the proposed change in reserves would represent a strengthening.

VII. Concluding Remarks

The authors believe that the concept of the coefficient of estimation is useful in improving the clarity of statements made about a reserve estimate. The clarity is made possible because the actuary is using a fixed reference point (i.e., a landscape) against which various reserving statements and/or comparative statements are made. Having described and illustrated a process for bringing such clarity, we must conclude this paper with a series of remarks that need to be considered as an actuary uses this tool:

A. Emphasis on **t**. The reader will note the insistence on mentioning **t** at every point of the construction. This is an essential point of emphasis as the condition of reserves can be assessed only contemporaneously. All other statements about a reserve that make use of later development are made at a later time are statements about the runoff.

B. Uncertainty. Even though the coefficient of estimation is a useful tool – in that it gives both the actuary and a user an opportunity to understand the texture of the underlying probabilities and the associated uncertainty, using the coefficient of estimation does not eliminate the inherent uncertainty of reserve estimates.

C. Distribution Choices. The authors acknowledge that no two actuaries need select the same underlying distribution for a line of business. However, whichever distribution is used by the actuary, he needs to identify the rationale for such choice.

D. Standard of Materiality. We need to emphasize again that no two actuaries will necessarily come up with the same standard of materiality. While the actuary has this freedom to select a standard of materiality, the obvious consequent duty is that the actuary needs to make an appropriate disclosure whenever he changes the standard of materiality.

E. Convolutions. Even though the discussion above dealt with a single line of business, all observations and methods are equally applicable to a convolution distribution of two of more underlying loss distributions.

F. The Opining Actuary. The actuary who actually opines on the reasonableness of a given reserve is now in a position to actually set that reserve in the framework of the historically indicated reserve and the **CE** associated with that distribution.

G. The Reviewing Actuary. The constructions described in this paper make it possible to more clearly delineate the work of an actuary in constructing a reserve estimate and associated statements and the work of a actuary charged with reviewing the work of another.

H. Direct and Net Reserves. All constructions and observations apply equally to both direct and net experience. The underlying distribution for the direct case, although not necessarily so, can be expected to be different from the underlying distribution for the net case.

I. Reinsurance. All constructions and observations apply equally to reinsurance experience. We should note, however, that in the case of reinsurance applications the distributions can be expected to exhibit greater skew.

J. Adequacy. A high **CE**, by itself, does not necessarily imply that a high level of adequacy may be attached to the associated reserve estimate. Over time, the claims situation may change so that adequacy can be measured only against what is known at time **t**. The converse is true in the case of a low **CE**. These comments represent a special case of the general condition that actuaries should not rely exclusively on the size of the associated **CE** in evaluating the instantaneous adequacy that can be attached to a reserve estimate.

The authors believe that careful application of the coefficient of estimation can help illuminate the difficult task of making statements about reserve estimates. Perhaps over time it will be possible to identify benchmarks by line of business as well as other materiality benchmarks. Such benchmarks can emerge by company, by line of business, and/or by industry segment or in total. All such developments are capable of advancing casualty actuarial practice such that users of reserve estimates may be able to place greater reliance on the work of the actuary.

* * *

Table A1, page 1					
Cumulative Frequency Distribution of IBNR Commerical Auto Liability					
Interval		**Cumulative**	**Interval**		**Cumulative**
>	≤	**Frequency**	>	≤	**Frequency**
1,255,810	1,281,586	5.1%	2,183,778	2,209,555	24.9%
1,281,586	1,307,363	5.4%	2,209,555	2,235,332	25.5%
1,307,363	1,333,140	6.1%	2,235,332	2,261,109	26.2%
1,333,140	1,358,917	6.4%	2,261,109	2,286,886	26.8%
1,358,917	1,384,694	7.0%	2,286,886	2,312,663	27.5%
1,384,694	1,410,471	7.4%	2,312,663	2,338,440	28.5%
1,410,471	1,436,248	7.8%	2,338,440	2,364,216	29.2%
1,436,248	1,462,025	8.3%	2,364,216	2,389,993	29.9%
1,462,025	1,487,802	8.7%	2,389,993	2,415,770	30.9%
1,487,802	1,513,579	9.2%	2,415,770	2,441,547	31.5%
1,513,579	1,539,356	9.6%	2,441,547	2,467,324	32.1%
1,539,356	1,565,132	10.2%	2,467,324	2,493,101	32.8%
1,565,132	1,590,909	10.7%	2,493,101	2,518,878	33.4%
1,590,909	1,616,686	11.1%	2,518,878	2,544,655	34.0%
1,616,686	1,642,463	11.7%	2,544,655	2,570,432	34.7%
1,642,463	1,668,240	12.2%	2,570,432	2,596,209	35.5%
1,668,240	1,694,017	12.7%	2,596,209	2,621,986	36.1%
1,694,017	1,719,794	13.2%	2,621,986	2,647,762	36.8%
1,719,794	1,745,571	13.8%	2,647,762	2,673,539	37.5%
1,745,571	1,771,348	14.3%	2,673,539	2,699,316	38.2%
1,771,348	1,797,125	14.9%	2,699,316	2,725,093	39.6%
1,797,125	1,822,901	15.6%	2,725,093	2,750,870	40.2%
1,822,901	1,848,678	16.2%	2,750,870	2,776,647	40.8%
1,848,678	1,874,455	16.9%	2,776,647	2,802,424	41.6%
1,874,455	1,900,232	17.5%	2,802,424	2,828,201	42.3%
1,900,232	1,926,009	18.1%	2,828,201	2,853,978	42.9%
1,926,009	1,951,786	18.7%	2,853,978	2,879,755	43.5%
1,951,786	1,977,563	19.3%	2,879,755	2,905,531	44.1%
1,977,563	2,003,340	19.9%	2,905,531	2,931,308	44.7%
2,003,340	2,029,117	20.5%	2,931,308	2,957,085	45.3%
2,029,117	2,054,894	21.0%	2,957,085	2,982,862	45.9%
2,054,894	2,080,671	21.6%	2,982,862	3,008,639	46.5%
2,080,671	2,106,447	22.3%	3,008,639	3,034,416	47.2%
2,106,447	2,132,224	23.0%	3,034,416	3,060,193	47.8%
2,132,224	2,158,001	23.6%	3,060,193	3,085,970	48.3%
2,158,001	2,183,778	24.2%	3,085,970	3,111,747	49.0%

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Table A1, page 2
Cumulative Frequency Distribution of IBNR
Commerical Auto Liability

Interval >	Interval ≤	Cumulative Frequency	Interval >	Interval ≤	Cumulative Frequency
3,111,747	3,137,524	49.6%	4,039,715	4,065,492	69.3%
3,137,524	3,163,301	50.2%	4,065,492	4,091,269	69.7%
3,163,301	3,189,077	50.8%	4,091,269	4,117,046	70.1%
3,189,077	3,214,854	51.3%	4,117,046	4,142,823	70.6%
3,214,854	3,240,631	51.9%	4,142,823	4,168,600	71.2%
3,240,631	3,266,408	52.5%	4,168,600	4,194,377	71.7%
3,266,408	3,292,185	53.1%	4,194,377	4,220,154	72.1%
3,292,185	3,317,962	53.6%	4,220,154	4,245,931	72.6%
3,317,962	3,343,739	54.2%	4,245,931	4,271,707	73.0%
3,343,739	3,369,516	54.9%	4,271,707	4,297,484	73.4%
3,369,516	3,395,293	55.4%	4,297,484	4,323,261	73.7%
3,395,293	3,421,070	56.4%	4,323,261	4,349,038	74.1%
3,421,070	3,446,847	56.9%	4,349,038	4,374,815	74.5%
3,446,847	3,472,623	57.5%	4,374,815	4,400,592	74.8%
3,472,623	3,498,400	58.3%	4,400,592	4,426,369	75.2%
3,498,400	3,524,177	58.9%	4,426,369	4,452,146	75.5%
3,524,177	3,549,954	59.4%	4,452,146	4,477,923	75.9%
3,549,954	3,575,731	59.9%	4,477,923	4,503,700	76.4%
3,575,731	3,601,508	60.5%	4,503,700	4,529,477	76.7%
3,601,508	3,627,285	61.1%	4,529,477	4,555,253	77.1%
3,627,285	3,653,062	61.6%	4,555,253	4,581,030	77.6%
3,653,062	3,678,839	62.1%	4,581,030	4,606,807	77.9%
3,678,839	3,704,616	62.6%	4,606,807	4,632,584	78.2%
3,704,616	3,730,392	63.1%	4,632,584	4,658,361	78.6%
3,730,392	3,756,169	63.6%	4,658,361	4,684,138	78.9%
3,756,169	3,781,946	64.1%	4,684,138	4,709,915	79.2%
3,781,946	3,807,723	64.6%	4,709,915	4,735,692	79.5%
3,807,723	3,833,500	65.1%	4,735,692	4,761,469	79.8%
3,833,500	3,859,277	65.6%	4,761,469	4,787,246	80.1%
3,859,277	3,885,054	66.1%	4,787,246	4,813,022	80.3%
3,885,054	3,910,831	66.6%	4,813,022	4,838,799	80.6%
3,910,831	3,936,608	67.0%	4,838,799	4,864,576	80.9%
3,936,608	3,962,385	67.5%	4,864,576	4,890,353	81.5%
3,962,385	3,988,162	68.0%	4,890,353	4,916,130	81.8%
3,988,162	4,013,938	68.4%	4,916,130	4,941,907	82.1%
4,013,938	4,039,715	68.8%	4,941,907	4,967,684	82.4%
Mean = 3,486,577			Standard Deviation = 1,754,637		

Table A2, page 1					
Table of Coefficients of Estimation					
Commerical Auto Liability					
Interval		Coeff. Of	Interval		Coeff. Of
>	≤	Estimation	>	≤	Estimation
1,255,810	1,281,586	8.7	2,183,778	2,209,555	42.7
1,281,586	1,307,363	9.3	2,209,555	2,235,332	43.8
1,307,363	1,333,140	10.4	2,235,332	2,261,109	44.9
1,333,140	1,358,917	11.0	2,261,109	2,286,886	46.0
1,358,917	1,384,694	12.0	2,286,886	2,312,663	47.2
1,384,694	1,410,471	12.7	2,312,663	2,338,440	48.9
1,410,471	1,436,248	13.4	2,338,440	2,364,216	50.1
1,436,248	1,462,025	14.2	2,364,216	2,389,993	51.3
1,462,025	1,487,802	15.0	2,389,993	2,415,770	53.0
1,487,802	1,513,579	15.8	2,415,770	2,441,547	54.0
1,513,579	1,539,356	16.6	2,441,547	2,467,324	55.1
1,539,356	1,565,132	17.5	2,467,324	2,493,101	56.2
1,565,132	1,590,909	18.3	2,493,101	2,518,878	57.3
1,590,909	1,616,686	19.1	2,518,878	2,544,655	58.4
1,616,686	1,642,463	20.0	2,544,655	2,570,432	59.5
1,642,463	1,668,240	20.9	2,570,432	2,596,209	60.8
1,668,240	1,694,017	21.7	2,596,209	2,621,986	61.9
1,694,017	1,719,794	22.7	2,621,986	2,647,762	63.1
1,719,794	1,745,571	23.6	2,647,762	2,673,539	64.4
1,745,571	1,771,348	24.6	2,673,539	2,699,316	65.5
1,771,348	1,797,125	25.5	2,699,316	2,725,093	68.0
1,797,125	1,822,901	26.7	2,725,093	2,750,870	69.0
1,822,901	1,848,678	27.7	2,750,870	2,776,647	70.1
1,848,678	1,874,455	28.9	2,776,647	2,802,424	71.4
1,874,455	1,900,232	30.0	2,802,424	2,828,201	72.5
1,900,232	1,926,009	31.0	2,828,201	2,853,978	73.6
1,926,009	1,951,786	32.1	2,853,978	2,879,755	74.6
1,951,786	1,977,563	33.1	2,879,755	2,905,531	75.7
1,977,563	2,003,340	34.1	2,905,531	2,931,308	76.7
2,003,340	2,029,117	35.1	2,931,308	2,957,085	77.8
2,029,117	2,054,894	36.1	2,957,085	2,982,862	78.8
2,054,894	2,080,671	37.1	2,982,862	3,008,639	79.8
2,080,671	2,106,447	38.2	3,008,639	3,034,416	80.9
2,106,447	2,132,224	39.4	3,034,416	3,060,193	81.9
2,132,224	2,158,001	40.4	3,060,193	3,085,970	82.9
2,158,001	2,183,778	41.5	3,085,970	3,111,747	84.1

Table A2, page 2

Table of Coefficients of Estimation

Commerical Auto Liability

Interval >	Interval ≤	Coeff. Of Estimation	Interval >	Interval ≤	Coeff. Of Estimation
3,111,747	3,137,524	85.1	4,039,715	4,065,492	118.8
3,137,524	3,163,301	86.1	4,065,492	4,091,269	119.5
3,163,301	3,189,077	87.1	4,091,269	4,117,046	120.3
3,189,077	3,214,854	88.1	4,117,046	4,142,823	121.1
3,214,854	3,240,631	89.1	4,142,823	4,168,600	122.1
3,240,631	3,266,408	90.1	4,168,600	4,194,377	122.9
3,266,408	3,292,185	91.0	4,194,377	4,220,154	123.6
3,292,185	3,317,962	92.0	4,220,154	4,245,931	124.6
3,317,962	3,343,739	93.0	4,245,931	4,271,707	125.2
3,343,739	3,369,516	94.1	4,271,707	4,297,484	125.9
3,369,516	3,395,293	95.0	4,297,484	4,323,261	126.5
3,395,293	3,421,070	96.7	4,323,261	4,349,038	127.1
3,421,070	3,446,847	97.6	4,349,038	4,374,815	127.8
3,446,847	3,472,623	98.6	4,374,815	4,400,592	128.4
3,472,623	3,498,400	100.0	4,400,592	4,426,369	129.0
3,498,400	3,524,177	101.0	4,426,369	4,452,146	129.6
3,524,177	3,549,954	101.9	4,452,146	4,477,923	130.2
3,549,954	3,575,731	102.7	4,477,923	4,503,700	131.1
3,575,731	3,601,508	103.8	4,503,700	4,529,477	131.6
3,601,508	3,627,285	104.7	4,529,477	4,555,253	132.2
3,627,285	3,653,062	105.6	4,555,253	4,581,030	133.1
3,653,062	3,678,839	106.6	4,581,030	4,606,807	133.6
3,678,839	3,704,616	107.4	4,606,807	4,632,584	134.2
3,704,616	3,730,392	108.3	4,632,584	4,658,361	134.8
3,730,392	3,756,169	109.2	4,658,361	4,684,138	135.3
3,756,169	3,781,946	110.0	4,684,138	4,709,915	135.8
3,781,946	3,807,723	110.8	4,709,915	4,735,692	136.3
3,807,723	3,833,500	111.7	4,735,692	4,761,469	136.8
3,833,500	3,859,277	112.5	4,761,469	4,787,246	137.3
3,859,277	3,885,054	113.3	4,787,246	4,813,022	137.8
3,885,054	3,910,831	114.2	4,813,022	4,838,799	138.3
3,910,831	3,936,608	115.0	4,838,799	4,864,576	138.8
3,936,608	3,962,385	115.8	4,864,576	4,890,353	139.8
3,962,385	3,988,162	116.6	4,890,353	4,916,130	140.3
3,988,162	4,013,938	117.3	4,916,130	4,941,907	140.9
4,013,938	4,039,715	118.1	4,941,907	4,967,684	141.3

Chart A1

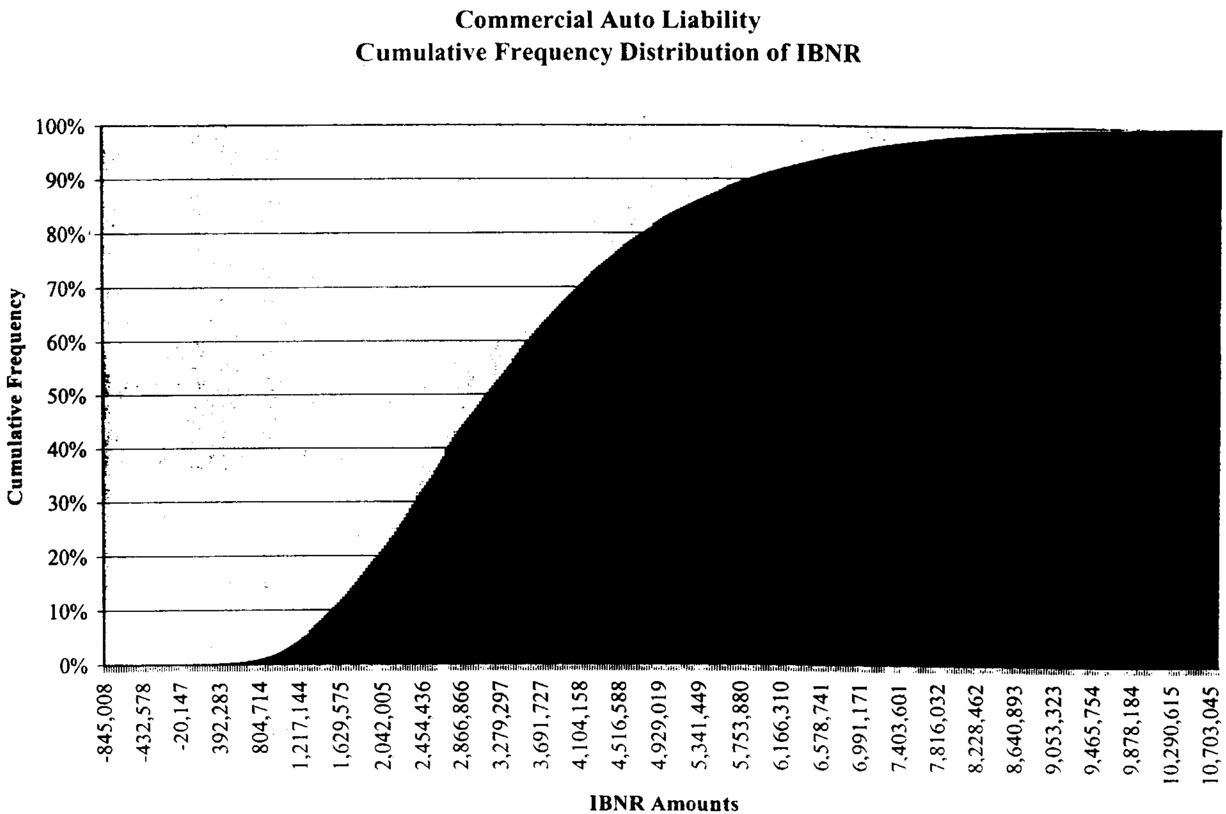

Commercial Auto Liability
Cumulative Frequency Distribution of IBNR
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
Cumulative Frequency
-845,008
-432,578
-20,147
392,283
804,714
1,217,144
1,629,575
2,042,005
2,454,436
2,866,866
3,279,297
3,691,727
4,104,158
4,516,588
4,929,019
5,341,449
5,753,880
6,166,310
6,578,741
6,991,171
7,403,601
7,816,032
8,228,462
8,640,893
9,053,323
9,465,754
9,878,184
10,290,615
10,703,045
IBNR Amounts

Chart A2

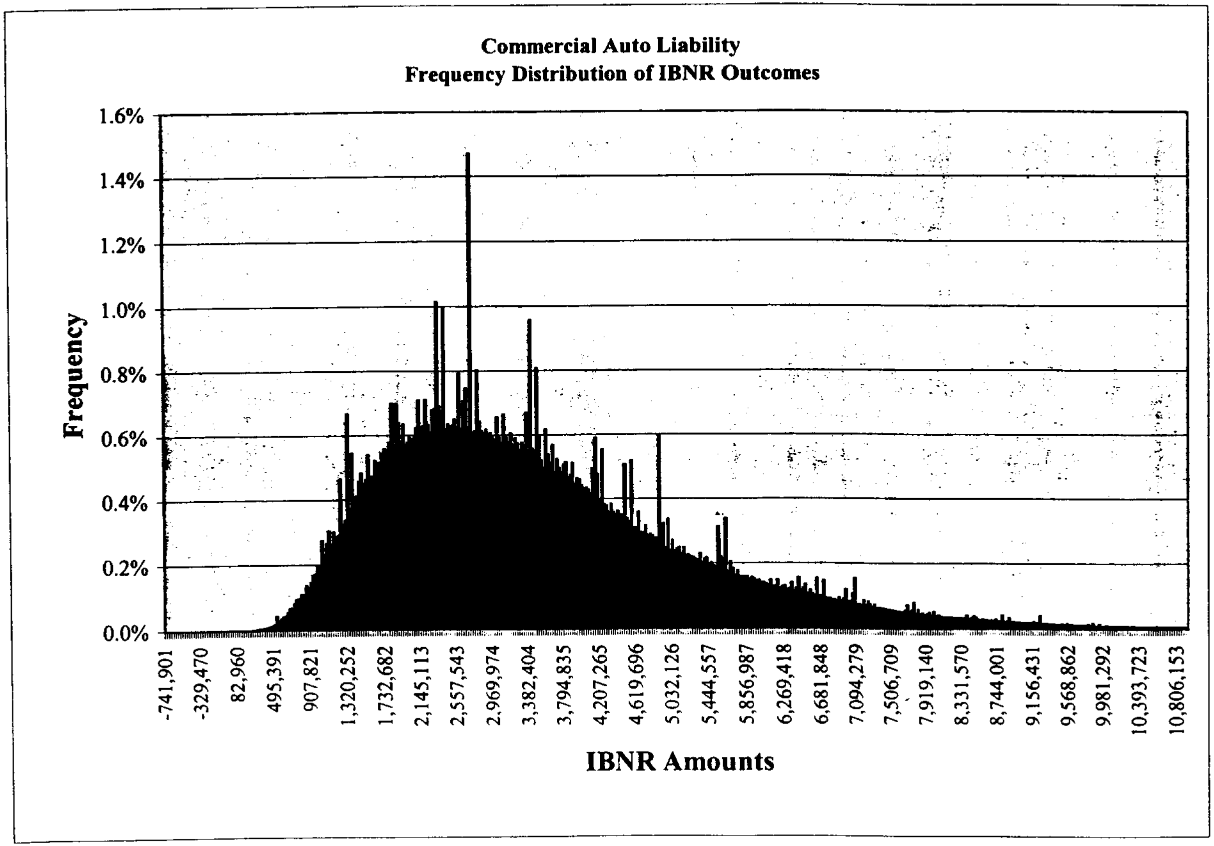

Commercial Auto Liability
Frequency Distribution of IBNR Outcomes
Frequency
1.6%
1.4%
1.2%
1.0%
0.8%
0.6%
0.4%
0.2%
0.0%
-741,901
-329,470
82,960
495,391
907,821
1,320,252
1,732,682
2,145,113
2,557,543
2,969,974
3,382,404
3,794,835
4,207,265
4,619,696
5,032,126
5,444,557
5,856,987
6,269,418
6,681,848
7,094,279
7,506,709
7,919,140
8,331,570
8,744,001
9,156,431
9,568,862
9,981,292
10,393,723
10,806,153
IBNR Amounts

Chart A3

Commercial Auto Liability
Graphic Representation of the Coefficients of Estimation

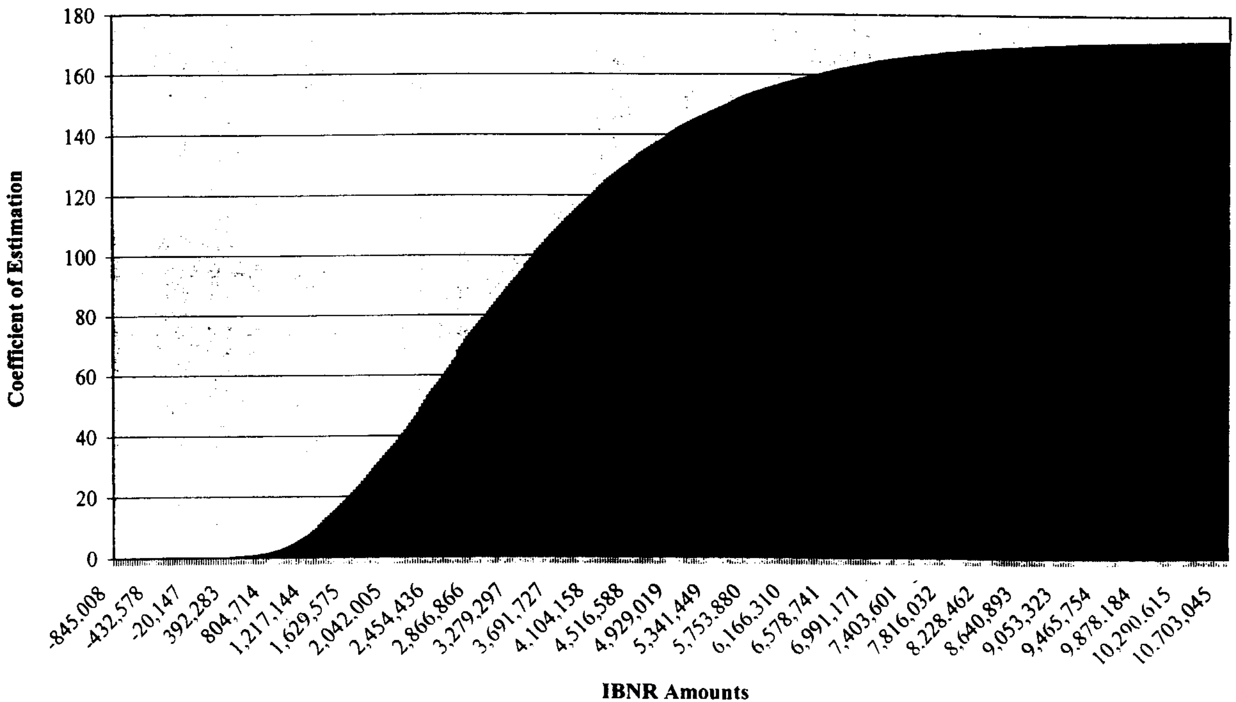
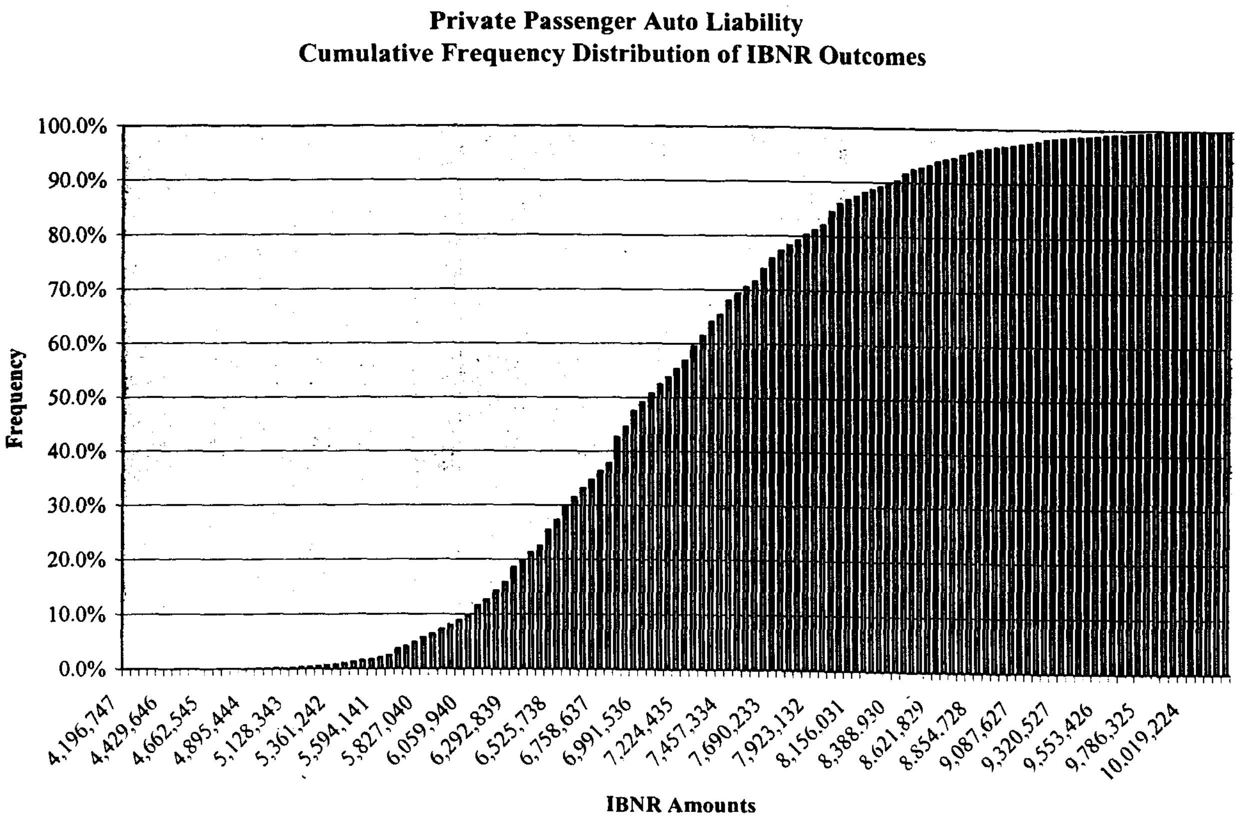


Table B1					
Cumulative Frequency Distribution Private Passenger Auto Liability					
Interval		**Cumulative Frequency**	**Interval**		**Cumulative Frequency**
>	≤		>	≤	
5,757,171	5,803,751	4.8%	7,434,044	7,480,624	68.0%
5,803,750	5,850,330	5.7%	7,480,624	7,527,204	69.3%
5,850,330	5,896,910	6.4%	7,527,204	7,573,784	70.5%
5,896,910	5,943,490	7.2%	7,573,783	7,620,363	71.6%
5,943,490	5,990,070	8.0%	7,620,363	7,666,943	73.9%
5,990,070	6,036,650	8.8%	7,666,943	7,713,523	75.9%
6,036,650	6,083,230	9.7%	7,713,523	7,760,103	77.3%
6,083,229	6,129,809	11.5%	7,760,103	7,806,683	78.4%
6,129,809	6,176,389	12.6%	7,806,682	7,853,262	79.2%
6,176,389	6,222,969	14.3%	7,853,262	7,899,842	80.3%
6,222,969	6,269,549	15.8%	7,899,842	7,946,422	81.2%
6,269,549	6,316,129	18.6%	7,946,422	7,993,002	82.0%
6,316,128	6,362,708	19.8%	7,993,002	8,039,582	84.5%
6,362,708	6,409,288	21.3%	8,039,582	8,086,162	86.0%
6,409,288	6,455,868	22.5%	8,086,161	8,132,741	86.8%
6,455,868	6,502,448	25.4%	8,132,741	8,179,321	87.4%
6,502,448	6,549,028	27.3%	8,179,321	8,225,901	88.1%
6,549,027	6,595,607	29.7%	8,225,901	8,272,481	88.7%
6,595,607	6,642,187	31.4%	8,272,481	8,319,061	89.2%
6,642,187	6,688,767	33.1%	8,319,060	8,365,640	89.7%
6,688,767	6,735,347	34.7%	8,365,640	8,412,220	90.3%
6,735,347	6,781,927	36.3%	8,412,220	8,458,800	91.6%
6,781,927	6,828,507	37.8%	8,458,800	8,505,380	92.4%
6,828,506	6,875,086	42.6%	8,505,380	8,551,960	92.8%
6,875,086	6,921,666	44.6%	8,551,960	8,598,540	93.2%
6,921,666	6,968,246	47.4%	8,598,539	8,645,119	93.9%
6,968,246	7,014,826	49.1%	8,645,119	8,691,699	94.2%
7,014,826	7,061,406	50.7%	8,691,699	8,738,279	94.6%
7,061,405	7,107,985	52.4%	8,738,279	8,784,859	95.3%
7,107,985	7,154,565	53.8%	8,784,859	8,831,439	95.6%
7,154,565	7,201,145	55.3%	8,831,438	8,878,018	96.1%
7,201,145	7,247,725	56.9%	8,878,018	8,924,598	96.3%
7,247,725	7,294,305	59.5%	8,924,598	8,971,178	96.5%
7,294,305	7,340,885	61.5%	8,971,178	9,017,758	96.8%
7,340,884	7,387,464	64.1%	9,017,758	9,064,338	97.0%
7,387,464	7,434,044	65.4%	9,064,337	9,110,917	97.1%
Mean = 7,148,286			Standard Deviation = 899,038		

Table B2					
Table of Coefficients of Estimation					
Private Passenger Auto Liability					
Interval		**Coeff. Of**	**Interval**		**Coeff. Of**
>	≤	**Estimation**	>	≤	**Estimation**
5,757,171	5,803,751	9.0	7,434,044	7,480,624	126.4
5,803,750	5,850,330	10.6	7,480,624	7,527,204	128.9
5,850,330	5,896,910	11.8	7,527,204	7,573,784	131.1
5,896,910	5,943,490	13.3	7,573,783	7,620,363	133.1
5,943,490	5,990,070	14.9	7,620,363	7,666,943	137.4
5,990,070	6,036,650	16.4	7,666,943	7,713,523	141.1
6,036,650	6,083,230	18.1	7,713,523	7,760,103	143.7
6,083,229	6,129,809	21.5	7,760,103	7,806,683	145.7
6,129,809	6,176,389	23.5	7,806,682	7,853,262	147.3
6,176,389	6,222,969	26.5	7,853,262	7,899,842	149.2
6,222,969	6,269,549	29.4	7,899,842	7,946,422	150.9
6,269,549	6,316,129	34.5	7,946,422	7,993,002	152.5
6,316,128	6,362,708	36.8	7,993,002	8,039,582	157.1
6,362,708	6,409,288	39.5	8,039,582	8,086,162	159.8
6,409,288	6,455,868	41.8	8,086,161	8,132,741	161.3
6,455,868	6,502,448	47.2	8,132,741	8,179,321	162.5
6,502,448	6,549,028	50.7	8,179,321	8,225,901	163.8
6,549,027	6,595,607	55.1	8,225,901	8,272,481	164.9
6,595,607	6,642,187	58.4	8,272,481	8,319,061	165.8
6,642,187	6,688,767	61.6	8,319,060	8,365,640	166.8
6,688,767	6,735,347	64.4	8,365,640	8,412,220	167.8
6,735,347	6,781,927	67.5	8,412,220	8,458,800	170.2
6,781,927	6,828,507	70.2	8,458,800	8,505,380	171.7
6,828,506	6,875,086	79.2	8,505,380	8,551,960	172.4
6,875,086	6,921,666	82.8	8,551,960	8,598,540	173.2
6,921,666	6,968,246	88.2	8,598,539	8,645,119	174.5
6,968,246	7,014,826	91.3	8,645,119	8,691,699	175.1
7,014,826	7,061,406	94.2	8,691,699	8,738,279	175.8
7,061,405	7,107,985	97.4	8,738,279	8,784,859	177.1
7,107,985	7,154,565	100.0	8,784,859	8,831,439	177.8
7,154,565	7,201,145	102.8	8,831,438	8,878,018	178.6
7,201,145	7,247,725	105.7	8,878,018	8,924,598	179.0
7,247,725	7,294,305	110.6	8,924,598	8,971,178	179.5
7,294,305	7,340,885	114.3	8,971,178	9,017,758	179.8
7,340,884	7,387,464	119.2	9,017,758	9,064,338	180.2
7,387,464	7,434,044	121.6	9,064,337	9,110,917	180.6

Chart B1



Private Passenger Auto Liability
Cumulative Frequency Distribution of IBNR Outcomes
100.0%
90.0%
80.0%
70.0%
60.0%
50.0%
40.0%
30.0%
20.0%
10.0%
0.0%
Frequency
4,196,747
4,429,646
4,662,545
4,895,444
5,128,343
5,361,242
5,594,141
5,827,040
6,059,940
6,292,839
6,525,738
6,758,637
6,991,536
7,224,435
7,457,334
7,690,233
7,923,132
8,156,031
8,388,930
8,621,829
8,854,728
9,087,627
9,320,527
9,553,426
9,786,325
10,019,224
IBNR Amounts

Chart B2

Private Passenger Auto Liability
Frequency Distribution of IBNR Outcomes

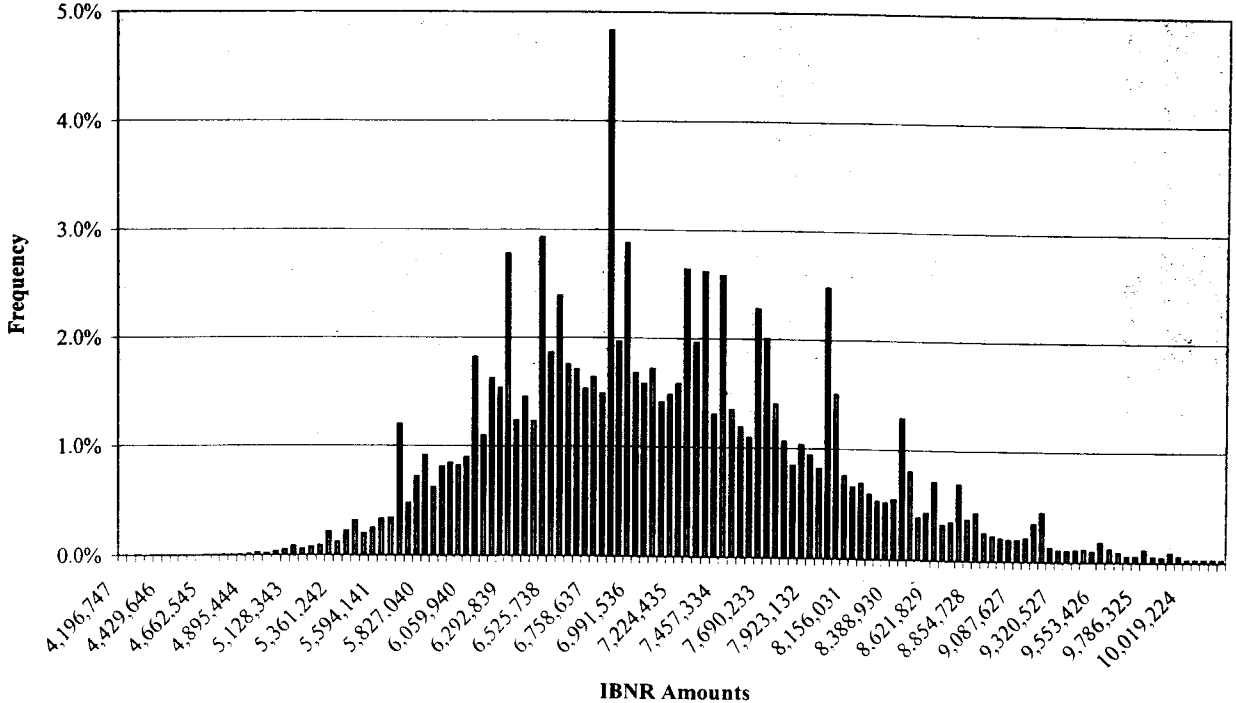


Chart B3

Private Passenger Auto Liability
Graphic Representation of the Coefficient of Estimation