An Analysis of the Underwriting Risk for DFA Insurance Company

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by

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Abstract

The DFA Insurance Company (DFAIC) is a fictional insurance company created by the CAS for the 2001 Dynamic Financial Analysis (DFA) Call for Papers. Those who respond to the call are expected to use DFA to answer specific questions about DFAIC's capital adequacy, capital allocation and reinsurance strategy. This paper is a response to that call

Acknowledgment

The author would like to give special thanks to Dr. Eugene Gaydos A DFA analysis of an insurance company such as DFAIC involves a myriad of details that require special attention Dr Gaydos did the underlying analyses that led to the parameterization of the models. He gathered the primary and supplemental data needed for this analysis, he wrote the spreadsheet that produced the final answers, and in many other ways he contributed to the overall success of the project.

Note

The theoretical backing for the methodology in this paper is provided in "The Cost of Financing Insurance" which is also published in this issue of the *CAS Forum*. Excel spreadsheets supporting these papers can be downloaded from the CAS Web Site: http://www.casact.org/pubs/forum/01spforum/meyers/index.htm

1. Introduction

In the mid-1990's the Casualty Actuarial Society coined the term "Dynamic Financial Analysis, " or "DFA" for short. Susan Szkoda [7], in her five part article beginning in the May 1997 *Actuarial Review*, defines DFA as "a process for analyzing the financial condition of an insurance entity. Financial condition refers to the ability of the entity's capital and surplus to adequately support future operations through a currently unknown future environment. ... In a very real sense, DFA requires the actuary to evolve into a financial risk manager."

In the ensuing years, the CAS has sponsored a number of special interest seminars, call paper programs, and research projects on DFA. Initially, those activities dealt with developing a model of insurance companies and getting the right data to support the model. As time passed, there was more focus on the specific insurer problems that DFA can solve. Some of those problems are in the 2001 CAS call for papers titled "Dynamic Financial Analysis, A Case Study." The call for papers presents participants with a specific actuarial situation, including a company description and financial statements. This paper is a response to the call.

Here, verbatim, is the description of the company and the specific actuarial situation provided by the call.

- ***Description of the Situation:***

The CEO of your company is considering the acquisition of DFAInsurance Company (DFAIC or the Company) as a stand-alone insurer. DFAIC is a privately held company and has not yet been contacted about this interest, and cannot be contacted until after your analysis is concluded. However, publicly available financial statements for the Company are available for the 1999 year and they are attached. The Company's last insurance department examination occurred in 1996 and there were no material issues. The Company has an unqualified actuarial opinion.

- ***Description of the Company's Business:***

The Company has an "A" rating from A.M. Best and it has maintained this rating for at least the past five years. It operates through the independent agency system and believes it has very strong relationships with its agency plant.

- ***Underwriting Profile***

The Company is licensed in all 50 states, but is primarily concentrated in the Northeast and the Midwest. The Company considers itself a "regional" company in these two geographic areas. Because of this focus, the Company has limited exposure to severe catastrophes. However, it does have exposure to less severe but more frequent retained catastrophe losses.

The Company writes a balanced book of both personal and main street commercial insurance coverages.

The Company has minimal exposure to asbestos and environmental exposures.

- ***Asset Classes***

The Company's cash and invested asset portfolio is approximately 70% fixed income, 12% equity and 18% cash.

The fixed income portfolio is approximately 80% in tax-exempt municipal issues and 20% in a mixture of Corporate and Government bonds. The Municipals have an average maturity of 10.5 years and an average yield of 6%. The Corporate and Government bonds have an average maturity of 4 years and an average yield of 8%. The equity portfolio is invested with a target return of the S&P 500.

- ***Reinsurance***

The Company maintains reinsurance to limit shock and catastrophic losses from a single event. The largest net aggregate amount insured in any one risk (excluding Workers Compensation) is $1 million. Excess of loss is used to protect property risks above $1,000,000 up to $20 million per risk, $50 million per occurrence. For casualty and Workers Compensation risks, an excess of loss treaty provides coverage above $500,000 up to $50.5 million.

The Company has a catastrophe cover of 90% of $150 million excess of $50 million for any single event. This limits the Company's net pre-tax PML for a catastrophe over a 100 year return period to 10% of surplus.

All of the Company's reinsurers are rated "A," or better, and there are no known problems with reinsurance recoverable.

- ***Questions the CEO would like addressed:***

1. Is the Company adequately capitalized? Is there excess capital? How much capital should the Company hold as a stand-alone insurer?
2. How should the capital be allocated to line of business?
3. What is the return distribution for each line of business and is it consistent with the risk for the line?
4. Should the Company buy more or less reinsurance? What type? How efficient is its current reinsurance program?
5. How efficient is the asset allocation?

2. Outline of the Analysis

The analysis will proceed in the following steps.

Section 3 describes how we calculated the aggregate loss distribution from its component claim severity and claim count distributions. With the aggregate loss distribution, we will then discuss the adequacy of DFAIC's capital.

Section 4 gives the capital allocations by line of business. We will also allocate capital to support outstanding losses from prior accident years. We will use these capital allocations to calculate the cost of financing for the individual lines of insurance.

Section 5 will use the results of Section 4 to calculate the cost of financing for the individual lines of insurance with the current reinsurance program. For the sake of comparison, we will calculate the cost of financing with alternative reinsurance programs, including the program of no reinsurance. We will then recommend a reinsurance program.

Section 6 will use the results of Section 5 to calculate target combined ratios that, if obtained, will lead DFAIC to make its target return on capital.

Included with this paper is a spreadsheet that takes the capital allocations described in Section 4 and derives the results in Sections 5 and 6. The spreadsheet will allow the reader to modify many of the assumptions made in Sections 5 and 6.

This paper focuses on DFAIC's underwriting risk. We will not attempt to quantify its asset risk or make any recommendations on how DFAIC should alter its investment strategy.

This paper will describe the capital measurement and allocation methodology in a "how-to-do-it" mode. Readers who desire a fuller description of this paper's methodology, including its economic rationale, should first read Meyers [4].

3. Capital Adequacy

The first step in evaluating an insurer's capital adequacy is to determine its aggregate loss distribution. The aggregate loss distribution can be thought of as a set of loss scenarios, where a "loss" is the sum of all the individual line of insurance losses from: (1) all claims from the current accident year; and (2) unsettled claims from prior accident years.

The following simulation algorithm explains our model of DFAIC's losses. Explanatory notes follow the description of the simulation algorithm.

Simulation Algorithm to Generate Loss Scenarios for DFAIC

Step

1. Select a random β from a distribution with $E[1/\beta] = 1$ and $Var[1/\beta] = b$.
2. For each covariance group, i, select random percentile p_i from a uniform (0,1) distribution.
3. For each covariance group, i, line of insurance, h, and accident year, y, with uncertain claim payments, do the following:
   - Select $\alpha_{ihy} = p_i^{th}$ percentile of a distribution with $E[\alpha_{ihy}] = 1$ and $Var[\alpha_{ihy}] = g_{ihy}$.
   - Select random claim count, K_{ihy} from a distribution with mean $\alpha_{ihy}\lambda_{ihy}$ where λ_{ihy} is the expected claim count for line of insurance h and accident year y in covariance group i.
   - For each i, h, and y, select random claim size, Z_{ihyk}, for $k = 1,\ldots,K_{ihy}$.
4. Set $X_{ihy} = \sum_{k=1}^{K_{ihy}} Z_{ihyk}$ = Loss for covariance group i, line h, and accident year y.
5. Set $X = \sum_{i} \sum_{h \in G_i} \sum_{y} X_{ihy} / \beta$ = Loss for DFAIC.

Notes on the Simulation Algorithm

- β has an inverse gamma distribution, as originally described by Heckman and Meyers [3]. The variance, b, is called the mixing parameter. b describes the uncertainty in future claim severity. As described in Meyers [4] the random multiplier, β, causes correlation between the lines of insurance.

- The various lines of insurance are classified into "covariance groups." The lines of insurance within each covariance group are those that we expect to move together over time. Table 3.1 below, gives the assignment of lines of insurance to covariance groups.

- By selecting the parameter $\alpha_{ihy} = p_i^{th}$ percentile of a distribution with $E[\alpha_{ihy}] = 1$ and $Var[\alpha_{ihy}] = g_{ihy}$ we are making "high" or "low" claim counts in all lines of a covariance group simultaneously.

- We based the selection of the parameters, g_{ih_y}, on an analysis of the data of several insurers that report their data to ISO. We used the estimation methodology described in Meyers [5]. Although the results based on this data had the greatest influence on the final parameter selections, data from Schedule P of insurer annual statements provided supplementary information.

- For most lines of business, we derived the claim severity distributions from data reported to ISO.

- We obtained a workers compensation size of loss distribution from an independent state rating bureau. Using (1) claim payout patterns; (2) aggregate loss payout patterns; and (3) the general intuition that later-settling claims are also larger claims, we were able to select size of loss distributions for the current and prior accident years that were consistent with the available data. We used this size of loss distribution for all states.

- We used a catastrophe model to generate a hurricane size-of-loss distribution. The call for papers did not give the necessary exposure information to run a cat model, but we have done analyses on insurer catastrophe exposure. See Insurance Services Office [2] for the complete analysis. We selected the catastrophe size-of-loss distribution from an insurer that has a similar geographic distribution to DFAIC. We made a scaling adjustment so that the 100-year loss was close to 10% of DFAIC's capital, as specified in the call for papers.

- We obtained the expected total losses by estimating the average loss ratio, projecting premium to the year 2000, and then multiplying the projected premium times the average loss ratio.

- We used a negative binomial distribution to describe the claim count distribution. We obtained the expected claim count dividing the expected loss by the expected claim severity. As described in Meyers [5], the same methodology that yields estimates for the g_{ih_y} parameters also gives the variance parameters of the claim count distributions.

- *In spite of the loss model's description as a simulation process, we did not use* simulation to calculate the aggregate loss statistics described below. Instead, we used Fourier inversion, as described by Heckman and Meyers [3] and Meyers [6]. The aggregate loss statistics calculated by the Fourier methodology are identical to what we would expect to obtain by simulation if we repeat the simulation several thousand times. The advantage of the Fourier methodology is that DFAIC's aggregate loss distributions can be calculated in a few seconds on current personal computers. Our loss model for DFAIC has 50 different line/accident year segments. In the analysis below, we need to calculate the marginal cost of capital by removing each *line/accident year segment from DFAIC and calculating the aggregate loss* distribution for the remaining losses. We do the calculation for each reinsurance strategy. The very fast calculation made possible by the Fourier methods is what makes this kind of analysis operationally possible.

Table 3.1

DFAIC Aggregate Loss Model Input

Line of Insurance	Covariance Group	Prior Accident Years	Source of Size-of-Loss Distribution Data
Property Catastrophe	1	0	Catastrophe Model
Allied Lines	2	1	ISO Basic Group 2 Commercial Property
Fire	2	1	ISO Basic Group 1 Commercial Property
Homeowners	2	4	Mixture of ISO HO property and liability
Commercial Auto	3	4	ISO Countrywide Commercial Auto Liability
Private Passenger Auto	3	6	ISO Countrywide Private Passenger Automobile Liability
Auto Physical Damage	3	1	ISO Countrywide Auto Physical Damage – Mixture of personal and commercial
General Liability	4	6	ISO Premises/Operations Liability
Products Liability	4	6	ISO Countrywide Products Liability
Commercial MultiPeril	4	6	Mixture of ISO Countrywide Premises/ Operations and Commercial Property
Workers Compensation	5	4	Independent State Rating Bureau

Our analysis of DFAIC's aggregate loss distribution did not include all lines of insurance. Table 3.2 contrasts the percentage of written premium for the included and excluded lines.

Table 3.2

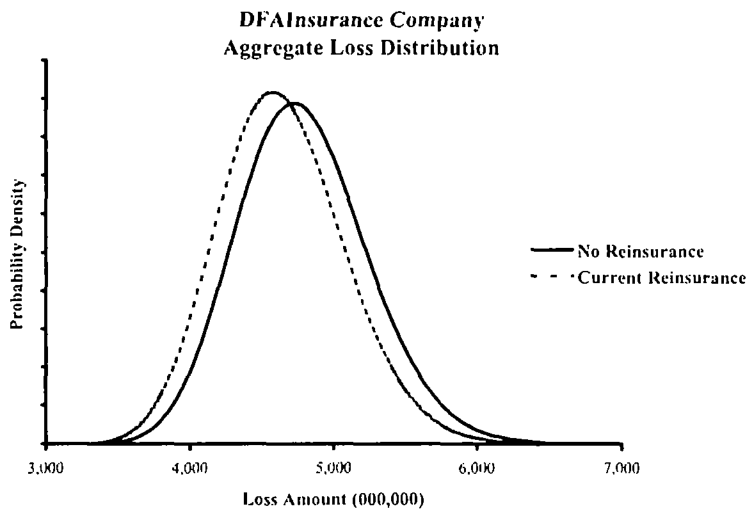
Lines Included/Excluded in DFAIC Aggregate Loss Analysis

Lines Included	%DWP	Lines Excluded	%DWP
Allied*	0.76%	Inland Marine	2.14%
Fire	0.66%	Earthquake	0.04%
Homeowners*	13.77%	Burglary	0.00%
Commmercial Auto Liability	7.01%	Special Liability	
Personal Auto Liability	24.67%	(Ocean Mar, Aircraft, B&M)	0.91%
Auto Physical Damage	22.48%	Other Liability Claims Made	0.03%
Other Liability Occurrence	2.61%	Reinsurance	0.27%
Product Liability Occurrence	0.05%	Fidelity/Surety	0.95%
CMP*	14.26%	Other (Credit, A&H)	0.21%
Workers Compensation	9.18%		
Total	**95.45%**	**Total**	**4.55%**

* A portion of the property losses was allocated to catastrophes.

We calculated the aggregate loss distribution for the current reinsurance strategy and for no reinsurance. Chart 3.1 shows the resulting probability density functions for each aggregate loss distribution.

Chart 3.1



Tables 3.3 and 3.4 give some loss statistics and various percentiles of the aggregate loss distributions with and without reinsurance. Tables 3.3 and 3.4 also include a recently developed measure of risk called the Tail Value at Risk. The Tail Value at Risk (TVaR) is a member of a class of "Coherent Measures of Risk," developed in a paper by Philippe Artzner, Freddy Delbaen, Jean-Marc Eber and David Heath [1]. Meyers [4] further describes this measure.

To calculate the TVaR, first select an α-value such as 99%. Then calculate the α^{th} percentile, otherwise known as the Value at Risk (VaR_α), of the insurer's aggregate loss distribution. The $TVaR_\alpha$ is the average of all the aggregate losses greater than VaR_α.

Following Meyers [4], we define the capital needed to support the insurer's losses as:

$$\text{Insurer's Capital}_\alpha = TVaR_\alpha - \text{Insurer's Expected Loss.} \qquad (3.1)$$

Table 3.3

Aggregate Loss Distribution
DFAInsurance Company's Current Reinsurance Strategy

Aggregate Mean	4,670,320,245
Aggregate Standard Deviation	441,528,312

Percentile/ α-Level	Value at Risk (VaR_α)	Tail Value at Risk ($TVaR_\alpha$)	$TVaR_\alpha$ Implied Capital
0.00%	0	4,670,320,245	0
5.00%	3,981,884,307	4,714,308,377	43,988,131
10.00%	4,118,916,712	4,750,885,242	80,564,997
15.00%	4,214,710,884	4,785,122,592	114,802,347
20.00%	4,292,757,132	4,818,268,948	147,948,703
25.00%	4,361,055,273	4,850,985,309	180,665,064
30.00%	4,423,437,646	4,883,724,871	213,404,626
35.00%	4,482,123,634	4,916,856,052	246,535,807
40.00%	4,538,587,601	4,950,719,808	280,399,562
45.00%	4,593,932,131	4,985,665,883	315,345,638
50.00%	4,649,081,266	5,022,083,793	351,763,548
55.00%	4,704,902,848	5,060,437,614	390,117,369
60.00%	4,762,308,463	5,101,313,239	430,992,994
65.00%	4,822,363,037	5,145,491,698	475,171,453
70.00%	4,886,440,632	5,194,074,153	523,753,908
75.00%	4,956,493,675	5,248,716,088	578,395,843
80.00%	5,035,597,008	5,312,116,293	641,796,048
85.00%	5,129,246,166	5,389,199,164	718,878,919
90.00%	5,249,256,922	5,490,689,078	820,368,832
92.50%	5,327,867,949	5,558,560,374	888,240,129
95.00%	5,431,481,121	5,649,433,290	979,113,045
95.50%	5,457,233,034	5,672,240,507	1,001,920,262
96.00%	5,485,511,269	5,697,378,336	1,027,058,091
96.50%	5,516,957,471	5,725,441,319	1,055,121,073
97.00%	5,552,500,699	5,757,291,825	1,086,971,580
97.50%	5,593,562,931	5,794,251,579	1,123,931,334
98.00%	5,642,491,579	5,838,505,673	1,168,185,428
98.50%	5,703,609,532	5,894,086,653	1,223,766,408
99.00%	5,786,406,345	5,969,867,978	1,299,547,733
99.50%	5,920,196,054	6,093,354,285	1,423,034,040
99.90%	6,201,613,212	6,356,439,702	1,686,119,456
99.95%	6,312,968,410	6,461,547,653	1,791,227,407
99.99%	6,553,948,422	6,690,713,812	2,020,393,567

Table 3.4

Aggregate Loss Distribution
DFAInsurance Company without any Reinsurance

Aggregate Mean	4,803,449,179
Aggregate Standard Deviation	451,811,506

Percentile/ α-Level	Value at Risk (VaR_α)	Tail Value at Risk ($TVaR_\alpha$)	$TVaR_\alpha$ Implied Capital	$TVaR_\alpha$ Implied Capital w/Reins	Difference in Capital
0.00%	0	4,803,449,179	0	0	0
5.00%	4,098,518,418	4,848,514,188	45,065,009	43,988,131	1,076,877
10.00%	4,239,075,826	4,885,960,910	82,511,731	80,564,997	1,946,735
15.00%	4,337,267,437	4,921,001,937	117,552,758	114,802,347	2,750,412
20.00%	4,417,229,633	4,954,919,320	151,470,141	147,948,703	3,521,439
25.00%	4,487,179,430	4,988,391,368	184,942,189	180,665,064	4,277,125
30.00%	4,551,051,966	5,021,882,806	218,433,627	213,404,626	5,029,001
35.00%	4,611,125,381	5,055,771,150	252,321,971	246,535,807	5,786,164
40.00%	4,668,912,417	5,090,405,549	286,956,369	280,399,562	6,556,807
45.00%	4,725,543,728	5,126,143,947	322,694,768	315,345,638	7,349,130
50.00%	4,781,966,292	5,163,384,814	359,935,635	351,763,548	8,172,087
55.00%	4,839,068,865	5,202,602,738	399,153,559	390,117,369	9,036,191
60.00%	4,897,784,402	5,244,396,801	440,947,622	430,992,994	9,954,628
65.00%	4,959,202,157	5,289,565,494	486,116,315	475,171,453	10,944,862
70.00%	5,024,727,025	5,339,234,548	535,785,369	523,753,908	12,031,461
75.00%	5,096,354,650	5,395,096,402	591,647,223	578,395,843	13,251,380
80.00%	5,177,227,531	5,459,910,283	656,461,104	641,796,048	14,665,056
85.00%	5,272,961,845	5,538,711,398	735,262,219	718,878,919	16,383,301
90.00%	5,395,633,669	5,642,467,530	839,018,351	820,368,832	18,649,519
92.50%	5,475,984,579	5,711,860,388	908,411,209	888,240,129	20,171,080
95.00%	5,581,892,641	5,804,783,090	1,001,333,911	979,113,045	22,220,866
95.50%	5,608,215,928	5,828,107,815	1,024,658,636	1,001,920,262	22,738,375
96.00%	5,637,122,398	5,853,817,515	1,050,368,336	1,027,058,091	23,310,245
96.50%	5,669,268,283	5,882,521,205	1,079,072,026	1,055,121,073	23,950,953
97.00%	5,705,604,046	5,915,102,021	1,111,652,842	1,086,971,580	24,681,262
97.50%	5,747,584,639	5,952,913,814	1,149,464,635	1,123,931,334	25,533,301
98.00%	5,797,612,008	5,998,195,243	1,194,746,064	1,168,185,428	26,560,636
98.50%	5,860,110,958	6,055,079,158	1,251,629,979	1,223,766,408	27,863,571
99.00%	5,944,797,553	6,132,662,637	1,329,213,458	1,299,547,733	29,665,725
99.50%	6,081,703,000	6,259,162,825	1,455,713,646	1,423,034,040	32,679,607
99.90%	6,370,035,159	6,529,082,838	1,725,633,659	1,686,119,456	39,514,203
99.95%	6,484,300,353	6,637,118,106	1,833,668,927	1,791,227,407	42,441,520
99.99%	6,732,087,246	6,873,187,638	2,069,738,459	2,020,393,567	49,344,892

The spreadsheet included with this paper gives the correlation matrices for the lines/accident year combinations of DFAIC, with and without reinsurance.

With the aggregate loss distribution in hand, we now turn to discussing the adequacy of DFAIC's capital. Ideally, we would like to have capital adequacy standards that enable us to select an α-level that corresponds to a given rating. While such standards may evolve in the future, we do not believe that standards exist yet. We therefore accept the unqualified actuarial opinion that DFAIC's capital is adequate. We also accept that DFAIC is entitled to the rating of A given to it by the A.M. Best company.

DFAIC's capital is $1,604,297,000. By examining Table 3.3 we see that this corresponds to an α-level between 99.5% and 99.9%. However, in constructing DFAIC's aggregate loss distribution, we ignored lines of insurance that account for almost 5% of the premium. We also ignored asset risk. With more than $500 million invested in stocks, a drop in asset values in the range of $50 to 100 million appears possible. At the time of this writing, the S&P 500 stock index has recently dropped from over 1,500 to below 1,200. With this in mind, we judgmentally set an α-level of 99.0%, as our standard for adequate capital for the modeled lines/accident year combinations. We will use that standard in the work below.

4. Allocating Capital

We allocate capital to the 50 line/accident year combinations in proportion to their marginal capital. To do that we need to calculate the $TVaR_{99\%}$ for DFAIC 50 times, removing each combination, in turn, from the calculation. Because of the reduction in risk due to pooling, the sum of marginal capitals for each combination will add up to less than the total capital. Thus, we need to multiply each marginal capital by a pooling factor to force the total capital to equal the sum of the allocated capitals. Meyers [4] provides the economic rationale for the pooling factor.

For the long-tailed lines for which DFAIC incurs losses in 2000, there will be uncertainty in the loss payments made in 2001, 2002 and even in later years. Thus, DFAIC will have to allocate capital for the accident year 2000 in 2001, 2002, and so on.

To make those allocations in future years, we must make a business plan for future years and allocate capital according to that plan. In our underwriting risk model for DFAIC, we allow for uncertainty in future loss reserves for seven years. We assume that DFAIC's business plan is to continue its present writings. If it also does not change its reinsurance plan, the allocations to line/accident year combinations over the next seven years will not change. If DFAIC decides to change its reinsurance plan in 2000, the allocations from prior accident years will still reflect the old reinsurance plan. For example, in calendar year 2000, there will be one year under the new plan, and six under the old plan. In calendar year 2001, there will be two years under the new plan, and five years under the old plan. If we introduce a new reinsurance plan, we must do a new allocation for each of seven years.

The spreadsheet that accompanies this paper contains capital allocations for four different reinsurance strategies. In this paper we exhibit two of those strategies – the current reinsurance plan and no reinsurance.

Table 4.1

Capital Allocations for Accident Year 2000 – Current Reinsurance Strategy

Line\Cal Year	2000	2001	2002	2003	2004	2005	2006
CAT	5,109,553	0	0	0	0	0	0
Allied	2,080,280	448,489	0	0	0	0	0
Fire	1,196,144	259,202	0	0	0	0	0
HO	59,783,298	13,620,819	1,834,222	814,426	390,224	0	0
CAL	82,133,539	53,923,373	30,202,957	14,514,549	6,154,661	0	0
PAL	319,844,532	74,634,107	16,882,652	6,088,341	2,215,292	794,825	304,133
APHD	223,086,578	43,362,982	0	0	0	0	0
OLOC	11,623,350	8,892,618	6,220,151	4,163,600	2,606,682	1,577,708	920,056
PLOC	180,894	143,389	108,976	80,123	54,912	36,474	24,125
CMP	125,859,211	47,323,486	20,457,481	13,436,588	8,388,555	5,104,263	2,930,801
WC	40,204,443	25,091,661	11,229,472	2,327,101	882,433	0	0
Other Acc. Years	433,555,463	1,031,847,606	1,212,611,822	1,258,123,006	1,278,854,972	1,292,034,462	1,295,368,617
Total	1,299,547,733	1,299,547,733	1,299,547,733	1,299,547,733	1,299,547,733	1,299,547,733	1,299,547,733

Table 4.2

Capital Allocations for Accident Year 2000 – No Reinsurance

Line\Cal Year	2000	2001	2002	2003	2004	2005	2006
CAT	10,825,505	0	0	0	0	0	0
Allied	2,755,376	598,084	0	0	0	0	0
Fire	2,088,443	462,445	0	0	0	0	0
HO	60,117,157	13,630,159	1,840,641	818,455	392,988	0	0
CAL	81,662,436	53,041,665	29,712,369	14,286,974	6,062,589	0	0
PAL	316,846,970	73,410,423	16,620,268	5,996,813	2,183,421	784,336	300,435
APHD	221,395,415	42,680,569	0	0	0	0	0
OLOC	13,151,693	10,512,690	7,509,305	5,071,918	3,205,067	1,960,704	1,150,798
PLOC	211,935	174,496	133,368	98,647	67,786	45,025	29,757
CMP	130,892,569	52,918,138	24,550,980	16,509,345	10,465,673	6,470,773	3,824,253
WC	43,339,968	27,208,148	12,230,581	2,536,807	962,906	0	0
Other Acc. Years	440,585,900	1,051,512,645	1,235,019,786	1,283,022,192	1,305,450,556	1,319,792,066	1,323,908,215
Total	1,313,047,863	1,326,149,463	1,327,617,298	1,328,341,150	1,328,790,987	1,329,052,903	1,329,213,458

Here are some observations on Tables 4.1 and 4.2.

- The current reinsurance strategy allocations in Table 4.1 correspond to the aggregate loss distribution in Table 3.3. The total capital for the current reinsurance strategy is equal to that implied by $TVaR_{99\%}$ in Table 3.3.
- If DFAIC changes over to no reinsurance, by 2006 we will allocate no capital to the line/accident year combinations affected by the current reinsurance. The total capital in 2006 for the no-reinsurance strategy is equal to that implied by $TVaR_{99\%}$ in Table 3.4.
- As removing the reinsurance affects more and more accident years, the total capital needed increases from $1,299,547,733 needed with reinsurance to $1,329,213,458 needed in 2006 with no reinsurance.

5. The Cost of Financing Insurance

Ultimately, the policyholders must bear the cost of capital and/or reinsurance. Investment earnings on the capital reduce that cost to some extent. In this section, we calculate the expected profit needed in 2000 for the insurer to make its overall expected return on capital.

Let $A_k(t)$ be the capital allocated to line of insurance k in calendar year 2000 + t. For example we see in Table 4.1 that for k = allied lines we have that $A_k(0)$ = $2,080,280 and $A_k(1)$ = $448,489. DFAIC needs $2,080,280 at the beginning of 2000 to support its allied lines losses from accident year 2000, and it needs $448,489 to support its allied lines losses from accident year 2000 at the beginning 2001. If DFAIC gets a 7% return on its invested assets the company can release $2,080,280 × 1.07 – 448,489 = $1,777,411 to its investors at the end of 2000. Let i be the return on invested assets, $R_k(0)$ be the Net Cost of Reinsurance, calculated as (Price – Expected Recovery)×(1 – Corporate Income Tax Rate) payable for line k at the beginning of 2000. Let $Rel_k(t)$ be the capital released at the beginning of calendar year 2000 + t. Then following Meyers [4], Table 5.1 gives the schedule for releasing capital.

Table 5.1

Schedule for Releasing Capital

Time	Financial Support Allocated at Time t	Amount Released at Time t
0	$A_k(0) + R_k(0)$	0
1	$A_k(1)$	$Rel_k(1) = A_k(0)(1+i) - A_k(1)$
---	---	---
t	$A_k(t)$	$Rel_k(t) = A_k(t-1)(1+i) - A_k(t)$
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We give the schedule for releasing capital for DFAIC for i = 7% in the following tables. These tables are also available on the spreadsheet included with this paper.

Table 5.2

Schedule for Releasing Capital at the End of the Year with Current Reinsurance Strategy

Line\Cal Year	2000	2001	2002	2003	2004	2005	2006
CAT	5,467,222	0	0	0	0	0	0
Allied	1,777,411	479,883	0	0	0	0	0
Fire	1,020,672	277,347	0	0	0	0	0
HO	50,347,310	12,740,055	1,148,191	481,211	417,540	0	0
CAL	33,959,513	27,495,052	17,802,615	9,375,906	6,585,488	0	0
PAL	267,599,543	62,975,842	11,976,096	4,299,233	1,575,538	546,330	325,422
APHD	195,339,656	46,398,391	0	0	0	0	0
OLOC	3,544,366	3,294,950	2,491,962	1,848,370	1,211,442	768,091	984,460
PLOC	50,168	44,450	36,481	30,819	22,282	14,902	25,814
CMP	87,345,870	30,178,650	8,452,916	5,988,594	3,871,490	2,530,761	3,135,957
WC	17,927,093	15,618,605	9,688,434	1,607,565	944,204	0	0

Table 5.3

Schedule for Releasing Capital at the End of the Year with No Reinsurance

Line\Cal Year	2000	2001	2002	2003	2004	2005	2006
CAT	11,583,290	0	0	0	0	0	0
Allied	2,350,168	639,950	0	0	0	0	0
Fire	1,772,190	494,816	0	0	0	0	0
HO	50,695,199	12,743,630	1,151,031	482,759	420,497	0	0
CAL	34,337,142	27,042,213	17,505,261	9,224,473	6,486,970	0	0
PAL	265,615,835	61,928,885	11,786,873	4,233,169	1,551,925	538,804	321,465
APHD	194,212,524	45,668,209	0	0	0	0	0
OLOC	3,559,621	3,739,274	2,963,038	2,221,885	1,468,718	947,155	1,231,354
PLOC	52,275	53,343	44,057	37,766	27,506	18,420	31,840
CMP	87,136,910	32,071,428	9,760,204	7,199,326	4,727,497	3,099,474	4,091,950
WC	19,165,618	16,882,137	10,549,915	1,751,477	1,030,310	0	0

Let e be DFAIC's expected pretax return on equity. Then, following Meyers [4], the cost of financing, $\Delta P_k(0)$, necessary for the insurer to make its expected rate of return is given by:

$$\Delta P_k(0) = \underbrace{A_k(0) - \sum_{t=1}^{\infty} \frac{Rel_k(t)}{(1+e)^t}}_{\text{Cost of Capital}} + \underbrace{R_k(0)}_{\substack{\text{Net Cost}\\ \text{of Reinsurance}}} \tag{5.1}$$

We will calculate the Net Cost of Reinsurance by first specifying an expected loss ratio, ELR_k. We then have:

$$R_k(0) = E[\text{Recovery in Line } k] \times \left(\frac{1}{ELR_k} - 1 \right) \times (1 - \text{Corporate Income Tax Rate}) \tag{5.2}$$

Then setting e = 15%, ELR_k = 50% for the catastrophe line; ELR_k = 65% for all other lines; the corporate income tax rate = 35% and applying Equation 5.1 to the entries in Table 5.2 --- we get the following table for the cost of financing with the current reinsurance strategy.

Table 5.4

The Cost of Financing Insurance with the Current Reinsurance Strategy

Line of Business	Cost of Capital	Net Cost of Reinsurance	Cost of Financing
CAT	355,447	2,857,770	3,213,217
Allied	171,845	1,042,073	1,213,918
Fire	98,890	1,168,697	1,267,587
HO	5,132,037	18,183	5,150,221
CAL	11,472,953	0	11,472,953
PAL	28,056,063	0	28,056,063
APHD	18,142,158	0	18,142,158
OLOC	2,050,057	1,796,206	3,846,263
PLOC	34,826	37,875	72,701
CMP	13,907,103	4,171,525	18,078,628
WC	5,046,888	5,986,520	11,033,408
Total	84,468,267	17,078,848	101,547,115

Doing the same calculation with the entries in Table 5.3, we get the following table for the cost of financing with no reinsurance.

Table 5.5

The Cost of Financing Insurance with No Reinsurance

Line of Business	Cost of Capital	Net Cost of Reinsurance	Cost of Financing
CAT	753,079	0	753,079
Allied	227,857	0	227,857
Fire	173,257	0	173,257
HO	5,156,459	0	5,156,459
CAL	11,346,968	0	11,346,968
PAL	27,753,784	0	27,753,784
APHD	17,983,232	0	17,983,232
OLOC	2,407,721	0	2,407,721
PLOC	41,975	0	41,975
CMP	15,108,299	0	15,108,299
WC	5,458,493	0	5,458,493
Total	86,411,124	0	86,411,124

We also considered two additional reinsurance strategies. We summarize the results in the following table.

Table 5.6

Cost of Financing Insurance for Four Reinsurance Strategies

Reinsurance Strategy	Cost of Capital	Net Cost of Reinsurance	Cost of Financing
Current Reinsurance	84,468,267	17,078,848	101,547,115
No Reinsurance	86,411,124	0	86,411,124
Cat Reinsurance Only 90% of Loss over $50 M	85,922,455	3,835,282	89,757,738
Liability Reinsurance Only	84,905,169	12,010,309	96,915,478

Comments

DFAIC is paying a net cost of $17,078,848 for its reinsurance in order to save $86,411,124 – $84,468,267 = $1,942,857 for its cost of capital. We recommend that DFAIC stop buying reinsurance. Qualitatively, this makes sense for a well-diversified insurer writing more than $2.5 billion in premium with more than $5.3 billion in assets, and no significant catastrophe potential.

However, we offer one qualification to this conclusion. The decision to purchase reinsurance is usually made by upper level management who are sensitive to the needs of the insurer's investors. *If* the investors value stability in earnings, they will demand a higher return on capital if the reinsurance coverage is dropped. In that case, the cost of financing reinsurance will not be reduced by the as much as the above analysis indicates. The following table gives the return on capital that makes all four of the above reinsurance strategies equivalent.

Table 5.7

Return on Capital for Four Reinsurance Strategies

Reinsurance Strategy	Return on Capital	Cost of Financing
Current Reinsurance	15.00%	101,547,115
No Reinsurance	16.59%	101,547,115
Cat Reinsurance Only 90% of Loss over $50 M	16.24%	101,547,115
Liability Reinsurance Only	15.49%	101,547,115

Whether or not investors will demand these returns is debatable. Financial theory tells us that investors will not demand a higher return if the risk removed by reinsurance is diversifiable. We leave it at that.

6. Target Combined Ratios

The final step in this analysis is to calculate target combined ratios for each line of insurance. These targets will take into account the cost of financing insurance, investment income derived from writing the insurance and expenses. We made the following assumptions (simplified for the purpose of this paper.)

- Losses are paid at the midpoint of the year.
- Losses are discounted at DFAIC's return on invested assets when calculating the Actuarial Present Value (APV) of the losses.
- Loss Adjustment Expenses (LAE) are a percentage of the expected loss and are paid at the same time as the losses.
- Other Expenses are a percentage of premium.

Table 6.1

Target Combined Ratios with Current Reinsurance Strategy

	CAT	Allied	Fire	HO	CAL	PAL
E[Loss]	18,645,163	19,915,510	13,083,761	437,032,492	190,819,744	743,842,606
APV[Loss]	18,024,960	18,938,164	12,441,681	411,201,278	162,654,461	698,375,986
LAE%	13.31%	8.10%	5 90%	12.10%	13.90%	13.40%
LAE	2,481,671	1,613,156	771,942	52,880,932	26,523,944	99,674,909
APV of LAE	2,399,122	1,533,991	734,059	49,755,355	22,608,970	93,582,382
Other Expense%	32.42%	31.10%	37 40%	30.70%	30.00%	22.80%
Other Expense	11,339,468	9,788,634	8,363,087	206,484,365	84,315,593	242,180,428
Cost of Financing	3,213,217	1,213,918	822,369	5,146,510	11,472,953	28,056,063
Cost of Financing%	9.19%	3.86%	3.68%	0.77%	4.08%	2.64%
Premium	34,976,767	31,474,707	22,361,196	672,587,508	281,051,978	1,062,194,859
Target Comb Ratio	92.82%	99.50%	99.36%	103.54%	107.33%	102.21%

	APHD	OLOC	PLOC	CMP	WC	Total
E[Loss]	540,201,933	50,547,922	817,783	457,887,696	346,008,816	2,818,803,427
APV[Loss]	513,691,728	38,679,364	579,805	417,569,543	309,668,745	2,601,825,715
LAE%	9.25%	25.10%	25.10%	17.20%	13.00%	13.15%
LAE	49,968,679	12,687,528	205,263	78,756,684	44,981,146	370,545,855
APV of LAE	47,516,485	9,708,520	145,531	71,821,961	40,256,937	340,063,314
Other Expense%	23.70%	27.70%	27.70%	36.40%	22.30%	27.54%
Other Expense	179,955,489	20,012,253	309,204	289,951,641	103,245,083	1,155,945,243
Cost of Financing	18,142,158	3,846,263	81,718	17,227,296	9,811,669	99,034,133
Cost of Financing%	2.39%	5.32%	7.32%	2.16%	2.12%	2.36%
Premium	759,305,859	72,246,400	1,116,258	796,570,442	462,982,433	4,196,868,405
Target Comb Ratio	101.43%	115.23%	119.35%	103.77%	106.75%	103.54%

Table 6.2

Target Combined Ratios with No Reinsurance

	CAT	Allied	Fire	HO	CAL	PAL
E[Loss]	18,645,163	19,915,510	13,083,761	437,032,492	190,819,744	743,842,606
APV[Loss]	18,024,960	18,938,164	12,441,681	411,201,278	162,654,461	698,375,986
LAE%	13.31%	8.10%	5.90%	12.10%	13.90%	13.40%
LAE	2,481,671	1,613,156	771,942	52,880,932	26,523,944	99,674,909
APV of LAE	2,399,122	1,533,991	734,059	49,755,355	22,608,970	93,582,382
Other Expense%	32.42%	31.10%	37.40%	30.70%	30.00%	22.80%
Other Expense	10,159,271	9,343,547	7,975,279	206,488,772	84,261,600	242,091,154
Cost of Financing	753,079	227,857	173,257	5,156,459	11,346,968	27,753,784
Cost of Financing%	2.40%	0.76%	0.81%	0.77%	4.04%	2.61%
Premium	31,336,432	30,043,560	21,324,276	672,601,865	280,871,999	1,061,803,307
Target Comb Ratio	99.84%	102.76%	102.38%	103.54%	107.38%	102.24%
	APHD	**OLOC**	**PLOC**	**CMP**	**WC**	**Total**
E[Loss]	540,201,933	50,547,922	817,783	457,887,696	346,008,816	2,818,803,427
APV[Loss]	513,691,728	38,679,364	579,805	417,569,543	309,668,745	2,601,825,715
LAE%	9.25%	25.10%	25.10%	17.20%	13.00%	13.15%
LAE	49,968,679	12,687,528	205,263	78,756,684	44,981,146	370,545,855
APV of LAE	47,516,485	9,708,520	145,531	71,821,961	40,256,937	340,063,314
Other Expense%	23.70%	27.70%	27.70%	36.40%	22.30%	27.54%
Other Expense	179,906,123	19,461,110	293,977	288,738,882	101,995,716	1,150,715,431
Cost of Financing	17,983,232	2,407,721	41,975	15,108,299	5,458,493	86,411,124
Cost of Financing%	2.37%	3.43%	3.96%	1.90%	1.19%	2.07%
Premium	759,097,567	70,256,715	1,061,288	793,238,686	457,379,890	4,179,015,584
Target Comb Ratio	101.45%	117.71%	124.10%	104.05%	107.78%	103.85%

The target combined ratios provide a tool to evaluate the line of business's financial performance. This tool reflects the line's contribution to DFAIC's total risk.

7. Conclusions

We give our responses to the questions the CEO would like addressed.

1. Is the Company adequately capitalized? Is there excess capital? How much capital should the Company hold as a stand-alone insurer?

   Response – We accept the current capital as adequate, with no excess capital. We find that the quantitative standard implied by the Tail Value at Risk evaluated at the 99% threshold works for DFAIC.

2. How should the capital be allocated to line of business?

   Response – We allocated capital in proportion to the marginal capital implied by the Tail Value at Risk evaluated at the 99% threshold. Tables 4.1 and 4.2 give the results, for the current reinsurance and no-reinsurance strategy.

3. What is the return distribution for each line of business and is it consistent with the risk for the line?

   Response – We defined the cost of financing insurance as the total of the allocated cost of capital and the net cost of reinsurance. These costs are consistent with the risk for each line of insurance. Tables 5.4 and 5.5 give the dollar costs for the current reinsurance and no-reinsurance strategies. Tables 6.1 and 6.2 give the target combined ratios implied by these costs of financing insurance for the two strategies.

4. Should the Company buy more or less reinsurance? What type? How efficient is its current reinsurance program?

   Response – We conclude that DFAIC should not buy any reinsurance. DFAIC is a well-diversified insurer with little catastrophe exposure. The company will save 15% of its cost of financing reinsurance by not buying reinsurance. We might modify this conclusion if DFAIC's investors would demand a higher return on capital when DFAIC's management drops the reinsurance.

5. How efficient is the asset allocation?

   This paper does not address that question.

References

1. Philippe Artzner, Freddy Delbaen, Jean-Marc Eber and David Heath, "Coherent Measures of Risk", Math. Finance 9 (1999), no. 3, 203-228
http://www.math.ethz.ch/~delbaen/ftp/preprints/CoherentMF.pdf

2. Insurance Services Office, "A Half Century of Hurricane Experience", 2000.

3. Philip E. Heckman and Glenn G. Meyers, "The Calculation of Aggregate Loss Distributions from Claim Severity Distributions and Claim Count Distributions" *PCAS LXX*, 1983.
http://www.casact.org/pubs/proceed/proceed83/83022.pdf

4. Glenn Meyers, "The Cost of Financing Insurance", Published simultaneously with this paper.

5. Glenn Meyers, "Estimating Between Line Correlations Generated by Parameter Uncertainty", *CAS Forum*, Summer 1999.
http://www.casact.org/pubs/forum/99sforum/99sf197.pdf

6. Glenn Meyers, "Discussion of Aggregation of Correlated Risk Portfolios by Shaun Wang", *PCAS LXXXVI*, 1999.
http://www.casact.org/pubs/proceed/proceed99/99705.pdf

7. Susan Szkoda, "How DFA Can Help the Property/Casualty Industry", *The Actuarial Review*, Volume 24, No. 1 edition, 1997.
http://www.casact.org/pubs/actrev/may97/dfapt1.htm