Misapplications of Internal Rate of Return Models in Property/Liability Insurance Ratemaking

Trent R. Vaughn, FCAS, MAAA

MISAPPLICATIONS OF INTERNAL RATE OF RETURN MODELS IN PROPERTY/LIABILITY INSURANCE RATEMAKING

Abstract

This paper describes two common misapplications of internal rate of return (IRR) models in property/liability insurance ratemaking. These misapplications have contributed to the popular belief that the fair premium is heavily dependent on supporting surplus, leading casualty actuaries to devote much time and attention to techniques of surplus allocation. In a correct property/liability pricing application, premium is scarcely impacted by changes in supporting surplus.

1. INTRODUCTION

The internal rate of return (IRR) model has been widely utilized for P/L insurance ratemaking, both for regulatory purposes and internal pricing studies. The National Council on Compensation Insurance (NCCI), for example, has extensively utilized IRR models for workers compensation rate filings. Feldblum (1992) describes and discusses NCCI's IRR model in depth.

The IRR method determines the fair premium by equating the internal rate of return with the cost of equity capital. Most practical applications of the IRR method accomplish this task by performing two steps independently. In step one, the user specifies the cost of equity capital, r_e. Feldblum describes several approaches to determining r_e, including the CAPM, the Gordon Growth Model, and an analysis of historical returns in the industry. In step 2, the user calculates the premium that equates the IRR with the selected cost of equity capital.

Myers and Cohn (1987) have developed an alternative discounted cash flow model. The Myers/Cohn (M/C) technique determines the fair premium for a P/L insurance policy according to the following formula:

Fair premium = PV of expected loss and expense

+ PV of the tax burden on the insurer's underwriting and investment income

The original M/C model ignored default risk, implicitly assuming that the insurer holds enough surplus to reduce the probability of ruin to a negligible level.

In a 1990 article in the *Journal of Risk and Insurance*, J. David Cummins compared and contrasted the IRR and M/C models. In particular, Cummins demonstrated that the models are nearly equivalent in a one-period (that is, two-date) ratemaking application. Section 2 of this paper provides a demonstration that is similar to that of Cummins.[1] In doing so, Section 2 highlights the first misapplication of most practical IRR models: failing to recognize the relationship between the cost of equity capital and the amount of supporting surplus.

Section 3 extends the original Cummins demonstration by pointing out the second misapplication: confusion between the average and marginal investment strategy. Lastly, Section 4 closes with three related topics: (1) problems with the IRR model in a multi-period setting, (2) the concept of "notional surplus", and (3) dealing with default risk and convexity.

2. MISAPPLICATION ONE: FAILING TO RECOGNIZE THE RELATIONSHIP BETWEEN THE COST OF EQUITY CAPITAL AND THE AMOUNT OF SUPPORTING SURPLUS

In the application of the IRR model, the internal rate of return varies inversely with the amount of supporting surplus. For instance, let's assume that we've allocated $1,000 of surplus to an insurance contract with a 10% cost of equity capital.[2] Given the premium for the policy, the expected loss amount, and the expected investment return, we can calculate the IRR – let's say it's equal to the 10% hurdle rate: that is, this is an acceptable risk.

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[1] The demonstration in Section 2 has clarified some of the assumptions in the Cummins paper and slightly modified the approach.

[2] Also assume that this $1,000 of surplus is greater than or equal to some minimum solvency requirement, S_M. This assumption will be clarified later in the paper.

Now, let's assume that the amount of supporting surplus is increased to $2,000. At this level, the new IRR will decrease; this new IRR will, in fact, fall below the 10% hurdle rate. The risk is no longer acceptable.

Unfortunately, this type of analysis is plagued by the first misapplication: it correctly recognizes that the IRR varies inversely with supporting surplus, but fails to recognize that the cost of equity capital does too. In fact, once this misapplication is corrected, the IRR premium is essentially equivalent to the M/C premium.[3] We will illustrate this result with a one-period ratemaking model, both with and without federal income taxes. Section 4 extends the discussion to multi-period models.

One Period Model in the Absence of Taxes

Assume a one-period insurance ratemaking model in the absence of federal income taxes. The insurer collects a premium of P at time 0, in exchange for assuming an expected loss and expense amount of L at time 1. The insurer's shareholders have committed S of surplus at time 0. The insurer then invests the premium and surplus funds, P+S, in financial assets with an expected return of r_A. At the end of of the period, the difference between assets and losses will be returned to the shareholders.

As in the Myers/Cohn model, we will assume that the probability of insolvency is zero. That is, let S_M be the amount of capital required to ensure that the assets will exceed losses at time 1 in all states of the world. We assume that the actual surplus committed by shareholders is greater than or equal to S_M; that is $S>=S_M$.

[3] The relationship is exact in a one-period ratemaking model with no taxes. This will be demonstrated subsequently in the paper.

In the absence of taxes, the Myers/Cohn formula reduces to: fair premium = discounted value of expected losses and expenses. In symbols, we have $P = PV(L)$. Note that the fair premium in this case does not depend on the amount of surplus, S, provided that $S>=S_M$.[4]

In order to calculate the IRR, we need to determine the cash flows to and from the insurance shareholders at the beginning and end of the period. At time 0, the shareholders commit S of capital; at time 1, the shareholders receive the difference between the assets and the losses and expenses, or $(P+S)(1+r_A) - L$. Thus, IRR is the solution of the following equation:

$$-S + [(P+S)(1+r_A)-L] / (1+IRR) = 0. \quad (2.1)$$

Solving this equation for IRR and equating to the cost of equity capital, r_e, gives us the following:

$$[(P+S)(1+r_A)-L]/S - 1 = IRR = r_e. \quad (2.2)$$

Lastly, by solving equation (2.2) for P, we have the fair premium according to the IRR method:

$$P = [(r_e-r_A)S + L] / (1+r_A) \quad (2.3)$$

In the actual application formula (2.3), most IRR models make two important assumptions. First, it is often assumed that the insurer invests in super-safe government debt; hence, $r_A = r_f$, where r_f is the risk-free rate of interest. Second, because the shareholders bear the underwriting risk of insurance, the models generally assume that $r_e > r_f$. Together, these two assumptions imply that the cost of equity capital is greater than the expected investment return (that is, $r_e > r_A$).

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[4] This statement is not necessarily true in the presence of taxes or bankruptcy costs.

In order to determine the relationship between premium and supporting surplus in the IRR model, we calculate the first derivative of premium with respect to surplus in formula (2.3): $dP/dS = (r_e - r_A) / (1 + r_A)$. As shown in the preceding paragraph, under the standard IRR assumptions $r_e > r_A$. Thus, $dP/dS > 0$, implying that fair premium in the IRR model is directly proportional to supporting surplus, even in the absence of federal income taxes and default risk.

Hence, the M/C model and the IRR model apparently provide contradictory results. By digging a little deeper, however, we will find that the discrepancy results from a common misapplication of the IRR model. Specifically, most practical applications of the IRR model implicitly assume that the cost of equity capital, r_e, is independent of the amount of supporting surplus. In reality, we demonstrate below that the cost of equity capital is inversely related to supporting surplus, assuming that P,L, and r_A are held constant.

In a 1968 *Proceedings of the Casualty Actuarial Society* paper, Ferrari proposed viewing the P/L insurer as a levered equity trust. In other words, Ferrari visualized the insurer as borrowing funds from policyholders, then investing the combined policyholder and shareholder funds in financial assets. This levered equity trust analogy points out that the shareholders of a P/L insurer hold a *residual claim* on the insurer's assets. By decreasing the amount of supporting surplus – for a fixed P,L, and r_A – we increase the insurer's *financial leverage*.[5] Increasing financial leverage creates a riskier position for shareholders, since their residual claim on the firm becomes more volatile.[6] This increased risk is reflected, in turn, by a higher cost of equity capital.

In a classic financial paper, Modigliani and Miller, or "MM", derived a well-known formula describing the relationship between financial leverage and the cost of equity capital. Specifically, MM's proposition II formula states:

[5] Financial leverage is the ratio of the discounted value of liabilities to supporting surplus.
[6] For a simple and clear mathematical demonstration of the relationship between leverage and volatility, see Brealey and Myers (1996), pp. 451-454.

$$r_e = r_A + (D/E)(r_A - r_D), \tag{2.4}$$

where r_e is the cost of equity capital, r_A is the expected return on assets, r_D is the expected return on debt, and D/E is the financial leverage ratio in terms of market (or present) values.

In the one-period insurance example of this section, the r_D term in the MM formula is given by $r_D = L/P - 1$.[8] The financial leverage ratio is given by PV(L)/S. This gives us the following formula for the cost of equity capital to the P/L insurer.[9]

$$r_e = r_A + [PV(L)/S][r_A - (L/P) + 1] \tag{2.5}$$

By solving equation (2.1) for the internal rate of return, we have the corresponding formula for the IRR:

$$IRR = r_A + (P/S)[r_A - (L/P) + 1] \tag{2.6}$$

Lastly, by comparing formulas (2.5) and (2.6), we see that r_e = IRR if and only if P = PV(L). Thus, the IRR model and the DCF model provide a consistent answer. In the absence of taxes and default risk, the fair premium equals the discounted value of expected losses and expenses.

One-Period Insurance Ratemaking Model in the Presence of Taxes

[7] In their original proof, MM utilized four simplifying assumptions: (1) no costs of bankruptcy, (2) risk-free debt, (3) no signaling opportunities, and (4) no agency costs. The risk-free debt assumption would seem to rule out our insurance example, where actual losses and expenses are variable. Fortunately, relaxing the assumption that debt is risk-free will not change the MM results; see, for instance, pages 462-464 of Weston and Copeland.

[8] In other words, the cost of debt is the expected underwriting loss as a percentage of the policyholder premium.

Next, we extend the one-period ratemaking model to incorporate federal income taxes. In order to incorporate federal taxes into any DCF model, the user must first specify (either implicitly or explicitly) the applicable assumptions regarding three key items:

(1) The relationship between the expected return on bonds and stocks of equivalent risk; or, in financial theory, the selected version of "debt and taxes."

(2) The insurer's asset allocation.

(3) The convexity structure of the corporate tax code.[10]

Most DCF models in practical use make the following assumptions regarding these items:[11]

(1) The expected (or required) return on risk-free common stock equals the interest rate on risk-free government debt. In other words, bonds and stocks of identical risk offer the same expected return. Brealey and Myers (1996) refer to this as the MM "corrected" theory of debt and taxes.

(2) The insurer invests only in risk-free government (i.e. taxable) debt.

(3) The insurer's expected tax liability equals the product of the corporate tax rate and the insurer's expected taxable income.[12]

In order to maintain consistency with current models, we will maintain these assumptions in this section. Moreover, we will also assume that r_L is the appropriate discount rate for expected losses and expenses,[13] and T_c is the marginal corporate tax rate.

Under these assumptions, the Myers/Cohn formula for fair premium is as follows:

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[9] Note that this formula is very similar to the well-known Ferrari formula. The major difference is that the MM formula refers to cash flows and market values, while Ferrari's formula focuses on accounting values.

[10] For a discussion of the role of convexity in insurance pricing, see Vaughn (1999).

[11] These assumptions are also consistent with the assumptions made in the original Myers and Cohn paper.

[12] In financial terms, this is equivalent to specifying a linear (not convex) corporate tax code.

[13] For many P/L lines, indemnity losses possess very little systematic risk. As such, the risk-free rate is often used as an acceptable approximation for r_L.

Fair Premium = Present Value of Expected Losses and Expenses

+ Present Value of Tax on Investment Income

+ Present Value of Tax on Underwriting Income

OR

$$P = L/(1+r_L) + [(P+S)r_fT_c]/(1+r_f) + PT_c/(1+rf) - LT_c/(1+r_L) \qquad (2.7)$$

Solving equation (2.7) for P gives us:

$$P = L/(1+r_L) + Sr_fT_c/[(1+r_f)(1-T_c)] \qquad (2.8)$$

Under the same assumptions, the IRR is given by the solution of the following formula:

$$-S + \{(P+S)(1+r_f) - L - T_c[(P+S)r_f + (P-L)]\}/(1+IRR) = 0. \qquad (2.9)$$

Solving equation (2.9) for the IRR and setting equal to the cost of equity capital, r_e, gives us:

$$\{(P+S)(1+r_f) - L - T_c[(P+S)r_f + (P-L)]\}/S - 1 = IRR = r_e \qquad (2.10)$$

The original MM formula for the cost of equity capital (discussed in the previous section) ignored taxes. In a 1963 paper, Modigliani and Miller revised their analysis to accommodate corporate taxes. This MM "corrected" formula for the cost of equity capital is: $r_e = r_A + (1-T_c)(D/E)(r_A-r_D)$. In our insurance example, this formula translates to:

$$r_e = r_f + (1-T_c)\{[L/(1+r_L)]/S\}(r_f-r_L) \qquad (2.11)$$

Thus, by substituting the formula for r_e in equation (2.11) into equation (2.10) and solving for P, we have:

$$P = L/(1+r_L) + Sr_fT_c/[(1+r_f)(1-T_c)] + [(L/S)(r_f-r_L)]/[(1+r_L)(1+r_f)] \qquad (2.12)$$

Note that equation (2.12) is equivalent to equation (2.8) with the exception of the additional (third) term on the right-hand-side. Yet, visual inspection of the formula reveals that the amount of this additional term is negligible compared to the total premium. Hence, the fair premium in the IRR model very closely approximates the fair premium in the Myers/Cohn model, even in the presence of taxes -- provided that the cost of equity capital in the IRR model is correctly calculated.

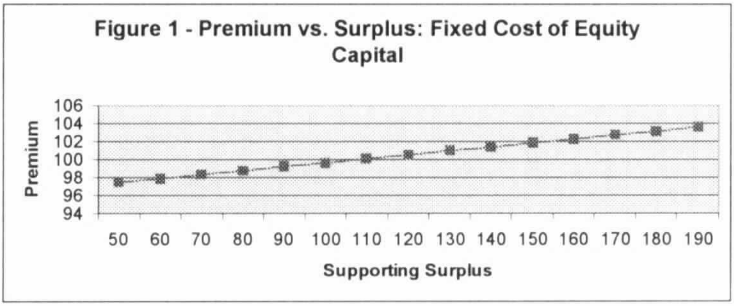
An Illustrative Example

For purposes of illustration, let's put some numbers on the one-period model of this section. Assume the following values for each of the necessary variables:

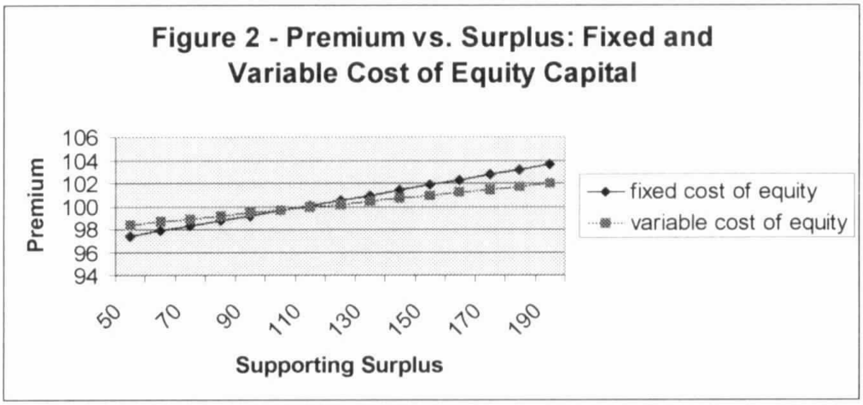
$L = \$100$, $r_f = 5\%$, $r_L = 3\%$, $S = \$100$, $T_c = 35\%$

The fair premium according to the Myers/Cohn model is given by equation (2.8) and equals \$99.65. The fair premium according to the IRR model, given by equation (2.12), equals \$99.67 -- a negligible difference from the M/C premium.[14]

Furthermore, let's examine the sensitivity of the IRR premium to changes in the amount of supporting surplus using the values assumed above. First, assume – as in most practical IRR models – that the cost of equity capital is fixed regardless of the amount of supporting surplus. Figure 1 displays the fair premium as a function of supporting surplus under this assumption:



Yet, by correcting this first misapplication the slope of this graph will change significantly. Specifically, utilizing formula (2.11) to calculate the cost of equity capital gives us a "flatter" relationship between premium and supporting surplus. Figure 2 graphically demonstrates the resulting premium for both approaches.



[14] Cummins (1990, pp. 90-91) notes that the two models are exactly equivalent if and only if $r_L = r_f$. In terms of formula (2.12), note that if $r_L = r_f$, the third term drops off and the two formulas are identical.

Note that the IRR premium is still highly dependent on supporting surplus. In the next section, however, we will further flatten the graph by correcting the second misapplication.

3. THE SECOND MISAPPLICATION: CONFUSION BETWEEN AVERAGE AND MARGINAL INVESTMENT STRATEGIES

In the previous section, we assumed an all-taxable-bond asset allocation and an MM "corrected" theory of debt and taxes. In the MM "corrected" model, there is a very strong tax disadvantage to corporate lending. Given this tax disadvantage, an all-taxable-bond portfolio would be highly suboptimal.[15] Under these assumptions, a value-maximizing insurer would invest a substantial amount of the available funds in municipal bonds and/or common stock.

As an illustration, let's maintain the Section 2 assumptions regarding "debt and taxes" and convexity: that is, an MM "corrected" world with a linear tax code. Assume, however, that the insurer allocates the P+S of available funds as follows: invest P in risk-free taxable bonds, and invest S in common stock of equivalent systematic risk (that is, "zero-beta" common stock).

In the MM "corrected" world, both the taxable bonds and the common stock will be priced to offer an expected (pre-tax) return of r_f. Interest payments from the taxable bonds will still be taxed at the full corporate rate of 35%. The effective tax rate on the common stock will be less than 35%, owing to two provisions of the corporate tax code: (1) only 30% of the dividends on common stock are taxed, and (2) unrealized capital gains escape taxation entirely.

Now, recall equation (2.12) for the fair premium according to the IRR rule in the presence of federal income taxes. Ignoring the negligible third term on the right-hand-side, this formula can be described in words as follows:

[15] For a further discussion, see Vaughn (1998).

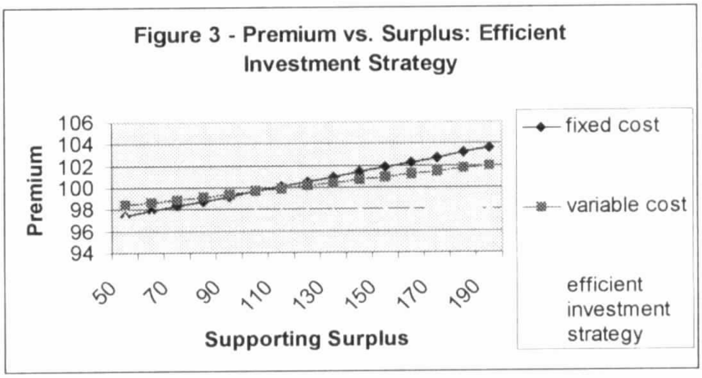
Fair premium = PV of expected losses and expenses

+ PV of tax liability on investment income from policyholders surplus[16]

If the policyholders surplus is invested in zero-beta common stocks, the second term in this formula is greatly reduced. For instance, let's assume that the effective tax rate on common stocks is $T^* = 10\%$. Under this assumption, the IRR premium is as follows:

$$P = L/(1+r_L) + Sr_fT^*/[(1+r_f)(1-T^*)]$$

Finally, be applying this formula to our illustrative example, we can see how the fair premium varies according to supporting surplus. Figure 3 displays this relationship with the Figure 2 curves also shown for comparison.



Note that by recognizing this more efficient investment strategy, the fair premium becomes even less sensitive to changes in supporting surplus. In fact, the fair premium is approximately equal to the present

[16] Remember: to derive formula (2.12) we assumed an MM "corrected" world. Under these assumptions, the expected investment income on the policyholders premium is offset by the expected underwriting loss.

value of expected losses and expenses, regardless of the surplus allocation. In other words, surplus allocation is irrelevant to the insurance pricing problem, even in the presence of federal income taxes.[17]

Interestingly, many IRR models in current use already assume that the insurer invests in some combination of taxable and tax-favored securities. Yet, premium in these models is still highly sensitive to supporting surplus. So, where do these models go wrong?

Here, the problem is confusion between average and marginal investment strategy. For instance, many IRR models begin by calculating the *average* investment return and *average* tax rate for the insurer's (or, in the case of the NCCI model, the industry's) current investment portfolio.

The mistake occurs when the supporting surplus is varied. For instance, assume that the minimum surplus requirement is S_M, and the current surplus allocation is S_1. If we increase the surplus allocation to S_2, the marginal surplus, S_2-S_1, is assumed to be invested to earn the average return subject to the average tax rate. In reality, however, this entire marginal surplus would be invested in tax-favored securities, and would be taxed at a much lower rate than the company's average tax rate. In other words, the marginal investment strategy differs significantly from the average investment strategy.

4. OTHER CONSIDERATIONS

The Multi-Period Context

The examples and discussion in this paper have assumed a single-period ratemaking model. In real world insurance ratemaking applications, loss and expense payments extend well beyond one year. In this case,

[17] Brealey and Myers (1996) describe two other common theories of "debt and taxes": (1) The Miller theory, and (2) A compromise theory. Vaughn (1998) demonstrates that there is an optimal asset allocation for each of these theories that eliminates the problem of double taxation and sets the fair premium equal to the discounted value of expected losses and expenses.

one must specify not only the surplus allocation, but also the timing of the surplus release throughout the life of the policy.

As discussed earlier, the appropriate cost of equity capital in the IRR model depends on the ratio of PV(L) to S. In a multi-period context, however, this ratio generally varies by period. As such, there is no one "cost of equity capital" to compare to the IRR.[18] Hence, in multi-period scenarios, the IRR model quickly becomes intractable.

Fortunately, the M/C model looks not at equity cash flows, but at the individual components. Thus, the M/C model can be easily extended to the multi-period scenario. Moreover, by incorporating an optimal investment strategy, the fair premium in the M/C model will simply equal the discounted value of expected losses and expenses, regardless of the surplus allocation or timing of surplus release. Details are provided in Vaughn (1998).

"Notional Surplus" and Minimum Surplus Allocation

The method presented in Sections 2 and 3 assumed that the insurer's entire surplus is allocated as part of the ratemaking process. In other words, the sum of the surplus allocated to individual policies equals the total surplus actually held by the insurer. Recall that for every policy we assumed that there exists some minimum surplus requirement S_M. In practice, this S_M depends on the marginal risk of the policy in relation to the rest of the insurance portfolio. The actual surplus allocated to the policy, S, was generally assumed to be greater than this minimum amount. Moreover, provided that $S >= S_M$, the resulting premium was shown to be essentially independent of the surplus actually allocated – provided the two misapplications are corrected.

[18] Taylor (1994) describes the specific circumstances under which the cost of equity capital will be constant for each period.

Unfortunately, in the incorrect application of the IRR model, premium is heavily dependent on supporting surplus. Many actuaries recognize (and are troubled by) the implicit penalty associated with excess surplus in these models. They may reason, "If we hold more surplus for a given policy (or line) than the market dictates, then we will be penalized for this excess surplus." Hence, in order to reduce this penalty, actuaries may establish a "notional surplus" account.

The notional surplus concept proceeds as follows. First, allocate to each policy only the minimum surplus required, S_M. Next, define S_T as the sum of these S_M's across all policies. The difference between the total surplus actually held by the company, S_A, and the sum of the S_M's is earmarked in a "notional surplus" account (that is, notional surplus = $S_A - S_T$). Furthermore, the assumption is made that the entire notional surplus is invested in tax-favored securities that will earn the shareholders' required rate of return. In this manner, the amount of notional surplus will have no impact on the insurer's pricing decisions.

The surplus allocation problem then becomes one of determining the minimum surplus required for each policy; that is, one must select the S_M's by policy (or line). Unfortunately, unless the two misapplications discussed above are corrected, the fair premium will still be heavily dependent on the selection of the S_M's. As such, a notional surplus methodology is a step in the right direction, but it doesn't eliminate the need to correct the two misapplications.

Default Risk and Convexity

This paper also highlighted two important assumptions inherent in all DCF models (both IRR and M/C). First, these models implicitly assume that the insurer holds enough surplus to reduce default risk to a negligible level. Second, the expected tax payment is calculated as the product of the corporate tax rate and expected taxable income; this is equivalent to a linear, not a convex, tax code.

These two assumptions, rarely explicitly stated, are made for one reason: simplicity. Within the framework of any DCF model, it is very difficult to incorporate default risk or convexity.

Fortunately, these assumptions are reasonable for most P/C lines. Most insurers carefully manage the total risk of the business to ensure a very low probability of default. Moreover, any taxable losses on the business can generally be absorbed relatively quickly via tax carryovers – thereby eliminating the tax costs of convexity.

Yet, the assumptions may not be appropriate for lines of insurance with extremely volatile or skew aggregate loss distributions. For these lines, it may take many years for a worse-case loss to be absorbed by carryovers – or, worse yet, such a loss may even threaten the solvency of the company. In this case, one may need to utilize a contingent claims analysis (CCA) approach, which explicitly allows for the incorporation of default risks and convexity costs.[19]

5. CONCLUSION

Sections 2 and 3 prove the following two points within the context of a one-period ratemaking model: (1) the IRR model is nearly equivalent to the M/C model, and (2) fair premium is essentially independent of supporting surplus.

Section 4 extends the discussion to a multi-period ratemaking model. In a multi-period context, the cost of equity capital will generally vary by period; as a result, the IRR model becomes intractable. The M/C model, however, works very well even in multi-period scenarios. In this case, the fair premium can be shown to equal the present value of expected losses and expenses.

[19] For these lines, most insurers utilize reinsurance (or one of the newer cat hedging tools, such as cat options) to reduce the costs of default risk and convexity. If so, the net costs (e.g. transaction costs) of the reinsurance should be included in the P/L premium. For a further discussion, see Vaughn (1999).

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