The

CASUALTY ACTUARIAL SOCIETY FORUM

SPRING 2001

CASUALTY ACTUARIAL SOCIETY FORUM

Spring 2001
Including the Dynamic Financial Analysis Call Papers

CASUALTY ACTUARIAL SOCIETY
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NOTICE

The Casualty Actuarial Society is not responsible for statements or opinions expressed in the papers in this publication. These papers have not been reviewed by the CAS Committee on Review of Papers.

The Casualty Actuarial Society *Forum*
Spring 2001 Edition
Including the Dynamic Financial Analysis Call Papers

To CAS Members.

This is the Spring 2001 Edition of the Casualty Actuarial Society *Forum*. It contains an introduction with background information on the 2001 Dynamic Financial Analysis Program from the Committee on Dynamic Financial Analysis, and eight Dynamic Financial Analysis Papers.

The Casualty Actuarial Society *Forum* is a nonrefereed journal printed by the Casualty Actuarial Society. The viewpoints published herein do not necessarily reflect those of the Casualty Actuarial Society.

The CAS *Forum* is edited by the CAS Committee for the Casualty Actuarial Society *Forum* Members of the committee invite all interested persons to submit papers on topics of interest to the actuarial community. Articles need not be written by a member of the CAS, but the paper's content must be relevant to the interests of the CAS membership Members of the Committee for the Casualty Actuarial Society *Forum* request that the following procedures be followed when submitting an article for publication in the *Forum*.

1 Authors should submit a camera-ready original paper and two copies.
2. Authors should not number their pages
3. All exhibits, tables, charts, and graphs should be in original format and camera-ready.
4 Authors should avoid using gray-shaded graphs, tables, or exhibits Text and exhibits should be in solid black and white.
5 Authors should submit an electronic file of their paper using a popular word processing software (e.g., Microsoft Word, WordPerfect) for inclusion on the CAS Web Site.

The CAS *Forum* is printed periodically based on the number of call paper programs and articles submitted The committee publishes two to four editions during each calendar year.

All comments or questions may be directed to the Committee for the Casualty Actuarial Society *Forum*.

Sincerely,

Dennis L. Lange

Dennis L Lange, CAS *Forum* Chairperson

The 2001 CAS Dynamic Financial Analysis Call Papers
Presented at the
2001 Dynamic Financial Analysis Seminar
June 7-8, 2001
The Boston Park Plaza Hotel
Boston, Massachusetts

The Spring 2001 Edition of the CAS *Forum* is a cooperative effort of the CAS *Forum* Committee and the CAS Committee on Dynamic Financial Analysis.

The CAS Dynamic Financial Analysis Committee presents for discussion eight papers prepared in response to its Call for 2001 Dynamic Financial Analysis Papers.

This Forum includes papers that will be discussed by the authors at the 2001 CAS Dynamic Financial Analysis Seminar, June 7-8, in Boston, Massachusetts

Introduction and Background Information

Dynamic Financial Analysis Call Papers

2001 Call for Papers—
Dynamic Financial Analysis, A Case Study

Committee on Dynamic Financial Analysis

2001 Call for Papers
Dynamic Financial Analysis, A Case Study

In this call paper program, participants were presented with a specific actuarial situation, including a company description and financial statements, and were asked to write a paper describing their approach and solution to the situation By giving all participants a common starting point, we hoped to (1) encourage creative problem solving by participants using Dynamic Financial Analysis (DFA), (2) demonstrate the range of DFA approaches and models to the CAS membership, and (3) illustrate how appropriate capital levels can be determined using DFA

This call focused on the application of DFA approaches to a given situation in order to illustrate how appropriate capital levels can be determined using DFA Each participant was expected to determine appropriate capital levels for the given insurance company based on standards used by ratings agencies, regulators, or financial markets The capital standard that was adopted for this study was left to the participant but should have been defended as to the appropriateness of its use It was expected that each paper would include the following

- Description of measures of risk and reward used in evaluation,
- Description of strategies considered,
- Description of the model used,
- Description of analytical process, and,
- Interpretation of model results/evaluation of strategies

The specific situation, company description, and financial statements are described below Participants were responsible for selecting appropriate risk and reward measures to either optimize or control

- ***Description of the Situation***
  The CEO of your company is considering the acquisition of DFA Insurance Company (DFAIC or the Company) as a stand-alone insurer DFAIC is a privately held company and has not yet been contacted about this interest, and cannot be contacted until after your analysis is concluded However, publicly available financial statements for the Company are available for the 1999 year and they are attached The Company's last insurance department examination occurred in 1996 and there were no material issues The Company has an unqualified actuarial opinion

- ***Description of the Company's Business***
  General – The Company has an "A" rating from A M Best and it has maintained this rating for at least the past five years It operates through the independent agency system and believes it has very strong relationships with its agency plant

  Underwriting Profile – The Company is licensed in all 50 states, but is primarily concentrated in the Northeast and the Midwest The Company considers itself a "regional" company in these two geographic areas Because of this focus, the Company has limited exposure to severe catastrophes However, it does have exposure to less severe but more frequent retained catastrophe losses

  The Company writes a balanced book of both personal and main street commercial insurance coverages

  The Company has minimal exposure to asbestos and environmental exposures

  Asset Classes – The Company's cash and invested asset portfolio is approximately 70% fixed income, 12% equity and 18% cash

  The fixed income portfolio is approximately 80% in tax-exempt municipal issues and 20% in a mixture of Corporate and Government bonds The Municipals have an average maturity of 10 5 years and an average yield of 6% The Corporate and Government bonds have an average maturity of 4 years and an average yield of 8%

  The equity portfolio is invested with a target return of the S&P 500

Reinsurance – The Company maintains reinsurance to limit shock and catastrophic losses from a single event The largest net aggregate amount insured in any one risk (excluding Workers Compensation) is $1 million Excess of loss is used to protect property risks above $1,000,000 up to $20 million per risk, $50 million per occurrence For casualty and Workers Compensation risks, an excess of loss treaty provides coverage above $500,000 up to $50 5 million

The Company has a catastrophe cover of 90% of $150 million excess of $50 million for any single event This limits the Company's net pre-tax PML for a catastrophe over a 100 year return period to 10% of surplus

All of the Company's reinsurers are rated "A," or better, and there are no known problems with reinsurance recoverable

- ***Questions the CEO would like addressed:***

1 Is the Company adequately capitalized? Is there excess capital? How much capital should the Company hold as a stand-alone insurer?
2 How should the capital be allocated to line of business?
3 What is the return distribution for each line of business and is it consistent with the risk for the line?
4 Should the Company buy more or less reinsurance? What type? How efficient is its current reinsurance program?
5 How efficient is the asset allocation?

- ***Financial Statements***

The financial statements are included in this section

Balance Sheet
1999

ASSETS		LIABILITIES & SURPLUS	
Bonds	3,324 007	Losses	1,908 774
Preferred Stock	327,805	Reins Payable on Paid L&LAE	(1,618)
Common Stock	236,120	LAE	421,387
Mortgage Loans - First	2,233	Contingent Comm & Other Charges	32,057
Mortgage Loans - Other Than First	0	Other Expenses	14,349
Mortgage Loans On RE	0	Taxes, Licenses & Fees	18,691
Real Estate - Occupied	30,479	Federal & Foreign Income Taxes	5,835
Real Estate - Other	1,555	Borrowed Money	0
Collateral Loans	0	Interest	0
Cash and Short Term	869,870	Unearned Premium	985,422
Other Invested Assets	0	Dividends Unpaid - Stockholders	180,000
Receivables	330	Dividends Unpaid - Policyholders	6,732
Aggregate Write-Ins	0	Funds Held Reins Treaties	1,577
Sub-total - Cash & Invested Assets	4,792,399	Amounts Withheld for Others	5,491
Prems/Agents Bal in Collection	183,104	Remittances and Items Unallocated	0
Prems/Agents Bal not due	262,029	Provision for Reinsurance	2,806
Accrued Retro Prems	0	Excess Stat Reserve	7,052
Funds Held for Reins	0	Net Adj Asset/Liab Due to For Exchg	0
Bills Receivable	61	Drafts Outstanding	186,209
Reins Recov on L&LAE Payments	49,600	Payable to Parent, Sub & Affil	0
FIT Recoverable	1,378	Payable for Securities	2,010
Guarantee Funds Receivable	3,370	Liability Amounts Held A&H	0
EDP Equipment	8,292	Capital Notes & Interest	0
Int, Div, RE Income Due & Accrued	61,515	Aggregate Write-ins	0
Receivable from Affiliates	0	Total Liabilities	3,776,776
Equities/Deposits in Pools & Assoc	19,324	Agg Write-ins	0
Amts Receivable A&H Plans	0	Common Capital Stock	43,652
Other Assets Non-admitted	0	Preferred Capital Stock	0
Aggregate Write-Ins	0	Agg Write-ins	0
		Surplus Notes	0
		Gross Paid In & Contributed Surplus	356,341
		Unassigned Funds (Surplus)	1,204,304
		Treasury Stock-Common	0
		Treasury Stock-Preferred	0
		Policyholders Surplus	1,604,297
Total Assets	5,381,073	Total - Liabilities & Surplus	5,381,073

Income Statement & Surplus Reconciliation
1999

INCOME STATEMENT	
Premiums Earned	2,353,625
Losses Incurred	1,586,511
Loss Expenses Incurred	191,923
Other Underwriting Expense Incurred	693,794
Agg Write-Ins	0
Total Underwriting Deductions	2,472,228
Net Underwriting G/L	(118,604)
Net Investment Income Earned	337,232
Net Realized Capital G/L	14,156
Net Investment G/L	351,388
Net G/L Agents/Prem Bal Chrgd Off	(16,667)
Fin & Svc Chrgs Not In Premiums	10,531
Agg Write-ins	0
Total Other Income	(6,136)
Net Income Before Policyholder Dividends	226,648
Dividends to Policyholders	12,169
Net Income after Policyholder Dividends	214,478
Federal & Foreign Income Tax Incurred	28,080
Net Income	186,398
SURPLUS RECONCILIATION	
Surplus as Regards Policyholders - Prior Year	1,663,322
Net Income	186,398
Net Unrealized Capital G/L	8,958
Change in Non-Admitted Assets	29,898
Change in Provision for Reinsurance	(41)
Change In Foreign Exchange Adjustment	0
Change in Excess Statutory Reserve	5,069
Change In Surplus Notes	0
Capital Changes Paid In	0
Capital Changes Transferred from Surplus	0
Capital Changes Transferred to Surplus	0
Surplus Adjustment Paid In	39,436
Surplus Adjustment Transferred to Capital	0
Surplus Adjustment Transferred from Capital	0
Net Remittances fr/to Home Office	0
Dividends to Stockholders	(319,181)
Change in Treasury Stock	0
Extraordinary Amounts of Taxes Prior Years	(9,562)
Aggregate Write-ins	0
Change in Policyholders Surplus for Year	(59,025)
Surplus as Regards Policyholders - Current Year	1,604,297

Underwriting Exhibit
1999

Line of Business	Direct Written Premium	Net Written Premium	Unearned Premium	Unpaid Loss	Unpaid LAE
Fire	16,879	14,794	6,556	10,495	934
Allied Lines	19,444	17,698	7,630	5,776	295
Farm Mp	3,209	931	(8)	3,146	715
Home Mp	349,884	322,732	181,636	107,444	27 636
Comm Mp	365,703	338,019	164,745	346,772	159,202
Mortg Guar	0	0	0	0	0
Ocean Marine	14,238	8,323	3,120	4,025	107
Inland Marine	55,083	49,780	25,604	8,957	1 367
Finan Guar	0	0	0	0	0
Medical Mal	0	0	0	0	0
Med Mal Occur	0	0	0	0	0
Med Mal Clms	0	0	0	0	0
Earthquake	1,021	964	521	35	1
Group A&H	5,374	2,746	0	1,277	30
Credit A&H	0	0	0	0	0
Other A&H	0	0	0	0	0
Workers' Comp	235,311	201,213	85,323	503,298	51,984
Other Liab	0	0	0	0	0
Oth Liab Occur	66,986	41,873	22,329	86,214	30,643
Oth Liab Clms	730	174	61	15	0
Prod Liab	0	0	0	0	0
Pr Liab Occur	1,264	1,193	604	511	513
Pr Liab Clms	0	0	0	0	0
Auto Liab	0	0	0	0	0
Priv Pass Auto	632,585	593,660	211,134	583,148	104,939
Comm Auto Liab	179 781	164,226	77,721	201,119	34,558
Auto Phys Dam	576 628	556,295	181,332	26,166	5,832
Aircraft	8,041	5,008	2,354	862	0
Fidelity	4,160	3,984	2,740	1,471	419
Surety	20,128	18,910	11,771	3,954	1,807
Glass	0	0	0	0	0
Burglary	120	115	56	14	7
Blr & Mach	1,151	81	65	52	1
Credit	0	0	0	0	0
International	0	0	0	0	0
Rein 30A	5,835	7,068	120	10,415	394
Rein 30B	993	454	5	3,368	0
Rein 30C	7	7	0	239	0
Rein 30D	0	0	0	0	0
Agg Write-Ins	0	0	0	0	0
Total	2,564,555	2,350,245	985,422	1,908,774	421,387
					2,330 161

Underwriting Exhibit
1999
Summary By Schedule P Line of Business

Line of Business	Direct Written Premium	Net Written Premium	Unearned Premium	Unpaid Loss	Unpaid LAE	Unpaid L&LAE
Homeowners	353,093	323,663	181,628	110,590	28,351	138,941
PP Auto Liability	632,585	593,660	211,134	583,148	104,939	688,087
Comm Auto Liability	179,781	164,226	77,721	201,119	34,558	235,677
Workers Compensation	235,311	201,213	85,323	503,298	51,984	555,282
CMP	365,703	338,019	164,745	346,772	159,202	505,974
Med Mal Occ	0	0	0	0	0	0
Med Mal CM	0	0	0	0	0	0
Special Liability	23,430	13,412	5,539	4,939	108	5,047
Other Liab Occ	66,986	41,873	22,329	86,214	30,643	116,857
Other Liab CM	730	174	61	15	0	15
International	0	0	0	0	0	0
Reins A	5,835	7,068	120	10,415	394	10,809
Reins B	993	454	5	3,368	0	3,368
Reins C	7	7	0	239	0	239
Products Liab Occ	1,264	1,193	604	511	513	1,024
Products Liab CM	0	0	0	0	0	0
All Other	698,837	665,283	236,213	58,146	10,695	68,841
Total	2,564,555	2,350,245	985,422	1,908,774	421,387	2,330,161

Expense Exhibit

		Loss Adjustment Expense	Underwriting Expense	Investment Expense	Total Expense
Loss Adjustment Expense	Direct	47,155	0	0	47,155
	Assumed	32,908	0	0	32,908
	Ceded	29,377	0	0	29,377
	Net	50,685	0	0	50,685
Commission & Brokerage	Direct	0	365,369	0	365,369
	Re Assumed	0	181,194	0	181,194
	Re Ceded	0	210,479	0	210,479
	Dir Cont	0	36,329	0	36,329
	Assmd Cont	0	14,286	0	14,286
	Ced Cont	0	14,398	0	14,398
	P&M Fees	0	0	0	0
	Net	0	372,303	0	372,303
Allowances to Mgrs & Agents		133	2,815	0	2,949
Advertising		464	3,284	7	3,754
Boards, Bureaus & Assoc		165	20,258	4	20,428
Survey & Undwtg Repts		7	7,190	0	7,197
Audit of Assureds Recs		28	415	0	443
Salaries		85,289	119,732	3,004	208,024
Payroll Taxes		5,237	9,338	0	14,576
Employee Welfare		14,392	19,625	489	34,505
Insurance		127	1,167	4	1,298
Directors' Fees		75	275	115	465
Travel		7,686	9 056	313	17,055
Rent		9,431	10,080	229	19,738
Equipment		8,310	26,229	498	35,035
Printing & Stationery		2,972	6,068	258	9,298
Postage & Telephone		5,850	12,689	378	18,917
Legal & Auditing		1,513	3,123	156	4,792
Sub-Total		141,677	251,343	5,453	398,473
State & Local Ins Tax		0	36,641	0	36,641
Ins Dept Lic & Feess		0	6,512	0	6,512
Guar Assn Assessments		0	1,186	0	1,186
Other Taxes, Lic & Fees		0	1,407	8	1,414
Total Taxes Lic & Fees		0	45,746	8	45 754
Real Estate Expenses		0	1,565	4,396	5,961
Real Estate Taxes		344	643	678	1,665
Reimb by Unin A&H Plans		0	0	0	0
Agg Write-ins		(782)	22,196	838	22,251
Total Expense Incurred		191,923	693,794	11,374	897,093
Unpaid Exp - Curr Year		421,387	65,097	0	486,484
Unpaid Exp - Prior Year		449,965	124,678	289	574,932
Amts Rec Unins A&H - PY		0	0	0	0
Amts Rec Unins A&H - CY		0	0	0	0
Total Expense Paid		220 501	753,376	11 663	985,540

Invested Assets

Bonds	Book	Market	Cost	Par	Statement
Government	295,845	307,907	296,888	294,950	295,845
States, Terr & Possessions	679,828	708,972	683,448	685,835	679,804
Political Subdivisions	596,515	623,300	601,997	588,635	596,515
Special Rev & Assessment	1,345,010	1,418,650	1,352,424	1,348,199	1,344,870
Public Utility	54,172	53,623	53,820	54,350	52,685
Industrial	355,647	365,201	355,713	354,924	354,286
Parent, Subs & Affiliated	0	0	0	0	0
Total Bonds	3,327,018	3,477,653	3,344,291	3,326,893	3,324,007
Preferred Stock					
Public Utility	50,009	49,771	49,835		
Bank Trust Ins Company	71,728	72,484	71,728		
Industrial	206,787	205,550	206,807		
Parent, Subs & Affiliated	0	0	0		
Total Preferred Stock	328,524	327,805	328,369		
Common Stock					
Public Utility	2,480	3,318	2,480		
Bank Trust Ins Company	6,829	11,621	6,829		
Industrial	120,400	165,920	120,403		
Parent, Subs & Affiliated	12,331	55,261	12,331		
Total Common Stock	142,040	236,120	142,043		
Total Stock	470,564	563,925	470,412		
Total Bonds & Stock	3,797,582	4,041,578	3,814,703		

Fixed Income Investments

By Maturity & Type

	<1	1-5	5-10	10-20	20+	Total	Publicly Traded	Private Placements
US Government	13,251	151,213	111,025	6,884	7,315	289,688	289,688	0
Other Government	4,754	4,205	1,232	2,719	0	12,910	12,910	0
States, Terr	9,517	179,211	198,062	238,575	54,438	679,804	679,804	0
Pol Sub	15,354	211,837	120,850	151,537	96,938	596,515	596,515	0
Spec Rev	38,070	310,794	369,310	565,668	61,228	1,345,070	1,345,070	0
Pub Util	7,984	30,644	11,013	3,045	0	52,685	52,685	0
Indust & Misc	164,603	91,810	186,242	39,206	9,090	490,951	490,951	0
Credit Tennants	0	0	0	0	0	0	0	0
PSA	0	0	0	0	0	0	0	0
Total	253,533	979,713	997,735	1,007,634	229,009	3 467,623	3,467,623	0

Fixed Income Investments

By Type & Class

US Government	<1	1-5	5-10	10-20	20+	Total	Publicly Traded	Private Placements
Class 1	13,251	151,213	111,025	6,884	7,315	289,688	289,688	0
Class 2	0	0	0	0	0	0	0	0
Class 3	0	0	0	0	0	0	0	0
Class 4	0	0	0	0	0	0	0	0
Class 5	0	0	0	0	0	0	0	0
Class 6	0	0	0	0	0	0	0	0
Other Government								
Class 1	4 754	0	0	0	0	4,754	4,754	0
Class 2	0	297	1 232	2,719	0	4,249	4,249	0
Class 3	0	3 907	0	0	0	3,907	3,907	0
Class 4	0	0	0	0	0	0	0	0
Class 5	0	0	0	0	0	0	0	0
Class 6	0	0	0	0	0	0	0	0
States, Terr								
Class 1	9 517	163,087	182,426	221,346	28,546	604,924	604,924	0
Class 2	0	0	15 636	17 229	25 891	58 756	58 756	0
Class 3	0	16,124	0	0	0	16,124	16,124	0
Class 4	0	0	0	0	0	0	0	0
Class 5	0	0	0	0	0	0	0	0
Class 6	0	0	0	0	0	0	0	0
Pol Sub								
Class 1	15,354	203,463	103,677	151,537	92 079	566,109	566,109	0
Class 2	0	8 374	17,173	0	4 858	30,406	30,406	0
Class 3	0	0	0	0	0	0	0	0
Class 4	0	0	0	0	0	0	0	0
Class 5	0	0	0	0	0	0	0	0
Class 6	0	0	0	0	0	0	0	0
Spec Rev								
Class 1	37,930	285,526	326,760	502 801	56,228	1,209 245	1,209,245	0
Class 2	0	15,232	38,225	59,953	0	113,410	113,410	0
Class 3	140	5,791	2,315	2,914	5 000	16 160	16,160	0
Class 4	0	830	2,010	0	0	2 840	2,840	0
Class 5	0	3,415	0	0	0	3,415	3,415	0
Class 6	0	0	0	0	0	0	0	0
Pub Util								
Class 1	2,984	5,000	2,914	3,045	0	13,942	13,942	0
Class 2	5,000	23,886	2,599	0	0	31 485	31,485	0
Class 3	0	1,758	5,500	0	0	7,258	7,258	0
Class 4	0	0	0	0	0	0	0	0
Class 5	0	0	0	0	0	0	0	0
Class 6	0	0	0	0	0	0	0	0
Indust & Misc								
Class 1	157,480	36 548	86,783	13,683	8,558	303,052	303,052	0
Class 2	5,484	42,247	87,956	25 244	0	160,930	160,930	0
Class 3	1,639	7,068	11,503	278	533	21 020	21,020	0
Class 4	0	5,948	0	0	0	5 948	5 948	0
Class 5	0	0	0	0	0	0	0	0
Class 6	0	0	0	0	0	0	0	0
Credit Tennants								
Class 1	0	0	0	0	0	0	0	0
Class 2	0	0	0	0	0	0	0	0
Class 3	0	0	0	0	0	0	0	0
Class 4	0	0	0	0	0	0	0	0
Class 5	0	0	0	0	0	0	0	0
Class 6	0	0	0	0	0	0	0	0
PSA								
Class 1	0	0	0	0	0	0	0	0
Class 2	0	0	0	0	0	0	0	0
Class 3	0	0	0	0	0	0	0	0
Class 4	0	0	0	0	0	0	0	0
Class 5	0	0	0	0	0	0	0	0
Class 6	0	0	0	0	0	0	0	0
Total								
Class 1	241 270	844,835	813,566	899,297	192,727	2,991 714	2,991,714	0
Class 2	10 484	90,037	162,822	105,145	30,749	399,237	399,237	0
Class 3	1,779	34,648	19,317	3,192	5 533	64,469	64 469	0
Class 4	0	6 778	2,010	0	0	8,788	8,788	0
Class 5	0	3 415	0	0	0	3,415	3,415	0
Class 6	0	0	0	0	0	0	0	0

Schedule P Summary
Summary

	NET				DIRECT & ASSUMED			
	Earned Premium	Paid L&LAE	Un-paid L&LAE	Ultimate L&LAE	Earned Premium	Paid L&LAE	Un-paid L&LAE	Ultimate L&LAE
Prior			166730				320307	
1990	1596494	1151101	30721	1181823	2470794	1796516	58076	1854594
1991	1737209	1163693	34067	1197760	2638630	1724810	58950	1783760
1992	1775609	1191475	41619	1233093	2673873	1710394	75891	1786284
1993	1879739	1186961	49137	1236098	2779441	1715327	92777	1808106
1994	1979732	1247456	73462	1320918	2903689	1697453	134762	1832216
1995	2107454	1464370	114253	1578622	3071550	2051512	211112	2262622
1996	2209212	1421609	167455	1589064	3273222	1974366	279221	2253589
1997	2276816	1480345	278784	1759129	3302666	2013850	474069	2487721
1998	2335012	1323943	463891	1787833	3362358	1779580	741119	2520698
1999	2353628	1019829	910056	1929885	3372517	1445408	1464449	2909855
	20 250,905	12 650 782	2,330 175	14,814,225	29 848 740	17 909 016	3 910 733	21 499 445

Ultimate Incurred Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	823916	1104131	1122146	1122467	1128326	1118885	1110486	1106679	1105497	1109923
1991	1159523	1160612	1151034	1148713	1134153	1124437	1115902	1112973	1112987	
1992	1266258	1242862	1217274	1186469	1171824	1157443	1150694	1148914		
1993	1310212	1265073	1233699	1189098	1160604	1144456	1142139			
1994	1359920	1314575	1275997	1236108	1213314	1204485				
1995	1570363	1547449	1526151	1487374	1464921					
1996	1562228	1528197	1503007	1470565						
1997	1666724	1658363	1626278							
1998	1677599	1645660								
1999	1759654									

Paid Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	459707	736944	869785	953428	1009607	1037505	1056334	1066110	1075164	1080414
1991	475583	740927	870282	958701	1010178	1043323	1060304	1072111	1080292	
1992	511605	787838	918887	1000836	1056514	1080877	1099026	1109015		
1993	523717	790482	920709	1004739	1050583	1078352	1095248			
1994	546303	830218	969956	1052257	1107812	1134153				
1995	711681	1036367	1191990	1300682	1355550					
1996	700904	1045166	1216549	1310046						
1997	830287	1204860	1359498							
1998	841694	1203813								
1999	902204									

Bulk & IBNR Reserves

	12	24	36	48	60	72	84	96	108	120
1990	319090	169339	110800	78364	58387	40421	28341	20788	15012	17692
1991	403557	201955	137372	89485	59786	40995	27533	19812	17963	
1992	441817	236282	145782	81560	56691	37910	27530	22241		
1993	472302	255327	161941	91444	54149	32017	25539			
1994	463389	232103	148843	82294	50484	37437				
1995	441776	233818	141054	82171	49085					
1996	452418	220372	134492	73931						
1997	435651	204904	113925							
1998	438671	190284								
1999	458213									

Reported Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	504828	934792	1011346	1044103	1069939	1078464	1082145	1085891	1090485	1092231
1991	755966	958857	1013662	1059228	1074367	1083442	1088369	1093161	1095024	
1992	824441	1006580	1071492	1104909	1115133	1119533	1123164	1126673		
1993	837910	1009746	1071758	1097654	1106455	1112439	1116600			
1994	896531	1082472	1127154	1153814	1162830	1167048				
1995	1128587	1313631	1385097	1405203	1415836					
1996	1109810	1307825	1368515	1396634						
1997	1231073	1453459	1512353							
1998	1238928	1455378								
1999	1301441									

Schedule P   Home
Home

	NET				DIRECT & ASSUMED			
	Earned Premium	Paid L&LAE	Un-paid L&LAE	Ultimate L&LAE	Earned Premium	Pa d L&LAE	Un-paid L&LAE	Ultimate L&LAE
Pnor			857				4109	
1990	182960	131875	637	132512	267430	187383	1530	188914
1991	196945	138220	298	138518	294480	195859	589	196450
1992	215241	171036	195	171231	319753	244972	522	245494
1993	232865	172846	1858	174704	345422	249482	3708	253187
1994	242880	174440	2042	176482	372249	249711	4566	254278
1995	259957	242440	4348	246788	407206	351214	7145	358362
1996	284348	213878	5683	219561	446710	305739	9918	315656
1997	304432	264007	13638	277644	463207	378179	22200	400382
1998	320240	214763	23968	2387ᴊ1	483897	294224	38743	332966
1999	324779	208513	85414	293927	501395	300317	129528	429844
	2,564 647	1 932,018	138 938	2 070 098	3 901 749	2 757 080	222 558	2 975 533

Ultimate Incurred Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	31169	123086	121828	121312	120960	120786	120687	120986	120907	120685
1991	126731	130028	127583	126730	125640	127269	126636	126266	125893	
1992	157558	159071	158104	159525	157525	157873	157124	156249		
1993	163692	163139	161354	161677	160495	160421	159270			
1994	167469	164228	163903	163628	161827	159595				
1995	230837	229624	227953	226813	226454					
1996	202686	201266	202338	200922						
1997	259065	260110	256783							
1998	222746	221905								
1999	268705									

Paid Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	76159	108951	114280	115908	116748	117938	118845	119859	120060	120052
1991	78459	109374	115786	119285	121939	124971	125535	125587	125596	
1992	107795	140993	146309	150711	153028	154570	155469	156047		
1993	106531	143622	148888	153620	156320	157590	157510			
1994	105504	142706	148931	154062	155933	157630				
1995	165492	206802	212794	218489	222294					
1996	132687	178721	189628	195514						
1997	187523	234064	243693							
1998	157251	199246								
1999	190205									

Bulk & IBNR Reserves

	12	24	36	48	60	72	84	96	108	120
1990	16747	6046	3291	1622	1006	1017	1114	756	610	518
1991	22076	6873	3360	1789	650	979	671	419	89	
1992	20808	5358	3974	2577	1807	1457	659	122		
1993	20824	6695	3856	2499	1486	1254	402			
1994	26343	6902	4454	3171	1956	357				
1995	23799	7374	3868	1363	889					
1996	25567	7015	3966	1276						
1997	27611	9315	4424							
1998	25832	5823								
1999	30318									

Reported Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	14422	117040	118537	'.119690	119954	119769	119553	120230	120297	120167
1991	104655	123153	124223	124941	124990	126290	125985	125847	125804	
1992	136750	153713	154130	156948	155718	156416	156465	156127		
1993	142868	156444	157498	159178	159009	159167	158868			
1994	141126	157326	159449	160457	159871	159238				
1995	207038	222250	224085	225450	225765					
1996	177119	194251	198372	199646						
1997	231454	250795	252359							
1998	196914	216082								
1999	238387									

Schedule P PPA
PPA

	NET				DIRECT & ASSUMED			
	Earned Premium	Paid L&LAE	Un-paid L&LAE	Ultimate L&LAE	Earned Premium	Paid L&LAE	Un-paid L&LAE	Ultimate L&LAE
Prior			7298				106598	
1990	335080	312946	4066	317012	520059	458661	10976	469639
1991	380573	314109	2349	316458	558734	434597	8806	443404
1992	419726	345629	3669	349298	619903	452844	15133	467977
1993	518045	373975	4924	378900	717146	470528	14253	484783
1994	576354	389123	10584	399706	779551	473475	24942	498419
1995	582729	404411	22652	427063	805111	512703	54563	567267
1996	618378	412208	40134	452342	859327	535228	71107	606335
1997	640517	403115	86525	489639	872722	524373	135351	659722
1998	629561	349946	170166	520112	834695	450637	249365	700006
1999	601444	180893	335722	516615	699219	240595	486256	726851
	5,302 407	3,486 355	688 089	4 167 145	7,266 467	4 553 641	1 177,350	5 624,403

Ultimate Incurred Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	235477	292601	293854	290788	292953	292894	291327	290965	291114	292586
1991	315929	306678	298835	295904	295276	292241	292301	292545	291313	
1992	363803	353503	342560	327715	324699	322260	320430	319877		
1993	413786	398749	387106	366160	353813	347647	346747			
1994	448442	413814	393299	373652	366441	364089				
1995	444678	434244	413134	400496	394819					
1996	455012	435873	420218	418199						
1997	467927	462347	452051							
1998	500040	477071								
1999	471228									

Paid Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	77087	168734	222770	254634	275564	282970	286341	287228	288283	288722
1991	79765	171913	226588	259418	275298	283357	286679	288712	289106	
1992	84913	190748	252451	285703	305307	310900	314584	316362		
1993	98716	208521	273674	310892	328984	338098	342021			
1994	102797	225148	289638	325387	346017	354008				
1995	120279	253104	318140	358551	373180					
1996	123944	261435	338422	379957						
1997	147317	303231	369873							
1998	163301	315852								
1999	155240									

Bulk & IBNR Reserves

	12	24	36	48	60	72	84	96	108	120
1990	98588	45059	22914	11012	5770	4310	1942	1244	1285	3048
1991	128830	51419	26920	10293	7259	2509	1985	1787	1133	
1992	142842	70827	32690	12381	6733	4429	2682	1426		
1993	174297	88315	53137	25044	11201	4460	3168			
1994	176867	79397	46819	20048	8270	4167				
1995	144802	73809	35777	15572	8221					
1996	155447	66134	32536	15439						
1997	147862	61175	32130							
1998	151913	55373								
1999	146225									

Reported Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	136889	247542	270940	279776	287183	288584	289385	289721	289829	289538
1991	187099	255259	271915	285611	288017	289732	290316	290758	290180	
1992	220961	282676	309870	315334	317966	317831	317748	318451		
1993	239489	310434	333969	341116	342612	343187	343579			
1994	271575	334417	346480	353604	358171	359922				
1995	299876	360435	377357	384924	386598					
1996	299565	369739	387682	40276C						
1997	320065	401172	419921							
1998	348127	421698								
1999										

Schedule P CAT
CAT

	NET				DIRECT & ASSUMED			
	Earned Premium	Paid L&LAE	Un-paid L&LAE	Ultimate L&LAE	Earned Premium	Paid L&LAE	Un-paid L&LAE	Ultimate L&LAE
Prior			2702				3715	
1990	121206	91802	406	92208	189409	144831	600	145432
1991	128555	92839	856	93695	204400	150188	1735	151927
1992	124279	79946	1339	81285	201211	126849	2074	128920
1993	123834	76712	2154	78866	198350	125331	3528	128861
1994	131029	92997	3977	96973	211733	149183	7193	156377
1995	143954	108255	9735	117990	229659	171916	19590	191508
1996	140865	95172	15902	111074	223683	143438	24484	167918
1997	152933	93358	34418	127777	238474	136977	54960	191939
1998	159345	70197	65478	135675	252011	111602	103884	215485
1999	163010	39154	98710	137864	270259	65089	165210	230297
	1,389 010	840 432	235 677	1 073 407	2 219 189	1 325 402	386 973	1 708 664

Ultimate Incurred Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	103752	91893	94238	90342	88504	86644	86210	86043	86110	85808
1991	97383	97424	92733	89450	89114	87362	86527	86704	86450	
1992	95589	86374	79188	78775	75922	75681	75573	75554		
1993	94625	86782	80340	78332	74121	73995	73827			
1994	100822	104601	100696	92725	89616	88182				
1995	122179	123418	119793	117433	111201					
1996	113679	111402	106610	104985						
1997	116224	116830	119508							
1998	123729	124085								
1999	125756									

Paid Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	16943	36378	58582	70462	78094	81467	83683	84618	85253	85419
1991	17128	36960	56873	69814	79020	82355	83888	85315	85635	
1992	15792	34328	49699	59062	66911	71248	73105	74273		
1993	15233	33379	49302	59438	65955	69586	71762			
1994	19119	41258	61135	73195	80738	84369				
1995	24246	53612	75796	93280	101944					
1996	21620	49388	73057	89645						
1997	23613	57681	86166							
1998	27102	60959								
1999	31039									

Bulk & IBNR Reserves

	12	24	36	48	60	72	84	96	108	120
1990	43521	27080	17110	8994	4757	2089	1283	726	425	156
1991	55125	30534	15215	7554	4407	2162	580	449	202	
1992	52945	25383	11683	6647	2140	1066	692	154		
1993	55010	30680	14366	5819	2192	865	510			
1994	48978	28808	19863	8013	3019	1045				
1995	54679	30816	17538	9470	2775					
1996	53285	26834	12875	5020						
1997	52032	21680	11979							
1998	57150	29154								
1999	54703									

Reported Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	60231	64913	77128	81348	83747	84555	84927	85317	85685	85652
1991	42258	66890	77518	81896	84707	85200	85947	86255	86248	
1992	42644	60991	67505	72128	73782	74615	74881	75400		
1993	39615	56102	65974	70513	71929	73130	73317			
1994	51844	75793	80833	84712	86597	87137				
1995	67500	92602	102255	107963	108426					
1996	60394	84768	93735	99965						
1997	64192	95150	107529							
1998	66579	94931								
1999										

Schedulo P WC
WC

	NET				DIRECT & ASSUMED			
	Earned Premium	Paid L&LAE	Un-paid L&LAE	Ultima'e L&LAE	Earned Premium	Paid L&LAE	Un-paid L&LAE	Ultimate L&LAE
Prior			78036				106378	
1990	204428	156599	18644	175243	310843	258698	27518	286216
1991	228937	170807	22199	193006	333848	258255	31958	290213
1992	245414	151145	25927	177073	361994	236442	39362	275807
1993	243405	138859	28236	167095	381489	233877	48020	281895
1994	257538	138945	29623	168568	357778	170405	44659	215065
1995	273523	123364	36253	159616	330575	148058	54122	202178
1996	276367	111114	42808	153921	357529	130814	64141	194955
1997	254069	105571	57325	162896	346474	129930	105424	235354
1998	222788	88125	78934	167058	322711	120501	125566	246068
1999	209180	48578	137297	185875	350794	76524	240165	316689
	2 415 649	1 233 107	555,280	1 710,351	3 454 035	1 763 504	887 309	2,544 440

Ultimate Incurred Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	237143	174455	182141	189341	189543	186321	180851	179232	176318	179289
1991	183183	198016	207766	210445	203707	201418	197066	191672	193143	
1992	203009	207190	205272	193977	192942	185360	180604	180741		
1993	196681	197471	189212	179989	172715	166559	166470			
1994	201220	195241	178265	172908	161405	158958				
1995	198383	183219	174096	162395	155457					
1996	177561	175394	163821	146620						
1997	175661	166151	152291							
1998	159221	151739								
1999	167356									

Paid Loss & ALAE

	12	24	36	48	60	72	84	98	108	120
1990	35126	82507	111848	130402	141543	147667	153294	156684	159190	161313
1991	38322	91252	121899	140972	152440	161352	165654	169236	171687	
1992	37356	86873	115335	131775	143216	148687	153199	155774		
1993	35233	77467	102068	120431	128284	134727	139327			
1994	34668	75349	102135	115720	124387	130418				
1995	34389	76001	98458	112317	120176					
1996	32455	72467	93806	105008						
1997	36129	75092	96765							
1998	33953	75818								
1999	37710									

Bulk & IBNR Reserves

	12	24	36	48	60	72	84	96	108	120
1990	70862	42977	30756	32418	27645	21922	15538	12541	8179	10544
1991	94118	54332	47691	41085	29834	24446	18619	12286	12533	
1992	111354	70926	53555	35145	30735	21885	15183	13910		
1993	114435	73432	52197	34810	26032	16988	14680			
1994	121669	70009	41174	31344	19751	16720				
1995	112516	57527	39464	26749	17419					
1996	96487	57303	41963	23429						
1997	86849	49691	27381							
1998	79853	37648								
1999	81687									

Reported Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	166281	131478	151385	156925	161898	164399	165315	166691	168139	168745
1991	89065	143684	160075	169360	173873	176972	178447	179386	180610	
1992	91655	136264	151717	158832	162207	163475	165421	166831		
1993	82446	124039	137015	145179	146683	149571	151790			
1994	79551	125232	137091	141562	141654	142238				
1995	85867	125692	134632	135646	138038					
1996	81074	118091	121858	123191						
1997	89012	116460	124910							
1998	79368	114091								
1999										

Schedule P CMP
CMP

	NET				DIRECT & ASSUMED			
	Earned Premium	Paid L&LAE	Un-paid L&LAE	Ultimate L&LAE	Earned Prem um	Paid L&LAE	Un paid L&LAE	Ultimate L&LAE
Pnor			30014				42451	
1990	268061	172490	6017	178507	483879	325526	13193	338717
1991	291919	170949	7500	178449	537660	313992	14519	328513
1992	274640	158064	9217	167282	409287	276272	16823	293097
1993	246992	139220	10615	149835	449394	271493	21117	292610
1994	249014	155661	24854	180515	476958	281876	49882	331757
1995	279522	204369	34205	238575	517417	368901	65087	433988
1996	285944	165684	49406	215091	526508	274108	86549	360656
1997	291659	163198	67729	230928	532719	279822	124208	404030
1998	312958	141202	100238	241439	561943	233964	175979	409944
1999	333786	115588	166178	281747	586434	191492	309420	500910
	2 834 495	1,586,405	505 973	2,062 368	5 171 997	2,817 446	919,228	3 694 220

Ultimate Incurred Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	116941	159279	164026	165374	166951	164740	165179	163981	165389	165814
1991	178674	170576	168527	172508	166693	164591	162938	164399	165802	
1992	184975	170748	166017	160701	156753	152659	154144	153881		
1993	162510	152915	150070	142577	137894	135851	135555			
1994	162102	159861	162836	161429	162919	165538				
1995	215829	220404	227334	225469	219830					
1996	200189	198951	197587	195372						
1997	216892	216868	208735							
1998	223208	220713								
1999	258721									

Paid Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	53345	93378	109723	124477	136250	144479	150445	153448	157898	160070
1991	57386	91261	107908	123718	135289	143442	150291	154320	158688	
1992	56720	87647	101284	115979	127967	135136	141522	145149		
1993	48704	78127	92073	104895	113236	120282	125630			
1994	50966	84838	103902	118267	133850	141842				
1995	78161	118181	146293	171203	187494					
1996	63526	103621	133062	148531						
1997	74185	122874	144416							
1998	83350	125455								
1999	101986									

Bulk & IBNR Reserves

	12	24	36	48	60	72	84	96	108	120
1990	50447	38781	28656	19859	14350	8591	7246	4671	3740	2777
1991	79523	48152	35577	24870	14345	9357	4787	3965	3348	
1992	86097	52461	37055	20904	13105	7997	7269	5731		
1993	72108	47185	32424	19468	11602	7097	5788			
1994	65905	38678	29713	17684	15606	13558				
1995	74134	52068	37983	23754	16595					
1996	85289	53101	35545	23456						
1997	87921	51496	31286							
1998	86114	52298								
1999	94552									

Reported Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	66494	120498	135370	145515	152801	156149	157933	159310	161649	163037
1991	99151	122424	132950	147638	152348	155234	158151	160434	162456	
1992	98878	118287	128962	139797	143648	144662	146875	148150		
1993	90402	105730	117646	123109	126292	128754	129767			
1994	96197	121183	133125	143745	147313	151980				
1995	141695	168336	189351	201715	203235					
1996	114800	145850	162042	171916						
1997	128971	165372	177449							
1998	137094	168415								
1999										

Schedule P Spcl_Liab
Spcl_Liab

	NET				DIRECT & ASSUMED			
	Earned Premium	Paid L&LAE	Un-paid L&LAE	Ultimate L&LAE	Earned Premium	Paid L&LAE	Un-paid L&LAE	Ultimate L&LAE
Prior			63				1014	
1990	11284	10350	0	10350	26660	24913	9	24922
1991	11963	5524	0	5524	22149	11497	14	11512
1992	7853	2584	0	2584	13061	4704	3	4707
1993	720	1565	0	1565	3242	5882	0	5882
1994	757	147	3	150	4349	2491	6	2497
1995	4485	4738	-115	4623	13337	10461	-106	10354
1996	5630	3795	669	4464	14176	8784	706	9489
1997	6195	3894	499	4393	20282	10134	715	10848
1998	4370	4085	1705	5790	13613	10164	3672	13836
1999	9225	3457	2239	5696	23560	11032	6513	17544
	62 482	40 139	5 063	45 139	154 429	100,062	12 546	111,591

Ultimate Incurred Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	9249	6967	8391	10317	10317	10208	10208	10206	10206	10206
1991	4970	5916	6086	6076	6048	6045	5257	5258	5258	
1992	2893	2580	2561	2551	2559	2462	2462	2462		
1993	497	491	471	475	1562	1558	1537			
1994	524	636	981	155	127	129				
1995	3479	3554	4730	4910	4601					
1996	3135	3605	3856	4411						
1997	3490	4226	4327							
1998	2423	5452								
1999	4464									

Paid Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	2624	5741	6665	10183	10202	10204	10204	10206	10206	10206
1991	1954	4073	5987	5984	6005	6044	5258	5258	5258	
1992	1563	2531	2551	2557	2557	2462	2462	2462		
1993	330	420	459	471	1558	1558	1537			
1994	323	505	638	89	113	126				
1995	619	2463	4077	4503	4715					
1996	1675	2571	3138	3743						
1997	1909	3115	3831							
1998	1160	3748								
1999	2278									

Bulk & IBNR Reserves

	12	24	36	48	60	72	84	96	108	120
1990	1501	783	720	110	110	2	2	0	0	0
1991	1150	650	35	46	13	0	0	0	0	
1992	509	7	5	1	1	0	0	0		
1993	27	19	3	3	3	0	0			
1994	60	53	65	34	-1	0				
1995	351	188	284	131	-56					
1996	240	445	239	442						
1997	764	361	197							
1998	242	960								
1999	381									

Reported Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	7748	6184	7671	10207	10207	10206	10206	10206	10206	10206
1991	3820	5266	6051	6030	6035	6045	5257	5258	5258	
1992	2384	2553	2556	2552	2558	2462	2462	2462		
1993	470	472	468	472	1559	1558	1537			
1994	464	583	916	121	128	129				
1995	3128	3366	4446	4779	4657					
1996	2895	3160	3617	3969						
1997	2726	3865	4130							
1998	2181	4492								
1999										

Schedule P OL_OCC
OL_OCC

	NET				DIRECT & ASSUMED			
	Earned Premium	Paid L&LAE	Un-paid L&LAE	Ultimate L&LAE	Earned Premium	Paid L&LAE	Un-paid L&LAE	Ultimate L&LAE
Prior			42889				47769	
1990	30235	12910	890	13800	64942	24651	4125	28777
1991	30355	13096	1008	14104	65030	22833	1576	24410
1992	23685	14413	1234	15647	55256	27196	2068	29264
1993	20320	10459	1354	11813	57590	18647	2149	20799
1994	22629	11501	2256	13757	62070	18106	3427	21530
1995	28984	15411	4280	19691	64549	22115	7151	29266
1996	22210	15208	5233	20441	65755	29267	8690	37959
1997	35295	11500	12567	24067	67685	16897	23343	40241
1998	37183	5248	17584	22831	74693	8167	32855	41021
1999	38718	3103	27581	30684	74071	4343	54224	58567
	289 614	112,849	116,856	186,835	651,641	192 222	187 377	331,834

Ultimate Incurred Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	54414	13721	15324	14387	13934	13289	11633	11497	11661	12173
1991	18846	17371	17329	14299	14560	12644	12087	12994	12440	
1992	16236	18092	16455	15620	14589	14887	13996	13995		
1993	18362	16030	14797	11981	10731	9585	10317			
1994	16047	16153	15484	11961	11872	11984				
1995	23491	19258	19169	17681	17805					
1996	22985	21344	20733	17923						
1997	22349	20149	21320							
1998	22523	20149								
1999	28440									

Paid Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	938	2626	5943	7171	8243	9797	10020	10406	10658	11330
1991	1462	3249	4989	7365	8004	9353	10018	10659	11492	
1992	1528	3883	6372	9346	11397	11712	12523	12823		
1993	756	2063	5261	6316	7432	7983	9047			
1994	1129	3065	5928	7361	8538	9863				
1995	3130	6029	9157	12243	13839					
1996	4210	7347	9915	13081						
1997	1513	4327	9360							
1998	1100	3491								
1999	1756									

Bulk & IBNR Reserves

	12	24	36	48	60	72	84	96	108	120
1990	9707	8192	6815	4963	3774	2377	1093	784	659	611
1991	8755	9461	7595	3627	2995	1394	880	862	698	
1992	11070	9521	5796	3642	1739	1054	1008	959		
1993	12767	8372	5297	3043	1543	1237	1058			
1994	10305	7769	5833	1946	1550	1571				
1995	14502	8334	4915	3284	2070					
1996	13492	8957	5839	3570						
1997	14081	9861	5633							
1998	14531	9152								
1999	21369									

Reported Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	44707	5529	8509	9424	10160	10912	10540	10713	11002	11562
1991	10091	7910	9734	10672	11565	11250	11207	12132	11742	
1992	5166	8571	10659	11978	12850	13833	12988	13036		
1993	5595	7658	9500	8938	9188	8348	9259			
1994	5742	8384	9651	10015	10322	10413				
1995	8989	10924	14274	14397	15735					
1996	9493	12387	14894	14353						
1997	8268	10288	15687							
1998	7992	10997								
1999										

Schedule P Reins_A
Reins_A

	NET				DIRECT & ASSUMED			
	Earned Premium	Paid L&LAE	Un-paid L&LAE	Ultimate L&LAE	Earned Premium	Paid L&LAE	Un-paid L&LAE	Ultimate L&LAE
Prior			4134				7323	
1990	1204	1874	6	1880	2540	3568	54	3622
1991	0	15	-138	-123	2209	886	-221	665
1992	0	160	-2	158	919	2049	-3	2046
1993	0	184	3	187	655	838	10	849
1994	0	917	-113	804	744	2091	-12	2079
1995	6715	4654	679	5333	9096	7280	794	8074
1996	9784	5269	805	6074	13415	9261	1211	10473
1997	14779	7600	2618	10218	20089	12082	3133	15215
1998	18946	5087	3529	8618	24984	6732	4399	11131
1999	10243	6825	-711	6114	19020	33825	4834	38659
	61 671	32 585	10 810	39 281	93 651	78 612	21,522	92 813

Ultimate Incurred Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	17165	1230	3126	2552	2744	2009	1862	1946	1969	1870
1991	0	0	0	0	0	0	14	15	-124	
1992	0	0	0	0	0	138	140	132		
1993	0	0	0	0	354	422	183			
1994	0	176	576	1081	1006	795				
1995	3657	4716	5568	5447	5314					
1996	4101	6042	5956	6006						
1997	9551	9014	10188							
1998	8243	8599								
1999	5870									

Paid Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	960	2037	2269	2168	1753	1806	1808	1865	1865	1865
1991	0	0	0	0	0	0	14	14	14	
1992	0	0	0	0	0	134	134	134		
1993	0	0	0	0	123	177	180			
1994	0	2	203	745	844	909				
1995	901	2653	4083	4394	4635					
1996	1082	3954	5061	5203						
1997	4877	6217	7573							
1998	3357	5072								
1999	6629									

Bulk & IBNR Reserves

	12	24	36	48	60	72	84	96	108	120
1990	478	567	467	460	729	73	35	71	93	38
1991	0	0	0	0	0	0	0	1	-2	
1992	0	0	0	0	0	-1	2	-2		
1993	0	0	0	0	13	85	-11			
1994	0	174	167	10	28	-68				
1995	1358	914	584	330	433					
1996	1296	839	608	389						
1997	2453	1143	988							
1998	2804	1509								
1999	-221									

Reported Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	16687	663	2659	2092	2015	1936	1827	1875	1876	1832
1991	0	0	0	0	0	0	14	14	-122	
1992	0	0	0	0	0	137	138	134		
1993	0	0	0	0	341	337	194			
1994	0	2	409	1071	978	863				
1995	2299	3802	4984	5117	4881					
1996	2805	5203	5348	5617						
1997	7098	7871	9200							
1998	5439	7090								
1999										

Schedule P Reins_B
Reins_B

	NET				DIRECT & ASSUMED			
	Earned Premium	Paid L&LAE	Un-paid L&LAE	Ultimate L&LAE	Earned Premium	Paid L&LAE	Un-paid L&LAE	Ultimate L&LAE
Prior			0				0	
1990	0	0	0	0	0	0	0	0
1991	0	0	0	0	0	0	0	0
1992	0	0	0	0	0	0	0	0
1993	0	0	0	0	0	0	0	0
1994	0	765	127	892	0	765	127	892
1995	4277	4426	354	4780	4595	4823	365	5188
1996	5545	4650	660	5310	6582	5025	687	5712
1997	7556	8832	482	9314	8677	9585	525	10110
1998	8407	3191	1401	4592	10177	3481	1610	5091
1999	981	101	345	446	1555	301	458	759
	26 766	21 965	3 369	25,334	31,586	23 980	3,772	27 752

Ultimate Incurred Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	0	0	0	0	0	0	0	0	0	0
1991	0	0	0	0	0	0	0	0	0	
1992	0	0	0	0	0	0	0	0		
1993	0	0	0	0	0	0	0			
1994	166	475	869	988	899	892				
1995	4383	4507	4355	4412	4780					
1996	4417	4966	4958	5310						
1997	8600	9239	9314							
1998	4637	4592								
1999	446									

Paid Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	0	0	0	0	0	0	0	0	0	0
1991	0	0	0	0	0	0	0	0	0	
1992	0	0	0	0	0	0	0	0		
1993	0	0	0	0	0	0	0			
1994	136	389	481	706	737	765				
1995	2307	3534	4073	4164	4426					
1996	1872	3809	4479	4650						
1997	4473	8006	8832							
1998	1676	3191								
1999	101									

Bulk & IBNR Reserves

	12	24	36	48	60	72	84	96	108	120
1990	0	0	0	0	0	0	0	0	0	0
1991	0	0	0	0	0	0	0	0	0	
1992	0	0	0	0	0	0	0	0		
1993	0	0	0	0	0	0	0			
1994	0	13	31	70	73	72				
1995	786	525	35	3	228					
1996	1020	286	153	310						
1997	1429	348	-110							
1998	1492	551								
1999	3									

Reported Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	0	0	0	0	0	0	0	0	0	0
1991	0	0	0	0	0	0	0	0	0	
1992	0	0	0	0	0	0	0	0		
1993	0	0	0	0	0	0	0			
1994	166	462	838	918	826	820				
1995	3597	3982	4320	4409	4552					
1996	3397	4680	4805	5000						
1997	7171	8891	9424							
1998	3145	4041								
1999										

	NET				DIRECT & ASSUMED			
	Earned Premium	Paid L&LAE	Un-paid L&LAE	Ultimate L&LAE	Earned Premium	Paid L&LAE	Un-paid L&LAE	Ultimate L&LAE
Prior			0				0	
1990	0	0	0	0	0	0	0	0
1991	0	0	0	0	0	0	0	0
1992	0	0	0	0	0	0	0	0
1993	0	0	0	0	0	0	0	0
1994	0	1	0	1	0	1	0	1
1995	219	336	240	576	219	336	240	576
1996	171	0	0	0	171	0	0	0
1997	70	0	0	0	73	0	0	0
1998	66	0	0	0	70	0	0	0
1999	7	0	0	0	7	0	0	0
	533	337	240	577	540	337	240	577

Ultimate Incurred Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	0	0	0	0	0	0	0	0	0	0
1991	0	0	0	0	0	0	0	0	0	
1992	0	0	0	0	0	0	0	0		
1993	0	0	0	0	0	0	0			
1994	0	1	1	1	1	1				
1995	165	306	446	886	576					
1996	0	1	0	0						
1997	4	0	0							
1998	4	0								
1999	0									

Paid Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	0	0	0	0	0	0	0	0	0	0
1991	0	0	0	0	0	0	0	0	0	
1992	0	0	0	0	0	0	0	0		
1993	0	0	0	0	0	0	0			
1994	0	1	1	1	1	1				
1995	60	208	269	305	336					
1996	0	0	0	0						
1997	0	0	0							
1998	0	0								
1999	0									

Bulk & IBNR Reserves

	12	24	36	48	60	72	84	96	108	120
1990	0	0	0	0	0	0	0	0	0	0
1991	0	0	0	0	0	0	0	0	0	
1992	0	0	0	0	0	0	0	0		
1993	0	0	0	0	0	0	0			
1994	0	0	0	0	0	0				
1995	84	39	61	489	157					
1996	0	1	0	0						
1997	4	0	0							
1998	4	0								
1999	0									

Reported Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	0	0	0	0	0	0	0	0	0	0
1991	0	0	0	0	0	0	0	0	0	
1992	0	0	0	0	0	0	0	0		
1993	0	0	0	0	0	0	0			
1994	0	1	1	1	1	1				
1995	81	267	385	397	419					
1996	0	0	0	0						
1997	0	0	0							
1998	0	0								
1999										

Schedule P: PL_OCC
PL_OCC

	NET				DIRECT & ASSUMED			
	Earned Premium	Paid L&LAE	Un-paid L&LAE	Ultimate L&LAE	Earned Premium	Paid L&LAE	Un-paid L&LAE	Ultimate L&LAE
Prior			72				77	
1990	496	363	38	401	620	528	38	566
1991	364	37	5	42	455	25	5	30
1992	282	7	0	8	352	-28	0	-28
1993	228	28	63	91	285	28	116	144
1994	196	12	0	12	249	-2	0	-2
1995	176	73	0	73	224	56	7	62
1996	129	2	51	53	218	-26	82	57
1997	140	43	267	310	204	8	405	414
1998	891	1086	249	1335	1095	1050	399	1449
1999	1485	11	281	292	1810	20	457	477
	4,387	1,662	1,026	2,617	5,512	1,659	1,586	3,169

Ultimate Incurred Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	437	52	191	146	147	176	183	181	179	396
1991	0	111	102	72	65	31	33	33	42	
1992	150	104	31	31	11	1	1	9		
1993	174	59	51	45	36	25	88			
1994	132	48	38	28	8	11				
1995	52	61	97	189	67					
1996	51	42	182	53						
1997	63	197	307							
1998	1942	1303								
1999	289									

Paid Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	7	-2	17	25	25	176	178	179	179	358
1991	-16	10	27	27	30	31	33	33	37	
1992	1	1	1	1	1	1	1	8		
1993	14	19	21	25	25	25	25			
1994	3	8	8	8	8	11				
1995	2	11	67	67	67					
1996	1	2	2	2						
1997	0	7	40							
1998	1036	1054								
1999	11									

Bulk & IBNR Reserves

	12	24	36	48	60	72	84	96	108	120
1990	8	15	35	0	10	0	0	0	0	0
1991	0	75	50	20	10	0	0	0	0	
1992	126	100	30	20	10	0	0	0		
1993	150	40	30	20	11	0	0			
1994	51	40	30	20	0	0				
1995	50	40	31	121	0					
1996	50	40	180	51						
1997	51	191	204							
1998	200	204								
1999	203									

Reported Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	431	37	156	146	137	176	183	181	179	396
1991	0	36	52	52	55	31	33	33	42	
1992	24	4	1	11	1	1	1	9		
1993	24	19	21	25	25	25	88			
1994	81	8	8	8	8	11				
1995	2	21	66	68	67					
1996	1	2	2	2						
1997	12	6	103							
1998	1742	1099								
1999										

Schedule P: Property - Short-Tail Lines
Property - Short-Tail Lines

	NET				DIRECT & ASSUMED			
	Earned Premium	Paid L&LAE	Un-paid L&LAE	Ultimate L&LAE	Earned Premium	Paid L&LAE	Un-paid L&LAE	Ultimate L&LAE
Prior			685				875	
1990	441540	259892	17	259910	604612	367757	35	367789
1991	467598	258097	-10	258087	619665	336678	-31	336638
1992	464489	268491	40	268527	602137	339094	-91	339000
1993	493330	273113	-70	273042	625868	339221	-124	339098
1994	499335	282947	109	283058	638010	349351	-28	349323
1995	522913	351893	1622	353514	689562	453649	2154	455801
1996	559841	394629	6106	400733	759148	532730	11646	544379
1997	569171	419227	2716	421943	732060	515663	3805	519466
1998	620257	441013	639	441654	782489	539058	4647	543701
1999	660625	413621	56984	470604	843802	521850	67322	589176
	5,299,099	3,382,923	68,838	3,431,072	6,897,353	4 295,051	90 210	4,384 367

Ultimate Incurred Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	18169	240747	239027	237908	242273	241818	242368	241642	241644	241096
1991	233807	234494	232073	233229	233050	232836	233043	233087	232770	
1992	242045	245220	247086	247574	246824	246124	246220	246014		
1993	259685	249437	250298	249862	248883	248393	248145			
1994	252996	259341	259047	257554	257193	254311				
1995	323230	324138	329456	321243	324017					
1996	378412	369311	376748	370764						
1997	386898	393232	391454							
1998	408883	410052								
1999	428379									

Paid Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	196518	238594	237688	237998	241185	241001	241516	241617	241572	241079
1991	201123	230835	230225	232118	232153	232418	232934	232997	232779	
1992	205937	240834	244885	245702	246130	246027	246027	245983		
1993	218200	246864	248963	248651	248666	248326	248209			
1994	231858	256951	256956	256716	256646	254211				
1995	282095	313769	318783	321166	322444					
1996	317852	363851	385979	364712						
1997	348748	390248	388949							
1998	368408	409927								
1999	375249									

Bulk & IBNR Reserves

	12	24	36	48	60	72	84	96	108	120
1990	27233	-161	38	-1072	238	40	90	-5	21	0
1991	13980	459	929	201	273	148	11	43	-38	
1992	16066	1699	994	245	421	23	35	-59		
1993	22684	589	631	738	66	31	-56			
1994	13211	260	694	-46	232	15				
1995	14715	2184	514	905	554					
1996	20245	-383	588	549						
1997	14794	-357	-187							
1998	18536	-2388								
1999	28993									

Reported Loss & ALAE

	12	24	36	48	60	72	84	96	108	120
1990	-9064	240908	238991	238980	242037	241778	242276	241647	241623	241096
1991	219827	234035	231144	233028	232777	232688	233032	233044	232806	
1992	225979	243521	246092	247329	246403	246101	246185	246073		
1993	237001	248848	249667	249124	248817	248382	248201			
1994	249785	259081	258353	257600	256961	254298				
1995	308515	321954	328942	320338	323463					
1996	358167	369694	376160	370215						
1997	372104	393589	391641							
1998	390347	412440								
1999										

Preliminary Due Diligence of DFA Insurance Company

Raju Bohra, ACAS and Thomas E. Weist

Preliminary Due Diligence of DFA Insurance Company

Raju Bohra, ACAS
American Re-Insurance Company

Thomas Weist
American Re-Insurance Company

Abstract

This paper is a DFA case study of a hypothetical insurance company, DFAIC The study was completed using American Re-Insurance's proprietary DFA model The company data used was provided in the Call Paper request The study evaluated capital adequacy, capital allocation, and underwriting performance issues Also, strategies regarding asset allocation and reinsurance structures were tested

In keeping with the case study format of the call request, the paper was written as a presentation to management with a cover letter and a technical appendix This format illustrates how recommendations from a DFA analysis can be effectively presented The presentation highlights the importance of understanding management's success criteria and quantifying management's measure and tolerance of risk

The technical document discusses how limited data can be used to parameterize a DFA model and what additional data would be needed to expand the analysis

Acknowledgements

This study and paper could not have been completed without the guidance of Donald Mango, the assistance of Timothy Lu, and the support of American Re-Insurance's DFA model development team

MEGAGroup Insurance *Memorandum*

To: Mr. Joseph Merger, CEO
From: Raju Bohra, Thomas Weist
Re: Analysis of DFAIC Acquisition

We have completed our preliminary due diligence of MEGAGroup's potential acquisition of DFA Insurance Company (DFAIC) Attached please find a presentation of the results along with a technical document detailing our methodology

This study was performed using the Dynamic Financial Analysis (DFA) model licensed by MEGAGroup from American Re-Insurance The model comprehensively reflects variability in both capital market conditions and liability results This study represents the type of dividends our company can expect from its investment in DFA modeling

KEY RESULTS

- DFAIC's capitalization exceeds levels generally required for solvency However, solvency analysis only reflects extreme ruin probabilities From an investor's perspective, the company has a material level of potential capital loss
- The reinsurance structure should include an accident-year stop loss (AYSL) cover with limits of 10% excess of a 70% LALAE ratio This would provide valuable protection to the company's net results
- The asset allocation should be changed to increase the company's level of equity holdings to 22% It is currently 12% on a market basis. An increase in equity exposure would provide a favorable risk/reward tradeoff
- Return distributions for each of DFAIC's lines of business have been calculated based on our allocation of capital. On a risk adjusted return basis, both the PPA and CA liability lines are performing worse than the company average.

The purpose of this study is to provide quantitative support to management The study was completed given the data provided When additional information is available a more comprehensive study can be performed The types of additional data that would be useful are listed in the technical document More importantly, however, a greater understanding of management's objectives and risk tolerances would greatly facilitate future modeling and result analysis This issue is illustrated in the presentation

We feel this study provides a good starting point for discussion A basic sense of how the answers vary in response to changing assumptions and risk tolerances can be seen from the various risk return charts included in this study

Analysis of DFA Insurance Company (DFAIC)

Using Dynamic Financial Analysis

Introduction

- MEGAGroup's Strategic Analysis Department evaluated the potential acquisition of DFA Insurance Company
- The analysis was performed using Dynamic Financial Modeling techniques.
- Data for the study was basically limited to publicly available information
- The study can be refined with additional data
- A technical document detailing the methodology used for the study is attached

Scope of Study

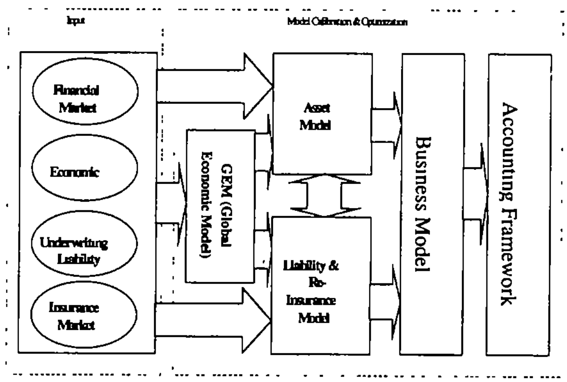
- Questions:
  - Is the company adequately capitalized?
  - How efficient is the reinsurance structure?
  - How efficient is the asset allocation?
  - How should capital be allocated to lines?
  - What is the return distribution for each line?

DFA Model Used

- A simulation model, licensed from American Re-Insurance Company, reflecting variability in economic, capital market, and liability conditions

- The model includes the following modules
  - Economic module to generate future states of economic variables and capital market conditions
  - Asset module to price current asset portfolio and implement target investment strategy
  - Liability module to project loss, expense and premium results
  - Reinsurance module to model the impact of all reinsurance terms
  - Accounting module to bring together all balances, cash flows and accruals into an accounting framework and reflect taxes

DFA Model Structure



Assumptions

- Due to limited data, certain assumptions were made
  - Projected loss ratios were based on historical averages
  - Stated reserves were used, assuming no deficiency or redundancy However, variability was introduced
  - Premium growth and loss trends were modeled to be flat
  - Base target asset allocation was set to the current allocation
- Other assumptions regarding parameterization of the model are discussed in the technical document
- These assumptions do not materially impact the study conclusions However, additional data would allow expansion of the scope and detail of the study

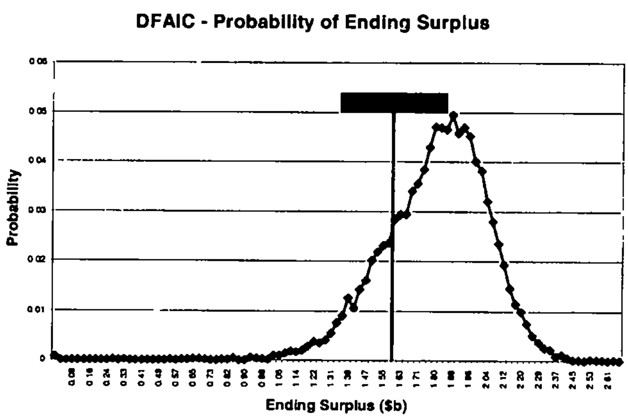
Capital Adequacy

Capital Adequacy

- Capital adequacy is a measure of a company's ability to pay all potential obligations.
- Surplus is exposed to the following risks.
  - Asset risk - default or drop in the value of assets
  - Interest rate risk - drop in asset values due to interest rate change
  - Credit risk - default on recoverable from agents and reinsurers
  - Reserve risk - reserves develop adversely
  - Premium risk - current business losses worse than plan

Capital Adequacy

- Rating agencies use a schedule of risk charges to compute capital requirements, then rank companies based on relative capital strength
- Using DFA, a complete probability graph of a company's ending surplus can be calculated. Using this graph, more detailed capital adequacy measures can be developed
- A probability graph of DFAIC one-year ending surplus is displayed on the following slide



Capital Adequacy

- As the graph shows, there is a very small probability DFAIC's surplus will be insufficient. However, the probability of surplus decline is significant
- Measures used to quantify probability of insolvency are
  - Probability of Ruin probability that surplus will be exhausted
  - Expected Policyholder Deficit quantifies degree to which surplus may be exhausted relative to expected loss
    Above measures are analogous to rating agency calculations
- Measure used to quantify probability of surplus decline:
  - Expected Default Loss Rate quantifies degree to which surplus may be reduced relative to initial capital
    Analogous to bond default rates in the capital markets

Capital Adequacy

DFAIC Capital Safety Levels

Risk Measure	**Level Implied by Current Capital**
Probability of Ruin	1 in 10,000 years or 0.01%
Expected Policyholder Deficit	0.5% EPD
Expected Default Loss Rate	2.66% EDLR

Capital Adequacy

- This study indicates that the company is very well capitalized. However, further data concerning extreme loss events are needed before the results of this study can be considered definitive
- The Probability of Ruin and EPD measures calculated are well within the thresholds rating agencies generally associate with highly capitalized insurers
- The results of this study would provide statistical support for raising the company's rating or allowing a release of capital during rating agency discussions

Capital Adequacy

- From an investor viewpoint, probability of surplus decline is as important as insolvency risk
- The Expected Default Loss Rate (EDLR) calculated for the company is 2.66%
- *Moody's Investor Service* categorizes bonds with a one-year default rate of 2.66% as speculative ("junk") grade.
- To reduce the company's EDLR to 1% ("investment") an additional $200mil of capital would be needed
- Alternatively, additional reinsurance, particularly a stop-loss cover, would reduce the EDLR to 1% This strategy is discussed in the following section

Reinsurance Structure

Reinsurance Structure

- Reinsurance analysis is based on examining the risk return trade-off of various alternative structures.
- Rationally priced reinsurance provides a reduction of risk at the cost of expected return (margin). The alternatives in this study were priced using internal pricing models.
- The key issue is the risk tolerance of the buyer:
  - Less reinsurance generally increases expected return.
  - More reinsurance will reduce risk for risk-averse buyers.
  - Need to determine the point where the trade-off is favorable.
- It is important to define a risk measure in alignment with the buyer's risk tolerance.

Reinsurance Structures

- For this study, four risk measures were analyzed for the current structure and the alternative structures:
  - *Standard deviation of ending surplus*: (Analytical) This measure is used in classical investment portfolio analysis. However, it is not a good measure of downside risk which is the focus of reinsurance.
  - *Probability of surplus decline > 25%*: (Regulatory) Such a decline would probably trigger regulatory action.
  - *Probability of surplus decline > 10%*: (Rating agency) Such a decline would probably trigger a rating downgrade.
  - *Probability of surplus decline > 0%*: (Investor) Such a decline would be analogous to a loss of principal on an investment.

Reinsurance Structure

- Accident year stop loss (AYSL) covers were tested since they address the following issues facing DFAIC:
  - Volatility in net results
  - Sizeable catastrophe net PML after current reinsurance
  - Frequency of small retained weather losses
  - Significant probability of surplus decline
- Four reinsurance structures were tested:
  - Current program
  - Three AYSL: 10%x70%, 20%x70%, 30%x70% (Loss and ALAE)
- Insufficient data was available to test alternatives to the company's excess of loss and catastrophe covers.

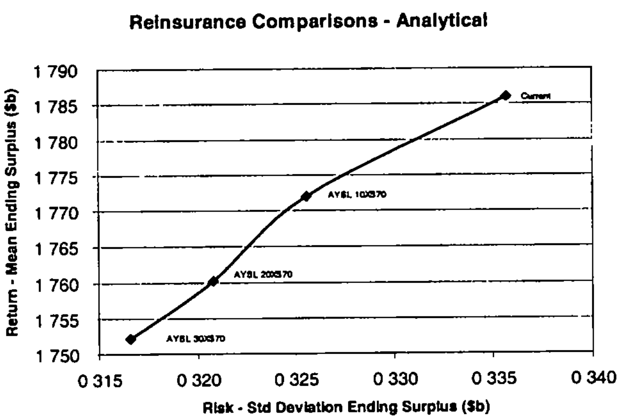
Reinsurance Structure

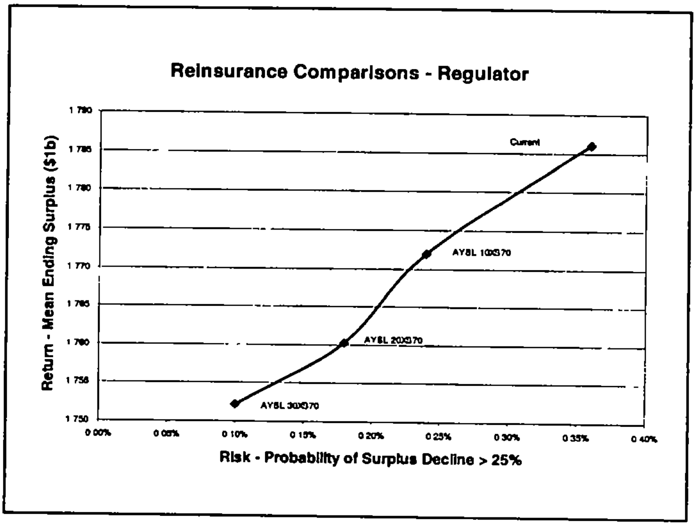
- The following charts show the risk return trade-off for the four structures under the various risk tolerance levels

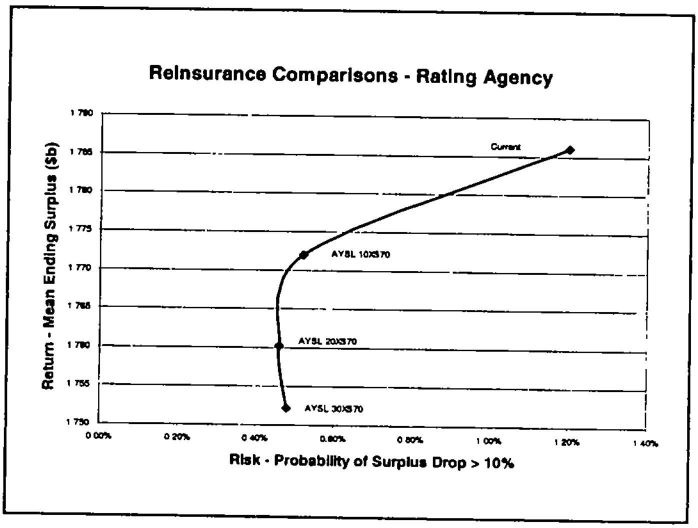
- Also following is a table of ratios that quantify the risk return trade-off. The ratio is defined as:

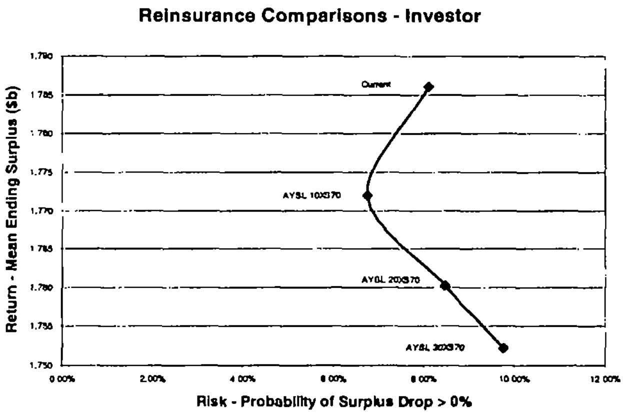
  (Pct change in return measure)
  (Pct change in risk measure)

- A ratio below 100% is favorable as relatively more risk protection is being afforded than expected margin charged. Negative ratios are unfavorable as risk is increasing.




Reinsurance Comparisons - Regulator
Return - Mean Ending Surplus ($1b)
Risk - Probability of Surplus Decline > 25%
Current


Reinsurance Comparisons - Rating Agency
Return - Mean Ending Surplus ($b)
Risk - Probability of Surplus Drop > 10%
Current
AYSL 10XS70
AYSL 20XS70
AYSL 30XS70



Reinsurance Structure

	Model Results					Trade-off Ratios			
Reinsurance Alternatives	Avg. Surplus	St.Dev. Surplus	Drop >25%	Drop >10%	Drop >0%	Analytic Level	Regulate Level	Rating Level	Invest Level
Current Structure	$1.786b	$0.336b	0.36%	1.20%	8.12%	Base	Base	Base	Base
AYSL 10x70	$1 772b	$0.326b	0 24%	0.52%	6.76%	26.1%	2.4%	1.4%	4.7%
AYSL 20x70	$1.760b	$0 321b	0 18%	0 46%	8.48%	45 6%	2 6%	5.7%	(2.6)
AYSL 30x70	$1.752b	$0 317	0.10%	0 48%	9.76%	34.8%	1.0%	(10.5)	(3 0)

Reinsurance Structures

- As the exhibits show, the "best" reinsurance structure depends on the selected risk tolerance.
- At lower risk tolerances (e.g. Analytical, Regulatory, and Rating) buying additional reinsurance is almost always favorable since the focus is on extreme events.
- At an Investor level, the focus is on protecting against surplus decline. Here the high costs of excessive reinsurance will be evident.
- The study indicates that purchasing AYSL coverage up to 10% would make sense, but greater coverage is too costly except for a very low risk tolerance level.

Asset Allocation

Asset Allocation

- The DFA model was used to evaluate varying the asset allocation using a risk return framework.
- For insurers, diversification possibilities exist if movements in capital market prices are assumed uncorrelated with changes in liability results.
- Six strategies were evaluated:
  - Current asset allocation, stock holding equal 12%
  - Stock holdings of 17%, 22%, 27%, 32%, and 37%.

Asset Allocation

- Each strategy was run through the model over five years.
- The return measure used was five year ending surplus.
- The risk measure used was standard deviation of surplus in keeping with classical investment portfolio analysis.
- An allocation is efficient if its return cannot be increased without increasing risk.

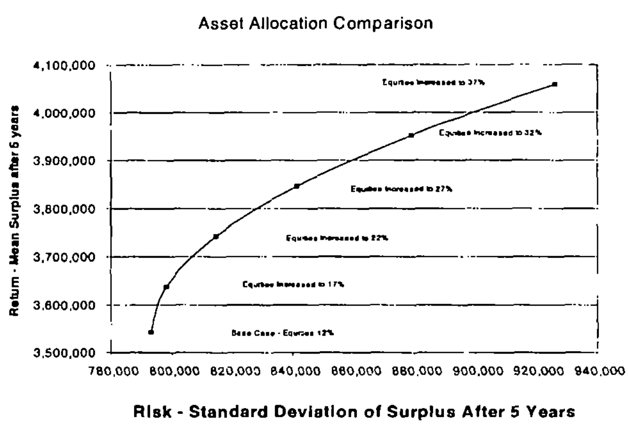
Asset Allocation

- The following chart shows the risk-return trade off for the six asset allocations.

- Like the reinsurance analysis, trade-off ratios were calculated . The ratio is defined as:

  (Pct change in return measure)
  (Pct change in risk measure)

- For this comparison, ratios *above* 100% are favorable as relatively more expected return is being afforded than risk . Negative ratios reflect inefficient portfolios.



Asset Allocation

Allocation Strategy	Model Results: Average Surplus	Model Results: Std Dev Surplus	Trade-off Ratio
Current 12%	$3.542b	$0.793b	Base
Equities 17%	$3.638b	$0.798b	403%
Equities 22%	$3.741b	$0.814b	140%
Equities 27%	$3.846b	$0.841b	85%
Equities 32%	$3.952b	$0.879b	61%
Equities 37%	$4.059b	$0.926b	50%

Asset Allocation

- As the exhibits show, the current allocation is efficient. However, increasing the allocation of equities to 22% provides a favorable risk return trade-off.

- Other aspects of investment strategy that can be evaluated in this manner include:
  - Duration of the fixed income portfolio
  - Average credit risk of asset portfolio
  - Mix of taxable and non-taxable holdings
  - Impact introducing securities with callability risks

Capital Allocation & LOB Return Distributions

Capital Allocation

- Profitability across line of business can be measured by risk adjusted ROE.
- The capital allocation to lines is based on the relative contribution of each line to the company's overall risk.
- ROE by line is then calculated using the formula:

$$\left(\text{Net Income} \big/ \text{Allocated Capital} \right)$$

- Each line can now be evaluated on a common basis.

Capital Allocation

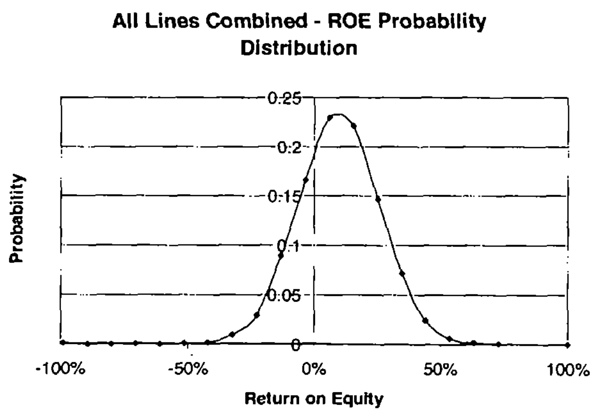
- However, the variability of a line depends on the order in which lines are analyzed. Often the diversification benefit of new business is not distributed to existing business.
- Also, most by-line risk measures usually do not add up to the total company risk measure.
- This often leads to an allocation of surplus which is not conducive to the stability of the group.
- In this study, Game Theory techniques were used to alleviate these problems, yielding an allocation that is order-independent, additive, and stable.

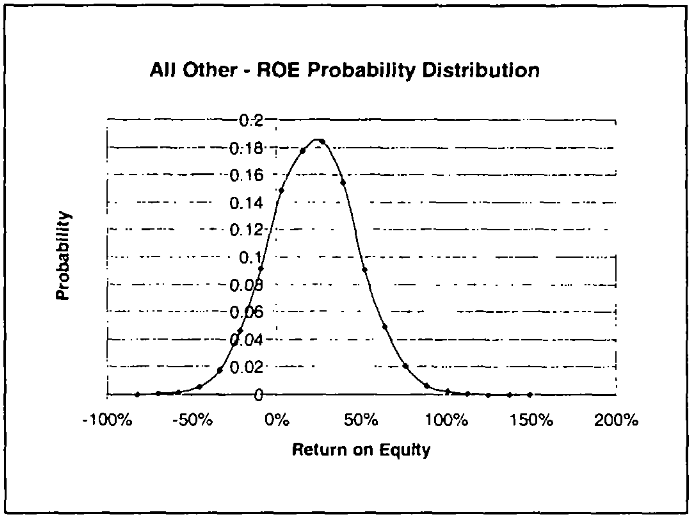
Capital Allocation

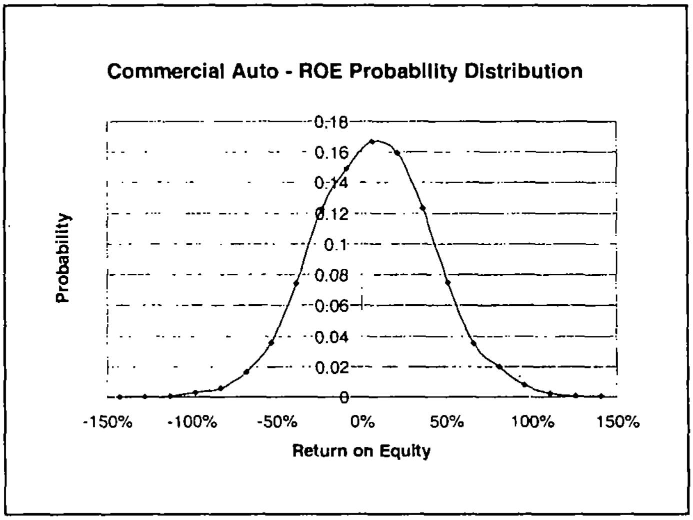
- The next exhibit displays the ROE results for each line.
- Following the chart are graphs displaying the distribution of ROE for each line.
- Based on the model results, both PPA and CA Liability are performing worse than the company average.
- Homeowners has the greatest variability of results due to catastrophe exposure.

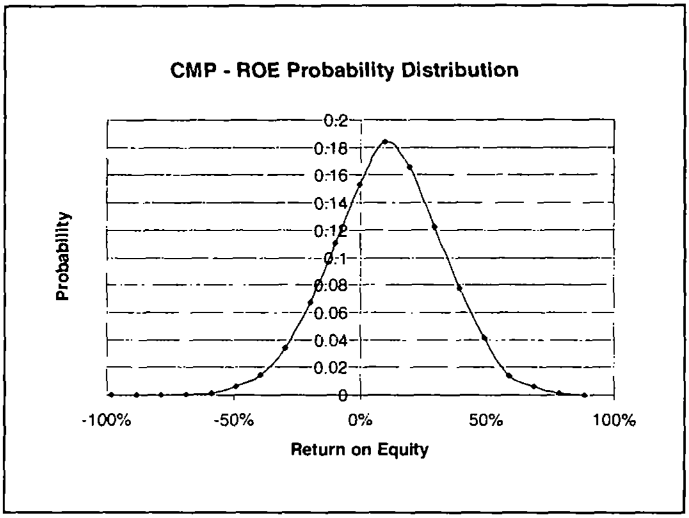
Capital Allocation and ROE

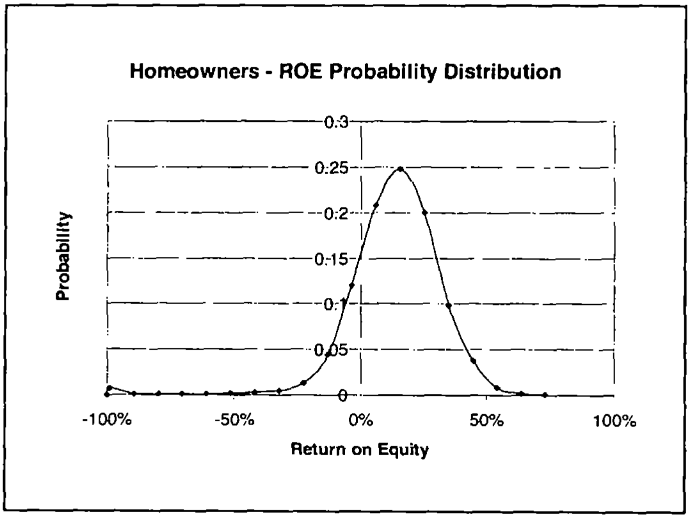
Line of Business	Net Earned Premium	Average Net Income	Allocated Capital	Average ROE	Std Dev. ROE
All-Other	$60.9m	$6.0m	$38.2m	15.7%	25.8%
CA Liab	$162.5m	($0.3m)	$71.3m	(0.5%)	34.9%
CMP	$335.0m	$12.8m	$223.4m	5.7%	22.0%
Home	$344.0m	$9 1m	$216.5m	4.2%	163.6%
PPA Liab	$602.1m	($23.2m)	$386.7m	(6 0%)	20.5%
Short Property	$659.0m	$56 9m	$485.1m	11.7%	13 2%
Workers Comp.	$208.7	$8.2m	$182.9m	4.5%	19.8%
ALL LINES	$2,372.1m	$69.5m	$1,604.1m	4.3%	27.2%

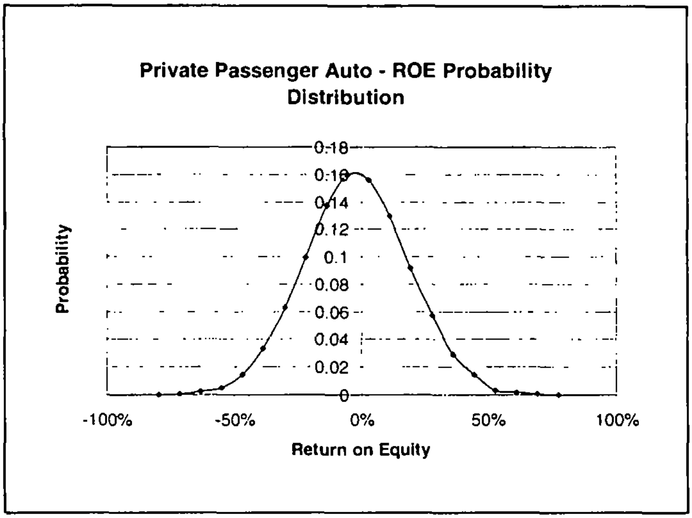



All Other - ROE Probability Distribution
Probability
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-100%
-50%
0%
50%
100%
150%
200%
Return on Equity

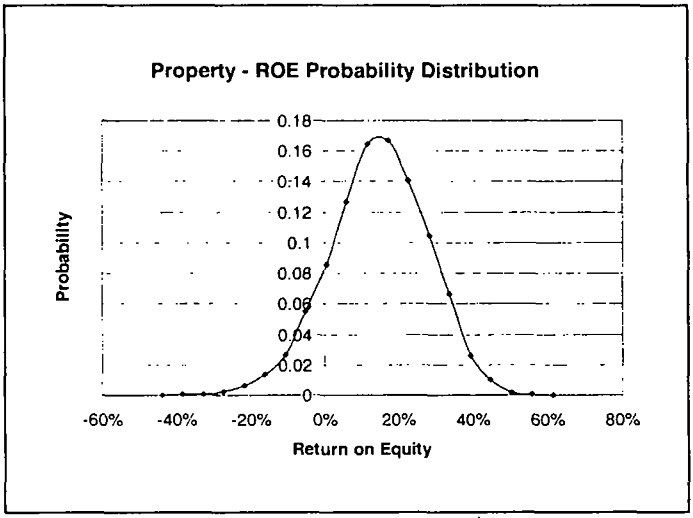

Commercial Auto - ROE Probability Distribution
Probability
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-150%
-100%
-50%
0%
50%
100%
150%
Return on Equity


CMP - ROE Probability Distribution
Probability
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-100%
-50%
0%
50%
100%
Return on Equity

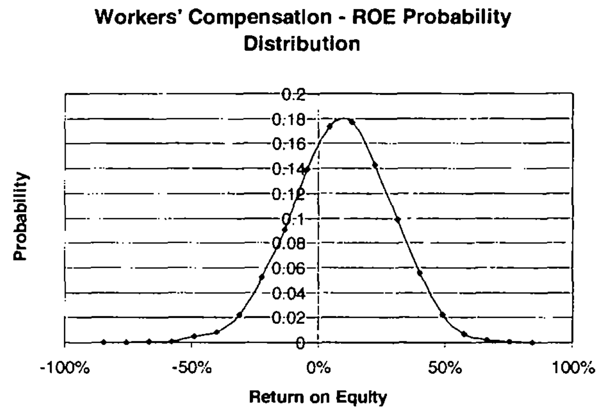

Homeowners - ROE Probability Distribution
Probability
0.3
0.25
0.2
0.15
0.1
0.05
0
-100%
-50%
0%
50%
100%
Return on Equity



Private Passenger Auto - ROE Probability Distribution
Probability
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-100%
-50%
0%
50%
100%
Return on Equity



Property - ROE Probability Distribution
Probability
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-60%
-40%
-20%
0%
20%
40%
60%
80%
Return on Equity



Next Steps

- Gain better understanding of management's return criteria and risk tolerance.
- Gather additional data to expand scope and detail of study.
- Calculate a probability distribution of the economic value of DFAIC to evaluate the proposed purchase price.

ANALYSIS OF DFA Insurance Company
Technical Document

SCOPE

We dynamically modeled the entire asset and liability structure of the company. Asset/Liability integration occurs through the use of links to a common economic model.

The starting point for the analysis was the year-end balance sheet, and other financial statements including Schedule D and P.

ECONOMIC MODELING

The economic scenario generator models relationships among economic variables with stochastic difference equations. The equations were calibrated using historical data. The economic model is multi-period and captures risks both within and across time[1].

User inputs specify the current economic environment and expectations for long-term median trends, i.e. mean reversion parameters. The model then generates plausible time series outcomes for each variable for future economies using simulation.

The following are the environmental variables of the economic model. We have also noted the data sources used to parameterize the initial state of the model.

1. Money Supply Growth
The M2 Growth statistic is taken from the Ibbotson database as of 10/1999. The data in the database is collected monthly. Instead of annualizing the 10/1999 value, we calculated the annual M2 growth over the latest 12 months.

2. Monetary Velocity Growth
V2 growth is calculated from M2 Growth and GDP Growth. The formula is

$$\boxed{V2\_Growth = (1 + M2\_Growth)\cdot(1 + GDP\_Growth) - 1}$$

[1] Berger, A., and Madsen, C., "A Comprehensive System for Selecting and Evaluating DFA Model Parameters," *CAS Forum*, Summer 1999, Dynamic Financial Analysis Call Paper Program, p. 51.

3. GDP Growth (Real)
GDP growth is modeled as real GDP Growth. This is the number customarily quoted in the financial press. Usually GDP is indexed to a base year to adjust for inflation.

For our model we used the GDP growth statistic released by the Commerce Department as of the 1st quarter, 2000. We used the trailing 4 quarters of GDP growth rather than the latest annualized growth figure.

4. Inflation
The Consumer Prince Index was selected as a measure of inflation in the economy. We used the figures available is as of May 2000. We calculated the inflation rate over the previous 12-months rather than annualizing the latest monthly data.

5. S&P 500 Earnings Growth
The *S&P 500* web site posts several statistics and estimates for the S&P 500 companies. The projected growth rate for the group is 19% next year.

6. S&P 500 Earnings Yield
Earnings Yield is calculated as 1 / Price-to-Earnings Ratio, which is a statistic readily available on the S&P web site. Price over the trailing 12-month earnings is the customary way of calculating the P/E ratio. Per S&P, this P/E ratio is 27.87.

7. S&P 500 Dividends Yield
This statistics also came from the S&P web site.

8. Interest Rate and Yield Curve
The interest rate and yield curve was based on the on the US Treasury Yield Curve.

Initial Economic Parameters:

Economic Variable	**Current**	**Mean Reversion**		**Interest Rates**	**Current**	**Mean Reversion**
				3 months	0.0582	0.0488
M2 Growth	0.060	0.060		6 months	0.0621	
V2 Growth	-0.015	-0.015		1 year	0.0610	
GDP Growth	0.045			2 years	0.0629	
Inflation	0.035	0.035		3 years	0.0634	
Equity Earn Growth	0.100	0.100		5 years	0.0616	0.0548
Equity Earn Yield	0.040	0.040		7 years	0.0607	
Equity Div Yield	0.015	0.015		10 years	0.0597	
				20 years	0.0598	
				30 years	0.0598	0.0598

ASSET MODELING

In setting up the company's asset portfolio, limited data from Schedule D was available. Ideally, for analyses where investment strategy is relevant, more detailed asset information by specific holding would be provided.

Based on the data given, an asset portfolio was constructed by creating broad asset classes. The modeled portfolio was set to match the company portfolio with respect to:

1) Asset allocation (cash, bonds, equities, and other)
2) Taxable and tax-exempt holdings
3) Average duration of bonds (years to maturity set to term)
4) Average credit quality
5) Coupon rates that result in expected bond income

Fixed income valuation is performed using market yields based on the projected yield curve adjusted for a credit spread. Equities were modeled as the S&P 500 index in the economic model.

Initial Invested Asset Portfolio

Asset Class	Market Value	Term (=YTM)	Coupon Rate	Yield Spread*
Cash	$869,870			
Common Stock	$236,120			
Preferred Stock	$327,085			
UST 1YR	$10,173	1	8.00%	
UST 3 YR	$262,238	3	8.00%	
UST 12YR	$14,199	12	8.00%	
MUNI 1YR	$94,154	1	6.00%	-0.11%
MUNI AA 5YR	$698,317	5	6.00%	-1.64%
MUNI AA 12YR	$1,641,392	12	6.00%	-1.40%
MUNI AAA 20YR	$212,602	20	6.00%	-1.28%
CORP 1YR	$13,653	1	8.00%	0.20%
CORP AA 2YR	$122,455	2	8.00%	0.30%
CORP AA 5YR	$197,255	5	8.00%	0.37%
CORP AA 10YR	$51,314	10	8.00%	0.52%
JUNK MUNI 5YR	$6,255	5	6.00%	6.25%

**Yield spreads were based on market rates. For municipal bonds the yield spread was set below zero to reflect tax-exempt status.*

LIABILITY MODELING

The objective of liability modeling is to reflect the impact of the company's liabilities on capital and profitability evaluation, asset/liability management, and reinsurance structure analysis.

The model captures the following basic aspect of the liabilities:

1) Expected values of losses and reserves and variability
2) Correlation between liability groups
3) Payment patterns for liabilities with variability
4) Premiums and expenses reflecting collection, earning, and payment patterns
5) Trends for losses, premiums, and expenses

Methodology is described below for both prospective business and existing reserves.

Prospective Business

The company was modeled assuming level premium writings and losses based on historical averages. If a business plan is available, prospective modeling should reflect the company's projections to some degree. The company's business was grouped according to Schedule P lines. Ideally, the model should group business on a basis that reflects how the company manages its operations.

Generally, business should first be modeled on a direct or gross basis. The impact of the current reinsurance structure on direct results would then be modeled to arrive at net results. In this case, all business was modeled on a net-of-reinsurance basis. This was done due to a lack of detailed information about the current reinsurance structure. In using historical net data, an implicit assumption was made that the reinsurance structure has not changed over time.

Premiums and Expenses

For this study, premium and expenses were modeled as non-stochastic variables. Premiums were set flat and expenses were set uniformly across all lines. Since projected premiums and expenses were not adjusted for changes in exposure or inflation, no loss trend was applied to projected losses. Also, in this analysis, losses include ALAE.

Losses (including ALAE)

Losses for each line can be modeled either in aggregate or by separate frequency and severity components. Data concerning large losses would be needed to perform separate frequency and severity modeling.

In this case, losses were modeled in aggregate using Schedule P loss ratios. Historical rate change, exposure, and trend information would be needed to bring the information to current levels.

Losses for each line of business were fit to a lognormal distribution. For this study, the homeowners line was split into catastrophe and non-catastrophe groups. The catastrophe bracket was parameterized using the fact that the company's net PML for a 1-in-100 year catastrophe event is $160mil. This provided the 99th percentile of the net catastrophe distribution. A rough estimate of the average retained catastrophe loss was made using net and direct Schedule P losses for homeowners. Based on these two points a lognormal distribution was fit. The non-catastrophe homeowners line was parameterized net of the modeled catastrophe losses.

Prospective Business Parameters

	Historical Data Parameters			Lognormal Fitted Params.	
LINE	Mean	Std. Dev.	C.V.	Mu	Sigma
HO – xCAT	212,326	9,000	0.042	12.265	0.043
HO – CAT	25,000	18,866,213	754.649	3.501	3.640
PPA – Liab	439,214	42,993	0.098	12.989	0.096
CAT – Liab	115,850	11,514	0.099	11.655	0.102
WorkComp	145,123	24,806	0.171	11.872	0.171
Comml MP	221,173	28,879	0.131	12.299	0.132
Short Prop	387,456	50,295	0.130	12.860	0.130
All Other	37,140	6,639	0.179	Use Normal Distribution	

Correlation

Next correlation of losses between lines of business was estimated based on historical loss ratios. The empirical results were highly volatile as would be expected using limited data. The average correlation across all lines was slightly under 25%. This was selected as the correlation parameter between all lines of business. The catastrophe group was assumed to be uncorrelated with the other lines.

No correlation was assumed between prospective lines of business and run-off of existing reserves. If the setting of reserves depends in large part on expected or historical planned loss ratios, then reserve development may co-vary with movements in the prospective losses. This can be particularly evident in long tail lines and reinsurance. Sufficient information was not available to model this effect.

Payment Patterns

Payment patterns for each line of business were based on the historical loss development shown in Schedule P. Payment pattern variability was introduced using a method that applies variability to the reserve disposal rates using a symmetrical beta distribution. The variability used in the beta distribution was based on the variability in the historical development patterns.

Existing Reserves

The stated reserves for each line were assumed to be accurate. No attempt was made to test the reserves for adequacy. A recent reserve study would have been useful in this regard.

Payout patterns for the reserves were calculated using the accident year payment patterns in Schedule P data and converting to a stream of future calendar year payments.

Although the expected value of the reserve liability was set to the carried amount, the possibility of adverse or favorable reserve development was introduced. In other words, the ultimate reserve amount was modeled as random variable with an expected value set to the held reserve.

Reserve variability by line was modeled using a method analogous to the payment pattern variability method described above. For this purpose, the modeled variability in each age-to-ultimate development factor was used to get a distribution of ultimate losses for each accident year. Paid losses to date were subtracted from the modeled accident year ultimate losses to arrive at a distribution of reserves. This methodology has the desirable quality of decreasing reserve variability as accident year maturity increases.

REINSURANCE MODEL

The purpose of reinsurance modeling is to reflect the impact of the current and proposed reinsurance structures on the results of the company.

To parameterize the module, detailed information about a reinsurance structure is required, including:

1) Coverage terms for each cover, e.g. retentions, limits, etc.
2) Rates, commissions, and profit-sharing terms
3) Subject business definitions
4) Inuring relationships
5) Cash flow impacts, e.g. collection and payment schedules

The model is capable of handling losses in aggregate or on a claim level depending on the detail of liability modeling. Claim level losses and reinsurance terms can be specified on a per-claim or occurrence basis.

For DFAIC, the reinsurance structure was not described in detail. However, as the liability modeling was on a net basis, the base case results of the model can be said to model the current structure implicitly.

Alternative Structures

The study was not able to consider changes to the company's excess of loss retention or catastrophe program retention. This would have required additional data such as a large loss listing or output distributions from a catastrophe model such as RMS.

Alternative reinsurance structures on top of the current structure were modeled, however. This was done in the form of accident year stop loss (AYSL) covers. The model was used to evaluate the ability of these covers to reduce the volatility of the company's net results at an acceptable price.

Three levels of AYSL coverage were considered: 10%, 20% and 30% coverage in excess of a 70% loss and ALAE ratio. Prices for these treaties were estimated using a pricing model for AYSL reinsurance. The pricing reflected not only expected losses and expenses, but also a risk load based on the variability of ceded losses. The DFA model will have to be updated to the extent actual market prices are different.

Reinsurance Alternatives ($000)

	Alternative #1	Alternative #2	Alternative #3
Attachment	70.0%	70.0%	70.0%
Limit	10.0%	20.0%	30.0%
Ceded Premium	$35,000	$50,000	$60,000
Expected Loss	$16,088	$17,248	$17,867
Std. Dev. Loss	$42,196	$50,147	$56,684

CAPITAL ADEQUACY

Capital adequacy is a prospective measure of the expected value and volatility of a company's surplus. Regulatory and Rating agencies are concerned with the probability of insolvency of a company. This can be evaluated by analyzing the outcome probability distribution of surplus. For regulatory agencies, this is usually done on a statutory basis. Rating agencies often make some economic adjustments to surplus.

Two measures were used to calculate capital adequacy from an insolvency perspective:

Probability of Ruin - This measure reflects the probability that the company will have negative surplus under simulated conditions. Often a "safety level" is selected. This represents the percentile of ruin. Then a required surplus number is calculated that results in probability of insolvency below the safety level. The required surplus is compared against the actual surplus to measure capital adequacy.

Expected Policyholder Deficit (EPD) - This measure reflects not only the probability of insolvency but also the severity of insolvency under simulated conditions. EPD is often stated as a percentage of expected loss.

$$\text{EPD Ratio} = \frac{\text{Expected [min(0,simulated ending surplus)]}}{\text{Expected [Losses]}}$$

The distribution of surplus can also be used to determine the probability of surplus decline. A measurement of this type would be of interest to an investor concerned with the preservation of investment principal. A measure of this type is Expect Default Loss Rate on Surplus[2] (EDLR). EDLR can be used to evaluate adequacy of capital in a manner analogous to how bonds are evaluated by rating agencies based on their default rates.

Expected Default Loss Rate – This measure reflects of the probability of any surplus decline. This is stated relative to current surplus.

$$\text{EDLR Ratio} = \frac{\text{Expected [min(0,current surplus - simulated ending surplus)]}}{\text{Current Surplus}}$$

CAPITAL ALLOCATION

Capital allocation to lines of business should be based on the relative contribution of each line's risk to the company's total risk. In this study, capital allocation was performed using the relative variability of net income by line of business.

A method based on Game Theory[3] techniques was employed to fairly allocate capital. This ensured the surplus requirements by line added to the company total. Also, it resulted in diversification benefits being equitably shared among all lines.

The allocation scheme was based on Shapley value calculations. Since the risk measure used was variance of results, the Shapley value is represented by:

Shapley Value = Var[division] + Cov[Rest of company, division]

When the Shapely value is compared to the formula for marginal variance,

Marginal variance = Var[division] + 2 x Cov[Rest of company, division]

the Shapley value splits the covariance evenly among divisions.

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[2] Mango, D., "Capital Allocation and Adequacy Using Dynamic Financial Analysis," *CAS Forum*, Summer 2000, Dynamic Financial Analysis Call Paper Program, p. 55.
[3] See Mango [2].

DFA Insurance Company Case Study, Part I: Reinsurance and Asset Allocation

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Thomas S. McIntyre, FCAS, MAAA and
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Dynamic Financial Analysis
DFA Insurance Company Case Study
Part I: Reinsurance and Asset Allocation

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Abstract

As a result of published papers, shared research and call paper programs such as this one, the technical specifications behind Dynamic Financial Analysis (DFA) have been well developed. This has led to a high level of convergence among many of the different concepts, models and processes behind DFA. The next logical step in promoting DFA is to show how these models and processes can be implemented to produce value to the insurance industry, its policyholders and its shareholders.

This paper has been submitted in response to the Committee on Dynamic Financial Analysis 2001 Call for Papers. The authors have applied dynamic financial analysis to DFA Insurance Company (DFAIC) to address the efficiency and effectiveness of DFAIC's reinsurance programs and asset allocation strategies. The DFA model used for this analysis was the Swiss Re Investors Financial Integrated Risk Management (FIRM™) System. This paper is Part 1 of a two-part submission. Part 2 deals with using DFA to explore capital adequacy and capital allocation issues.

Dynamic Financial Analysis
DFA Insurance Company Case Study
Part I: Reinsurance and Asset Allocation

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Preface

Dynamic Financial Analysis (DFA) is still fairly new to a property-casualty insurance industry whose roots can be traced back to the 17th Century and earlier. As such it is not surprising that the industry is cautious about a technology that purports to look at their business in a whole new way. The Casualty Actuarial Society, being active in the formulation and development of DFA, has classified it as:

> *"a systematic approach to financial modeling in which financial results are projected under a variety of possible scenarios, showing how outcomes might be affected by changing internal and/or external conditions[1]."*

As a result of published papers, shared research and call paper programs such as this one, the technical specifications behind DFA have been well developed. This has led to a high level of convergence among many of the different concepts, models and processes behind DFA. Unfortunately, while the details of DFA are better understood, the industry is still scratching its collective head on what to do with this new technology.

Part of the problem has to due with the fact that DFA is considered to be a modeling tool, one that can be used to supplement existing tools. While a modeling tool is essential for implementing dynamic financial analysis, it is just one element of a much grander picture. More than a model, dynamic financial analysis is a way of thinking that weaves through the entire operations of an insurance company. Effective dynamic financial analysis calls for dedicated and knowledgeable professionals who are trained in the intricacies of DFA and enabled to identify and take advantage of current industry and company inefficiencies. DFA promotes moving from existing structures designed to evaluate and reward the individual pieces of the business to a structure that encourages and rewards the evaluation of strategic decisions in a holistic, total company framework.

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[1] Casualty Actuarial Society Dynamic Financial Analysis Website, DFA Research Handbook, http://www.casact.org/RESEARCH/DFA

For these reasons we were excited to embrace this call paper program exercise. While the original concept may have been designed to evaluate different DFA modeling techniques and the resulting analyses as they relate to a common problem and common data, we decided it was a perfect opportunity to show how DFA might work in the insurance company of tomorrow. The ultimate benefit to the company is not just the final answer, but rather the increased understanding and the common grounds of communication that comes from going through the DFA process.

The proposed situation involves DFA Insurance Company (DFAIC), a multi-line property-casualty insurance company that is unknowingly the target of a potential acquisition. The analysis was conducted from the point of view of the acquiring company. We will define the acquiring company, Falcon, as a newly capitalized holding company that is organized and structured to run its business in a holistic manner. As such Falcon has a financial risk management unit led by its Chief Risk Officer (CRO) who reports directly to the CEO. The CEO has asked that the following questions about DFAIC be addressed:

1. Is the Company adequately capitalized? Is there excess capital? How much capital should the Company hold as a stand-alone insurer?

2. How should the capital be allocated to line of business?

3. What is the return distribution for each line of business and is it consistent with the risk for the line?

4. Should the Company buy more or less reinsurance? What type? How efficient is its current reinsurance program?

5. How efficient is the asset allocation?

In a traditional insurance company these questions would be farmed out to different business units within the organization. These units would include but not be limited to the actuarial department, the reinsurance department and the investment department. Each unit would perform their stand-alone analysis and report back to the CEO using terminology and metrics appropriate to their assigned task. The CEO would be left to assimilate all the individual analyses and use her professional judgment and insights to build a complete picture of the attractiveness of the potential acquisition.

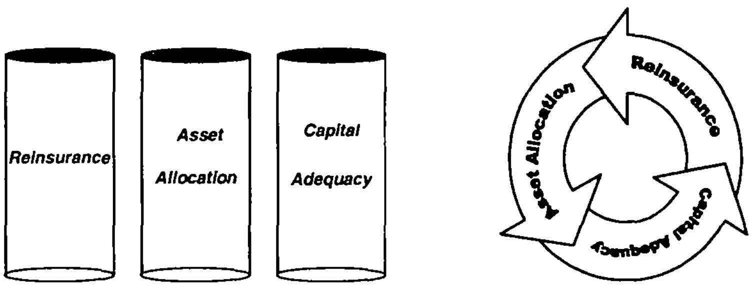
Falcon, however, is organized in such a way that the complete analysis can be performed within the financial risk management unit with input from professionals in each of the above-mentioned departments. The results of the analysis can thus be presented to the CEO using a single set of terminology and metrics that consider both the individual and joint dynamics of the issues in question.

Due to the scope and breadth of the required analysis, we will present the DFA study in two papers. This paper will deal with the efficiency of the reinsurance and asset allocation strategies and a sister paper will concentrate on capital allocation and capital adequacy issues. Note that despite breaking the analysis up into two papers, the overall analysis is the result of a common DFA model and process.

DFA, being holistic, allows a company to deal with all of its major strategic decisions simultaneously within a single framework. As such it is not unusual to have an analysis that continuously revisits these strategic levers in what we call the DFA spiral. This is in contrast to the traditional approach in which these strategic decisions are evaluated each in their individual silos. Figure 1 gives a graphical picture of these two different approaches.

Figure 1

Traditional Analysis **Dynamic Financial Analysis**



Unfortunately, a paper does not easily lend itself to a spiral analysis, so for the sake of convenience we will first complete a single loop around the DFA spiral holding the strategic decisions that relate to other sections constant. This will allow us to show how DFA can be used to deal with individual strategic initiatives but still within a holistic framework. We will then begin a second loop taking into consideration the strategic initiatives suggested as a result of the initial loop. This will allow us to identify and discuss the additional opportunities that result from simultaneous changes to two or more strategic initiatives.

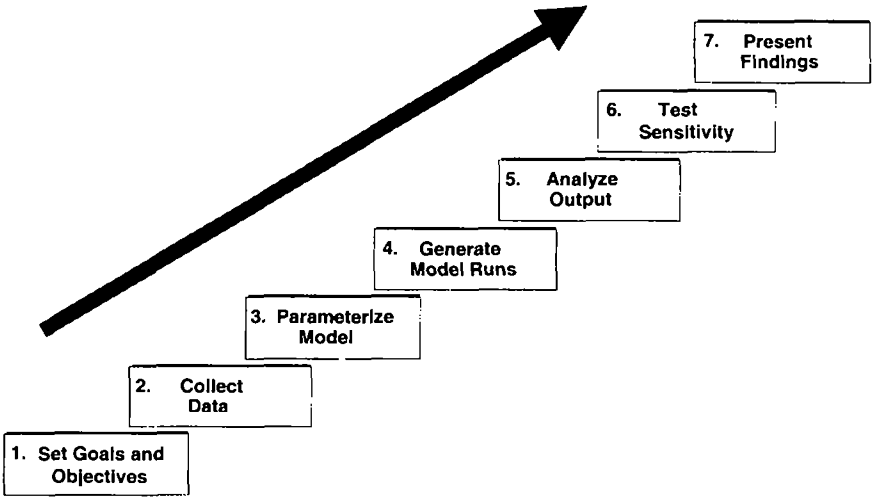
This paper concentrates on the reinsurance structure and the asset allocation strategy. While information concerning capital adequacy will be stated, the interested reader should refer to the sister paper "Dynamic Financial Analysis, DFA Insurance Company Case Study, Part II: Capital Adequacy and Capital Allocation" for a detailed description of the methodology used in the development of these numbers.

The DFA Process

The DFA process refers to a high-level overview of how current strategies are evaluated and how strategic alternatives are developed. We have outlined, in Figure 2, the DFA process that was used for the DFAIC analysis.

Figure 2

The Dynamic Financial Analysis Process



The remainder of this paper will explore the assumptions and model details that we used in performing DFA on DFA Insurance Company. We will run through the seven steps of the DFA process and describe the work performed in each step. We will conclude this paper with some final thoughts on this exercise and on DFA in general.

Introduction to DFAIC

DFAIC is a privately held property-casualty insurance company operating in all fifty states, with business concentrations in the northeast and mid-west. The company writes personal and "main-street" commercial coverage through independent agents and maintains an "A" rating from A.M. Best. The company's northeast and mid-west concentrations limit their exposure to severe catastrophes[2].

DFAIC's underwriting results have deteriorated recently, but the 1999 combined ratio, 105.1, was slightly better than the industry average, 107.8[3]. DFAIC's balance sheet appears slightly leveraged versus the current industry average, but its premium-to-surplus and reserves-to-surplus ratios of 1.47 and 1.45, respectively, are low by historical standards. The company cedes a relatively small portion (8.5%) of its direct written premium to a combination of excess-of-loss (XOL), per risk excess and catastrophe reinsurance contracts.

DFAIC's invested assets are reportedly weighted toward tax-exempt municipal bonds (56%), with smaller allocations to government and corporate bonds and equities. The company's cash position is unusually large at 18% of invested assets, versus the industry average, 4.0%[4]. Investment income earned in 1999 was 7.0% of terminal invested assets.

Step 1: Goals and Objectives

The DFA process starts with a thorough discussion and understanding of the goals, objectives, constraints and risk tolerance of the company. This step determines the metrics that will be most important in evaluating alternative strategic initiatives. It also tends to be a valuable exercise as it helps management think through, focus on, and communicate exactly those items that are most important to them as a company. These items are stated in terms of financial statement results and, once determined, provide a common set of metrics that can be applied to all of the company's financial strategic decisions.

There is no limit to the number and types of possible objective functions that can be used for evaluating strategic initiatives. Some simple objective functions might be defined as expected surplus (policyholder surplus, shareholders' equity, or economic value) for the reward measure, and the standard deviation of the surplus for the measure of risk. Alternatively, downside risk measures can be substituted for standard deviation or company-specific risk/reward functions can be defined.

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[2] We have assumed that DFAIC has no substantial earthquake exposure.

[3] 1999 industry excluding state funds combined ratio after policyholder dividends. "Best's Aggregates & Averages, Property-Casualty U.S.", 2000 Edition, p. 119.

[4] "Best's Aggregates & Averages, Property-Casualty U.S.", 2000 Edition, p. 122.

In the case of the potential acquisition of DFAIC, the goals and objectives for the analysis would be set by Falcon's board and senior management team[5]. As such, Falcon's objective is to undertake strategies that will maximize the economic value of the company at the end of a five-year time horizon.

The five-year horizon was chosen since it is consistent with Falcon senior management's business planning horizon and it allows them to benefit from time diversification. This also gives Falcon a competitive advantage over those companies that are forced to operate on a year-to-year basis due to shareholder, regulatory or company-imposed constraints.

Extending the time horizon beyond the company's planning horizon increases the risk that business does not develop as planned, and can thus reduce the effectiveness of the analysis. Risk is defined as the standard deviation of economic value, as Falcon management believes that this is an indicator of true economic risk.

One criticism of economic value as an insurance company objective is that it does not reflect statutory or regulatory constraints. Further, it is not part of the required annual financial reporting of insurance companies and is therefore not standardized or completely understood. Thus, a second risk measure, which was treated as a constraint, dealt with the reality of statutory reporting and regulatory oversight. This was reflected in the calculation of the probability of the statutory surplus falling below a minimum threshold.

Step 2: Data Collection

Data collection and evaluation is a time-consuming but important part of DFA. Since DFA deals with all financial aspects of insurance company operations the data requirements can be significant[6]. Published financial information, similar to data used in this case study, is readily available from organizations such as A.M. Best, One Source, shareholder reports, the SEC and numerous other sources. These sources streamline the data gathering and data entry required to feed (parameterize) DFA models. However, analyses based solely upon public databases and published financial information risk misinterpretation of events, trends, and on-going company operations. As such, DFA studies limited to public data are sub-optimal and if not carefully implemented and documented, can lead to inappropriate conclusions.

In this section, we discuss some of the problems of using public data for the DFAIC case study. However, we must be careful to separate the real pitfalls of public data from the

[5] Since no specific guidelines for measuring the effectiveness or efficiency of the strategic initiatives have been communicated in the instructions to the case study, we were free to elect measures that were in accordance with the holistic nature of our newly capitalized holding company.

[6] The data collection for DFAIC was made simple in that it was completely provided by the CAS with instructions that no additional information would be made available. In the case of DFAIC only a small portion of the plethora of publicly available data was provided.

artificial ones introduced by the fictitious public data created for DFAIC. Artificial data problems are those leading to questions that could be resolved with additional public information[7]. The information provided for DFAIC is more reflective of the limitations one might experience dealing with non-U.S. companies, where publicly available data is much more limited.

Despite the fact that this DFAIC case study does not require a formal data collection process, there are still the important chores of data evaluation and reconciliation of the DFAIC data. We have undertaken such processes and found a number of areas where additional data and research would be warranted. Since no additional research could be undertaken, we will note a few of these irregularities here and then make reasonable assumptions to correct for these inconsistencies.

A solid data evaluation process requires the reconciliation of the provided information to the company's last reported financial statements. In putting together this reconciliation, we found that the investment income earned on the portfolio is inconsistent with the stated asset allocation. The asset allocation for DFAIC is purported to be 70% fixed-income, four-fifths of which is in tax-exempt bonds. DFAIC's large allocation to tax-exempt bonds, given the lower yields on these securities, is inconsistent with its reported above-average investment income (7% versus 5% industry average[8]). In order to reconcile the stated asset allocation to the reported investment income, the tax-exempt bonds would have to be earning a book yield in excess of 7.5%. Since market yields on tax-exempt municipal bonds were in the range of 3%-6% during 1999, the tax-exempt holdings of DFAIC would have to contain a large unrealized capital gain. However, the reported book and market values on these holdings are very close to each other: $3,327M and $3,478M, respectively. It is unlikely that the tax-exempt bonds could be providing such a contribution to investment income.

This inconsistency raises some serious concerns regarding data quality of the DFAIC asset portfolio. Examination of Schedule D reveals that over half of the bond holdings of DFAIC are classified in the "Special Rev & Assessment" category. This category typically contains taxable structured bonds (mortgage-backed and asset-backed securities). Since tax-exempt holdings are not specifically categorized in Schedule D, it is likely that some or all of the bond holdings of DFAIC that were reported to be tax-exempt bonds are actually taxable bonds.

Further evidence to support this theory lies in the fact that if all bonds were assumed to be taxable, the calculated investment income would reconcile with the reported investment income results. If complete statutory records for DFAIC were available, analysis of the Schedule D details would resolve any doubt about this inconsistency. Since no such details were available, professional judgment must guide us on how we should model DFAIC's current asset portfolio. Thus, given that the investment income, and book and market values of the asset portfolio can be directly traced to the provided

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[7] Because DFAIC is not a real company and it would have been impractical for the CAS to provide a complete DFAIC annual statement, there is no such additional information (e.g., prior years' annual statements, annual statement schedules).

[8] 1999 industry excluding state funds investment yield. "Best's Aggregates & Averages, Property-Casualty U.S.", 2000 Edition, p. 117.

financial statements, the most logical and consistent course of action is to assume that entire fixed-income portfolio of DFAIC is taxable.

In contrast to the artificial data problems introduced by fictional data, there were other issues raised by the reported DFAIC data that, if the option were available, could be resolved by talking to management. Prominent among those issues were (1) DFAIC's 18% allocation to cash and short-term investments (industry average is 4%) and; (2) reported changes in case reserve adequacy. These are the types of data analyses that are required at the front end of the DFA process to assure the robustness of the ultimate DFA findings.

Step 3: Model Parameterization

Model parameterization refers to how the asset and liability variables are assumed to behave over the forecast horizon. Assumptions concerning the general economic and capital market environment as well as the specific assets and liabilities of DFAIC need to be parameterized. These assumptions can have a substantial impact on the evaluation and the recommended strategies. In the modeling world this risk is referred to as "parameter risk." The impact of parameter risk can be investigated and better understood through sensitivity testing.

Economic and capital market assumptions are an important part of any quantitative assessment of the potential rewards and risks associated with alternative strategic business decisions. These assumptions need to reflect both recent conditions and longer-term relationships inherent in the economy and capital markets. The simulations based on these assumptions should comprise a reasonable set of future paths that, while consistent with historical observations, reflect a prospective view of economic and capital market expectations and uncertainties.

The model that we used to generate our DFA economic and capital market simulations (FIRM™ Asset Model) differs from traditional mean/variance models in that economic variables, including interest rates and inflation, are explicitly modeled using accepted and rigorously tested stochastic processes. Capital market returns are then generated on a consistent basis with the underlying economic environment. This type of model has the following advantages over traditional mean/variance models:

- the explicit modeling of both economic and capital market variables;
- the ability to incorporate mean reversion in yields, providing for control over the serial correlation of capital market returns over time;
- multi-period simulation capabilities; and
- additional flexibility in modeling asset categories such as mortgage-backed securities and other securities with embedded options.

The economic and capital market parameterization process involved identifying and selecting asset classes that best represented the homogeneous groups of invested assets available to DFAIC. The twelve asset classes we defined and modeled were:

- Cash Equivalents
- Government Bonds (1-5 years)
- Government Bonds (5-10 years)
- Government Bonds (10-30 years)
- Corporate Bonds (1-5 years)
- Corporate Bonds (5-10 years)
- Corporate Bonds (10-30 years)
- Municipal Bonds (1-5 years)
- Municipal Bonds (5-10 years)
- Municipal Bonds (10-30 years)
- Common Stock
- Preferred Stock

The economic and capital market simulation model required assumptions concerning the initial levels of interest rates, inflation rates, real GDP growth, equity earnings growth, equity P/E levels, and the dividend payout ratio together with a set of long-term levels to which the initial levels will revert over time. In setting the long-term levels, the goal was to produce risk premiums between asset classes that are consistent with historical data[9].

For our DFAIC study, we have set long-term levels equal to the initial market conditions as of our model start date (1/1/2000). This avoids bias with respect to expected price appreciation or depreciation due to interest movements or changing P/E ratios over the time horizon. Initial market conditions together with the assumed mean levels for are shown in Table 1.

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[9] For example, the spread between cash and inflation is historically about 2% and the risk premium for long government bonds over cash is about 2%.

Table 1: Initial and Mean Interest Rate and Share Assumptions

Variable	Initial Conditions 1/1/2000[10]	Mean Levels
Government Yields:		
3-Month Interest Rate	5.53%	5.53%
1-Year Interest Rate	6.19%	
3-Year Interest Rate	6.34%	
5-Year Interest Rate	6.39%	
10-Year Interest Rate	6.36%	
30-Year Interest Rate	6.56%	6.56%
Corporate Yields:		
3-Month Interest Rate	6.16%	6.16%
1-Year Interest Rate	6.70%	
3-Year Interest Rate	6.99%	
5-Year Interest Rate	7.11%	
10-Year Interest Rate	7.28%	
30-Year Interest Rate	7.65%	7.65%
Municipal Yields:		
3-Month Interest Rate	3.91%	3.91%
1-Year Interest Rate	4.09%	
3-Year Interest Rate	4.54%	
5-Year Interest Rate	4.79%	
10-Year Interest Rate	5.22%	
30-Year Interest Rate	5.99%	5.99%
Expected Price Inflation	2.5%	2.5%
Expected Real GDP	2.5%	2.5%
S&P 500 P/E Ratio	32	32
S&P 500 Earnings Growth		9.0%
S&P 500 Dividend Payout Ratio	40%	40%

The returns for cash equivalents, bonds and common stock are directly controlled by the initial and mean assumptions shown in Table 1.

[10] Source: Bloomberg

Cash Equivalent returns are the accumulation of 1-month government interest rates over time. Government Bond returns are a function of the applicable interest rate level, the change in the rate and the bond maturity. Corporate and Municipal Bond returns are modeled as a proxy to the US Single A corporate and the insured general obligation municipal bond markets respectively. They are calculated similarly to government bond returns. Corporate yields are modeled at a stochastic spread to government yields and municipal yields are modeled as a stochastic ratio to the government yields. Reported market yields on corporate bonds are adjusted to reflect historical defaults[11]. Common Stock returns are modeled as a proxy to the S&P 500 index. The returns are composed of capital gains/losses plus dividends[12].

Table 2 shows the expected annual (arithmetic) and annualized compound (geometric) returns for each of the twelve modeled asset classes.

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[11] This is based on the 10-year cumulative default study for Single A bonds provided by Moodys. A 50% recovery rate on defaults is assumed.

[12] Because we are assuming that long-term mean P/E ratios are equal to initial P/E ratios, valuation changes are not reflected in the risk premium between stocks and bonds. Thus the modeled equity risk premium is less than the historical average (6-7%), but is in-line with the historical average when adjusted for valuation changes.

Table 2: Simulated Five-Year Return Statistics[13]

Asset Class	Expected Annual Return	Annual Std Dev	Annualized Compound Return	Annualized Compound Std Dev
Cash Equivalents	5.9%	1.9%	5.9%	1.4%
US Gov't Bonds (1-5)	6.5%	3.5%	6.5%	0.8%
US Gov't Bonds (5-10)	6.9%	6.7%	6.7%	1.8%
US Gov't Bonds (10-30)	7.4%	10.7%	6.9%	3.3%
US Corporate Bonds (1-5)	7.2%	3.6%	7.2%	0.9%
US Corporate Bonds (5-10)	7.6%	6.8%	7.4%	1.9%
US Corporate Bonds (10-30)	8.0%	10.8%	7.5%	3.3%
US Municipal Bonds (1-5)	4.9%	3.2%	4.8%	0.7%
US Municipal Bonds (5-10)	6.1%	7.8%	5.8%	2.0%
US Municipal Bonds (10-30)	7.0%	11.8%	6.4%	3.2%
US Stock	10.8%	20.0%	9.3%	7.6%
Preferred Stock	8.3%	12.6%	7.7%	4.2%

The operations of insurance companies differ from other industries for a number of reasons. Prominent among these is the receipt of payment for a product before the magnitude or timing of the product's costs are known to the company (insurer). A reserve must be established to account for this contingent obligation. The importance of liabilities to the operations of an insurance company implies a similarly important role to an appropriate insurance company DFA model. Such items as payment patterns, loss ratios and reserves, expense ratios, and premiums are examples of obvious inputs to a DFA model. Further, one must apply these and other inputs within the context of other considerations such as line of business, whether we are generating results gross or net of reinsurance, or whether these parameters are applied to business already written or business to be written at some future time. This section will not focus on the details but rather present a general overview of the parameterization process for losses and liabilities as well as some of the more interesting particulars.

Our study of DFAIC's current reinsurance program and how it compares to alternative programs does not include loss portfolio transfers or other retrospective coverage. Hence existing business, with its attendant loss and unearned premium reserves, is modeled on a net of reinsurance basis. New business, however, is modeled on a gross basis. This allows us to vary prospective reinsurance strategies and compare the consequences of differing strategies. Since a principle focus of our paper is the current reinsurance program and its possible alternatives, we begin with a brief discussion of

[13] Expected annual return statistics are arithmetic averages and are indicative of risk and return expectations over a one-year holding period. Annualized compound return statistics are geometric averages and reflect the impact of time diversification over the five-year holding period.

DFAIC's current reinsurance program and its implications for parameterizing our DFA model.

DFAIC's current reinsurance program includes excess of loss coverage for property, liability, and workers compensation risks, as well as coverage for catastrophes. In order to model the effects of these and alternative treaties, we generated individual large losses and occurrences on a gross of reinsurance basis. This necessitates the development of both frequency and severity probability distributions within the context of a collective risk model. Both company-specific and industry experience were gathered and analyzed for this purpose. Once the collective risk model was ready, individual large losses and catastrophes were generated stochastically and reinsurance covers were applied to obtain simulated losses net of reinsurance. Normally, company management would be consulted before finalizing company specific assumptions such as reinsurance arrangements or the frequency and severity of large losses and catastrophes.

In setting up our model, we condensed DFAIC's business into five distinct lines: Workers Compensation, Auto Liability (both personal and commercial), Property (homeowners and CMP property coverage), General Liability (other liability, product liability, special liability, and CMP liability coverage), and All Other (predominantly auto physical damage). Segregation of business into these five lines allows for the effective modeling of reinsurance programs without burying results within a mass of detail. Each of these five lines is assigned a set of descriptive parameters to appropriately model its constituent line of business. Needed parameterizations relate to such items as premiums, losses (including loss adjustment expenses), other expenses, and payment patterns, as well as their stochastic properties. A preliminary step in our analysis involved restating historical results to be consistent with our five modeled lines of business[14].

Projections of expected future premiums and loss ratios are in part based upon our assumed future business plans for DFAIC. An analysis of DFAIC's Schedule P reveals a recent deterioration in underwriting results and earned premium levels. Such a situation might indicate past DFAIC rate reductions made in an attempt to maintain market share within a competitive environment. Falcon's business plan would be to raise rates thereby restoring loss ratios to DFAIC historical levels in three to five years. Anticipated effects of this business plan are reflected in our parameterization of future written premium levels.

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[14] E.g. CMP results were segregated into property or casualty and allocated to our Property or General Liability lines of business, respectively.

Table 3: Projected Growth Rates for Written Premium

	Workers Comp	Auto Liability	Property	General Liability	All Other
2000	-2.5%	-2.5%	-2.5%	-2.5%	-2.5%
2001	-2.5%	-2.5%	-2.5%	-2.5%	-2.5%
2002	-2.5%	-2.5%	-2.5%	-2.5%	-2.5%
2003	0%	0%	0%	0%	0%
2004	2.5%	2.5%	2.5%	2.5%	2.5%

DFAIC's simulated losses have been modeled in two pieces, core and large. Briefly, losses are categorized as large or core depending on magnitude. Large losses are simulated through a collective risk model, while core losses, specifically core loss ratios, are generated through a mean-reverting, momentum-driven random walk.

The model user determines the appropriate mean reversion factor, momentum factor and long term average core loss ratio. Considerations in selecting such parameter values might include an anticipated underwriting cycle or other market change. The actual simulated core loss ratio is generated from a user-selected distribution having a mean and a variance defined by the user. A blind algorithmic approach to selecting these parameters is not appropriate. As is true throughout the parameterization process, simulated results must be constantly checked to verify the reasonableness of results. For example, the variance of simulated, total loss ratios was checked against estimates of loss ratio volatility obtained from historical company results. Modeled gross accident year loss ratios by year and by line of business are shown in Table 3.

Table 4: Accident Year Loss Ratios by Line of Business[15]

	Year 1	Year 2	Year 3	Year 4	Year 5	Standard Deviation
Workers Comp	85%	81%	77%	77%	77%	18%
Auto Liab	92%	85%	81%	79%	79%	12%
Home/CMP-Prop	78%	75%	75%	75%	75%	8%
All Other	68%	65%	64%	64%	64%	8%
GL/CMP-Liab	66%	61%	59%	58%	59%	11%

The above statistics do not include the effects of catastrophes

The timing of loss payments is as important as their magnitude. Payment patterns were estimated using DFAIC Schedule P loss triangles and industry results. We derived two sets of payment patterns that were separately applied to existing reserves and new business for each of the five lines of business. The consolidated reserve run-off pattern and accident year payment pattern for DFAIC are shown in Exhibits 1 and 2.

[15] The standard deviations actually increase with time due to the diffusion process used to model loss ratios. Intuitively, one would expect volatility of projections to increase with the time horizon.

Exhibit 1 DFAIC s Consolidated Reserve Run-off Pattern

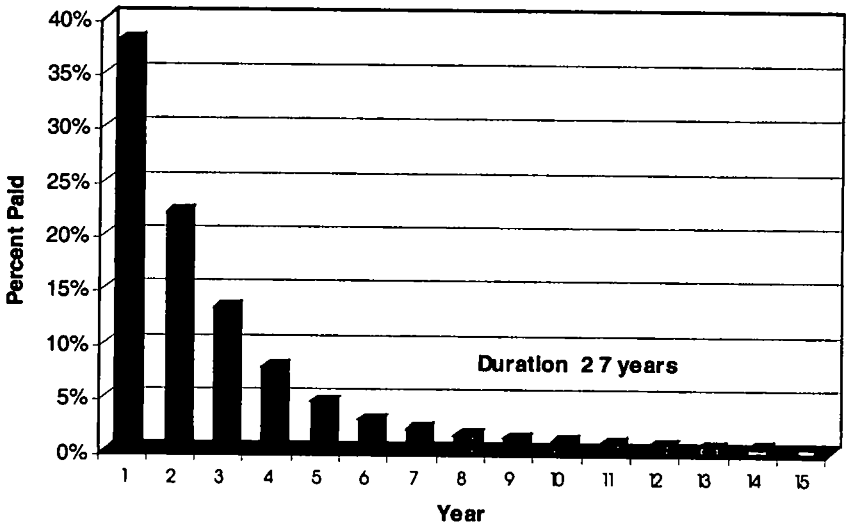
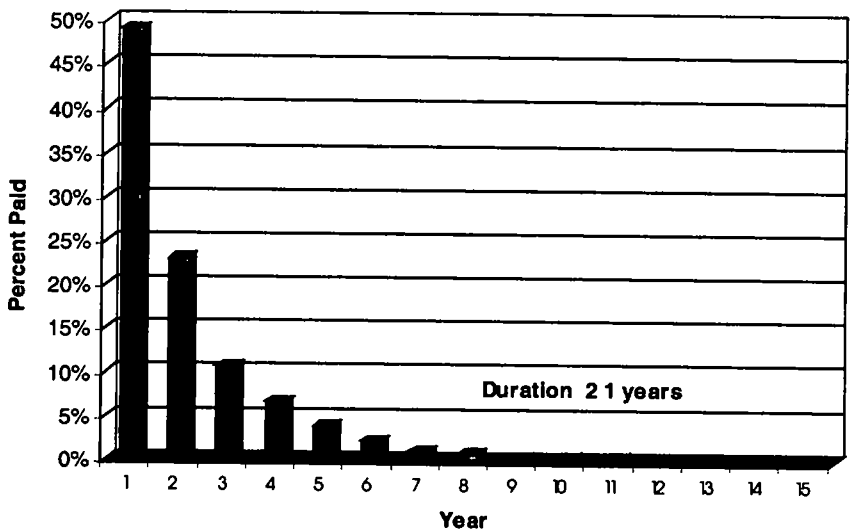


Exhibit 2 DFAIC s Consolidated Accident Year Payment Pattern



Expenses, other than the loss adjustment expenses already incorporated into the loss ratios, were modeled as both fixed and variable. Actual values were again obtained through a combination of company-specific and industry statistics.

We have already discussed some of the randomness modeled into the projected core loss ratios. Further randomness is introduced to the model through the sensitivity of losses, expenses, and premiums to unexpected changes in the level of inflation. For DFAIC, we modeled the losses and fixed expenses as being immediately and fully responsive to unexpected changes in the level of inflation while premiums were partially responsive after a one-year time lag. Inflation sensitivity introduces a stochastic element affecting loss ratios, expenses, premiums, and payment patterns.

The simulations include a reasonable level of positive correlation between lines of business as indicated in Table 5.

Table 5: Ultimate Loss Ratio Correlation Coefficients

	WC	Auto	Property	GL	Other
WC	1.0				
Auto	0.3	1.0			
Property	0.4	0.4	1.0		
GL	0.4	0.5	0.6*	1.0	
Other	0.2	0.3	0.3	0.3	1.0

**Note that the GL/Property correlation coefficient is artificially inflated because CMP loss ratios are a component of the loss ratios for both lines.*

Such positive correlation between lines of business is commonly accepted. It is probably the result of several factors, including changes to overall pricing levels in the insurance market and unanticipated inflation impacting the loss ratios of all lines of business.

Assumptions concerning correlation between lines of business are part of a series of parameter assumptions important within the context of building an appropriate DFA model. Because of our inability to access DFAIC for further information, it is especially important that our assumptions are reasonable both in isolation and in conjunction with other assumptions. For example, our collective risk model for generating workers compensation losses gross of reinsurance appears reasonable when compared to industry and available DFAIC statistics. But just as important, when we used this loss model to develop pricing for the current workers compensation excess of loss cover, the indicated reinsurance premium was comparable to that indicated by DFAIC Annual Statement exhibits. Such observed consistencies build confidence in the model and its assumed parameter values.

The analysis of alternative reinsurance structures is a key component in our DFA analysis. Such analyses are meaningless if not carried out under consistent and proper assumptions. In the particular case of the workers compensation loss model, we subsequently used this model to assist in pricing alternative reinsurance arrangements. If the same loss model is not used to price current and alternative reinsurance structures, then perceived differences in the efficiencies of these structures might be a function of different underlying loss models as opposed to true differences in efficiencies. Inconsistencies in actual reinsurance coverage and related premiums available in the market surely exist. Our focus here, however, is to seek more efficient reinsurance structures, not over/under priced coverage.

Developing reasonable and consistent parameter assumptions for a DFA model is challenging at best, and can be particularly difficult when dealing with reinsurance arrangements. It is important to continually test for the reasonableness of assumptions both in isolation and in tandem with other assumptions.

Step 4: Running the Model

In order to become comfortable with a particular modeling system for implementing DFA, one must understand the system's underlying methodology and how that particular methodology will impact the results of the analysis. By DFA model methodology we refer to the specific technical implementation of the DFA process.

Whereas the general DFA process has become fairly standardized, there are still a number of different methodologies that are used in the technical implementation of a DFA model. Since the technical implementation of a model can have a significant impact on the results of an analysis, it is imperative that the users of a model sign off on the technical implementations and understand how the model's methodology will impact the analysis. The risk that model results are specific to a particular DFA methodology is referred to as "model risk." This is a difficult risk to evaluate; due to the time, effort and expense of performing DFA, it is often impractical to duplicate the analysis using different DFA modeling systems. As such, users should look for systems that provide a significant amount of flexibility and whose underlying fixed methodologies are consistent with their views of the insurance and financial markets.

At Swiss Re Investors, we developed our Financial Integrated Risk Management (FIRM™) System as the modeling tool backing our DFA process. The FIRM System, like most DFA systems, uses simulation techniques to model both the assets and liabilities of an insurance company. The projected cash flows are transformed into future balance sheets and income statements that reflect GAAP, statutory, tax and economic viewpoints. The simulations are generated by a series of stochastic differential equations that are designed to allow the model user to reflect a full range of distributions, dynamics and relationships with respect to the underlying stochastic variables. The FIRM system is designed to allow a high level of flexibility in describing how the underlying stochastic variables behave in an attempt to minimize model risk. This increase in flexibility, however, has the result of moving a significant burden from the model, to the model user and the model assumptions[16].

Whereas the majority of the technical calculations are generated in Step 4 of the DFA process, after gaining an appropriate level of understanding of the modeling system, there is little the DFA professional is required to do in this step other than to tell the computer to begin processing.

Step 5: Analysis

The DFA process does not end with the running of the model; rather, the "analysis" phase within the DFA process begins. Dynamic financial analysis models generate large amounts of pro forma financial statement data. The Swiss Re Investors FIRM System, for example, generates financial statement details on a GAAP, statutory, tax and economic basis for each year and simulation. Since we are running a five-year horizon and 1000 simulations, we end up with over 20,000 individual pro forma financial statements. Thus, being able to work with such a large amount of data and condense it into a clear and concise analysis is key to successful DFA.

DFAIC's existing reinsurance programs include traditional forms of excess of loss, per risk excess and catastrophe coverage. As such, its ceded reinsurance program is fairly typical for a company of its size. The company ceded approximately 8% of its prior year's premium; it is not an extensive consumer of reinsurance when viewed relative to written premium. However, the company's seemingly modest reinsurance program generated over $200 million in ceded premium in the prior year versus a statutory net income of $186 million, so it is material to their operations.[17]

Like many of its peers' ceded reinsurance programs, DFAIC's is designed to manage volatility of each of its various LOBs (or small combinations thereof). I.e., it is a "silo" approach to ceded reinsurance purchasing. It should come as no surprise that a company management structure aligned with LOB will incent managers to purchase reinsurance that does not recognize the diversification that exists by simply writing multiple LOBs. In fact it would be unnatural to expect a line manager to act in a manner inconsistent with LOB results (e.g., accept highly volatile LOB results), even if it is in the best interest of the company. Many large insurers have gravitated toward centralized reinsurance purchasing to address this inefficiency. Curiously, small companies may be well equipped to make similar changes because of their limited resources. If one person must wear many hats, the management structure may already be centralized.

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[16] Interested readers can find additional information on the mechanics of the Swiss Re Investors FIRM System by referring to the previous Swiss Re Investors DFA papers listed in the references to this paper.

[17] Unfortunately, the Schedule P data provided for the case study included the impact of inter-company pooling, so DFAIC's actual 1999 ceded losses could not be determined.

Line-oriented management of ceded reinsurance will likely lead to a program that guards against large individual claims even in years where actual losses (in total) are lower than originally anticipated and/or in reinsurance that fails to recognize the diversification benefits of writing multiple LOBs. But this is nothing new, and it certainly does not require a DFA model to recognize that diversification exists whenever a company writes two or more LOBs.[18] We will show that the inefficiency goes beyond a missed opportunity (failing to recognize the diversification already present in their business), since for DFAIC their existing program actually impairs (slightly) certain capital adequacy measures.

We will demonstrate that Falcon's enterprise-level philosophy to managing risk with reinsurance is by far the most important element to building the efficient program that the company seeks. Given that DFAIC is large and well capitalized, by focusing on company-wide results rather than LOB results DFAIC could eliminate most of its current reinsurance programs without any significant increase in risk to the consolidated company loss ratio. We will also show that additional improvement can be achieved through new reinsurance structures that embrace enterprise-level rather than LOB-level reinsurance strategies. Thus, reinsurance in the "new" DFAIC will truly become a mechanism by which the enterprise forgoes part of its expected return[19], in exchange for protection from events that jeopardize overall stability.

Before we begin, a discussion of the modeling of the reinsurance program and the alternatives is required. Almost any reinsurance program can be made to look exceptionally good or bad within a DFA model by simply mis-pricing the coverage. We modeled DFAIC's existing reinsurance program which has a large component of per occurrence excess of loss coverage attaching at $500,000 in combination with a per risk excess cover on commercial property[20] and a property catastrophe cover attaching at $50 million. We created an alternative reinsurance structure wherein the per occurrence and per risk covers are replaced with an accident year aggregate stop loss contract covering 75% of 20 loss ratio points excess of 80.

In deriving the prices for the various reinsurance contracts, we erred on the side of conservatism specifically to avoid making the current program look bad or the alternative look good. That is, we priced the current program at a rate that we believe is slightly biased toward the low end of the reasonable range, thereby increasing overall ceded loss ratios. Likewise, we priced the aggregate contract in the alternative program toward

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[18] Unless the LOBs are perfectly positively correlated, some diversification will be achieved.

[19] Throughout this case study we have assumed the company cannot achieve a gain by purchasing under-priced reinsurance.

[20] We assumed that DFAIC's $20 million excess $1 million per risk excess cover applied exclusively to property in its CMP program. In doing so, we implicitly assumed that there was no homeowner exposure above $1 million. If this were a "live" DFA study this assumption might deserve additional consideration. Further, we assumed that the $50 million occurrence aggregate had a minimal impact on the coverage.

the higher end of the reasonable range[21]. Our intent was to make it slightly more difficult to discard the current program in favor of the alternative aggregate coverage.

Finally, the required capital under the current and alternative reinsurance programs is roughly one-third of DFAIC's statutory surplus.[22] Actual capital in excess of required capital was the only capital constraint that we imposed on the alternative reinsurance programs considered for DFAIC. We note that a capital-oriented approach, where alternative programs are judged by changes in the company's required capital, could have been employed to evaluate alternative reinsurance programs. Below we present one interesting reinsurance finding based on a capital-oriented approach, but we chose to define "efficiency" of the company's reinsurance program in terms of the stability of loss ratios and the shift in the company's efficient frontier for the remainder of the case study.

Risk Based Capital (RBC) was introduced by the NAIC in the 1990s to supplement the then-existing solvency early warning tests. More recently, rating agencies including A.M. Best and S&P have introduced their own brand of capital adequacy ratio. The underlying tenet of these ratios is that the combined charges for various risk factors provide guidance as to the amount of capital required by an organization. The ratio of actual capital to capital required (as determined by the risk factor charges) is the "capital adequacy ratio". We can use these ratios to compare the expected performance of alternative reinsurance programs within the context of a DFA model.

Working with the NAIC RBC factors, we calculated the probability of DFAIC's actual capital falling below the required capital level at any time in a five-year period under three scenarios: (1) no reinsurance; (2) current reinsurance; and (3) the alternative reinsurance program. The results are presented in the Table 6.

Table 6: Probability of Actual Capital Falling below the Required RBC Level

Scenario	Probability*
No Reinsurance	1.2%
Current Reinsurance	1.6%
Alternative Reinsurance	0.4%

** Cumulative probability over five years.*

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[21] The accident year aggregate cover provides approximately $375 million of coverage for approximately $94 million, a rate on line of roughly 25%. Considering the duration of the expected payments, we believe that this is a reasonably conservative price for the contract.

[22] Philbrick, Stephen and Robert Painter, "DFA Insurance Company Case Study, Part II: Capital Adequacy and Capital Allocation," *Casualty Actuarial Society Forum*, Summer 2001. Arlington, VA: Casualty Actuarial Society.

First, observe that these probabilities are very low. DFAIC is a strongly capitalized company with an RBC ratio of over 300%[23]. Second, it is not surprising that the alternative reinsurance program reduces the likelihood of "impairing" surplus because it results in a more stable distribution of net loss ratios (see Table 7 below). Finally, the interesting conclusion is that DFAIC actually *increases* its chance of impairing surplus by purchasing its existing reinsurance program!

This somewhat odd finding occurs because the company "trades dollars" with its reinsurer, not because reinsurance is over priced[24]. In other words, DFAIC reinsures losses that on average occur every year; they incur additional expenses (e.g., the reinsurer's profit) which increases their probability of failing the capital adequacy test. Capital testing, as in the RBC example above, can be used to choose between alternative reinsurance programs. However, we took a slightly different approach, by screening potential reinsurance programs based on loss ratio variability, then comparing selected programs based on economic risk/reward.

Note that we could just as easily have compared several alternative reinsurance programs based on the risk/reward analysis, using the economic value of surplus, statutory surplus, GAAP equity or some other metric, without reviewing loss ratio variability. In fact, screening reinsurance programs based on loss ratio variability arguably is not DFA because although the process includes the impact of the simulated economic conditions (i.e., inflation) on losses and premium, changes in asset values are ignored. Nonetheless, we have included it to emphasize that there are many ways to use DFA (in this case a single DFA model) to conduct such an analysis. Another, perhaps more important motivation for using the loss ratio analysis was that without a thorough understanding of the key drivers of our results, our analysis may be subject to criticism. As we will see the loss ratio analysis provides that understanding.

We screened several alternative reinsurance programs for possible use in the case study including two variants of DFAIC's existing program substituting $1 and $5 million retentions on the per occurrence contracts. We also considered and ultimately settled upon an accident year aggregate cover in place of all of the company's non-catastrophe coverage. To illustrate the process, the net loss ratios from accident year 2 and the corresponding standard deviations of net losses are summarized in Table 7.

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[23] DFAIC's statutory surplus is more than three times the minimum surplus, according to the risk based capital formula. Under the more conservative assumptions underlying the rating agency capital adequacy ratios, this ratio drops to roughly two times the minimum.

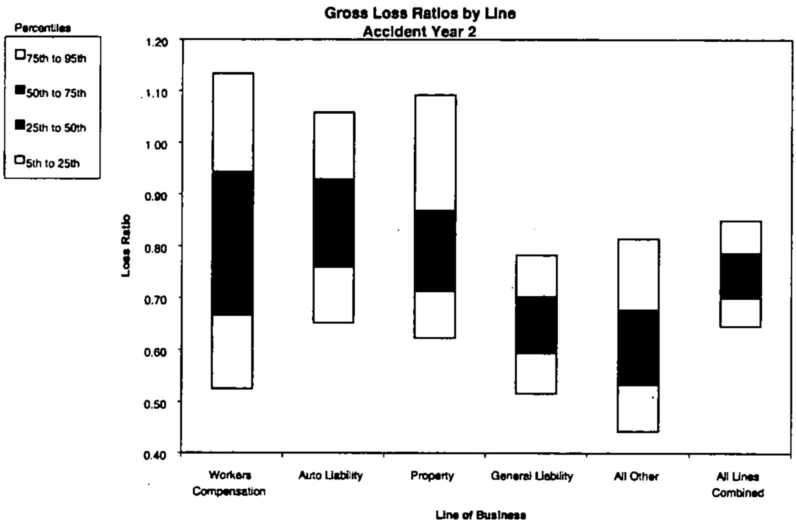
[24] The ultimate ceded loss ratio modeled for the current program (including the catastrophe contract) was 80.6% versus 80.0% for the alternative program. The alternative program's ceded payments were much more volatile than under the current program but its duration was also much longer. We assumed that the longer duration adequately compensated the reinsurer for the increased volatility.

Table 7: DFAIC's Net Loss Ratio and Standard Deviation of Net Losses (Acc Yr 2)

Reinsurance Structure	Description	Net Loss Ratio	Standard Deviation of Net Losses
--	Gross	81.4	136,172
1	Current	82.9	118,807
2	Current w/ $1M Ret.	82.8	119,517
3	Current w/ $5M Ret.	84.7	121,849
4	Aggregate	83.7	91,027

The holistic approach underlying structure 4 is primarily responsible for the improvement in net loss ratio standard deviations. That is, by focusing on the company's overall loss ratio and seeking to reinsure only those losses that in the aggregate (across LOBs), exceed acceptable limits, DFAIC improves the efficiency of its ceded reinsurance program. Exhibit 3 shows the variability in gross loss ratios by LOB and the corresponding reduction in variability of the overall loss ratio. Hence, DFAIC achieves most of its efficiency gain by merely recognizing the diversifying effect of writing more than one LOB and by purchasing reinsurance that recognizes this characteristic of their business.

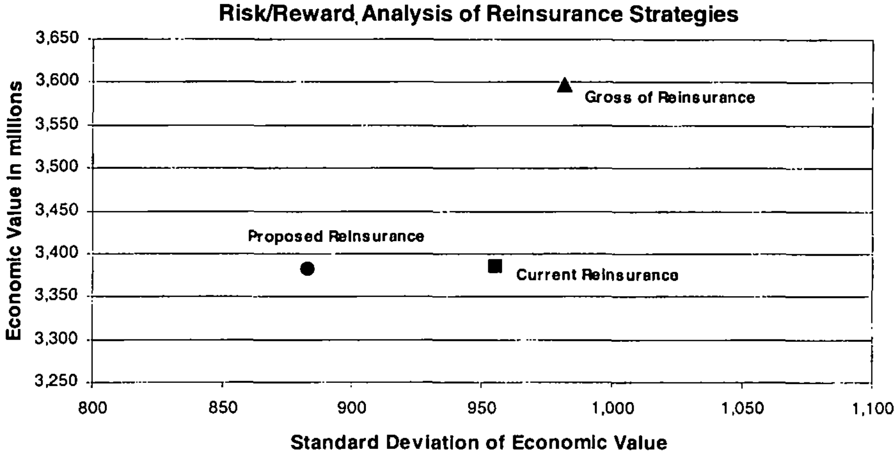
Exhibit 3: DFAIC's Gross Loss Ratios by Line of Business



Referring to Table 7, we compared net loss ratios to determine if the various programs were reasonably priced. Then we eliminated structures 2 and 3 from further consideration in the case study because they were not significantly different from the existing program. Finally, we selected structure 4 as the alternative structure for the case study because it produced a significant reduction in net losses' variability. The alternative reinsurance structure (structure 4) replaced the company's per risk and per occurrence coverage with an accident year aggregate stop loss; the catastrophe coverage was unchanged.

In our third and final approach, we reviewed the risk/reward profile of the current and alternative reinsurance programs. The process is illustrated herein using one alternative to the current reinsurance program, but there is no limit on the number of such alternatives that could be considered. Our risk/reward analysis is based on the economic value of the company's surplus (reward) and the standard deviation of the same (risk). We plot these figures on a simple graph with risk on the X-axis and reward on the Y-axis (see Exhibit 4). Points that are up (greater reward) and to the left (lower risk) are preferable to those that are down and to the right (lower reward/greater risk).

Exhibit 4: Risk/Reward Plot of Alternative Reinsurance Programs



First observe that the gross result, being the highest and furthest to the right, provides the greatest return but at the greatest risk. This is consistent with our pricing assumption that the company cannot achieve an economic gain through cheap reinsurance. Of course, we could easily relax that assumption if market conditions justified it, but for the purposes herein we have not. Second, observe that current and alternative reinsurance programs have similar costs[25], but the alternative program has a significantly lower risk. That is, the alternative program produces roughly the same economic value but it does so more consistently. Hence the alternative program is more economically efficient than DFAIC's existing reinsurance.

Finally, we return to the CEO's reinsurance questions: do we have enough reinsurance, is it efficient and what types should we consider? The company's likelihood of impairing its capital adequacy ratio (not probability of ruin) is very low (see Table 6) even without reinsurance, so it could be argued that from a rigid economic point of view, reinsurance is unnecessary. Such an approach might be deemed reckless by regulators and/or rating agencies, or management might prefer more stable earnings, so some reinsurance might be warranted. Based on the capital adequacy ratios, the current program provides sufficient coverage, however the alternative program also provides

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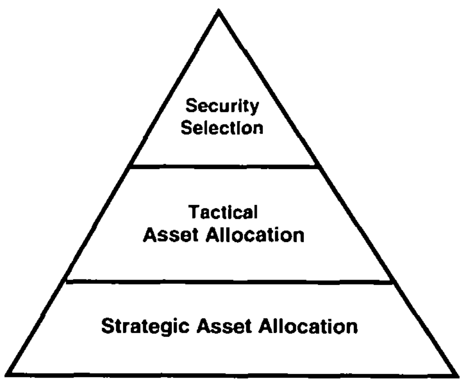
[25] The programs have similar costs in terms of the economic values that they produce even though the ceded premium in the alternative program is roughly 25% lower than for the existing program. This is because they have been priced to yield a similar overall return to the reinsurer. Hence, the reinsurer's rate of return is much larger on average under the alternative program to compensate for the increased volatility of the ceded losses.

such coverage and is superior based on the RBC test and in the economic risk/reward analysis. Hence, we would recommend that DFAIC adopt the alternative reinsurance program.

The analysis of loss ratios presented in the parameterization section of this paper hints at the reasons for preferring the alternative program. DFAIC's existing reinsurance program essentially covers each LOB individually. In doing so, it does not account for the diversifying effect of writing more than one LOB. In fact, it even provides coverage for large claims when aggregate losses in a particular line are lower than expected.

Thus far our discussion has focused on reinsurance, holding the company's asset allocation constant, but dynamic financial analysis can also be used to evaluate and set strategic asset allocation (SAA) guidelines for property-casualty insurance companies. Strategic asset allocation is the basis of a sound investment process that includes tactical asset allocation and security selection (see Figure 3). We will demonstrate that the company's reinsurance and asset strategies are interdependent and that by adopting the alternative reinsurance program DFAIC can alter its asset strategy to improve returns and reduce risk in both economic and statutory terms. Furthermore, our analysis of reinsurance and asset allocation will rely upon identical risk/reward metrics rather than traditional, but not comparable, strategy specific measures (e.g., loss ratios, return on assets, etc.).

Figure 3: Investment Process



Strategic asset allocation sets the investment targets, ranges, operational constraints and investment restrictions that are part of a company's investment policy statement. Tactical asset allocation (TAA), on the other hand, allows for shifts in the strategic asset allocation targets, subject to the strategic ranges, based on short- to intermediate-term economic and market outlooks. The goal of TAA is to outperform the results that would be achieved from strict adherence to the SAA. Security selection refers to the buying and selling of specific securities. Whereas tactical asset allocation attempts to add value by correctly adding to or reducing the amounts placed into individual asset classes, security selection attempts to add value by outperforming the benchmark indexes used to proxy the individual asset classes.

In this paper we deal with only the strategic asset allocation component of the investment process, as DFA is not an appropriate tool for performing tactical asset allocation or security selection. While many strategic questions could be addressed, given the limitations of information about DFAIC, we will concentrate on the following three major strategic investment issues:

- the target fixed-income duration;
- the target allocation to equities; and
- The target split between taxable and tax-exempt bonds.

To address these issues we will make use of the optimization algorithm within the FIRM system to identify DFAIC's efficient frontier[26]. The objective function will be the one discussed in step 1 of the DFA process. Our first efficient frontier will be subject to the continuation of the current reinsurance program. Exhibit 5 shows DFAIC's economic value efficient frontier at the end of five years along with the position of their current asset allocation strategy.

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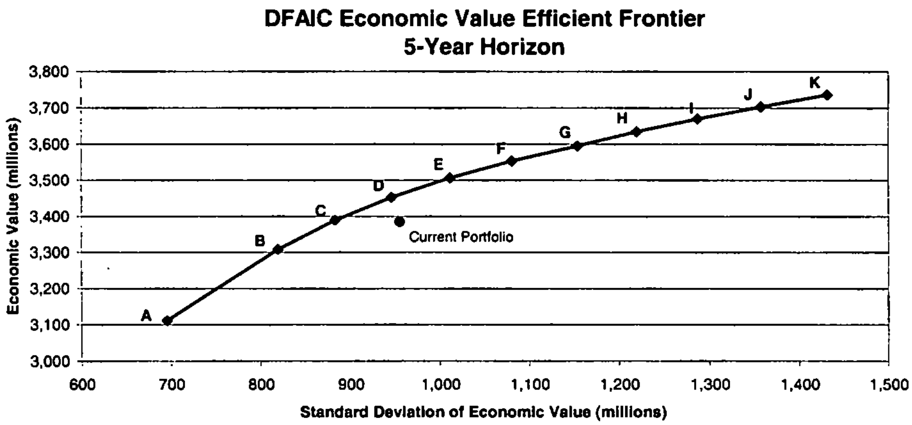
[26] See Markowitz.

Exhibit 5: DFAIC's Economic Value Efficient Frontier

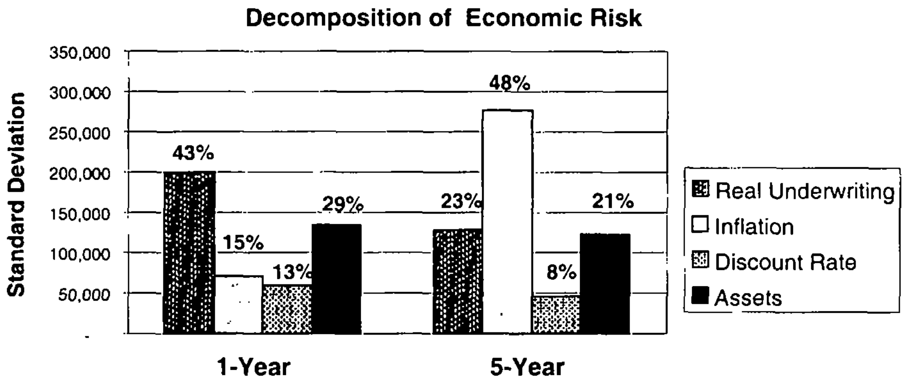
The economic value efficient frontier for DFAIC shows a low-risk, investment strategy (Strategy A) that consists of short duration, taxable fixed-income securities and no equities. Moving up the efficient frontier into higher return/higher risk strategies involves lengthening the duration of the fixed-income portfolio, moving into tax-exempt securities and increasing allocations to equities.



	Current	A	B	C	D	E	F	G	H	I	J	K
Fixed Income:												
Taxable	88.0	100.0	95.4	84.6	72.7	60.3	51.7	38.0	29.3	20.3	7.7	2.0
Tax-Exempt	-	-	-	4.0	10.0	18.9	20.0	35.1	37.1	39.5	47.3	48.0
Duration	5.0	0.3	2.1	2.6	3.2	4.1	4.0	6.1	6.0	6.0	6.6	6.4
Equity:	11.2	-	4.5	11.4	17.3	20.8	28.3	26.9	33.6	40.2	45.0	50.0

To help understand the asset strategies on DFAIC's economic value efficient frontier it is necessary to understand DFAIC's risk exposures. Again, DFA can be used to do this through a technique called decomposition of risk. By applying decomposition of risk techniques we can identify the impact that various factors have on DFAIC's economic risk. We can then use this information to gain insights into the logic behind the strategies recommended by the economic value efficient frontier. Exhibit 6 shows the impact of real underwriting , inflation, discount rates, and asset returns on the economic risk of DFAIC over one-year and five-year time horizons for the current investment strategy[27].

Exhibit 6: Decomposition of Economic Risk



As can be seen from Exhibit 6, the impact of the four risk factors on economic risk depends on the time horizon. Neither inflation nor discount rates are significant risk factors over short time periods. The major risk to DFAIC over a one-year horizon is, not surprisingly, real underwriting uncertainty. The picture changes dramatically when considering a five-year time horizon. Underwriting risk tends to diversify over time whereas inflation risk will tend to accumulate. Thus, inflation uncertainty becomes the biggest risk to the economic well-being of DFAIC over the long term.

This explains why low-duration, fixed-income securities appear as the low-risk investment strategy on the five-year economic efficient frontier. Low-duration, cash equivalent investments tend to move hand-in-hand with inflation, helping to offset the impact of unexpected inflation. Unfortunately, low-duration fixed-income strategies result in low yields and low expected returns. Thus following a low-risk investment strategy is an expensive way of reducing the economic risk of DFAIC.

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[27] Underwriting volatility typically includes the impact of inflation but for the purposes of asset strategy it is helpful to separate underwriting volatility into the amount due to inflation and the amount due to loss uncertainty net of inflation. Loss uncertainty net of inflation is assumed independent of asset strategy but loss uncertainty as a result of unexpected inflation is a risk that can be reduced through strategic asset allocation.

This also helps explain the role of equities across the efficient frontier. Equities, in addition to their higher expected returns, provide for a long-term inflation hedge. When the additional diversification benefits of equities are considered, it becomes clear why the addition of equities together with a reduction of the fixed-income duration results in a higher-reward, lower-risk investment strategy.

These results may be surprising to those who advocate duration-matching strategies as a way to minimize risk. Duration matching is predicated on the fact that interest rate sensitivity is the major source of economic risk. This is true for many financial instruments such as bonds where the future cash flows are fixed and certain. The liability characteristics of DFAIC, however, are anything but fixed and certain. Instead they are subject to substantial underwriting uncertainty as well as the whims of unexpected inflation. Because of the significant correlation between interest rates and inflation, changes in interest rates will typically be accompanied by changes in inflation rates. Further, higher inflation rates will lead to higher loss payments which will counteract the economic benefit of a higher discount rate. Thus, controlling only the interest rate risk through a duration matching strategy, when liabilities are inflation sensitive, is an ineffective and inappropriate way of controlling financial risk and can lead to an unintended and severe exposure to unexpected inflation[28].

The final issue to explore concerning DFAIC's economic value efficient frontier is the role of tax-exempt investments. As there were no tax statements provided for DFAIC, information concerning their tax position had to be gathered from their statutory filings. Before serious tax planning can occur, we would want additional information concerning DFAIC's tax reserves, net operating loss carryforwards (NOLs) and capital loss carryforwards. For this study, we assumed no operating loss or capital loss carryforwards and we estimated tax reserves to be a constant ratio to calculated statutory reserves.

The traditional approach for determining DFAIC's optimal allocation to tax-exempt investments is to adjust the tax-exempt allocation to the point that equates the regular tax liability to the alternative minimum tax liability under the company's deterministic budgeted forecast. This methodology for tax management planning can lead to an inefficient allocation to tax-exempt securities since it fails to take into consideration the volatility of the company's projected profitability and the changing relationship between taxable and tax-exempt yields over time.

A much more robust approach to determining the optimal tax-exempt allocation for DFAIC can be identified through the use of dynamic financial analysis. The yield relationships between taxable and tax-exempt fixed-income securities were first simulated based on a combination of historical yield analysis and current market conditions. Similarly, DFAIC's operating results were simulated based on their historical loss performance and current business plans. In this way the after-tax investment income penalty that results from holding tax-exempt securities in unprofitable years can be evaluated against the after-tax investment income advantage of holding tax-exempt

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[28] Inflation sensitivity is a parameter in the Swiss Re FIRM system. Different inflation sensitivity assumptions will result in different efficient investment strategies. If liabilities are assumed to be insensitive to inflation, duration-matching strategies may be more effective at mitigating risk.

securities in profitable years. The model also determines whether the alternative minimum tax (AMT) is required and factors the AMT penalty into the analysis accordingly.

As a result, a prospective model of DFAIC's tax liabilities under many possible scenarios was evaluated. The optimization model found the allocation to tax-exempt securities that maximizes DFAIC's reward objective within the bounds of the company's risk tolerance. Based on our model of DFAIC and the assumed future business plans, the probabilities of negative taxable income for each of the next five years under the current asset investment strategy are indicated in Table 8.

Table 8: DFAIC's Taxable Income

	Mean (000s)	Probability of Negative
Year 1	47,791	30%
Year 2	128,251	21%
Year 3	196,870	18%
Year 4	227,064	13%
Year 5	264,481	10%

The above table is consistent with the loss ratio improvements built into Falcon's business plan assumptions for DFAIC. The increasing expected income levels combined with the decreasing probability of negative income results suggests that tax-exempt investments should very well have a role in the investment strategy for DFAIC over the five-year planning horizon.

Now, assuming that Falcon management is happy with DFAIC's current risk tolerance level, the investment strategy can be adjusted to that indicated by Strategy D (see Exhibit 5). This would suggest increasing the equity allocation from 11.2% to 17.3%, reducing the fixed-income duration from 4.9 to 3.2 and allocating 10% of the fixed-income portfolio into tax-exempt securities.

The move to Strategy D results in a $67.8M increase in economic reward over the five-year horizon without any additional increase in economic risk. The next step is to examine the statutory implications of such a strategy. Exhibit 7 shows the impact to DFAIC's statutory surplus under both the current asset strategy and Strategy D.

Exhibit 7: DFAIC's Statutory Surplus Comparisons

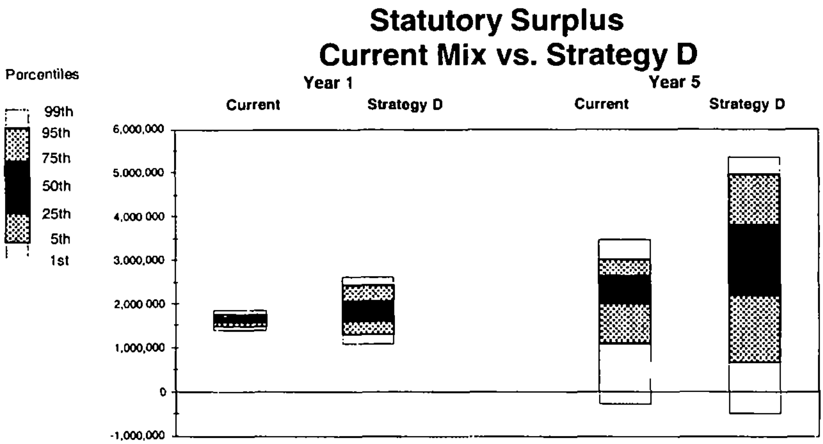
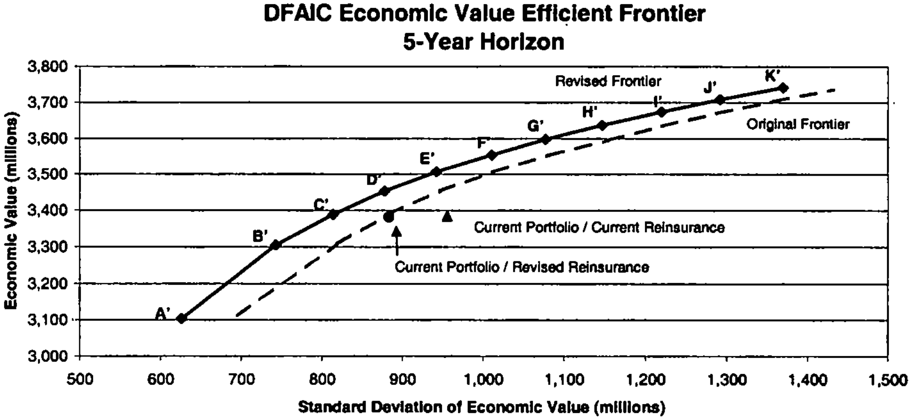


Exhibit 7 shows that while there is no increased economic risk from moving to Strategy D, there is additional statutory risk over both a one- and five-year horizon. Management is left to decide whether the increased economic reward is great enough to compensate for the increased statutory risk.

We are thus left with determining whether this is the optimal investment strategy on the efficient frontier. Stated another way, does Strategy D result in the greatest value added to DFAIC given their objectives and risk tolerance? This issue is addressed in "Beyond the Frontier: Using a DFA Model to Derive the Cost of Capital", by Daniel Isaac and Nathan Babcock.

The final part of our DFAIC analysis is to examine the impact on the investment guidelines under the revised reinsurance program. To do this we generated a second efficient frontier assuming the reinsurance program was based on implementing an accident year aggregate cover in place of DFAIC's existing program. Exhibit 8 shows both the revised and the original efficient frontiers.

Exhibit 8: DFAIC's Economic Efficient Frontier (Revised Reinsurance)



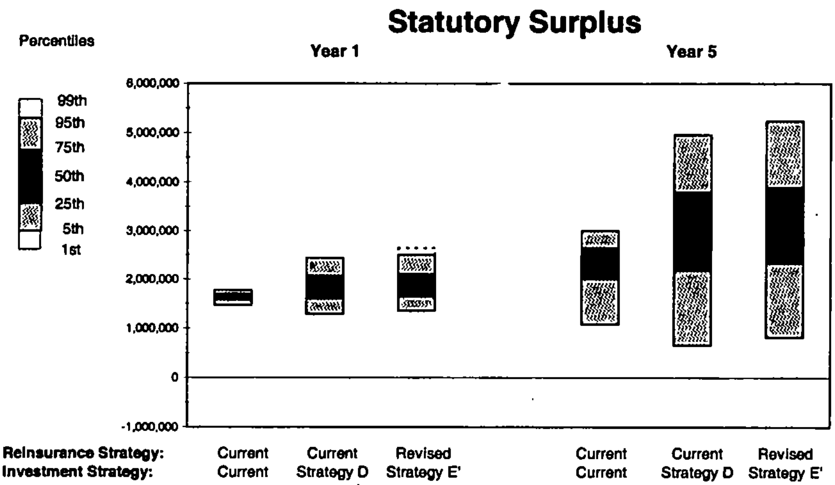
	Current	A'	B'	C'	D'	E'	F'	G'	H'	I'	J'	K'
Fixed Income:												
Taxable	88.0	100.0	87.9	80.3	68.7	57.7	44.4	36.7	29.8	18.3	7.3	2.0
Tax-Exempt	-	-	1.7	9.6	17.3	22.0	32.9	34.2	34.2	43.4	47.8	48.0
Duration	5.0	0.3	1.6	3.1	4.0	4.5	5.6	5.5	5.4	6.5	6.6	6.4
Equity:	11.2	-	10.4	10.2	14.0	20.3	22.7	29.1	36.0	38.3	44.8	50.0

Exhibit 8 shows that the revised reinsurance program pushes the efficient frontier for DFAIC up and to the left. This is extremely desirable from Falcon management's perspective as this means that DFAIC can experience higher economic rewards at lower economic risk levels. The individual strategies that make up the revised efficient frontier tell a similar story to those on the original frontier. The overall lower risk profile as a result of the revised reinsurance structure, however, allows DFAIC to move to a more aggressive asset strategy without any more economic risk than the company is currently experiencing.

Again assuming that Falcon management is happy with DFAIC's current risk tolerance level, under the revised reinsurance program the investment strategy can be adjusted to that indicated by Strategy E'. This would suggest increasing the equity allocation from 11.2% to 20.3%, reducing the fixed-income duration from 4.9 to 4.5 and allocating 22% of the fixed-income portfolio into tax-exempt securities. The net effect of moving to the revised reinsurance program and an asset strategy in line with that suggested by Strategy E' is an additional expected economic benefit of more than $121M over the five-year planning horizon.

Returning again to the statutory implications, of this combined reinsurance and investment strategy. Exhibit 9 shows the impact to DFAIC's statutory surplus relative to the current asset strategy of changing only the investment strategy and of changing both the reinsurance and investment strategy.

Exhibit 9: DFAIC's Statutory Surplus Comparisons



By simultaneously increasing the efficiency of their reinsurance strategy and investment strategy, DFAIC accomplishes a better economic risk/reward profile and is able to achieve a better statutory profile at the end of the five-year horizon. Thus by considering DFAIC's business holistically, our analysis indicates that we can implement a revised reinsurance strategy and take a more aggressive asset strategy, resulting in an expected economic benefit and improved long-term statutory results.

Finally, using the Tail Conditional Expectation (1-Year) approach from our sister paper ("DFA Insurance Company Case Study, Part II: Capital Adequacy and Capital Allocation ", by S. Philbrick and R. Painter), we found that the new reinsurance program coupled with asset strategy E', increased the company's required capital by only 6%. Thus, DFAIC's actual capital is still significantly above the required minimum level. Additional details and changes in required capital under other capital adequacy measures (e.g., RBC capital adequacy ratios) can be found in that paper.

Step 6: Sensitivity Testing

Sensitivity testing is required to ascertain that the conclusions are not the product of a particular set of assumptions or the result of a particular set of random scenarios. This step in the DFA process requires the testing of key input factors such as renewal rates, inflation and interest rate sensitivity of future premiums and liability payments, changes in capital market equilibrium assumptions, and variability of loss ratios. Sensitivity testing highlights the major factors affecting each business segment and the degree to which those factors affect each segment. Each factor needs to be tested independently, and relevant factors should be tested in tandem. Sensitivity testing allows for the assessment of the individual as well as collective impact of modifying key factors by business segment.

Since the underlying framework for DFA is simulation, sensitivity testing should include research into the number of simulations required to assure that the results of the analysis are robust. The required number of simulations will depend on many factors such as whether the analysis is dealing with relative comparisons or absolute levels. The metrics used for the objectives and constraints will also impact the required number of simulations. For example, downside risk measures typically require more simulations than simple standard deviations. There is no magic number or formula that tells exactly how many simulations are required for a particular analysis, so the user is left to ascertain, through sensitivity testing, that the findings of the DFA study are robust and can be easily reproduced.

Step 7: Presentation of Findings

The importance of the presentation of DFA findings should not be underestimated. While the DFA professional has the benefit of months of analysis in developing understanding of the problems, issues and solutions, they must summarize and present their findings to the senior management or Board of the company briefly and succinctly. This is no easy undertaking. The presentation of the DFA study should do more than show the numbers and present the conclusions, rather the presentation should tell a story. The story should review the highlights of each step of the DFA process and lay out the logic that went into the analysis in such a way that the conclusions become evident before they are revealed. It is important to keep in mind that the value of DFA is not just in the answer but also in the increased understanding of the issues that lead to the answer and ultimate decision.

Conclusion

Armed with the DFA results, the CEO of Falcon is ready to move on to negotiation stage of the acquisition with the knowledge that his holistic approach to insurance company management can produce a better, more efficient DFAIC. However, the competitive advantage of a holistic approach to insurance company management anticipated by our fictional CEO might be short-lived. The ability to perform holistic analysis through DFA has largely been made possible by the recent advancements in computing power and speed. These advancements, combined with sophisticated DFA models and dedicated DFA professionals, have brought the power of Dynamic Financial Analysis to within reach of all interested property-casualty companies.

One final note: The results of this DFA study, while raising some general insurance industry issues, are specific to the objectives, characteristics and assumptions that we used for DFAIC. DFA is not a trivial endeavor. Even given a good DFA modeling system, the analysis that is performed can be poor. A good DFA analysis will tie the conclusions to the assumptions in a clear and concise manner. The impact of alternative strategic initiatives will be explained in such a way that someone who is unfamiliar with the details of DFA will still be able to follow, understand and ultimately accept the stated conclusions. While the undertaking is not trivial, the potential efficiencies that can be gained through a holistic approach to property-casualty insurance management can be significant to those who are willing to supply the effort.

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DFA Insurance Company Case Study, Part II: Capital Adequacy and Capital Allocation

Stephen W. Philbrick, FCAS, MAAA and
Robert A. Painter

Dynamic Financial Analysis
DFA Insurance Company Case Study
Part II: Capital Adequacy and Capital Allocation

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Abstract

This paper has been submitted in response to the Committee on Dynamic Financial Analysis 2001 Call for Papers. The authors have applied dynamic financial analysis to DFA Insurance Company (DFAIC) to address capital adequacy and capital allocation issues. The DFA model used for this analysis was the Swiss Re Investors Financial Integrated Risk Management (FIRM™) System. This paper is Part 2 of a two-part submission. Part 1 deals with using DFA to explore reinsurance efficiency and asset allocation issues.

This paper explores different general risk measures used in the past to judge capital adequacy. This overview of various risk measures will incorporate the concept of coherent risk measures. It introduces a practical method for using Tail Conditional Expectation (TCE) as a measure of capital adequacy. We will look at the adequacy of DFAIC's capital position using the TCE risk measure along with other more widely accepted regulatory and rating agency capital adequacy measures for different reinsurance/asset allocation strategies.

Additionally, we will discuss different risk measures associated with capital allocation, including TCE, along with different allocation procedures. This section will also explore the idea of allocating capital to assets. Different allocation methods will be discussed and the Shapley Value method, found in game theory, will be applied to two different risk measures to allocate DFAIC's current capital to line of business and to assets.

Dynamic Financial Analysis
DFA Insurance Company Case Study
Part II: Capital Adequacy and Capital Allocation

By Stephen W. Philbrick, FCAS, MAAA,
and Robert A. Painter

Preface

Dynamic Financial Analysis (DFA) is still fairly new to a property-casualty insurance industry whose roots can be traced back to the 17th Century and earlier. As such it is not surprising that the industry is cautious about a technology that purports to look at their business in a whole new way. The Casualty Actuarial Society, being active in the formulation and development of DFA, has classified it as:

> *"a systematic approach to financial modeling in which financial results are projected under a variety of possible scenarios, showing how outcomes might be affected by changing internal and/or external conditions[1]."*

As a result of published papers, shared research and call paper programs such as this one, the technical specifications behind DFA have been well developed. This has led to a high level of convergence among many of the different concepts, models and processes behind DFA. Unfortunately, while the details of DFA are better understood, the industry is still scratching its collective head on what to do with this new technology.

Part of the problem has to do with the fact that DFA is mainly considered to be a modeling tool, one that can be used to supplement existing tools. While a modeling tool is essential for implementing dynamic financial analysis, it is just one element of a much grander picture. More than a model, dynamic financial analysis is a way of thinking that weaves through the entire operations of an insurance company. Effective dynamic financial analysis calls for dedicated and knowledgeable professionals who are trained in the intricacies of DFA and enabled to identify and take advantage of current industry and company inefficiencies. DFA promotes moving from existing structures designed to evaluate and reward the individual pieces of the business to a structure that encourages and rewards the evaluation of strategic decisions in a holistic, total company framework.

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[1] Casualty Actuarial Society Dynamic Financial Analysis Website, DFA Research Handbook, http://www.casact.org/RESEARCH/DFA

For these reasons we were excited to embrace this call paper program exercise. While the original concept may have been designed to evaluate different DFA modeling techniques and the resulting analyses as they relate to a common problem and common data, we decided it was a perfect opportunity to show how DFA might work in the insurance company of tomorrow. The ultimate benefit to the company is not just the final answer, but rather the increased understanding and the common grounds of communication that comes from going through the DFA process.

The proposed situation involves DFA Insurance Company (DFAIC), a multi-line property-casualty insurance company that is unknowingly the target of a potential acquisition. The analysis was conducted from the point of view of the acquiring company. We will define the acquiring company, Falcon, as a newly capitalized holding company that is organized and structured to run its business in a holistic manner. Falcon has a financial risk management unit led by its Chief Risk Officer (CRO) who reports directly to the CEO. The CEO has asked that the following questions about DFAIC be addressed:

1. Is the Company adequately capitalized? Is there excess capital? How much capital should the Company hold as a stand-alone insurer?

2. How should the capital be allocated to line of business?

3. What is the return distribution for each line of business and is it consistent with the risk for the line?

4. Should the Company buy more or less reinsurance? What type? How efficient is its current reinsurance program?

5. How efficient is the asset allocation?

In a traditional insurance company these questions would be farmed out to different business units within the organization. These units would include but not be limited to the actuarial department, the reinsurance department and the investment department. Each unit would perform their stand-alone analysis and report back to the CEO using terminology and metrics appropriate to their assigned task. The CEO would be left to assimilate all the individual analyses and use professional judgment and insights to build a complete picture of the attractiveness of the potential acquisition.

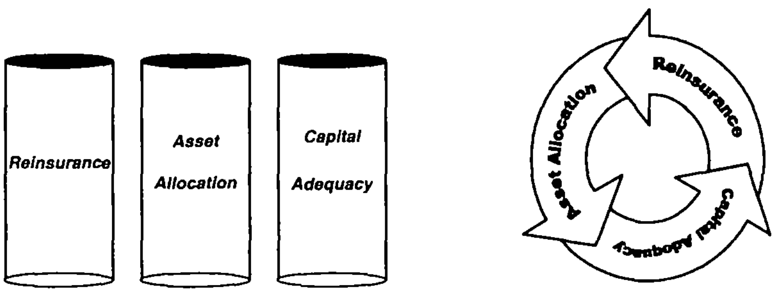
Falcon, however, is organized in such a way that the complete analysis can be performed within the financial risk management unit with input from professionals in each of the departments mentioned above. The results of the analysis can thus be presented to the CEO using a single set of terminology and metrics that consider both the individual and joint dynamics of the issues in question.

Due to the scope and breadth of the required analysis, we will present the DFA study in two papers. This paper will deal with the capital adequacy and capital allocation issues and a sister paper will concentrate on reinsurance and asset allocation strategy issues. Note that despite breaking the analysis up into two papers, the overall analysis is the result of a common DFA model and process.

DFA, being holistic, allows a company to deal with all of its major strategic decisions simultaneously within a single framework. As such it is not unusual to have an analysis that continuously revisits these strategic levers in what we call the DFA spiral. This is in contrast to the traditional approach in which these strategic decisions are evaluated each in their individual silos. Figure 1 gives a graphical picture of these two different approaches.

Figure 1

Traditional Analysis **Dynamic Financial Analysis**



Unfortunately, a paper does not easily lend itself to a spiral analysis, so for the sake of convenience we will first complete a single loop around the DFA spiral, holding the strategic decisions that relate to other sections constant. This will allow us to show how DFA can be used to deal with individual strategic initiatives but still within a holistic framework. We will then begin a second loop taking into consideration the strategic initiatives suggested as a result of the initial loop. This will allow us to identify and discuss the additional opportunities that result from simultaneous changes to two or more strategic initiatives.

This paper concentrates on capital adequacy and capital allocation issues. While information concerning revisions to the reinsurance program and asset allocation will be stated, the interested reader should refer to the sister paper "Dynamic Financial Analysis, DFA Insurance Company Case Study, Part I: Reinsurance and Asset Allocation" [11] for a detailed description of the methodology used in the development of these numbers.

Roadmap

This paper will:

- Set forth the seven steps of The DFA Process—an approach to think about DFA.
- Discuss several risk measures, then use a TCE measure, which satisfies the axioms for a coherent risk measure.
- Apply a DFA approach to a specific case study—the DFAIC hypothetical company supplied by the CAS.

First, the DFA Process will be described. The steps of this process will be used throughout the rest of the paper to organize the discussion.

The next section will begin with a general discussion of capital adequacy. This will be followed by a brief discussion of prior work on this issue and the direction taken in recent research. Next, we will discuss three measures of capital adequacy, and then discuss the general concepts underlying any risk measure.

Next, we will discuss three capital adequacy measures used by regulators and rating agencies. We will then explain why Tail Conditional Expectation (TCE) is selected as the measure of risk over the other three choices. Because the concept of TCE may be new to many readers, and it is the selected method in this paper, we will go into that measure in somewhat more detail than the other two methods. Then we will summarize the results of each of the capital adequacy measures for DFAIC.

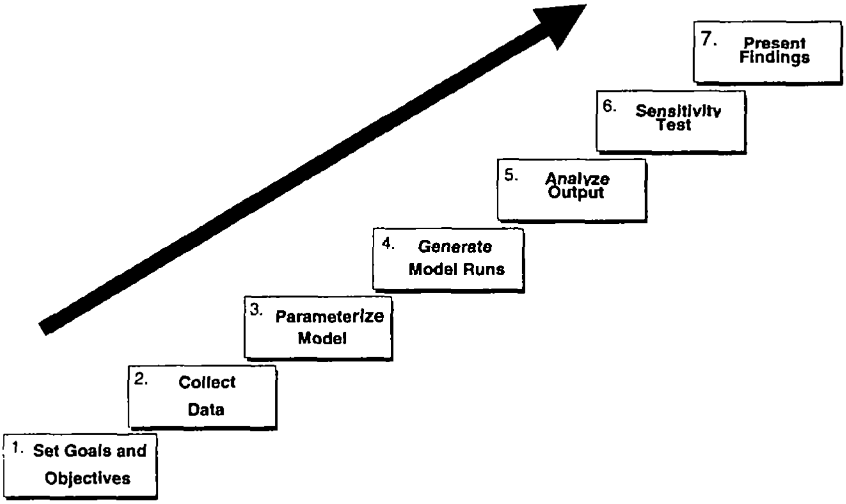
Finally, we will discuss the concept of capital allocation, and show how a TCE measure can be used to allocate capital to segments of DFAIC.

The DFA Process

The DFA Process refers to a high-level overview of how a DFA model can be brought to bear on a specific problem [13]. We have outlined, in Figure 2, the DFA process that we used for our analysis of DFAIC.

Figure 2

The Dynamic Financial Analysis Process



It is critical to understand that DFA is more than just a model. The development of a computer model can be viewed as "step zero" of the process. It is a necessary step, but it represents the development of a tool, rather than the DFA process itself. The DFA process starts with a thorough discussion and understanding of the goals, objectives, constraints and risk tolerance of a company. This step determines the metrics that will be most important in evaluating alternative strategic initiatives. It also tends to be a valuable exercise as it helps management think through, focus on, and communicate exactly those items that are most important to them as a company. These items are stated in terms of financial statement results and, once determined, provide a common set of metrics that can be applied to all of the company's financial strategic decisions.

Steps 2 through 4 of the DFA process depend on the specifics of the DFA modeling system that is being used for the analysis. Whereas a common DFA process allows for effective and efficient sharing of concepts and ideas, it could be argued that different modeling methodologies and assumptions are healthy in order to address the potential problem of model bias (model risk) and assumption bias (parameter risk).

In order to become comfortable with a particular modeling system for implementing DFA, one must understand both the methodology that underlies the system and how that particular methodology will impact the results of the analysis. By DFA model methodology we refer to the specific technical implementation of the DFA process.

Whereas the general DFA process has become fairly standardized, there are still a number of different methodologies that are used in the technical implementation of a DFA model. Since the technical implementation of a model can have a significant impact on the results of an analysis, it is imperative that the users of a model sign off on the technical implementations and understand how the specific model methodology will impact the analysis. The risk that model results are specific to a particular DFA methodology is referred to as "model risk." This is a difficult risk to evaluate; due to the time, effort and expense of performing DFA, it is often impractical to duplicate the analysis using different DFA modeling systems. As such, users should look for systems that provide a significant amount of flexibility and whose underlying fixed methodologies are consistent with their views of the insurance and financial markets.

At Swiss Re Investors, we developed our Financial Integrated Risk Management (FIRM™) System as the modeling tool backing our DFA process. The FIRM System, like most DFA systems, uses simulation techniques to model both the assets and liabilities of an insurance company. The projected cash flows are transformed into future balance sheets and income statements that reflect GAAP, statutory, tax and economic viewpoints. The simulations are generated by a series of stochastic differential equations that are designed to allow the model user to reflect a full range of distributions, dynamics and relationships with respect to the underlying stochastic variables. The tool is designed to allow a high level of flexibility in describing how the underlying stochastic variables behave in an attempt to minimize model risk. This increase in flexibility, however, has the result of moving a significant burden from the model, to the model builder and the model assumptions. Interested readers can find additional information on the mechanics of the Swiss Re Investors FIRM System by referring to our previous CAS DFA call papers.

Assumptions and model parameterization are closely tied to methodology in that they also deal with the technical details of DFA. DFA model assumptions refer to how the asset and liability variables are assumed to behave over the forecast horizon. The major difference between methodology and assumptions is that assumptions can be changed whereas methodology, within a particular system, is generally fixed. Assumptions used in DFA modeling can have a substantial impact on the recommended strategies. In the modeling world this risk is referred to as "parameter risk." The impact of parameter risk can be substantially reduced through the use of sensitivity testing and by having the analysis performed by experienced DFA professionals.

Steps 5 and 6 of the DFA process relate to analysis and sensitivity testing. While there is still some connection to the modeling system used for the analysis, the effectiveness of these steps are more a function of the DFA professional. Even given a good DFA modeling system, the analysis performed can be poor. A good DFA analysis will tie the conclusions to the assumptions in a clear and concise manner. The impact of alternative strategic initiatives will be explained in such a way that someone who is unfamiliar with the details of DFA will still be able to follow, understand and ultimately accept the stated conclusions. Sensitivity testing is required to ascertain that the conclusions are not the product of a particular set of assumptions or the result of a particular set of random scenarios.

Finally, the presentation of the DFA study (step 7) should do more than show the numbers and present the conclusions. Rather, the presentation should tell a story. The story should review the highlights of each step of the DFA process and lay out the logic that went into the analysis in such a way that the conclusions become evident before they are revealed. It is important to keep in mind that the value of DFA is not just in the answer but also in the increased understanding of the issues that lead to the answer and ultimate decision.

The remainder of this paper will explore the assumptions and model details that we used in performing our DFA on DFAIC. Several of the steps are identical to the steps in our sister paper on reinsurance and asset allocation. Rather than repeat those steps, we refer the reader to the discussion in that paper. In this paper, we will discuss the aspects that are unique to the adequacy and allocation analysis. For easy reference, the discussion of the parameterization of DFAIC will be included as Appendix A and B.

Capital Adequacy

Adequacy of capital is critical to a consumer of insurance products. In many companies, the product is delivered at the time of purchase. While a consumer, for example, may have some legitimate interest in the ongoing solvency of a manufacturing company to provide access to spare parts, an insurance product is, at its core, a promise to deliver in the future. The ability to make good on its promises is critical to the insurance company.

The actuarial literature contains many papers on the subject of capital adequacy. The CAS commissioned an annotated bibliography of relevant research papers on the subject. The bibliography is contained in a report by Brender, Brown and Panjer [10]. This report was completed in July 1992. This year was a good year for capital adequacy research for another reason—the CAS issued a call for papers on Insurer Financial Solvency. Those papers are contained in the 1992 Discussion Papers on Insurer Financial Solvency [1]. The early work on capital adequacy focused on the underwriting side of the balance sheet. Over time, various papers have incorporated more sophisticated treatment of assets. [2], [13], [22], [29], [33] This has proceeded through:

- Recognition of investment income (acknowledging the existence of assets, but treating assets as largely fixed)
- Recognition of asset variability, but treatment of asset variability as independent of underwriting variability
- Recognition of asset volatility as well as the economic interdependencies between assets and liabilities

While analytic and simulation techniques have both been used in a variety of papers, the complex nature of the interactions of assets over time and of the relationship between assets and liabilities virtually requires a simulation approach, typically embodied in a Dynamic Financial Analysis (DFA) model. A recent paper by Mango and Mulvey [27] describes a DFA approach to the capital adequacy and allocation problems.

The evolution of capital adequacy has proceeded in another dimension as well. In addition to more sophisticated handling of assets, the analysis of the risk measure has become more refined. Early papers concentrated on the probability of ruin, that is, the probability that the firm would become insolvent. While this is clearly an important issue, it emphasizes the owners of the firm over other interested parties. More recent research has extended this concept in two ways:

1. Recognition that the amount of insolvency, not just the probability, matters to policyholders, or at least to the insolvency funds that must pay in the event of insolvency. As a consequence, regulators are interested in the cost of insolvency, not just the likelihood. [12]

2. Formal recognition that firms care about surplus reduction even when it doesn't result in insolvency. While this isn't a new idea, more sophisticated DFA models can be used to analyze reductions in surplus of less than 100%. These options are useful for examining the likelihood of ratings downgrades.

Discussion of Risk Measures

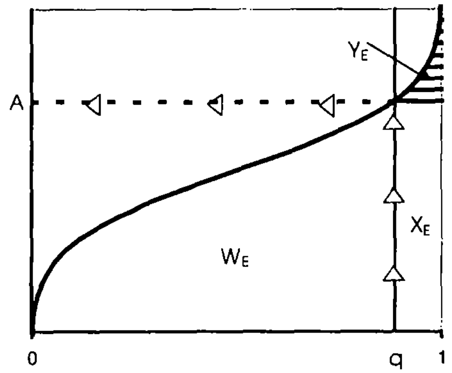
The risk measures we will discuss in this section by no means define the universe of possible risk measures. These are some the prominent measures that have emerged in the literature. There is no single measure that is recognized as the best, but some have appealing properties that make them more relevant to the discussion of capital adequacy.

Probability of Ruin, or Ruin Theory, is probably the most intuitive risk measure when discussing capital adequacy: how likely is it that I will be able to stay in business over a given time period? This paper defines Probability of Ruin in its most general sense: the probability that a given variable or event is below some defined limit over a defined period of time. This measure is dependent on the target company selecting a fixed minimum capital limit where they would define themselves as "ruined". This is a binary process where either the company is ruined or not ruined—there is no contemplation of degree of ruin in this risk measure. It is necessary to emphasize that that selection of risk variable and risk limit and tolerance levels should be based on the individual circumstances and goals of the company. Mango[27]

Probability of Ruin is closely associated with Value at Risk (VaR), a concept that originates from the banking industry. For banks, VaR would be the maximum amount the bank could potentially lose over a time period in which they could not react to market conditions. This might be the amount they could lose from financial positions left open overnight while the bank is closed. In an insurance context, the concept of the company not being able to react to market conditions has been ignored due to the much longer time frames being evaluated in solvency analysis.

Figure 3 shows the inverted cumulative distribution of results for a given financial variable. The Y-axis measures the magnitude of the financial variable. The X-axis is the percentile of the corresponding financial result. Given a risk tolerance criterion of α, α is defined as 1-q. Following the arrows up from q to the intersection with the distribution and over to A, the VaR is the dollar equivalent for a given risk tolerance α.

Figure 3



$A = F^{-1}(q)$	; A = VaR
$\int_{F(A)}^{1} [F^{-1}(x) - A]\, dx = Y_E$	; A = Point where Y_E = EPD Tolerance

A second approach commonly used to measure capital adequacy is Expected Policyholder Deficit (EPD). Whereas Ruin Theory only takes into account the probability of insolvency, EPD considers the magnitude of ruin. EPD incorporates the fact that not all insolvencies are the same. Regulators, policyholders, and debtholders care about the amount by which the company will not be able to fully meet its obligations. As a result, the criterion for this risk measure is defined by a tolerable amount of obligations that will not be met. This EDP criterion can be stated as either a dollar amount or as a percentage of total obligations, and is represented in Figure 3 as the shaded area Y_E. EDP and the distribution can be expressed in terms of many different financial variables. In Figure 3, total obligations are equal to $W_E + X_E + Y_E$. Point A, as defined by the tolerance area Y_E, is the level of obligation that the company can handle without being in a "deficit position".

The two prior measures are intuitively appealing, but were developed ad hoc. The likelihood that a company might become insolvent seems like a logical risk measure. Similarly, the extension to the cost, rather than simply the probability of insolvency seems like an obvious improvement. Nevertheless, neither approach was developed using the axiomatic approach of mathematics—to first identify desirable properties of a measure, then mathematically search for measures that meet the criteria. In recent years, researchers have taken this approach. A thorough discussion of the selection of the axioms, and the resulting measures, called coherent risk measures is beyond the scope of this paper. However, because we use a coherent risk measure as a critical part of our analysis, and the concept is still relatively new to many people, Appendix C contains a brief introduction to the concept of coherent risk measures, including the underlying axioms.

The third approach used to measure capital adequacy is a coherent risk measure, Tail Conditional Expectation (TCE). [3], [4], [5], [30]. Tail Conditional Expectation combines the ideas behind VaR and EPD into a single measure. In order to calculate the TCE result, a TCE risk tolerance criterion must first be selected. The VaR tolerance is a function of a selected percentile along the x-axis, whereas EPD tolerance is a function of a selected area. The TCE tolerance is conceptually similar to the VaR tolerance in that it is based on selecting an appropriate point along the x-axis. In Figure 4 the TCE tolerance[2] is equal to $1 - q = \alpha$. Referring to Figure 4, again the sum of all potential events is equal to $W_T + X_T + Y_T$. All results to the right of the vertical line, defined by the TCE tolerance α, are considered "tail events". The sum of these tail events is equal to $X_T + Y_T$. The average tail event is equal to the Tail Conditional Expectation. Graphically, the TCE is equal to the height of the $X_T + Z_T$ such that the area of $(X_T + Z_T)$ equals the area of $(X_T + Y_T)$.

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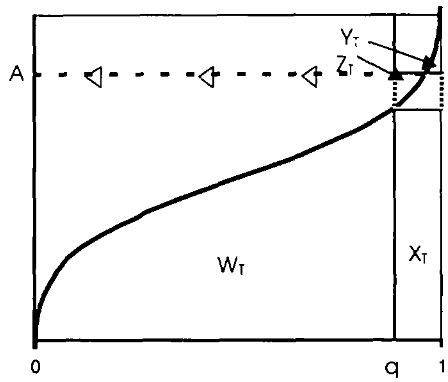
[2] For a VaR tolerance of α_v and a TCE tolerance of α_t, if $\alpha_v = \alpha_t$ and $F^{-1}(x)$ is a continuously increasing function, then TCE Required Capital ≥ VaR Required Capital

Figure 4



$$A = \frac{\int_q^1 F^{-1}(x)dx}{1-q} \quad ; A = TCE$$

While these three approaches differ in important ways, there is a common theme. In each case, the analysis of capital adequacy proceeds in these four steps:

1. Select a Financial Variable
2. Select a Time Frame
3. Select a Measure
4. Select a Criterion

Financial Variable

The main decision for the financial variable is how much of the balance sheet to incorporate—whether the emphasis will be on liabilities or both assets and liabilities. In the former case, aggregate losses may be the financial variable; in the latter case, surplus. Secondary considerations:

- Should all liabilities be modeled or just loss and LAE?
- Should the accounting basis be statutory valuation, GAAP valuation, or some other basis?

Time Frame

The time frame represents the period of time over which the analysis is performed. In principle, this can be unlimited. Some work in ruin theory looks at unlimited time horizons, but this requires assumptions about future business that are unrealistic if interpreted as true projections about infinite time horizons.

For time periods other than unlimited, it may be necessary to clarify what is meant by the time frame. For example, does a one-year time frame mean that balance sheets and income statements are simply projected forward one year? Or does it mean that one additional year of new business is written, and then all outstanding liabilities are run off? A third alternative (common in valuation exercises) is to project one year's worth of business, including both new and renewal business, and then to include renewal business only, along with the liability runoff, for a specified number of renewal periods, or until the renewal business becomes de minimis. Any projection should clarify which basis is being used.

Typical time frames for insurance companies are one, three, and five years. Projecting beyond five years becomes speculative.

Measure

The simplest measure is the Financial Variable itself (along with its associated distribution). Other measures, such as EPD and TCE, can be formed as a function of the distribution of the variable of interest.

Criterion

Finally, one must specify a critical value of the measure. Generally, this value will be used as a binary separator to distinguish between acceptable and unacceptable levels of capital.

Application

The generic approach described above applies to each of the three common approaches to capital adequacy:

- Ruin Theory - The financial variable is surplus. However, early historical approaches treated assets as if they were a constant, and treated liabilities as the only random variable. More recently, both assets and surplus are handled as random variables. The time frame can be unlimited in some circumstances, but it is typically a relatively short period of time (before runoff) in DFA studies. The measure is the surplus itself, considered as a random variable. The criterion is some suitably small value such as 0.01 or 0.005, representing the probability that the financial variable can be less than zero in the selected time frame.

- Expected Policyholder Deficit - The financial variable is usually the aggregate liability distribution. The time frame typically ranges from one to five years. The measure is the EPD, which can be expressed as a function of the aggregate loss distribution. In words, it is the average loss amount in excess of the assets of the company, averaged over those situations in which the liabilities exceed the assets (that is, the company is technically insolvent). This amount can be expressed in dollars, or it can be expressed as a ratio to the expected liabilities to put it on a comparable basis across companies.

- Tail Conditional Expectation - The financial variable is typically aggregate liabilities, although surplus can be used. The time frame typically ranges from one to five years. The measure is TCE, which can be expressed as a function of the aggregate loss distribution. In words, it is the average aggregate loss amount (from ground up, rather than excess of some amount as in the case of EPD) for loss scenarios satisfying a criterion. As is the case with EPD, it can be expressed as a dollar amount, or it can be expressed as a ratio to total liabilities or total assets.

Introduction to DFAIC

DFAIC is the hypothetical company provided by the CAS for this exercise. This company is a privately held property-casualty company operating in all fifty states, writing personal lines and "main street" commercial coverages through independent agents. Key financial values:

- Current Assets — 5.381 billion
    - Total Fixed Income (Average Maturity) — 4.193 billion (7.4yrs)
    - Total Equity — 0.564 billion
    - Current Liabilities — 3.777 billion
    - Current Booked Loss+LAE Reserves — 2.330 billion

- Current Statutory Surplus 1.604 billion
- Previous Year Net Earned Premium Volume 2.409 Billion
- Projected Combined Ratio (Year 1) 107%

DFAIC currently holds per risk and per occurrence covers on all lines of business, along with a property CAT treaty. In total, the company cedes approximately 8% of premium.

Step 1:Goals and Objectives

The goal for the capital adequacy section of the analysis is to answer the first question in the Preface:

Is the Company adequately capitalized? Is there excess capital?

Our assignment is to determine how much capital the company should carry, as a theoretical exercise, and compare it to the capital requirements according to regulatory and rating agencies. The company will carry the largest of the alternative amounts. If the required capital exceeds the current amount of capital on its balance sheet, the company will consider various ways to increase the actual capital or decrease the need for capital. If the actual capital exceeds the necessary capital, the acquiring company can release the excess capital to the owners, or consider whether additional risk can be taken on. This could be in the form of increased writings, more aggressive asset risks, or reduced reinsurance.

Steps 2-4:Data Collection, Parameterization and Model Runs

- The data collection phase is discussed in Step 2 of our sister paper.
- The parameterization is discussed in Appendix A and Appendix B, although certain aspects of the parameterization are discussed in the allocation section of this paper.
- The generation of the model runs is discussed in Step 4 of our sister paper

Steps 5-7:Analyze Output, Sensitivity Test, Present Findings

We will look at the following three different commonly accepted capital adequacy measures to help us analyze DFAIC's capital adequacy: the NAIC's Risk Based Capital(RBC) [34], A.M. Best's Absolute Capital Adequacy Ratio(BCAR) [9], and Standard & Poor's Capital Adequacy Ratio(CAR) [40]. Additionally, we will develop a fourth capital adequacy measure based on Tail Conditional Expectation (TCE). The formulas behind the NAIC, Best, and S&P measures can be found below in Figure 5.

Risk Based Capital

The Risk Based Capital is one of the means the NAIC uses to monitor capital adequacy. Set forth in the early 1990's, the NAIC RBC Model Act specifies responsibilities for both the regulator and insurer [15]. These responsibilities are triggered when the RBC Ratio (RBC Adjusted Statutory Surplus/Risk Based Capital) falls below 100%. The degree and severity of action increases as this ratio decreases. [34]

Best's Net Required Capital

The Best's capital adequacy model is somewhat similar in structure to the RBC model. Some of the key differences between the two models are the following:

- Best's model is interactive (manual adjustments can be made to the outcome),
- it takes into account the quality of loss reserves,
- it explicitly considers quality of reinsurer, and
- it explicitly considers CAT risk. [8],[9]

Best's does make adjustments to the numerator of the Absolute Capital Adequacy Ratio for many different factors; for this discussion we will assume that these adjustments net out to zero. As a result, we will limit our discussion to the denominator of the ratio, the Net Required Capital (NCR). Best's model self-admittedly produces a significantly higher NCR number than RBC's minimum solvency requirement. In the late 1990's, Best recalibrated its loss reserve and premium risk factors to recognize the concept of Expected Policyholder Deficit (EPD). Generally, a company is considered "Vulnerable" if its Absolute Capital Adequacy Ratio is below 100%.

S&P CAR

The CAR calculation is one element that goes into the S&P Rating. The S&P process considers many of the same variables as both RBC and the Best capital adequacy model. As a general rule, a CAR of greater that 125% is considered "Strong". [40]

Figure 5: Capital Adequacy Formulas

RBC $= R_0 + (R_1^2 + R_2^2 + (.5 \times R_3)^2 + [(.5 \times R_3) + R_4]^2 + R_5^2)^{1/2}$

R_0 = Noncontrolled Assets and Growth Risk
R_1 = Fixed Income Investment Risk
R_2 = Equity Investment Risk
R_3 = Receivables Risk
R_4 = Net Loss&LAE Reserve Risk
R_5 = Net Written Premium Reserve Risk

Bests Absolute Capital Adequacy Ratio = Adjusted Surplus / Net Required Capital

Net Required Capital $= (B1^2 + B2^2 + B3^2 \cdot (.5xB4)^2 + ((.5xB4) + B5)^2 + B6^2 + B7^2)^{1/2}$

B1 = Fixed Income Securities
B2 = Equity Securities
B3 = Interest Rate
B4 = Credit
B5 = Loss&LAE Reserves
B6 = Net Written Premium
B7 = Off Balance Sheet

S&P CAR =

$$\frac{\text{Total Adjusted Capital - Asset Related Risk Charges - Credit Related Risk Charges}}{\text{Underwriting Risk + Reserve Risk + Other Business Risk}}$$

Total Adjusted Capital = Statutory Surplus +/- Loss Reserve Deficiency + Time Value of Money

TCE Required Capital Method

A graphical representation of and the method for calculating TCE Required Capital are presented in Figure 6 and Figure 7, respectively. Briefly, the TCE risk measure is applied to a distribution of simulated estimates of Required Assets to Cover Liabilities[3] at the end of Year 1 ($\tilde{A}_1$). $\tilde{A}_1$ is synonymous to simulated Statutory Surplus at the end of year 1 (individual simulated results) minus the Average Assets at the end of year 1.

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[3] This also takes into account of the volatility of assets.

The calculation of Statutory Surplus for this adequacy measure is on a basis where the company reserves to the exact ultimate at the end of year 1. This perfect knowledge adjusts both existing reserves and one year of new business to their ultimate undiscounted levels.

We have selected a one-year time frame for this measure because most regulatory measures tend to be over a one-year time horizon. Unlike many other measures that only take into account underwriting results, statutory surplus takes into account the volatility of both assets and liabilities, along with the interactions between the two.

Once a distribution of Required Assets to Cover Liabilities at the end of year 1 ($\tilde{A}_1$) is generated, the TCE risk measure is applied. First, a TCE Tolerance is selected. This selected tolerance (1% in this discussion[4]) represents the largest 1% of all potential outcomes for the financial variable $\tilde{A}_1$. For ease of discussion, these large tail events will be called "Large Losses"[5]. Looking to Figure 6, the events defined by the tolerance are equal to $X_T + Y_T$. The Average "Large Loss" is equal to the TCE Required Assets ($A_{1(TCE)}$). From Figure 6, this is equal to the height of $Z_T + X_T$, where the area of $(Z_T + X_T)$ equals the area of $(Y_T + X_T)$, which equals the sum of all "Large Losses". Finally, TCE Required Capital is the difference between TCE Required Assets ($A_{1(TCE)}$) and the Expected Liabilities at the end of year 1 ($E[L_1]$).

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[4] This 1% tolerance is the level we selected for DFAIC. More work needs to be done to explore appropriate tolerance levels for different company risk profiles. A company should select its own tolerance based on an understanding the individual risks it faces.

[5] "Large Loss" is a misnomer to the extent that asset volatility and other influences contribute to the tail event.

Figure 6

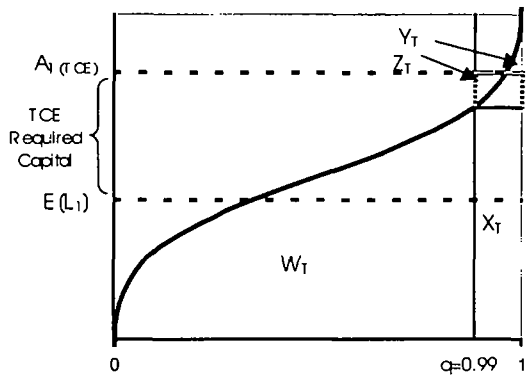

Figure 6 Identities:

1) Total Loss = $W_T + X_T + Y_T$

2) Total "Large Loss" = $X_T + Y_T$

3) Tolerance = 1 - q = 1 - 0.99 = 0.01

4) $Z_T = Y_T$

$$\text{TCE Required Assets} = A_{1\,(TCE)} = \frac{\int_q^1 F^{-1}(x)dx}{1-q}$$

The TCE Required Capital Method emphasizes the tail of the distribution which differs it from standard deviation or variance of financial variables. It specifically concentrates on the scenarios that might be specifically detrimental to solvency. These types of threat scenarios are the reason companies carry capital.

However, the TCE Required Capital amount produced from our DFA model does not take into account all events that could in real life initiate a tail event. For example, our model does not specifically simulate reinsurance credit default, and we have not adjusted results for such contingencies. The three common capital adequacy measures discussed above do attempt to take into account reinsurance credit issues. Our TCE Required Capital estimate should be adjusted upwards for such a potential event. There are many other occurrences, such as embezzlement and fraud, which should also be considered when determining an appropriate level of capitalization. Our DFA model, along with these four capital adequacy measures, does not adjust for such occurrences.

Figure 7: TCE Required Capital Method

Step 1: $\tilde{A}_1 = E[A_1] - \tilde{S}_1$ Step 2: Select a TCE tolerance Step 3: Given a TCE Tolerance, Calculate a TCE Required Assets = $A_{1(TCE)}$ Where F(x) is a function of $\tilde{A}_1$ Step 4: TCE Required Capital = $A_{1(TCE)} - E(L_1)$
<u>Where:</u> $\tilde{S}_1$ = Statutory Surplus at the End of Year 1 Individual Simulation (where it is assumed that the company correctly projects and books ultimate loss with perfect knowledge of future economic influences on payments) $E(A_1)$ = Expected Value of Total Assets at the End of Year 1 $A_{1(TCE)}$ = TCE Required Assets $E(L_1)$ = Expected Value of Total Liabilities at the End of Year 1

The DFA model runs produced the estimates of required capital found in Table 1 for the described capital adequacy measures. Before analyzing this model output, it is especially important to note that these outputs are the result of thousands of stochastic simulations. Adequate modeling of the tail is especially important for the TCE Required Capital measures. Additionally, the modeler should run enough stochastic simulations to produce robust output. The number of simulations should be selected such that the level of sampling error is within an acceptable range. The level of sampling error is determined through sensitivity testing. (Step 6 of the "DFA Process")

Table 1

Estimates of Required Capital (Amounts in $Millions)				
	Best's Net Required Capital	Risk-Based Capital	2 x Risk-Based Capital	TCE Required Capital
End of Year 1	1,223	494	988	805

DFAIC currently holds 1.6 billion in statutory surplus. The Best's calculations suggest a required capital of slightly over 1.2 billion. (It should be emphasized that not all aspects of the Best's formulas are public; this calculation represents an estimate based upon what is known about the formula.) The RBC value is much lower, but the RBC value is not intended to produce an acceptable capital requirement. A company carrying the RBC amount would not be immediately shut down, but it would find itself under intense regulatory scrutiny. This company decides to carry at least twice the RBC value to keep the regulators happy. In this instance, double the RBC amount is still less than the number indicated by the Best's calculations.

The company also looks at the S&P formula. The mean S&P Capital Adequacy Ratio at the end of the year will be 265, using their present capital, projected to year-end. This is well above the S&P limit of 125.

If there were no rating agencies or regulatory authorities, the company would be comfortable with the TCE Required Capital indication of 0.8 billion. That this value is lower than the regulatory and rating agency values either indicates that those formulas are slightly more conservative than the assumptions selected for the TCE calculation, or that the riskiness of DFAIC is lower than companies of comparable size and underwriting mix. The regulatory and ratings agency formulas attempt to reflect some of the specific aspects of each company, but also reflect industry averages to some extent. Additionally, the TCE Required Capital estimate did not adjust for quality of reinsurance issues; making an adjustment for this should increase the TCE Required Capital. Also, the TCE Required Capital has been calculated in a DFA/ALM framework which considers the interactions and co-movements of the assets and liabilities. These interactions and co-movements can have diversifying effects which will soften the blows of tail events driven by inflation, especially when the company is maintaining a buy and hold fixed income strategy. These interactions can only be captured in an integrated DFA/ALM modeling process. The regulatory and agency measures do not, and realistically can not, incorporate the diversification benefits between assets and liabilities. This effect is more apparent when looking over a longer time horizon. However, even over this very short one-year time horizon there is a slight effect.

After considering all of the risk measures, the company concludes that it will be able to reduce the carried capital by a significant amount without impairing the adequacy of the capital, either as measured by the external (regulatory and rating agency) entities, or by the internal calculation.

As a result, DFAIC looks into alternative reinsurance and asset allocation strategies. All of these alternative strategies are discussed in our sister paper. Ultimately the company decides to explore replacing its current per occurrence reinsurance program with a more efficient aggregate cover. Additionally, in conjunction with this change in reinsurance program, DFAIC decides to increase its asset exposure by increasing its equity allocation from 11% to 20%.

Under this revised reinsurance/asset strategy the different estimates of required capital are the following:

Table 2

Estimates of Required Capital (Amounts in $Millions End of Year 1)				
	Best's Net Required Capital	Risk-Based Capital	2 x Risk-Based Capital	TCE Required Capital
Current Strategy	1,223	494	988	805
Revised Strategy	1,238	532	1,064	839
Percent Change	+1.2%	+7.7%	+7.7%	+4.2

The change in regulatory and agency adequacy measures increased almost solely due to the increase in allocation to equities. The liability components of these formulas remained almost constant; these measures were unable to react to a new, more efficient reinsurance cover. As stated earlier the TCE Required Capital measure is driven by tail scenarios. Comparing the tail "Large Loss" simulations for DFAIC shows that the TCE Required Capital reacts to the change in reinsurance and asset allocation differently than the regulatory and agency measures. The analysis of scenarios showed that the TCE Required Capital reacted in a way consistent with what really occurred. The TCE Required Capital increase was driven by the more aggressive asset strategy, but this increase was dampened by the revised, more efficient, reinsurance structure.

Capital Allocation

Roadmap

The capital allocation section will start out with an introduction, discussing some of the controversy surrounding the concept of allocation, and resolving the issue by noting that capital allocation is better thought of as an approach to allocate the cost of shared capital. We will then discuss some of the prior research in this area, highlighting the work on marginal surplus, which led to variance-covariance measures. Next, we will discuss the axiomatic development leading to a Shapley value calculation, and show how this equates to the variance-covariance measure, under an assumption of an overall risk measure based upon standard deviation. As we did in the prior section, we will adopt a coherent risk measure, TCE. This measure will be implemented in a DFA model, and applied to the hypothetical company DFAIC. We will outline the goals of the approach, summarize the required parameterization of the DFA model, discuss certain aspects of the model runs, and then analyze the output of the DFA model, concluding with some observation of how the TCE allocation compares with other classical approaches.

Introduction

In one respect, the issue of capital allocation is as controversial a subject as there is within the actuarial profession. For many subjects, there may be disagreement among professionals as to the best approach, or formula or distribution to use in certain circumstances. However, in the case of capital allocation, there are professionals arguing, not about the best formula, but whether it should be done at all. [6] The opponents to capital allocation have an excellent point—all of the capital of a legal entity is available to pay the claims of any line of business or policy. It is arguably misleading to allocate surplus to a line, as that amount does not serve as a limit on the company's obligation to pay claims.

The proponents of capital allocation usually aren't interested in the assignment of an amount of capital to a line as an end product, but rather as an intermediate result, as part of an exercise to determine required rates of return for a line, policy or block of business.

The resolution may be to realize that the goal of the exercise isn't allocation of capital, but allocation of the **cost** of capital, as Stefan Bernegger[6] called it.

[6] This comment was made at an internal company actuarial meeting

When an insurance company writes a policy, a premium is received. A portion of this policy can be viewed as the loss component. When a particular policy incurs a loss, the company can look to three places to pay the loss. The first place is the loss component (together with the investment income earned) of the policy itself. In many cases, this will not be sufficient to pay the loss. The second source is unused loss components of other policies. In most cases, these two sources will be sufficient to pay the losses. In some years, it will not, and the company will have to look to a third source, the surplus, to pay the losses.

The entire surplus is available to every policy to pay losses in excess of the aggregate loss component. Some policies are more likely to create this need than others are, even if the expected loss portions are equal. Roughly speaking, for policies with similar expected losses, we would expect the policies with a large variability of possible results to require more contributions from surplus to pay the losses. We can envision an insurance company instituting a charge for the access to the surplus. This charge should depend, not just on the likelihood that surplus might be needed, but on the amount of such a surplus call. We can think of a capital allocation method as determining a charge to each line of business that is dependant on the need to access the surplus account. Conceptually, we might want to allocate a specific cost to each line for the right to access the surplus account. In practice though, we tend to express it by allocating a portion of surplus to the line, and then requiring that the line earn (on average) an adequate return on surplus. Lines with more of a need for surplus will have a larger portion allocated to them, and hence will have to charge more to the customers to earn an adequate rate of return on the surplus. Effectively, this will create a charge to each line for its fair share of the overall cost of capital.

Step 1:Goals and Objectives

The CEO's question related to allocation was,

> *How should the capital be allocated to line of business?*

We now realize that this is the intermediate goal—our ultimate goal is the determination of a charge to a line (or policy) for the access to capital. The opening sentence of the abstract in Kreps [23] embodies this concept—that the determination of allocated capital is intermediate to determining the charge for capital (risk load):

> *The return on the marginal surplus committed to support the variability of a proposed reinsurance contract is used to derive an appropriate risk load for reinsurers.*

Kreps selected a ruin theory based risk measure:

> *For example, if the distribution is Normal, then a z of 3.1 is a 1/1000 probability, and an amount of surplus given as above will cover the actual losses 999 years out of 1000 years, on average.*

While the risk measure is formally a ruin theory measure, he assumed a particular distributional form, so that the risk measure is also a standard deviation measure[7]. Gogol [18] and Mango [26] note a problem with this measure. As Mango says:

> *However, problems arise when these marginal methods are used to calculate risk loads for the renewal of accounts in a portfolio. These problems can be traced to the order dependency of the marginal risk load methods.*

Both arrived at the same solution, in terms of a formula: the risk load should be proportional to the variance of the additional contract plus the covariance of the contract with the rest of the portfolio. This contrasts with the Kreps approach, which effectively produces a risk load proportional to the variance plus twice the covariance. While the results were the same, the approaches were different. Gogol proved his result as a theorem using return on surplus assumptions [19]. Mango applied a game theoretic approach as outlined in papers by Lemaire [24], [25]. In brief, Mango and Lemaire applied an approach called the Shapley value.

The marginal approach to surplus requirements can be thought of as follows:

Given a company writing a block of business, consider the addition of a new contract. Calculate the surplus requirements for the portfolio without the new contract, and then with the new contract. The increase in required surplus represents the marginal surplus required by the addition of the contract. The risk load, or capital charges, can be made proportional to the marginal surplus. We can think of this process as a "last-in" process. That is, how much capital is needed if this contract is the last one added to the portfolio. The Shapley value can be thought of as a logical extension to this concept. Rather than treating every contract as if it were the last one in, calculate the marginal surplus requirement over all orders of entry. That is, how much surplus would be required if it were the first one in (sometimes called the stand-alone approach), how much would be required if it were the second contract written, the third, etc.? Then the surplus requirement is calculated as the average over all possible orders of entry.

It is important to note that, while this is a convenient way of explaining how the calculation can be done, it isn't a description of how the formula was derived. Similar to the way the TCE approach was developed, Shapley selected a few desirable axioms, and derived the result from the axioms. Thus, the resulting value is not arbitrary, but the result of a theoretically sound basis. The calculation of the Shapley value can get cumbersome, particularly for a large number of contracts or lines of business. Mango's insight was to show that the formula based upon the variance and covariance is equivalent to the Shapley value [26]. Thus, this formula produces a theoretically sound approach to capital allocation, if one accepts the overall standard deviation risk measure for the entire portfolio.

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[7] There is potential confusion in the terminology of the risk measure. Kreps' risk measure is proportional to standard deviation at the portfolio level, but is a function of the variance and the covariance at the contract level. Thus, describing the risk measure as a standard deviation, variance, or covariance-based measure could be accurate, depending on whether the measure is viewed at the level of the total company portfolio, or the individual portfolios, represented by either contracts or lines of business.

However, as we have discussed earlier, the standard deviation measure does not conform to the coherence axioms for risk measures. The TCE measure does satisfy those axioms. Consequently, when we chose to allocate the capital to each line of business, we chose the TCE measure as the risk measure. We aren't aware of a simplification to the calculation parallel to the one shown by Mango, so we applied the Shapley method to the TCE measure. We used the formula in Lemaire [24].

Steps 2-3:Data Collection and Parameterization

In setting up our model, we condensed DFAIC's business into five distinct lines: Workers Compensation, Auto Liability (both personal and commercial), Property (homeowners and CMP property coverage), General Liability (other liability, product liability, special liability, and CMP liability coverage), and All Other Miscellaneous Lines (predominantly auto physical damage). For ease of discussion, we will refer to the combined miscellaneous lines as Auto Physical Damage (APD). Segregation of business into these five lines allows for the effective modeling of reinsurance programs without burying results within a mass of detail. Each of these five lines is assigned a set of descriptive parameters to appropriately model its constituent line of business. Needed parameterizations relate to such items as premiums, losses (including loss adjustment expenses), other expenses, and payment patterns, as well as their stochastic properties. A preliminary step in our analysis involved restating historical results to be consistent with our five modeled lines of business[6]. Table 3 summarizes some the attributes that define these five modeled lines of business.

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[6] E.g. CMP results were segregated into property or casualty and allocated to our Property or General Liability lines of business, respectively.

Table 3: Key Liability Values

Line of Business	Previous Year Net Earned Premium (Millions)	Average Accident Year Duration (Years)	Current Booked Loss+LAE Reserves (Millions)	Modeled Mean Year 1 Combined Ratio
Workers Comp	209	3.9	555	113
Auto Liab	764	2.4	924	120
Home/CMP(Prop)	525	1.3	316	106
Auto Phys Dam	671	0.9	83	94
GL/CMP(Liab)	239	3.8	452	96
All Lines Total	2,409	2.1	2,330	107

See Appendix A for a more detailed explanation of the Liability parameterization.

Model parameterization refers to how the asset and liability variables are assumed to behave over the forecast horizon. Economic and capital market assumptions are an important part of any quantitative assessment of the potential rewards and risks associated with alternative strategic business decisions. The model that we used to generate our DFA economic and capital market simulations (FIRM™ Asset Model) differs from traditional mean/variance models in that economic variables, including interest rates and inflation, are explicitly modeled using accepted and rigorously tested stochastic processes. Details of the economic and capital market model parameterization can be found in Appendix B along with Step 3: Parameterization in our sister paper. DFAIC currently holds approximately 11% of its invested assets in equities. The majority of the remainder is invested in high quality fixed income instruments with an average maturity of approximately 7 years.

DFAIC's current reinsurance program includes excess of loss coverage for property, liability, and workers compensation risks, as well as coverage for catastrophes. In order to model the effects of these and alternative treaties, we generated individual large losses and occurrences on a gross of reinsurance basis. This necessitates the development of both frequency and severity probability distributions within the context of a collective risk model. Both company-specific and industry experience were gathered and analyzed for this purpose. Once the collective risk model was parameterized, individual large losses and catastrophes were generated stochastically and reinsurance covers were applied to obtain simulated losses, by line of business, net of reinsurance.

Step 4:Model Runs

The model runs needed for capital allocation are much more extensive and complex than those needed to determine capital adequacy, especially when using the Shapley value allocation method.

The Shapley value method, as discussed above, compares the marginal differences in some risk measure from adding a single individual to a coalition of individuals. For Shapley, the risk measure must be calculated the number of times indicated by the formula in Figure 8. For capital allocation, the DFAIC model has 6 individuals: 5 lines of business along with an allocation to assets. Therefore Shapley requires 63 different risk measure calculations.

Figure 8

$$\sum_{i=1}^{n} \binom{n}{i}$$

The number of required calculations grows exponentially as the number of individuals grows linearly. The DFA model becomes very large as the desired level of detailed allocation increases. As can be imagined, this can become expensive in terms of required computer runtime and the amount of memory needed to store the model output.

Shapley allocates to individual parts of the company by comparing the company "with and without" all combinations of the individual parts. From a practical perspective, how does one look at an insurance company without a line of business, or more interestingly, without assets. The method used in this paper adapts this "with and without" concept to looking at the company with and without the volatility associated with a certain line of business or asset portfolio. In real life companies use reinsurance to manage their liability risk, and adjust their asset allocations to manage their asset risk. This is the approach this method has taken for looking at a company with and without an individual source of risk.

The DFA model has been parameterized to sequentially reinsure away all combinations of the five lines of business. We have applied loss portfolio transfers to the lines of business to remove the risk from the existing business, and have applied aggregate covers to reinsure away the future business. All reinsurance treaties have been priced on an economically neutral basis to mimic the company as if the business had never been written.

The first instinct of many might be to define minimum asset risk as investing all assets at the risk free rate. However, this is not true in an ALM/DFA framework, where risk is defined by the combined impacts of both assets and liabilities and their interactions. This method defines the minimum asset risk portfolio as the least risky portfolio on the economic efficient frontier. (See our sister paper for a full discussion of the efficient frontier) The minimum economic risk asset portfolio has been calculated for each of the 31 line of business combinations. (See Figure 8 where n=5)

In the past, the volatility of assets has often not been recognized when discussing both capital adequacy and capital allocation. Many of the previous allocation methods concentrate on the risk associated with their respective line of business losses. In a DFA framework, where the entire balance sheet is holistically modeled, the contribution of asset volatility to surplus volatility can recognized. Historically, P&C insurance has thought of assets very differently than it has thought of liabilities. In fact, the differences when considering balance sheet risk are almost non-existent. Assets, like workers comp or auto liability, are just another element of the overall riskiness of the company. The allocation of capital to assets is a realization that the investment department is required to produce a higher return for a more risky investment strategy.

This is best described through a heuristic example. Table 4 displays the capital required for two different asset strategies where the underwriting is held fixed.

Table 4

Asset Strategy	Required Capital
Less Risky Investment Strategy	$100
More Risky Investment Strategy	$200

The company currently is operating under the less risky investment strategy. If the company does not allocate capital to its investment department then the $100 would be split up between the lines of business. The investment managers then decide to move to a more risky investment strategy that doubles the total capital required by the company. The line of business managers are not going to accept this increase in capital allocated to their lines along with the increased return to premium they will be forced to produce to hit their target return on surplus. The investment department should be allocated a portion of this capital on which they should be forced to meet a target return.

Steps 5-7:Analyze Output, Sensitivity Test, Present Findings

The Shapley Value allocation method has been selected to allocate DFAIC's capital for their current net of reinsurance position. Table 5 shows the results of this allocation for two different risk measures: Standard Deviation of Statutory Surplus at the end of Year 5, and TCE Required Capital discussed in the capital adequacy section of this paper. For comparative purposes, Table 5 also includes results using the Marginal "Last-In" allocation method.

The Standard Deviation of Statutory Surplus measure considers the volatility of surplus 5 years in the future assuming DFAIC maintains its historical reserving practices and has a normal responsiveness to unexpected inflation. The TCE Required Capital measure, as discussed in previous sections, looks at the required capital at the end of 1 year assuming the company immediately reserves to ultimate loss with perfect knowledge of the impact of future economic variables on loss payments. The TCE Required Capital has again been calculated using a 1% tolerance for each of the 63 Shapley combinations.

Table 5: Capital Allocation Results

Allocation Method	**Shapley Value**	Marginal Last-In	Shapley Value
Allocation Risk Measure	**TCE Required Capital Method**	TCE Required Capital Method	Standard Deviation of Statutory Surplus End of Year 5
Capital Allocation Center			
Workers Comp	**38%**	43%	14%
Auto Liab	**24%**	28%	34%
Home/CMP(Prop)	**9%**	5%	15%
Auto Phys Dam	**1%**	-5%	15%
GL/CMP(Liab)	**18%**	20%	11%
Assets	**10%**	9%	11%

The allocation of capital to assets is comparable between all measures. The percentage allocation to assets would increase if the company were to more aggressively invest its required and excess capital.

The Marginal "Last-In" allocation method only evaluates the marginal risk addition to the business as a whole. The Shapley value allocation method builds on the Marginal "Last-In" concept by considering all possible combinations of entry. One of the most striking results presented in Table 5 is the negative allocation to Auto Physical Damage (APD) for the marginal allocation of TCE Required Capital. The APD line of business is very profitable, not very volatile, and makes up approximately one quarter of DFAIC's book of business. The magnitude of the "Large Losses", in the TCE calculation, is dampened when the large and fairly certain expected profit from the APD line is added to the tail scenarios generated by the more volatile and less profitable lines of business. This results in an overall decrease in the TCE Required Capital when this line is added.

But, is it appropriate to analyze the marginal impact of a line of business being added to the business as a whole? The axioms supporting the Shapley value method would say no. As a stand alone, the APD line of business would require capital to operate. These two scenarios, the marginal impact of a line to the business as a whole and the stand alone, produce the extremes of the potential results. Shapley takes both of these extremes into account, along with all other potential combinations.

Table 6 displays the normalized percentage allocation to the individual lines of business for the portion of capital assigned to liabilities for both of the risk measures using the Shapley value allocation method. Additionally, some key loss metrics are shown. The selection of risk measure is dependant on what a company considers risk. The company should hold a total amount of capital, and allocate its capital, in a manner consistent with its definition of risk.

The percentage of capital allocated via the Standard Deviation measure aligns closely with the percentage of loss exposure from the 5 years of new business (Expected Accident Year Loss & LAE) and the existing reserves (Current Booked Loss & LAE Reserves). In fact, for all lines, the allocation percentage falls within the range of the new business and existing reserve percentages. In addition to the magnitude of loss potential, the measure is to a lesser extent driven by the volatility of the individual lines. For example, the Auto Physical Damage (APD) line accounts for 24% of the new business loss exposure but is assigned a slightly lower percentage of capital. DFAIC's APD line has been modeled with the least loss ratio volatility of any of the lines. This is the factor that dampens the allocation of capital to 17%.

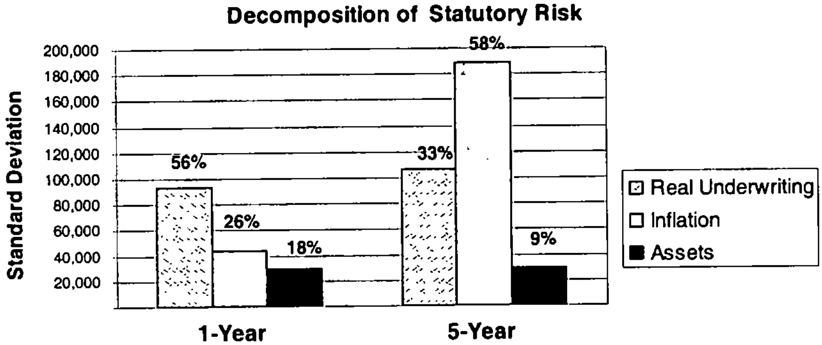
The Standard Deviation method looks at risk as uncertainty of all potential losses, whether good or bad. In contrast, the TCE Required Capital Method concentrates on those extreme tail events that can cause insolvency. Referring to the results in Table 6, the TCE Required Capital Method allocation is very different from the Standard Deviation of Statutory Surplus allocation. The capital is being allocated to the longer tailed lines. In fact, the allocation seems to be driven by the duration of the individual line of business, but dampened by the overall magnitude of the line of business. For example, workers compensation, the longest tailed line is allocated a portion of the capital much greater than its corresponding portion of expected loss exposure. The longer tailed lines, workers compensation and general liability, have increased their allocations over the standard deviation method, while the shorter tailed lines have decreased their allocations.

Table 6: Line of Business Shapley Value Allocation Analysis

Capital Allocation Center	Standard Deviation of Statutory Surplus End of Year 5 (Normalized)	TCE Required Capital Method (Normalized)	Expected Accident Year Loss & LAE	Current Booked Loss & LAE Reserves	Average Accident Year Duration (Years)
Workers Comp	16%	42%	9%	24%	3.9
Auto Liab	38%	27%	37%	40%	2.4
Home/CMP(Prop)	17%	11%	22%	14%	1.3
Auto Phys Dam	17%	1%	24%	4%	0.9
GL/CMP(Liab)	12%	21%	8%	19%	3.8
Assets	X	X	X	X	X

DFAIC holds per occurrence and per risk reinsurance across all lines along with a CAT cover: $500,000 retention for all major lines, except property, which has a $1,000,000 retention. The company has covered most of its exposure from real severity (where real severity is severity in real dollars which strips out the impact of unexpected inflation) through reinsurance. Most people do not think about decomposing severity into non-inflation based and inflation based components. Additionally, due to the law of large numbers, it is difficult to grossly misestimate frequency of large occurrences for a company of this large size. As a result, one of the greatest profitability/surplus exposures the company faces is from mispricing the policy due to increased nominal severity driven by unexpected inflation. Unexpected inflation impacts all sizes of loss. Therefore the majority of its impact is retained by DFAIC and not ceded to its reinsurer. A decomposition of DFAIC's risk to statutory surplus is displayed in Figure 9. (See Correnti [13] for more discussion of decomposition of risk.)

Figure 9: Decomposition of Statutory Risk



The power of unexpected inflation does not discriminate based on the size of the company. Unexpected inflation does not diversify away. In fact, it cumulates over time. As seen in Figure 9, the contribution of inflation to the overall balance sheet volatility is significant even when looking at the company over a one-year time horizon. As the time horizon extends, the risk from real underwriting (real severity + frequency) diversifies and the contribution of unexpected inflation begins to dominate the risk landscape. Though inflation is currently at relatively low levels, we can not be lulled into believing that the inflation levels of the early 1980's will never return.

At first glance, a 42% allocation to workers compensation seems outrageous. In analyzing the tail events (worst 1% of all simulated combined lines results) this allocation begins to look less outrageous. Again, the company has defined "risk" as tail events that can yield insolvency. For analyzing these tail events, this tail risk seems to be largely driven by unexpected inflation. The average annualized compound inflation rate over a five year period for all modeled simulations is 2.4%, which is in line with the current CPI. The same statistic for the worst 1% of all simulations is 9%. Those lines with the longest duration, workers compensation and general liability, have the greatest exposure to unexpected inflation. As a result, the longer tailed lines are receiving a proportionally large amount of the capital allocated to them.

This capital allocation exercise is all about how a company defines risk. They must select a risk measure that is consistent with how they define risk.

Conclusion

In this paper we have:

- Chosen a measure of risk (TCE) that is consistent with reasonable standard, as expressed by the axioms for coherent risk measures
- Chosen an allocation method, using the TCE risk measure, and an allocation approach (Shapley) consistent with reasonable axioms for allocations.
- Chosen a risk variable (statutory surplus) that incorporates the effects of both asset and liability variability.

After we made these choices, we analyzed a hypothetical company DFAIC in a DFA framework.

We chose a DFA framework because:

- Interactions between line of business results are generally too complex to be modeled analytically
- Modeling the simultaneous impact of economic variables on multiple categories of assets as well as on liability payments is too complex to handle analytically
- Calculation of a risk measure such as TCE requires a simulation approach if the underlying components are modeled using simulation
- We wish to allocate the cost of risk to the assets as well as to each line of business.

We concluded:

- That DFAIC is currently adequately capitalized. Moreover, we have a measure of the amount of excess capital that can be released to the owners, and a framework to analyze changes to required capital levels as a result of changes to the reinsurance program, asset mix, or underwriting plans.
- That the allocation of capital to line, and hence the required cost of capital to be built into the rating structure differs from the values under other approaches. If our competitors continue to use traditional methods, we will be able to be more competitive in lines where our risk exposure is less.

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Appendix A: Liability Parameterization

Our study of DFAIC's current reinsurance program and how it compares to alternative programs does not include loss portfolio transfers or other retrospective coverage. Hence existing business, with its attendant loss and unearned premium reserves, is modeled on a net of reinsurance basis. New business, however, is modeled on a gross basis. This allows us to vary prospective reinsurance strategies and compare the consequences of differing strategies. Since a principle focus of our paper is the current reinsurance program and its possible alternatives, we begin with a brief discussion of DFAIC's current reinsurance program and its implications for parameterizing our DFA model.

DFAIC's current reinsurance program includes excess of loss coverage for property, liability, and workers compensation risks, as well as coverage for catastrophes. In order to model the effects of these and alternative treaties, we generated individual large losses and occurrences on a gross of reinsurance basis. This necessitates the development of both frequency and severity probability distributions within the context of a collective risk model. Both company-specific and industry experience were gathered and analyzed for this purpose. Once the collective risk model was ready, individual large losses and catastrophes were generated stochastically and reinsurance covers were applied to obtain simulated losses net of reinsurance. Normally, company management would be consulted before finalizing company specific assumptions such as reinsurance arrangements or the frequency and severity of large losses and catastrophes.

In setting up our model, we condensed DFAIC's business into five distinct lines: Workers Compensation, Auto Liability (both personal and commercial), Property (homeowners and CMP property coverage), General Liability (other liability, product liability, special liability, and CMP liability coverage), and All Other Miscellaneous Lines(predominantly auto physical damage). For ease of discussion, we will refer to the other miscellaneous lines as Auto Physical Damage (APD). Segregation of business into these five lines allows for the effective modeling of reinsurance programs without burying results within a mass of detail. Each of these five lines is assigned a set of descriptive parameters to appropriately model its constituent line of business. Needed parameterizations relate to such items as premiums, losses (including loss adjustment expenses), other expenses, and payment patterns, as well as their stochastic properties. A preliminary step in our analysis involved restating historical results to be consistent with our five modeled lines of business[9].

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[9] E.g. CMP results were segregated into property or casualty and allocated to our Property or General Liability lines of business, respectively.

Projections of expected future premiums and loss ratios are in part based upon our assumed future business plans for DFAIC. An analysis of DFAIC's Schedule P reveals a recent deterioration in underwriting results and earned premium levels. Such a situation might indicate past DFAIC rate reductions made in an attempt to maintain market share within a competitive environment. Falcon's business plan is to raise rates thereby restoring loss ratios to DFAIC historical levels in three to five years. Anticipated effects of this business plan are reflected in our parameterization of future written premium levels.

Table 1A: Projected Growth Rates for Written Premium

	Workers Comp	Auto Liability	Property	General Liability	APD
2000	-2.5%	-2.5%	-2.5%	-2.5%	-2.5%
2001	-2.5%	-2.5%	-2.5%	-2.5%	-2.5%
2002	-2.5%	-2.5%	-2.5%	-2.5%	-2.5%
2003	0%	0%	0%	0%	0%
2004	2.5%	2.5%	2.5%	2.5%	2.5%

DFAIC's simulated losses have been modeled in two pieces, core and large. Briefly, losses are categorized as large or core depending on magnitude. Large losses are simulated through a collective risk model, while core losses, specifically core loss ratios, are generated through a mean-reverting, momentum-driven random walk.

The model user determines the appropriate mean reversion factor, momentum factor and long term average core loss ratio. Considerations in selecting such parameter values might include an anticipated underwriting cycle or other market change. The actual simulated core loss ratio is generated from a user-selected distribution having a mean and a variance defined by the user. A blind algorithmic approach to selecting these parameters is not appropriate. As is true throughout the parameterization process, simulated results must be constantly checked to verify the reasonableness of results. For example, the variance of simulated, total loss ratios was checked against estimates of loss ratio volatility obtained from historical company results.

Table 2A: Accident Year Loss&LAE Ratios by Line of Business[10]

	Year 1	Year 2	Year 3	Year 4	Year 5	Standard Deviation
Workers Comp	85%	81%	77%	77%	77%	18%
Auto Liab	92%	85%	81%	79%	79%	12%
Home/CMP-Prop	78%	75%	75%	75%	75%	8%
Auto Phys Dam	68%	65%	64%	64%	64%	8%
GL/CMP-Liab	66%	61%	59%	58%	59%	11%

The above statistics do not include the effects of catastrophes

The timing of loss payments is as important as their magnitude. Payment patterns were estimated using DFAIC Schedule P loss triangles and industry results. We derived two sets of payment patterns that were separately applied to existing reserves and new business for each of the five lines of business. The consolidated reserve run-off pattern and accident year payment pattern for DFAIC are shown in Figure 1A and 2A.

[10] The standard deviations actually increase with accident year due to the diffused nature of our modeling process. Intuitively, one would expect volatility of projections to increase with the time horizon.

Figure 1A: DFAIC's Consolidated Reserve Run-off Pattern

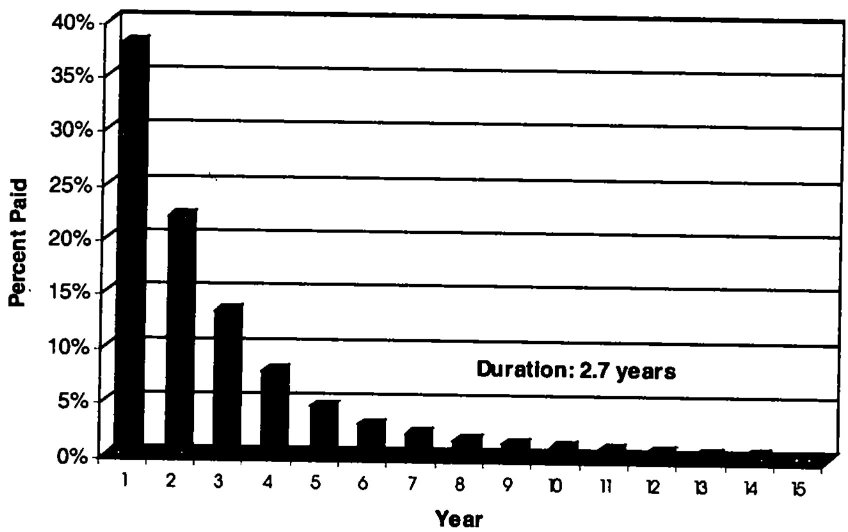
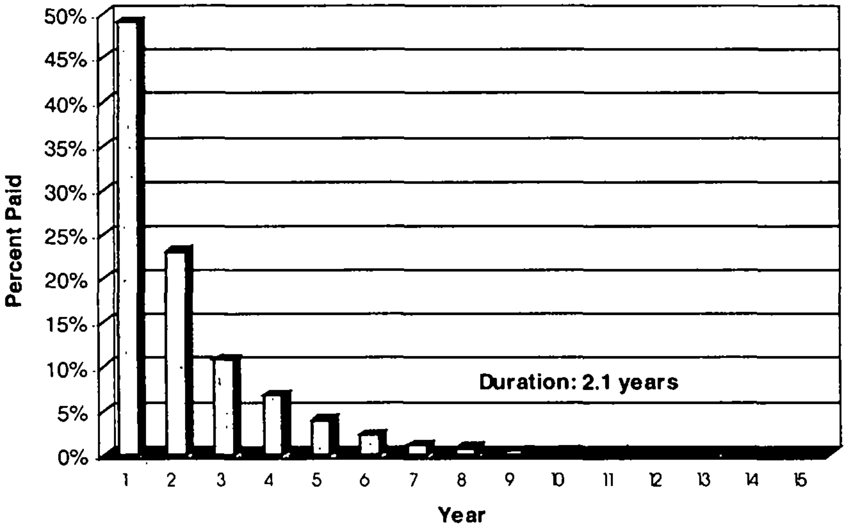


Figure 2A: DFAIC's Consolidated Accident Year Payment Pattern



Expenses, other than the loss adjustment expenses already incorporated into the loss ratios, were modeled as both fixed and variable. Actual values were again obtained through a combination of company specific and industry wide statistics.

We have already discussed some of the randomness modeled into the projected core loss ratios. Further randomness is introduced to the model through the sensitivity of losses, expenses, and premiums to unexpected changes in the level of inflation. For DFAIC, we modeled losses and fixed expenses as immediately and fully responsive to unexpected changes in the level of inflation while premiums were partially responsive after a one-year time lag. Inflation sensitivity introduces a stochastic element affecting loss ratios, expenses, premiums, and payment patterns.

The simulations include a reasonable level of positive correlation between lines of business as indicated in Table 3A.

Table 3A: Ultimate Loss Ratio Correlation Coefficients

	WC	Auto	Property	GL	APD
WC	1.0				
Auto	0.3	1.0			
Property	0.4	0.4	1.0		
GL	0.4	0.5	0.6	1.0	
APD	0.2	0.3	0.3	0.3	1.0

Such positive correlation between lines of business is commonly accepted. It is probably the result of several factors including changes to overall pricing levels in the insurance market and unanticipated inflation impacting the loss ratios of all lines of business.

Assumptions concerning correlation between lines of business are part of a series of parameter assumptions important within the context of building an appropriate DFA model. Because of our inability to access DFAIC for further information, it is especially important that our assumptions are reasonable both in isolation and in conjunction with other assumptions. For example, our collective risk model for generating workers compensation losses gross of reinsurance appears reasonable when compared to industry and available DFAIC statistics. But just as important, when we used this loss model to develop pricing for the current workers compensation excess of loss cover, the indicated reinsurance premium was comparable to that indicated by DFAIC Annual Statement exhibits. Such observed consistencies build confidence in the model and its assumed parameter values.

Appendix B: Economic and Capital Market Parameterization

The model that we used to generate our DFA economic and capital market simulations (FIRM™ Asset Model) differs from traditional mean/variance models in that economic variables, including interest rates and inflation, are explicitly modeled using accepted and rigorously tested stochastic processes. Capital market returns are then generated on a consistent basis with the underlying economic environment. This type of model has the following advantages over traditional mean/variance models:

- the explicit modeling of both economic and capital market variables;
- the ability to incorporate mean reversion in yields, providing for control over the serial correlation of capital market returns over time;
- multi-period simulation capabilities; and
- additional flexibility in modeling asset categories such as mortgage-backed securities and other securities with embedded options.

The economic and capital market parameterization process involved identifying and selecting asset classes that best represented the homogeneous groups of invested assets available to DFAIC. The twelve asset classes we defined and modeled were:

- Cash Equivalents
- Government Bonds (1-5 years)
- Government Bonds (5-10 years)
- Government Bonds (10-30 years)
- Corporate Bonds (1-5 years)
- Corporate Bonds (5-10 years)
- Corporate Bonds (10-30 years)
- Municipal Bonds (1-5 years)
- Municipal Bonds (5-10 years)
- Municipal Bonds (10-30 years)
- Common Stock
- Preferred Stock

The economic and capital market simulation model required assumptions concerning the initial levels of interest rates, inflation rates, real GDP growth, equity earnings growth, equity P/E levels, and the dividend payout ratio together with a set of long-term levels to which the initial levels will revert over time. In setting the long-term levels, the goal was to produce risk premiums between asset classes that are consistent with historical data[11].

For our DFAIC study, we have set long-term levels equal to the initial market conditions as of our model start date (1/1/2000). This avoids bias with respect to expected price appreciation or depreciation due to interest movements or changing P/E ratios over the time horizon. Initial market conditions together with the assumed mean levels for are shown in Table 1B.

[11] For example, the spread between cash and inflation is historically about 2% and the risk premium for long government bonds over cash is about 2%.

Table 1B: Initial and Mean Interest Rate and Share Assumptions

Variable	Initial Conditions 1/1/2000[12]	Mean Levels
Government Yields:		
3-Month Interest Rate	5.53%	5.53%
1-Year Interest Rate	6.19%	
3-Year Interest Rate	6.34%	
5-Year Interest Rate	6.39%	
10-Year Interest Rate	6.36%	
30-Year Interest Rate	6.56%	6.56%
Corporate Yields:		
3-Month Interest Rate	6.16%	6.16%
1-Year Interest Rate	6.70%	
3-Year Interest Rate	6.99%	
5-Year Interest Rate	7.11%	
10-Year Interest Rate	7.28%	
30-Year Interest Rate	7.65%	7.65%
Municipal Yields:		
3-Month Interest Rate	3.91%	3.91%
1-Year Interest Rate	4.09%	
3-Year Interest Rate	4.54%	
5-Year Interest Rate	4.79%	
10-Year Interest Rate	5.22%	
30-Year Interest Rate	5.99%	5.99%
Expected Price Inflation	2.5%	2.5%
Expected Real GDP	2.5%	2.5%
S&P 500 P/E Ratio	32	32
S&P 500 Earnings Growth		9.0%
S&P 500 Dividend Payout Ratio	40%	40%

The returns for cash equivalents, bonds and common stock are directly controlled by the initial and mean assumptions shown in Table 1B.

[12] Source: Bloomberg

Cash Equivalent returns are the accumulation of 1-month government interest rates over time. Government Bond returns are a function of the applicable interest rate level, the change in the rate and the bond maturity. Corporate and Municipal Bond returns are modeled as a proxy to the US Single A corporate and the insured general obligation municipal bond markets respectively. They are calculated similarly to government bond returns. Corporate yields are modeled at a stochastic spread to government yields and municipal yields are modeled as a stochastic ratio to the government yields. Reported market yields on corporate bonds are adjusted to reflect historical defaults[13]. Common Stock returns are modeled as a proxy to the S&P 500 index. The returns are composed of capital gains/losses plus dividends[14].

Table 2B shows the expected annual (arithmetic) and annualized compound (geometric) returns for each of the twelve modeled asset classes.

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[13] This is based on the 10-year cumulative default study for Single A bonds provided by Moodys. A 50% recovery rate on defaults is assumed.

[14] Because we are assuming that long-term mean P/E ratios are equal to initial P/E ratios, valuation changes are not reflected in the risk premium between stocks and bonds. Thus the modeled equity risk premium is less than the historical average (6-7%), but is in-line with the historical average when adjusted for valuation changes.

Table 2B: Simulated Five-Year Return Statistics[15]

Asset Class	Expected Annual Return	Annual Std Dev	Annualized Compound Return	Annualized Compound Std Dev
Cash Equivalents	5.9%	1.9%	5.9%	1.4%
US Gov't Bonds (1-5)	6.5%	3.5%	6.5%	0.8%
US Gov't Bonds (5-10)	6.9%	6.7%	6.7%	1.8%
US Gov't Bonds (10-30)	7.4%	10.7%	6.9%	3.3%
US Corporate Bonds (1-5)	7.2%	3.6%	7.2%	0.9%
US Corporate Bonds (5-10)	7.6%	6.8%	7.4%	1.9%
US Corporate Bonds (10-30)	8.0%	10.8%	7.5%	3.3%
US Municipal Bonds (1-5)	4.9%	3.2%	4.8%	0.7%
US Municipal Bonds (5-10)	6.1%	7.8%	5.8%	2.0%
US Municipal Bonds (10-30)	7.0%	11.8%	6.4%	3.2%
US Stock	10.8%	20.0%	9.3%	7.6%
Preferred Stock	8.3%	12.6%	7.7%	4.2%

[15] Expected annual return statistics are arithmetic averages and are indicative of risk and return expectations over a one-year holding period. Annualized compound return statistics are geometric averages and reflect the impact of time diversification over the five-year holding period.

Appendix C: Coherent Risk Measures

This paper assumes that, when choosing a risk measure, the risk measure should be coherent. That is, it should satisfy the axioms of coherence. While this concept is not new, it is relatively new, and has not been discussed in detail in the Casualty Actuarial Society literature. Consequently, while we will refer readers to the original papers for the complete explication of the concepts, we would like to summarize some of the key points here, with specific application to the concept of risk measures used for the determination of surplus levels.

Actuaries have developed a number of risk measures over time and debated the merits of the alternatives. The discussion of standard deviation versus variance versus covariance has been discussed in a number of places [19],[20],[29],[37],[38]. However, until recently, the measures were generally developed ad hoc. Very recently, several researchers [3],[4],[5] have approached the problem by defining a set of axioms, and then examining the set of risk measures that satisfy the axioms.

Let:

- X_i represent portfolios of risks. (Think of it as the liabilities of a particular insurance company).
- α be some constant
- $\rho(\bullet)$ be a function that assigns a value of risk to a portfolio

Axiom T—Translation invariance

$\rho(\mathbf{X} + \alpha\,\mathbf{r}) = \rho(\mathbf{X}) + \alpha.$

That is, if a constant loss is added to a portfolio of risks, the required surplus for the combined portfolio is increased by the amount of the constant loss[16].

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[16] The careful read will note that this formula differs from the formula in the papers by Artzner et al. In those papers the right side contains a negative α, rather than positive. This is because their paper is written in terms of assets. Adding a risk free asset to a portfolio of risky assets **reduces** the surplus requirement. In actuarial convention, we express losses using positive numbers. If we expressed them in terms of negative values, then the Artzner formulation would apply, and we would interpret the constant as adding a positive amount, an asset, to a set of liabilities. In that case, the addition of a positive amount to a set of losses (expressed as negatives) would reduce the surplus requirement.

Axiom S—Subadditivity

$\rho(\mathbf{X}_1+\mathbf{X}_2) \leq \rho(\mathbf{X}_1) + \rho(\mathbf{X}_2)$

That is, if we have two separate portfolios of risks, the surplus requirement for the combined portfolio is no larger than the sum of the surplus requirements for each portfolio.

Axiom PH—Positive Homogeneity

$\rho(\alpha\mathbf{X})= \alpha\ \rho(\mathbf{X})$

If we have a surplus requirement for a portfolio of risks, increasing each risk by a scalar increases the aggregate surplus requirement by the same scalar. As an alternative example, we could say that, if two researchers analyze a portfolio, and one expresses the amounts in dollars, and the other in thousands of dollars, the resulting surplus measures should differ by the same factor, a factor of 1,000.

Axiom M—Monotonicity

For $\mathbf{X}_1<\mathbf{X}_2$, $\rho(\mathbf{X}_1)<\rho(\mathbf{X}_2)$

If a particular portfolio is completely dominated by another portfolio, that is, if for every quantile, the loss amount in the second distribution exceeds the value of the first, the risk measure will be greater for the second[17].

These axioms do not appear to be overly restrictive. However, as various papers have shown [3],[4],[5],[30] traditional risk measures such as VaR (probability of ruin), EPD, standard deviation measures, and variance measures fail one or more of these axioms. The rationale for the TCE measure is that it does satisfy these four axioms.

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[17] As noted in the previous footnote, this formula differs from that in Artzner et al. The inequality sign is reversed. The reasoning is the same. In actuarial convention, we tend to express losses in positive amounts, as well as surplus requirements. Given two portfolios of risks, one of which dominates the other, the inequality sign will be reversed, depending on whether the losses are expressed as positive or negative values. As long as the researcher is careful with the sign convention, the values of the surplus requirements should work out correctly.

DFA—The Value of Risk

Stavros Christofides and Andrew D. Smith

DFA – The Value of Risk

STAVROS CHRISTOFIDES AND ANDREW D. SMITH

ABSTRACT

This paper describes the use of a Dynamic Financial Analysis (DFA) model to answer questions on capitalisation, business and asset strategy in the case of a US P&C insurer, in the framework of maximising stockholder wealth.

We measure this wealth by applying a risk measure on the individual stochastic cash flows from the DFA model, in preference to more conventional approaches based, for example, on historic betas The risk translates into value by two mechanisms:

1. For systematic risk, we use a multiple-factor arbitrage-free pricing approach. This is calibrated to be consistent with the prices that our stochastic macroeconomic model generates. We implement these ideas using explicit deflator processes.
2. Both systematic and non-systematic risks generate frictional costs, which we model explicitly. These costs are of vital importance to insurance, yet are often overlooked in DFA analysis. We allocate these frictional costs back to each simulation so as to produce realistic, rather than idealistic, financial statements which then enable us to look at capitalisation issues as well as valuation ones.

Our approach to risk definition is consistent with the recent findings of the CAS Risk Premium Project – see Butsic et al (2000)

KEYWORDS

Dynamic Financial Analysis; Capital, Risk, Return; Financial Economics, Reinsurance; Catastrophe Exposure; Investment Strategy; Systematic Risk, Non-Systematic Risk; Frictional Costs; Operational Risk; Capital Allocation;

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DFA – The Value of Risk

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EXECUTIVE SUMMARY

The following report outlines a detailed analysis of DFA Insurance Company (DFAIC) using Dynamic Financial Analysis (DFA) This is a brief summary of our conclusions – our full report provides more details and supporting evidence

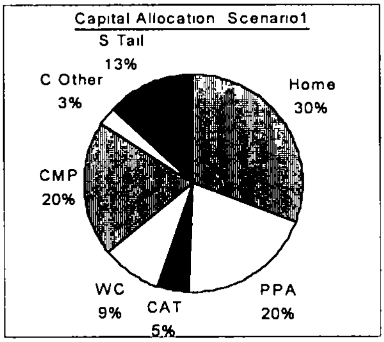
Is DFAIC adequately capitalised?

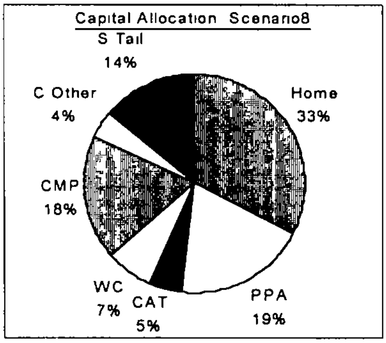
We believe that the company can reduce its capital by at least $100m without increasing its risk to financial impairment We measure this impairment by estimating the probability distribution of the minimum surplus to premium ratio over a five year projection period This capital release is part of a new strategy that has reduced asset risk with all equity investments replaced by bonds and a more aggressive insurance strategy which eliminates all class excess of loss reinsurances

We have demonstrated that this new strategy (Scenario 8) is just as financially sound as the existing one (Scenario 1) which has been good enough to ensure the company maintained its A rating from A M Best over the last five years The new strategy increases dividends to shareholders by around $65m pa on a reduced capital base without increasing risk to policyholders or management

How should the capital be allocated to line of business?

We have allocated capital within DFAIC according to the risks to which it is exposed The risk costs for each class include the class specific systematic and non systematic costs and an apportionment of the frictional costs associated with the capital account Our allocation of capital is shown in the pie charts below for Scenarios 1 and Scenario 8 Scenario 8 has lower maintained surplus holds no equity investments and places no class of business reinsurances





What is the return distribution of each business and is it consistent with the risk for the line?

We have estimated the distribution of emerging profits, gross of frictional costs, for each line of business. From these distributions, we have identified two components of the cost of capital, relating to systematic risk and also to frictional costs.

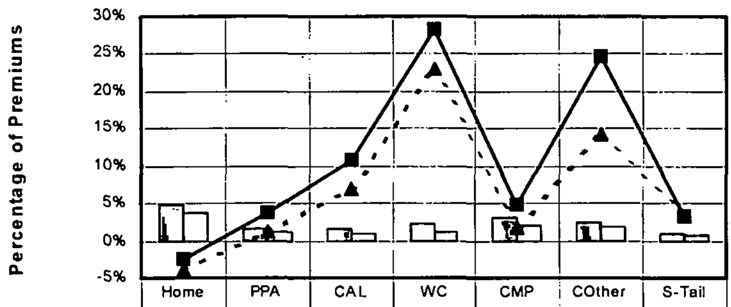
The chart shows the mean profit, and the associated capital costs, expressed as a proportion of net premium income. The two lines show the 'ideal profits' for the two strategies, Scenario 1 and Scenario 8. The blocks show the cost of capital for each class for the two strategies. These class costs consist of the class specific risk costs and the allocated capital account frictional costs.

We can see that the Home class is destroying significant value for shareholders; this warrants management attention. The class results are poor, even after investment income, and the cost of capital is large thanks to the catastrophe exposures.

Although PPA and CMP are currently generating profits, these are not creating value because the cost of capital exceeds the profits generated. However, scenario 8 improves profitability by reducing reinsurance costs, and also reduces the cost of capital by more prudent investment. The restructuring we have suggested would then transform PPA and CMP into value-creating classes of business.

CAL, WC and C Other produce profits, which comfortably exceed their cost of capital under Scenarios 1 and 8. Our class risk allocations include the capital account frictional costs.

Risk and Return by Class



	Home	PPA	CAL	WC	CMP	COther	S-Tail
Risk Costs Scen1	4.8%	1.7%	1.5%	2.3%	3.2%	2.4%	1.0%
Risk Costs Scen8	3.7%	1.1%	1 0%	1.2%	2.0%	1.7%	0.8%
Ideal ProfitScen1	-3.9%	1.2%	6 9%	23.0%	1.8%	14.2%	3.2%
Ideal ProfitScen8	-2 4%	3 7%	10.7%	28.2%	4.9%	24.7%	3.2%

Should the company buy more or less reinsurance?

DFAIC, under the assumed base scenario (Scenario 1), currently pays $145m per annum in class excess of loss reinsurance premiums. In return it receives, on average, $75m of reinsurance recoveries, and a reduction of $3m in internal frictional costs. There is negligible impact on systematic risk.

The analysis also shows that DFAIC can terminate these class reinsurances without impacting its financial strength, as measured by its ability to maintain its surplus to premium ratios at levels that are almost identical to those it achieves with the benefit of these reinsurances. The company can therefore achieve this change in strategy without requiring additional capital. Allowing for the additional frictional costs and tax, the shareholders will see an average increase in dividends of just over $30m pa throughout the plan period.

In the case of the catastrophe reinsurance protection, the analysis (Scenario 3) indicates that the annual savings will be around $8m pa, after allowing for the $4m increase in frictional costs. The impairment analysis, in this case, indicates a weakening financial position, which will require additional capital. We have not attempted to identify this amount, as it is unlikely that such a change in catastrophe reinsurance protection will be considered prudent or justified by external analysts in the short term. This is an area for future DFA analysis.

How efficient is the asset allocation?

DFAIC currently invests 35% of its free assets (surplus) in equities. Allowing for higher returns and also higher risk-based operational costs, this strategy increases mean profits by $43m over a five-year period compared to a bond strategy. The corresponding increase in cost of capital for DFAIC is $94m. Therefore, the equity strategy is destroying value.

There may be some arguments for establishing bond portfolios that more closely replicate the liabilities, however our analysis indicates that this is of little value in the context of the avoidable operational costs imposed on DFAIC by the current equity exposure.

1. INTRODUCTION

Insurance managers, insurance regulators and analysts have long recognised the potential value of asset-liability modelling (ALM) for P&C companies. Examples of ALM or Solvency models can be traced back to the late seventies and early eighties. In the nineties the subject was given a new name, DFA or Dynamic Financial Analysis, and this has now entered the P&C vocabulary as the process for financial risk evaluation of P&C insurers.

The early P&C ALM models progressed knowledge but failed to deliver much in answering real problems. There were two main reasons for these limitations. Firstly, there were poor links between assets and liabilities, often due to poor economic scenario generators. Secondly, there was no clear consensus on how to interpret the mass of outputs. These are the two key factors behind the development of the approach described in this paper.

The DFA approach adopted concentrates on the key variables and attempts to maintain economic soundness in how assets and liabilities are modelled and how the results are interpreted. The resulting DFA framework for risk pricing recognises and quantifies systematic and non-systematic risk as recommended by the latest research on the subject by the CAS's own Risk Premium Project (RPP) Phase I and II Report –see Butsic et al (2000).

The resulting framework enables the valuation and ranking of alternative management strategies and also provides a more realistic approach to the investigation of financial impairment and risk sensitive capital requirement questions. The methodology extends to the allocation of both systematic and non-systematic risk costs to individual DFA simulations or to classes of business. These risk cost allocations are often the main objective of 'capital allocation' and are derived directly and coherently from DFA outputs.

This paper describes both the theoretical background and practical implementation of this new approach to DFA modelling using the study case selected for such a purpose by the CAS Committee on DFA.

The next section introduces the case study, describes the main features of the DFA model used for the analysis and how this was calibrated for DFA Insurance Company. Section 3 contains the technical details that underpin the analysis. This section contains some new material. Section 4 presents the results of the DFA analysis of DFAIC and demonstrates how the theory of the earlier section can help to turn the huge volume of DFA data output into a few key 'value' measures that can be used in the decision making progress. The final Section contains some concluding remarks and is followed by a list of References and a number of Appendices.

2. DFAIC AND DFA MODEL CALIBRATION

OVERVIEW OF DFA INSURANCE COMPANY

DFAIC is a US P&C insurance company licensed in all 50 states, which writes a balanced book of personal and mainstream commercial business. It has a primary concentration in the Northeast and Midwest and has enjoyed a rating of 'A' from A.M. Best for at least the past five years.

The company has minimal exposure to asbestos and environmental exposures and limited exposure to severe catastrophes. It maintains reinsurance protections to limit losses to $1million from individual risk and buys catastrophe reinsurance cover of 90% of $150m excess of $50m for any single event, which limits the pre-tax PML exposure over a 100 year return period to 10% of surplus, or roughly $160m.

Around 70% of the assets are invested in fixed income securities, most in tax-exempt municipal bonds. Of the remaining 30%, 18% are in cash and 12% in equities.

The financial information available shows that in 1999 the company had net premium earnings of just over $2.3billion and a Surplus of just over $1.6billion, or 70% of its net annual premium.

OVERVIEW OF THE DFA MODEL

The DFA model used for the analysis is a multi-line, multi-period, multi-scenario stochastic plan generator, implemented in C++ for speed. For this exercise we used annual periods and simulated financial statements for five years.

Economic scenarios are pre-generated and include the usual asset returns, split into income and gains as well as mid-year and end-year deflators and twenty year term structures for inflation and interest rates The deflators are used in the interpretation to quantify systematic risk and the term structures are used to set fair premiums and set claim reserves. For this exercise a 20-year term-structure was considered sufficient to cover the claims run-off period. More information on the economic scenario generator (TSM or The Smith Model) is given in Appendix A. For more details on deflators, see Jarvis et al (2001)

The DFA model projects premium amounts using indices of exposure and rating adequacy and a fair premium adjustment. The adjustment allows for the impact of any changes in inflation and interest earning expectations over the period of the exposures covered by the premium. Claim amounts are adjusted for any (earned) exposure changes and are simulated with anticipated claims inflation for the class in the case of reserves or actual inflation for the class in the case of payments. Projected loss ratios are the result of simulated premium and loss figures rather than a simulated variable, as is often the case with DFA models.

Three types of claims are used. Small claims are modelled in aggregate using one of the many available distributions. Large claims are modelled by a frequency and a severity distribution. Finally, peril losses are modelled at company, or market, level and then allocated to affected classes. A base year is used to define the required parameters and numbers and amounts for subsequent year simulations are calculated taking account of exposure and inflation changes.

Reinsurance modelling is available at both class and company level. For this particular analysis we only used the class excess of loss and the catastrophe excess of loss reinsurance facilities.

Expenses are modelled at class level, with four sets of parameters, a commission rate, and a dollar amount and by two percentages that apply to gross premiums and claim payments. Each of these variables can vary by year.

Assets backing liabilities are held in notional funds for each class, with the balance or surplus held in a capital account. Each of these funds can invest in any of the available asset classes, which include cash, equities, index linked bonds and bonds of various durations.

A single tax rate is used and there is a facility for tax deferral on unrealised capital gains. The model accommodates a large number of dividend strategies, including varying amounts that may or may not be inflated, variable amounts based on percentages of post-tax profits or varying amounts that pay any surplus in excess of a premium ratio. This option allows the company to maintain a level of surplus to premium over its plan period and so attempt to maintain its actual rating in the market place. It is possible under this strategy for the actual surplus ratio to fall below the set target, which allows us to investigate the impact that a particular business strategy may have on the probability that such a ratio falls to a level that may lead to a downgrade of the company rating

CALIBRATING THE DFA MODEL FOR DFAIC

Capital Structure, Taxes and Dividends

The capital, or surplus, was taken to be $1604m as at the beginning of the projection period and this was made of issued capital and retained earnings. It was assumed that there was no subordinated debt and that all dividends are paid to shareholders. A tax rate of 20% was used to reflect the low tax paying position of the company, with its high level of tax efficient municipal bond holdings. This is an area where a more US-specific tax treatment would be warranted in a real case exercise.

The amount of shareholder dividends paid in 1999 appeared close to the overall investment returns less policyholder dividends and taxes. Policyholder surplus reduced by $60m, or 3% of annual net premiums, as a result and the year-end premium to surplus ratio increased from 1.4 to 1.47. As this did not impact the company's rating from A.M. Best, a premium to surplus level of 1.43 was taken as reflecting the required level of surplus to be maintained through the plan period.

The economic outlook for the plan period, as projected by the underlying scenario generator, indicated lower average investment returns for the whole of the projection period. The most likely impact of reduced investment earnings will be reduced dividend payments. For the purposes of the evaluation, we defined a dividend strategy for DFAIC designed to safeguard its rating using the premium to surplus ratio of 1.43 discussed above, which is equivalent to maintaining a 'solvency ratio' (ratio of surplus to premiums) of 70%.

It is also assumed that in case of overall losses there will be no dividend payments or capital injections. In such cases, the surplus ratio will increase (the solvency ratio will decrease) and in more extreme cases, or following a series of poor results, the deterioration in surplus may lead to a ratings downgrade. This approach is used to test the resilience of the plans and ultimately the capital requirements of the company.

Classes, Premiums and losses
There were seven classes of business with annual premiums exceeding $150m and a number of much smaller classes with aggregate net premiums of less that $65m. We grouped these smaller classes together for the modelling ending with just seven classes, Home, PPA, CAL, WC, CMP, Commercial Other and Short-Tail. Premium, loss and loss payment pattern characteristics were then obtained from the financial data supplied.

For the purposes of the evaluation, we assumed that the company exposures are stable, with growth in premiums and claims costs arising purely as a result of economic variables, such as inflation and interest expectations affecting premiums and claims amounts. This implicitly assumes that future prices are being set to maintain the 'premium adequacy' at the base year (Year 2000) levels. Pricing cycles and price-volume changes could be included in the modelling but this was not considered necessary for this analysis.

Expenses And Allocations
Commission and expense figures were only available in aggregate and these were allocated to the above classes of business using broad assumptions, checked for overall reasonableness only. These allocations do not have a significant impact on the overall projections or results, except in that they limit what can be said with any degree of confidence in regard to the actual pricing adequacy of any of these classes. It is, however, still possible to make useful comments on the required risk-sensitive performance requirements for these classes.

For the modelling, loss related expenses were included in the loss projections. Commissions and other expenses by class were then modelled by two class specific percentages. These percentages were set for the whole of the projection period.

Assets And Allocations
Detailed asset information was available in aggregate form. The DFA model actually maintains invested funds for each class of business and the capital account (policyholder surplus) separately to facilitate better matching of assets and liabilities, if required. Choosing to mis-match assets and liabilities in this way may result in an increase in any systematic risk associated with the particular class of business and may well be a strategy that could be investigated in our framework.

The assets were grouped into seven main classes, cash, equities and bonds of durations of 1, 3, 5, 10 and 30 years. The initial invested funds for each class were estimated from the financial information and then allocated to the available asset classes broadly to reflect the term of the liabilities. Equity investments were assumed to be from shareholder funds (surplus). These initial allocations were deemed to reflect the company asset strategy and were maintained through the projection period. The actual amounts and allocations are given in Table 11, Appendix B

Large and peril losses and Reinsurances

The company buys a significant level of excess of loss and catastrophe reinsurance. We have modelled these reinsurances for all classes except for the short tail class. We estimated reinsurance premiums from the financial information and used the limits of reinsurance purchased to help us select a likely large loss frequency and severity distribution for each class that provided a reasonable match to both the cost of the reinsurances and the amount of cover purchased.

We made an assumption that the price of these reinsurances is around twice expected risk cost. This may be considered relatively expensive cover. Clearly, assuming that these reinsurances are priced at below risk or expected cost will result in a clear benefit emerging from the purchase of the reinsurance, particularly if it is assumed, as will often be the case with such modelling, that there is no resulting credit risk associated with such low reinsurance costs.

In the case of the catastrophe cover, we used the amount of cover and the indicated PML exposure information to identify an appropriate set of loss generating parameters. Here the cost is assumed to carry a heavier risk loading of 2.66 times expected risk cost. This value is equivalent to pricing the catastrophe reinsurance using the Wang proportional hazards transform with a risk aversion index of 1 6. See Christofides (1998) for more details of this approach and a justification for the choice of the loading factor.

In all cases, the Poisson distribution was used to generate the number of large claims and catastrophe occurrences. The loss amounts were generated by a new distribution, which we call the Parbull, and which is a Pareto with a Weibull tail. This distribution has three parameters, the usual two parameters of the Pareto, a scale and a shape, and the value at which the Weibull takes over.

Catastrophe exposures, losses and reinsurance costs, were assumed to fall 80% on the Home class with the other 20% on the CMP class. The average annual catastrophe retained losses are approximately 10% of premium for the Home class and 2.5% of premium for CMP. This is a key assumption as it has a significant impact on the class results. The company has significant exposures in the North East, where the coastal region has a high hurricane exposure. With our limited knowledge of the US market, we assumed that most of this exposure falls on the Home account. The choice of affected classes and allocation was selected to demonstrate the implications of such losses on the risk characteristics of the affected classes more easily, rather than reflect the actuality at class level. The overall company catastrophe risk impact is not affected by this allocation.

The main parameter assumptions for the DFAIC calibration are given in Appendix B.

3. MODELLING MARKET INTERACTIONS

EMPIRICAL NATURE OF DFA MODELS

Our discussion of the DFA modelling process so far has been largely empirical. There is no general theory to tell us whether large loss distributions should be Pareto, lognormal, gamma or some other family. The decision is a matter of historical data, and in the absence of data, experienced guesswork.

This empirical aspect identifies a number of possible problems with a model. Other authors analysing the same data are likely to built significantly different models. Given *different* data sets, but relating to the same company, two analysts' models would diverge still further. It is clear that our calibration is subject to significant model and parameter error.

In many cases, there is little that can be done about this error, other than to acknowledge its existence and exercise caution in interpreting model output. To model the parameter error itself requires the construction of meta-models in which the parameters themselves are treated stochastically. Vast arrays of meta-parameters proliferate further, rapidly exhausting the degrees of freedom in the data. This way madness lies.

SAMPLING ERROR AND OPTIMISATION

There is one situation where a purely empirical approach to model estimation can be more dangerous. This arises in situations where one or more players are competing – for example, in capital markets and premium rates (both direct and reinsurance). In this case, a reasonable prior view would be that competitive pressures cause convergence in profitability between alternative investments or lines of business.

If this prior view is not reflected in a DFA model, we risk overestimating the extent of any capital allocation opportunities. For example, let us suppose (naively) that competitive pressures forced 10 lines of business towards equal expected profit margins as a proportion of premium. An examination of historic data is unlikely to show equal actual profitability; sampling error causes variations in the estimated results by line.

If we ignored the prior convergence view, we would estimate one line as being more profitable than the others. This would be an example of a *non-competitive* model. We would allocate most, if not all, of the company's premium capacity to this most profitable line. We would overestimate aggregate projected profitability, and we would mistakenly forsake diversification in favour of hoped-for profits. To avoid such misleading results, it is important to consider the effect of competition in a more structured way.

MODELLING COMPETITIVE EFFECTS

An ambitious way to model competitive effects would be to construct explicit models of current and potential future competitors and their actions. In an insurance context, this many require a model of many dozens of firms; capital markets have millions of participants. Such models quickly become unmanageable.

Is there a practical alternative to simple models that assume no competition? The other extreme is to use economic models based on *perfect competition*. Economists have built these models to describe the effect of a many parties competing with each other. In this case, major structural simplifications apply which avoid the need separately to model each individual participant in a perfectly competitive market.

Competitive models contain many other useful pieces of information. For example, competitive market models imply predictive theories of how markets will price certain products. We use this pricing information to estimate the effect of strategic choices on the price of an insurer's share.

The use of competitive models creates biases in the opposite direction from non-competitive models We underestimate profit opportunities. A competitive market provides no profitable niches; every cash flow is fairly priced. There is nothing to be gained from smart resource allocation. The best strategy is to diversify as far as possible and to track market resource allocation decisions.

Whether we want to model a non-competitive, or a perfectly competitive, situation will depend on the characteristics of the markets we wish to model – that is, how competitive we think the market is. It also depends on the outputs we wish to examine; if we need to estimate future market prices, there is no practical alternative to the use of competitive models. On the other hand, if we model all markets as competitive, then the optimal strategy becomes a foregone conclusion – simply conform to a peer average. We must identify some competitive failures if DFA modelling is to be of any value.

MARKET PRICING AND DEFLATORS

Probably the best contenders for the competitive market approach, in an insurance context, are the capital markets. This does not imply we think capital markets are perfect. There are specialist securities firms who have competitive advantages in terms of information or execution, who can extract excess profits from capital markets by proprietary trading. However, most of these institutions are not insurers, and in particular, DFAIC is not one of them. It is prudent to assume DFAIC faces competitive capital markets.

Such an approach provides the added boon of a market pricing capability, which we have implemented using deflators. Our competitive asset model (TSM) explains traded asset prices in terms of their future cash flow distributions. We can also use the deflators to interpolate market pricing, thus valuing cash flows for which a market price is not directly observable. In this case, the competitive market framework provides a risk-sensitive equilibrium value for that cash flow stream.

This is vitally important for evaluating different corporate strategies. Our deflator approach provides market values for strategic alternatives. It is calibrated to replicate market prices of traded assets, so market consistency is guaranteed. The result of the modelling process is a clear ranking of attractiveness of different strategies, according to the value the market would put on DFAIC should it adopt that strategy

To create meaningful valuations, the cash flow model needs to be good enough. Deflators are widely used in the pricing of financial products such as options Unlike financial products, there is no contractual formula linking insurance profit streams to capital market inputs. The links are via actuarial formulas containing all sorts of estimated parameters and leaving out all sorts of remote contingencies. We investigate some of these contingencies in the final section of this chapter.

OLIGOPOLY PROFIT

The next step from a competitive market model is to model some forms of market imperfection. This intermediate step is an *oligopoly*. An oligopoly may provide economic profits, for example because of barriers to entry, economies of scale, regulatory capture, or niches of asymmetric information. The oligopoly profit is an explicit adjustment between a competitive market price (for example, for reinsurance) and a price used in a DFA model. Such a modelling structure ensures that modelled premium rates respond appropriately, for example, to a change in interest rates or in inflation expectations.

Simpler modelling approaches, for example based on loss ratios, do not respond in the right way to changes in the economic outlook. Instead, oligopoly profits in the loss ratio approach become implicit items. Even when inputs appear consistent, the implied oligopoly profit under the loss ratio approach is the difference between two large numbers in the calibration, and may behave erratically unless deliberate thought is given to the issue.

In capital markets we have modelled oligopoly profits to be zero. In insurance markets we recognise a number of specific imperfections, which impact prices. As optimal corporate strategies are driven by deviations from perfect markets, our DFA approach involves optimising the impact of these imperfections.

SYSTEMATIC AND NON-SYSTEMATIC RISK

Deflators provide competitive capital market pricing for any cash flows, including insurance cash flows. Consistent with capital market theory, this methodology implies a reward for investors who are exposed to systematic risk, that is, market risk that remains even in a diversified portfolio.

This mechanism provides no reward for non-systematic, that is diversifiable or specific risk. There may be investors who face high diversification costs. Will these investors bid up the price of specific risk? No, investors with high diversification costs will favour investments, such as pooled vehicles, that are already diversified. Such investors will see as unattractive any insurance shares carrying material specific risk. Insurance shares will instead be sold at higher prices to investors who face lower diversification costs.

Nevertheless, there is a widely held conviction in the insurance community that specific risks should carry some (non-zero) price. This is manifested in pricing practices such as standard deviation loads or proportional hazards transformations. It is also implicit in most approaches to capital allocation, which often look at percentiles, put option prices or other measures of total volatility, without distinguishing the systematic and non-systematic components.

We make the distinction between perfection in capital markets, in contrast to the impact of well documented distorting costs embedded in insurance pricing. An insurer may well enjoy some form of competitive advantage in its core markets, where it has bought its way through entry barriers, building customer relationships, branding, developing specific expertise and managing relations with third parties such as regulators, distributors and analysts. It is less plausible to believe that international capital markets, with far lower barriers to entry, offer any special terms to insurers. This point is commonly misunderstood; for example, we often encounter the misconception that risk loadings in insurance markets necessarily imply a mis-pricing of traded financial securities.

FRICTIONAL COSTS AND RISK LOADING

The fact that deflators do not associate a premium with non-systematic risks has some important consequences. For example, in their ground breaking 1958 paper, Modigliani & Miller demonstrated that the way that a firm was financed, either using debt or equity, made no fundamental difference to the value that a market would place on a firm. Their argument showed that swapping equity capital for bond capital just increased the gearing of the firm and hence the return required by equity holders. They concluded that the capital structure of the firm was irrelevant to the firm's valuation. A similar argument explains that, within the context of perfect capital markets, changes in investment strategy would similarly leave unchanged the market value of an insurance company.

Modigliani and Miller considered a simple model of a company, which ignored a number of items that are important in practice. They are sometimes called *frictional costs*, or *operational risks*

Examples of frictional costs include:

- Future business terms being sensitive to credit risk.
- Project disruption & wastage of unbudgeted flows.
- Optimistic plans survive longer in uncertain world
- Convex tax formulas – not able to use tax losses.
- Back office / processing expense which is convex in transaction flow.
- Capital raising, distribution, restructuring costs.
- Double taxation of income on retained surplus.
- Operational risk of cash misuse.
- Management time opportunity cost.

Frictional costs may in the past have been given little attention because they have been regarded as small, compared for example to claim payments. More dangerously, future frictional costs are also often ignored within the planning process and even within DFA models. The model projections are overly optimistic. Although most actuarial models do not allow explicitly for frictional costs, there may be implicit allowance inside a hurdle rate of shareholder return that seems puzzlingly high or in the use of total risk measures in an efficient frontier construction.

We prefer, instead, to build an explicit model of frictional costs. We allow management a constrained choice within a family of convex functions, each of which relates frictional costs to profit. This enables us not only to measure current frictional costs, but also to understand the impact of possible risk mitigation initiatives, such as asset-liability matching or reinsurance. In so far as they can minimize the frictional costs the management can then influence the market value of the firm.

MODELLING FRICTIONAL COST

Our model for frictional costs is an extended proportional hazards approach. It is based on 'ideal profit' as an independent variable. We define and relate true profit, ideal profit and frictional cost as follows:

true profit = ideal profit – frictional costs

The ideal profit is a measure coming out of a business plan or DFA model, which may contain optimism, either in parameter estimates or in cash flows omitted from the model

We model frictional costs as a function of the ideal profit. This function is determined by a combination of:

- management choices, relating to softer decisions on how they run their business
- market constraints on the minimised frictional costs for various aspects of the business.

We would usually expect frictional cost functions to be a convex function of ideal profit, so that businesses whose profits are more risky also attract higher frictional costs. We would expect them to be minimised for some finite value of profit, and to increase more steeply on the left than on the right. This is because unexpectedly low ideal profits typically generate more frictional costs than unexpectedly high ideal profits.

We would expect frictional cost families to give at least the following flexibilities to managers:

- managers should be able to translate the frictional cost function by a scalar, so that an addition of a constant (risk free) amount to the profit would not affect the frictional costs
- managers should be able to choose between risk tolerant cost functions that are more or less flat but high, compared to risk averse cost functions. A risk averse cost function would have a lower minimum but would increase faster if ideal profits moved away from that minimum.

There are many possible choices of frictional cost function families that satisfy these criteria. Our chosen functions are of the form:

$$\theta(x) = \lambda \int_{-\infty}^{x} G(y)^{1-\lambda} dy + \int_{x}^{\infty} \left[(1-\lambda)G(y)^{-\lambda} + \lambda G(y)^{1-\lambda} - 1\right] dy$$

where:

- x is the ideal profit
- θ is the frictional cost
- λ is a risk loading parameter between 0 and 1, and is a determined by the overall level of costs in the market. $\lambda = 0$ corresponds to zero risk loading; $\lambda = 1$ implies that all risks are priced at their maximum value.
- y is a dummy integration variable
- $G(y)$ is a function which increases from 0 to 1 as y moves from $-\infty$ to ∞. The increase is not necessarily strict, nor continuous.

Management Decision Process

The softer management decisions are assumed to affect the choice of the function G. In our model, they can choose this function in the knowledge of the distribution of ideal profit. However, management cannot peek ahead to the actual outcome of ideal profit.

We assume that management will choose the optimal profit to minimise the market value of the frictional cost In other words, given a value of λ between 0 and 1, management are assumed to pick a function G to minimise the expectation:

$$\mathbf{E}[\, D\theta(X)\,]$$

where D is the state price deflator.

We can solve this optimisation problem as follows. Let $F(x)$ be the cumulative probability function of the ideal profit X. Let $D(x)$ be the expected deflator, conditional on the ideal profit taking the value x. Then the value of the frictional cost is given by

$$\int_{-\infty}^{\infty} D(x)\theta(x)dF(x)$$

On simplification, we finally obtain the following frictional cost:

$$\mathbf{E}[D\theta(X)] = \int_{-\infty}^{\infty}\left\{\lambda G(x)^{1-\lambda}\left[\int_{-\infty}^{\infty} D(y)dF(y)\right] - \left[1-(1-\lambda)G(x)^{-\lambda}\right]\left(\int_{-\infty}^{x} D(y)dF(y)\right)\right\}dx$$

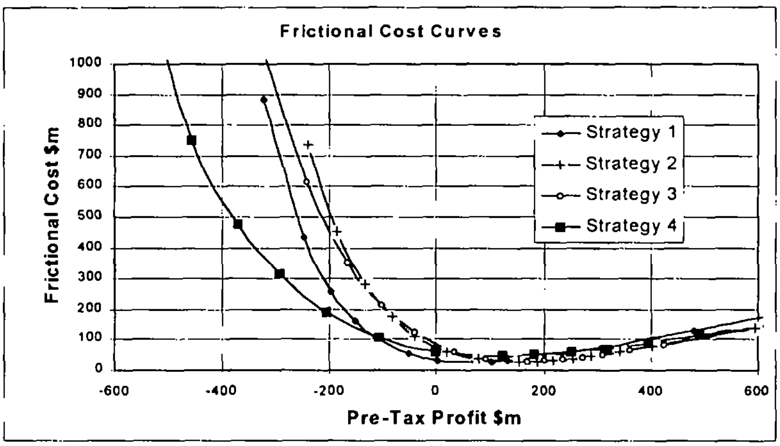
We seek to choose G to minimise this quantity. The optimum is achieved when

$$G(x) = \frac{\int_{-\infty}^{x} D(y)dF(y)}{\int_{-\infty}^{\infty} D(y)dF(y)}$$

We recognise this as the cumulative distribution function of G under the risk neutral law.

The typical shape of the frictional cost functions selected for use in the analysis of DFAIC is shown in Figure 1.

Figure 1: Typical Frictional Cost Curves from the DFA model



The minimised frictional cost is then

$$\mathbf{E}[D\theta(X)] = \int_{-\infty}^{\infty} D(y)dF(y)\left[\int_{-\infty}^{\infty} xdG(x) - xd\left[G(x)^{1-\lambda}\right]\right]$$

and so

$$\mathbf{E}[DX - D\theta(X)] = \int_{-\infty}^{\infty} D(y)dF(y)\int_{-\infty}^{\infty} xd\left[G(x)^{1-\lambda}\right]$$

Thus, the mean value of the realistic profit is equal to the mean idealistic profit, but under an adjusted risk neutral probability law. The second adjustment involves raising the cumulative distribution function to a power of 1-λ. This always has the effect of increasing the cumulative distribution function, or, alternatively, of shifting it to the left. This is equivalent to the proportional hazards transform proposed by Wang (1995). Wang's version has some sign changes relative to ours, as he deals with insurance losses where we deal with overall profit. Our analysis has shown how Wang's method deals with the non-systematic component of risk, as represented by frictional costs Our analysis, in using a risk neutral law, generalises Wang's work to cover both systematic and non-systematic risks.

Allocation of Risk Cost by Line

We now move on to the allocation of frictional costs by line of business. We do not seek to allocate the costs on each simulation by line of business. Instead, it is the deflated value we allocate. Thus, we can either allocate total frictional costs by simulation, or by line of business, but not by both at once.

Our approach requires a decomposition of ideal profit into the sum of a number of components, one for each line of business, and one for the capital account. The ideal profit for each line should add up to the ideal profit for the total.

The idea then is to allocate the total frictional cost according to the marginal impact of each line of business.

Let $y(x)$ denote the conditional mean of the line 1 profit y conditional on the total profit being x. Our expression for the allocated frictional cost is given by the integral:

$$\int_{-\infty}^{\infty} D(y)dF(y)\left[\int_{-\infty}^{\infty} y(x)dG(x) - y(x)d\left[G(x)^{1-\lambda}\right]\right]$$

It is clear from this expression that the total of the allocated costs for each line gives the value of frictional costs for the business as a whole, as it should.

4. DFA ANALYSIS OF DFAIC

This section presents the main results of the DFA analysis of DFAIC. These results are based on a set of assumptions made from very limited data and with no access to management. The analysis could be improved with more information and with access to management. Such information may have a significant impact on absolute values, such as an estimate of the market value of DFAIC, but may have less of an impact on the risk costs calculated or their allocation.

RECENT PERFORMANCE AND FUTURE PLAN ASSUMPTIONS

The company experienced an operating ratio of around 105.5% and paid dividend to stockholders in excess of $300m, reducing policyholder surplus in the year by nearly $60m. Accident Year losses for 1999 looked somewhat higher than the more developed positions of the earlier accident years and this may indicate some initial redundancy in the most recent claims provisions

The 1999 accident year net loss and loss expense ratio was over 7% points higher than the revenue year figure. We have assumed that the opening balance sheet claims reserves as well as all future claims reserves are set on a best-estimate basis without any margins. Any surplus in the opening loss reserves will be 'lost' as it will be assumed paid as a loss or loss expense. This is an area that would receive much more attention in practice. In a real DFA analysis, a reserve review would often be a necessary first step of the DFA exercise.

The company has cash balances totalling 18% of assets, which seems a little high for a company with a relatively diversified portfolio and with relatively low exposure to catastrophe losses The bond portfolio also appears to be of longer duration than the insurance liabilities it is supporting Equity investments are almost insignificant, at 12% of overall assets. The impact of increasing the equity investments and moving the bond portfolio nearer to the duration of the liabilities will be considered in the analysis section.

The company buys a considerable amount of reinsurance, to provide protection to relatively low levels of retention at class of business level. A cursory review of gross and net accident year loss ratios did not indicate any significant smoothing or benefit from these reinsurances at company level. The impact of the main excess of loss reinsurance treaties will be investigated to see whether these reinsurances actually reduce the company results variability or simply protect class of business results at a real cost to shareholders.

THE BASE SCENARIO

The select 'Plan Scenario' has parameter selections that result, on average, in an operating ratio that is below that reported in 1999 before allowing for frictional costs. Once allowance is made for these costs, the average operating result is in line with the base figures.

For the purposes of the exercise, it is assumed that the company continues to write the same volume of business at the same fair premium levels, all protected with the same reinsurance arrangements at similar costs. Expense and commission rates remain the same throughout the five-year projection period and asset allocation as initially derived for each 'fund' are rebalanced at the end of every year to their initial percentages.

Finally, any surplus in excess of a premium to surplus ratio of 1.43 (70% solvency ratio) is paid as dividends to shareholders. This dividend strategy is selected to test the ability of the company to maintain its market rating and fund its inflationary premium growth without recourse to shareholders. Clearly, such a policy results in variable flows of dividends that could be zero, in cases where results are poor or surplus adequacy levels are recovering.

Some summary plan statistics of the base scenario are given in Appendix B

ALTERNATIVE SCENARIOS FOR DFA ANALYSIS

In order to test alternative strategies, a number of other sets of assumptions, or scenarios, were selected to demonstrate the use of the DFA model and explore some of the questions listed.

Eight different scenarios are used in the analysis These are as follows.
Scenario 1: Base or assumed Plan with the reinsurance and asset strategies as in 1999
Scenario 2: Base but with no Class Excess of Loss Reinsurance
Scenario 3. Base but with no Class or Catastrophe Reinsurance
Scenario 4: Base but with reinsurance at risk cost (cover at risk or expected loss costs)
Scenario 5. Base but with 100% of surplus in Equities rather than 35%
Scenario 6: Base but with all investments in bonds with mean terms matching the liabilities
Scenario 7: Base, no reinsurance, surplus in equities (Scenario 3+Scenario 5)
Scenario 8: Base, lower capital, no class reinsurance, investments in bonds

Looking at these alternatives, Scenario 4 will be better than Scenario 1, if cheaper reinsurance can be purchased without an increase in credit risk and should provide a benchmark for evaluating the other reinsurance alternatives. Scenarios 5 to 7 are intended to help evaluate the asset or investment strategy. Scenario 7 should prove to be a high risk one. Scenario 8 was developed after some initial evaluation of the results of the earlier scenarios.

EVALUATION OF ALTERNATIVE STRATEGIES

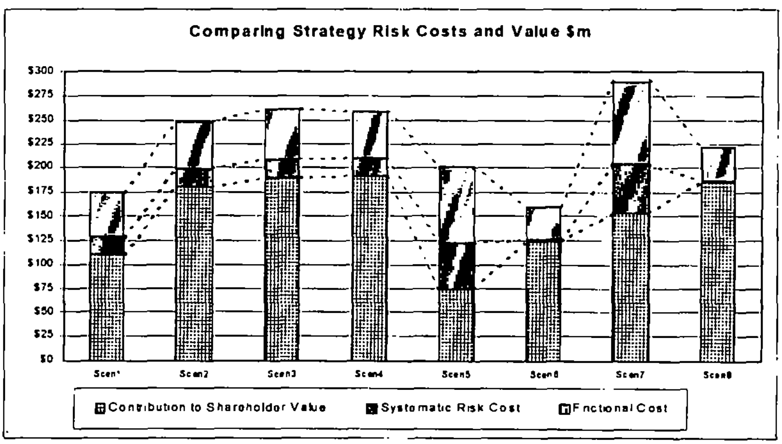
Results from the economic evaluation of the above scenarios, using 2,500 simulations over the five-year projection period, are shown in Table 1. The calculations are based on the pre-tax profit values, adjusted for systematic and non-systematic risk, using a frictional cost index of $\lambda = 0.33$. The impact of such a choice is considered later. The 'P-TP@rfr' row shows the average value of the 'plan period Pre-Tax Profits discounted at the risk free rates. The CSV (Contribution to Shareholder Value) line shows the result of adjusting the earlier discounted values for both systematic and operational risk, using the method outlined in Section 2. As these calculations are based on the pre-tax profit figures as the independent variable, the CSV is a gross of tax value. Post tax values are discussed later when we discuss the value of dividends and of the company. In the meantime these gross figures are sufficient for the first evaluation of alternative strategies and for risk cost allocations. The values shown are the averages over the five year period.

Table 1: Scenario Value and Risk Cost comparisons

Value	Scen1	Scen2	Scen3	Scen4	Scen5	Scen6	Scen7	Scen8
P-TP @rfr	175,123	248,122	261,594	257,945	202,009	159,400	290,307	221,456
Sys Cost	18,593	19,218	19,257	18,687	48,725	1,574	50,617	1,381
Frictional Cost	46,567	48,969	53,027	47,490	79,948	33,532	85,632	34,543
Total Risk$	65,160	68,187	72,284	66,176	128,673	35,106	136,249	35,924
CSV (Gross)	109,963	179,936	189,310	191,769	73,337	124,294	154,057	185,532

The following figure shows the results much more clearly.

Figure 2. Scenario Value and Risk Cost comparisons



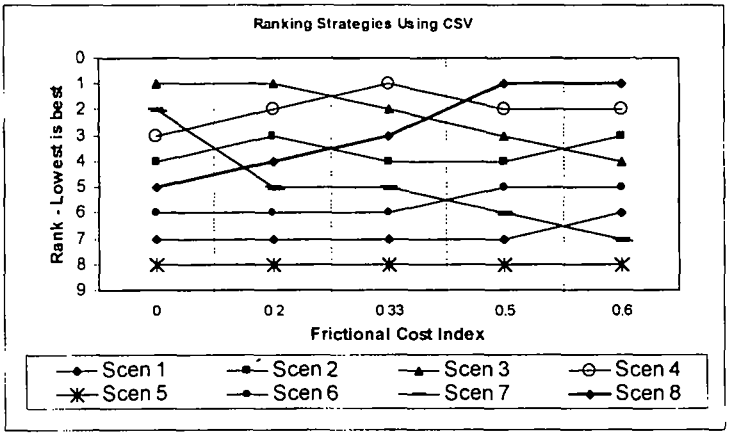
Both Table 1 and Figure 2 show results that are in line with expectations. It is expected, for example, that cheap, good quality, reinsurance (Scenario 4) should be more valuable than expensive reinsurance (Scenario 1). This is confirmed. The results also indicate that, in terms of shareholder value, reinsurance at much above risk cost (Scenario 1) is not of value to shareholders who may be much better off with less reinsurance (Scenarios 2) or no reinsurance at all (Scenario 3).

These observations are, however, based purely on the shareholders' perspective. Later analysis will consider what these strategies may do to the security of the policyholder and also the interest of the other key stakeholder, the manager, who may also lose if the company loses its rating as a result of strategies that are of benefit to well diversified shareholders.

Looking at the next four scenarios, it is clear that investing the surplus in higher risk assets, such as equities, increases the average return but may also significantly increase both systematic and non-systematic risk costs. This strategy (Scenario 5) is shown to be less valuable than any other of the tested strategies. Investing the surplus in bonds reduces average returns but increases value (Scenario 6). Scenario 7 is a high risk one. There in no reinsurance and the surplus is invested in equities. The average return is now maximised, but this strategy generates high risk costs. We will see later that it is also a high risk strategy for policyholders and managers (Table 7). Scenario 8 was developed by reviewing the earlier results and attempts to show a realistic practical strategy that management can adopt to maintain the financial strength, or rating, of the company whilst improving returns to shareholders. This strategy has lower initial capital, no class excess of loss reinsurances or any equity investments.

The calculations assume a frictional cost index of 0.33. Figure 3 and Table 2 show the sensitivity of these conclusions to the choice of index. At a value of 0, we only allow for systematic costs.

Figure 3: Impact of the Frictional Cost Index on Strategy value contribution



The actual dollar value impact of varying this key index is shown more clearly in Table 2.

Table 2: Impact of the Frictional Cost index on Shareholder Value Contributions

CSV $k	Impact of λ on Value Contribution							
λ	Scen1	Scen2	Scen3	Scen4	Scen5	Scen6	Scen7	Scen8
0.00	156,530	228,904	242,337	239,259	153,285	157,826	239,689	220,075
0.20	131,413	202,477	213,993	213,621	109,478	140,100	192,952	201,786
0.33	109,963	179,936	189,310	191,769	73,337	124,294	154,057	185,532
0.50	71,245	139,323	143,624	152,420	11,611	93,729	86,700	154,228
0.60	38,869	105,436	104,446	119,607	-36,382	65,904	33,375	125,841

Table 2 shows that the Scenario 8, which was developed after the initial evaluation of the earlier scenarios, does create more value once we allow for non-systematic risk. Scenario 4 starts better but begins to lose as we increase the frictional costs. Scenario 4 is the one that assumes that reinsurance can be bought at risk cost, which is clearly unrealistic. Scenario 8 has starting surplus that is $100m less than the other scenarios as explained earlier.

RISK COST AND CAPITAL ALLOCATIONS

The interpretation methodology can be extended to the allocation of risk costs to the underlying classes of business, including the capital account.

Table 3: Overall Risk Cost Allocations – Systematic and Non-Systematic Risk

Value	Scen1	Scen2	Scen3	Scen4	Scen5	Scen6	Scen7	Scen8
Capital	38,504	40,341	38,796	39,829	111,311	5,300	115,430	4,636
Home	8,082	7,751	13,641	7,952	4,355	10,411	8,519	10,173
PPA	5,276	5,711	5,097	5,228	3,603	5,430	2,940	6,010
CAT	1,298	1,581	1,418	1,283	966	1,219	960	1,512
WC	2,349	2,652	2,400	2,331	1,834	1,899	1,423	2,192
CMP	5,445	5,577	6,769	5,405	3,732	5,634	4,239	5,791
OC	785	1,304	1,100	770	581	709	680	1,246
S-Tail	3,420	3,270	3,063	3,378	2,292	4,503	2,058	4,365
Total Risk $	65,160	68,187	72,284	66,176	128,673	35,106	136,249	35,924

Table 3 shows allocation of the Frictional cost values shown in Table 1. The impact of the high equity investment of the capital account (surplus) can be easily seen in the results for Scenarios 5, and 7. The impact of removing the catastrophe reinsurance protection can be seen in the Home and CMP risk cost increases for Scenario 3. It is interesting to note that in the case of Scenario 7, the high equity investment increases overall risk charges with a greater proportion now falling on the capital account. In other words, variability from this source is more significant than variability from the liabilities. These costs can be subdivided further as shown in Table 4.

Table 4: Risk Cost Allocations – Frictional Costs

Value	Scen1	Scen2	Scen3	Scen4	Scen5	Scen6	Scen7	Scen8
Capital	21,550	22,630	20,986	22,453	64,225	4,965	65,840	4,342
Home	7,841	7,507	13,445	7,763	4,114	10,197	8,350	9,955
PPA	4,980	5,529	4,916	5,013	3,306	5,241	2,873	5,943
CAT	1,148	1,414	1,252	1,162	815	1,091	818	1,370
WC	2,223	2,521	2,269	2,269	1,707	1,880	1,405	2,173
CMP	4,784	4,897	6,101	4,794	3,071	5,082	3,682	5,222
OC	705	1,283	1,080	742	501	650	684	1,249
S-Tail	3,336	3,186	2,979	3,294	2,207	4,426	1,981	4,288
Total Risk $	46,567	48,969	53,027	47,490	79,948	33,532	85,632	34,543

The risk cost allocations can be used to derive the benchmarks needed for relative performance measurement. For example, they can be used to 'allocate capital'. It is instructive to tabulate the risk costs to premiums to see what they indicate Table 5 shows such an analysis.

Table 5: Total Risk Costs to Premiums

Value	Scen1	Scen2	Scen3	Scen4	Scen5	Scen6	Scen7	Scen8
Capital	1.54%	1.52%	1.45%	1.54%	4.46%	0.21%	4.33%	0.18%
Home	2.34%	2.18%	3.67%	2.20%	1.26%	3.02%	2.29%	2.86%
PPA	0.82%	0.84%	0.75%	0.79%	0.56%	0.84%	0.43%	0.88%
CAT	0.75%	0.84%	0.75%	0.71%	0.56%	0.71%	0.51%	0.80%
WC	1.10%	1.07%	0.97%	1.01%	0.86%	0.89%	0.58%	0.89%
CMP	1.53%	1.47%	1.77%	1.46%	1.05%	1.58%	1.11%	1.53%
OC	1.17%	1.40%	1.18%	0.96%	0.87%	1.06%	0.73%	1.34%
S-Tail	0.49%	0.47%	0.44%	0.48%	0.33%	0.64%	0.29%	0.62%
Total to Prm	2.61%	2.58%	2.71%	2.56%	5.15%	1.41%	5.11%	1.36%

The percentages shown for the capital, or surplus, account are the risk costs associated with the investment of the surplus as a percentage of the overall premium. An alternative approach would be to allocate the capital associated frictional costs to the classes of business. These risk cost allocations can be turned into risk-sensitive profit targets by class of business, which can in turn, be expressed as target loss or operating ratios. All the information necessary to do this, such as claim payment and premium receipt patterns, is available from the DFA calibration but these results would be highly dependent on the accuracy of the expense cost assumptions. As both commissions and expenses were allocated from overall figures, based on no more than inspired guesswork, this has not been done for this analysis

Whether and how overall risk costs should be allocated to classes of business is a matter of choice. For example, looking at Scenario 5, it is clear that the increase in investment risk, whilst increasing overall risk costs actually decreases the class of business allocations. Our class of business risk cost allocations for cost of capital or capital allocation purposes, would include the apportionment of the frictional costs associated with the capital (surplus) account. What is also happening here is that this higher risk strategy actually requires a higher level of capital in order to provide the same level of security to the policyholders. This is considered in the next section.

CAPITAL EVALUATION

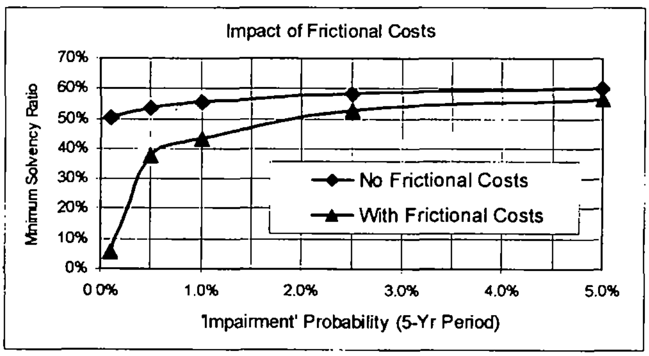
Insurance companies need to maintain a level of surplus which is considered sufficient by regulators and market security analysts and which provides a minimum necessary degree of protection to policyholders. Holding excess capital dilutes the returns to shareholders and may encourage managers to take on projects or business that they may otherwise consider unattractive.

Identifying the 'correct or optimal' amount of capital for an insurance company is a particularly demanding task as so much depends on future utilisation and management competence, as well as market conditions, competition and many other factors that are often outside the control of managers. This optimal amount of capital should also reflect the financial exposures the company faces, both from the type of business it underwrites, its investment strategy and also its degree of geographic and business diversification. In practice, financial impairment, or insolvency in extreme cases, of insurance companies is often associated with mis-management or operational risk.

Traditional capital evaluations using DFA or ALM or Solvency models, have tended to concentrate on financial impairment or probability of ruin using outputs from these models without any attempt to allow for such operational risks. The result is that increasingly remote probability levels have to be used in order to derive capital requirement values that look believable in the context of market experience and practice. The incorporation of frictional costs which include operational risk, changes impairment assessment and provides a basis for capital evaluation using DFA that was previously unavailable.

The process requires that the frictional costs be allocated to individual simulations. The family of cost functions described in Section 2 facilitate these associations using the pre-tax profit values as the 'ideal profit' variable. The results produce impairment probability estimates that appear much more realistic and useable. The following diagram illustrates the differences between the raw DFA output and the frictional cost adjusted results for the Base scenario.

Figure 4: Impairment assessment comparison



The two lines plot the probability that surplus, at some time during the 5-year projection period, as measured relative to annual premiums, drops to levels that may result in the company losing its security rating. In the absence of frictional costs, the surplus rarely drops below a 50% ratio (premium to surplus ratio of 2). The diagram shows the significant impact of frictional costs in these evaluations It shows, for example, that there is a 1% chance that solvency drops below 43% (premium to surplus ratio of 2.3).

This may well result in a downgrade of the company. It is a useful benchmark to adopt as defining the optimal level capital of DFAIC, consistent with its assumed plan strategy, to help investigate the capital implications implicit in the other strategies described above. The following table shows the results of such an impairment evaluation of all these strategies.

Table 6: Impairment impact of alternative strategies

Impairment	Minimum Solvency Ratio During Plan (5-yr) Period							
Probability	Scen1	Scen2	Scen3	Scen4	Scen5	Scen6	Scen7	Scen8
0.25%	28%	28%	21%	31%	18%	30%	16%	23%
0.5%	38%	41%	31%	42%	21%	38%	21%	37%
1.0%	43%	46%	38%	48%	25%	48%	29%	43%
2.5%	53%	54%	51%	56%	34%	58%	38%	52%
5.0%	57%	58%	56%	60%	42%	61%	45%	55%
10.0%	60%	61%	60%	63%	48%	64%	51%	58%
25.0%	65%	66%	65%	67%	56%	67%	59%	60%

The first thing to note is that Scenario 5 and Scenario 7 are the ones with a greater chance of impairment as measured by the minimum surplus to premium ratio at, say, the 1.0% level. For both of these scenarios, the surplus ratio is as low as 25% and 29% respectively, compared to the Plan 70% level that is assumed to be the level required to maintain the company A.M. Best, A rating. In practice, ratings may be impacted by changes of 20% in the 'surplus' amount backing the rating. In the example above, this will occur with a probability of 5% under the assumed Plan strategy (Scenario 1) and a disturbing probability of 25% in the case of Scenario 5, where 100% of the policyholder surplus is invested in equities.

Keeping to the 1% benchmark level, the results show that Scenario 2 (no class reinsurance) requires less capital as more premium and profit, are retained. This is not the case with the catastrophe reinsurance cover, as is shown by the results for Scenario 3. Clearly, this reinsurance increases the amount of capital required under the capital benchmark assumption in order to increase the minimum surplus ratio from the estimated 38% to the required 43% of the current scenario. By far the best option is Scenario 4, which assumes reinsurance protections at risk cost. This is, however, a somewhat artificial Scenario and unlikely to be available to DFAIC. It does help to demonstrate that with class reinsurances, much depends on the assumed price against the benefits implicit in the DFA large loss parameters.

Scenario 8 is the one developed by a preliminary analysis of the other scenarios. Its impairment shape, as indicated by the results in Table 6, is very close to the base scenario, Scenario 1. The select scenario has a lower surplus level, 60% of premium compared to the 70% of premium assumed for all the other scenarios and yet it is no more susceptible to impairment on this basis than Scenario 1.

The DFA model can be used to help identify the level of capital required to meet the impairment objective identified above for any of the other scenarios. The scenario in question is run with varying initial surplus levels and the 1% impairment level percentages are identified. Simple numerical approximation usually produces the exact capital level associated with the scenario. In such an exercise it is important to remember to adjust the dividend strategy to each new level of capital.

This is now demonstrated with Scenario 5 In this scenario the surplus is wholly invested in equities, compared to the current 35%, so the task is to identify the new surplus level that is required to support such a change in investment strategy. We define three new scenarios, identical to Scenario 5 but with initial surplus set at 85%, 100% and 125% of 1999 premiums, remembering also to amend the dividend strategy ratios in each case. The results are shown in the following table.

Table 7: Impairment table for capital evaluation

Impairment Probability	Min Solvency Ratio (5-yr)				
	Base	Scen 5 (70%)	Cap 85%	Cap 100%	Cap 125%
0.1%	6%	7%	15%	21%	26%
0.5%	38%	21%	27%	32%	43%
1.0%	43%	25%	31%	39%	49%
2.5%	53%	34%	43%	51%	63%
5.0%	57%	42%	52%	60%	76%
10.0%	60%	48%	59%	69%	86%
25.0%	65%	56%	68%	80%	99%

The results indicate that the required level of capital needed to support the particular change in investment strategy is somewhere between 100% and 125% of annual premium. The actual answer turns out to be 111% of premium. This is a near 60% increase in surplus, which would be very difficult to justify to shareholders.

The analysis does not stop at the identification of the surplus required to support a new strategy, whilst maintaining the previous or desired level of impairment criteria. Each new capital level will generate a different flow of dividends and these will need to be valued in order to see whether such a change in strategy is of benefit to shareholders. This is relatively easy to do, using deflators.

THE VALUE OF DFAIC

A publicly quoted company has an on-going market valuation in its capitalisation value. The methodology presented in this paper values variable cash flows in a manner that is consistent with the way the market values such flows.

One way to value a company, which is often implicit in multiplier approaches used in practice, is to project the stream of dividends and then risk discount the mean flows to present value. The DFA methodology described in this paper facilitates these calculations by using deflators and frictional risk cost adjustments to 'stochastically discount' the individual projected dividend streams.

DFA models are not intended to project over the longer term, with typical projection periods in practice ranging from three to seven years. These models are capable of valuing both the dividend streams and the retained end surplus at the end of the projection period. It is then possible to use these values, together with some simple assumptions, to estimate market values of the study company.

The following simple example illustrates how this can be done in practice.

Define a variable M as the ratio of market value to surplus, that is:

Market Value = M * Surplus

At the beginning of the projection period we have a value for the surplus. Now use the DFA model to project the dividends over the plan period and also the value of the ending surplus. Both the dividends and the ending surplus are already adjusted for frictional costs, so all that remains is to apply deflators to these values to obtain their market value at the beginning of the projection period.

We can now express the current market value as the value of the dividends and the market value of the ending surplus. This enables us to deduce the implicit multiple M, which we can then multiply by the current surplus to get an estimate of the company market value.

Using the Base assumptions, the DFA model estimates the present (deflated) value over the five-year projection period at $310m. The present (deflated) value of the end surplus is $1,431m. The assumed initial surplus was taken as $1,604m.

The multiplier M is then 1.79 [310/(1604-1431)], which estimates the market value of DFAIC at $2,874m. This estimate is dependent on the dividend stream projected and the particular choice of the frictional cost index.

The DFA model outputs that are used to estimate the likely market value of DFAIC rely on both the input business assumptions, particularly those relating to profitability levels as well as the choice of frictional cost index. DFA derived values that look abnormally high or low may simply indicate a poor calibration of the model business assumptions or a poor choice of the frictional cost index.

We prefer to view the availability of a market value as a very helpful element of the DFA calibration process. The approach described above is simply reversed to ensure that the level of 'ideal profits' being projected and the frictional cost index being used to adjust these 'ideal profits' to more realistic values are consistent with the company market capitalisation

The frictional cost index provides a link between the company plan, as defined by the 'ideal profit' to a market view of value. This helps to illustrate that the appropriate frictional costs index for a company, at a given time, will be highly dependent on the 'quality' of the underlying Plan and the markets' view on the likelihood that plan profits will be delivered In turn, this is influenced by the markets' assessment of the quality of the management team. We have demonstrated that in certain cases it is possible to identify the frictional cost index that provides the link between the company plan and the market's valuation of the company

Often, a company plan will be improved during the planning process until it meets an expected level of performance. This is sometimes achieved by reducing projected future losses or expenses. Unless such improvements are justified by changes in strategy the only real change may simply be the removal of some costs from these plans. Such plan changes are unlikely to be reflected in immediate increases in the market value of the company unless the managers convince the market of their viability.

In our formulation, what has happened is that the 'ideal profits' have been increased and we simply need to increase the frictional cost level to reflect any un-justified increase in these profits

5. CONCLUSION

In this paper we have seen that DFA can be a practical, powerful and flexible strategic management tool.

In particular we have described how DFA can help management with the:

- Evaluation of capital requirements
- Evaluation of capital utilisation and risk allocations
- Evaluation of asset and reinsurance strategies
- Identification of appropriate dividend strategies
- Identification of Shareholder Value Contributions

In order to achieve such functionality, the model has to have.

- A sound economic scenario generator
- Proper economic linkages between the liability and asset developments
- A methodology for turning the huge volume of outputs to summary information

Finally, we are aware that there is still a significant amount of scepticism as to whether DFA models, particularly complex ones, can be truly valuable or practical tools. Our experience has convinced us of the value that a focused DFA analysis can bring – we hope this paper will encourage many more to build and learn from DFA models.

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APPENDIX A: DESCRIPTION OF THE DFA MODEL

BASIC FEATURES

The model is a multi-line, multi-period, multi-currency, multi-strategy, multi-asset stochastic plan generator. It has small, large and peril losses, class excess, stop loss and whole account catastrophe excess of loss reinsurances, flexible investment and dividend strategies.

Economic files are pre-generated using a proprietary economic scenario generator (TSM – see next appendix) and include term structures and deflators. Returns for eight asset classes, including cash, stocks and bonds of various durations are available, split into income and gains.

DFA Scenarios are characterised by an economic file and a strategy file. The strategy file contains the user inputs that describe the company financials and plan characteristics. The model can run a number of scenarios at the same time These scenarios may use different strategy assumptions, for example in the amount of reinsurance to be bought, or may use different economic files to test the sensitivity of results from random economic assumptions. There is complete flexibility in the number of scenarios, simulations, seed numbers and the level of raw simulation data and summary statistical data that is saved. A Results Analyser facilitates the calibration and evaluation processes.

INPUTS

There are four main types of inputs required in order to run the model.

1: Company capital structure, accounting currency, assets, dividend and asset strategies, taxation rate and details of any whole account reinsurances.

2: Class of business details including patterns for receipt and earning of premiums, distributions to generate future losses and pay future and outstanding claims amounts, indices to generate future premiums and loss volumes, large loss frequency and severity distributions and reinsurance details, including the share of any whole account cover costs. Commission and expense information as well as the investment policy for the class policyholder funds is also required.

3: Peril losses (catastrophe events). Distributions for the frequency and severity of each such peril event, the loss amount main currency, the payment pattern and cost allocation to classes of business, for each of these peril events

4: Simulation control inputs, including scenarios to run, number of simulations, seed number (if required), name and format of the outputs database and the level of information to be output.

CAPITAL ACCOUNT, TAXES AND DIVIDENDS

Capital comprises issued, retained earnings and subordinated debt. A facility exists for consolidating an investment by year and simulation.

A single tax rate is input. Tax calculated is assumed paid at the end of the year. The user specifies the rate at which capital gains are to be realised for tax purposes. The model keeps track of unrealised gains and deferred taxes.

A number of alternative dividend strategies are accommodated, including no dividends, fixed dividends, fixed dividends in real terms, dividends calculated as % of post-tax profits and dividends calculated to reduce solvency or surplus margin to specified %.

ASSETS AND CURRENCIES

The model is a multi-currency one. Each class of business has a currency and this may or may not be the company accounting currency. This feature was not used in the DFAINC analysis as the company was assumed to have no exposures or investments in currency except the US$.

Assets classes for each economy include cash, equities, index linked bonds and government bonds of durations 1, 3, 5, 10 and 30 years. Asset mix for both shareholder funds as well as policyholder funds by class of business is specified for the projection period and can be in any of the available currencies and asset types. The model rebalances each 'fund' at each projection point.

PREMIUM AND LOSS CALCULATIONS

Premium income is determined as a product of a volume and pricing adequacy (cycle) with a fair premium and inflation adjustment, including any super-imposed inflation for the class of business·

The amount of variability of the first plan year premiums can be controlled by the user by class·

The theoretical 'fair' premium for a class for each period, is calculated taking account of the conditional expectations of future cash flows and the time value of money, allowing both for the claims payment and premium receipt patterns and inflation and interest term structures from the economic file. Premium receipt and earning patterns allow for multi-year policies

Losses for each class of business include normal or attritional losses, individual large losses and a share of any event or catastrophe losses.

Attritional losses are based on a user selected base year loss distribution for each class. Actual losses for a particular class/year/simulation are calculated from simulated values of the base year distribution, indexed by volume and inflation changes since the base year.

Large individual losses for each class are projected for each year using a loss frequency and loss severity distribution with the frequency adjusted for volume changes and the severity for inflation changes.

Event, peril or catastrophe losses for any number of perils (storms, flood, etc) are modelled at company level and allocated to classes according to an initial percentage and then adjusted for volume changes and inflation. Each peril event has its own payment pattern and a main loss currency.

Loss payments, before claims inflation (stochastic consumer price inflation plus a stochastic class specific super-inflation component), are determined by a payment pattern by class with random variability in the payments determined by an error distribution applied to the disposal rate by development year. Actual loss payments are subject to claims inflation including any class specific superimposed inflation, at the time of payment.

Loss and loss adjustment reserves are set at each evaluation point and for each accident year and class, taking account of pre-inflated amounts at the time and expected class specific inflation at the time, using the CPI term structure projected by the ESG, of expected payment. For each class, claims reserves may be discounted or undiscounted and may or may not contain margins defined by a percentage.

CORRELATIONS, REINSURANCE AND STATISTICAL DISTRIBUTIONS

Correlation between classes is generated implicitly as a result of a number of contributory influences, including premium market price indices, the impact of inflation on claims and the occurrence of catastrophe or peril losses that impact more than one class of business. The overall correlations resulting from these assumptions are then validated for reasonableness during the calibration process.

Reinsurance modelling is available at both class of business and overall company level, with excess of loss and stop loss in the case of classes and catastrophe excess of loss for the overall account. More complex reinsurances are modelled externally, before inflation, and net distributions so derived are used directly in the DFA model to allow for timing and inflation impacts to be evaluated.

Many statistical distributions are available through a distributions dynamic link library (dll) including the usual standard ones, such as normal, lognormal, poisson, pareto, weibull, extreme value as well as a number of user defined options and a new distribution, the Parbull, which is a pareto with a weibull tail and is described by three parameters, a (pareto) scale, a (pareto) shape and the point, or large value, at which the pareto tail becomes a weibull one. This distribution has been found particularly useful in modelling catastrophe event and large claim loss amounts.

THE ECONOMIC SCENARIO GENERATOR (TSM)
The Smith Model (TSM) is a proprietary macro-economic model calibrated to major world economies. It is a comprehensive, coherent, innovative and robust economic scenario generator.

It describes interest rates, inflation, exchange rates and equity returns (split between income and capital gains). Where inflation-linked bonds have been issued, these too are modelled.

The building block for The Smith Model is the numeraire, which is an economic cash flow quantity, which is modelled statistically. Examples of numeraires include currencies, inflation indices and equity dividend indices. Numeraires are treated within The Smith Model in an entirely symmetric manner. No single accounting unit holds a central role; any numeraire can be expressed in terms of any other numeraire.

Term Structures within financial markets consist of traded claims on future numeraires. For example, bonds denominated in various currencies can be considered to be future claims on that currency. Every different redemption date defines a different bond, which is modelled separately. This gives rise to a 'term structure' of interest rates, which describes how bond yields vary by term to redemption. Similarly, inflation linked bonds are considered as future claims on an inflation index. Even equities can be thought of as a special kind of bond whose cash flows are linked to a dividend numeraire - but this bond market is the least developed of all because investors only trade perpetuities

It is an efficient market, arbitrage free model. It generates asset prices by equating the supply of different investments to the demand of a representative investor. The equilibrium construction enables us to model risk and return consistently. The model can output the state price deflator, a weight which when applied to each simulation translates from the 'true' probability measure to the risk neutral version.

This enables market-consistent valuations to be assigned to awkwardly constructed cash flows; for example, insurance benefits or statutory profit. It is based on a Levy process, which in any time interval has both a large number of small jumps and also a small number of large jumps. These jumps apply to all asset classes, including interest rates, currencies and equities. However, the large jumps are more noticeable in some markets than others. It is these large jumps that capture the failure of traditional hedging techniques. It is implemented fully in continuous time

APPENDIX B: DFAIC DFA ASSUMPTIONS AND RESULTS

INPUT ASSUMPTIONS

Table 8: Starting balances, premium and expense ratios

	Loss and Loss Adjustment Reserves as at 12/99							
Acc Yr	All	Home	PPA	CAL	WC	CMP	COther	STail
1990 & Prior	196749	1494	11364	3108	96680	36031	48072	0
1991	34077	298	2349	856	22199	7500	875	0
1992	41579	195	3669	1339	25927	9217	1232	0
1993	49207	1858	4924	2154	28236	10615	1420	0
1994	74124	2042	10584	3977	29623	24854	2273	771
1995	114253	4348	22652	9735	36253	34205	5438	1622
1996	167455	5683	40134	15902	42806	49406	7418	6106
1997	278784	13638	86525	34418	57325	67729	16433	2716
1998	463891	23968	170166	65478	78934	100238	24468	639
1999	910056	85414	335722	98710	137297	166178	29751	56984
Total	2330175	138938	688089	235677	555280	505973	137380	68838

Other Balances 12/99	All	Home	PPA	CAL	WC	CMP	COther	STail
UPR	985422	181628	211134	77721	85323	164745	28658	236213
Agents Balances	445133	82045	95373	35108	38542	74418	12945	106702
RI due	49609	2958	14649	5018	11822	10772	2925	1466
Drafts	186209	11103	54987	18833	44374	40433	10978	5501
Funds for Inv	3007064	246666	844187	292106	634613	625961	161146	202385

Ratios	All	Home	PPA	CAL	WC	CMP	COther	STail
Earn Yr1		50.0%	68 5%	56.8%	67.8%	54.4%	64.5%	64 5%
Prem Receipt Yr1		74 7%	83.9%	78.6%	80.8%	78.0%	79.8%	84.0%
Com RatioNet		15 0%	14.0%	14.0%	9.0%	20.0%	20.0%	18.0%
U/W ExpenseNet		15.6%	12.0%	16 8%	19.2%	19.2%	13.2%	18.0%
Com Ratio Gr		13 9%	13.2%	12.8%	7.8%	18.6%	14.3%	18.0%
U/W ExpenseGr		14.5%	11 3%	15.4%	16 6%	17.8%	9.4%	18.0%

Table 9. Summary statistics from the economic (TSM) simulations.

Averages	Inflation	Cash	Equity	1Yr B	3Yr B	5Yr B	10Yr B	30Yr B
2000	1 6%	5 1%	8.5%	5.1%	5.1%	5 1%	5.2%	5.4%
2001	1.6%	5 1%	8.0%	5.1%	5.1%	5 1%	5.1%	5.2%
2002	1 6%	5.1%	8.4%	5.1%	5.1%	5 1%	5.2%	5.3%
2003	1 6%	5.1%	8.0%	5.1%	5.1%	5 1%	5.1%	5.1%
2004	1 6%	5 1%	8.5%	5.1%	5.1%	5 1%	5.2%	5.3%

Stats Yr2002	Inflation	Cash	Equity	1Yr B	3Yr B	5Yr B	10Yr B	30Yr B
St Dev	0 9%	0.6%	14.1%	0.6%	1.0%	1.4%	2 3%	4.7%
Skewness	5.0%	23.8%	38.8%	29 3%	16.7%	12.7%	12.6%	20.5%
1% Percentile	-0.6%	3.9%	-20 2%	3.9%	2.9%	2.0%	0 0%	-5.1%
5% Percentile	0.1%	4.2%	-12 8%	4.2%	3.6%	2.9%	1 6%	-2.1%
50% Percentile	1 6%	5 1%	7.1%	5.1%	5.1%	5 1%	5 1%	5.1%
95% Percentile	3 2%	6 1%	33.4%	6.1%	6.8%	7 4%	9 0%	13.3%
99% Percentile	3 8%	6 5%	45.4%	6.5%	7.5%	8.4%	10 7%	17.3%

Table 10: Class calibration assumptions

Loss Basis		Home	PPA	CAL	WC	CMP	COther	S-Tail
Base Premium		361086	645127	182675	239438	371117	90529	679254
Target N L/R		80.0%	80.0%	75.0%	72.5%	72.5%	67.5%	65.0%
Target RI Xol Pm		10000	39000	15000	34000	22000	25000	0
Parbull shape		1.6	1.5	1.5	1.5	1.4	1.5	0
Parbull scale		250	250	250	250	250	250	0
Parbull large		5000	15000	15000	15000	5000	15000	0
Parbull Mean		620	700	700	700	734	700	0
Number Large		38	60	23	51	46	38	0
Gross Large		23560	42000	16100	35700	33764	26600	0
Retained Large		18620	23400	8970	19890	23414	14820	0
Cats allocation		0.8	0	0	0	0.2	0	0
Retained Cats		33512	0	0	0	8378	0	0
Cat Premium		16000	0	0	0	4000	0	0
Attritional base		210981	441651	109748	123421	201414	24452	432459
Coeff Variation		7.5%	5.0%	5.0%	7.5%	7.5%	10.0%	5.0%
Log Par Attritional		12.257	12.997	11.605	11.721	12.210	10.099	12.976

Pre-Inflated Loss and Loss Adjustment Cumulative payment patterns								
Development Year		Home	PPA	CAL	WC	CMP	COther	S-Tail
1		70.2%	34.0%	27.3%	24.3%	39.1%	29.5%	87.6%
2		89.3%	66.0%	50.2%	49.5%	56.0%	41.2%	99.8%
3		94.6%	81.1%	71.5%	61.1%	68.1%	63.4%	99.3%
4		97.0%	90.2%	84.4%	68.4%	74.5%	77.1%	98.3%
5		97.9%	93.9%	90.7%	73.6%	83.4%	82.1%	99.5%
6		98.6%	96.8%	95.2%	79.0%	84.1%	83.1%	100.0%
7		98.7%	98.4%	96.7%	79.7%	91.4%	87.7%	100.0%
8		99.8%	98.6%	98.0%	82.2%	93.2%	91.8%	100.0%
9		99.7%	99.0%	98.8%	85.7%	94.7%	94.4%	100.0%
10		99.4%	98.3%	99.4%	86.8%	95.7%	95.5%	100.0%
Ultimate		100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%

Table 11: Base Asset Allocation

Asset Allocation	Cap Acc	Home	PPA	CAL	WC	CMP	COther	STail
Invested Amount $m	1,605	246	844	292	634	625	161	202
Adjusted Cash	0.0%	20.0%	20.0%	20.0%	15.0%	20.0%	20.0%	25.0%
Stocks	35.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
One Year	0.0%	5.0%	5.0%	10.0%	10.0%	5.0%	5.0%	30.0%
5 Year	5.0%	25.0%	25.0%	25.0%	25.0%	40.0%	25.0%	45.0%
10 Year	10.0%	50.0%	40.0%	35.0%	25.0%	15.0%	25.0%	0.0%
20 Year	45.0%	0.0%	10.0%	10.0%	10.0%	10.0%	20.0%	0.0%
30 Year	5.0%	0.0%	0.0%	0.0%	15.0%	10.0%	5.0%	0.0%

SAMPLES OF SUMMARY OUTPUTS

Table. 12: Base Scenario sample statistics

Year 2000	Average	StDev	Skew	1%	25%	50%	75%	99%
uwPremiumWritten	2400220	20904	-0.007	2351219	2385894	2400532	2414449	2450377
plPremiumEarned	2405048	12799	-0.006	2374785	2396187	2405293	2413764	2435062
PlClaimsIncd	1770595	75331	0.831	1619260	1721246	1765984	1813548	1967819
PcLossRatio	73.6%	3.0%	0.948	67.7%	71.7%	73.4%	75.3%	81.5%
plCommissionEarned	380745	1975	-0.007	376117	379393	380774	382088	385485
PlcfExpenses	328541	1670	-0.007	324675	327391	328570	329661	332503
plProfitOperating	-74833	71405	-0.957	-264724	-116222	-70055	-28135	67692
pcOperatingRatio	103.1%	3.0%	0.950	97.2%	101.2%	102.9%	104.8%	111.0%
PlProfitPre	130098	155833	-4.149	-353384	71478	149894	222726	375393
plFrictionalCost	-53707	65573	-17.110	-215091	-59451	-39883	-29959	-26545
plProfitPost	104079	124667	-4.149	-282707	57182	119915	178181	300314
diviDeclPaid	60094	63177	0.891	0	0	43410	104517	231547
bsRetainedProfit	87584	91153	-9.151	-239107	91087	110326	123081	147985
bsShareCapital	1560700	0	0.000	1560700	1560700	1560700	1560700	1560700
pcSolvencyRatio	68.7%	3.8%	-9.023	54.9%	69.1%	70.0%	70.0%	70.0%
pcReturnOnCapital	7.7%	13.4%	-16.480	-24.2%	4.4%	9.1%	13.5%	22.9%

Year 2002	Average	StDev	Skew	1%	25%	50%	75%	99%
uwPremiumWritten	2497660	52980	0.044	2379413	2462131	2497575	2532438	2621087
plPremiumEarned	2478598	47924	0.031	2369439	2446263	2478451	2510168	2592612
plClaimsIncd	1814255	74578	0.263	1651212	1762010	1812505	1861023	2008594
pcLossRatio	73.2%	2.9%	0.341	67.0%	71.3%	73.1%	75.1%	80.5%
plCommissionEarned	393187	7446	0.028	376502	388108	393113	398120	410725
plcfExpenses	338319	6324	0.026	323976	334029	338338	342530	353171
plProfitOperating	-67163	71445	-0.321	-248886	-113289	-65377	-19754	89037
pcOperatingRatio	102.7%	2.9%	0.342	96.4%	100.8%	102.6%	104.6%	110.0%
plProfitPre	157482	143995	-1.518	-289853	87820	173342	249467	433754
plFrictionalCost	-51561	43274	-7.154	-186881	-59385	-38444	-28628	-25449
plProfitPost	125985	115196	-1.518	-231883	70256	138673	199574	347003
diviDeclPaid	92435	84142	0.624	0	0	83161	153356	306995
bsRetainedProfit	153278	115614	-8.057	-293461	140048	175469	202649	267040
bsShareCapital	1560700	0	0.000	1560700	1560700	1560700	1560700	1560700
pcSolvencyRatio	68.6%	4.5%	-9.021	51.1%	69.9%	70.0%	70.0%	70.0%
pcReturnOnCapital	9.1%	8.9%	-2.141	-20.4%	5.2%	10.2%	14.5%	25.0%

Year 2004	Average	StDev	Skew	1%	25%	50%	75%	99%
uwPremiumWritten	2597184	73499	0.012	2425501	2546307	2597353	2646529	2772055
plPremiumEarned	2577289	67944	0.014	2419730	2530246	2577521	2621790	2743855
plClaimsIncd	1886714	88397	0.469	1698149	1827210	1885738	1941250	2102865
pcLossRatio	73.2%	3.0%	0.843	66.8%	71.2%	73.1%	74.9%	80.8%
plCommissionEarned	408557	10562	0.010	383809	401359	408637	415417	434390
plcfExpenses	351181	8971	0.006	330103	345147	351234	356997	373098
plProfitOperating	-69163	76667	-0.795	-265417	-114267	-67683	-18451	97298
pcOperatingRatio	102.7%	3.0%	0.840	96.3%	100.7%	102.6%	104.4%	110.2%
plProfitPre	168058	163430	-1.770	-332893	92188	184091	271421	475625
plFrictionalCost	-56408	51799	-8.982	-207115	-65350	-42888	-31175	-27766
plProfitPost	134446	130744	-1.770	-266314	73751	147273	217137	380500
diviDeclPaid	101272	93503	0.650	0	0	90898	170727	336902
bsRetainedProfit	218162	127586	-6.120	-227882	191991	241702	280795	364876
bsShareCapital	1560700	0	0.000	1560700	1560700	1560700	1560700	1560700
pcSolvencyRatio	68.5%	4.6%	-7.558	50.8%	69.8%	70.0%	70.0%	70.0%
pcReturnOnCapital	9.3%	10.5%	-4.139	-23.0%	5.1%	10.3%	15.2%	26.9%

Table 13: Class of Business Operating Results

Year 2000	Average values over 2,500 simulations							
Item	Total Ins	Home	PPA	CAL	WC	CMP	C Other	S Tail
uwPremiumWritten	2400220	334501	605209	167381	205069	344522	65411	678128
plPremiumEarned	2405048	335836	613519	166280	213455	340354	61998	673606
plClaimsIncd	1770595	269219	502894	124601	144720	246526	41620	441016
pcLossRatio	73.6%	80.2%	82.0%	74 9%	67.8%	72 4%	67.1%	65.5%
plCommissionEarned	380745	50293	86126	23203	19299	68138	12436	121249
plcfExpenses	328541	44142	61985	23565	34640	54950	6870	102388
plProfitOperating	-74833	-27819	-37486	-5089	14796	-29260	1072	8953
pcOperatingRatio	103 1%	108.3%	106.1%	103.1%	93.1%	108.6%	98.3%	98.7%

Year 2000	Standard deviations from 2,500 simulations							
Item	Total Ins	Home	PPA	CAL	WC	CMP	C Other	S Tail
uwPremiumWritten	20904	2821	8809	1379	1720	2844	584	5381
plPremiumEarned	12799	1391	5916	754	1109	1503	330	3470
plClaimsIncd	75331	37038	33073	9787	13956	18586	7123	30887
pcLossRatio	3.0%	11.0%	5.0%	5.8%	6.4%	5.4%	11 4%	4.6%
plCommissionEarned	1975	199	831	105	100	297	66	625
plcfExpenses	1670	174	598	107	180	240	37	528
plProfitOperating	71405	36938	31083	9621	13649	18286	7085	30675
pcOperatingRatio	3.0%	11.0%	5.0%	5 8%	6 4%	5 4%	11.4%	4.6%

Year 2000	1st percentile (1%) from 2,500 simulations							
Item	Total Ins	Home	PPA	CAL	WC	CMP	C Other	S Tail
uwPremiumWritten	2351219	327993	584652	164199	201101	337961	64063	665714
plPremiumEarned	2374785	332627	599714	164540	210896	336886	61238	665599
plClaimsIncd	1619260	210167	427766	103748	115792	206693	26134	373603
pcLossRatio	67.7%	62.7%	70.8%	62.9%	54.5%	60.9%	42.1%	55 4%
plCommissionEarned	376117	49835	84188	22960	19068	67452	12284	119808
plcfExpenses	324675	43740	60590	23319	34224	54397	6786	101171
plProfitOperating	-264724	-137489	-113528	-30801	-17686	-74689	-16466	-63513
pcOperatingRatio	97 2%	90.8%	95.0%	91 0%	79 8%	97 1%	73.3%	88.6%

Year 2000	99th percentile (99%) from 2,500 simulations							
Item	Total Ins	Home	PPA	CAL	WC	CMP	C Other	S Tail
uwPremiumWritten	2450377	341192	626250	170652	209149	351267	66797	690890
plPremiumEarned	2435062	339134	627650	168068	216086	343920	62781	681838
plClaimsIncd	1967819	379983	584073	150776	178570	292720	59164	514312
pcLossRatio	81.5%	112.7%	94.1%	90.3%	82.9%	85.8%	95.4%	76 2%
plCommissionEarned	385485	50765	88110	23452	19537	68844	12593	122731
plcfExpenses	332503	44556	63412	23819	35067	55519	6957	103639
plProfitOperating	67692	30794	30754	14972	42865	9826	16314	76243
pcOperatingRatio	111.0%	140.9%	118 3%	118 4%	108.2%	121.9%	126.6%	109.4%

Table 14: Plan period results – Base Scenario (Scen 1)

Base Scenario	Revenue Year				
Average values	2000	2001	2002	2003	2004
UwPremiumWritten	2400220	2449568	2497660	2546847	2597184
PlPremiumEarned	2405048	2430564	2478598	2527378	2577289
PlClaimsIncd	1770595	1780205	1814255	1850992	1886714
PcLossRatio	73.6%	73.2%	73.2%	73.2%	73.2%
PlCommissionEarned	380745	385699	393187	400787	408557
PlcfExpenses	328541	332046	338319	344682	351181
PlProfitOperating	-74833	-67386	-67163	-69083	-69163
PcOperatingRatio	103.1%	102.8%	102 7%	102 7%	102 7%
PlProfitPre	130098	145088	157482	156764	168058
PlFrictionalCost	-53707	-52119	-51561	-53257	-56408
PlProfitPost	104079	116071	125985	125411	134446
DiviDeclPaid	60094	83927	92435	93702	101272
BsRetainedProfit	87584	119728	153278	184988	218162
BsShareCapital	1560700	1560700	1560700	1560700	1560700
PcSolvencyRatio	68.7%	68.6%	68.6%	68.5%	68.5%
PcReturnOnCapital	7.7%	8.3%	9.1%	8.9%	9.3%

Base Scenario	Revenue Year				
Standard Deviations	2000	2001	2002	2003	2004
UwPremiumWritten	20904	44231	52980	62734	73499
PlPremiumEarned	12799	34868	47924	57474	67944
PlClaimsIncd	75331	74632	74578	81619	88397
PcLossRatio	3.0%	3.0%	2.9%	2.9%	3.0%
PlCommissionEarned	1975	5404	7446	8930	10562
PlcfExpenses	1670	4583	6324	7582	8971
PlProfitOperating	71405	72568	71445	73728	76667
PcOperatingRatio	3.0%	3 0%	2.9%	2.9%	3.0%
PlProfitPre	155833	152335	143995	153316	163430
PlFrictionalCost	65573	64686	43274	52453	51799
PlProfitPost	124667	121868	115196	122653	130744
DiviDeclPaid	63177	78372	84142	86758	93503
BsRetainedProfit	91153	109268	115614	121915	127586
BsShareCapital	0	0	0	0	0
PcSolvencyRatio	3.8%	4.4%	4.5%	4.6%	4.6%
PcReturnOnCapital	13.4%	18.9%	8.9%	10.1%	10.5%

Table 15: Scenario Results comparison – Year 2000

Year 2000 Averages	Scen 1	Scen 2	Scen 3	Scen 4	Scen 5	Scen 6	Scen 7	Scen 8
uwPremiumWritten	2,400,220	2,545,070	2,565,070	2,485,703	2,400,220	2,400,220	2,565,070	2,545,070
plClaimsIncd	1,770,595	1,845,638	1,852,388	1,770,595	1,770,595	1,770,595	1,852,388	1,845,638
pcLossRatio	73.6%	72.4%	72.1%	71.1%	73.6%	73.6%	72.1%	72.4%
plProfitOperating	-74,833	-5,026	8,224	10,650	-74,833	-74,833	8,224	-5,026
pcOperatingRatio	103.1%	100.2%	99.7%	99.6%	103.1%	103.1%	99.7%	100.2%
plProfitPre	130,098	201,921	210,401	217,710	122,672	126,853	206,833	193,731
plFrictionalCost	-53,707	-54,913	-60,108	-53,765	-94,100	-38,162	-96,078	-39,079
plProfitPost	104,079	161,537	168,320	174,168	98,138	101,482	165,466	154,985
dvDeclPaid	60,094	35,024	33,734	67,251	88,654	47,063	63,441	139,473
bsRetainedProfit	87,584	170,113	178,186	150,517	53,084	98,019	145,625	59,112
pcSolvencyRatio	68.7%	68.0%	67.8%	68.8%	67.2%	69.1%	66.5%	59.7%
pcReturnOnCapital	7.7%	11.7%	12.0%	12.8%	6.9%	7.0%	11.7%	11.8%

Year 2000 St Dev	Scen 1	Scen 2	Scen 3	Scen 4	Scen 5	Scen 6	Scen 7	Scen 8
uwPremiumWritten	20,904	21,688	21,688	21,296	20,904	20,904	21,688	21,688
PlClaimsIncd	75,331	79,434	90,228	75,331	75,331	75,331	90,228	79,434
PcLossRatio	3.0%	3.0%	3.4%	2.9%	3.0%	3.0%	3.4%	3.0%
plProfitOperating	71,405	75,334	86,602	71,258	71,405	71,405	86,602	75,334
pcOperatingRatio	3.0%	3.0%	3.4%	2.9%	3.0%	3.0%	3.4%	3.0%
PlProfitPre	155,833	158,839	180,960	155,963	255,462	130,360	261,730	132,329
plFrictionalCost	65,573	66,005	82,920	65,571	75,119	75,765	79,291	76,048
DvDeclPaid	63,177	49,504	48,885	66,360	110,013	44,526	93,588	65,998
bsRetainedProfit	91,153	102,935	123,222	89,038	135,787	86,692	155,692	74,764
pcSolvencyRatio	3.8%	4.1%	4.9%	3.6%	5.7%	3.6%	6.1%	2.9%
pcReturnOnCapital	13.4%	12.7%	18.4%	12.7%	17.8%	37.9%	17.8%	50.4%

Year 2000 Skew	Scen 1	Scen 2	Scen 3	Scen 4	Scen 5	Scen 6	Scen 7	Scen 8
PcLossRatio	0.948	0.825	1.495	0.948	0.948	0.948	1.495	0.825
plProfitOperating	-0.957	-0.839	-1.511	-0.962	-0.957	-0.957	-1.511	-0.839
PlProfitPre	-4.149	-3.984	-5.094	-4.138	-1.269	-11.656	-1.464	-11.106
plFrictionalCost	-17.110	-16.274	-15.826	-17.077	-5.979	-27.882	-6.291	-26.926
DvDeclPaid	0.891	1.507	1.571	0.778	1.307	0.716	1.721	-0.385
bsRetainedProfit	-9.151	-6.986	-7.778	-9.599	-3.819	-18.776	-3.408	-25.060
pcSolvencyRatio	-9.023	-6.791	-7.598	-9.510	-3.811	-18.456	-3.383	-25.732
pcReturnOnCapital	-16.480	-14.240	-22.009	-15.472	-2.871	-46.792	-3.294	-47.696

Year 2000 99th-tile	Scen 1	Scen 2	Scen 3	Scen 4	Scen 5	Scen 6	Scen 7	Scen 8
PcLossRatio	81.5%	80.4%	82.3%	78.7%	81.5%	81.5%	82.3%	80.4%
pcOperatingRatio	111.0%	108.2%	109.9%	107.2%	111.0%	111.0%	109.9%	108.2%
PlProfitPre	-353,384	-308,069	-360,059	-266,230	-674,462	-192,890	-626,045	-143,058
plFrictionalCost	-215,091	-230,304	-264,078	-215,268	-355,141	-142,564	-357,656	-153,000
PlProfitPost	-282,707	-246,456	-288,047	-212,984	-539,570	-154,312	-500,836	-114,446
pcSolvencyRatio	54.9%	53.2%	51.2%	55.8%	43.5%	59.7%	43.0%	53.9%
pcReturnOnCapital	-24.2%	-20.8%	-24.7%	-17.8%	-50.5%	-12.6%	-46.2%	-9.9%

An Analysis of the Underwriting Risk for DFA Insurance Company

Glenn G. Meyers, FCAS, MAAA

An Analysis of the Underwriting Risk for DFA Insurance Company

by

Glenn Meyers

Insurance Services Office, Inc.

Abstract

The DFA Insurance Company (DFAIC) is a fictional insurance company created by the CAS for the 2001 Dynamic Financial Analysis (DFA) Call for Papers. Those who respond to the call are expected to use DFA to answer specific questions about DFAIC's capital adequacy, capital allocation and reinsurance strategy. This paper is a response to that call

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Note

The theoretical backing for the methodology in this paper is provided in "The Cost of Financing Insurance" which is also published in this issue of the *CAS Forum*. Excel spreadsheets supporting these papers can be downloaded from the CAS Web Site: http://www.casact.org/pubs/forum/01spforum/meyers/index.htm

1. Introduction

In the mid-1990's the Casualty Actuarial Society coined the term "Dynamic Financial Analysis, " or "DFA" for short. Susan Szkoda [7], in her five part article beginning in the May 1997 *Actuarial Review*, defines DFA as "a process for analyzing the financial condition of an insurance entity. Financial condition refers to the ability of the entity's capital and surplus to adequately support future operations through a currently unknown future environment. ... In a very real sense, DFA requires the actuary to evolve into a financial risk manager."

In the ensuing years, the CAS has sponsored a number of special interest seminars, call paper programs, and research projects on DFA. Initially, those activities dealt with developing a model of insurance companies and getting the right data to support the model. As time passed, there was more focus on the specific insurer problems that DFA can solve. Some of those problems are in the 2001 CAS call for papers titled "Dynamic Financial Analysis, A Case Study." The call for papers presents participants with a specific actuarial situation, including a company description and financial statements. This paper is a response to the call.

Here, verbatim, is the description of the company and the specific actuarial situation provided by the call.

- ***Description of the Situation:***

The CEO of your company is considering the acquisition of DFAInsurance Company (DFAIC or the Company) as a stand-alone insurer. DFAIC is a privately held company and has not yet been contacted about this interest, and cannot be contacted until after your analysis is concluded. However, publicly available financial statements for the Company are available for the 1999 year and they are attached. The Company's last insurance department examination occurred in 1996 and there were no material issues. The Company has an unqualified actuarial opinion.

- ***Description of the Company's Business:***

The Company has an "A" rating from A.M. Best and it has maintained this rating for at least the past five years. It operates through the independent agency system and believes it has very strong relationships with its agency plant.

- ***Underwriting Profile***

The Company is licensed in all 50 states, but is primarily concentrated in the Northeast and the Midwest. The Company considers itself a "regional" company in these two geographic areas. Because of this focus, the Company has limited exposure to severe catastrophes. However, it does have exposure to less severe but more frequent retained catastrophe losses.

The Company writes a balanced book of both personal and main street commercial insurance coverages.

The Company has minimal exposure to asbestos and environmental exposures.

- ***Asset Classes***

The Company's cash and invested asset portfolio is approximately 70% fixed income, 12% equity and 18% cash.

The fixed income portfolio is approximately 80% in tax-exempt municipal issues and 20% in a mixture of Corporate and Government bonds. The Municipals have an average maturity of 10.5 years and an average yield of 6%. The Corporate and Government bonds have an average maturity of 4 years and an average yield of 8%. The equity portfolio is invested with a target return of the S&P 500.

- ***Reinsurance***

The Company maintains reinsurance to limit shock and catastrophic losses from a single event. The largest net aggregate amount insured in any one risk (excluding Workers Compensation) is $1 million. Excess of loss is used to protect property risks above $1,000,000 up to $20 million per risk, $50 million per occurrence. For casualty and Workers Compensation risks, an excess of loss treaty provides coverage above $500,000 up to $50.5 million.

The Company has a catastrophe cover of 90% of $150 million excess of $50 million for any single event. This limits the Company's net pre-tax PML for a catastrophe over a 100 year return period to 10% of surplus.

All of the Company's reinsurers are rated "A," or better, and there are no known problems with reinsurance recoverable.

- ***Questions the CEO would like addressed:***

1. Is the Company adequately capitalized? Is there excess capital? How much capital should the Company hold as a stand-alone insurer?
2. How should the capital be allocated to line of business?
3. What is the return distribution for each line of business and is it consistent with the risk for the line?
4. Should the Company buy more or less reinsurance? What type? How efficient is its current reinsurance program?
5. How efficient is the asset allocation?

2. Outline of the Analysis

The analysis will proceed in the following steps.

Section 3 describes how we calculated the aggregate loss distribution from its component claim severity and claim count distributions. With the aggregate loss distribution, we will then discuss the adequacy of DFAIC's capital.

Section 4 gives the capital allocations by line of business. We will also allocate capital to support outstanding losses from prior accident years. We will use these capital allocations to calculate the cost of financing for the individual lines of insurance.

Section 5 will use the results of Section 4 to calculate the cost of financing for the individual lines of insurance with the current reinsurance program. For the sake of comparison, we will calculate the cost of financing with alternative reinsurance programs, including the program of no reinsurance. We will then recommend a reinsurance program.

Section 6 will use the results of Section 5 to calculate target combined ratios that, if obtained, will lead DFAIC to make its target return on capital.

Included with this paper is a spreadsheet that takes the capital allocations described in Section 4 and derives the results in Sections 5 and 6. The spreadsheet will allow the reader to modify many of the assumptions made in Sections 5 and 6.

This paper focuses on DFAIC's underwriting risk. We will not attempt to quantify its asset risk or make any recommendations on how DFAIC should alter its investment strategy.

This paper will describe the capital measurement and allocation methodology in a "how-to-do-it" mode. Readers who desire a fuller description of this paper's methodology, including its economic rationale, should first read Meyers [4].

3. Capital Adequacy

The first step in evaluating an insurer's capital adequacy is to determine its aggregate loss distribution. The aggregate loss distribution can be thought of as a set of loss scenarios, where a "loss" is the sum of all the individual line of insurance losses from: (1) all claims from the current accident year; and (2) unsettled claims from prior accident years.

The following simulation algorithm explains our model of DFAIC's losses. Explanatory notes follow the description of the simulation algorithm.

Simulation Algorithm to Generate Loss Scenarios for DFAIC

Step

1. Select a random β from a distribution with $E[1/\beta] = 1$ and $Var[1/\beta] = b$.
2. For each covariance group, i, select random percentile p_i from a uniform (0,1) distribution.
3. For each covariance group, i, line of insurance, h, and accident year, y, with uncertain claim payments, do the following:
   - Select $\alpha_{ihy} = p_i^{th}$ percentile of a distribution with $E[\alpha_{ihy}] = 1$ and $Var[\alpha_{ihy}] = g_{ihy}$.
   - Select random claim count, K_{ihy} from a distribution with mean $\alpha_{ihy}\lambda_{ihy}$ where λ_{ihy} is the expected claim count for line of insurance h and accident year y in covariance group i.
   - For each i, h, and y, select random claim size, Z_{ihyk}, for $k = 1,\ldots,K_{ihy}$.
4. Set $X_{ihy} = \sum_{k=1}^{K_{ihy}} Z_{ihyk}$ = Loss for covariance group i, line h, and accident year y.
5. Set $X = \sum_{i} \sum_{h \in G_i} \sum_{y} X_{ihy} / \beta$ = Loss for DFAIC.

Notes on the Simulation Algorithm

- β has an inverse gamma distribution, as originally described by Heckman and Meyers [3]. The variance, b, is called the mixing parameter. b describes the uncertainty in future claim severity. As described in Meyers [4] the random multiplier, β, causes correlation between the lines of insurance.

- The various lines of insurance are classified into "covariance groups." The lines of insurance within each covariance group are those that we expect to move together over time. Table 3.1 below, gives the assignment of lines of insurance to covariance groups.

- By selecting the parameter $\alpha_{ihy} = p_i^{th}$ percentile of a distribution with $E[\alpha_{ihy}] = 1$ and $Var[\alpha_{ihy}] = g_{ihy}$ we are making "high" or "low" claim counts in all lines of a covariance group simultaneously.

- We based the selection of the parameters, g_{ihy}, on an analysis of the data of several insurers that report their data to ISO. We used the estimation methodology described in Meyers [5]. Although the results based on this data had the greatest influence on the final parameter selections, data from Schedule P of insurer annual statements provided supplementary information.

- For most lines of business, we derived the claim severity distributions from data reported to ISO.

- We obtained a workers compensation size of loss distribution from an independent state rating bureau. Using (1) claim payout patterns; (2) aggregate loss payout patterns; and (3) the general intuition that later-settling claims are also larger claims, we were able to select size of loss distributions for the current and prior accident years that were consistent with the available data. We used this size of loss distribution for all states.

- We used a catastrophe model to generate a hurricane size-of-loss distribution. The call for papers did not give the necessary exposure information to run a cat model, but we have done analyses on insurer catastrophe exposure. See Insurance Services Office [2] for the complete analysis. We selected the catastrophe size-of-loss distribution from an insurer that has a similar geographic distribution to DFAIC. We made a scaling adjustment so that the 100-year loss was close to 10% of DFAIC's capital, as specified in the call for papers.

- We obtained the expected total losses by estimating the average loss ratio, projecting premium to the year 2000, and then multiplying the projected premium times the average loss ratio.

- We used a negative binomial distribution to describe the claim count distribution. We obtained the expected claim count dividing the expected loss by the expected claim severity. As described in Meyers [5], the same methodology that yields estimates for the g_{ihy} parameters also gives the variance parameters of the claim count distributions.

- *In spite of the loss model's description as a simulation process, we did not use* simulation to calculate the aggregate loss statistics described below. Instead, we used Fourier inversion, as described by Heckman and Meyers [3] and Meyers [6]. The aggregate loss statistics calculated by the Fourier methodology are identical to what we would expect to obtain by simulation if we repeat the simulation several thousand times. The advantage of the Fourier methodology is that DFAIC's aggregate loss distributions can be calculated in a few seconds on current personal computers. Our loss model for DFAIC has 50 different line/accident year segments. In the analysis below, we need to calculate the marginal cost of capital by removing each *line/accident year segment from DFAIC and calculating the aggregate loss* distribution for the remaining losses. We do the calculation for each reinsurance strategy. The very fast calculation made possible by the Fourier methods is what makes this kind of analysis operationally possible.

Table 3.1

DFAIC Aggregate Loss Model Input

Line of Insurance	Covariance Group	Prior Accident Years	Source of Size-of-Loss Distribution Data
Property Catastrophe	1	0	Catastrophe Model
Allied Lines	2	1	ISO Basic Group 2 Commercial Property
Fire	2	1	ISO Basic Group 1 Commercial Property
Homeowners	2	4	Mixture of ISO HO property and liability
Commercial Auto	3	4	ISO Countrywide Commercial Auto Liability
Private Passenger Auto	3	6	ISO Countrywide Private Passenger Automobile Liability
Auto Physical Damage	3	1	ISO Countrywide Auto Physical Damage – Mixture of personal and commercial
General Liability	4	6	ISO Premises/Operations Liability
Products Liability	4	6	ISO Countrywide Products Liability
Commercial MultiPeril	4	6	Mixture of ISO Countrywide Premises/ Operations and Commercial Property
Workers Compensation	5	4	Independent State Rating Bureau

Our analysis of DFAIC's aggregate loss distribution did not include all lines of insurance. Table 3.2 contrasts the percentage of written premium for the included and excluded lines.

Table 3.2

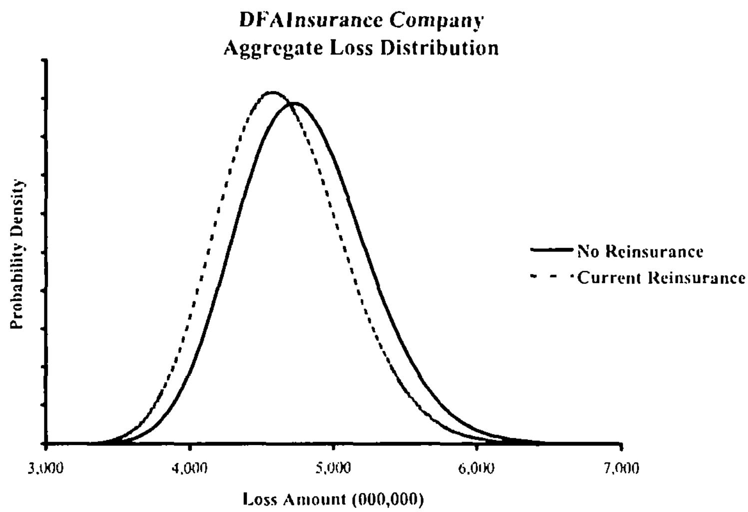
Lines Included/Excluded in DFAIC Aggregate Loss Analysis

Lines Included	%DWP	Lines Excluded	%DWP
Allied*	0.76%	Inland Marine	2.14%
Fire	0.66%	Earthquake	0.04%
Homeowners*	13.77%	Burglary	0.00%
Commmercial Auto Liability	7.01%	Special Liability	
Personal Auto Liability	24.67%	(Ocean Mar, Aircraft, B&M)	0.91%
Auto Physical Damage	22.48%	Other Liability Claims Made	0.03%
Other Liability Occurrence	2.61%	Reinsurance	0.27%
Product Liability Occurrence	0.05%	Fidelity/Surety	0.95%
CMP*	14.26%	Other (Credit, A&H)	0.21%
Workers Compensation	9.18%		
Total	**95.45%**	**Total**	**4.55%**

* A portion of the property losses was allocated to catastrophes.

We calculated the aggregate loss distribution for the current reinsurance strategy and for no reinsurance. Chart 3.1 shows the resulting probability density functions for each aggregate loss distribution.

Chart 3.1



Tables 3.3 and 3.4 give some loss statistics and various percentiles of the aggregate loss distributions with and without reinsurance. Tables 3.3 and 3.4 also include a recently developed measure of risk called the Tail Value at Risk. The Tail Value at Risk (TVaR) is a member of a class of "Coherent Measures of Risk," developed in a paper by Philippe Artzner, Freddy Delbaen, Jean-Marc Eber and David Heath [1]. Meyers [4] further describes this measure.

To calculate the TVaR, first select an α-value such as 99%. Then calculate the α^{th} percentile, otherwise known as the Value at Risk (VaR_α), of the insurer's aggregate loss distribution. The $TVaR_\alpha$ is the average of all the aggregate losses greater than VaR_α.

Following Meyers [4], we define the capital needed to support the insurer's losses as:

$$\text{Insurer's Capital}_\alpha = TVaR_\alpha - \text{Insurer's Expected Loss.} \tag{3.1}$$

Table 3.3

Aggregate Loss Distribution
DFAInsurance Company's Current Reinsurance Strategy

Aggregate Mean	4,670,320,245
Aggregate Standard Deviation	441,528,312

Percentile/ α-Level	Value at Risk (VaR_α)	Tail Value at Risk ($TVaR_\alpha$)	$TVaR_\alpha$ Implied Capital
0.00%	0	4,670,320,245	0
5.00%	3,981,884,307	4,714,308,377	43,988,131
10.00%	4,118,916,712	4,750,885,242	80,564,997
15.00%	4,214,710,884	4,785,122,592	114,802,347
20.00%	4,292,757,132	4,818,268,948	147,948,703
25.00%	4,361,055,273	4,850,985,309	180,665,064
30.00%	4,423,437,646	4,883,724,871	213,404,626
35.00%	4,482,123,634	4,916,856,052	246,535,807
40.00%	4,538,587,601	4,950,719,808	280,399,562
45.00%	4,593,932,131	4,985,665,883	315,345,638
50.00%	4,649,081,266	5,022,083,793	351,763,548
55.00%	4,704,902,848	5,060,437,614	390,117,369
60.00%	4,762,308,463	5,101,313,239	430,992,994
65.00%	4,822,363,037	5,145,491,698	475,171,453
70.00%	4,886,440,632	5,194,074,153	523,753,908
75.00%	4,956,493,675	5,248,716,088	578,395,843
80.00%	5,035,597,008	5,312,116,293	641,796,048
85.00%	5,129,246,166	5,389,199,164	718,878,919
90.00%	5,249,256,922	5,490,689,078	820,368,832
92.50%	5,327,867,949	5,558,560,374	888,240,129
95.00%	5,431,481,121	5,649,433,290	979,113,045
95.50%	5,457,233,034	5,672,240,507	1,001,920,262
96.00%	5,485,511,269	5,697,378,336	1,027,058,091
96.50%	5,516,957,471	5,725,441,319	1,055,121,073
97.00%	5,552,500,699	5,757,291,825	1,086,971,580
97.50%	5,593,562,931	5,794,251,579	1,123,931,334
98.00%	5,642,491,579	5,838,505,673	1,168,185,428
98.50%	5,703,609,532	5,894,086,653	1,223,766,408
99.00%	5,786,406,345	5,969,867,978	1,299,547,733
99.50%	5,920,196,054	6,093,354,285	1,423,034,040
99.90%	6,201,613,212	6,356,439,702	1,686,119,456
99.95%	6,312,968,410	6,461,547,653	1,791,227,407
99.99%	6,553,948,422	6,690,713,812	2,020,393,567

Table 3.4

Aggregate Loss Distribution
DFAInsurance Company without any Reinsurance

Aggregate Mean	4,803,449,179
Aggregate Standard Deviation	451,811,506

Percentile/ α-Level	Value at Risk (VaR_α)	Tail Value at Risk ($TVaR_\alpha$)	$TVaR_\alpha$ Implied Capital	$TVaR_\alpha$ Implied Capital w/Reins	Difference in Capital
0.00%	0	4,803,449,179	0	0	0
5.00%	4,098,518,418	4,848,514,188	45,065,009	43,988,131	1,076,877
10.00%	4,239,075,826	4,885,960,910	82,511,731	80,564,997	1,946,735
15.00%	4,337,267,437	4,921,001,937	117,552,758	114,802,347	2,750,412
20.00%	4,417,229,633	4,954,919,320	151,470,141	147,948,703	3,521,439
25.00%	4,487,179,430	4,988,391,368	184,942,189	180,665,064	4,277,125
30.00%	4,551,051,966	5,021,882,806	218,433,627	213,404,626	5,029,001
35.00%	4,611,125,381	5,055,771,150	252,321,971	246,535,807	5,786,164
40.00%	4,668,912,417	5,090,405,549	286,956,369	280,399,562	6,556,807
45.00%	4,725,543,728	5,126,143,947	322,694,768	315,345,638	7,349,130
50.00%	4,781,966,292	5,163,384,814	359,935,635	351,763,548	8,172,087
55.00%	4,839,068,865	5,202,602,738	399,153,559	390,117,369	9,036,191
60.00%	4,897,784,402	5,244,396,801	440,947,622	430,992,994	9,954,628
65.00%	4,959,202,157	5,289,565,494	486,116,315	475,171,453	10,944,862
70.00%	5,024,727,025	5,339,234,548	535,785,369	523,753,908	12,031,461
75.00%	5,096,354,650	5,395,096,402	591,647,223	578,395,843	13,251,380
80.00%	5,177,227,531	5,459,910,283	656,461,104	641,796,048	14,665,056
85.00%	5,272,961,845	5,538,711,398	735,262,219	718,878,919	16,383,301
90.00%	5,395,633,669	5,642,467,530	839,018,351	820,368,832	18,649,519
92.50%	5,475,984,579	5,711,860,388	908,411,209	888,240,129	20,171,080
95.00%	5,581,892,641	5,804,783,090	1,001,333,911	979,113,045	22,220,866
95.50%	5,608,215,928	5,828,107,815	1,024,658,636	1,001,920,262	22,738,375
96.00%	5,637,122,398	5,853,817,515	1,050,368,336	1,027,058,091	23,310,245
96.50%	5,669,268,283	5,882,521,205	1,079,072,026	1,055,121,073	23,950,953
97.00%	5,705,604,046	5,915,102,021	1,111,652,842	1,086,971,580	24,681,262
97.50%	5,747,584,639	5,952,913,814	1,149,464,635	1,123,931,334	25,533,301
98.00%	5,797,612,008	5,998,195,243	1,194,746,064	1,168,185,428	26,560,636
98.50%	5,860,110,958	6,055,079,158	1,251,629,979	1,223,766,408	27,863,571
99.00%	5,944,797,553	6,132,662,637	1,329,213,458	1,299,547,733	29,665,725
99.50%	6,081,703,000	6,259,162,825	1,455,713,646	1,423,034,040	32,679,607
99.90%	6,370,035,159	6,529,082,838	1,725,633,659	1,686,119,456	39,514,203
99.95%	6,484,300,353	6,637,118,106	1,833,668,927	1,791,227,407	42,441,520
99.99%	6,732,087,246	6,873,187,638	2,069,738,459	2,020,393,567	49,344,892

The spreadsheet included with this paper gives the correlation matrices for the lines/accident year combinations of DFAIC, with and without reinsurance.

With the aggregate loss distribution in hand, we now turn to discussing the adequacy of DFAIC's capital. Ideally, we would like to have capital adequacy standards that enable us to select an α-level that corresponds to a given rating. While such standards may evolve in the future, we do not believe that standards exist yet. We therefore accept the unqualified actuarial opinion that DFAIC's capital is adequate. We also accept that DFAIC is entitled to the rating of A given to it by the A.M. Best company.

DFAIC's capital is $1,604,297,000. By examining Table 3.3 we see that this corresponds to an α-level between 99.5% and 99.9%. However, in constructing DFAIC's aggregate loss distribution, we ignored lines of insurance that account for almost 5% of the premium. We also ignored asset risk. With more than $500 million invested in stocks, a drop in asset values in the range of $50 to 100 million appears possible. At the time of this writing, the S&P 500 stock index has recently dropped from over 1,500 to below 1,200. With this in mind, we judgmentally set an α-level of 99.0%, as our standard for adequate capital for the modeled lines/accident year combinations. We will use that standard in the work below.

4. Allocating Capital

We allocate capital to the 50 line/accident year combinations in proportion to their marginal capital. To do that we need to calculate the $TVaR_{99\%}$ for DFAIC 50 times, removing each combination, in turn, from the calculation. Because of the reduction in risk due to pooling, the sum of marginal capitals for each combination will add up to less than the total capital. Thus, we need to multiply each marginal capital by a pooling factor to force the total capital to equal the sum of the allocated capitals. Meyers [4] provides the economic rationale for the pooling factor.

For the long-tailed lines for which DFAIC incurs losses in 2000, there will be uncertainty in the loss payments made in 2001, 2002 and even in later years. Thus, DFAIC will have to allocate capital for the accident year 2000 in 2001, 2002, and so on.

To make those allocations in future years, we must make a business plan for future years and allocate capital according to that plan. In our underwriting risk model for DFAIC, we allow for uncertainty in future loss reserves for seven years. We assume that DFAIC's business plan is to continue its present writings. If it also does not change its reinsurance plan, the allocations to line/accident year combinations over the next seven years will not change. If DFAIC decides to change its reinsurance plan in 2000, the allocations from prior accident years will still reflect the old reinsurance plan. For example, in calendar year 2000, there will be one year under the new plan, and six under the old plan. In calendar year 2001, there will be two years under the new plan, and five years under the old plan. If we introduce a new reinsurance plan, we must do a new allocation for each of seven years.

The spreadsheet that accompanies this paper contains capital allocations for four different reinsurance strategies. In this paper we exhibit two of those strategies – the current reinsurance plan and no reinsurance.

Table 4.1

Capital Allocations for Accident Year 2000 – Current Reinsurance Strategy

Line\Cal Year	2000	2001	2002	2003	2004	2005	2006
CAT	5,109,553	0	0	0	0	0	0
Allied	2,080,280	448,489	0	0	0	0	0
Fire	1,196,144	259,202	0	0	0	0	0
HO	59,783,298	13,620,819	1,834,222	814,426	390,224	0	0
CAL	82,133,539	53,923,373	30,202,957	14,514,549	6,154,661	0	0
PAL	319,844,532	74,634,107	16,882,652	6,088,341	2,215,292	794,825	304,133
APHD	223,086,578	43,362,982	0	0	0	0	0
OLOC	11,623,350	8,892,618	6,220,151	4,163,600	2,606,682	1,577,708	920,056
PLOC	180,894	143,389	108,976	80,123	54,912	36,474	24,125
CMP	125,859,211	47,323,486	20,457,481	13,436,588	8,388,555	5,104,263	2,930,801
WC	40,204,443	25,091,661	11,229,472	2,327,101	882,433	0	0
Other Acc. Years	433,555,463	1,031,847,606	1,212,611,822	1,258,123,006	1,278,854,972	1,292,034,462	1,295,368,617
Total	1,299,547,733	1,299,547,733	1,299,547,733	1,299,547,733	1,299,547,733	1,299,547,733	1,299,547,733

Table 4.2

Capital Allocations for Accident Year 2000 – No Reinsurance

Line\Cal Year	2000	2001	2002	2003	2004	2005	2006
CAT	10,825,505	0	0	0	0	0	0
Allied	2,755,376	598,084	0	0	0	0	0
Fire	2,088,443	462,445	0	0	0	0	0
HO	60,117,157	13,630,159	1,840,641	818,455	392,988	0	0
CAL	81,662,436	53,041,665	29,712,369	14,286,974	6,062,589	0	0
PAL	316,846,970	73,410,423	16,620,268	5,996,813	2,183,421	784,336	300,435
APHD	221,395,415	42,680,569	0	0	0	0	0
OLOC	13,151,693	10,512,690	7,509,305	5,071,918	3,205,067	1,960,704	1,150,798
PLOC	211,935	174,496	133,368	98,647	67,786	45,025	29,757
CMP	130,892,569	52,918,138	24,550,980	16,509,345	10,465,673	6,470,773	3,824,253
WC	43,339,968	27,208,148	12,230,581	2,536,807	962,906	0	0
Other Acc. Years	440,585,900	1,051,512,645	1,235,019,786	1,283,022,192	1,305,450,556	1,319,792,066	1,323,908,215
Total	1,313,047,863	1,326,149,463	1,327,617,298	1,328,341,150	1,328,790,987	1,329,052,903	1,329,213,458

Here are some observations on Tables 4.1 and 4.2.

- The current reinsurance strategy allocations in Table 4.1 correspond to the aggregate loss distribution in Table 3.3. The total capital for the current reinsurance strategy is equal to that implied by $TVaR_{99\%}$ in Table 3.3.
- If DFAIC changes over to no reinsurance, by 2006 we will allocate no capital to the line/accident year combinations affected by the current reinsurance. The total capital in 2006 for the no-reinsurance strategy is equal to that implied by $TVaR_{99\%}$ in Table 3.4.
- As removing the reinsurance affects more and more accident years, the total capital needed increases from $1,299,547,733 needed with reinsurance to $1,329,213,458 needed in 2006 with no reinsurance.

5. The Cost of Financing Insurance

Ultimately, the policyholders must bear the cost of capital and/or reinsurance. Investment earnings on the capital reduce that cost to some extent. In this section, we calculate the expected profit needed in 2000 for the insurer to make its overall expected return on capital.

Let $A_k(t)$ be the capital allocated to line of insurance k in calendar year 2000 + t. For example we see in Table 4.1 that for k = allied lines we have that $A_k(0)$ = \$2,080,280 and $A_k(1)$ = \$448,489. DFAIC needs \$2,080,280 at the beginning of 2000 to support its allied lines losses from accident year 2000, and it needs \$448,489 to support its allied lines losses from accident year 2000 at the beginning 2001. If DFAIC gets a 7% return on its invested assets the company can release \$2,080,280 × 1.07 – 448,489 = \$1,777,411 to its investors at the end of 2000. Let i be the return on invested assets, $R_k(0)$ be the Net Cost of Reinsurance, calculated as (Price – Expected Recovery)×(1 – Corporate Income Tax Rate) payable for line k at the beginning of 2000. Let $Rel_k(t)$ be the capital released at the beginning of calendar year 2000 + t. Then following Meyers [4], Table 5.1 gives the schedule for releasing capital.

Table 5.1

Schedule for Releasing Capital

Time	Financial Support Allocated at Time t	Amount Released at Time t
0	$A_k(0) + R_k(0)$	0
1	$A_k(1)$	$Rel_k(1) = A_k(0)(1+i) - A_k(1)$
---	---	---
t	$A_k(t)$	$Rel_k(t) = A_k(t-1)(1+i) - A_k(t)$
---	---	---

We give the schedule for releasing capital for DFAIC for i = 7% in the following tables. These tables are also available on the spreadsheet included with this paper.

Table 5.2

Schedule for Releasing Capital at the End of the Year with Current Reinsurance Strategy

Line\Cal Year	2000	2001	2002	2003	2004	2005	2006
CAT	5,467,222	0	0	0	0	0	0
Allied	1,777,411	479,883	0	0	0	0	0
Fire	1,020,672	277,347	0	0	0	0	0
HO	50,347,310	12,740,055	1,148,191	481,211	417,540	0	0
CAL	33,959,513	27,495,052	17,802,615	9,375,906	6,585,488	0	0
PAL	267,599,543	62,975,842	11,976,096	4,299,233	1,575,538	546,330	325,422
APHD	195,339,656	46,398,391	0	0	0	0	0
OLOC	3,544,366	3,294,950	2,491,962	1,848,370	1,211,442	768,091	984,460
PLOC	50,168	44,450	36,481	30,819	22,282	14,902	25,814
CMP	87,345,870	30,178,650	8,452,916	5,988,594	3,871,490	2,530,761	3,135,957
WC	17,927,093	15,618,605	9,688,434	1,607,565	944,204	0	0

Table 5.3

Schedule for Releasing Capital at the End of the Year with No Reinsurance

Line\Cal Year	2000	2001	2002	2003	2004	2005	2006
CAT	11,583,290	0	0	0	0	0	0
Allied	2,350,168	639,950	0	0	0	0	0
Fire	1,772,190	494,816	0	0	0	0	0
HO	50,695,199	12,743,630	1,151,031	482,759	420,497	0	0
CAL	34,337,142	27,042,213	17,505,261	9,224,473	6,486,970	0	0
PAL	265,615,835	61,928,885	11,786,873	4,233,169	1,551,925	538,804	321,465
APHD	194,212,524	45,668,209	0	0	0	0	0
OLOC	3,559,621	3,739,274	2,963,038	2,221,885	1,468,718	947,155	1,231,354
PLOC	52,275	53,343	44,057	37,766	27,506	18,420	31,840
CMP	87,136,910	32,071,428	9,760,204	7,199,326	4,727,497	3,099,474	4,091,950
WC	19,165,618	16,882,137	10,549,915	1,751,477	1,030,310	0	0

Let e be DFAIC's expected pretax return on equity. Then, following Meyers [4], the cost of financing, $\Delta P_k(0)$, necessary for the insurer to make its expected rate of return is given by:

$$\Delta P_k(0) = \underbrace{A_k(0) - \sum_{t=1}^{\infty} \frac{Rel_k(t)}{(1+e)^t}}_{\text{Cost of Capital}} + \underbrace{R_k(0)}_{\substack{\text{Net Cost}\\ \text{of Reinsurance}}} \tag{5.1}$$

We will calculate the Net Cost of Reinsurance by first specifying an expected loss ratio, ELR_k. We then have:

$$R_k(0) = E[\text{Recovery in Line } k] \times \left(\frac{1}{ELR_k} - 1 \right) \times (1 - \text{Corporate Income Tax Rate}) \tag{5.2}$$

Then setting e = 15%, ELR_k = 50% for the catastrophe line; ELR_k = 65% for all other lines; the corporate income tax rate = 35% and applying Equation 5.1 to the entries in Table 5.2 --- we get the following table for the cost of financing with the current reinsurance strategy.

Table 5.4

The Cost of Financing Insurance with the Current Reinsurance Strategy

Line of Business	Cost of Capital	Net Cost of Reinsurance	Cost of Financing
CAT	355,447	2,857,770	3,213,217
Allied	171,845	1,042,073	1,213,918
Fire	98,890	1,168,697	1,267,587
HO	5,132,037	18,183	5,150,221
CAL	11,472,953	0	11,472,953
PAL	28,056,063	0	28,056,063
APHD	18,142,158	0	18,142,158
OLOC	2,050,057	1,796,206	3,846,263
PLOC	34,826	37,875	72,701
CMP	13,907,103	4,171,525	18,078,628
WC	5,046,888	5,986,520	11,033,408
Total	84,468,267	17,078,848	101,547,115

Doing the same calculation with the entries in Table 5.3, we get the following table for the cost of financing with no reinsurance.

Table 5.5

The Cost of Financing Insurance with No Reinsurance

Line of Business	Cost of Capital	Net Cost of Reinsurance	Cost of Financing
CAT	753,079	0	753,079
Allied	227,857	0	227,857
Fire	173,257	0	173,257
HO	5,156,459	0	5,156,459
CAL	11,346,968	0	11,346,968
PAL	27,753,784	0	27,753,784
APHD	17,983,232	0	17,983,232
OLOC	2,407,721	0	2,407,721
PLOC	41,975	0	41,975
CMP	15,108,299	0	15,108,299
WC	5,458,493	0	5,458,493
Total	86,411,124	0	86,411,124

We also considered two additional reinsurance strategies. We summarize the results in the following table.

Table 5.6

Cost of Financing Insurance for Four Reinsurance Strategies

Reinsurance Strategy	Cost of Capital	Net Cost of Reinsurance	Cost of Financing
Current Reinsurance	84,468,267	17,078,848	101,547,115
No Reinsurance	86,411,124	0	86,411,124
Cat Reinsurance Only 90% of Loss over $50 M	85,922,455	3,835,282	89,757,738
Liability Reinsurance Only	84,905,169	12,010,309	96,915,478

Comments

DFAIC is paying a net cost of $17,078,848 for its reinsurance in order to save $86,411,124 – $84,468,267 = $1,942,857 for its cost of capital. We recommend that DFAIC stop buying reinsurance. Qualitatively, this makes sense for a well-diversified insurer writing more than $2.5 billion in premium with more than $5.3 billion in assets, and no significant catastrophe potential.

However, we offer one qualification to this conclusion. The decision to purchase reinsurance is usually made by upper level management who are sensitive to the needs of the insurer's investors. *If* the investors value stability in earnings, they will demand a higher return on capital if the reinsurance coverage is dropped. In that case, the cost of financing reinsurance will not be reduced by the as much as the above analysis indicates. The following table gives the return on capital that makes all four of the above reinsurance strategies equivalent.

Table 5.7

Return on Capital for Four Reinsurance Strategies

Reinsurance Strategy	Return on Capital	Cost of Financing
Current Reinsurance	15.00%	101,547,115
No Reinsurance	16.59%	101,547,115
Cat Reinsurance Only 90% of Loss over $50 M	16.24%	101,547,115
Liability Reinsurance Only	15.49%	101,547,115

Whether or not investors will demand these returns is debatable. Financial theory tells us that investors will not demand a higher return if the risk removed by reinsurance is diversifiable. We leave it at that.

6. Target Combined Ratios

The final step in this analysis is to calculate target combined ratios for each line of insurance. These targets will take into account the cost of financing insurance, investment income derived from writing the insurance and expenses. We made the following assumptions (simplified for the purpose of this paper.)

- Losses are paid at the midpoint of the year.
- Losses are discounted at DFAIC's return on invested assets when calculating the Actuarial Present Value (APV) of the losses.
- Loss Adjustment Expenses (LAE) are a percentage of the expected loss and are paid at the same time as the losses.
- Other Expenses are a percentage of premium.

Table 6.1

Target Combined Ratios with Current Reinsurance Strategy

	CAT	Allied	Fire	HO	CAL	PAL
E[Loss]	18,645,163	19,915,510	13,083,761	437,032,492	190,819,744	743,842,606
APV[Loss]	18,024,960	18,938,164	12,441,681	411,201,278	162,654,461	698,375,986
LAE%	13.31%	8.10%	5.90%	12.10%	13.90%	13.40%
LAE	2,481,671	1,613,156	771,942	52,880,932	26,523,944	99,674,909
APV of LAE	2,399,122	1,533,991	734,059	49,755,355	22,608,970	93,582,382
Other Expense%	32.42%	31.10%	37.40%	30.70%	30.00%	22.80%
Other Expense	11,339,468	9,788,634	8,363,087	206,484,365	84,315,593	242,180,428
Cost of Financing	3,213,217	1,213,918	822,369	5,146,510	11,472,953	28,056,063
Cost of Financing%	9.19%	3.86%	3.68%	0.77%	4.08%	2.64%
Premium	34,976,767	31,474,707	22,361,196	672,587,508	281,051,978	1,062,194,859
Target Comb Ratio	92.82%	99.50%	99.36%	103.54%	107.33%	102.21%
	APHD	**OLOC**	**PLOC**	**CMP**	**WC**	**Total**
E[Loss]	540,201,933	50,547,922	817,783	457,887,696	346,008,816	2,818,803,427
APV[Loss]	513,691,728	38,679,364	579,805	417,569,543	309,668,745	2,601,825,715
LAE%	9.25%	25.10%	25.10%	17.20%	13.00%	13.15%
LAE	49,968,679	12,687,528	205,263	78,756,684	44,981,146	370,545,855
APV of LAE	47,516,485	9,708,520	145,531	71,821,961	40,256,937	340,063,314
Other Expense%	23.70%	27.70%	27.70%	36.40%	22.30%	27.54%
Other Expense	179,955,489	20,012,253	309,204	289,951,641	103,245,083	1,155,945,243
Cost of Financing	18,142,158	3,846,263	81,718	17,227,296	9,811,669	99,034,133
Cost of Financing%	2.39%	5.32%	7.32%	2.16%	2.12%	2.36%
Premium	759,305,859	72,246,400	1,116,258	796,570,442	462,982,433	4,196,868,405
Target Comb Ratio	101.43%	115.23%	119.35%	103.77%	106.75%	103.54%

Table 6.2

Target Combined Ratios with No Reinsurance

	CAT	Allied	Fire	HO	CAL	PAL
E[Loss]	18,645,163	19,915,510	13,083,761	437,032,492	190,819,744	743,842,606
APV[Loss]	18,024,960	18,938,164	12,441,681	411,201,278	162,654,461	698,375,986
LAE%	13.31%	8.10%	5.90%	12.10%	13.90%	13.40%
LAE	2,481,671	1,613,156	771,942	52,880,932	26,523,944	99,674,909
APV of LAE	2,399,122	1,533,991	734,059	49,755,355	22,608,970	93,582,382
Other Expense%	32.42%	31.10%	37.40%	30.70%	30.00%	22.80%
Other Expense	10,159,271	9,343,547	7,975,279	206,488,772	84,261,600	242,091,154
Cost of Financing	753,079	227,857	173,257	5,156,459	11,346,968	27,753,784
Cost of Financing%	2.40%	0.76%	0.81%	0.77%	4.04%	2.61%
Premium	31,336,432	30,043,560	21,324,276	672,601,865	280,871,999	1,061,803,307
Target Comb Ratio	99.84%	102.76%	102.38%	103.54%	107.38%	102.24%
	APHD	**OLOC**	**PLOC**	**CMP**	**WC**	**Total**
E[Loss]	540,201,933	50,547,922	817,783	457,887,696	346,008,816	2,818,803,427
APV[Loss]	513,691,728	38,679,364	579,805	417,569,543	309,668,745	2,601,825,715
LAE%	9.25%	25.10%	25.10%	17.20%	13.00%	13.15%
LAE	49,968,679	12,687,528	205,263	78,756,684	44,981,146	370,545,855
APV of LAE	47,516,485	9,708,520	145,531	71,821,961	40,256,937	340,063,314
Other Expense%	23.70%	27.70%	27.70%	36.40%	22.30%	27.54%
Other Expense	179,906,123	19,461,110	293,977	288,738,882	101,995,716	1,150,715,431
Cost of Financing	17,983,232	2,407,721	41,975	15,108,299	5,458,493	86,411,124
Cost of Financing%	2.37%	3.43%	3.96%	1.90%	1.19%	2.07%
Premium	759,097,567	70,256,715	1,061,288	793,238,686	457,379,890	4,179,015,584
Target Comb Ratio	101.45%	117.71%	124.10%	104.05%	107.78%	103.85%

The target combined ratios provide a tool to evaluate the line of business's financial performance. This tool reflects the line's contribution to DFAIC's total risk.

7. Conclusions

We give our responses to the questions the CEO would like addressed.

1. Is the Company adequately capitalized? Is there excess capital? How much capital should the Company hold as a stand-alone insurer?
   Response – We accept the current capital as adequate, with no excess capital. We find that the quantitative standard implied by the Tail Value at Risk evaluated at the 99% threshold works for DFAIC.

2. How should the capital be allocated to line of business?
   Response – We allocated capital in proportion to the marginal capital implied by the Tail Value at Risk evaluated at the 99% threshold. Tables 4.1 and 4.2 give the results, for the current reinsurance and no-reinsurance strategy.

3. What is the return distribution for each line of business and is it consistent with the risk for the line?
   Response – We defined the cost of financing insurance as the total of the allocated cost of capital and the net cost of reinsurance. These costs are consistent with the risk for each line of insurance. Tables 5.4 and 5.5 give the dollar costs for the current reinsurance and no-reinsurance strategies. Tables 6.1 and 6.2 give the target combined ratios implied by these costs of financing insurance for the two strategies.

4. Should the Company buy more or less reinsurance? What type? How efficient is its current reinsurance program?
   Response – We conclude that DFAIC should not buy any reinsurance. DFAIC is a well-diversified insurer with little catastrophe exposure. The company will save 15% of its cost of financing reinsurance by not buying reinsurance. We might modify this conclusion if DFAIC's investors would demand a higher return on capital when DFAIC's management drops the reinsurance.

5. How efficient is the asset allocation?
   This paper does not address that question.

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The Cost of Financing Insurance

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The Cost of Financing Insurance

by

Glenn Meyers

Insurance Services Office, Inc.

Abstract

This paper uses Dynamic Financial Analysis (DFA), to attack one of the longest-running problems in actuarial science — that of determining the appropriate profit loading for a line of insurance. For an insurance company, the cost of financing insurance is its (dollar) cost of capital plus the net cost of its reinsurance. The profit loading for a line of insurance is the cost of financing allocated to the line of insurance. Important considerations in determining this allocation include: (1) how much does the line contribute to the need for capital; and (2) how long must the insurer hold capital to support the uncertainty in its underwriting results.

Introduction

This paper uses the recently coined actuarial discipline, Dynamic Financial Analysis (DFA), to attack one of the longest-running problems in actuarial science – that of determining the appropriate profit loading for a line of insurance. Susan Szkoda [8], in her five-part article beginning in the May 1997 *Actuarial Review*, defines DFA as "a process for analyzing the financial condition of an insurance entity. Financial condition refers to the ability of the entity's capital and surplus to adequately support future operations through a currently unknown future environment. ... In a very real sense, DFA requires the actuary to evolve into a financial risk manager."

In this paper, I will attempt to derive a logically consistent method for using DFA to determine the profit loading on a line of insurance. I will then apply the method to one hypothetical insurer.

The ABC Insurance Company is a multiline insurance company. Its goal is to obtain an above-average return on equity by setting profitability targets for each of its underwriting divisions that reflect the cost of capital needed to support each division's contribution to the overall underwriting risk. If ABC expects an underwriting division's long-term results to fall below its target, the company intends to get out of that line of insurance.

ABC's management wants to use the following considerations as input into its decisions.

- How much capital must the company hold? While ABC's management recognizes the important role of regulators and rating agencies in determining an insurer's capital, the managers feel that controlling the insurer's risk, as measured by its statistical distribution of outcomes, provides a meaningful yardstick for setting profitability targets.

- How long must the company hold capital? The company may not know its underwriting results of its liability lines of insurance for several years. As long as there is uncertainty in the final result, the company must hold some capital. The profitability targets for each line of insurance should reflect the cost of holding capital until all claims are settled.

- How much investment income does the insurance operation generate? As the insurer is holding capital for the contingency of unusually high losses, it is also earning investment income on its capital. The profitability targets for each line of insurance should also reflect the investment earnings generated by each line of business.

- How closely correlated are the losses in the various lines of insurance? The textbook illustrations of the economic value of insurance often assume that insured accidents are independent events. Positive correlation increases the amount of capital needed and hence its cost. The profitability targets for each line of insurance should reflect this cost

- What is the effect of reinsurance? In place of raising capital, an insurer may rely on reinsurance to provide security for its ability to pay losses. The effect of reinsurance is to replace part of the cost of capital with the cost of reinsurance. The profitability targets should reflect both the cost and benefit of reinsurance for each line of insurance.

I define the cost of financing an insurance company as the combined cost of capital and the net cost of reinsurance (that is, the premium less the expected reinsurance recovery). The ABC Insurance Company wants to allocate its cost of financing back to its individual underwriting divisions.

ABC will add the allocated cost of financing insurance to the expected losses and the other allocated expenses to obtain target combined ratios for each underwriting division in the company.

2. Outline

The final product of this analysis will be a table of target combined ratios for underwriting divisions of the ABC Insurance Company. As we move toward that end, I will cover a number of actuarial and financial concepts. Here are the highlights of our trip.

- Section 3 discusses the concept of capital and the insurer's aggregate loss distribution. The typical insurer writes several lines of insurance and so we must get the distribution of the sum of the random losses from each line. That means we must consider the possibility that the losses in each line are correlated.

- Section 4 introduces the concept of measures of risk. The section begins with four axioms that risk measures should satisfy. Next I state a theorem that characterizes all risk measures that satisfy these axioms. I then discuss how well some of the popular actuarial risk measures fit into this axiomatic framework.

- Section 5 discusses the cost of capital. We express the amount of needed capital in terms of the insurer's chosen measure of risk. The insurance company's investors provide this capital — at a cost. The policyholder must ultimately pay the cost of providing this capital. This section gives a formula to allocate the cost of capital to the various underwriting divisions — which in turn must decide how to allocate their allocated cost of capital to their individual policyholders.

- Section 6 discusses the effect of long-tailed lines of insurance. An insurer does not know the underwriting result for the typical liability line for insurance several years. As long as there is uncertainty in the final result, the insurer must hold some capital. This capital has a cost. This section shows how to allocate the cost back to the appropriate underwriting division.

- Section 7 discusses reinsurance. In place of raising capital, an insurer may rely on reinsurance to provide security for its ability to pay losses. The effect of reinsurance is to replace the cost of capital with the net cost of reinsurance. Introducing reinsurance forces us to move from the very specific concept of the cost of capital to the more general concept of the cost of financing insurance.

- Sections 8 and 9 put all the pieces together to calculate the cost of financing insurance for each underwriting division. We will calculate the cost of financing with and without reinsurance, and for two different measures of risk.

- Section 10 translates the results into target combined ratios.
- Section 11 finishes the paper with some concluding remarks.

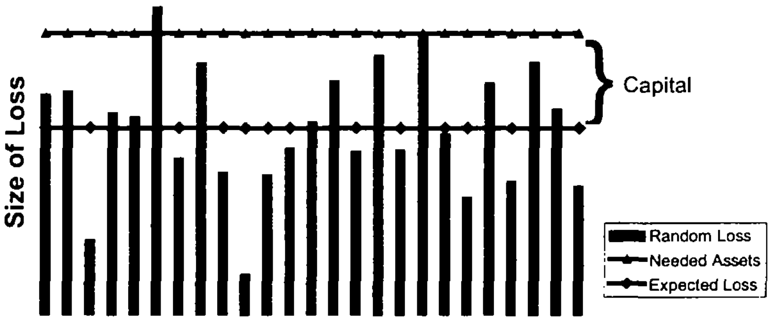
I am writing this paper to provide a conceptual overview of how to apply DFA to the management of underwriting risk. A comprehensive DFA analysis on a real insurance company involves a myriad of details that, if considered here, would make the underlying concepts harder to grasp. Therefore, I have made a number of simplifications, the most important of which is the model of the insurer's losses.

3. Capital and the Distribution of an Insurer's Aggregate Losses

The first step in our analysis will be to determine how much capital an insurer needs to be "reasonably" certain that it can pay its claims. Often, the insurer will be able to pay its claims from the expected loss portion of its premium income. But in some years losses are above average, and the insurer needs additional capital to make good its pledge to its insureds. Although the insureds would like to be absolutely certain that the insurer has enough capital to pay its claims, in practice, insureds are willing to allow for the "rare" possibility that the insurer will have insufficient funds. Chart 3.1 illustrates the idea.

We will further refine our notion of "rare" in the next section.

Chart 3.1



The total assets needed to cover losses is equal to the sum of:
(1) the expected loss, which comes from the premium, plus
(2) the capital which comes from the insurer's investors.

We need to consider the insurer's distribution of aggregate losses when determining the amount of capital needed. The most common description of an insurer's aggregate losses is the collective risk model. That model describes the insurer's losses in terms of a random claim count and a random claim size for each line of insurance. The model allows us to account for several features of the insurer's business including inflation, deductibles, policy limits, and reinsurance.

Conceptually, the easiest way to implement the collective risk model is to perform a Monte Carlo simulation. There are practical problems in doing this because the simulations can take a considerable amount of time. If the insurer wants to consider a number of alternative strategies that involve purchasing reinsurance and/or modifying its book of business, the time needed to do the computations can limit the number of alternatives the company can consider. There are faster ways to perform collective risk model calculations, but those methods rely on advanced mathematical techniques.

In writing this paper, I have chosen to move most of the problems to the background by building a simplified aggregate loss model. The model consists of four lines of insurance. We will describe the aggregate loss distribution for each line of insurance by a normal distribution with mean μ and standard deviation σ. The lines will have varying risk and loss payment characteristics. There will be an additional catastrophe loss that occurs with a low probability. With that simplified model, we can perform the necessary convolutions to sum the random losses and instantaneously calculate the various statistics needed to do the financial analysis.

The example we will follow throughout this paper will be the ABC Insurance Company. For the accident year 2002, it expects to pay $250 million in losses. For prior accident years it holds reserves totaling $227 million. The following table presents the outstanding liabilities for each line of insurance.

Table 3.1
By Line Loss Statistics for ABC Insurance Company

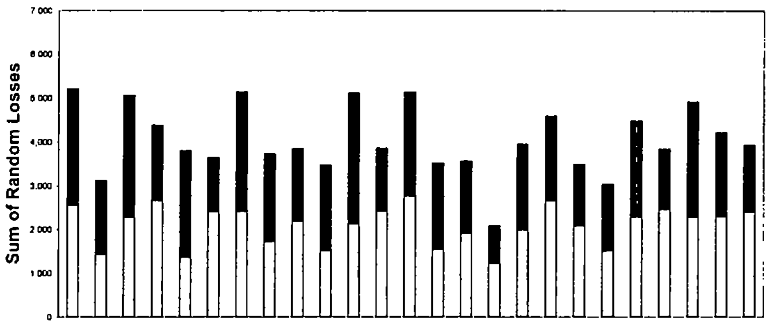
Outstanding Loss + ALAE Parameters

Line & AY	μ	σ/μ	σ
GL-1998	2,000,000	0.270	540,000
GL-1999	10,000,000	0.180	1,800,000
GL-2000	25,000,000	0.120	3,000,000
GL-2001	45,000,000	0.090	4,050,000
GL-2002	70,000,000	0.060	4,200,000
PL-1998	5,000,000	0.300	1,500,000
PL-1999	15,000,000	0.200	3,000,000
PL-2000	30,000,000	0.150	4,500,000
PL-2001	50,000,000	0.100	5,000,000
PL-2002	70,000,000	0.080	5,600,000
Auto-2000	10,000,000	0.140	1,400,000
Auto-2001	35,000,000	0.080	2,800,000
Auto-2002	70,000,000	0.050	3,500,000
Prop-2002	35,000,000	0.090	3,150,000
Cat-2002	5,000,000	7.000	35,000,000

The catastrophe loss distribution consists of a loss of $250 million with probability 0.02, and a loss of zero with probability 0.98.

An important consideration when analyzing aggregate loss distributions is correlation. Consider an example with independent random losses X_1 and X_2, each with mean 2000 and standard deviation 500. Chart 3.2 shows a plot of the sum of X_1 and X_2 for 25 random selections.

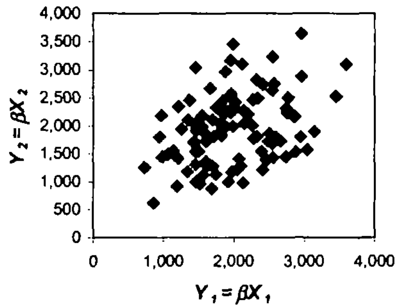
Chart 3.2
Uncorrelated Losses



An insurer covering X_1+X_2 would need assets slightly over \$5,000 to cover the losses shown.

Now, let's complicate the example by first taking a random multiplier, β, of 0.7, 1.0, or 1.3. (The corresponding probabilities of β are 1/6, 2/3, and 1/6 respectively.) Next we take X_1 and X_2 as above and then set $Y_1 = \beta X_1$ and $Y_2 = \beta X_2$. Chart 3.3 shows a plot of 100 randomly selected pairs (Y_1, Y_2). As Chart 3.3 illustrates, Y_1 and Y_2 are correlated.

Chart 3.3

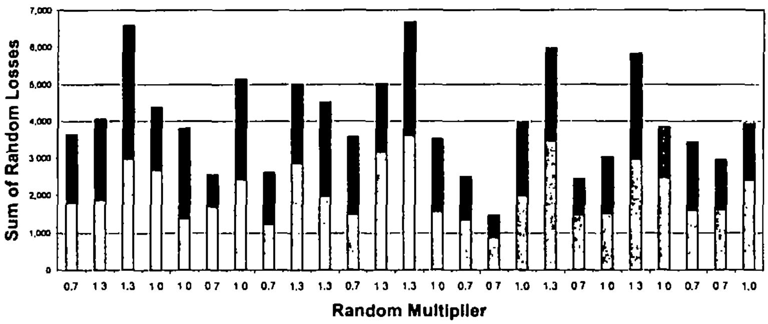


Large values of Y_1 (=βX_1) are statistically associated with large values of Y_2 (=βX_2). Hence Y_1 and Y_2 are positively correlated.

Adding a pair of correlated random losses produces a more volatile distribution than adding a pair of similar but uncorrelated random losses. Chart 3.4 shows the effect of adding the Y's corresponding to the X's in Chart 3.2.

As Charts 3.2 and 3.4 clearly illustrate, an insurer would need more assets to cover Y_1+Y_2 than it would need to cover X_1+X_2. Now since E[β] =1, we have that E[Y_1+Y_2] = E[X_1+X_2]. Hence the insurer would need to get more capital from its investors to cover Y_1+Y_2 than it would need to cover X_1+X_2.

Chart 3.4
Correlated Losses



An insurer covering Y_1+Y_2 would need assets well over $6,000 to cover the losses shown. That is noticeably higher than the assets needed to cover the losses X_1+X_2 shown in Chart 3.2.

Now let's apply this random multiplier idea to our model of the noncatastrophe losses of the ABC Insurance Company. For a given $b > 0$, choose random multipliers:

$\beta = 1 - \sqrt{3b}$ with probability 1/6
$\beta = 1$ with probability 2/3
$\beta = 1 + \sqrt{3b}$ with probability 1/6

We have that $E[\beta] = 1$ and $Var[\beta] = b$.

We will apply the random multiplier, β, to all of ABC's noncatastrophe losses. Setting $b = 0$ forces ABC's non-catastrophe losses to be independent. Increasing b results in a greater volatility of ABC's total noncatastrophe losses. Table 3.2 gives some aggregate statistics for ABC's noncatastrophe losses over a range of b's.

Table 3.2
Aggregate Statistics for ABC's Noncatastrophe Losses

b	Standard Deviation	99th Percentile
0.00	12,899,868	502,009,504
0.01	48,948,040	577,282,947
0.02	68,010,402	612,585,449
0.03	82,794,437	639,672,796

4. Measures of Risk

4.1 Introduction

The discussion in the previous section of the assets needed to cover an insurer's potential losses has two implicit assumptions:

1. The amount of needed capital increases with the volatility of the insurer's losses.
2. It is unreasonable to require an amount of capital sufficient to cover all potential losses.

In this section, we discuss some rules for determining how much assets and capital an insurer needs to cover its losses. These rules will depend on the insurer's aggregate loss distribution. Other valid considerations, such as the quality and reputation of the insurer's management, are beyond the scope of this paper.

Most of the ideas in this section come from the paper "Coherent Measures of Risk" by Philippe Artzner, Freddy Delbaen, Jean-Marc Eber and David Heath [2]. Their paper considers the problem of setting margin requirements on an organized exchange. This problem is similar to that of setting capital requirements for insurance companies.

This paper was written for an academic audience with extensive training in probability theory. Some actuaries will have some difficulty digesting the paper itself. In this section, I will attempt to describe the paper's ideas in language that is familiar to most actuaries.

Artzner [3] has written another paper on the subject that casualty actuaries might find more accessible.

4.2 A Motivation for the Definition of Coherence

Consider the following set of ten scenarios, each with associated losses X_1, X_2, X_3 and X_4.

Table 4.1

Scenario	X_1	X_2	X_1+X_2	$X_3 = 2*X_1$	$X_4 = X_1+1$
1	1.00	0.00	1.00	2.00	2.00
2	2.00	0.00	2.00	4.00	3.00
3	3.00	0.00	3.00	6.00	4.00
4	4.00	1.00	5.00	8.00	5.00
5	3.00	2.00	5.00	6.00	4.00
6	2.00	3.00	5.00	4.00	3.00
7	1.00	4.00	5.00	2.00	2.00
8	0.00	3.00	3.00	0.00	1.00
9	0.00	2.00	2.00	0.00	1.00
10	0.00	1.00	1.00	0.00	1.00
Maximum Loss	4.00	4.00	5.00	8.00	5.00

We can think of the X_i's as random variables representing the losses of the i^{th} risk. In our examples, we shall assume that each scenario is equally likely. Let us define a measure of risk for X_i as

$$\rho(X_i) = Maximum(X_i),$$

where the maximum is taken over all ten scenarios.

That measure of risk fulfills the needs of an insurance regulator who wishes to require that the insurer have sufficient assets, quantified by $\rho(X)$, to cover the losses incurred in each of the scenarios. Premiums paid by the insureds may supply some of the assets. The remainder of the assets must be supplied as insurer capital.

Using Table 4.1 as an aid, the reader should be able to verify that the measure of risk, ρ, satisfies the following axioms.

1. Subadditivity – For all random losses X and Y,
$$\rho(X+Y) \le \rho(X)+\rho(Y).$$
2. Monotonicity – If $X \le Y$ for each scenario, then,
$$\rho(X) \le \rho(Y).$$
3. Positive Homogeneity – For all $\lambda \ge 0$ and random losses X,
$$\rho(\lambda X) = \lambda \rho(X).$$
4. Translation Invariance – For all random losses X and constants α,
$$\rho(X+\alpha) = \rho(X)+\alpha.$$

A measure of risk that satisfies these four axioms is called a **coherent measure of risk**.

The axioms describe what appear to be reasonable properties for a measure of risk.

- Subadditivity reflects the diversification of portfolios or that "a merger does not create extra risk" [5, page 5] and [2, page 5]. This is a natural requirement consistent with the role of insurance. In general, we expect mergers to reduce the risk.
- Monotonicity means that if $X < Y$ for every scenario, the assets needed to support X are less than the assets needed to support Y.
- Positive homogeneity is a limiting case of subadditivity, representing what happens when there is precisely no diversification effect [5, p. 4].

The "Standard Deviation" criterion sets the measure as the expected value of the loss plus a predetermined multiple of the standard deviation. For the scenarios listed in Table 4.3 below we have:

$$X_1 \le X_2$$
$$\rho(X_1) \equiv E[X_1] + 2 \cdot StDev[X_1] = 5.83$$
$$\rho(X_2) \equiv E[X_2] + 2 \cdot StDev[X_2] = 5.00$$

As this example shows, the Standard Deviation criterion violates the monotonicity axiom.

Table 4.3

Scenario	X_1	X_2
1	1.00	5.00
2	2.00	5.00
3	3.00	5.00
4	4.00	5.00
5	5.00	5.00
6	5.00	5.00
7	4.00	5.00
8	3.00	5.00
9	2.00	5.00
10	1.00	5.00
$E[Loss]$	3.00	5.00
$StDev[Loss]$	1.41	0.00
$E[Loss]+2*StDev[Loss]$	5.83	5.00

Note the following.

Proposition 4.1

The Standard Deviation criterion is subadditive.

Proof

Let $\sigma_X^2 = \mathrm{Var}[X]$, $\sigma_Y^2 = Var[Y]$, $\sigma_{X+Y}^2 = Var[X+Y]$ and $r_{XY} = Corr[X,Y]$. Then:

$$\begin{aligned} E[X+Y] + T\sigma_{X+Y} &= E[X+Y] + T\sqrt{\sigma_X^2 + 2r_{XY}\sigma_X\sigma_Y + \sigma_Y^2} \\ &\le E[X+Y] + T\sqrt{\sigma_X^2 + 2\sigma_X\sigma_Y + \sigma_Y^2} \\ &= E[X+Y] + T\sqrt{(\sigma_X + \sigma_Y)^2} \\ &= E[X] + T\sigma_X + E[Y] + T\sigma_Y. \end{aligned}$$

■

4.3 Other Measures of Risk

It turns out that many common measures of risk used by actuaries are not coherent. Consider the following examples.

Define the "Value at Risk" or *VaR* as the smallest loss greater than a predetermined percentile of the loss distribution. This measure is similar to "Probability of Ruin" measures that actuaries have long discussed.

If our measure of risk, $\rho(X)$, is the 85^{th} percentile of the random loss X, we have for the scenarios listed in Table 4.2 below:

$$0 = \rho(X_1) + \rho(X_2) < \rho(X_1 + X_2) = 1.$$

As this example shows, the Value at Risk criterion violates the subadditivity axiom.

Table 4.2

Scenario	X_1	X_2	X_1+X_2
1	0.00	0.00	0.00
2	0.00	0.00	0.00
3	0.00	0.00	0.00
4	0.00	0.00	0.00
5	0.00	0.00	0.00
6	0.00	0.00	0.00
7	0.00	0.00	0.00
8	0.00	0.00	0.00
9	0.00	1.00	1.00
10	1.00	0.00	1.00
VaR@85%	0.00	0.00	1.00

So far, I have demonstrated that two popular statistical measures on solvency standards are not coherent. Let me now turn to a more general description of coherent measures of risk.

4.4 The Representation Theorem

Let Ω denote a finite set of scenarios. Let X be the loss incurred by the insurer under a particular business plan. We associate each loss with an element of Ω.

The representation theorem [2, Proposition 4.1, and 5, Proposition 2.1], stated here without proof, says that a measure of risk, ρ, is coherent *if and only if* there exists a family, $\mathcal{P}$, of probability measures defined on Ω such that

$$\rho(X) = \sup\{E_{\mathbf{P}}[X] | \mathbb{P} \in \mathcal{P}\}. \quad (4.1)$$

One way to construct a family of probability measures on Ω is to take a collection $\mathcal{A} = \{A_i\}_{i=1}^{m}$ of subsets of Ω with the property that $\bigcup_{i=1}^{m} A_i = \Omega$. Let n_i be the number of elements in A_i. Assume that all elements in Ω are equally likely. We then define the probability measure, $\mathbf{P}_i$, on the elements $\omega \in \Omega$ as the conditional probability, given that the element is in the set A_i, and 0 otherwise. That is:

$$\mathbb{P}_i(\omega) = \begin{cases} \frac{1}{n_i} & \text{if } \omega \in A_i \\ 0 & \text{if } \omega \notin A_i \end{cases}.$$

The authors [3, p. 16] refer to the collection of probability measures, $\mathcal{P}$, on the set of scenarios as "generalized scenarios."

Let's look at an example. The following table gives a set of scenarios and associated losses.

Table 4.4

Scenario	X
1	0
2	2
3	2
4	6

Let $A_1 = \{1,2\}$ and $A_2 = \{3,4\}$. We then calculate the expected values

$$E_{\mathbf{P}_1}[X] = 1 \text{ and } E_{\mathbf{P}_2}[X] = 4.$$

The associated coherent measure of risk, $\rho_A(X)$, is then given by

$$\rho_A(X) = \sup\left\{E_{\mathbf{P}_i}[X] \middle| i = 1,2\right\} = 4.$$

We can similarly construct a second coherent measure of risk, $\rho_B(X)$, on the scenarios in Table 4.4 with the subsets $B_i = \{i\}$. In that case we have $\rho_B(X) = 6$.

We can impose varying degrees of conservatism on coherent measures of risk by varying the choice of generalized scenarios.

4.5 A Proposal for a Measure of risk

The paper by Artzner *et. al.* finishes with a proposal for a measure of risk that actuaries should find easy to implement. Let's start with the formal definition of the Value at Risk (*VaR*). Let α be a selected probability (for example, 99%). Then

$$VaR_\alpha(X) = \inf\{x | \Pr\{X \leq x\} > \alpha\}$$

As demonstrated in section 4.3, *VaR* is not a coherent measure of risk.

We now define the proposed measure in terms of the *VaR*. We call this measure the Tail Conditional Expectation (*TCE*) or Tail Value at Risk (*TVaR*).

$$TCE_\alpha(X) \equiv TVaR_\alpha(X) \equiv E[X | X \geq VaR_\alpha(X)]$$

The *TVaR* is linked to a well-known criterion in recent CAS literature for solvency — the Expected Policyholder Deficit (*EPD*). See, for example, [1]. *EPD(t)* is defined as the expected loss over a predetermined threshold *t*. It turns out that

$$TailVaR_\alpha(X) = VaR_\alpha(X) + \frac{EPD(VaR_\alpha(X))}{1-\alpha}.$$

I will now demonstrate that the *TVaR* is coherent under some common conditions.
For any subset A of Ω, let n_A be the number of elements in A. Define the probability measure

$$\mathbb{P}_A(\omega) = \begin{cases} \frac{1}{n_A} & \text{if } \omega \in A \\ 0 & \text{if } \omega \notin A \end{cases}.$$

Proposition 4.2

If each element of Ω is equally likely, then the *TVaR* is a coherent measure of risk.

Proof

Let n be the number of elements in Ω. Denote the various values of X by $x_1 \le x_2 \le \ldots \le x_n$. Let k be the integer with $0 \le k < n$ such that $\alpha \in \left[\frac{k}{n}, \frac{k+1}{n}\right.$.

Since $\Pr\{X \le x_{k+1}\} \ge \frac{k+1}{n} > \alpha$ and $\Pr\{X < x_{k+1}\} \le \frac{k}{n} \le \alpha$ we have that $VaR_\alpha(X) = x_{k+1}$.

Let $\mathcal{A}$ be the family of subsets of Ω with exactly $n-k$ elements. Define the family of measures $\mathcal{P} = \{\mathbb{P}_A\}_{A \in \mathcal{A}}$. By Equation 4.1, $\rho(X) = \sup\{E_{\mathbf{P}_A}[X] | A \in \mathcal{A}\}$ is a coherent measure of risk.

For any scenario, $\omega \in A$, $\Pr\{W = \omega | \omega \in A\} = \frac{1}{n-k}$.

Let $AMax$ be the member of $\mathcal{A}$ with the $n-k$ largest elements; i.e., $\{x_{k+1}, x_{k+2}, \ldots, x_n\}$.

Then

$$\begin{aligned} TVaR_\alpha(X) &= \mathrm{E}\left[X | X \ge VaR_\alpha(X)\right] \\ &= \frac{x_{k+1} + x_{k+2} + \ldots + x_n}{n-k} \\ &= E_{\mathbf{P}_{AMax}}[X]. \end{aligned}$$

For any other set $A \in \mathcal{A}$, $\mathrm{E}_{\mathbf{P}_A}[X] \le \mathrm{E}_{\mathbf{P}_{AMax}}[X]$.

Thus $TVaR_\alpha(X) = \sup\{\mathrm{E}_{\mathbf{P}_A}[X] | A \in \mathcal{A}\}$ and the measure of risk is coherent.

■

In the examples below, we will use the Tail Value at Risk as our measure of risk. We will also show the results for the Standard Deviation measure. The Standard Deviation measure satisfies three of the four coherence axioms. It has the added advantage of being computationally faster. Also, the Tail Value at Risk is a new measure of risk. In my experience, whenever one proposes a new actuarial technique, there are always those who want to compare the new with the old – regardless of the justification for the change.

The Tail Value at Risk does address a common complaint that many actuaries have made about the Standard Deviation measure. The complaint is that the Standard Deviation measure penalizes the potential for unusually good results — up-side risk — as well as the potential for unusually bad results — down-side risk. The Tail Value at Risk is sensitive only to down-side risk.

5 The Cost of Capital

We will use a measure of risk, $\rho(X)$, to determine the assets needed to cover the random insured loss, X. Of that amount, $\rho(X)$, the insured's premium supplies the expected value, $E(X)$. The remainder, $C(X) \equiv \rho(X) - E(X)$, must come from the investors in the insurance company. We call $C(X)$ the insurer's capital. The insurer places that capital at risk for the purpose of covering losses in excess of $E(X)$.

For the examples in this paper, we will use $\rho(X) = TVar_\alpha(X)$ with $\alpha = 99\%$. Another insurer might set its capital by using a 99.5% $TVaR$ level or set it equal to 2 times its aggregate standard deviation. The insurer's policyholders might demand different standards for those insurers. While such standards are rarely so explicit in the real world, the rating agencies clearly have a more subjective version of this kind of standard in mind. Note the names they give to their ratings. For example, we have the "Best's Capital Adequacy Rating" and the Standard and Poor's rating of "Claims Paying Ability."

In return for placing their capital at risk, investors seek a target (that is, expected) rate of return at least as high as other investments of comparable risk. Exactly how high that rate of return is can be a matter of considerable debate. We could appeal to a financial theory such as the Capital Asset Pricing Model (CAPM). The CAPM tells us that investors will demand a higher return if the insurer's financial results move with the stock market. For example, a property insurer whose principal exposure is to natural hazards might find that its returns are independent of the market. A casualty insurer whose principal exposure is in long-tailed lines, such as workers compensation, may find that its returns are highly correlated with other segments of the stock market.

The insights of financial theory, while having an attractive rationale, have been difficult to quantify. An insurer might incorporate those insights into a target rate of return by selecting a peer group of insurers that the company expects to have similar returns and are comparably rated by the rating agencies. Such an analysis would subject these insights to the reality test of a benchmark.

Rightly or wrongly, setting a target rate of return is a routine exercise done by insurer boards of directors.

For the examples in this paper, we will use a target rate of return, denoted by e, equal to 12%.

Ultimately, the policyholders must bear the cost of providing necessary capital through the premiums they pay for the insurance. The insurer now faces the question of allocating that cost back to a diverse set of policyholders. One way of doing this is to allocate the capital to groups of policyholders (called underwriting divisions) within the company and compare their expected (dollar) return to their allocated capital. Each underwriting division then has the responsibility of obtaining the insurer's overall rate of return on its allocated capital. The underwriting division strives to earn that target through its underwriting and pricing activities.

In allocating capital to an underwriting division, we should convince ourselves that the resulting decisions implied by our allocation method make economic sense. By making "economic sense" we mean that insuring the policies in a given underwriting division does not decrease the insurer's expected rate of return.

Some mathematics will make this argument clearer. Let:

$C \equiv C(X) =$ Capital needed to support X.

$X_k =$ Random loss for the k^{th} underwriting division.

$\Delta C_k \equiv C(X) - C(X - X_k) =$ Marginal capital for the k^{th} underwriting division.

$\Delta P_k =$ Expected profit for the k^{th} underwriting division.

$P = \sum_k \Delta P_k$.

Proposition 5.1

Including the insurance policies in underwriting division k increases the overall expected rate of return if and only if underwriting division k's expected rate of return on its marginal capital is greater than the insurer's overall rate of return.

Proof

$$\frac{P - \Delta P_k}{C - \Delta C_k} < \frac{P}{C} \Leftrightarrow P \cdot \Delta C_k < C \cdot \Delta P_k \Leftrightarrow \frac{P}{C} < \frac{\Delta P_k}{\Delta C_k}$$

■

Proposition 5.1 places a lower bound on an underwriting division's expected profit for it to be economically viable with its insurance company. One might expect that it is all right to set a profitability target so that each underwriting division's expected return on its marginal capital is equal to the insurer's overall return on capital. But alas, life is not so simple. Consider the following proposition.

Proposition 5.2

Let an insurer's capital, C, be determined by a subadditive measure of risk, ρ. Then:

$\sum_k \Delta C_k \leq C$ that is, the sum of the marginal capitals is less than or equal to the original capital.

Proof

I first offer the proof when there are two underwriting divisions.

$$\begin{aligned}
&\Delta C_1 + \Delta C_2 \\
&= \rho(X_1 + X_2) - E(X_1 + X_2) - (\rho(X_2) - E(X_2)) + \rho(X_1 + X_2) - E(X_1 + X_2) - (\rho(X_1) - E(X_1)) \\
&= 2\rho(X_1 + X_2) - E(X_1 + X_2) - (\rho(X_1) + \rho(X_2)) \\
&\leq 2\rho(X_1 + X_2) - E(X_1 + X_2) - \rho(X_1 + X_2) \text{ (by subadditivity)} \\
&= \rho(X_1 + X_2) - E(X_1 + X_2) \\
&= C
\end{aligned}$$

If there are three underwriting divisions, apply the logic in the above proof to (X_1+X_2) and to X_3. Next use the result from the proof directly on (X_1+X_2) to get the final result for three underwriting divisions.

Proceed inductively to get the result for 4, 5, ... underwriting divisions.

■

Since it is the job of insurers to diversify risk, the inequality of Proposition 5.2 should be strict. That is, the sum of the marginal capitals should be strictly less than the total capital. That requirement leads us to the following proposition.

Proposition 5.3

If the sum of the marginal capitals is less than the total capital, and the insurer expects to make a return, $e = P/C$, then at least some of its underwriting divisions must have a return on its marginal capital greater than e.

Proof

Assume that $\frac{\Delta P_k}{\Delta C_k} = \frac{P}{C} \equiv e$ for all underwriting divisions, k. Then:

$$P = \sum_k \Delta P_k = \frac{P}{C} \sum_k \Delta C_k < P \quad (!)$$

This contradiction means that we must have $\Delta P_k / \Delta C_k > e$ for at least one k.

■

Suppose an insurer has a choice of continuing its business in one of two underwriting divisions j and k. In its analysis of market prices, the insurer finds that it can expect to make profits of ΔP_j and ΔP_k for underwriting divisions j and k, respectively. Furthermore, it calculates that it must retain ΔC_j and ΔC_k of capital for underwriting divisions j and k, respectively. From a financial point of view, it makes sense for the insurer to favor the underwriting division that has the larger return on marginal capital.

Over time, each underwriting division in the company will come under similar scrutiny, with the ultimate result that each underwriting division will expect the same return, d, on marginal capital.

Let A_k be the capital allocated to the underwriting division k. Then:

$$\frac{\Delta P_k}{A_k} = e \text{ and } \frac{\Delta P_k}{\Delta C_k} = d. \qquad (5.1)$$

Hence $\sum_j \Delta P_j = d \sum_j \Delta C_j = e \sum_j A_j = eC$, and thus $e = d \frac{\sum_j \Delta C_j}{C}$. (5.2)

Substituting Equation 5.1 into Equation 5.2 yields:

$$\frac{\Delta P_k}{A_k} = \frac{\Delta P_k}{\Delta C_k} \frac{\sum_j \Delta C_j}{C}. \tag{5.3}$$

Solving Equation 5.3 for A_k yields:

$$A_k = \Delta C_k \frac{C}{\sum_j \Delta C_j}. \tag{5.4}$$

We now recap the chain of reasoning in this section.

1. We started with the assumption that we want to derive an insurer's required capital from a subadditive measure of risk. A subadditive measure of risk is desirable because it reflects the benefits of diversification.

2. The policyholders must ultimately bear the cost of providing the insurer's capital. How much of that cost each policyholder must bear becomes an issue. In this paper, I have chosen to allocate the cost to insurer defined underwriting divisions. (In principle, the underwriting divisions could be individual policyholders.)

3. The method I have chosen to allocate the cost of capital to the underwriting divisions is to allocate the insurer's capital to underwriting divisions and then apply the insurer's selected rate of return to the allocated capital. (I chose the capital allocation method because it is conventional and not because it is fundamentally necessary.)

4. Proposition 5.1 limits our choice of capital allocation methods. If we require an underwriting division to "carry its own weight," the capital allocated to the underwriting division can be no less than its marginal capital.

5. Proposition 5.3 tells us that setting the allocated capital equal to the marginal capital will not lead to the insurer's recovering its cost of capital from the underwriting divisions.

6. We make the additional assumption that in the long run, insurers will structure their books of business so that their return on marginal capital is the same for all underwriting divisions. That assumption leads to a capital allocation formula, Equation 5.4, that amounts to multiplying the marginal capital for each underwriting division by a constant factor.

I should point out that other long run assumptions, such as those made by Game Theory, lead to different capital allocation formulas. See Delbaen and Denault [5], and Mango [7] for additional details.

6. Allocating Capital to Support Outstanding Loss Reserves

The insurer's pledge to pay losses can be a long-term commitment. As time goes on, the insurer pays some losses and the uncertainty in future loss payments declines. Therefore the insurer can release some of the original capital allocated to an underwriting division, for a given accident year, can be released.

In the current year, the insurer will have its capital supporting the outstanding losses from prior accident years. In this section, we apply the logic described in Section 5 and allocate capital to outstanding loss reserves. We calculate the reduction in needed capital when the outstanding losses are removed from the insurance company, and then allocate the capital in proportion to the marginal capital of each underwriting division and each loss reserve. Keep in mind that when establishing target rates of return for the current year, we must consider how much capital the insurer will allocate to the outstanding losses in future years. To do that, the insurer needs a plan for its future business.

Allocation of capital has been actively discussed in the Casualty Actuarial Society over the past several years. The classic "Kenney Rule" was a rule of thumb for capital adequacy. It simply stated that an insurer was adequately capitalized if its premium to capital ratio was two to one. Insurers could easily apply such a rule by line of insurance by setting the allocated capital supporting an underwriting division by dividing its premium by two.

A problem with such an allocation is that it does not recognize variability in the length of time, by line of insurance, that insurers must hold capital. Russell Bingham [4] recognizes that problem. His solution is to allocate capital in proportion to the reserve to capital ratio. That allocation is a step in the right direction. One might expect that a larger reserve would indicate a larger uncertainty in the reserve, and hence the insurer should allocate more capital to the larger reserve. However, the size of the reserve might not be proportional to the risk it contributes to the insurer. Consider the case where the insurer knows for certain that it will have to pay a fixed amount A at some time t in the future. The insurer sets up a loss reserve for this fixed amount A but needs no additional capital to support it. Conversely, suppose the insurer will have to pay a claim of an uncertain amount at time t in the future. Suppose further that the expected payment is equal to A. The insurer sets up a loss reserve equal to this expected amount, A, but will have to hold additional capital because of the uncertain amount of the claim. If the insurer were to allocate capital in proportion to reserves, it would allocate the same amount to each of those claim reserves. The approach I have taken in another paper, Meyers [7], is to use claim severity distributions that vary by settlement lag. That is a further step in the right direction because it recognizes variability in the loss reserve. However, the claim severity distributions, derived from claims settled after a given time, do not recognize the additional information that may be available at the time of the reserve evaluation. Work done by Taylor [9] for the CAS Committee on the Theory of Risk addresses the problem of additional knowledge. That approach may move the problems further toward the ultimate solution.

7. Reinsurance

An insurer can reduce the amount of capital it needs by buying reinsurance. When buying reinsurance, the insurer faces a transaction cost (that is, the reinsurance premium less the provision for expected loss) that replaces a portion of the capital. Note that the insurer does not need to know the reinsurer's pricing assumptions. The insurer can, and perhaps should, use its own estimate of the reinsurer's expected loss to back out the reinsurance transaction cost.

Taxes play an important role in the transaction costs of reinsurance. The insurer deducts reinsurance costs from its taxable income. Capital, whether raised externally or from retained earnings, is subject to corporate income tax. Vaughan [10] points out that the tendency for reinsurance to stabilize insurer income also provides tax advantages. That gives reinsurance an advantage as a provider of insurer financing.

8. The Cost of Financing Insurance

Ultimately, an insurer must be able to pay its financing costs out of the premiums charged to the insureds and from the returns on invested assets. We now determine how much of those financing costs should come from premium. The first step is to decide on a target return on equity. Typically, an insurer's board of directors makes that decision based on considerations described in Section 5.

Investors provide the capital to the insurer. In return, they expect to receive a cash flow reflecting:

1. Premium income
2. Payments to reinsurers
3. Investment income
4. Loss and expense payments
5. Income from the capital that is released as liabilities either expire or become certain

Premium income and payments to reinsurers contain provisions for losses and expenses. It will simplify matters to remove loss and expense payments from our immediate attention by taking expected values and allowing the actual losses in (4) to cancel out the expected loss provisions in (1) and (2). That simplification allows us to concentrate on the cash flow of insurer capital and the net cost of reinsurance, that is, the cost of financing insurance. Investors provide capital to the insurer. After netting out the insurer's loss and expense payments the investors receive a cash flow reflecting:

1. Income from the profit provision in the premium
2. Payments of the net costs to reinsurers
3. Investment income from the capital held for uncertain liabilities
4. Income from the capital that is released as liabilities either expire or become certain

The insurer makes its targeted return on capital if the present value of that cash flow, evaluated *at the targeted return on capital*, is equal to the invested capital. If we allow that:

1. The insurer collects the profit provision in the premium immediately.
2. The insurer makes its reinsurance payments immediately.
3. The insurer determines its necessary capital at the beginning of the year and holds that capital at the end of the year. The insurer then releases capital not needed for the next year. The insurer simultaneously releases investment income on the invested capital.

Then the profit provision necessary for the insurer to make its targeted return on equity is equal to:

Capital + Reinsurance Transaction Costs – Present Value of Released Capital.

To get the profit provision for each underwriting division we need to calculate the marginal cost of capital and the transaction costs for reinsurance for: (1) each underwriting division; and (2) each outstanding loss reserve. We now examine the calculations in some detail.

Table 8.1

Component for Accident Year y	Symbol
Capital investment for current calendar year y+t Note: The insurer needs the capital to cover claims from the current year as well as claims incurred in prior years. The capital also covers business projected for accident years, up to and including year y+t.	$C(t)$
Capital needed in calendar year y+t if the insurer removes underwriting division/accident year k	$C_k(t)$
Marginal Capital for underwriting division/accident year k in calendar year y+t	$\Delta C_k(t) = C(t) - C_k(t)$
Sum of marginal capitals in calendar year y+t	$SM(t)$
Capital allocated to underwriting division/accident year k for calendar year y+t	$A_k(t) = C(t)\Delta C_k(t)/SM(t)$
Transaction costs for underwriting division k's reinsurance (for current accident year only)	$R_k(0)$
Profit provision for underwriting division k	$\Delta P_k(0)$
Insurer's return on its investments	i
Insurer's target return on capital	e

The capital allocated to a given time period earns interest until the beginning of the next period. At that time, the insurer releases a portion of the capital either to pay for losses or to return to the investors.

Table 8.2

Time	Financial Support Allocated at Time t	Amount Released at Time t
0	$A_k(0) + R_k(0)$	0
1	$A_k(1)$	$Rel_k(1) = A_k(0)(1+i) - A_k(1)$
---	---	---
t	$A_k(t)$	$Rel_k(t) = A_k(t-1)(1+i) - A_k(t)$
---	---	---

Then:

$$\Delta P_k(0) = \underbrace{A_k(0) - \sum_{t=1}^{\infty} \frac{Rel_k(t)}{(1+e)^t}}_{\text{Cost of Capital}} + \underbrace{R_k(0)}_{\text{Net Cost of Reinsurance}} . \tag{8.1}$$

Equation 8.1 gives the profit provision for underwriting division k.

I selected $\alpha = 99\%$ as the threshold to determine the ABC Insurance Company's capital using the Tail Value at Risk. I selected $T = 2.185$ as the multiple using the Standard Deviation measure of risk. The reason for the odd multiple, T, is that it will force equality in the necessary capital for two examples given below. The basic loss statistics are given in Table 3.1. I applied a covariance generator, $b = 0.03$, to the non-catastrophe losses.

Tables 8.3 and 8.4 show the results of the capital allocation calculations for the Tail Value at Risk (TVaR) measure and the Standard Deviation measure of risk respectively. (Note that for the Standard Deviation measure of risk, the allocation percentages are the same no matter what multiplier is used. So I omitted the multiplier in the calculations.)

Note that we allocate capital to outstanding losses from prior years. In future years, we will need to allocate capital to outstanding losses from the current year. And we must fund the cost of that capital from the current year's premiums. The capital allocated to outstanding losses in future years will, in part, depend upon future writings. To keep it simple (and to save paper) I assumed that future writings are the same as past writings.

Table 8.3

Capital Allocation Calculation for Tail Value at Risk

Calendar Year 2002 Line & AY	E[OS Loss] w/o Line & AY	VaR[OS Loss] w/o Line & AY	TVaR[OS Loss] w/o Line & AY	Marginal TVaR Capital	% Allocated Capital
GL-1998	475,000,000	720,000,512	773,855,722	206,015	0.118%
GL-1999	467,000,000	711,998,997	764,994,608	1,067,129	0.610%
GL-2000	452,000,000	697,000,933	748,373,602	2,688,136	1.537%
GL-2001	432,000,000	677,000,063	726,214,789	4,846,948	2.771%
GL-2002	407,000,000	651,999,362	698,687,861	7,373,876	4.216%
PL-1998	472,000,000	716,999,867	770,515,190	546,547	0.312%
PL-1999	462,000,000	706,999,494	759,373,602	1,688,136	0.965%
PL-2000	447,000,000	691,999,076	742,630,697	3,431,041	1.962%
PL-2001	427,000,000	671,999,821	720,525,337	5,536,401	3.165%
PL-2002	407,000,000	652,000,766	698,381,454	7,680,283	4.391%
Auto-2000	467,000,000	712,000,685	765,021,207	1,040,530	0.595%
Auto-2001	442,000,000	687,000,559	737,398,147	3,663,590	2.095%
Auto-2002	407,000,000	652,000,474	698,804,347	7,257,390	4.149%
Prop-2002	442,000,000	687,000,812	737,354,017	3,707,720	2.120%
Cat-2002	472,000,000	639,672,796	646,894,524	124,167,213	70.993%
Combined/Total	**477,000,000**	**721,999,255**	**776,061,737**	**174,900,954**	**100.000%**

At this point, it will be helpful to connect the equations in Table 8.1 with the numbers in Table 8.3. Here are some illustrated calculations.

- Calendar year y = 2002
- Capital needed for calendar year 2002 = $C(0)$ = 776,061,737 – 477,000,000 = 299,061,737.
- Capital needed in calendar year 2002 if we remove (k =) GL underwriting division/accident year 2002 = $C_k(0)$ = 698,687,861 – 407,000,000 = 291,687,861.
- Marginal capital for (k =) GL underwriting division/accident year 2002 is $\Delta C_k(0)$ = 299,061,737 – 291,687,861 = 7,373,876.
- The sum of the marginal capitals, $SM(0)$, is equal to 174,900,954.
- The percentage of capital allocated to (k=) GL underwriting division/accident year 2002 is $\Delta C_k(0)/SM(0)$ = 4.216%.
- At the beginning of calendar year 2002, ABC has unpaid losses from accident year 2001. Following the procedure outlined above, we calculate that the percentage of capital allocated to (k=) GL underwriting division/accident year 2001 = 2.771%.
- Since we are assuming that future writings are the same as past writings, we have that for (k=) GL underwriting division/accident year 2002, $\Delta C_k(1)/SM(1)$ is also equal to 2.771%. If ABC planned to change future writings, we would need an accident year 2003 version of Table 8.3.

Table 8.4 gives the underwriting division/accident year allocations for the Standard Deviation measure of risk.

Table 8.4

Capital Allocation Calculation for Standard Deviation Measure of Risk

Calendar Year 2002 Line & AY	**E[OS Loss] w/o Line & AY**	**Std[OS Loss] w/o Line & AY**	**Marginal Std[OS Loss]**	**% Allocated Capital**
GL-1998	475,000,000	89,571,750	316,618	0.387%
GL-1999	467,000,000	88,297,121	1,591,247	1.947%
GL-2000	452,000,000	85,915,067	3,973,301	4.862%
GL-2001	432,000,000	82,760,946	7,127,422	8.721%
GL-2002	407,000,000	78,907,221	10,981,147	13.436%
PL-1998	472,000,000	89,088,446	799,922	0.979%
PL-1999	462,000,000	87,479,134	2,409,235	2.948%
PL-2000	447,000,000	85,067,393	4,820,976	5.899%
PL-2001	427,000,000	81,933,929	7,954,439	9 733%
PL-2002	407,000,000	78,817,625	11,070,744	13 546%
Auto-2000	467,000,000	88,304,587	1,583,782	1.938%
Auto-2001	442,000,000	84,364,647	5,523,722	6.759%
Auto-2002	407,000,000	78,942,392	10,945,976	13.393%
Prop-2002	442,000,000	84,351,933	5,536,435	6.774%
Cat-2002	472,000,000	82,794,437	7,093,932	8.680%
Combined/Total	**477,000,000**	**89,888,369**	**81,728,899**	**100.000%**

Perhaps the more striking comparison between the measures of risk is in the capital allocated to the catastrophe underwriting division.

We now continue the calculations described in Table 8.1 and 8.2.

Table 8.5

Needed Tail Value at Risk Allocated Capital at the Beginning of Each Calendar Year for Accident Year 2002

Line\Cal Year	2002	2003	2004	2005	2006
General Liability	12,608,532	8,287,757	4,596,421	1,824,675	352,263
Products Liability	13,132,455	9,466,647	5,866,709	2,886,530	934,536
Auto	12,409,354	6,264,344	1,779,193	0	0
Property	6,339,801	0	0	0	0
Catastrophe	212,312,521	0	0	0	0
Other OS Losses	42,259,075	275,042,989	286,819,413	294,350,533	297,774,938
TVaR Capital	299,061,737	299,061,737	299,061,737	299,061,737	299,061,737

We continue the illustrative calculations.

- The capital allocated to (k=) GL underwriting division/accident year 2002, $A_k(0)$, is equal to the total capital for calendar year 2002, (299,061,737), times the corresponding allocation percentage from Table 8.3, (4.216%) and is equal to 12,608,532.
- The capital allocated to (k=) GL underwriting division/accident year 2002, $A_k(1)$, is equal to the total capital for calendar year 2003, (299,061,737), times the corresponding allocation percentage from Table 8.3, (2.771%) and is equal to 8,287,757.
- Other OS Losses refers to outstanding losses from other accident years.

Table 8.6 gives the capital allocations for the Standard Deviation Measure of risk.

Table 8.6

Needed Standard Deviation Allocated Capital at the Beginning of Each Calendar Year for Accident Year 2002

Line\Cal Year	2002	2003	2004	2005	2006
General Liability	26,387,924	17,127,344	9,547,925	3,823,801	760,840
Products Liability	26,603,226	19,114,682	11,584,905	5,789,442	1,922,229
Auto	26,303,408	13,273,618	3,805,861	0	0
Property	13,304,169	0	0	0	0
Catastrophe	17,046,865	0	0	0	0
Other OS Losses	86,750,647	146,880,595	171,457,548	186,782,997	193,713,170
Std Dev Capital	196,396,239	196,396,239	196,396,239	196,396,239	196,396,239

- The total capital for the Standard Deviation measure of risk, 196,396,239, is given by the standard deviation, 89,888,369, (from Table 8.4) times our selected multiplier, 2.185.

The next step is to calculate how much capital the insurer can release at the end of each year.

For each underwriting division, the insurer:

1. Receives allocated capital (Tables 8.5 and 8.6)
2. Earns interest on that capital (here assumed to be 6%)
3. Releases capital not needed for the next year

Tables 8.7 and 8.8 give the results of those calculations.

Table 8.7

Schedule for Releasing Tail Value at Risk Capital at the End of Each Calendar Year for Accident Year 2002

Line\Cal Year	2002	2003	2004	2005	2006
General Liability	5,077,287	4,188,601	3,047,532	1,581,892	373,399
Products Liability	4,453,755	4,167,937	3,332,182	2,125,185	990,609
Auto	6,889,571	4,861,011	1,885,945	0	0
Property	6,720,189	0	0	0	0
Catastrophe	225,051,272	0	0	0	0

Here is a sample calculation:

- The amount of capital released for General Liability at the end of 2002 is equal to 12,608,532 *times* 1.06 *minus* 8,287,757 = 5,077,287.

Table 8.8

Schedule for Releasing Standard Deviation Capital at the End of Each Calendar Year for Accident Year 2002

Line\Cal Year	2002	2003	2004	2005	2006
General Liability	10,843,856	8,607,059	6,297,000	3,292,388	806,491
Products Liability	9,084,738	8,676,658	6,490,557	4,214,579	2,037,563
Auto	14,607,994	10,264,174	4,034,213	0	0
Property	14,102,419	0	0	0	0
Catastrophe	18,069,676	0	0	0	0

Now that we have calculated the schedule for releasing capital, we can then apply Equation 8.1 to calculate the cost of capital (that is, profit) that must be supplied by the policyholders. We set e = 12.00%. Here are the results:

Table 8.9

Cost of Capital by Underwriting Division

	TVaR Capital	Std Dev Capital
General Liability	1,349,742	2,812,338
Products Liability	1,548,761	3,120,415
Auto	1,040,404	2,206,546
Property	339,632	712,723
Catastrophe	11,373,885	913,225
Total	15,652,425	9,765,247

Note the relative size of the catastrophe cost of capital in the two measure of risks.

9. The Cost of Financing Insurance When Using Reinsurance

We have seen that, the effect of reinsurance is to replace part of the cost of capital with the net cost of reinsurance. In this section, we will apply the equations in Tables 8.1 and 8.2 to the ABC Insurance Company when it purchases catastrophe insurance covering losses in excess of $50 million.

Insurers deduct the cost of reinsurance, including the reinsurer's expenses and profit, from taxable income. The net cost of the reinsurance is then equal to:

$$\text{Expected Reinsurance Recovery} \times \left(\frac{1}{ELR} - 1\right) \times (1 - \text{Tax Rate}),$$

where *ELR* is the reinsurer's expected loss ratio. I set the tax rate equal to 35%.

As in the last section, we now calculate the total cost of financing ABC's insurance portfolio for the two measures of risk. The following tables, corresponding to the tables in Section 8, show the calculations with catastrophe reinsurance.

Table 9.1

Capital Allocation Calculation for Tail Value at Risk with Catastrophe Reinsurance

Calendar Year 2002 Line & AY	E[OS Loss] w/o Line & AY	VaR[OS Loss] w/o Line & AY	TVaR[OS Loss] w/o Line & AY	Marginal TVaR Capital	% Allocated Capital
GL-1998	471,000,000	639,782,921	651,919,381	622,782	0.386%
GL-1999	463,000,000	629,143,638	641,288,266	3,253,897	2.015%
GL-2000	448,000,000	609,167,042	621,332,646	8,209,517	5.083%
GL-2001	428,000,000	582,538,754	594,738,644	14,803,519	9.166%
GL-2002	403,000,000	549,931,480	562,138,364	22,403,799	13.872%
PL-1998	468,000,000	635,724,080	647,866,024	1,676,139	1.038%
PL-1999	458,000,000	622,167,113	634,332,646	5,209,517	3.226%
PL-2000	443,000,000	601,704,202	613,925,904	10,616,259	6.573%
PL-2001	423,000,000	575,284,274	587,535,781	17,006,382	10.530%
PL-2002	403,000,000	548,708,303	561,006,009	23,536,154	14.573%
Auto-2000	463,000,000	629,247,167	641,388,772	3,153,392	1.952%
Auto-2001	438,000,000	596,263,226	608,424,852	11,117,311	6.883%
Auto-2002	403,000,000	550,393,465	562,573,065	21,969,098	13.602%
Prop-2002	438,000,000	596,089,957	608,259,153	11,283,010	6.986%
Cat-2002	472,000,000	639,672,796	646,894,524	6,647,640	4.116%
Combined/Total	**473,000,000**	**642,406,295**	**654,542,163**	**161,508,417**	**100.000%**

Table 9.2

Capital Allocation Calculation for Standard Deviation with Catastrophe Reinsurance

Calendar Year 2002 Line & AY	E[OS Loss] w/o Line & AY	Std[OS Loss] w/o Line & AY	Marginal Std[OS Loss]	% Allocated Capital
GL-1998	471,000,000	82,747,196	342,628	0.419%
GL-1999	463,000,000	81,365,727	1,724,096	2.110%
GL-2000	448,000,000	78,774,354	4,315,470	5.281%
GL-2001	428,000,000	75,321,804	7,768,019	9.505%
GL-2002	403,000,000	71,065,812	12,024,012	14.713%
PL-1998	468,000,000	82,223,788	866,036	1.060%
PL-1999	458,000,000	80,477,319	2,612,505	3.197%
PL-2000	443,000,000	77,848,965	5,240,859	6.413%
PL-2001	423,000,000	74,412,155	8,677,669	10.618%
PL-2002	403,000,000	70,966,316	12,123,508	14.835%
Auto-2000	463,000,000	81,373,829	1,715,995	2.100%
Auto-2001	438,000,000	77,080,436	6,009,388	7.353%
Auto-2002	403,000,000	71,104,861	11,984,962	14.665%
Prop-2002	438,000,000	77,066,521	6,023,303	7.370%
Cat-2002	472,000,000	82,794,437	295,387	0.361%
Combined/Total	**473,000,000**	**83,089,824**	**81,723,836**	**100.000%**

Table 9.3
Needed Tail Value at Risk Allocated Capital with Catastrophe Reinsurance at the Beginning of Each Calendar Year for Accident Year 2002

Line\Cal Year	2002	2003	2004	2005	2006
General Liability	25,182,800	16,639,770	9,227,838	3,657,516	700,033
Products Liability	26,455,614	19,115,879	11,933,116	5,855,714	1,884,050
Auto	24,694,178	12,496,319	3,544,543	0	0
Property	12,682,572	0	0	0	0
Catastrophe	7,472,223	0	0	0	0
Other OS Losses	85,054,777	133,290,195	156,836,667	172,028,934	178,958,080
TVaR Capital	181,542,163	181,542,163	181,542,163	181,542,163	181,542,163

Table 9.4
Needed Standard Deviation Allocated Capital with Catastrophe Reinsurance at the Beginning of Each Calendar Year for Accident Year 2002

Line\Cal Year	2002	2003	2004	2005	2006
General Liability	26,710,263	17,255,957	9,586,429	3,829,925	761,117
Products Liability	26,931,284	19,276,662	11,642,098	5,803,446	1,923,821
Auto	26,623,518	13,349,315	3,811,929	0	0
Property	13,380,227	0	0	0	0
Catastrophe	656,175	0	0	0	0
Other OS Losses	87,240,698	131,660,229	156,501,708	171,908,793	178,857,226
Std Dev Capital	181,542,163[1]	181,542,163	181,542,163	181,542,163	181,542,163

Table 9.5
Schedule for Releasing Tail Value at Risk Capital with Catastrophe Reinsurance at the End of Each Calendar Year for Accident Year 2002

Line\Cal Year	2002	2003	2004	2005	2006
General Liability	10,053,998	8,410,318	6,123,993	3,176,933	742,035
Products Liability	8,927,071	8,329,716	6,793,389	4,323,006	1,997,093
Auto	13,679,509	9,701,556	3,757,216	0	0
Property	13,443,526	0	0	0	0
Catastrophe	7,920,556	0	0	0	0

Table 9.6
Schedule for Releasing Standard Deviation Capital with Catastrophe Reinsurance at the End of Each Calendar Year for Accident Year 2002

Line\Cal Year	2002	2003	2004	2005	2006
General Liability	11,056,921	8,704,886	6,331,690	3,298,604	806,784
Products Liability	9,270,499	8,791,164	6,537,178	4,227,832	2,039,250
Auto	14,871,613	10,338,345	4,040,644	0	0
Property	14,183,040	0	0	0	0
Catastrophe	695,545	0	0	0	0

[1] The Standard Deviation Capital Multiplier of 2.185 was selected so that the capital required for the TVaR capital is equal to the standard deviation capital for the catastrophe reinsurance case.

Table 9.7

The Cost of Financing Insurance with Catastrophe Reinsurance

	Tail Value at Risk Capital			Standard Deviation Capital		
	Cost of Capital	Net Cost of Reinsurance	Cost of Financing	Cost of Capital	Net Cost of Reinsurance	Cost of Financing
General Liability	2,702,376	0	2,702,376	2,837,645	0	2,837,645
Products Liability	3,128,662	0	3,128,662	3,148,768	0	3,148,768
Auto	2,071,998	0	2,071,998	2,227,575	0	2,227,575
Property	679,423	0	679,423	716,798	0	716,798
Catastrophe	400,298	2,600,000	3,000,298	35,152	2,600,000	2,635,152
Total	8,982,757	2,600,000	11,582,757	8,965,938	2,600,000	11,565,938

Compare Table 9.7 with Table 8.9. Note that the cost of financing insurance for ABC decreases with the reinsurance when we measure risk by the Tail Value at Risk, while it increases with this reinsurance when we measure risk by the standard deviation.

Now, anyone familiar with real-world catastrophe reinsurance knows that the price of catastrophe reinsurance can vary widely from time to time. When prices go down, insurers buy more reinsurance, and when prices go up they buy less. That behavior is consistent with this model of ABC Insurance Company. Consider the following tables, where we calculate the level of reinsurance that minimizes the cost of financing insurance.

Table 9.8

Optimal Level of Reinsurance when Risk is Measured by the Tail Value at Risk

Reinsurance ELR	Optimal Cat Limit	Cost of Financing
50.00%	40,976,282	11,572,039
60.00%	29,012,942	10,639,167
70.00%	21,219,679	9,943,190
80.00%	14,660,548	9,404,707
90.00%	8,136,819	8,974,210

Table 9.9

Optimal Level of Reinsurance when Risk is Measured by the Standard Deviation

Reinsurance ELR	Optimal Cat Limit	Cost of Financing
50.00%	No Limit	9,765,247
60.00%	No Limit	9,765,247
70.00%	212,024,801	9,748,414
80.00%	119,610,539	9,551,183
90.00%	52,467,114	9,254,743

Although the two measures of risk both indicate, qualitatively, observed reinsurance purchasing behavior, the quantitative results are quite different. The Tail Value at Risk indicates a greater use of catastrophe reinsurance – for ABC Insurance Company. My own sense of the reinsurance market leads me to hypothesize that the Tail Value at Risk will provide a better explanation of reinsurance market behavior. Research could test my hypothesis by applying this methodology to real insurers and seeing to what extent insurers follow the indicated behavior.

10. Target Combined Ratios

All that remains is to express our results in terms of target combined ratios for the ABC Insurance Company. To do that, we need to make the following additional expense assumptions.

Table 10.1

Underwriting Expense Factors

Underwriting Division	ULAE % of Loss	Other Expense % of Premium
General Liability	10.00%	30.00%
Product Liability	10.00%	30.00%
Auto	7.00%	30.00%
Property	7.00%	30.00%
Catastrophe	7.00%	30.00%

We also need the actuarial present value (APV) of the losses for each of the underwriting divisions.

Table 10.2

Underwriting Division	Expected Loss	APV of Loss
General Liabililty	70,000,000	63,637,691
Products Liability	70,000,000	62,720,330
Auto	70,000,000	65,547,100
Property	35,000,000	33,995,005
Catastrophe	5,000,000	4,856,429
Total	250,000,000	230,756,556

I derived the following target combined ratios using the expense factors from Table 10.1 and the cost of financing insurance from the Tail Value at Risk part of Table 9.7.

Table 10.3

Target Combined Ratios for Tail Value at Risk with Catastrophe Reinsurance

	General Liability	Products Liability	Auto	Property	Catastrophe
E[Loss]	70,000,000	70,000,000	70,000,000	35,000,000	5,000,000
APV[Loss]	63,637,691	62,720,330	65,547,100	33,995,005	4,856,429
ULAE%	10.00%	10.00%	7.00%	7.00%	7.00%
ULAE	7,000,000	7,000,000	4,900,000	2,450,000	350,000
APV of LAE	6,363,769	6,272,033	4,588,297	2,379,650	339,950
Other Expense%	30.00%	30.00%	30.00%	30.00%	30.00%
Other Expense	31,158,787	30,909,011	30,946,026	15,880,320	3,512,862
Cost of Financing	2,702,376	3,128,662	2,071,998	679,423	3,000,298
Cost of Financing%	2.60%	3.04%	2.01%	1.28%	25.62%
Premium	103,862,622	103,030,037	103,153,422	52,934,399	11,709,539
Target Comb Ratio	104.14%	104.74%	102.61%	100.75%	75.69%
Overall Comb Ratio	102.51%				

Lest we forget – in Section 3, I stressed the importance of correlation. Recall that we generated correlations in the noncatastrophe underwriting divisions using random β's with variance $b = 0.03$. Changing $b = 0.03$ to $b = 0.01$ reduces the overall needed capital from 181,542,163 to 119,199,301. The following table gives the corresponding changes in the target combined ratios.

Table 10.4

Target Combined Ratios for
Tail Value at Risk with Catastrophe Reinsurance and $b = 0.01$

	General Liability	Products Liability	Auto	Property	Catastrophe
E[Loss]	70,000,000	70,000,000	70,000,000	35,000,000	5,000,000
APV[Loss]	63,637,691	62,720,330	65,547,100	33,995,005	4,856,429
ULAE%	10.00%	10.00%	7.00%	7.00%	7.00%
ULAE	7,000,000	7,000,000	4,900,000	2,450,000	350,000
APV of LAE	6,363,769	6,272,033	4,588,297	2,379,650	339,950
Other Expense%	30.00%	30.00%	30.00%	30.00%	30.00%
Other Expense	30,731,258	30,433,530	30,610,823	15,772,387	3,542,252
Cost of Financing	1,704,808	2,019,207	1,289,858	427,582	3,068,875
Cost of Financing%	1 66%	1.99%	1.26%	0.81%	25.99%
Premium	102,437,525	101,445,101	102,036,078	52,574,625	11,807,507
Target Comb Ratio	105.17%	105 90%	103.41%	101.23%	75.31%
Overall Comb Ratio	103.37%				

11. Concluding Remarks

In constructing the example for ABC Insurance Company, I made two important simplifications that were not mentioned above. First, I did not consider asset risk. And second, I minimized the effort in the selection of solvency thresholds.

In our exercise, ABC assets earned a fixed rate of interest of 6%. If ABC invested in higher-yielding assets with variable returns, the company would have to have more assets, and hence more capital. That observation suggests a need to allocate capital between the underwriting and investment operations. I suggest making such an allocation by first calculating how much capital the company requires with fixed-rate investments, and then calculating how much capital the company requires with the actual investments. The difference between the two will yield the marginal capital for the investment operation.

The most influential determinants of insurer capital are the state regulators and the rating agencies. To take a first crack at determining a solvency threshold, we could determine appropriate capital by consulting regulators and rating agencies. We would then back the threshold out of that capital.

If we were to do the exercise on several insurers, we should then be able to reach a consensus on the appropriate threshold.

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Reserve Estimates Using Bootstrapped Statutory Loss Information

William C. Scheel

Reserve Estimates Using Bootstrapped Statutory Loss Information

by
William C. Scheel[1]

Abstract

The reserving methodology described in this paper produces minimum sufficiency levels for reserves that are risk adjusted both for uncertainty in claims payments and uncertainty in investments. The minimum sufficiency level is derived from measurements of correlation and other statistical properties of link ratios. These statistics are found using bootstrap methods. Because the approach relies on bootstrap methods, there is no explicit measurement of either process or parameter risk that ordinarily appears in dynamic financial analysis.

Introduction

The information in a property/casualty loss triangle is highly aggregated; individual claims information is lost during the summation processes both for accident and calendar periods. Ordinarily, bootstrap methods would be applied to raw claims information rather than to an aggregation such as the loss triangles found in Schedule P of the annual statement. However, published information about individual claims experience for companies is non-existent.

The paper describes how bootstrap methods can be applied to public loss information to produce range estimates for future losses.[2] This reserving methodology could use any of the popular reserving methods appearing in the literature. However, the focus of the paper is primarily on the use of bootstrapping to obtain adjustments both for uncertainty in claims payments and uncertainty in investments. The choice among the plethora of reserving methods was kept as simple as possible to illustrate more important principles. The chain ladder reserving method was used. The methods used in this paper are strictly mechanical; no actuarial judgment arises.

A correlation matrix for all lines of business evolves from the method of bootstrapping. Other statistics are derived during the same bootstrap process that produces estimates of correlation factors. The reserves that are estimated have adjustment for the correlation among lines of business, claims payments uncertainty and investment uncertainty. This

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[1] William C. Scheel, Ph.D., is President of DFA Technologies, LLC This paper was submitted in response to the 2001 Call for Papers, Dynamic Financial Analysis, A Case Study. A companion paper entitled "Valuing An Insurance Enterprise" also was submitted. The author gratefully acknowledges the insight of both William J. Blatcher and Gerald S. Kirschner in spotting several of the author's errors during the unfolding of this paper.

[2] The data used in this paper were provided to authors participating in the 2001 DFA Seminar Call Papers contest held by the Casualty Actuarial Society. They include statutory Schedule P information for a hypothetical insurance company.

differs from conventional mono-line reserving approaches that often do not adjust for either source of uncertainty except by actuarial judgment.

The paper also introduces a new approach for reserve valuation that is tightly coupled to optimization methods applied to investment portfolios. It is difficult to separate where reserving leaves off and dynamic financial analysis (DFA) begins; in this regard they are inseparable.

Valuation Steps

There are six steps in the first phase of the reserving method:

1. Perform a bootstrap of link ratios for ultimate loss. Loss development factors neither were directly measured nor bootstrapped. Both the unfolding of ultimate loss and its relationship to paid loss were chosen as the bootstrap objects. The ultimate loss triangle contains potentially useful information not found in the paid loss triangle—it includes actuarial judgment.
2. Use bootstrapped ultimate link ratios to derive statistics including correlation coefficients, means and standard errors. Track the proportion of loss payments to ultimate loss as part of the bootstrap sampling of ultimate loss links.
3. Use the correlation matrices and statistics obtained in step 2 and simulate future ultimate development period links for each line of business using multinormal methods.
4. Apply the simulated ultimate link ratios to the latest loss triangle diagonal. The ultimate losses for the forecast period are obtained.
5. Perform a second-stage simulation using the probability distribution of paid-to-ultimate ratios (also derived as part of the bootstrap process in step 1).[3] The probability distribution for these ratios is a by-product of the ultimate link bootstrapping. The paid/ultimate ratios were tracked (and bootstrapped) during the bootstrap of the ultimate loss triangle. Each line of business has a probability distribution of these paid/ultimate ratios. It is used to simulate a payment proportion for the simulated ultimate losses. Forward period cash flows for each scenario in step 3 are obtained.
6. Use the cash flows determined in step 5 to calculate annuity-equivalent values for future loss cash flow. Do this at each forward calendar period. There is an annuity-equivalent valuation at each point in time that includes future estimated losses from that point in time onward. Repeat this step for each scenario. This produces a distribution of annuity-equivalent values or present values of future losses. These annuity distributions are discounted reserves. The discount rate is conservative. It could be zero.

[3] The probability distribution of the paid/ultimate loss ratio is a conditional one. The ratio was measured conditional on the bootstrapped ultimate link. Recall that the ultimate link ratios were bootstrapped. Each bootstrap sample involved resampling among accident periods. This was done independently for each development period link. The profile of resampled accident periods used for this ultimate link ratio bootstrapping was also used in connection with the payments triangle to calculate the ratio of paid to ultimate. There was a direct matching of accident periods for the paid and ultimate triangles in this process.

One might stop here. The distribution of the present value of future paid losses provides necessary information for a reserve range in which uncertainty in loss payment is recognized.[4] The distribution could be used to obtain ex ante estimates of reserves for future fiscal periods.[5]

But, a second phase that extends the measurement of uncertainty is useful, so we will not stop with just the uncertainty in claims payments. The distribution obtained in step 6 reflects only this source of uncertainty.[6] The second phase attempts to adjust the reserve levels for uncertainty in asset accumulations needed to back them. This secondary analysis seeks the sufficiency level for reserves.

Sufficiency Levels

Chance-constrained ranges can be set on the present value of future loss payments using the results of Step 6. Managerial judgment could be used to choose a percentile of this distribution. Because the percentile is a sample estimate, a conservative approach would pick the upper confidence level for the percentile. This choice is called the *minimum sufficiency target*. The present value of future payments (discounted reserves) is nominally sufficient to pay claims amounts within defined levels of confidence and sampling error. This result is a target, not the actual minimum sufficiency level because the target is risk adjusted *only* for uncertainty in claims payments. The target has a specified probability of sufficiency; but only to the extent of the amount of the liability for claims payment. The target is conditional on no risk in investment returns.[7]

The minimum sufficiency target for period t includes claims paid in period t and subsequent development periods, t+1, t+2, ... The target is a hurdle rate expressed as an end-of-period value.[8] Were assets at time t to equal the minimum sufficiency target, the

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[4] The distribution of the present value of future paid losses can be used to answer questions such as "What is the range in values within a 90 percent confidence band?" or "What is the loss level with a probability of no more than 0.05 of being exceeded (0.95 percentile)?" These and other chance-constrained questions concerning loss reserves can be answered using this distribution.

[5] The valuations for future periods do not include future business. There are many extensions of this reserving approach that can be done with DFA methods. One important extension is to include new business development. Others include separation among various sources of loss, such as allocated and unallocated loss adjustment, uncertainty in both frequency and severity of loss and the effects of reinsurance on loss transfer.

[6] It is not the intent of this paper to engage in a discussion of what uncertainties should properly be reflected in loss reserves. Suffice it to note that it still is a regulatory failure when an insurer set its reserves adequately in the sense that reserves for future claims payments were deemed to have a 95 percent chance of covering payments; but, unfortunately, the insurer's assets dwindled to insufficient levels. Policyholders or stockholders end up taking the fall anyway. When the original liability was established, it reflected uncertainty only in the magnitude of payments, not uncertainty in the ability to meet those payments. It is a moot issue both to the policyholder and to the stockholder whether insolvency occurred because the insurer cannot pay either an expected or unexpected loss payment.

[7] The minimum sufficiency target still has investment risk; so, it is a target. The target is not immunized because it involves discount assumptions. However, as a practical matter it might have been discounted at a riskless or near riskless rate and also be an immunized target sufficiency level.

[8] It is assumed that payments are end-of-period amounts for the purpose of this analysis.

liability would be covered at the indicated confidence level. Suppose that the distribution of required assets at time t were known. This target distribution could be discounted to get the distribution of beginning-of-period required assets. The discounted distribution is the premium distribution for a single premium deferred annuity. A confidence level associated with this asset distribution is referred to as the *minimum sufficiency confidence level*.

The minimum sufficiency level of assets funds future claims payments within specified levels of confidence. Both the minimum sufficiency level and targets are percentiles of probability distributions.[9]

Determination of Sufficiency Levels

The future claims are expressed as a present value using a conservative rate of discount. The minimum sufficiency target is an amount derived from this distribution of present values. Simulated link ratios lead to forecast-period cash flow estimation, and these cash flows are the source of the present value determinations.[10]

Determination of Link-Ratio Correlation Matrices

The period-to-period changes in estimated ultimate loss were bootstrapped in a special way so that a line-of-business correlation matrix could be obtained for each link ratio. A bootstrap sample of developed claims is drawn. This is done from the set of accident periods that can be used for the t^{th} development factor.

Table 1: Feasible Region for Bootstrap Sampling of a Link Ratio
the shaded area can be used for bootstrapping of the link for the 36-48 month development period. A bootstrap sample involves drawing with replacement from this region to create a pair of columns in which the rows are randomly sampled many times with replacement from the original set. The sampling scheme that unfolds for one line of business is used for the other lines too. For example if the rows in the region were numbered {1,2,3,..., 7}, a sampling scheme for the 36-48 month link could be {1,1,3,5,7,4,4} The corresponding column pairs from each line of business would be used and from them a link factor for the 36-48 development period for each line would be calculated. This technique of bootstrapping in a synchronous fashion from a multivariate sample space is reviewed in Scheel, et al [2000] and Laster [1998].

The derivation of the other development period links is done independently. For example, the bootstrap sample for the 72-84 month development period might use a sampling scheme of accident periods {4,2,2,1}. Table 1: Feasible Region for Bootstrap Sampling of a Link Ratio
illustrates this sampling scheme. But, other lines of business also would have this same replacement sampling for determining their 72-84 month link for this bootstrap sample.

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[9] Because the percentiles have sampling error, the sufficiency amounts are really confidence limits on the percentile. Respectively, they are the lower and upper confidence limits for the minimum sufficiency and target sufficiency percentiles.
[10] Statutory discounts might be at zero rates of interest.

Each development period link is an independent sample, and there is no scaling problem associated with exposure volumes in the various accident periods.

This sampling method is repeated many times to obtain many values for each calendar period link ratio within the sample space. The entire set of bootstrap samples can be used to derive statistics for the ratios. All of the link ratios for different calendar periods can be obtained by using the available accident periods for each transition link within the loss triangle.[11]

The bootstrap samples for different lines of business can be used to calculate all of the needed statistics for links. They also can be used to calculate line-of-business correlation coefficients for the links. Standard errors for these various statistics can be computed using bootstrap methods.

Correlations among lines of business are measured using the experimental sample space. In this case, the bootstrap samples being drawn in a synchronous fashion for all lines of business is that sample space. From a computational standpoint there is a great deal of housekeeping required, but the methods for obtaining a correlation matrix and estimates of the mean and standard deviation for a link are straightforward.[12]

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[11] Links for calendar periods 8 and 9 are not obtained from bootstrap sampling because of the sparse number of usable accident periods. Links for these periods are based on the actual loss triangle information and not bootstrap samples of it. The links for any forecast periods beyond 9 use actual $link_9$. The bootstrap sampling uses a decreasing number of accident years when calculating $link_1$, $link_2$, ..., $link_7$ for the transition in ultimate loss estimates.

[12] Calculations and simulations for this study were done using Microsoft Excel 2000. Multivariate normal simulations were performed with Excel 2000 and a DLL written with Compaq Visual Fortran Version 6.5. The multinormal simulation relies on a Cholesky factorization of the covariance matrix. See Rubinstein [1981] for a discussion of the multinormal simulation methods. Non-linear optimization was done with Frontline Systems Premium Solver Plus version 3.5, an add-in for Excel.

Table 1: Feasible Region for Bootstrap Sampling of a Link Ratio

	12	24	36	48	60	72	84	96	108	120
1990	92,906	123,086	121,828	121,312	120,960	120,786	120,667	120,986	120,907	120,685
1991	126,731	130,026	127,583	126,730	125,640	127,269	126,636	126,266	125,893	
1992	157,558	159,071	158,104	159,525	157,525	157,873	157,124	156,249		
1993	163,692	163,139	161,354	161,677	160,495	160,421	159,270			
1994	167,469	164,228	163,903	163,628	161,827	159,595				
1995	230,837	229,624	227,953	226,813	226,454					
1996	202,686	201,266	202,338	200,922						
1997	259,065	260,110	256,783							
1998	222,746	221,905								
1999	268,705									

Table 2: Example of Portions of a Bootstrap Sample in Shaded Regions

	12	24	36	48	60	72	84	96	108	120
1990	92,906	123,086	121,828	121,312	120,960	160,421	159,270	120,986	120,907	120,685
1991	126,731	130,026	121,828	121,312	125,640	127,269	126,636	126,266	125,893	
1992	157,558	159,071	158,104	159,525	157,525	127,269	126,636	156,249		
1993	163,692	163,139	163,903	163,628	160,495	120,786	120,667			
1994	167,469	164,228	202,338	200,922	161,827	159,595				
1995	230,837	229,624	161,354	161,677	226,454					
1996	202,686	201,266	161,354	161,677						
1997	259,065	260,110	256,783							
1998	222,746	221,905								
1999	268,705									

The 36-48 month link for the bootstrap sample in the shaded region of Table 2 is 0.99941. The 72-84 month link is .99547. Although these are members of the same bootstrap sample, the links for a development period are independent replacement sampling processes. The ratio of paid loss to ultimate loss for any development period also can be calculated for this same bootstrapped sample. It would use the same set of accident periods, but payment information for them is found in the payments triangle.

The correlation matrix for one of the development period links is shown in Table 1: Statistics for $Link_1$. This table also includes general statistics for the paid-to-ultimate loss ratio. This ratio is developed during the bootstrap process along with the ultimate link factors. The distribution of the paid/ultimate ratio is used during a second-stage simulation to provide the transition from ultimate to paid loss. The second stage produces payment pattern variation; whereas, the first simulation stage works with the ultimate link ratios. During a second stage simulation a paid/ultimate ratio is determined and simulated cash flow is obtained for paid loss. This simulation methodology is discussed in detail later in the paper

Simulation Using Link Ratios

Links were simulated and applied to the most recent diagonal of the ultimate loss triangle to obtain forecast period ultimate losses. The means, standard deviations and correlation matrices used for the simulation are shown in Table 1: Statistics for $Link_1$. The links among lines of business were assumed to be multivariate normal with no serial correlation. Each simulation of a link was done independent of other link simulations; all were multivariate normal simulations.[13]

Each simulation had 3,000 trials so that a sample of 3,000 cash flows over a forecast period of 10 years was available for calculating the present value distributions used in subsequent analysis.

[13] See Rubinstein [1981] for a discussion of how the multivariate normal simulation is done. The algorithm used in this study is the IMSL fortran subroutine **DRNMVN**.

Table 1: Statistics for Link$_l$

	Line of Business Correlation Matrix									
	Home	PPA	CAL	WC	CMP	Spcl_Liab	OL_OCC	Reins_A	Reins_B	Reins_C
PPA	0.0089									
CAL	-0.0031	-0.0028								
WC	-0.0075	-0.0067	0.0016							
CMP	0.0119	0.0104	-0.0026	-0.0103						
Spcl_Liab	-0.0157	-0.0147	0.0108	0.0072	-0.0146					
OL_OCC	-0.0212	-0.0187	0.0049	0.0179	-0.0269	0.0244				
Reins_A	-0.0539	-0.0454	0.0172	0.0369	-0.0583	0.0684	0.1030			
Reins_B	-0.0047	-0.0132	0.0103	0.0059	-0.0043	-0.0670	0.0354	0.0052		
Reins_C	0 0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
Property_ShortTail	0.0026	0.0023	-0.0006	-0.0020	0.0031	-0.0027	-0.0050	-0.0146	-0.0042	0.0000

Line of Business	Ultimate Link1		Paid/Ultimate Ratio1	
	Expected Value	Standard Deviation	Expected Value	Standard Deviation
Home	1.0353	0.0795	0.8822	0.0159
PPA	0.9935	0.0713	0.5829	0.0431
CAL	0.9715	0.0475	0.4263	0.0370
WC	0.9585	0.0784	0.4345	0.0337
CMP	1.0177	0.1014	0.5408	0.0233
Spcl_Liab	1.1854	0.3476	0.7765	0.0877
OL_OCC	0.8596	0.1921	0.2174	0.0584
Reins_A	0.9823	0.3608	0.5491	0.1999
Reins_B	1.4092	0.5176	0.7865	0.0474
Reins_C	1.2188	0.0000	0.5461	0.2491
Property_ShortTail	1.0040	0.0270	0.9861	0.0075

Paid Loss Distributions

The variation in speed of payments is a source of uncertainty. Both this uncertainty and uncertainty in ultimate loss must be reflected in cash flow simulations during the forecast period. The distribution of the ratio of paid-to-ultimate also is by-product of the bootstrapping methods. Just as each bootstrap sample produces a link ratio for a development period, the same bootstrap sample develops the ratio of paid to ultimate. The ratio uses the same bootstrap sample accident periods as the ultimate link except that the same sampled accident periods are extracted from the paid loss triangle. The numerator of the paid/ultimate ratio is found in the bootstrapped accident periods of the paid triangle; the denominator is found in the ultimate triangle. The average of the ratios is used as that bootstrap sample's paid/ultimate ratio. The result of all bootstraps is the source of the conditional probability distribution of paid loss. The conditional operator here applies to paid loss given the ultimate loss linkage for the development period.

Payment Pattern Simulation

The ultimate-to-paid transition for cash flow determination occurs in a two-stage simulation. The first stage produces the ultimate link factors for all calendar period transitions. The second stage simulation produces a payment pattern in the form of paid/ultimate.

Each line of business has a set of bootstrap samples that represent a set of payment patterns in the form of paid/ultimate ratios. Once the change in ultimate loss estimates is determined from the first-stage simulation, a payment pattern is chosen during the second stage. In other words, the bootstrap samples of payment patterns are the source for a second stage simulation.

This second-stage simulation adjusts paid losses both for uncertainty in ultimate loss and for uncertainty in the speed of claims payments. The effect is simulation of a payment pattern associated with each ultimate loss level derived in the first-stage simulation. Finally, the cash flows for present value analysis can be assembled from the forecasted diagonals of the simulated paid loss triangle.

Discounting Simulated Paid Loss

Statistics for these present values are shown in Table 2: Statistics for Discounted Paid Losses. The 0.9 percentile of the distributions in this table are the minimum sufficiency targets used in subsequent optimizations. For example, the minimum sufficiency *target* at the end of period 1 would be $1,798,921. The minimum sufficiency target secularly declines, dropping to $484,940 by period 5 and $47,013 by period 10. As previously noted, all cash flows for losses were assumed to have occurred at the end of the period.

The distributions are risk adjusted only for uncertainty in the ultimate loss and variation in the speed of payments. Nevertheless, these results provide ranges for reserve estimation. Conventional reserve practice, both statutory and generally accepted accounting, is to use a point estimate of future paid losses as the basis for liability determination. The values shown in Table 2: Statistics for Discounted Paid Losses provide ranges and other chance-constrained values of what might be considered the conventional GAAP estimates.[14]

[14] A five-percent discount rate was used for present-value calculations of the paid loss cash flows.

Table 2: Statistics for Discounted Paid Losses

All Lines Statistics Table	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9	Period 10
Mean	1,798,921	1,282,873	896,766	650,352	484,940	369,549	278,479	199,570	124,023	47,013
Standard Deviation	91,557	55,185	37,405	21,528	12,388	8,818	6,451	4,571	2,833	845
Median	1,798,343	1,281,362	895,937	649,768	484,449	369,120	278,192	199,447	123,920	46,980
5 percentile	1,649,169	1,195,019	837,174	615,604	465,394	355,541	268,231	192,349	119,508	45,660
10 percentile	1,682,185	1,214,548	850,383	623,497	469,507	358,660	270,411	193,797	120,432	45,956
25 percentile	1,737,040	1,244,796	870,057	634,645	476,207	363,456	274,039	196,427	122,083	46,435
75 percentile	1,860,955	1,319,754	920,838	664,331	492,981	375,434	282,777	202,531	125,786	47,557
90 percentile	***1,913,878***	***1,352,208***	***944,923***	***678,150***	***500,876***	***380,722***	***286,849***	***205,513***	***127,761***	***48,135***
95 percentile	1,948,887	1,374,690	960,082	686,303	506,422	384,138	289,258	207,396	128,899	48,491

Treatment of Incomplete Information

Some of the lines of business had incomplete Schedule P information. Some lines had either a few accident periods or accident periods with few or no losses. Only lines of business with at least fifty percent of completed ultimate and payments cells were used.[15]

In a few cases, the information provided was invalid—ultimate loss for some cells of the ultimate triangle did equal the sum of paid losses and reserves. Ad hoc methods were used in the cleansing of these few imbalanced cells. In general, the ultimate figures were taken to be valid and the paid loss was adjusted with reference to experience in near-by calendar periods. It is not likely that these adjustments had a material impact on the results.

Optimization

The distribution of present-valued claims payments was used to define target sufficiency levels. There is no risk adjustment in these levels for uncertainty in asset growth. We now turn to the interesting question of how such uncertainty might be recognized in the determination of reserves.

Reflecting Investment Uncertainty in Reserves

The distribution of present values for end-of-period valuations for future claims provides the means for assigning fair value to such claims given a conservative growth in investments backing them. The target sufficiency level constitutes a type of financial immunization. Because the target sufficiency is reckoned at a risk-free rate, the company could bank this level of assets and be assured of claims payment with the level of confidence used to determine the targets. Because there is little or no interest rate risk in the target sufficiency, the liability could be commuted; it is an actuarially fair value within defined confidence limits of the loss modeling mechanism.[16]

It remains for investment risk to be similarly bounded so that sufficient funds will exist at this target sufficiency level. The sought-for objective is an asset level at beginning-of-period that will grow to the required target with confidence. The main purpose of this study is not to eschew a particular asset modeling methodology. Any model can be used provided it can generate investment scenarios. This study uses prior work that derives

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[15] The treatment of immature lines of business suffers from the problems plaguing any study using non-parametric methods. These approaches, including bootstrapping, rely on the availability of underlying data. Parametric procedures under these limitations also have a hard time determining appropriate choices for probability distributions or their parameters.

Another approach to handling this problem of unavailable or missing data would be to substitute "pure-play" data available from other companies or reference sources. These data would serve as proxies for the missing information and would have to be adjusted to the exposure volumes in existence for lines of business where such proxy data were deployed.

[16] The sufficiency target is the reinsurance pure premium for risk transfer at a level of confidence defined in the analysis. It includes risk margins for variation in loss payments, but no allowance for volatility in investment of those premiums from the discount rate of five-percent.

estimates of a covariance matrix for a mixture of assets [Scheel, et al, 2000]. The description of this database appears in Appendix A: Review of Data Sources. Other asset scenario models, which are based on time-dependent functions such as multi-factor models with mean reversion, could have been used. The approach described in this paper would remain unchanged even if another method of asset scenario generation had been deployed.

Investment returns were simulated using a bootstrapped estimation of the covariance matrix and expected values using monthly returns data for the 20-year period 1/1/1980-12/31/99. Multinormal simulation methods were used in the simulation; they were identical to the ones used for simulating calendar period links. Annualized rates of return were generated from the monthly data by assuming no serial correlation and compounding simulated monthly returns. Various statistics relating to this simulation appear in Table 3: Statistics for Simulated Asset Scenarios. This table shows investment performance for ten annual periods used in the study.

Table 3: Statistics for Simulated Asset Scenarios

Annualized Return	EAFEU	INTLUHD	S&P5	USTB	R_MID	HIYLD	CONV	LBCORP	LBGVT	LBMBS
Expected Value	0.1647	0.0958	0.1495	0.0659	0.1659	0.1018	0.1181	0.0940	0.0917	0.0964
Standard Deviation	0.1803	0.0814	0.1625	0.0084	0.1788	0.0904	0.1124	0.0859	0.0592	0.0587
0.25 Percentile	0.0375	0.0383	0.0330	0.0605	0.0392	0.0393	0.0406	0.0354	0.0509	0.0553
0.50 Percentile	0.1557	0.0958	0.1393	0.0658	0.1512	0.0968	0.1116	0.0909	0.0892	0.0953
0.75 Percentile	0.2764	0.1479	0.2553	0.0715	0.2828	0.1611	0.1932	0.1487	0.1317	0.1353

Correlation Matrix for Proxy Assets										
EAFEU	1.0000	0.4240	0.4124	-0.0297	0.3891	0.2689	0.3975	0.1823	0.1836	0.1150
INTLUHD	0.4240	1.0000	0.0075	0.0233	-0.0102	0.1407	-0.0253	0.2748	0.3301	0.2578
S&P5	0.4124	0.0075	1.0000	-0.0624	0.9435	0.4616	0.9313	0.3204	0.2374	0.2477
USTB	-0.0297	0.0233	-0.0624	1.0000	-0.0276	0.0130	-0.0918	0.1791	0.2622	0.2317
R_MID	0.3891	-0.0102	0.9435	-0.0276	1.0000	0.5063	0.9465	0.3156	0.2328	0.2549
HIYLD	0.2689	0.1407	0.4616	0.0130	0.5063	1.0000	0.5214	0.6655	0.5243	0.5248
CONV	0.3975	-0.0253	0.9313	-0.0918	0.9465	0.5214	1.0000	0.3314	0.2409	0.2500
LBCORP	0.1823	0.2748	0.3204	0.1791	0.3156	0.6655	0.3314	1.0000	0.9041	0.8112
LBGVT	0.1836	0.3301	0.2374	0.2622	0.2328	0.5243	0.2409	0.9041	1.0000	0.8424
LBMBS	0.1150	0.2578	0.2477	0.2317	0.2549	0.5248	0.2500	0.8112	0.8424	1.0000

Legend: EAFEU international equities; INTLHDG international fixed income; S&P5 large cap domestic equities; USTB cash; RMID mid-cap domestic equities, HIYLD high yield debt, CONV convertible securities, LBCORP corporate bonds, LBGOVT government bonds, LBMBS mortgage backed securities. Additional information about the proxy assets is in Appendix A: Review of Data Sources.

Table 3 illustrates the statistical properties of the annualized asset scenarios for just one of the annual periods in the analysis. However, because each annual period's asset scenarios were independently calculated from the same multinormal distribution of returns, the statistical properties for other periods were approximately the same. A small sample of some of the investment scenarios appears in Appendix B: Example of Asset Scenarios for an Annual Period.

Optimization Methods

Non-linear optimization was used. The optimizer posits trial solution set of weights for the investments. All of the simulated investment scenarios were weighted with this investment profile, and a portfolio return was calculated for each scenario. The result is a distribution of portfolio returns for a period. The portfolio return for each scenario is a discount rate that can be used to determine beginning-of-period sufficiency requirements.

The minimum sufficient asset level (beginning-of-period) can be calculated using the portfolio discount rate applied to the (end-of-period) target sufficiency level. When this is done for each investment scenario, a distribution of minimum sufficiency levels is obtained. That distribution then is used to choose a chance-constrained minimum sufficient level. It is a reserve that is risk-adjusted both for uncertainty in claim payments and in investment return.

The minimum sufficiency level (beginning-of-period) is returned to the optimizer as the objective value. The non-linear optimizer continues to posit different investment weights until a minimum for this objective value is found. Such an optimized minimum is the risk-adjusted reserve being sought.

Optimization Constraints

The optimizer was given a standard set of feasibility constraints for investments; all component asset weights were constrained to lie between 0 and 1 and the weights must add to 1. No short sales were allowed.

Optimization Objective Function Calculation

The optimization objective function was the present value of the target sufficiency level. It was minimized by the optimizer. The objective value was calculated for each trial solution of the optimizer using a separate instance of Excel.[17]

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[17] The computational method used to derive the objective function values involved use of a separate instance of Excel as a COM object for the Excel instance running Solver. Although these are programming issues, they are important to the study and warrant some explanation. When the optimizer supplies the workbook with a trial solution for the portfolio weights, it recalculates the workbook. This recalculation is supposed to produce a value for the objective function cell.

The goal cell contained a cell function, a call to an Excel macro that must be within the same workbook as Solver. This macro has restrictions on what it can do with the workbook cells while it is executed during a recalculation of the workbook. The macro only can read sections of the workbook, it cannot modify the contents of any cell during its execution. It only can return a value to the cell from which the macro was called. Although this limits what might be done while Solver executes, COM objects running in separate processes provide exceptional flexibility that ordinarily would be missing were just the solver workbook to

Reserves

The minimum sufficiency reserve levels for each period are shown in Table 4: Statistics for Reserve Minimum Sufficiency Levels and Optimal Investment Portfolios. The reserve level is "Minimum Beginning of Period" values. It is the amount, which with confidence .9, will grow to the "Required End-of-Period" value—the target level for sufficiency. The weights for components of the optimized proxy portfolio also appear in the table. Statistics for the end-of-period portfolio values are shown.

For example, an asset level of $1,591,549 at the beginning of period 1 is the nominal amount needed to provide for payments of claims in this period and fund the present value of all future claims in periods 2, 3,.... The target level declines as the magnitude of future claims payments dwindles over time. For example, by period 5, the target sufficiency has declined to $296,149, and by period 9 it drops to $37,711.

There is a .1 probability of assets not growing to the target sufficiency level.[18] Further, that target level has a .2 probability of being inadequate for claims payment because it

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be used. The calculation of the objective value given the weights, for example, is complex. However, it can be easily done in its own instance of Excel. This instance is being controlled by the macro of the workbook running solver.

The objective cell macro uses, as an argument, the reference to the cell range containing the weights being suggested by the optimizer for the current trial solution. The fact that an argument was used in the macro call is extremely important...it assures that the macro function will not be executed until after the optimizer has written the trial solution weights to the referenced cell area Because the macro can read cells within the Solver workbook, the macro can copy the weights into the separate instance of Excel. Previously, that instance was also provided the sufficiency target, rates of return for simulated asset scenarios and other information about confidence levels. The separate instance is recalculated and the results are available to the macro for return to the objective function cell.

The separate instance if Excel has all of the information it needs to perform its own calculation. This calculation is driven by the Solver macro after it has done the necessary setup in the separate instance. The investment returns for all scenarios contained in the separate workbook are weighted by the trial solution set of weights. The recalculation of this instance develops the distribution of present values for the sufficiency target. Finally, the upper confidence limit for the percentile of that distribution is calculated. The percentile is binomially distributed. With adequate numbers of investment scenarios, the upper confidence limits for the percentile can be calculated using normal approximation methods. The 2,500 investment scenarios used in this calculation were more than adequate for a normal approximation.

To summarize, the Solver goal cell is recalculated along with other cells in the workbook during a trial solution. The weights are passed through the macro to a separate instance of Excel. The separate instance is recalculated by the macro to produce the answer that is returned to the goal cell. Solver does not know that a separate computational environment was used to derive the complex goal calculation.

Of course, this calculation is repeated many times as the optimizer tests trial combinations of the weights. Furthermore, it is done for each period in the analysis. This trick of using a separate instance of Excel and COM techniques is useful for deriving complex calculations associated with optimizer objective function calculations. It is essential when these calculations require multiple workbook recalculation or involve their own macros that may be difficult to otherwise order correctly within the cell recalculation hierarchy used by Excel during a workbook recalculation.

[18] The probability of inadequacy is actually more conservative because the stated percentile of the distribution was adjusted for sampling error in measuring that stated percentile. So, the minimum

was based on the sufficiency payment target set at a confidence level of .8. Higher confidence levels could have been used both for the payments and investments.

Release of Capital in Reserves

The assets, before payment of claims, were ***expected*** to grow from $1,591,549 to $1,770,939 given the optimized portfolio weights shown in the table. This expected value for asset growth is ***higher*** than the target minimum sufficiency level of $1,683,785. The chance-constrained level imposes a higher standard than expected value; there only can be a .1 probability of the growth being inadequate, which was the confidence level chosen for objective function valuation. The built-in margins in the reserve are a source of expected capital release as the reserves are released.

A higher volume of initial assets is required to assure the confidence levels sought.[19] As a result, the higher initial reserve level that must be maintained is ***expected*** to grow to be more than is required. Of course, it may not grow at the expected rate, but the optimization sets a higher confidence level so that the reserve will be sufficient both with respect to claims and investment experience (asset growth confidence is 0.9). There is an expectation of a favorable release of the contingency margins, both claims and investment, in this reserving method.

A local optimum is shown for the minimum reserve in the first row of Table 4: Statistics for Reserve Minimum Sufficiency Levels and Optimal Investment Portfolios. Other weights of assets produce different local optima. In general, the other local optima are similar. The variations are discussed in detail in the section "Variation among Local Minima".

Investment Portfolio Rebalancing

Table 4: Statistics for Reserve Minimum Sufficiency Levels and Optimal Investment Portfolios shows the changes that would occur were portfolio rebalancing to track the changes in the optimal portfolios each period. It will be noted at a later point in the paper that the result shown in the table is a local optimum. Other solutions of the non-linear optimizer produce different optimum values. A high volatility in portfolio composition could be found among optimizer solutions to what amounts to the same problem. Many different portfolios could lead to approximately the same optimized solution; the local optima, although clustered, were constructed from rather different portfolio compositions.

This makes generalizations about changes in portfolio composition over time very difficult to make. Even the rank order of asset weights was highly volatile. In general, higher risk investments give way to lower risk ones such as U.S. Treasury bills and debt.

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sufficiency level was an empirical observation somewhat lower than the observed .1 percentile of the distribution. A similar conservative adjustment was made in the choice of the empirically determined target sufficiency level. Both adjustments to the stated percentiles were done because of sampling error in measuring them.

[19] There are two confidence levels: (1) that the investments will grow to a target sufficiency level, and (2) that the target sufficiency level will be adequate to fund future claims assuming only a risk-free level of return thereafter.

The Sharpe ratio, which is measure of risk-adjusted return, also reflects a rebalancing scheme that tends to move from higher risk to lower risk.

An intuitive explanation for this rebalancing scheme is that the portfolio cannot be placed at risk during later periods when losses have less chance of being recouped. Higher risk in the early periods, however, may be an acceptable tradeoff both because there is a longer period to recover early investment losses and because the higher expected returns assist in meeting the higher demands for cash flow at early stages of claims development.

Table 4: Statistics for Reserve Minimum Sufficiency Levels and Optimal Investment Portfolios

	Period 1	*Period 2*	*Period 3*	*Period 4*	*Period 5*	*Period 6*	*Period 7*	*Period 8*	*Period 9*	*Period 10*
Min Sufficiency Level	1,591,549	1,064,347	680,513	445,919	296,149	198,432	129,609	78,977	37,711	0
Required EOP target assets	1,683,785	1,128,672	722,708	471,934	313,611	210,181	137,047	83,439	39,879	0
EAFEU	.067	.073	.146	.208	.009	.144	.011	.063	.015	
INTLUHD	.189	.039	.126	.198	.166	.134	.154	.226	.036	
S&P5	.163	.023	.0	.147	.063	.0	.084	.082	.039	
USTB	.067	.465	.063	.167	.341	.141	.199	.058	.559	
R_MID	.050	.0	.097	.083	.034	.040	.023	.119	.023	
HIYLD	.122	.148	.002	.004	.0	.012	.179	.098	.159	
CONV	.036	.055	.131	.033	.067	.140	.061	.041	.003	
LBCORP	.032	.068	.023	.032	.046	.033	.025	.105	.0	
LBGVT	.123	.011	.325	.116	.096	.205	.107	.046	.148	
LBMBS	.151	.117	.088	.011	.179	.152	.157	.164	.017	
Expected Return	1,770,939	1,162,538	759,151	499,672	323,787	219,901	142,558	87,867	40,883	
Standard Deviation	105,186	40,051	44,333	33,730	11,656	11,316	6,513	5,284	1,176	
0.1 Percentile	1,635,357	1,112,691	704,420	457,192	308,825	205,665	134,405	81,043	39,420	
0.2 Percentile	1,683,802	1,128,682	722,719	471,996	313,612	210,183	137,059	83,447	39,880	
0.25 Percentile	1,699,140	1,135,369	729,442	478,066	315,443	211,864	138,069	84,296	40,067	
0.5 Percentile	1,767,097	1,160,296	756,702	498,949	323,608	219,678	142,282	87,783	40,827	
0.75 Percentile	1,842,846	1,189,563	787,478	520,928	331,522	227,456	146,862	91,338	41,673	
0.8 Percentile	1,859,232	1,197,558	795,862	526,399	333,689	229,322	148,179	92,295	41,892	
0.9 Percentile	1,908,091	1,215,841	816,402	542,054	338,831	234,158	151,193	94,726	42,410	
Sharpe ratio	.712	.697	.759	.727	.688	.740	.682	.697	.580	

Effect of Using Line-of-Business Correlated Links

The bootstrap method used in this study produced a correlation matrix for each link factor. The correlations were small and very often insignificant. This can be seen in the correlation matrix for one of the links shown in Table 1: Statistics for $Link_1$. However, the measured correlations were more often positive than negative. For example, Spcl_Liab and OL_OCC have positive correlation with Reins_A exceeding 0.07. In general, small positive correlations were found for many other lines and for other development periods. It is reasonable to characterize the ultimate links for lines of business in this study as being generally uncorrelated, but occasionally having isolated pockets of positively correlated loss among certain lines.

The experimental results were recast so that line-of-business independence was assumed. The same expected values and variances were used, but the multinormal link ratios were simulated with a zero correlations among the lines. The minimum sufficiency levels were calculated using the same investment scenarios. So, the only difference in treatment was the removal of the generally slight positive correlation. The results appear in Table 5: Effects of Removal of By-Line Correlation. This table should be compared with Table 4: Statistics for Reserve Minimum Sufficiency Levels and Optimal Investment Portfolios.

Table 5: Effects of Removal of By-Line Correlation

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9	Period 10
Min Sufficiency Level	1,586,858	1,067,462	682,914	444,120	295,781	198,493	129,681	78,428	37,512	0
Required EOP target assets	1,680,031	1,129,561	723,563	471,676	313,792	210,037	137,006	83,388	39,792	0
EAFEU	.027	.226	.251	.111	.082	.172	.092	.060	.054	
INTLUHD	.139	.124	.071	.018	.112	.142	.146	.036	.118	
S&P5	.012	.156	.025	.059	.050	.069	.038	.003	.041	
USTB	.002	.007	.057	.402	.452	.004	.117	.738	.371	
R_MID	.142	.087	.097	.0	.090	.014	.011	.020	.034	
HIYLD	.004	.073	.103	.030	.004	.093	.091	.005	.072	
CONV	.072	.049	.011	.089	.008	.157	.132	.006	.003	
LBCORP	.023	.120	.250	.011	.010	.047	.220	.0	.002	
LBGVT	.001	.124	.099	.087	.006	.148	.104	.079	.212	
LBMBS	.579	.034	.035	.193	.186	.154	.050	.052	.093	
Expected Return	1,761,302	1,206,609	768,268	487,332	324,459	221,731	143,432	84,694	41,001	
Standard Deviation	97,050	93,454	54,729	19,170	12,377	14,050	7,681	1,602	1,453	
0.1 Percentile	1,637,518	1,091,963	700,940	462,841	308,686	204,174	133,796	82,668	39,139	
0.2 Percentile	1,680,084	1,129,562	723,598	471,678	313,795	210,037	137,009	83,389	39,792	
0.25 Percentile	1,693,157	1,141,484	731,828	474,425	315,561	211,698	138,070	83,591	40,029	
0.5 Percentile	1,759,289	1,202,290	765,699	486,883	324,270	221,319	143,077	84,669	40,934	
0.75 Percentile	1,826,979	1,268,604	803,468	499,658	332,806	230,934	148,617	85,766	41,966	
0.8 Percentile	1,843,977	1,285,234	812,933	502,672	334,830	233,406	150,128	86,021	42,200	
0.9 Percentile	1,889,191	1,329,506	837,534	511,794	340,621	239,733	153,711	86,799	42,873	
Sharpe ratio	.724	.735	.735	.735	.734	.721	.682	.683	.697	

The positive correlation seen in this study generally increased the minimum sufficiency levels. Chance-constrained reserves must be higher in the presence of correlation among lines of business. The company should have a higher level of capital attribution for the collection of correlated lines of business than what would be needed were they to be independent. But, the effect for the company in this study was small. For example, the beginning (Period 1) minimum sufficiency level dropped from $1,591,549 to $1,586,858. Comparison of Table 4: Statistics for Reserve Minimum Sufficiency Levels and Optimal Investment Portfolios and Table 5: Effects of Removal of By-Line Correlation discloses that there generally are higher minimum sufficiency levels when correlation in the ultimate links is considered. However, the effect is modest and not always consistent. For example, periods 2,3 and 7 have modestly higher minimum sufficiency levels when independence was assumed among the lines of business.

Caveats Regarding this Study

Variation among Local Minima

The optimization problem requires non-linear methods. Many combinations of asset weightings are likely to yield the same objective value, and the optimizer will produce varying optima for the same problem. These local optima arise when the optimizer randomly seeds different paths to a solution.

A separate test of the optimization procedures was done to better understand the nature of the local minima. There was variation in the answers produced by the optimizer when all aspects of the problem were held constant except one: the optimizer was seeded in different ways at the start of the optimization by giving it different starting values for the portfolio allocation. Optimum values found by the optimizer are dependent on many empirical properties of the data being optimized—generalizations are difficult.

Variation among local optima of minimum sufficiency levels was studied for two of the periods. The results are summarized in Table 6: Variation in Local Optima.

Table 6: Variation in Local Optima

Values of Optimization	Variation Observed
Objective function (minimum sufficiency level)	Based on 266 replications of the optimization using different seeding for portfolio allocations, the local optima had modest variation.[20]

[20] The period means are higher than reported in other Tables because this test of local optima was based on different confidence levels. But, the overall results are insensitive to the choice of sufficiency probabilities; the variance in local optima is small.

Values of Optimization	Variation Observed
	1 1,805,772 51,733 3 832,612 22,321 From an operational standpoint, the standard deviations are small; sufficiency estimates are substantially unchanged regardless of the local optimum chosen by the optimizer.
Asset allocation weights	There was high volatility in portfolio allocation among the local optima.[21] Even rank shifts among asset category weights were large. In the following table, statistics for rank order of asset appearance in Period 1 results are given. Observe that the mean ranks are very close and standard deviations of the ranks are high. This indicates a high volatility in the ranking of any given asset category among the local optima. Period 1 Asset Category Ranks EAFEU 5.60 2.28 INTLUHD 5.37 2.37 S&P5 5.50

[21] The fact that many different portfolio allocations result in similar objective values was indirectly observed in Scheel, et al [2000]. In that study, efficient frontiers often were found not to be particularly efficient in a forecast sense. Off-frontier points in that study had portfolio weights, which produced results comparable to those of on-frontier points having different weights. A similar insensitivity to portfolio allocation was found in this study, because many different portfolio allocations resulted in local optima that differed by insubstantial amounts.

Values of Optimization	Variation Observed	
		2.69
	USTB	
		4.91
		2.63
	R_MID	
		5.45
		2.80
	HIYLD	
		5.23
		3.08
	CONV	
		5.39
		3.22
	LBCORP	
		5.63
		3.27
	LBGVT	
		6.64
		3.23
	LBMBS	
		5.28
		2.67

The results shown in Table 6: Variation in Local Optima show that at least for this study, the variation in the minimum sufficiency levels is small; the local optima appear to be clustered. However, there were numerous portfolio profiles with some rank order stability but still considerable differences in weight magnitudes. Although the optimization methods seem to be reasonably robust from an operational standpoint, this may not generally be true for other empirical datasets.

Other Assumptions and Limitations

Because this study was focused on methodology and not on precision of the actual valuations of reserves, some shortcuts were made. The following limitations may be important if greater precision is desired:

1. No adjustments for uncertainty respecting inflation were made. In fact, no business scenario generation, other than investment returns, was done in this analysis.

2. The possibility of future inflation differing from expected inflation was not considered. There was no common economic tie binding future loss projections and future asset valuations.
3. Tax frictions were not analyzed.
4. The finesse of actuarial reserving involves many considerations such as actuarial judgment, the appropriate reserving model to be used with aggregate loss triangle information, and many other loss reserve details.
5. The reserving method deployed was a simple average chain ladder approach; some actuaries may think it naive, but other reserving methods could be applied using the approach laid out here.
6. Claim frequency was not studied.
7. The implicit assumption is that the proxy asset portfolio used in this study is a reasonable representation of assets used to back reserves. Other asset proxies and approaches to investment scenario generation might yield materially different results.
8. The optimizer constraint set did not limit the extent of portfolio allocations for specific asset classes such as equities or international securities within the proxy portfolio. In general, the mix of assets in the optimal portfolios was under 20 percent equities. Allocations to mortgaged backed securities got as high as 18 percent. These and some allocations to convertible and high yield bonds may not be consistent with statutory limitations on such asset classes. These asset class-specific constraints were deemed to be beyond the scope of this study; however, appropriate changes could be made in the constraints set to limit the asset allocation weight for a class of investments.

Appendix A: Review of Data Sources

Liabilities

The source of financial information used for this paper is the data provided to participants in the CAS Call Paper program for the 2001 DFA Seminar. The data consisted primarily of Schedule P information.

Proxy Assets

This paper uses monthly time series of asset class total returns. A selection of broad asset classes typical of P&C insurance company asset portfolios was chosen for examination. The time series all begin January 1, 1970. However, certain asset classes (e.g., mortgage backed securities) do not have a history that extends back this far. For these classes the time series were backfilled to the January 1, 1970 start date by an investment consultant. The backfill process was based on a consideration of the market conditions of the time (e.g. interest rates, fixed income spreads, inflation expectations) and how the particular sector would have performed given those market conditions.

Table 6 Asset Components

Class	Code	Source	Start Date
International Equities	EAFEU	MSCI EAFE Index	1/1970
International Fixed Income	INTLHDG	JP Morgan Non-US Traded Index	1/1970
Large Cap Domestic Equities	S&P5	S&P 500 Index	1/1970
Cash	USTB	90 Day US Treasury Bill	1/1970
Mid Cap Domestic Equities	RMID	S&P Mid Cap 400 Index	1/1982
Mid Cap Domestic Equities	RMID	S&P Mid Cap 400 Index	1/1982
High Yield	HIYLD	CSFB High Yield Bond Index	1/1986

Class	Code	Source	Start Date
International Equities	EAFEU	MSCI EAFE Index	1/1970
Convertible Securities	CONV	CSFB Convertible Index	1/1982
Corporate Bonds	LBCORP	Lehman Brothers Corporate Bond Index	1/1973
Government Bonds	LBGOVT	Lehman Brothers Government Bond Index	1/1973
Mortgage Backed Securities	LBMBS	Lehman Brothers Mortgage Backed Securities Index	1/1986

Appendix B: Example of Asset Scenarios for an Annual Period

Specimen Annualized Returns

Scenario	EAFEU	INTLHDG	S&P5	USTB	R_MID	HIYLD	CONV	LBCORP	LBGVT	LBMBS
1	-0.0027	0.0623	-0.0158	0.0802	0.0022	-0.0290	0.0297	0.0129	0.0757	0.0647
2	0.1580	-0.1165	0.1401	0.0621	0.1595	0.1326	0.1681	0.0600	0.0645	0.0513
3	0.3420	0.0894	0.2213	0.0707	0.1533	0.1743	0.1572	0.1284	0.1234	0.1386
4	-0.1184	0.0985	-0.1215	0.0656	-0.0601	0.0159	-0.0607	0.0657	0.0823	0.0778
5	0.2593	0.0544	0.2091	0.0625	0.2001	0.2195	0.1990	0.1681	0.1452	0.1144
6	0.0529	0.1815	-0.0815	0.0640	-0.1208	-0.0084	-0.0478	-0.0194	0.0714	0.0716
7	0.0352	0.0257	0.4263	0.0561	0.3197	0.1684	0.2330	0.0785	0.0928	0.1219
8	-0.0263	-0.0213	0.1301	0.0654	0.1293	0.2067	0.1132	0.1714	0.1567	0.1576
9	0.0622	0.0899	-0.0052	0.0630	-0.0161	0.0749	0.0622	0.0689	0.0691	0.0900
10	-0.1113	0.0308	-0.0285	0.0687	-0.0737	-0.0644	-0.0015	-0.0181	0.0659	0.0232
11	0.4261	0.0867	0.3720	0.0594	0.3460	0.1973	0.2650	0.1624	0.1031	0.1247
12	0.3469	0.0412	0.2036	0.0610	0.2292	0.2230	0.1441	0.1319	0.0908	0.1537
13	0.1085	0.0008	0.0505	0.0662	0.1505	-0.0178	0.0839	-0.0024	0.0036	0.0287
14	0.1889	0.1957	0.0605	0.0611	-0.0622	0.1085	-0.0074	0.2271	0.2147	0.1889
15	0.7624	0.1837	0.4632	0.0765	0.6217	0.2408	0.3528	0.2786	0.2308	0.1944
16	0.4232	0.1025	0.0522	0.0760	-0.0179	-0.0002	0.0297	0.1637	0.1714	0.1520
17	0.2868	0.1125	0.1013	0.0718	0.0766	0.1287	0.1499	0.0728	0.0873	0.1445
18	0.2166	0.0479	0.3057	0.0728	0.2180	0.0654	0.1420	0.0621	0.1379	0.0866
19	-0.0120	0.0196	0.2636	0.0705	0.2442	0.0827	0.1536	0.0235	0.0456	0.0105
20	0.2230	0.0416	0.0891	0.0515	0.1085	0.2218	0.1033	0.1687	0.1087	0.0765
21	-0.0172	-0.0164	-0.0093	0.0747	0.0470	0.0555	0.0214	-0.0214	0.0483	0.0812
22	-0.0505	0.1184	0.0768	0.0765	0.0528	0.0757	0.0090	0.1442	0.1424	0.1624
23	0.0845	-0.0059	0.0561	0.0692	0.0730	-0.0704	0.0608	0.0045	0.0513	0.0376
24	0.2747	0.1734	0.2725	0.0582	0.2774	0.0052	0.2158	0.0623	0.1011	0.0947
25	0.1503	0.1671	-0.0037	0.0650	0.0446	0.0852	0.0275	0.1147	0.1007	0.0244

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Valuing An Insurance Enterprise

William C. Scheel

Valuing An Insurance Enterprise

by
William C. Scheel[1]

Abstract

The valuation methodology described in this paper follows from minimum sufficiency levels for reserves. The valuation is risk adjusted both for uncertainty in claims payments and uncertainty in investments. Attribution of capital is inherent in the method of determining minimum sufficiency levels. Value of an enterprise is seen as consisting of two parts: (1) current asset levels beyond what is required for minimum reserve sufficiency; and (2) capital release that is expected by virtue of the chance-constrained properties of the conservative minimum sufficiency levels. The valuation of an insurance enterprise in a runoff mode seeks to know the capital required to support the runoff of the enterprise and the probability distribution of the release of excess capital back to shareholders for each of the forecast periods. Because the approach relies on bootstrap methods, there is no explicit measurement of either process or parameter risk that ordinarily appears in dynamic financial analysis.

Introduction

The *Minimum Sufficiency Level* is defined to be the level of assets necessary to fund future claims payments with a specified level of confidence [Scheel, 2001]. This level is risk adjusted both with respect to uncertainty in the stream of future claims payments and uncertainty in the returns on assets needed to fund those claims payments. Risk adjustment is in the form of chance-constrained confidence; confidence that investments will grow to a target minimum sufficiency level and that the target will be sufficient to immunize future claims payments.

This paper describes a valuation procedure based on minimum sufficiency levels. It seeks to establish:

1. The valuation of the insurance enterprise.
2. Capital requirements for a runoff of the enterprise.
3. The probability distribution of capital release for each of the forecast periods.

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[1] William C. Scheel, Ph.D., is President of DFA Technologies, LLC. This paper was submitted in response to the 2001 Call for Papers, Dynamic Financial Analysis, a Case Study. A companion paper entitled "Reserve Estimates Using Bootstrapped Statutory Loss Information" also was submitted. The author gratefully acknowledges the wisdom of both William J. Blatcher and Gerald S. Kirschner in correcting several of the author's mental blocks in deriving this paper.

The difference between the market value of assets and the minimum sufficiency level is excess value beyond what is required for claims against the enterprise. The minimum sufficiency level of assets contains capital attribution; it is the risk-adjusted amount that will fund future claims obligations.[2] We refer to this difference between market value and minimum sufficiency level as the *current excess value*. Remaining value lies in the difference between asset accrual and claims payments over time; this value is referred to in this paper as capital release. Future capital release is a random variable and only can be measured with a probability distribution. We can speak of chance-constrained values—values that confidently lie under a threshold or within a range.

Both the nature of the distribution of capital release and how it may change over time are the foundations of enterprise valuation.

Aquisition Value

Table 1: Current Excess Value describes the initial valuation of the enterprise.[3]

Table 1: Current Excess Value

Source of Value	*Amount*
Current market value	5,534,719
Less:	
Current min sufficiency level	1,591,549
Current ultimate loss for lines not analyzed	2,565
Net Current excess value	3,940,605
PV E(capital release) (@.05)	297,109

[2] The concept of capital attribution used in this paper avoids the accounting distinction between liabilities and earmarked surplus. The author rejects the concept that equity that has been segregated is still equity; rather, it is a liability in the sense that the real liability has been misstated during the course of accounting ministrations. Whether there are different legal attributes afforded liabilities and segregated surplus is irrelevant to this paper. This paper is concerned with risk-adjusted measures of future obligations. The author refers to them as liabilities even if an accountant does not.

[3] The data used in this paper were provided to authors participating in the 2001 DFA Seminar Call Papers contest held by the Casualty Actuarial Society. They include Schedule P information for a hypothetical insurance company and other financial statements.

The table indicates that a large proportion of current value is excess; it could be distributed to stockholders as a dividend without impairment to the enterprise. The minimum sufficiency level is risk-adjusted and will provide for future obligations. We now turn to a discussion about what this minimum sufficiency level is and how it is determined.

Distribution of Minimum Sufficiency Levels

The minimum sufficiency level shown in Table 1: Current Excess Value is the amount, which with a confidence level of 0.9, will grow through investments and be sufficient to cover future claims payments.[4] This study assumes that investment returns are described by a multinormal distribution.[5] The investment data are those used in one of the author's prior articles [Scheel, et al, 2000]; they are summarized in Appendix A: Review of Data Sources. The proxy investment choices cover a broad range, including fixed obligations, collateralized mortgage obligations, foreign and domestic bonds and equities.

A full description of how the distribution of minimum sufficiency levels was determined appears in Scheel [2001]. A brief summary is given here. Non-linear optimization methods were used to evaluate portfolio weights and determine a reserve that is risk-adjusted both for uncertainty in claims amounts and uncertainty in investments; this reserve is the minimum sufficiency level. Managerial decision-making established an acceptable level of confidence in a probabilistic sense. Within these levels of confidence, the minimum sufficiency level of assets will grow to a target amount that will both fund claims for the period and immunize (within a specified confidence level) the company both from fluctuations in investment return and remaining claims. The minimum sufficiency level is a risk-adjusted reserve that contains capital attribution. Additional capital is needed only to assure margins beyond those already built into the minimum sufficiency level, or for other risk-bearing purposes.[6]

Targets for Sufficiency

Targets for the required growth levels were obtained from simulations of correlated link ratios for ultimate losses and payment patterns.[7] They were applied to current loss triangle diagonals to provide estimates of: (a) changes in ultimate loss, and (b) the relationship between paid and ultimate loss for each accident period during the forecast development periods. The statistical foundation for the simulation was the use of bootstrap methods applied to loss triangles.

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[4] Detailed descriptions of how sufficiency levels were calculated appear in the companion paper [Scheel, 2001].

[5] The hypothetical insurance company has invested assets but their efficacy was not examined in this paper. Rather, the analysis considers current assets to have been rebalanced into the proxy portfolio. The paper describes in detail how an optimal portfolio was constructed and rebalanced over time using the rich set of securities in the proxy portfolio.

[6] This study did not include all joint costs associated with claims and, therefore, overstates net current excess value. Only claims costs included in Schedule P paid losses have been considered.

[7] Correlation among lines of business was considered for the determination of each link. However, correlation among different development periods either within a line of business or among lines of business was not considered.

The initial step involved multivariate bootstrapping using the link ratios for ultimate and paid loss triangles. This bootstrap was done in multivariate fashion to measure correlation among lines of business in ultimate links. The next step used the estimates of means and covariances obtained from the bootstrapping in a multinormal simulation. This simulation produced ultimate link ratios for forecasting changes in ultimate loss. Then, a secondary simulation elicited the speed of claims payment. These simulations were the source of cash flow during the forecast period.[8]

The forecast period for paid loss cash flow extended ten years; these paid losses were discounted at a risk-free rate. Many scenarios were derived for loss payment cash flow. These were discounted and the result was a probability distribution of end-of-period target sufficiency levels. A chance-constrained target was measured with this probability distribution; it is referred to as the *target sufficiency level.*

The target sufficiency level was the upper confidence level associated with the .8 percentile of the present value of future claims cash flow. In this case, the upper .9 confidence point of the percentile was used.[9] Other risk tolerances would lead to different levels; but the fundamental approach taken to firm valuation would be unchanged.

The target sufficiency level is similar to a conventional GAAP reserve calculation because cash flow is discounted.[10] A conservative interest rate was used in the discounting of future claims obligations. Cash flow measurement for paid losses followed from simulations of paid-loss/ultimate ratios. First, link ratios were simulated for transition in ultimate loss estimates across calendar periods. Then, payments were generated based on the simulated ultimate loss.

Sufficiency Levels for Investments

The second phase of the reserve determination is the translation of the target (end-of-period) sufficiency level into a beginning-of-period sum required for investment. This sum is risk-adjusted for investment uncertainty. It is the *minimum sufficiency level*, and non-linear optimization methods are used to calculate it. It is an invested amount that grows with income to the target sufficiency level within a managerial-selected confidence level.

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[8] This study did not deal with unearned premiums or any other accrual items. The only source of cash flow was assumed to be claims payments as they are reflected in estimates of ultimate loss.

[9] The percentile is binomially distributed. Its confidence band is a function of sample size. Simulations used in this paper were always at least 2,500 iterations so the normal approximation could be used to evaluate the confidence band for percentiles. See John C. Freund, *Mathematical Statistics*, 1971, Prentice-Hall, Inc., p. 276.

[10] Reserves are not always discounted for GAAP. In fact, the GAAP rules can be interpreted as either allowing or not allowing discounting. But, were discounting to be demanded, the target sufficiency levels are an abstraction from the probability distribution of GAAP reserves.

During the second phase, investment scenarios were generated using an asset model.[11] During a trial solution, a profile of weights is tested by the optimizer. The trial profile is applied to every simulated scenario to ascertain a portfolio return for it. The asset weights in the profile were constrained by the optimizer to eliminate the possibility of short sales.[12]

The simulated portfolio return is a growth factor for the invested sum, the minimum sufficiency level. In a converse manner, it can be used as a discount rate to convert the target sufficiency level into the required minimum sufficiency level. The present value is the minimum sufficiency amount that is a required; it is a beginning-of-period amount. It leads to the end-of-period target level within prescribed confidence levels. The distribution of these present values is obtained for all asset scenarios using the trial profile provided by the optimizer. A chance-constrained limit of this distribution was returned to the optimizer as an objective value.

The optimizer repeats the process with different sets of proxy investment weights until the objective function is found to be a minimum. In summary, the optimizer minimized the invested sum need to provide risk-adjusted growth to a target sufficiency level. The objective function for the optimizer is a confidence level of the probability distribution of the discounted value of the target sufficiency level. There is a simulated set of returns for all asset categories, and apportionment among them is given by the optimizer. Because (a) we know the end-of-period target and (b) we have a simulated set of portfolio returns, the discounted beginning-of-period invested amount can be ascertained for any desired confidence level.[13] We refer to this risk-adjusted reserve as the *minimum sufficiency level.*

Capital Attribution

The minimum sufficiency levels contain attribution of capital. The release of that capital is of interest because it can be a source of future stockholder dividends.

Capital may be released when minimum sufficiency levels grow in an ***expected*** fashion that leads to sums greater than the target sufficiency level. There is an additional expected capital release because expected claims are less than the conservative target sufficiency level that immunizes the company.

Capital release during period t is defined by (1.1).

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[11] The choice of an asset model was not particularly important for this paper; any asset model that produces investment scenarios for a broad spectrum of securities could work. The one used here was a multi-variate normal simulation. Of course, different models for investment and claims scenario generation would produce different valuations.

[12] Investments were not constrained to limits imposed by regulation. For example, no constraints were set on the proportion of assets invested in equities. These and other similar limitations could be added to the constraints used by the optimizer.

[13] It was assumed that claims were incident at the end of each period. In this analysis, periods were calendar years.

$$SR_t = MSL_t(1 + p_t) - (C_t + MSL_{t+1}) \qquad (1.1)$$

where:

SR_t = capital released at the end of period t,

MSL_t = minimum sufficiency level at the beginning of period t,

p_t = portfolio return during period t,

C_t = claims payments during period t.

Capital release, SR_t, is a random variable. We now turn to procedures used to estimate the probability distribution for capital release each period.

Distribution of Capital Release

The capital release random variable is a function of two other random variables that are independent: investment growth, p_t, and paid losses, C_t. The distribution of each of them is found through modeling. The asset model provides scenarios for invested assets. The liability model provides scenarios for losses. The minimum sufficiency levels are determined using both the investment scenario and the paid loss generators.

The steps are:

1. Randomly generate an investment scenario for the period. Assume the investment portfolio backing the minimum sufficiency level is apportioned according to the optimized profile used to measure it. Determine the period's return, p_t, for the weighted portfolio.
2. The result of step 1 is the growth rate of minimum sufficiency assets. This end-of-period value, $MSL_t(1 + p_t)$, is used to pay the period's claims[14] and fund next period's minimum sufficiency level.
3. Paid claims for the period, C_t, are obtained from the liability simulator. A payments scenario is randomly generated.[15]
4. The results of steps (1)-(3) are used in equation (1.1) to calculate SR_t, a simulated observation of capital release.
5. Steps (1)-(4) are repeated many times to build up the distribution of capital release for the t^{th} period.
6. This method is extended through remaining periods of the analysis.

[14] This study simplifies the analysis by assuming claim payments occur at the end of the period.

[15] The loss generator could be any of the conventional ones deployed in popular dynamic financial analysis models. But, it should the same one used to evaluated the minimum sufficiency levels. The one used in this study relies on bootstrapping and involves no other data than published information. See Scheel [2000] Because it relies on bootstrap methods, there is no explicit measurement of either process or parameter risk that ordinarily appears in dynamic financial analysis

The results are shown in Table 2: Distribution of Capital Release. The discounted present value of the expected capital release is one of the items shown in Table 1: Current Excess Value.

Table 2: Distribution of Capital Release

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9
Mean	133,441	59,222	51,999	39,575	17,900	15,222	8,490	6,766	4,395
Standard Deviation	113,283	45,983	48,344	36,444	13,694	12,471	7,220	5,709	2,895
Median	127,131	58,012	50,623	37,486	17,596	14,711	8,308	6,558	4,360
5 percentile	-42,909	-15,214	-23,636	-16,731	-4,230	-4,442	-3,078	-1,956	-347
10 percentile	-6,434	1,953	-8,050	-4,987	402	-195	-905	-535	678
25 percentile	54,018	27,826	18,292	13,848	8,234	6,438	3,511	2,681	2,431
75 percentile	208,578	89,464	83,842	63,171	27,064	23,719	13,385	10,634	6,361
90 percentile	280,925	118,269	114,969	88,253	35,390	31,348	17,719	14,450	8,200
95 percentile	325,144	135,291	134,078	101,404	41,021	35,717	20,375	16,522	9,103

What is the Source of Expected Capital Release?

This is an interesting question. If we were to hold a minimum sufficiency level equal to the expected value of the runoff,.would there be no expected capital release? What is the foundation for an expectation of capital release?

Ex ante, the minimum sufficiency levels are conservative, chance-constrained values. As defined in this paper, there are two sources of such conservatism: (1) the target sufficiency level is higher than the expected present value of losses and (2) the beginning level assets is higher than the expect level needed to achieve this deferred target. If the first target was based on the expected present value of claims (and, the expected value was a riskless rate of return during the holding period) *and* if the value of assets held were expected to yield this target amount, there would be no *expectation* of capital release.

The expected source of capital release arises from contingency margins both in the target sufficiency levels and in the required assets backing them—the minimum sufficiency levels. Were such levels to be based strictly on expectations, capital release still could occur. But it would arise from fortuitous events—there would be no expectation of capital release. It means that in an expectation sense the minimum sufficiency levels set at fair value (expected) levels have no expectation of being either excessive or deficient. Sufficiency, at least in the context of this paper and the setting of reserves, requires a higher standard. It requires that there be an expectation of capital release. This expectation is the foundation of insurer solidity.

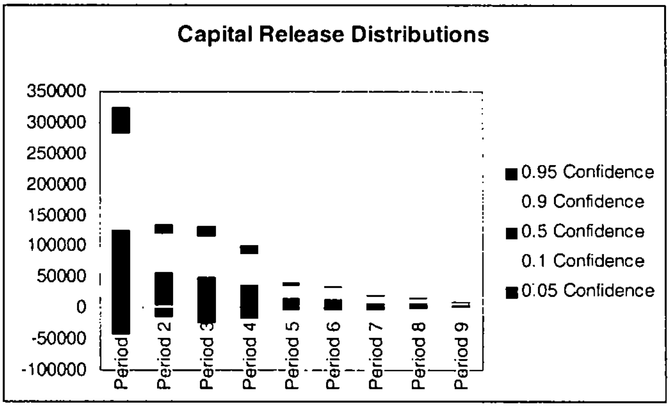
This result becomes clearer if we switch from expected value to median value. Were the distributions to be normally distributed, the mean expectancy and median merge. Under these circumstances, it becomes apparent that either capital release or deficiency has a 50:50 chance of emerging. Neither median-based nor expected value-based estimates seem to be reasonable standards. A regulatory context of sufficiency seeks solidity of the enterprise and the paramount preservation of policyholder interests. This standard imposes conservative chance-constrained levels. It is this conservatism in using high confidence levels that leads to an expectation of capital release.

Value of the Enterprise

The value of the enterprise consists of the current excess value and subsequent capital release. Table 1: Current Excess Value identifies these sources of value. The contribution to enterprise value from future ***expected*** capital release is discounted and added to initial excess value to produce the total net present value. As can be seen in the table, this enterprise value is a high percentage of the current market value of assets.

Capital release will not unfold as expected. Other ways of expressing value are to examine percentiles of the capital release distributions. The capital release percentiles are shown in Figure 1: Value of the Enterprise. Both the Figure and Table show that there is slight positive skewness in the distributions. The median value is less than the expected value for all periods. Because there is a 50:50 chance that capital release will be lower than expected, the valuation of the enterprise might be considered somewhat less. However, risk lovers may see great value in the enterprise if windfall probabilities end up causing some of the high capital release values that lie in the tails of these distributions.

Figure 1: Value of the Enterprise



Caveats Regarding this Study

The following shortcuts were made:

1. Ordinarily a DFA analysis would use the existing assets of an enterprise so that rebalancing could trace their disposition. The specific assets held would have to be modeled as part of the investment scenario generation process. This may require modeling other business climate aggregates that are thought to impact on investment returns for these securities. This study assumes that a rich set of investment aggregates serve as a proxy for the real assets. The focus of this study was on the rebalancing that might be required were this set of investment proxies to be used as actual investments. Maintenance of the existing portfolio or how it might be rebalanced was beyond the scope of this study. The implicit assumption is that all assets, valued at market, could be reinvested immediately in the proxy portfolio used in this study.
2. No adjustments for uncertainty respecting inflation were made. In fact, no business scenario generation, other than investment returns, was done in this analysis.
3. The possibility of future inflation differing from expected inflation was not considered. There was no common economic tie binding future loss projections and future asset valuations.

4. Future business writings were not considered.
5. Tax frictions were not analyzed.
6. Financial statement generation was limited to cash flow analysis. Acquisition and integration of an insurance entity would necessitate the modeling of consolidated statements and many aspects of line-of-business integration were the parent company also an insurer. Consideration of these effects on total risk bearing may have a material impact on valuation.
7. The study did not attempt to harvest uncertainty in non-claims accruals or financial accounts other than claims and investments. The total risk of the enterprise might be materially affected were other sources of uncertainty to be considered.
8. Administration and other expenses were ignored.
9. The optimizer constraint set did not limit the extent of portfolio allocations for specific asset classes such as equities or international securities within the proxy portfolio. In general, the mix of assets in the optimal portfolios was under 20 percent equities. Allocations to mortgaged backed securities got as high as 18 percent. These and some allocations to convertible and high yield bonds may not be consistent with statutory limitations on such asset classes. These asset class-specific constraints were deemed to be beyond the scope of this study; however, appropriate changes could be made in the constraints set to limit the asset allocation weight for a class of investments.

Appendix A: Review of Data Sources

This paper uses monthly time series of asset class total returns. A selection of broad asset classes typical of P&C insurance company asset portfolios was chosen for examination. The time series all begin January 1, 1970. However, certain asset classes (e.g. mortgage backed securities) do not have a history that extends back this far. For these classes the time series were backfilled to the January 1, 1970 start date by an investment consultant. The backfill process was based on a consideration of the market conditions of the time (e.g. interest rates, fixed income spreads, inflation expectations) and how the particular sector would have performed given those market conditions. The Start Date in Table 3 refers to the date historical data begins.

Table 3 Asset Components

Class	Code	Source	Start Date
International Equities	EAFEU	MSCI EAFE Index	1/1970
International Fixed Income	INTLHDG	JP Morgan Non-US Traded Index	1/1970
Large Cap Domestic Equities	S&P5	S&P 500 Index	1/1970
Cash	USTB	90 Day US Treasury Bill	1/1970
Mid Cap Domestic Equities	RMID	S&P Mid Cap 400 Index	1/1982
High Yield	HIYLD	CSFB High Yield Bond Index	1/1986
High Yield	HIYLD	CSFB High Yield Bond Index	1/1986
Convertible Securities	CONV	CSFB Convertible Index	1/1982
Convertible Securities	CONV	CSFB Convertible Index	1/1982
Corporate Bonds	LBCORP	Lehman Brothers Corporate Bond Index	1/1973

Class	Code	Source	Start Date
International Equities	EAFEU	MSCI EAFE Index	1/1970
Government Bonds	LBGOVT	Lehman Brothers Government Bond Index	1/1973
Mortgage Backed Securities	LBMBS	Lehman Brothers Mortgage Backed Securities Index	1/1986

Appendix B: Examples of Capital Release Scenarios

	Minimum Sufficency Level (2)	Target Sufficiency Level (3)	Paid Claims (4)	Optimized Portfolio Return (5)	Simulated EOP Value (6)=(2)*[1+(5)]	Next Period's MSL (7)	Capital Release (8)=(6)-[(4)+(7)]
Period 1							
1	1,591,549	1,683,785	571,004	0.1512	1,832,128	1,064,347	**196,777**
2	1,591,549	1,683,785	550,142	0.1563	1,840,355	1,064,347	**225,866**
3	1,591,549	1,683,785	600,991	0.0353	1,647,748	1,064,347	**-17,590**
4	1,591,549	1,683,785	598,510	0.2693	2,020,096	1,064,347	**357,238**
5	1,591,549	1,683,785	510,979	0.2408	1,974,870	1,064,347	**399,544**
Period 2							
1	1,064,347	1,128,672	448,891	0.0244	1,090,309	680,513	**-39,095**
2	1,064,347	1,128,672	400,138	0.0264	1,092,403	680,513	**11,751**
3	1,064,347	1,128,672	455,988	0.1262	1,198,640	680,513	**62,139**
4	1,064,347	1,128,672	402,754	0.1181	1,190,084	680,513	**106,817**
5	1,064,347	1,128,672	436,252	0.1090	1,180,324	680,513	**63,559**
Period 3							
1	680,513	722,708	225,438	0.0137	689,836	445,919	**18,479**
2	680,513	722,708	233,072	0.0765	732,601	445,919	**53,610**
3	680,513	722,708	281,247	0.0854	738,658	445,919	**11,493**
4	680,513	722,708	273,722	0.0969	746,454	445,919	**26,813**
5	680,513	722,708	292,232	0.1032	750,721	445,919	**12,570**
Period 4							
1	445,919	471,934	156,646	-0.0228	435,732	296,149	**-17,063**
2	445,919	471,934	159,586	0.2777	569,748	296,149	**114,013**
3	445,919	471,934	159,446	0.0511	468,689	296,149	**13,094**
4	445,919	471,934	158,879	0.0589	472,172	296,149	**17,144**
5	445,919	471,934	193,995	0.0627	473,873	296,149	**-16,271**

	Minimum Sufficency Level (2)	Target Sufficiency Level (3)	Paid Claims (4)	Optimized Portfolio Return (5)	Simulated EOP Value (6)=(2)*[1+(5)]	Next Period's MSL (7)	Capital Release (8)=(6)-[(4)+(7)]
Period 5							
1	296,149	313,611	109,267	0 0560	312,726	198,432	**5,028**
2	296,149	313,611	111,052	0 0961	324,596	198,432	**15,113**
3	296,149	313,611	110,854	0 1495	340,437	198,432	**31,152**
4	296,149	313,611	112,634	0 1691	346,232	198,432	**35,165**
5	296,149	313,611	106,550	0 1364	336,550	198,432	**31,568**
Period 6							
1	198,432	210,181	69,061	0 1341	225,036	129,609	**26,366**
2	198,432	210,181	71,070	0 1125	220,760	129,609	**20,082**
3	198,432	210,181	69,934	0 1268	223,588	129,609	**24,046**
4	198,432	210,181	77,007	0 2471	247,473	129,609	**40,857**
5	198,432	210,181	69,786	0 0557	209,484	129,609	**10,089**
Period 7							
1	129,609	137,047	51,764	0 0315	133,689	78,977	**2,949**
2	129,609	137,047	59,632	0 1125	144,189	78,977	**5,581**
3	129,609	137,047	51,172	0 1607	150,434	78,977	**20,285**
4	129,609	137,047	55,113	0 1085	143,669	78,977	**9,579**
5	129,609	137,047	59,218	0 1393	147,666	78,977	**9,471**
Period 8							
1	78,977	83,439	47,517	0 2188	96,254	37,711	**11,026**
2	78,977	83,439	42,505	0 1290	89,162	37,711	**8,946**
3	78,977	83,439	45,269	0 1612	91,705	37,711	**8,725**
4	78,977	83,439	45,651	0 0859	85,763	37,711	**2,400**
5	78,977	83,439	43,289	0 0134	80,037	37,711	**-964**
Period 9							
1	37,711	39,879	32,317	0 0686	40,297	0	**7,980**
2	37,711	39,879	37,275	0 0670	40,239	0	**2,963**
3	37,711	39,879	39,955	0 1242	42,394	0	**2,439**
4	37,711	39,879	37,788	0 0741	40,507	0	**2,719**
5	37,711	39,879	32,851	0 1188	42,192	0	**9,341**

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