CASUALTY ACTUARIAL SOCIETY FORUM

Winter 2002
Including the Ratemaking Discussion Papers

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The Casualty Actuarial Society *Forum*
Winter 2002 Edition
Including the Ratemaking Discussion Papers

To CAS Members:

This is the Winter 2002 Edition of the Casualty Actuarial Society *Forum*. It contains six Ratemaking Discussion Papers, two committee reports, and three additional papers.

The Casualty Actuarial Society *Forum* is a nonrefereed journal printed by the Casualty Actuarial Society. The viewpoints published herein do not necessarily reflect those of the Casualty Actuarial Society.

The CAS *Forum* is edited by the CAS Committee for the Casualty Actuarial Society *Forum*. Members of the committee invite all interested persons to submit papers on topics of interest to the actuarial community. Articles need not be written by a member of the CAS, but the paper's content must be relevant to the interests of the CAS membership. Members of the Committee for the Casualty Actuarial Society *Forum* request that the following procedures be followed when submitting an article for publication in the *Forum*:

1. Authors should submit a camera-ready original paper and two copies.
2. Authors should not number their pages.
3. All exhibits, tables, charts, and graphs should be in original format and camera-ready.
4. Authors should avoid using gray-shaded graphs, tables, or exhibits. Text and exhibits should be in solid black and white.
5. Authors should submit an electronic file of their paper using a popular word processing software (e.g., Microsoft Word and WordPerfect) for inclusion on the CAS Web Site.

The CAS *Forum* is printed periodically based on the number of call paper programs and articles submitted. The committee publishes two to four editions during each calendar year.

All comments or questions may be directed to the Committee for the Casualty Actuarial Society *Forum*.

Sincerely,

Dennis L. Lange

Dennis L. Lange, CAS *Forum* Chairperson

The 2002 CAS Ratemaking Discussion Papers
Presented at the
2002 Ratemaking Seminar
March 7-8, 2002
The Tampa Marriott Waterside
Tampa, Florida

The Winter 2002 Edition of the CAS *Forum* is a cooperative effort between the CAS *Forum* Committee and the Committee on Ratemaking.

The CAS Committee on Ratemaking presents for discussion six papers prepared in response to its Call for 2002 Ratemaking Discussion Papers.

This Forum includes papers that will be discussed by the authors at the 2002 CAS Seminar on Ratemaking, March 7-8, in Tampa, Florida.

Ratemaking Discussion Papers

Committee Reports

Additional Papers

Pricing Aggregate and Credit Risk for Risk Sharing Entities

John D. Deacon, FCAS

ABSTRACT

This paper recognizes that entering into a risk-sharing financial arrangement with another entity creates credit risk. One can use a distribution of outcomes to price both aggregate and credit risk. This paper presents a way to price aggregate and credit risk for deals in which another entity is contractually liable for losses.

INTRODUCTION

Can you say "balance"? When arranging any risk-sharing deal between an insurance company and another entity, the insurance company can be extending credit, whether it knows it or not. Pricing the aggregate risk in concert with credit risk must be considered to avoid exposing the insurance carrier to risks that were not realized, quantified, or priced for. A financial arrangement can quickly get out of balance when these two risks are not priced and evaluated together. I will introduce concepts that are fundamental to this study, discuss pricing of aggregate risk, and offer a method for pricing credit risk when aggregate cover is provided.

FUNDAMENTALS

The scope of this paper is small risk sharing entities (RSE), which generally do not have much equity to commit. The analysis in this paper can be applied to any size entity, however. A few typical entities that fall into the category of RSE are: group and single parent captives, risk retention groups, large insureds with large deductibles (or annual deductibles), and large insureds on retrospective rating plans. 'Small' is considered to have annual losses less than $10 million. Attempts can be made to price aggregate cover for entities with less than $1 million expected annual losses, but I argue that the losses for this entity will be too volatile to provide aggregate cover.

To price a deal with an RSE requires balance. Since these entities do not have much equity, they generally have an interest in having an annual cap on losses they will be responsible for. Entering into a risk-sharing deal with an RSE usually exposes the insurance carrier to another form of risk, credit risk.

Typically, an RSE will do business with a primary insurance carrier, taking advantage of the strength, flexibility, services, expertise, national presence, or product line rate/form/rule filings. The RSE needs the insurance carrier for one of these reasons, or a primary insurance carrier would not be involved. One of the risks for the primary carrier is that RSE losses will exceed the aggregate limit. When RSE losses exceed the aggregate limit, the RSE is relieved of responsibility for losses above that level which are now the responsibility of the insurance carrier. The insured and the claimant may not even be aware of the business arrangement, since the purpose of insurance is to indemnify those damaged as a result of doing their business. The primary company is ultimately responsible for paying claims to claimants, which makes it even more important that they structure deals to protect themselves and ensure their solvency.

Even if the all the premiums are sufficient to cover all the losses on a gross basis, it is certainly conceivable to mis-structure and mis-price a deal so that one entity or the other will certainly lose financially. This can happen quite easily. We propose keys in this paper to avoid financial disasters.

PRICING AGGREGATE - Simulation

What can actuaries do when we are pricing the aggregate charge for an RSE? We can use simulation. Two alternatives for a simulation approach to pricing aggregate cover:

1) simulation of frequency and severity and
2) simulation of loss ratios

Simulation of frequency and severity should be the more precise method as long as the parameters and loss distributions are accurate. Those are critical conditions, however. This method is not for discussion in this paper. The results from this first type of simulation should be theoretically the same as those from simulation of loss ratios.

Another possible way to use simulation in pricing aggregate cover is simulation of loss ratios. Simulation of loss ratios involves determining the distribution of expected outcomes based on the expected amount and variance of how losses relate to premium historically. One immediate caveat is that we need to on-level premiums and losses so that we are measuring true variance and not just variance from changing prices or trends.

Simulation of loss ratios is much quicker and more convenient than simulation of frequency and severity. The claim and severity distributions don't need to be determined explicitly. Loss ratio simulation should nonetheless provide a reasonable approximation of the variance of the outcomes on a prospective basis if on-leveling has been done. We can easily read loss ratios and their corresponding percentiles from the simulation output and relate them to key parameters of the deal. For instance, if the aggregate attachment is 90% of premium we can determine the likelihood that the RSE losses will exceed the agg in terms of percentile.

To help illustrate, the table below shows some assumptions for a possible deal and the relationships between simulated loss ratios and their likelihood of occurring, given the inputs. Let's assume that this is a homogeneous deal for General Liability coverage where the RSE retains $500k per occurrence and also has and aggregate cap of 90% of gross premium (%GP) on annual occurrence-limited losses. We have already simulated loss ratios given the assumptions.

Assumptions	
Loss Ratio Distribution	Lognormal
On_Level Limited Expected LR (MEAN)	60%
Standard Deviation of Lim LR (STD DEV)	21%
Aggregate Attachment (%GP)	90%

Percentile	LR	Agg Loss % GP	Probability Weighted Expected Agg Loss
5%	30%	0%	0.0%
10%	34%	0%	0.0%
15%	37%	0%	0.0%
20%	40%	0%	0.0%
25%	42%	0%	0.0%
30%	45%	0%	0.0%
35%	47%	0%	0.0%
40%	49%	0%	0.0%
45%	51%	0%	0.0%
50%	54%	0%	0.0%
55%	56%	0%	0.0%
60%	59%	0%	0.0%
65%	61%	0%	0.0%
70%	65%	0%	0.0%
75%	68%	0%	0.0%
80%	72%	0%	0.0%
85%	77%	0%	0.0%
90%	84%	0%	0.0%
95%	96%	6%	0.3%
99%	121%	31%	1.6%

TOT COST % Gr Prem	1.8%

For example, we would say that a 77% loss ratio is at the 85th percentile. Or, there is a 15% chance that the loss ratio will be higher than 77%.

The loss ratio for a given percentile can be seen as the average for a range. I equate the change in percentile of a given range with the probability of that (average) loss ratio occurring. These agg losses (loss ratio minus agg attachment) are weighted against these probabilities. For example, at a 96% loss ratio the expected agg losses are 6% but these are not certain. They need to be multiplied by the probability of that loss ratio occurring in that range. So multiply 6% by 5% (.95-.90 = the difference in the probability from 90%ile to 95%ile) to get 0.3%, expected agg loss for that outcome.

We would perform the same calculations (to determine expected agg losses) using the output from simulation of frequency and severity in pricing the agg as we do with simulation of loss ratios.

Using simulation to price aggregate cover is not the only method. Table M is a traditional way insurance companies price annual aggregate cover for workers' compensation (WC). The NCCI has developed Table M which uses 'size group' and 'entry ratio' to determine expected agg losses in pricing aggregate cover. Table M contains ranges of expected losses to determine the size group. The entry ratio refers to the relationship of the aggregate attachment point and the expected losses. When one looks up the size group and entry ratio, the table returns the expected aggregate loss as a percentage of RSE expected losses.

The table M charge was developed using all workers compensation business. There are adjustments to be made for differences in severity and whether or not the RSE has a cap on individual claims and others. So then, is Table M applicable to RSEs? This question needs to be evaluated. Table M is the benchmark and is the starting point for RSEs but may not be applicable for a few reasons: 1) homogeneity 2) risk sharing and 3) pricing agg for lines of business other than WC. I do not wish to destroy the credibility of Table M, as the theory is solid, but only to offer alternatives to pricing aggregate cover.

<u>PRICING CREDIT RISK</u>

Once we have determined the distribution of aggregate losses, we have another type of risk our hands, maybe without even realizing it. This risk is credit risk. As mentioned earlier, RSEs are generally not very well capitalized.

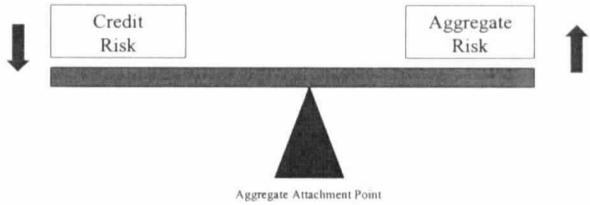
We now introduce the need to achieve balance in extending aggregate cover to an RSE. Earlier I spoke of determining ways to avoid financial disaster. Financial disaster can happen a few ways.

One example of financial disaster would be that the aggregate is set so high that the RSE is not able to pay for their losses (because the RSE does not have enough equity) if they were to reach the agg. In this case the primary carrier must pay the losses to the claimants anyway, and will encounter a credit loss. If the carrier does not charge appropriately for the credit risk, it will lose financially.

Another example of a financial disaster would be for the primary carrier to provide agg cover at a low level because the RSE can only afford to pay for losses up to a certain point above premiums. In this case the primary carrier can have very high agg losses since the likelihood of losses exceeding the agg is high. I would consider agg risk to be 'high' when either there exists a 20% chance or higher of an agg loss or if expected agg losses are larger than 10% of GP. These are very **rough** guidelines and can vary greatly from deal to deal. If the carrier does not charge appropriately for the agg risk, it will lose financially.

We would consider a deal to be balanced when we have both an aggregate attachment point that will be reached only infrequently (roughly less than 20% of the time and 10% of GP) but at the same time the RSE needs to be able to afford losses at that level.

Basics

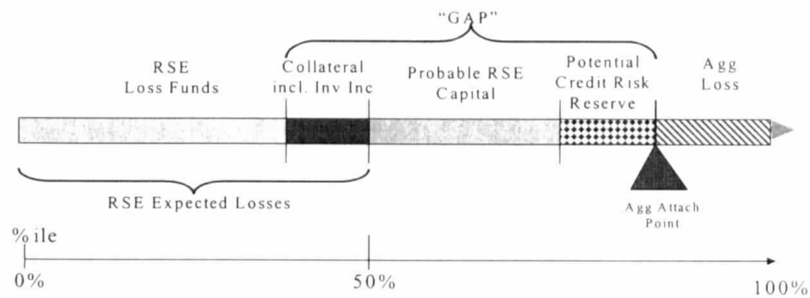


•Reducing risk of agg losses by increasing agg attachment increases credit risk,
•Reducing risk of credit losses by decreasing attachment increases risk of agg losses

The first illustration "Basics" shows how the balance between agg and credit risk works. Pushing on one makes the other pop up.

The second illustration "Possible Outcomes and How They are Covered" is a more detailed illustration of the risks at play. There are a few key sections of this graph to explain. The illustration shows a number line of all of the different outcomes of RSE losses. If losses are low, they are covered by the loss funds (portion of premium for losses, the non-expense piece only). If losses are high enough, they will exceed the agg and be the responsibility of the primary carrier or reinsurer providing the agg cover. The difference between the agg and loss fund is often referred to as the "gap". The gap is the maximum amount the RSE can actually lose, since it uses loss funds from premium to cover at least a portion of its liabilities.

Possible Outcomes and How They Are Covered



If losses exceed RSE loss funds, they will need to be covered by some other funds. Loss funds earn investment income until used to pay losses, and these investment gains could also be used to pay losses. The primary carrier should absolutely hold collateral (to protect against credit losses) at least up to expected losses of the RSE, preferably up to the agg if possible. In the illustration above, RSE loss funds are insufficient to cover RSE expected losses. This is natural for the same reason that it is common for insurance companies to write profitably above a 100% combined ratio. The reason they can do that is investment income.

In the above example, the RSE expected losses are at the 50th percentile of the distribution of outcomes. It is common when using a Lognormal distribution that the expected loss outcome is at 55th percentile or higher, due to the nature of a long-tailed distribution.

If losses are greater than expected, how will those losses get paid? The answer is the same way that they are paid when the premiums of a primary carrier are insufficient. They must be paid with equity or surplus. So, it becomes necessary to determine whether the RSE can even afford to pay for the "gap", if losses were to reach the agg. Even if the RSE can afford to pay for the "gap", there is some chance that they will not, for whatever reason. For the risk that the RSE will be unable or unwilling to pay, the insurance carrier will have to carry and charge for a credit risk reserve.

Now it is time to estimate the amount of credit risk of the primary carrier. While there are probably an unlimited number of ways to do this, I believe there is some value in having a simple model.

The steps for measuring credit risk for one contract period are as follows:

1. Determine the size of the gap
2. Determine the likelihood of the losses occurring in the gap
3. Determine how much capital the RSE has
4. Estimate the likelihood of receiving reimbursement for losses out of this surplus

I will now explain each of these steps in turn.

Determining the size of the gap should be the easiest step. All that is required is to subtract the loss funds from the aggregate attachment point (agg) which are both known values. The agg is often represented as a % of gross premium, which is 90% in the example. Specifying the agg attachment point in this manner is highly recommended since a set dollar amount can become inadequate quickly with any growth in exposures in the program from the time the pricing work is done to when final exposures are known. For that matter, increases/decreases in price adequacy throughout the contract year can also improve/deteriorate the level of agg protection. These issues should be addressed when setting up the deal. In the example, the "gap" is 35% of gross premium (GP).

Determining the likelihood of losses occurring in the gap can be accomplished by referring to the simulation of losses or loss ratios. The actuary can read percentiles of the distribution that relate to the endpoints of the gap (beginning = level of loss funds %GP, end = aggregate attachment %GP). Since the loss funds (55% GP) are at about the 50th percentile, the probability of losses occurring in the gap (or higher) is about 50% = 1-.50.

In order to evaluate the surplus of the RSE, or whether the RSE will even be able to pay if losses occur in the "gap", we can analyze the financial information of the RSE. Generally speaking, the equity or surplus must be able to take a financial hit in the amount of the gap (and then some since equity needs to cover multiple years of exposure). The equity must be greater than the gap. If it isn't the deal is not really financially viable since the RSE will not be able to pay their liabilities and have some surplus left over to stay in business. This is a straightforward high-level check to make sure the RSE is capable of covering its risk. In the example, the RSE is barely capable of paying for losses if they reach the agg since dollars of gap are close to equity. The step of determining the amount of RSE equity can be fairly difficult since:

1. Accurate financial information may be tough to get (but should be required)
2. Assets and liabilities are constantly changing so this information can become quickly out of date, especially with a small RSE
3. The financial information may not have been audited
4. The RSE may have business with other insurance carriers, which makes any individual carriers "stake" of the RSEs equity extremely difficult to determine.

Since the RSE equity and surplus can bounce around, it behooves the actuary of the primary company to be in tune with the way that the RSE books liabilities and budget for any under reserving through adjusted surplus or conservative estimates, we would recommend the former if possible. It is possible to structure a deal so that the RSE will be unable to pay, so the onus is on the actuary of the primary company to recognize when this is happening and make alternative recommendations.

The final element to evaluate is the likelihood that the RSE will pay (or be able to pay) its liabilities. I name this creditworthiness. Creditworthiness can be determined from financial analysis of the RSE balance sheet and operations. Evaluation of this element is not for the scope of this paper. The number is critical and must be estimated, since ignoring it implies 100% creditworthiness, which is an optimistic and impossible assumption. Every entity has some likelihood of being unable or unwilling to pay its liabilities.

Resulting from this exercise is a dollar and %GP estimate of credit risk that needs to be priced for and managed. For the example, this is $116,361 or 1.7% of GP. This risk also needs to be funded. It should ideally be charged back to the RSE on an expected basis since it is a real risk that the primary carrier takes on. There exist circularity issues with this method since if the price of credit risk is added in with the expenses, the loss funds will be lower. So it is an iterative process. I will not go through the iteration here.

Structure of the deal, key parameters, and Credit Pricing

Program		Value	Formula
Est Annual Gross Premium (GP)	A	$7,000,000	given
Per Occurrence limit	B	$500,000	given
Annual Aggregate Attachment point	C	90%	given
Program Expenses incl. reins.	D	45%	given
"GAP"	E	35%	C-H
"GAP"	F	$2,482,721	AXE

RSE Information			
RSE Expected Loss	G	60%	given
RSE Loss Fund	H	55%	1-D
RSE Equity	I	$3,000,000	given
Creditworthiness	J	0.375	given
Probability of Losses above loss funds	K	50%	from sim table

Credit Risk Pricing			
Probability Expected Credit Loss	L	$116,371	A*E*K-I*J
Probability Expected Credit Loss	M	1.7%	L/A
Discount factor for time value of money	N	0.86	3 years at 5%
Indicated Price for Credit Risk	O	1.4%	M*N

Assumptions with this method of evaluating credit risk:

1. Measuring credit risk can be done with a simple model
2. Reliant on estimates of probabilities from the simulation
3. Liabilities on the RSE financial statements are adequately stated (mentioned earlier); i.e. all equity is allocated to prospective contract period.

This exercise could be applied to older contract periods in much the same manner. The distribution of outcomes (i.e. the variance) should decrease as the period matures.

CONCLUSION

When pricing a financial arrangement with an RSE, the actuary must be cognizant of the different nuances of the deal. The actuary must recognize that RSEs do not usually have a lot of equity to commit, so that entering into a deal with an RSE can create credit risk. Knowing where the attachment point is with respect to the expected losses and quantifying the relative position of these key elements is critical to a well-structured arrangement. If the credit risk and agg risk are not priced in concert, the arrangement can quickly get out of balance, and the primary carrier could have great financial losses.

In most cases, entering into a deal with an RSE also means extending credit. For small RSEs without much surplus, this can be true. In order to price for credit risk, we should consider the amount the RSE has at risk, the likelihood of losses exceeding loss funds, the amount of equity the RSE has, and the likelihood the RSE will pay losses when they exceed loss funds. Using this information, the actuary can recognize, quantify, and price for credit risk and aggregate risk using the same distribution of outcomes.

Managing Commercial Lines Pricing Levels in a Loss Cost Environment

Lisa A. Hays, FCAS, MAAA

Managing Commercial Lines Pricing Levels in a Loss Cost Environment

Lisa A. Hays, F.C.A.S., M.A.A.A.

Abstract

The Percent of Loss Cost statistic (PoLC) is an effective tool, either alone or in conjunction with standard renewal pricing reports, to measure changes in commercial lines price levels in a loss cost environment. This paper demonstrates the calculations and definitions associated with the PoLC statistic. A case study for workers' compensation is presented which demonstrates a practical application of how PoLC can be used to segment a book of business when implementing indicated rate changes. Finally, sample reports are developed to monitor pricing results versus stated goals.

[The opinions expressed by the author are solely her own and are not attributable to any organization with which the author has been affiliated. The author would like to thank Kevin Kelley, Kim Mullins, Mike Sullivan, and the CAS team of reviewers for their helpful comments on earlier drafts.]

Managing Commercial Lines Pricing Levels in a Loss Cost Environment

One challenge facing today's commercial lines actuary is to accurately measure pricing changes in a book of business. In commercial lines, the actuary cannot simply file a base rate change and feel confident that the intended change will be the implemented change. Underwriters have many judgment rating tools at their disposal including access to multiple companies and schedule rating plans that enable them to match the pricing of a policy to its exposure, or to match the pricing of a policy to a competitor's quote. Thus, the implemented rate change may not be equal to the filed or intended rate change.

Correctly estimating the actual pricing change is an important task for both pricing and planning. Anticipated rate changes are typically used for business production and loss ratio plans. If these changes are inaccurate, one may find (12 or 15 months later) that the planned results differ from the actual results due to the difference in the planned and actual pricing levels. Estimated rate impacts are generally the basis for the calculation of premium onlevel factors. Inaccuracy of these onlevel factors can have a material impact in the calculation of rate-level indications.

In order to measure the actual impact of a rate filing or pricing change, it is necessary to develop methods for tracking the actual change in the total price level, which includes measuring changes in the impact of all the rating factors as well as base rate changes. These changes can then be more accurately reflected in business plans and in subsequent rate indications.

How are these changes tracked today?

Today many companies are able to measure the pricing level changes on their renewal book of business by performing a **cash-to-cash** comparison on policies that were renewed. It is a fairly simple process – the policies that renewed are matched up to their expiring terms and the premiums are directly compared after adjusting for some obvious differences such as unequal policy term lengths. The pros and cons of this method are:

- PRO – cash-to-cash is easy for the underwriters to understand and to implement on a policy-by-policy basis.
- PRO – when measured over the entire book of business it should provide an adequate measure of the rate plus exposure change for the book.
- PRO – the data necessary to perform the comparison is fairly basic and should be available without extensive manipulation.
- CON – on an individual policy basis or with smaller segments of the book, significant exposure changes will distort the results.
- CON – only renewal business that is retained is evaluated, new business and lost renewals are excluded.

It is possible, although potentially difficult depending on data availability, to develop a renewal increase report that adjusts for exposure changes. That would certainly address the first drawback of the cash-to-cash reports; however, it would still only capture pricing changes for renewal business while ignoring new business.

For new business, some companies have monitored discretionary credit/debit usage and the amount of change from one time period to another; however, this is only one of several pricing/rating factors affecting the overall price change. The pricing picture is only complete when all factors are included.

Is there a better way?

For lines of business where rating bureaus promulgate loss costs, there is a valuable pricing measurement tool that appears to be under-utilized in the management of commercial lines insurance operations, the Percent of Loss Cost (PoLC) statistic.

PoLC Definition

Most rating algorithms start with basic limits bureau loss costs and apply a multitude of factors to compute the premium. Depending on the line of business, the list of potential factors includes (but is not limited to);

1.) Increased Limits Factor
2.) Deductible Factor
3.) Experience Modification Factor
4.) Package Modification Factor – the rating bureau generally files a suggested package mod to reflect the decreased expense in issuing a commercial multi-peril policy
5.) Loss Cost Multiplier – defined here as the (expense multiplier) * (company deviation)
6.) Schedule Rating Factor – generally a subjective factor used to capture risk characteristics not already accounted for in the rating algorithm, such as quality of management, dispersion of risk, etc.
7.) Company-specific deviations to territorial, class, or other relativities.
8.) Renewal Credits – can be based on a combination of loss experience and/or the number of policy terms that the insured has been a customer

The PoLC is the ratio of the collected premium to the underlying bureau loss cost dollars. The loss cost dollars are calculated by multiplying the published loss costs by the exposures and represent the amount of premium the bureaus estimate is needed to cover projected losses and loss adjustment expenses. The first decision to make is which rating factors should be included in the calculation of the underlying loss costs. Some factors, such as the increased limits factor and the deductible factor, should always be considered part of the underlying loss costs since they are promulgated by the rating bureau and are objective factors used to quantify the expected loss costs. Likewise, obviously judgmental factors such as schedule rating should not be included in the underlying loss cost. However, it is less clear how package mods and experience mods, for example, should be treated in the formula.

If a company uses bureau-promulgated package mods with no modification, they should be included in the loss cost. However, if the company has filed package mods that are materially different than the bureau, the revised mods (or at least the difference from the bureau level) should be tracked as a deviation to loss costs and monitored over time.

Although experience rating plans themselves are considered to be objective, in practice, there are situations where the use of schedule credit may double-count a risk characteristic underlying the experience mod. For this reason, it is useful to track the experience mod as part of the PoLC statistic, but retain the ability to exclude it for ad hoc analysis.

As a general rule of thumb, rating factors that result from the pricing actuaries' or the field underwriters' judgment should be captured and tracked via the PoLC statistic as a deviation from the bureau loss costs.

For the purpose of this paper, the PoLC is defined as the aggregation of the loss cost multiplier, schedule rating factor, experience modification factor, package modification factor, and any company-filed deviations from the bureau loss costs. A PoLC of 120(%) means the collected written premium was 20% more than what the bureau has filed for estimated loss costs. The general formula for the PoLC when all rating factors are multiplicative is:

$$PoLC = \frac{\text{Collected Written Premium}}{\text{Loss Costs * Exposures}} = \frac{\text{Loss Costs * Exposures * LCM * OTHR * PKG * SRP * EXPER}}{\text{Loss Costs * Exposures}}$$

LCM = Loss Cost Multiplier (including company deviations)
OTHR = Modification Factor for company-specific deviations such as territory, class, or renewal credits
PKG = Package Modification Factor
SRP = Schedule Rating Modification Factor (1 + credit/debit)
EXPER = Experience Rating Modification Factor

"Rate" states, or states that have not converted to loss costs, can also be included in the calculation by estimating the LCM. This can be determined by using the underwriting expenses and profit load assumed in the bureau rate filing, and converting these to a loss cost multiplier. Then, the rates are divided by the LCM to compute the underlying loss costs.

In a Perfect World

Ideally, the PoLC is calculated by comparing the collected written premium to the loss cost dollars in effect at a chosen point in time (the base year). These indexed loss costs are calculated as the product of the exposures in the experience period and the loss costs in effect for the base year. The computation of an 'Indexed PoLC' facilitates comparison between years by capturing underlying base loss cost changes as well as changes in all of the modification factors. The calculation is a simple one, assuming that the loss costs from the base year are accessible and you have a program that can re-rate the current exposures with the base loss costs. For policies written in 1999:

$$\text{Indexed PoLC}_{1999} = \frac{\text{Collected Written Premium}_{1999}}{(\text{Exposures}_{1999} \text{ * Base Loss Costs})}$$

If the indexed PoLCs were 90%, 97% and 105% for policies with effective dates in 1998, 1999, and 2000 respectively the computed pricing changes for 1999 and 2000 are:

1999 change = (97% / 90%) – 1 = +7.8%

2000 change = (105% / 97%) – 1 = +8.2%

The +7.8% change for 1999 could be due to a change in loss costs, company deviation, schedule rating, or any other rating factor affecting the overall premium. Because the same exposures are used for the calculation of both the collected premium and the indexed loss cost dollars, the exposures cancel out and it becomes possible to use the indexed PoLC to measure true pricing changes from year to year. In other words, the change in indexed PoLC measures the change in price per exposure over the entire book; thus, it addresses both of the Cons listed for the cash-to-cash renewal reports.

Unfortunately, most companies do not have the capability to re-rate or extend exposures in this manner. If that is the case, it is still beneficial to understand the changes in all factors other than the loss costs, and to quantify the change in loss costs separately.

Calculation of Components

If re-rating or extending exposures are not viable options, there is another way to compute the underlying loss costs and the impact of each of the rating components for the PoLC statistic. Exhibit 1 demonstrates the calculations for a 5-record Commercial Auto database. This same calculation can be applied to more extensive databases. This example is for Commercial Auto where schedule rating and experience modifications are additive. The rating formula for a single vehicle and a single coverage is:

$$\text{Written Premium} = \text{Loss Cost} * \text{LCM} * \text{OTHR} * (\text{SRP} + \text{EXPER} - 1)$$

The loss cost should be the only field not readily available since the other fields are required for statistical reporting purposes; therefore, the first step is to calculate the Loss Cost (LC) for each record in your database:

$$\text{LC} = \frac{\text{Written Premium}}{\text{LCM} * \text{OTHR} * (\text{SRP}+\text{EXPER} - 1)}$$

At this point, you can compute the PoLC for a segment of business by adding the written premiums and comparing them to the sum of the loss costs.

$$\text{PoLC} = \frac{\text{Written Premium}}{\text{LC}}$$

Although this indicates where you are pricing your book relative to bureau loss costs, it does not quantify how much each of the rating elements is impacting the PoLC. The contributions by rating element become important in using the PoLC information to formulate pricing guidelines for the field underwriters.

The first component to quantify is the LCM, which also includes any filed company deviations. To determine the impact of the LCM, create a new field called 'LC_LCM' which is the LC multiplied by the LCM for each record:

$$\text{LC\_LCM} = \text{LC} * \text{LCM}.$$

To calculate the average loss cost multiplier for the entire book of business, sum LC_LCM for all records and divide by the sum of the loss costs. This is simply the weighted average Loss Cost Multiplier using the Loss Cost as weights.

$$\text{Average Loss Cost Multiplier} = \frac{\text{LC\_LCM}}{\text{LC}}$$

The average expense mod is computed using the newly calculated 'LC_LCM' field as a base. A new field, LC_LCM_OTHR is then calculated and the sum of

LC_LCM_OTHR is compared to the sum of LC_LCM to compute the average expense mod.

$$LC\_LCM\_OTHR = LC\_LCM * OTHR = LC * LCM * OTHR$$

$$\text{Average 'Other' Mod} = \frac{LC\_LCM\_OTHR}{LC\_LCM}$$

Again, this is the weighted average 'Other' Mod factor using the product of the Loss Costs and the Loss Cost Multiplier as weights.

Because schedule and experience rating are additive in this example, the base, LC_LCM_OTHR, will be the same for each average modification factor. New fields, LC_LCM_OTHR_SRP and LC_LCM_OTHR_EXPER are calculated as follows:

$$LC\_LCM\_OTHR\_SRP = LC\_LCM\_OTHR * SRP$$

$$LC\_LCM\_OTHR\_EXPER = LC\_LCM\_OTHR * EXPER$$

and the average factors are computed:

$$\text{Average SRP Mod} = \frac{LC\_LCM\_OTHR\_SRP}{LC\_LCM\_OTHR}$$

$$\text{Average EXPER Mod} = \frac{LC\_LCM\_OTHR\_EXPER}{LC\_LCM\_OTHR}$$

Note that for individual records, it is mathematically equivalent to use the factors alone in the PoLC calculation:

$$PoLC = LCM * OTHR * (SRP + EXPER - 1)$$

This is also true for a segment of business using weighted factors as computed above.

Workers' Compensation Case Study

The first step in using PoLC to manage pricing levels is to correlate the PoLC levels with loss experience so that target PoLC levels can be established. A sample analysis for Workers' Compensation is shown in Exhibit 2. The exhibit shows WC loss ratios and claim frequencies by PoLC range. For this company, the results for business priced below 65 PoLC have been consistently poor relative to the average. For policy year (PY) 2000, the loss ratio relativity is 1.321 for this segment compared to the total reported loss ratio. Results in the 66 to 75 PoLC range have deteriorated, showing a loss ratio relativity of 1.077 and a frequency relativity of 1.207 for PY 2000. While the projected ultimate loss ratios for business in the 76 to 145 PoLC range have deteriorated from PY 1997 to PY 2000, there has been little variation across this range within each individual year. Results at PoLC levels of 146 and above have been consistently worse than average. This may indicate that the underwriters are able to do a better job of matching price to exposure in the 76 to 145 PoLC range than above and below it.

After the loss ratios by PoLC range have been calculated, you can use the rate indication to determine the necessary rate action for each PoLC range (Exhibit 3). In this example, the overall rate indication is +20% as computed using standard actuarial methods. After allocating the rate increase to PoLC range using the loss ratio relativities from the prior step, it shows an indicated increase of +51-59% for the 'Less than .65' range. Over half of the premium is from the PoLC ranges with an indicated increase of +5% (the 76-145 range). If one were to file for an increase in loss costs and/or loss cost multiplier of +5%, the underwriters could essentially renew this business 'as is' - i.e. use the same schedule credit, company, etc. assuming that updates to the experience modification factors would net to a negligible change.

Further segmentation is necessary to determine a plan of action for the ranges with significantly different indicated rate changes. In this example, the '1.46 and Above' range is a mix with 35% of the category being comprised of accounts with experience mods less than 125. As you can see from Exhibit 3, the indicated increase on this business is in line with the +5% that was selected for the overall rate change. The remaining business has an indication ~+33-37% and are policies that generate experience debits greater than 25%. Since they are likely larger policies, underwriters may be tempted to follow market pricing and price them in a lower rated company or with unwarranted schedule credit, thereby partially offsetting the impact of the debit. Instead of substantially increasing the base rates, it may be more appropriate to evaluate the use of company rating tiers or schedule credit and correct individual policies.

At the opposite end, the policies at PoLC levels below 75% appear to be significantly under-priced. In an effort to write the best risks, the underwriters may have double-counted the risks' prior profitable experience by applying too much schedule credit for characteristics already captured in the experience mod (probably a credit). Although there may be some classes or segments where this price level is appropriate, in general this problem will need to be corrected by individual risk pricing and underwriting and not by across-the-board base rate increases. Based on the loss correlation analysis, new business pricing guidelines should be established that limit or specify the types of business that can be written at a PoLC less than 75% or over 145%.

The above analysis, when conducted on a countrywide basis, assumes that the underlying loss cost inadequacy or redundancy is the same across states and industry segments. Companies that write business in a limited number of states or industry segments may find this assumption to be reasonable; however, other companies may find it necessary to review the PoLC and loss ratio correlations by industry group or by state.

Setting Goals and Monitoring Results

Exhibit 4 shows a sample PoLC monitoring report for Workers' Compensation that could be produced at various levels of detail including countrywide, industry segment, state, or profit center. In this example, the PoLC increased from 84.7% in 1999 to 88.2% in 2000 with most of the change coming from a reduction in SRP credits. Upon closer examination it is evident that new business pricing in 2000 did not improve in comparison to the overall average for 1999; however, the renewal business price level relative to loss costs increased by 6.7% (PoLC increased from 84.7% to 90.1%).

Given that the rate indication was +20% and that this PoLC report is <u>not</u> indexed with loss costs from a base year, if a 5% loss cost increase were filed effective 1/1/2001, the PoLC goal for 2001 policies to achieve rate adequacy would be:

$$88.2\% * (1.20 / 1.05) = 100.8\%.$$

In this case, where changes to the underlying loss costs are not reflected in the PoLC statistic, the adjustment of (1.20 / 1.05) represents the amount of pricing increase that needs to come from factors other than the underlying loss cost change.

The goal of 100.8% can apply to both new and renewal business; however, since rather large increases were selected at either end of the PoLC ranges, the average 2001 PoLC statistic could be impacted by low policy retention in these ranges. For example, non-renewing a significant portion of the policies in the < 75 PoLC range would increase the average PoLC for the book of business, even if the pricing change on the remaining policies was flat. In situations where targeted price changes vary significantly, it is probably better to use the PoLC report to monitor new business and to use renewal price increase reports to monitor the implementation of a segmented pricing plan.

Exhibit 4 shows an example of tying the selected rate changes by PoLC range for Workers' Compensation to an existing renewal increase report. For each policy, the current PoLC is computed and matched to the selected rate change for its PoLC range. If the renewal increase is calculated on a cash-to-cash basis (not adjusted for exposure changes) the selected rate change should be increased by the expected average exposure change. The 'Target Renewal Premium' is then computed by multiplying the written premium by the selected rate and exposure changes for each policy in the database. On the cash-to-cash report, for policies that were renewed and retained, the actual renewal premium is compared to the expiring premium to compute the renewal increase. The renewal increase goal is the target renewal premium divided by the expiring premium for the policies that renewed.

Mapping the goals to individual policies as opposed to publishing an overall average goal will yield a more accurate measure of actual vs. target pricing levels. For example, if policies with large targeted increases are cancelled or non-renewed, the goal will automatically adjust downward for the lost policies and there will not be a 'penalty' by comparing the achieved pricing change to an overall goal. As mentioned earlier, on a policy-by-policy basis the results vs. goal may not track well due to large exposure changes; however, on a countrywide or state level, the overall exposure change should be close to the expected average built into the goals. Obviously, the ideal is to compute the goals and the actual renewal price change excluding the impact of exposure changes – especially if reports by field underwriter or agency are to be produced.

Caveats

The case studies and examples provided in this paper assume that the underlying loss costs are inadequate or redundant by the same percentage amount across states, industry groups and effective years. If the loss cost redundancy for a state (or industry group) differs significantly from the countrywide average and if the mix of business is shifting either into or out of the state, an adjustment for this mix shift should be made before comparing the countrywide PoLC statistic from one year to the next. Likewise, state-to-state comparisons within the same year should recognize differences in underlying loss cost adequacy. For example, if the loss costs in State A are 10% more adequate than the loss costs in State B, business priced at a PoLC of 100% in State A is equivalent to a PoLC of 110% in State B.

Summary

The PoLC statistic can be a powerful tool for quantifying pricing changes for lines of business that rely on bureau loss costs. Un-indexed, it measures the change in usage of company tiers, schedule credits, and experience rating plans over time. An indexed PoLC also incorporates underlying loss cost changes and completes the pricing picture. Correlating PoLC with loss experience provides another method of segmenting a book of business and establishing pricing goals more appropriate for the risk as opposed to implementing across-the-board rate changes. Tying PoLC ranges with renewal pricing goals should reduce adverse selection and help to improve retention of business that is already adequately priced since that business will no longer subsidize inadequately priced insureds and will receive lower than average price increases. In short, incorporating PoLC into a company's pricing strategy can result in more accurate and responsive assessments of pricing changes and therefore, enhance the ability to attain profitability in a competitive commercial lines marketplace.

Percent of Loss Cost Example
Commercial Auto

Record #	Collected Wr. Prem	LCM Loss Cost Multiplier	OTHR 'Other' Mod	SRP Schedule Rating Mod	EXPER Experience Mod	LC Loss Cost	LC_LCM LC * LCM	LC_LCM_OTHR LC*LCM*OTHR	LC_LCM_OTHR_SRP LC*LCM*OTHR*SRP	LC_LCM_OTHR_EXPER LC*LCM*OTHR*EXPER	PoLC
1	1,000.00	1.40	1.00	0.90	1.05	751.88	1,052.63	1,052.63	947.37	1,105.26	133.0%
2	750.00	1.60	1.00	0.75	0.80	852.27	1,363.64	1,363.64	1,022.73	1,090.91	88.0%
3	800.00	1.55	0.95	1.00	1.00	543.29	842.11	800.00	800.00	800.00	147.3%
4	600.00	1.35	1.00	1.00	1.00	444.44	600.00	600.00	600.00	600.00	135.0%
5	450.00	1.50	1.00	0.80	0.90	428.57	642.86	642.86	514.29	578.57	105.0%
Total	3,600.00					3,020.46	4,501.23	4,459.13	3,884.38	4,174.74	119.2%

Assume the rating formula: Collected Written Premium = Loss Cost * LCM * OTHR * (SRP + EXPER - 1)

Percent of Loss Cost (PoLC) =	119.2%	= 3,600.00 / 3,020.46	= Written Premium / LC
Average Loss Cost Multiplier (LCM) =	1.490	= 4,501.23 / 3,020.46	= LC_LCM / LC
Average 'Other' Modification (OTHR) =	0.991	= 4,459.13 / 4,501.23	= LC_LCM_OTHR / LC_LCM
Average Schedule Rating Mod (SRP) =	0.871	= 3,884.38 / 4,459.13	= LC_LCM_OTHR_SRP / LC_LCM_OTHR
Average Experience Mod (EXPER) =	0.936	= 4,174.74 / 4,459.13	= LC_LCM_OTHR_EXPER / LC_LCM_OTHR

Double-check: 119.2% = 1.49 * 0.991 * (0.871 + 0.936 - 1)

Since Experience and Schedule Rating are additive in this example, they are compared to the same base, LC_LCM_OTHR

Exhibit 1

Exhibit 2

Workers' Compensation
Loss Ratio Analysis by PoLC Range

Policy Year 2000 as of 12/2000

PoLC Range	Earned Prem	Pctg of Total Earned Prem	Projected Ultimate Loss Ratio	Loss Ratio Relativity	Ultimate Claim Counts	Avg Claim Frequency per $1000 Earned Premium	Frequency Relativity
Less than 65%	32,715,625	20.7%	92	1.321	9,684	0.296	1.441
66 to 75	17,812,500	11.3%	75	1.077	4,418	0.248	1.207
76 to 85	14,364,000	9.1%	60	0.862	2,183	0.152	0.740
86 to 95	11,720,625	7.4%	62	0.890	1,894	0.162	0.787
96 to 105	16,957,500	10.7%	56	0.804	2,578	0.152	0.740
106 to 115	13,715,625	8.7%	60	0.862	2,151	0.157	0.763
116 to 125	10,723,125	6.8%	62	0.890	1,853	0.173	0.841
126 to 135	9,975,000	6.3%	56	0.804	1,772	0.178	0.865
136 to 145	9,226,875	5.8%	60	0.862	1,402	0.152	0.740
146 and Above	20,662,500	13.1%	73	1.048	4,496	0.218	1.059
Total	157,873,375	100.0%	70	1.000	32,430	0.205	1.000
Avg PoLC	**0.870**						

Policy Year 1999 as of 12/2000

PoLC Range	Earned Prem	Pctg of Total Earned Prem	Projected Ultimate Loss Ratio	Loss Ratio Relativity	Ultimate Claim Counts	Avg Claim Frequency per $1000 Earned Premium	Frequency Relativity
Less than 65%	72,500,000	22.2%	83	1.297	19,793	0.273	1.369
66 to 75	37,500,000	11.5%	70	1.094	8,663	0.231	1.159
76 to 85	28,800,000	8.8%	55	0.860	4,687	0.163	0.816
86 to 95	23,500,000	7.2%	53	0.828	3,504	0.149	0.748
96 to 105	34,000,000	10.4%	54	0.844	5,177	0.152	0.764
106 to 115	27,500,000	8.4%	51	0.797	4,331	0.158	0.790
116 to 125	21,500,000	6.6%	56	0.875	3,296	0.153	0.769
126 to 135	20,000,000	6.1%	55	0.860	3,360	0.168	0.843
136 to 145	18,500,000	5.7%	57	0.891	2,855	0.154	0.774
146 and Above	43,500,000	13.3%	66	1.032	9,592	0.221	1.106
Total	327,300,000	100.0%	64	1.000	65,257	0.199	1.000
Avg PoLC	**0.832**						

Policy Year 1998 as of 12/2000

PoLC Range	Earned Prem	Pctg of Total Earned Prem	Projected Ultimate Loss Ratio	Loss Ratio Relativity	Ultimate Claim Counts	Avg Claim Frequency per $1000 Earned Premium	Frequency Relativity
Less than 65%	75,500,000	23.3%	78	1.264	25,670	0.340	1.301
66 to 75	41,200,000	12.7%	64	1.037	11,124	0.270	1.033
76 to 85	36,500,000	11.3%	55	0.891	8,395	0.230	0.880
86 to 95	21,500,000	6.6%	56	0.908	4,945	0.230	0.880
96 to 105	29,500,000	9.1%	52	0.843	6,490	0.220	0.842
106 to 115	25,000,000	7.7%	53	0.859	5,250	0.210	0.804
116 to 125	20,000,000	6.2%	50	0.810	4,000	0.200	0.765
126 to 135	19,500,000	6.0%	53	0.859	4,485	0.230	0.880
136 to 145	17,400,000	5.4%	56	0.908	3,306	0.190	0.727
146 and Above	38,200,000	11.8%	63	1.021	11,078	0.290	1.110
Total	324,300,000	100.0%	62	1.000	84,743	0.261	1.000
Avg PoLC	**0.799**						

Policy Year 1997 as of 12/2000

PoLC Range	Earned Prem	Pctg of Total Earned Prem	Projected Ultimate Loss Ratio	Loss Ratio Relativity	Ultimate Claim Counts	Avg Claim Frequency per $1000 Earned Premium	Frequency Relativity
Less than 65%	74,500,000	23.6%	75	1.261	23,468	0.315	1.273
66 to 75	39,750,000	12.6%	58	0.975	9,739	0.245	0.990
76 to 85	35,500,000	11.2%	54	0.908	7,988	0.225	0.909
86 to 95	20,500,000	6.5%	53	0.891	4,100	0.200	0.808
96 to 105	28,900,000	9.1%	50	0.841	5,636	0.195	0.788
106 to 115	25,000,000	7.9%	52	0.874	5,125	0.205	0.828
116 to 125	19,100,000	6.0%	51	0.857	4,107	0.215	0.869
126 to 135	17,600,000	5.6%	54	0.908	3,344	0.190	0.768
136 to 145	16,900,000	5.3%	53	0.891	3,803	0.225	0.909
146 and Above	38,200,000	12.1%	61	1.026	10,887	0.285	1.152
Total	315,950,000	100.0%	59	1.000	78,194	0.247	1.000
Avg PoLC	**0.797**						

Workers' Compensation
Calculation of Indicated Rate Increases by PoLC Range

Pctg of Loss Cost Pricing Range	PY 2000 Loss Ratio Relativity (From Exhibit 2)	PY 1999 Loss Ratio Relativity	PY 1998 Loss Ratio Relativity	PY 1997 Loss Ratio Relativity	Indication Based on PY 2000 Relativities	Indication Based on PY 1999 Relativities	Indication Based on PY 1998 Relativities	Indication Based on PY 1997 Relativities	Selected Rate Change
Less than .65	1.321	1.297	1.264	1.261	58.6%	55.7%	51.7%	51.3%	55.0%
.66 to .75	1.077	1.094	1.037	0.975	29.3%	31.3%	24.5%	17.0%	25.0%
.76 to .85	0.862	0.860	0.891	0.908	3.4%	3.2%	7.0%	8.9%	5.0%
.86 to .95	0.890	0.828	0.908	0.891	6.9%	-0.6%	8.9%	6.9%	5.0%
.96 to 1.05	0.804	0.844	0.843	0.841	-3.5%	1.3%	1.1%	0.9%	5.0%
1.06 to 1.15	0.862	0.797	0.859	0.874	3.4%	-4.3%	3.1%	4.9%	5.0%
1.16 to 1.25	0.890	0.875	0.810	0.857	6.9%	5.0%	-2.8%	2.9%	5.0%
1.26 to 1.35	0.804	0.860	0.859	0.908	-3.5%	3.2%	3.1%	8.9%	5.0%
1.36 to 1.45	0.862	0.891	0.908	0.891	3.4%	6.9%	8.9%	6.9%	5.0%
1.46 and Above	1.048	1.032	1.021	1.026	25.8%	23.8%	22.5%	23.1%	23.5%
Total	1.000	1.000	1.000	1.000	20.0%	20.0%	20.0%	20.0%	20.0%

Detail Analysis for '1.46 and Above' Range.

Year	Range	*Earned Premium*	Projected *Ultimate* Loss Ratio	*Loss Ratio* Relativity	Indication Based on *L/R* Relativities
2000	> 146, Exper Mod < 1.25	7,231,875	61.0	0.88	5.1%
	> 146, Exper Mod > 1.25	13,430,625	79.5	1.14	36.9%
1999	> 146, Exper Mod < 1.25	15,225,000	55.0	0.86	3.2%
	> 146, Exper Mod > 1.25	28,275,000	71.9	1.12	34.9%
1998	> 146, Exper Mod < 1.25	13,370,000	53.0	0.86	3.1%
	> 146, Exper Mod > 1.25	24,830,000	68.4	1.11	33.0%
1997	> 146, Exper Mod < 1.25	13,370,000	52.0	0.87	4.9%
	> 146, Exper Mod > 1.25	24,830,000	65.8	1.11	32.8%

Percent of Loss Cost Report
Workers' Compensation

Total

Effective Year	Quarter	Written Premium (000's)	Loss Costs (000's)	PoLC	LCM	OTHR	SRP	EXPER
1999	1	88,500	104,468	84.7	1.10	0.998	0.848	0.910
1999	2	79,600	96,117	82.8	1.08	0.997	0.874	0.880
1999	3	85,500	98,813	86.5	1.11	0.999	0.867	0.900
1999	4	73,700	86,921	84.8	1.11	0.998	0.860	0.890
1999	Total	327,300	386,319	84.7	1.10	0.998	0.862	0.895

Total

Effective Year	Quarter	Written Premium (000's)	Loss Costs (000's)	PoLC	LCM	OTHR	SRP	EXPER
2000	1	87,615	99,437	88.1	1.09	0.998	0.895	0.905
2000	2	77,575	91,054	85.2	1.06	0.998	0.910	0.885
2000	3	81,588	91,485	89.2	1.09	0.999	0.900	0.910
2000	4	69,550	76,514	90.9	1.10	0.998	0.920	0.900
2000	Total	316,328	358,491	88.2	1.08	0.998	0.905	0.900

Percent of Loss Cost Report
Workers' Compensation

New

Effective Year	Quarter	Written Premium (000's)	Loss Costs (000's)	PoLC	LCM	OTHR	SRP	EXPER
2000	1	23,500	29,083	80.8	1.07	0.998	0.855	0.885
2000	2	22,000	26,438	83.2	1.06	0.998	0.874	0.900
2000	3	23,000	27,341	84.1	1.05	0.999	0.867	0.925
2000	4	20,000	22,833	87.6	1.09	0.998	0.880	0.915
2000	Total	88,500	105,696	83.7	1.07	0.998	0.868	0.906

Renewal

Effective Year	Quarter	Written Premium (000's)	Loss Costs (000's)	PoLC	LCM	OTHR	SRP	EXPER
2000	1	64,115	70,354	91.1	1.10	0.998	0.911	0.913
2000	2	55,575	64,616	86.0	1.06	0.998	0.925	0.879
2000	3	58,588	64,144	91.3	1.11	0.999	0.913	0.904
2000	4	49,550	53,681	92.3	1.10	0.998	0.937	0.894
2000	Total	227,828	252,795	90.1	1.09	0.998	0.921	0.898

Renewal Price Increase Goals
Workers' Compensation

Policy	Experience Mod > 125%	Expiring Premium	PoLC	Target Renewal Price Chg	Target Renewal Exposure Chg	Target Renewal Premium
101112	N	5,000	120	5%	3%	5,408
123456	N	2,500	130	5%	3%	2,704
212223	N	25,000	70	25%	3%	32,188
345678	Y	30,000	150	35%	3%	41,715
567891	N	7,500	110	5%	3%	8,111

and so on...

Policy level detail sums into summary reports for policies that renewed.

Renewal Price Increase Report
Workers' Compensation

State	(A) Expiring Premium (000's)	(B) Renewing Premium	(C) Target Renewal Premium	(B)/(A) - 1 Renewal Price Change	(C)/(A) - 1 Goal
AL	5,000	5,375	5,300	7.5%	6.0%
AR	1,500	1,620	1,620	8.0%	8.0%
CA	2,500	2,650	2,638	6.0%	5.5%
CO	2,000	2,200	2,240	10.0%	12.0%

and so on...

Mining Insurance Data to Promote Traffic Safety and Better Match Rates to Risk

Gregory L. Hayward,
FCAS, MAAA, FCIA, CPCU

Mining Insurance Data To Promote Traffic Safety and Better Match Rates to Risk

By Greg Hayward, FCAS, MAAA, FCIA, CPCU

Abstract

Operating or riding in a vehicle is one of the most dangerous things the typical person does on a regular basis. This paper describes how one company is using new technologies and techniques to mine massive amounts of vehicle crash statistics. In 1998, the company invested in new data mart technology that opened the door to more sophisticated analysis of real world insurance claims data by vehicle, by driver, and by geographic area. This paper will discuss the new data mart and illustrate some data mining tools. Four examples will be used to illustrate how the data is being mined to promote safety and better match rates to risk. These include vehicle safety, dangerous intersections, child passenger safety, and teenage driver safety.

Introduction

The CAS Constitution challenges us to advance the body of knowledge of actuarial science. The Actuarial Code of Professional Conduct obligates us to fulfill the profession's responsibility to the public and to uphold the reputation of the actuarial profession. The CAS Statement of Principles Regarding Property and Casualty Ratemaking emphasizes to us how important it is that actuaries derive rates that protect the insurance system's financial soundness and promote equity and availability for insurance consumers. These three objectives are repeated in the ASOP #12 concerning Risk Classification. The purpose of this paper is to expand our actuarial horizons by focusing on using new technologies and techniques to mine massive amounts of insurance data not only to promote financial soundness, equity, and availability but also to promote traffic safety.

Operating or riding in a vehicle is one of the most dangerous things the typical person does on a regular basis. A vehicle crash with injury occurs on average every ten seconds in the United States. More than half of us will be involved in an injurious vehicle crash during our lifetime. Over 3.2 million people are seriously injured or killed every year in vehicle crashes. Over 375,000 children (age 0 to 15) are among those who are killed or injured. Our profession is uniquely positioned with the skills, the data, and new technology that can make a positive difference for the benefit of all society. By embracing new technologies, we can open up opportunities for actuaries not only to better match rates to risks, but also to serve as broader based problem solvers working to reduce risks.

Data Mart Technology

Data is at the heart of nearly everything actuaries do. We desire data that is accurate, appropriate, and comprehensive. As noted in ASOP #23, such data is seldom, if ever, available to our complete satisfaction. No matter how much of this four-letter word we have, we crave more. The good news is, with today's technology, there is no excuse to be data-starved. If you have not recently reinvented your actuarial data base technology, it's time to think again.

In 1998, my company embraced new data technology by investing in a state-of-the-art SP2 data mart. At the time this data mart was built, it was the largest commercial DB2 database parallel processor in the United States. It consists of 130 nodes with sixteen 18.2 GB disk drives per node. There are four processors per node for a total of 520 processors and 2,080 disk drives. This adds up to a total of 37 terabytes of storage capacity. (A terabyte=1,000 gigabytes = 10^12 bytes) It would take a stack of CD's higher than the CN Tower, the world's tallest building, to match the storage capacity of this SP2 technology. Exhibit 1 provides a photo of the data mart.

As of July 2001, this data mart contained roughly 14 terabytes of transaction level data. This includes nearly all policy related and claims related transactions, allowing us the flexibility to slice and dice the data with multivariate analysis in ways that were never before possible.

The following suggestions are drawn from the lessons we learned in the experience of setting up a comprehensive data mart. It is critical that you have an in-depth understanding of the data that will populate your data mart, along with the hardware and software needed to store and process the data. Design your system to be as flexible as possible, plan for growth, and expect frequent changes. Take the time to determine: (1) the specific data elements that you will need, (2) at what point in the business cycle the data will become available, (3) how frequently the data will be refreshed, and (4) the number of years the data will be kept. When selecting data elements, be inclusive yet avoid the urge to include everything except the kitchen sink. Develop a data dictionary that contains the source of the data, the specific coding system, the data edits, and the data validations. Review your data loading procedures to partition data among the disk drives in a way that optimizes the access speeds. Establish rigorous quality control procedures and make sure you comply with privacy laws. Never think you are finished upgrading your system. Appreciate and protect the value of the investment that your company has in its data.

Data Mining Tools

Giving actuaries access to terabytes of detailed data was like putting kids in a candy store. It opened up endless opportunities to do actuarial work that was previously impossible. It has been, by far, the biggest structural change I have seen in my twenty-plus years of actuarial work. At the same time, we faced some major challenges, such as figuring out where to start and what tools to use. It required learning new skills and training our actuarial staff how to access and use the new data mart. As with any change, some have embraced the new technology more quickly than others. Some have merely done what they had previously done in a more efficient manner. Others have been more creative and have realized the power of this technology by doing new and more sophisticated multivariate analyses.

We evaluated numerous tools and decided to use SAS as our primary interface to the DB2 database. Exhibit 2 illustrates the main menu our actuaries use to access the data mart.

Using SAS for the desktop, we created basic lookup programs for frequently used applications. These queries produce common data elements such as exposures, premiums, claim frequency, claim severity, pure premium, expenses, loss ratios, etc. By selecting the desired combinations of states, type of vehicle, coverage, driver class, vehicle, etc. these buttons provide data lookups for many variations. Exhibits 3 and 4 provide illustrations of how actuaries obtain the data for frequently used applications such as limits analysis and loss development triangles. These data queries can be tailored to any combination of attributes and data elements on the data mart.

A very useful technique is to download this data into an EXCEL pivot table. To facilitate this each of these applications has a button programmed for this purpose. Exhibit 5 illustrates how a pivot table was created so that the actuary can quickly analyze how the data elements change for various combinations of variables. Significant correlations and risk relationships can be discovered that were previously not visible using traditional ratemaking techniques.

Two questions encountered in data mining are: (1) how to identify the most significant and predictive risk characteristics and eliminate the insignificant ones and (2) once identified, how to use them in ratemaking procedures.

The term "data mining" is often used to refer to a specific group of computer-intensive techniques (such as neural networks, decision trees, and association rules) which are used to detect patterns and relationships in large volumes of data. In this paper the term is used in a more general way. It is beyond the scope of this paper to discuss the various data mining software on the market or to endorse one over another. However, for purposes of illustration, Exhibit 6 provides a brief summary of the use of the data mart in conjunction with step-wise regression analysis as one way to identify the most significant risk characteristics.

After the most significant risk characteristics are identified, various ratemaking methods can be utilized to match the rates to the risks. It's beyond the scope of this paper to provide an exhaustive discussion of how such methods can utilize the data mart. For illustrative purposes, one such method is to begin with a traditional univariate analysis of the loss ratios for each significant risk characteristic. The initial indicated rate factors are calculated to equalize the loss ratios within that risk characteristic, for example, the age of the principal operator. The univariate analysis is repeated for each of the risk characteristics. The resulting initial rate factors become the seed for an iterative multivariate rate factor analysis that uses the data mart. The premium for every combination of risk characteristics is recomputed using the rating factors for the initial seeds. The loss ratios for each risk characteristic are then recomputed based on the summation of the premiums and losses in the individual cells from the data mart. These loss ratios are used to calculate the next iteration of indicated factors. The iterative process is repeated until the indications stabilize. Exhibit 7 provides an algebraic illustration of how the data mart can be used with this ratemaking procedure.

Putting the Technology to Use

As actuaries doing ratemaking, how many of us are viewed as heartless bearers of bad news about rate increases? Yes, it is important to mine the data to match rates to risk through cost based pricing, but is there more that can be done? The following are four examples of mining massive amounts of insurance data to not only better match rates to risk, but also to promote traffic safety. Each is a work in progress as we continue to refine the analysis and do additional multivariate studies. Each has received considerable attention from the media. [1]

Vehicle Safety

Even though all vehicles are made to meet federal motor vehicle standards, consumers are demanding and manufacturers are delivering advanced safety features. Manufacturers have recognized that "safety sells" and they are attempting to differentiate their vehicle's safety features to promote sales.

The insurance industry has long been very active in promoting vehicle safety. The Insurance Institute for Highway Safety performs crash tests and provides the public with valuable safety information. [2] While crash tests are important, insurance claims data captures how well vehicles perform in the real world laboratory. Insurance data measures the frequency and severity of crashes of all types and medical treatments associated with those crashes.

How well a vehicle performs in the real world depends on how all of the safety features interact in the event of a crash. These include among others the braking systems, the head and belt restraints, the types of airbags, the visibility including daytime running lights and rear brake lights, how well the crumple zones absorb the crash energy, and how well the passenger cage protects the occupants. Overall vehicle safety is not based on these items in isolation, but instead on their interaction during actual crashes.

Effective January 1, 2001, the company implemented a new Vehicle Safety Discount that was based on an analysis of claims experience from millions of crashes. Exhibit 8 provides a summary of the 2000 model year vehicles that qualified for the maximum vehicle safety discount.

The analysis was based on the most recent three years of claims experience by make and model of car. The claims experience was adjusted for distributional bias by using the loss ratio method with adjustments for age and gender related class differences. The experience was then adjusted to recognize fixed expenses. In multivariate analysis, sparsity of data is an important consideration. In accordance with ASOP #25, a credibility procedure was used when a particular model's experience was not fully credible. The standard for full credibility was based on 30,000 vehicles (for 3 years or 10,000 per year). Partial credibility was assigned based on the square root rule. The complement of credibility was assigned to the loss experience for a similar group of vehicles; however, the maximum credibility assigned to any similar grouping was 50%. Any credibility not assigned to the vehicle's claims experience or the similar group's claims experience was assigned to the average for all cars.

An issue from this analysis that received considerable media attention was the vehicle safety ratings by body style. Each body style had some models that qualified for the maximum vehicle safety discount of 40%. Contrary to some of the media coverage, very few SUV models qualified for the maximum vehicle safety discount.

The following table provides a distribution of the vehicle safety discounts applicable to the medical and personal injury protections premiums.

	Vehicle Safety Discounts for 2000 Models		
Body Style	40%	30%	20%
2-door cars	6 models	23 models	51 models
4-door cars	24	22	62
2-wd pickups	5	12	9
4-wd pickups	3	12	8
Station wagons	2	14	4
SUV's	3	30	24
Vans	11	9	11

The feedback we have received from consumers has been very positive and underscores consumers' interest in vehicle safety when purchasing vehicles. Each of the major auto manufacturers has been in contact with us to review the claims experience for the vehicles they produce. This has prompted some very healthy discussions about ways to further improve vehicle safety.

As a brief example, roughly two-thirds of all injuries involve neck sprains and strains. This amounts to billions of dollars of treatments for neck injuries each year. The federal standards for head restraints have not been updated in over 30 years. Some manufacturers have developed safer head restraints that are showing positive results in our data analysis. The National Highway Traffic Safety Administration (NHTSA) is now considering a proposed rule change for head restraints. [3]

Using these data mining techniques we hope to work with manufacturers, NHTSA, and others to promote vehicle safety to reduce injuries in automobile accidents while at the same time better matching rates to risks.

Dangerous Intersections

The screeching of tires, followed by the sound of metal smashing metal, happens on average every 11 seconds at an intersection somewhere in the United States. A significant number of them are deadly. The National Safety Council reported 8,514 fatal crashes at intersections in 1999. [4] Roughly one-third of all injury accidents occur at intersections. Rear-end collisions are the most frequent at intersections because vehicles often are required to stop. Side-impact collisions are also a common event with frontal collisions being less prevalent at intersections.

Taking advantage of our terabytes of data and data mining tools, we sought to identify the most dangerous intersections in the United States and Ontario. Unlike traditional traffic engineering studies, which focus on the number of vehicles passing through intersections, this study mines the insurance data in order to focus on driver behaviors and places greater emphasis on safety-driven solutions to intersection problems. We used the data mining tools to sort through every claim, determining where each one happened, and to add them street by street. Our initial analysis counted all accidents the same, whether they were fatal or just fender-benders. We have now developed a crash severity index that considers two levels of property damage severity and the presence of injuries in the crash. A two-year period (1999 and 2000) was utilized. The claims were adjusted to a common baseline using the company's percentage of vehicles insured by area.

In June 2001 the company released the latest results of this analysis for the ten most crash-prone intersections in the United States. Separate lists were created for 38 states, the District of Columbia, and Ontario. The national top ten list is displayed in Exhibit 9.

The Company offers grants up to $120,000 per intersection to the communities on the national list and up to $20,000 per intersection to municipalities on state lists to fund engineering studies and low-cost improvements. Roughly 100 grants have been issued for approximately $2.4 million in the first phase of this program. The second phase will make another $5 million available for studies and improvements at intersections on the new lists. Some of the recommendations have been as follows:

- Add traffic signals and/or improve location of existing signals
- Install better traffic signal timing to prevent rear-end collisions
- Designate left turn-only lanes and allow only protected left turns
- Improve visibility of signals by making them brighter and larger
- Provide larger street signs
- Give pedestrians more time to cross
- Install skid-resistant pavement
- Improve lighting at intersections
- Relocate driveways that are too close to intersections

It is too early to claim success, but there is early evidence that the number of crashes has been reduced at locations where intersection design improvements were implemented as a result of safety studies we funded. Using these data mining techniques, we hope to work with traffic safety officials and others to make our intersections safer while at the same time providing valuable information in matching rates to risk.

Child Safety

Motor vehicle crashes remain the leading cause of death and disability in the United States for children over the age of one. In an effort to improve this situation, my company has partnered with The Children's Hospital of Philadelphia to do a comprehensive investigation of how and why children are injured or killed in crashes. This study is the largest single research project in the country devoted exclusively to pediatric motor vehicle injury.

Partners for Child Passenger Safety is led by a multidisciplinary research team of internationally recognized experts in medicine, biomechanics, engineering, health education, advocacy and behavioral science. While the study is on-going, preliminary findings have already been published in some prestigious and scientific journals including the Journal of the American Medical Association. [5]

The Partners study is unprecedented in size and scope. The study uses crash database analysis in conjunction with in-depth telephone interviews, near real time on-site crash investigations, and computer crash simulations. The study is examining the entire range of crash and injury severity, from the most minor to the most severe. That helps to explain why children are injured in some crashes but not in others.

The following are some preliminary findings regarding the injuries children are sustaining:

- Sixty-four percent of significant injuries sustained by children are to the head (8% face, 6% neck/back/spine, 8% upper extremity, 6% abdomen, 5% lower extremity, and 3% chest).
- Fifteen percent of children come into contact with something in the vehicle.
- Forty percent made contact with the back of the seat in front of the child (34% with the door, window, or side panel; 20% with broken glass; 15% with a loose object in the vehicle; 13% with the dashboard or windshield; and 13% made contact with another occupant).

The following are some preliminary findings about why children are injured:

- Children not restrained are three times more likely to sustain significant injury
- Eighty-two percent of car seats are being misused in some way ranging from minor misuse such as not being tightly fastened to gross misuse such as not even being attached to the vehicle
- Eighty-three percent of children between the ages of 4 and 8 are inappropriately restrained in adult seat belts
- Sixteen percent of children age 12 and under are inappropriately seated in the vehicle front seat
- Thirty percent of infants are incorrectly turned forward facing in their car seat before age one

Exhibit 10 shows that children in seat belts were 3.5 times more likely to suffer a significant injury in a crash than children in car seats or booster seats. In particular, children in seat belts were 4 times more likely to suffer significant head injury than those in car or booster seats. It is clear that premature graduation to adult seat belts places young children at risk.

There is hope that the Partners for Child Passenger Safety project will produce recommendations that will significantly reduce deaths and injuries to children in auto accidents.

Teenage Drivers

Traffic safety for teenage drivers is a major health problem that has received far too little attention. Automobile crashes are the number one killer of young people between age 15 and 20. Over 6,000 teenagers die every year in traffic accidents and another 600,000 are injured. Per mile traveled, teen drivers have the highest crash risk of any age group.

Our data mart contains valuable information on millions of teenage drivers. We have been able to mine this insurance data in the development and support of programs aimed at the teenage driver problem. In cooperation with the American Driver and Traffic Education Association, we have developed and experimented with a new program aimed at an agent/parent/driver team to focus on gaining experience and having a safe attitude.

The agent/parent/driver team program is called Steer Clear™ and is aimed at two powerful deterrents to auto crashes: experience and attitude. Experience comes with time, practice, and exposure to a variety of driving situations. Attitude comes with being patient, not being aggressive or taking unsafe risks, and not becoming distracted. The program provides a period in which driving is supervised during the safest hours of the day under controlled circumstances, allowing young drivers to gain experience under safer conditions. The program includes an 11-minute video, a magazine, and requires the teen driver to complete a pre-trip and post-trip log for 30 trips of at least 15 minutes each. Supervised trips are also logged by the passenger. The trips are to cover a variety of driving experiences in varying weather conditions including every-day trips to work or school, running errands, social trips with passengers, nighttime trips, dusk/dawn trips, and highway trips. Upon completion of the program there is a quiz and a personal meeting with their insurance agent. Although not mandatory, the program also offers a parent/driver agreement form.

We are carefully mining the data that has come from this experiment. As shown in Exhibit 11, the preliminary results are encouraging. Teenagers completing the Steer Clear program have roughly a 20% better claim frequency at each age level.

Using these data mining techniques, we hope to refine these teenage driver programs to save as many lives and prevent as many injuries as possible. In the process we are providing a ratemaking tool to better match rates to risk as those completing the program receive a substantial discount.

Conclusion

Today's technology provides us better access to data and better data mining tools than ever before. These tools are extremely valuable in our quest to match rates to risk in accordance with our ratemaking principles and standards of practice. However, we can go beyond that to use these tools to promote safety for one of the most dangerous things the typical person does on a regular basis. This paper has discussed how one company is using this technology to mine insurance data to both promote traffic safety and better match rates to risk. Four examples were discussed including vehicle safety, dangerous intersections, child passenger safety, and teenage driver safety.

I hope that this paper has provided helpful information about how technology is changing the way actuaries make rates and has challenged you to find ways in which actuaries can be of further benefit to society as a whole. The value and recognition of our profession will be significantly enhanced if we wisely use our skills, data, and technology to promote financial soundness, equity, availability, and safety.

References

[1.] Major Media Coverage of the Four Analyses

(a) Vehicle Safety

The New York Times carried a front-page story about the new vehicle safety discount on November 28, 2000. Each of the major nightly news networks carried stories about the analysis. It also received coverage in more than 200 media outlets including U.S. News and World Report, Wall Street Journal, Associated Press, National Public Radio, Los Angeles Times, Washington Post, Miami Herald, Dallas Morning News, Boston Herald, Detroit News, and Bloomberg News.

(b) Dangerous Intersections

NBC's "Dateline" covered the Dangerous Intersections analysis in both June 1999 and June 2001. Major stories also appeared both times in USA Today. The analysis received the attention of daily newspapers representing two-thirds of the readership of all of the dailies in the United States. In addition, an estimated 80 million television viewers across the U.S. saw at least a portion of the video news release.

(c) Child Safety

Both the "Today Show" and "Dateline" have featured stories about the analysis. This study of child passenger safety has received coverage in more than 500 media outlets including CNN's Headline News, MSNBC, USA Today, The New York Times, Los Angeles Times, Chicago Tribune, and Parents Magazine.

(d) Teenage Driver Safety

The video portion of the teenage driver safety program has received two awards. Other media coverage has primarily occurred in the two states in which the experiment occurred (Arizona and Maryland).

[2.] The Insurance Institute for Highway Safety, www.hwysafety.org, Vehicle Ratings

[3.] National Highway Traffic and Safety Administration, Federal Motor Vehicle Standard 202, www.nhtsa.dot.gov

[4.] National Safety Council, www.ncs.gov, Injury Facts, 2000 Edition

[5.] Published Research regarding the Partners for Child Passenger Safety Study

(a) "The Danger of Premature Graduation to Seat Belts for Young Children," Pediatrics, June 2000, Volume 105, No. 6

(b) "Factors Influencing Pediatric Injury in Side Impact Collisions," Advancement of Automotive Medicine, October 2000

(c) "The Exposure of Children to Airbags," Advancement of Automotive Medicine, October 2000

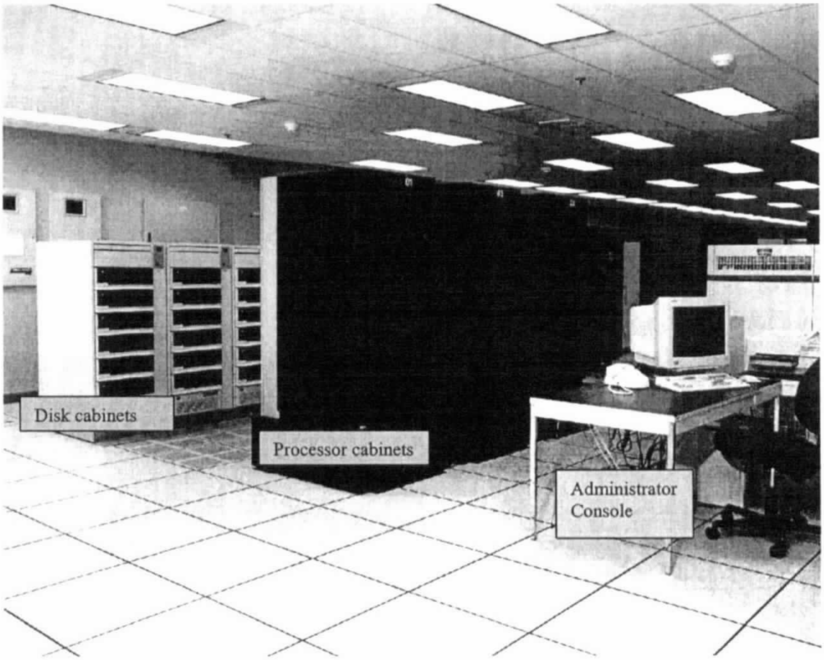
(d) "Misuse of Booster Seats," Injury Prevention, December 2000

(e) "Partners for Child Passenger Safety: A Unique Child-Specific Crash Surveillance System," Accident Analysis & Prevention, May 2001

(f) "Seat Belt Syndrome: A Case Report and Review of the Literature," Pediatric Emergency Care, in press

Exhibit 1

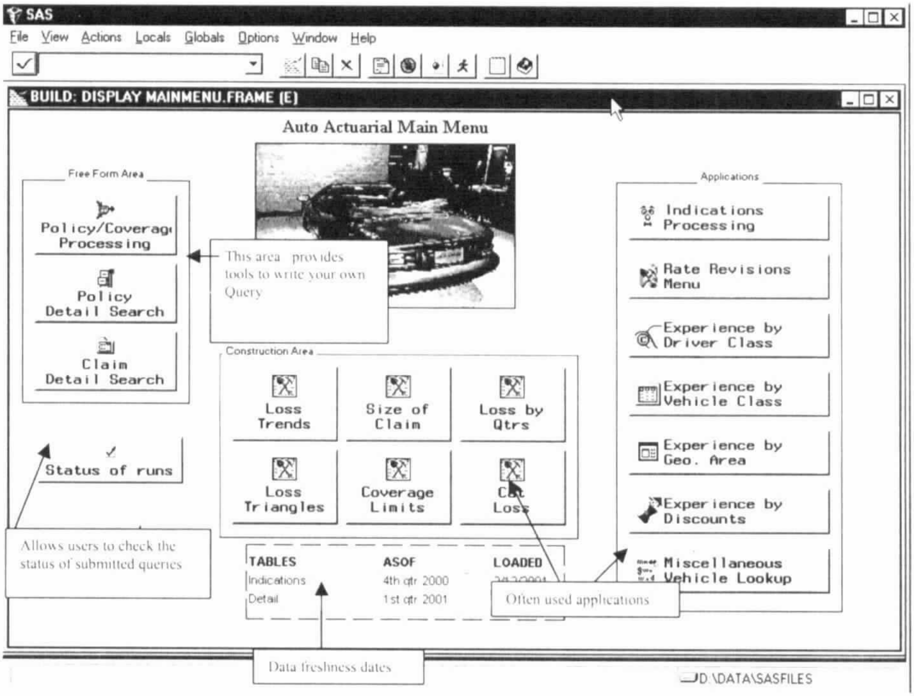
Photo of the Data Mart



This photo shows some of the cabinets that house the processors and disk storage units of the SP2 data mart. Each disk cabinet contains drawers of disc drives where the data is stored. In the event a disk fails, the offending disk can be swapped out and reloaded from a backup, normally with no disruption to the person using the system. The processor cabinets contain the processors, memory, and local disk for the operating system. The processors are grouped into modules for easy access and replacement. A standard RS/6000 workstation is at the center of the SP2 data mart management system. It is the focal point for systems administration. Each actuary's workstation is connected to the data mart through the company's network.

Exhibit 2

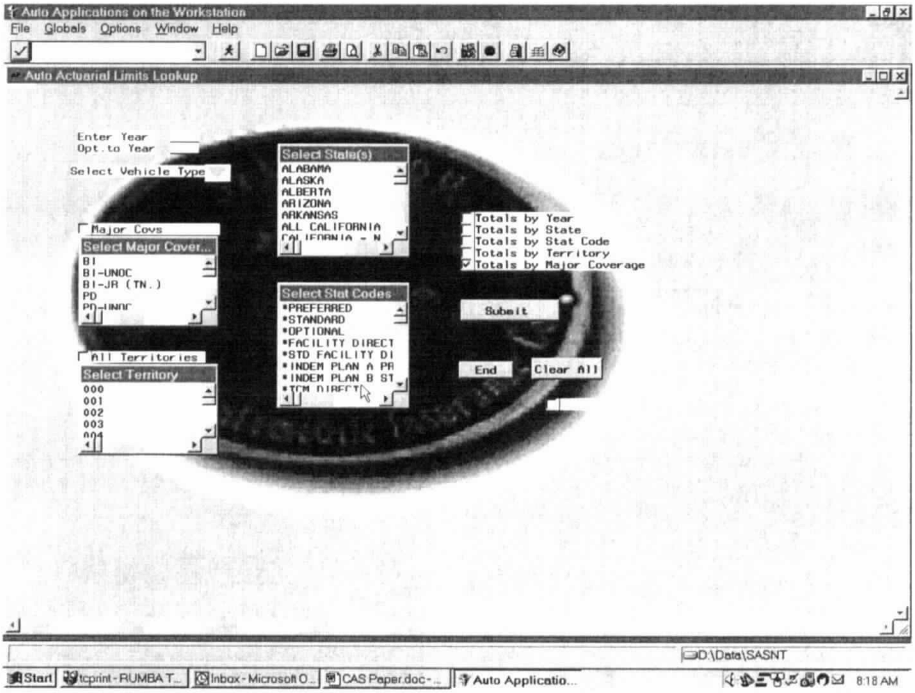
Main Data Mart Menu



This is the main menu that actuaries use to access the data mart. Various buttons have been created to provide the actuary a means to quickly and flexibly download the data needed to do ratemaking analyses and to conduct data mining research.

Exhibit 3

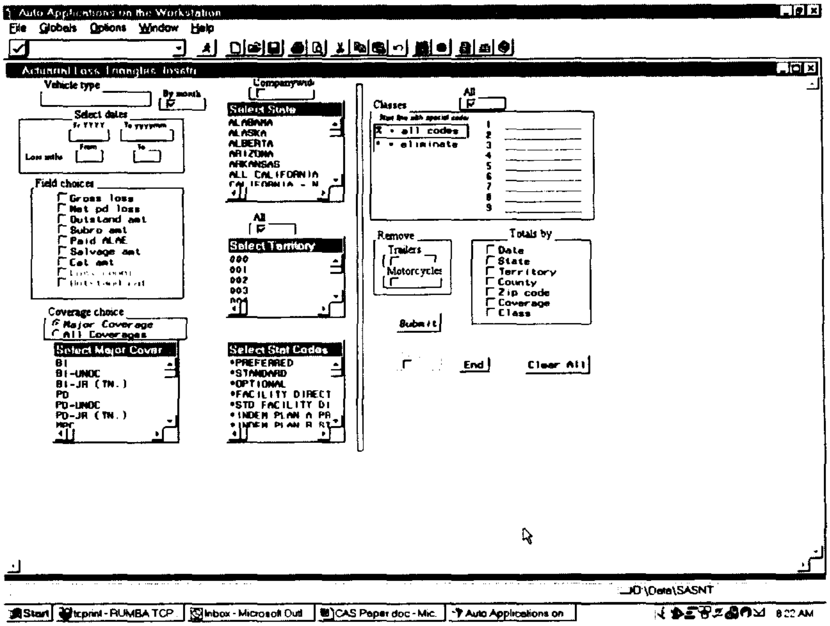
Illustration of the Data Menu for Limits



It is common for actuaries to review data by limit of coverage when doing ratemaking analysis. This menu provides the actuary a means to quickly and flexibly download data from the data mart by limit for various analyses. Similar menus were constructed for other information that is frequently needed for ratemaking such as size of claim, catastrophe experience, age of driver, vehicle usage, accident and conviction records, make and model of vehicle, deductibles, geographic areas, and miscellaneous vehicles.

Exhibit 4

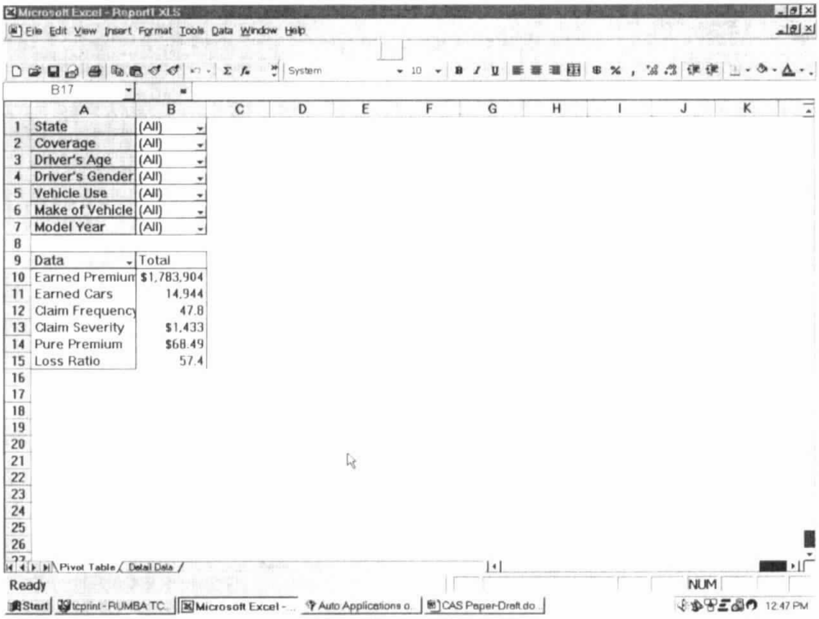
Illustration of the Data Menu for Loss Triangles



It is common for actuaries to sort various claim and claim expense data into loss triangles. This menu provides the actuary a means to quickly and flexibly download data from the data mart for various analyses. Similar menus were constructed for other frequently used ratemaking applications such as premium, loss, and expense trends.

Exhibit 5

Illustration of Using Pivot Tables



	A	B
1	State	(All)
2	Coverage	(All)
3	Driver's Age	(All)
4	Driver's Gender	(All)
5	Vehicle Use	(All)
6	Make of Vehicle	(All)
7	Model Year	(All)
8		
9	Data	Total
10	Earned Premium	$1,783,904
11	Earned Cars	14,944
12	Claim Frequency	47.8
13	Claim Severity	$1,433
14	Pure Premium	$68.49
15	Loss Ratio	57.4

This pivot table has been populated with detailed information for seven variables from the data mart. These seven variables are state, coverage, driver's age, driver's gender, vehicle use, make of vehicle, and the model year. These variables can be selected in any combination desired. For example, for the "state" button, the actuary can select a specific state or any combination of states. The same is true for each of the seven variables. Once a selection is made for each of the variables, the pivot table provides six data elements. Any variable or data element in the data mart can be put into this type of pivot table.

This tool is primarily used for exploratory data analysis. It provides a means to quickly sort, filter, and summarize data with the click of a few buttons. The dimensionality and flexibility of the analysis is limited only by the creativeness of the actuary and the credibility of the data.

Identifying Significant Risk Characteristics in the Data Mart, A Step-Wise Regression Approach

Identify the target variable you wish to predict. In identifying significant risk characteristics in an insurance environment, this might be claim frequency, loss ratio, or pure premium.

Transform the target variable in order to facilitate the predictions. When dealing with characteristics and insurance performance of individual exposure units, a large percentage of the observations will have a frequency, loss ratio, or pure premium value of zero. The purpose of this target variable transformation is to make the variable more Normally distributed, and improve the model predictions over the entire range of values. Examples might involve logarithms or roots, for example: (1.00 + square root of loss ratio).

Search the available information about the exposure units loaded onto the data mart - sources might include any available internal insurance statistical data files based upon the company's own data collection, both before and after the risks were insured, as well as demographic information obtained from external sources, and consumer report information.

Explore the available data items for quality and usability as independent variables – consider missing or invalid values, extreme values, and accuracy of coding. Sometimes it helps to review relationships to the target variable based upon univariate analysis.

Develop derived variables that are relationships between two variables or derivatives of single variables. A simple example is that the birth year could be used to derive the age of the applicant during each calendar year.

Derive meaningful intervals for some variables. For example, it might be useful to divide applicants into several different age intervals, instead of keeping all of the different integer values of age. Sometimes this is a good way to deal with missing and extreme values.

In this manner, a pool of candidate risk characteristics (independent variables) is developed. It is not unusual to include hundreds of risk characteristics in the pool of candidates.

Identifying Significant Risk Characteristics in the Data Mart, A Step-Wise Regression Approach

Next, a step-wise multivariate regression analysis might proceed as follows:

All of the candidate variables are tested, and the one which explains the greatest amount of variation in the (transformed) target variable is selected as the first independent variable.

Next, all of the remaining candidate variables are tested, assuming that the first independent variable will also be used. The one that explains the most variation in the target variable, above and beyond what is explained by the first variable alone, is selected as the second variable.

This process is repeated. Tests are performed to determine whether some previously added variables should be deleted, since, for example, two selected independent variables might be highly correlated. Correlations are determined after each step. Various statistical tests are performed to help determine whether each variable is a significant contributor.

Eventually, adding more variables does not significantly improve prediction of the dependent target variable.

Exhibit 7

An Iterative Multivariate Rate Factor Analysis Procedure

Let X_i = indicated rate factor for the x variable, i^{th} class
Y_j = indicated rate factor for the y variable, j^{th} class
Z_k = indicated rate factor for the z variable, k^{th} class
L_{ijk} = losses in the ijk^{th} cell from the data mart
P_{ijk} = premium in the ijk^{th} cell from the data mart

Note: For ease of illustration, this exhibit assumes a three variable analysis of multiplicative related variables. The process is the same for additional variables and can be modified for other than multiplicative relationships.

In the iterative process:

$$X^1_i = X_i \cdot \frac{\sum_i L_{ijk}}{\sum_i P_{ijk}} \Bigg/ \frac{\sum_{ijk} L_{ijk}}{\sum_{ijk} P_{ijk}}$$

$$Y^1_j = Y_j \cdot \frac{\sum_j L_{ijk}}{\sum_j P_{ijk}} \Bigg/ \frac{\sum_{ijk} L_{ijk}}{\sum_{ijk} P_{ijk}}$$

$$Z^1_k = Z_k \cdot \frac{\sum_k L_{ijk}}{\sum_k P_{ijk}} \Bigg/ \frac{\sum_{ijk} L_{ijk}}{\sum_{ijk} P_{ijk}}$$

Repeat the process replacing X^1_i for X_i, Y^1_j for Y_j, Z^1_k for Z_k and

Replacing P_{ijk} with $P_{ijk} \cdot \frac{X^1_i}{X_i} \cdot \frac{Y^1_j}{Y_j} \cdot \frac{Z^1_k}{Z_k}$

Repeat the process until the indications stabilize.

Exhibit 8

Vehicle Safety Discount

54 Models Received the Maximum Discount for Model Year 2000*

Acura RL 4 door model

Audi A4 4-door model

Audi A6 4-door and Station Wagon

BMW 528i, 540i, 740i, 750i, M5 4 door models

Chevrolet Corvette 2dr and Convertible models

Chevrolet Express 2500 and 3500 Vans

Chevrolet Silverado 2500 4wd pickup

Chevrolet Silverado 3500 2wd and 4wd pickups

Chevrolet Suburban C1500 and K1500 SUV's

Chrysler Town and Country Van

Dodge B2500 Ram Van/Wagon

Dodge Ram 3500 2wd pickup

Ford Econoline E150, E250, and E350 vans

Ford F150 4wd pickup

Ford F250 and F350 2wd pickups

GMC Savana 1500, 2500, and 3500 vans

GMC Sierra 1500 2wd pickup

GMC Yukon XL C1500 SUV

Infiniti Q45 4-door model

Jaguar S-type, VDP, XJ8, XJR 4-door models

Jaguar XK8 and XKR 2-door models

Lexus GS 300, GS 400, and LS 400 4-door models

Mercedes-Benz E320, E430, and E55 AMG 4-door models

Pontiac Montana van

SAAB 9-3 Convertible

SAAB 9-3 and 9-5 4-door models

Volvo C70 Convertible

Volvo S40, S70, and S80 4-door models

Volvo V70 Station Wagon

- **Medical Payments and Personal Injury Protection Coverage Premiums Discounted 40%**

Exhibit 9

Top 10 Most Dangerous Intersections

City/State	Location	Danger Index*
1. Pembroke Pines, Fla.	Flamingo Road and Pines Boulevard	2568
2. Philadelphia, Penn.	Red Lion Road and Roosevelt Boulevard	2317
3. Philadelphia, Penn.	Grant Avenue and Roosevelt Boulevard	2204
4. Phoenix, Ariz.	7th Street and Bell Road	2089
5. Tulsa, Okla.	51st Street and Memorial Drive	2000
6. Tulsa, Okla.	71st Street and Memorial Drive	1995
7. Phoenix, Ariz.	19th Avenue and Northern Avenue	1975
8. Plano, Tex.	State Highway 121 and Preston Road	1937
9. Metairie, La.	Clearview Parkway and Veterans Memorial Boulevard	1925
10. Sacramento, Calif.	Fair Oaks Boulevard and Howe Avenue	1912

*** The danger index is determined by the number of crashes at various intersections, how many of those crashes involved injury, and the severity of those crashes. It is adjusted to account for the percentage of vehicles insured by the Company in areas where the intersections are located.**

Exhibit 10

Child Passenger Safety

Premature Graduation To Seat Belts Places Young Children At Risk

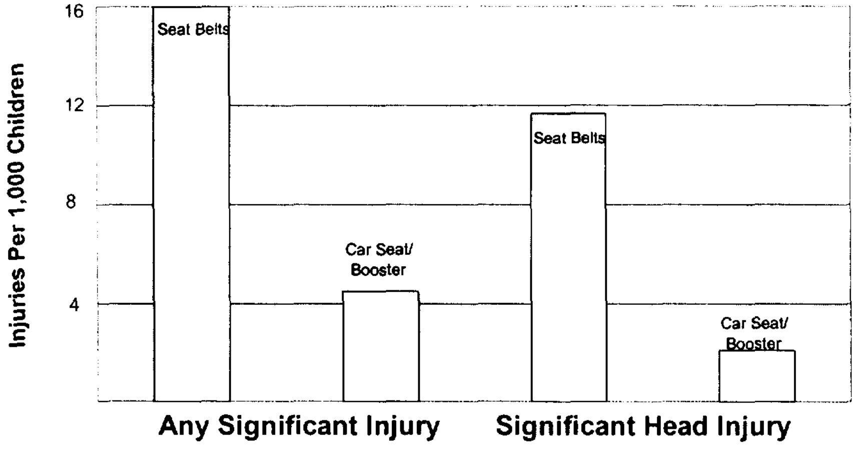
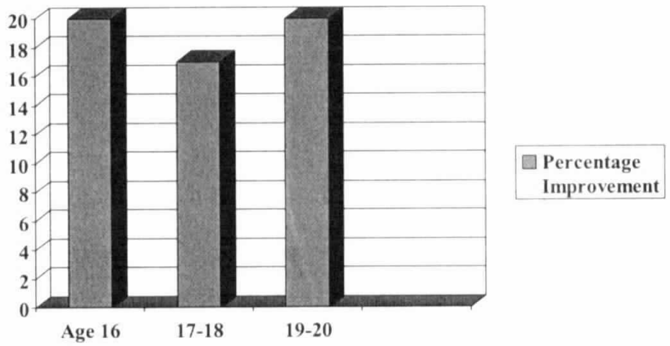


Exhibit 11

Teenage Drivers

Percentage Reduction In Claim Frequency of Those Completing the Steer Clear Program, Among Different Age Categories



Dependence Models and the Portfolio Effect

Donald F. Mango, FCAS, MAAA and
James C. Sandor, ACAS, MAAA

Dependence Models and the Portfolio Effect

Donald Mango, FCAS and James Sandor, ACAS
American Re-Insurance

Abstract

This paper describes efforts to estimate the "portfolio effect" — the diversification benefit from assembling a portfolio – by simulating the implied portfolio-level capital safety standard for various contract-level capital safety standards. The results showed that apparently aggressive contract-level capital standards still implied conservative portfolio-level capital safety standards. Taken at face value, this would have had a dramatic impact on pricing decisions.

However, the method used to generate the simulated contract outcomes — the Normal copula — was found to generate asymptotically independent tail samples, thus understating the tail of the portfolio outcome distribution. Tail-based risk measures were, therefore, understated as well.

This provides compelling evidence why actuaries must utilize alternative dependence models beyond the Normal copula.

Key words: dependence models, Normal copula, portfolio effect, capital allocation.

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Dependence Models and the Portfolio Effect

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1. Introduction

Many re-insurers assess the price of their business using some form of contract-level capital allocation — e.g., ruin threshold, marginal standard deviation, expected policyholder deficit ("EPD") [1], or tail conditional expectation ("TCE") [4]. Practical application of any of these capital measures in contract pricing requires (i) stochastically modeled contract outcome distributions, and (ii) a selected "safety standard" for that risk measure (e.g., 99^{th} percentile for ruin threshold). The more stringent the safety standard, the more capital will be allocated. But what contract safety standard should a company use? And what *portfolio* safety standard does that contract-level standard imply? In other words, what is the "**portfolio effect**" — the diversification benefit of writing a contract as a part of a large portfolio rather than on its own?

This paper presents results of a simulation study of the relationship between contract-level safety standards and the implied portfolio safety standard for the expected policyholder deficit and ruin threshold risk measures. The study uses a simulation model of a portfolio of reinsurance contracts programmed in the S-Plus language[1].

Using a standard technique for generating a multivariate log-Normal sample with correlation — the "**Normal copula**" — across a reasonable range of inputs, apparently aggressive contract-level safety standards roll up to prudent portfolio-level safety standards. Similarly, more conservative contract-level safety standards roll up to extremely conservative portfolio safety standards.

Taken at face value, these results challenge popular thinking about reinsurance pricing using capital allocation, as the portfolio effect is greater than anticipated. However, upon deeper analysis, it appears the effect may be overstated due to limitations in the dependence modeling implicit in the Normal copula. In other words, an accepted "standard" actuarial simulation technique understated the tail of the portfolio outcome distribution. Tail-based risk measures were, therefore, understated as well.

This provides compelling evidence why actuaries must utilize alternative dependence models beyond the Normal copula.

This paper proceeds through six additional sections. Section 2 presents an overview of the study. Section 3 provides details of the study, and Section 4 describes in depth all the calculations for a single example iteration of the study. Section 5 explains the results

[1] The results of the study in Excel pivot tables, as well as the S-Plus script file (program), will be posted on the CAS website.

of the study. Section 6 addresses the concept of dependence modeling. Section 7 gives conclusions.

2. Overview of the Study

This study was prompted by efforts at American Re to calculate risk-based capital amounts for individual contracts. The methods tested were expected policyholder deficit ("EPD") and ruin threshold. To calculate contract capital using a particular method, a contract-level safety standard is needed. An EPD standard based in part on A.M. Best information was available at the portfolio level, and there were several popular anecdotal ruin thresholds (e.g., 99^{th} percentile).

Clearly the portfolio-level standard would be too conservative to use at the contract level, due to diversification — the elusive portfolio effect. But how much should the standard be relaxed at the contract level? And is there any way to tie the selected contract standard to the portfolio standard? In order to make an informed decision, and to test the assumptions, we conducted the simulation study described below.

The results were surprising and contrary to our expectations. Because of the widespread use of similar techniques in re-insurance (and some primary insurance), we felt it would be beneficial to put the details and results of the study into the public domain. It is our hope that this study will prompt deeper discussion about choices of a dependence model with respect to diversification and capital allocation.

3. Details of the Study

The impact of these four variables was studied:

A. Individual contract expected loss
B. Aggregate portfolio standard deviation
C. Inter-contract correlation measure
D. Contract-level risk measure standard

For each iteration of the simulation, we selected a value for each of these variables.

A. Individual Contract Expected Loss
We modeled the portfolio as comprised of identical contracts of various expected loss amounts. We tested seven different individual contract expected loss amounts:

$5M, $10M, $15M, $25M, $50M, $75M, and $100M

We assumed the entire portfolio of contracts had a total expected loss of **$1 Billion**. Given that amount, the choice of an average contract size determines how many contracts of that size make up the portfolio. For example, there would be two hundred \$5M contracts, one hundred \$10M contracts, etc.

B. Aggregate Portfolio Standard Deviation
We tested aggregate portfolio coefficients of variation ("CV's") of **0.29, 0.32, and 0.36**. We considered these to be reasonable values for the overall portfolio variability. Given the \$1B total expected loss, these CV's determined the portfolio standard deviation.

C. Inter-contract Correlation Measure
We tested three different inter-contract correlation levels for input to the multivariate Normal copula: **15%, 20%, and 25%**. Since the process involves generating Normal samples, then exponentiating these to derive log-Normal samples, these measures in fact represent the correlation between the log of the contract outcomes. We assumed this correlation was constant between all contracts.

D. Contract-level Risk Measure Standard
We tested the following levels for EPD and ruin threshold:

EPD: **20%, 15%, 10%, 7.5%, and 5%**
Ruin: **15%, 12.5%, 10%, 7.5%, and 5%**

Contract Loss Distributions
Given:

A. *Contract Expected Loss* (hence number of contracts),
B. *Aggregate Portfolio Standard Deviation*, and
C. *Inter-contract Correlation,*

individual contract variance is uniquely defined. Aggregate portfolio variance is the sum over the entire covariance matrix. The diagonal elements of the covariance matrix are the individual contract variance (assumed constant). Each off-diagonal element is the individual contract variance multiplied by the inter-contract correlation (assumed constant). Thus, for N contracts,

Contract Variance = v
Aggregate Portfolio Variance = V
Inter-contract correlation = ρ

$$V = Nv + \rho N(N-1)v = v[N + \rho N(N-1)]$$
$$v = V / [N + \rho N(N-1)]$$

We assumed a log-Normal distribution for the individual contracts, because it is a skewed distribution that represents aggregate contract loss distributions reasonably well. It is also straightforward to generate correlated log-Normal samples using the multivariate Normal distribution. We determined the μ and σ parameters for the log-Normal using moment matching.

Contract-level Capital
For each iteration, we selected a Total Asset amount ***A*** for each contract, based on either EPD or ruin threshold. For example, a ruin threshold of 99% (1% ruin probability) for a log-Normal distribution with known parameters is simply the 99^{th} percentile of that distribution. This amount would be ***A***.

A is composed of premium and capital. For purposes of the study, we assumed the premium amount was the individual contract expected loss amount, implying contract capital $C = A - E[L]$.

Implied Portfolio Capital
The implied portfolio capital is the sum of the calculated individual contract capital amounts. The portfolio expected loss is the sum of individual contract expected loss amounts. The sum of these two items gives the portfolio asset amount. In order to determine what risk measure standard this total asset amount corresponds to, we needed to determine an aggregate portfolio loss distribution. We did this using simulation.

Using the μ and σ parameters and the selected inter-contract correlation, we generated 5,000 samples from a multivariate Normal distribution with the number of variables equal to the implied number of contracts. Log-Normal samples were then created by exponentiating the generated Normal samples. For each iteration, the sum of these log-Normal sampled loss amounts is the simulated portfolio total loss.

The 5,000 iterations produce an empirical portfolio aggregate loss distribution. We could then calculate the risk measures using this distribution. Portfolio ruin probability is estimated as the number of iterations where portfolio loss exceeded the total portfolio assets divided by the total number of iterations. Portfolio EPD is the expected value of the amount by which portfolio loss exceeded the total assets.

4. Detailed Explanation of an Example Iteration

This section provides details of a single example iteration of the study. As stated above, for each set of simulations we selected a different scenario from each of four variables:

A. Contract Expected Loss (7 possibilities)
B. Aggregate portfolio standard deviation (3)
C. Inter-contract correlation (3)
D. Contract-level risk measure standard (5)

In the actual study, the simulation was repeated 315 times (7 × 3 × 3 × 5). For this example, we will select one value from each of the above variables.

A. Contract Expected Loss
In this case, we will use **$10M** as our individual contract expected loss. Since we are keeping our aggregate expected loss fixed ($1B), this individual contract expected loss implies 100 contracts.

B. Aggregate portfolio standard deviation
We used different CV scenarios to come up with our implied aggregate portfolio standard deviation. Here we will use a **0.32** CV, which implies an overall portfolio standard deviation of $320M.

C. Inter-contract correlation
For this example we will use **0.20**. We already have the first moment of our individual contract loss distribution by assumption ($10M). Our selection of inter-contract correlation, combined with our assumption with respect to aggregate portfolio standard deviation, implies a unique second moment for our individual contract loss distribution.

Contract Variance = v
Aggregate Portfolio Variance = $V = (320M)^2$
Inter-contract correlation = $\rho = 0.20$
Number of Contracts = $N = 100$

$$V = Nv + \rho N(N-1)v = v[N + \rho N(N-1)]$$
$$(320M)^2 = v[(100) + (0.20)(100)(99)]$$
$$v = (320M)^2 / (2080)$$
$$v = (7.016M)^2$$

An intuitive way to visualize this is to picture our 100 × 100 covariance matrix, which represents our entire portfolio of contracts. By assumption, the sum of this matrix must add up to $(320M)^2$. In the case of independence, only the diagonal of our covariance matrix would be populated and our individual contract variance would simply be equal to V / N. As we increase the correlation, the variance along the diagonal becomes diluted as we spread more and more of the total variance to the off-diagonal cells in our matrix. The $\rho N(N-1)$ term in the above expression represents the strength of this dilution.

D. Contract Loss Distributions
Our individual contracts have an expected loss of $10M with a variance of $(7.016M)^2$. This implies a contract coefficient of variation of 0.7016. Since we are assuming a log-Normal distribution for individual contracts, we can solve for the σ parameter by using the following relationship.

$$CV = \sqrt{e^{\sigma^2} - 1}$$
$$\sigma = \sqrt{\ln(CV^2 + 1)} \qquad (1)$$

where CV is the coefficient of variation.

The σ parameter for our contracts is the square root of $\ln[(0.7016)^2 + 1]$ which is equal to 0.6327. Similarly, we can solve for the μ parameter using the following relationship.

$$E[L] = e^{\left(\mu+\frac{\sigma^2}{2}\right)}$$

$$\mu = \ln(E[L]) - \frac{\sigma^2}{2} \qquad (2)$$

where $E[L]$ is the expected loss.

For our example, we know the expected loss is \$10M and we know σ is equal to 0.6327. This implies

$$\mu = \ln(\$10M)-(0.5)(0.6327)^2 = 15.918.$$

Contract-level risk measure and safety standard

We'll examine the above individual contract at a 10% expected policyholder deficit. A similar exercise can be performed for probability of ruin. We will take this result for the individual contract, and multiply it by the number of contracts in our portfolio to get total implied capital for our portfolio.

The deficit (D) for a given contract with a certain amount of assets (A) allocated to it, and an uncertain loss amount (X) can be defined as:

$$D = \begin{cases} X - A & X > A \\ 0 & X \le A \end{cases}$$

If $f(x)$ is the density function for the loss variable X, then the expectation of the deficit, D, is:

$$\begin{aligned} E(D) &= \int_{-\infty}^{\infty} D \cdot f(x)dx \\ &= \int_{0}^{A}(0)f(x)dx + \int_{A}^{\infty}(x-A)f(x)dx \\ &= \int_{A}^{\infty}(x-A)f(x)dx \\ &= \int_{A}^{\infty}(x)f(x)dx - \int_{A}^{\infty}(A)f(x)dx \end{aligned}$$

$$= -\int_0^A (x)f(x)dx + \int_0^A (x)f(x)dx + \int_A^\infty (x)f(x)dx - A[1-F(A)]$$

$$= \left[\int_0^A (x)f(x)dx + \int_A^\infty (x)f(x)dx\right] - \left[\int_0^A (x)f(x)dx + A[1-F(A)]\right]$$

$$= E(X) - E(X \wedge A)$$

where ***E*(*X*^*A*)** is the expected value of ***X***, limited to ***A***.

Typically, the expected deficit is expressed as a percent of expected loss, ***E*(*X*)**. This gives us:

$$EPD\% = \frac{E(X) - E(X \wedge A)}{E(X)} = 1 - \frac{E(X \wedge A)}{E(X)} \qquad (4)$$

For our specific example we have an EPD percent of 10% and the log-Normal parameters of our individual contract loss distribution μ and σ are 15.918 and 0.6327, respectively. We need to solve for ***A***, individual contract assets. This can be determined either via simulation or through numerical methods. In our case, ***A*** is equal to approximately $16.2M.

Implied Portfolio Capital and Safety standard

Continuing our example, assuming we write 100 identical $10M contracts, the implied portfolio capital would be 100 × $16.2M or $1.62B. The final question is, "How 'safe' is the portfolio?"

To create the distribution of losses for our portfolio, we simulated from a multivariate Normal distribution using our individual contract parameters and the selected correlation ρ, in this case 20%. In this example, since we have 100 contracts, each iteration of the simulation produces a vector of length 100. This vector is exponentiated, then summed. This procedure is repeated 5000 times to produce the loss distribution for our portfolio.

Using this loss distribution for the portfolio, it is a simple exercise to solve for EPD% in the above expression using A = $1.62B. For our example, using the simulation results from the study, this was equal to 0.0067 or 0.67%.

5. Results of the Study

Pivot Table of Results
Given the number of dimensions in motion here (four), the best way to assess the results is with a pivot table. A Microsoft Excel 97 file with pivot tables of results for ruin threshold and EPD is posted on the CAS website (www.casact.org). The pivot table allows the user to select aggregate portfolio CV and inter-contract correlation values. The tables then display contract expected loss down the column, and contract-level risk measure across the row. The table itself shows the resulting portfolio risk measure.

Tables 1 - 6 show the EPD and ruin threshold results for selected aggregate portfolio CV's and inter-contract correlation values.

S-Plus Script
S-Plus is a statistical programming environment produced by Insightful Software (www.insightful.com). The S language was first developed by Bell Labs. S-Plus is used extensively in the statistical community. It is a vector-based language with substantial statistical and simulation capabilities. It handles large amounts of data well, and runs large-scale simulations quickly. The script file is also on the CAS website for others to use or modify and extend the analysis.

Implications
For a range of reasonable input assumptions, aggressive contract-level safety standards (e.g., 10% EPD) appear to roll up to prudent portfolio-level safety standards. Similarly, more conservative contract-level safety standards (e.g., 1% EPD) roll up to extremely conservative portfolio safety standards.

For example, if the company wished to hold capital commensurate with an A rating (roughly corresponding to an EPD of 0.5%), they could use the study results to support contract safety standards anywhere from 5.0% to 10.0%. Similar examples could be found using ruin threshold. The implications of implementing such contract safety standards in pricing are dramatic. These results were far from those expected by underwriters and pricing actuaries. They were also far from the standard in use at the time of the study (1% EPD). Implementing even the most conservative standard — 5% EPD — would have represented a dramatic shift.

Whenever indications deviate dramatically from current figures, both sets are called into question. The same phenomenon occurred here; the divergence of indications from current values led us to backtrack and analyze each component step in the simulation study. The range of input assumptions held up under further review. However, the seemingly innocuous choice of simulation method did not.

6. Dependence Modeling

As described previously, our approach to generating multivariate log-Normal samples relied upon the Normal copula and linear correlation. This approach qualifies as de facto "standard practice" for many simulation exercises carried out by North American actuaries. We are familiar with the multivariate Normal distribution and linear correlation from our exam syllabus, and software products to generate samples from this distribution are widely available (e.g., Microsoft Excel with @Risk, S-Plus). That makes it familiar and convenient, but is it any good? Does it produce appropriate results?

Risk and capital measures focus on the tails of distributions, so simulation techniques should reasonably model aggregation risk as reflected in the tail of the portfolio distribution. We relied on the Normal copula to model that risk. Our results were to a large extent a function of the mechanics of the Normal copula and its implicit *dependence model.*

The concept of *stochastic dependence measures* is not on the North American actuarial syllabus yet. Correlation is, but correlation is only one measure from this broader and more general class. Quoting Embrechts et al [2]:

> "Some of the confusion [surrounding correlation] may arise from the literary use of the word to cover any notion of dependence. To a mathematician correlation is only one particular measure of stochastic dependence among many. It is the canonical measure in the world of multivariate normal distributions, and more generally for spherical and elliptical distributions. However, empirical research in finance and insurance shows that the distributions of the real world are seldom in this class." [2, p. 2]

In other words, linear correlation completely describes the dependence relationship among the variables for the classes of elliptical and spherical distributions, of which the multivariate Normal is a member. However, most skewed distributions – including the log-Normal – are not members of these classes. So the dependence relationship between individual variables in a multivariate distribution from non-elliptical and non-spherical classes is *not fully described* by the linear correlation matrix.

Asymptotic Tail Independence in the Normal Copula

Of particular concern to actuaries performing simulation studies is the asymptotic tail independence of the Normal copula. Section 4.4 of Embrechts et al [2] discusses this at length. Summarizing their conclusions:

> "Thus the Gaussian [Normal] copula gives asymptotic independence, provided that $\rho <$ 1. Regardless of how high a correlation we choose, if we go far enough into the tail, extreme events appear to occur independently in each margin." [2, p.19]

This is an alarming conclusion. Most actuarial risk measures focus on the tails. Any multivariate simulation exercise that systematically generates essentially independent tail samples will understate aggregate tail probabilities and, thus, understate the risk measure. The very portion of the curve we are focusing on is not being modeled properly by this "familiar and convenient" method.

A simple example will help reinforce this important concept. Figure 1 shows the plot of a 5000 point sample generated from a bivariate Normal distribution with $\mu = (12, 12)$, $\sigma = (0.5, 0.5)$, and correlation = 70%.

Figure 1

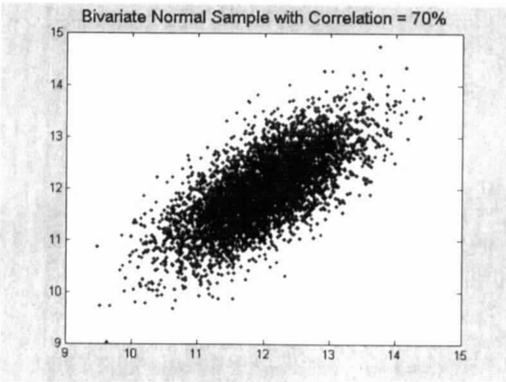
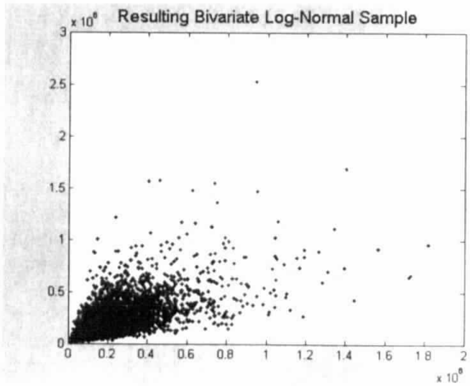


Figure 2



The sample correlation is 70.3%, showing the sample size is significant and the multivariate Normal generation algorithm is reasonably accurate. The correlation is also noticeable from the plot by the clustering of the points along the 45-degree line in an ellipsoid shape.

Figure 2 shows the plot of the bivariate log-Normal sample generated from the Normal sample by exponentiating every point. What is immediately apparent visually is the divergence of the points away from the 45-degree line. The divergence appears to grow wider as the magnitude of the generated loss amounts increases. This demonstrates the asymptotic tail independence of the bivariate log-Normal distribution.

Analytic measures of dependence fare no better. Apparently the simple act of exponentiating did not preserve the correlation, as the 70% sample correlation for the Normal sample drops to 64.3% for the log-Normal. Embrechts et al [2] explain why:

> "Linear correlation has the serious deficiency that it is *not* invariant under non-linear strictly increasing transformations." [2, Section 3.2]

If we perform this demonstration using more variables, the impact of the tail independence would be even more pronounced.

Other Copulas

Copulas are multivariate uniform-(0,1) distributions with a defined dependence relationship. Frees and Valdez [3] provide this definition:

> "*To define a copula, begin as you might in a simulation study by considering p uniform* (on the unit interval) random variables, u_1, u_2, ..., u_p. Here, *p* is the number of outcomes you wish to understand. Unlike many simulation applications, we do not assume that u_1, u_2, ..., u_p are independent; yet they may be related. This relationship is described through their joint distribution function
>
> $$C(u_1, u_2, \ldots, u_p) = \Pr(U_1 \le u_1, U_2 \le u_2, \ldots, U_p \le u_p).$$
>
> Here, *we call the function C a copula.*" [3, p.2]

If the multivariate distribution is continuous, the copula is unique. Per Embrechts et al [2], if it is unique, the copula can be interpreted as the *dependence structure*. Since the multivariate Normal is continuous, its copula is unique and, therefore, the dependence structure is unique and completely defined by the linear correlation. If we are using the Normal copula, there is no way to generate any more tail dependence than we have seen. The asymptotic tail independence is a fundamental characteristic of the Normal copula itself, and makes it a poor choice for many simulation studies. If actuaries want different dependence relationships, they must employ *different copulas*.

Embrechts et al [2], Frees and Valdez [3] and Venter [5] discuss several promising alternative copulas. Many of the explanations are steeped in difficult statistical language that hampers the communication effort to broad actuarial audiences. To facilitate wider acceptance and use of these copulas in the North American actuarial community, actuaries need to become more familiar with alternative dependence measures. In addition, both algorithms and demonstration software need to be placed in the public domain.

Alternative Dependence Measures: other copulas require measures of dependence besides linear correlation: for example, rank correlation, Kendall's tau, and comonotonicity. See Embrechts et al [2] for an extensive discussion of these measures.

North American actuaries need to understand these new measures, how they are calculated, how they might be estimated from insurance data, how they measure tail dependence in particular, and how they compare with correlation. Of perhaps primary importance is "plain English" translations of the often complex formulas, to help actuaries develop an intuitive comfort level. Also critical are techniques that evaluate the appropriateness of various copulas for the particular study. Venter [5] presents several measures focusing on tail dependence, which is relevant to risk and capital measurement.

Algorithms and Software: Perhaps the Normal copula enjoys such widespread use in part because of its prevalence in so many software packages. Linear correlation can be calculated in a spreadsheet. Well-documented, widely available software implementations of new dependence measures and copulas would substantially increase their use and facilitate further research.

7. Conclusion

This paper has presented compelling evidence for alternative dependence models to the Normal copula. Many of the listed references provide detailed explanations of these models, but often from a statistical perspective that is difficult for a broad audience to grasp. There is a need for publication of survey papers to translate these often difficult statistical concepts into terms accessible to a broader audience. Equally important is the need for public domain demonstration software, giving practical examples of the measurement and use of these methods.

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Table 1
EPD Summary
29% Portfolio CV, 15% Correlation

Portfolio CV	28.6%
Correlation	15.0%

Average of Portfolio EPD	Contract EPD				
Contract Size ($000's)	20.0%	15.0%	10.0%	7.5%	5.0%
5,000	4.40%	1.56%	0.28%	0.07%	0.02%
10,000	4.80%	1.64%	0.28%	0.12%	0.01%
15,000	5.48%	1.79%	0.33%	0.10%	0.02%
25,000	5.95%	2.14%	0.51%	0.12%	0.02%
50,000	7.02%	3.01%	0.75%	0.33%	0.06%
75,000	8.50%	3.60%	1.13%	0.41%	0.09%
100,000	9.16%	4.55%	1.48%	0.62%	0.14%

Table 2
EPD Summary
32% Portfolio CV, 20% Correlation

Portfolio CV	32.1%
Correlation	20.0%

Average of Portfolio EPD	Contract EPD				
Contract Size ($000's)	20.0%	15.0%	10.0%	7.5%	5.0%
5,000	6.04%	2.78%	0.73%	0.17%	0.06%
10,000	6.02%	2.68%	0.67%	0.28%	0.04%
15,000	6.59%	2.75%	0.73%	0.25%	0.05%
25,000	6.98%	3.30%	0.81%	0.35%	0.08%
50,000	7.96%	3.73%	1.33%	0.51%	0.11%
75,000	9.14%	4.37%	1.54%	0.75%	0.16%
100,000	10.11%	5.15%	1.98%	0.85%	0.35%

Table 3
EPD Summary
36% Portfolio CV, 25% Correlation

Portfolio CV	35.7%
Correlation	25.0%

Average of Portfolio EPD	Contract EPD				
Contract Size ($000's)	20.0%	15.0%	10.0%	7.5%	5.0%
5,000	6.94%	3.67%	1.18%	0.32%	0.10%
10,000	7.45%	3.47%	1.23%	0.50%	0.12%
15,000	7.87%	3.67%	1.19%	0.56%	0.19%
25,000	8.29%	3.99%	1.29%	0.68%	0.10%
50,000	8.83%	4.36%	1.72%	0.71%	0.27%
75,000	9.57%	5.20%	2.14%	1.13%	0.29%
100,000	10.48%	5.79%	2.34%	1.26%	0.33%

Table 4

Ruin Summary

29% Portfolio CV, 15% Correlation

Portfolio CV	**28.6%**
Correlation	**15.0%**

Average of Portfolio Ruin	Contract Ruin				
Contract Size	**15.0%**	**12.5%**	**10.0%**	**7.5%**	**5.0%**
5,000	2.58%	1.42%	0.50%	0.12%	0.08%
10,000	2.76%	1.20%	0.48%	0.28%	0.00%
15,000	3.22%	1.26%	0.52%	0.16%	0.04%
25,000	3.38%	1.52%	0.90%	0.18%	0.02%
50,000	3.78%	1.98%	0.88%	0.48%	0.08%
75,000	4.86%	2.26%	1.32%	0.38%	0.10%
100,000	4.58%	2.94%	1.52%	0.66%	0.12%

Table 5

Ruin Summary

32% Portfolio CV, 20% Correlation

Portfolio CV	**32.1%**
Correlation	**20.0%**

Average of Portfolio Ruin	Contract Ruin				
Contract Size	**15.0%**	**12.5%**	**10.0%**	**7.5%**	**5.0%**
5,000	4.22%	2.60%	1.44%	0.30%	0.14%
10,000	3.94%	2.46%	1.20%	0.54%	0.12%
15,000	4.20%	2.32%	1.20%	0.42%	0.10%
25,000	4.68%	3.06%	1.22%	0.56%	0.12%
50,000	5.00%	2.92%	1.82%	0.74%	0.18%
75,000	6.00%	3.22%	1.98%	0.98%	0.22%
100,000	6.18%	4.02%	2.62%	0.94%	0.46%

Table 6

Ruin Summary

36% Portfolio CV, 25% Correlation

Portfolio CV	**35.7%**
Correlation	**25.0%**

Average of Portfolio Ruin	Contract Ruin				
Contract Size	**15.0%**	**12.5%**	**10.0%**	**7.5%**	**5.0%**
5,000	5.00%	4.06%	1.86%	0.58%	0.26%
10,000	5.34%	3.14%	2.32%	0.90%	0.24%
15,000	5.98%	3.58%	2.16%	0.94%	0.32%
25,000	6.40%	3.80%	1.86%	1.18%	0.22%
50,000	5.94%	3.70%	2.20%	1.02%	0.46%
75,000	6.84%	4.74%	2.94%	1.68%	0.42%
100,000	7.52%	5.04%	2.96%	1.76%	0.50%

Reinventing Risk Classification—A Set Theory Approach

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Abstract

Risk Classification represents one of the most important and controversial topics of actuarial science. It is covered broadly throughout the Casualty Actuarial Society's exam syllabus. The importance and persistence of this topic is also reflected in the long array of papers that permeate the casualty actuarial literature. Most of the recent work on Risk Classification has focused on automobile insurance coverage, which is principally responsible for bringing the issue into the public debate. However, Risk Classification impacts on all types of insurance coverage and has ramifications beyond the world of insurance.

Risk Classification starts necessarily as a subjective process. The characteristics along which risks are delineated are intuitive at best. Traditional treatments of Risk Classification in the actuarial literature, in our view, do not provide the tools to move beyond intuition. In this paper, we will review the common definitions of Risk Classification by quickly glancing through two reference materials on the subject: the American Academy of Actuaries Risk Classification Statement of Principles [2] and Robert Finger's chapter on Risk Classification in the Foundations of Casualty Actuarial Science textbook [6]. Then, by building on the existing definitions, we will look to establish a more rigorous and consistent treatment of the subject. At the core of our treatment will be a non-traditional definition of the notion of *class*. We will borrow terminology from Set Theory[1] to help us in this endeavor. We will not only define more rigorously such concepts as ***homogeneity*** and *separation* but we will also integrate them into the very definition of Risk Classification. A method of Risk Classification will emerge as a natural byproduct of our definitions. This method, which may be described as what Venter [10, p. 345] terms a "credibility only" method, will provide an alternative to using arithmetic functions in Risk Classification schemes. To illustrate our newly

[1] Familiarity with elementary Set Theory, although not required, is helpful in order to understand the material presented herein. For an introduction to Set Theory, see Gilbert and Gilbert [7].

defined precepts of Risk Classification, we will construct a specific model using simulated observations. We will introduce a set of statistics that will allow us to make inferences about our model. Also, we will propose measures for assessing the relative efficiency of competing schemes and suggest procedures for validating a classification scheme. Finally, we hope that this paper will provide ideas to actuaries looking to build a Risk Classification scheme from scrap.

Introduction

Let us introduce an example where rates are being sought to provide professional liability coverage to actuaries. A *classification scheme* is proposed, which groups actuaries based on two criteria or *classification variables*: "area of practice" and "years of experience." "Area of practice" is subdivided into two (mutually exclusive) bands or *risk characteristics*: Life, Non-Life while "years of experience" is subdivided into two (mutually exclusive) *bands*: 10 or fewer, 11 or more. Four *cells* or sets of actuaries will emerge out of this arrangement: Life actuaries with 10 or fewer years of experience, Life actuaries with 11 or more years of experience, Non-Life actuaries with 10 or fewer years of experience, Non-Life actuaries with 11or more years of experience.

Why are we pooling actuaries into various cells in the first place? Couldn't we charge a single rate to all actuaries based on their combined experience? Taking this approach, we would run the risk of charging the same rates to groups that have fundamentally different loss propensities[2]. This would create subsidies across groups that carry both economic and social consequences. Conversely, are we to presume that actuaries across these cells have different loss propensities by virtue of our having separated them in this manner? Should we proceed to calculate rates for each cell based on its respective experience? Wouldn't we then run the risk of charging different rates to groups that essentially have the same loss propensity? This too might create subsidies with dire economic and social consequences. What if we had instead devised a classification scheme that grouped actuaries according to whether they were left-handed or right-

[2] Loss propensity may refer to either the probability distribution of the claim process for a cell or to the parameters and functions of parameters of the probability distributions within a cell.

handed and according to whether they sported bifocals or contact lenses (assuming all actuaries wear one or the other eye-device but not both)? Besides from lacking intuitive appeal, what separates the latter scheme from the former? Perhaps, we need to take a step back and ask ourselves what exactly is Risk Classification or its purpose. Let's look to the literature for guidance.

Current Definitions

The American Academy of Actuaries, Risk Classification Statement of Principles defines Risk Classification as "[the process of] grouping risks with similar risk characteristics for the purpose of setting prices. [2, p.1]" "Risk Classification", the Statement adds, "is intended simply to group individual risks having reasonably similar expectations of loss. [2, p.1]"

Robert Finger defines Risk Classification as the "formulation of different premiums for the same coverage based on group characteristics. [6, p.231]"

Discussion

Both the above definitions are intuitively appealing. However, in our opinion, they leave open certain key questions. For instance, the Statement's definition does not directly address the question of whether the risks across cells need to have different loss propensities. Finger, while implying in his definition that Risk Classification should recognize differences amongst cells, does not elaborate on how those differences might be recognized. The mere grouping of risks with similar characteristics, as suggested by the American Academy of Actuaries, seems like a rather incomplete goal of Risk Classification. We agree with Finger that Risk Classification must entail the emergence of differences amongst cells of risks. Otherwise, there would not be a need to classify in the first place. However, in our opinion, there should not be a presumption that any chosen risk characteristics, however intuitive, will result in cells that have different loss propensities. Before charging different rates to risks across different cells, it seems that one would need to be reasonably certain that the cells have different loss propensities. We believe that Risk

Classification should avoid two mistakes: charging the same rates to pools of risks that have fundamentally dissimilar loss propensities or charging different rates to pools of risks that have fundamentally similar loss propensities. The goal of risk classification should be to arrive at rates that closely represent the loss propensity of every risk while avoiding these the two types of mistakes.

Let's return to the classification scheme for actuaries introduced earlier. We should keep in mind that it may be appropriate to devise a different classification scheme[3] for each aspect of the claim process. For instance, the rating variables along which frequency is analyzed need not coincide with those used for severity. For simplicity, let's assume we are looking at only one aspect of the claim process and that aspect alone determines the differences (if any) in the cost of coverage between cells. Let's assume a probability model is initially derived for each cell based on the respective observations in each cell. Let's finally make the assumption that the models all have the same functional form and only their parameter values may differ. Let's review the following four scenarios:

Scenario 1: In the first scenario, the parameters underlying the models for life and non-life actuaries with 10 or fewer years of experience, respectively, can't be differentiated. We will say of these cells that they are compatible[4]. Under this scenario, life and non-life actuaries with 11 or more years of experience, respectively, are also compatible. Finally, under this scenario both life and non-life actuaries with 10 or fewer years of experience, respectively, are compatible with their more experienced counterparts. This scenario is illustrated in the chart shown in figure 1 below. Has Risk Classification been successful under this scenario? Can the process even be called Risk Classification? Do any of the cells defined above constitute *classes*? More importantly, should the observations of all or any of the cells be joined for the purpose of estimating the parameters of the models?

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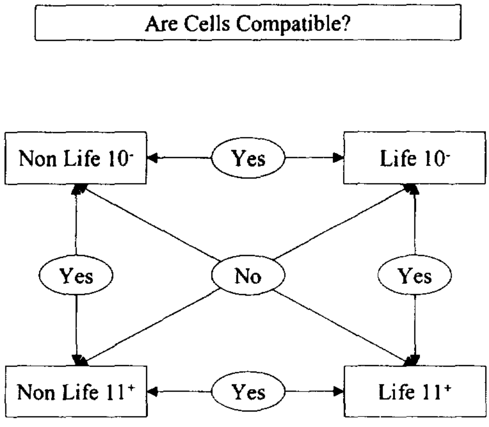
[3] If the processes are independent as is often assumed, it makes sense to classify them separately.

[4] This narrow definition of compatibility assumes symmetry. That is, given two cells C_i and C_k, if C_i compatible with C_k, this definition implies that C_k compatible with C_i and vice versa. This will not be the case in our general definition provided later in this paper. Also, a cell is compatible with itself by definition.

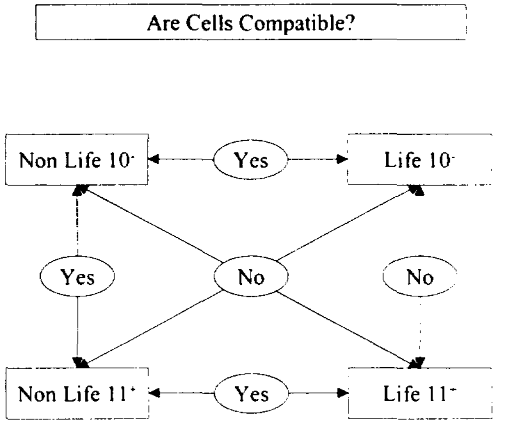
Figure 1: Compatibility[5] Chart for Scenario 1



Scenario 2: In the second scenario, it is found that the parameters underlying the models for life actuaries with 10 or fewer years of experience and those with 11 or more years of experience can be differentiated. We will say of these two cells that they are incompatible. Under the second scenario, it is found that life and non-life actuaries, respectively, who fall in the same experience group are compatible. It is also found that non-life actuaries with 10 or fewer years of experience are compatible with their more experienced counterparts. The compatibility chart is shown in figure 2 below. To what degree has Risk Classification been successful under this scenario? Do any or some of the cells defined above constitute *classes*? Should any of the cells be joined for the purpose of estimating the parameters of these models? If so, which?

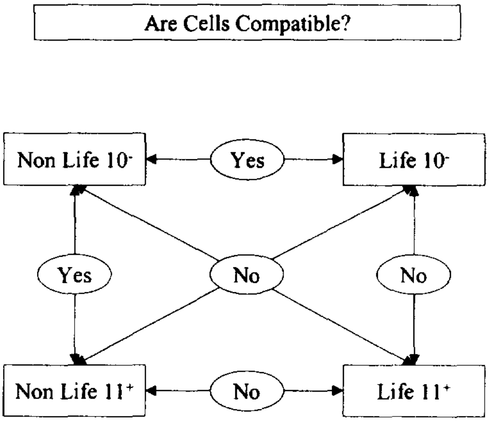
[5] Life actuaries are not compared with non-life actuaries falling in opposite experience groups, as these groups do not share any common characteristics. These comparisons would be irrelevant in the context of the given classification scheme. These pairs of cells will be defined later as non-adjacent and are incompatible by definition.

Figure 2: Compatibility Chart for Scenario 2



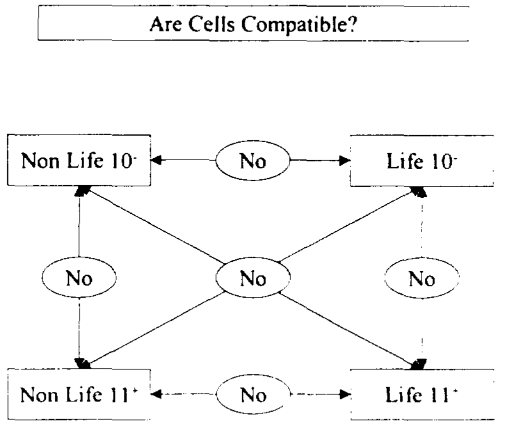
Scenario 3: In the third scenario, it is found that life actuaries with 10 or fewer years of experience and those with 11 or more years of experience are incompatible. Also, under this scenario, it is found that life and non-Life actuaries who fall in the 10 or fewer years of experience group are compatible while life and non-life actuaries who fall in the 11 or more years of experience group are incompatible. Finally, it is found that non-life actuaries with 10 or fewer years of experience are compatible with their more experienced counterparts. The compatibility chart is shown in figure 3 below. To what degree has Risk Classification been successful under this scenario? Do any or some of the cells defined above constitute *classes*? Should any of the cells be joined for the purpose of the parameters of these models? If so, which?

Figure 3: Compatibility Chart for Scenario 3



Scenario 4: Finally, in the fourth scenario, all pairs of cells are found incompatible. The compatibility chart is shown in figure 4 below. Is this the only scenario under which Risk Classification has been successful? Is this the only scenario in which the cells defined by the classification scheme constitute classes? Should any of the cells be joined for the purpose of estimating the parameters of these models? If so, which?

Figure 4: Compatibility Chart for Scenario 4



We have raised several questions in reviewing the preceding scenarios. Let's see how these questions could be answered from the perspective of the AAA's Statement of Principles and Robert Finger's chapter on the subject. Based on our understanding of the Statement of Principles, the pooling of actuaries suggested in our example would fit the AAA's definition of Risk Classification even before any of the scenarios are considered. Remember that the American Academy of Actuaries' Statement of Principles simply defines Risk Classification as "a grouping of Risk with similar risk characteristics." The Statement of Principles is silent on the issue of whether, and which cells should be joined for the purpose of estimating costs. The Statement of Principles does list credibility among three statistical considerations in designing a Classification scheme. Under this consideration, the Academy suggests that "it is desirable that each of the classes in a risk classification scheme be *large enough* to allow credible statistical predictions about that class...Accurate predictions for small, narrowly defined classes often can be made by appropriate statistical analysis of the experience for broader grouping of *correlative classes*. [2, p.10]" This implies that the parameters of a cell with a small number of observations may be estimated by joining it with other cells, while the parameters of a cell with a large number of observations may be based on that cell alone.

Would our grouping satisfy Robert Finger's definition of Risk Classification under the first scenario? Under that scenario, the grouping would be unable to formulate statistically different premiums based on the characteristics of each cell of actuaries. What about the second scenario where only one pair of cells shows differences in the parameters of their models, or the third scenario? Would our grouping fit Finger's definition under these scenarios? Finger is also silent on the issue of whether, when, and which cells should be joined for the purpose of estimating the models' parameters. Similarly to the American Academy of Actuaries Statement of Principles, Finger mentions credibility as one of four actuarial criteria for selecting rating variables. This criterion requires that "a rating group ... be large enough to measure costs with sufficient accuracy. [6, p.237]"

The notion of credibility, as presented in Finger [6] and the American Academy of Actuaries [2] and for that matter in most actuarial papers on Risk Classification and Ratemaking, is used in what Philbrick [8, p. 214] calls "[its] familiar sense (as opposed to its technical meaning) [as] almost a synonym for confidence." "[This] terminology", Venter [11, p. 382] tells us "is misleading if it implies that the credibility weight is an inherent property of the data. " Our definition of credibility, unlike that of Finger and of the Academy, will be analogous to the technical meaning of credibility as presented in Philbrick [8, p. 214] that is credibility is " the appropriate weight to be given to a statistic of the experience in question relative to other experience."

We view the *grouping* of the actuaries into the four cells as no more than the posing of a pair of hypotheses, which roughly state:

1) Actuaries within the same cell share the same loss propensity.
2) Actuaries across different cells have different loss propensities.

We will refer to the first and second hypotheses as the *homogeneity* and *separation*[6] hypotheses, respectively. Merely setting the hypotheses does not make them true. Merely selecting classification variables and risk characteristics that seem intuitive and reasonable does not mean that the resulting cells will satisfy the hypotheses. For, intuition and reasonableness remain only subjective concepts.

Homogeneity: It may be difficult to prove directly that all risks within a cell have the same loss propensity. However, this hypothesis may be proven false if one or more risk characteristics are found such that risks *within a cell can be subdivided to define new sub-cells and the risks across the newly defined sub-cells* have different loss propensities. Theoretically, there are an infinite number of risk characteristics that could be used to separate risks within a cell. In reality, most potential risk characteristics are either unknown or simply unfeasible to use. Hence, one is limited to a handful of characteristics from which to choose. When a classification scheme is proposed, one may test homogeneity by introducing additional characteristics (known and feasible) to see whether the risks across the newly defined sub-cells have different loss propensities. For instance, we may introduce pension as an additional area of practice by which to pool non-life actuaries. If no such characteristics emerge, we may assume the homogeneity hypothesis to hold. Alternatively, we may simply assume that a given classification scheme provides the smallest and finest pooling of risks and no further subdivision of the cells is possible. Therefore, the homogeneity hypothesis would hold by default.

Separation: This hypothesis can be tested by successively comparing the compatibility of different pairs of cells. We assume that a test or a statistic can be devised to answer the question of compatibility between pairs of cells. For instance, given a range of values of a chosen statistic, we may conclude that two given cells are incompatible and, therefore, their parameters need to be estimated independently of each other. Conversely, for values of the chosen statistic that fall outside the range, we would conclude that two given cells are compatible. Then, the law of large numbers dictates that the observations across both cells

[6] This concept is somewhat different than the one introduced by Michael Walters who, in his 1981 Dorweiler prize-winning paper Risk Classification Standards, defines separation as "a measure of whether classes are sufficiently different in their expected losses to warrant the setting of different premium rates [12, p. 11]."

provide a better estimate of the parameters underlying the statistical models of these cells rather than just the observations in each individual cell. If all pairs of cells were incompatible, we would then accept the separation hypothesis. Then the parameters underlying each cell in the classification scheme would be estimated by relying solely on the observations from that cell. If the separation hypothesis were rejected, then one of several alternatives could be accepted. These alternative hypotheses range from finding that all pairs of cells are compatible (no need to classify at all) to finding various combinations of cells that are compatible. For example, given a cell C and a set $C_{Compatible}$ representing the union of all cells that are compatible with C excluding C itself, the estimates of the parameters of C would be derived from observations taken from C together with those taken from $C_{Compatible}$.

Credibility: When the separation hypothesis fails, the new estimates of C based on observations taken from C together with those taken from $C_{Compatible}$ can also be viewed as the credibility weighted average of the estimates based on observations from C alone with estimates based on observations taken from $C_{Compatible}$. The new estimates E_{New} of the parameters of C might then be expressed as follows:

$$E_{New} = Z \times E_{Compatible\ Cells} + (1 - Z) \times E_{Old} \qquad (1)$$

where $E_{Compatible\ Cells}$ are the estimates based on $C_{Compatible}$, E_{Old} are the estimates based on C, and Z is the credibility weight assigned to the observations from $C_{Compatible}$. The value of Z may be calculated from the values of E_{Old}, $E_{Compatible\ Cells}$, and E_{New}. Seldom will we be interested in the value of Z if E_{New} is already known. Ultimately, if we know the value of Z, E_{Old}, and $E_{Compatible\ Cells}$, we want to be able to calculate E_{New} via credibility formula (1) above rather than through an additional estimation based on the collective data from C and $C_{Compatible}$. We can derive Z from the statistical assumptions made about the cells. For instance, if we assume the observations from C are from a normally distributed population with mean μ and variance σ^2, the population mean is estimated by the sample mean E_{Old}. If the separation hypothesis fails, we conclude that the observation from $C_{Compatible}$ are also from a normally distributed population with mean μ and variance σ^2 with the population mean estimated by the sample mean $E_{Compatible\ Cells}$, and the estimate E_{New} of the mean of the population C is given by formula (1) above. The credibility weight Z attached to the mean of the observations from $C_{Compatible}$ is given in Venter [11, p 381] as:

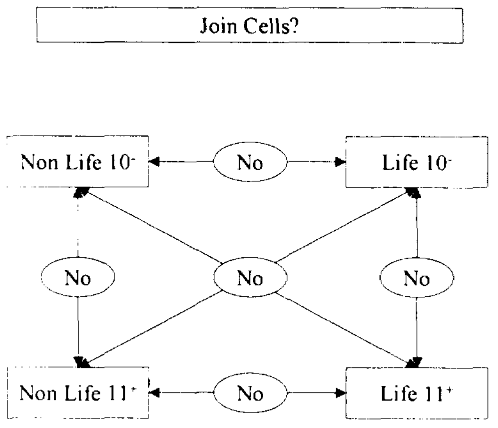
$$Z = n^{-1}/(n^{-1} + m^{-1}) = t^2/(s^2 + t^2) \qquad (2),$$

where m and n represent the number of observations in $C_{Compatible}$ and C, respectively, while s^2 and t^2 are the variances of the means of the observations in $C_{Compatible}$ and C, respectively.

Joining the cells

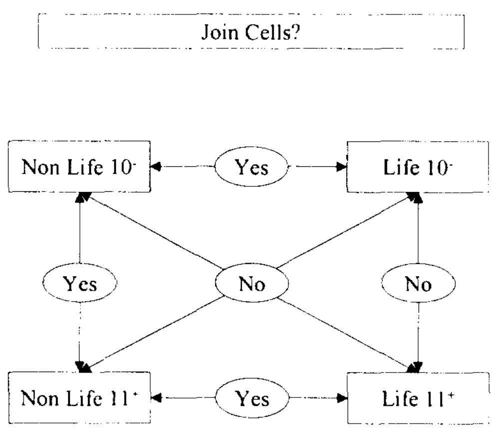
The manner in which cells should be joined for each of the scenarios introduced earlier is shown in figures 5 through 8 below. Under the fourth scenario, the separation hypothesis is true. In that scenario, the parameters underlying the probability model for life actuaries with 10 or fewer years of experience would be estimated based solely on the experience of that cell. The same would apply to the remaining three cells. This is illustrated in figure 5 below:

Figure 5: How to Join Cells in Scenario 4



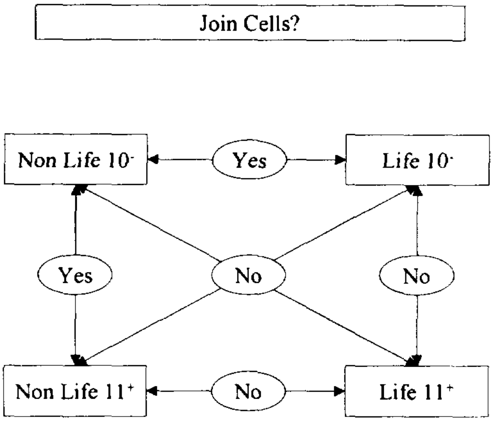
Scenarios one through three represent various alternative hypotheses to the separation hypothesis. In the second scenario, the estimates of the parameters of the models of all four cells would involve other cells as shown in figure 6 below:

Figure 6: How to Join Cells in Scenario 2



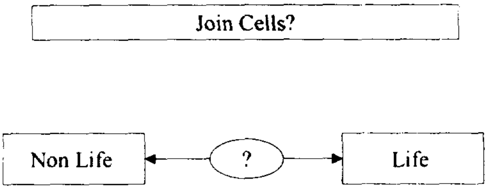
In the third scenario, the cells would be joined as shown in figure 7:

Figure 7: How to Join Cells in Scenario 3



Under the first scenario, all pairs of cells are compatible except for those that are incompatible by definition. To borrow terminology from regression analysis, we may say that the risk characteristics are insignificant and the classification scheme needs to be reconstructed. An alternative classification scheme is provided by dropping one of the rating variables. For instance, by dropping the years of experience variable, we would compare Life versus Non-Life actuaries as shown in figure 8. Alternatively, by dropping the area of practice variable, actuaries with 10 or fewer years of experience would be compared to those with 11or more. If these pairs of new cells are found to be compatible again, then all rating variables are dropped and the original four cells are merged into one to make one set of parameter estimates. If the two new cells are incompatible, then parameters are estimated from each new cell separately.

Figure 8



Defining Risk Classification

Although the definition of various terms that were introduced earlier should be obvious from the context in which they were introduced, let's now attempt to formally define several of these terms.

Risk: "Individual or entity covered by financial security systems [1, p. 2]."

Risk Characteristic: Attribute that identifies a risk or group of risks.

Classification Variable: Categorization or set of risk characteristics consisting of two or more such characteristics. Within a classification variable, risk characteristics define mutually exclusive sets of risks. In other words, a risk can't be identified by more than one characteristic within a classification variable.

Classification Dimension: Number of classification variables used.

Classification Cell: Set of risks sharing all the same risk characteristics.

Adjacent Cells: Two cells C_j and C_k are adjacent if they have exactly D-1 common characteristics where D represents the dimension of the classification scheme.

Non-adjacent Cells: *Two cells C_j and C_k are non-adjacent if they have fewer than D-1 common* characteristics where D represents the dimension of the Classification scheme.

Cell Universe Ω: Set of all cells defined by the classification variables and risk characteristics.

Classification sample: All observations generated by the risk universe for the process under examination (i.e. frequency, severity). Assume the classification sample is made up of N observations, each observation x_i may be seen as a realization of a random variable X_i where $i = 1,2,...,N$.

Models: Probability Distributions $F_{X_k}(x)$ underlying the random variables in a classification sample. The models underlying the random variables in a classification cell share the same functional form and the same parameters. The parameters of an a priori model for a cell C_j are based on observations from that cell only. The parameters of an a posteriori model for a cell C_j are based on the observations from the class $\{C_j\}$ (see below for a definition of class).

Compatibility: C_k *is compatible with* C_j *if there is a "reasonable probability" that* the observations in cell C_k could have come from the a priori model (or from a model with the same parameters as) for cell C_j. Technically, the a priori models underlying each cell may have different functional forms. For instance, the models underlying cells C_j and C_k may be Poisson and Negative Binomial, respectively. For the purpose of assessing compatibility of cell C_k to cell C_j, one asks whether the observations from cell C_k could have come from a Poisson distribution with parameters as in C_j. It is up to the modeler to devise an appropriate test or a set of statistics that can be used to answer the question of compatibility and to define *reasonable probability*. By definition, a cell is compatible with itself.

Incompatibility: C_k is incompatible with C_j if C_k is not compatible with C_j. By definition, we will require that non-adjacent cells be incompatible with one another.

Relation R from Ω to Ω: Non-empty set of ordered pairs (C_j, C_k) such that C_k is compatible with C_j. If $(C_j, C_k) \in R$, we write $C_k R C_j$. If $(C_j, C_k) \notin R$, we write $C_k \not{R} C_j$. If two cells C_j and C_k are non-adjacent, then by definition $C_k \not{R} C_j$ and $C_j \not{R} C_k$

A very important type of relation in set theory is an equivalence relation, which is defined as one having the following three properties:

1) Reflexivity: This property holds that any cell in the cell universe is compatible with itself. We write:

$C_k R C_j$ when $j = k$ or $(C_j, C_j) \in R \, \forall j$.

2) Symmetry: Given any two cells C_j and C_k, if C_j is compatible with C_k then C_k is compatible with C_j and vice versa. We write: $C_k R C_j \Leftrightarrow C_j R C_k$

3) Transitivity: Given any three cells C_j, C_k, and C_l in the cell universe, if C_j is compatible with C_k, and C_k compatible with C_l, it follows that C_j is compatible with C_l. We write:

$C_k R C_j \; and \; C_l R C_k \Rightarrow C_l R C_j$

By definition, the first property always holds for the relation R from Ω to Ω. However, R need be neither symmetric nor transitive. In order words, R need not be an equivalence relation. In Appendix D, we provide an example of an asymmetric relation.

Class $\{C_j\}$: Set of all cells that are compatible with C_j. All cells $C_k \in \Omega$ s.t. $(C_j, C_k) \in R$. Each cell within a classification scheme defines its own class.

Credibility: Weights assigned to the a priori parameter estimates of the cells in a class $\{C_j\}$ in order to come up with the a posteriori parameter estimates for the cell C_j.

Classification Scheme: Process of defining the risks to be covered in a classification scheme, the classification variables and the risk characteristics, the statistical models of the cells, and the rules of compatibility.

Empirical Distribution of the Classification Sample: The empirical distribution of the classification is given by $F_N(x) = \dfrac{N_x}{N}$ where N_x represents the number of X_k 's such $x_k \leq x$.

Fitted Distribution of the Classification Sample: This distribution is given by $F_Y(x) = \dfrac{1}{N}\sum_{i=1}^{N} F_{X_i}(x)$, where $F_{X_i}(x)$ represents the a posteriori probability distribution underlying the random variable X_i. If the $F_{X_i}(x)$'s are identical for all the random variables in a cell, then we can write $F_Y(x) = \dfrac{1}{N}\sum_{j=1}^{n} N_j F_j(x)$, where $F_j(x)$ represents the a posteriori probability distribution underlying the random variables in cell C_j, N_j the number of observations in cell C_j, and n the number of cells in the cell universe.

Illustration

Let's use the classification example presented in our introduction to illustrate our definitions:

Risk: Each actuary represents a risk

Risk Characteristic: Life, Non-Life, 10 or fewer years of experience, 11 or more years of experience.

Classification Variable: Area of practice (life or non-life), Years of experience (10 or fewer, 11 or more).

Classification Dimension: 2.

Classification Cell: For example, life actuaries with 10 or fewer years of experience represent a classification cell.

Adjacent Cells: Two cells are adjacent if they have at least one common characteristic. For instance, Life actuaries with 10 or fewer years of experience and Non-life actuaries with 10 or fewer are adjacent cells.

Non-adjacent Cells: Two cells that have no common characteristics. Life actuaries with 10 or fewer years of experience and Non-life actuaries with 11 or more years of experience are non-adjacent cells.

Cell Universe Ω: Life actuaries with 10 or fewer years of experience, Life actuaries with 11 or more years of experience, Non-Life actuaries with 10 or fewer years of experience, Non-Life actuaries with 11or more years of experience.

Model: $f(x) = \dfrac{(\lambda d)^x e^{-\lambda d}}{x!}$

Compatibility: $C_k R C_j$ if $\Pr ob(\lambda_j = \lambda_k) \geq .9$.

Let's assume information is collected as per table 1 below:

Table 1

Actuaries	Exposure Units	# of Claims
Life_10⁻	5,000	20
Life_11⁺	10,000	48
Non-Life_10⁻	15,000	88
Non-Life_11⁺	25,000	161

We assume the number of claims in each cell is modeled by a Poisson distribution. The density function of the Poisson distribution is given by: $f(x) = \frac{(\lambda d)^x e^{-\lambda d}}{x!}$ where d is the number of exposure units and λ is the average number of claims per exposure unit. The maximum likelihood estimates $\hat{\lambda}$ of the λ's for each cell of actuary is obtained by dividing the number of claims by the number of exposure units and are shown in table 2 below:

Table 2: MLE Estimates

Actuaries	Exposure Units	# of Claims	MLE Estimate
Life_10⁻	5,000	20	.0040
Life_11⁺	10,000	48	.0048
Non-Life_10⁻	15,000	88	.0059
Non-Life_11⁺	25,000	161	.0064

Recall the two hypotheses introduced in the discussion above.

1) Actuaries within the same cell share the same loss propensity.
2) Actuaries across different cells have different loss propensities.

We will assume that the first hypothesis is true. The second hypothesis can be tested using the following statistic to compare in succession the λ's for pairs of cells C_j and C_k: $\hat{R}_0 = \frac{\hat{\lambda}_j - \hat{\lambda}_k}{\sqrt{\frac{\hat{\lambda}_j}{d_j} + \frac{\hat{\lambda}_k}{d_k}}}$ where $\hat{\lambda}_j$ *and* $\hat{\lambda}_k$ represent the MLE for cells C_j and C_k, respectively, and d_j *and* d_k represent the exposure units in cells C_j and C_k, respectively. If $\hat{\lambda}_j$ *and* $\hat{\lambda}_k$ are equal (we will refer to the equality of the λ's as a sub-hypothesis) in which case we will say that cells C_j and C_k are compatible, then $\hat{R}_0 \to N(0,1)$. In other words, $\hat{R}_0$ has the standard normal distribution if cells A and B are compatible. This fact is proven in detail in Appendix A. $\hat{R}_0$ may be thought of as a measure of the distance between the λ's of the two models. For values of $\hat{R}_0$ falling within a given range we will accept the sub-hypothesis that $\hat{\lambda}_j$ *and* $\hat{\lambda}_k$ are equal, while we will reject that sub-hypothesis for values of $\hat{R}_0$ falling outside that range. For instance at a 90% confidence level, the range of real numbers for which we will accept the hypothesis is (-1.645,1.645). We need only calculate $\hat{R}_0$ for adjacent cells. By definition, non-adjacent cells are not compatible. We must reject all the sub-hypotheses in order to accept the main hypothesis. The following two figures 9, 10, and 11show, respectively, the values of $\hat{R}_0$ for the relevant pairs of cells, whether a sub-hypothesis has been accepted or rejected, and whether cells are compatible:

Figure 9: $\hat{R}_0$ values

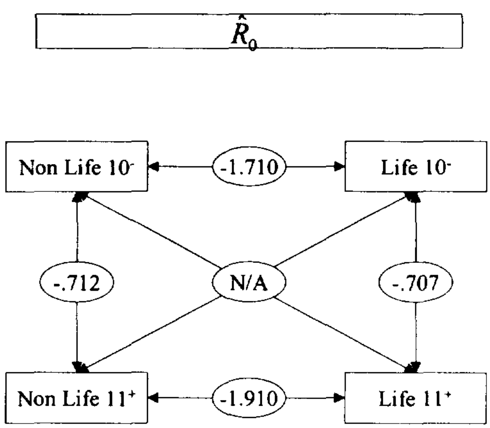


Figure 10: Test Results

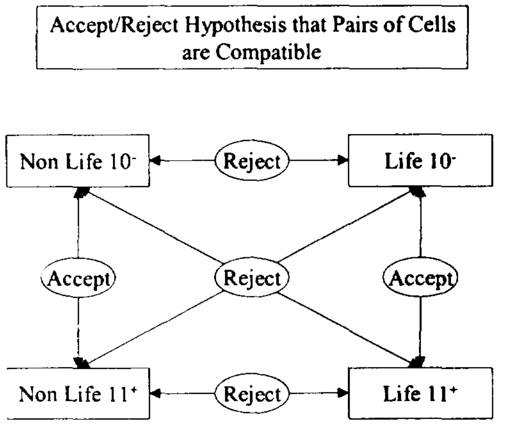
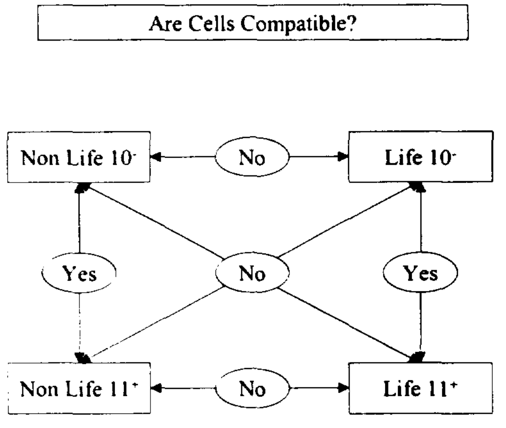


Figure 11: Compatibility Chart



Relation R from Ω to Ω: {(Life_10$^-$,Life_10$^-$), (Non-Life_10$^-$,Non-Life_10$^-$), (Life_11$^+$,Life_11$^+$), (Non-Life_11$^+$,Non-Life_11$^+$) (Life_10$^-$,Life_11$^+$), (Life_11$^+$,Life_10$^-$), (Non-Life_10$^-$,Non-Life_11$^+$), (Non-Life_11$^+$,Non-Life_10$^-$)}

Classes:

{Life_10$^-$} ={Life_10$^-$, Life_11$^+$}

{Life_11$^+$} ={Life_10$^-$, Life_11$^+$}

{Non-Life_10$^-$} ={Non-Life_10$^-$, Non-Life_11$^+$}

{Non-Life_11$^+$} ={Non-Life_10$^-$, Non-Life_11$^+$}

Credibility[7]:

{Life_10$^-$} ={1/3 Life_10$^-$, 2/3 Life_11$^+$}

{Life_11$^+$} ={1/3 Life_10$^-$, 2/3 Life_11$^+$}

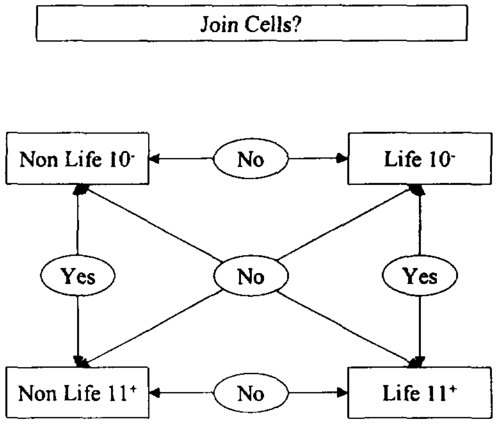
{Non-Life_10$^-$} ={3/8 Non-Life_10$^-$, 5/8 Non-Life_11$^+$}

{Non-Life_11$^+$} ={3/8 Non-Life_10$^-$, 5/8 Non-Life_11$^+$}

Based on figure 11 above, the compatible cells will be joined as shown in figure 12 below in order to produce new estimates of the λ 's. Another way of viewing this is that the new estimates for each cell will be a credibility weighted average of the original estimates of other compatible cells where the weights are given by the relative exposure units of each cell.

[7] For the Poison model the credibility weights for each cell in a class equal the number of exposures in a cell divided by total number of exposures in a class. The derivation is shown in Appendix C.

Figure 12: How to Join Cells



The re-estimated λ 's are as per table 3 below:

Table 3: Revised MLE Estimates

Actuaries	Exposure Units	# of Claims	Initial MLE Estimate	Revised MLE Estimate
Life_10⁻	5,000	20	.0040	.0045
Life_11⁺	10,000	48	.0048	.0045
Non-Life_10⁻	15,000	88	.0059	.0062
Non-Life_11⁺	25,000	161	.0064	.0062

The separation hypothesis has been rejected. The alternative hypothesis that is being accepted here is that both life and non-life actuaries, respectively, have the same loss propensity (expected number of loss per unit of exposure) regardless of their years of experience and that life and non-life actuaries have distinct loss propensities. Hence, the experience of all life actuaries across all years of experience will be combined to arrive at a single estimate of the average claim per exposure unit and the same will be done for non-life actuaries. If, for example, the severity of claims for all actuaries were constant, all life actuaries and all non-life actuaries, respectively, would be charged the same rates. If the separation hypothesis had been accepted, each cell of actuaries would be charged a different rate. In particular, more experienced actuaries would be charged a higher rate than less experienced ones. The number of claims were simulated from two Poisson distributions for which the actual λ 's are shown in table 4 below. If the main hypothesis had been erroneously accepted, it would lead to subsidies from more experienced actuaries to less experienced ones.

Table 4

Actuaries	Exposure Units	# of Claims	Initial MLE Estimate	Revised MLE Estimate	Actual
Life_10$^-$	5,000	20	.0040	.0045	.0050
Life_11$^+$	10,000	48	.0048	.0045	.0050
Non-Life_10$^-$	15,000	88	.0059	.0062	.0060
Non-Life_11$^+$	25,000	161	.0064	.0062	.0060

Classification Efficiency

Given a classification scheme, we would like to be able to measure its performance. Classification efficiency is an oft-used notion of performance, which Robert Finger defines as "a measure of a classification system's accuracy [6, p.250]." "A perfect classification system," Finger adds, "would produce the same variability as the insured population. [6, p.250]" Then Finger settles on the squared ratio of the classification system's coefficient of variation (CV) to that of the underlying population as a measure of efficiency. Finger, after observing that, "...the variability [of the insured population] is unknowable," goes on to calculate the efficiency factor for an automobile classification example based on an assumed coefficient of variation of 1.00 for the insured population. This CV of 1.00 is also assumed by Robert Bailey who in his 1960 paper, Any Room Left for Skimming the Cream, uses a similar measure of classification efficiency to Robert Finger's.

The procedure we have outlined makes assumptions about not only the variability but also the actual probability distribution of the underlying population by providing a fitted distribution $F_Y(x)$ to the sample's empirical distribution $F_N(x)$. We could compare the empirical CV of the insured population to that of the fitted distribution. Like Finger and Bailey, we could use some ratio of the CV's as a measure of efficiency. However, the comparison of CV's provides only a limited picture of a classification scheme's accuracy. We know or have assumed too much about the insured population to rely only on CV ratios to assess the accuracy of the classification scheme. Traditional measures of goodness-of-fit may be more appropriate to evaluate the fit of the assumed distribution vis-à-vis the empirical sample distribution. Two measures immediately come to mind: the Chi Square and the Kolmogorov-Smirnov statistics.

Efficiency, however, should not be thought of as an absolute measure. We would slightly alter Finger's definition of efficiency to read: "efficiency is a measure of a classification system's relative accuracy." What we are in fact measuring is the accuracy of one scheme relative to another. The task becomes one of selecting the classification scheme that best represents the underlying population amongst competing schemes. Each efficiency measure may give a different ranking of the goodness of fit of classification schemes. The modeler may take into account other considerations when making a judgment as to which classification scheme to use.

Validation

Measures of efficiency help us choose the best amongst competing models. However, even the best model might give a poor representation of the data. Validation helps us decide whether the chosen model will be relevant or valid in some future period for which a forecast is sought. If a model fails to validate, we need to rethink the classification scheme and start the process over.

A procedure that could be used to validate a classification scheme consists of randomly selecting out of each cell a percentage of the observations, say 90%, and re-estimate the parameters of the cells through the same process used for the full data set. One then checks to see whether the parameters for each cell fall within an acceptable confidence band of the parameters estimated using the full data set. One may also compare the fitted distribution derived from a 90% sample of the data with that derived from the full data set to see whether the two are "close". This process can be repeated several hundreds or thousands of times using a new random sample each time. A large percentage of the models based on the 90% random samples being consistent with that based on the full data set would tend to validate the original model. One of the problems with this procedure is that the compatibility of cells will likely depend on the number of observations in the cells. A reduced sample size may affect the compatibility relationships and the composition of the classes. Other validation procedures such as "train and test" and those based on "bootstrap" may be adapted to our classification problem.

When trying to validate a model through the procedure mentioned above, one may need to develop confidence intervals for the original estimates of the parameters of the models so that one could gauge whether the estimates based on the re-sampled data are within an acceptable range of the original estimates. For instance, the standard error of λ in our example is given by $\sqrt{\frac{\hat{\lambda}}{d}}$ and a $k\%$ confidence interval for λ is defined by the interval $\hat{\lambda} \pm z_{(1+k)/2}\sqrt{\frac{\hat{\lambda}}{d}}$, where $z_{(1+k)/2}$ is the $(1+k)/2$ th quantile of the standard normal distribution. The derivation of this interval is shown in Appendix B. The standard error and the 90% confidence interval for λ are shown in table 5 below. A classification scheme that is successfully validated would ensure Predictive Stability, which is one of three actuarial criteria listed by the American Academy of Actuaries in designing a classification scheme.

Table 5: Confidence Interval for λ

Actuaries	Exposure Units	# of Clms	Initial MLE Estimate	Revised MLE Estimate	Std Error of λ	90% Confidence Interval	Actual
Life_10⁻	5,000	20	.0040	.0045	.00055	(.0036,0054)	.0050
Life_11⁺	10,000	48	.0048	.0045	.00055	(.0036,0054)	.0050
Non-Life_10⁻	15,000	88	.0059	.0062	.00039	(.0056,0068)	.0060
Non-Life_11⁺	25,000	161	.0064	.0062	.00039	(.0056,0068)	.0060

Practical Considerations

Earlier in the paper, we stated that separate classification schemes should be used for different aspects of the claim process. The claim process may be decomposed into a frequency and severity component and these components can be further decomposed into more sub-components. We believe that whenever possible such decomposition may provide a better understanding of the entire claim process. Finally, given how hard it is to find, manipulate, and make inferences about models representing single components of the claim process, the task gets only more daunting when these components are compounded.

Often in insurance problems, there is a need to adjust data for trend and development. Adjustments made to a body of data may cause that data to violate the assumptions of a model. For instance, a Poisson random variable multiplied by a constant is no longer Poisson. If adjustments are made to the data, the model needs to be adjusted accordingly. There may be ways to define the models to see whether any adjustments are appropriate in the first place and the magnitude of such adjustments.

Areas of development

The procedures we have outlined rely on finding good models to represent the probability of random events in a classification cell. There is an extensive library of such models in the literature. In addition, the ability to test the compatibility of cells in a classification scheme is an equally important feature of the procedures presented above. In the illustration, we presented a statistic that allowed us to test the equality of the expected claim per exposure of two Poisson distributions. A number of statistics are available to test hypotheses of the Normal and, by extension, the Lognormal distributions. Various tests and statistics need to be developed in order to make inferences about other distributions, such as the Gamma, Pareto, or the Negative Binomial, that are often used in insurance problems. Distribution of test statistics may also be obtained through simulation rather than heavy-handed calculus.

However, it may not be always feasible to come up with models to represent the probability of events in a cell. Perhaps, there is an even greater role to be played by non-parametric distribution functions and non-parametric approaches to hypothesis testing such as those based on "bootstrap" and "permutation." See Efron and Tibshirani [5] for a discussion of these topics.

Conclusion

The American Academy of Actuaries [2, p. 2] states that the three primary purposes of Risk Classification should be to:

1) protect the insurance program's financial soundness;
2) be fair; and
3) permit economic incentives to operate and thus encourage widespread availability of coverage.

Our definition of Risk Classification is derived out of the very concept of fairness. It is a concept that requires that the same rates not be charged to pools of risks that have fundamentally dissimilar loss propensities or that different rates be charged to pools of risks that have fundamentally similar loss propensities. We believe that the first and third purposes are direct byproducts of the second. The Academy also lists three statistical considerations: homogeneity, credibility and predictive stability. Our definition of credibility differs from that of the Academy. Credibility, as we have defined it, can't be a goal into itself. In lieu of credibility, we would substitute separation as one of the statistical considerations of a classification scheme. If we take this liberty, the purposes and considerations inherent in our definition of risk classification are consistent with those of the American Academy of Actuaries. Our definition provides a definite methodology by which these goals and considerations are met. In addition, nothing in the way we have defined risk classification should preclude us from taking into account other considerations listed by the American Academy of Actuaries including the operational and acceptability considerations.

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APPENDIX A

Let A and B represent two cells, each with frequency distribution defined by a Poisson model with parameters λ_A and λ_B representing the expected number of occurrences per unit of exposure d_i where $d_i \in N \; and \; 1 \le d_i \le d_{\max} \; for \; all \; i's$. Assume that $m \; and \; n$ observations represented by random variables X_i are made for cells A and B, respectively. Their Poisson models are set as follows:

Random Variable	*Number of Occurrences*	*Exposures Units*	*Mean*	*Variance*
X_1	x_1	d_1	$\lambda_A d_1$	$\lambda_A d_1$
X_2	x_2	d_2	$\lambda_A d_2$	$\lambda_A d_2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
X_m	x_m	d_m	$\lambda_A d_m$	$\lambda_A d_m$
X_{m+1}	x_{m+1}	d_{m+1}	$\lambda_B d_{m+1}$	$\lambda_B d_{m+1}$
X_{m+2}	x_{m+2}	d_{m+2}	$\lambda_B d_{m+2}$	$\lambda_B d_{m+2}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
X_{m+n}	x_{m+n}	d_{m+n}	$\lambda_B d_{m+n}$	$\lambda_B d_{m+n}$

The maximum likelihood estimator $\hat{\lambda}_A$ of λ_A is obtained by maximizing the likelihood function

$$L = \frac{\lambda_A^{\sum_{i=1}^{m} x_i} \prod_{i=1}^{m} d_i^{x_i} e^{-\lambda_A \sum_{i=1}^{m} d_i}}{\prod_{i=1}^{m} x_i!}$$

$$\ln(L) = \ln(\lambda_A) \sum_{i=1}^{m} x_i + \sum_{i=1}^{m} x_i \ln(d_i) - \lambda_A \sum_{i=1}^{m} d_i - \sum_{i=1}^{m} \ln(x_i!)$$

$$\frac{d \ln(L)}{d\lambda_A} = \frac{1}{\lambda_A} \sum_{i=1}^{m} x_i - \sum_{i=1}^{m} d_i$$

$$\frac{d\ln(L)}{d\lambda_A}=0\Rightarrow\hat{\lambda}_A=\frac{\sum_{i=1}^{m}x_i}{\sum_{i=1}^{m}d_i}$$

$$Similarly, \hat{\lambda}_B=\frac{\sum_{i=m+1}^{m+n}x_i}{\sum_{i=m+1}^{m+n}d_i}$$

$\hat{\lambda}_A$ *and* $\hat{\lambda}_B$ are realizations of random variables $\hat{\Lambda}_A=\frac{\sum_{i=1}^{m}X_i}{\sum_{i=1}^{m}d_i}$ *and* $\hat{\Lambda}_B=\frac{\sum_{i=m+1}^{m+n}X_i}{\sum_{i=m+1}^{m+n}d_i}$

$$E(\hat{\Lambda}_A)=\lambda_A \qquad Var(\hat{\Lambda}_A)=\frac{\lambda_A}{\sum_{i=1}^{m}d_i}$$

Also,

$$E(\hat{\Lambda}_B)=\lambda_B \qquad Var(\hat{\Lambda}_B)=\frac{\lambda_B}{\sum_{i=m+1}^{m+n}d_i}$$

Let $\delta=E(\hat{\Lambda}_A)-E(\hat{\Lambda}_B)=\lambda_A-\lambda_B$. Let's define

$R_\delta=\dfrac{\hat{\Lambda}_A-\hat{\Lambda}_B-\delta}{\sqrt{\dfrac{\lambda_A}{\sum_{i=1}^{m}d_i}+\dfrac{\lambda_B}{\sum_{i=m+1}^{m+n}d_i}}}$ and $\hat{R}_\delta=\dfrac{\hat{\Lambda}_A-\hat{\Lambda}_B-\delta}{\sqrt{\dfrac{\hat{\Lambda}_A}{\sum_{i=1}^{m}d_i}+\dfrac{\hat{\Lambda}_B}{\sum_{i=m+1}^{m+n}d_i}}}$

For $\delta=0$, we have $R_0=\dfrac{\hat{\Lambda}_A-\hat{\Lambda}_B}{\sqrt{\dfrac{\lambda_A}{\sum_{i=1}^{m}d_i}+\dfrac{\lambda_B}{\sum_{i=m+1}^{m+n}d_i}}}$ and $\hat{R}_0=\dfrac{\hat{\Lambda}_A-\hat{\Lambda}_B}{\sqrt{\dfrac{\hat{\Lambda}_A}{\sum_{i=1}^{m}d_i}+\dfrac{\hat{\Lambda}_B}{\sum_{i=m+1}^{m+n}d_i}}}$

Equation 1

We may write $\hat{R}_\delta=\sqrt{\dfrac{\dfrac{\lambda_A}{D_A}+\dfrac{\lambda_B}{D_B}}{\dfrac{\hat{\Lambda}_A}{D_A}+\dfrac{\hat{\Lambda}_B}{D_B}}}\,R_\delta$

Definition 1[4, p. 216]

A sequence of random variables, $X_1, X_2, \ldots$, converges in distribution to a random variable X if

$$\lim_{n\to\infty} F_{X_n}(x) = F_X(x),$$

at all points x where $F_X(x)$ is continuous.

We write $X_n \xrightarrow{L} X$

Definition 2 [4, p. 213]

A sequence of random variables, $X_1, X_2, \ldots$, converges in probability to a random variable X if, for every $\varepsilon > 0$,

$$\lim_{n\to\infty} P\{|X_n - X| \ge \varepsilon\} \to 0$$

We write $X_n \xrightarrow{P} X$

We will show that $R_\delta, \hat{R}_\delta, R_0, \hat{R}_0 \xrightarrow{L} Z$, where Z has the standard normal distribution $N(0,1)$.

Then $\hat{R}_0$ can be used to test the null hypothesis

H₀: $\lambda_A = \lambda_B$ versus the alternative

H₁: $\lambda_A \neq \lambda_B$

We accept H₀ at the p confidence level if $|\hat{R}_0| \le z_{(1+p)/2}$,

and we reject H₀ if $|\hat{R}_0| > z_{(1+p)/2}$.

If the null hypothesis is accepted, we conclude that the claim frequency per unit of exposure underlying cells A and B are equal and we estimate one λ for both cells based on the joint experience of the two. If the null hypothesis is rejected, we say that say that cells A and B have different expected claim frequencies, which are estimated with parameters λ_A and λ_B.

We first show that $R_\delta \xrightarrow{L} Z$.

$$R_\delta = \frac{\hat{\Lambda}_A - \hat{\Lambda}_B - \delta}{\sqrt{\dfrac{\lambda_A}{\sum_{i=1}^{m} d_i} + \dfrac{\lambda_B}{\sum_{i=m+1}^{m+n} d_i}}} = \frac{\dfrac{\sum_{i=1}^{m} X_i}{\sum_{i=1}^{m} d_i} - \dfrac{\sum_{i=m+1}^{m+n} X_i}{\sum_{i=m+1}^{m+n} d_i} - \lambda_A + \lambda_B}{\sqrt{\dfrac{\lambda_A}{\sum_{i=1}^{m} d_i} + \dfrac{\lambda_B}{\sum_{i=m+1}^{m+n} d_i}}}$$

$$R_\delta = \frac{\dfrac{\sum_{i=m+1}^{m+n} d_i \sum_{i=1}^{m} X_i - \sum_{i=1}^{m} d_i \sum_{i=m+1}^{m+n} X_i - \sum_{i=1}^{m} d_i \sum_{i=m+1}^{m+n} d_i \lambda_A + \sum_{i=1}^{m} d_i \sum_{i=m+1}^{m+n} d_i \lambda_B}{\sum_{i=1}^{m} d_i \sum_{i=m+1}^{m+n} d_i}}{\dfrac{\sum_{i=1}^{m} d_i \sum_{i=m+1}^{m+n} d_i}{\sum_{i=1}^{m} d_i \sum_{i=m+1}^{m+n} d_i} \sqrt{\dfrac{\sum_{i=m+1}^{m+n} d_i \lambda_A + \sum_{i=1}^{m} d_i \lambda_B}{\sum_{i=1}^{m} d_i \sum_{i=m+1}^{m+n} d_i}}}$$

$$R_\delta = \frac{\sum_{i=m+1}^{m+n} d_i \sum_{i=1}^{m} X_i - \sum_{i=1}^{m} d_i \sum_{i=m+1}^{m+n} X_i - \sum_{i=1}^{m} d_i \sum_{i=m+1}^{m+n} d_i \lambda_A + \sum_{i=1}^{m} d_i \sum_{i=m+1}^{m+n} d_i \lambda_B}{\sqrt{\sum_{i=1}^{m} d_i (\sum_{i=m+1}^{m+n} d_i)^2 \lambda_A + \sum_{i=m+1}^{m+n} d_i (\sum_{i=1}^{m} d_i)^2 \lambda_B}}$$

Let $D_A = \sum_{i=1}^{m} d_i$ and $D_B = \sum_{i=m+1}^{m+n} d_i$

We may rewrite $R_\delta = \dfrac{\sum_{i=1}^{m} D_B X_i - \sum_{i=m+1}^{m+n} D_A X_i - \sum_{i=1}^{m} D_B d_i \lambda_A + \sum_{i=m+1}^{m+n} D_A d_i \lambda_B}{\sqrt{\sum_{i=1}^{m} D_B{}^2 d_i \lambda_A + \sum_{i=m+1}^{m+n} D_A{}^2 d_i \lambda_B}}$

We define:

$$U_i = \begin{matrix} D_B X_i & i = 1,2,\cdots,m \\ -D_A X_i & i = m+1, m+2, \cdots m+n \end{matrix} \quad and \quad u_i = \begin{matrix} D_B x_i & i = 1,2,\cdots,m \\ -D_A x_i & i = m+1, m+2, \cdots m+n \end{matrix}$$

We then have:

$$\mu_i = E(U_i) = \begin{matrix} D_B d_i \lambda_A & i = 1,2,\cdots,m \\ -D_A d_i \lambda_B & i = m+1, m+2, \cdots, m+n \end{matrix}$$

$$\sigma^2{}_i = Var(U_i) = \begin{matrix} D_B{}^2 d_i \lambda_A & i = 1,2,\cdots,m \\ D_A{}^2 d_i \lambda_B & i = m+1, m+2, \cdots, m+n \end{matrix}$$

We again rewrite $R_\delta = \dfrac{\sum_{i=1}^{m+n} U_i - \sum_{i=1}^{} \mu_i}{\sqrt{\sum_{i=1}^{m+n} \sigma_i{}^2}}$

The Central Limit Theorem - Lindeberg [9, p. 282] provides that the distribution of R_δ converges to the standard normal distribution if the following condition is met:

$$Q = \underset{m+n\to\infty}{Lim} \frac{1}{s^2_{m+n}} \sum_{i=1}^{m+n} \sum_{|u_{il} - \mu_i| > \varepsilon_{m+n}} (u_{il} - \mu_i)^2 p_{il} = 0,$$

where $s_{m+n}^2 = \sum_{i=1}^{m+n} \sigma^2{}_i$ and $p_{il} = \Pr ob(U_i = u_{il})$.

$$p_{il} = \Pr ob(U_i = u_{il}) = \begin{cases} \Pr ob(D_B X_i = u_{il})\ for\ i = 1,2,\dots,m \\ \Pr ob(-D_A X_i = u_{il})\ for\ i = m+1, m+2,\dots,m+n \end{cases}$$

$$p_{il} = \begin{cases} \Pr ob(X_i = u_{il}/D_B)\ for\ i = 1,2,\dots,m \\ \Pr ob(X_i = -u_{il}/D_A\ for\ i = m+1, m+2,\dots,m+n \end{cases}$$

$$p_{il} = \Pr ob(X_i = x_{il})\ where \begin{cases} x_{il} = u_{il}/D_B\ for\ i = 1,2,\dots,m \\ x_{il} = -u_{il}/D_A\ for\ i = m+1, m+2,\dots,m+n \end{cases}$$

$$p_{il} = \begin{cases} \dfrac{(\lambda_A d_i)^{x_{il}} e^{-\lambda_A d_i}}{x_{il}!}\ for\ i = 1,2,\dots,m \\ \dfrac{(\lambda_B d_i)^{x_{il}} e^{-\lambda_B d_i}}{x_{il}!}\ for\ i = m+1, m+2,\dots,m+n \end{cases}$$

$$Q = \lim_{m+n\to\infty} \frac{1}{s_{m+n}^2} \sum_{i=1}^{m+n} \sum_{|u_{il}| > \varepsilon s_{m+n}} u_{il}^2\, p_{il+\mu_i} = 0$$

$$p_{il+\mu_i} = \Pr ob(U_i = u_{il} + \mu_i) = \begin{cases} \Pr ob(D_B X_i = u_{il} + \mu_i)\ for\ i = 1,2,\dots,m \\ \Pr ob(-D_A x_i = u_{il} + \mu_i)\ for\ i = m+1, m+2,\dots,m+n \end{cases}$$

$$p_{il+\mu_i} = \Pr ob(X_i = x_{il} + \chi_i)\ where \begin{cases} \chi_i = \mu_i/D_B = E(X_i)\ for\ i = 1,2,\dots,m \\ \chi_i = -\mu_i/D_A = E(X_i)\ for\ i = m+1, m+2,\dots,m+n \end{cases}$$

$$p_{il+\mu_i} = \begin{cases} \dfrac{(\lambda_A d_i)^{x_{il}+\chi_i} e^{-\lambda_A d_i}}{(x_{il}+\chi_i)!} = \dfrac{(\lambda_A d_i)^{\chi_i} x_{il}!}{(x_{il}+\chi_i)!} \dfrac{(\lambda_A d_i)^{x_{il}} e^{-\lambda_A d_i}}{x_{il}!}\ for\ i = 1,2,\dots,m \\ \dfrac{(\lambda_B d_i)^{x_{il}+\chi_i} e^{-\lambda_B d_i}}{(x_{il}+\chi_i)!} = \dfrac{(\lambda_B d_i)^{\chi_i} x_{il}!}{(x_{il}+\chi_i)!} \dfrac{(\lambda_B d_i)^{x_{il}} e^{-\lambda_B d_i}}{x_{il}!}\ for\ i = m+1, m+2,\dots,m+n \end{cases}$$

$$p_{il+\mu_i} \le \begin{cases} (\lambda_A d_i)^{\chi_i} \dfrac{(\lambda_A d_i)^{x_{il}} e^{-\lambda_A d_i}}{x_{il}!}\ for\ i = 1,2,\dots,m \\ (\lambda_B d_i)^{\chi_i} \dfrac{(\lambda_B d_i)^{x_{il}} e^{-\lambda_B d_i}}{x_{il}!}\ for\ i = m+1, m+2,\dots,m+n \end{cases}$$

$$Q \le \lim_{m+n\to\infty} \frac{1}{s_{m+n}^2} \left\{ \sum_{i=1}^{m} \sum_{|u_{il}| > \varepsilon s_{m+n}} u_{il}^2 (\lambda_A d_i)^{\chi_i} \frac{(\lambda_A d_i)^{x_{il}} e^{-\lambda_A d_i}}{x_{il}!} + \sum_{i=m+1}^{m+n} \sum_{|u_{il}| > \varepsilon s_{m+n}} u_{il}^2 (\lambda_B d_i)^{\chi_i} \frac{(\lambda_B d_i)^{x_{il}} e^{-\lambda_B d_i}}{x_{il}!} \right\}$$

$$Q \le \lim_{m+n\to\infty} \frac{1}{s_{m+n}^2} \left\{ \sum_{i=1}^{m} \sum_{|u_{il}| > \varepsilon s_{m+n}} \frac{x_{il}^2}{D_B^2} (\lambda_A d_i)^{\chi_i} \frac{(\lambda_A d_i)^{x_{il}} e^{-\lambda_A d_i}}{x_{il}!} + \sum_{i=m+1}^{m+n} \sum_{|u_{il}| > \varepsilon s_{m+n}} \frac{x_{il}^2}{D_A^2} (\lambda_B d_i)^{\chi_i} \frac{(\lambda_B d_i)^{x_{il}} e^{-\lambda_B d_i}}{x_{il}!} \right\}$$

$$Q \le \underset{m+n\to\infty}{Lim} \frac{1}{s_{m+n}^{2}} \left\{ \sum_{i=1}^{m} \frac{(\lambda_A d_i)^{x_i}}{D_B^2} \sum_{|x_{il}| > \frac{\varepsilon_{m+n}}{D_B}} x_{il}^{2} \frac{(\lambda_A d_i)^{x_{il}} e^{-\lambda_A d_i}}{x_{il}!} + \sum_{i=m+1}^{m+n} \frac{(\lambda_B d_i)^{x_i}}{D_A^2} \sum_{|x_{il}| > \frac{\varepsilon_{m+n}}{D_A}} x_{il}^{2} \frac{(\lambda_B d_i)^{x_{il}} e^{-\lambda_B d_i}}{x_{il}!} \right\}$$

$$Q \le \underset{m+n\to\infty}{Lim} \frac{1}{s_{m+n}^{2}} \left\{ \sum_{i=1}^{m} \frac{(\lambda_A d_i)^{x_i}}{D_B^2} \sum_{|x_{il}| > \frac{\varepsilon_{m+n}}{D_B}} x_{il} \frac{(\lambda_A d_i)^{x_{il}} e^{-\lambda_A d_i}}{(x_{il}-1)!} + \sum_{i=m+1}^{m+n} \frac{(\lambda_B d_i)^{x_i}}{D_A^2} \sum_{|x_{il}| > \frac{\varepsilon_{m+n}}{D_A}} x_{il} \frac{(\lambda_B d_i)^{x_{il}} e^{-\lambda_B d_i}}{(x_{il}-1)!} \right\}$$

$$Q \le \underset{m+n\to\infty}{Lim} \frac{1}{s_{m+n}^{2}} \left\{ \begin{aligned} & \sum_{i=1}^{m} \frac{(\lambda_A d_i)^{x_i}}{D_B^2} \sum_{|x_{il}| > \frac{\varepsilon_{m+n}}{D_B}+1} (x_{il}+1) \frac{(\lambda_A d_i)^{x_{il}+1} e^{-\lambda_A d_i}}{x_{il}!} \\ & + \sum_{i=m+1}^{m+n} \frac{(\lambda_B d_i)^{x_i}}{D_A^2} \sum_{|x_{il}| > \frac{\varepsilon_{m+n}}{D_A}+1} (x_{il}+1) \frac{(\lambda_B d_i)^{x_{il}+1} e^{-\lambda_B d_i}}{x_{il}!} \end{aligned} \right\}$$

$$Q \le \underset{m+n\to\infty}{Lim} \frac{1}{s_{m+n}^{2}} \left\{ \begin{aligned} & \sum_{i=1}^{m} \frac{(\lambda_A d_i)^{x_i}}{D_B^2} \sum_{|x_{il}| > \frac{\varepsilon_{m+n}}{D_B}+1} x_{il} \frac{(\lambda_A d_i)^{x_{il}+1} e^{-\lambda_A d_i}}{x_{il}!} + \sum_{|x_{il}| > \frac{\varepsilon_{m+n}}{D_B}+1} \frac{(\lambda_A d_i)^{x_{il}+1} e^{-\lambda_A d_i}}{x_{il}!} \\ & + \sum_{i=m+1}^{m+n} \frac{(\lambda_B d_i)^{x_i}}{D_A^2} \sum_{|x_{il}| > \frac{\varepsilon_{m+n}}{D_A}+1} x_{il} \frac{(\lambda_B d_i)^{x_{il}+1} e^{-\lambda_B d_i}}{x_{il}!} + \sum_{|x_{il}| > \frac{\varepsilon_{m+n}}{D_A}+1} \frac{(\lambda_B d_i)^{x_{il}+1} e^{-\lambda_B d_i}}{x_{il}!} \end{aligned} \right\}$$

$$Q \le \underset{m+n\to\infty}{Lim} \frac{1}{s_{m+n}^{2}} \left\{ \begin{aligned} & \sum_{i=1}^{m} \frac{(\lambda_A d_i)^{x_i}}{D_B^2} \sum_{|x_{il}| > \frac{\varepsilon_{m+n}}{D_B}+1} \frac{(\lambda_A d_i)^{x_{il}+1} e^{-\lambda_A d_i}}{(x_{il}-1)!} + \sum_{|x_{il}| > \frac{\varepsilon_{m+n}}{D_B}+1} \frac{(\lambda_A d_i)^{x_{il}+1} e^{-\lambda_A d_i}}{x_{il}!} \\ & + \sum_{i=m+1}^{m+n} \frac{(\lambda_B d_i)^{x_i}}{D_A^2} \sum_{|x_{il}| > \frac{\varepsilon_{m+n}}{D_A}+1} \frac{(\lambda_B d_i)^{x_{il}+1} e^{-\lambda_B d_i}}{(x_{il}-1)!} + \sum_{|x_{il}| > \frac{\varepsilon_{m+n}}{D_A}+1} \frac{(\lambda_B d_i)^{x_{il}+1} e^{-\lambda_B d_i}}{x_{il}!} \end{aligned} \right\}$$

$$Q \le \underset{m+n\to\infty}{Lim} \frac{1}{s_{m+n}^{2}} \left\{ \begin{aligned} & \sum_{i=1}^{m} \frac{(\lambda_A d_i)^{x_i}}{D_B^2} \sum_{|x_{il}| > \frac{\varepsilon_{m+n}}{D_B}+2} \frac{(\lambda_A d_i)^{x_{il}+2} e^{-\lambda_A d_i}}{x_{il}!} + \sum_{|x_{il}| > \frac{\varepsilon_{m+n}}{D_B}+1} \frac{(\lambda_A d_i)^{x_{il}+1} e^{-\lambda_A d_i}}{x_{il}!} \\ & + \sum_{i=m+1}^{m+n} \frac{(\lambda_B d_i)^{x_i}}{D_A^2} \sum_{|x_{il}| > \frac{\varepsilon_{m+n}}{D_A}+2} \frac{(\lambda_B d_i)^{x_{il}+2} e^{-\lambda_B d_i}}{x_{il}!} + \sum_{|x_{il}| > \frac{\varepsilon_{m+n}}{D_A}+1} \frac{(\lambda_B d_i)^{x_{il}+1} e^{-\lambda_B d_i}}{x_{il}!} \end{aligned} \right\}$$

$$Q \le \lim_{m+n\to\infty} \frac{1}{s^2_{m+n}} \left\{ \begin{array}{l} \displaystyle\sum_{i=1}^{m} \frac{(\lambda_A d_i)^{\chi_i}}{D_B^2} (\lambda_A d_i)^2 \sum_{|x_{il}|>\frac{\varepsilon_{m+n}}{D_B}+2} \frac{(\lambda_A d_i)^{x_{il}} e^{-\lambda_A d_i}}{x_{il}!} + (\lambda_A d_i) \sum_{|x_{il}|>\frac{\varepsilon_{m+n}}{D_B}+1} \frac{(\lambda_A d_i)^{x_{il}} e^{-\lambda_A d_i}}{x_{il}!} \\ \displaystyle + \sum_{i=m+1}^{m+n} \frac{(\lambda_B d_i)^{\chi_i}}{D_A^2} (\lambda_B d_i)^2 \sum_{|x_{il}|>\frac{\varepsilon_{m+n}}{D_A}+2} \frac{(\lambda_B d_i)^{x_{il}} e^{-\lambda_B d_i}}{x_{il}!} + (\lambda_B d_i) \sum_{|x_{il}|>\frac{\varepsilon_{m+n}}{D_A}+1} \frac{(\lambda_B d_i)^{x_{il}} e^{-\lambda_B d_i}}{x_{il}!} \end{array} \right\}$$

$$Q \le \lim_{m+n\to\infty} \frac{1}{s^2_{m+n}} \left\{ \begin{array}{l} \displaystyle\sum_{i=1}^{m} \frac{(\lambda_A d_i)^{\chi_i}}{D_B^2} (\lambda_A d_i)^2 \sum_{|x_{il}|>\frac{\varepsilon_{m+n}}{D_B}+2} \frac{(\lambda_A d_i)^{x_{il}} e^{-\lambda_A d_i}}{x_{il}!} + (\lambda_A d_i) \sum_{|x_{il}|>\frac{\varepsilon_{m+n}}{D_B}+1} \frac{(\lambda_A d_i)^{x_{il}} e^{-\lambda_A d_i}}{x_{il}!} \\ \displaystyle + \sum_{i=m+1}^{m+n} \frac{(\lambda_B d_i)^{\chi_i}}{D_A^2} (\lambda_B d_i)^2 \sum_{|x_{il}|>\frac{\varepsilon_{m+n}}{D_A}+2} \frac{(\lambda_B d_i)^{x_{il}} e^{-\lambda_B d_i}}{x_{il}!} + (\lambda_B d_i) \sum_{|x_{il}|>\frac{\varepsilon_{m+n}}{D_A}+1} \frac{(\lambda_B d_i)^{x_{il}} e^{-\lambda_B d_i}}{x_{il}!} \end{array} \right\}$$

$$Q \le \lim_{m+n\to\infty} \frac{1}{s^2_{m+n}} \left\{ \begin{array}{l} \displaystyle\sum_{i=1}^{m} \frac{(\lambda_A d_i)^{\chi_i}}{D_B^2} (\lambda_A d_i)^2 \sum_{|x_{il}|>\frac{\varepsilon_{m+n}}{D_B}+2} \frac{(\lambda_A d_i)^{x_{il}} e^{-\lambda_A d_i}}{x_{il}!} + (\lambda_A d_i) \sum_{|x_{il}|>\frac{\varepsilon_{m+n}}{D_B}+1} \frac{(\lambda_A d_i)^{x_{il}} e^{-\lambda_A d_i}}{x_{il}!} \\ \displaystyle + \sum_{i=m+1}^{m+n} \frac{(\lambda_B d_i)^{\chi_i}}{D_A^2} (\lambda_B d_i)^2 \sum_{|x_{il}|>\frac{\varepsilon_{m+n}}{D_A}+2} \frac{(\lambda_B d_i)^{x_{il}} e^{-\lambda_B d_i}}{x_{il}!} + (\lambda_B d_i) \sum_{|x_{il}|>\frac{\varepsilon_{m+n}}{D_A}+1} \frac{(\lambda_B d_i)^{x_{il}} e^{-\lambda_B d_i}}{x_{il}!} \end{array} \right\}$$

$$Q \le \lim_{m+n\to\infty} \frac{1}{s^2_{m+n}} \left\{ \begin{array}{l} \displaystyle\sum_{i=1}^{m} \frac{(\lambda_A d_i)^{\chi_i+2}}{D_B^2} \Pr ob\left[|x_{il}| > \frac{\varepsilon_{m+n}}{D_B} + 2\right] + \frac{(\lambda_A d_i)^{\chi_i+1}}{D_B^2} \Pr ob\left[|x_{il}| > \frac{\varepsilon_{m+n}}{D_B} + 1\right] \\ \displaystyle + \sum_{i=m+1}^{m+n} \frac{(\lambda_B d_i)^{\chi_i+2}}{D_A^2} \Pr ob\left[|x_{il}| > \frac{\varepsilon_{m+n}}{D_A} + 2\right] + \frac{(\lambda_B d_i)^{\chi_i+1}}{D_A^2} \Pr ob\left[|x_{il}| > \frac{\varepsilon_{m+n}}{D_A} + 1\right] \end{array} \right\}$$

Using Chebyshev's inequality, we obtain:

$$Q \le \lim_{m+n\to\infty} \frac{1}{s^2_{m+n}} \left\{ \begin{array}{l} \displaystyle\sum_{i=1}^{m} \frac{(\lambda_A d_i)^{\chi_i+2}}{D_B^2} \frac{E(x_{il})}{\frac{\varepsilon_{m+n}}{D_B}} + \frac{(\lambda_A d_i)^{\chi_i+1}}{D_B^2} \frac{E(x_{il})}{\frac{\varepsilon_{m+n}}{D_B}} \\ \displaystyle + \sum_{i=m+1}^{m+n} \frac{(\lambda_B d_i)^{\chi_i+2}}{D_A^2} \frac{E(x_{il})}{\frac{\varepsilon_{m+n}}{D_A}} + \frac{(\lambda_B d_i)^{\chi_i+1}}{D_A^2} \frac{E(x_{il})}{\frac{\varepsilon_{m+n}}{D_A}} \end{array} \right\}$$

$$Q \le \lim_{m+n\to\infty} \frac{1}{s^2_{m+n}} \left\{ \begin{array}{l} \displaystyle\sum_{i=1}^{m} \frac{(\lambda_A d_i)^{\chi_i+2}}{D_B^2} \frac{E(x_{il})}{\frac{\varepsilon_{m+n}}{D_B}} + \frac{(\lambda_A d_i)^{\chi_i+1}}{D_B^2} \frac{E(x_{il})}{\frac{\varepsilon_{m+n}}{D_B}} \\ \displaystyle + \sum_{i=m+1}^{m+n} \frac{(\lambda_B d_i)^{\chi_i+2}}{D_A^2} \frac{E(x_{il})}{\frac{\varepsilon_{m+n}}{D_A}} + \frac{(\lambda_B d_i)^{\chi_i+1}}{D_A^2} \frac{E(x_{il})}{\frac{\varepsilon_{m+n}}{D_A}} \end{array} \right\}$$

$$Q \le \lim_{m+n\to\infty} \frac{1}{s_{m+n}^2} \left\{ \begin{array}{l} \sum_{i=1}^{m} \frac{(\lambda_A d_i)^{\chi_i+2}}{D_B} \frac{E(x_{il})}{\varepsilon s_{m+n}} + \frac{(\lambda_A d_i)^{\chi_i+1}}{D_B} \frac{E(x_{il})}{\varepsilon s_{m+n}} \\ + \sum_{i=m+1}^{m+n} \frac{(\lambda_B d_i)^{\chi_i+2}}{D_A} \frac{E(x_{il})}{\varepsilon s_{m+n}} + \frac{(\lambda_B d_i)^{\chi_i+1}}{D_A} \frac{E(x_{il})}{\varepsilon s_{m+n}} \end{array} \right\}$$

$$Q \le \lim_{m+n\to\infty} \frac{1}{s_{m+n}^2} \left\{ \sum_{i=1}^{m} \frac{(\lambda_A d_i)^{\chi_i+3}}{D_B \varepsilon s_{m+n}} + \frac{(\lambda_A d_i)^{\chi_i+2}}{D_B \varepsilon s_{m+n}} + \sum_{i=m+1}^{m+n} \frac{(\lambda_B d_i)^{\chi_i+3}}{D_A \varepsilon s_{m+n}} + \frac{(\lambda_B d_i)^{\chi_i+2}}{D_A \varepsilon s_{m+n}} \right\}$$

Recall that $s_{m+n}^2 = \sum_{i=1}^{m} D_B{}^2 d_i \lambda_A + \sum_{i=m+1}^{m+n} D_A{}^2 d_i \lambda_B$

Hence, $s_{m+n}^2 \ge \sum_{i=1}^{m} D_B{}^2 d_i \lambda_A$ and $s_{m+n}^2 \ge \sum_{i=m+1}^{m+n} D_A{}^2 d_i \lambda_B$

$s_{m+n}^2 \ge mn^2 \lambda_A$ and $s_{m+n}^2 \ge m^2 n \lambda_B$

since $1 \le d_i \le d_{max}$ for all $i = 1,2,\dots,m+n$,

and $D_A \ge m$ and $D_B \ge n$

$$Q \le \lim_{m+n\to\infty} \frac{1}{s_{m+n}^2} \sum_{i=1}^{m} \frac{(\lambda_A d_i)^{\chi_i+3}}{D_B \varepsilon s_{m+n}} + \frac{(\lambda_A d_i)^{\chi_i+2}}{D_B \varepsilon s_{m+n}} + \frac{1}{s_{m+n}^2} \sum_{i=m+1}^{m+n} \frac{(\lambda_B d_i)^{\chi_i+3}}{D_A \varepsilon s_{m+n}} + \frac{(\lambda_B d_i)^{\chi_i+2}}{D_A \varepsilon s_{m+n}}$$

$$Q \le \lim_{m+n\to\infty} \frac{1}{mn^2 \lambda_A} \sum_{i=1}^{m} \frac{(\lambda_A d_{max})^{\chi_i+3}}{n\varepsilon\sqrt{mn^2\lambda_A}} + \frac{(\lambda_A d_{max})^{\chi_i+2}}{n\varepsilon\sqrt{mn^2\lambda_A}} + \frac{1}{m^2 n \lambda_B} \sum_{i=m+1}^{m+n} \frac{(\lambda_B d_{max})^{\chi_i+3}}{m\varepsilon\sqrt{m^2 n\lambda_B}} + \frac{(\lambda_B d_{max})^{\chi_i+2}}{m\varepsilon\sqrt{m^2 n\lambda_B}}$$

Define M such that

$M \ge (\lambda_A d_{max})^{\chi_i+3} + (\lambda_A d_{max})^{\chi_i+2}$ and also $M \ge (\lambda_B d_{max})^{\chi_i+3} + (\lambda_B d_{max})^{\chi_i+2}$

We then have, $Q \le \lim_{m+n\to\infty} \frac{M}{n^4 \varepsilon\sqrt{m\lambda_A}} + \frac{M}{m^4 \varepsilon \sqrt{n\lambda_B}} = 0$

Therefore, the Lindeberg condition is met, and $R_\delta \xrightarrow{L} Z$.

We now show that $\hat{R}_\delta \xrightarrow{L} Z$.

From equation 1, we have $\hat{R}_\delta = \sqrt{\dfrac{\frac{\lambda_A}{D_A} + \frac{\lambda_B}{D_B}}{\frac{\hat{\Lambda}_A}{D_A} + \frac{\hat{\Lambda}_B}{D_B}}} R_\delta$.

The following theorems and statements can be found in or easily verified from Rohatgi [9].

Theorem 1 [9, p.253]

If $X_n \xrightarrow{L} X$ and $Y_n \xrightarrow{P} a$, a constant, then

$$Y_n X_n \xrightarrow{L} aX \qquad \text{if } c \neq 0$$

Theorem 2 [9, p. 245]

Let $X_n \xrightarrow{P} X$ and g be a continuous function defined on $\Re$, then $g(X_n) \xrightarrow{P} g(X)$ as $n \to \infty$.

Corollary 1 [9, p. 245]

$X_n \xrightarrow{P} c$, where c is a constant $\Rightarrow g(X_n) \xrightarrow{P} g(c)$, g being a continuous function.

Statement 1

$X_n \xrightarrow{P} a \Rightarrow \dfrac{a}{X_n} \xrightarrow{P} 1$, where a is a constant.

We first show that $\dfrac{\hat{\Lambda}_A}{D_A} + \dfrac{\hat{\Lambda}_B}{D_B} \xrightarrow{P} \dfrac{\lambda_A}{D_A} + \dfrac{\lambda_B}{D_B}$.

We will show that $K = \underset{m+n\to\infty}{Lim} \Pr ob\left[\left|\dfrac{\hat{\Lambda}_A}{D_A} + \dfrac{\hat{\Lambda}_B}{D_B} - (\dfrac{\lambda_A}{D_A} + \dfrac{\lambda_B}{D_B})\right| > \varepsilon\right] = 0$.

$$\Pr ob\left[\left|\frac{\hat{\Lambda}_A}{D_A} + \frac{\hat{\Lambda}_B}{D_B} - (\frac{\lambda_A}{D_A} + \frac{\lambda_B}{D_B})\right| > \varepsilon\right] = \Pr ob\left[\left|D_B\hat{\Lambda}_A + D_A\hat{\Lambda}_B - D_B\lambda_A - D_A\lambda_B\right| > \varepsilon D_A D_B\right]$$

$$\Pr ob\left[\left|D_B\hat{\Lambda}_A + D_A\hat{\Lambda}_B - D_B\lambda_A - D_A\lambda_B\right| > \varepsilon D_A D_B\right] = \Pr ob\left[\left(D_B\hat{\Lambda}_A + D_A\hat{\Lambda}_B - D_B\lambda_A - D_A\lambda_B\right)^2 > \varepsilon^2 D_A^2 D_B^2\right]$$

Using the Chebyshev inequality, we have

$$K \le \underset{m+n\to\infty}{Lim} \frac{E(D_B\hat{\Lambda}_A + D_A\hat{\Lambda}_B - D_B\lambda_A - D_A\lambda_B)^2}{\varepsilon^2 D_A^2 D_B^2} = \underset{m+n\to\infty}{Lim} \frac{Var(D_B\hat{\Lambda}_A + D_A\hat{\Lambda}_B)}{\varepsilon^2 D_A^2 D_B^2} = \underset{m+n\to\infty}{Lim} \frac{D_B^2 \dfrac{\lambda_A}{D_A} + D_A^2 \dfrac{\lambda_B}{D_B}}{\varepsilon^2 D_A^2 D_B^2}$$

$$K \le Lim \frac{D_B^3\lambda_A + D_A^3\lambda_B}{\varepsilon^2 D_A^3 D_B^3} = \underset{m+n\to\infty}{Lim} \frac{D_B^3\lambda_A}{\varepsilon^2 D_A^3 D_B^3} + \underset{m+n\to\infty}{Lim} \frac{D_A^3\lambda_B}{\varepsilon^2 D_A^3 D_B^3} = \underset{m+n\to\infty}{Lim} \frac{\lambda_A}{D_A^3} + \underset{m+n\to\infty}{Lim} \frac{\lambda_B}{D_B^3} = 0$$

By application of statement 1, we find that

$$\frac{\frac{\lambda_A}{D_A}+\frac{\lambda_B}{D_B}}{\frac{\hat{\Lambda}_A}{D_A}+\frac{\hat{\Lambda}_B}{D_B}} \xrightarrow{P} 1$$

By corollary 1, we find

$$\sqrt{\frac{\frac{\lambda_A}{D_A}+\frac{\lambda_B}{D_B}}{\frac{\hat{\Lambda}_A}{D_A}+\frac{\hat{\Lambda}_B}{D_B}}} \xrightarrow{P} 1$$

and finally, by application of theorem 1,

$$\hat{R}_S = \sqrt{\frac{\frac{\lambda_A}{D_A}+\frac{\lambda_B}{D_B}}{\frac{\hat{\lambda}_A}{D_A}+\frac{\hat{\lambda}_B}{D_B}}}\, R_S \xrightarrow{l} N(0,1)$$

The proof is complete.

APPENDIX B

Confidence Interval for λ

Let $\hat{\lambda}$ be the MLE of λ for Poisson distributed random variables X_i, $i = 1,2,\ldots,n$, with means λd_i and variances λd_i where d_i is the number of exposure units associated with X_i. Let x_i, $i = 1,2,\ldots,n$ be the realization of the random variables X_i.

Then, $\hat{\lambda} = \dfrac{\sum_{i=1}^{N} x_i}{\sum_{i=1}^{N} d_i}$ is the realization of a random variable $\hat{\Lambda}$, where $\hat{\Lambda} = \dfrac{\sum_{i=1}^{N} X_i}{\sum_{i=1}^{N} d_i}$.

$\hat{g} = \dfrac{\hat{\lambda} - \lambda}{\sqrt{\dfrac{\hat{\lambda}}{\sum_{i=1}^{N} d_i}}}$ is the realization of the random variable $\hat{G} = \dfrac{\hat{\Lambda} - \lambda}{\sqrt{\dfrac{\hat{\Lambda}}{\sum_{i=1}^{N} x_i}}}$

Using the definition introduced in appendix A, we will show $\hat{G} \xrightarrow{L} Z$ where Z has the standard normal distribution, and a $k\%$ confidence interval for λ is $\hat{\lambda} \pm z_{(1+k)/2} \sqrt{\dfrac{\hat{\lambda}}{\sum_{i=1}^{N} d_i}}$ where $z_{(1+k)/2}$ is the $(1+k)/2$ th quantile of the standard normal distribution.

Let's now prove that $\hat{G} \xrightarrow{L} Z$.

Let $G = \dfrac{\hat{\Lambda} - \lambda}{\sqrt{\dfrac{\lambda}{\sum_{i=1}^{N} x_i}}}$. Observe that $\hat{G} = \sqrt{\dfrac{\lambda}{\hat{\lambda}}} \dfrac{\hat{\lambda} - \lambda}{\sqrt{\dfrac{\lambda}{\sum_{i=1}^{N} d_i}}} = \sqrt{\dfrac{\lambda}{\hat{\lambda}}} G$

We first prove $G \xrightarrow{L} Z$

We rewrite $G = \dfrac{\dfrac{\sum_{i=1}^{N} X_i}{\sum_{i=1}^{N} d_i} - \lambda}{\sqrt{\dfrac{\lambda}{\sum_{i=1}^{N} d_i}}} = \dfrac{\sum_{i=1}^{N} X_i - \lambda d_i}{\sqrt{\sum_{i=1}^{N} \lambda d_i}} = \dfrac{\sum_{i=1}^{N} X_i - E(X_i)}{\sqrt{\sum_{i=1}^{N} Var(X_i)}}$

The Central Limit Theorem- Linderberg [9, p.282] states that the distribution of G converges to the standard normal distribution provided that the following condition is met:

$$L = \underset{n\to\infty}{Lim} \frac{1}{s_N^2} \sum_{i=1}^{N} \sum_{|x_{il}-\mu_i|>\varepsilon s_n} (x_{il} - \mu_i)^2 p_{il} = 0,$$

where $\mu_i = E(X_i), s_n^2 = \sum_{i=1}^{n} Var(X_i), and\ p_{il} = \Pr ob(X_i = x_{il})$.

$$L = \underset{N\to\infty}{Lim} \frac{1}{s_N^2} \sum_{i=1}^{N} \sum_{|x_{il}|>\varepsilon s_N} x_{il}^2 p_{il+\mu_i}$$

$$p_{il+\mu_i} = \frac{(\lambda d_i)^{x_{il}+\mu_i} e^{-\lambda d_i}}{(x_{il}+\mu_i)!} = \frac{(\lambda d_i)^{\mu_i} x_{il}!}{(x_{il}+\mu_i)!} \frac{(\lambda d_i)^{x_{il}} e^{-\lambda d_i}}{x_{il}!} \le (\lambda d_i)^{\mu_i} \frac{(\lambda d_i)^{x_{il}} e^{-\lambda d_i}}{x_{il}!}$$

$$L \le \underset{N\to\infty}{Lim} \frac{1}{s_N^2} \sum_{i=1}^{N} \sum_{|x_{il}|>\varepsilon s_N} x_{il}^2 (\lambda d_i)^{\mu_i} \frac{(\lambda d_i)^{x_{il}} e^{-\lambda d_i}}{x_{il}!} = \underset{N\to\infty}{Lim} \frac{1}{s_N^2} \sum_{i=1}^{N} (\lambda d_i)^{\mu_i} \sum_{|x_{il}|>\varepsilon s_N} x_{il}^2 \frac{(\lambda d_i)^{x_{il}} e^{-\lambda d_i}}{x_{il}!}$$

$$L \le \underset{N\to\infty}{Lim} \frac{1}{s_N^2} \sum_{i=1}^{N} (\lambda d_i)^{\mu_i} \sum_{|x_{il}|>\varepsilon s_N} x_{il} \frac{(\lambda d_i)^{x_{il}} e^{-\lambda d_i}}{(x_{il}-1)!} = \underset{N\to\infty}{Lim} \frac{1}{s_N^2} \sum_{i=1}^{N} (\lambda d_i)^{\mu_i} \sum_{|x_{il}|>\varepsilon s_N+1} (x_{il}+1) \frac{(\lambda d_i)^{x_{il}+1} e^{-\lambda d_i}}{x_{il}!}$$

$$L \le \underset{N\to\infty}{Lim} \frac{1}{s_N^2} \sum_{i=1}^{N} (\lambda d_i)^{\mu_i} \left[\sum_{|x_{il}|>\varepsilon s_N+1} x_{il} \frac{(\lambda d_i)^{x_{il}+1} e^{-\lambda d_i}}{x_{il}!} + \sum_{|x_{il}|>\varepsilon s_N+1} \frac{(\lambda d_i)^{x_{il}+1} e^{-\lambda d_i}}{x_{il}!} \right]$$

$$L \le \underset{N\to\infty}{Lim} \frac{1}{s_N^2} \sum_{i=1}^{N} (\lambda d_i)^{\mu_i} \left[\sum_{|x_{il}|>\varepsilon s_N+1} \frac{(\lambda d_i)^{x_{il}+1} e^{-\lambda d_i}}{(x_{il}-1)!} + \sum_{|x_{il}|>\varepsilon s_N+1} \frac{(\lambda d_i)^{x_{il}+1} e^{-\lambda d_i}}{x_{il}!} \right]$$

$$L \le \underset{N\to\infty}{Lim} \frac{1}{s_N^2} \sum_{i=1}^{N} (\lambda d_i)^{\mu_i} \left[\sum_{|x_{il}|>\varepsilon s_N+2} \frac{(\lambda d_i)^{x_{il}+2} e^{-\lambda d_i}}{x_{il}!} + \sum_{|x_{il}|>\varepsilon s_N+1} \frac{(\lambda d_i)^{x_{il}+1} e^{-\lambda d_i}}{x_{il}!} \right]$$

$$L \le \underset{N\to\infty}{Lim} \frac{1}{s_N^2} \sum_{i=1}^{N} (\lambda d_i)^{\mu_i} \left[(\lambda d_i)^2 \sum_{|x_{il}|>\varepsilon s_N+2} \frac{(\lambda d_i)^{x_{il}} e^{-\lambda d_i}}{x_{il}!} + (\lambda d_i) \sum_{|x_{il}|>\varepsilon s_N+1} \frac{(\lambda d_i)^{x_{il}} e^{-\lambda d_i}}{x_{il}!} \right]$$

$$L \le \underset{N\to\infty}{Lim} \frac{1}{s_N^2} \sum_{i=1}^{N} (\lambda d_i)^{\mu_i+2} \Pr ob\left[|x_{il}| > \varepsilon s_N + 2\right] + (\lambda d_i)^{\mu_i+1} \Pr ob\left[|x_{il}| > \varepsilon s_N + 1\right]$$

Using Chebyshev's inequality, we obtain:

$$L \le \underset{N\to\infty}{Lim} \frac{1}{s_N^2} \sum_{i=1}^{N} (\lambda d_i)^{\mu_i+2} \frac{E(x_{il})}{\varepsilon s_N + 2} + (\lambda d_i)^{\mu_i+1} \frac{E(x_{il})}{\varepsilon s_N + 1} = \underset{N\to\infty}{Lim} \frac{1}{s_N^2} \sum_{i=1}^{N} \frac{(\lambda d_i)^{\mu_i+3}}{\varepsilon s_N + 2} + \frac{(\lambda d_i)^{\mu_i+2}}{\varepsilon s_N + 1}$$

$$L \le \underset{N\to\infty}{Lim} \frac{1}{s_N^2} \sum_{i=1}^{N} \frac{(\lambda d_i)^{\mu_i+3} + (\lambda d_i)^{\mu_i+2}}{\varepsilon s_N}$$

Since $1 \le d_i \le d_{\max}$ for all $i = 1,2,\ldots,N$.

$s_N^2 = \sum_{i=1}^{N} d_i \lambda \ge N\lambda$ and $(\lambda d_i)^{\mu_i+3} + (\lambda d_i)^{\mu_i+2} \le M$

Therefore, $L \le \underset{N\to\infty}{Lim} \frac{1}{N\lambda} \sum_{i=1}^{N} \frac{M}{\varepsilon\sqrt{N\lambda}} = \underset{N\to\infty}{Lim} \frac{1}{\lambda} \frac{M}{\varepsilon\sqrt{N\lambda}} = 0$

Hence, the Linderberg condition is satisfied and $G \xrightarrow{L} Z$.

We now show that $\hat{G} \xrightarrow{L} Z$.

$$\hat{G} = \sqrt{\frac{\lambda}{\hat{\lambda}}} G$$

We first show $\hat{\lambda} \xrightarrow{P} \lambda$.

We will show that $K = \underset{N \to \infty}{Lim} \Pr ob \left| \left| \hat{\lambda} - \lambda \right| > \varepsilon \right| = 0$

Using Chebyshev's inequality,

$$K \le \underset{N \to \infty}{Lim} \frac{E(\hat{\lambda} - \lambda)^2}{\varepsilon^2} = \underset{N \to \infty}{Lim} \frac{Var(\hat{\lambda})}{\varepsilon^2} = \underset{N \to \infty}{Lim} \frac{\lambda}{\varepsilon^2 \sum_{i=1}^{N} d_i} \le \underset{N \to \infty}{Lim} \frac{\lambda}{\varepsilon^2 N} = 0.$$

By application of statement 1 of Appendix A, we find

$$\frac{\lambda}{\hat{\lambda}} \to 1$$

By application of corollary 1,

$$\sqrt{\frac{\lambda}{\hat{\lambda}}} \to 1$$

Finally, by application of theorem 1

$$\hat{G} = \sqrt{\frac{\lambda}{\hat{\lambda}}} G \to N(0,1).$$

Our proof is complete.

APPENDIX C

Assume there are n cells in a class, and the MLE estimate $\hat{\lambda}_j$ of cell C_j, for $j = 1,2,\ldots n$, is given by

$\hat{\lambda}_j = \dfrac{\sum_{i=1}^{N_j} x_i}{\sum_{i=1}^{N_j} d_i}$ where N_j is the number of observations in cell C_j. The MLE estimate for the class is given

by $\hat{\lambda} = \dfrac{\sum_{i=1}^{N} x_i}{\sum_{i=1}^{N} d_i}$ where N is the total number of observations across all cells such that $\sum_{j=1}^{n} N_j = N$.

We rewrite $\hat{\lambda}$ as:

$$\hat{\lambda} = \frac{\sum_{i=1}^{N_1} x_i + \sum_{i=N_1+1}^{N_2} x_i + \ldots + \sum_{i=N_{n-1}+1}^{N_n} x_i}{\sum_{i=1}^{N_1} d_i + \sum_{i=N_1+1}^{N_2} d_i + \ldots + \sum_{i=N_{n-1}+1}^{N_n} d_i}$$

$$\hat{\lambda} = \frac{\sum_{i=1}^{N_1} x_i}{\sum_{i=1}^{N} d_i} \frac{\sum_{i=1}^{N_1} d_i}{\sum_{i=1}^{N_1} d_i} + \frac{\sum_{i=N_1+1}^{N_2} x_i}{\sum_{i=1}^{N} d_i} \frac{\sum_{i=N_1+1}^{N_2} d_i}{\sum_{i=N_1+1}^{N_2} d_i} + \ldots + \frac{\sum_{i=N_{n-1}+1}^{N_n} x_i}{\sum_{i=1}^{N} d_i} \frac{\sum_{i=N_{n-1}+1}^{N_n} d_i}{\sum_{i=N_{n-1}+1}^{N_n} d_i}$$

$$\hat{\lambda} = \hat{\lambda}_1 \frac{\sum_{i=1}^{N_1} d_i}{\sum_{i=1}^{N} d_i} + \hat{\lambda}_2 \frac{\sum_{i=N_1+1}^{N_2} d_i}{\sum_{i=1}^{N} d_i} + \ldots + \hat{\lambda}_n \frac{\sum_{i=N_{n-1}+1}^{N_n} d_i}{\sum_{i=1}^{N} d_i}$$

APPENDIX D

Assume two cells C_1 and C_2 with the following ten observations:

C_1	C_2
10.154	8.508
11.510	8.100
5.453	11.707
13.239	8.772
10.065	14.156
(2.307)	6.953
17.625	7.612
13.242	10.633
14.319	7.463
7.619	5.546

The observations in C_1 are assumed to come from a Normal distribution with cumulative probability function F_1, while those in C_2 are assumed to come from a Lognormal distribution with cumulative probability function F_2.

Compatibility is defined as follows:

> Given two cells C_i and C_j, C_i is compatible to C_j if: $F_j(x_{ik}) > 0$ *for all* $k = 1,2,\ldots n$, where x_{ik} is an observation from C_i, n the number of observations in C_i, and F_j the cumulative probability function for cell C_j.

Since $F_1(x_{2k}) > 0$ *for all* $k = 1,2,\ldots 10$, we say that C_2 is compatible to C_1. However, $F_2(-2.307) = 0$, therefore we say that C_1 is not compatible to C_2.

On the Practical Multiline Excess of Loss Pricing

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On the practical multiline excess of loss pricing

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On the practical multiline excess of loss pricing

Abstract

More and more ceding companies are asking for global protections of their portfolios. One example is the protection by the reinsurer of two (or more) lines, e.g. fire and motor third party liability. Clearly this allows the insurance company to optimally balance its portfolio and to pay the lowest reinsurance premium. In this paper we analyse how to price an excess of loss treaty covering multiple lines.

Keywords

Excess of loss, multiple cover, payment pattern, stability clause, capital allocation, cost of capital, cash flow model, multiline aggregate deductible, sliding scale, profit commission.

1. Introduction

Insurance companies are corporations and, as such, they are willing to buy reinsurance for the same reasons that corporations buy insurance. These reasons include the fact that entities are not able to diversify insurable risks. They will therefore demand some compensation for their risk-averseness. This compensation may take different forms :

- higher wages for employees and managers
- lower rates for clients
- more allocated capital by the shareholders
- buying some (re)insurance.

The latter is observed on the market and we will discuss in this paper the pricing of some particular reinsurance treaties.
More and more insurance companies are trying to optimize their reinsurance structure. They are looking for a global protection with their reinsurers. One of these global solutions is to cover two lines simultaneously. Clearly this allows to take better advantage of the diversification of an insurance portfolio. Thus a better reinsurance cover follows.
Let us take an example. Assume a fire treaty existing of three layers :

- Layer 1 (Fire) : 2500 xs 1000 with three reinstatements at 100%.
- Layer 2 (Fire) : 3000 xs 3000 with two reinstatements at 100%.
- Layer 3 (Fire) : 4000 xs 6000 with one reinstatement at 100%.

Assume a MTPL (Motor Third Party Liability) treaty existing of three layers :

- Layer 1 (MTPL) : 3000 xs 2000 with unlimited free reinstatements.
- Layer 2 (MTPL) : 5000 xs 5000 with unlimited free reinstatements.
- Layer 3 (MTPL) : ∞ xs 10000 with unlimited free reinstatements.

North American readers may be surprised to see layers with unlimited free reinstatements, as well as an unlimited layer. This is in fact common practice in Europe, and in particular in Belgium, at least for Motor Third Party Liability covers. Property covers are always limited and General Liability covers are usually limited.
An alternative solution might be to keep Layers 2 and 3 for Fire and MTPL and to create a global treaty with alternative Layer 1bis (Fire) and Layer 1bis (MTPL) :

- Layer 1bis (Fire) : 2500 xs 500 with unlimited free reinstatements,
- Layer 1bis (MPTL) : 4000 xs 1000 with unlimited free reinstatements.

with a global annual aggregate deductible of, say, 1000 (Ribeaud (2000) calls it a multiline aggregate deductible). So, for the working layer we combine Fire and MTPL and, as it is a working layer, we impose a large annual aggregate deductible in order to avoid a huge amount of claims to be paid by the reinsurer and high premiums to be paid by the insurer. Note that Layer 1bis (Fire) and Layer 1bis (MTPL) are one treaty. One global premium is asked for that cover. We now have three treaties :

- Fire with two layers : 3000 xs 3000 and 4000 xs 6000.

- MTPL with two layers : 5000 xs 5000 and ∞ xs 10000.

- Global, which is affected by claims hitting Layer 1bis (Fire) and Layer 1bis (MTPL) with a global (multiline) annual aggregate deductible of 1000.

This global treaty is exactly the kind of treaty we want to price in this paper.
Throughout the paper we will use a numerical example in order to apply the models and formulae that will be derived.
The rest of the paper is organized as follows. Section 2 presents the general model we will work with as well as the particular distributions that will be used in the numerical example. Section 3 recalls the use of the Panjer's algorithm as well as the use of lattice distributions. Section 4 presents the detailed model we will work with, i.e. reinsurance liabilities with potential clauses. Section 5 shows how to mix both lines and obtains expected values required for the cash flow model that is presented in section 6. Section 7 discusses the use of clauses making the reinsurance premium random. Section 8 gives the conclusion.

2. General model

From now on we will adopt the traditional convention that treaties are yearly based, which is common practice.
We will work within the collective risk model. In this model, claims arise anonymously from the portfolio. It is assumed that the losses are identically distributed and mutually independent. It is also assumed that they are independent of the number of claims, which is a random variable (typically a Poisson distribution).
Working with the collective risk model is not a limitation, as other models may be used, e.g. the individual risk model. In this model it is assumed that each risk has a (known) chance to produce at least one claim during the coverage period. It is also assumed that the loss distribution, in case of a claim, is known for each risk.
Let us define

- X_i as the i^{th} claim amount of type Fire,

- Y_i as the i^{th} claim amount of type MTPL.

It is assumed that the X_i's are independent and identically distributed as well as the Y_i's. X_i's and Y_i's are assumed to be mutually independent. We also define

- N as the number of claims of type Fire,

- M as the number of claims of type MTPL.

We assume that N and M are independent and that N and the X_i's on the one hand and M and the Y_i's on the other hand are also independent.
We are then able to build two collective risk models :

$$\begin{aligned} S &= X_1 + \cdots + X_N, \\ T &= Y_1 + \cdots + Y_M, \end{aligned}$$

where S denotes the aggregate fire claims and T denotes the aggregate MTPL claims.
Let us assume that the distributions of X, Y, N and M have been estimated, possibly based on past data, as follows

- the distribution of the fire claim amounts, X, is Pareto with parameters $A = 400$ and $\alpha = 1.50$. The distribution of the MTPL claim amounts, Y is Pareto with parameters $A = 700$ and $\alpha = 2.50$. Let us recall the cumulative density distribution of a Pareto distribution ($X \sim Pa(A,\alpha)$) :

$$\begin{aligned} F_X(x) &= 0 \quad \text{if } x \leq A, \\ &= 1 - \left(\frac{x}{A}\right)^{-\alpha} \quad \text{if } x > A. \end{aligned}$$

- the distribution of the fire claim numbers, N is Poisson with parameter $\lambda = 2.5$. The distribution of the MTPL claim numbers, M is Poisson with parameter $\lambda = 5$. Let us recall the probability function of a Poisson distribution ($N \sim Po(\lambda)$) :

$$\mathbb{P}[N = n] = p(n) = e^{-\lambda}\frac{\lambda^n}{n!} \quad , \quad n = 0, 1, \ldots$$

3. Practical calculations for the required distributions

In general, the actuary knows the behaviour of the claims losses. He has fitted, based on past data, a continuous distribution for X and Y. Furthermore, he assumes that N and M are Poisson distributed because he chose to work within the collective risk model.
First we have to obtain a discretization of the claims distributions. Indeed we will use Panjer's algorithm (see Panjer (1981)) that works with lattice distributions. For the distribution of S, we have :

$$\begin{aligned} f_S(0) &= e^{-\lambda(1-f_X(0))}, \\ f_S(s) &= \lambda\sum_{i=1}^{s}\frac{i}{s}f_X(i)f_S(s-i) \quad , \quad s = 1, 2, \ldots \end{aligned}$$

where f_X (resp. f_S) denotes the probability density function of X (resp. S) and λ is the parameter of the distribution of N. We observe that the Panjer's algorithm needs a discrete distribution. Therefore a continuous distribution may not be used as such and has to be discretized. Moreover it will be most convenient to obtain a discrete version of the continuous distribution which will be of lattice type, that is with non-negative masses on points of the type $x = kh$, $k = 0, 1, \ldots$ with $h > 0$. h is called the span. When the span is different from 1, a simple change of money (divide losses by h) allows to use the Panjer's algorithm optimally with respect to computing-time.
We immediately observe that the smaller the span, the better the precision of the discretization. However, the smaller the span, the longer the computing-time. The user should make a choice regarding the step in order to obtain a good precision and a sufficiently low computing-time. There are various methods for obtaining a lattice distribution from a general distribution. I choose to work with the easiest method : the rounding method (see Gerber and Jones (1976)). Let us choose a span h. The rounding method simply accumulates the original mass of a random variable X around the mass points of the lattice distribution (X_{dis}) as follows :

$$\begin{aligned} f_{X_{dis}}(0) &= F_X\left(\frac{h}{2} - 0\right), \\ f_{X_{dis}}(xh) &= F_X\left(xh + \frac{h}{2} - 0\right) - F_X\left(xh - \frac{h}{2} - 0\right) \quad , \quad x = 1, 2, \ldots \end{aligned}$$

For the particular case of a Pareto distribution ($X \sim Pa(A,\alpha)$) we obtain

$$
\begin{aligned}
f_{X_{dis}}(A) &= 1 - \left(\frac{A+h/2}{A}\right)^{-\alpha}, \\
f_{X_{dis}}(A+xh) &= \left(\frac{A-xh/2}{A}\right)^{-\alpha} - \left(\frac{A+xh/2}{A}\right)^{-\alpha} \quad, \quad x = 1,2,\dots
\end{aligned}
$$

We choose to work with a lattice step $h = 20$.
The first masses points of the lattice distributions for our numerical example are

x	400	425	450	475	500	525	...
$\mathbb{P}[X = x]$	0.0451	0.0807	0.0699	0.0611	0.0537	0.0475	...
y	700	725	750	775	800	825	...
$\mathbb{P}[Y = y]$	0.0433	0.0790	0.0702	0.0626	0.0560	0.0503	...

Table 1: Lattice version of the original distributions

Using the Panjer's algorithm we are able to obtain the aggregate claims distributions of S and T :

x	0	25	50	75	100	125
$\mathbb{P}[S = x]$	0.0919	0.0185	0.0179	0.0174	0.0169	0.0164
$\mathbb{P}[T = x]$	0.0084	0.0033	0.0036	0.0039	0.0041	0.0044

Table 2: Aggregate claims distributions

Note that these distributions concern the ceding company whereas we are interested in the pricing of reinsurance covers. This will be discussed in the next section.

4. Detailed model

4.1. Attachment points and covers

Let us now define the liability of an excess of loss reinsurer i.r.o. the claims. Let us denote

- $P_{Fire} = 500$ as the deductible of the Fire claims,
- $P_{MTPL} = 1000$ as the deductible of the MTPL claims,
- $L_{Fire} = 2500$ as the cover of the Fire claims,
- $L_{MTPL} = 4000$ as the cover of the MTPL claims.

We obtain the reinsurer's liability for the individual claims as follows :

$$
\begin{aligned}
X_i^{Re} &= \min(L_{Fire}, \max(0, X_i - P_{Fire})), \\
Y_i^{Re} &= \min(L_{MTPL}, \max(0, X_i - P_{MTPL})).
\end{aligned}
$$

The aggregate liability of the reinsurer is :

$$
\begin{aligned}
S^{Re} &= X_1^{Re} + \dots + X_N^{Re}, \\
T^{Re} &= T_1^{Re} + \dots + T_M^{Re}.
\end{aligned}
$$

The distribution of the reinsurer's liability for the individual claims and for the aggregate claims is

x	0	25	50	75	100	125	
$\mathbb{P}[X=x]$	0.3105	0.0475	0.0423	0.0379	0.0340	0.0307	...
$\mathbb{P}[Y=x]$	0.6026	0.0235	0.0216	0.0199	0.0183	0.0170	...
$\mathbb{P}[S=x]$	0.1784	0.0212	0.0201	0.0192	0.0183	0.0175	...
$\mathbb{P}[T=x]$	0.1371	0.0161	0.0158	0.0154	0.0151	0.0148	...

Table 3: Reinsurer's claims and aggregate claims distributions

4.2. Long-tailed business and inflation

We now have to introduce the fact that, in an insurance context, claims are not paid outright. Especially in excess of loss reinsurance where large claims are involved, it may be very long before a claim is finally settled. Thus, we have to introduce this notion and a companion thereof : the future inflation. We will follow the presentation of Walhin et al. (2001).

We will assume that the payments of the claims occur at times $t_0, t_2, \ldots, t_n$ according to a given claims payment pattern : $c_{Fire}(t_0), \ldots, c_{Fire}(t_n)$ or $c_{MTPL}(t_0), \ldots, c_{MTPL}(t_n)$ where t_n is the time of final settlement. We will furthermore assume that the payments arise, on average, in the middle of the year, i.e. $t_j = j + 0.5$, $j = 0, 1, \ldots, n$.

The claims payment pattern is supposed to be estimated by using past data and adjusted for potential changes in the future payment patterns, e.g. due to changes in legislation or in the claims management.

Let us assume that the MTPL claims are completely settled in $n = 7$ years whereas the fire claims are completely settled in two years. We use the following payment patterns :

t	0	1	2	3	4	5	6	7
c_{Fire}	50%	40%	10%	0%	0%	0%	0%	0%
c^{Σ}_{Fire}	50%	90%	100%	100%	100%	100%	100%	100%
c_{MTPL}	5%	10%	10%	10%	25%	25%	10%	5%
c^{Σ}_{MTPL}	5%	15%	25%	35%	60%	85%	95%	100%

Table 4: Payment patterns

where c^{Σ} denotes the cumulative claims pattern payment.

Moreover the future payments will undergo future inflation. Indeed the losses X_i are assumed not to include any future inflation. Let us define an inflation index : $inf_{Fire}(t_0), \ldots, inf_{Fire}(t_n)$ and $inf_{MTPL}(t_0), \ldots, inf_{MTPL}(t_n)$. The future payments for a loss X_i or Y_i then read :

$$\begin{aligned} X_i(j+0.5) &= c_{Fire}(t_j) X_i \frac{inf_{Fire}(t_j)}{inf_{Fire}(t_0)} \quad , \quad j = 0, 1, \ldots, n, \\ Y_i(j+0.5) &= c_{MTPL}(t_j) Y_i \frac{inf_{MTPL}(t_j)}{inf_{MTPL}(t_0)} \quad , \quad j = 0, 1, \ldots, n. \end{aligned}$$

The future inflation will be modelled by a geometric growth and we furthermore assume the future inflation index to be constant between two times $t_j = j$, $j = 0, 1, \ldots, n$:

$$\frac{inf_{Fire}(j)}{inf_{Fire}(j-1)} - 1 = 3\%, \; j = 1, 2, \ldots, n,$$

$$
\begin{aligned}
inf_{Fire}(j+0.5) &= inf_{Fire}(j)\,,\ j=0,1,\ldots,n,\\
\frac{inf_{MTPL}(j)}{inf_{MTPL}(j-1)}-1 &= 3.5\%\,,\ j=1,2,\ldots,n,\\
inf_{MTPL}(j+0.5) &= inf_{MTPL}(j)\,,\ j=0,1,\ldots,n.
\end{aligned}
$$

Future claims payments then read

	t	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5
	X_i								
$X_i(t)$	400	200.00	164.80	42.44	0	0	0	0	0
	425	212.50	175.10	45.09	0	0	0	0	0
	450	225.00	185.40	47.74	0	0	0	0	0
	475	237.50	195.70	50.39	0	0	0	0	0
	Y_i								
$Y_i(t)$	700	35.00	72.45	74.99	77.61	200.82	207.85	86.05	44.53
	725	36.25	75.04	77.66	80.38	207.99	215.27	89.12	46.12
	750	37.50	77.63	80.34	83.15	215.16	222.69	92.19	47.71
	775	38.75	80.21	83.02	85.93	222.33	230.11	95.27	49.30

Table 5: Future claims payments (inflation only)

As we are interested in large losses, it is commonly observed on the market that this category of losses undergoes a higher inflation than usual. One speaks of the superimposed inflation. For the future payments, it is then more adequate to use another index, including inflation and superimposed inflation : $supinf_{Fire}(t_0),\ldots,supinf_{Fire}(t_n)$ or $supinf_{MTPL}(t_0),\ldots,supinf_{MTPL}(t_n)$. The future payments for a loss X_i or Y_i then read :

$$
\begin{aligned}
X_i(t_j) &= c_{Fire}(t_j)X_i\frac{supinf_{Fire}(t_j)}{supinf_{Fire}(t_0)}\quad,\quad j=0,1,\ldots,n,\\
Y_i(t_j) &= c_{MTPL}(t_j)Y_i\frac{supinf_{MTPL}(t_j)}{supinf_{MTPL}(t_0)}\quad,\quad j=0,1,\ldots,n.
\end{aligned}
$$

Let us assume that the future inflation and superimposed inflation is modelled by a geometric growth :

$$
\begin{aligned}
\frac{supinf_{Fire}(j)}{supinf_{Fire}(j-1)}-1 &= 3\%\,,\ j=1,2,\ldots,n,\\
supinf_{Fire}(j+0.5) &= supinf_{Fire}(j)\,,\ j=0,1,2,\ldots,n,\\
\frac{supinf_{MTPL}(j)}{supinf_{MTPL}(j-1)}-1 &= 5\%\,,\ j=1,2,\ldots,n,\\
supinf_{MTPL}(j+0.5) &= supinf_{MTPL}(j)\,,\ j=0,1,2,\ldots,n,
\end{aligned}
$$

that is we assume no superimposed inflation for the fire claims and 1.50% of superimposed inflation for the MTPL claims.

Future claims payments then read

	t	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5
	X_i								
$X_i(t)$	400	200.00	164.80	42.44	0	0	0	0	0
	425	212.50	175.10	45.09	0	0	0	0	0
	450	225.00	185.40	47.74	0	0	0	0	0
	475	237.50	195.70	50.39	0	0	0	0	0
	Y_i								
$Y_i(t)$	700	35.00	75.50	77.18	81.03	212.71	223.35	93.81	49.25
	725	36.25	76.13	79.93	83.93	220.31	231.33	97.16	51.01
	750	37.50	78.75	82.69	86.82	227.91	239.30	100.51	52.77
	775	38.75	81.38	85.44	89.72	235.50	247.28	103.86	54.53

Table 6: Future claims payments (including superimposed inflation)

It is also interesting to define the cumulative payments for a loss X_i or Y_i as :

$$\begin{aligned} X_i^{\Sigma}(j+0.5) &= \sum_{k=0}^{j} X_i(k+0.5) \quad , \quad j=0,1,\ldots,n, \\ Y_i^{\Sigma}(j+0.5) &= \sum_{k=0}^{j} Y_i(k+0.5) \quad , \quad j=0,1,\ldots,n. \end{aligned}$$

The evolution of the cumulative payments for the reinsurer for a loss X_i or Y_i then reads :

$$\begin{aligned} X_i^{\Sigma Re}(j+0.5) &= \min(L_{Fire}, \max(0, X_i^{\Sigma}(j+0.5) - P_{Fire})) \quad , \quad j=0,1,\ldots,n, \\ Y_i^{\Sigma Re}(j+0.5) &= \min(L_{MTPL}, \max(0, Y_i^{\Sigma}(j+0.5) - P_{MTPL})) \quad , \quad j=0,1,\ldots,n. \end{aligned}$$

Within our numerical example we have

	t	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5
	X_i								
	⋮								
$X_i^{\Sigma}(t)$	500	250.00	456.00	509.05	509.05	509.05	509.05	509.05	509.05
	525	262.50	478.80	534.50	534.50	534.50	534.50	534.50	534.50
	550	275.00	501.60	559.95	559.95	559.95	559.95	559.95	559.95
	575	287.50	524.40	585.40	585.40	585.40	585.40	585.40	585.40
	⋮								
	Y_i								
	⋮								
$Y_i^{\Sigma}(t)$	3000	150.00	465.00	795.75	1143.04	2054.67	3011.88	3413.91	3624.97
	3025	151.25	468.88	802.38	1152.56	2071.79	3036.98	3442.36	3655.18
	3050	152.50	472.75	809.01	1162.09	2088.91	3062.08	3470.81	3685.39
	3075	153.75	476.63	815.64	1171.61	2106.03	3087.18	3499.25	3715.60
	⋮								
	X_i								
$X_i^{\Sigma Re}(t)$	500	0	0	9.04	9.04	9.04	9.04	9.04	9.04
	525	0	0	34.50	34.50	34.50	34.50	34.50	34.50
	550	0	1.60	59.95	59.95	59.95	59.95	59.95	59.95
	575	0	24.40	85.40	85.40	85.40	85.40	85.40	85.40
	⋮								
	Y_i								
	⋮								
$Y_i^{\Sigma Re}(t)$	3000	0	0	0	143.04	1054.67	2011.88	2413.91	2624.97
	3025	0	0	0	152.56	1071.79	2036.98	2442.26	2655.18
	3050	0	0	0	162.09	1088.91	2062.08	2470.81	2685.39
	3075	0	0	0	171.61	1106.03	2087.18	2499.26	2715.60
	⋮								

Table 7: Cumulative insurer's and reinsurer's payments

We show the evolution of the figures from 500 for Fire claims and from 3000 for MTPL claims in order to see figures different from 0 for the reinsurer's payments.

4.3. TECHNICAL RESERVES

In an ideal situation the claims manager is able to calculate exact reserves for a loss X_i or Y_i:

$$\begin{aligned} RX_i(j+0.5) &= X_i^{\Sigma}(n+0.5) - X_i^{\Sigma}(j+0.5) \quad , \quad j=0,1,\ldots,n, \\ RY_i(j+0.5) &= Y_i^{\Sigma}(n+0.5) - Y_i^{\Sigma}(j+0.5) \quad , \quad j=0,1,\ldots,n. \end{aligned}$$

Within our numerical example, we have

	t	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5
	X_i								
	⋮								
$RX_i(t)$	500	259.05	53.05	0	0	0	0	0	0
	525	272.00	55.70	0	0	0	0	0	0
	550	284.95	58.35	0	0	0	0	0	0
	575	297.90	61.00	0	0	0	0	0	0
	⋮								
	Y_i								
	⋮								
$RY_i(t)$	3000	3474.97	3159.97	2829.22	2481.93	1570.30	613.09	211.07	0
	3025	3503.93	3186.31	2852.80	2502.62	1583.39	618.20	212.82	0
	3050	3532.89	3212.64	2876.38	2523.30	1596.48	623.31	214.58	0
	3075	3561.85	3238.97	2899.95	2543.98	1609.56	628.42	216.34	0
	⋮								

Table 8: Ideal reserves

However there may be systematic deviations from these exact reserves. Let us assume that we have observed a pattern of deviation of the incurred loss (overstatement or understatement) : $d_{Fire}(t_0), \ldots, d_{Fire}(t_n)$ or $d_{MTPL}(t_0), \ldots, d_{MTPL}(t_n)$ where $d(t_j) = 100\%$ if there is no deviation of reservation at time t_j. The incurred loss and the outstanding, for a loss X_i or Y_i, may now be defined as follows :

$$\begin{aligned} IX_i(j+0.5) &= d_{Fire}(j+0.5)X_i^{\Sigma}(n+0.5) \quad , \quad j=0,1,\ldots,n, \\ RX_i(j+0.5) &= IX_i(j+0.5) - X_i^{\Sigma}(j+0.5) \quad , \quad j=0,1,\ldots,n, \\ IY_i(j+0.5) &= d_{MTPL}(j+0.5)Y_i^{\Sigma}(n+0.5) \quad , \quad j=0,1,\ldots,n, \\ RY_i(j+0.5) &= IY_i(j+0.5) - Y_i^{\Sigma}(j+0.5) \quad , \quad j=0,1,\ldots,n. \end{aligned}$$

Let us assume that the overstatement pattern is given by

t	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5
d_{Fire}	100%	100%	100%	100%	100%	100%	100%	100%
d_{MTPL}	125%	125%	125%	125%	105%	105%	100%	100%

Table 9: Overstatement pattern

We then have the evolution of the outstanding and incurred losses :

	t	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5
	X_i								
	⋮								
$IX_i(t)$	500	509.05	509.05	509.05	509.05	509.05	509.05	509.05	509.05
	525	534.50	534.50	534.50	534.50	534.50	534.50	534.50	534.50
	550	559.95	559.95	559.95	559.95	559.95	559.95	559.95	559.95
	575	585.40	585.40	585.40	585.40	585.40	585.40	585.40	585.40
	⋮								
	⋮								
$RX_i(t)$	500	259.05	53.05	0	0	0	0	0	0
	525	272.00	55.70	0	0	0	0	0	0
	550	284.95	58.35	0	0	0	0	0	0
	575	297.90	61.00	0	0	0	0	0	0
	⋮								
	Y_i								
	⋮								
$IY_i(t)$	3000	4531.22	4531.22	4531.22	4531.22	3806.22	3806.22	3624.97	3624.97
	3025	4568.98	4568.98	4568.98	4568.98	3837.94	3837.94	3655.18	3655.18
	3050	4606.74	4606.74	4606.74	4606.74	3869.66	3869.66	3685.39	3685.39
	3075	4644.50	4644.50	4644.50	4644.50	3901.38	3901.38	3715.60	3715.60
	⋮								
	⋮								
$RY_i(t)$	3000	4381.22	4066.22	3735.47	3388.18	1751.55	794.34	211.07	0
	3025	4417.73	4100.10	3766.59	3416.41	1766.15	800.96	212.82	0
	3050	4454.24	4133.99	3797.72	3444.65	1780.75	807.58	214.58	0
	3075	4490.75	4167.87	3828.85	3472.88	1795.34	814.20	216.34	0
	⋮								

Table 10: Insurer's reserves and incurred losses with overstatement

From the evolution of the incurred losses, it is now possible to derive the evolution of the incurred losses for the excess of loss reinsurer :

$$
\begin{aligned}
IX_i^{Re}(j+0.5) &= \min(L_{Fire}, \max(0, IX_i(j+0.5) - P_{Fire})) \quad , \quad j = 0, 1, \ldots, n. \\
IY_i^{Re}(j+0.5) &= \min(L_{MTPL}, \max(0, IY_i(j+0.5) - P_{MTPL})) \quad , \quad j = 0, 1, \ldots, n.
\end{aligned}
$$

Within our numerical example we have

	t	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5
	X_i								
$IX_i^{Re}(t)$	0	9.04	9.04	9.04	9.04	9.04	9.04	9.04	9.04
	25	34.50	34.50	34.50	34.50	34.50	34.50	34.50	34.50
	50	59.95	59.95	59.95	59.95	59.95	59.95	59.95	59.95
	75	85.40	85.40	85.40	85.40	85.40	85.40	85.40	85.40
	⋮								
	Y_i								
	⋮								
$IY_i^{Re}(t)$	3000	3531.21	3531.25	3531.25	3531.25	2806.22	2806.22	2624.97	2624.97
	3025	3568.97	3568.97	3568.97	3568.97	2837.94	2837.94	2655.18	2655.18
	3050	3606.73	3606.73	3606.73	3606.73	2869.66	2869.66	2685.39	2685.39
	3075	3644.50	3644.50	3644.50	3644.50	2901.38	2901.38	2715.60	2715.60
	⋮								

Table 11: Reinsurer's incurred losses

Our aim is to obtain the distribution of the paid claims and the distribution of the loss reserves at times $j + 0.5$, $j = 0, 1, \ldots, n$. This will allow us to obtain average values and so a cash flow model will be built in order to find the net present value of the business. This will allow us to determine if the business is worth the value or not. However before obtaining these distributions, we first have to consider some clauses that may affect the claims individually or in the aggregate.

It should be clear that the extension to multiple insurance lines is immediate. However, for educational purposes, we will limit ourselves to the methodology for two lines only.

4.4. Stability clause

If the attachment point (P) of the treaty is fixed, the reinsurer will take all future inflation during the development of the claim for his own account. Indeed once the loss is exceeds the attachment point, all future increases (except the part of the loss exceeding the cover of the treaty) due to inflation are borne by the reinsurer only. In order to protect themselves against this kind of possible moral hasard, reinsurers have introduced the stability clause. With this clause the reinsurer is willing to optimally share the future inflation between the ceding company and himself. There are several variants of the stability clause (see e.g. Gerathewohl (1980) for details). In this paper, and in particular in our numerical application, we will work with the so-called "date of payment" stability clause. When this clause is applied, the attachment point and/or the cover of the treaty are indexed each year with the following ratio

$$ratio = \frac{\text{sum of actual payments}}{\text{sum of adjusted payments}},$$

where adjusted payments means that each payment is discounted to the inception of the treaty with use of a conventional index, let us say the inflation index. The interested reader

is referred to Walhin et al. (2001) for further details.
We thus arrive at future attachment points and covers :
$P_{Fire}(t_0), P_{Fire}(t_1), \ldots, P_{Fire}(t_n)$, $P_{MTPL}(t_1), P_{MTPL}(t_2), \ldots, P_{MTPL}(t_n)$,
$L_{Fire}(t_0), L_{Fire}(t_1), \ldots, L_{Fire}(t_n)$ and $L_{MTPL}(t_1), L_{MTPL}(t_2), \ldots, L_{MTPL}(t_n)$ instead of singles P_{Fire}, P_{MTPL}, L_{Fire} and L_{MTPL}.
In accordance with the hypotheses on inflation, we will assume that $P_{Fire}(j+0.5) = P_{Fire}(j)$, $P_{MTPL}(j+0.5) = P_{MTPL}(j)$, $L_{Fire}(j+0.5) = L_{Fire}(j)$ and $L_{MTPL}(j+0.5) = L_{MTPL}(j)$, $j = 0, 1, \ldots, n$.
The evolution of the cumulative paid loss and incurred loss, for a loss X_i or Y_i, for the reinsurer now reads :

$$\begin{aligned}
X_i^{\Sigma Re}(j+0.5) &= \min(L_{Fire}(j+0.5), \max(0, X_i^{\Sigma}(j+0.5) - P_{Fire}(j+0.5))) \quad , \quad j = 0, 1, \ldots, n, \\
Y_i^{\Sigma Re}(j+0.5) &= \min(L_{MTPL}(j+0.5), \max(0, Y_i^{\Sigma}(j+0.5) - P_{MTPL}(j+0.5))) \quad , \quad j = 0, 1, \ldots, n, \\
IX_i^{Re}(j+0.5) &= \min(L_{Fire}(j+0.5), \max(0, IX_i(j+0.5) - P_{Fire}(j+0.5))) \quad , \quad j = 0, 1, \ldots, n, \\
IY_i^{Re}(j+0.5) &= \min(L_{MTPL}(j+0.5), \max(0, IY_i(j+0.5) - P_{MTPL}(j+0.5))) \quad , \quad j = 0, 1, \ldots, n.
\end{aligned}$$

When the claim is finally settled, both situations lead to the same repartition of the loss between the insurer and the reinsurer. The only difference is in the evolution of the cash flows.
Let us assume that the date of payment stability clause is applied to the attachment point and to the limit of the MTPL claims with a margin of 10%, i.e. the payments will be adjusted only if the claims index shows an evolution larger than the margin (see Walhin et al. (2001) for formulae details or Gerathewohl (1980) for further general details on the subject). The selected index is the claims index. It is also assumed that the application of the stability clause is based on incurred losses, that is, outstanding losses are used, and discounted as if they were payments. The attachment point and limit for the Fire claims are fixed, which is not illogical since Fire is not long-tail business. The evolution of the attachment point and limit for the MTPL claims is the following :

t_j	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5
P_{MTPL}	1000	1000	1000	1086.58	1108.74	1124.33	1129.94	1131.99
L_{MTPL}	4000	4000	4000	4346.30	4434.96	4497.32	4519.75	4527.95

Table 12: Evolution of the MTPL layer with stability clause

The payments and incurred losses of the reinsurer now read

	t	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5
	X_i								
	⋮								
$Y_i^{\Sigma Re}(t)$	3000	0	0	0	56.46	945.93	1887.55	2283.97	2492.98
	3025	0	0	0	65.99	963.05	1912.65	2312.42	2523.19
	3050	0	0	0	75.51	980.17	1937.75	2340.87	2553.40
	3075	0	0	0	85.03	997.29	1962.84	2369.32	2583.61
	⋮								
	X_i								
	⋮								
$IY_i^{Re}(t)$	3000	3531.21	3531.25	3531.25	3444.64	2697.48	2681.89	2495.03	2492.95
	3025	3568.97	3568.97	3568.97	3482.40	2729.20	2713.61	2525.24	2523.19
	3050	3606.73	3606.73	3606.73	3520.16	2760.92	2745.33	2555.45	2553.40
	3075	3644.50	3644.50	3644.50	3557.92	2792.64	2777.05	2585.66	2583.61
	⋮								

Table 13: Reinsurer's payments and incurred losses with stability clause (MTPL only)

4.5. Interests sharing clause / Loss adjustment expenses clause

When the claims development is long, it is expected that legal interests will have to be paid. The longer the claims development is, the higher the legal interests are. Once again for moral hazard reasons it may be tempting from the reinsurer's point of view to share the legal interests proportionnally between the cedent and the reinsurer. This is the aim of the interests sharing clause which is common practice, e.g. in Belgium.

The *interests sharing clause states that the legal interests have to be shared between the* ceding company and the reinsurer according to the pro rata liability of the reinsurer in the total liability of the loss excluding the legal interests. This means that the legal interests have to be excluded from the incurred loss before the application of the treaty. Afterwards they are divided between the ceding company and the reinsurer in accordance with the pro rata liability of both parties in the loss. Let us assume that on average a proportion δ_{Fire} or δ_{MTPL} of the incurred loss represents the interests. Note that it is reasonable to assume that this proportion is a function of the loss. However, in practice, it is extremely difficult to estimate the average proportion of the legal interests in such a way that it does not seem necessary to assume a varying proportion. Nevertheless it is possible to work within an extended model. The interested reader is referred to Walhin et al. (2001) for further details.

A common practice on North American markets is that loss adjustment expenses undergo the same treatment as the legal interests in Belgium, i.e. they are also shared on a pro rata basis between the insurer and the reinsurer. These expenses may thus be treated exactly as are the legal interests, within the loss adjustment expenses clause.

We will assume an interests sharing clause only for the MTPL claims and we assume that the portion of interests in the losses is $\delta = 15\%$.

The payments and incurred losses of the reinsurer now read

	t	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5
	X_i	0	1	2	3	4	5	6	7
	⋮								
$Y_i^{\Sigma Re}(t)$	3000	0	0	0	0	750.27	1689.14	2084.57	2293.22
	3025	0	0	0	0	767.39	1714.24	2113.02	2323.43
	3050	0	0	0	0	784.51	1739.33	2141.47	2353.64
	3075	0	0	0	0	801.63	1764.43	2169.92	2383.85
	⋮								
	X_i								
	⋮								
$IY_i^{Re}(t)$	3000	3354.74	3354.74	3354.74	3252.89	2501.82	2483.48	2295.63	2293.22
	3025	3392.50	3392.50	3392.50	3290.65	2533.54	2515.20	2325.84	2323.43
	3050	3430.26	3430.26	3430.26	3328.41	2565.26	2546.92	2356.05	2353.64
	3075	3468.02	3468.02	3468.02	3366.17	2596.98	2578.63	2386.26	2383.85
	⋮								

Table 14: Reinsurer's payments and incurred losses with interests sharing clause (MTPL only)

4.6. Lattice distributions

Most probably the random variables derived above are not of lattice type. So it is necessary to make a rearithmetization of them. This is done again with the rounding method.
With the lattice version of the payments and incurred losses, we will be able to apply Panjer's algorithm in order to obtain the aggregate claims / incurred losses for each development year.

As an example, here are some rearithmetized distributions :

	t	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5
	x								
$\mathbb{P}[X_i^{\Sigma Re}(t) = x]$	0	0.752	0.400	0.310	0.310	0.310	0.310	0.310	0.310
	25	0.017	0.038	0.048	0.048	0.048	0.048	0.048	0.048
	50	0.015	0.034	0.042	0.042	0.042	0.042	0.042	0.042
	75	0.014	0.031	0.038	0.038	0.038	0.038	0.038	0.038
	$\vdots$								
$\mathbb{P}[IX_i^{Re}(t) = x]$	0	0.310	0.310	0.310	0.310	0.310	0.310	0.310	0.310
	25	0.046	0.046	0.046	0.046	0.046	0.046	0.046	0.046
	50	0.042	0.042	0.042	0.042	0.042	0.042	0.042	0.042
	75	0.038	0.038	0.038	0.038	0.038	0.038	0.038	0.038
	$\vdots$								
$\mathbb{P}[Y_i^{\Sigma Re}(t) = x]$	0	0.999	0.997	0.990	0.981	0.919	0.802	0.733	0.686
	25	0.000	0.000	0.001	0.001	0.005	0.009	0.014	0.017
	50	0.000	0.000	0.000	0.001	0.002	0.008	0.000	0.016
	75	0.000	0.000	0.000	0.001	0.004	0.008	0.013	0.000
	$\vdots$								
$\mathbb{P}[IY_i^{Re}(t) = x]$	0	0.255	0.255	0.255	0.407	0.626	0.648	0.686	0.686
	25	0.056	0.056	0.056	0.000	0.022	0.000	0.017	0.017
	50	0.000	0.000	0.000	0.040	0.020	0.020	0.016	0.016
	75	0.050	0.050	0.050	0.037	0.000	0.018	0.000	0.000
	$\vdots$								

Table 15: Rearithmetized reinsurer's payments and incurred losses distributions

4.7. Clauses limiting the liability of the reinsurer

There are two clauses which may limit the liability of the reinsurer in an excess of loss treaty. The annual aggregate limit (Aal_{Fire} or Aal_{MTPL}) on the one hand is the maximal aggregate loss the reinsurer will pay. The annual aggregate deductible (Aad_{Fire} or Aad_{MTPL}) on the other hand is a deductible on the aggregate loss of the reinsurer. Both annual clauses may coexist. In such a case the aggregate loss of the reinsurer reads :

$$
\begin{aligned}
S_{X^{\Sigma Re}}(j+0.5) &= \min(Aal_{Fire}, \max(0, \sum_{i=1}^{N} X_i^{\Sigma Re}(t+0.5) - Aad_{Fire})) \quad , \quad j = 0,1,\ldots,n, \\
S_{Y^{\Sigma Re}}(t+0.5) &= \min(Aal_{MTPL}, \max(0, \sum_{i=1}^{M} Y_i^{\Sigma Re}(t+0.5) - Aad_{MTPL})) \quad , \quad j = 0,1,\ldots,n, \\
S_{IX^{Re}}(t+0.5) &= \min(Aal_{Fire}, \max(0, \sum_{i=1}^{N} IX_i^{Re}(t+0.5) - Aad_{Fire})) \quad , \quad j = 0,1,\ldots,n, \\
S_{IY^{Re}}(t+0.5) &= \min(Aal_{MTPL}, \max(0, \sum_{i=1}^{M} IY_i^{Re}(t+0.5) - Aad_{MTPL})) \quad , \quad j = 0,1,\ldots,n.
\end{aligned}
$$

Let us assume that there is no annual aggregate deductible and no annual aggregate limit for the separate treaties :

$$\begin{aligned} Aad_{Fire} &= 0, \\ Aad_{MTPL} &= 0, \\ Aal_{Fire} &\rightarrow \infty, \\ Aal_{MTPL} &\rightarrow \infty. \end{aligned}$$

We have the following distributions

	t	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5
	x								
$\mathbb{P}[S_{X^{Re}}(t) = x]$	0	0.538	0.223	0.178	0.178	0.178	0.178	0.178	0.178
	25	0.023	0.021	0.021	0.021	0.021	0.021	0.021	0.021
	50	0.021	0.020	0.020	0.020	0.020	0.020	0.020	0.020
	75	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019
$\mathbb{P}[S_{IX^{Re}}(t) = x]$	0	0.178	0.178	0.178	0.178	0.178	0.178	0.178	0.178
	25	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021
	50	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020
	75	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019
$\mathbb{P}[S_{Y^{Re}}(t) = x]$	0	0.999	0.987	0.953	0.908	0.667	0.371	0.263	0.208
	25	0.000	0.001	0.002	0.005	0.017	0.017	0.018	0.018
	50	0.000	0.001	0.002	0.003	0.008	0.016	0.001	0.017
	75	0.000	0.001	0.002	0.004	0.015	0.015	0.017	0.001
$\mathbb{P}[S_{IY^{Re}}(t) = x]$	0	0.024	0.024	0.024	0.051	0.154	0.172	0.208	0.208
	25	0.007	0.007	0.007	0.000	0.017	0.000	0.018	0.018
	50	0.001	0.001	0.001	0.011	0.016	0.017	0.017	0.017
	75	0.006	0.006	0.006	0.010	0.002	0.016	0.001	0.001

Table 16: Reinsurer's aggregate payments and incurred losses

5. Global distributions and global expected values

As we are interested in a global treaty combining Fire and MTPL claims, we have to obtain the global distributions of :

$$\begin{aligned} S_{(X+Y)^{\Sigma Re}}(j+0.5) &= \min(Aal, \max(0, S_{X^{\Sigma Re}}(j+0.5) + S_{Y^{\Sigma Re}}(j+0.5) - Aad)) \quad , \quad j = 0, 1, \ldots, n, \\ S_{(IX+IY)^{Re}}(j+0.5) &= \min(Aal, \max(0, S_{IX^{Re}}(j+0.5) + S_{IY^{Re}}(j+0.5) - Aad)) \quad , \quad j = 0, 1, \ldots, n, \end{aligned}$$

where Aal is a multiline annual aggregate limit and Aad is a multiline annual aggregate deductible.
We will assume that there is an annual aggregate deductible on the global treaty (multiline aggregate deductible) :

$$\begin{aligned} Aad &= 1000, \\ Aal &\rightarrow \infty. \end{aligned}$$

Note that Ribeaud (2000) used the terminology "Multiline aggregate deductible" / "Multiline aggregate limit".
These distributions are easily obtained by convolutions because for our model we assumed mutual independencies .
Note that in case of dependencies between the claim amounts or between the claim frequencies, algorithms exist, giving the joint distributions of $(S_{X^{\Sigma R_e}}, S_{Y^{\Sigma R_e}})$ or $(S_{IX^{R_e}}, S_{IY^{R_e}})$. See e.g. Walhin and Paris (2000a) for the first case of dependency and Walhin and Paris (2000b) for the second case of dependency. Having the joint distributions, it then becomes immediate to obtain the distributions of $S_{X^{\Sigma R_e}} + S_{Y^{\Sigma R_e}}$ or $S_{IX^{R_e}} + S_{IY^{R_e}}$.
Within our numerical example we obtain

	t	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5
	x								
$\mathbb{P}[S_{X^{\Sigma R_e}}(t) + S_{Y^{\Sigma R_e}}(t) = x]$	0	0.537	0.221	0.170	0.162	0.119	0.066	0.047	0.037
	25	0.023	0.021	0.021	0.020	0.017	0.011	0.009	0.008
	50	0.021	0.020	0.019	0.019	0.015	0.011	0.006	0.008
	75	0.019	0.019	0.019	0.018	0.016	0.011	0.008	0.005
	100	0.018	0.018	0.018	0.018	0.014	0.010	0.008	0.007
	125	0.016	0.017	0.017	0.017	0.015	0.010	0.008	0.007
	⋮								
$\mathbb{P}[S_{IX^{\Sigma R_e}}(t) + S_{IY^{\Sigma R_e}}(t) = x]$	0	0.004	0.004	0.004	0.009	0.028	0.031	0.037	0.037
	25	0.002	0.002	0.002	0.001	0.006	0.004	0.008	0.008
	50	0.001	0.001	0.001	0.003	0.006	0.007	0.008	0.008
	75	0.002	0.002	0.002	0.003	0.004	0.006	0.005	0.005
	100	0.002	0.002	0.002	0.002	0.006	0.007	0.007	0.007
	125	0.001	0.001	0.001	0.003	0.006	0.007	0.007	0.007
	⋮								

Table 17: Global payment and incurred losses distributions

As we will use a cash flow model that is introduced in section 6 (investment decision process) we are interested in obtaining the expected values of the future payments and outstanding.
The incremental payments are

$$\begin{aligned} Paid(0.5) &= S_{(X+Y)^{\Sigma R_e}}(0.5), \\ Paid(j+0.5) &= S_{(X+Y)^{\Sigma R_e}}(j+0.5) - S_{(X+Y)^{\Sigma R_e}}(t_{j-0.5}) \quad , \quad j = 1, 2, \ldots, n, \end{aligned}$$

and the loss reserves are

$$Reserve(j+0.5) = S_{(IX+IY)^{R_e}}(j+0.5) - S_{(X+Y)^{\Sigma R_e}}(j+0.5) \quad , \quad j = 0, 1, \ldots, n.$$

This is the situation where the reinsurer follows the information given by the cedent. Another situation might be that the reinsurer books the ultimate loss in such a way that he avoids overstatement and / or understatement of the ceding company's reserves. In this case the loss reserves read :

$$Reserve(j+0.5) = S_{(X+Y)^{\Sigma R_e}}(n+0.5) - S_{(X+Y)^{\Sigma R_e}}(j+0.5) \quad , \quad j = 0, 1, \ldots, n.$$

We are now able to obtain the average aggregate payments and average aggregate reserves for the reinsurer :

- paid losses :

$$PL(j+0.5) = -\mathbb{E}Paid(j+0.5) \quad , \quad j = 0, 1, \ldots, n.$$

- reserve :

$$RES(j+0.5) = \mathbb{E}Reserve(j+0.5) \quad , \quad j = 0, 1, \ldots, n.$$

Let us assume that the share of the reinsurer in the treaty is 20%. It is indeed common practice that several reinsurers take a share in a given treaty. Unless the ceded risk is really small, a cedent would not accept to work with only one reinsurer for solvency reasons.
The following table gives the expected aggregate payments and loss reserves of the reinsurer (for a share of 20%). We assume that the reinsurer follows the reserves of the cedent. Furthermore we will assume that all cash flows related to losses happen in the middle of the year.

t	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5
$-PL(t)$	27.19	59.78	21.70	5.78	35.10	76.38	49.29	30.04
$RES(t)$	533.50	473.72	452.01	387.61	192.03	110.69	30.41	0

Table 18: Expected aggregate payments and loss reserves of the reinsurer

Let us assume that the estimated premium income is 50000. This information is important as the reinsurance premium is usually expressed as a percentage of the cedent's premium income. One traditionally speaks of a rate.
By adding up the payments we immediately arrive at the technical rate (TR) :

$$TR = \frac{305.25}{20\% \times 50000} = 3.05\%.$$

This rate is not satisfactory because it does not take into account the investment income the reinsurer can obtain on loss reserves. On the other hand neither does it take into account the cost of reserving (in particular when there is overstatement). Finally, it does not take into account the fact that the total payment is a sum of different cash flows. This is the reason why we introduce the following cash flow model.

6. The cash flow model

This section is adapted from Walhin et al. (2001).

When a reinsurer wants to write business he has to provide a solvency margin, or some allocated capital : C. Let us assume that the return after tax which the shareholders demand from this capital is coc. We call coc the cost of capital. It can be derived e.g. via the CAPM (Capital Asset Pricing Model, see e.g. Brealey and Myers (2000)) where $coc = r_F + \beta P_R$. r_F is the risk-free rate and P_R is the risk premium of the market. β measures the systematic risk, i.e. market sensitivity, associated to the investment.

In the present paper we assume the same cost of capital whatever the type of business is. This is clearly a simplifying hypothesis. One may be tempted to work within a more general model where each line of business has its own cost of capital. For example it is clear that cat business hardly correlated with the market, implying that the cost of capital for cat business should be about the risk free rate. Independently the required capital for writing cat business is large due to the high volatility of this kind of business and the risk of large deviations.

In our case we have two types of business to analyse : Motor Third Party Liability and Fire. Even if we had two different costs of capital, it is really not clear how we could use them. As we mix both types of business, we have to use one cost of capital, possibly some (weighted) average of the above-mentioned costs of capital. The present multiline cover shows a limitation of working with different costs of capital. There is clearly room for further research at this point.

Traditionally we say that the business is worth the value if the net present value of all future cash flows, including capital allocation and release, is positive. A nil value implies that the requirements of the shareholders are just fulfilled. A positive value implies some creation of value for the shareholders. In the latter case we have the following inequality :

$$0 < \sum_{j=0}^{n} \frac{CF(t_j)}{(1+coc)^{t_j}}.$$

We will use the cash flow model in this way and say that a treaty is acceptable if the net present value of all future cash flows, including the variations in allocated capital, is positive. Let us note that if the firm is not financed exclusively through equity capital but also through some debt or hybrid capital, coc becomes a weighted average cost of capital (see e.g. Brealey and Myers (2000) for details). This however is obviously not very important for insurers and reinsurers who are essentially financed through equity capital. We will assume the cost of capital to be $coc = 11\%$.

We have three types of cash flows related to losses :

- paid losses

$$PL(j+0.5) = -\mathbb{E}Paid(j+0.5) \quad , \quad j = 0, 1, \ldots, n.$$

- variation of the loss reserve : $VR(j+0.5)$, $j = 0, 1, \ldots, n$:

$$\begin{aligned} RES(j+0.5) &= \mathbb{E}Reserve(j+0.5) \quad , \quad j = 0, 1, \ldots, n, \\ VR(0.5) &= -RES(0.5), \\ VR(j+0.5) &= RES(j+1.5) - RES(j+0.5) \quad , \quad j = 1, 2, \ldots, n. \end{aligned}$$

- investment income on reserve : $IR(j+0.5)$, $j = 0, 1, \ldots, n$:

$$\begin{aligned} IR(0.5) &= 0, \\ IR(j+0.5) &= rRES(j-0.5) \quad , \quad j = 1, \ldots, n. \end{aligned}$$

We logically assume that investment income on the reserves are paid with a one year delay. We will assume that the interest rate obtained on the loss reserve is $r = 5\%$. We observe a limitation of our model. It is not possible to account for two different interest rates on the loss reserves (note that it would be possible if there were no clauses on the global distribution, which seldom is the case).
We can now define the aggregate cash flow at the middle of the year :

$$CF(j+0.5) = PL(j+0.5) + VR(j+0.5) + IR(j+0.5) \quad , \quad j = 0, 1, \ldots, n.$$

We will assume that all the other cash flows occur at the beginning of the year : $t_j = j$, $j = 0, 1, \ldots, n+1$. These cash flows are :

- commercial premium ($CP(j)$).
  The premium may be thought to be incepted at time 0. This is not always the case. Often there is a minimum deposit premium at time 0. The balance is paid at time 1. We do not take into account (but it is not difficult to do so) the fact that the minimum deposit premium is often paid in different instalments (one quarter every three months or one half every six months). Moreover we will see in section 7 that premium adjustments may be necessary. Thus premium cash flows at times other than 0 and 1 are not excluded. We will assume that there is a minimum and deposit premium of 80% of the expected commercial reinsurance premium. By deposit we mean that 80% of the premium is paid at time $t = 0$ whereas the balance is paid at time $t = 1$. By minimum we mean that at least the reinsurance rate times 80% of the premium income (estimated by the cedent) will be paid. In case the actual premium income is lower than 80% of the estimated premium income, the minimum and deposit premium is due. We assume that the estimated premium income will be the actual one.
- brokerage ($B(j)$).
  Brokerage, if any, is traditionally a percentage of the commercial premium. It will thus be deducted at times premiums are paid. We will assume that brokerage is 10%.
- retrocession ($R(j)$).
  Cost of retrocession, if any, is not the premium paid to the retrocessionnaire but rather the expected value of this premium minus the aggregate loss paid by the retrocessionnaire. A possible modelization is a percentage of the commercial premium minus a fraction of the paid losses. The first percentage is the traditional rate demanded by the retrocessionnaire on commercial premiums. The latter fraction represents the share of the average claims the retrocessionnaire is expected to pay. We will assume that retrocession costs (premiums) are 3% of the commercial premium. We assume that on average 2% of the losses are paid by the retrocession (this is assumed to be estimated with the developped model). In other words we cede 2% of the losses to the retrocession and the premium we are asked for that risk is 3% of the commercial premium.

- administrative expenses ($AE(j)$).
  Administrative expenses may be of two types : fixed expenses and proportional expenses. The fixed expenses represent the fixed costs of the reinsurer (including the fixed costs of the priced treaty) whereas the proportional costs represent the costs directly associated with the management of the treaty. We assume that these proportional expenses are based on the paid losses (note that this is just an assumption that can be easily modified). It is not illogical to admit that the expenses will be paid during the course of the treaty (think of the accounting and claims management of the treaty). So there may be a cash flow of expenses for all times j. We will assume that administrative expenses are 5 for the fixed part and 4% of the paid losses each year (the proportional administrative expenses are assumed to be paid at the end of the year).
- variation in the allocated capital ($VC(j)$).
  As announced in the previous section, some capital has to be allocated in order to run the business. However, at last at the end of the development, this allocated capital is released to the shareholders. In practice, the allocation rule may be such that the allocated capital is given back after x years or in function of the evolution of the loss reserves. So there will be variations in the allocated capital, exactly as there are in the loss reserves. Within our numerical example the allocated capital, $C(j)$, $j = 0, 1, \ldots, n+1$ is assumed to be 1.25 times the standard deviation of the ultimate aggregate claims, i.e. $\sqrt{Var(1-\gamma)S_{(X+Y)^{ERe}}(n+0.5)}$ where γ is the fraction of the claims paid by the retrocessionnaire. We assume e.g. that the capital allocation is based on the standard deviation premium principle (see Walhin et al. (2001) for further details). We make the hypothesis that capital has to be allocated during three years. See Walhin et al. (2001) for further details on capital allocation.
- investment income on the allocated capital ($IC(j)$).
  As allocated capital is mobilized, an auto-remuneration of this capital is possible. Indeed the mobilized capital will be invested and will produce an investment income. Moreover one might think that this auto-remuneration is higher than the remuneration on the loss reserves because the latter are probably invested in risk-free assets. So, while capital is allocated there is a cash flow of investment income on it at a return rate $l = 7\%$.

We are then able to define the cash flows at integer times :

$$CF(j) = CP(j) + B(j) + R(j) + AE(j) + VC(j) + IC(j) \quad , \quad j = 0, 1, \ldots, n+1.$$

The problem of taxes remains to be treated. In order to find the tax we first have to define the taxable profit at times j and $j + 0.5$:

$$\begin{aligned} TaxProfit(j) &= CP(j) + B(j) + R(j) + AE(j) + IC(j) \quad , \quad j = 0, 1, \ldots, n+1, \\ TaxProfit(j+0.5) &= PL(j-0.5) + VR(j-0.5) + IR(j-0.5) \quad , \quad j = 0, 1, \ldots, n. \end{aligned}$$

The tax cash flows are then

$$\begin{aligned} Tax(j) &= \tau TaxProfit(j) \quad , \quad j = 0, 1, \ldots, n+1, \\ Tax(j+0.5) &= \tau TaxProfit(j+0.5) \quad , \quad j = 0, 1, \ldots, n. \end{aligned}$$

where $\tau = 30\%$ is an average tax rate. It assumes all cash flows, including financial return, to be taxed at the same rate. This is obviously not always true and specific corrections are easy to include in the model according to the tax regime of the reinsurer's domicile.
The treaty will be acceptable if

$$\sum_{j=0}^{n+1} \frac{CF(j) - Tax(j)}{(1+coc)^j} + \sum_{j=0}^{n} \frac{CF(j+0.5) - Tax(j+0.5)}{(1+coc)^{j+0.5}} > 0.$$

The following table gives the cash flow model with the technico-financial premium. This table takes into account a reinsurer's share of 20%.

t	0	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5
TFP	**294.69**	0	0	0	0	0	0	0	0
PL	0	−27.19	−59.78	−21.70	−5.78	−35.10	−76.38	−49.29	−30.04
VR	0	−533.50	59.78	21.70	64.40	195.59	81.33	80.28	30.41
IR	0	0	26.67	23.69	22.60	19.38	9.60	5.53	1.52
$CF(j)$	294.69								
$CF(j+0.5)$	0	−560.69	26.67	23.69	81.22	179.87	14.56	36.53	1.89
$\frac{CF(j)}{(1+coc)^j}$	294.69	0	0	0	0	0	0	0	0
$\frac{CF(j+0.5)}{(1+coc)^{j+0.5}}$	0	−532.18	22.81	18.25	56.37	112.46	8.20	18.54	0.86
NPV	**0**								

Table 19: Cash flow model for the technico-financial premium

The technico-financial premium (TFP) is 294.69.
The technico-financial rate is thus given by

$$TFR = \frac{294.69}{50000 \times 20\%} = 2.95\%.$$

It may seem surprising that the technico-financial premium is so close to the technical premium. This is due to the fact that there is a lot of overstatement by the ceding company and that overstatement is followed by the reinsurer. We will make some sensitivity analysis on this aspect.

We now obtain the commercial premium :

j	0	1	2	3	4	5	6	7	8
$j+0.5$	0	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5
CP	384.22	96.05	0	0	0	0	0	0	0
AE	−5	−1.09	−2.39	−0.87	−0.23	−1.40	−3.06	−1.97	−1.20
B	−38.42	−9.61	0	0	0	0	0	0	0
R	−11.53	−2.34	1.20	0.43	0.12	0.70	1.53	0.99	0.60
PL	0	−27.19	−59.78	−21.70	−5.78	−35.10	−76.38	−49.29	−30.04
VR	0	−533.50	59.78	21.70	64.40	195.59	81.33	80.28	30.41
IR	0	0	26.67	23.69	22.60	19.38	9.60	5.53	1.52
VC	−497.94	0	0	497.94	0	0	0	0	0
IC	0	34.86	34.86	34.86	0	0	0	0	0
$CF(j)$	−168.67	117.88	33.66	532.36	−0.12	−0.70	−1.53	−0.99	−0.60
$CF(j+0.5)$	0	−560.69	26.67	23.68	81.22	179.87	14.56	36.53	1.89
$TaxPr(j)$	329.27	117.88	33.66	34.42	−0.12	−0.70	−1.53	−0.99	−0.60
$TaxPr(j+0.5)$	0	−560.69	26.67	23.69	81.22	179.87	14.56	36.53	1.89
$Tax(j)$	98.78	35.36	10.10	10.33	−0.03	−0.21	−0.46	−0.30	−0.18
$Tax(j+0.5)$	0	−168.21	8.00	7.11	24.36	53.96	4.37	10.96	0.57
$\frac{CF(j)-Tax(j)}{(1+coc)^j}$	−267.45	74.34	19.12	381.71	−0.05	−0.29	−0.57	−0.33	−0.18
$\frac{CF(j+0.5)-Tax(j+0.5)}{(1+coc)^{j+0.5}}$	0	−372.53	15.97	12.77	39.46	78.72	5.74	12.98	0.61
NPV	0								

Table 20: Cash flow model for the commercial premium

The total commercial premium is then

$$384.22 + 96.05 = 480.27,$$

which produces a rate of

$$\frac{480.27}{50000 \times 20\%} = 4.80\%.$$

Summarizing we have the following rates

TR	3.05%
TFR	2.95%
CR	4.80%

Table 21: Rates

It is now easy to provide some sensitivity analyses. Let us compare the rates for different multiline aggregate deductibles (MAD). We will also give the rate in the case where there is no overstatement for the MTPL claims :

	with overstatement			without overstatement		
MAD	TR	TFR	CR	TR	TFR	CR
1000	3.05%	2.95%	4.80%	3.05%	2.56%	4.35%
2000	1.90%	1.89%	3.55%	1.90%	1.58%	3.18%
3000	1.13%	1.13%	2.69%	1.13%	0.92%	2.40%

Table 22: Sensitivity analysis 1

We observe the effect of the overstatement on the technico-financial rate. The effect of the multiline aggregate deductible is equally important. Note that it would be difficult to obtain these rates without the comprehensive model we use.

Let us now assume that there is an annual aggregate deductible for the MTPL and Fire claims of $Aad_{Fire} = Aad_{MTPL} = 500$. To compensate, the multiline aggregate deductible becomes $Aad = 500$. We obtain :

TR	2.65%
TFR	2.60%
CR	4.29%

Table 23: Sensitivity analysis 2

7. Special clauses

It is often observed in excess of loss treaties that the reinsurance premium is a function of the excess of loss amounts. In these situations, governed by typical clauses, the reinsurance premium is a random variable :

$$P_{Re} = P^{Init} + P^{Rand}.$$

where P^{init} denotes the initial premium, which is not random whereas P^{Rand} denotes the random part of the premium.
The clauses are

- Paid reinstatements
- Sliding scale premium
- Profit commission.

The practical pricing proceeds in two steps. The first one is easy : we merely calculate the commercial premium necessary to cover the treaty if there is no "random" clause. We then obtain the evolution of paid losses, loss reserves, investment income on loss reserves, allocated capital, investment income on allocated capital and administrative expenses. There is no reason to believe that these elements will be different in the cash flow model with "random clause". We now move to the second step, i.e. the cash flow model with the "random" clause. The previous elements are fixed. Other elements may vary : premiums, brokerage, retrocession, and taxes. The process will be iterative. As a first guess we choose an initial premium (or one limit of the scale in the case of a sliding scale). According to the evolution of the incurred losses, this premium will be split in several premiums in the future, i.e.

- $CP(0) = P^{Init}$ (or, more exactly, the minimum and deposit premium, the balance of it which will be paid in $t = 1$) for a treaty with paid reinstatements. $CP(j) =$ future adjustments for reinstatements due to incurred losses hitting the layer for $j = 1, 2, \ldots, n+1$.

- $CP(0) = P^{Init} = P_{min}$ (or, more exactly, the minimum deposit premium, the balance of which will be paid in $t = 1$) for a treaty with sliding scale. $CP(j) =$ future adjustments for $j = m, m+1, \ldots, n+1$ where m is the first year for which a premium adjustment is contractually agreed.

- $CP(0) = P^{Init}$ (or, more exactly, the minimum deposit premium, the balance of which will be in $t = 1$) for a treaty with profit commission. $CP(j) =$ future adjustments for profit commission for $j = m, m+1, \ldots, n+1$ where m is the first year for which a premium adjustment is contractually agreed.

With this pattern of premium payments, we immediately obtain the pattern of brokerage, retrocession and as a result the pattern of tax. We are then able to calculate the net present value of the business. If it is positive we try a new premium lower than the previous one. If it is negative we try a new premium higher than the previous one. The trial and error scheme is continued until the net present value of the business is 0.
The interested reader will find more details in Walhin et al. (2001).
We now present the pricing for the case of a sliding scale. We always assume the same conditions. The sliding scale has a minimum rate $R_{min} = 3.75\%$, a loading $f = \frac{100}{70}$ and we look for the maximum rate R_{max}. We also assume that the first premium adjustement is foreseen after three years. The solution is given by $R_{max} = 5.91\%$. The following table gives the cash flows related to the commercial premium :

j	0	1	2	3	4	5	6	7	8
CP	300.00	75.00	0	180.28	−10.08	−29.61	−1.51	−7.42	−0.10

Table 24: *Cash flow related to the commercial premium with a sliding scale*

We observe the particular pattern of premium payment. At time $t = 0$, 80% of the minimum premium is paid. At time $t = 1$, 20% of the minimal premium is paid. There are no adjustments until time $t = 3$. At that time a huge positive adjustment is needed after which smaller negative adjustments follow. This shows an important fact for the sliding scale : a fraction of the premium may be paid late and this must have an influence on the pricing.

In the next table we give R_{max} in function of R_{min} and the first time for premium adjustments (m) :

R_{min}/m	1	2	3	4	5
4.50%	4.98%	5.04%	5.12%	5.21%	5.31%
4.25%	5.12%	5.24%	5.39%	5.55%	5.75%
4.00%	5.27%	5.44%	5.65%	5.89%	6.17%
3.75%	5.41%	5.64%	5.91%	6.24%	6.60%

Table 25: Sensitivity analysis 3

This table confirms what was said above. We observe a dramatic effect of the variable first year of premium adjustment. This aspect is however traditionally neglected by reinsurers when pricing sliding scale covers.
Many more sensitivity analyses are possible : see Walhin et al. (2001) for more analyses in the single branch pricing.

8. Conclusion

We have shown in this paper that a comprehensive methodology is of great help when pricing excess of loss treaties, even multiline treaties.
All the elements of a pricing are combined in a unique tool : actuarial elements (the severities $X, Y, \ldots$, the frequencies $N, M, \ldots$, the clauses, the retrocession), financial elements (the financial advantage when claims are paid long after the premium instalment, the remuneration of the shareholders at the cost of capital, the use of a cash flow model), economic elements (inflation, superinflation) and commercial elements (brokerage, administrative expenses).
The Panjer's algorithm is a powerful tool we often use (in fact as many times as there are periods between claims payments in our model) in order to find the aggregate situation of one line in the future. Obviously this has a computing cost which is really low nowadays. The aggregate claims distribution of the multiline is simply obtained by convolution.
The notion of cost of capital has been used in order to provide a fair price for the shareholders.
A lot of parameters are necessary in order to run our model. Note that these parameters would also be necessary within a simplified model. In case some parameters are difficult to estimate, our methodology provides a solution in the sense that it easily allows for sensitivity analyses.

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Final Report of the Advisory Committee on Enterprise Risk Management

CAS Advisory Committee on Enterprise Risk Management

FINAL REPORT

OF THE ADVISORY COMMITTEE

ON ENTERPRISE RISK MANAGEMENT

November 5, 2001

Casualty Actuarial Society
Advisory Committee on Enterprise Risk Management

Final Report

CONTENTS

APPENDICES

Casualty Actuarial Society
Advisory Committee on Enterprise Risk Management

Final Report

I. Executive Summary

The Advisory Committee on Enterprise Risk Management (the Committee) has conducted a thorough and systematic assessment of the research and education needs of the CAS on the subject of Enterprise Risk Management (ERM) – the centerpiece of which was a formal survey of the CAS membership – and has developed a set of specific recommendations to meet those needs. Given the importance of this subject to the future of the CAS, the Committee adopted an aggressive timetable for its work. This report presents our charge, our work process, our results, our recommendations, and our additional thoughts. Our recommendations are highlighted in this Executive Summary.

To guide our work, we developed a working definition of ERM for CAS purposes, and a conceptual "ERM Framework" that specifies the Risk Types covered by ERM and the sequential steps of the Risk Management Process. Central to our definition and Framework is the notion that ERM is not merely a defensive process, but a proactive value creation tool.

In the area of research, we recommend that 12 specific topics within ERM be the subject of focused research. Each of these topics is assigned a priority, which varies according to industry focus. The recommended research methods for each topic are identified. We also recommend that a standing ERM Research Committee be formed to direct and monitor this research, and to take responsibility for related tasks such as maintaining an updated ERM bibliography (an initial bibliography has been drafted by the Committee), providing advice and content to the CAS committees that plan ERM-related seminars and workshops, partnering with other professional organizations as appropriate, developing ERM messages for the CAS to communicate internally and externally, and designating ERM media spokespersons.

In the area of education, we recommend a series of approaches in specific areas to close the gap between current and desired level of knowledge within each element of the ERM Framework. These approaches range from exam syllabus treatment to annual ERM seminars, to tracks/sessions within existing seminars, to self-study guidance, depending on the subject area within the Framework, and reflect the expressed preferences of the CAS membership regarding education methods. These recommendations can be carried out through existing CAS committees, as specified in our recommendations – no new committee or other organizational change is required. To assist these committees, we have drafted a complete set of "Learning Objectives" for each element of the ERM Framework.

We also provide recommendations for enhancing CAS visibility in the ERM arena.

The Committee respectfully requests that our recommendations be promptly approved. There is currently a leadership void among the various professional organizations with interest in ERM, and the CAS is well positioned to assume this leadership role. The ERM approach also presents an opportunity to create a compelling business-relevant whole of the currently unconnected parts of the casualty actuarial discipline, and in the process aid the recruitment of candidates who are asking thoughtful questions about the relevance of the CAS to their careers. Finally, it is evident from our research that the CAS membership is looking for clear and assertive movement in the direction we are recommending.

II. Our Charge

The CAS Executive Council (EC) established the Advisory Committee on Enterprise Risk Management (the Committee) in the summer of 2000 with the following charge:

> "The CAS Advisory Committee on Enterprise Risk Management is to identify research and education that the CAS should undertake in the area of enterprise risk management. The Committee is to recommend methods, priorities, and timetables to the EC for implementing that research and education, but is not expected to carry the work out itself. The Committee should learn about and monitor efforts by other CAS committees and recommend any additional efforts it considers appropriate to be undertaken by existing or new committees and task forces. A broad definition of enterprise risk management will be used by the Committee in determining the scope of its work."

The CAS received a very large number of volunteers for this Committee. The Committee was ultimately staffed by Mike Belfatti, Martin Cauchon, Ed Davenport, Kevin Dickson, Chuck Emma, John Kollar, John Kryczka, Marc-Andre Lefebvre, Larry Marcus, Jerry Miccolis (chair), Chris Nelson, Andrew Rippert, Joe Wallen, Bill Yit, and Ted Zubulake. Mary Frances Miller was assigned to the Committee as EC Liaison.

III. Our Work Process and Results

The Committee met approximately every two months between August 2000 and October 2001. Significant work was performed between full Committee meetings by various short-duration, special-purpose subcommittees.

During the course of its work, the Committee communicated with such other CAS committees as the DFA Committee, the Syllabus Committee, and the Continuing Education Committee. The Committee worked extensively with the Committee on Special Interest Seminars to plan and staff the CAS's April 2001 special interest seminar "Understanding the Enterprise Risk Management Process", and made a presentation on our work in progress at the CAS Annual Meeting in November 2000. The Committee also engaged in some ongoing dialogue with the other CAS Advisory Committees: Securitization/Risk Financing, Valuation of P/C Insurance Companies, and Asset/Liability Management & Investment Policy.

Committee Goals

The Committee began its work by establishing the interim goals it needed to achieve in order to meet its charge. The goals established were to:

- Define ERM for the purposes of the Committee's work
- Develop a framework within which to identify ERM research and education needs of the CAS
- Outline the ERM knowledge level desired of CAS members
- Determine the current state of the ERM practice/knowledge among CAS members
- Specify learning objectives
- Identify research needs to close the gap between the desired level of knowledge and the current state of knowledge of CAS members
- Identify education needs to close the gap
- Recommend – for both research and education – the methods, priorities, timetable, and industry focus needed to close the gap
- Prepare an initial ERM bibliography
- Identify the implications of ERM on CAS policy and standards, and recommend additional efforts for existing and new CAS committees/task forces to further CAS research and education efforts on ERM and to increase CAS visibility in ERM

ERM Working Definition

For the purposes of its work, the Committee developed the following definition:

> "ERM is the process by which organizations in all industries assess, control, exploit, finance, and monitor risks from all sources for the purpose of increasing the organization's short and long term value to its stakeholders."

Important elements of this definition include the dual nature of risk (i.e., as both threat to be controlled and opportunity to be exploited), the ultimate objective of value creation, and the relevance of the CAS to industries beyond insurance. These themes are expanded upon in Section V: Concluding Comments.

ERM Conceptual Framework

To guide its discussion of ERM research and education needs, the Committee developed its conceptual framework for ERM. This framework consisted of two dimensions. The first dimension outlined the Risk Types (i.e., sources of risk) encompassed by our view of ERM:

- Hazard Risk
- Financial Risk
- Operational Risk
- Strategic Risk

The second dimension outlined the sequential steps in the Risk Management Process:

- Establishing Context
- Identifying Risks
- Analyzing/Quantifying Risks
- Integrating Risks
- Assessing/Prioritizing Risks
- Treating/Exploiting Risks
- Monitoring And Reviewing

An elaboration on the Risk Types and the Risk Management Process may be found in Appendix A.

Gap Analysis

The key series of steps in the Committee's work was to identify the role of the actuary in the ERM process, and to determine the current level of knowledge, the source of that knowledge, and the desired level of knowledge of the CAS membership as respects ERM. The Committee referred to these steps collectively as the "Gap Analysis."

The Committee undertook several approaches to performing the GAP Analysis, including:

- Review of the CAS Exam Syllabus and the plans of the Syllabus Committee
- Review of CAS continuing education activities and future plans
- Review of other relevant surveys such as the CAS Non-traditional Practice Area Survey, the CAS CEO Survey, and the Tillinghast ERM Benchmarking Survey
- Discussions with other professional organizations such as RIMS, GARP, IAFE, and the SOA
- Discussions among the Committee members

The Committee concluded that these approaches, alone, were insufficient to answer the specific questions relevant to the Gap Analysis.

The Committee decided to conduct a survey of the CAS membership as the primary vehicle to complete the Gap Analysis.

CAS Membership Survey

An eight-page, 21-question, self-administered questionnaire was developed by the Committee and approved by the EC at its March 28, 2001 meeting. (A draft of the survey was circulated to other CAS Advisory Committee chairs to determine if a joint survey covering their collective subject areas would be desirable. It was decided, given the size of the questionnaire and the fact that so much of its content was relevant only to ERM, that the idea of a single multi-subject questionnaire was not feasible.) A total of 3,021 questionnaires were sent, via e-mail, to Fellows, Associates, and Affiliates of the CAS on April 24, 2001. A total of 298 completed questionnaires were returned to the CAS office by May 25, representing a response rate that exceeded our expectations for a survey of this type. A complete Survey Report (including an executive summary of the results, a description of the survey methodology, a profile of the respondents, a detailed presentation of the results, and a copy of the survey questionnaire) may be found in Appendix B.

Results of Gap Analysis

Based on the results of the membership survey and the other avenues that we explored, the Committee completed the Gap Analysis. The Committee developed grids that mapped our ERM Framework in two dimensions, i.e., each of the four Risk Types was mapped on one dimension and each of the seven steps in the Risk Management Process was mapped on the other. For each cell in this grid, the Committee examined the CAS membership's current level of knowledge against their desired level of knowledge.
A key result of the Gap Analysis is summarized in the table below. The figures in the table represent the percentage of survey respondents who feel that it is important for all CAS members to know about, or be expert in, the various aspects of ERM – as contained in the ERM framework – by 2005. The figures in bold italics represent areas in which the respondents indicated the greatest relative ***gap*** between this desired level and their current level of knowledge.

Process Step	**Risk Type**			
	Hazard	Financial	Operational	Strategic
Establish Context	*72%*			
Identify Risks	89%	***79%***	53%	54%
Analyze/Quantify Risks	91%	***81%***	51%	54%
Integrate Risks	***69%***			
Assess/Prioritize Risks	85%	***73%***	46%	51%
Treat/Exploit Risks	75%	***64%***	41%	44%
Monitor & Review	***61%***			

As is evident from the table, the greatest relative education and research needs are in the area of financial risk.

The full research and education implications of our Gap Analysis are contained in Section IV: Our Recommendations.

Bibliography

The Committee learned that Tillinghast had been engaged by the Institute of Internal Auditors (IIA) to conduct a study of ERM trends and best practices across multiple industries. Tillinghast's work for the IIA included the preparation of a comprehensive bibliography on ERM. The Committee approached the IIA about the possibility of the IIA sharing the bibliography with the CAS. The IIA agreed, with the stipulation that the CAS would share any future updates of the bibliography with the IIA. The IIA ERM bibliography was expanded upon and categorized by the Committee, and is presented in Appendix C. This represents an initial ERM bibliography for CAS purposes, to be maintained and updated per the discussion in Section IV: Our Recommendations.

Learning Objectives

The Committee decided to begin the build-out of the ERM Framework that it had established by specifying the subjects that would need to be mastered by the "ERM expert" within each element of the Framework. The resulting "Learning Objectives" were developed by the Committee and are presented in Appendix D. It is our intent that these Learning Objectives be used by the Syllabus Committee per the discussion in Section IV: Our Recommendations.

IV. Our Recommendations

The Committee's recommendations are presented separately for research and for education, below. Recommendations on increasing CAS visibility on ERM are presented thereafter.

Research

Based on the results of the Gap Analysis, the Committee developed the following table that depicts our:

- Prioritization of the ERM research needs of the CAS, and
- Recommendations as to the methods by which the identified research needs should be met.

Research Topic	Priority (A=highest) & Industry Focus			Research Method		
	P/C Industry	Other Financial Services	Other	Funded Research (CAS, AERF)	Call Paper Pgms.	Survey Existing (incl. Non-CAS) Research
ERM Overview	A	A	A	✓	✓	✓
Value Creation through ERM	A	B	B	✓	✓	✓
Risk Quantification						
Financial	A	A	A			✓
Operational	A	B	C		✓	✓
Strategic	A	B	C		✓	✓
Risk Correlation	A	B	C	✓		✓
Risk Integration	A	B	C	✓		✓
Establishing Risk Tolerances	B	B	C		✓	✓
Practical Approaches to Optimization & Risk/Reward Metrics & Marginal Portfolio Contributions	B	C	C		✓	✓
Risk Monitoring Tools	C	D	D		✓	✓
Risk Treatments	C	D	D			✓
Pricing, Reserving, Reporting of Integrated Products	C	D	D		✓	✓

The Committee also recommends that the CAS EC and/or Board form a standing ERM Research Committee, with the following charge:

- Direct and monitor research per the table above
- Expand, update and maintain the ERM bibliography initiated by the Committee (see Appendix C), adding (Organize per the ERM Framework "grid", and annotate):
  - Books
  - Articles
  - Editorials
  - Web sites
  - Sources of data
  - Sources of models and software
  - Other relevant professional organizations
    - — ERM activities
    - — Membership requirements
    - — Meetings
- Work with other ERM-relevant CAS committees to coordinate ERM research activities and provide advice and content for ERM education vehicles; these committees include:
  - Syllabus Committee
  - Program Planning Committee
  - Committee on Special Interest Seminars
  - DFA Committee
  - DFA Seminar Committee[1]
  - Valuation, Finance & Investments Committee
- Stay abreast of other organizations' ERM-related activities (e.g., SOA, AAA, IAA, GARP, RIMS, IAFE, NAIC)
  - Build partnerships where appropriate
  - Explore joint committees
- Develop internal and external CAS communication messages on ERM to be delivered through various media such as the CAS web site
- Designate ERM "subject matter experts" as media spokespersons

Education

Based on the results of the Gap Analysis, the Committee developed the table below that depicts the Committee's assessment of the ERM education needs of CAS members by the year 2005. This also reflects the preferences expressed by the CAS membership (through the ERM Survey) regarding the means by which the education is provided. The key to this table is as follows:

[1] This dialogue has already begun.

Key to Table Below:		
Code	**Desired Knowledge Level**	**Education Implication**
1	All CAS members should be expert in this area	On exam syllabus – core subject (similar to ratemaking, reserving)
2	...	On exam syllabus – moderate treatment (similar to accounting)
3	All CAS members need to know about this area	On exam syllabus – light treatment (similar to underwriting)
4	...	On exam syllabus – very light introductory treatment (similar to claims)
5	...	Continuing Ed – annual ERM seminar (similar to CLRS)
6	Some CAS members should know about this area	Continuing Ed – special interest and/or limited attendance seminars (similar to M&A)
7	...	Continuing Ed – Special tracks/sessions within existing CAS (and non-CAS) meetings/seminars
8	...	Self-study/on-line courses/university courses (CAS to maintain bibliography)
9	Outside the scope of CAS	N/A
Note: Any exam syllabus item (codes 1 - 4) also carries continuing education/self-study implications (codes 5 – 8); any continuing education item (codes 5 – 7) also carries self-study implications (code 8)		

Depth of ERM Knowledge Within CAS Desired by 2005:

ERM Overview **2**

Process Step	**Risk Type**			
	Hazard	Financial	Operational	Strategic
Establish Context	**3**			
Identify Risks	**5**	**6**	**7**	**7**
Analyze/Quantify Risks	**1**	**2**	**4**	**4**
Integrate Risks	**2**			
Assess/Prioritize Risks	**4**	**4**	**5**	**5**
Treat/Exploit Risks	**4**	**4**	**6**	**6**
Monitor & Review	**5**			

The Committee also recommends that the CAS:

- Add a paper that presents a thorough introduction to ERM ("ERM Overview") to the Part 8 (or Part 5) syllabus as soon as it is available (see Research Recommendations above)
- Immediately begin to provide continuing education opportunities per the table above
  - To provide on-going learning for new FCAS's
  - To provide "catch-up" learning for veteran FCAS's
- Specifically add the following sessions to CAS meetings and other seminars, as soon as content is available (see Research Recommendations above):
  - "ERM Overview"
  - "Value Creation through ERM"
- Incorporate into the current planning for the 2005 syllabus:
  - The ERM Learning Objectives[2] (see Appendix D)
  - The Committee's specific exam recommendations per the above table
- Publish the ERM bibliography (see Appendix C) and all updates

The Committee strongly supports the following current educational activities of the CAS:

- The eight-module online course on financial risk management
- The resurrection, in 2002, of the initial 1999 special interest seminar on Financial Risk Management
- The planned reconfiguration (and renaming) of the annual DFA Seminar to cover a broader and more business-relevant range of ERM topics[3]

It is assumed that the ERM Research Committee, once authorized and staffed, will be solicited for advice and content regarding each of the seminars, conferences and workshops cited above.

CAS Visibility

The Committee recommends that the CAS:

- Develop (with assistance from the future ERM Research Committee), and internally and externally publicize, the "CAS position" on ERM
- Publish the Executive Summary and/or some form of this report
- Invite a broad (i.e., including non-CAS) audience to its ERM-related seminars and workshops
- Work through the CAS and AAA Media Committees to proactively promote the actuary's role in ERM (with subject matter experts to be designated by the future ERM Research Committee)

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[2] Note that the Learning Objectives in Appendix D are expressed in terms of the knowledge required of an ***expert*** in ERM. For syllabus purposes, the required knowledge level should be calibrated to conform to the "Desired Depth of Knowledge" within the table at the bottom of the preceding page for each element of the ERM Framework.

[3] As noted earlier, this dialogue has already begun.

V. Concluding Comments

In the preceding sections, we have attempted to present our analyses, findings and recommendations in a crisp and concise manner, to promote prompt review and action by the CAS leadership. In this section we would like to provide some additional context and rationale for our recommendations and, in the process, convey a bit of the passion that the Committee members feel regarding the importance of the CAS's role in ERM.

ERM as a Value Creation Tool

Embedded in our definition of ERM and in our ERM Framework is the notion that the objective of ERM is not simply to protect the organization from threats, but to proactively create value. ERM does this by:

- Systematically identifying the material risks (both threats and opportunities) relevant to the organization's business objectives
- Rigorously analyzing the organization's capital requirements to help achieve financial efficiency (for the owners) while protecting solvency (for the customers)
- Evaluating strategies (e.g., capital allocation, asset/liability management, insurance/reinsurance/hedging, operational changes) to find the optimal combination to improve growth and return prospects (i.e., optimization under uncertainty)
- Exploiting the natural hedges, portfolio effects and operational efficiencies of integrated risk management (including opportunities to undertake value-creating ventures that may not have been accepted under less sophisticated risk analysis)
- Enhancing stability (i.e., reducing volatility) of results, which attracts higher ratings, valuations and, for publicly traded companies, stock prices

ERM as a Unifying Framework

ERM has substantial potential as the broad conceptual framework that unifies the many varied parts of the actuarial discipline. Beyond its core sub-disciplines of pricing and reserving, the actuarial discipline spans such subject areas as risk modeling[4], capital management, asset/liability management, reinsurance, financial performance measurement, accounting, and portfolio management. As should be evident from the preceding discussion on "ERM as a Value Creation Tool", ERM provides a logical structure to link these subject areas together in a compelling way to form an integrated whole. In so doing, ERM addresses critical business issues such as growth, return, consistency and value creation. It expresses risk not just as threat but as opportunity – the reason that business is conducted in the first place. Furthermore, the convergence of financial services industries creates more demand for the type of "cross-silo" risk treatment that ERM represents.

[4] The CAS's work to develop DFA as the insurance industry's risk modeling template has been admirable, but DFA is losing ground to simpler alternatives from the banking sector that are, in our opinion, dangerously ill-suited to the insurance industry. DFA is in need of a more robust platform that can showcase its superior and multi-faceted business applicability.

Properly unified, these component strengths should make CAS members the primary candidates for Chief Risk Officer, one of the more exciting and rewarding career opportunities to have emerged in recent years. It should accordingly be the case that the actuarial knowledge base is perceived as a very valuable commodity in industries well beyond insurance and financial services.

ERM as a Recruitment Tool

CAS candidates and recent Associates and Fellows are increasingly asking thoughtful and troubling questions about the relevance of CAS membership to their careers. They cite the curricula for the Chartered Financial Analyst (CFA) designation and the Financial Risk Management (FRM) designation, for example, as being more business-relevant in the current environment and offering more growth potential. ERM provides an opportunity to re-engage these young professionals in actuarial science as the most logical path to playing a significant role in a business culture that is clearly embracing ERM as a modern management discipline.

Filling the Leadership Void

There are a number of professional disciplines and organizations that are quite active in ERM. These include the other actuarial bodies (SOA, IAA and LOMA, in particular) as well as the Global Association of Risk Professionals (which grants the FRM designation), the Association of Investment Management and Research (which grants the CFA designation), the Institute of Internal Auditors, and the Association of Certified Public Accountants. Currently, while each of these organizations has targeted ERM as a growth area, none has assumed a clear leadership position in the development or promulgation of ERM as a discipline. This may change shortly, particularly within the financial services industry as that industry converges. The CAS appears to be well positioned to assume that leadership role, as our scope of risks is already quite broader than most of those other organizations, and our structural simulation-based Dynamic Financial Analysis tools and techniques, properly expanded, may be well suited to accommodate the comprehensive risk modeling requirements of ERM. Clearly, in whatever role the CAS plays in the ERM movement, cooperation and partnering with a number of these other organizations is advisable.

The flip side, of course, is that if the CAS does not strongly stake a claim to ERM, the CAS may find itself defined very narrowly in the current business environment, and its status diminished.

The Membership Wants to Go There

The enthusiasm for ERM among the CAS membership is evident from, for example:

- The number of volunteers for this Committee, which was more than twice as many as could be accommodated.

- The number of responses to the ERM Survey, which was 150% of the level typical for membership surveys of this type. And the level of interest expressed through the responses to the survey.
- The high level of attendance at the first CAS-sponsored ERM Seminar in April 2001. And the positive feedback from attendees at the seminar.

It is apparent that a clear and assertive statement by the CAS leadership that the CAS intends to be a major player in the ERM movement would be well received by the membership.

Do We Really Have a Choice?

Even those CAS members who do not clearly see ERM as important to the future of the CAS will require ERM research and education as the world evolves around them. Traditional pricing actuaries will increasingly be asked to price integrated products. Traditional reserving actuaries may not otherwise be qualified, for example, to certify the reserves of a captive that provides integrated coverage for property/casualty, financial and employee benefits risks. Traditional corporate actuaries need to know how to respond to rating agencies' use of their own ERM models in their analyses of companies' capital adequacy.

Finally, there are those who believe that the CAS would have "missed its calling" if some other profession or organization becomes the predominant source of Chief Risk Officers of the future – they see it as the obligation of the CAS to properly prepare its membership to thrive in this ERM environment we find ourselves in.

ERM Framework

The ERM Framework is organized by Risk Type and by sequential steps within the Risk Management Process.

Risk Types

- *Hazard* risks, such as:
  - Liability suits (e.g., operations, products, environmental)
  - Fire and other property damage
  - Windstorm and other natural perils (including catastrophes)
  - Theft and other crime
  - Personal injury, disease, disability (including work-related injuries and diseases)
  - Business interruption
- *Financial* risks, such as:
  - Price (e.g. asset value, interest rate, foreign exchange, commodity)
  - Liquidity (e.g. cash flow, call risk, opportunity cost)
  - Credit (e.g. default, downgrade).
  - Inflation/purchasing power
  - Hedging/basis risk
- *Operational* risks, such as:
  - Business operations (e.g. customer satisfaction, human resources, product development, capacity, efficiency, product/service failure, trademark/brand erosion)
  - Empowerment (e.g., leadership, change readiness)
  - Information technology (e.g. relevance, availability)
  - Integrity (e.g., management fraud, reputation)
  - Information/business reporting (e.g., budgeting and planning, accounting information, pension fund, investment evaluation, taxation)
- *Strategic* risks, such as:
  - Competition
  - Customer wants
  - Demographic and social/cultural trends
  - Technological innovation
  - Capital availability
  - Regulatory and political trends

Risk Management Process

- *Establishing Context* – Achieving a full understanding of the present conditions in which the organization operates; this includes understanding the external context (e.g., organization/environment relationship, stakeholder communication policies), the internal context (e.g., business objectives, oversight structure, key performance indicators), and the risk management context (e.g., units covered, degree of coordination throughout organization).

- *Identifying Risks* – Documenting the conditions and events that represent material threats to the organization's achievement of its objectives or represent areas to exploit for competitive advantage.

- *Analyzing/Quantifying Risks* – Calibrating and, wherever possible, creating probability distributions of outcomes for each material risk.

- *Integrating Risks* – Aggregating all risk distributions, reflecting correlations and portfolio effects, and expressing results in terms of impact on the organization's key performance indicators (i.e., the "aggregate risk profile").

- *Assessing/Prioritizing Risks* – Determining the contribution of each risk to the aggregate risk profile, and prioritizing accordingly.

- *Treating/Exploiting Risks* – Developing strategies for controlling or exploiting the various risks.

- *Monitoring and Reviewing* – Continual gauging of the risk environment and the performance of the risk management strategies.

The Framework "Grid"

Some Risk Management Process steps apply to each Risk Type individually, and some, to all Risk Types in the aggregate, according to the following grid, which the Committee used to guide our work and organize our findings.

Process Step	**Risk Type**			
	Hazard	Financial	Operational	Strategic
Establish Context				
Identify Risks				
Analyze/Quantify Risks				
Integrate Risks				
Assess/Prioritize Risks				
Treat/Exploit Risks				
Monitor & Review				

SURVEY ON

ENTERPRISE RISK MANAGEMENT

REPORT

Compiled by CAS Office
July 26, 2001

EXECUTIVE SUMMARY

- The response level was high (298 respondents vs. an expected 200 for surveys of this type).

- The profile of survey respondents may be biased in favor of those more interested in ERM (thus the responses to certain opinion questions may exhibit a bias toward enthusiasm for ERM), but any potential bias is tempered by the fact that the response level was high overall (indicating that such enthusiasm is more representative than would otherwise be implied).

- Respondents from vendors (i.e., brokers, agents, consultants and similar organizations: the "vendor group") tend to be more familiar with ERM, but respondents from the industry (P/C insurance and reinsurance companies: the "industry group") tend to spend more of their time on ERM projects.

- ERM knowledge was gained primarily through self-initiative (e.g., on-the-job learning, from others in the same company, self-study of the literature).

  — Sources of ERM learning are mostly CAS-related (and mostly through seminars).

- For those respondents having hands-on involvement with ERM projects:

  — With respect to risk identification, the industry group is much less likely to bridge from property/casualty risks to other hazard risks (e.g., health, safety, HR-related risks) than is the vendor group.

  — All respondent groups are substantially involved in all risk management process steps, across all types of risk, as summarized in the table below (percentages in the table represent the percentage of respondents that indicated that they have been personally involved in the particular aspect of an ERM project).

<table>
<tr><th rowspan="2">Process Step</th><th colspan="4">Risk Type</th></tr>
<tr><th>Hazard</th><th>Financial</th><th>Operational</th><th>Strategic</th></tr>
<tr><td>Identify Risks</td><td>80%</td><td>74%</td><td>51%</td><td>51%</td></tr>
<tr><td>Analyze/Quantify Risks</td><td>83%</td><td>47%</td><td>38%</td><td>34%</td></tr>
<tr><td>Integrate Risks</td><td colspan="4">59%</td></tr>
<tr><td>Assess/Prioritize Risks</td><td>51%</td><td>50%</td><td>35%</td><td>35%</td></tr>
<tr><td>Treat/Exploit Risks</td><td colspan="4">61%</td></tr>
<tr><td>Monitor & Review</td><td colspan="4">35%</td></tr>
</table>

  — Professionals most often involved in ERM projects are P/C actuaries, accountants, brokers, financial analysts, risk managers and underwriters.

  — Relatively few respondents serve as the "integrator" of risks from the various sources.

— Project leadership:

  — Respondents among the vendor group are equally likely to be the project manager or the technical analyst; respondents among the industry group are more than twice as likely to be the technical analyst than to be the project manager.

  — For projects involving the vendor group, the project leader is often a P/C actuary, a broker or an MBA/management consultant; for projects involving the industry group, the project leader is often a P/C actuary or an underwriter.

  — Interestingly, none of the projects in which the respondents were involved were led by internal audit staff – this is at odds with evidence from other sources (e.g., the recent ERM survey by the Institute of Internal Auditors and client experience of the committee members).

- There is a lack of knowledge of important tools and concepts such as economic capital, Economic Value Added, Expected Policyholder Deficit, Extreme Value Theory, options pricing theory, Risk Adjusted Return on Capital, risk mapping and Value at Risk – even among those respondents who rate themselves as expert in or highly familiar with ERM; Net Present Value and Dynamic Financial Analysis are the only tools/concepts of which respondents have a great deal of knowledge.

- With respect to their ability to model risks, respondents rated themselves highest with respect to P/C hazard risks (as expected) but lowest with respect to non-P/C hazard risks (lower even than for operational or strategic risks).

- Respondents believe CAS members should play a significant role in ERM, from project leader to technical analyst to risk integrator.

- 94% of respondents believe it is important for CAS members to increase their knowledge of ERM; 83% believe it is important to increase their *own* ERM knowledge.

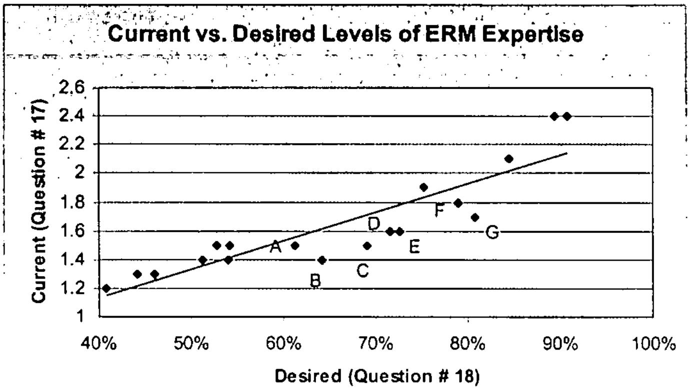
- Respondents believe CAS members should apply their ERM skills beyond the P/C insurance industry, particularly in the broader financial services industry.

- There is a very strong desire by respondents to obtain their ERM learning from CAS sources (followed closely by on-the-job learning) rather than from other professional organizations.

  — Among the other organizations, the AIMR (Association for Investment Management and Research, grantor of the CFA – Chartered Financial Analyst – designation) ranks highest.

  — The preferred CAS learning vehicle is seminars.

  — The respondents' view of the gap between desired and current level of knowledge – by risk type and risk management process – is summarized in the graph below. The x-axis of this graph shows the percentage of respondents who felt that it was important for *all* CAS members to know about, or be expert in, the various aspects of ERM as contained in our ERM framework (Question #18). The y-axis shows the average score for the *current* level of ERM expertise among respondents (Question #17: low=1, high=3). In general, there was rough alignment between desired and current level of knowledge. The seven labeled points below the "line of alignment" indicate areas where the current level of expertise is relatively further behind the desired level (see key to labels below the graph). These areas represent the high-priority areas of focus for future CAS research and education, according to respondents. Note that these areas relate primarily to financial risk.



Key to labels A-G:

Process Step	Risk Type			
	Hazard	Financial	Operational	Strategic
Establish Context	D			
Identify Risks		F		
Analyze/Quantify Risks		G		
Integrate Risks	C			
Assess/Prioritize Risks		E		
Treat/Exploit Risks		B		
Monitor & Review	A			

Finally, it should be noted that the responses to certain questions suggest that:

- some respondents may have equated ERM with DFA, and may not have considered the broader scope that ERM implies; and
- it is likely that, among those that are experienced in ERM, the focus of that experience is within the P/C insurance industry (as opposed to other industries).

SURVEY METHODOLOGY

Designing the Questionnaire

An eight-page, 21-item self-administered questionnaire (see Appendix) was developed by the CAS Advisory Committee on Enterprise Risk Management and approved by the CAS Executive Council.

Conducting the Survey

A total of 3,021 questionnaires were sent as an e-mail attachment to Fellows, Associates, and Affiliates of the CAS on April 24, 2001. In addition, the survey could be completed online through the CAS Web Site. Respondents were asked to complete the survey by May 25, 2001.

Data Analysis

A total of 298 (9.9%) completed questionnaires were returned to the CAS Office. A total of 258 surveys (87%) were completed electronically through the Web Site. Responses to survey questions were compiled, coded, and entered into a database. The responses were then analyzed using a statistical analysis software package (SPSS).

Respondent Groups

The data for responses to survey questions includes results for all respondents, as well as 7 separate groups based on key demographic categories. Therefore, for most questions there are 8 columns of respondents. The respondent groups, with the number of respondents for each category included in parentheses, are as follows:

1. **All respondents** (298 total)
2. **Vendor Group**, i.e., employment type of Broker/Agent, Consultant, or Organization Serving Insurance (91)
3. **Industry Group**, i.e., employment type of Property/Liability Insurance Company or Reinsurance Company (192)
4. **Other**, i.e., employment type of Academic, Government, Life, Accident & Health Ins., Retired, or Other (15)
5. **Fellows** (202)
6. **Associates** (87)
7. **Experts**, i.e., Respondents who answered a or b to question 4 (expert in ERM or familiar with ERM) (86)
8. **Non-experts**, i.e., Respondents who answered c, d, or e to question 4 (some understanding of ERM, not very familiar with ERM, or non-interest in ERM). (211)

RESPONDENT PROFILE

CAS Membership Status

Response	Frequency	Percent
FCAS	202	67.8
ACAS	87	29.2
Affiliate	6	2.0
Blank	3	1.0
Total	298	100.0

Year Designation Was Attained

Response	Frequency	Percent
2001	5	1.7
2000	6	2.0
1999	1	0.3
1998	7	2.3
1997	4	1.3
1996	9	3.0
1995	2	0.7
1994	4	1.3
1993	4	1.3
1992	2	0.7
1990	3	1.0
1989	4	1.3
1988	4	1.3
1987	4	1.3
1986	1	0.3
1984	2	1.7
1983	4	1.3
1982	1	0.3
1980	1	0.3
1979	4	1.3
1978	2	0.7
1977	1	0.3
1976	1	0.3
1975	2	0.7
1974	1	0.3
1972	1	0.3
1970	1	0.3
1969	1	0.3
Blank	216	73.5
Total	298	100.0

Type of Employment:

Response	Frequency	Percent
Academic	2	0.7
Broker/Agent	23	7.7
Consultant	62	20.8
Government	5	1.7
Life, Accident, and Health Insurance	1	0.3
Organization Serving Insurance Business	6	2.0
Property/Liability Insurance	149	50.0
Reinsurance	43	14.4
Retired	2	0.7
Other	5	1.7
Total	298	100.0

Level of knowledge, involvement, and interest in ERM:

Response	Frequency	Percent
I consider myself somewhat of an expert in ERM and devote a considerable portion of my time to ERM projects.	17	5.7
I am familiar with ERM and have been involved with some ERM projects.	69	23.2
I have some understanding of the ERM concept, but have never been involved with an ERM project.	115	38.6
I am not very familiar with ERM, but am interested to learn about it.	91	30.5
I have no interest in ERM.	5	1.7
Blank	1	0.3
Total	298	100.0

Level of knowledge, involvement, and interest in ERM by Demographic Profile

	All Respondents	Vendor Group	Industry Group	Other	Fellows	Associates
Expert in ERM and devote considerable portion of time to ERM	5.7	8.8	4.2	6.7	5.4	6.9
Familiar with ERM and been involved with ERM projects.	23.2	31.9	19.3	20.0	23.8	20.7
Understanding of ERM but have never been involved in ERM project.	38.6	41.8	38.0	26.7	38.1	37.9
Not very familiar with ERM, but interested to learn about it.	30.5	15.4	37.5	33.3	30.7	32.2
Have no interest in ERM	1.7	1.1	1.0	13.3	1.5	2.3
Blank	0.3	1.1	0.0	0.0	0.5	0.0

RESULTS

Question 3:
Which of the following describes how you attained your knowledge of Enterprise Risk Management (ERM)?

For question 3, respondents were asked to rank the order of importance (1 through 8) of various ways that they attained their knowledge of ERM. The responses were scored in inverse order, with a response of 1 getting a score of 8, a response of 2 getting a score of 7 and so forth, with a response of 8 getting a score of 1. The system used to rank the responses was to sum the scores. The options for how respondents attained their knowledge of ERM are ordered from most important to least important, based on all respondents. The table cells include the rank and the score for each response.

	All	Vendor Group	Industry Group	Other	Fellows	Associates	Experts	Non-experts
Through on-the-job learning.	1 1088	2 375	1 651	1 62	1 703	1 357	1 589	1 499
From others in my company.	2 951	1 391	2 514	3 46	2 618	2 296	2 427	2 524
Through self-study of the literature.	3 792	3 253	3 490	2 49	3 581	3 180	3 337	3 445
Through seminars or courses.	4 485	4 169	5 307	7 9	4 347	6 122	4 196	6 289
From CAS exam materials.	5 445	6 95	4 326	5 24	5 280	4 154	5 133	5 312
I have no current knowledge of ERM.	6 432	5 104	6 296	4 32	6 272	5 152	8 0	4 424
Other	7 257	8 83	7 157	6 17	7 185	7 63	7 77	7 180
From specific readings/textbooks.	8 232	7 90	8 137	8 5	8 166	8 47	6 113	8 119

Written responses to "Through seminars or courses presented by: (please specify)"

- CAS (21 responses)
- 2001 CAS Special Interest Seminar on ERM (7 responses)
- CAS Spring Meeting (2000 or 2001) (4 responses)
- CANE Meeting (March 2000 or March 2001) (3 responses)
- CAS Ratemaking Seminar (2 responses)
- RIMS (2 responses)
- CAS Annual Meeting (2000)
- CCA
- D&T
- D'Arcy limited attendance seminar on finance
- DFA Seminars
- GARP

- IRRM
- MAF
- MBA degree
- My firm
- Oliver Wyman Co.
- Others outside my company
- Princeton University
- Reinsurance Broker
- Risk Conferences (British group)
- SOA meeting sessions
- TMA and IPCQ
- Various
- Various seminar firms
- VT Captive Association

Written responses to "From specific readings/textbooks. (please specify)"
- CAS Publications (Insurance in the Next Century, Proceedings) (5 responses)
- Various readings, articles (4)
- Business Insurance (3)
- Industry journals and periodicals (3)
- Trade Press (3)
- CFO Magazine (2)
- National Underwriter (2)
- Risk Management (2)
- Any Peter Drucker books
- Arthur Andersen
- Australian RM Best Practices
- Best's Review
- Business Finance
- Call papers
- CFA Syllabus
- CPCU curriculum
- FRM exam material
- Swiss Re articles
- Tillinghast's Emphasis Magazine
- Various books on Value at Risk
- Various concerning worker's comp

Written responses to "Other (please specify)"
- Brokers (reinsurance) (2)
- Committee work (CAS committees, RBC committees) (2)
- This survey (2)
- RiskMail listserv (2)
- Another company's Risk Management guidelines' table of contents
- CAS activities
- Clients

- Consultants
- DFA work/analysis
- Discussions with CEO's & RMs
- GARP
- Internet
- Others not in my company
- Own philosophy
- Self study
- Software Development and Modeling Research
- Speakers/lecturers

Questions 5 – 9

Note: Questions 5 through 9 were answered only by respondents that indicated that they considered *themselves somewhat of an expert in ERM and devoted a considerable portion of their time to ERM* projects, or were familiar with ERM and had been involved with some ERM projects. There are five columns of respondents for questions 5 through 9. The respondent groups, with the number of respondents for each category included in parentheses, are as follows:

1. **All respondents** (86 total)
2. **Vendor Group**, i.e., employment type of Broker/Agent, Consultant, or Organization Serving Insurance (37)
3. **Industry Group**, i.e., employment type of Property/Liability Insurance Company or Reinsurance Company (45)
4. **Fellows** (59)
5. **Associates** (24)

Question 5:

Approximately what percentage of your time over the past 12 months have you been involved in ERM projects?

	All Respondents	Vendor Group	Industry Group	Fellows	Associates
Over 75%	9.3	10.8	8.9	6.8	16.7
Between 50 and 75%	5.8	8.1	4.4	6.8	4.2
About 50%	5.8	5.4	6.7	6.8	4.2
Between 25 and 50%	17.4	13.5	20.0	10.2	33.3
Less than 25%	41.9	48.6	33.3	42.4	37.5
Blank	19.8	13.5	26.7	27.1	4.2

Question 6:
Which of the following aspects of ERM projects have you personally been involved in?

The percentages in the table represent the percentage of respondents that indicated that they have been personally involved in the particular aspect of an ERM project.

a. Identification of risks on an enterprise-wide scale:

	All Respondents	Vendor Group	Industry Group	Fellows	Associates
Hazard risk – property/casualty	80.2	83.8	77.8	76.3	87.5
Hazard risk – other (e.g., health, safety, HR-related)	20.9	32.4	8.9	16.9	29.2
Financial risk	74.4	70.3	75.6	72.9	75.0
Operational risk	51.2	51.4	51.1	50.8	54.2
Strategic risk	51.2	51.4	48.9	52.5	45.8

b. Analysis/quantification of hazard risk:

	All Respondents	Vendor Group	Industry Group	Fellows	Associates
Property/casualty hazard risks	82.6	86.5	80.0	79.7	87.5
Other types of hazard risk	14.0	13.5	13.3	10.2	25.0

Written responses to "Other types of hazard risk (please specify)"

- Investment (3 responses)
- Legal (2)
- Weather (2)
- A&H
- Asset
- DCAT/DST
- Financial and technological
- FX, interest rates, commodity prices, credit
- Life/health
- Liquidity
- Political
- Power
- Product contamination
- Tax
- Worker's safety and related

c. Analysis/quantification of financial risks:

	All Respondents	Vendor Group	Industry Group	Fellows	Associates
Credit risk	39.5	37.8	37.8	35.6	50.0
Foreign exchange risk	29.1	24.3	33.3	23.7	45.8
Interest rate risk	46.5	45.9	42.2	44.1	50.0
Liquidity risk	26.7	24.3	24.4	27.1	25.0
Other	15.1	16.2	15.6	15.3	8.3

Written responses to "Other (please specify)"

- Commodity price risk (4 responses)
- Equity market risk (2)
- ALM
- Asset risk
- Capital adequacy risk
- Equity valuation risk
- Long-tail line severity
- Medical inflation and stock market yields
- Rate adequacy
- Stocks, corporate bonds, munis
- Underwriting risk

d. Analysis/quantification of operational risks:

	All Respondents	Vendor Group	Industry Group	Fellows	Associates
Operations	38.4	37.8	40.0	37.3	41.7
Information Technology	18.6	16.2	22.2	22.0	8.3
Integrity	10.5	8.1	13.3	13.6	0.0
Information risk	9.3	5.4	13.3	11.9	0.0
Other	9.3	13.5	6.7	10.2	8.3

Written responses to "Other (please specify)"

- Bad faith allegations
- Claims operations and legislative risks
- Data quality risk/issues
- Income fluctuation risk
- Product recall, inventory
- Product warranty

e. Analysis/quantification of strategic risk:

	All Respondents	Vendor Group	Industry Group	Fellows	Associates
Competition	33.7	29.7	35.6	28.8	41.7
Customer wants	14.0	13.5	13.3	13.6	16.7
Technological innovation	16.3	16.2	15.6	22.0	0.0
Capital availability	32.6	24.3	37.8	32.2	29.2
Regulatory	33.7	35.1	31.1	33.9	33.3
Political	19.8	24.3	15.6	20.3	16.7
Other	8.1	13.5	4.4	6.8	8.3

Written responses to "Other (please specify)"

- Asset allocation and capital requirements
- Capital investment projects
- Disposable income
- Each of the three on various projects
- Market channel risk
- Obsolescence/senescence risk
- Reputation

f. Integrated risk analysis including the following categories:

	All Respondents	Vendor Group	Industry Group	Fellows	Associates
Integration of various types of P/C hazard risks faced by an entity (e.g., property catastrophe risk with automobile liability).	59.3	70.3	48.9	55.9	66.7
Integration of P/C hazard risks with other types of hazard risk faced by an entity (e.g., workers compensation with health risk).	26.7	29.7	20.0	18.6	45.8
Integration of hazard risk, financial risk, operational risk, and/or strategic risk.	59.3	67.6	48.9	57.6	58.3

g. Assessment or prioritization of risks faced by an entity.

	All Respondents	Vendor Group	Industry Group	Fellows	Associates
Hazard	51.2	64.9	44.4	50.8	50.0
Financial	50.0	62.2	42.2	47.5	54.2
Operational	34.9	45.9	26.7	30.5	45.8
Strategic	34.9	45.9	26.7	32.2	37.5

h.

	All Respondents	Vendor Group	Industry Group	Fellows	Associates
Recommending ways to treat or exploit risks that have been identified, quantified, and assessed.	60.5	73.0	51.1	59.3	66.7

i.

	All Respondents	Vendor Group	Industry Group	Fellows	Associates
Monitoring of changes in the risk environment and performance of the risk management processes.	34.9	37.8	33.3	35.6	33.3

Question 7:
What other types of practitioners have you worked with in an ERM project?

The percentages in the table represent the percentage of respondents that indicated that they have worked with the type of practitioner in an ERM project.

	All Respondents	Vendor Group	Industry Group	Fellows	Associates
Academics	19.8	18.9	17.8	15.3	29.2
Accountants	55.8	56.8	53.3	52.5	58.3
Actuaries - life/health	18.6	18.9	13.3	13.6	33.3
Actuaries - pension	8.1	10.8	4.4	6.8	12.5
Actuaries - P/C	70.9	70.3	71.1	71.2	70.8
Brokers	46.5	59.5	37.8	44.1	54.2
Economists	27.9	32.4	22.2	22.0	41.7
Financial analysts	48.8	37.8	55.6	42.4	62.5
HR professionals	14.0	18.9	11.1	11.9	16.7
Internal auditors	20.9	16.2	20.0	22.0	16.7
IT professionals	20.9	21.6	20.0	23.7	12.5
Lawyers	17.4	18.9	13.3	16.9	16.7
Marketing professionals	17.4	18.9	13.3	18.6	12.5
MBA/Mgmt consultant	25.6	32.4	15.6	23.7	33.3
Operations experts	15.1	16.2	15.6	18.6	4.2
Risk managers	39.5	51.4	31.1	30.5	58.3
Risk specialists	19.8	24.3	15.6	13.6	37.5
Strategy/org. experts	16.3	16.2	15.6	16.9	12.5
Underwriters	47.7	35.1	60.0	45.8	50.0
Others	10.5	13.5	8.9	6.8	16.7

Written responses to "Other (please specify)"

- Investment Managers, Professionals, Chief Investment Officer (3 responses)
- Cat Modelers
- Claims Experts
- Earth Sciences, Engineers
- ERM Consultants
- Executives: CEO, CFO
- Geologists / Chemists
- Meteorologists
- Project Managers
- Statisticians

Question 9:
For those ERM projects in which you were one of several practitioners involved with the ERM process, in general which practitioner served as the project leader?

The percentages in the table represent the percentage of respondents that indicated that the type of practitioner served as the project leader in an ERM project.

	All Respondents	Vendor Group	Industry Group	Fellows	Associates
Academics	0.0	0.0	0.0	0.0	0.0
Accountants	4.7	2.7	4.4	3.4	8.3
Actuaries – life/health	1.2	2.7	0.0	0.0	4.2
Actuaries – pension	0.0	0.0	0.0	0.0	0.0
Actuaries – P/C	15.1	10.8	17.8	16.9	8.3
Brokers	5.8	10.8	2.2	5.1	4.2
Economists	1.2	2.7	0.0	1.7	0.0
Financial analysts	4.7	0.0	8.9	3.4	8.3
HR professionals	0.0	0.0	0.0	0.0	0.0
Internal auditors	0.0	0.0	0.0	0.0	0.0
IT professionals	1.2	0.0	2.2	1.7	0.0
Lawyers	0.0	0.0	0.0	0.0	0.0
Marketing professionals	0.0	0.0	0.0	0.0	0.0
MBA/Mgmt consultant	5.8	10.8	2.2	5.1	8.3
Operations experts	0.0	0.0	0.0	0.0	0.0
Risk managers	2.3	2.7	2.2	1.7	4.2
Risk specialists	0.0	0.0	0.0	0.0	0.0
Strategy/org. experts	2.3	0.0	4.4	3.4	0.0
Underwriters	9.3	0.0	15.6	10.2	8.3
Others	9.3	5.4	13.3	10.2	4.2
Blank	37.1	51.3	26.7	37.3	41.7

Written responses to "Other (please specify)"

- CFO (3)
- Actuaries and/or accountants
- Chief Investment Officer
- Product managers
- Project Managers/Deal Structurers
- Various
- Various personnel at clients

Question 8:
Which of the following best describes your primary role in the ERM projects that you have been involved with?

	All Respondents	Vendor Group	Industry Group	Fellows	Associates
Project leader	25.6	32.4	20.0	25.4	25.0
Primary technical analyst for all risks	12.8	13.5	13.3	11.9	16.7
Primary technical analyst for hazard risk, but a secondary role in other risks	29.1	18.9	37.8	23.7	37.5
"Integrator" of all risks.	9.3	5.4	11.1	10.2	8.3
Other	14.0	16.2	11.1	18.6	4.2
Blank	9.3	13.5	6.7	10.2	8.3

Written responses to "Other (please specify)"

- Changed over time. Now mostly internal consultant
- Committee participant
- Consultant
- General team member
- Group member in identifying risks – no analysis
- Involved with partial ERM projects only
- Management role overseeing results
- Manager of primary tech analyst for hazard risk
- Manager of project leader
- Manager/decision authority over technical actuarial analysts
- Participant in data collection
- Peer Reviewer
- Secondary analyst
- Subject matter expert for information risk
- Team member for general identification of risk
- Various

Note: Questions 10 through 20 were answered by all respondents. Therefore, there are ten columns of respondents for questions 10 through 20.

Question 10:
Please indicate your level of understanding of the following, using the scale 1 = no or low understanding, 2 = medium understanding, 3 = high understanding.

The data in the table cells represents the average response for each category. A response of 1 was inserted for those items left blank.

	All	Vendor Group	Industry Group	Other	Fellows	Associates	Experts	Non-experts
Basel Capital Accord	1.11	1.12	1.08	1.08	1.10	1.14	1.26	1.05
Causal modeling	1.27	1.35	1.28	1.28	1.29	1.23	1.47	1.19
Credit Risk Models (e.g., Credit metrics.)	1.40	1.45	1.38	1.34	1.41	1.37	1.57	1.34
Dynamic Financial Analysis (DFA)	2.07	2.27	2.12	1.78	2.07	2.03	2.40	1.93
Economic capital	1.59	1.70	1.64	1.72	1.64	1.45	1.87	1.48
Economic Cost of Ruin or Expected Policy-holder Deficit	1.77	1.87	1.87	1.48	1.81	1.66	1.97	1.68
Economic Value Added (EVA)	1.50	1.71	1.58	1.54	1.55	1.34	1.74	1.40
Extreme Value Theory (EVT)	1.25	1.30	1.31	1.22	1.23	1.24	1.43	1.18
Fuzzy logic	1.17	1.23	1.20	1.08	1.18	1.15	1.23	1.15
Generalized Auto-regressive Hetero-skedastic models	1.13	1.24	1.10	1.10	1.12	1.16	1.16	1.12
Net Present Value (NPV)	2.61	2.78	2.69	2.48	2.65	2.48	2.74	2.55
Options Pricing Theory (e.g., Black-Scholes model)	1.78	2.01	1.84	1.92	1.72	1.85	2.03	1.67
Real Options	1.34	1.55	1.31	1.46	1.33	1.33	1.52	1.26
Return on Risk Adjusted Capital–RORAC	1.52	1.43	1.64	1.82	1.56	1.39	1.78	1.41
Risk Adjusted Return on Capital–RAROC	1.60	1.65	1.72	1.90	1.66	1.44	1.95	1.45
Risk Mapping	1.21	1.50	1.16	1.04	1.20	1.18	1.47	1.11
Strengths, Weaknesses, Opportunities, Threats (SWOT) Analysis	1.46	1.81	1.53	1.30	1.50	1.33	1.66	1.36
System Dynamics models	1.08	1.18	1.03	1.00	1.05	1.08	1.14	1.05
Value at Risk (VAR)	1.54	1.84	1.56	1.38	1.51	1.55	1.95	1.37
Tail VAR	1.27	1.48	1.23	1.30	1.25	1.26	1.57	1.14
Other ERM-relevant metrics, models, concepts	1.09	1.09	1.08	1.08	1.09	1.10	1.19	1.05

Written responses to "Other ERM-relevant metrics, models and concepts (please specify)"

- CAPT
- Catastrophe models
- Correlation/Copulas
- Hedging via futures and options
- Marginal analysis
- Probability of ruin
- Statistical modeling
- Utility based return measures

Question 11:
Please indicate your level of ability to quantify/model the following types of risk, using the scale 1 = no or low ability, 2 = medium ability, 3 = high ability.

The data in the table cells below represent the average response for each category. A response of 1 was inserted for those items left blank.

	All	Vendor Group	Industry Group	Other	Fellows	Associates	Experts	Non-experts
hazard – property/casualty	2.46	2.41	2.47	2.26	2.46	2.46	2.47	2.45
hazard – non p/c (e.g., health, safety, HR-related)	1.30	1.45	1.25	1.44	1.26	1.37	1.40	1.26
financial	1.69	1.63	1.70	1.76	1.70	1.68	1.94	1.59
operational	1.34	1.43	1.32	1.36	1.33	1.37	1.42	1.30
strategic	1.38	1.40	1.36	1.50	1.39	1.36	1.51	1.33

Question 12:
In the future, which of the following best describes the role you believe CAS members should play in the ERM process?

The percentages in the table represent the percentage of respondents that indicated that CAS members should play the particular role in the ERM process.

	All	Vendor Group	Industry Group	Other	Fellows	Associates	Experts	Non-experts
As the project leader	50.0	50.5	49.5	53.3	51.0	49.4	66.3	43.1
As a project team member with primary responsibility for all risk quantification work	60.4	63.7	58.3	66.7	58.9	64.4	69.8	56.4
As a project team member with primary responsibility for all hazard risk quantification work	46.0	51.6	41.7	66.7	48.0	42.5	50.0	44.1
As a project team member with primary responsibility for all property/casualty risk quantification work	53.7	50.5	53.1	80.0	55.4	50.6	51.2	54.5
As a risk "integrator"	43.3	48.4	40.1	53.3	46.5	37.9	61.6	35.5
Other	5.7	8.8	3.6	13.3	5.9	4.6	9.3	4.3
None	2.0	1.1	1.6	13.3	1.5	3.4	3.5	1.4

Written responses to "Other (please specify)"

- Any of the above for which qualified.
- As the writer of responses to requests for proposals to perform ERM work, encompassing all aspects of the proposed analysis (although other team members would be equally qualified to do this).
- Client
- None, if they're smart.
- Risk Consultant
- Subject matter expert based on job experience.

Question 13:
How would you rate the relative importance of increasing your knowledge of ERM?

	All	Vendor Group	Industry Group	Other	Fellows	Associates	Experts	Non-experts
Low importance	16.8	15.4	17.2	20.0	16.8	16.1	10.5	19.4
Medium importance	47.3	37.4	52.1	46.7	51.0	37.9	40.7	50.2
High importance	33.6	45.1	28.1	33.3	29.7	43.7	44.2	28.9
Blank	2.3	2.2	2.6	0.0	2.5	2.3	4.7	1.4

Question 14:
How important is it for CAS members to become better trained in ERM?

	All	Vendor Group	Industry Group	Other	Fellows	Associates	Experts	Non-experts
Low importance	6.4	8.8	5.2	6.7	7.4	4.6	8.1	5.7
Medium importance	51.7	44.0	55.2	53.3	52.5	49.4	37.2	57.8
High importance	39.9	45.1	37.5	40.0	37.1	46.0	50.0	35.5
Blank	2.0	2.2	2.1	0.0	3.0	0.0	4.7	0.9

Question 15:
Please indicate how involved actuaries should be with respect to ERM in the following industry groups, using the scale 1 = no or low involvement, 2 = medium involvement, 3 = high involvement.

The data in the table cells below represent the average response for each category. A response of 1 was inserted for those items left blank.

	All	Vendor Group	Industry Group	Other	Fellows	Associates	Experts	Non-experts
Property/casualty insurance industry only	2.72	2.52	2.77	2.92	2.68	2.86	2.74	2.72
All financial services	2.04	2.04	2.02	2.34	1.98	2.20	2.13	2.00
Any industry	1.76	1.83	1.62	1.64	1.74	1.77	1.83	1.72

Question 16:
Please rank the relative importance of the following potential sources of ERM education for current CAS members, using a scale 1 = no or low importance, 2 = medium importance, 3 = high importance.

The data in the table cells below represent the average response for each category. A response of 1 was inserted for those items left blank.

	All	Vendor Group	Industry Group	Other	Fellows	Associates	Experts	Non-experts
CAS Special Interest Seminars	2.43	2.29	2.42	2.58	2.45	2.45	2.34	2.46
CAS Examination Syllabus	1.87	1.73	1.89	1.48	1.89	1.84	1.83	1.88
Other professional organizations' seminars/syllabi	1.34	1.62	1.26	1.28	1.33	1.38	1.48	1.29
Business school	1.54	1.70	1.54	1.66	1.52	1.57	1.57	1.52
Self-study	2.07	2.25	1.97	2.14	2.08	2.06	2.28	1.98
On-the-job learning	2.23	2.45	2.18	2.12	2.13	2.45	2.42	2.15
Other	1.15	1.18	1.11	1.12	1.16	1.13	1.09	1.18

Written responses to "Other professional organizations' seminars/syllabi (please specify)"

- CFA (13 responses)
- GARP (5)
- CPCU (4)
- RIMS (2)
- AAA
- Any related
- Association of Financial Professionals
- CAS Part 8
- CCA
- Financial institutions / economists
- Institute of Actuaries
- Risk Conferences
- SOA
- Specialties as needed
- Topics at CAS conventions
- University courses

Written responses to "Other (please specify)"

- Sessions at May & November meetings (3 responses)
- All CAS meetings and seminars – not just special interest seminars
- Call Papers / Forum
- Can CAS compile a bibliography?
- Continuing education
- Coordinating with other people doing similar work
- Online Course
- Post fellowship certification program
- Sessions at DFA seminar

Question 17:
On the risk type / process grid below, please indicate the level of expertise you currently possess, using the scale 1 = no or low level of expertise, 2 = medium level of expertise, and 3 = high level of expertise.

A total of 25 respondents did not provide an answer for question 17. These respondents are not included in the data analysis below. The demographic breakdown for these respondents follows:

Employment type of Broker/Agent, Consultant, or Organization Serving Insurance	6
Employment type of Property/Liability Insurance Company or Reinsurance Company	19
Employment type of Academic, Government, Life, Accident & Health Ins., Retired, or Other	0
Fellows	20
Associates	5
Respondents who answered a of b to question 4	9
Respondents who answered c, d, or e to question 4	16

The data in the table cells below represent the average response for each category. A response of 1 was inserted for those items left blank.

	All	Vendor Group	Industry Group	Other	Fellows	Associates	Experts	Non-experts
Establish Context (all risk types)	1.64	1.94	1.57	1.44	1.60	1.66	2.05	1.48
Identify Risks – Hazard	2.41	2.69	2.30	2.10	2.40	2.39	2.66	2.31
Identify Risks – Financial	1.81	1.95	1.77	1.56	1.86	1.67	2.08	1.70
Identify Risks – Operational	1.50	1.82	1.46	1.40	1.54	1.38	1.66	1.43
Identify Risks – Strategic	1.52	1.76	1.48	1.52	1.56	1.39	1.83	1.39
Analyze/Quantify Risks - Hazard	2.41	2.68	2.28	2.02	2.41	2.39	2.55	2.36
Analyze/Quantify Risks – Financial	1.68	1.87	1.64	1.44	1.69	1.62	1.99	1.55
Analyze/Quantify Risks – Operational	1.36	1.52	1.34	1.04	1.38	1.34	1.45	1.32
Analyze/Quantify Risks – Strategic	1.38	1.52	1.34	1.40	1.39	1.37	1.56	1.30
Integrate Risks (all risk types)	1.48	1.84	1.36	1.20	1.46	1.52	1.91	1.31
Assess/Prioritize Risks - Hazard	2.13	2.53	2.01	1.56	2.15	2.07	2.38	2.03
Assess/Prioritize Risks – Financial	1.56	1.76	1.53	1.38	1.61	1.41	1.86	1.44
Assess/Prioritize Risks – Operational	1.32	1.54	1.29	1.40	1.35	1.24	1.45	1.27
Assess/Prioritize Risks – Strategic	1.37	1.68	1.33	1.20	1.40	1.30	1.61	1.27
Treat/Exploit Risks - Hazard	1.92	2.33	1.77	1.46	1.88	1.98	2.19	1.81
Treat/Exploit Risks - Financial	1.41	1.59	1.35	1.30	1.41	1.38	1.69	1.30
Treat/Exploit Risks - Operational	1.22	1.37	1.20	1.08	1.24	1.20	1.35	1.17
Treat/Exploit Risks - Strategic	1.27	1.40	1.24	1.20	1.29	1.24	1.47	1.19
Monitor & Review (all risk types)	1.48	1.58	1.43	1.20	1.49	1.46	1.70	1.39

Question 18:
On the risk type/ process grid below, please indicate the level of knowledge you believe is necessary for CAS members to have by the year 2005. Use the scale below:

1 = All CAS members should be expert in this area.
2 = All CAS members need to know about this area and some should be expert.
3 = Some CAS members should know about this area.
4 = Outside the scope of CAS and should remain so.

A total of 41 respondents did not provide an answer for question 18. These respondents are not included in the data analysis below. The demographic breakdown for these respondents follows:

Employment type of Broker/Agent, Consultant, or Organization Serving Insurance	11
Employment type of Property/Liability Insurance Company or Reinsurance Company	27
Employment type of Academic, Government, Life, Accident & Health Ins., Retired, or Other	3
Fellows	26
Associates	14
Respondents who became members within the last five years	2
Respondents who became members more than five years ago	6
Respondents who answered a of b to question 4	14
Respondents who answered c, d, or e to question 4	27

Establish Context (all risk types)

	All	Vendor Group	Industry Group	Other	Fellows	Associates	Experts	Non-experts
All CAS members should be expert in this area.	9.7	10.0	9.1	16.7	7.4	9.6	16.7	7.1
All CAS members need to know about this area and some should be expert.	58.8	52.5	63.0	41.7	58.5	61.6	59.7	58.2
Some CAS members should know about this area.	26.1	30.0	23.0	41.7	27.8	24.7	18.1	29.3
Outside the scope of CAS and should remain so.	1.2	2.5	0.6	0.0	1.1	1.4	1.4	1.1
Blank	4.3	5.0	4.2	0.0	5.1	2.7	4.2	4.3

Identify Risks – Hazard

	All	Vendor Group	Industry Group	Other	Fellows	Associates	Experts	Non-experts
All CAS members should be expert in this area.	41.6	48.8	38.2	41.7	39.2	46.6	54.2	37.0
All CAS members need to know about this area and some should be expert.	46.3	40.0	49.7	41.7	48.9	39.7	38.9	48.9
Some CAS members should know about this area.	8.9	8.8	8.5	16.7	9.7	8.2	5.6	10.3
Outside the scope of CAS and should remain so.	1.6	1.3	1.8	0.0	1.1	2.7	1.4	1.6
Blank	1.6	1.3	1.8	0.0	1.2	2.7	0.0	2.1

Identify Risks – Financial

	All	Vendor Group	Industry Group	Other	Fellows	Associates	Experts	Non-experts
All CAS members should be expert in this area.	10.5	13.8	9.7	0.0	8.5	15.1	13.9	9.2
All CAS members need to know about this area and some should be expert.	67.3	62.5	69.1	75.0	71.0	57.5	70.8	65.8
Some CAS members should know about this area.	19.1	20.0	18.2	25.0	17.6	23.3	13.9	21.2
Outside the scope of CAS and should remain so.	1.6	2.5	1.2	0.0	1.7	1.4	1.4	1.6
Blank	1.6	1.3	1.8	0.0	1.2	2.7	0.0	2.1

Identify Risks – Operational

	All	Vendor Group	Industry Group	Other	Fellows	Associates	Experts	Non-experts
All CAS members should be expert in this area.	3.9	6.3	3.0	0.0	2.8	6.8	5.6	3.3
All CAS members need to know about this area and some should be expert.	47.9	45.0	49.1	50.0	46.0	52.1	45.8	48.4
Some CAS members should know about this area.	42.0	40.0	42.4	50.0	44.9	37.0	43.1	41.8
Outside the scope of CAS and should remain so.	4.3	6.3	3.6	0.0	5.1	1.4	4.2	4.3
Blank	2.0	2.6	1.8	0.0	1.2	2.7	1.4	2.1

Identify Risks – Strategic

	All	Vendor Group	Industry Group	Other	Fellows	Associates	Experts	Non-experts
All CAS members should be expert in this area.	4.3	6.3	3.6	0.0	4.0	5.5	5.6	3.8
All CAS members need to know about this area and some should be expert.	48.6	50.0	49.1	33.3	46.6	53.4	50.0	47.8
Some CAS members should know about this area.	41.2	37.5	41.2	66.7	43.8	35.6	40.3	41.8
Outside the scope of CAS and should remain so.	3.5	3.8	3.6	0.0	4.0	2.7	2.8	3.8
Blank	2.3	2.6	2.4	0.0	1.7	2.7	1.4	2.7

Analyze/Quantify Risks – Hazard

	All	Vendor Group	Industry Group	Other	Fellows	Associates	Experts	Non-experts
All CAS members should be expert in this area.	51.0	61.3	46.7	41.7	48.3	56.2	61.1	46.7
All CAS members need to know about this area and some should be expert.	37.0	26.3	42.4	33.3	39.8	32.9	31.9	39.1
Some CAS members should know about this area.	7.8	10.0	6.1	16.7	8.5	5.5	5.6	8.7
Outside the scope of CAS and should remain so.	1.2	0.0	1.8	0.0	1.1	1.4	0.0	1.6
Blank	3.1	2.6	3.0	8.3	2.3	4.1	1.4	3.8

Analyze/Quantify Risks – Financial

	All	Vendor Group	Industry Group	Other	Fellows	Associates	Experts	Non-experts
All CAS members should be expert in this area.	13.2	16.3	12.7	0.0	11.9	15.1	15.3	12.0
All CAS members need to know about this area and some should be expert.	65.4	65.0	66.1	58.3	67.6	61.6	66.7	65.2
Some CAS members should know about this area.	17.5	16.3	17.0	33.3	16.5	19.2	18.1	17.4
Outside the scope of CAS and should remain so.	1.2	1.3	1.2	0.0	1.7	0.0	0.0	1.6
Blank	2.7	1.3	3.0	8.3	2.3	4.1	0.0	3.8

Analyze/Quantify Risks – Operational

	All	Vendor Group	Industry Group	Other	Fellows	Associates	Experts	Non-experts
All CAS members should be expert in this area.	5.4	3.8	6.7	0.0	3.4	11.0	4.2	6.0
All CAS members need to know about this area and some should be expert.	44.0	48.8	43.6	16.7	43.2	46.6	43.1	44.0
Some CAS members should know about this area.	43.2	40.0	42.4	75.0	45.5	38.4	47.2	41.8
Outside the scope of CAS and should remain so.	3.9	5.0	3.6	0.0	5.1	0.0	4.2	3.8
Blank	3.5	2.6	3.6	8.3	2.9	4.1	1.4	4.3

Analyze/Quantify Risks – Strategic

	All	Vendor Group	Industry Group	Other	Fellows	Associates	Experts	Non-experts
All CAS members should be expert in this area.	5.1	3.8	6.1	0.0	4.0	8.2	5.6	4.9
All CAS members need to know about this area and some should be expert.	47.5	53.8	46.7	16.7	46.6	50.7	48.6	46.7
Some CAS members should know about this area.	41.6	37.5	41.2	75.0	43.2	37.0	43.1	41.3
Outside the scope of CAS and should remain so.	3.1	2.5	3.6	0.0	4.0	1.4	1.4	3.8
Blank	2.7	2.6	2.4	8.3	2.3	2.7	1.4	3.2

Integrate Risks (all risk types)

	All	Vendor Group	Industry Group	Other	Fellows	Associates	Experts	Non-experts
All CAS members should be expert in this area.	7.0	6.3	7.9	0.0	4.0	12.3	6.9	7.1
All CAS members need to know about this area and some should be expert.	57.2	52.5	60.0	50.0	59.1	53.4	56.9	57.1
Some CAS members should know about this area.	27.6	28.8	25.5	50.0	27.3	28.8	29.2	27.2
Outside the scope of CAS and should remain so.	1.2	2.5	0.6	0.0	1.7	0.0	1.4	1.1
Blank	7.0	10.1	6.1	0.0	8.0	5.5	5.6	7.6

Assess/Prioritize Risks – Hazard

	All	Vendor Group	Industry Group	Other	Fellows	Associates	Experts	Non-experts
All CAS members should be expert in this area.	28.8	37.5	24.8	25.0	25.0	27.0	40.3	23.9
All CAS members need to know about this area and some should be expert.	54.1	46.3	58.2	50.0	60.8	41.1	38.9	60.3
Some CAS members should know about this area.	13.6	12.5	13.3	25.0	10.8	19.2	18.1	12.0
Outside the scope of CAS and should remain so.	1.6	1.3	1.8	0.0	1.7	1.4	0.0	2.2
Blank	2.0	2.6	1.8	0.0	1.7	1.4	2.8	1.6

Assess/Prioritize Risks – Financial

	All	Vendor Group	Industry Group	Other	Fellows	Associates	Experts	Non-experts
All CAS members should be expert in this area.	8.9	12.5	7.9	0.0	7.4	12.3	11.1	7.6
All CAS members need to know about this area and some should be expert.	62.6	57.5	64.8	66.7	66.5	56.2	61.1	63.6
Some CAS members should know about this area.	25.7	27.5	24.2	33.3	22.7	30.1	26.4	25.5
Outside the scope of CAS and should remain so.	1.2	1.3	1.2	0.0	1.7	0.0	0.0	1.6
Blank	1.6	1.3	1.8	0.0	1.7	1.4	1.4	1.6

Assess/Prioritize Risks – Operational

	All	Vendor Group	Industry Group	Other	Fellows	Associates	Experts	Non-experts
All CAS members should be expert in this area.	1.9	3.8	1.2	0.0	0.6	5.5	4.2	1.1
All CAS members need to know about this area and some should be expert.	43.2	43.8	43.0	41.7	42.6	46.6	38.9	44.6
Some CAS members should know about this area.	47.5	43.8	49.1	50.0	48.3	45.2	48.6	47.3
Outside the scope of CAS and should remain so.	5.4	6.3	4.8	8.3	6.8	1.4	5.6	5.4
Blank	2.0	2.6	1.8	0.0	1.7	1.4	2.8	1.6

Assess/Prioritize Risks – Strategic

	All	Vendor Group	Industry Group	Other	Fellows	Associates	Experts	Non-experts
All CAS members should be expert in this area.	2.7	3.8	1.8	8.3	2.3	4.1	6.9	1.1
All CAS members need to know about this area and some should be expert.	47.5	47.5	47.9	41.7	47.7	49.3	44.4	48.4
Some CAS members should know about this area.	42.4	42.5	42.4	41.7	41.5	42.5	41.7	42.9
Outside the scope of CAS and should remain so.	5.4	3.8	6.1	8.3	6.8	2.7	4.2	6.0
Blank	2.0	2.6	1.8	0.0	1.7	1.4	2.8	1.6

Treat/Exploit Risks – Hazard

	All	Vendor Group	Industry Group	Other	Fellows	Associates	Experts	Non-experts
All CAS members should be expert in this area.	23.3	27.5	21.8	16.7	18.8	32.9	29.2	20.7
All CAS members need to know about this area and some should be expert.	49.8	48.8	50.9	41.7	56.3	38.4	40.3	53.8
Some CAS members should know about this area.	21.8	16.3	23.0	41.7	19.9	24.7	23.6	21.2
Outside the scope of CAS and should remain so.	2.3	3.8	1.8	0.0	2.3	2.7	1.4	2.7
Blank	2.7	3.8	2.4	0.0	2.9	1.4	5.6	1.6

Treat/Exploit Risks – Financial

	All	Vendor Group	Industry Group	Other	Fellows	Associates	Experts	Non-experts
All CAS members should be expert in this area.	5.4	7.5	4.8	0.0	2.8	11.0	8.3	3.8
All CAS members need to know about this area and some should be expert.	57.2	53.8	60.0	41.7	60.8	52.1	51.4	59.8
Some CAS members should know about this area.	32.7	32.5	30.9	58.3	31.3	32.9	33.3	32.6
Outside the scope of CAS and should remain so.	2.3	3.8	1.8	0.0	2.3	2.7	2.8	2.2
Blank	2.3	2.6	2.4	0.0	2.9	1.4	4.2	1.6

Treat/Exploit Risks – Operational

	All	Vendor Group	Industry Group	Other	Fellows	Associates	Experts	Non-experts
All CAS members should be expert in this area.	1.6	3.8	0.6	0.0	0.6	4.1	4.2	0.5
All CAS members need to know about this area and some should be expert.	38.1	33.8	41.2	25.0	36.4	45.2	31.9	40.2
Some CAS members should know about this area.	50.2	50.0	49.1	66.7	51.7	45.2	48.6	51.1
Outside the scope of CAS and should remain so.	7.4	8.8	6.7	8.3	8.5	4.1	9.7	6.5
Blank	2.7	3.8	2.4	0.0	2.9	1.4	5.6	1.6

Treat/Exploit Risks – Strategic

	All	Vendor Group	Industry Group	Other	Fellows	Associates	Experts	Non-experts
All CAS members should be expert in this area.	2.3	3.8	1.8	0.0	1.7	4.1	5.6	1.1
All CAS members need to know about this area and some should be expert.	40.5	36.3	44.2	16.7	39.8	45.2	36.1	41.8
Some CAS members should know about this area.	48.2	52.5	44.2	75.0	48.3	45.2	48.6	48.4
Outside the scope of CAS and should remain so.	5.8	3.8	6.7	8.3	7.4	2.7	4.2	6.5
Blank	3.1	3.8	3.0	0.0	2.9	2.7	5.6	2.1

Monitor & Review (all risk types)

	All	Vendor Group	Industry Group	Other	Fellows	Associates	Experts	Non-experts
All CAS members should be expert in this area.	1.9	1.3	2.4	0.0	1.7	2.7	5.6	0.5
All CAS members need to know about this area and some should be expert.	56.4	53.8	59.4	33.3	54.5	60.3	50.0	58.7
Some CAS members should know about this area.	32.3	28.8	32.1	58.3	34.7	27.4	29.2	33.7
Outside the scope of CAS and should remain so.	4.7	10.0	1.8	8.3	4.0	5.5	8.3	3.3
Blank	4.7	6.3	4.2	0.0	5.1	4.1	6.9	3.8

Question 19:
To which of the following organizations do you belong?
The percentages in the table represent the percentage of respondents that indicated that they belong to the particular organization.

	All	Vendor Group	Industry Group	Other	Fellows	Associates	Experts	Non-experts
Global Association of Risk Professionals (GARP)	2.3	5.5	1.0	0.0	2.5	1.1	5.8	0.9
Risk and Insurance Management Society (RIMS)	1.7	4.4	0.5	0.0	2.0	0.0	4.7	0.5
Association for Investment Management and Research (AIMR)	1.7	1.1	2.1	0.0	2.5	0.0	1.2	1.9
American Institute of Certified Public Accountants (AICPA)	0.7	0.0	1.0	0.0	1.0	0.0	1.2	0.5
International Association of Financial Engineers (IAFE)	0.7	1.1	0.5	0.0	0.0	1.1	1.2	0.5
Society of Actuaries (SoA)	3.7	7.7	1.6	6.7	3.0	3.4	4.7	3.3
Other	16.4	23.1	13.5	13.3	15.8	17.2	17.4	15.6
None	40.6	33.0	43.2	53.3	41.6	41.4	36.0	42.7

Written responses to "Other (please specify)"

- Society of CPCU's (6 responses)
- CIA (5)
- ARIA (4)
- Institute of Actuaries (3)
- AFIR (2)
- ASTIN (2)
- CCA (2)
- Institute of Actuaries of Australia (2)
- Society of Insurance Research (2)
- ASPA
- CAJPA
- International Anti-Fad Management Society
- National Association of Insurance Women
- PARMA
- PRIMA
- RAA

Question 20:
Which of the following designations do you hold?
The percentages in the table represent the percentage of respondents that indicated that they hold the particular designation.

	All	Vendor Group	Industry Group	Other	Fellows	Associates	Experts	Non-experts
MBA	4.0	3.3	3.6	13.3	4.5	3.4	4.7	3.8
Ph.D.	2.7	3.3	2.1	6.7	2.5	3.4	4.7	1.9
Financial Risk Manager (FRM)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Associate in Risk Management (ARM)	4.0	6.6	2.6	6.7	4.0	3.4	4.7	3.8
Chartered Property Casualty Underwriter (CPCU)	7.4	8.8	6.8	6.7	9.4	2.3	3.5	9.0
Chartered Financial Analyst (CFA)	0.7	2.2	0.0	0.0	0.5	0.0	1.2	0.5
Fellow or Associate of Society of Actuaries (FSA/ASA)	5.4	7.7	3.6	13.3	5.0	5.7	9.3	3.8
Certified Public Accountants (CPA)	0.7	0.0	1.0	0.0	0.5	1.1	1.2	0.5
Associate in Reinsurance (ARe)	3.4	2.2	4.2	0.0	4.0	2.3	3.5	3.3
Other	12.4	14.3	12.0	6.7	8.9	17.2	7.0	14.7
None	36.2	29.7	37.5	60.0	36.6	37.9	31.4	38.4

Written responses to "Other (please specify)"

- MS (including applied math, psychology, economics and statistics) (7)
- FCIA (5)
- MA in Mathematics (2)
- AIM
- ARP
- Associate in Insurance Accounting and Finance (AIAF)
- Associate in Regulation and Compliance (ARC)
- Associate in Underwriting
- B.Sc.
- Broker license
- Certified Professional Insurance Woman
- ChFC
- CLU
- FCA
- FIA
- FIAA

Question 21:
Given the Advisory Committee's charge to identify research and education that the CAS should undertake in the area of ERM, what other information would you like to share to assist the Committee in fulfilling this charge?

Research

- Investigate what exists first.

- More work needs to be done on correlation/dependency structures. In particular, many financial risks are affected by macroeconomic factors. A better understanding of econometrics would be valuable for better projections of the overall risk profile.

- Literature search for firm-wide approaches to risk management, especially PCAS and "Collective Risk," AFIR, and other actuarial papers.

- Research should be undertaken in the statistical and business aspects of dealing with ERM so that actuaries will have some tools to fall back upon.

- ERM means different things to different people. A simple ERM could be a "basket aggregate" of insurance coverages to the more sophisticated "enterprise" which would include Financial (foreign exchange), Human Capital (employee recruitment/retention), Legal (Internet Liability, Professional liability, contract liability), Natural (ice storms), Operational (supplier failure), Political (confiscation/nationalization), Technological (cyber property), and the Insurance risk wrapped together. It is not just a simple ERM topic, it will need to be identified as to the scope of what you want to undertake. Also, the subject is difficult to sell to a Risk Manager. It is the CFO and the CEO who will be able to have the larger perspective in order to grasp this concept. ERM is slowly getting some attention. One of the problems is the high cost of modeling the individual client's risk. Unless there is some payback to the client they are reluctant to engage such a large project. We also have some tremendous competition from our CFA folks who seem to be able to do it cheaper. Remember that ERM is one or two steps away from a DFA.

- Committee on Theory of Risk research agenda eventually gets around to exploring the relationship between catastrophes and financial economics. It is important to get there, and subsequently review the steps we took on the way there.

- I think ERM should be treated as any other actuarial discipline where certain actuaries should become experts and the rest have a basic knowledge of the subject. A personal lines or BOP actuary does not need to be an expert in ERM from the customer's standpoint; this is an area that pertains to a certain insured group and those that price and analyze that group are the only necessary experts.

- Should link up with a GARP or IAFE and develop research program.

- My impression is that the CAS approach is primarily academic at this time and not yet an aggressive business approach worthy of CEO and Board attention. Key research should be some examples in which casualty actuarial contributions were crucial to survival and growth of an organization.

- Correlation issues between risks.

- Parameter risk will kill you. Don't expect usage to outpace practicality.

- Despite Shaun Wang's excellent work, I think more should be done in the area of estimating correlation structures and generating correlated variates. In particular, the measurement of correlation between indicated loss reserves and economic variables needs to be developed.

- The ERM Special Interest Seminar was very good. It was great to see actuaries thinking about operational risks in a broad context.

- CAS should work with finance academics on valuation models for inefficient markets (i.e., those where arbitrage-free pricing models don't apply).

- Like Black-Scholes, Property Catastrophe, and other models that are useful for business, we should have an ERM Lite model available for CAS use. I believe Lite versions should suffice on most occasions, until one wants to get the more detailed examination, in which case consultants who have broad knowledge in the specific model and industry expertise are invaluable.

- Research and take a position on the various competing ways to determine capital adequacy in the P/C industry.

- Quantification and modeling of risk. Integrated risk modeling.

Education

- Don't add to exams until there's good material, and other material is removed.

- I think this is a great area for P/C actuaries to be involved in.

- Option Theory -- Risk structures to exploit knowledge have often included options, some explicit and some implicit. Recognizing and anticipating the incentives these options create for the different parties involved is necessary for proper exploitation.

- ERM should be included as an exam topic, and seminars should be offered if there is enough interest to support them. While this topic would seem to be an ideal area of concentration for a casualty actuary, the actual performance of this function is often based either on the interest of the individual actuary or the political set-up of the company management. Many actuaries are happily and fully employed doing other tasks. Many companies focus their listening based on top management preferences rather than formal education qualifications.

- The education of actuaries will continue to break apart into specialties. ERM could be such a specialty. Most CAS members do compliance work and have no benefit in studying ERM rather than something else.

- While this has been developed since my time as a student, I believe that the Actuarial Control Cycle course taught by Macquarie and other universities for the Institute of Actuaries of Australia covers a range of enterprise risk management topics and could be worthy of consideration by the committee.

- Actual ERM deals are very few and do not represent a significant source of income; therefore, they should not be heavily emphasized in the CAS educational process.

- Not all actuaries will be involved (or will posses the skills to be capable of being involved) in ERM. We are not generalists but specialists and some people will not develop beyond certain limitations/interests, and that is OK. Keep in mind that most people that take the time to respond to this survey will likely have a passing interest in ERM so the results will be skewed towards a higher need than actually exists in the market today and in the near future. Think about it. DFA was recently added to the syllabus and people still work in the area--self study and on the job training is FINE for very specific areas. Many will not go beyond being rate filing actuaries, ever.

- I would like to learn much more about Enterprise Risk.

- Should link up with a GARP or IAFE and develop certificate program.

- The Procrustean bed of a uniform approach to enterprise risk management means that less capable consultants can perform engagements resembling enterprise risk management. All organizations are not alike. Clear description of the variety of organizational goals and diagnostic tools to decide risk management strategies are most needed for CAS education. Processes should not be emphasized until we have a foundation in place.

- I believe that the CAS should make the members aware of enterprise risk issues, but should NOT try to become an educator. The CAS has already wandered too far from actuarial core competencies. There is plenty of information and educational resources available through treasury and financial seminars to facilitate the learning process for those who seek expertise in this area. This is not an area for ALL actuaries.

- We need a lot of education. Basel Capital, ECOR, EVA, EVT, etc., are all unfamiliar concepts that we need to know. We need more management training too so that we can add value to the operations of any company - not just Insurance or Financial Services Companies.

- See your question 16. You ask about "CAS special interest seminars", but make no reference to 'normal' meetings and seminars. I predict that if you hold a special interest seminar on ERM, you will get some of the members. If instead, you hold an introductory session at the spring and fall meetings and the ratemaking and reserving seminars, you will get dozens, if not hundreds of people at each session.

- I recently attended a luncheon where H. Felix Kloman spoke briefly about this very subject. I think he would be a very good speaker/educator about the identification/integration of risk analysis. I think RIMS might also be a very good source for education/seminars.

- What really struck me after attending the special interest seminar in San Francisco, was how it seems many people know about this topic, but it seems the education is seriously lacking. How does one go about doing an ERM project, start to finish? I have no clue and would like very much to know.

- The CAS can't be all things to all people. CAS actuaries may decide to pursue this path, but it is not for everyone. I wouldn't put the info on the ACAS exams as these are a building block, not specifics. General mention on the FCAS exams should provide a start.

- Emphasize simple models for assessing hazard/financial risk that can be set up, run, and rerun quickly. Sacrifice mathematical sophistication in favor of more understandable models. "Black box" DFA models are to be discouraged.

- The Advisory Committee should issue a call for Forum papers on enterprise risk management, including a paper that surveys all methods for quantifying and assessing risks (per question 10). The survey should include general description, data requirements, strengths, and weaknesses.

- In an overloaded syllabus which has historically trained "generalists", I believe that other than mild knowledge and testing of students is beyond the scope of the formal education process. This is an advanced topic, needed by only the senior-most company officers / or consulting actuaries.

- I think the necessary education to produce an enterprise risk manager is far more encompassing than what the CAS should offer. Investment and accounting expertise can be picked up in other organizations. We can't expect to put students though 8 years of exams learning the fundamentals of how to work with property casualty type contingencies, only to need to tack on a few more to try and teach non-insurance accounting and business skills to students. Business skills that teach the wide range of items that need to be considered in the ERM process can't be taught in a syllabus type context. Furthermore, not every actuary needs to be skilled at the level of detail required to be a top-notch ERM. I have no problems in adding something like this material to the syllabus (very brief) so students are exposed to this area of practice. However given how poorly the CAS tests over DFA, I think learning about ERM would probably be best handled by special interest seminars. Disclaimer: I took Part 8 this spring, it was a poor exam, yes I'm still bitter.

- Question #18 – I think could be better categorized as some should know about area and some should be expert in area. I think it is unrealistic to think all should know about the subject. There are still other areas of expertise that are needed and should be specialized in as well.

- Seminars should present cutting edge topics. This year's ERM seminar presented a good perspective from Ford, but insurance companies aren't doing anything, except providing some capital markets products. I haven't seen anything that is truly combining P&C risk with some other type.

- We could use some better "practitioner" literature on how to combine DFA-type analysis of hazard risks with operational, financial and strategic risks.

APPENDIX

Casualty Actuarial Society Membership Survey on Enterprise Risk Management

Introduction
The CAS Board of Directors established the CAS Advisory Committee on Enterprise Risk Management to identify research and education that the CAS should undertake in the area of enterprise risk management. A key step in the Committee's work is to identify the current level of knowledge, the source of that knowledge, and the desired level of knowledge of the CAS membership as respects enterprise risk management. Your participation in this survey will greatly assist the Committee in its efforts.

Please complete and return the survey even if you are not presently involved in the area of Enterprise Risk Management. This alone provides valuable information to the committee. You should be able to complete the survey in about 15 minutes.

Background
We define and explain some terms and concepts to help you to better understand the questions in the survey.

Enterprise Risk Management
The Advisory Committee's working definition of *Enterprise Risk Management* is:
"The process by which organizations in all industries assess, control, exploit, finance and monitor risks from all sources for the purpose of increasing the organization's short and long term value to its stakeholders."

Note the term "*exploit*" risk. This term is used to highlight the fact that the ERM process can be used not just to mitigate or transfer risk, but also to take advantage of the risks present within a firm, and their relationships to the firm's environment.

Types of Risk
The advisory committee categorizes the types of risk that are subject to Enterprise Risk Management as follows:

- *Hazard* – traditional property/casualty risk, including catastrophic loss, business interruption risk, and environmental risk; health and safety risk; human resources related risk.
- *Financial* – price (e.g., interest rate, foreign exchange, commodity); liquidity (e.g., cash flow, opportunity cost); credit (e.g., default).
- *Operational* – operations (e.g., customer satisfaction, human resources, product development, capacity, efficiency, product/service failure, trademark/brand erosion); empowerment (e.g., leadership, change readiness); information technology (e.g., relevance, availability); integrity (e.g., management fraud, reputation); information risk – business reporting (e.g., budgeting and planning, accounting information, pension fund, investment evaluation, taxation).
- *Strategic* – competition, customer wants, technological innovation, capital availability, regulatory, political, etc.

Enterprise Risk Management Process

The Advisory Committee considers the Enterprise Risk Management process as comprising the following steps:

- *Establish Context* – includes understanding the strategic (external) context (e.g., organization/environment relationship, stakeholder communication policies, "Strengths, Weaknesses, Opportunities and Threats" (SWOT) analysis), the organizational (internal) context (e.g., goals, objectives, oversight structure, common language and criteria), and the risk management context (e.g., units covered, coordination throughout organization).

- *Identify Risks* – includes documenting the conditions and events that represent material threats to the organization's achievement of its strategic objectives or represent areas to exploit for competitive advantage.

- *Analyze/Quantify Risks* – includes creating probability distributions of outcomes for each material risk.

- *Integrate Risks* – includes aggregating all risk distributions, reflecting correlations and portfolio effects, and expressing results in terms of the organization's common language and criteria.

- *Assess/Prioritize Risks* – includes both quantitative and qualitative determination of the contribution of each risk to the aggregate risk profile.

- *Treat/Exploit Risks* – includes both operational and financial responses.

- *Monitor and Review* – includes continual gauging of the risk environment and the performance of the risk management processes.

About the Survey

Your participation in this survey will greatly assist the Committee in determining the current level of activity of CAS members in Enterprise Risk Management (ERM), and where the research and education needs are most critical.

In completing the survey, please understand that the Advisory Committee recognizes that most CAS members are involved in the analysis of property/casualty hazard risk and that this can be construed as a part of Enterprise Risk Management work. However, what the Advisory Committee is looking for is how involved the CAS membership is in the identification, quantification, and treatment of all types of risk in an enterprise-wide context.

Please return the survey by May 25, 2001 to:

Casualty Actuarial Society,
1100 N. Glebe Rd, #600
Arlington, VA, 22201
Fax to: 703-276-3108
E-mail to: office@casact.org

CAS Membership Survey on Enterprise Risk Management

1. Please indicate your type of employment (*please check one*):
   a. ____ Academic
   b. ____ Broker/Agent
   c. ____ Consultant
   d. ____ Government
   e. ____ Life, Accident, and Health Insurance
   f. ____ Organization Serving Insurance Business
   g. ____ Property/Liability Insurance
   h. ____ Reinsurance
   i. ____ Retired
   j. ____ Other

2. Please indicate your CAS membership status (*please check one*):

   a. ____ Fellow ________ Year Attained

   b. ____ Associate ________ Year Attained

   c. ____ Affiliate ________ Year Attained

3. Which of the following describes how you attained your knowledge of Enterprise Risk Management (ERM)?
   (*Please identify and rank all that apply in order of importance, with 1 = most important.*)

   a. ____ From others in my company.

   b. ____ From CAS Examination materials.

   c. ____ Through seminars/courses presented by: (*please specify*) ____________________

   d. ____ Through self-study of the literature.

   e. ____ Through on-the-job learning.

   f. ____ From specific readings/textbooks. (*please specify*) ____________________

   g. ____ Other (*please specify*) ____

   h. ____ I have no current knowledge of ERM.

4. Which of the following best describes your level of knowledge, involvement, and interest in ERM? (*Please check one.*)

   a. ____ I consider myself somewhat of an expert in ERM and devote a considerable portion of my time to ERM projects.

   b. ____ I am familiar with ERM and have been involved with some ERM projects.

   c. ____ I have some understanding of the ERM concept, but have never been involved with an ERM project.

   d. ____ I am not very familiar with ERM, but am interested to learn about it.

   e. ____ I have no interest in ERM.

Note: If you answered C, D, or E to question 4, please proceed to question 10 on page 5. Otherwise, please continue to question 5.

5. Approximately what percentage of your time over the past 12 months have you been involved in ERM projects? (*Please check one.*)

   a. ____ Over 75%
   b. ____ Between 50% and 75%
   c. ____ About 50%
   d. ____ Between 25% and 50%
   e. ____ Less than 25%

6. Which of the following aspects of ERM projects have you personally been involved in? (*Please check all that apply.*)

   a. Identification of risks on an enterprise-wide scale:

      ____ Hazard risk – property/casualty
      ____ Hazard risk – other (e.g., health, safety, HR-related)
      ____ Financial risk
      ____ Operational risk
      ____ Strategic risk

   b. Analysis/quantification of hazard risk:

      ____ Property/casualty hazard risks
      ____ Other types of hazard risk (*please specify*) ______________________________

c. Analysis/quantification of financial risks:

____ Credit risk
____ Foreign exchange risk
____ Interest rate risk
____ Liquidity risk
____ Other (*please specify*) ______________________________

d. Analysis/quantification of operational risks:

____ Operations
____ Information Technology
____ Integrity
____ Information risk
____ Other (*please specify*) ______________________________

e. Analysis/quantification of strategic risk:

____ Competition
____ Customer wants
____ Technological innovation
____ Capital availability
____ Regulatory
____ Political
____ Other (*please specify*) ______________________________

f. Integrated risk analysis including the following categories:

____ Integration of various types of property/casualty hazard risks faced by an entity (e.g., property catastrophe risk with automobile liability).

____ Integration of property/casualty hazard risks with other types of hazard risk faced by an entity (e.g., workers compensation with health risk).

____ Integration of hazard risk, financial risk, operational risk, and/or strategic risk.

g. Assessment or prioritization of risks faced by an entity.

____ hazard
____ financial
____ operational
____ strategic

h. ____ Recommending ways to treat or exploit risks that have been identified, quantified, and assessed.

i. ____ Monitoring of changes in the risk environment and performance of the risk management processes.

7. What other types of practitioners have you worked with in an ERM project? (*Please check all that apply.*)

   a. ____ Academics
   b. ____ Accountants
   c. ____ Actuaries - life/health
   d. ____ Actuaries - pension
   e. ____ Actuaries - property/casualty
   f. ____ Brokers
   g. ____ Economists
   h. ____ Financial analysts
   i. ____ HR professionals
   j. ____ Internal auditors
   k. ____ IT professionals
   l. ____ Lawyers
   m. ____ Marketing professionals
   n. ____ MBAs/Management consultants
   o. ____ Operations experts
   p. ____ Risk managers
   q. ____ Risk specialists (e.g., currency risk expert)
   r. ____ Strategy and organization experts
   s. ____ Underwriters
   t. ____ Others (*please specify*) ________________________________

8. Which of the following best describes your primary role in the ERM projects that you have been involved with? (*Please check one.*)

   a. ____ Project leader. (*If checked, please proceed to Question 10 on the next page.*)

   Part of a team of practitioners with my role being:

   b. ____ Primary technical analyst for all risks.
   c. ____ Primary technical analyst for hazard risk, but a secondary role in other risks.
   d. ____ "Integrator" of all risks.
   e. ____ Other (*please specify*) ________________________________

9. For those ERM projects in which you were one of several practitioners involved with the ERM process, in general which practitioner served as the project leader? (*Please check only one.*)

   a. ____ Academics
   b. ____ Accountants
   c. ____ Actuaries - life/health
   d. ____ Actuaries - pension
   e. ____ Actuaries - property/casualty
   f. ____ Brokers

g. ____ Economists
h. ____ Financial analysts
i. ____ HR professionals
j. ____ Internal auditors
k. ____ IT professionals
l. ____ Lawyers
m. ____ Marketing professionals
n. ____ MBAs/Management consultants
o. ____ Operations experts
p. ____ Risk managers
q. ____ Risk specialists (e.g., currency risk expert)
r. ____ Strategy and organization experts
s. ____ Underwriters
t. ____ Others (*please specify*) ____

10. Please indicate your level of understanding of the following, using the scale 1 = no or low understanding, 2 = medium understanding, 3 = high understanding.

a. ____ Basel Capital Accord
b. ____ Causal modeling
c. ____ Credit Risk Models (e.g., Credit metrics.)
d. ____ Dynamic Financial Analysis (DFA)
e. ____ Economic capital
f. ____ Economic Cost of Ruin (ECOR) or Expected Policyholder Deficit (EPD)
g. ____ Economic Value Added (EVA)
h. ____ Extreme Value Theory (EVT)
i. ____ Fuzzy logic
j. ____ Generalized Autoregressive Heteroskedastic (GARCH) models
k. ____ Net Present Value (NPV)
l. ____ Options Pricing Theory (e.g., Black-Scholes model)
m. ____ Real Options
n. ____ Return on Risk Adjusted Capital (RORAC)
o. ____ Risk Adjusted Return on Capital (RAROC)
p. ____ Risk Mapping
q. ____ Strengths, Weaknesses, Opportunities, and Threats (SWOT) Analysis
r. ____ System Dynamics models
s. ____ Value at Risk (VAR)
t. ____ Tail VAR
u. ____ Other ERM-relevant metrics, models and concepts (*please specify*) ___________

11. Please indicate your level of ability to quantify/model the following types of risk, using the scale 1 = no or low ability, 2 = medium ability, 3 = high ability.

a. ____ hazard – property/casualty
b. ____ hazard – non property/casualty (e.g., health, safety, HR-related)
c. ____ financial
d. ____ operational
e. ____ strategic

12. In the future, which of the following best describes the role you believe CAS members should play in the ERM process? (*Please check all that apply.*)

   a. ____ As the project leader.
   b. ____ As a project team member with primary responsibility for all risk quantification work.
   c. ____ As a project team member with primary responsibility for all hazard risk quantification work.
   d. ____ As a project team member with primary responsibility for all property/casualty risk quantification work.
   e. ____ As a risk "integrator".
   f. ____ Other (*please specify*) ________________________________
   g. ____ None

13. How would you rate the relative importance of increasing your knowledge of ERM?

   a. ____ Low importance
   b. ____ Medium importance
   c. ____ High importance

14. How important is it for CAS members to become better trained in ERM?

   a. ____ Low importance
   b. ____ Medium importance
   c. ____ High importance

15. Please indicate how involved actuaries should be with respect to ERM in the following industry groups, using the scale 1 = no or low involvement, 2 = medium involvement, 3 = high involvement.

   a. ____ Property/casualty insurance industry only
   b. ____ All financial services
   c. ____ Any industry

16. Please rank the relative importance of the following potential sources of ERM education for current CAS members, using a scale 1 = no or low importance, 2 = medium importance, 3 = high importance.

   a. ____ CAS Special Interest Seminars
   b. ____ CAS Examination Syllabus
   c. ____ Other professional organizations' seminars/syllabi (*please specify*) ____________
   d. ____ Business school
   e. ____ Self-study
   f. ____ On-the-job learning
   g. ____ Other (*please specify*) ________________________________

17. On the risk type / process grid below, please indicate the level of expertise **you currently possess**, using the scale 1 = no or low level of expertise, 2 = medium level of expertise, and 3 = high level of expertise.

Process Step*	Risk Type*			
	Hazard	Financial	Operational	Strategic
Establish Context (all risk types)				
Identify Risks				
Analyze/Quantify Risks				
Integrate Risks (all risk types)				
Assess/Prioritize Risks				
Treat/Exploit Risks				
Monitor & Review (all risk types)				

* Refer to definitions in the Background section.

18. On the risk type/ process grid below, please indicate the level of knowledge you believe is **necessary for CAS members to have by the year 2005**. Use the scale below:

5 = All CAS members should be expert in this area.
6 = All CAS members need to know about this area and some should be expert.
7 = Some CAS members should know about this area.
8 = Outside the scope of CAS and should remain so.

Process Step*	Risk Type*			
	Hazard	Financial	Operational	Strategic
Establish Context (all risk types)				
Identify Risks				
Analyze/Quantify Risks				
Integrate Risks (all risk types)				
Assess/Prioritize Risks				
Treat/Exploit Risks				
Monitor & Review (all risk types)				

* Refer to definitions in the Background section.

19. To which of the following organizations do you belong?

a. ____ Global Association of Risk Professionals (GARP)
b. ____ Risk and Insurance Management Society (RIMS)
c. ____ Association for Investment Management and Research (AIMR)
d. ____ American Institute of Certified Public Accountants (AICPA)
e. ____ International Association of Financial Engineers (IAFE)
f. ____ Society of Actuaries (SoA)
g. ____ Other (*please specify*) ____________________
h. ____ None

20. Which of the following designations do you hold?

a. ____ MBA
b. ____ Ph.D.
c. ____ Financial Risk Manager (FRM)
d. ____ Associate in Risk Management (ARM)
e. ____ Chartered Property Casualty Underwriter (CPCU)
f. ____ Chartered Financial Analyst (CFA)
g. ____ Fellow or Associate of Society of Actuaries (FSA/ASA)
h. ____ Certified Public Accountants (CPA)
i. ____ Associate in Reinsurance (ARe)
j. ____ Other (*please specify*) ________________________________
k. ____ None

21. Given the Advisory Committee's charge to identify research and education that the CAS should undertake in the area of ERM, what other information would you like to share to assist the Committee in fulfilling this charge?

Research:________________________________

____

____

____

____

Education:________________________________

____

____

____

____

Optional
Name:________________________ Company________________________

Please return the survey by May 25, 2001. Thank you for your participation.

APPENDIX C

ERM Bibliography

Books

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A Sampling of Risk-Related Regulatory, Rating Agency and Corporate Governance Guidelines and Requirements

There are a number of regulatory, rating agency and corporate governance guidelines and regulations that ERM programs and policies need to consider. The more prominent of these are described below.

- General Industry
  - Cadbury Report, et al (UK) — the London Stock Exchange has adopted a set of principles — the Combined Code — that consolidates previous reports on corporate governance by the Cadbury, Greenbury and Hampel committees. This code, effective for all accounting periods ending on or after December 23, 2000 (and with a lesser requirement for accounting periods ending on or after December 23, 1999), makes directors responsible for establishing a sound system of internal control and reviewing its effectiveness, and reporting their findings to shareholders. This review should cover all controls, including operational and compliance controls and risk management. The Turnbull Committee issued guidelines in September 1999 regarding the reporting requirement for non-financial controls.
  - Dey Report (Canada) — commissioned by the Toronto Stock Exchange and released in December 1994, it requires companies to report on the adequacy of internal control. Following that, the clarifying report produced by the Canadian Institute of Chartered Accountants, "Guidance on Control" (CoCo report, November 1995) specifies that internal control should include the process of risk assessment and risk management. While these reports have not forced Canadian listed companies to initiate an ERM process, they do create public pressure and a strong imperative to do so. In actuality, many companies have responded by initiating ERM processes.
  - Australia/New Zealand Risk Management Standard — a common set of risk management standards issued in 1995 that call for a formalized system of risk management and for reporting to the organization's management on the performance of the risk management system. While not binding, these standards create a benchmark for sound management practices that includes an ERM system.

- Financial Services Industry
  - Basel Committee:
    - The Basel Committee on Banking Regulation and Supervisory Practices was established in 1974 (originally called the Cooke Committee) in response to the erosion of capital in leading global banks. The committee meets under the auspices of the Bank for International Settlements (BIS) but is not part of the BIS. The committee consists of representatives from the central banks/supervisory authorities of the G10 countries + Luxembourg. The committee has no legal authority, but the governments of the representatives on the committee have always legislated to make the recommendations part of their own national law. The standards set by the committee are widely

regarded to be best practice and a large number of other countries that are not formally represented on the committee have implemented the proposals. In the U.S., the Federal Reserve has adopted the Basel Capital Accord ("Basel I" - see below).

— "Basel I" — the 1988 Basel Capital Accord established a framework to calculate a minimum capital requirement for banks. The Accord focused on credit risk and was crude in its recognition of the relative risk of different loans. A number of amendments were made to the Accord (prior to "Basel II" - see below), the most significant of which is the market risk amendment in 1996; this extended the 1988 Accord to cover market risk and allowed for the use of internal models to quantify regulatory capital.

— "Basel II" — in 1999 the Basel Committee issued a draft proposal for a new accord and accepted comment. Based on feedback, the Committee issued a revised proposal in 2001 for review and comment. In this New Basel Capital Accord, proposed for implementation in 2004, among other changes a capital charge for operational risk is included as part of the capital framework. The charge reflects the Committee's "realization that risks other than market and credit" can be substantial. Operational risk is defined as "the risk of direct or indirect loss resulting from inadequate or failed internal processes, people and systems or from external events". The new capital adequacy framework is proposed to apply to insurance subsidiaries of banks and may apply to insurance companies as insurance and banking activities converge.

- OSFI (Canada) — the Office of the Supervisor of Financial Institutions supervisory framework defines "inherent risk" to include credit risk, market risk, insurance risk, operational risk, liquidity risk, legal and regulatory risk and strategic risk. It states that "Where independent reviews of operational management and controls have not been carried out or where independent risk management control functions are lacking, OSFI will, under normal circumstances, make appropriate recommendations or direct that appropriate work be done."
- FSA (UK) — the Financial Services Authority (FSA – the recently created regulator of all UK financial services businesses) is introducing a system of risk based supervision which will create a single set of prudential requirements organized by risk rather than by type of business. Regulated businesses will have to demonstrate that they have identified all material risks and have adequate systems and financial resources to manage and finance such risks, including market risk, credit risk, operational risk and insurance risk. There is also likely to be a requirement for formal documentation of the whole process in a format that is readily accessible to the FSA.

- Insurance Industry
  - A.M. Best — in its *Enterprise Risk Model: A Holistic Approach to Measuring Capital Adequacy*, A.M. Best describes its VaR-based method for determining the adequacy of capital for rating purposes. The report states: "The Enterprise Risk Model is a modular system designed to capture all risks, including noninsurance and non-U.S. related risks. VaR methodologies are somewhat controversial in insurance circles, but they are the standard for other financial-services organizations. More importantly, A.M. Best believes that VaR-based methodologies provide a more accurate assessment of risk and required capital, since they use observable market metrics. Beyond its application in the rating process, the model can also be a useful tool for financial managers, since the VaR framework provides a natural springboard to other applications, including risk-adjusted return on capital (RAROC) and dynamic financial analysis (DFA). The Enterprise Risk Model quantifies the risk to the future surplus – net worth – of an organization arising from a change in underlying risk variables, such as credit risk, insurance risk, interest rate risk, market risk and foreign exchange risk. The model also quantifies the benefits of diversification as it takes a macro view of the correlations among risks within an organization...Like other VaR-based models, it is calibrated to measure the risks over a defined holding period – one year -- for a given level of statistical confidence – 99%."
  - Moody's — in its *One Step in the Right Direction: The New C-3a Risk-Based Capital Component*, June 2000, Moody's Investors Service states that it will use the new method devised by the NAIC and the American Academy of Actuaries for measuring a life insurance company's C-3a (interest-rate) risk, as it incorporates a cash-flow testing requirement for annuity and single premium life products and is more consistent with industry advances in dynamic cash-flow testing: "…the revised calculation is a more accurate barometer of the amount of capital required to support an insurer's interest-sensitive business, as it explicitly incorporates asset-liability mismatches in determining the appropriate amount of required regulatory capital for a company. Consequently, the new calculation should help discourage companies from taking unwarranted asset-liability risk."
  - S&P — in its *Revised Risk-Based Capital Adequacy Model for Financial Products Companies* Standard & Poor's states: "Standard & Poor's Insurance Capital Markets Group has developed a new, risk-based capital adequacy model to analyze the credit, financial market, and operational risks of companies that are offering products or are using sophisticated risk management techniques that are not considered under the existing Rating Group's capital models. The model will also determine these companies' capital adequacy. The primary application of the model will be to analyze specialized financial product companies (FPCs) that are subsidiaries of insurance companies or that are credit enhanced by insurance companies.... The model may also be applied to portions of insurance companies that control or mitigate their risks to a greater extent than is implied by the capital charges applied in the standard life/health capital adequacy model, which bases charges for interest-rate risk and credit risk on industry averages and liability types rather than company-specific exposure."

- NAIC — The National Association of Insurance Commissioners:
  - Risk Based Capital (RBC) — Following a detailed examination of the growing diversity of business practices of insurance companies conducted in 1990, the NAIC concluded that minimum capital requirements placed on companies needed to be increased to protect consumers. The NAIC adopted life/health risk-based capital requirements in December 1992 and adopted Property/Casualty risk-based capital requirements in December 1993. Although risks involved in these two segments of the industry are very different, the NAIC was able to develop a consistent two-step approach to setting risk-based capital requirements for individual companies:
    - Step 1 involves the calculation of a company's capital requirement and total adjusted capital, based on formulas developed by NAIC for each industry.
    - Step 2 calls for comparison of a company's total adjusted capital against the risk-based capital requirement to determine if regulatory action is called for, under provisions of the Risk-Based Capital for Insurers Model Act. The model law sets the points at which a commissioner is authorized and expected to take regulatory action.
  - Interest rate risk — the NAIC's Life Risk-Based Capital Working Group, in conjunction with the American Academy of Actuaries Life Risk-Based Capital Task Force, has finalized the development of an improved method for measuring a company's interest-rate risk. The method, which is effective for the year-end 2000 statements, "incorporates a cash-flow testing requirement for annuity and single premium life products and makes the RBC C-3a calculation more consistent with recent industry advances in dynamic cash-flow testing...The task force has recognized the need to accurately incorporate these additional risks into the RBC formula. They have stated that equity indexed annuities (EIAs) and variable products with secondary guarantees will be incorporated in a future C-3a update. This would be consistent with the task force's goal of upgrading C-3a from a measure of interest-rate risk to a more complete measure of asset/liability risk."
- APRA (Australia) — a feature of ongoing reforms to the regulation of general insurers is a layer of four standards covering the subjects of capital adequacy, liability valuation, reinsurance arrangements and operational risk. The Australian Prudential Regulation Authority (APRA) is implementing an approach based on development of, and compliance with, a range of risk management strategies. These strategies will need to deal with the myriad interlocking risks involved in managing a general insurance company. Each company will need to have its strategy agreed upon by APRA and will then be responsible for managing compliance. APRA has made it clear that an internal enterprise risk model with appropriate specifications will go a long way toward meeting compliance objectives.

ERM Learning Objectives

INTRODUCTION

These Learning Objectives are expressed in terms of the knowledge required of an ***expert**** in enterprise risk management (ERM).

The Learning Objectives are organized within the sequential steps of the Risk Management Process:

- *Establishing Context* – Achieving a full understanding of the present conditions in which the organization operates; this includes understanding the external context (e.g., organization/environment relationship, stakeholder communication policies), the internal context (e.g., business objectives, oversight structure, key performance indicators), and the risk management context (e.g., units covered, degree of coordination throughout organization).

- *Identifying Risks* – Documenting the conditions and events that represent material threats to the organization's achievement of its objectives or represent areas to exploit for competitive advantage.

- *Analyzing/Quantifying Risks* – Calibrating and, wherever possible, creating probability distributions of outcomes for each material risk.

- *Integrating Risks* – Aggregating all risk distributions, reflecting correlations and portfolio effects, and expressing results in terms of impact on the organization's key performance indicators (i.e., the "aggregate risk profile").

- *Assessing/Prioritizing Risks* – Determining the contribution of each risk to the aggregate risk profile, and prioritizing accordingly.

- *Treating/Exploiting Risks* – Developing strategies for controlling or exploiting the various risks.

- *Monitoring and Reviewing* – Continual gauging of the risk environment and the performance of the risk management strategies.

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* Note: With regard to the examination syllabus through 2005, the level of knowledge need not be at the "expert" level for all subject areas. Hence, for syllabus purposes during this period, the required knowledge level should be calibrated to conform to the "Desired Depth of Knowledge" within the table on the following page, for each element of the ERM Framework.

DESIRED DEPTH OF ERM KNOWLEDGE

Key to Table Below:		
Code	**Desired Knowledge Level**	**Education Implication**
1	All CAS members should be expert in this area	On exam syllabus – core subject (similar to ratemaking, reserving)
2	...	On exam syllabus – moderate treatment (similar to accounting)
3	All CAS members need to know about this area	On exam syllabus – light treatment (similar to underwriting)
4	...	On exam syllabus – very light introductory treatment (similar to claims)
5	...	Continuing Ed – annual ERM seminar (similar to CLRS)
6	Some CAS members should know about this area	Continuing Ed – special interest and/or limited attendance seminars (similar to M&A)
7	...	Continuing Ed – Special tracks/sessions within existing CAS (and non-CAS) meetings/seminars
8	...	Self-study/on-line courses/university courses (CAS to maintain bibliography)
9	Outside the scope of CAS	N/A

Note: Any exam syllabus item (codes 1 - 4) also carries continuing education/self-study implications (codes 5 – 8); any continuing education item (codes 5 – 7) also carries self-study implications (code 8)

Depth of ERM Knowledge Within CAS Desired by 2005:

ERM Overview **2**

Process Step	Risk Type			
	Hazard	Financial	Operational	Strategic
Establish Context	3			
Identify Risks	5	6	7	7
Analyze/Quantify Risks	1	2	4	4
Integrate Risks	2			
Assess/Prioritize Risks	4	4	5	5
Treat/Exploit Risks	4	4	6	6
Monitor & Review	5			

LEARNING OBJECTIVES

In addition to the requirements cited in the Learning Objectives specific to each Risk Management Process step below, the ERM expert should have a working knowledge of:

- Economics
- Business finance and accounting
- Statistics and stochastic modeling
- Project management

I. Establishing Context

This first step in the Risk Management Process involves achieving a full understanding of the present conditions in which the organization operates. This includes understanding the external context (e.g., organization/environment relationship, stakeholder communication policies), the internal context (e.g., business objectives, oversight structure, key performance indicators), and the risk management context (e.g., units covered, degree of coordination throughout organization).

Regarding this step, the expert should be able to:

- Identify the key business issues in the organization's industry, including growth prospects, degree of competition, barriers to entry, supply and demand levels, product differentiation, price elasticity, regulatory environment, etc.
- Analyze the organization's competitive position within its industry.
- Articulate the organization's mission/vision/strategic objectives
- Identify the organization's various specific business objectives and constraints (e.g., financial, social, political, legal) and the interplay among them.
- Identify the organization's business model, management and governance structure, decision-making processes and systems.
- Interpret the organization's financial statements and key performance indicators.
- Evaluate the practical implications of the major stakeholders' (e.g., shareholders, employees, clients) expectations of the organization.
- Determine the organization's key assets.
- Conduct a strengths/weaknesses/opportunities/threats (SWOT) analysis.
- Elicit and describe the organization's risk management objectives.
- Describe the organization's risk control processes.
- Formulate risk management mission statement, policies and guidelines.

II. Identifying Risks

This second step in the Risk Management Process involves documenting the conditions and events that represent material threats to the organization's achievement of its objectives or represent areas to exploit for competitive advantage.

The scope of risks includes the following Risk Types:

- *Hazard* risks, such as:
  - Liability suits (e.g., operations, products, environmental)

- Fire and other property damage
  - Windstorm and other natural perils (including catastrophes)
  - Theft and other crime
  - Personal injury, disease, disability (including work-related injuries and diseases)
  - Business interruption
- *Financial* risks, such as:
  - Price (e.g. asset value, interest rate, foreign exchange, commodity)
  - Liquidity (e.g. cash flow, call risk, opportunity cost)
  - Credit (e.g. default, downgrade).
  - Inflation/purchasing power
  - Hedging/basis risk
- *Operational* risks, such as:
  - Business operations (e.g. customer satisfaction, human resources, product development, capacity, efficiency, product/service failure, trademark/brand erosion)
  - Empowerment (e.g., leadership, change readiness)
  - Information technology (e.g. relevance, availability)
  - Integrity (e.g., management fraud, reputation)
  - Information/business reporting (e.g., budgeting and planning, accounting information, pension fund, investment evaluation, taxation)
- *Strategic* risks, such as:
  - Competition
  - Customer wants
  - Demographic and social/cultural trends
  - Technological innovation
  - Capital availability
  - Regulatory and political trends

Regarding this step, the expert should be able to:

- Generate a comprehensive list of risks that may affect the organization's objectives, using a systematic application of appropriate risk detection techniques, such as:
  - Expert interviewing
  - Site inspections
  - Checklists
  - Document and data reviews
  - Scenario analysis
- Identify the area of impact (i.e., on earnings, cash flow, etc.) of each risk.
- Identify the possible causes and scenarios underlying each risk.
- Qualitatively determine the materiality of each risk in the context of the organization's objectives, and considering the potential correlation with other hazard, financial, strategic and operational risks.
- Select and rank order the risks for further analysis.
- Classify the risks in a manner that is meaningful to their mitigation, for example:
  - Separate the risks that can be simply and immediately mitigated from those that require a substantial capital outlay or a change in strategic direction.
  - Determine those risks requiring rigorous quantification and modeling.

III. Analyzing and Quantifying Risks

This third step in the Risk Management Process involves calibrating and, wherever possible, creating probability distributions of outcomes for each material risk.

Regarding this step, the expert should be able to:

- Identify and access the sources of relevant external data on the quantification of risks relevant to the organization's industry.
- Collect and organize the necessary internal data on the quantification of risks unique to the organization.
- Understand and apply appropriate risk quantification approaches, depending on the nature and availability of data and expert input, including but not limited to:
  - Probability distribution fitting to historical data
  - Extreme Value Theory
  - Regression over variables that affect risk
  - Causal modeling
  - Influence diagrams
  - Fuzzy logic
  - Delphi method
  - Judgement
- Express the risks in terms of a probability distribution of outcomes.
- Assess the effectiveness of existing control measures (managerial, technical, procedural, financial, insurance, etc.).
- Modify the probability distributions as appropriate to reflect the impact of existing control measures.
- Provide estimates of the timing and duration of the outcomes in the context of the organization's objectives and strategies.
- Determine the present value of the future stream of contingent financial outcomes.
- Validate the qualitative rank ordering of risks (from process step II) under various quantitative risk expressions, such as:
  - In the context of the organization's objectives and strategies, using the organization's key performance metrics (e.g., net operating earnings, probability of ruin, growth in embedded value).
  - In the context of impact on the organization's social and other non-financial objectives (e.g., commitment to the community, commitment to employees).
- Determine a suitable model for the particular business situation:
  - Identify, through the application of statistical tests, the frequency and severity probability distributions and parameters that best fit the data.
  - Assess the variability of the parameters and the goodness of fit of the model, to determine the confidence that should be given to the model output in making decisions.
  - Consider the quality and credibility of the data.
  - Conduct sensitivity testing of the models and assumptions.
  - In the absence of an actuarial or other established quantitative model or data for a given situation, make reasonable judgments using sound business logic.

IV. Integrating Risks

This fourth step in the Risk Management Process involves aggregating all risk distributions, reflecting correlations and portfolio effects, and expressing results in terms of impact on the organization's key performance indicators (i.e., the "aggregate risk profile").

Regarding this step, the expert should be able to:

- Determine and document the correlations and causes of interaction among the various hazard, financial, operational and strategic risks, using appropriate qualitative and quantitative techniques including but not limited to:
  - Influence diagrams/event tree analysis
  - Decomposition analysis
  - Analysis of variance
  - Multiple regression analysis
  - Econometric methods
  - Neural networks
- Understand different models and types of analysis commonly used to aggregate risks, know their common features and differences, and apply them appropriately.
- Identify and describe the importance of reliances, assumptions and other simplifications (such as the inclusion or exclusion of certain risks) in the model or analysis that could have a material effect on the results.
- Create an aggregate risk profile (i.e., an aggregate probability distribution) of all material risks, reflecting the probability distributions of the individual risks and their correlations, using appropriate techniques, including but not limited to:
  - Monte Carlo simulation
  - Statistical convolution
  - Causal modeling
  - Dynamic Financial Analysis (DFA) and other structural simulation models
  - Mean/variance/covariance (MVC) and other statistical analytic models
- Create an aggregate risk distribution in terms of the organization's key performance metrics.

V. Assessing/Prioritizing Risks

This fifth step in the Risk Management Process involves determining the contribution of each risk to the aggregate risk profile, and prioritizing accordingly.

Regarding this step, the expert should be able to:

- Determine the appropriate *risk* measures and organizational tolerances against these measures, such as:
  - Solvency-related measures (related to the "tail" of the distribution)
    - — Value at Risk (VaR)
    - — Tail VaR
    - — Probability of ruin/default

    - Expected Policyholder Deficit (EPD)/Economic Cost of Ruin (ECOR)
    - Shortfall risk
    - Risk Based Capital (RBC)
    - Insurance Regulatory Information System (IRIS) tests
    - Rating agency models
    - Basel Capital Accord measures
  - Volatility measures (related to the "center" of the distribution)
    - Variance/semi-variance
    - Standard deviation/downside standard deviation
    - Mean average deviation
    - Below-Target-Risk (BTR)
  - Qualitative measures, such as the impact on:
    - The community
    - Employees
    - Brand reputation and image
    - Investor perceptions
- Determine the *reward* measures and benchmarks appropriate to the organization's objectives, such as:
  - Expected operating earnings
  - Growth in book value
  - Growth in Embedded Value
  - Economic Value Added (EVA)
  - Total return on equity
  - Return on Risk Adjusted Capital (RORAC)
  - Risk Adjusted Return on Capital (RAROC)
- Use appropriate techniques to determine the marginal contribution of each risk to the aggregate risk profile.
- Perform stress testing to determine whether and to what degree the importance of individual risks varies under different risk and reward metrics.
- Prioritize risks according to their marginal impact on the aggregate risk profile.
- Decompose the impact of each high-priority risk in order to inform its treatment.

VI. Treating/Exploiting Risks

This sixth step in the Risk Management Process involves developing strategies for controlling or exploiting the various risks.

Regarding this step, the expert should be able to:

- Identify and evaluate the various financial, operational and strategic techniques to avoid, control, transfer/finance or capitalize on the various types and combinations of risk; such techniques include but are not limited to:
  - Exposure avoidance
  - Loss prevention
  - Loss reduction
  - Segregation of exposure units
  - Contractual transfer

- Insurance/reinsurance
- Risk retention vehicles (e.g., captives, pools)
- Use of financial markets (e.g., lines of credit, derivatives, securitization), including:
  - Arbitrage
  - Futures vs. forward contracts
  - Spot and forward markets
  - Options and swaps
  - Exotic options
  - Foreign exchange

- Evaluate alternate risk financing tools and techniques such as:
  - Special risk insurance solutions
  - Funding alternatives
  - Derivatives
  - Contingent capital

  and their impact on results, such as:
  - Reducing Cost of Risk (COR),
  - Stabilizing COR over time,
  - Strengthening the balance sheet,
  - Optimizing tax position,
  - Leveraging risk-bearing capacity.
- Incorporate the evaluation of key strategies in a consistent, comprehensive model; these strategies include but are not limited to:
  - Capital structure (i.e., financial leverage)
  - Capital allocation
  - Exposure management
  - Asset allocation
  - Reinsurance
- Evaluate strategies by using an optimization framework, which includes:
  - Determining the risk and reward characteristics of each strategy
  - Comparing the strategies in an "efficient frontier" analysis
  - Determining optimal strategies that maximize the organization's objectives and satisfy its constraints
- Develop a comprehensive risk management strategy and rationale that includes:
  - Evaluating the various options for managing risks.
  - Determining which risks should be controlled and which risks should be exploited for competitive advantage given the nature of the risk and the organization's capabilities.
  - Constructing an integrated plan of action for control and exploitation of these risks.
  - Demonstrating the business case for the risk management strategy, using sound business logic.
- Develop a comprehensive decision framework by which the organization can evaluate new threats/opportunities in a consistent manner.

VII. Monitoring & Reviewing

This seventh step in the Risk Management Process involves continual gauging of the risk environment and the performance of the risk management strategies.

Regarding this step, the expert should be able to:

- Track changes in the business context and risk environment, by:
  - Keeping current with organizational priorities/objectives.
  - Continually updating the organization's risk profile.
  - Keeping current with regulatory requirements.
  - Keeping current with risk management best practices.
  - Timely detection of future threats and opportunities.
- Measure the performance of the implemented strategies, by:
  - Tracking results against reward measures.
  - Measuring departures from expected results against volatility constraints.
  - Tracking compliance with legal and regulatory requirements.
- Track changing risk factors and use different processes for identifying and sourcing risks such as:
  - "Fish bone" diagrams
  - Run charts
  - Samplings
- Back-test models and assumptions, and make appropriate adjustments.
- Revise strategies as appropriate.
- Determine how often risk models and analyses need to be updated.

Survey on Management Data and Information Report

CAS Committee on Management Data and Information

SURVEY ON MANAGEMENT DATA AND INFORMATION

Introduction

On behalf of the Committee on Management Data and Information (MDI), the CAS recently distributed a survey to provide the Committee with information that will enhance its role in providing resources on management data to the CAS membership. The distribution of respondents included 68.5% from insurance or reinsurance companies, 13.5% from consultants, 6.7% from brokerages or agencies, 5.1% from organizations serving insurance, and the remaining 6.2% from other areas. This report will discuss the key findings of the survey.

Data Management Time

Question #1 asked respondents what percentage of their time did they spend on specific data management activities. The two categories where we seem to spend most of our data management activities are "extraction and manipulation of data" (with 40.5% spending over 10% of their time) and "assisting in the design of new analytical tools" (with 29.8% spending over 10% of their time). The other areas, in decreasing order of time spent are: assuring the quality of internal data, mining internal data, data extraction, assisting in the design of new data collection and processing systems, obtaining data from external sources, responding on data issues, and providing input on data collection. What was perhaps most interesting was the number of write-in responses for "other." It is clear that there is a wide range of activities we consider to fall in the realm of "data management," including project management, underwriting, and information systems.

Privacy/Ownership of Data

Sixty-four percent of the respondents reported spending no time on this issue, with another 31.5% spending 1-5% of their time. Not surprisingly, persons working for organizations serving insurance or "other" employers such as banks or financial institutions each had 11% of the respondents spending 6-10% of their time in this area.

Data Management Priorities

Question #3 asked you to indicate the importance of specific data management issues. The percentage of respondents rating each issue as either important or of great importance is as follows: ease of access to internal data systems (90.4%), having a detailed and accurate data dictionary (88.7%), developing the skills necessary to identify, extract, and manipulate data (86.5%), actuarial input in system design (84.3%), having knowledge of traditional external data sources (68.0%), having the ability to perform data mining on internal data (63.5%), having knowledge of non-traditional external data sources (61.8%), having current knowledge of reporting rule changes (60.7%), cost considerations associated with data collection and extraction (52.8%), understanding emerging data technologies (48.3%), ease of integration of external and internal data sources (39.4%), issues of intellectual property and ownership (23.0%). Thus, while we don't seem to spend a significant amount of time on data management issues as shown in question #1, we do place a high value on data management skills and abilities.

Data Management Activities

Many of you have at one time been involved in development of specifications or design for a database or reporting system. Fewer of you have been involved with development of test data or definition of data edits.

Performance of Data Management

Question #5 asked you to identify who performs specific data management activities in your company. Actuaries and actuarial students perform most of the ratemaking and reserving functions. Non-actuaries perform most reporting activities.

Data Management Education

In the last five years, 61.8% of the respondents had attended at least one CAS Ratemaking Seminar. Only 20.2% of the respondents had attended a presentation of the MDI Committee. Thus, roughly two-thirds of attendees at Ratemaking Seminars do not attend the data track presentations. The Limited Attendance Seminar on Emerging Technologies was attended by 17.4% of the respondents.

Future Education Activities

Question #7 asked how important it was that the CAS become more involved in data management education. The respondents rated publishing papers and including sessions at CAS meetings as the most important activities. Developing textbooks or syllabus material was not rated as very important.

Use of Online Catalog/Bibliography

Roughly 80% of respondents were aware that the CAS Web Site provides an online catalog containing a bibliography of reference materials. Of those, 80% found it to be useful. There were many suggested additions or enhancements to the catalog, most referencing better search and index features. These suggestions will be forwarded to the Committee on Online Services.

Use of Data Management Committee

Question #9 asked respondents to rate the importance of activities that the MDI Committee could provide assistance with. Activities that over 50% of respondents rated as important or of great importance include dealing with data that is incomplete or of poor quality (76.4%), developing procedures for ensuring data quality (65.7%), demonstrating techniques for reconciliation of financial and statistical data (57.9%), and keeping up with state-of-the-art in data management (56.7%). Less important were testing techniques, compiling data for research, information on developing new systems, assistance with Web-enabled applications, and providing information on the latest software.

CAS Role in Data Management

Question #10 asked in which activities should the CAS take an active role. Only one-third of the respondents thought the CAS should serve as a data repository, while 60% thought the CAS should provide assistance by directing persons to other resources.

Additional Comments

The respondents provided many additional comments. There is clearly a wide range of thoughts regarding how and to what extent actuaries should be involved in data management activities. For the most part however, the majority of respondents agreed that the CAS should not provide data collection services nor should its members become technology experts. In fact, it is often a struggle to convince our non-actuarial colleagues that we have general business expertise and that we are more than just "number crunchers." On the other hand, this committee believes that as generalists having knowledge about many areas of insurance, we should also have some basic understanding of data quality, data reconciliation, and other key data management issues. This is consistent with many respondents thinking that the CAS should direct members to useful sources. We can act as a bridge between business partners and information systems programmers for example. Regarding education, while we agree with the respondents that a textbook would become quickly outdated, we disagree that data management should not be on the syllabus. We strongly believe that some amount of fundamental data management information should be included in the syllabus, especially Actuarial Standard of Practice #23 on Data Quality. We appreciate the time and effort taken by the respondents to this survey and will use this information to improve on the services provided by both this Committee and the CAS in data management efforts.

SURVEY ON

MANAGEMENT DATA AND INFORMATION

REPORT

Compiled by CAS Office
May 31, 2001

EXECUTIVE SUMMARY

The Casualty Actuary Society's Committee on Management Data and Information addresses actuarial issues on property and casualty insurance data and information systems. The CAS Membership Survey on Management Data and Information was intended to provide the Committee with information that will enhance its role in providing resources on management data to members of the Society. The following are the key findings of the survey:

- When asked how much of their time was spent in various data management activities, respondents reported that "extraction and manipulation of data" (40.5% of respondents spent over 10% of their time) and "assisting in the design of new analytical tools" (29.8% of respondents spent over 10% of their time) were the most time-consuming activities.

- Nearly two-thirds of all respondents (64%) reported that they do not spend any time on issues related to privacy/ownership of data About a third (31.5%) spend 1-5% of their time on these issues.

- When asked to rate the importance of data management activities, respondents rated "ease of access to internal data systems" as the most important, with over nine in ten respondents (90.4%) rating the activity "important" or of "great importance." Nearly nine in ten of the respondents (88.7%) rated "having a detailed and accurate data dictionary" as "important" or of "great importance."

- Almost three-fourths of respondents (71.9%) have recently been involved in the "production of ad-hoc reports from the database," while over two-thirds (64%) have recently been involved in the "development of specifications for a report or report system."

- When asked to indicate who performed data management activities for their company, over three-fourths of the respondents reported that actuaries performed ratemaking/pricing and reserve setting (83.1% and 76.4%, respectively).

- Nearly two-thirds of all respondents (61.8%) have attended at least one CAS Ratemaking Seminar in the last five years. Of those that reported attending the CAS Ratemaking Seminar, about a third (32.7%) attended the Management Data and Information Committee's presentation.

- When asked about the importance of future data management education activities, nearly two-thirds of respondents (63.5%) indicated that "publishing papers on data management/data quality" was "important" or of "great importance." About the same percentage (62.9%) indicated that "publishing papers on new applications" was "important" or of "great importance."

- More than eight in ten of the respondents (81.5%) indicated that they are aware that the CAS Web Site provides an Online Catalog that contains a bibliography of reference materials. Of those that are aware of the Catalog, 80% have found it useful.

- When asked about the importance of various activities the Data Management Committee is considering undertaking to provide assistance to actuaries, over three-fourths (76.4%) of the respondents indicated that assistance in "dealing with data that is incomplete or of poor quality" was "important" or of "great importance." Nearly two-thirds (65.7%) indicated that assistance in "developing procedures for insuring data quality" was "important" or of "great importance."

- When asked about the importance of various activities in which the CAS could take an active role, over three-fifths of the respondents (60.5%) indicated that "providing assistance by steering requestors" to other sources was "important" or of "great importance." Just over a quarter of respondents (28.1) felt that "serving as a data repository" was "important" or of "great importance."

SURVEY METHODOLOGY

Designing the Questionnaire
A ten-page, 78-item self-administered questionnaire (see Appendix) was developed by the CAS Committee on Management Data and Information and approved by the CAS Executive Council.

Conducting the Survey
A total of 3,021 questionnaires were sent as an e-mail attachment to Fellows, Associates, and Affiliates of the CAS on February 5, 2001. In addition, the survey could be completed online through the CAS Web Site. Respondents were asked to complete the survey by March 2, 2001.

Data Analysis
A total of 178 (5.9%) completed questionnaires were returned to the CAS Office. A total of 153 surveys (86%) were completed electronically through the Web Site. Responses to survey questions were compiled, coded, and entered into a database. The responses were then analyzed using a statistical analysis software package (SPSS).

For each question, responses are provided for all that completed the survey. In addition, the responses are reported for select groups of respondents: insurance company actuaries, consulting actuaries, organizations serving insurance and other. The groups were divided based on the response to the question "Which of the following best describes your company, organization or department?" The first group includes those that indicated "Property/Liability Insurance Company" or "Reinsurance Company." The second group includes "Consulting Actuary" and "Insurance Broker or Agent." The third group includes "Organization Serving Insurance" and the fourth group includes all others. Thus, there are five tables of responses for each question – one for all respondents followed by four more for the break-out groups.

Responses to Open-ended Questions
The survey contained several open-ended questions that asked respondents to write-in their responses. Where responses to open-ended questions are summarized in the report, a number precedes each response. This identification number represents the specific survey on which the comments were written. This allows those reading the report to track the written comments of a particular respondent, if desired.

RESULTS

Question a):
What is your highest CAS designation?

Response	Frequency	Percent
FCAS	128	71.9
ACAS	46	25.8
Blank	4	2.2
Total	178	100.0

Question b):
How many years of actuarial experience do you have?

Response	Frequency	Percent
0-5	8	4.5
6-10	31	17.4
11-20	84	47.2
21+	55	30.9
Total	178	100.0

Question c):
Which of the following best describes your company, organization or department?

Response	Frequency	Percent
Property/Liability Insurance Company	101	56.7
Reinsurance Company	21	11.8
Life, Accident, and Health Insurance	0	0.0
Insurance Broker or Agent	12	6.7
Banks, Financial Institutions	1	0.6
Organization Serving Insurance	9	5.1
Academic	1	0.6
Government	5	2.8
Consulting Actuary	24	13.5
Retired	0	0.0
Other	2	1.1
Blank	2	1.1
Total	178	100.0

Responses to "Other":
(6) Monopolistic state fund.
(185) Multi-line (L&NL insurer & reinsurer).

Question 1: Data Management Time
What percentage of your time do you spend on the following activities?

a) Assisting in the design of new data collection/processing systems (that support operations, policy issuance, claims, billing, etc.).

All Respondents

Response	Frequency	Percent
0%	48	27.0
1-5%	96	53.9
6-10%	25	14.0
11-24%	7	3.9
25+%	2	1.1
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
0%	25	20.5
1-5%	71	58.2
6-10%	21	17.2
11-24%	4	3.3
25+%	1	.8
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
0%	16	44.4
1-5%	15	41.7
6-10%	3	8.3
11-24%	1	2.8
25+%	1	2.8
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
0%	3	33.3
1-5%	3	33.3
6-10%	1	11.1
11-24%	2	22.2
25+%	0	0.0
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
0%	3	33.3
1-5%	6	66.7
6-10%	0	0.0
11-24%	0	0.0
25+%	0	0.0
Total	9	100.0

b) Assisting in the design of new analytical tools (that support pricing, reserving, underwriting, etc.).

All Respondents

Response	Frequency	Percent
0%	10	5.6
1-5%	58	32.6
6-10%	56	31.5
11-24%	30	16.9
25+%	23	12.9
Blank	1	0.6
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
0%	5	4.1
1-5%	39	32.0
6-10%	41	33.6
11-24%	19	15.6
25+%	18	14.8
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
0%	3	8.3
1-5%	9	25.0
6-10%	10	27.8
11-24%	11	30.6
25+%	3	8.3
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
0%	1	11.1
1-5%	4	44.4
6-10%	2	22.2
11-24%	0	0.0
25+%	1	11.1
Blank	1	11.1
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
0%	0	0.0
1-5%	5	55.6
6-10%	3	33.3
11-24%	0	0.0
25+%	1	11.1
Total	9	100.0

c) Assuring the quality of internal data being compiled by existing systems.

All Respondents

Response	Frequency	Percent
0%	27	15.2
1-5%	82	46.1
6-10%	45	25.3
11-24%	19	10.7
25+%	5	2.8
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
0%	14	11.5
1-5%	51	41.8
6-10%	37	30.3
11-24%	16	13.1
25+%	4	3.3
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
0%	8	22.2
1-5%	18	50.0
6-10%	6	16.7
11-24%	3	8.3
25+%	1	2.8
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
0%	1	11.1
1-5%	7	77.8
6-10%	1	11.1
11-24%	0	0.0
25+%	0	0.0
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
0%	3	33.3
1-5%	5	55.6
6-10%	1	11.1
11-24%	0	0.0
25+%	0	0.0
Total	9	100.0

d) Providing input on collection of data including cost/benefit analysis.

All Respondents

Response	Frequency	Percent
0%	48	27.0
1-5%	101	56.7
6-10%	23	12.9
11-24%	5	2.8
25+%	0	0.0
Blank	1	0.6
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
0%	33	27.0
1-5%	71	58.2
6-10%	13	10.7
11-24%	4	3.3
25+%	1	0.8
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
0%	10	27.8
1-5%	18	50.0
6-10%	8	22.2
11-24%	0	0.0
25+%	0	0.0
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
0%	2	22.2
1-5%	6	66.7
6-10%	1	11.1
11-24%	0	0.0
25+%	0	0.0
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
0%	1	11.1
1-5%	6	66.7
6-10%	1	11.1
11-24%	1	11.1
25+%	0	0.0
Total	9	100.0

e) Extraction and manipulation of data.

All Respondents

Response	Frequency	Percent
0%	20	11.2
1-5%	39	21.9
6-10%	47	26.4
11-24%	43	24.2
25+%	29	16.3
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
0%	12	9.8
1-5%	23	18.9
6-10%	33	27.0
11-24%	30	24.6
25+%	24	19.7
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
0%	5	13.9
1-5%	10	27.8
6-10%	8	22.2
11-24%	11	30.6
25+%	2	5.6
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
0%	1	11.1
1-5%	5	55.6
6-10%	3	33.3
11-24%	0	0.0
25+%	0	0.0
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
0%	2	22.2
1-5%	1	11.1
6-10%	3	33.3
11-24%	1	11.1
25+%	2	22.2
Total	9	100.0

f) Training co-workers how to properly identify data elements to be extracted, as well as data extraction techniques.

All Respondents

Response	Frequency	Percent
0%	55	30.9
1-5%	85	47.8
6-10%	23	12.9
11-24%	13	7.3
25+%	1	0.6
Blank	1	0.6
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
0%	32	26.2
1-5%	59	48.4
6-10%	18	14.8
11-24%	12	9.8
25+%	1	0.8
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
0%	15	41.7
1-5%	17	47.2
6-10%	3	8.3
11-24%	0	0.0
25+%	0	0.0
Blank	1	2.8
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
0%	5	55.6
1-5%	3	33.3
6-10%	1	11.1
11-24%	0	0.0
25+%	0	0.0
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
0%	3	33.3
1-5%	4	44.4
6-10%	1	11.1
11-24%	1	11.1
25+%	0	0.0
Total	9	100.0

g) Mining internal data (e.g., new product development, identifying data correlations, neural networks).

All Respondents

Response	Frequency	Percent
0%	70	39.3
1-5%	73	41.0
6-10%	19	10.7
11-24%	9	5.1
25+%	6	3.4
Blank	1	0.6
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
0%	44	36.1
1-5%	53	43.4
6-10%	13	10.7
11-24%	6	4.9
25+%	5	4.1
Blank	1	0.8
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
0%	17	47.2
1-5%	12	33.3
6-10%	4	11.1
11-24%	2	5.6
25+%	1	2.8
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
0%	3	33.3
1-5%	4	44.4
6-10%	1	11.1
11-24%	1	11.1
25+%	0	0.0
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
0%	5	55.6
1-5%	3	33.3
6-10%	1	11.1
11-24%	0	0.0
25+%	0	0.0
Total	9	100.0

h) Obtaining data from external sources (e.g., industry, government, economic statistics).

All Respondents

Response	Frequency	Percent
0%	34	19.1
1-5%	110	61.8
6-10%	25	14.0
11-24%	8	4.5
25+%	0	0.0
Blank	1	0.6
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
0%	26	21.3
1-5%	72	59.0
6-10%	17	13.9
11-24%	6	4.9
25+%	0	0.0
Blank	1	0.8
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
0%	3	8.3
1-5%	27	75.0
6-10%	4	11.1
11-24%	2	5.6
25+%	0	0.0
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
0%	2	22.2
1-5%	4	44.4
6-10%	3	33.3
11-24%	0	0.0
25+%	0	0.0
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
0%	3	33.3
1-5%	6	66.7
6-10%	0	0.0
11-24%	0	0.0
25+%	0	0.0
Total	9	100.0

i) Responding on data issues (e.g., regulatory, bureaus, AM Best).

All Respondents

Response	Frequency	Percent
0%	71	39.9
1-5%	80	44.9
6-10%	20	11.2
11-24%	6	3.4
25+%	1	0.6
Blank	0	0.0
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
0%	43	35.2
1-5%	59	48.4
6-10%	14	11.5
11-24%	5	4.1
25+%	1	0.8
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
0%	20	55.6
1-5%	13	36.1
6-10%	2	5.6
11-24%	1	2.8
25+%	0	0.0
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
0%	3	33.3
1-5%	2	22.2
6-10%	4	44.4
11-24%	0	0.0
25+%	0	0.0
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
0%	4	44.4
1-5%	5	55.6
6-10%	0	0.0
11-24%	0	0.0
25+%	0	0.0
Total	9	100.0

j) Other management and data activities:

- (3) Project management (25%+).
- (6) Recreating hardcopy reports in S.S. (11-24%).
- (8) Client P&C filings (11-24%).
- (8) Client loss reserve opinions (6-10%).
- (10) Creation and modification of data models (1-5%).
- (11) Rate filing (11-24%).
- (11) Internal reports (11-24%).
- (11) External reports (25%+).
- (13) Management of loss cost reviews and policy form development (25%+).
- (38) Management Information Reports (1-5%).
- (45) Manage staff who verify and report stat data (6-10%).
- (45) Manage staff who extract & summarize information.
- (50) Management-level communications (25%+).
- (64) Analysis (25%+).
- (73) Product management (11-24%).
- (75) Various reporting to keep management informed (1-5%).
- (78) Reports to Boards / Action plans (25%+).
- (78) Accounting / Annual statement (6-10%).
- (78) Pension plans (11-24%).
- (96) Data warehouse (11-24%).
- (109) Assisting in the conversion to the new information system (25%+).
- (129) Developing group data standards (1-5%).
- (131) Reconciling data from one system to another (1-5%).
- (150) Design and modification of data models (6-10%).
- (156) Reinsurer data requests (11-24%).
- (156) Pricing/Reserving reports (11-24%).
- (162) Head technology management committee (1-5%).
- (165) Data stewardship – reaching consensus on definitions (6-10%).
- (185) Risk management (11-24%).
- (185) Manage life & health underwriting unit (25%+).
- (185) General executive/management (11-24%).
- (185) Management of actuarial department (6-10%).
- (185) Underwriting (25%+).
- (186) Creating and overseeing the population of planning databases (6-10%).
- (186) Overseeing the creation of data warehousing capabilities (6-10%).
- (188) Working with vendors to supply good data (1-5%).

Question 2: Privacy/Ownership of Data

a) What percentage of your time do you spend on issues related to privacy/ownership of data?

All Respondents

Response	Frequency	Percent
0%	114	64.0
1-5%	56	31.5
6-10%	5	2.8
11-24%	0	0.0
25+%	1	0.6
Blank	2	1.1
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
0%	83	68.0
1-5%	33	27.0
6-10%	3	2.5
11-24%	0	0.0
25+%	1	0.8
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
0%	21	58.3
1-5%	15	41.7
6-10%	0	0.0
11-24%	0	0.0
25+%	0	0.0
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
0%	4	44.4
1-5%	4	44.4
6-10%	1	11.1
11-24%	0	0.0
25+%	0	0.0
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	**Frequency**	**Percent**
0%	4	44.4
1-5%	4	44.4
6-10%	1	11.1
11-24%	0	0.0
25+%	0	0.0
Total	9	100.0

b) Please identify the privacy issues you have recently dealt with and the percentage of time spent on each issue:

- (3) Release of employer data (11-24%).
- (8) Expert Witness on Loss Reserving Techniques (1-5%).
- (10) Should individual applications "own" (have exclusive write access to) specific corporate data elements? (1-5%).
- (29) Access to individual claims data by journalists (1-5%).
- (31) Regulatory requests for individual company data (1-5%).
- (43) Fair credit reporting act (1-5%).
- (43) Protecting insurer data from regulatory release (1-5%).
- (50) Claimant confidentiality issues (1-5%).
- (51) Confidentiality issues (1-5%).
- (51) Proprietary issues (1-5%).
- (53) GLB act (1-5%).
- (58) Internal firewalls (1-5%).
- (59) Ownership of aggregated data from several clients (1-5%).
- (66) Client listings (1-5%).
- (67) Reporting private data to reinsurers (1-5%).
- (69) Cat model vendors (1-5%).
- (69) Client exposure data and operating plans (1-5%).
- (85) Banking and insurance secrecy – no sharing of data (1-5%).
- (91) Limiting output to keep account lists private (1-5%).
- (101) Actuarial reports on other companies (1-5%).
- (106) Ownership of and access to prospecting info (1-5%).
- (106) Extranet access to customer loss data (1-5%).
- (106) Ownership of and access to customer profitability (1-5%).
- (116) Confidentiality of examination information (6-10%).
- (126) Ownership of credit score data (1-5%).
- (132) Working with product manager on implications of GLB (1-5%).
- (140) Discussion over ownership of policy data with company (1-5%).
- (150) Should applications "own" specific data elements? (1-5%).
- (152) Non-sharing of client specific information (1-5%).
- (159) Use of client loss experience for another client (1-5%).
- (161) Contractual issues with client (1-5%).
- (162) Following new privacy regulations (1-5%).
- (165) Graham-Leach Bliley (1-5%).
- (170) Intellectual property issues (1-5%).

- (170) Password and security issues (1-5%).
- (171) Business unit data accessibility by other units (6-10%).
- (173) Credit scoring (1-5%).
- (174) Input necessary for modeling cat data (1-5%).
- (176) GLB issues – sending third-party aggregate data (1-5%).

Question 3: Data Management Priorities
Please indicate the importance of the following data management issues.

a) Actuarial input in system design.

All Respondents

Response	Frequency	Percent
Little Importance	2	1.1
Somewhat Important	19	10.7
Neutral	7	3.9
Important	87	48.9
Great Importance	63	35.4
Blank	0	0.0
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Little Importance	1	0.8
Somewhat Important	15	12.3
Neutral	5	4.1
Important	60	49.2
Great Importance	41	33.6
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	3	8.3
Neutral	1	2.8
Important	17	47.2
Great Importance	15	41.7
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	0	0.0
Neutral	0	0.0
Important	5	55.6
Great Importance	4	44.4
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Little Importance	1	11.1
Somewhat Important	0	0.0
Neutral	1	11.1
Important	4	44.4
Great Importance	3	33.3
Total	9	100.0

b) Ease of access to your internal data systems.

All Respondents

Response	Frequency	Percent
Little Importance	1	0.6
Somewhat Important	7	3.9
Neutral	8	4.5
Important	80	44.9
Great Importance	81	45.5
Blank	1	0.6
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	4	3.3
Neutral	4	3.3
Important	59	48.4
Great Importance	55	45.1
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
Little Importance	1	2.8
Somewhat Important	2	5.6
Neutral	2	5.6
Important	15	41.7
Great Importance	15	41.7
Blank	1	2.8
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	0	0.0
Neutral	0	0.0
Important	1	11.1
Great Importance	8	88.9
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	0	0.0
Neutral	2	22.2
Important	5	55.6
Great Importance	2	22.2
Total	9	100.0

c) Having a detailed and accurate data dictionary including fields, definitions, etc.

All Respondents

Response	Frequency	Percent
Little Importance	1	0.6
Somewhat Important	9	5.1
Neutral	9	5.1
Important	93	52.2
Great Importance	65	36.5
Blank	1	0.6
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	2	1.6
Neutral	5	4.1
Important	70	57.4
Great Importance	45	36.9
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
Little Importance	1	2.8
Somewhat Important	5	13.9
Neutral	3	8.3
Important	11	30.6
Great Importance	15	41.7
Blank	1	2.8
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	1	11.1
Neutral	0	0.0
Important	7	77.8
Great Importance	1	11.1
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	0	0.0
Neutral	1	11.1
Important	4	44.4
Great Importance	4	44.4
Total	9	100.0

d) Having knowledge of traditional external data sources (publications, historical data vendors).

All Respondents

Response	Frequency	Percent
Little Importance	4	2.2
Somewhat Important	20	11.2
Neutral	33	18.5
Important	97	54.5
Great Importance	24	13.5
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Little Importance	3	2.5
Somewhat Important	18	14.8
Neutral	27	22.1
Important	63	51.6
Great Importance	11	9.0
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
Little Importance	1	2.8
Somewhat Important	0	0.0
Neutral	3	8.3
Important	21	58.3
Great Importance	11	30.3
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	1	11.1
Neutral	1	11.1
Important	6	66.7
Great Importance	1	11.1
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	1	11.1
Neutral	2	22.2
Important	5	55.6
Great Importance	1	11.1
Total	9	100.0

e) Having knowledge of non-traditional external data sources (e.g., Internet, Web Sites).

All Respondents

Response	Frequency	Percent
Little Importance	7	3.9
Somewhat Important	19	10.7
Neutral	41	23.0
Important	91	51.1
Great Importance	19	10.7
Blank	1	0.6
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Little Importance	7	5.7
Somewhat Important	17	13.9
Neutral	29	23.8
Important	58	47.5
Great Importance	10	8.2
Blank	1	0.8
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	1	2.8
Neutral	7	19.4
Important	20	55.6
Great Importance	8	22.2
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	1	11.1
Neutral	1	11.1
Important	6	66.7
Great Importance	1	11.1
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	0	0.0
Neutral	3	33.3
Important	6	66.7
Great Importance	0	0.0
Total	9	100.0

f) Ease of integration of external and internal data sources.

All Respondents

Response	Frequency	Percent
Little Importance	13	7.3
Somewhat Important	32	18.0
Neutral	63	35.4
Important	61	34.3
Great Importance	9	5.1
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Little Importance	11	9.0
Somewhat Important	25	20.5
Neutral	44	36.1
Important	39	32.0
Great Importance	3	2.5
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
Little Importance	2	5.6
Somewhat Important	4	11.1
Neutral	17	47.2
Important	9	25.0
Great Importance	4	11.1
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	0	0.0
Neutral	1	11.1
Important	8	88.9
Great Importance	0	0.0
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	2	22.2
Neutral	1	11.1
Important	4	44.4
Great Importance	2	22.2
Total	9	100.0

g) Issues of intellectual property and ownership.

All Respondents

Response	Frequency	Percent
Little Importance	24	13.5
Somewhat Important	36	20.2
Neutral	76	42.7
Important	29	16.3
Great Importance	12	6.7
Blank	1	0.6
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Little Importance	18	14.8
Somewhat Important	21	17.2
Neutral	60	49.2
Important	18	14.8
Great Importance	4	3.3
Blank	1	0.8
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
Little Importance	4	11.1
Somewhat Important	12	33.3
Neutral	10	27.8
Important	6	16.7
Great Importance	4	11.1
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	2	22.2
Neutral	3	33.3
Important	2	22.2
Great Importance	2	22.2
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Little Importance	1	11.1
Somewhat Important	1	11.1
Neutral	3	33.3
Important	2	22.2
Great Importance	2	22.2
Total	9	100.0

h) Developing the skills necessary to identify, extract and manipulate data.

All Respondents

Response	Frequency	Percent
Little Importance	3	1.7
Somewhat Important	9	5.1
Neutral	12	6.7
Important	102	57.3
Great Importance	52	29.2
Blank	0	0.0
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Little Importance	1	0.8
Somewhat Important	8	6.6
Neutral	6	4.9
Important	69	56.6
Great Importance	38	31.1
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
Little Importance	2	5.6
Somewhat Important	1	2.8
Neutral	4	11.1
Important	21	58.3
Great Importance	8	22.2
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	0	0.0
Neutral	1	11.1
Important	6	66.7
Great Importance	2	22.2
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	0	0.0
Neutral	1	11.1
Important	4	44.4
Great Importance	4	44.4
Total	9	100.0

i) Keeping up-to-date on reporting rule changes that change the substance of data collection (e.g., recent loss expense definition change).

All Respondents

Response	Frequency	Percent
Little Importance	9	5.1
Somewhat Important	20	11.2
Neutral	41	23.0
Important	77	43.3
Great Importance	31	17.4
Blank	0	0.0
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Little Importance	5	4.1
Somewhat Important	13	10.7
Neutral	35	28.7
Important	51	41.8
Great Importance	18	14.8
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
Little Importance	3	8.3
Somewhat Important	6	16.7
Neutral	4	11.1
Important	13	36.1
Great Importance	10	27.8
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	0	0.0
Neutral	0	0.0
Important	7	77.8
Great Importance	2	22.2
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Little Importance	1	11.1
Somewhat Important	1	11.1
Neutral	2	22.2
Important	4	44.4
Great Importance	1	11.1
Total	9	100.0

j) Having the ability to perform data mining on internal data.

All Respondents

Response	Frequency	Percent
Little Importance	6	3.4
Somewhat Important	18	10.1
Neutral	40	22.5
Important	68	38.2
Great Importance	45	25.3
Blank	1	0.6
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Little Importance	4	3.3
Somewhat Important	8	6.6
Neutral	26	21.3
Important	51	41.8
Great Importance	33	27.0
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
Little Importance	1	2.8
Somewhat Important	7	19.4
Neutral	11	30.6
Important	8	22.2
Great Importance	8	22.2
Blank	1	2.8
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	1	11.1
Neutral	2	22.2
Important	4	44.4
Great Importance	2	22.2
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Little Importance	1	11.1
Somewhat Important	2	22.2
Neutral	1	11.1
Important	4	44.4
Great Importance	1	11.1
Total	9	100.0

k) Understanding emerging data technologies.

All Respondents

Response	Frequency	Percent
Little Importance	7	3.9
Somewhat Important	22	12.4
Neutral	63	35.4
Important	66	37.1
Great Importance	20	11.2
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Little Importance	4	3.3
Somewhat Important	13	10.7
Neutral	46	37.7
Important	46	37.7
Great Importance	13	10.7
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
Little Importance	2	5.6
Somewhat Important	7	19.4
Neutral	9	25.0
Important	15	41.7
Great Importance	3	8.3
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Little Importance	1	11.1
Somewhat Important	0	0.0
Neutral	3	33.3
Important	2	22.2
Great Importance	3	33.3
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	2	22.2
Neutral	3	33.3
Important	3	33.3
Great Importance	1	11.1
Total	9	100.0

l) Cost considerations associated with data collection and extraction.

All Respondents

Response	Frequency	Percent
Little Importance	7	3.9
Somewhat Important	29	16.3
Neutral	45	25.8
Important	73	41.0
Great Importance	21	11.8
Blank	2	1.1
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Little Importance	7	5.7
Somewhat Important	15	12.3
Neutral	32	26.2
Important	53	43.4
Great Importance	14	11.5
Blank	1	0.8
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	10	27.8
Neutral	8	22.2
Important	13	36.1
Great Importance	4	11.1
Blank	1	2.8
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	2	22.2
Neutral	1	11.1
Important	4	44.4
Great Importance	2	22.2
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	2	22.2
Neutral	4	44.4
Important	2	22.2
Great Importance	1	11.1
Total	9	100.0

m) Other:

- (47) Quality of data not reported outside the company – Important.
- (111) Availability of software (e.g., Excel, Access) – Great importance
- (111) Ability of systems to handle large amounts of data – Great importance

Question 4: Data Management Activities
Which of following data management activities have you been involved in?

All Respondents

	Recently	Earlier In Your Career	Never
a) Development of specifications for design of a database.	47.2	50.6	9.0
b) Selection of data elements to be captured in a database.	59.0	44.4	5.1
c) Development of specifications for a report or report system.	64.0	39.9	6.2
d) Production of ad-hoc reports from the database.	71.9	35.4	3.9
e) Development of test data.	34.3	43.8	25.8
f) Definition of data edits.	29.8	44.4	28.7
g) Development of procedures to insure data quality.	45.5	40.4	19.7

Insurance Company Actuaries

	Recently	Earlier In Your Career	Never
a) Development of specifications for design of a database.	53.3	45.9	9.0
b) Selection of data elements to be captured in a database.	63.1	40.2	4.9
c) Development of specifications for a report or report system.	67.2	40.2	4.9
d) Production of ad-hoc reports from the database.	74.6	34.4	4.1
e) Development of test data.	36.9	41.8	26.2
f) Definition of data edits.	32.8	41.8	28.7
g) Development of procedures to insure data quality.	49.2	38.5	18.0

Consulting Actuaries

	Recently	Earlier In Your Career	Never
a) Development of specifications for design of a database.	30.6	63.9	8.3
b) Selection of data elements to be captured in a database.	50.0	55.6	2.8
c) Development of specifications for a report or report system.	58.3	38.9	8.3
d) Production of ad-hoc reports from the database.	66.7	41.7	2.8
e) Development of test data.	27.8	52.8	22.2
f) Definition of data edits.	22.2	50.0	30.6
g) Development of procedures to insure data quality.	41.7	33.3	30.6

Organizations Serving Insurance

	Recently	Earlier In Your Career	Never
a) Development of specifications for design of a database.	44.4	55.6	11.1
b) Selection of data elements to be captured in a database.	66.7	33.3	11.1
c) Development of specifications for a report or report system.	55.6	44.4	11.1
d) Production of ad-hoc reports from the database.	44.4	44.4	11.1
e) Development of test data.	33.3	44.4	22.2
f) Definition of data edits.	22.2	55.6	22.2
g) Development of procedures to insure data quality.	33.3	55.6	11.1

Other (Banks, Financial Institutions, Academic, Government, Other)

	Recently	Earlier In Your Career	Never
a) Development of specifications for design of a database.	44.4	55.6	0.0
b) Selection of data elements to be captured in a database.	33.3	66.7	0.0
c) Development of specifications for a report or report system.	66.7	33.3	0.0
d) Production of ad-hoc reports from the database.	88.9	11.1	0.0
e) Development of test data.	33.3	33.3	33.3
f) Definition of data edits.	33.3	33.3	33.3
g) Development of procedures to insure data quality.	33.3	66.7	11.1

h) Other:

- (7) Decision if/how to use a collected field – Earlier in your career.
- (10) Design and development of logical/physical data models – Recently.
- (54) Investigation of data quality problems – Recently.
- (74) Testing/Error checking a new report system – Recently.
- (78) Develop OLAP tools – Earlier in your career.
- (111) Testing data systems – Earlier in your career.
- (125) Specifications for new collection procedures – Recently

- (132) Helping to move from one data system to another – Recently.
- (140) Selection of vendors for data management – Recently, Earlier in your career.
- (140) Oversight of vendors – Earlier in your career.
- (150) Designing of logical/physical data models – Recently.
- (165) Data stewardship – Recently.
- (165) DMV reporting process development – Recently.
- (170) Design of business intelligence – Recently.

Question 5: Performance of Data Management
Please indicate who performs the following data management activities for your company.

Note: The percentages in the table represent the percentage of respondents that reported that the particular group performed the activity (e.g., 83.1% of the respondents indicated that actuaries perform data management activity for ratemaking/pricing).

All Respondents

Data Management Activity	Actuaries	Actuarial Students	Non-Actuaries	External Consultants
a) Ratemaking/pricing	83.1	58.4	38.2	5.6
b) Reserve setting	76.4	45.5	20.8	9.6
c) Reserve opinions	70.8	11.8	2.8	17.4
d) Underwriting reports	37.1	26.4	65.2	1.1
e) Marketing reports	13.5	10.1	72.5	1.7
f) Claims management	16.3	6.7	71.3	1.7
g) Financial analysis and investments	40.4	11.2	76.4	6.2
h) Financial reporting	41.0	19.7	75.3	3.4
i) Rate regulation	46.6	32.6	47.8	4.5
j) Statistical agency reporting	20.8	15.2	64.6	3.4

Insurance Company Actuaries

Data Management Activity	Actuaries	Actuarial Students	Non-Actuaries	External Consultants
a) Ratemaking/pricing	91.0	64.8	44.3	5.7
b) Reserve setting	88.5	52.5	25.4	9.8
c) Reserve opinions	80.3	10.7	2.5	20.5
d) Underwriting reports	45.9	35.2	84.4	0.8
e) Marketing reports	13.1	12.3	90.2	0.8
f) Claims management	19.7	9.0	88.5	0.8
g) Financial analysis and investments	45.1	12.3	88.5	4.1
h) Financial reporting	49.2	23.8	91.0	0.8
i) Rate regulation	53.3	40.2	61.5	2.5
j) Statistical agency reporting	21.3	16.4	81.1	2.5

Consulting Actuaries

Data Management Activity	Actuaries	Actuarial Students	Non-Actuaries	External Consultants
a) Ratemaking/pricing	63.9	41.7	19.4	2.8
b) Reserve setting	50.0	33.3	11.1	5.6
c) Reserve opinions	55.6	16.7	5.6	8.3
d) Underwriting reports	19.4	8.3	19.4	2.8
e) Marketing reports	16.7	8.3	36.1	2.8
f) Claims management	11.1	2.8	36.1	2.8
g) Financial analysis and investments	27.8	11.1	44.4	5.6
h) Financial reporting	19.4	8.3	41.7	5.6
i) Rate regulation	27.8	8.3	16.7	8.3
j) Statistical agency reporting	13.9	2.8	27.8	5.6

Organizations Serving Insurance

Data Management Activity	Actuaries	Actuarial Students	Non-Actuaries	External Consultants
a) Ratemaking/pricing	88.9	77.8	33.3	0.0
b) Reserve setting	44.4	22.2	0.0	0.0
c) Reserve opinions	33.3	11.1	0.0	0.0
d) Underwriting reports	11.1	11.1	22.2	0.0
e) Marketing reports	11.1	0.0	33.3	0.0
f) Claims management	0.0	0.0	11.1	0.0
g) Financial analysis and investments	22.2	11.1	66.7	0.0
h) Financial reporting	44.4	33.3	33.3	0.0
i) Rate regulation	66.7	55.6	33.3	11.1
j) Statistical agency reporting	44.4	55.6	44.4	0.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Data Management Activity	Actuaries	Actuarial Students	Non-Actuaries	External Consultants
a) Ratemaking/pricing	44.4	22.2	44.4	22.2
b) Reserve setting	55.6	22.2	22.2	22.2
c) Reserve opinions	44.4	11.1	0.0	22.2
d) Underwriting reports	22.2	0.0	22.2	0.0
e) Marketing reports	11.1	0.0	11.1	0.0
f) Claims management	11.1	0.0	33.3	11.1
g) Financial analysis and investments	55.6	0.0	44.4	33.3
h) Financial reporting	22.2	0.0	33.3	22.2
i) Rate regulation	11.1	11.1	11.1	11.1
j) Statistical agency reporting	11.1	11.1	22.2	11.1

k) Other:

- (45) Product/Loss cost custom work; performed by actuaries, actuarial students, and non-actuaries.
- (45) Development of underwriting tools; performed by actuaries, actuarial students, and external consultants.
- (45) New coverages and forms; performed by performed by actuaries, actuarial students, and external consultants.
- (116) Financial examinations; performed by actuaries, actuarial students, and external consultants.

Question 6: Data Management Education

a) How many times have you attended the CAS Ratemaking Seminar in the last 5 years?

All Respondents

Response	**Frequency**	**Percent**
5	7	3.9
4	3	1.7
3	9	5.1
2	33	18.5
1	58	32.6
0	68	38.2
Total	178	100.0

Insurance Company Actuaries

Response	**Frequency**	**Percent**
5	4	3.3
4	3	2.5
3	4	3.3
2	22	18.0
1	42	34.4
0	47	38.5
Total	122	100.0

Consulting Actuaries

Response	**Frequency**	**Percent**
5	1	2.8
4	0	0.0
3	5	13.9
2	7	19.4
1	10	27.8
0	13	36.1
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
5	2	22.2
4	0	0.0
3	0	0.0
2	3	33.3
1	3	33.3
0	1	11.1
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
5	0	0.0
4	0	0.0
3	0	0.0
2	1	1.1
1	1	1.1
0	7	77.8
Total	9	100.0

b) How many times have you attended the Management Data and Information Committee's presentation at the CAS Ratemaking Seminar in the last 5 years?

All Respondents

Response	Frequency	Percent
5	1	0.6
4	1	0.6
3	1	0.6
2	5	2.8
1	28	15.7
0	142	79.8
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
5	0	0.0
4	1	0.8
3	1	0.8
2	3	2.5
1	21	17.2
0	96	78.7
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
5	0	0.0
4	0	0.0
3	0	0.0
2	1	2.8
1	4	11.1
0	31	86.1
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
5	1	11.1
4	0	0.0
3	0	0.0
2	0	0.0
1	2	22.2
0	6	66.7
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
5	0	0.0
4	0	0.0
3	0	0.0
2	1	11.1
1	1	11.1
0	7	77.8
Total	9	100.0

c) How many times have you attended the Emerging Technology Seminar?

All Respondents

Response	Frequency	Percent
2	2	1.1
1	29	16.3
0	147	82.6
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
2	1	0.8
1	21	15.6
0	100	82.0
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
2	0	0.0
1	5	13.9
0	31	86.1
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
2	1	11.1
1	2	22.2
0	6	66.7
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
2	0	0.0
1	1	11.1
0	8	88.9
Total	9	100.0

Question 7: Future Education Activities
Please indicate the importance of each of the following ways in which the CAS could become involved in data management education.

a) Developing a textbook or other reference resource.

Response	Frequency	Percent
Little Importance	42	23.6
Somewhat Important	38	21.3
Neutral	41	23.0
Important	45	25.3
Great Importance	7	3.9
Blank	5	2.8
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Little Importance	29	23.8
Somewhat Important	28	23.0
Neutral	28	23.0
Important	28	23.0
Great Importance	6	4.9
Blank	3	2.5
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
Little Importance	9	25.0
Somewhat Important	5	13.9
Neutral	10	27.8
Important	11	30.6
Great Importance	0	0.0
Blank	1	2.8
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Little Importance	2	22.2
Somewhat Important	1	11.1
Neutral	1	11.1
Important	3	33.3
Great Importance	1	11.1
Blank	1	11.1
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Little Importance	1	11.1
Somewhat Important	3	33.3
Neutral	2	22.2
Important	3	33.3
Great Importance	0	0.0
Total	9	100.0

b) Sponsoring seminars or workshops.

Response	Frequency	Percent
Little Importance	11	6.2
Somewhat Important	34	19.1
Neutral	33	18.5
Important	86	48.3
Great Importance	11	6.2
Blank	3	1.7
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Little Importance	10	8.2
Somewhat Important	20	16.4
Neutral	23	18.9
Important	58	47.5
Great Importance	9	7.4
Blank	2	1.6
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
Little Importance	1	2.8
Somewhat Important	11	30.6
Neutral	5	13.9
Important	19	52.8
Great Importance	0	0.0
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	2	22.2
Neutral	1	11.1
Important	3	33.3
Great Importance	2	22.2
Blank	1	11.1
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	0	0.0
Neutral	3	33.3
Important	6	66.7
Great Importance	0	0.0
Total	9	100.0

c) Including this subject in the CAS Syllabus.

Response	Frequency	Percent
Little Importance	46	25.8
Somewhat Important	27	15.2
Neutral	57	32.0
Important	32	18.0
Great Importance	13	7.3
Blank	3	1.7
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Little Importance	31	25.4
Somewhat Important	18	14.8
Neutral	40	32.8
Important	21	17.2
Great Importance	10	8.2
Blank	2	1.6
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
Little Importance	13	36.1
Somewhat Important	4	11.1
Neutral	10	27.8
Important	8	22.2
Great Importance	1	2.8
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Little Importance	2	22.2
Somewhat Important	1	11.1
Neutral	3	33.3
Important	1	11.1
Great Importance	1	11.1
Blank	1	11.1
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	3	33.3
Neutral	3	33.3
Important	2	22.2
Great Importance	1	11.1
Total	9	100.0

d) Including sessions at CAS Meetings.

Response	Frequency	Percent
Little Importance	4	2.2
Somewhat Important	31	17.4
Neutral	32	18.0
Important	90	50.6
Great Importance	18	10.1
Blank	3	1.7
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Little Importance	4	3.3
Somewhat Important	18	14.8
Neutral	19	15.6
Important	65	53.3
Great Importance	14	11.5
Blank	2	1.6
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	11	30.6
Neutral	9	25.0
Important	15	41.7
Great Importance	1	2.8
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	2	22.2
Neutral	0	0.0
Important	4	44.4
Great Importance	2	22.2
Blank	1	11.1
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	0	0.0
Neutral	2	22.2
Important	6	66.7
Great Importance	1	11.1
Total	9	100.0

e) Publishing papers on data management/data quality.

Response	Frequency	Percent
Little Importance	8	4.5
Somewhat Important	23	12.9
Neutral	30	16.9
Important	98	55.1
Great Importance	15	8.4
Blank	4	2.2
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Little Importance	6	4.9
Somewhat Important	14	11.5
Neutral	20	16.4
Important	69	56.6
Great Importance	11	9.0
Blank	2	1.6
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
Little Importance	2	5.6
Somewhat Important	7	19.4
Neutral	4	11.1
Important	20	55.6
Great Importance	2	5.6
Blank	1	2.8
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	2	22.2
Neutral	2	22.2
Important	3	33.3
Great Importance	1	11.1
Blank	1	11.1
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	0	0.0
Neutral	3	33.3
Important	5	55.6
Great Importance	1	11.1
Total	9	100.0

f) Publishing papers on new applications, incorporating new data sources.

Response	Frequency	Percent
Little Importance	9	5.1
Somewhat Important	19	10.7
Neutral	34	19.1
Important	92	51.7
Great Importance	20	11.2
Blank	4	2.2
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Little Importance	7	5.7
Somewhat Important	12	9.8
Neutral	26	21.3
Important	64	52.5
Great Importance	11	9.0
Blank	2	1.6
Total	36	100.0

Consulting Actuaries

Response	Frequency	Percent
Little Importance	2	5.6
Somewhat Important	7	19.4
Neutral	3	8.3
Important	17	47.2
Great Importance	6	16.7
Blank	1	2.8
Total	122	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	0	0.0
Neutral	0	0.0
Important	6	66.7
Great Importance	2	22.2
Blank	1	11.1
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	0	0.0
Neutral	4	44.4
Important	4	44.4
Great Importance	1	11.1
Total	9	100.0

g) Other:

- (43) What's happening in data standards arena; important.
- (110) Paper call; important.
- (110) Have a survey; important.
- (110) Look at data quality for cat models; important.
- (146) Define scope of FCAS to include insurance data; great importance.
- (148) Feedback/review of existing software/sources; important.
- (148) Review or referral to non-CAS published papers; important.
- (175) Presentations at IASA; important.

Question 8:
Use of Online Catalog/Bibliography

a) Are you aware that the CAS Web Site provides you with an Online Catalog that contains a bibliography of reference materials?

All Respondents

Response	Frequency	Percent
Yes	145	81.5
No	30	16.9
Blank	3	1.7
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Yes	94	77.0
No	25	20.5
Blank	3	2.5
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
Yes	32	88.9
No	4	11.1
Total	122	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Yes	8	88.9
No	1	11.1
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Yes	9	100.0
No	0	0.0
Total	9	100.0

b) If yes, is the Online Catalog of any use to you?

Response	Frequency	Percent
Yes	116	80.0
No	29	20.0
Total	145	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Yes	69	56.6
No	25	20.5
Blank	28	23.0
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
Yes	30	83.3
No	3	8.3
Blank	3	8.3
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Yes	7	77.8
No	0	0.0
Blank	2	22.2
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Yes	8	88.9
No	1	11.1
Total	9	100.0

c) What can be added to make the data management section of the Online Catalog more useful?

- (9) When I search the CAS Web Site I often get many references leading nowhere. Maybe include a synopsis on each found search item.
- (10) Subsection for improving information quality through proper coding/interface standards: "Code Complete", Steve McConnell, Microsoft Press, 1993; "Microsoft Windows User Experience", Microsoft Press, 1999.
- (34) More user-friendly.
- (43) References to IDMA and ACORD, if not already there.
- (44) I LOVE it. However, some links between the download library and the online catalog are not provided, although they should be. (I am a member of COOS).
- (48) More complete.
- (56) Better topical searches.
- (58) Provide access to 1) All CAS papers, 2) Important Papers (published by actuaries or others). Have a much better index. It's rarely worth my time to try and find something online. Provide access to useful information like ISO/NCCI filings, trend reports, etc. Also AM Bests reports. I realize this would be expensive, but I think if you included it as an option and charged members extra for it, over-time you could collect the costs. And actuaries would be even more likely to pull a decent rabbit out of the hat, thus enhancing the profession long-term.
- (60) I know it's out there, but except for one instance, have not used it too much. I think we have perused it looking for specific items such as Use of Credit or Vehicle Symboling papers. So maybe grouping or cross referencing available papers by topic for easier use. Also some of the papers are mere power-point presentations of the slides. These are just highlight or bullet point information, and a broader summary of the presentation would generally be more helpful if the topic was to the point I was looking for.
- (99) Better indexing / search features. I often can't find what I'm looking for, even if I know it's there.
- (134) I have had trouble obtaining the right references when looking up a specific topic. I ended up using the index to the Proceedings at the library.
- (140) Outlines of data elements to be considered in developing a new data management system. Discussion of the paths and uses of data within an insurance operation, highlighting the importance of accuracy in various elements depending on the ultimate user.
- (165) Basic texts on data management.

Question 9: Use of Data Management Committee
The Committee is considering undertaking several activities in order to help actuaries deal with data management issues. Please indicate the importance of the following areas that the Committee may be able to provide assistance to actuaries.

a) Keeping up with the state-of-the art in data management.

All Respondents

Response	Frequency	Percent
Little Importance	9	5.1
Somewhat Important	28	15.7
Neutral	35	19.7
Important	80	44.9
Great Importance	21	11.8
Blank	5	2.8
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Little Importance	8	6.6
Somewhat Important	19	15.6
Neutral	18	14.8
Important	59	48.4
Great Importance	15	12.3
Blank	3	2.5
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
Little Importance	1	2.8
Somewhat Important	7	19.4
Neutral	9	25.0
Important	15	41.7
Great Importance	3	8.3
Blank	1	2.8
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	0	0.0
Neutral	3	33.3
Important	4	44.4
Great Importance	2	22.2
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	1	11.1
Neutral	4	44.4
Important	2	22.2
Great Importance	1	11.1
Blank	1	11.1
Total	9	100.0

b) *Demonstrating how to get users involved in developing new systems.*

All Respondents

Response	Frequency	Percent
Little Importance	15	8.4
Somewhat Important	29	16.3
Neutral	48	27.0
Important	65	36.5
Great Importance	13	7.3
Blank	8	4.5
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Little Importance	9	7.4
Somewhat Important	21	17.2
Neutral	34	27.9
Important	46	37.7
Great Importance	7	5.7
Blank	5	4.1
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
Little Importance	6	16.7
Somewhat Important	5	13.9
Neutral	7	19.4
Important	14	38.9
Great Importance	3	8.3
Blank	1	2.8
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	1	11.1
Neutral	2	22.2
Important	4	44.4
Great Importance	1	11.1
Blank	1	11.1
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	1	11.1
Neutral	4	44.4
Important	1	11.1
Great Importance	2	22.2
Blank	1	11.1
Total	9	100.0

c) Developing procedures for insuring data quality.

All Respondents

Response	Frequency	Percent
Little Importance	6	3.4
Somewhat Important	21	11.8
Neutral	30	16.9
Important	93	52.2
Great Importance	24	13.5
Blank	4	2.2
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Little Importance	5	4.1
Somewhat Important	14	11.5
Neutral	22	18.0
Important	61	50.0
Great Importance	17	13.9
Blank	3	2.2
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
Little Importance	1	2.8
Somewhat Important	3	8.3
Neutral	8	22.2
Important	21	58.3
Great Importance	3	8.3
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	0	0.0
Neutral	0	0.0
Important	7	77.8
Great Importance	2	22.2
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	3	33.3
Neutral	0	0.0
Important	3	33.3
Great Importance	2	22.2
Blank	1	11.1
Total	9	100.0

d) Dealing with data that is incomplete or of poor quality.

All Respondents

Response	Frequency	Percent
Little Importance	2	1.1
Somewhat Important	14	7.9
Neutral	21	11.8
Important	91	51.1
Great Importance	45	25.3
Blank	5	2.8
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Little Importance	2	1.6
Somewhat Important	11	9.0
Neutral	17	13.9
Important	57	46.7
Great Importance	32	26.2
Blank	3	2.5
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	2	5.6
Neutral	4	11.1
Important	23	63.9
Great Importance	7	19.4
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	0	0.0
Neutral	0	0.0
Important	6	66.7
Great Importance	3	33.3
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	1	11.1
Neutral	0	0.0
Important	3	33.3
Great Importance	3	33.3
Blank	2	22.2
Total	9	100.0

e) Demonstrating techniques for reconciliation of financial and statistical data.

All Respondents

Response	Frequency	Percent
Little Importance	7	3.9
Somewhat Important	18	10.1
Neutral	46	25.8
Important	81	45.5
Great Importance	22	12.4
Blank	4	2.2
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Little Importance	6	4.9
Somewhat Important	14	11.5
Neutral	29	23.8
Important	55	45.1
Great Importance	15	12.3
Blank	3	2.5
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
Little Importance	1	2.8
Somewhat Important	4	11.1
Neutral	13	36.1
Important	14	38.9
Great Importance	4	11.1
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	0	0.0
Neutral	2	22.2
Important	6	66.7
Great Importance	1	11.1
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	0	0.0
Neutral	1	11.1
Important	5	55.6
Great Importance	2	22.2
Blank	1	11.1
Total	9	100.0

f) Demonstrating techniques for testing reports, report systems, etc.

All Respondents

Response	Frequency	Percent
Little Importance	8	4.5
Somewhat Important	24	13.5
Neutral	58	32.6
Important	71	39.9
Great Importance	13	7.3
Blank	4	2.2
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Little Importance	7	5.7
Somewhat Important	17	13.9
Neutral	39	32.0
Important	46	37.7
Great Importance	10	8.2
Blank	3	2.5
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
Little Importance	1	2.8
Somewhat Important	6	16.7
Neutral	13	36.1
Important	13	36.1
Great Importance	3	8.3
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	0	0.0
Neutral	4	44.4
Important	5	55.6
Great Importance	0	0.0
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	1	11.1
Neutral	2	22.2
Important	5	55.6
Great Importance	0	0.0
Blank	1	11.1
Total	9	100.0

g) Providing information on the latest software.

All Respondents

Response	Frequency	Percent
Little Importance	17	9.6
Somewhat Important	29	16.3
Neutral	62	34.8
Important	59	33.1
Great Importance	7	3.9
Blank	4	2.2
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Little Importance	10	8.2
Somewhat Important	20	16.4
Neutral	41	33.6
Important	43	35.2
Great Importance	5	4.1
Blank	3	2.5
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
Little Importance	7	19.4
Somewhat Important	6	16.7
Neutral	13	36.1
Important	9	25.0
Great Importance	1	2.8
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	1	11.1
Neutral	4	44.4
Important	3	33.3
Great Importance	1	11.1
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	2	22.2
Neutral	3	33.3
Important	3	33.3
Great Importance	0	00.0
Blank	1	11.1
Total	9	100.0

h) Demonstrating techniques for implementation and use of Web enabled applications.

All Respondents

Response	Frequency	Percent
Little Importance	17	9.6
Somewhat Important	27	15.2
Neutral	60	33.7
Important	57	32.0
Great Importance	13	7.3
Blank	4	2.2
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Little Importance	13	10.7
Somewhat Important	16	13.1
Neutral	41	33.6
Important	42	34.4
Great Importance	7	5.7
Blank	3	2.5
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
Little Importance	4	11.1
Somewhat Important	6	16.7
Neutral	13	36.1
Important	10	27.8
Great Importance	3	8.3
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	1	11.1
Neutral	4	44.4
Important	2	22.2
Great Importance	2	22.2
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	2	22.2
Neutral	2	22.2
Important	3	33.3
Great Importance	1	11.1
Blank	1	11.1
Total	9	100.0

i) Compiling data for research.

All Respondents

Response	Frequency	Percent
Little Importance	14	7.9
Somewhat Important	27	15.2
Neutral	52	29.2
Important	59	33.1
Great Importance	21	11.8
Blank	5	2.8
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Little Importance	10	8.2
Somewhat Important	18	14.8
Neutral	39	32.0
Important	38	31.1
Great Importance	14	11.5
Blank	3	2.5
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
Little Importance	1	2.8
Somewhat Important	8	22.2
Neutral	10	27.8
Important	12	33.3
Great Importance	5	13.9
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Little Importance	2	22.2
Somewhat Important	0	0.0
Neutral	2	22.2
Important	4	44.4
Great Importance	0	0.0
Blank	1	11.1
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Little Importance	1	11.1
Somewhat Important	1	11.1
Neutral	1	11.1
Important	3	33.3
Great Importance	2	22.2
Blank	1	11.1
Total	9	100.0

j) Compiling data for general membership use.

All Respondents

Response	Frequency	Percent
Little Importance	18	10.1
Somewhat Important	24	13.5
Neutral	51	28.7
Important	56	31.5
Great Importance	23	12.9
Blank	6	3.1
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Little Importance	13	10.7
Somewhat Important	13	10.7
Neutral	38	31.1
Important	38	31.1
Great Importance	16	13.1
Blank	4	3.3
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
Little Importance	1	2.8
Somewhat Important	8	22.2
Neutral	9	25.0
Important	12	33.3
Great Importance	5	13.9
Blank	1	2.8
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Little Importance	2	22.2
Somewhat Important	1	11.1
Neutral	3	33.3
Important	3	33.3
Great Importance	0	0.0
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Little Importance	2	22.2
Somewhat Important	0	0.0
Neutral	1	11.1
Important	3	33.3
Great Importance	2	22.2
Blank	1	11.1
Total	9	100.0

k) Other:

- (6) How to get "tribal wisdom" about the data + system documented; great importance.
- (34) Make real data available for research; great importance.
- (78) Expense information survey (departmental); great importance.
- (110) Study data quality for cat models; great importance.

Question 10: CAS Role in Data Management

Please indicate the importance of each of the following activities where the CAS could take an active role.

a) Serving as a data repository, in general.

All Respondents

Response	Frequency	Percent
Little Importance	51	28.7
Somewhat Important	21	11.8
Neutral	51	28.7
Important	37	20.8
Great Importance	13	7.3
Blank	5	2.8
Total	178	100.0

Insurance Company Actuaries

Response	**Frequency**	**Percent**
Little Importance	37	30.3
Somewhat Important	17	13.9
Neutral	36	29.5
Important	22	18.0
Great Importance	7	5.7
Blank	3	2.5
Total	122	100.0

Consulting Actuaries

Response	**Frequency**	**Percent**
Little Importance	11	30.6
Somewhat Important	2	5.6
Neutral	8	22.2
Important	9	25.0
Great Importance	5	13.9
Blank	1	2.8
Total	36	100.0

Organizations Serving Insurance

Response	**Frequency**	**Percent**
Little Importance	2	22.2
Somewhat Important	1	11.1
Neutral	2	22.2
Important	3	33.3
Great Importance	0	0.0
Blank	1	11.1
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	**Frequency**	**Percent**
Little Importance	1	11.1
Somewhat Important	1	11.1
Neutral	4	44.4
Important	2	22.2
Great Importance	1	11.1
Total	9	100.0

b) Serving as data repository for research committees.

All Respondents

Response	Frequency	Percent
Little Importance	21	11.8
Somewhat Important	31	17.4
Neutral	44	24.7
Important	62	34.8
Great Importance	15	8.4
Blank	5	2.8
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Little Importance	15	12.3
Somewhat Important	19	15.6
Neutral	33	27.0
Important	43	35.2
Great Importance	9	7.4
Blank	3	2.5
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
Little Importance	3	8.3
Somewhat Important	9	25.0
Neutral	8	22.2
Important	10	27.8
Great Importance	5	13.9
Blank	1	2.8
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Little Importance	2	22.2
Somewhat Important	1	11.1
Neutral	1	11.1
Important	4	44.4
Great Importance	0	0.0
Blank	1	11.1
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Little Importance	1	11.1
Somewhat Important	2	22.2
Neutral	2	22.2
Important	3	33.3
Great Importance	1	11.1
Total	9	100.0

c) Providing assistance by steering requestors to other sources.

All Respondents

Response	Frequency	Percent
Little Importance	12	6.7
Somewhat Important	18	10.1
Neutral	38	21.3
Important	85	47.8
Great Importance	22	12.4
Blank	3	1.7
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Little Importance	8	6.6
Somewhat Important	13	10.7
Neutral	26	21.3
Important	56	45.9
Great Importance	16	13.1
Blank	3	2.5
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
Little Importance	4	11.1
Somewhat Important	3	8.3
Neutral	8	22.2
Important	17	47.2
Great Importance	4	11.1
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	1	11.1
Neutral	2	22.2
Important	5	55.6
Great Importance	1	11.1
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Little Importance	0	0.0
Somewhat Important	1	11.1
Neutral	1	11.1
Important	6	66.7
Great Importance	1	11.1
Total	9	100.0

d) Serving no role in this area.

All Respondents

Response	Frequency	Percent
Little Importance	73	41.0
Somewhat Important	3	1.7
Neutral	78	43.8
Important	5	2.8
Great Importance	7	3.9
Blank	12	6.7
Total	178	100.0

Insurance Company Actuaries

Response	Frequency	Percent
Little Importance	43	35.2
Somewhat Important	3	2.5
Neutral	57	46.7
Important	3	2.5
Great Importance	6	4.9
Blank	10	8.2
Total	122	100.0

Consulting Actuaries

Response	Frequency	Percent
Little Importance	21	58.3
Somewhat Important	0	0.0
Neutral	13	36.1
Important	1	2.8
Great Importance	1	2.8
Total	36	100.0

Organizations Serving Insurance

Response	Frequency	Percent
Little Importance	3	33.3
Somewhat Important	0	0.0
Neutral	4	44.4
Important	1	11.1
Great Importance	0	0.0
Blank	1	11.1
Total	9	100.0

Other (Banks, Financial Institutions, Academic, Government, Other)

Response	Frequency	Percent
Little Importance	5	55.6
Somewhat Important	0	0.0
Neutral	3	33.3
Important	0	0.0
Great Importance	0	0.0
Blank	1	11.1
Total	9	100.0

11. Additional Comments:

- (4) The SoA takes a real leadership role as a repository of data (mortality studies and so forth). Maybe the CAS should consider this as well.

- (9) I am not interested in the technical aspects of new/emerging technologies – that is what my IT area should be doing. I also don't think it is reasonable to have the syllabus cover specific current technologies. I think it should cover things like (1) how to check for reasonableness; and, (2) how to verify against other sources.

- (23) I hope there is a strong link between the CAS Committee and the Insurance Data Management Association. The IDMA can both contribute knowledge and gain knowledge in the area of data management.

- (37) I can't imagine that my company would ever view the CAS as a legitimate source of information on data management or technologies. The CAS should focus their energies on areas where they are the acknowledged experts (e.g. ratemaking, reserving, DFA, etc.). This doesn't mean that individual actuaries shouldn't be involved with data management or technology issues in their jobs. It just means that the CAS as a professional organization shouldn't focus their time there. Leave data management and technology issues to the ITS professional organizations.

- (56) The CAS should focus on the call paper programs and seminars. The CAS should NOT become a provider of data, data services or data collection/manipulation standards.

- (57) I would rank publishing textbooks up higher except that it seems that the information could become quickly out of date. Maybe this committee could review some of the books written for general database and data management and recommend some of the better ones. Many issues regarding data management are not insurance specific and there is always new material being published in this area.

  Where/how does IDMA fit in with this committee - should that be considered an additional resource for insurance data issues?

- (58) I feel it is of no long-term benefit and may even be a detriment to the maintenance/prestige of the actuarial profession to concentrate resources around the issue of designing data collection systems. Although actuaries often do (and should) contribute input as to what data needs to be collected, the actual design of systems is not an actuarial issue. If we spend a lot of time designing computer systems, we run the risk of being regarded as no more than programmers. And, in fact, I've actually seen it happen that a competent actuary gets disregarded because he's seen as a "great data resource person" but not someone who can help with insurance analysis or management. I'm personally very opposed to devoting the Casualty ACTUARIAL Society's resources on COMPUTER issues.

  On the other hand, I think it would be a benefit if the CAS could provide more data (not just theory papers) to its members, even if the CAS charged the members for access.

- (60) As for documenting data quality and coding issues - how much time will be wasted developing intuitive ideas? I don't know how much non-intuitive issues need to be brought to the forefront, although I'm sure there are a few.

  Data Issues are extremely important, but all issues boil down to a company level, so are a lot of generalities or guidelines for good data a necessity? I think most actuaries can differentiate between good data and bad data inside their company. It's more of an internal company issue in understanding your data sources, what they mean and what the pitfalls in the data are. Unfortunately that is a tough thing for people new to a company to develop immediately, so their natural thoughts are that any data is good - I'll take what I can get, and try to use it.

Certainly w/ the electronic age and downloading data - there is often an assumption among younger workers that I got data, now lets go...not spending time to verify the data, or have the background to know intuitively the data looks wrong. This is always a problem.

As for Other Data Sources - A book?? Scary thought - how fast do these become outdated. I would assume an online source would be much more preferential and much more easily updated for new sources. I think the idea is not a bad one it's just how it's accomplished.

Syllabus material - this is not unlike DFA - a very untestable subject. I think a few meaningless papers use to be on the old Ratemaking exam. Data sources are transforming every 5 or 10 years, so it is hard to stay current. But in general I don't see this as a very practical subject for the syllabus.

- (78) I would love to know the average cost of annual statement production, winning rate approval, processing a policy, and other items only obtainable via surveys.

- (83) Actuaries and statisticians were heavily involved in MIS long before MIS was an acronym. 15 years ago, it was a prerequisite for an actuarial student to be able to extract and manipulate raw data with mainframe languages such as Basic, Fortran, SAS, or FOCUS. Sadly, I've seen much of the data extraction/data manipulation functions leave the actuarial realm and become part of traditional IT departments, with armies of DBAs maintaining oversized Oracle and DB2 tables.

  Most experienced actuaries can glance at a table of raw data and instantly know far more about the data, how it should be arranged, and how it can be used than "owners" of the data residing in IT. I'm concerned that younger actuaries may not be getting the type of training we received.

- (98) I would have answered this survey differently if you had asked how much my staff is involved in these activities. Much of the data work is done by students. Are you sending this to students?

  I wasn't sure about your ranking system. To me, "neutral" seems like a lower ranking than "little importance", yet you give it a ranking of between "important" and "somewhat important".

- (117) At Travelers, each department has different data management requirements, and so my response reflects my unit's data issues and probably is not representative of other departments.

- (121) The CAS should be very careful not to let itself evolve into a statistical agent.

- (129) Section 6 is left blank since I work in an international group head office and these activities are performed in the local countries.

- (140) As legacy systems begin to crumble, and new insurance operations come online, some actuaries will soon be participating in the design and (re)building of data management systems. The sheer number of data elements that need to be considered can be staggering. Also, few actuaries (or anyone else) truly understand the complex path of data within insurance organizations, who the final users are, and why certain elements are critical to certain users. Providing information that would help to close this gap is important.

- (144) I think the Committee's role is more to obtain existing materials rather than develop actuarial-specific papers. I expect that database administrators, data warehouse architects and developers of accounting systems have addressed most of these issues already. Nevertheless, in my experience actuaries are considered the data experts from the business (as opposed to Information Technology) point of view, despite no formal education.

- (146) As part of the debate on the value of the FCAS designation, I believe we provide the most value in touting our ability to recognize and properly analyze insurance data. Actuaries should become more involved in the entire data management process for insurance companies, from system design to data quality. It is important that actuaries are able to develop these skills and that actuaries are able to trade on these skills within their organizations.

- (165) Good survey except that I didn't know how to answer 10d. I think we should have a small role, but should not compile a general data repository for the membership.

- (167) You are headed into dangerous waters here. I am not convinced that the CAS needs to be tremendously active here nor am I convinced that the CAS is the most qualified organization to develop these skill sets in their members. Please do not overstep the bounds that the CAS has set for itself: "to advance the body of knowledge of actuarial science applied to property, casualty, and similar exposures, to establish and maintain standards of qualification for membership, to promote and maintain high standards of conduct and competence for the members, and to increase the awareness of actuarial science." I am having trouble justifying some of the activities suggested above relative to this standard. Do we need data management competency - absolutely, is the CAS the right vehicle for us to develop those skills - I'm not convinced.

- (183) Actuaries in my company spend far, far too much time scrubbing data.

APPENDIX

CAS Membership Survey
Management Data and Information

The Casualty Actuary Society's Committee on Management Data and Information addresses actuarial issues on property and casualty insurance data and information systems. The following survey is intended to provide the Committee with information that will enhance our role in providing resources on management data to members of the Society. For your convenience, an electronic version of the Survey can be completed on the CAS Web Site at http://www.casact.org/research/datasurv.htm. Please return the survey no later than March 2, 2001.

Your personal assistance in completing the Survey will be greatly appreciated.

YOUR ROLE AS AN ACTUARY

a) What is your highest CAS designation? FCAS ☐ ACAS ☐

b) How many years of actuarial experience do you have?
0-5 ☐ 6-10 ☐ 11-20 ☐ 21+ ☐

c) Which of the following best describes your company, organization or department?

Property-Liability Insurance Company	☐	Organization Serving Insurance	☐
Reinsurance Company	☐	Academic	☐
Life, Accident & Health Insurance	☐	Government	☐
Insurance Broker or Agent	☐	Consulting Actuary	☐
Banks, Financial Institutions	☐	Retired	☐
Other (Please specify.)	☐	____________________	

1. Data Management Time

What percentage of your time do you spend on the following activities? (Check the appropriate percentage.)

	0%	1-5%	6-10%	11-24%	25%+
a) Assisting in the design of new data collection/ processing systems (that support operations, policy issuance, claims, billing, etc.).	☐	☐	☐	☐	☐

	0%	1-5%	6-10%	11-24%	25%+
b) Assisting in the design of new analytical tools (that support pricing, reserving, underwriting, etc.).	☐	☐	☐	☐	☐
c) Assuring the quality of internal data being compiled by existing systems.	☐	☐	☐	☐	☐
d) Providing input on collection of data including cost/benefit analysis.	☐	☐	☐	☐	☐
e) Extraction and manipulation of data.	☐	☐	☐	☐	☐
f) Training co-workers how to properly identify data elements to be extracted, as well as data extraction techniques.	☐	☐	☐	☐	☐
g) Mining internal data (e.g., new product development, identifying data correlations, neural networks).	☐	☐	☐	☐	☐
h) Obtaining data from external sources (e.g., industry, government, economic statistics).	☐	☐	☐	☐	☐
i) Responding on data issues (e.g., regulatory, bureaus, AM Best).	☐	☐	☐	☐	☐

j) Other management and data activities. (Please specify and check the appropriate percentage.)

1.____________________	☐	☐	☐	☐	☐
2.____________________	☐	☐	☐	☐	☐
3.____________________	☐	☐	☐	☐	☐
4.____________________	☐	☐	☐	☐	☐
5.____________________	☐	☐	☐	☐	☐

2. Privacy/Ownership of Data:

	0%	1-5%	6-10%	11-24%	25%+
a) What percentage of your time do you spend on issues related to privacy/ownership of data?	☐	☐	☐	☐	☐

b) Please identify the privacy issues you have recently dealt with and the percentage of time spent on each issue.

	0%	1-5%	6-10%	11-24%	25%+
1.______________________	☐	☐	☐	☐	☐
2.______________________	☐	☐	☐	☐	☐
3.______________________	☐	☐	☐	☐	☐
4.______________________	☐	☐	☐	☐	☐

3. Data Management Priorities

Please indicate the importance of the following data management issues.

	Little Importance	Somewhat Important	Neutral	Important	Great Importance
a) Actuarial input in system design.	☐	☐	☐	☐	☐
c) Ease of access to your internal data systems.	☐	☐	☐	☐	☐
d) Having a detailed and accurate data dictionary including fields, definitions, etc.	☐	☐	☐	☐	☐
e) Having knowledge of traditional external data sources (publications, historical data vendors).	☐	☐	☐	☐	☐
e) Having knowledge of non-traditional external data sources (e.g., Internet, Web Sites).	☐	☐	☐	☐	☐
f) Ease of integration of external and internal data sources.	☐	☐	☐	☐	☐

	Little Importance	Somewhat Important	Neutral	Important	Great Importance
g) Issues of intellectual property and ownership.	☐	☐	☐	☐	☐
h) Developing the skills necessary to identify, extract and manipulate data.	☐	☐	☐	☐	☐
i) Keeping up-to-date on reporting rule changes that change the substance of data collection (e.g., recent loss expense definition change).	☐	☐	☐	☐	☐
j) Having the ability to perform data mining on internal data.	☐	☐	☐	☐	☐
k) Understanding emerging data technologies.	☐	☐	☐	☐	☐
l) Cost considerations associated with data collection and extraction.	☐	☐	☐	☐	☐
m) Other (Please specify.)					
1.________________	☐	☐	☐	☐	☐
2.________________	☐	☐	☐	☐	☐
3.________________	☐	☐	☐	☐	☐
4________________	☐	☐	☐	☐	☐
5.________________	☐	☐	☐	☐	☐

4. Data Management Activities

Which of following data management activities have you been involved in? (Check all that apply.)

	Recently	Earlier In Your Career	Never
a) Development of specifications for design of a database.	☐	☐	☐
b) Selection of data elements to be captured in a database.	☐	☐	☐
c) Development of specifications for a report or report system.	☐	☐	☐

	Recently	Earlier In Your Career	Never
d) Production of ad-hoc reports from the database.	☐	☐	☐
e) Development of test data.	☐	☐	☐
f) Definition of data edits.	☐	☐	☐
g) Development of procedures to insure data quality.	☐	☐	☐
h) Other (Please specify.)			
1.____________________	☐	☐	☐
2.____________________	☐	☐	☐
3.____________________	☐	☐	☐
4.____________________	☐	☐	☐

5. Performance of Data Management

Please indicate who performs the following data management activities for your company. (Check all that apply.)

Data Management Activity	Actuaries	Actuarial Students	Non-Actuaries	External Consultants
a) Ratemaking/pricing	☐	☐	☐	☐
b) Reserve setting	☐	☐	☐	☐
c) Reserve opinions	☐	☐	☐	☐
d) Underwriting reports	☐	☐	☐	☐
e) Marketing reports	☐	☐	☐	☐
f) Claims management	☐	☐	☐	☐
g) Financial analysis and investments	☐	☐	☐	☐
h) Financial reporting	☐	☐	☐	☐

Data Management Activity	Actuaries	Actuarial Students	Non-Actuaries	External Consultants
i) Rate regulation	☐	☐	☐	☐
j) *Statistical agency reporting*	☐	☐	☐	☐
k) Other (Please specify.)				
1.____________________	☐	☐	☐	☐
2.____________________	☐	☐	☐	☐
3.____________________	☐	☐	☐	☐
4.____________________	☐	☐	☐	☐

6. Data Management Education

a) How many times have you attended the CAS Ratemaking Seminar in the last 5 years?

5☐ 4☐ 3☐ 2☐ 1☐ 0☐

b) How many times have you attended the Management Data and Information Committee's presentation at the CAS Ratemaking Seminar in the last 5 years?

5☐ 4☐ 3☐ 2☐ 1☐ 0☐

c) How many times have you attended the Emerging Technology Seminar?

2☐ 1☐ 0☐

7. Future Education Activities

Please indicate the importance of each of the following ways in which the CAS could become involved in data management education.

	Little Importance	Somewhat Important	Neutral	Important	Great Importance
a) Developing a textbook or other reference resource.	☐	☐	☐	☐	☐
b) Sponsoring seminars or workshops.	☐	☐	☐	☐	☐
c) Including this subject in the CAS Syllabus.	☐	☐	☐	☐	☐
d) Including sessions at CAS Meetings.	☐	☐	☐	☐	☐
e) Publishing papers on data management/data quality.	☐	☐	☐	☐	☐
f) Publishing papers on new applications, incorporating new data sources.	☐	☐	☐	☐	☐
g) Other (Please specify.)					
1.____________________	☐	☐	☐	☐	☐
2.____________________	☐	☐	☐	☐	☐
3.____________________	☐	☐	☐	☐	☐

I. Use of Online Catalog/Bibliography

a) Are you aware that the CAS Web Site provides you with an Online Catalog that contains a bibliography of reference materials? Yes ☐ No ☐

b) If yes, is the Online Catalog of any use to you? Yes ☐ No ☐

c) What can be added to make the data management section of the Online Catalog more useful?

1.____

2.____

3.____

Use of Data Management Committee

he Committee is considering undertaking several activities in order to help actuaries deal with data ıanagement issues. Please indicate the importance of the following areas that the Committee may ꞓ able to provide assistance to actuaries.

	Little Importance	Somewhat Important	Neutral	Important	Great Importance
ı) Keeping up with the state-of-the art in data management.	☐	☐	☐	☐	☐
›) Demonstrating how to get users involved in developing new systems.	☐	☐	☐	☐	☐
:) Developing procedures for insuring data quality.	☐	☐	☐	☐	☐
I) Dealing with data that is incomplete or of poor quality.	☐	☐	☐	☐	☐
ꞓ) Demonstrating techniques for reconciliation of financial and statistical data.	☐	☐	☐	☐	☐
) Demonstrating techniques for testing reports, report systems, etc.	☐	☐	☐	☐	☐
ȷ) Providing information on the latest software.	☐	☐	☐	☐	☐
ı) Demonstrating techniques for implementation and use of Web enabled applications.	☐	☐	☐	☐	☐
) Compiling data for research.	☐	☐	☐	☐	☐
) Compiling data for general membership use.	☐	☐	☐	☐	☐
‹) Other (Please specify.)					
1.________________	☐	☐	☐	☐	☐
2.________________	☐	☐	☐	☐	☐
3.________________	☐	☐	☐	☐	☐

10. CAS Role in Data Management

Please indicate the importance of each of the following activities where the CAS could take an active role.

	Little Importance	Somewhat Important	Neutral	Important	Great Importance
a) Serving as a data repository, in general.	☐	☐	☐	☐	☐
b) Serving as data repository for research committees.	☐	☐	☐	☐	☐
c) Providing assistance by steering requestors to other sources.	☐	☐	☐	☐	☐
d) Serving no role in this area.	☐	☐	☐	☐	☐
e) Other (Please specify.)					
1.________________	☐	☐	☐	☐	☐
2.________________	☐	☐	☐	☐	☐
3.________________	☐	☐	☐	☐	☐
4.________________	☐	☐	☐	☐	☐

11. Additional Comments:

____

____

____

____

____

____

Thank you for taking the time to participate in this very important survey.

Please complete the following:

NAME: ______________________________
TITLE: ______________________________
COMPANY/ORGANIZATION ______________________________
MAILING ADDRESS: ______________________________
____
TELEPHONE NUMBER ______________________________
E-MAIL ADDRESS ______________________________

PLEASE RETURN THE COMPLETED SURVEY TO:

CASUALTY ACTUARIAL SOCIETY
ATTN: DATA SURVEY
1100 NORTH GLEBE ROAD
SUITE 600
ARLINGTON, VIRGINIA 22201
FAX: 703-276-3108

A Characterization of Life Expectancy with Applications to Loss Models

Daniel R. Corro

A Characterization of Life Expectancy with Applications to Loss Models

Dan Corro
National Council on Compensation Insurance, Inc.
September, 2001

Abstract:

In its usual (one-dimensional) form, a loss model is just a distribution of nonnegative real numbers $[0,\infty)$. *This note establishes necessary and sufficient conditions for a differentiable function to equal the life expectancy of some loss model. Examples are provided to illustrate the shape of the life expectancy function of several common loss models. The characterization is used to define a general class of loss models flexible enough to cover the Pareto, Lognormal, Weibull, and Gamma densities. Finally, the approach is extended to model multi-dimensional survivorship.*

I. Introduction

In general, life expectancy can be expressed as a simple descriptive statistic. The usual functional forms used to describe loss distributions, namely cumulative density functions [CDFs], probability density functions [PDFs], and hazard rate functions generally demand some processing to visualize and often require fitting parameters to an assumed form for calculation purposes.

On the other hand, the formal nature of CDFs and PDFs and hazard rates are apparent. A differentiable function $F(t)$ on $[0,\infty)$ is a CDF of a loss model exactly when:

$$F(0)=0, \quad \frac{dF}{dt}\geq 0, \quad and \quad \lim_{t\to\infty} F(t)=1.$$

An integrable function, $f(t)$, on $[0,\infty)$ is a PDF of a loss model exactly when:

$$f(t)\geq 0, \quad and \quad \int_0^\infty f(t)dt=1.$$

Similarly, an integrable function, $h(t)$, on $[0,\infty)$ is a hazard rate function of a loss model exactly when:

$$h(t)\geq 0, \quad and \quad \int_0^\infty h(t)dt=\infty.$$

The main result is that a differentiable function $\rho(t)>0$ on $[0,\infty)$ is a life expectancy function [LEF] of a loss model exactly when:

$$\frac{d\rho}{dt}\geq -1, \quad and \quad \int_0^\infty \frac{1}{\rho(t)}dt=\infty.$$

When working with insurance data, "claim life expectancy" can often be regarded as a reserve and conversely a reserve as a life expectancy (c.f. [4]). In practice, reserves may be related with claim survival data to the extent that closed, i.e. "dead", cases are characterized by having no reserves.

It is evident from the discussion below how a life expectancy function completely determines the loss model. Because life expectancy is often easier to determine than the CDF, PDF or hazard function, being able to recognize such functions may come in handy. Examples show that the graph of the life expectancy function is simpler than those of the CDF or PDF functional forms used to define some popular loss models. Also, bivariate loss models pose many technical difficulties; however, these observations on life expectancy are readily extended to higher dimensions (c.f. [5]).

II. Notation and Background

Let $f(t)$ denote an integrable function on the nonnegative real numbers $[0,\infty)$ satisfying:

$$\int_0^\infty f(t)dt = 1$$

Regard $f(t)$ as a probability density of failure times and define the function:

$$S(t) = 1 - \int_0^t f(s)ds = \int_t^\infty f(s)ds$$

As is customary, we refer to $S(t)$ as the *survival function*, $f(t)$ as the *probability density function [PDF]*, *F(t)*=1-*S(t)* as the *cumulative density function[CDF]*, and t as "time." We also let *T* denote the random variable for the distribution of survival times and $\mu = E(T)$ the mean duration or life expectancy, which we assume throughout to be finite and nonzero. Survival analysis refers to the following function:

$$h(t) = \frac{f(t)}{S(t)}$$

as the *hazard rate function* or sometimes as the *force of mortality*. The hazard rate function measures the instantaneous rate of failure at time *t* and can be expressed as a limit of conditional probabilities:

$$h(t) = \lim_{\Delta t \to 0} \frac{\Pr\{t \le T < t + \Delta t \mid T \ge t\}}{\Delta t}$$

There are many well-known relationships and interpretations of hazard rate functions (refer to Allison[1] for a particularly succinct discussion).

It is convenient to recall that if we set

$$g(t) = \int_0^t h(s)ds, \text{ then } S(t) = e^{-g(t)}.$$

Let's fix *t* and restrict our attention to values of time *w* > *t*. The conditional probability of survival to *w*, given survival to *t*, is $S_t(w) = \frac{S(w)}{S(t)}$. In this context (see [2]), the *expectation of life at time t*, given survival to time *t*, is just:

$$\rho(t) = \frac{\int_t^\infty (w-t)f(w)dw}{\int_t^\infty f(w)dw} = \int_t^\infty S_t(w)dw = \int_t^\infty \frac{S(w)}{S(t)}dw$$

Observe that under our assumptions, $\rho(0) = \mu > 0$ and the function $\rho(t)$ is well defined for all t>0. We also observe that for any a<b with $S(a) > 0$, we have the relation:

$$\rho(a)S(a) = \int_a^\infty S(t)dt = \int_a^b S(t)dt + \int_b^\infty S(t)dt$$

$$\leq \int_a^b S(a)dt + \int_b^\infty S(t)dt = S(a)(b-a) + \rho(b)S(b)$$

$$\Rightarrow \text{(read "implies")} \qquad a + \rho(a) \leq b + \frac{\rho(b)S(b)}{S(a)} \leq b + \rho(b),$$

with strict inequality exactly when $S(b) < S(a)$.

Not surprisingly, there are formal relationships between hazard, $h(t)$, and life expectancy, $\rho(t)$, as in:

Proposition 1:

$$1 + \frac{d\rho}{dt} = h(t)\rho(t)$$

Proof: This is straightforward from the above definitions--see [2].

Proposition 2: *For any differentiable function, $\varphi(t)$, on $[0,\infty)$, the following are equivalent:*

i) $a, b \in [0,\infty), a \leq b \quad \Rightarrow \quad a + \varphi(a) \leq b + \varphi(b)$

ii) $\dfrac{d\varphi}{dt} \geq -1 \quad on\,[0,\infty)$

Proof: Consider the function $\psi(t) = \varphi(t) + t$, then ψ is non-decreasing on $[0,\infty)$ if and only $\dfrac{d\psi}{dt} = \dfrac{d\varphi}{dt} + 1 \geq 0$ on $[0,\infty)$; the result follows.

So we now let $\varphi(t) > 0$ be a differentiable function on $[0,\infty)$ such that $\frac{d\varphi}{dt} \geq -1$ on $[0,\infty)$. From Proposition 1, it is natural to consider the loss model defined via its hazard function, as above, by:

$$h(t) = h_\varphi(t) = \frac{1 + \frac{d\varphi}{dt}}{\varphi(t)} \geq 0 \; on\,[0,\infty)$$

Keeping the above notation, we have:

$$h(t) = \frac{1}{\varphi(t)} + \frac{d \ln \varphi(t)}{dt} \Rightarrow g(t) = \int_0^t h(w)dw = \int_0^t \frac{dw}{\varphi(w)} + \ln\left(\frac{\varphi(t)}{\varphi(0)}\right)$$

$$\Rightarrow S(t) = e^{-g(t)} = \frac{\varphi(0)e^{-\int_0^t \frac{dw}{\varphi(w)}}}{\varphi(t)}$$

$$\Rightarrow \rho(t) = \int_t^\infty \frac{S(v)}{S(t)}dv = \varphi(t)\int_t^\infty \frac{e^{-\int_t^v \frac{dw}{\varphi(w)}}}{\varphi(v)}dv$$

Regard t as fixed and use the change of variable:

$$u(v) = \int_t^v \frac{dw}{\varphi(w)} \Rightarrow du = \frac{dv}{\varphi(v)}$$

At the limits of integration we have

$v = t$ corresponds to $u = 0$ and $v = \infty$ corresponds to $u = \int_t^\infty \frac{dw}{\varphi(w)}$.

It follows that:

$$\rho(t) = \varphi(t) \int_0^{\int_t^\infty \frac{dw}{\varphi(w)}} e^{-u}du = \varphi(t)\left(1 - e^{-\int_t^\infty \frac{dw}{\varphi(w)}}\right) \leq \varphi(t)$$

Which means that the life expectancy function, or, can be characterized as the smallest solution to the differential equation (Proposition 1) that relates hazard with life expectancy.

Since clearly

$$\int_0^\infty \frac{dw}{\varphi(w)} = \infty$$

$$\Leftrightarrow \text{(read "if and only if") } \int_t^\infty \frac{dw}{\varphi(w)} = \infty \text{ for all } t \in [0,\infty),$$

it follows that:

$$\rho(t) = \varphi(t) \Leftrightarrow \int_0^\infty \frac{dw}{\varphi(w)} = \infty$$

and we have established the main result of this paper, which is stated as the following Proposition:

Proposition 3: *A differentiable function* $\rho(t) > 0$ *on* $[0, \infty)$ *is a life expectancy of a loss model exactly when:*

$$\frac{d\rho}{dt} \geq -1, \quad and \quad \int_0^\infty \frac{1}{\rho(t)} dt = \infty$$

In this paper we will refer to a function $\rho(t)$ as an LEF exactly when it is a life expectancy of a loss model. The remainder of this paper consists primarily of applying the Proposition 3 characterization of LEFs. Conceptually, the "local" derivative constraint relates to a limitation that at any time no more "deaths" can occur than the number then "living" while the "global" integral constraint requires the model to account for all lives.

Example: Suppose $\varphi(t) = t^2 + 1$. Then $\frac{d\varphi}{dt} = 2t \geq -1$ when $t \in [0, \infty)$ and we can define

$$h(t) = h_\varphi(t) = \frac{1 + \frac{d\varphi}{dt}}{\varphi(t)} = \frac{2t+1}{t^2+1}$$

The reader can readily verify that in this case we have:

$$g(t) = \ln(t^2 + 1) + \tan^{-1}(t)$$

$$S(t) = \frac{1}{(t^2+1)e^{\tan^{-1}(t)}}$$

$$\rho(t) = (t^2+1)\left(1 - e^{\tan^{-1}(t) - \frac{\pi}{2}}\right) = \varphi(t)\left(1 - e^{\tan^{-1}(t) - \frac{\pi}{2}}\right) < \varphi(t)$$

We see that $\varphi(t) = t^2 + 1$ is **not** the life expectancy of any loss model.

III. Examples of Life Expectancy Functions

In this section we show what the life expectancy looks like for several of the most commonly used loss models.

Example III.1. *Pareto density with parameters* $a > 1, b > 0$. In this example, define

$$f(a, b; t) = ab^a (b+t)^{-a-1}.$$

Then (see, e.g. [6], pp. 222-223)

$$S(t) = \left(\frac{b}{b+t}\right)^a, \; h(t) = \frac{a}{b+t} \quad \text{and} \quad \rho(t) = \frac{b+t}{a-1}.$$

The Pareto density is characterized by a linear LEF. Note that for the Pareto loss model:

$$\frac{d\rho}{dt} \equiv \frac{1}{a-1} \quad \text{and} \quad \lim_{t \to \infty} \rho(t) = \infty .$$

Example III.2. *Lognormal density with parameters* $\mu, \sigma > 0$. In this example, define

$$f(\mu,\sigma;t) = \frac{e^{\frac{-1}{2\sigma^2}(\ln t - \mu)^2}}{t\sigma\sqrt{2\pi}}$$

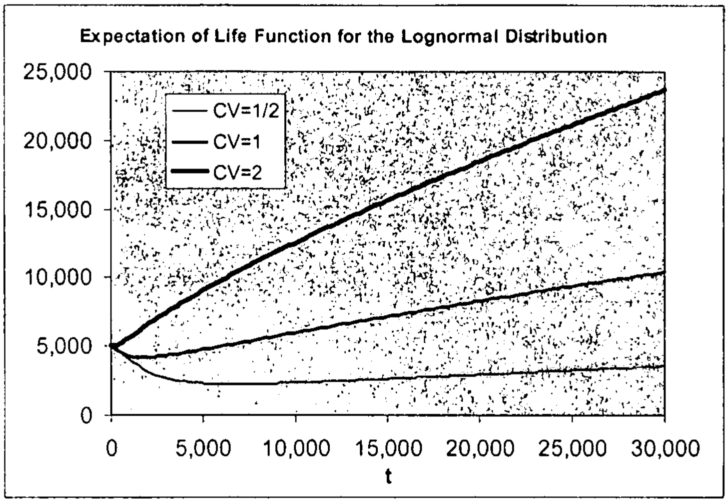
(see, e.g. [6], pp. 229-230). It can be shown that for a Lognormal loss model:

$$\lim_{t \to \infty} h(t) = 0; \qquad \lim_{t \to \infty} \rho(t) = \infty; \qquad \lim_{t \to \infty} \frac{d\rho}{dt} = \infty .$$

The coefficient of variation, CV, is defined as the ratio of the standard deviation to the mean; it is a convenient and dimensionless measure of variation. We leave to the reader the verification that the parameters for a Lognormal density with mean M and coefficient of variation C can be determined from:

$$\sigma = \sqrt{\ln(C^2+1)} \qquad \mu = \ln(M) - \frac{\sigma^2}{2}$$

The following chart shows the LEF's for a Lognormal loss model, expressed as above as a function $\varphi(t)$ of "time" t and with a constant mean, $\varphi(0) = 5{,}000$, and for CV = ½, 1, and 2, respectively.



Example III.3. *Weibull density with parameters* $a, b > 0$. In this example, define

$$f(a,b;t) = abt^{b-1}e^{-at^b}.$$

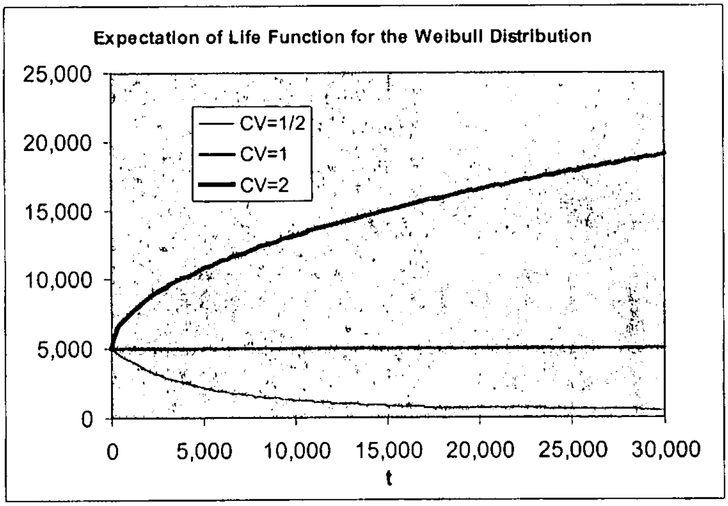
Then (see, e.g. [6] pp. 231-232)

$$S(t) = e^{-at^b}; \; h(t) = abt^{b-1}; \quad \text{and} \quad \mu = \frac{\Gamma\left(\frac{1}{b}\right)}{ba^{\frac{1}{b}}}$$

For a Weibull density we have:

$$\lim_{t\to\infty} \rho(t) = \frac{1}{\lim_{t\to\infty} h(t)} = \begin{cases} \infty & b < 1 \\ \frac{1}{a} & b = 1 \\ 0 & b > 1 \end{cases}$$

The following chart shows the LEF's for a Weibull loss model with mean of 5,000 and coefficients of variation = ½, 1, and 2, respectively. Note that a Weibull loss model with CV = 1 is an exponential density (case b=1), characterized by a constant LEF.



Example III.4. *Gamma density with parameters* $a, b > 0$. In this example, define

$$f(a,b;t) = \frac{a^b t^{b-1} e^{-at}}{\Gamma(b)}.$$

Then (see, e.g. [6] pp. 226-227)

$$S(t) = 1 - \Gamma(b; at); \quad \text{and} \quad \mu = \frac{b}{a}$$

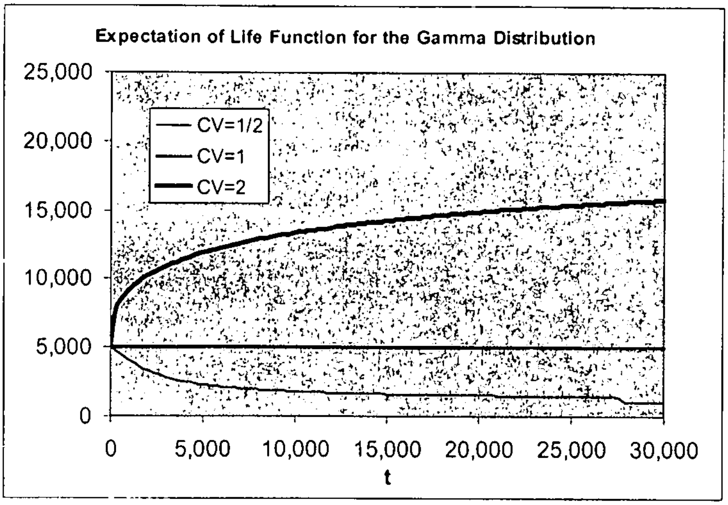
It can be shown (see, e.g. [7] pp. 86-87) that for a Gamma loss model:

$$\lim_{t \to \infty} h(t) = a \quad \text{and} \quad \lim_{t \to \infty} \rho(t) = \frac{1}{a}.$$

We leave to the reader the verification that the parameters for a Gamma density with mean M and coefficient of variation C can be determined from:

$$a = \frac{1}{C^2 M} \quad \text{and} \quad b = \frac{1}{C^2}$$

The following chart shows the expectation of life for a Gamma loss model with mean of 5,000 and coefficients of variation = ½, 1, and 2, respectively. Note that a Gamma loss model with CV = 1 is again an exponential density (case b=1).



This section concludes with two general examples.

Example III.5. *Piecewise linear functions*

In each of the above loss models the graph of the expectation of life function is rather flat, exhibiting at most one relative maximum or minimum. This suggests that such curves can be successfully approximated by fairly simple functions, e.g. by piecewise linear functions with rather few pieces. Consider any positive, continuous, piecewise linear function on $[0,\infty)$ with finitely many pieces. Then the rightmost slope must be nonnegative, so the integral over $[0,\infty)$ diverges. It is intuitively clear (and easy to prove) each of the "corners" of such a function can be approximated to any desired tolerance by a smooth curve that matches the slopes of the corner's two sides while keeping its derivative within the range of those two slopes. It follows from our findings that a positive, continuous piecewise linear function on $[0,\infty)$ represents the expectation of life of a loss model exactly when all its (finitely many) slopes are ≥ -1. This is a very simple criterion to accommodate when fitting empirical data to a piecewise linear representation.

Example III.6. *Rational functions*

Another natural choice of "simple" functions, these differentiable, is the set of rational functions. We consider first the case of a ratio of two first degree polynomials:

$$\varphi(t) = \varphi(b,c,d;t) = \frac{bt+c}{t+d} \qquad t \in [0,\infty)$$

We claim that the following are necessary and sufficient conditions for $\varphi(t)$ to be LEF of a loss model on $[0,\infty)$ with positive mean:

$$(RF1) \quad c>0,\, d>0,\, b \geq Max\left(\frac{c}{d}-d,0\right).$$

We will abuse notation somewhat and use RF1 to denote both these conditions and the class of functions they determine. To verify the claim, observe first that:

$$-1 \leq \frac{d\varphi}{dt} = \frac{(t+d)b-(bt+c)}{(t+d)^2} \Leftrightarrow (t+d)^2 \geq c-db$$

which holds for all $t \geq 0$ exactly when $d^2 \geq c-db$.

Assume first that $\varphi(t)$ satisfies conditions RF1, then clearly $\varphi(t)$ is differentiable and positive on $[0,\infty)$ and we have just verified that its derivative is ≥ -1. We also have:

$$b=0 \Rightarrow \frac{1}{\varphi(t)} = \frac{t+d}{c} \Rightarrow \int_0^\infty \frac{dt}{\varphi(t)} = \frac{1}{c}\left[\frac{t^2}{2}+dt\right]_0^\infty = \infty$$

$$b>0 \Rightarrow \lim_{t\to\infty}\frac{1}{\varphi(t)} = \lim_{t\to\infty}\frac{t+d}{bt+c} = \frac{1}{b} > 0 \Rightarrow \int_0^\infty \frac{dt}{\varphi(t)} = \infty$$

and so conditions RF1 suffice to make $\varphi(t)$ an LEF.

Convesely, if $\varphi(t)$ is an LEF, then being well defined on $[0,\infty)$ forces d>0 and clearly:

$$0 < \mu = \varphi(0) = \frac{c}{d} \Rightarrow c > 0$$
$$\lim_{t\to\infty} \varphi(t) = b \Rightarrow b \geq 0$$

and the observation on $\frac{d\varphi}{dt}$ implies that conditions RF1 hold.

Ratios of linear terms are a rather restricted class of functions, not even including linear functions. So we consider next the case of a second-degree polynomial divided by a linear term:

$$\varphi(t) = \varphi(a,b,c,d;t) = \frac{at^2 + bt + c}{t + d} \qquad t \in [0,\infty)$$

Two simple lemmas are useful here:

Lemma: *For a>0, b>0 the quadratic* $at^2 + bt + c$ *has a positive root if and only if* $b \leq -2\sqrt{ac}$.

Proof: Assume first that $b \leq -2\sqrt{ac}$, then $b^2 - 4ac \geq 0$, and from the quadratic formula,

$$r = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{|b| + \sqrt{b^2 - 4ac}}{2a} > 0$$

is a positive root. Conversely, if there is a positive root, the quadratic formula implies that

$$-b + \sqrt{b^2 - 4ac} > 0 \Rightarrow |b| = \sqrt{b^2} > \sqrt{b^2 - 4ac} > b \Rightarrow b < 0,$$

and it follows that

$$b^2 - 4ac \geq 0 \Rightarrow b^2 \geq 4ac \Rightarrow |b| \geq 2\sqrt{ac} \Rightarrow b = -|b| \leq -2\sqrt{ac}$$

and the lemma is established.

Lemma: $\frac{d\varphi}{dt} = a + \frac{bd - ad^2 - c}{(t+d)^2} \qquad \frac{d^2\varphi}{dt^2} = -2\frac{bd - ad^2 - c}{(t+d)^3}$

Proof: This is just a straightforward calculation:

$$\begin{aligned}\frac{d\varphi}{dt} &= \frac{(t+d)(2at+b) - (at^2 + bt + c)}{(t+d)^2} \\ &= \frac{at^2 + 2adt + ad^2 - ad^2 + bd - c}{t^2 + 2dt + d^2} \\ &= a + \frac{bd - ad^2 - c}{(t+d)^2}\end{aligned}$$

and the lemma is clear.

We claim that the following are necessary and sufficient conditions for $\varphi(t)$ to be the LEF of a loss model on $[0,\infty)$ with positive mean:

$$(RF2) \quad a \geq 0, c > 0, d > 0, b \geq Max\left(\frac{c}{d} - d, -2\sqrt{ac}\right)$$

We have already verified this for $a = 0$, so we assume $a \neq 0$.

Assume first that $\varphi(t)$ satisfies conditions (*RF2*), then clearly $\varphi(t)$ is differentiable and the above observations assure that $\varphi(t)$ is positive on $[0,\infty)$ with derivative ≥ -1. Also,

$$\int_0^\infty \frac{dt}{\varphi(t)} = \int_0^\infty \frac{t+d}{at^2+bt+c}dt = \left(\frac{1}{2a}\right)\int_0^\infty \frac{2at+b}{at^2+bt+c}dt + \left(\frac{2ad-b}{2a}\right)\int_0^\infty \frac{1}{at^2+bt+c}dt.$$

We observe that the first integral diverges:

$$\int_0^\infty \frac{2at+b}{at^2+bt+c}dt = \left[\ln(at^2+bt+c)\right]_0^\infty = \infty,$$

while the right hand integral is finite:

$$at^2+bt+c \geq at^2 - 2\sqrt{ac}\,t + c = \left(\sqrt{a}\,t - \sqrt{c}\right)^2 = u(t)^2$$

$$\Rightarrow$$

$$\int_0^\infty \frac{1}{at^2+bt+c}dt = \int_0^{\frac{1+\sqrt{c}}{\sqrt{a}}} \frac{1}{at^2+bt+c}dt + \int_{\frac{1+\sqrt{c}}{\sqrt{a}}}^\infty \frac{1}{at^2+bt+c}dt$$

$$\leq \int_0^{\frac{1+\sqrt{c}}{\sqrt{a}}} \frac{1}{at^2+bt+c}dt + \frac{1}{\sqrt{a}}\int_1^\infty \frac{du}{u^2} = \int_0^{\frac{1+\sqrt{c}}{\sqrt{a}}} \frac{1}{at^2+bt+c}dt + \frac{1}{\sqrt{a}} < \infty$$

$$\Rightarrow \int_0^\infty \frac{dt}{\varphi(t)} = \infty$$

and (*RF2*) is sufficient to make $\varphi(t)$ an LEF.

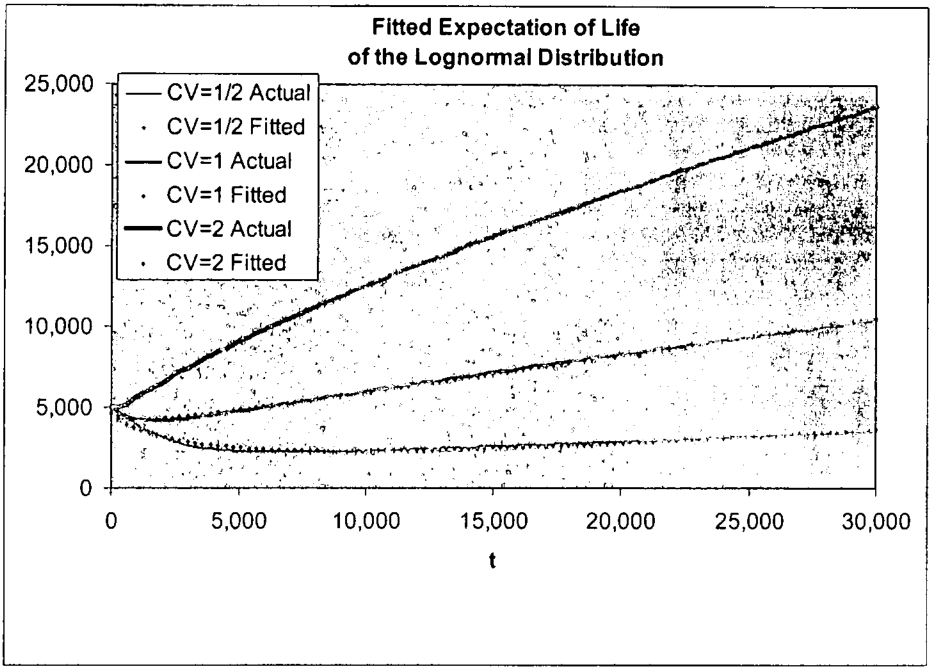
Conversely, if $\varphi(t)$ is an LEF, then being well defined on $[0,\infty)$ again forces d>0 and clearly:

$$0 < \mu = \varphi(0) = \frac{c}{d} \Rightarrow c > 0$$

$$0 \leq \lim_{t\to\infty}\varphi(t) = \lim_{t\to\infty}\frac{at^2+bt+c}{t+d} = \lim_{t\to\infty}\frac{2at+b}{1} \Rightarrow a \geq 0$$

and the lemmas imply that conditions (*RF2*) hold.

These constraints can be imposed when fitting data. Since this class of functions includes any linear expectancy function, it covers the Pareto and exponential cases. The following graphs show how the RF2 class of functions approximates the Lognormal Example III.2:



$RF2$ Fit to Lognormal				
Parameters	a	b	c	d
CV=½	0.1079	332	18,814,530	4,175
CV=1	0.2337	3,739	4,388,204	938
CV=2	0.4245	22,265.8	85,152,045	18013.2

Observe that while the tail behavior seems closely fit, the *RF2* approximation is not particularly good for CV=2 near t=0. This is because the *RF2* class of functions is not adept at fitting a slope at or near −1 over an interval. The Lognormal density shows few failures near t=0, corresponding to the thin right-hand tail of the corresponding normal density. There are various approaches to dealing with this (the next section illustrates restricting or renormalizing the loss interval); we conclude this section with a refinement of the formula. Consider broadening *RF2* by eliminating the derivative constraint:

$$(\overline{RF2}) \quad a \geq 0, c > 0, d > 0, b \geq -2\sqrt{ac}\,.$$

Let

$$\alpha = \alpha(a,b,c,d) = \begin{cases} \sqrt{\dfrac{ad^2 - bd + c}{a+1}} - d & ad^2 - bd + c \geq 0 \\ -d & ad^2 - bd + c \leq 0 \end{cases}$$

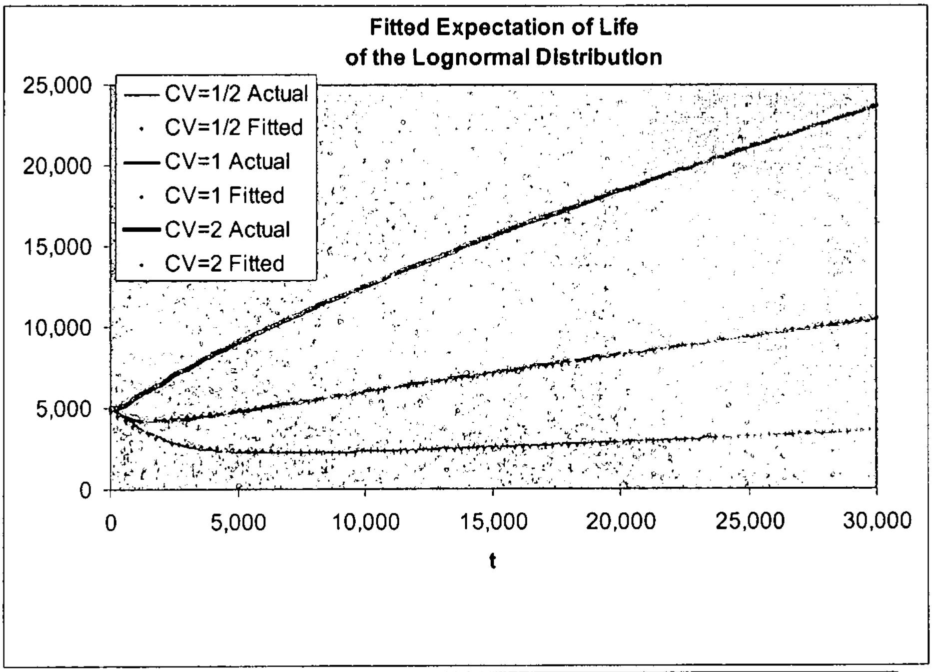
From an earlier lemma, $\dfrac{d^2\varphi}{dt^2}$ has the same sign as the constant $ad^2 - bd + c$, which implies that α is the largest value of t, if any, for which $\dfrac{d\varphi}{dt} = -1$. We can now define:

$$\overline{\varphi}(t) = \overline{\varphi}(a,b,c,d;t) = \begin{cases} \varphi(\alpha) + (\alpha - t) & t \leq \alpha \\ \varphi(t) & t \geq \alpha \end{cases}$$

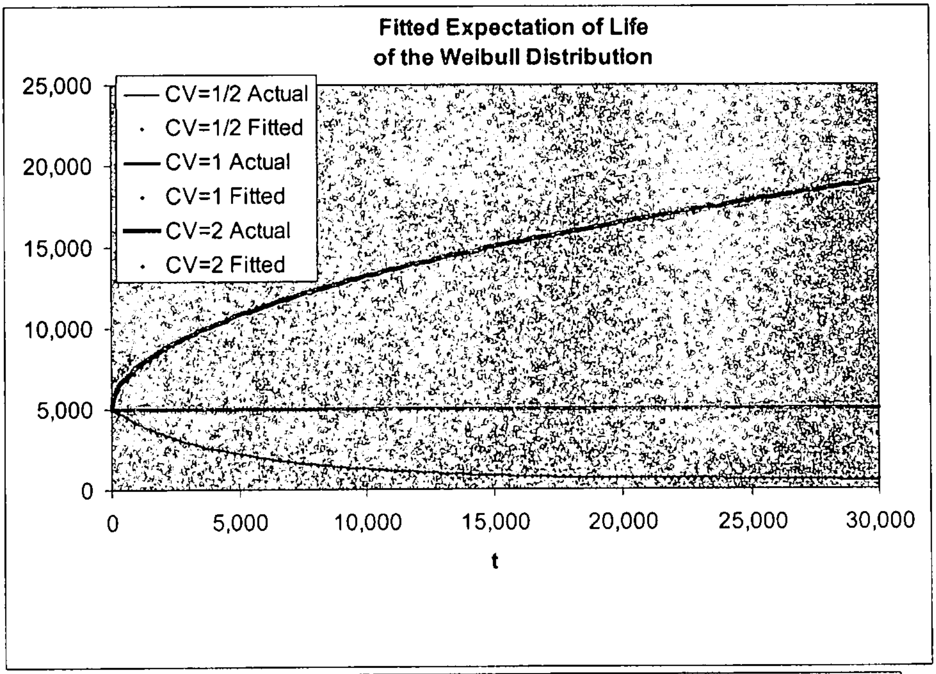
Then $\overline{\varphi}(t)$ is a differentiable function and our observations show $\overline{\varphi}(t)$ is an LEF. As no surprise, we note that:

$$\begin{aligned} (\overline{RF2})\ \&\ (\overline{\varphi} = \varphi) &\Leftrightarrow (\overline{RF2})\ \&\ (\alpha \leq 0) \\ &\Leftrightarrow (\overline{RF2})\ \&\left(\sqrt{\frac{ad^2 - bd + c}{a+1}} \leq d\right) \\ &\Leftrightarrow (\overline{RF2})\ \&\left(ad^2 - bd + c \leq ad^2 + d^2\right) \\ &\Leftrightarrow (\overline{RF2})\ \&\left(\frac{c}{d} - d \leq b\right) \Leftrightarrow (RF2) \end{aligned}$$

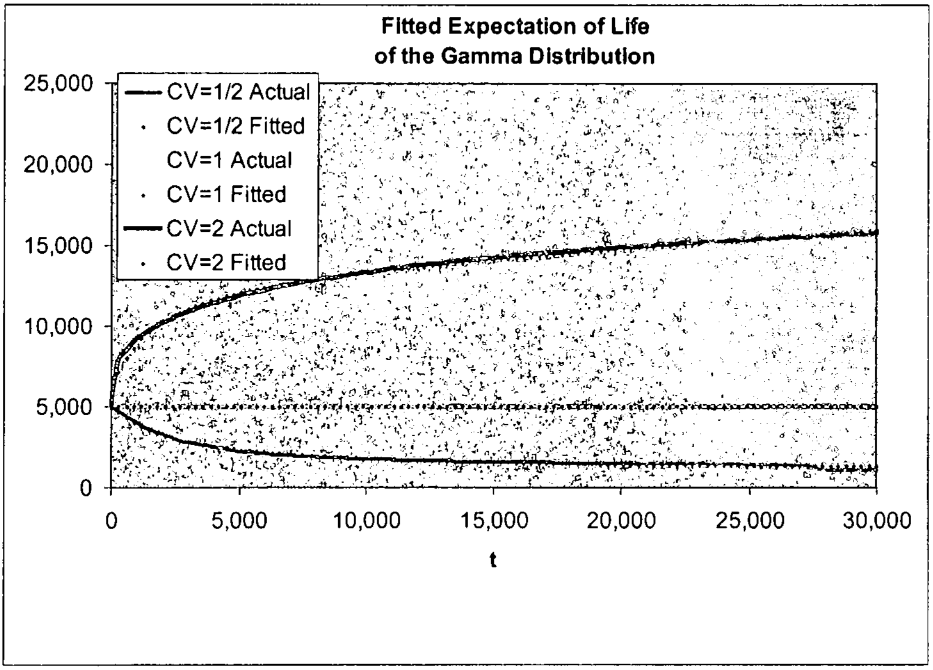
We conclude this section with charts illustrating how well the class of functions $\overline{RF2}$ is able to approximate Examples III.2, III.3, and III.4. We arrived at these estimates by first fitting the form $\varphi(t)$ without the derivative constraint on parameter b (using the SAS PROC NLIN procedure) and then using $\overline{\varphi}(t)$ as the fitted LEF.



$\overline{RF2}$ **Fit to Lognormal**				
Parameters	a	b	c	d
CV=½	0.0867	919	9,728,662	1,845.1
CV=1	0.2313	3,685.7	1,968,510	387.5
CV=2	0.4245	22,265.8	85,152,045	18,013.2



$\overline{RF2}$ Fit to Weibull				
Parameters	a	b	c	d
CV=½	0.0289	-1,081.5	27,621,056	5,479
CV=1	0	5,000	5,000	1
CV=2	0.2339	13,948.5	22,940,996	4,040.4



$\overline{RF2}$ Fit to Gamma				
Parameters	a	b	c	d
CV=½	0	998.9	11,943,663	2,306.2
CV=1	0	5,000	5,000	1
CV=2	0.098	13,558.6	9,600,852	1,636.5

IV. Limited Loss Models

In the previous discussion we have referred to loss models as essentially equivalent to continuous probability densities on $[0,\infty)$. The astute reader will have noticed a rather clumsy slight of hand as regards loss models of finite support, i.e., for which there is an upper loss limit L such that $f(t)=0$ for $t>L$. We have implicitly assumed that life expectancy $\rho(t)>0$ on $[0,\infty)$, which in effect means that there is no maximum loss. Consider, then, any probability density on $[0,\infty)$ with survival function, $S(t)$, and expectation of life function $\rho(t)$. We have:

$$\frac{dS}{dt}=-f(t)\le 0 \Rightarrow S \text{ nonincreasing} \Rightarrow S^{-1}(\{0\})=[L,\infty)$$

for some L. Note that we may have $L = \infty$ (and $[\infty,\infty) = \phi$ is empty). We find, therefore, from the definition of the expectation of life function that:

$$\rho(t) = \int_t^\infty \frac{S(v)}{S(t)} dv \Rightarrow \{t \mid \rho(t) = 0\} = S^{-1}(\{0\}) = [L,\infty)$$

and the reader can easily verify that this observation, together with our previous arguments, enables us to refine our main result somewhat:

Proposition 3: *A differentiable function* $\rho(t)$ *on* $[0,\infty)$ *is a life expectancy of a loss model exactly when:*

$$\{t \mid \rho(t) = 0\} = [L,\infty), \quad \frac{d\rho}{dt} \geq -1, \quad and \quad \int_0^L \frac{1}{\rho(t)} dt = \infty .$$

When $L < \infty$ is finite, the (continuous) loss models we have considered still demand that the probability of meeting or exceeding L is 0. It is more convenient when dealing with limited losses to consider an alternative formulation. By a limited loss model, we mean a probability density on $[0,1]$ that is a combination of a continuous density on $[0,1)$ and a point mass at $\{1\}$ that may have a positive probability. This corresponds to the case when all losses may not exceed a particular maximum value. It is convenient to use that maximum value as the unit for expressing loss amounts. In effect, this amounts to a change of variable $x = \frac{t}{L}$ and the point mass at $\{1\}$ corresponds to the probability that a loss hits the per occurrence loss limit. For convenience, we further require that $S(t) > 0$ on $[0,1)$ (see [3] for a more complete discussion, where these models are related to "hazard functions with finite support").

In this case, some of the arithmetic is simplified, as we have fewer improper integrals to worry about.

A differentiable function, $F(t)$, on $[0,1)$ is a CDF of a limited loss model exactly when:

$$F(0) = 0, \quad \frac{dF}{dt} \geq 0, \quad \text{and} \quad \lim_{t \to 1} F(t) \leq 1$$

An integrable function $f(t)$ on $[0,1]$ is a PDF of a limited loss model exactly when:

$$f(t) \geq 0, \quad \text{and} \quad \int_0^1 f(t) dt = 1 - f(1)$$

Similarly, any nonnegative integrable function $h(t)$ on $[0,1)$ is a hazard rate function of a limited loss model. Observe that Propositions 1 and 2 apply in this context, when restricted to the open interval $(0,1)$, and we have:

Proposition 3A: *A differentiable function* $\rho(t)$ *on* $[0,1)$ *is a life expectancy of a limited loss model exactly when:*

$$\rho(t) > 0, \quad \frac{d\rho}{dt} \geq -1, \quad and \quad \int_0^1 \frac{1}{\rho(t)} dt = \infty$$

Proof: Let $\varphi(t) > 0$ be a differentiable function on $[0,1)$ such that $\frac{d\varphi}{dt} \geq -1$ on $[0,1)$ and consider the limited loss model determined via its hazard function, as above, by:

$$h(t) = h_\varphi(t) = \frac{1 + \frac{d\varphi}{dt}}{\varphi(t)} \geq 0 \ on\ [0,1)$$

Keeping the above notation, we have, just as before:

$$h(t) = \frac{1}{\varphi(t)} + \frac{d \ln \varphi}{dt} \Rightarrow g(t) = \int_0^t h(w) dw = \int_0^t \frac{dw}{\varphi(w)} + \ln\left(\frac{\varphi(t)}{\varphi(0)}\right)$$

$$\Rightarrow S(t) = e^{-g(t)} = \frac{\varphi(0) e^{-\int_0^t \frac{dw}{\varphi(w)}}}{\varphi(t)}$$

$$\Rightarrow \rho(t) = \int_t^1 \frac{S(v)}{S(t)} dv = \varphi(t) \int_t^1 \frac{e^{-\int_t^v \frac{dw}{\varphi(w)}}}{\varphi(v)} dv$$

Similar to before, using the change of variable

$$u(v) = \int_t^v \frac{dw}{\varphi(w)} \Rightarrow du = \frac{dv}{\varphi(v)}$$

$v = t$ corresponds to $u = 0$ and $v = 1$ corresponds to $u = \int_t^1 \frac{dw}{\varphi(w)}$

$$\Rightarrow \rho(t) = \varphi(t) \int_0^{\int_t^1 \frac{dw}{\varphi(w)}} e^{-u} du = \varphi(t)\left(1 - e^{-\int_t^1 \frac{dw}{\varphi(w)}}\right) \leq \varphi(t)$$

Which means that here too the LEF is the smallest solution to the differential equation (Proposition 1) that relates hazard with life expectancy and it follows that:

$$\rho(t) = \varphi(t) \Leftrightarrow \int_0^1 \frac{dw}{\varphi(w)} = \infty$$

and we have established the sufficiency of the conditions to be a LEF. For the necessity, it only remains to observe that $S(t) > 0$ on $[0,1)$ implies that $\rho(t) > 0$ on $[0,1)$, completing the proof.

Note that evidently:

$$\lim_{t \to 1} \rho(t) = 0$$

for the LEF of any limited loss model, even though we did not need to make that an explicit requirement in the statement of Proposition 3A.

V. Application to Multi-Dimensional Loss Models

One significant advantage of expectation of life is that it is rather simple to generate empirical data in multi-dimensional contexts. Given a database of individual claim information, it would be reasonable to expect to be able to identify closed cases and to be able to identify claims whose paid costs exceed a fixed amount x and whose ALAE exceeds a fixed amount y. Taking the average benefits =MeanCost(x,y) and average ALAE costs =MeanALAE(x,y) over that set of claims leads to another pair of positive numbers (U,V) = (U(x,y),V(x,y)) = (MeanCost(x,y)-x,MeanALAE(x,y)-y). Because we are considering closed cases, (U,V) can be regarded as a life expectancy or "reserve" vector. The association of (x,y) with (U,V) is a vector field which is termed an "expected survival" vector field in [5]. The correlation between claim costs, ALAE, and claim closure is all captured in that vector field.

Similarly, we could let x represent indemnity benefits and y medical benefits on Workers' Compensation claims. A good model of the survival vector field might help in the determination of case reserves or in modeling loss development.

It follows that an understanding of what type of functions can reasonably model life expectancy can be helpful in producing multi-dimensional survival models. It can be shown that these models are more flexible than traditional multi-variate loss models (see [5]). The use of piecewise linear functions to approximate life expectancy is straightforward, just noting, as above, the condition that the partial derivatives (where they exist) exceed or equal -1 and that the function be nonnegative sufficiently far from the origin.

To illustrate, we conclude this paper by presenting a model for using rational functions, as above, to approximate life expectancy in two dimensions. Begin with the observation that, formally:

$$\frac{a_{11}x^2 + \beta_1 xy + a_{12}y^2 + b_{11}x + b_{12}y + c_{11}}{x + y + d_{11}} = \frac{a_{11}x^2 + (b_{11} + \beta_1 y)x + (a_{12}y^2 + b_{12}y + c_{11})}{x + (y + d_{11})}$$
$$= \varphi(a_{11}, b_{11} + \beta_1 y, (d_{11} + y)\varphi(a_{12}, b_{12}, c_{11}, d_{11}; y), d_{11} + y; x)$$

Consider, therefore, the following two-dimensional vector field on the positive quadrant in the xy-plane:

$$U(x,y)=\hat{\varphi}(a_{11},b_{11}+\beta_1 y,(d_{11}+y)\varphi(a_{12},b_{12},c_{11},d_{11};y),d_{11}+y;x)$$
$$V(x,y)=\hat{\varphi}(a_{21},b_{21}+\beta_2 x,(d_{21}+x)\varphi(a_{22},b_{22},c_{21},d_{21};x),d_{21}+x;y) \quad x,y\in[0,\infty)$$

and then the vector field defined by:

$$\eta(x,y)=\left(\frac{-1+\frac{\partial U}{\partial x}}{U(x,y)}\ ,\ \frac{-1+\frac{\partial V}{\partial y}}{V(x,y)}\right) \qquad x,y\in[0,\infty)$$

We claim that the following conditions suffice to assure that η is a hazard vector field as defined in [5] (or what amounts to the same, that (U,V) is an expected survival vector field as defined there):

$$a_{j1}\ge 0,\ b_{j1}\ge -2\sqrt{a_{j1}c_{j1}},c_{j1}>0,\ d_{j1}>0,$$
$$a_{j2}\ge 0,b_{j2}\ge 2\sqrt{a_{j2}c_{j1}},\beta_j\ge -2\sqrt{a_{j1}a_{j2}} \qquad j=1,2$$

From the above, and the obvious symmetry in x and y, all is clear except to verify that these conditions assure that:

$$b_{11}+\beta_1 y\ge -2\sqrt{a_{11}(a_{12}y^2+b_{12}y+c_{11})} \quad \text{for all } y\ge 0$$

To see this, first note that

$$a_{12}y^2+b_{12}y+c_{11}\ge a_{12}y^2+2\sqrt{a_{12}c_{11}}\,y+c_{11}=\left(\sqrt{a_{12}}+\sqrt{c_{11}}\right)^2$$

And it follows that:

$$\begin{aligned}
&b_{11}+\beta_1 y+2\sqrt{a_{11}(a_{12}y^2+b_{12}y+c_{11})}\\
&\ge b_{11}+\beta_1 y+2\sqrt{a_{11}}\sqrt{(\sqrt{a_{12}}\,y+\sqrt{c_{11}})^2}\\
&=\ b_{11}+\beta_1 y+2\sqrt{a_{11}}(\sqrt{a_{12}}\,y+\sqrt{c_{11}})\\
&=\left(b_{11}+2\sqrt{a_{11}c_{11}}\right)+\left(\beta_1+2\sqrt{a_{11}a_{12}}\right)y\ge 0
\end{aligned}$$

and the result follows. In practice, however, the recommendation is to fit data without constraints and then make any ad hoc adjustments needed to assure the use of a valid LEF.

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Can Long Tailed Lines of Business Really Afford Higher Loss Ratios?

Jonathan P. Evans, FCAS, MAAA

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Abstract

Perhaps the most commonly accepted principle of modern property and liability insurance is that longer tailed lines of business are able to operate profitably at higher loss ratios, or almost equivalently higher combined ratios, than short tailed lines. A combined ratio of 120% might be devastating to an auto physical damage line of business but quite healthy for per occurrence excess liability reinsurance. However, this maxim may be eroding due to three real world forces:

1. The requirement that property and casualty insurers generally hold loss reserve liabilities at full undiscounted values.
2. The requirement that additional surplus capital be held to support risk in loss reserves on top of surplus held to support current writings.
3. The demands of investors, insurance executives, and modern capital markets that profits be high enough to support all invested capital at a cost per unit of capital judged to be commensurate with the perceived exposure to risk.

All of these factors may push necessary loss ratios for longer loss payment duration lines down to the levels necessary for short loss payment duration lines. In concrete terms, it may be that a per occurrence excess liability reinsurance line requires a combined ratio on the order of 95%, just like an auto physical damage line, to produce an equivalent return on invested capital. In this paper we review some modeling results for different sets of assumptions and examine this issue, but do not attempt to ultimately resolve it.

Note: Henceforth we shall use the terms "long duration" and "short duration" to refer to lines of business whose average times from policy inception to loss payments are long and short, respectively.

Caveat and Disclaimer: It is not the intent of this paper to strongly advocate the ultimate validity of a specific profitability model or specific values for model parameters such as surplus requirements. It does intend to show that within the range of different models and parameter assumptions, which may be appropriate according to contemporary actuarial practices and standards, there are frequent cases where longer duration lines require underwriting profit provisions equal to or greater than those for short duration lines.

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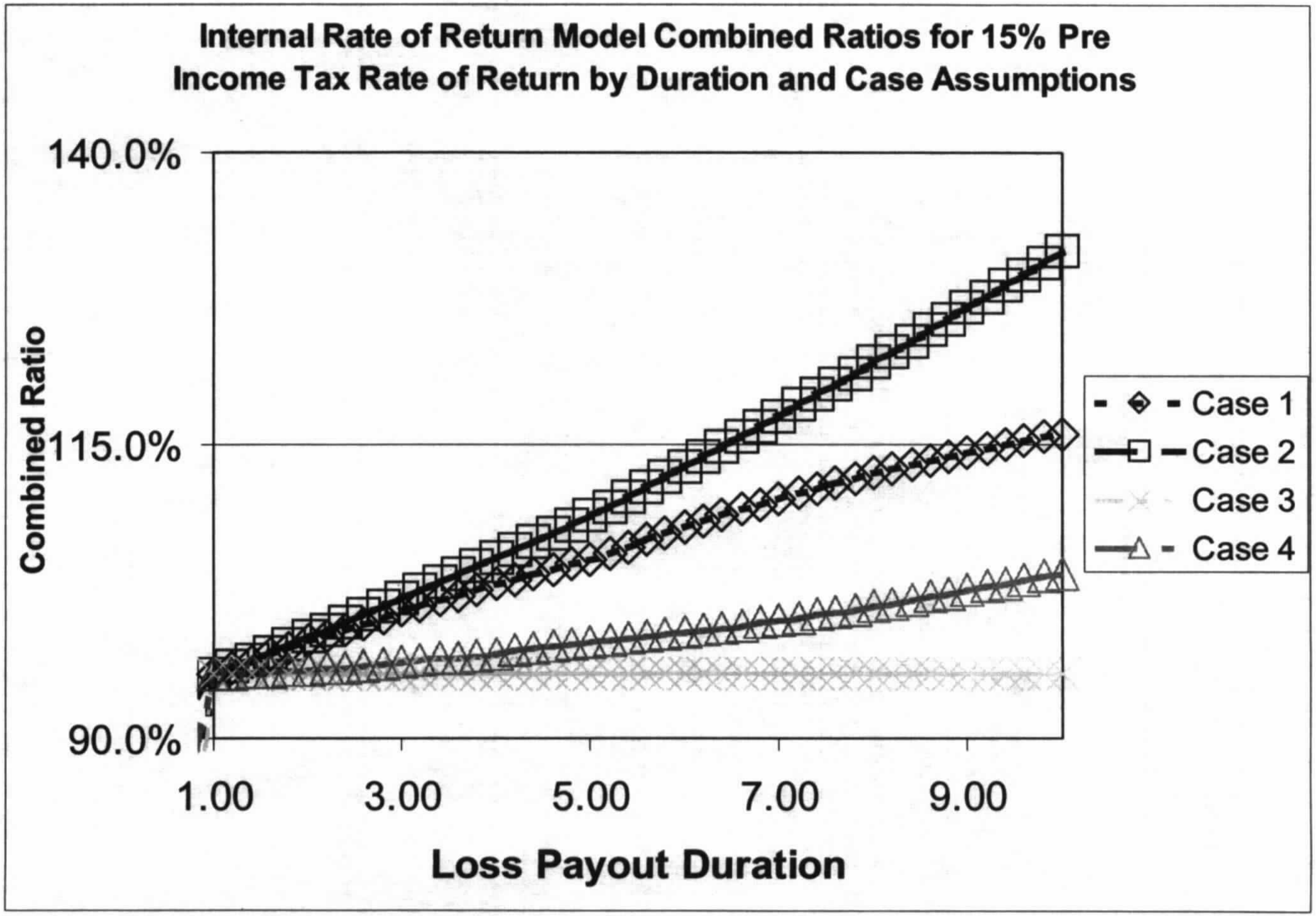
Some Results from an Internal Rate of Return Analysis

Consider the following results from a simple internal rate of return analysis (See Appendix I for modeling details):

Combined Ratios Necessary to Produce 15% Internal Rate of Return Before Income Taxes Under Different Assumptions

Case	Surplus Requirement	Loss Reserve Requirement	5.5 Years Undiscounted Loss Payout Duration	1.5 Years Undiscounted Loss Payout Duration
1	Released After Premium Earned	Undiscounted	106.2%	97.0%
2	Released After Premium Earned	Discounted	110.8%	97.1%
3	Held Until Loss Reserves Paid	Undiscounted	95.5%	95.5%
4	Held Until Loss Reserves Paid	Discounted	98.8%	95.6%

The traditional perspective is that Case 1 most accurately represents reality. Here we clearly see a higher combined ratio tolerance for the long duration line. However, Cases 3 or 4 may be closer to reality, for reasons which we will address subsequently. In Case 3 both lines must produce the same combined ratio to achieve their profitability objective.


Internal Rate of Return Model Combined Ratios for 15% Pre Income Tax Rate of Return by Duration and Case Assumptions
140.0%
115.0%
90.0%
Combined Ratio
1.00
3.00
5.00
7.00
9.00
Loss Payout Duration
Case 1
Case 2
Case 3
Case 4

Some Results from a Calendar Year Analysis

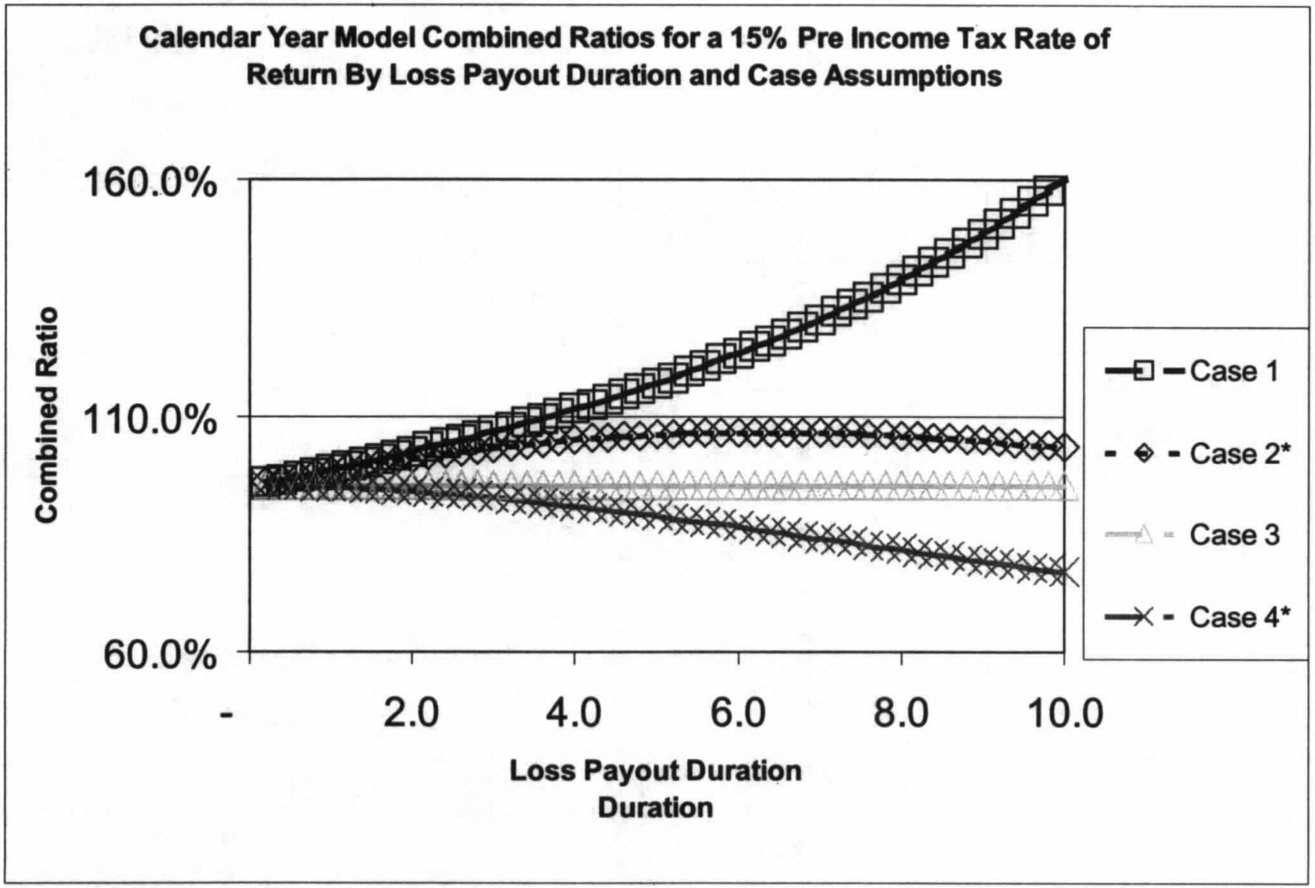
Now, we will alter our case assumptions slightly and consider results for an ongoing steady state calendar year analysis (See Appendix II for modeling details):

Combined Ratios Necessary to Produce 15% Calendar Year Return Before Income Taxes Under Different Assumptions

Case	5.5 Years Undiscounted Loss Payout Duration	1.5 Years Undiscounted Loss Payout Duration
1	120.0%	100.5%
2*	106.3%	99.7%
3	95.3%	95.3%
4*	87.8%	94.6%

Cases 1 and 3 embody basically the same surplus and reserve assumptions as in the previous internal rate of return analysis. Cases 2* and 4* are different from Cases 2 and 4 in the previous section. In Cases 2* and 4* loss reserves are actually held at undiscounted values in addition to surplus, but the loss reserve equity due to discount is included in the calculation of invested capital.

These results are even more stark. In Case 4*, where loss reserve equity is recognized as adding to invested capital and surplus is held to support loss reserves in addition to current writings, the allowed combined ratio is actually lower for the long duration line !


Calendar Year Model Combined Ratios for a 15% Pre Income Tax Rate of Return By Loss Payout Duration and Case Assumptions
160.0%
110.0%
60.0%
Combined Ratio
-
2.0
4.0
6.0
8.0
10.0
Loss Payout Duration
Duration
Case 1
Case 2*
Case 3
Case 4*

Loss Reserves

Higher loss ratios for longer duration lines are tolerated based on the justification that, from the calendar year perspective, the large reserves which build up to support long duration lines generate large amounts of investment income. Almost equivalently, from the internal rate of return perspective, one can say that much more investment income is earned between the time premium is collected and when losses are paid. This justification may be flawed. It ignores the additional cost of capital for large amounts of discount equity in the loss reserves. Loss reserves for long duration lines are generally held at undiscounted nominal values under both U.S. statutory accounting and GAAP.

It is generally true that return on equity and related profitability objectives set by insurance executives typically refer to return on GAAP equity, which excludes loss reserve discount equity, or a similar measure of return on invested capital. However, U.S. federal income tax accounting does consider equity in loss reserves. Such concerns are taken into account for valuations of books of loss reserves during acquisitions. They are also present in the minds of managers of long duration excess reinsurance companies.

Insurance companies must actually carry assets sufficient to cover nominal loss reserve liabilities in addition to their capital held as policyholder surplus and as deferred acquisition expense equity in their unearned premium reserves. In an economic sense the excess of nominal loss reserves over present value loss reserves is an additional contribution of invested capital by the insurer. The capital implicit in these nominal reserves demands much more profit be made to produce an overall rate of return consistent with the cost of invested capital.

Surplus Capital

Another consideration with regard to invested capital is the required level of statutory surplus held. Traditionally this required level has been set at a fixed ratio to yearly written or earned premium. This standard is regulated by the first NAIC IRIS test. Alone, it would imply that no surplus is needed for loss reserves. However, the recent addition of a Risk Based Capital (RBC) test requirement by the NAIC regulates that surplus also be required to support loss reserves. Although the RBC test does account for discount in its reserve risk component, this test is compared to an adjusted surplus where even tabular reserves are adjusted to nominal values. The RBC is therefore a requirement for assets in addition to undiscounted reserves and will generally be positive even in the case of a pure runoff portfolio. RBC generally results in a surplus requirement less than extending a leverage ratio to reserves in addition to premium. However, RBC will be higher, relative to annual premium levels, for a company writing long duration business. This requirement can add another large amount of invested capital, which must be supported at an appropriate cost per unit.

Even beyond the requirement imposed by RBC, credit rating agencies and financial analysts would be wary of large loss reserves unsupported by capital. Although the NAIC IRIS test 1 does not distinguish between different lines of business, many financial analysts do. It is not uncommon to see companies use different premium to surplus ratios, with long duration lines having lower leverage ratios, when doing internal allocations of surplus. For example, the overall premium to surplus ratio might be 1.00 with a 1.50 ratio for property lines and a 0.75 ratio for liability lines.

The argument may be made that the discount equity in the nominal loss reserves acts as a sufficient amount of capital at risk. However, if reserves are underestimated in any of a number of ways – neglect of IBNR, implicit discounting, etc. – this risk buffer may easily prove to be nonexistent. The discount buffer itself is highly sensitive to the effects of inflation and varying investment returns. Relying on this discount equity as the only risk buffer for loss reserves is often an unsuitable solution.

The Demands of Investors

Modern investment analysts and capital markets will recognize the total invested capital value of a company. If profits are not competitive with investments in the same broad category of risk, market forces will require divestiture or restructuring of operations.

It may be argued that recognition of a larger amount of capital leads to recognition of lower risk, and hence less pressure on profitability targets, due to a reduced cost per unit of capital. This argument is somewhat relevant when the comparison is a highly capitalized long duration line of business versus a minimal capitalization of the same long duration line. The same long duration line has the same underwriting obligations, and therefore the same volatility in its underwriting liabilities whether it is highly capitalized or minimally capitalized. More capital is likely to reduce the risk per unit of capital and hence the cost.

This same argument is usually not applicable if the relevant comparison is the larger capital invested in long duration insurance lines versus short duration lines. A long duration line, with its build up of volatile loss reserves or equivalently from the individual policy perspective the longer delay in reporting or payment of claims brings additional risk not present in a short duration line. The capital in both loss reserve discount and surplus supporting loss reserves may in fact be a reasonable requirement to cushion the extra risk at the same cost per unit of capital.

There is another point about cost of capital, aside from the arguments of changing risk and cost per unit of capital which might accompany changes in requirements for capital, or just changes in the recognition of total invested capital. It is probably unrealistic to expect markets, analysts, and possibly even executives to quickly adjust their targeted rates of return for such subtleties. That is to say any of these parties is very likely to fix on a standard such as: "Insurance operations should return 15% on investment." They are likely to apply the same standard of 15% to a larger amount of recognized capital, at least over the short term, for a specific company or a specific line of business.

It is difficult to dismiss the possibility that more absolute dollars of profit must be made to support a much larger capital base at roughly the same cost per unit of capital, for a long duration line.

The Risk in Large Loss Reserves

At this point it is warranted, based on the above discussions of surplus and required rates of return, to briefly consider in more detail the issue of risk in loss reserves. There is frequently a confusion that mature loss reserves for older accident or policy years are always less volatile than losses for current writings or reserves for more recent years. In some cases, where there is no possibility of pure IBNR and most claim cases have been closed, this may be true. It is often not true if risk is measured by an appropriate relative measurement such as the coefficient of variation of loss reserves. The confusion arises because older, mature accident or policy years are usually less volatile relative to their ultimate total losses. However, most of these ultimate losses have already been paid and are not being held as loss reserve liabilities. Relative to their loss reserves, older, mature accident or policy years may easily be as volatile as recent years' or next year's writings.

Consider the following hypothetical example (See Appendix IV for details):

	Coefficients of Variation		
Years After Policy Inception	**Total Accident/Policy Year Losses**	**Incremental Losses**	**Calendar Year Loss Reserves**
1	*15.7%*	7.4%	17.5%
2	*15.1%*	9.0%	18.9%
3	*14.4%*	11.0%	20.6%
4	*13.5%*	13.5%	22.5%
5	*12.4%*	16.6%	24.7%
6	*11.0%*	20.3%	27.4%
7	*9.3%*	24.8%	30.9%
8	*7.2%*	30.4%	36.0%
9	*4.6%*	37.3%	45.6%
10	*0.0%*	45.6%	NA
Correlated Totals		16.2%	17.2%

In the example above the total calendar year reserves of a company have a coefficient of variation, at 17.2%, which is higher than for the ultimate of a single accident/policy year's losses at inception, which is 16.2%. What declines over time is the coefficient of variation for the ultimate total losses for a given accident/policy year.

Inadequate Profits Versus Operating Losses, a Possible Mitigating Factor

A possible mitigating factor for the dangers of running long duration lines at high loss ratios may be found by examining what happens when loss ratios are high. Consider the internal rate of return results when we revisit Case 3 with a 120% combined ratio (See Appendix V for modeling details):

Internal Rates of Return Before Income Taxes Corresponding to 120% Combined Ratios for Case 3.

Case	Surplus Requirement	Loss Reserve Requirement	5.5 Years Undiscounted Loss Payout Duration	1.5 Years Undiscounted Loss Payout Duration
3	Held Until Loss Reserves Paid	Undiscounted	5.17%	-8.43%

Similarly, here are the calendar year results when we revisit Case 3 with a 120% combined ratio (See Appendix V for modeling details):

Calendar Year Returns Before Income Taxes Implied by a 120% Combined Ratio for Case 3 Assumptions

Case	5.5 Years Undiscounted Loss Payout Duration	1.5 Years Undiscounted Loss Payout Duration
3	7.08%	-3.70%

Previously, we had shown that both the calendar year and internal rate of return models indicated a combined ratio of slightly over 95% was needed for a pre-tax return of 15% in Case 3. When we change the combined ratio to 120% we see a consistent difference in both models between the different loss payout durations. The long duration line still produces a gross profit, although lower than our 15% target. The short duration line actually produces an operating loss.

A partial explanation of the insurance industry's general tolerance of higher loss or combined ratios for long duration lines may be that the consequence is only an inadequate rate of return, rather than actual dollar losses as would be the consequence for a short duration line.

Implications for Actuarial Practice

Actuarial practitioners doing profitability analyses, with emphasis on loss reserve payout durations, should take special care with the following considerations:

- What exactly is total invested capital ? What asset components such as unearned premium reserve equity, statutory surplus, loss reserve discount equity, etc. should be included in invested capital ?
- For what periods of time after policy inception must invested capital remain committed, and to what specific lines/exposures is invested capital allocated ? When exactly must capital be contributed and when exactly can it be released from corporate assets ?
- What is an appropriate rate of rate of return on invested capital ? Does this rate apply to all the components of invested capital or just a fraction of total invested capital ? Does this rate differ for different components ?

These questions are not new. There has been much discussion of these considerations by actuaries doing profitability analyses. However, as we have shown, differences in how these considerations are addressed by modeling assumptions may dramatically and qualitatively alter results for long duration lines of business. Specifically, it may change the relative performance benchmarks of long duration versus short duration lines of business

Conclusion

In this paper we have raised the question of whether long duration lines of business can run higher loss ratios than short duration lines and be equally profitable. We have shown that this principle is dependent on assumptions about invested capital and its associated cost per unit. Some common assumptions about these two considerations, which lead to higher loss ratio tolerances for long duration lines, may not be valid in the real world. These assumptions may be inconsistent with regulatory requirements, demands of investors, or perhaps even financial economic theory. The acceptance of higher loss ratios for long duration lines may be partially explained by the property that such cases tend to produce lower rates of return but not actual dollar operating losses. It is beyond the scope of this paper to propose a definite solution or take a specific stance on this issue. It is clear that in the insurance industry there is a great deal of confusion and disagreement about which assumptions should be used for profitability modeling. There are sets of assumptions, which are not entirely outlandish, implying that long duration lines of business should produce loss ratios equal to, or even below, loss ratios for short duration lines of business.

Appendix I

An internal Rate of Return Model

Here are some details of this specific IRR model:

- Time is measured in discrete years and each transaction is at year beginning or equivalently last year end.
- Premium is collected at year 0.
- Losses are reported and paid at the same time.
- All underwriting expenses are a fixed 30% of premium.
- All underwriting expenses are paid at year 0 and correspond to an investment of capital for equity in the unearned premium reserve for the time between year 0 and year 1.
- Initial surplus is an investment of capital equal to 50% of premium or equivalently 50% of the initial unearned premium reserve.
- Depending on the case assumptions surplus in subsequent years is either 0 or 50% of loss reserves.
- Depending on the case assumptions loss reserves are either held at discounted or undiscounted values.
- Invested assets correspond to the total of loss reserves, unearned premium reserves, and surplus.
- Investment income is a fixed 5% of the prior year's invested assets.
- The underwriting profit provision, or equivalently the loss ratio or combined ratio, is chosen to produce a 15% internal rate of return before income taxes.
- Income taxes are not explicitly modeled, but they could be reasonably modeled as a factor adjustment to the internal rate of return. (i.e. If income tax is 30% we are solving for a 10.5% after tax rate of return.)
- Although we have fixed premium and solved for loss ratio, the same underwriting profit provisions result if loss cost is fixed and we solve for premium.
- Although we have modeled all underwriting expenses as a variable, that is a fixed percentage of premium, we could have modeled fixed dollar expenses as a deduction to the loss cost resulting in an adjustment to the resulting loss ratio. The combined ratio would be unaffected.

Case 1 - 5.5 Year Loss Payment Duration

Surplus	Released
Loss Reserves	Nominal
UW provision	-6.2%

	0	1	2	3	4	5	6	7	8	9	10
Premium Collected	1000	0	0	0	0	0	0	0	0	0	0
Expense Ratio	30.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Loss Ratio	76.2%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus	500	-	-	-	-	-	-	-	-	-	-
Invested Capital	800	-	-	-	-	-	-	-	-	-	-
UEPR	1000	0	0	0	0	0	0	0	0	0	0
Nominal Loss Reserve	-	686	610	534	457	381	305	229	152	76	-
Total Invested Assets	1,500	686	610	534	457	381	305	229	152	76	-
Expense Payments	300	-	-	-	-	-	-	-	-	-	-
Incremental Loss Payout Pattern	0.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%
Incremental Loss Payout	-	76	76	76	76	76	76	76	76	76	76
Investment Income Rate	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%
Investment Income	-	75	34	30	27	23	19	15	11	8	4
Release of Earnings	(800)	813	34	30	27	23	19	15	11	8	4
IRR	15.0%										
Discount Factor	1.000	0.870	0.756	0.658	0.572	0.497	0.432	0.376	0.327	0.284	0.247
Cash Flow of Earnings and Capital	(800)	707	26	20	15	11	8	6	4	2	1
Discounted Loss Reserve		542	493	441	387	330	270	208	142	73	-

Case 1 - 1.5 Year Loss Payment Duration

Surplus	Released
Loss Reserves	*Nominal*
UW provision	3.0%

	0	1	2	3	4	5	6	7	8	9	10
Premium Collected	1000	0	0	0	0	0	0	0	0	0	0
Expense Ratio	30.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Loss Ratio	67.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus	500	-	-	-	-	-	-	-	-	-	-
Invested Capital	800	-	-	-	-	-	-	-	-	-	-
UEPR	1000	0	0	0	0	0	0	0	0	0	0
Nominal Loss Reserve	-	335	-	-	-	-	-	-	-	-	-
Total Invested Assets	1,500	335	-	-	-	-	-	-	-	-	-
Expense Payments	300	-	-	-	-	-	-	-	-	-	-
Incremental Loss Payout Pattern	0.0%	50.0%	50.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Incremental Loss Payout	-	335	335	-	-	-	-	-	-	-	-
Investment Income Rate	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%
Investment Income	-	75	17	-	-	-	-	-	-	-	-
Release of Earnings	(800)	905	17	0	0	0	0	0	0	0	0
IRR	15.0%										
Discount Factor	1.000	0.870	0.756	0.658	0.572	0.497	0.432	0.376	0.327	0.284	0.247
Cash Flow of Earnings and Capital	(800)	787	13	-	-	-	-	-	-	-	-
Discounted Loss Reserve		319	-	-	-	-	-	-	-	-	-

Case 2 - 5.5 Year Loss Payment Duration

Surplus	Released
Loss Reserves	Discounted
UW provision	-10.8%

	0	1	2	3	4	5	6	7	8	9	10
Premium Collected	1000	0	0	0	0	0	0	0	0	0	0
Expense Ratio	30.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Loss Ratio	80.8%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus	500	-	-	-	-	-	-	-	-	-	-
Invested Capital	*800*	-	-	-	-	-	-	-	-	-	-
UEPR	1000	0	0	0	0	0	0	0	0	0	0
Nominal Loss Reserve	-	727	646	566	485	404	323	242	162	81	(0)
Total Invested Assets	1,500	574	522	467	410	350	286	220	150	77	-
Expense Payments	300	-	-	-	-	-	-	-	-	-	-
Incremental Loss Payout Pattern	0.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%
Incremental Loss Payout	-	81	81	81	81	81	81	81	81	81	81
Investment Income Rate	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%
Investment Income	-	75	29	26	23	21	17	14	11	8	4
Release of Earnings	(800)	920	0	0	0	0	0	0	0	0	0
IRR	15.0%										
Discount Factor	1.000	0.870	0.756	0.658	0.572	0.497	0.432	0.376	0.327	0.284	0.247
Cash Flow of Earnings and Capital	(800)	800	-	-	-	-	-	-	-	-	-
Discounted Loss Reserve		574	522	467	410	350	286	220	150	77	-

Case 2 - 1.5 Year Loss Payment Duration

Surplus	Released
Loss Reserves	Discounted
UW provision	2.9%

	0	1	2	3	4	5	6	7	8	9	10
Premium Collected	1000	0	0	0	0	0	0	0	0	0	0
Expense Ratio	30.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Loss Ratio	67.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus	500	-	-	-	-	-	-	-	-	-	-
Invested Capital	800	-	-	-	-	-	-	-	-	-	-
UEPR	1000	0	0	0	0	0	0	0	0	0	0
Nominal Loss Reserve	-	335	-	-	-	-	-	-	-	-	-
Total Invested Assets	1,500	320	-	-	-	-	-	-	-	-	-
Expense Payments	300	-	-	-	-	-	-	-	-	-	-
Incremental Loss Payout Pattern	0.0%	50.0%	50.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Incremental Loss Payout	-	335	335	-	-	-	-	-	-	-	-
Investment Income Rate	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%
Investment Income	-	75	16	-	-	-	-	-	-	-	-
Release of Earnings	(800)	920	0	0	0	0	0	0	0	0	0
IRR	15.0%										
Discount Factor	1.000	0.870	0.756	0.658	0.572	0.497	0.432	0.376	0.327	0.284	0.247
Cash Flow of Earnings and Capital	(800)	800	-	-	-	-	-	-	-	-	-
Discounted Loss Reserve		320	-	-	-	-	-	-	-	-	-

Case 3 - 5.5 Year Loss Payment Duration

Surplus	Held
Loss Reserves	Nominal
UW provision	4.5%

	0	1	2	3	4	5	6	7	8	9	10
Premium Collected	1000	0	0	0	0	0	0	0	0	0	0
Expense Ratio	30.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Loss Ratio	65.5%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus	500	295	262	229	197	164	131	98	66	33	-
Invested Capital	800	295	262	229	197	164	131	98	66	33	-
UEPR	1000	0	0	0	0	0	0	0	0	0	0
Nominal Loss Reserve	-	590	524	459	393	328	262	197	131	66	-
Total Invested Assets	1,500	884	786	688	590	491	393	295	197	98	-
Expense Payments	300	-	-	-	-	-	-	-	-	-	-
Incremental Loss Payout Pattern	0.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%
Incremental Loss Payout	-	66	66	66	66	66	66	66	66	66	66
Investment Income Rate	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%
Investment Income	-	75	44	39	34	29	25	20	15	10	5
Release of Earnings	(800)	625	77	72	67	62	57	52	47	43	38
IRR	15.0%										
Discount Factor	1.000	0.870	0.756	0.658	0.572	0.497	0.432	0.376	0.327	0.284	0.247
Cash Flow of Earnings and Capital	(800)	544	58	47	38	31	25	20	16	12	9
Discounted Loss Reserve		466	423	379	332	284	232	178	122	62	-

Case 3 - 1.5 Year Loss Payment Duration

Surplus	Held
Loss Reserves	Nominal
UW provision	4.5%

	0	1	2	3	4	5	6	7	8	9	10
Premium Collected	1000	0	0	0	0	0	0	0	0	0	0
Expense Ratio	30.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Loss Ratio	65.5%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus	500	164	-	-	-	-	-	-	-	-	-
Invested Capital	800	164	-	-	-	-	-	-	-	-	-
UEPR	1000	0	0	0	0	0	0	0	0	0	0
Nominal Loss Reserve	-	328	-	-	-	-	-	-	-	-	-
Total Invested Assets	1,500	491	-	-	-	-	-	-	-	-	-
Expense Payments	300	-	-	-	-	-	-	-	-	-	-
Incremental Loss Payout Pattern	0.0%	50.0%	50.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Incremental Loss Payout	-	328	328	-	-	-	-	-	-	-	-
Investment Income Rate	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%
Investment Income	-	75	25	-	-	-	-	-	-	-	-
Release of Earnings	(800)	756	188	0	0	0	0	0	0	0	0
IRR	15.0%										
Discount Factor	1.000	0.870	0.756	0.658	0.572	0.497	0.432	0.376	0.327	0.284	0.247
Cash Flow of Earnings and Capital	(800)	658	142	-	-	-	-	-	-	-	-
Discounted Loss Reserve		312	-	-	-	-	-	-	-	-	-

Case 4 - 5.5 Year Loss Payment Duration

Surplus	Held
Loss Reserves	Disounted
UW provision	1.2%

	0	1	2	3	4	5	6	7	8	9	10
Premium Collected	1000	0	0	0	0	0	0	0	0	0	0
Expense Ratio	30.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Loss Ratio	68.8%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus	500	310	275	241	206	172	138	103	69	34	-
Invested Capital	800	310	275	241	206	172	138	103	69	34	-
UEPR	1000	0	0	0	0	0	0	0	0	0	0
Nominal Loss Reserve	-	619	551	482	413	344	275	206	138	69	-
Total Invested Assets	1,500	799	720	639	556	470	382	291	197	100	-
Expense Payments	300	-	-	-	-	-	-	-	-	-	-
Incremental Loss Payout Pattern	0.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%
Incremental Loss Payout	-	69	69	69	69	69	69	69	69	69	69
Investment Income Rate	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%
Investment Income	-	75	40	36	32	28	24	19	15	10	5
Release of Earnings	(800)	707	50	48	46	45	43	41	40	38	36
IRR	15.0%										
Discount Factor	1.000	0.870	0.756	0.658	0.572	0.497	0.432	0.376	0.327	0.284	0.247
Cash Flow of Earnings and Capital	(800)	615	38	32	27	22	19	16	13	11	9
Discounted Loss Reserve		489	445	398	349	298	244	187	128	66	-

Case 4 - 1.5 Year Loss Payment Duration

Surplus	Held
Loss Reserves	Disounted
UW provision	4.4%

Time Period	0	1	2	3	4	5	6	7	8	9	10
Premium Collected	1000	0	0	0	0	0	0	0	0	0	0
Expense Ratio	30.0%										
Loss Ratio	65.6%										
Surplus	500	164	-	-	-	-	-	-	-	-	-
Invested Capital	800	164	-	-	-	-	-	-	-	-	-
UEPR	1000	0	0	0	0	0	0	0	0	0	0
Nominal Loss Reserve	-	328	-	-	-	-	-	-	-	-	-
Total Invested Assets	1,500	477	-	-	-	-	-	-	-	-	-
Expense Payments	300	-	-	-	-	-	-	-	-	-	-
Incremental Loss Payout Pattern	0.0%	50.0%	50.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Incremental Loss Payout	-	328	328	-	-	-	-	-	-	-	-
Investment Income Rate	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%
Investment Income	-	75	24	-	-	-	-	-	-	-	-
Release of Earnings	(800)	770	172	0	0	0	0	0	0	0	0
IRR	15.0%										
Discount Factor	1.000	0.870	0.756	0.658	0.572	0.497	0.432	0.376	0.327	0.284	0.247
Cash Flow of Earnings and Capital	(800)	670	130	-	-	-	-	-	-	-	-
Discounted Loss Reserve		313	-	-	-	-	-	-	-	-	-

Appendix II

A Calendar Year Rate of Return Analysis

Here are some details of this specific calendar year rate of return model:

- Loss reserves are held at nominal undiscounted values.
- All underwriting expenses are a fixed 30% of premium.
- All underwriting expenses are paid up front and correspond to an investment of capital for equity in the unearned premium reserve.
- Depending on the case assumptions surplus is 50% of either premium, or premium and loss reserves.
- The unearned premium reserve is equal to 50% of the premium.
- Loss reserves are equal to the product of the premium, loss ratio, and duration. (See Appendix III.)
- The discount factor for computing loss reserve discount equity is based on a uniform payout pattern lasting for twice the payout duration. (See Appendix III.)
- Depending on the case assumptions loss reserve discount equity is or is not included in invested capital.
- Invested assets correspond to the total of loss reserves, unearned premium reserves, and surplus.
- Investment Income is a fixed 5% of Invested Assets.
- The underwriting profit provision, or equivalently the loss ratio or combined ratio, is chosen to produce a 15% calendar year rate of return before income taxes.
- Income taxes are not explicitly modeled, but they could be reasonably modeled as a factor adjustment to the internal rate of return. (i.e. If income tax is 30% we are solving for a 10.5% after tax rate of return.)
- Although we have fixed premium and solved for loss ratio, the same underwriting profit provisions result if loss cost is fixed and we solve for premium.
- Although we have modeled all underwriting expenses as a variable, that is a fixed percentage of premium, we could have modeled fixed dollar expenses as a deduction to the loss cost resulting in an adjustment to the resulting loss ratio. The combined ratio would be unaffected.

Case 1 Premium leverage based on premium, with no loss reserve discount equity included in invested assets.
Case 2* Premium leverage based on premium, with loss reserve discount equity included in invested assets.
Case 3 Premium leverage based on premium and loss reserves, with no loss reserve discount equity included in invested assets.
Case 4* Premium leverage based on premium and loss reserves, with loss reserve discount equity included in invested assets.

Calendar Year Model for 5.5 Year Loss Payout Duration

Case	Target ROR	Premium	Expense Ratio	Investment Rate of Return	Duration	Leverage	Combined Ratio
1	15%	1,000	30%	5.0%	5.50	2.00	120.0%
2*	15%	1,000	30%	5.0%	5.50	2.00	106.3%
3	15%	1,000	30%	5.0%	5.50	2.00	95.3%
4*	15%	1,000	30%	5.0%	5.50	2.00	87.8%

Case	UW Margin	Loss Ratio	UEPR	UEPR Equity	Loss Reserves Held	Loss Reserve Discount Equity	Statutory Surplus
1	-20.00%	90%	500	150	4,950	-	500
2*	-6.34%	76%	500	150	4,199	660	500
3	4.75%	65%	500	150	3,589	-	2,294
4*	12.24%	58%	500	150	3,177	500	2,088

Case	Invested Assets	Invested Capital	UW income	Investment Income	Earnings	ROR
1	5,950	650	(200)	298	97	15.0%
2*	5,199	1,310	(63)	260	197	15.0%
3	6,383	2,444	48	319	367	15.0%
4*	5,765	2,738	122	288	411	15.0%

Calendar Year Model for 1.5 Year Loss Payout Duration

Case	Target ROR	Premium	Expense Ratio	Investment Rate of Return	Duration	Leverage	Combined Ratio
1	15%	1,000	30%	5.0%	1.50	2.00	100.5%
2*	15%	1,000	30%	5.0%	1.50	2.00	99.7%
3	15%	1,000	30%	5.0%	1.50	2.00	95.3%
4*	15%	1,000	30%	5.0%	1.50	2.00	94.6%

Case	UW Margin	Loss Ratio	UEPR	UEPR Equity	Loss Reserves Held	Loss Reserve Discount Equity	Statutory Surplus
1	-0.54%	71%	500	150	1,058	-	500
2*	0.26%	70%	500	150	1,046	49	500
3	4.75%	65%	500	150	979	-	989
4*	5.43%	65%	500	150	968	46	984

Case	Invested Assets	Invested Capital	UW income	Investment Income	Earnings	ROR
1	2,058	650	(5)	103	98	15.0%
2*	2,046	699	3	102	105	15.0%
3	2,468	1,139	48	123	171	15.0%
4*	2,453	1,180	54	123	177	15.0%

Appendix III

Loss Reserves Held at a Point in Time and Discount Factor

We will calculate the average ratio of outstanding loss reserves to the rate of losses currently being incurred. The motivation behind this is to show that without growth or decline in written exposures the product of the premium, loss ratio, and duration is a reasonable estimate of loss reserves.

Let f(t) be the probability density for the time between when a certain amount of exposure is earned and the time when the corresponding losses are paid. Let F(t) be the corresponding cumulative distribution for f(t). Let D be the undiscounted duration or average time to loss payment, which we shall refer to as the "duration". Hence the following integral relations hold:

$$\int_0^\infty f(t)dt = 1$$

$$D = \int_0^\infty t \cdot f(t)dt$$

$$F(s) = \int_0^s f(t)dt$$

We define v(t) to the rate at which exposure (measured in incurred losses) is earned at time t. Consequently we can calculate the average outstanding loss reserves R at time 0 based on previously earned exposure :

$$R = \int_0^\infty \upsilon(-t) \cdot [1 - F(t)]dt$$

Next we let v(t) follow a constant exponential rate of growth and solve the integral using integration by parts:

$$\upsilon(t) = e^{\alpha t}$$

$$R = \int_0^\infty e^{-\alpha t} \cdot [1 - F(t)]dt$$

$$= \left[\frac{e^{-\alpha t}}{-\alpha} [1 - F(t)] \right]_0^\infty - \int_0^\infty \frac{e^{-\alpha t}}{-\alpha} f(t)dt$$

$$= \frac{1}{\alpha} - \frac{M_T(-\alpha)}{\alpha}$$

$$= \frac{1 - M_T(-\alpha)}{\alpha}$$

where $M_T(\ )$ is the moment generating function of the density f(t). Finally, we can use L'Hospital's Rule to evaluate this expression for the steady state case, where growth is zero:

$$R = \left. \frac{M_T'(-\alpha)}{1} \right|_{\alpha=0} = E[T] = D$$

Since we defined our exposure to be 1 unit of loss per time period at time 0, the duration is a reasonable estimate of the ratio of outstanding loss reserves to the rate of losses incurred at a point in time, when there has been 0 growth for a long time prior.
Now we will address the issue of the average discount factor for loss reserves. If we denote the discount factor for dollars paid at time s, by the symbol a(s), the following expression holds.

$$PV(Loss\,\mathrm{Re}\,serves) =$$

$$\int_0^{\infty} v(-t)dt \int_0^{\infty} f(t+s)a(s)ds$$

We will set the discount factor to correspond to continually compounded interest, the loss payout density to be uniform between 0 and 2D, and the exposure to be uniformly earned at a rate of 1 :

$$a(s) = e^{-\beta s}$$

$$f(t) = \frac{1}{2D} \qquad t \in [0,2D]$$

$$f(t) = 0 \qquad t \notin [0,2D]$$

$$v(t) = 1$$

Now we can evaluate the present value of loss reserves:

$$PV(Loss\,\mathrm{Re}\,serves) = \int_0^{2D} dt \int_0^{2D-t} \frac{e^{-\beta s}}{2D} ds$$

$$= \int_0^{2D} \frac{1-e^{-\beta(2D-t)}}{2\beta D} dt$$

$$= \frac{1}{\beta} + \frac{e^{-2\beta D} - 1}{2\beta^2 D}$$

We can divide this by the nominal amount of reserves, which we have previously shown to be D, to get an overall discount factor:

$$\frac{PV(Loss\,\mathrm{Re}\,serves)}{Loss\,\mathrm{Re}\,serves} = \frac{\frac{1}{\beta} + \frac{e^{-2\beta D} - 1}{2\beta^2 D}}{D}$$

$$= \frac{1}{\beta D} + \frac{e^{-2\beta D} - 1}{2\beta^2 D^2}$$

Appendix IV

Hypothetical Demonstration of Volatility of Loss Reserves

- This demonstration uses a flat dollar amount reporting/payment pattern over 10 years after policy inception.
- Time is discrete and losses are reported and paid at the same time
- The number of claim counts reported/paid for a given policy year in a given calendar year after policy inception is Poisson distributed.
- The severity of claims is uniformly distributed between 0 and twice the average severity.
- As the policy year matures the expected number of claims reported/paid in a given calendar year decreases and their severity increases.
- The incremental dollar amounts of losses for different calendar periods after policy inception for the same policy year have a correlation coefficient of 50%. This is used to determine the total variance for the losses of a policy year.
- Similarly, the total dollar amounts of loss reserves for different accident/policy years have a 50% correlation. This is used to determine the total variance for the loss reserves of a calendar year, assuming no growth or decline in written exposure volume.

Hypothetical Demonstration of Loss Reserve Volatility

Correlation Coefficient of Incremental Losses for an Accident/Policy Year	50%
Correlation of Between Loss Reserves for Different Accident/Policy Years	50%

Incremental Policy/Accident Year Losses

Years After Policy Inception	Mean Incremental Losses on Policy	Poisson Frequency of Claims	Mean Claim Severity	Claim Severity Variance	Variance of Incremental Losses	Standard Deviation of Incremental Losses	Coefficient of Variation of Incremental Losses
1	1,000,000	200.0	5,000	2,083,333	5,416,666,667	73,598	7.4%
2	1,000,000	133.3	7,500	4,687,500	8,125,000,000	90,139	9.0%
3	1,000,000	88.9	11,250	10,546,875	12,187,500,000	110,397	11.0%
4	1,000,000	59.3	16,875	23,730,469	18,281,250,000	135,208	13.5%
5	1,000,000	39.5	25,313	53,393,555	27,421,875,000	165,596	16.6%
6	1,000,000	26.3	37,969	120,135,498	41,132,812,500	202,812	20.3%
7	1,000,000	17.6	56,953	270,304,871	61,699,218,750	248,393	24.8%
8	1,000,000	11.7	85,430	608,185,959	92,548,828,125	304,218	30.4%
9	1,000,000	7.8	128,145	1,368,418,407	138,823,242,188	372,590	37.3%
10	1,000,000	5.2	192,217	3,078,941,417	208,234,863,281	456,328	45.6%
Total	10,000,000	589.6	16,961	4,373,583,686	2,638,178,325,928	1,624,247	16.2%

Policy/Accident Year Reserves

Years After Policy Inception	Mean Loss Reserves	Variance of Loss Reserves	Standard Deviation of Loss Reserves	Coefficient of Variation of Loss Reserves
1	9,000,000	2,479,259,701,390	1,574,567	17.5%
2	8,000,000	2,291,258,965,740	1,513,691	18.9%
3	7,000,000	2,070,957,081,014	1,439,082	20.6%
4	6,000,000	1,816,070,055,659	1,347,616	22.5%
5	5,000,000	1,526,288,345,750	1,235,430	24.7%
6	4,000,000	1,204,964,483,487	1,097,709	27.4%
7	3,000,000	861,801,932,280	928,333	30.9%
8	2,000,000	517,081,159,368	719,084	36.0%
9	1,000,000	208,234,863,281	456,328	45.6%
10	-	-	-	NA
Calendar Year Totals	45,000,000	59,654,966,719,124	7,723,663	17.2%

Ultimate Losses

Years After Policy Inception	*Coefficient of Variation of Ultimate Accident/Policy Year Losses*
1	*15.7%*
2	*15.1%*
3	*14.4%*
4	*13.5%*
5	*12.4%*
6	*11.0%*
7	*9.3%*
8	*7.2%*
9	*4.6%*
10	*0.0%*

Appendix V

Calendar Year Model Case 3 at 120% Combined Ratio for 5.5 Year Loss Payout Duration

Case	Target ROR	Premium	Expense Ratio	Investment Rate of Return	Duration	Leverage	Combined Ratio
3	7.08%	1,000	30%	5.0%	5.50	2.00	120.0%

Case	UW Margin	Loss Ratio	UEPR	UEPR Equity	Loss Reserves Held	Loss Reserve Discount Equity	Statutory Surplus
3	-20.00%	90%	500	150	4,950	-	2,975

Case	Invested Assets	Invested Capital	UW income	Investment Income	Earnings	ROR
3	8,425	3,125	(200)	421	221	7.1%

Calendar Year Model Case 3 at 120% Combined Ratio for 1.5 Year Loss Payout Duration

Case	Target ROR	Premium	Expense Ratio	Investment Rate of Return	Duration	Leverage	Combined Ratio
3	-3.70%	1,000	30%	5.0%	1.50	2.00	120.0%

Case	UW Margin	Loss Ratio	UEPR	UEPR Equity	Loss Reserves Held	Loss Reserve Discount Equity	Statutory Surplus
3	-20.03%	90%	500	150	1,350	-	1,175

Case	Invested Assets	Invested Capital	UW income	Investment Income	Earnings	ROR
3	3,026	1,325	(200)	151	(49)	-3.7%

120% Combined Ratio for Case 3 - 5.5 Year Loss Payment Duration

Surplus	Held
Loss Reserves	Nominal
UW provision	-20.0%

	0	1	2	3	4	5	6	7	8	9	10
Premium Collected	1000	0	0	0	0	0	0	0	0	0	0
Expense Ratio	30.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Loss Ratio	90.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus	500	405	360	315	270	225	180	135	90	45	(0)
Invested Capital	800	405	360	315	270	225	180	135	90	45	(0)
UEPR	1000	0	0	0	0	0	0	0	0	0	0
Nominal Loss Reserve	-	810	720	630	540	450	360	270	180	90	(0)
Total Invested Assets	1,500	1,215	1,080	945	810	675	540	405	270	135	(0)
Expense Payments	300	-	-	-	-	-	-	-	-	-	-
Incremental Loss Payout Pattern	0.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%
Incremental Loss Payout	-	90	90	90	90	90	90	90	90	90	90
Investment Income Rate	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%
Investment Income	-	75	61	54	47	41	34	27	20	14	7
Release of Earnings	(800)	270	106	99	92	86	79	72	65	59	52
IRR	5.2%										
Discount Factor	1.000	0.951	0.904	0.860	0.817	0.777	0.739	0.703	0.668	0.635	0.604
Cash Flow of Earnings and Capital	(800)	257	96	85	75	66	58	51	44	37	31
Discounted Loss Reserve		640	582	521	457	390	319	245	167	86	-

120% Combined Ratio for Case 3 - 1.5 Year Loss Payment Duration

Surplus Held
Loss Reserves Nominal

UW provision -20.0%

	0	1	2	3	4	5	6	7	8	9	10
Premium Collected	1000	0	0	0	0	0	0	0	0	0	0
Expense Ratio	30.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Loss Ratio	90.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus	500	225	-	-	-	-	-	-	-	-	-
Invested Capital	800	225	-	-	-	-	-	-	-	-	-
UEPR	1000	0	0	0	0	0	0	0	0	0	0
Nominal Loss Reserve	-	450	-	-	-	-	-	-	-	-	-
Total Invested Assets	1,500	675	-	-	-	-	-	-	-	-	-
Expense Payments	300	-	-	-	-	-	-	-	-	-	-
Incremental Loss Payout Pattern	0.0%	50.0%	50.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Incremental Loss Payout	-	450	450	-	-	-	-	-	-	-	-
Investment Income Rate	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%
Investment Income	-	75	34	-	-	-	-	-	-	-	-
Release of Earnings	(800)	450	259	0	0	0	0	0	0	0	0
IRR	-8.4%										
Discount Factor	1.000	1.092	1.193	1.302	1.422	1.553	1.696	1.852	2.023	2.209	2.412
Cash Flow of Earnings and Capital	(800)	491	309	-	-	-	-	-	-	-	-
Discounted Loss Reserve		429	-	-	-	-	-	-	-	-	-

Bibliographic Note

There have been many papers published in the Proceedings of the Casualty Actuarial Society and Casualty Actuarial Society Forum which address underwriting profit provision models. By no means are the points discussed in this paper entirely original. However, the author does not endorse any particular pricing model presented in this paper or elsewhere. The reader interested in further information should consult the research sections of the Casualty Actuarial Society website: www.casact.org

Misapplications of Internal Rate of Return Models in Property/Liability Insurance Ratemaking

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MISAPPLICATIONS OF INTERNAL RATE OF RETURN MODELS IN PROPERTY/LIABILITY INSURANCE RATEMAKING

Abstract

This paper describes two common misapplications of internal rate of return (IRR) models in property/liability insurance ratemaking. These misapplications have contributed to the popular belief that the fair premium is heavily dependent on supporting surplus, leading casualty actuaries to devote much time and attention to techniques of surplus allocation. In a correct property/liability pricing application, premium is scarcely impacted by changes in supporting surplus.

1. INTRODUCTION

The internal rate of return (IRR) model has been widely utilized for P/L insurance ratemaking, both for regulatory purposes and internal pricing studies. The National Council on Compensation Insurance (NCCI), for example, has extensively utilized IRR models for workers compensation rate filings. Feldblum (1992) describes and discusses NCCI's IRR model in depth.

The IRR method determines the fair premium by equating the internal rate of return with the cost of equity capital. Most practical applications of the IRR method accomplish this task by performing two steps independently. In step one, the user specifies the cost of equity capital, r_e. Feldblum describes several approaches to determining r_e, including the CAPM, the Gordon Growth Model, and an analysis of historical returns in the industry. In step 2, the user calculates the premium that equates the IRR with the selected cost of equity capital.

Myers and Cohn (1987) have developed an alternative discounted cash flow model. The Myers/Cohn (M/C) technique determines the fair premium for a P/L insurance policy according to the following formula:

Fair premium = PV of expected loss and expense

+ PV of the tax burden on the insurer's underwriting and investment income

The original M/C model ignored default risk, implicitly assuming that the insurer holds enough surplus to reduce the probability of ruin to a negligible level.

In a 1990 article in the *Journal of Risk and Insurance*, J. David Cummins compared and contrasted the IRR and M/C models. In particular, Cummins demonstrated that the models are nearly equivalent in a one-period (that is, two-date) ratemaking application. Section 2 of this paper provides a demonstration that is similar to that of Cummins.[1] In doing so, Section 2 highlights the first misapplication of most practical IRR models: failing to recognize the relationship between the cost of equity capital and the amount of supporting surplus.

Section 3 extends the original Cummins demonstration by pointing out the second misapplication: confusion between the average and marginal investment strategy. Lastly, Section 4 closes with three related topics: (1) problems with the IRR model in a multi-period setting, (2) the concept of "notional surplus", and (3) dealing with default risk and convexity.

2. MISAPPLICATION ONE: FAILING TO RECOGNIZE THE RELATIONSHIP BETWEEN THE COST OF EQUITY CAPITAL AND THE AMOUNT OF SUPPORTING SURPLUS

In the application of the IRR model, the internal rate of return varies inversely with the amount of supporting surplus. For instance, let's assume that we've allocated $1,000 of surplus to an insurance contract with a 10% cost of equity capital.[2] Given the premium for the policy, the expected loss amount, and the expected investment return, we can calculate the IRR – let's say it's equal to the 10% hurdle rate: that is, this is an acceptable risk.

[1] The demonstration in Section 2 has clarified some of the assumptions in the Cummins paper and slightly modified the approach.

[2] Also assume that this $1,000 of surplus is greater than or equal to some minimum solvency requirement, S_M. This assumption will be clarified later in the paper.

Now, let's assume that the amount of supporting surplus is increased to $2,000. At this level, the new IRR will decrease; this new IRR will, in fact, fall below the 10% hurdle rate. The risk is no longer acceptable.

Unfortunately, this type of analysis is plagued by the first misapplication: it correctly recognizes that the IRR varies inversely with supporting surplus, but fails to recognize that the cost of equity capital does too. In fact, once this misapplication is corrected, the IRR premium is essentially equivalent to the M/C premium.[3] We will illustrate this result with a one-period ratemaking model, both with and without federal income taxes. Section 4 extends the discussion to multi-period models.

One Period Model in the Absence of Taxes

Assume a one-period insurance ratemaking model in the absence of federal income taxes. The insurer collects a premium of P at time 0, in exchange for assuming an expected loss and expense amount of L at time 1. The insurer's shareholders have committed S of surplus at time 0. The insurer then invests the premium and surplus funds, P+S, in financial assets with an expected return of r_A. At the end of of the period, the difference between assets and losses will be returned to the shareholders.

As in the Myers/Cohn model, we will assume that the probability of insolvency is zero. That is, let S_M be the amount of capital required to ensure that the assets will exceed losses at time 1 in all states of the world. We assume that the actual surplus committed by shareholders is greater than or equal to S_M; that is $S>=S_M$.

[3] The relationship is exact in a one-period ratemaking model with no taxes. This will be demonstrated subsequently in the paper.

In the absence of taxes, the Myers/Cohn formula reduces to: fair premium = discounted value of expected losses and expenses. In symbols, we have P = PV(L). Note that the fair premium in this case does not depend on the amount of surplus, S, provided that $S >= S_M$.[4]

In order to calculate the IRR, we need to determine the cash flows to and from the insurance shareholders at the beginning and end of the period. At time 0, the shareholders commit S of capital; at time 1, the shareholders receive the difference between the assets and the losses and expenses, or $(P+S)(1+r_A) - L$. Thus, IRR is the solution of the following equation:

$$-S + [(P+S)(1+r_A)-L] / (1+IRR) = 0. \quad (2.1)$$

Solving this equation for IRR and equating to the cost of equity capital, r_e, gives us the following:

$$[(P+S)(1+r_A)-L]/S - 1 = IRR = r_e. \quad (2.2)$$

Lastly, by solving equation (2.2) for P, we have the fair premium according to the IRR method:

$$P = [(r_e-r_A)S + L] / (1+r_A) \quad (2.3)$$

In the actual application formula (2.3), most IRR models make two important assumptions. First, it is often assumed that the insurer invests in super-safe government debt; hence, $r_A = r_f$, where r_f is the risk-free rate of interest. Second, because the shareholders bear the underwriting risk of insurance, the models generally assume that $r_e > r_f$. Together, these two assumptions imply that the cost of equity capital is greater than the expected investment return (that is, $r_e > r_A$).

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[4] This statement is not necessarily true in the presence of taxes or bankruptcy costs.

In order to determine the relationship between premium and supporting surplus in the IRR model, we calculate the first derivative of premium with respect to surplus in formula (2.3): $dP/dS = (r_e - r_A) / (1 + r_A)$. As shown in the preceding paragraph, under the standard IRR assumptions $r_e > r_A$. Thus, $dP/dS > 0$, implying that fair premium in the IRR model is directly proportional to supporting surplus, even in the absence of federal income taxes and default risk.

Hence, the M/C model and the IRR model apparently provide contradictory results. By digging a little deeper, however, we will find that the discrepancy results from a common misapplication of the IRR model. Specifically, most practical applications of the IRR model implicitly assume that the cost of equity capital, r_e, is independent of the amount of supporting surplus. In reality, we demonstrate below that the cost of equity capital is inversely related to supporting surplus, assuming that P,L, and r_A are held constant.

In a 1968 *Proceedings of the Casualty Actuarial Society* paper, Ferrari proposed viewing the P/L insurer as a levered equity trust. In other words, Ferrari visualized the insurer as borrowing funds from policyholders, then investing the combined policyholder and shareholder funds in financial assets. This levered equity trust analogy points out that the shareholders of a P/L insurer hold a *residual claim* on the insurer's assets. By decreasing the amount of supporting surplus – for a fixed P,L, and r_A – we increase the insurer's *financial leverage*.[5] Increasing financial leverage creates a riskier position for shareholders, since their residual claim on the firm becomes more volatile.[6] This increased risk is reflected, in turn, by a higher cost of equity capital.

In a classic financial paper, Modigliani and Miller, or "MM", derived a well-known formula describing the relationship between financial leverage and the cost of equity capital. Specifically, MM's proposition II formula states:

[5] Financial leverage is the ratio of the discounted value of liabilities to supporting surplus.

[6] For a simple and clear mathematical demonstration of the relationship between leverage and volatility, see Brealey and Myers (1996), pp. 451-454.

$$r_e = r_A + (D/E)(r_A - r_D), \quad (2.4)$$

where r_e is the cost of equity capital, r_A is the expected return on assets, r_D is the expected return on debt, and D/E is the financial leverage ratio in terms of market (or present) values.

In the one-period insurance example of this section, the r_D term in the MM formula is given by $r_D = L/P - 1$.[8] The financial leverage ratio is given by PV(L)/S. This gives us the following formula for the cost of equity capital to the P/L insurer.[9]

$$r_e = r_A + [PV(L)/S][r_A - (L/P) + 1] \quad (2.5)$$

By solving equation (2.1) for the internal rate of return, we have the corresponding formula for the IRR:

$$IRR = r_A + (P/S)[r_A - (L/P) + 1] \quad (2.6)$$

Lastly, by comparing formulas (2.5) and (2.6), we see that r_e = IRR if and only if P = PV(L). Thus, the IRR model and the DCF model provide a consistent answer. In the absence of taxes and default risk, the fair premium equals the discounted value of expected losses and expenses.

One-Period Insurance Ratemaking Model in the Presence of Taxes

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[7] In their original proof, MM utilized four simplifying assumptions: (1) no costs of bankruptcy, (2) risk-free debt, (3) no signaling opportunities, and (4) no agency costs. The risk-free debt assumption would seem to rule out our insurance example, where actual losses and expenses are variable. Fortunately, relaxing the assumption that debt is risk-free will not change the MM results; see, for instance, pages 462-464 of Weston and Copeland.

[8] In other words, the cost of debt is the expected underwriting loss as a percentage of the policyholder premium.

Next, we extend the one-period ratemaking model to incorporate federal income taxes. In order to incorporate federal taxes into any DCF model, the user must first specify (either implicitly or explicitly) the applicable assumptions regarding three key items:

(1) The relationship between the expected return on bonds and stocks of equivalent risk; or, in financial theory, the selected version of "debt and taxes."

(2) The insurer's asset allocation.

(3) The convexity structure of the corporate tax code.[10]

Most DCF models in practical use make the following assumptions regarding these items:[11]

(1) The expected (or required) return on risk-free common stock equals the interest rate on risk-free government debt. In other words, bonds and stocks of identical risk offer the same expected return. Brealey and Myers (1996) refer to this as the MM "corrected" theory of debt and taxes.

(2) The insurer invests only in risk-free government (i.e. taxable) debt.

(3) The insurer's expected tax liability equals the product of the corporate tax rate and the insurer's expected taxable income.[12]

In order to maintain consistency with current models, we will maintain these assumptions in this section. Moreover, we will also assume that r_L is the appropriate discount rate for expected losses and expenses,[13] and T_c is the marginal corporate tax rate.

Under these assumptions, the Myers/Cohn formula for fair premium is as follows:

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[9] Note that this formula is very similar to the well-known Ferrari formula. The major difference is that the MM formula refers to cash flows and market values, while Ferrari's formula focuses on accounting values.

[10] For a discussion of the role of convexity in insurance pricing, see Vaughn (1999).

[11] These assumptions are also consistent with the assumptions made in the original Myers and Cohn paper.

[12] In financial terms, this is equivalent to specifying a linear (not convex) corporate tax code.

[13] For many P/L lines, indemnity losses possess very little systematic risk. As such, the risk-free rate is often used as an acceptable approximation for r_L.

Fair Premium = Present Value of Expected Losses and Expenses

+ Present Value of Tax on Investment Income

+ Present Value of Tax on Underwriting Income

OR

$$P = L/(1+r_L) + [(P+S)r_f T_c]/(1+r_f) + PT_c/(1+rf) - LT_c/(1+r_L) \qquad (2.7)$$

Solving equation (2.7) for P gives us:

$$P = L/(1+r_L) + Sr_f T_c/[(1+r_f)(1-T_c)] \qquad (2.8)$$

Under the same assumptions, the IRR is given by the solution of the following formula:

$$-S + \{(P+S)(1+r_f) - L - T_c[(P+S)r_f + (P-L)]\}/(1+IRR) = 0. \qquad (2.9)$$

Solving equation (2.9) for the IRR and setting equal to the cost of equity capital, r_e, gives us:

$$\{(P+S)(1+r_f) - L - T_c[(P+S)r_f + (P-L)]\}/S - 1 = IRR = r_e \qquad (2.10)$$

The original MM formula for the cost of equity capital (discussed in the previous section) ignored taxes. In a 1963 paper, Modigliani and Miller revised their analysis to accommodate corporate taxes. This MM "corrected" formula for the cost of equity capital is: $r_e = r_A + (1-T_c)(D/E)(r_A-r_D)$. In our insurance example, this formula translates to:

$$r_e = r_f + (1-T_c)\{[L/(1+r_L)]/S\}(r_f-r_L) \qquad (2.11)$$

Thus, by substituting the formula for r_e in equation (2.11) into equation (2.10) and solving for P, we have:

$$P = L/(1+r_L) + Sr_fT_c/[(1+r_f)(1-T_c)] + [(L/S)(r_f-r_L)]/[(1+r_L)(1+r_f)] \qquad (2.12)$$

Note that equation (2.12) is equivalent to equation (2.8) with the exception of the additional (third) term on the right-hand-side. Yet, visual inspection of the formula reveals that the amount of this additional term is negligible compared to the total premium. Hence, the fair premium in the IRR model very closely approximates the fair premium in the Myers/Cohn model, even in the presence of taxes -- provided that the cost of equity capital in the IRR model is correctly calculated.

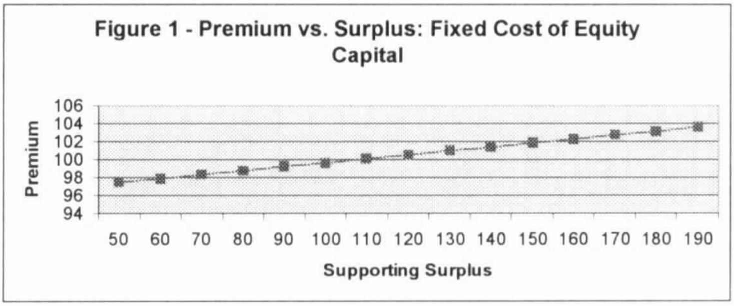
An Illustrative Example

For purposes of illustration, let's put some numbers on the one-period model of this section. Assume the following values for each of the necessary variables:

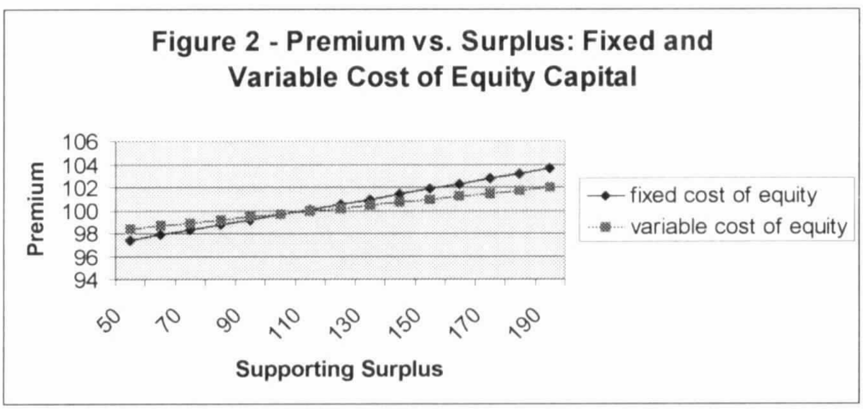
$L = \$100$, $r_f = 5\%$, $r_L = 3\%$, $S = \$100$, $T_c = 35\%$

The fair premium according to the Myers/Cohn model is given by equation (2.8) and equals \$99.65. The fair premium according to the IRR model, given by equation (2.12), equals \$99.67 -- a negligible difference from the M/C premium.[14]

Furthermore, let's examine the sensitivity of the IRR premium to changes in the amount of supporting surplus using the values assumed above. First, assume – as in most practical IRR models – that the cost of equity capital is fixed regardless of the amount of supporting surplus. Figure 1 displays the fair premium as a function of supporting surplus under this assumption:



Yet, by correcting this first misapplication the slope of this graph will change significantly. Specifically, utilizing formula (2.11) to calculate the cost of equity capital gives us a "flatter" relationship between premium and supporting surplus. Figure 2 graphically demonstrates the resulting premium for both approaches.



[14] Cummins (1990, pp. 90-91) notes that the two models are exactly equivalent if and only if $r_L = r_f$. In terms of formula (2.12), note that if $r_L = r_f$, the third term drops off and the two formulas are identical.

Note that the IRR premium is still highly dependent on supporting surplus. In the next section, however, we will further flatten the graph by correcting the second misapplication.

3. THE SECOND MISAPPLICATION: CONFUSION BETWEEN AVERAGE AND MARGINAL INVESTMENT STRATEGIES

In the previous section, we assumed an all-taxable-bond asset allocation and an MM "corrected" theory of debt and taxes. In the MM "corrected" model, there is a very strong tax disadvantage to corporate lending. Given this tax disadvantage, an all-taxable-bond portfolio would be highly suboptimal.[15] Under these assumptions, a value-maximizing insurer would invest a substantial amount of the available funds in municipal bonds and/or common stock.

As an illustration, let's maintain the Section 2 assumptions regarding "debt and taxes" and convexity: that is, an MM "corrected" world with a linear tax code. Assume, however, that the insurer allocates the P+S of available funds as follows: invest P in risk-free taxable bonds, and invest S in common stock of equivalent systematic risk (that is, "zero-beta" common stock).

In the MM "corrected" world, both the taxable bonds and the common stock will be priced to offer an expected (pre-tax) return of r_f. Interest payments from the taxable bonds will still be taxed at the full corporate rate of 35%. The effective tax rate on the common stock will be less than 35%, owing to two provisions of the corporate tax code: (1) only 30% of the dividends on common stock are taxed, and (2) unrealized capital gains escape taxation entirely.

Now, recall equation (2.12) for the fair premium according to the IRR rule in the presence of federal income taxes. Ignoring the negligible third term on the right-hand-side, this formula can be described in words as follows:

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[15] For a further discussion, see Vaughn (1998).

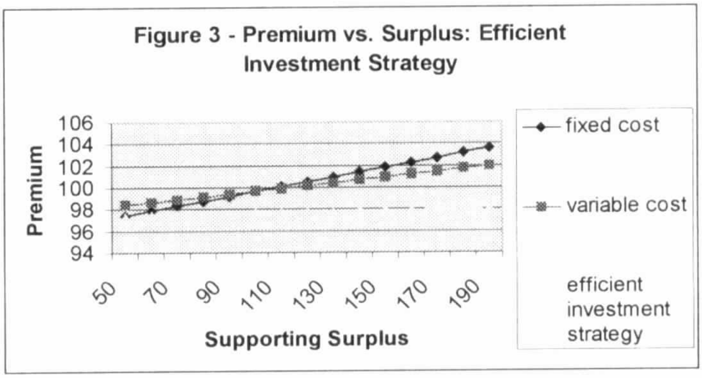
Fair premium = PV of expected losses and expenses

+ PV of tax liability on investment income from policyholders surplus[16]

If the policyholders surplus is invested in zero-beta common stocks, the second term in this formula is greatly reduced. For instance, let's assume that the effective tax rate on common stocks is T* = 10%. Under this assumption, the IRR premium is as follows:

$$P = L/(1+r_L) + Sr_fT^*/[(1+r_f)(1-T^*)]$$

Finally, be applying this formula to our illustrative example, we can see how the fair premium varies according to supporting surplus. Figure 3 displays this relationship with the Figure 2 curves also shown for comparison.



Note that by recognizing this more efficient investment strategy, the fair premium becomes even less sensitive to changes in supporting surplus. In fact, the fair premium is approximately equal to the present

[16] Remember: to derive formula (2.12) we assumed an MM "corrected" world. Under these assumptions, the expected investment income on the policyholders premium is offset by the expected underwriting loss.

value of expected losses and expenses, regardless of the surplus allocation. In other words, surplus allocation is irrelevant to the insurance pricing problem, even in the presence of federal income taxes.[17]

Interestingly, many IRR models in current use already assume that the insurer invests in some combination of taxable and tax-favored securities. Yet, premium in these models is still highly sensitive to supporting surplus. So, where do these models go wrong?

Here, the problem is confusion between average and marginal investment strategy. For instance, many IRR models begin by calculating the *average* investment return and *average* tax rate for the insurer's (or, in the case of the NCCI model, the industry's) current investment portfolio.

The mistake occurs when the supporting surplus is varied. For instance, assume that the minimum surplus requirement is S_M, and the current surplus allocation is S_1. If we increase the surplus allocation to S_2, the marginal surplus, S_2-S_1, is assumed to be invested to earn the average return subject to the average tax rate. In reality, however, this entire marginal surplus would be invested in tax-favored securities, and would be taxed at a much lower rate than the company's average tax rate. In other words, the marginal investment strategy differs significantly from the average investment strategy.

4. OTHER CONSIDERATIONS

The Multi-Period Context

The examples and discussion in this paper have assumed a single-period ratemaking model. In real world insurance ratemaking applications, loss and expense payments extend well beyond one year. In this case,

[17] Brealey and Myers (1996) describe two other common theories of "debt and taxes": (1) The Miller theory, and (2) A compromise theory. Vaughn (1998) demonstrates that there is an optimal asset allocation for each of these theories that eliminates the problem of double taxation and sets the fair premium equal to the discounted value of expected losses and expenses.

one must specify not only the surplus allocation, but also the timing of the surplus release throughout the life of the policy.

As discussed earlier, the appropriate cost of equity capital in the IRR model depends on the ratio of PV(L) to S. In a multi-period context, however, this ratio generally varies by period. As such, there is no one "cost of equity capital" to compare to the IRR.[18] Hence, in multi-period scenarios, the IRR model quickly becomes intractable.

Fortunately, the M/C model looks not at equity cash flows, but at the individual components. Thus, the M/C model can be easily extended to the multi-period scenario. Moreover, by incorporating an optimal investment strategy, the fair premium in the M/C model will simply equal the discounted value of expected losses and expenses, regardless of the surplus allocation or timing of surplus release. Details are provided in Vaughn (1998).

"Notional Surplus" and Minimum Surplus Allocation

The method presented in Sections 2 and 3 assumed that the insurer's entire surplus is allocated as part of the ratemaking process. In other words, the sum of the surplus allocated to individual policies equals the total surplus actually held by the insurer. Recall that for every policy we assumed that there exists some minimum surplus requirement S_M. In practice, this S_M depends on the marginal risk of the policy in relation to the rest of the insurance portfolio. The actual surplus allocated to the policy, S, was generally assumed to be greater than this minimum amount. Moreover, provided that $S >= S_M$, the resulting premium was shown to be essentially independent of the surplus actually allocated – provided the two misapplications are corrected.

[18] Taylor (1994) describes the specific circumstances under which the cost of equity capital will be constant for each period.

Unfortunately, in the incorrect application of the IRR model, premium is heavily dependent on supporting surplus. Many actuaries recognize (and are troubled by) the implicit penalty associated with excess surplus in these models. They may reason, "If we hold more surplus for a given policy (or line) than the market dictates, then we will be penalized for this excess surplus." Hence, in order to reduce this penalty, actuaries may establish a "notional surplus" account.

The notional surplus concept proceeds as follows. First, allocate to each policy only the minimum surplus required, S_M. Next, define S_T as the sum of these S_M's across all policies. The difference between the total surplus actually held by the company, S_A, and the sum of the S_M's is earmarked in a "notional surplus" account (that is, notional surplus = $S_A - S_T$). Furthermore, the assumption is made that the entire notional surplus is invested in tax-favored securities that will earn the shareholders' required rate of return. In this manner, the amount of notional surplus will have no impact on the insurer's pricing decisions.

The surplus allocation problem then becomes one of determining the minimum surplus required for each policy; that is, one must select the S_M's by policy (or line). Unfortunately, unless the two misapplications discussed above are corrected, the fair premium will still be heavily dependent on the selection of the S_M's. As such, a notional surplus methodology is a step in the right direction, but it doesn't eliminate the need to correct the two misapplications.

Default Risk and Convexity

This paper also highlighted two important assumptions inherent in all DCF models (both IRR and M/C). First, these models implicitly assume that the insurer holds enough surplus to reduce default risk to a negligible level. Second, the expected tax payment is calculated as the product of the corporate tax rate and expected taxable income; this is equivalent to a linear, not a convex, tax code.

These two assumptions, rarely explicitly stated, are made for one reason: simplicity. Within the framework of any DCF model, it is very difficult to incorporate default risk or convexity.

Fortunately, these assumptions are reasonable for most P/C lines. Most insurers carefully manage the total risk of the business to ensure a very low probability of default. Moreover, any taxable losses on the business can generally be absorbed relatively quickly via tax carryovers – thereby eliminating the tax costs of convexity.

Yet, the assumptions may not be appropriate for lines of insurance with extremely volatile or skew aggregate loss distributions. For these lines, it may take many years for a worse-case loss to be absorbed by carryovers – or, worse yet, such a loss may even threaten the solvency of the company. In this case, one may need to utilize a contingent claims analysis (CCA) approach, which explicitly allows for the incorporation of default risks and convexity costs.[19]

5. CONCLUSION

Sections 2 and 3 prove the following two points within the context of a one-period ratemaking model: (1) the IRR model is nearly equivalent to the M/C model, and (2) fair premium is essentially independent of supporting surplus.

Section 4 extends the discussion to a multi-period ratemaking model. In a multi-period context, the cost of equity capital will generally vary by period; as a result, the IRR model becomes intractable. The M/C model, however, works very well even in multi-period scenarios. In this case, the fair premium can be shown to equal the present value of expected losses and expenses.

[19] For these lines, most insurers utilize reinsurance (or one of the newer cat hedging tools, such as cat options) to reduce the costs of default risk and convexity. If so, the net costs (e.g. transaction costs) of the reinsurance should be included in the P/L premium. For a further discussion, see Vaughn (1999).

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