A Set of New Methods and Tools for Enterprise Risk Capital Management and Portfolio Optimization

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A SET OF NEW METHODS AND TOOLS FOR ENTERPRISE RISK CAPITAL MANAGEMENT AND PORTFOLIO OPTIMIZATION

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Abstract

The focus of this paper is on some new developments in the methodologies for enterprise risk management (ERM). The paper presents a set of new methods and tools, including (i) a universal risk measure for both assets and liabilities, (ii) a coherent method of determining the aggregate capital requirement for a firm, and (iii) a coherent method of allocating the cost of capital to individual business units. The discussed methods can be used for asset/loss portfolio optimization, and for quantifying the "value creation" of ERM. The paper also discusses some correlation models and methods for risk aggregation.

Introduction

The Casualty Actuarial Society (CAS) is presently promoting research in enterprise risk management and capital management. The current Call Paper Program focuses on analyzing, integrating, and optimizing the financial and insurance risks held by a financial institution or insurance company, so that capital may be efficiently deployed and consistently allocated, across the enterprise.

Recently, the CAS Advisory Committee on Enterprise Risk Management (ERM) recommended a conceptual "ERM framework," emphasizing that ERM should not solely be employed for defensive purposes, that is, to protect the firm's capital base against the "downside" of unexpected losses. ERM should also be employed for proactive purposes, that is, *to help manage the entire risk portfolio (including both assets and liabilities), and, ultimately, to enhance shareholder value.* It is believed that the pivotal role of ERM in "value creation" will become more evident in the near future.

The CAS conceptual "ERM framework" outlines a risk-management process that:

- Analyzes and quantifies risks, by obtaining and calibrating a probability distribution of outcomes for each major identified risk; then
- Integrates these major risks, by combining their outcome distributions, fully reflecting their correlations and portfolio effects; then
- Assesses and prioritizes these risks, by determining the contribution of each major identified risk to the firm's aggregate risk profile, and, in terms of their potential positive or negative impact to the firm's capital base; and then
- Optimizes the firm's aggregate risk profile, so that capital may be efficiently deployed and consistently allocated, across the global enterprise.

Outline and Focus of the Paper

Section 0. The growing pivotal role of ERM in the insurance industry
Section 1. A new universal risk measure for all assets and liabilities
Section 2. A coherent risk measure of required capital that captures overall loss distributions
Section 3. Allocating risk capital among the business units of the enterprise
Section 4. Aggregating correlated risks to produce an integrated risk profile of the firm
Section 5. Optimizing the "portfolio of the firm" to create new shareholder value

To meet the emerging ERM needs of the insurance industry, this paper presents a set of universal methods and tools that, taken together in a single framework, coherently analyzes, manages, integrates, prioritizes, and optimizes the capital requirements and risk-return trade-off of the firm.

In particular this paper presents:

- a universal risk measure for both assets and liabilities;
- a coherent method of determining the aggregate capital requirement for a firm;
- a coherent method of assessing the risk contribution, or the allocated cost of capital, of individual business units, so that RORAC (return on risk-adjusted capital) assessments can be made;
- aggregation methods for combining correlated risks;
- a proposed method for asset/loss portfolio optimization, and for quantifying the "value creation" of ERM.

Although I could have chosen to describe the detailed steps of some real-life ERM exercises, I decided to focus on a more urgent problem in that the industry lacks a sound, commonly agreed upon, methodology framework. To keep a reasonable scope for this paper, I will not detail an ERM exercise for a large financial institution. Instead I will present some new methodologies in risk measure, capital allocation, and portfolio optimization of the firm.

Please note that the methodologies discussed here are not exhaustive. Indeed, many un-mentioned issues deserve separate discussions, to name a few: (1) cost of capital for long-tailed liabilities, (2) soft invisible correlation, (3) diversification versus area of expertise, and (4) macro- and micro- risk dynamics. With these caveats, this paper hopes to present innovations that can be formalized later into a set of ERM best practices, enabling insurance companies to prosper and grow in their risk taking.

Section 0. The Growing Pivotal Role Of ERM In The Insurance Industry

In this section I give a brief overview of important issues related to insurance risk and capital management, which serves as practical background for the technical discussions in later sections.

Insurance Risk and Capital

Unlike manufacturing, "insurance" is unique in that "capital" is not "spent" producing durable goods or building factories. Instead, capital is used as a cushion against the risk that insurance premiums combined with investment income are not sufficient to pay future policyholder claims. As a general principle, insurance companies with higher risks should carry higher levels of cushion capital. The very nature of insurance thus illustrates the universal link between *capital* and *risk.*

It is also because of this direct link, insurance risk managers often refer to "risk" and "capital" interchangeably. For example, when industry professionals refer to the allocation of insurance company "capital," they really mean the allocation of "risk contributions" from various business units. Insurance company capital is not legally divisible, so all of the capital available at any given time supports all insurance policies. Theoretically, a single policy with unlimited cover can claim the entire capital base of the whole insurance company.

Historically, regulatory cushion capital was determined by a simple rule-of-thumb, based on premium-to-surplus ratios, or reserve-to-surplus ratios. These simple rules-of-thumb did not reflect the true economic risk realities of insurance. The National Association of Insurance Commissioners has since tried to better link regulatory cushion capital with risk by developing a Risk Based Capital (RBC) system. So far, however, the RBC system has not been very effective in the property/casualty sector. This calls for advanced enterprise risk modeling that better captures the major risks of an insurance company.

Aggregate Capital Requirement

The capital requirements of an insurance company should measure the aggregate risk of the company risk portfolio, by incorporating asset risks, liability risks, event risks, and operational business risks. Enterprise risk modeling must properly incorporate *all of these disparate risks* in order to present an accurate profile of firm-wide risk.

Knowing the capital requirements of the firm is the first step to improved capital management. Excess capital, if any, can be transferred from treasury (risk-free) instruments, and re-deployed for more productive returns. A shortfall in capital can be rebalanced by infusions of fresh capital, purchases of reinsurance, or, by trimming risks from the company portfolio.

The Basel Accord[1] in the banking industry has inspired some insurance regulators to promote better practices in capital and risk management. For instance, Allan Brender, a Senior Director at the Office of the Superintendent of Financial Institutions Canada (OSFI), recently stated that the ultimate goal of insurance regulation is actually to help insurers better manage their capital and risk.

An Integrated View of Insurance Company Risks

There are at least two different views of the insurance business.

The Traditional Underwriting View: Insurance is mainly an underwriting operation, financing investments that earn low, but stable returns, just like a bank deposit. The emphasis is on managing liabilities. Many insurance company executives from the last century were from an underwriting background, and they guarded their companies against investing in unfamiliar risk vehicles that were outside their familiar turf.

A Financial Investment View: Insurance premiums are collected and held before claims are paid out. This creates a cash-flow float. This float provides opportunities for investing in a wide array of investment risk vehicles. In other words, the underwriting operation is essentially a "mutual fund," providing money for investment with higher returns. The emphasis is on managing assets.

The "underwriting" and the "investment" viewpoints reflect the flipsides of managing liabilities and assets in the insurance enterprise. It is better to take a more integrated view of underwriting and investment risks, where liabilities and assets are calibrated to maximize the company overall risk-return trade-off. Warren Buffett is an example of taking an integrated view of insurance operations. He has criticized some companies for aggressively accumulating investment funds using the underwritten cash-flow float, by sacrificing underwriting standards that subsequently resulted in unanticipated big losses. As another example of taking an integrated approach, some insurance companies were successful in operating high return hedge funds, but with sound risk-management in place

[1] See http://www.bis.org/bcbs/aboutbcbs.htm

to control the aggregate risk limit.

State of Affairs for the P/C Insurance Sector

In the years just before 2001, it was widely acknowledged that the property/casualty industry was over-capitalized. Company management tended to retain massive amounts of excess capital, to support company insurance and credit ratings, and to fortify reserves to withstand unexpected catastrophes. However, this excess capital was not utilized effectively. Instead of seeking better investment opportunities, insurance executives used the excess capital to subsidize price cutting in insurance premiums, so as to gain or defend market share over competitors. Actuarial indicated premium rates were useless in such a cutthroat competitive environment. Years of irresponsible pricing led to huge underwriting losses by many insurance companies.

The events of September 11 were a wake up call to the insurance industry, destroying a significant portion of the excess capital. Insurers suddenly found themselves in a dangerously weak capital position, and became much more responsible in taking on more risks. Insurance companies are now showing a higher appreciation for improved measurements of both liability and asset risks.

On the liability side, Renaissance Re is a catastrophe reinsurer that has achieved 20% annual returns on equity over the last decade. Jim Stanard, Chairman and CEO of Renaissance Re, is an early pioneer in enterprise risk modeling (see Lowe and Stanard, 1989). More generally, on the asset side, some insurance companies are taking on new kinds of market and credit investments for improved returns. These are welcome movements toward a holistic approach to actively manage *all* liability and asset risks within the insurance company.

Market Perspective versus Company Perspective

When it comes to measuring risks, the market and the company may have two different perspectives. In a market setting, transacted insurance prices are additive. To an individual company, however, the cost of taking on twice the amount of a specified risk exposure may be more than double, due to increases in portfolio concentration. For a company, different portfolio combinations can result in different aggregate risks.

Within the CAS, a group of prominent researchers are vigorously debating insurance capital allocation issues. Most of the differences in opinion can be attributed to the apparent incompatibility of market and company perspectives. I would argue that we look at insurance risk capital from both perspectives. Indeed, the interplay between these two perspectives lays the future foundation for responsive insurance risk capital management.

Traditionally, insurance transactions were driven by long-term relationships. Nowadays traded and underwritten risks are more and more becoming commodities. Consumers are becoming more conscious about shopping for the best price. In a competitive market, it is increasingly difficult for individual insurers to differentiate their offerings by pricing alone. However, these individual insurers have ample room to improve their enterprise risk capital management, and improve their shareholder return, by optimizing those liabilities and assets comprising the overall "portfolio of the firm."

To insurance company shareholders and executives, managing the asset return is becoming just as important as managing the liability risk. But more crucially, they realize that the risk/return trade-off for the integrated "portfolio of the firm" determines the day-to-day valuation of the insurance company.

The ERM Process

To succeed, the ERM process needs to be openly mandated, monitored, and managed by the executive suite. Insurance companies can contain people who are used to old ways of doing things and are skeptical to the ERM exercise. Unless these people are provided with imperative "marching orders from above," it can be difficult to get timely cooperation from the managers of individual business units.

The very first phase of ERM involves classifying major risk factors and business segments, so that data can be gathered and analyses performed in the most efficient and logical manner. Each business unit may have a particular way of obtaining, storing, and analyzing risk data. These peculiarities should be documented when the business unit data is gathered.

In the second phase, the ERM process compiles major risk factors, including:

- Market Risks, like fluctuations in equity portfolio valuation
- Credit Risks, like bond defaults and reinsurance receivables
- Interest Rate Risks, like shifts in the yield curve
- Foreign Exchange Risks, like changes in Euro/US Dollar currency exchange rates
- Catastrophe Events and Mass Tort Liabilities, like Hurricane losses, asbestos claims
- Loss Development Uncertainty, like the future unwinding of loss reserve estimates
- Business and Pricing Risks, like softening or hardening of California WC market

- Operational Risks, like captive agent compliance issues.

The industry is now moving to a standardized system for major risk factors, like those listed in the forthcoming IAA Solvency Working Party Report.

A common set of future economic scenarios, representative of stressed and unstressed conditions, should be then applied to these risk factors, across all business units, to capture correlations and concentrations of risk that may unduly impact the capital base simultaneously.

The third phase is a qualitative evaluation of the data by the managers of the individual business units. We should not be surprised if the first, raw compilations of data do not fully portray the true opportunities or risks of a given business unit. The ERM exercise succeeds only if it incorporates the practical knowledge and expertise of the business managers.

The biggest hurdle to ERM adoption by the insurance industry is the lack of a commonly accepted ERM methodology. One common mistake by many companies is spending too much effort on non-significant risks while ignoring the more important business risk dynamics. Another big hurdle is the lack of consistency of competing methods for capital risk analysis. Different methods applied to the same data can produce very different results. Venter (2002) gives a concise critique of certain quantitative approaches to capital allocation.

To be effective, the ERM methodology needs to do more than just consistently evaluate the relative levels of risk and return within the insurance enterprise. The ERM methodology must also help risk managers to take specific actions to enhance the bottom-line results of the enterprise.

Section 1. A New Universal Risk Measure For All Assets and Liabilities

In this section I propose a framework for measuring financial and insurance risks. A two-factor model of risk-adjustment for all moments in a distribution and for parameter uncertainty provides a "fair value" for a given risk vehicle. This risk vehicle can be an asset or a liability, traded or underwritten, whose outcomes have a normal or non-normal distribution.

Standard Deviation Method

Consider a risky asset with a one-period time horizon. Assume that the asset return R has a normal distribution. In a competitive market, the Capital Asset Pricing Model (CAPM) asserts that

$$\mathrm{E}[R] = r + \lambda\, \sigma[R], \qquad \text{(eq-1.1)}$$

where r is the risk-free interest rate, and the parameter λ is the "market price of risk." In asset portfolio management, the parameter $\lambda=(\mathrm{E}[R]-r)/\sigma[R]$ is called the Sharpe Ratio.

On the insurance side, the pricing of a liability usually starts with objective loss data, then calculates an expected loss (burning cost), and then loads for risk margin and expenses. The standard-deviation method (eq-1.1) has traditionally been used in risk-adjustment for losses:

Fair Premium = (Expected Loss) + λ (Standard Deviation of Losses),

where λ is a loading multiplier, analogous to the above "market price of risk" for assets.

Despite its popularity, the standard-deviation loading method fails to reflect the skew of a loss distribution. In fact, standard-deviation loading may unwittingly penalize upside skew and ignore downside skew. This drawback of standard-deviation loading has motivated actuarial researchers to develop various alternatives over the last decade.

Consider a loss variable X with a general exceedance curve $G(x)=\Pr\{X>x\}$. The following transform is a direct extension of the standard-deviation method of loading:

$$G^*(x) = \Phi\left(\Phi^{-1}(G(x)) + \lambda\right), \qquad \text{(eq-1.2)}$$

Here Φ represents the standard normal cumulative distribution function, where the parameter λ extends the concept of "market price of risk" or the Sharpe Ratio. Wang (2000, 2001) derived transform (eq-1.2) in the context of reinsurance pricing by layer, and showed that (eq-1.2) recovers CAPM for pricing underlying assets and replicates the results of the Black–Scholes formula for pricing options. The Wang Transform (eq-1.2) was inspired by an earlier work of Venter (1991).

Note that (eq-1.2) can be applied to risks with both positive and negative values. The mean value under the transformed distribution is

$$\mathrm{E}^*(X) = -\int_{-\infty}^{0}[1 - g(G(x))]dx + \int_{0}^{+\infty} g(G(x))dx.$$

For a given loss variable X with objective loss exceedance curve $G(x)$, the Wang Transform (eq-1.2) produces a "risk-adjusted" loss exceedance curve $G^*(x)$. The mean value $\mathrm{E}^*[X]$, under distribution $G^*(x)$, defines a risk-adjusted "fair value" of X at time T, which can be further discounted to time zero, using the risk-free interest rate.

One important property of the Wang Transform (eq-1.2) is that normal and lognormal distributions are preserved:

- If G has a normal(μ,σ^2) distribution, G^* is also a normal distribution with $\mu^* = \mu+\lambda\sigma$ and $\sigma^* = \sigma$.
- For a loss with a normal distribution, the Wang Transform (eq-1.2) recovers the traditional standard-deviation loading, with the parameter λ being the constant multiplier.
- If G has a lognormal(μ,σ^2) distribution such that $\ln(X) \sim$ normal(μ,σ^2), G^* is another lognormal distribution with $\mu^* = \mu+\lambda\sigma$ and $\sigma^* = \sigma$.

For any computer-generated distribution, the Wang Transform (eq-1.2) is fairly easy to compute numerically. Many software packages have both Φ and Φ^{-1} as built-in functions. In Microsoft Excel, $\Phi(y)$ can be evaluated by NORMSDIST(y) and $\Phi^{-1}(y)$ can be evaluated by NORMSINV(y).

Unified Treatment of Assets and Liabilities

A liability with loss variable X can be viewed as a negative asset with gain variable $Y = -X$, and vice versa. Mathematically, a liability with a "market price of risk" λ, can be treated as a negative asset whose market price of risk is $-\lambda$. That is, the "market price of risk" will have the same value but opposite signs, depending upon whether a risk vehicle is treated as an asset or liability. For an asset with gain variable X, the Wang Transform (eq-1.2) has an equivalent representation:

$$F^*(x) = \Phi\left[\Phi^{-1}(F(x)) + \lambda\right] \qquad \text{(eq-1.3)}$$

where $F(x) = 1-G(x)$ is the cumulative distribution function (cdf) of X.

The following operations are equivalent:

1. Apply transform (eq-1.2) with λ to the exceedance curve $G(x)$ of the loss variable X,
2. Apply transform (eq-1.2) with $-\lambda$ to the exceedance curve $G(y)$ of the gain variable $Y=-X$, and
3. Apply transform (eq-1.3) with λ to the cdf $F(y)=1-G(y)$ of the gain variable $Y=-X$.

These equivalences ensure that the same price is obtained for both sides of a risk transaction.

Stock prices are often modeled by lognormal distributions, which implies that stock returns are modeled by normal distributions. Equivalent results can be obtained by applying the Wang Transform (eq-1.3) either to the stock price distribution, or, alternatively, to the stock return distribution.

A Variation of the Wang Transform

For normal distributions, the Wang Transform (eq-1.3) represents a location-shift while preserving the volatility. As a variation of the (eq-1.3), we can simultaneously apply a location-shift and a volatility-multiplier:

$$F^{*}(x)=\Phi\left[b\cdot\Phi^{-1}(F(x))+\lambda\right]. \qquad \text{(eq-1.4)}$$

When $F(x)$ has a normal(μ, σ^2) distribution, (eq-1.4) represents an adjustment of the volatility by $\sigma^*=\sigma/b$, and a shift in the mean by $\mu^*=\mu+\lambda\sigma$. For most applications we would like to have $0<b<1$, so that $\sigma^*=\sigma/b$ is greater than σ (in other words, the volatility is inflated). In an unpublished result, Major and Venter (1999) first fitted model (eq-1.4) to a set of observed CAT-layer prices. Butsic (1999) applied both a location-shift and a volatility-multiplier to a lognormal CAT-loss distribution.

Adjustment for Parameter Uncertainty

So far we have assumed that probability distributions for risks under consideration are known without ambiguity. Unfortunately, this is seldom the case in real-life risk modeling. Parameter uncertainty is part of reality in risk modeling. Even with the best data and technologies available today, there are parameter uncertainties in the modeling of insurance losses (see Kreps, 1997; Major, 1999).

Consider the classic sampling theory in statistics. Assume that we have m independent observations from a given population with a normal(μ,σ^2) distribution. Note that μ and σ are not directly observable, we can at best estimate μ and σ by the sample mean $\tilde{\mu}$ and sample standard deviation $\tilde{\sigma}$. As a result, when we make probability assessments regarding a future outcome, we effectively need to use a student-t distribution with k = $m-2$ degrees-of-freedom.

The Student-t distribution with k degrees-of-freedom has a density

$$f(t;k) = \frac{1}{\sqrt{2\pi}} \cdot c_k \cdot \left[1 + \frac{t^2}{k}\right]^{-(0.5k+1)}, \quad -\infty<t<\infty$$

where

$$c_k = \sqrt{\frac{2}{k}} \cdot \frac{\Gamma((k+1)/2)}{\Gamma(k/2)}$$

In terms of density at zero we have $f(0;k) = c_k \cdot \phi(0)$, where $\phi(0)$ is the standard normal density at x=0. Student-t has a lower density than standard normal at zero. As the degrees-of-freedom k increases, the factor c_k increases and approaches one:

k	3	4	5	6	7	8	9
c_k	0.921	0.940	0.952	0.959	0.965	0.969	0.973

The Student-t distribution can be generalized to having fractional degrees-of-freedom.

Following the statistical sampling theory that uses a Student-t distribution in place of a normal distribution, I suggest the following technique of adjusting for parameter uncertainty:

$$F^*(x) = Q\left(\Phi^{-1}(F(x))\right) \qquad \text{(eq-1.5)}$$

where Q has a Student-t distribution with degrees-of-freedom k.

Note that (eq-1.5) is an extension of the classic sampling theory, since there is no restriction imposed on the underlying distribution $F(x)$.

It may be argued that the adjustment (eq-1.5) represents a more objective view of the risk's probability distribution, instead of a form of profit loading. Empirical evidence suggests that market prices do often contain an adjustment for parameter uncertainty.

A Two-Factor Model

Let $G(x)$ be a best-estimate probability distribution, before adjustment for parameter uncertainty. The combination of parameter uncertainty adjustment in (eq-1.5) and pure risk adjustment using the Wang Transform in (eq-1.2) yields the following two-factor model:

$$G^*(y) = Q\left(\Phi^{-1}(G(y)) + \lambda\right) \qquad \text{(eq-1.6)}$$

where Q has a Student-t distribution with k degrees-of-freedom.

The two-factor model (eq-1.6) can also be written in terms of adjustments of local volatilities:

$$F^*(x) = Q\left[\Phi^{-1}(F(x)) + \lambda\right] = \Phi\left[b \cdot \Phi^{-1}(F(x)) + \lambda\right] \qquad \text{(eq-1.7)}$$

where the multiplier b depends on the value of $F(x)$, rather than being a constant.

As shown in Figure 1.1, the implied b-values in (eq-1.7) depend on the value of $F(x)$. In the middle range of a risk probability distribution, the implied b-values are closer to one, indicating a relatively smaller "volatility adjustment."

At the extreme tails of a risk probability distribution, the implied b-values deviate further below one, showing an increasing adjustment at the extreme tails. The extreme tails may represent many different pricing situations: deep out-of-the-money options, low-frequency but high-severity catastrophe losses, or, markets where risk vehicles are illiquid, benchmark data sparse, negotiations difficult, and the cost of keeping capital reserves is high.

If we choose Q as a Student-t without rescaling in the two-factor model (eq-1.7), the degrees-of-freedom will affect the simultaneous estimation of the Sharpe Ratio λ. To overcome this drawback, we can choose Q in (eq-1.7) being a rescaled Student-t distribution that matches the standard normal density at x=0. This rescaled Student-t distribution has a density function:

$$q(t;k) = \frac{1}{\sqrt{2\pi}} \cdot \left[1 + \frac{x^2}{k \cdot c_k^2}\right]^{-(0.5k+1)}, \quad -\infty < x < \infty$$

An advantage of using rescaled Student-t is to ensure a more robust estimate of the Sharpe Ratio λ. This can be useful to a fund manager comparing the Sharpe Ratio of risk vehicles from different asset classes.

Symmetry versus Asymmetry

Insurance risks are characterized by having skewed distributions. As Lane (2000) stated:

> "Any appraisal of the risks contained in insurance or reinsurance covers must take into account the fact that the statistical distribution of profit and loss outcomes may be severely skewed. Conventional risk measurement (i.e. the standard

deviation) deals with random outcomes that are symmetric in nature. Price volatility is usually viewed as symmetric. Event or outcome risk (a characteristic of insurance) is not. How is the asymmetry to be captured? What are the components of event risk and how they factor into price?"

Although the distributions Φ and Q are symmetric themselves, the one-factor Wang Transform (eq-1.3) and the two-factor model (eq-1.7) automatically reflect the skew in the input distribution $G(x)$. This ability to reflect the skew is an advantage over the standard deviation loading.

As an example, consider two bets X and Y with the following gain/loss probability distributions.

The bet X has a probability distribution of gain/loss:

x	−1	0	1	19
f(x)	0.29	0.6	0.1	0.01

The bet Y has a probability distribution of gain/loss:

y	−19	−1	0	1
f(y)	0.01	0.1	0.6	0.29

Both X and Y have the same mean=0 and variance=4. While X has an upside skew, Y has a downside skew.

1. Apply the Wang Transform (eq-1.3) with λ=0.4, to get fair values of $E^*[X]$= −0.33 and $E^*[Y]$= −0.52. Note that $E^*[X]$ −$E^*[Y]$= 0.19. As shown in the table below, for small values of lambda (say < 0.4), the one-factor Wang Transform (eq-1.3) differentiates slightly the upside skew from the downside skew. However, as the lambda value increases, this differentiating power increases "exponentially."

Lambda Value	**One-factor E*[X]**	**One-factor E*[Y]**	***Difference E*[X] - E*[Y]***
0.20	**-0.18**	**-0.23**	***0.05***
0.40	**-0.33**	**-0.52**	***0.19***
0.60	**-0.45**	**-0.90**	***0.45***
0.80	**-0.56**	**-1.39**	***0.83***
1.00	**-0.65**	**-2.01**	***1.36***

1.50	**-0.82**	**-4.27**	***3.44***
2.00	**-0.93**	**-7.47**	***6.55***
2.50	**-0.97**	**-11.14**	***10.16***

2. Apply a Student-t adjustment (eq-1.5) for parameter uncertainty with degrees-of-freedom k=6, to get fair values $E^*[X]$= 0.36 and $E^*[Y]$= −0.36. The Student-t adjustment (eq-1.6) clearly reflects the direction of the skew. We have $E^*[X] - E^*[Y]$= 0.72. As shown in the table below, the differentiating power decreases as the degrees-of-freedom increase.

Degrees of Freedom	Student-t E*[X]	Student-t E*[Y]	*Difference E*[X] - E*[Y]*
4	**0.56**	**-0.56**	***1.12***
5	**0.44**	**-0.44**	***0.88***
6	**0.36**	**-0.36**	***0.72***
7	**0.31**	**-0.31**	***0.62***
8	**0.27**	**-0.27**	***0.54***
9	**0.23**	**-0.23**	***0.46***
15	**0.14**	**-0.14**	***0.28***
20	**0.10**	**-0.10**	***0.20***

3. Apply the two-factor model (eq-1.7) with λ=0.4 and k=6, to get fair values of $E^*[X]$= −0.05 and $E^*[Y]$= −0.95. We have $E^*[X] - E^*[Y]$= 0.90, approximately equal to the combined differences, by separately using (eq-1.3) with λ=0.4, and using (eq-1.5) with 6 degrees-of-freedom.

Risk Premiums for Higher Moments

In classic CAPM where asset returns are assumed to follow multivariate normal distributions, the "market price of risk," $\lambda=(E[R]-r)/\sigma[R]$, represents the excess return per unit of volatility.

The classic CAPM has gone through important enhancements in modern finance and insurance research. In addition to risk premium associated with volatility, there is strong evidence of risk premium for higher moments (and for parameter uncertainty). This evidence has spurred extensions of classic CAPM, to include higher moments. In their recent paper, Kozik and Larson (2001) give a formal account of an n-moment CAPM. The authors offer insightful discussions on the risk premium for higher moments, pointing out that a three-moment CAPM significantly improves the fit of empirical financial data; however, there is little marginal gain by including higher moments beyond the third moment.

Obviously, the risk premium for higher moments has direct implications in pricing property catastrophe insurance, high excess-of-loss insurance layers, credit default risk, and deep-out-of-the-money options. From a risk management point of view, the cost of cushion capital increases with gearing and parameter uncertainty, as though they were extreme tail events.

The one-factor Wang Transform (eq-1.3), which can be viewed as an analog to the two-moment CAPM, does not produce sufficient risk adjustment at the extreme tails of the risk probability distribution.

The Student-t adjustment (eq-1.5) captures two opposing forces that often distort investors' rational behavior, namely *greed* and *fear*. Although investors may fear unexpected large losses, they desire unexpected large gains. As a result the tail probabilities are often inflated at both tails, with the magnitude of distortion increasing at the extremes. This distributional adjustment at both tails increases the kurtosis of the underlying distribution. The mean value of the transformed distribution under (eq-1.5) reflects the skew (asymmetry) of the underlying loss distribution.

The two-factor model (eq-1.7), however, as a combination of (eq-1.3) and (eq-1.5), provides risk premium adjustments not only for the second moment, but also for higher moments, and for parameter uncertainty.

The two-factor Wang Transform provides good fit to CAT-bond transaction data and corporate credit yield spreads (see Wang, 2002a). The parameter λ is directly linked to the Sharpe Ratio, a familiar concept to fund managers. With this universal pricing formula, investors can compare the risk/return trade-off of risk vehicles drawn from virtually any class of assets or liabilities.

Section 2. A Coherent Risk Measure Of Required Capital That Captures Overall Loss Distributions

In this section I propose a risk measure for measuring the capital requirements of the firm that goes beyond coherence. The popular VaR and the coherent Tail-VaR measures ignore information from a large part of the loss distribution. I propose a new coherent risk-measure that utilizes information from the entire loss distribution.

VaR as a Quantile Measure

Capital requirement risk-measures are used to decide the required levels of capital for a given risk portfolio, based on downside risk potential. A popular risk-measure for capital requirements in the banking industry is the Value-at-Risk (VaR), based on a percentile concept.

Consider a risk portfolio (e.g., investment portfolio, trading book, insurance portfolio) in a specified time-period (e.g., 10-day, one-year). Assume that the projected end-of-period aggregate loss (or shortfall) X has a probability distribution $F(x)$.

The Value-at-Risk is an amount of money such that the portfolio loss will be less than that amount with a specified probability α (e.g., α=99%):

$$\mathrm{VaR}(\alpha) = \mathrm{Min}\ \{x \mid F(x) \geq \alpha\}.$$

If the capital is set at $\mathrm{VaR}(\alpha)$, the probability of ruin will be no greater than $1-\alpha$. For computer-generated discrete distributions, it is possible that $\Pr\{X > \mathrm{VaR}(\alpha)\} < 1-\alpha$.

VaR, as a risk-measure, is only concerned with the frequency of shortfall, but not the size of shortfall. For instance, doubling the largest loss may not impact the VaR at all. From the perspective of company executives, the quantile "VaR" at the enterprise level may be a meaningful risk-measure, as the primary concern is the occurrence of shortfall. However, as a risk measure for capital requirement, VaR has limitations since it ignores the size of shortfall and it may exhibit inconsistencies when used for comparing risk portfolios.

Tail-VaR as a Coherent Risk-Measure

From a regulatory perspective, Professors Artzner, Delbaen, Eber, and Heath (1999) advocated a set of consistency rules for a risk-measure.

1. Subadditivity: For all random losses X and Y, $\rho(X+Y) \leq \rho(X) + \rho(Y)$.
2. Monotonicity: If $X \leq Y$ for each outcome, then $\rho(X) \leq \rho(Y)$.
3. Positive Homogeneity: For positive constant b, $\rho(bX) = b\rho(X)$.
4. Translation Invariance: For constant c, $\rho(X+c) = \rho(X) + c$.

They demonstrated that VaR does not satisfy these consistency rules. From a risk-management perspective, a consistent evaluation of the risks for business units and alternative strategies would require a coherent risk-measure other than VaR.

Artzner et al. (1999) proposed an alternative risk measure, called a "Conditional Tail Expectation" (CTE), also called the Tail-VaR. Letting α be a prescribed security level, Tail-VaR has the following expression (see Hardy, 2001):

$$\mathrm{CTE}(\alpha) = \mathrm{VaR}(\alpha) + \frac{\Pr\{X > \mathrm{VaR}(\alpha)\}}{1-\alpha} \cdot \mathrm{E}[X - \mathrm{VaR}(\alpha) \mid X > \mathrm{VaR}(\alpha)].$$

This lengthy expression is due to the complication that for computer-generated discrete distributions we may have $\Pr\{X > \mathrm{VaR}(\alpha)\} < 1 - \alpha$.

Tail-VaR reflects not only the frequency of shortfall, but also the expected value of shortfall. Tail-VaR is coherent, which makes it a superior risk-measure than VaR.

Recently there is a surge of interest in coherent risk-measures, evidenced in numerous discussions in academic journals and at professional conventions (see Yang and Siu, 2001; Meyers, 2001; among others). The Office of the Superintendent of Financial Institutions in Canada has put in regulation for the use of $\mathrm{CTE}(0.95)$ to determine the capital requirement for segregated fund risks.

The Tail-VaR, although being coherent, reflects only losses exceeding the quantile "VaR", and consequently lacks incentive for mitigating losses below the quantile "VaR". Moreover, Tail-VaR does not properly adjust for extreme low-frequency and high-severity losses, since it only accounts for the expected shortfall.

An Alternative Measure for Capital Requirement

Coherent risk-measure is by no means unique. The Wang Transform (eq-3) also satisfies the consistency rules of Artzner et al (1999). As an alternative to Tail-VaR, I propose a coherent risk-measure for capital requirements.

Definition 2.1 *For a loss (shortfall) variable X with distribution F, we define a new risk-measure for capital requirements as follows:*

1. *For a pre-selected security level α, let $\lambda = \Phi^{-1}(\alpha)$.*
2. *Apply the Wang Transform: $F^*(x) = \Phi[\Phi^{-1}(F(x)) - \lambda]$.*
3. *Set the capital requirement to be the expected value under F^*:*

$$WT(\alpha) = E^*[X].$$

For normal distributions, $WT(\alpha)$ is identical to $VaR(\alpha)$, the 100α-th percentile. For distributions other than normal, $WT(\alpha)$ may correspond to a percentile higher or lower than α, depending on the shape of the distribution.

When loss X has a log-normal distribution with $\ln(X) \sim Normal(\mu,\sigma^2)$, the WT-measure has a simple formula:

$$WT(\alpha)= \exp(\mu+\lambda\sigma+\sigma^2/2) \text{ with } \lambda = \Phi^{-1}(\alpha).$$

The $WT(\alpha)$ for the log-normal distribution corresponds to the percentile $\Phi(\lambda+\sigma/2)$, which is higher than α.

The following examples show that $WT(\alpha)$ improves differentiation at the extreme tails, and *provides the right incentives for risk management.*

Example 2.1. Consider two hypothetical portfolios with the following loss distributions.

Table 2.1. Loss Distributions for Portfolio A & Portfolio B

Portfolio A	
Loss x	**Prob f(x)**
\$0	***0.600***
\$1	***0.395***
\$5	***0.005***

Portfolio B	
Loss x	**Prob f(x)**
\$0	***0.600***
\$1	***0.398***
\$11	***0.002***

Table 2.2. Risk-Measures With α=0.99.

Portfolio	CTE(0.99)	WT(0.99)
A	\$3.00	\$2.59
B	\$3.00	\$3.89

At the security level α=0.99, given that a shortfall occurs, Portfolios A and B have the same expected shortfall. However, the maximal shortfall for Portfolio B (\$11) is more than double that for portfolio A (\$5). For most prudent individuals, Portfolio B constitutes a higher risk. Tail-VaR fails to recognize the differences between A and B. By contrast, WT(0.99) gives a higher capital requirement for Portfolio B (\$3.89) than for Portfolio A (\$2.59).

Example 2.2. Consider a risk portfolio with ten equally likely scenarios with loss amounts \$1, \$2, ..., \$10, respectively. Assume that all loss-scenarios can be eliminated though active risk management, except that the worst-case \$10 loss cannot be mitigated at all. Suppose a risk-manager is weighing the cost of active mitigation of risk against the benefit of capital relief. Tail-VaR would not encourage the active mitigation of risk, because there is no capital relief for removing losses below the worst-case loss. However, by removing all losses below \$10, WT(0.99) drops from \$9.71 to \$8.52, showing a \$1.19 capital relief. WT(0.95) drops from \$9.12 to \$6.42, showing a \$2.70 capital relief.

RORAC Calculations

It is common practice for risk-managers to calculate the return on risk-adjusted capital (RORAC) for a given standalone portfolio. For such an exercise, our new risk-measure can be used in calculating the expression denominator, that is, for calculating the RAC, or risk-adjusted capital.

Comment on the Threshold

Regardless of the choice of risk measure, say, VaR(α), TailVaR(α), or WT(α), the value of the parameter α has significant implications to the financial performance of the enterprise. From the regulatory perspective, it may create market inefficiencies when selecting too low or too high a value of α. The optimal value for α may well depend on alternative investment opportunities in other industries. Kreps (1998) explores similar ideas in the context of reinsurance pricing. The optimal value of α is an important subject that deserves further research.

Section 3. Allocating Risk Capital Among the Business Units of the Enterprise

In this section I propose a framework for measuring risk and allocating capital among the business units of the company, based on exponential tilting.

Variance-Based Risk Measure

From a company's portfolio perspective, doubling a risk exposure may more than double the risk contribution to the aggregate "portfolio of the firm," due to increased risk concentration. Traditionally the aggregate risk concentration is better measured by "*variance*" rather than "*standard deviation.*"

Because "variance" is based on the second moment, it also suffers the drawback of "standard deviation" in failing to differentiate upside skew from downside skew. This drawback of the "variance" measure, however, can also be overcome by a probability transform:

$$f^*(x) = \frac{f(x)\exp(\lambda x)}{E[\exp(\lambda X)]}, \qquad \text{(eq-3.1)}$$

which is called the Esscher Transform (see Gerber and Shiu, 1994).

When X has a Normal(μ,σ^2) distribution, the Esscher Transform gives another normal distribution with $\mu^*=\mu+\lambda\sigma^2$, and $\sigma^*=\sigma$. Thus, for normally distributed risks, the Esscher premium recovers the variance-loading method:

$$H_{Esscher}[X;\lambda] = E[X] + \lambda \cdot Var[X].$$

In other words, the Esscher Transform extends variance-based risk-adjustment to risks with non-normal distributions. This is analogous to how the one-factor Wang Transform (eq-1.3) extends standard-deviation loading to risks with non-normal distributions.

States of the World

Let Ω represent a collection of possible *states* of the world. Each state of the world ω contains multivariate risk factors or events that could potentially happen in a specified time period. For instance, the collection of events that have had happened in 2001 can be viewed as a realized *state* of the world. In the U.S. insurance market, some major events happened in 2001 included the terror attacks of September 11, the collapse of Enron, increasing mold claims in Texas, and a lower domestic interest-rate environment.

Different business units, or lines of business, within an insurance company were impacted differently by these events.

Exponential Tilting

Consider a risk X and a reference portfolio with aggregate risk Z. Here the reference portfolio may be a company portfolio, industry portfolio, or the financial impact of a selected risk factor. We define an exponential tilting of X induced by Z:

$$X^*(\omega) = X(\omega)\frac{\exp(\lambda Z(\omega))}{E[\exp(\lambda Z(\omega))]}, \text{ for every possible state } \omega \text{ in } \Omega.$$

We denote

$$H_\lambda[X,Z] = \frac{E[X \cdot \exp(\lambda Z)]}{E[\exp(\lambda Z)]}. \qquad \text{(eq-3.2)}$$

Remark: The theoretical foundation for "exponential tilting" is rooted in an equilibrium-pricing model of Buhlmann (1980, 1984). He considered an optimal risk exchange model where each participant aims to maximize his/her expected utility. Buhlmann showed that in the equilibrium the price for risk X has the same expression as (eq-3.2).

> **Example 3.1**: When X and Z have a bivariate normal distribution with correlation coefficient $\rho_{X,Z}$, the transformed variable X^* also has a normal distribution with
>
> $$\mu_X^* = \mu_X + \lambda\rho_{X,Z}\sigma_X\sigma_Z \text{ and } \sigma_X^* = \sigma_X.$$

Esscher Transform from Company Portfolio Perspective

For the aggregate risk of the reference portfolio, the exponential tilting gives

$$H_\lambda[Z,Z] = \frac{E[Z \cdot \exp(\lambda Z)]}{E[\exp(\lambda Z)]}.$$

This is exactly the Esscher premium, an extension of variance-based risk adjustment.

Systematic Risk for a Given Reference Portfolio

Let Z be the aggregate risk for a reference portfolio. Assume that risk X can be decomposed into two parts

$$X = X_{sys} + X_{non},$$

where

- X_{sys} (being co-monotone with Z) represents the systematic portion of X, and

- X_{non} (being uncorrelated with Z) represents the idiosyncrasy or non-systematic portion.
- By definition, X_{sys} and X_{non} are uncorrelated.

Note that the notion of "systematic risk" has a relative meaning, depending upon the reference portfolio Z.

It can be easily verified that

$$H_{\lambda}[X,Z] = E[X_{non}] + \frac{E[X_{sys} \cdot \exp(\lambda Z)]}{E[\exp(\lambda Z)]}. \qquad \text{(eq-3.3)}$$

In other words, exponential tilting induced by Z only adjusts for the non-diversifiable risk with respect to Z.

The following result can be found in Wang (2002b).

> **Theorem 3.1**: Let the reference portfolio be the market portfolio. Under the assumption that
>
> - Risk X_j is co-monotone with the aggregate risk Z,
> - The aggregate risk Z has a normal distribution with standard deviation $\sigma(Z)$,
>
> the exponential tilting (eq-3.2) is equivalent to the one-factor Wang Transform: $F^*(x) = \Phi\left[\Phi^{-1}(F(x)) + \lambda_0\right]$, where $\lambda_0=\lambda\sigma(Z)$ represents the market price per unit of risk.

Conceptually, when we enlarge a company portfolio so that it approaches the market portfolio, we can reasonably expect that the risk measure based on the company portfolio perspective should converge to that of market perspective. We have seen that "exponential tilting" facilitates such a natural transition; it produces the Esscher Transform for the company portfolio perspective, and produces the Wang Transform for the market perspective.

We shall show that exponential tilting lends itself to a coherent allocation of risk contributions of various business units.

Intra-Company Allocation of the Cost of Capital

Consider a company with n individual business units. In making strategic evaluations, firms often need to allocate the total cost of capital to different business units, or among lines of business. For such an allocation exercise, the correlation structure between various business units, or among lines of business, becomes critically important.

For every possible state ω in Ω, let $X_1(\omega)$, $X_2(\omega)$, ..., $X_n(\omega)$ represent the losses to n individual business units. The aggregate loss to the company is:

$$Z(\omega) = X_1(\omega) + X_2(\omega) + \ldots + X_n(\omega).$$

The correlations between $\{X_1(\omega), X_2(\omega), \ldots, X_n(\omega)\}$ are completely specified by their dependence of various states of the world. To describe such a correlation structure, we can use a representative sample of multivariate values based on historical data, and/or scenario-based simulations (see Section 4).

Assume that the aggregate capital requirement C_{aggr} has been given for the whole company. For instance, it can be based on a coherent risk measure such as WT(0.95), as in section 2.

We can solve out a number λ such that

$$C_{aggr} = H_\lambda[Z,Z] - E[Z]. \qquad \text{(eq-3.4)}$$

We propose the following allocation of cost of capital to individual business unit j (j=1, 2, ..., n):

$$C_j = H_\lambda[X_j,Z] - E[X_j]. \qquad \text{(eq-3.5)}$$

Obviously this allocation method is additive: $C_{aggr} = \sum_{j=1}^{n} C_j$.

Theorem 3.2. Assume that $X_1(\omega), X_2(\omega), \ldots, X_n(\omega)$ have a multivariate normal distribution with a covariance matrix:

$$\begin{pmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 & \cdots & \rho_{1n}\sigma_1\sigma_n \\ \rho_{12}\sigma_1\sigma_2 & \sigma_2^2 & \cdots & \rho_{2n}\sigma_2\sigma_n \\ \vdots & \vdots & \vdots & \vdots \\ \rho_{1n}\sigma_1\sigma_n & \rho_{2n}\sigma_2\sigma_n & \cdots & \sigma_n^2 \end{pmatrix}.$$

The allocation method is equivalent to the covariance method with

$$C_{aggr} = \lambda\sigma_{aggr}^2 = \lambda\sum_{i,j=1}^{n}\rho_{ij}\sigma_i\sigma_j, \quad \text{and} \quad C_j = \lambda\sum_{i=1}^{n}\rho_{ij}\sigma_i\sigma_j, \text{ for } j=1, 2, \ldots, n.$$

Remark:

1. Under multivariate normal assumptions, the allocation method as outlined in (eq-3.4) & (eq-3.5) is exactly the same as the covariance method.

2. For other than multivariate normal distributions, the allocation method (eq-3.4) & (eq-3.5) is superior to the covariance method in that it better reflects tail correlation and skew/kurtosis in the individual risk distributions.

Market Implied Cost of Capital

From the market perspective, a formula-based benchmark price $E^*[X_j]$ for insurance risk X_j implies the following economic capital for X_j:

$$\pi(X_j) = (E^*[X_j] - E[X_j])/\text{TEROE},$$

where TEROE is the *target excess return on equity*, over the risk-free rate r. Market benchmark pricing implies an aggregate capital requirement of $\pi(Z) = \pi(X_1) + \ldots + \pi(X_n)$.

Recall that C_{aggr} represents the aggregate capital requirement based on the company's own risk portfolio. It is useful to compare C_{aggr} with $\pi(Z)$, and C_j with $\pi(X_j)$, respectively. The relative sizes between C_{aggr} and $\pi(Z)$, C_j and $\pi(X_j)$, reveals the extent of company diversification, relative to an "average" industry portfolio.

Section 4. Aggregating Correlated Risks to Produce An Integrated Risk Profile of The Firm

The correlation structure between risks can significantly impact the aggregate portfolio risk, as well as the allocations of risk capital to business units. Here I discuss some useful correlation models.

Extreme Correlation

In many real-life situations, extreme correlation is often higher than what the linear correlation coefficient indicates. For instance, the terror attacks of September 11 resulted in big losses in many lines of business, including life insurance, property insurance, aviation insurance, and workers compensation. The collapse of Enron resulted in sudden increases in surety bond premium rates, which in turn forced the retailer K-Mart Stores to file for bankruptcy protection.

Normal Copula Is Sometimes Inadequate For Capital Allocation

One of the most popular correlation models is the normal copula, because (i) the correlation structure can be completely specified by a correlation matrix; and (ii) there are readily available simulation routines and software. Unfortunately, the normal copula does not give sufficient extreme correlations. Embrechts et al (1999) showed that a normal copula shows asymptotically zero-correlation at the extreme tails. An alternative, the student-t copula shows higher correlation at the tails. Mango and Sandor (2002) have cautioned against using the normal copula in capital allocation exercises. Venter (2001) analyzed various copulas using simulated catastrophe loss data.

The known drawbacks of the normal copula encourages the use of a statistical copula that properly incorporates a higher correlation at the extreme tails. This statistical copula can be empirically constructed from historical data, or modified with a set of stress tests embodied in scenarios.

Empirical Copula

There are numerous parametric copula models (see Frees and Valdz, 1998). Although a parametric copula can be fit to empirical multivariate data, the estimation of copula parameters often depends on the model choice of marginal distributions.

I propose using a type of empirical copula that is not affected by the choice of marginal distributions. When limited by historical multivariate data, we can supplement this empirical copula by scenario-based simulations.

Consider a sample of simultaneous observations $\{x_m, y_m\}$, $m=1, 2, \ldots, M$, of two variables X and Y.

Rank the values of x_m, $m=1, 2, \ldots, M$, in an ascending order. For each x_m we assign a sub-interval, $I(x_m)$, situated within $[0,1]$.

The lower boundary of the interval $I(x_m)$ is

$$\frac{\text{Number of observations strictly less than } x_m}{M}.$$

The upper boundary of the interval $I(x_m)$ is

$$1-\frac{\text{Number of observations strictly greater than } x_m}{M}.$$

For each x_m, $m=1, 2, \ldots, M$, we define u_m as the mid-point of the interval $I(x_m)$, Note that repetitive values of x_m will result in repetitive values of u_m.

We do the same for the values of y_m, $m=1, 2, \ldots, M$. Let v_m be the mid-point of the interval $I(y_m)$.

We call the sample discrete distribution $\{u_m, v_m\}$, $m=1, 2, \ldots, M$, an empirical copula induced by the sample $\{x_m, y_m\}$, $m=1, 2, \ldots, M$.

A simple instance of this empirical copula is implied in multivariate traded prices for multiple stocks or stock indices. Multivariate insurance data, however, is much less abundant. We must rely heavily on scenario-based simulations to generate the appropriate multivariate data.

Use Empirical Copula in Modeling and Combining Correlated Risks

Consider two risks W_1 and W_2 with marginal distributions, F_1 and F_2, respectively. To simulate a bivariate sample of W_1 and W_2 with their correlation structure as specified by the empirical copula $\{u_m, v_m\}$, $m=1, 2, \ldots, M$, the following method can be employed:

$$\left\{w_1(m) = F_1^{-1}(u_m),\ \ w_2(m) = F_2^{-1}(u_m)\right\},\quad m=1, 2, \ldots, M,$$

with each pair having a probability of $1/M$.

Using this simulation method, we can easily generate a sample of the aggregate risk: $W = W_1 + W_2$.

This aggregation method can be easily generalized to combining "n" risks.

Multivariate Normal Variance Mixture

Consider multivariate standard normal variables $(Z_1, Z_2, \ldots, Z_n)$ with correlation coefficients $\rho_{ij} = \text{corr}(Z_i, Z_j)$. Let B be a non-negative random variable. We define

$$(X_1, X_2, \ldots, X_n) = (BZ_1, BZ_2, \ldots, BZ_n).$$

Intuitively, this is a stochastic volatility model (that is, the variance itself is random). We say that $(X_1, X_2, \ldots, X_n)$ have a multivariate normal mixture distribution.

Normal variance mixtures preserve the linear correlation coefficients of the multivariate normal distribution:

$$\text{Corr}(X_i, X_j) = \text{Corr}(Z_i, Z_j).$$

Example 4.1. Let the multiplier B have a lognormal distribution with mean=1 and CV=γ.

Example 4.2. Let $B = \sqrt{\frac{k}{C}}$ where C has a Chi-square distribution with k degrees-of-freedom. Then $(X_1, X_2, \ldots, X_n)$ have a multivariate student-t distribution with k degrees-of-freedom. Frey et al (2001) compared the impacts on VaR calculations using student-t copula versus normal copula.

Section 5. Optimizing The "Portfolio Of The Firm" To Create New Shareholder Value

This section applies the two-factor Wang Transform to portfolio optimization, and to quantifying the value creation associated with portfolio strategies. This can be viewed as an extension of the Markowiz mean-variance framework.

The Sharpe Ratio and Mean-Variance Optimization

Let R be the return in the forthcoming time period on an asset portfolio. Let the benchmark portfolio be the risk-free rate r at the same time horizon. The Sharpe Ratio, as a measure of reward-to-variability ratio, is defined as

$$\lambda = \frac{E[R]-r}{\sigma[R]},$$

which also corresponds to the concept of the "market price of risk."

After its initial publication by economist William Sharpe in 1966, the Sharpe Ratio soon became a popular way for fund managers to calculate excess return for a given level of risk. As a simple rule of thumb, the higher the Sharpe ratio, the better the prospect of return, relative to a level of risk. Given some constraints of risk-tolerance set by the portfolio managers, an optimal portfolio can be constructed so as to maximize the prospective Sharpe Ratio for the aggregate portfolio.

In financial economics, the optimal asset portfolios lie on an efficient frontier, under the Markowiz mean-variance framework. For normally distributed risks, maximizing the Sharpe Ratio is consistent with the Efficient Frontier under the Markowiz framework.

The lambda parameter in the Wang Transform (eq-3) extends the Sharpe Ratio concept to assets/losses with non-normal distributions. With further incorporation of parameter uncertainty and higher moments, the two-factor Wang Transform enables an extension of the Markowiz mean-variance framework.

Portfolio Fair Economic Value

Consider a risk portfolio with a fixed time horizon, $[0, T]$. Suppose that we have projected a probability distribution, $F(x)$, of the aggregate profit/loss $X(T)$ at the end of the period for the whole portfolio.

With market benchmark parameters for the Sharpe Ratio λ and the Student-t degrees-of-freedom, we apply the two-factor Wang Transform

$$F^*(x) = Q\left[\Phi^{-1}(F(x)) + \lambda\right]. \qquad \text{(eq-5.1)}$$

We define fair economic value (EconVal) for the portfolio as the mean value under the transformed distribution, with further risk-free discounting for the fixed time horizon to present value as needed:

$$\text{EconVal} = \exp(-rT)\, E^*[X(T)]. \qquad \text{(eq-5.2)}$$

In real life most firms are operating under some sort of constraints (budgeting, capital requirement, and cash-flow liquidity). Insurance companies operate under stringent capital requirements imposed upon by regulators and rating agencies. A firm with excessive risk-taking may jeopardize its long-term viability, and incur substantial transaction costs when under financial stress.

Optimization Method #1

We can state our optimization problem as maximizing the expected value of the risk-adjusted returns *under certain operating constraints*. Maximizing expected profit *without constraint* would lead firms to engage in speculative investments with the highest risk-adjusted expected returns. One operating constraint is the probability of ruin within a given threshold. Another example of operating constraint is requiring the company to remain at a "AA" credit or insurance rating by the end of the given period.

The portfolio optimization process can then be formalized as maximizing the fair economic value in (eq-5.2) subject to some given operating constraints.

Based on the ERM analysis, a company may make strategic changes to its risk portfolio. The value creation can be simply quantified as

$$\text{ValCr} = \text{EconVal(Ending Portfolio)} - \text{EconVal (Initial Portfolio)}.$$

Optimization Method #2

Let X be the profit/loss distribution for the risk portfolio. Let the economic capital be determined by a pre-selected risk-measure, say, WT(α), for α=0.99, as in section 2. We can calculate the return on economic capital as:

$$R = X / \mathrm{WT}(\alpha). \qquad \text{(eq-5.3)}$$

We apply a two-factor Wang Transform in (eq-5.1) to the probability distribution of R, and calculate the Risk-Adjusted Return On Capital (*RAROC*) as its expected value.

Thus we can perform portfolio optimization by maximizing the *RAROC* under some operating constraints. By maximizing the RAROC in this way, on an enterprise-wide basis, ERM can lead to optimal decisions and shareholder value creation.

Conclusions

In this paper we have presented a set of methods and tools for measuring risks and allocating the cost of capital. These tools are inter-connecting parts of a common framework for enterprise risk capital management. A sound methodology framework lays the foundation for building a knowledge-based risk management system. To use these tools correctly in ERM practices, it is critical to first develop good risk metrics that captures the real risk dynamics of individual business units, and their inter-relations. Of course, many other issues remain to be addressed in future research. Stay tuned.

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Figure 1.1 Implied *b*-values using Student-t distribution
(Here "def" refers to degrees-of-freedom)