Surplus Allocation: A DFA Application

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BIOGRAPHY

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ABSTRACT

Surplus allocation has been requested from actuaries many times over the years. There are those who feel surplus allocation of any sort is incomprehensible. Since actuaries are asked to allocate surplus, we need to ensure the processes being used are sound. It is such a request from upper management that sent the author looking for the methods employed by others and pondering what additional methods could be constructed. This paper reviews reserve and duration based allocation methods and then ventures into devising an alternative method based on variation. A brief discussion is also included on what surplus amount should be used.

PURPOSE

The purpose of this paper is to share methods for surplus allocation with others, receive feedback on these methods, and promote further development. The author is a company actuary in the pursuit of answers for management. This project was begun to answer a question presented by the company's CFO. The questions raised were non-actuarially based but needed to be answered by someone with a financial understanding. Given the company's surplus, what is the optimal distribution of surplus by line of business? This will allow tracking, calculating, and determining profitability of each line of business on its own.

INTRODUCTION

This paper will review and analyze three methods of allocating surplus. The methods can be used to distribute current surplus by line of business. This is desired for many reasons including pricing activities, determining ROE by line of business, figuring premium-to surplus ratios by line of business, and distributing investment income to line of business.

Although many ways have been discussed to allocate surplus, there is no single standard accepted by everybody. California Proposition 103 used the proportion of loss and unearned premium reserve to allocate surplus. It has been suggested that surplus being used for pricing purposes should be allocated based on one's favorite risk load formula[1]. Other methods include allocating surplus in proportion to loss reserves, in proportion to duration, or based on the coefficient of variation in loss ratios. This paper will start with the simpler methods and venture into a variance-based method. The methods will discuss allocation by reserves, duration, and variation.

Keep in mind the allocation of surplus to line of business will not mean line of business independence, because the total amount of the surplus is still there to support the company as a whole. The standard deviation of the enterprise surplus or operating gain will always be less than the sum of the standard deviations by line of business, due to less than perfect correlation between the lines of business.

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[1] Suggested by Glenn G. Meyers via the CASNET.

METHODS

REVIEW OF SURPLUS

What is the purpose of surplus? Surplus is there for two purposes 1) to support insurance company operations and 2) to support other activities. The surplus allocation for the purposes of this paper deals with supporting insurance company operations. This is an amount necessary to cover risks such as the variation in liabilities at a point in time, as well as prepare for future needs. From a statutory view, as the company grows the expenses are realized immediately, while the premiums are earned over the course of the policy. If the company accelerates its growth, there will be a reduction of surplus to cover the current expenses. From a going-concern basis, the future liabilities also need to be considered in the surplus allocation.

> "Surplus [exists to] protect the insurer against several types of risk. Asset risk is the risk that financial assets will depreciate (e.g., bonds will default or stock prices will drop). Pricing risk is the risk that at policy expiration, incurred losses and expenses will be greater than expected. Reserving risk is the risk that loss reserves will not cover ultimate loss payments. Asset-liability mismatch risk is the risk that changes in interest rates will affect the market value of certain assets, such as bonds, differently than that of liabilities. Catastrophe risk is the risk that unforeseen losses, such as hurricanes or earthquakes, will depress the return realized by the insurer. Reinsurance risk is the risk that reinsurance recoverables will not be collected. Credit risk is the risk that agents will not remit premium balances or that insureds will not remit accrued retrospective premiums." [5]

RESERVE METHOD

Distributing surplus based on loss reserves and unearned premium reserves may be the easiest method. Allocating surplus according to the volume of business per line is a logical choice since surplus is committed when the policy is written and released when the loss is paid. If it is a stable book of business, the loss reserves and unearned premium reserves will remain relatively constant from year to year. California's proposition 103 used this method to allocate surplus to line of business for their calculations.

This method matches available surplus to line of business in proportion to reserves held. There are no tricky calculations or multiple iterations. The necessary information can be found in the annual actuarial report or the annual statement.

The method begins by listing the ultimate loss reserves needed by line of business and summing them for the enterprise. The same is done for the unearned premium reserve. These are shown in columns 1 and 2 of Table 1 below, while the sum is shown in column 3. For each line of business, take the respective reserve sum and divide by the enterprise sum. This gives the distribution of reserves by line of business, which can be applied to surplus. (See Table 1 or Exhibit 1.) Enterprise surplus can then be multiplied by the corresponding percentages to get the amount of surplus by line of business.

Table 1

	(1) Loss Reserves (000's)	(2) Unearned Premium (000's)	(3) Sum (000's)	(4) Dist.	(5) Surplus (000's)
Homeowners	66,900	27,277	94,177	7.3%	36,500
Personal Auto Liability	385,914	44,801	430,715	33.5%	167,500
Personal Auto Phys Dam	37,426	41,044	78,470	6.1%	30,500
Commercial Auto Liability	112,318	6,093	118,411	9.2%	46,000
Commercial Auto Phys Dam	3,599	2,218	5,817	0.5%	2,500
CPP Liability	141,808	51,320	193,128	15.0%	75,000
CPP Property Damage	63,106	68,577	131,683	10.2%	51,000
Other Liability	3,725	7,565	11,290	0.9%	4,500
Umbrella	1,394	316	1,710	0.1%	500
Workers Compensation	146,415	74,058	220,473	17.2%	86,000
Enterprise	962,607	323,269	1,285,876	100.0%	500,000

The reserve method is a quick and easy method to use, but there are several disadvantages to using this method. It does not consider the length of the reserve pay-out tail, adjustments in the reserve payment pattern, or the time value of money. All of these can cause variations or unexpected results, the precise thing surplus is there to cover. This is a static method. The distribution is determined based on an expected value at a point in time and does not consider future changes in the distribution by line of business.

The reserve method of allocation considers only the pricing and reserving risks. Larger amounts of surplus are allocated to the lines of business holding larger reserves. This method ignores the five other significant areas of variability referenced above in determining the surplus allocation. These five neglected risks include asset risk, asset-liability mismatch risk, catastrophe risk, reinsurance risk, and credit risk.

For more information on different reserve based methods reference "An Evaluation of Surplus Methods Underlying Risk Based Capital Calculations" by Michael Miller and Jerry Rapp (1992 Discussion Paper Program, Vol. 1).

DURATION METHOD

Many people on the CAS web site and CASNET touted duration as a means to allocate surplus. Duration allocation is perceived to be superior to loss and unearned premium reserve allocation since duration considers payment pattern changes and interest rate changes in the duration calculation. Longer tail lines receive relatively more surplus to cover the larger potential volatility in the payment pattern.

Duration is a time value weighted pay-out length. In other words, duration is a weighted average term to completion where the years are weighted by the present value of the related cash flows. [6]

$$\text{Duration} = \frac{\sum_{t=1}^{n} [(t \cdot CF_t) / (1 + y)^t]}{\sum_{t=1}^{n} [CF_t / (1 + y)^t]}$$

CF_t = Cash Flow in year t
y = yield to maturity
t = year of cash flow
n = number of years to maturity

Duration Example

Table 2

Calendar Year	(1) Projected Paid	(2) Year	(3) Present Value @ 6.5%	(4) (2) x (3) Weighted PV @ 6.5%
1997	350,000	0.5	339,151	169,576
1998	210,000	1.5	191,071	286,607
1999	60,000	2.5	51,260	128,150
2000	20,000	3.5	16,044	56,153
2001	7,500	4.5	5,649	25,421
Total	647,500		603,175	665,907

Macaulay Duration = 665,907 / 603,175 = 1.104

Modified Duration = 1.104 / 1.065 = 1.0366

Table 2 gives an example of a duration calculation. Column 1 is the amount projected to be paid in each calendar year. This includes payments from all accident years 1997 and prior. Column 2 shows that the duration is being examined from the beginning of 1997 since the average payment is expected to be paid half way through the year, assuming that in any calendar year the payments are uniform. The present value of column 1 at 6.5% is shown in column 3, while column 4 is (2) times (3). Column 4 gives a weighted present value based on the length until payment. The Macaulay duration is the sum of column 4 divided by the sum of column 3. The Modified duration is the Macaulay duration divided by 1 plus the interest rate used.

The duration method can be applied using a Dynamic Financial Analysis (DFA) model that incorporates changing discount rates, payment patterns, and inflation amounts by iteration in the calculation. Dynamo2 is an Excel based model developed by the actuarial consulting firm of Miller, Rapp, Herbers & Terry (MRH&T) used by the author. Further description of the model can be found in Appendix A.

The DFA model needed some programming additions to capture and calculate the necessary components for duration. Appendix B lays out the changes made to generate the payments by accident year and calendar year and to generate the interest rate.

With the necessary information obtained, formulas were inserted in the DFA model to calculate duration as shown in the example above. A sample iteration of this duration process for the homeowners line of business is shown in Exhibit 2. After the DFA ran 1,000 iterations (maximizing computing capacity), durations were selected equal to the means of the 1,000 durations by line of business.

Table 3 below shows the process of the duration method. Determining a distribution of surplus begins by normalizing the duration by line of business with the enterprise duration. Each line of business duration in column 1 is divided by the enterprise duration. Multiplying the resulting relativities in column 2 by the inverse of the company's premium-to-surplus ratio changes the relativities to the amount of surplus needed per dollar of premium. The next step is to apply the appropriate premium from column 4 to each line of business to arrive at the estimated surplus in column 5. From here a distribution may be determined by dividing line of business estimated surplus by the enterprise surplus. The resulting distribution in column 6 can then be used to spread the real surplus to line of business. This method is also outlined in Exhibit 3.

Table 3

	(1) Avg. Duration	(2) Relativity	(3) Surplus-to-Prem	(4) Premium (000s)	(5) Est. Surplus (000s)	(6) Dist.	(7) Surplus (000's)
Homeowners	1.8677	0.8309	0.4985	54,553	27,197	5.8%	29,000
Personal Auto Liab	2.0180	0.8978	0.5387	179,204	96,531	20.5%	102,500
Personal Auto Phys Dam	0.9251	0.4115	0.2469	164,175	40,539	8.6%	43,000
Comm'l Auto Liab	2.5823	1.1488	0.6893	24,370	16,798	3.6%	18,000
Comm'l Auto Phys Dam	1.3161	0.5855	0.3513	8,872	3,117	0.7%	3,500
CPP Liab	3.2253	1.4349	0.8609	102,640	88,367	18.7%	93,500
CPP Property	1.7051	0.7586	0.4551	137,154	62,424	13.2%	66,000
Other Liability	2.2799	1.0143	0.6086	15,130	9,208	1.9%	9,500
Umbrella	2.2278	0.9911	0.5947	631	375	0.1%	500
Workers Comp	3.2076	1.4270	0.8562	148,116	126,817	26.9%	134,500
Enterprise	2.2478	1.0000		834,844	471,373	100%	500,000

Prem / Surplus = 1.67 (Assumed)[2]
Surplus / Prem = 1 / (Prem / Surplus) = 0.60

(2) = (1) / [(1) Enterprise]
(3) = (2) * 0.60
(5) = (3) * (4)
(7) = 500,000 * (6)

In this presentation the Macaulay duration was used. The question may come up as to why use the Macaulay duration and not the Modified duration. Since this method deals with relative duration by dividing each line of business by the enterprise duration, it does

[2] The correct premium to surplus ratio is assumed to be 1.67.

not matter which one is used. If the modified duration were used, then all the durations would be divided by the same factor maintaining the same relativities between them.

In addition to the advantages listed above, there are a few disadvantages to using the duration method to allocate surplus. The duration method distributes surplus based on projected ultimate losses for past years and the payment pattern for those years. From that point of view, using duration has a run-off view point. It allocates surplus to lines of business in relation to how those lines will run-off and in relation to current premium volume. This covers the vulnerability to greater variation in the longer payout lines of business. This is a static view of business at a point in time. It does not consider future growth or changes in the mix of business going forward. For a company that plans on continuing to write business and grow, this might not be the best option. Surplus needs to be allocated for future premium growth. For statutory accounting, the expenses of writing policies are recognized immediately, while the premiums are earned over the course of the policies. This is why companies with accelerated growth may see surplus decline (statutory surplus, not market value surplus). "Rapid premium growth precedes nearly all of the major failures. Rapid growth is not harmful, per se. However, rapid premium growth reduces the margin for error in the operation of insurers." [4] Additional surplus is needed to cover the reduced margin of error for growing lines of business.

The duration method does a better job than the reserve method of considering the risks surplus is to protect against. The lines of business with longer pay-out patterns have higher durations. Here duration is a proxy for the riskiness of the long tail lines. Longer tailed lines are exposed to more interest rate and payment pattern changes incorporated in reserving risk, asset risk, and asset-liability mismatch risk. By using duration as a proxy, this allocation method covers these risks. Again, four risks surplus is to protect against are not even considered by this method (pricing risk, catastrophe risk, reinsurance risk, and credit risk).

Keep in mind that even though this model considers variation in the payment pattern, judicial or legislative changes that could effect payments are not considered. Such changes would create greater variability, but are difficult if not impossible to predict. These types of changes can not be foreseen on any method presented here.

VARIATION METHOD

The variation method is one that the author developed while working with the DFA model and trying to answer the CFO's questions. It is a forward-looking method on what may happen. Loss reserves are already set up to cover losses that have occurred. Surplus exists for unexpected events or variations from the norm. This method uses standard deviations on a comparable basis among lines of business to distribute surplus.

The variation method uses the calendar year operating gain by line of business from each iteration of the DFA, calculated by adding net underwriting profit to the investment return during the calendar year. To calculate such information, additions needed to be made to the DFA spreadsheet to capture interest earned by line of business. This is described in Appendix C.

Following the steps described in Appendix C, the investment return by line of business to be included in the operating profit was derived as the amount of reserves available for investment times the rate of return for the appropriate year. The calendar year net operating profit was calculated by adding this investment return and the calendar year net underwriting profit by line of business.

The next step was to compare the variation between lines of business. Using the variance of operating profit alone would give results that are difficult to compare between lines of business. Each line of business variance would be based on differing amounts of premium and number of policies. To put all lines of business on a comparable basis the operating gain needed to be normalized before determining the variance.

The net operating gain was divided by the net written premium for that line of business. This ratio is a unit of measure with the dollar units canceling out. This put all lines of business on a net operating gain per dollar of net written premium basis before the variance was determined.

As the steps of the variation method calculation are described, reference will be made to the portions of Exhibit 4 discussed in the text. Exhibit 4 shows this method laid out in its entirety. By capturing the operating gain by line of business for each calendar year, the

@Risk software calculates the standard deviation of each line and year over the 1,000 iterations.

Table 4 shows the results of the simulation. Columns (1) through (5) are the standard deviations of net operating gain per dollar of net written premium. This information was generated by the DFA model and @Risk. Appendix D lays out the credibility weightings of these standard deviations.

Table 4

Standard Deviation of Net Operating Gain
Per Dollar of Net Written Premium

	(1)	(2)	(3)	(4)	(5)
	1998	1999	2000	2001	2002
Home	0.4532	2.2769	0.6970	0.6594	0.4550
P Auto Liab	0.1574	0.1235	0.1461	0.1395	0.1536
P Auto Phys Dam	0.1199	0.9526	0.2524	0.2326	0.1184
C Auto Liab	1.3752	1.4673	1.6137	1.6697	1.7350
C Auto Phys Dam	0.4705	2.3967	0.7265	0.6892	0.5526
CPP Liab	0.0575	0.0646	0.1012	0.0772	0.0841
CPP Prop	0.5931	4.5416	1.1913	1.0898	0.5455
Other Liab	0.1688	0.2055	0.2183	0.2339	0.2391
Other Liab - Umbrella	0.8574	0.9288	1.1948	1.2197	1.3292
Workers Comp	0.1308	0.1456	0.1748	0.1856	0.2019
Personal	0.1255	0.7125	0.2001	0.1830	0.1159
Commercial	0.2088	1.4789	0.3995	0.3605	0.2009
Enterprise	0.1544	1.1122	0.2955	0.2666	0.1367

Table 5 below shows the remaining steps to determine the surplus allocation of the variation method. Dividing the credibility weighted standard deviations (Table 8, column 12, Appendix D) by the average standard deviation of the enterprise (Table 8, column 13, Appendix D) normalizes the credibility-weighted standard deviations. Multiplying the resulting relativities by the inverse of the company's premium-to-surplus ratio changes the relativities to the amount of surplus needed per dollar of premium. The next step is to apply the appropriate premium to each line of business to arrive at the estimated surplus (column 17 = (15) * (16)). Appropriate premium could include the year-end premium by line of business or the first year's projected premium. From here a distribution may be determined by dividing line of business estimated surplus by the enterprise estimated surplus. The resulting distribution in column 18 can then be used to spread the real surplus to line of business.

Table 5

	(12) Credibility Weighted Std Dev	(14) Relativity	(15) Surplus-to-Prem	(16) Premium (000s)	(17) Est. Surplus (000s)	(18) Dist.	(19) Surplus (000s)
Homeowners	0.8621	2.1932	1.3133	54,553	71,645	13.0%	65,000
Personal Auto Liab	0.1441	0.3665	0.2195	179,204	39,331	7.1%	35,500
Personal Auto Phys Dam	0.3569	0.9079	0.5436	164,175	89,252	16.2%	81,000
Comm'l Auto Liab	1.5628	3.9757	2.3807	24,370	58,017	10.5%	52,500
Comm'l Auto Phys Dam	0.9035	2.2985	1.3763	8,872	12,221	2.2%	11,000
CPP Liab	0.0770	0.1959	0.1173	102,640	12,041	2.2%	11,000
CPP Property	1.0756	2.7363	1.6385	137,154	224,728	40.8%	204,000
Other Liability	0.2133	0.5427	0.3250	15,130	4,917	0.9%	4,500
Umbrella	1.0980	2.7933	1.6727	631	1,055	0.2%	1,000
Workers Comp	0.1680	0.4273	0.2559	148,116	37,899	6.9%	34,500
Enterprise	0.3931	1.0000		834,844	551,095	100%	500,000

Prem / Surplus = 1.67 (assumed)[3]
Surplus / Prem = 1 / (Prem / Surplus) = 0.60

(15) = (14) * 0.60
(17) = (15) * (16)

[3] The correct premium to surplus ratio is assumed to be 1.67.

The distribution created by the variation method may raise some questions. Why is it that the property and physical damage coverages receive more surplus based on this method? The property and physical damage coverages are subject to catastrophes and therefore more variation from year to year. The variation is a result of both the frequency and the severity of catastrophes. The liability lines have the potential for high single occurrence pay-outs by policy, but the number of these are relatively consistent from year to year. The law of large numbers makes predicting the result for this line of business more consistent.

An example to look at is the amount of surplus allocated to Commercial Auto Physical Damage (CAPD) and CPP Liability (CPP Liab). As can be seen in Table 5, both of these lines are allocated $11,000,000 surplus, whereas the premium for CPP Liab is 11.5 times as large as that for CAPD. In CPP Liab, the law of large numbers helps smooth results, while CAPD is subject to catastrophes. The reinsurance in place also underlies these results.

Both liability and property lines of business for smaller companies are affected by variations in large losses from year to year. The author did not test which lines of business had more variability in large losses, attributing the major variations between lines of business to catastrophes. Changing reinsurance agreements or types of business written could reduce the impact of catastrophe losses.

This method contains many of the characteristics that are desired from a surplus allocation method. The length and amount of the tail are considered with varying payment patterns, incorporating reserving risk. The DFA model varies interest rates to include asset risk into the considerations. The varying interest rate is brought into the operating gain through the investment income. Using operating gains also reflects pricing risk by including variability of loss ratios embedded in the operating return and catastrophe risk by including simulated catastrophes in the underwriting results. The impact of asset-liability mismatch risk is included by varying the interest rates in the model as well as varying the ultimate loss and payment patterns included in the operating gains. The DFA model does not consider reinsurance risk or credit risk, but these could be incorporated based on distributions of uncollectability. Reinsurance risk may be considered negligible depending on the reinsurers' A.M. Best ratings.

This method goes beyond the first two methods and looks to the future. This is a going-concern method, which tries to reveal what the distribution by line of business should be going forward. To do this it incorporates the company's growth plans by line of business and the variability by line of business based on the growth plans and past experience. If the company is going to cut rates to grow more, then this is included in the variation in net operating gain per dollar of net written premium and figured into the standard deviation. Most company changes in growth, mix of business, or type of business are reflected in the operating gains as long as the DFA model is set up appropriately to reflect these changes.

Non-catastrophe reinsurance levels also influence the variability by line of business. On a net of reinsurance basis, as the threshold for excess of loss coverage is reduced, the variability of results also declines. "[A]ny risk which lowers the aggregate Exposure Ratio of the portfolio has added capacity to the portfolio."[9] The exposure ratio is the coefficient of variation (standard deviation / mean). As the variability decreases, the level of surplus needed for the line of business decreases freeing up surplus for other uses.

COMPARISION OF METHODS

The three different methods presented give widely varying results as can be seen in Table 6 below.

Table 6

	(1)	(2)	(3)	(4)
	Reserve Method	Duration Method	Variation Method	Driving Risks
Homeowners	7.3%	5.8%	13.0%	Catastrophe risk
Personal Auto Liability	33.5%	20.5%	7.1%	Pricing & Interest Rate
Personal Auto Phys Dam	6.1%	8.6%	16.2%	Catastrophe risk
Commercial Auto Liability	9.2%	3.6%	10.5%	Pricing & Interest Risk
Commercial Auto Phys Dam	0.5%	0.7%	2.2%	Catastrophe risk
CPP Liability	15.0%	18.7%	2.2%	Pricing & Interest Risk
CPP Property Damage	10.2%	13.2%	40.8%	Catastrophe risk
Other Liability	0.9%	1.9%	0.9%	Pricing & Interest Risk
Umbrella	0.1%	0.1%	0.2%	Pricing & Reinsurance
Workers Compensation	17.2%	26.9%	6.9%	Pricing & Interest Rate
Enterprise	100.0%	100.0%	100.0%	Catastrophe & Pricing risk

These variations are the result of the reasoning behind the methods. Looking at personal auto liability, the loss reserve method in column (1) allocates 33.5% to this line, whereas, the duration method apportions 20.5% and the variation method only 7.1%. The personal auto liability line of business has a consistent amount of losses every year and consistent sales growth producing higher reserves held. The reserve method reflects this explicitly. The duration method analysis notes that the payout pattern is weighted heavily to the earlier years. This does not allow much time for adverse development. The variation method looks at the reserves and the payout pattern, but also considers that from year to year the loss ratios are consistent. The ultimate personal auto liability losses can be reasonably estimated from year to year without much variation from expected. Therefore less surplus would be needed for unforeseen circumstances.

The homeowners line of business is another good example. With the payouts being quick and settlements rather fast, the level of reserves carried is relatively low. The reserve method looks only at the carried reserves to determine the allocation (7.3%). The duration method considers that the pay-out pattern is relatively short meaning less surplus is necessary (5.8%). Yet when losses are compared from year to year there is greater variation due to catastrophes. The surplus necessary to cover these greater variations is 13.0%.

The driving risks affecting surplus differs for each line of business. For example, catastrophes have more of an impact on property lines than liability coverages. There are certain sets of risks for each line of business that maintain significant influence on results. These driving risks are listed in column 4 of Table 6.

SURPLUS

What overall amount of surplus should be used? All of the methods discussed above allocate a stated surplus amount. There are a few different methods to determine how much surplus is to be distributed.

ACTUAL

The most straightforward method would be to use the company's actual surplus as of year-end. This amount would then be distributed back to line of business based on the method of choice. A few problems with this method would be if the company was over capitalized (under capitalized). If this were the case than too much (little) would be allocated. As stated toward the beginning of the paper, surplus is there to support insurance operations as well as other activities. The surplus to allocate should be the amount supporting the insurance operations.

Actual surplus also has many definitions to consider. If allocating actual surplus, is it market value, statutory value, or GAAP value? Should equity in the unearned premium reserve or the discounted amount from the loss reserve discount factor be included?

PREMIUM-SURPLUS RATIO

A second method pegs the surplus at a certain premium to surplus ratio (P/S). There are a variety of reasons and justifications for selecting a certain P/S ratio. The P/S ratio could be selected by management's desire not to exceed a certain P/S ratio, say 2:1. It could be pegged to match a certain peer group in the industry. A word of caution: P/S ratios can be manipulated from company to company.

OPERATING GAIN DISTRIBUTION

The amount of surplus needed by a company is based on its aversion to risk. Assume that a company's risk manager determines that they want to be 95% confident that the surplus doesn't decrease, or 90% confident the surplus decreases by no more than 10%. To do this the company would need to generate a distribution of the change in surplus for the year. Another alternative would be to use operating gain for the year. In both of these distributions the desired confidence level amount is found by referencing (1.00 – confidence level) corresponding to the cumulative percentages for the distribution. By choosing the corresponding amount from the distribution, it can be used to determine the desired amount of surplus for the company.

The net operating gains from the DFA model iterations used in the allocation process can be captured and set into a distribution. Using the example from above, the goal would be 90% confidence that surplus would decrease at most 10% in the given year. From the

1,000 iterations, the Enterprise operating gain for 1998 was captured with the resulting distribution shown in Table 7.

Table 7

Partial Distribution of Operating Gain

	Enterprise Operating Gain 1998
5% Perc =	(71,619,600)
10% Perc =	(50,000,000)
15% Perc =	(32,521,600)
20% Perc =	(10,258,950)
25% Perc =	(763,150)
30% Perc =	12,859,600
35% Perc =	38,245,700
40% Perc =	60,052,150
45% Perc =	84,517,300

This is a portion of the full distribution that increases in 5% increments up to 95%. This table communicates for example that 5% of the operating gain samples are less than $(71,619,600) and that 30% of the samples are less than $12,859,600. At a 90% confidence level $(50,000,000) is the operating gain. Similarly at a 95% confidence level the gain is $(71,619,000).

At the 90% confidence level, surplus would be decreasing at most $50,000,000 in the year. If the company started with only $50,000,000 this would not be a pleasant outcome. So a second constraint is necessary, that is 'what is the maximum proportion of surplus the company management is willing to lose in any year?' For example, management is willing to risk a decrease to be at most 10% of starting surplus. In other words, the \$(50,000,000) would equal (0.10) times the needed surplus. The surplus needed would be calculated as follows:

\$(50,000,000) / (0.10) = \$500,000,000 surplus needed

The calculated surplus is the theoretical amount needed to support business as a going-concern under the stated constraints. This amount should be used in the ROE and pricing calculations. Comparing this surplus to the enterprise surplus may indicate a redundancy or deficiency. If the calculated surplus is less than the company surplus, the redundancy isn't necessarily excess to squander. The total enterprise surplus may need to be maintained for statutory or regulatory purposes.

RUIN WITH ROE MEASURES

Many insurance companies are being evaluated from a financial viewpoint. The question that comes up is the level of ROE that the company wants to target. The level of surplus affects this ROE measure. Lower surplus translates into a higher leverage ratio increasing the potential ROE while generating a greater chance of ruin. To reduce the chance of ruin more surplus would be held, reducing the ROE. This puts the insurance company in a precarious position.

With the DFA model it is possible to test out different levels of surplus. One can begin with a certain level of surplus, capturing the appropriate values for ROE and ending surplus. Different levels of surplus translate into differing ROE averages. Accompanying each ROE average is a probability of ruin distribution. An optimization then has to be made on the risk and return trade off.

CONCLUSION

As the line between the financial industry and the insurance industry blurs, actuaries are becoming the financial leaders in the insurance industry. From a financial perspective there is a strong desire to allocate surplus to measure, track, and rate performance on a line of business basis. There are many ways to allocate surplus once the overall needed surplus amount is determined. Of the methods presented, the variation method incorporates the most characteristics desired from a surplus allocation method. However, this is just the starting point for others to build upon and to improve.

Distribution by Reserves

	(1) Loss Reserves	(2) Unearned Premiums	(3) Sum	(4) Distribution	(5) Estimated Surplus
Homeowners	66,900,470	27,277,000	94,177,470	7.3%	36,619,972
P Auto Liab	385,914,100	44,801,000	430,715,100	33.5%	167,479,280
P Auto Phys Dam	37,426,200	41,044,000	78,470,200	6.1%	30,512,356
C Auto Liab	112,318,300	6,093,000	118,411,300	9.2%	46,043,055
C Auto Phys Dam	3,599,136	2,218,000	5,817,136	0.5%	2,261,935
CPP Liab	141,808,400	51,320,000	193,128,400	15.0%	75,096,056
CPP Prop	63,105,810	68,577,000	131,682,810	10.2%	51,203,550
Other Liab	3,725,144	7,565,000	11,290,144	0.9%	4,390,060
Other Liab - Umbrella	1,394,024	316,000	1,710,024	0.1%	664,926
Workers Comp	146,415,200	74,058,000	220,473,200	17.2%	85,778,809
Personal	490,240,770	113,122,000	603,362,770	46.9%	234,611,608
Commercial	472,366,014	210,147,000	682,513,014	53.1%	265,388,392
Enterprise	962,606,784	323,269,000	1,285,875,784	100.0%	500,000,000

(1) Year end 1997 Net Loss Reserves
(2) Year end 1997 Unearned Premium Reserves
(4) (3) / Enterprise (3)
(5) (4) * Enterprise (5)

Exhibit 2

DURATION CALCULATION

Line of Business: Homeowners

Accident Years

Calendar Year	(1) 1988	(2) 1989	(3) 1990	(4) 1991	(5) 1992	(6) 1993	(7) 1994	(8) 1995	(9) 1996	(10) 1997	(11) Total	(12) Year	(13) Present Value	(14) Weighted PV
													6.53%	
1997	2,671	1,005,428	1,248,769	1,235,685	1,468,734	4,554,592	9,874,726	10,488,831	31,928,154	48,079,825	109,887,415	0.5	106,466,259	53,233,129
1998		1,071,315	2,672,012	1,972,956	1,901,699	4,189,929	7,045,897	7,282,894	35,960,250	83,542,241	145,639,193	1.5	132,455,615	198,683,422
1999		983,868	36,768	652,113	4,388,283	1,902,007	5,579,526	2,659,516	15,570,392	37,330,497	69,102,970	2.5	58,995,230	147,488,074
2000			36,833	996,306	716,188	2,585,618	4,265,356	1,372,445	8,610,860	12,484,401	31,068,007	3.5	24,897,839	87,142,436
2001				306,002	104,284	551,020	951,525	2,545,685	2,007,813	6,180,550	12,646,879	4.5	9,513,924	42,812,659
2002					102,661	739,430	585,785	1,066,333	1,142,832	1,595,912	5,232,953	5.5	3,695,313	20,324,222
2003						1,268,525	26,357	2,154,525	1,437,799	1,631,019	6,518,225	6.5	4,320,777	28,085,049
2004							26,357	5,142,536	43,174	308,906	5,520,973	7.5	3,435,391	25,765,432
2005								2,153,695	21,587	576,838	2,752,120	8.5	1,607,519	13,663,907
2006								1,245,852	21,587	55,009	1,322,448	9.5	725,096	6,888,409
2007								784,585		55,009	839,594	10.5	432,130	4,537,362
2008								532,565			532,565	11.5	257,303	2,958,989
2009								235,252			235,252	12.5	106,693	1,333,657
2010											0	13.5	0	0
2011											0	14.5	0	0
2012											0	15.5	0	0
2013											0	16.5	0	0
2014											0	17.5	0	0
2015											0	18.5	0	0
2016											0	19.5	0	0
2017											0	20.5	0	0
2018											0	21.5	0	0
2019											0	22.5	0	0
2020											0	23.5	0	0
2021											0	24.5	0	0
													346,909,087	632,916,747

(15) Macaulay Duration 1.8244

(16) Modified Duration 1.7126

(1) - (10) Calculated in the DFA
(11) Sum of columns (1) through (10)
(12) Year of payment assuming uniform over a given year
(13) (11) / [(1 + interest rate) ^ (12)]
(14) (13) * (12)
(15) Total (14) / Total (13)
(16) (15) / (1 + interest rate)

Duration Distribution

Duration

	Duration	Relativity	(S/P)	Premium	Estimated Surplus	Split
	(1)	(2)	(3)	(4)	(5)	(6)
Homeowners	1.8698	0.8023	0.4814	54,552,830	26,261,991	5.4%
P Auto Liab	2.0946	0.8988	0.5393	179,204,200	96,644,215	19.7%
P Auto Phys Dam	1.2064	0.5177	0.3106	164,174,860	50,994,823	10.4%
C Auto Liab	2.5792	1.1068	0.6641	24,369,986	16,183,090	3.3%
C Auto Phys Dam	1.4776	0.6340	0.3804	8,872,202	3,375,212	0.7%
CPP Liab	3.2283	1.3853	0.8312	102,639,868	85,311,534	17.4%
CPP Prop	1.8037	0.7740	0.4644	137,153,660	63,692,602	13.0%
Other Liab	2.6794	1.1498	0.6899	15,129,588	10,437,285	2.1%
Other Liab - Umbrella	2.6639	1.1431	0.6859	630,543	432,473	0.1%
Workers Comp	3.6035	1.5463	0.9278	148,116,140	137,419,704	28.0%
Personal	1.8861	0.8093		397,931,890	173,901,029	35.4%
Commercial	2.7944	1.1991		436,911,987	316,851,898	64.6%
Enterprise	2.3304	1.0000		834,843,877	490,752,928	

	Adjusted Surplus	P/S
	(7)	(8)
Homeowners	26,756,836	2.0388
P Auto Liab	98,465,245	1.8200
P Auto Phys Dam	51,955,699	3.1599
C Auto Liab	16,488,021	1.4780
C Auto Phys Dam	3,438,810	2.5800
CPP Liab	86,919,027	1.1809
CPP Prop	64,892,737	2.1135
Other Liab	10,633,951	1.4228
Other Liab - Umbrella	440,622	1.4310
Workers Comp	140,009,051	1.0579
Personal	177,177,781	2.2459
Commercial	322,822,219	1.3534
Enterprise	500,000,000	1.6697

(1) Duration
(2) Line Duration / Enterprise Duration
(3) (2) * Surplus/Premium ratio of 0.60 = (1/1.6697)
(4) Premium
(5) (3) * (4) = (Surplus/Premium) * (Premium) = Estimated Surplus
(6) (5) / (Enterprise 5)
(7) (6) * Enterprise
By Line: [(5) / Consumer or Commerical (5)] * Consumer or Commercial (7)
(8) (4) / (7)

Variation Distribution

Operating Gain per $ WP
Standard Deviation

	(1) 1998	(2) 1999	(3) 2000	(4) 2001	(5) 2002		(6) Expected Value	(7) Within Variance	(10) K factors
Home	0.4532	2.2769	0.6970	0.6594	0.4550		0.9083	0.4784	1.5015
P Auto Liab	0.1574	0.1235	0.1461	0.1395	0.1536		0.1440	0.0001	0.0004
P Auto Phys Dam	0.1199	0.9526	0.2524	0.2326	0.1184		0.3352	0.0984	0.3088
C Auto Liab	1.3752	1.4673	1.6137	1.6697	1.7350		1.5722	0.0175	0.0550
C Auto Phys Dam	0.4705	2.3967	0.7265	0.6692	0.5526		0.9671	0.5195	1.6302
CPP Liab	0.0575	0.0646	0.1012	0.0772	0.0641		0.0769	0.0002	0.0007
CPP Prop	0.5931	4.5416	1.1913	1.0898	0.5455		1.5923	2.2411	7.0333
Other Liab	0.1688	0.2055	0.2183	0.2339	0.2391		0.2131	0.0006	0.0020
Other Liab - Umbrella	0.8574	0.9288	1.1948	1.2197	1.3292		1.1060	0.0328	0.1028
Workers Comp	0.1308	0.1456	0.1748	0.1856	0.2019		0.1677	0.0007	0.0021
Personal	0.1255	0.7125	0.2001	0.1830	0.1159				
Commercial	0.2068	1.4789	0.3995	0.3605	0.2009	(13)			
Enterprise	0.1544	1.1122	0.2955	0.2666	0.1367	0.3931			
							(8)		
Expected Value	0.4384	1.3103	0.6316	0.6197	0.5415		0.7063		
							(9)		
Between Variance	0.1571	1.8391	0.2649	0.2702	0.2798		0.3186		

Credibility Factors	(11)	Operating Gain per $ WP Credibility Weighted Standard Deviations	(12)
Home	0.7691	Home	0.8621
P Auto Liab	0.9999	P Auto Liab	0.1441
P Auto Phys Dam	0.9418	P Auto Phys Dam	0.3569
C Auto Liab	0.9891	C Auto Liab	1.5628
C Auto Phys Dam	0.7541	C Auto Phys Dam	0.9035
CPP Liab	0.9999	CPP Liab	0.0770
CPP Prop	0.4155	CPP Prop	1.0756
Other Liab	0.9998	Other Liab	0.2133
Other Liab - Umbrella	0.9798	Other Liab - Umbrella	1.0980
Workers Comp	0.9996	Workers Comp	0.1680

	Operating Gain per $ WP Credibility Weighted Standard Deviation/ Standard Deviation of Enterprise (14)	(Std Dev ratio) * (Surplus/Prem ratio) (15)		Premium (16)
Home	2.1932	1.3133		54,552,816
P Auto Liab	0.3665	0.2195		179,199,580
P Auto Phys Dam	0.9079	0.5436		164,176,060
C Auto Liab	3.9757	2.3807		24,369,966
C Auto Phys Dam	2.2965	1.3763		8,872,197
CPP Liab	0.1959	0.1173		102,639,824
CPP Prop	2.7363	1.6385		137,153,600
Other Liab	0.5427	0.3250		15,129,588
Other Liab - Umbrella	2.7933	1.6727		630,538
Workers Comp	0.4273	0.2559		148,116,220
			Consumer	397,928,456
Premium/Surplus ratio	1.67		Commercial	436,911,953
			Enterprise	834,840,409
Surplus/Premium ratio	0.60 (assumed)			

	Estimated Surplus (17)	(18)	(19)	Surplus (19)	P/S (20)
Home	71,644,576	13.0%		65,002,064	0.8392
P Auto Liab	39,331,270	7.1%		35,684,679	5.0218
P Auto Phys Dam	89,251,941	16.2%		80,976,965	2.0274
C Auto Liab	58,017,003	10.5%		52,637,969	0.4630
C Auto Phys Dam	12,211,073	2.2%		11,078,926	0.8008
CPP Liab	12,040,677	2.2%		10,924,328	9.3955
CPP Prop	224,728,029	40.8%		203,892,415	0.6727
Other Liab	4,916,523	0.9%		4,460,688	3.3918
Other Liab - Umbrella	1,054,673	0.2%		956,889	0.6589
Workers Comp	37,898,862	6.9%		34,385,077	4.3078
Consumer	200,227,787	36.33%	181,663,708		2.1905
Commercial	350,866,839	63.67%	318,336,292		1.3725
Enterprise	551,094,626		500,000,000		1.6697

Notes:
(1) - (5) Standard Deviations of Operating gain per Dollar of Written Premium
(6) - Mean of years by line of business
(7) - Variance of years by line of business
(8) - Mean of (6)
(9) - Variance of Expected Values (6)
(10) - (7) / (9)
(11) - n / (n + K) = Z where n = # of years
(12) - (6) * (11) + (8) * (1 - (11))
(13) - Mean of Enterprise
(14) - (12) / (13)
Distribution of capital
(15) - (14) * selected surplus/premium ratio
(16) - Year end 1997 Premium
(17) - (15) * (16)
(18) - (17) / Enterprise (17)
(19) - (18) * Enterprise (19)
(20) - (16) / (19)

DFA MODEL USED

The following includes excerpts from the papers "Building a Public Access PC-Based DFA Model" (1997 CAS Summer Forum, Vol.2) [2] and "Using the Public Access DFA Model: A case Study" (1998 CAS Summer Forum) [3]. Both papers are used with the permission of the papers' authors.

The Dynamic Financial Analysis (DFA) model used in this paper is a public access model. **The actuarial consulting firm Miller, Rapp, Herbers & Terry (MRH&T) created Dynamo2**. **Dynamo2** is Excel based enabling the user to create calculations as needed.

Each iteration of the model starts with detailed underwriting and financial data showing the historical and current positions of the company. It randomly selects values for 4,387 stochastic variables, calculates the effect on the company of each of these selected values, and produces summary financial statements of the company for the next five years based on the combined effect of the random variables and other deterministic factors.

The model consists of several different modules, each of which calculates a component of the model indications. Separate modules are included for investments, catastrophes, underwriting, taxation, interest rates, and loss reserve development. The number of lines of business can be expanded or contracted to fit the needs of the user. The model used allows for ten different lines of business:

- Homeowners
- Private Passenger Auto Liability
- Private Passenger Auto Phys. Dam.
- Commercial Auto Liability
- Commercial Auto Phys. Dam.
- CPP - Liability
- CPP - Property
- Other Liability
- Umbrella
- Workers Compensation

For each line of business, the underwriting gain or loss is calculated separately for: 1) new business, 2) 1st renewal business and 3) 2nd and subsequent renewals. This division is provided to reflect the aging phenomenon, in which loss experience improves with the length of time a policyholder has been with a company. These three categories are then added to calculate underwriting results on a direct, ceded, and net basis.

The values for each simulation are shared among the different modules. Thus, if the random number generator produces a high value for the short term interest rate, this high interest rate is used in the investment module as well as the underwriting module. Similarly, a high value for catastrophes in the catastrophe module carries through to the reinsurance and underwriting modules.

The primary risks that are reflected in the model are:

- Pricing risk
- Loss reserve development risk
- Catastrpohe risk
- Investment risk

CAVEATS OF THE MODEL

Some factors, having a potentially significant impact on results, are omitted from the model because, in the opinion of the authors, they are beyond the scope of an actuarial analysis. For example, fraud by managers is a leading cause of insurance insolvency. Whether fraud is likely to occur (or is currently occurring) at a particular insurer, is not something an actuary is qualified to ascertain. Thus, any financial effects from fraudulent behavior are simply omitted from the model. Other examples of omitted factors that definitely could have a significant effect on insurance operations include a change in the tax code, repeal of the McCarran-Ferguson Act, a major shift in the application of a legal doctrine or the risk of a line of business being socialized by a state, province, or federal government. Thus, the range of possible outcomes from operating an insurance company is actually greater than a DFA model would indicate; the model is designed to account only for the risks that can be realistically quantified.

The values used as input in the model are derived from past experience and current operational plans. To the extent that something happens in the future that is completely out of line with past events, the model will be inaccurate. For example, the size of a specific catastrophe is based on a lognormal distribution with the parameter values based on experience over the period 1949 - 1995 (adjusted for inflation). However, if this process had been used just prior to 1992, the chance of two events occurring within the next 2 ½ years, both of which exceeded the largest previous loss by a factor of more than 2, would have been extremely small. However, Hurricane Andrew caused $15.5 billion in losses in August of 1992 and the Northridge earthquake caused $12.5 billion in insured losses in January 1994. The largest insured loss prior to that was Hurricane Hugo, which had caused $4.2 billion in losses in 1989. Also, if changes in any operations occur, then the results would not be valid.

The DFA model encompasses catastrophes, which have a significant impact on the property lines of business. The liability lines of business are more influenced by changes in public attitudes, and legislative or judicial changes. These changes are difficult if not impossible to model accurately. The variation method considers these to the extent that they are captured in historical data and variations.

The number of years used may affect the credibility of results. The DFA model results have a compounding effect from year to year (e.g., the first year results are used in the second year, the second year results are used in the third year, and so on). With nominal growth assumptions, this will result in larger variation for the more distant years. If ample simulations are run, then the distant years' variation becomes more stable.

When a significant legislative or judicial change occurs, the model should be adjusted to reflect such changes. The surplus allocation process should be run once again to incorporate these changes.

MODEL USAGE

Before relying on a DFA model for **any** purpose the user must be comfortable with the inputs and the outputs. This includes using it to allocate surplus.

Duration Adjustments to the DFA Model

The assumption was made that all payments would be made by the end of the twenty-first year for each accident year, however in the original model only five calendar years of payments were calculated for each accident year. These payments needed to be extended to twenty years past the last projected accident year. Extended payments were produced in the same fashion as done by the model for the first five years.

After projecting how much is going to be paid from each accident year for any calendar year, these payments are summed across all accident years for the appropriate calendar year. This generates projected calendar year paid amounts as in column 1 of Table 2. The total column in Exhibit 2 also shows calendar year paid amounts.

A discount rate was needed to find the present value of these calculations. The discount rate used came from the first year's projected investment information of the model. Dividends, coupon payments, and interest were summed and divided by the average book value amount invested in stocks, bonds, and cash over the year. This avoids both realized and unrealized capital gains or losses. By calculating a DFA discount rate, it allows the interest rate to vary with the projected economic conditions for each iteration.

For calculating present values, a uniform payment pattern during each year was assumed, giving an average payment mid-way through the year.

Investment Return Adjustments to the DFA Model

To capture interest earned by line of business, adjustments had to be made to the DFA model.

The pay-out rate of reserves was determined from the payment patterns already mentioned. The percentage of reserves available for investment over the course of the year is:

$$[\ \{ (1 - PP_T) + (1 - PP_{T+1}) \} / 2\ ] / (1 - PP_T)$$

Where: PP = Payment Pattern (Cumulative % Paid)
T = Beginning of Calendar Year
T+1 = End of Calendar Year

The division by 2 in the formula assumes the payment of reserves is made uniformly over the calendar year.

This calculation is done for each Calendar Year / Accident Year combination needed.

Using the above calculation, the amount available for investment was found by multiplying the percentage of reserves available by the appropriate reserves.

$$[\ [\ \{ (1 - PP_T) + (1 - PP_{T+1}) \} / 2] / (1 - PP_T)\ ] * \text{Reserves}$$

Where : When Calendar Year = Accident Year
Reserves = Reserves at the end of the Calendar Year

Otherwise
Reserves = Reserves at the beginning of the Calendar Year

Example: For a given line of business and accident year, 20% of the losses had been paid by the beginning of the current calendar year and 40% paid by the end. The year began with $5,000,000 in reserves for this particular line and accident year.

The amount available for investment is [{ (1-.20) + (1-.40) } /2]/(1-.20) = 0.875. In other words, 87.5% of the beginning of the year reserves are available for investment over the course of the year, or $5,000,000 * .875 = $4,375,000.

The method of calculating the rate of return used for this method was based on the market value return. The ending market value of the stocks, bonds, and cash were added to the sum of dividends, coupon payments, and interest received. The resulting amount was divided by the sum of the beginning market value of the stocks, bonds, and cash. A different rate of return was determined for each calendar year.

Credibility Weightings

In the Variation Method

Even after running 1,000 iterations, the information is not necessarily fully credible since this calculation deals with the standard deviation. The model itself should be fully credible, but the standard deviation deals with the number of samplings. If enough iterations are run, the standard deviations should be relatively stable from year to year. Due to computing limitations, only 1,000 iterations were run, which lacks full credibility.

Table 8 below lays out the credibility weighting of the standard deviations. Applying the Bühlmann credibility across the years and between the lines with the use of columns 6 through 10, credibility is determined by line of business and displayed in column 11. This process is shown explicitly in Exhibit 4. Giving credibility weight to the expected value for a line of business over the years (column 6) and the complement to the average of the expected values for all lines of business (column 8) results in a credibility weighted standard deviation of net operating gain per dollar of net written premium (column 12). This is the main factor in helping determine the distribution of surplus. The rest of the steps are similar to those used in the duration method. The Bühlmann credibility is further described in Appendix E.

Table 8

Standard Deviation of Net Operating Gain
Per Dollar of Net Written Premium

	(1) 1998	(2) 1999	(3) 2000	(4) 2001	(5) 2002	(6) Expected Value	(7) Within Variance	(10) K Factors
Home	0.4532	2.2769	0.6970	0.6594	0.4550	0.9083	0.4784	1.5015
P Auto Liab	0.1574	0.1235	0.1461	0.1395	0.1536	0.1440	0.0001	0.0004
P Auto Phys Dam	0.1199	0.9526	0.2524	0.2326	0.1184	0.3352	0.0984	0.3088
C Auto Liab	1.3752	1.4673	1.6137	1.6697	1.7350	1.5722	0.0175	0.0550
C Auto Phys Dam	0.4705	2.3967	0.7265	0.6892	0.5526	0.9671	0.5195	1.6302
CPP Liab	0.0575	0.0646	0.1012	0.0772	0.0841	0.0769	0.0002	0.0007
CPP Prop	0.5931	4.5416	1.1913	1.0898	0.5455	1.5923	2.2411	7.0333
Other Liab	0.1688	0.2055	0.2183	0.2339	0.2391	0.2131	0.0006	0.0020
Other Liab – Umbrella	0.8574	0.9288	1.1948	1.2197	1.3292	1.1060	0.0328	0.1028
Workers Comp	0.1308	0.1456	0.1748	0.1856	0.2019	0.1677	0.0007	0.0021
Personal	0.1255	0.7125	0.2001	0.1830	0.1159			
Commercial	0.2088	1.4789	0.3995	0.3605	0.2009		(13)	
Enterprise	0.1544	1.1122	0.2955	0.2666	0.1367		0.3931	
						(8)		
Expected Value[4]	0.4384	1.3103	0.6316	0.6197	0.5415	0.7083		
						(9)		
Between Variance[5]	0.1571	1.8391	0.2649	0.2702	0.2798	0.3186		

(6) = average of (1) to (5)

	(11) Credibility	(12) Cred. Wtd. Std Deviations
Home	0.7691	0.8621
P Auto Liab	0.9999	0.1441
P Auto Phys Dam	0.9418	0.3569
P Auto Liab	0.9891	1.5628
C Auto Phys Dam	0.7541	0.9035
CPP Liab	0.9999	0.0770
CPP Prop	0.4155	1.0756
Other Liab	0.9996	0.2133
Other Liab – Umbrella	0.9798	1.0980
Workers Comp	0.9996	0.1680

[4] The expected value is a straight average of the individual line of business data points.

[5] The between variance is the sum of the squared differences between the line of business data point and the expected value all divided by the number of lines of business.

BÜHLMANN CREDIBILITY

Bühlmann credibility is based on the formula n / (n + K) = Z

Where: n is the number of observations

K is the within variance / between variance

Z is the crebibility factor

The within variance is calculated within the same class or line of business across years or periods. In this application it would be the variance for a certain line of business over the 5 year period.

$$\frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n}$$

Xi is an individual observation

$\bar{X}$ is the average observation within the line of business

n is the 5 years

The between variance is calculated within the same year but across lines of business.

$$\frac{\sum_{i=1}^{m} (Y_i - \bar{Y})^2}{m}$$

Yi is an individual observation in the year

$\bar{Y}$ is the average observation for the year over all lines of business

m is the number of lines of business which is equal to 10 in our application

REFERENCES

[1] Calfarm v. Deukmejian, 48 Cal. 3d 805, 771 P.2nd 1247 (1989).

[2] D'Arcy, S. P., R. W. Gorvett, J. A. Herbers, T. E. Hettinger, S. G. Lehmann, and M. J. Miller, 1997, "Building a Public Access PC-Based DFA Model," *Casualty Actuarial Society Forum,* Fall 1997, Vol. 2, pp. 1-40.

[3] D'Arcy, S. P., R. W. Gorvett, T. E. Hettinger, and R. J. Walling, III, 1998, "Using the Public Access DFA Model: A Case Study," *Casualty Actuarial Society Forum,* Summer 1998, pp. 53-118.

[4] Ettlinger, K.H.; Hamilton, K.L.; and Krohm,G., *State Insurance Regulation* (First Edition), Insurance Institute of America, 1995, Chapter 8, pp. 209-231.

[5] Feldblum, S. "Pricing Insurance Policies: The Internal Rate of Return Model," May 1992.

[6] Ferguson, R. E., "Duration," *Proceedings of the Casualty Actuarial Society*, Vol. LXX, 1983, pp. 265-288.

[7] Miller, M. and Rapp, J., "An Evaluation of Surplus Methods underlying Risk Based Capital calculations," *1992 Discussion Paper Program,* Volume 1, pp. 1-122.

[8] Herzog, T. N., "Bühlmann's Approach," *Introduction to Credibility Theory* (Second Edition), 1996, pp. 69-98.

[9] Stone, J.M., "A Theory of Capacity and the Insurance of Catastrophe Risks," *The Journal of Risk and Insurance,* June 1973, Vol. XL No. 2, Part I, pp. 231-243, and September 1973, Vol. XL No. 3, Part II, PP. 339-355.