Management's Best Estimate of Loss Reserves

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Abstract

An economically rational way for management to set reserve estimates is to utilize the future change in the value of the company as a statistical decision function and then to choose the reserve estimate so as to minimize the average value of this function. The mean of the reserve distribution is almost surely too low as an outcome.

Introduction

Management is required[1] to provide a best estimate of loss reserves. In the opinion of this author, actuarial practices[2] strongly suggest the estimate be the mean value of the distribution of loss reserves. It will be argued that the problem of the estimate is best approached by statistical decision theory, and that all the usual statistical estimates can be produced in such a fashion. Further, there is an economic basis for choosing a decision function, which then determines the estimate. Desirable characteristics for a decision function are discussed, and a candidate function is proposed. A simplified example and a spreadsheet are provided. One general conclusion that emerges is that the mean is probably not a good estimate, as it is almost surely low.

Statement of Statutory Accounting Principles

The SSAP #55 effective January 1, 2001 says[3] in part "For each line of business and for all lines of business in the aggregate management shall record its best estimate of its liabilities ..." and "Management's analysis of the reasonableness of ... reserve estimates shall include an analysis of the amount of variability in the estimate." Not to put too much into a single word, but please note that it is "in the estimate" rather than "of the estimate." The author believes that actuaries have tended to place too much attention on the differences between different estimates and not enough on the variability of actual results.

SSAP 55 goes on to say "Management's range [of estimates] shall be realistic and, therefore, shall not include the set of all possible outcomes but only those outcomes that

[1] Statutory Statement of Accounting Principles #55 effective January 1, 2001
[2] ASOP # 36, especially section 3.6.3
[3] This and subsequent quotes in this section are taken from SSAP 55, page 55-6, sections 10 and 11.

are considered reasonable." In other words, weight scenarios by their probabilities: use a distribution. But how?

In the next section SSAP 55 says that in the case of a range with all values equally likely, choose the middle of the range; but if the equally likely values do not form a range "management should determine its best estimate of the liability." Again, there is not a lot of help here on how to actually do it. Let us see what the actuaries have to say.

Actuarial Standard of Practice

ASOP 36 – Statements of Actuarial Opinion Regarding Property/Casualty Loss and Loss Adjustment Expense Reserves - has quite a bit to say about reserves and uncertainty. We will assume that the hard work of evaluating trends, court climates, and other sources of both process and parameter uncertainty has been done and that we have a best candidate loss reserve distribution which includes all the outcomes and best estimates of their probabilities.

This is, of course, a major assumption seldom made explicit in practice but often used implicitly. For example, section 3.6.3 is entitled "Expected Value Estimate" and says "In evaluating the reasonableness of reserves, the actuary should consider one or more expected value estimates of the reserves, except when such estimates cannot be made based on available data and reasonable assumptions." Expected value, apart from the fact that you never actually expect to see it happen, is the mean of a distribution. So "one or more expected value estimates" is saying "find different ways of getting the mean of our unknown but implicitly present distribution."

The same section goes on to say "Other statistical values such as the mode (most likely value) or the median (50^{th} percentile) may not be appropriate measures for evaluating loss and loss adjustment expense reserves, such as when the expected value estimates can be significantly greater than these other estimates." Here the author sees the innate, carefully cultivated, and experientally substantiated conservatism of the actuaries expressed as "Let's go for the higher value." Curiously enough, this paper will argue that the mean itself is too low.

The section continues "The actuary may use various methods or assumptions to arrive at expected value estimates. In arriving at such expected value estimates, it is not necessary to estimate or determine the range of all possible values, nor the probabilities associated with any particular values." So, although ASOP wants mean value estimates, it does not want a distribution. Most of the techniques for doing reserve estimates, even including those which are used for sparse or missing data, have an underlying statistical model and an implied distribution. We are invited to use the models and forget the distributions.

The next section 3.6.4 is entitled "Range of Reasonable Reserve Estimates." It begins "The actuary may determine a range of reasonable reserve estimates that reflects the uncertainties associated with analyzing the reserves. A *range of reasonable estimates* is a range of estimates that could be produced by appropriate actuarial methods or alternative sets of assumptions that the actuary judges to be reasonable." Clearly, something like this needs to be in the ASOP in order to give the actuary room not to be forced by a formula. This is saying, look at least implicitly at several reasonable distributions (alternatively, models), get the mean from each one, and make a weighted choice. The author would prefer that the judgment calls be in the creation of the best predictive distribution. For example, if there are two equally valid distributions, weight them equally. The mean will be the average of the individual means. Of course, this does require going from no distributions to three.

This same section continues "The actuary may include risk margins in a range of reasonable estimates, but is not required to do so, except as required by ASOP No. 20. A range of reasonable estimates, however, usually does not represent the range of all possible outcomes." So, while allowing that there really is a distribution with outcomes of differing probabilities, the ASOP wants to make sure that the reserve estimate is well inside the range while not giving any guidance on how to get to a risk margin or what might be appropriate. Presumably, the margin should be for the risk that there will be, in the words of section 3.3.3, an "amount of adverse deviation that the actuary judges to be material with respect to the statement of actuarial opinion.." Unless the distribution is very narrow, this seems quite likely to be the case.

Section 3.6.5 on "Adverse Deviation" makes the same point, but does not suggest a risk margin. It only says "The actuary should consider whether the future paid amounts are subject to significant risks and uncertainties that could result in a material adverse deviation." Section 4.6(g) says that it is up to the actuary to include mention of this in the report . In addition, section 4.8 says "An actuary must be prepared to justify the use of any procedures that depart materially from those set forth in this standard and must include, in any actuarial communication disclosing the results of the procedures, an appropriate statement with respect to the nature, rationale, and effect of such departures." Perhaps this justification could be "and I plunked down another 35% because this line is all over the place."

The author's sense of the ASOP writing on uncertainty is that it gives management some idea of how much wiggle room there is in the creation of management's best estimate, at least according to the actuaries. But none of this actually gives an economic basis for how the estimate should be made.

Statistical Decision Theory – the short version

An economic basis for the creation of an estimate needs a way to combine a probability distribution of outcomes with an economic function describing the pain that will be felt when the realized random outcome differs from the estimate. The simplest recipe is to go for Least Pain, as follows:

(1) Create a pain function based on economic reality. This function will be the economic decision function which will drive the results. "Pain function" actually is the technical term because this function is meant to represent how unpleasant adverse outcomes of

various sorts may be. This function will depend on the estimate and the random variable representing possible outcomes. Typically it will be zero when random outcome and estimate are equal.

(2) Average the pain function over the probability distribution of outcomes for every fixed estimate.

(3) Find the value of your estimate which makes the average pain smallest.

It is easy enough to see that such a prescription satisfies SSAP 55, and that the description of the pain function represents management's logic in creating the estimate.

The context is in hand is setting the reserves and then a year later making adjustments for development on old years. More precisely, we assume that the assessment a year later is "correct" and represents a random realization of the underlying distribution.

What we will show next is that all the usual estimates can be represented as being derived from pain functions. The comparison of pain functions gives us a way to speak about the relevance of the estimates in a business and economic context. Pain functions can be thought of as negative utility functions.

Following that we will argue for the general characteristics of a pain function for loss reserving.

Mathematical Representation of the recipe

The general case is that we have a probability density function f(x) with support from 0 to infinity. We also have a pain function p(m,x) which is a function of the estimate m and the random variable x. We denote the average of p over f as P(m):

$$P(m) = E(p) = \int_0^\infty p(m,x) f(x) dx$$

It is reasonable to ask that the integral exists, and that p >= 0 everywhere. We want m to be such that P(m) is the smallest, so we choose the value for m which makes

$$0 = \frac{d}{dm} P(m) = \int_0^\infty \frac{\partial}{\partial m} p(m,x) f(x) dx$$

Sometimes in practice p will be discontinuous at x = m. In that case define

$$p(m,x) = \begin{cases} p_-(m,x) & \text{for } x \le m \\ p_+(m,x) & \text{for } x \ge m \end{cases}$$

which makes

$$\frac{dP(m)}{dm} = \int_0^m \frac{\partial p_-(m,x)}{\partial m} f(x)dx + \int_m^\infty \frac{\partial p_+(m,x)}{\partial m} f(x)dx$$
$$+ p_-(m,m) - p_+(m,m)$$

Usually the last difference is zero and in fact the individual terms are usually zero.

Notice that the scale and absolute value of P(m) do not enter into the calculation for m. You can add a constant and multiply by any constant and m does not change. The pain functions given are the only ones known to the author which give the usual statistics for all distributions.

Example: the mean

For the mean, the pain function is a quadratic about the estimate:

$$p(m,x)=(m-x)^2$$

The average pain is

$$P(x)=\int_0^\infty (m-x)^2 f(x)\,dx$$

In this particular case, we can do the integrals in terms of the mean and variance of the distribution:

$$\begin{aligned}P(m)&=\int_0^\infty \left(m^2-2mx+x^2\right)f(x)\,dx\\&=m^2-2m*mean+\left(Var+mean^2\right)\\&=Var+(m-mean)^2\end{aligned}$$

As a function of m, this clearly has a minimum when m equals the mean of the distribution.

At this point, we should pause and ask ourselves "Why do I want a quadratic decision function? What is so good about squared dollars?" The symmetry of the pain function about the estimate implies that for the reserves to come in lower than our estimate is as bad as having them come in higher. The quadratic form implies that two dollars low is four times as bad as one dollar low.

Example: the median

For the median, the pain function is linear about the estimate with equal slope on both sides, and with a discontinuity in the derivative at x = m:

$$\begin{aligned}p(m,x)&=abs(m-x)\\&=\begin{cases}m-x & \text{for x } \le m\\ x-m & \text{for x } \ge m\end{cases}\end{aligned}$$

Although this function is still symmetric about the estimate, it says that it is dollars, rather than squared dollars, that are of interest; and that two dollars off is only twice as bad as one dollar off. This has some plausibility.

For the evaluation, the partial derivative is

$$\frac{\partial \mathrm{p}(m,\mathrm{x})}{\partial m} = \begin{cases} 1 & \text{for } \mathrm{x} < m \\ -1 & \text{for } \mathrm{x} > m \end{cases}$$

So the equation to be solved for m is

$$\begin{aligned} 0 &= \int_0^m \mathrm{f(x)dx} - \int_m^\infty \mathrm{f(x)dx} \\ &= F(m) - \left[1 - F(m)\right] \end{aligned}$$

where F(x) is the cumulative distribution function. This requires

$$F(m) = \frac{1}{2}$$

That is to say, m is the median.

Example: the fixed percentile

For an arbitrary fixed percentile, the pain function is linear about the estimate, but with different slopes on the high and low side:

$$p(m,x) = \begin{cases} m - x & \text{for x } \leq m \\ \alpha(x - m) & \text{for x } \geq m \end{cases}$$

Again it is dollars that are of interest but here it is a factor of α worse for x to be high rather than low. Take α to be some constant, say 3.

For the evaluation, the partial derivative is

$$\frac{\partial \mathrm{p}(m,\mathrm{x})}{\partial m} = \begin{cases} 1 & \text{for } \mathrm{x} < m \\ -\alpha & \text{for } \mathrm{x} > m \end{cases}$$

So the equation to be solved for m is

$$\begin{aligned} 0 &= \int_0^m \mathrm{f(x)dx} - \int_m^\infty \alpha \mathrm{f(x)dx} \\ &= F(m) - \alpha\left[1 - F(m)\right] \end{aligned}$$

where again F(x) is the cumulative distribution function. This requires

$$F(m) = \frac{\alpha}{\alpha + 1}$$

That is to say, m is the α /□ α +1 percentile value. For $\alpha = 3$, this is the 75th percentile.

It will be argued that a decision function that gives more weight to the high side than the low is desirable for loss reserving. It is usually worse to come in above your estimate than below it.

Example: the mode

Here the decision function is one outside of some (preferably small) interval around the estimate, and zero within it. The economic interpretation of this decision function is that for the reserves to come in outside of this interval is equally bad no matter where it

happens. This means that high or low, just outside or very far away are all the same. This does not seem reasonable. On the other hand, this will provide the single best guess for an interval of given size to contain the result.
The pain function is

$$\mathrm{p}(m, x) = \begin{cases} 1 & \text{for } x \le m - \varepsilon \\ 0 & \text{for } m - \varepsilon < x < m + \varepsilon \\ 1 & \text{for } x \ge m + \varepsilon \end{cases}$$

and 2ε is the size of the interval. The average pain is the probability that the random variable is realized outside of the interval:

$$P(m) = F(m - \varepsilon) + [1 - F(m + \varepsilon)]$$

Setting the derivative to zero,

$$0 = f(m - \varepsilon) - f(m + \varepsilon) \approx -2\varepsilon \frac{df(m)}{dm}$$

For small interval, this says that the density function is a maximum at m. That is, m is the mode.

Fundamental considerations

It is clear that all of the usual estimates can be phrased as resulting from a particular choice of decision function and that infinitely many decision functions are possible. What is needed is to construct the decision function from the economic or other forces which impact the entity setting the reserve levels. This will be the appropriate decision function.

One possibility is a purely subjective estimate of how, say, the CFO feels about various sizes of future difference of reserves from the stated estimate. Slightly better would be to use the reserves committee as input, in a Delphi method.

Another possibility is to examine the fundamental economic consequences which result from the reserves (at least as appearing in next year's Annual Statement) coming in different from the reserve level which is currently set. A good candidate for the decision function is the **decrease in the net economic worth of the company** as a result of the reserve changes. While estimates of this may involve subjective judgments, at least something of definite and measurable economic value is being considered.

Interested parties who may affect the economic value of the company include policyholders, stockholders, agents, regulators, rating agencies, IRS, investment analysts, and lending institutions.

If the reserves come in slightly higher than the estimate, there perhaps is not much market reaction. The industry as a whole has had the reputation of being under-reserved. It may also be that some managements will like being slightly under-reserved because then they are able to have overstated earnings the previous year. However, if the increase in reserve levels is significant enough that surplus is significantly impacted then a number of effects come into play: The capacity to write business is impaired; the firm's credit rating may become impaired, increasing the cost of capital; rating agencies, investor analysts, and the IRS become more concerned; future renewals (the "goodwill" of the firm) become more problematical. And if the change is large enough then IRIS tests begin to be triggered and regulatory authorities are involved, which definitely will decrease the value of the firm. The economic consequences would seem to be rapid and non-linear in the reserve increase.

On the other hand suppose the reserves come in significantly below the estimate. This means that the company has been over-reserved and consequently[4] is less competitive than it could have been; that the IRS has ammunition for its audit; that dividends could have been larger; participatory plans could have been more generous; that there is a danger of losing future business from over-pricing. These effects would seem to be less immediately significant than the results from under-reserving, at least in the short term.

Each of these situations will generate a negative effect on the net worth of the company compared to its value if the reserves came in as stated. However, intuition suggests that the effect will be much stronger on the under-reserving side than on the over-reserving side, and will be non-linear, especially as particular analyst, rating agency, and regulatory tests reach trigger points.
The immediate consequence is that a symmetric pain function such as that for the mean would be over-estimating the negative impact of reserve decreases and consequently the estimate is intrinsically too low.

An approximation to the correct function

As a crude approximation which has some of the properties just suggested, consider a decision function which is quadratic around the estimator but linear (using the tangent to the parabola) at some value below it. Call it "semi-quadratic." This function is mathematically well-behaved, as the value and the first derivative are both continuous. This function will also satisfy that being high (in the outcome) is never better than being low and that the high side is quadratic while the low side is linear. The choice of distance below determines the slope of the line; the closer it is to the estimator the smaller the slope.

We make the decision function have the dimensions of dollars to mimic the economic value. S is the company surplus, since that is the appropriate scale for many tests.

The decision function is $$p(m,x) = \begin{cases} 2(m\text{-}x) - \alpha S & \text{for } x \le m - \alpha S \\ \dfrac{(x\text{-}m)^2}{\alpha S} & \text{for } x \ge m - \alpha S \end{cases}$$

[4] Assuming that the pricing and reserving actuaries actually talk with each other

The parameter α is dimensionless and reflects management attitude. The pain is equal to αS when the outcome is αS above or below the estimator. A small α reflects a relatively low pain for over-reserving and conversely a higher pain for under-reserving.

An α of 3% is used below, which says that a reserve change of 3% of surplus downward is the same pain as the same change upward. However, 10% under-reserved is about twice as bad as 10% over-reserved; 20% under-reserved is about 3.6 times as bad as 20% over-reserved. Again, as α gets smaller management is less tolerant of under-reserving.

The partial derivative is
$$\frac{\partial \mathrm{p}(m,\mathrm{x})}{\partial m} = \begin{cases} -2 & \text{for } \mathrm{x} \le m - \alpha \mathrm{S} \\ \dfrac{2(x-m)}{\alpha \mathrm{S}} & \text{for } \mathrm{x} \ge m - \alpha \mathrm{S} \end{cases}$$

and the equation to be solved is
$$0 = -\int_0^{m-\alpha S} \mathrm{f(x)dx} + \frac{1}{\alpha S}\int_{m-\alpha S}^{\infty} (\mathrm{x}-m)\,\mathrm{f(x)dx}$$

Or, $$F(m-\alpha S) = \frac{1}{\alpha S}\left\{\left[mean - F_1(m-\alpha S)\right] - m\left[1-F(m-\alpha S)\right]\right\}$$

Where $F_1(x)$ is the integral of xf(x) – the first moment distribution.

In order to work entirely with dimensionless variables, it is convenient to measure the estimate and the mean in units of surplus. Then the above equation holds with S = 1.

Just to make explicit the kind of results that might be seen, as an example take F(x) to be a lognormal distribution with known *mean* and coefficient of variation **c**. Both F(x) and $F_1(x)$ can be explicitly calculated in terms of the normal distribution function

$N(x) \equiv \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{z^2}{2}}dz$:

We have $$F(x) = N\left(\frac{\ln(x)-\mu}{\sigma}\right)$$

and $$F_1(x) = mean\, N\left(\frac{\ln(x)-\mu}{\sigma} - \sigma\right).$$

with the usual $\sigma = \sqrt{\ln(1+c^2)}$ and $\mu = \ln(mean) - \sigma^2/2$.

For a company which has a mean reserve to surplus ratio of 3.5 with a coefficient of variation in the reserves of 10%, an α of 3% results in setting the reserve estimator at 11.5% above the mean. This estimator is at about the 87.2% level of the reserve CDF. See the accompanying worksheet SemiQuadraticExample.xls for details and other values of α.

Why bother?

Almost everything actuarial that goes into actually setting reserves is assumed in this discussion. In particular, the explicit existence of a distribution is problematical. Not impossible, just hard.

In many companies the current reserve-setting process probably already done on a least-pain basis. However, the pain is not future pain but present pain. Reserves are set with an eye on what was set in the past and on current analyst expectations. The reserve committee is always playing catch-up. The procedure discussed here assumes that the reserves are set on old years (how the random variable comes in) with no concern for current politics – clearly a naïve assumption.

Also, the author is not aware of a line of business decision function that would make sense. Perhaps some of the capital allocation methodologies could be helpful.

Similarly, no one actually knows how the value of the company will decrease; but experienced players have some sense of it. There have been enough examples in the last few years to show that reserve changes can have significant impact. It is also clear that what may impact a given company's pain function may be quite different from another's, and that the emphasis may very well change from year to year. Allowing this would be a problem for regulators.

Still, it would be a useful exercise for the reserve committee at a company to get together and try to build, even crudely, their pain function for the year. They could perhaps begin with some standardized event (lose 10% of surplus) which has enough pain to work as a comparison with other possibilities, and then fill out both the high and low sides at a convenient and realistic set of values[5]. Then they could ask the actuary to do the numerical calculation for the estimate, and have a much better idea of what the increase in average pain would be from using an estimator not at the minimum if they choose to do so. And, in the process they would come to be able to explain how they arrived at their estimate. Another interesting exercise[6] would be to put the management incentive plan as an input to the pain function.

All of this makes reserving by formula (e.g., use the mean) impossible. But it really is, anyway.

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[5] For example, let loss 10% of surplus = 1 pain unit then make up the rest: lose 25% = 10; lose 20% = 8; lose 15% = 3; lose 5% = 0.5; lose 0 = 0; gain 5% = 0; gain 10% = 0.5; gain 15% = 1; gain 20% = 2.

[6] This is probably implicitly happening already.