DFA—The Value of Risk

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ABSTRACT

This paper describes the use of a Dynamic Financial Analysis (DFA) model to answer questions on capitalisation, business and asset strategy in the case of a US P&C insurer, in the framework of maximising stockholder wealth.

We measure this wealth by applying a risk measure on the individual stochastic cash flows from the DFA model, in preference to more conventional approaches based, for example, on historic betas The risk translates into value by two mechanisms:

1. For systematic risk, we use a multiple-factor arbitrage-free pricing approach. This is calibrated to be consistent with the prices that our stochastic macroeconomic model generates. We implement these ideas using explicit deflator processes.
2. Both systematic and non-systematic risks generate frictional costs, which we model explicitly. These costs are of vital importance to insurance, yet are often overlooked in DFA analysis. We allocate these frictional costs back to each simulation so as to produce realistic, rather than idealistic, financial statements which then enable us to look at capitalisation issues as well as valuation ones.

Our approach to risk definition is consistent with the recent findings of the CAS Risk Premium Project – see Butsic et al (2000)

KEYWORDS
Dynamic Financial Analysis; Capital, Risk, Return; Financial Economics, Reinsurance; Catastrophe Exposure; Investment Strategy; Systematic Risk, Non-Systematic Risk; Frictional Costs; Operational Risk; Capital Allocation;

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EXECUTIVE SUMMARY

The following report outlines a detailed analysis of DFA Insurance Company (DFAIC) using Dynamic Financial Analysis (DFA) This is a brief summary of our conclusions – our full report provides more details and supporting evidence

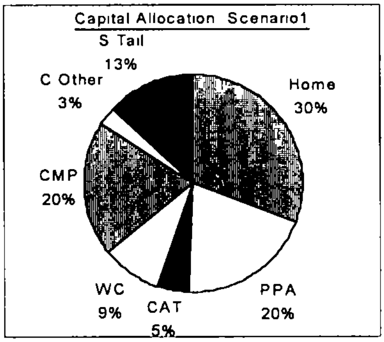
Is DFAIC adequately capitalised?

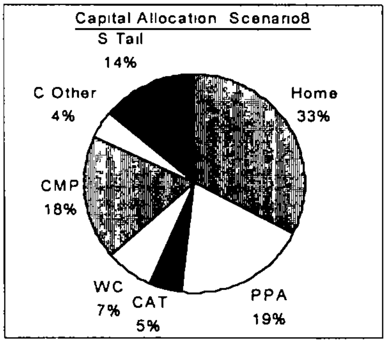
We believe that the company can reduce its capital by at least $100m without increasing its risk to financial impairment We measure this impairment by estimating the probability distribution of the minimum surplus to premium ratio over a five year projection period This capital release is part of a new strategy that has reduced asset risk with all equity investments replaced by bonds and a more aggressive insurance strategy which eliminates all class excess of loss reinsurances

We have demonstrated that this new strategy (Scenario 8) is just as financially sound as the existing one (Scenario 1) which has been good enough to ensure the company maintained its A rating from A M Best over the last five years The new strategy increases dividends to shareholders by around $65m pa on a reduced capital base without increasing risk to policyholders or management

How should the capital be allocated to line of business?

We have allocated capital within DFAIC according to the risks to which it is exposed The risk costs for each class include the class specific systematic and non systematic costs and an apportionment of the frictional costs associated with the capital account Our allocation of capital is shown in the pie charts below for Scenarios 1 and Scenario 8 Scenario 8 has lower maintained surplus holds no equity investments and places no class of business reinsurances





What is the return distribution of each business and is it consistent with the risk for the line?

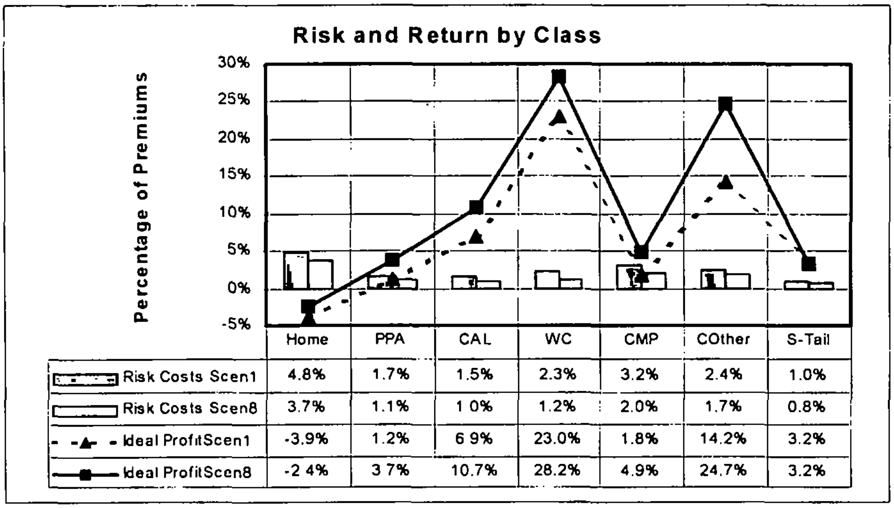
We have estimated the distribution of emerging profits, gross of frictional costs, for each line of business. From these distributions, we have identified two components of the cost of capital, relating to systematic risk and also to frictional costs.

The chart shows the mean profit, and the associated capital costs, expressed as a proportion of net premium income. The two lines show the 'ideal profits' for the two strategies, Scenario 1 and Scenario 8. The blocks show the cost of capital for each class for the two strategies. These class costs consist of the class specific risk costs and the allocated capital account frictional costs.

We can see that the Home class is destroying significant value for shareholders; this warrants management attention. The class results are poor, even after investment income, and the cost of capital is large thanks to the catastrophe exposures.

Although PPA and CMP are currently generating profits, these are not creating value because the cost of capital exceeds the profits generated. However, scenario 8 improves profitability by reducing reinsurance costs, and also reduces the cost of capital by more prudent investment. The restructuring we have suggested would then transform PPA and CMP into value-creating classes of business.

CAL, WC and C Other produce profits, which comfortably exceed their cost of capital under Scenarios 1 and 8. Our class risk allocations include the capital account frictional costs.



	Home	PPA	CAL	WC	CMP	COther	S-Tail
Risk Costs Scen1	4.8%	1.7%	1.5%	2.3%	3.2%	2.4%	1.0%
Risk Costs Scen8	3.7%	1.1%	1 0%	1.2%	2.0%	1.7%	0.8%
Ideal ProfitScen1	-3.9%	1.2%	6 9%	23.0%	1.8%	14.2%	3.2%
Ideal ProfitScen8	-2 4%	3 7%	10.7%	28.2%	4.9%	24.7%	3.2%

Should the company buy more or less reinsurance?

DFAIC, under the assumed base scenario (Scenario 1), currently pays $145m per annum in class excess of loss reinsurance premiums. In return it receives, on average, $75m of reinsurance recoveries, and a reduction of $3m in internal frictional costs. There is negligible impact on systematic risk.

The analysis also shows that DFAIC can terminate these class reinsurances without impacting its financial strength, as measured by its ability to maintain its surplus to premium ratios at levels that are almost identical to those it achieves with the benefit of these reinsurances. The company can therefore achieve this change in strategy without requiring additional capital. Allowing for the additional frictional costs and tax, the shareholders will see an average increase in dividends of just over $30m pa throughout the plan period.

In the case of the catastrophe reinsurance protection, the analysis (Scenario 3) indicates that the annual savings will be around $8m pa, after allowing for the $4m increase in frictional costs. The impairment analysis, in this case, indicates a weakening financial position, which will require additional capital. We have not attempted to identify this amount, as it is unlikely that such a change in catastrophe reinsurance protection will be considered prudent or justified by external analysts in the short term. This is an area for future DFA analysis.

How efficient is the asset allocation?

DFAIC currently invests 35% of its free assets (surplus) in equities. Allowing for higher returns and also higher risk-based operational costs, this strategy increases mean profits by $43m over a five-year period compared to a bond strategy. The corresponding increase in cost of capital for DFAIC is $94m. Therefore, the equity strategy is destroying value.

There may be some arguments for establishing bond portfolios that more closely replicate the liabilities, however our analysis indicates that this is of little value in the context of the avoidable operational costs imposed on DFAIC by the current equity exposure.

1. INTRODUCTION

Insurance managers, insurance regulators and analysts have long recognised the potential value of asset-liability modelling (ALM) for P&C companies. Examples of ALM or Solvency models can be traced back to the late seventies and early eighties. In the nineties the subject was given a new name, DFA or Dynamic Financial Analysis, and this has now entered the P&C vocabulary as the process for financial risk evaluation of P&C insurers.

The early P&C ALM models progressed knowledge but failed to deliver much in answering real problems. There were two main reasons for these limitations. Firstly, there were poor links between assets and liabilities, often due to poor economic scenario generators. Secondly, there was no clear consensus on how to interpret the mass of outputs. These are the two key factors behind the development of the approach described in this paper.

The DFA approach adopted concentrates on the key variables and attempts to maintain economic soundness in how assets and liabilities are modelled and how the results are interpreted. The resulting DFA framework for risk pricing recognises and quantifies systematic and non-systematic risk as recommended by the latest research on the subject by the CAS's own Risk Premium Project (RPP) Phase I and II Report –see Butsic et al (2000).

The resulting framework enables the valuation and ranking of alternative management strategies and also provides a more realistic approach to the investigation of financial impairment and risk sensitive capital requirement questions. The methodology extends to the allocation of both systematic and non-systematic risk costs to individual DFA simulations or to classes of business. These risk cost allocations are often the main objective of 'capital allocation' and are derived directly and coherently from DFA outputs.

This paper describes both the theoretical background and practical implementation of this new approach to DFA modelling using the study case selected for such a purpose by the CAS Committee on DFA.

The next section introduces the case study, describes the main features of the DFA model used for the analysis and how this was calibrated for DFA Insurance Company. Section 3 contains the technical details that underpin the analysis. This section contains some new material. Section 4 presents the results of the DFA analysis of DFAIC and demonstrates how the theory of the earlier section can help to turn the huge volume of DFA data output into a few key 'value' measures that can be used in the decision making progress. The final Section contains some concluding remarks and is followed by a list of References and a number of Appendices.

2. DFAIC AND DFA MODEL CALIBRATION

OVERVIEW OF DFA INSURANCE COMPANY

DFAIC is a US P&C insurance company licensed in all 50 states, which writes a balanced book of personal and mainstream commercial business. It has a primary concentration in the Northeast and Midwest and has enjoyed a rating of 'A' from A.M. Best for at least the past five years.

The company has minimal exposure to asbestos and environmental exposures and limited exposure to severe catastrophes. It maintains reinsurance protections to limit losses to $1million from individual risk and buys catastrophe reinsurance cover of 90% of $150m excess of $50m for any single event, which limits the pre-tax PML exposure over a 100 year return period to 10% of surplus, or roughly $160m.

Around 70% of the assets are invested in fixed income securities, most in tax-exempt municipal bonds. Of the remaining 30%, 18% are in cash and 12% in equities.

The financial information available shows that in 1999 the company had net premium earnings of just over $2.3billion and a Surplus of just over $1.6billion, or 70% of its net annual premium.

OVERVIEW OF THE DFA MODEL

The DFA model used for the analysis is a multi-line, multi-period, multi-scenario stochastic plan generator, implemented in C++ for speed. For this exercise we used annual periods and simulated financial statements for five years.

Economic scenarios are pre-generated and include the usual asset returns, split into income and gains as well as mid-year and end-year deflators and twenty year term structures for inflation and interest rates The deflators are used in the interpretation to quantify systematic risk and the term structures are used to set fair premiums and set claim reserves. For this exercise a 20-year term-structure was considered sufficient to cover the claims run-off period. More information on the economic scenario generator (TSM or The Smith Model) is given in Appendix A. For more details on deflators, see Jarvis et al (2001)

The DFA model projects premium amounts using indices of exposure and rating adequacy and a fair premium adjustment. The adjustment allows for the impact of any changes in inflation and interest earning expectations over the period of the exposures covered by the premium. Claim amounts are adjusted for any (earned) exposure changes and are simulated with anticipated claims inflation for the class in the case of reserves or actual inflation for the class in the case of payments. Projected loss ratios are the result of simulated premium and loss figures rather than a simulated variable, as is often the case with DFA models.

Three types of claims are used. Small claims are modelled in aggregate using one of the many available distributions. Large claims are modelled by a frequency and a severity distribution. Finally, peril losses are modelled at company, or market, level and then allocated to affected classes. A base year is used to define the required parameters and numbers and amounts for subsequent year simulations are calculated taking account of exposure and inflation changes.

Reinsurance modelling is available at both class and company level. For this particular analysis we only used the class excess of loss and the catastrophe excess of loss reinsurance facilities.

Expenses are modelled at class level, with four sets of parameters, a commission rate, and a dollar amount and by two percentages that apply to gross premiums and claim payments. Each of these variables can vary by year.

Assets backing liabilities are held in notional funds for each class, with the balance or surplus held in a capital account. Each of these funds can invest in any of the available asset classes, which include cash, equities, index linked bonds and bonds of various durations.

A single tax rate is used and there is a facility for tax deferral on unrealised capital gains. The model accommodates a large number of dividend strategies, including varying amounts that may or may not be inflated, variable amounts based on percentages of post-tax profits or varying amounts that pay any surplus in excess of a premium ratio. This option allows the company to maintain a level of surplus to premium over its plan period and so attempt to maintain its actual rating in the market place. It is possible under this strategy for the actual surplus ratio to fall below the set target, which allows us to investigate the impact that a particular business strategy may have on the probability that such a ratio falls to a level that may lead to a downgrade of the company rating

CALIBRATING THE DFA MODEL FOR DFAIC

Capital Structure, Taxes and Dividends

The capital, or surplus, was taken to be $1604m as at the beginning of the projection period and this was made of issued capital and retained earnings. It was assumed that there was no subordinated debt and that all dividends are paid to shareholders. A tax rate of 20% was used to reflect the low tax paying position of the company, with its high level of tax efficient municipal bond holdings. This is an area where a more US-specific tax treatment would be warranted in a real case exercise.

The amount of shareholder dividends paid in 1999 appeared close to the overall investment returns less policyholder dividends and taxes. Policyholder surplus reduced by $60m, or 3% of annual net premiums, as a result and the year-end premium to surplus ratio increased from 1.4 to 1.47. As this did not impact the company's rating from A.M. Best, a premium to surplus level of 1.43 was taken as reflecting the required level of surplus to be maintained through the plan period.

The economic outlook for the plan period, as projected by the underlying scenario generator, indicated lower average investment returns for the whole of the projection period. The most likely impact of reduced investment earnings will be reduced dividend payments. For the purposes of the evaluation, we defined a dividend strategy for DFAIC designed to safeguard its rating using the premium to surplus ratio of 1.43 discussed above, which is equivalent to maintaining a 'solvency ratio' (ratio of surplus to premiums) of 70%.

It is also assumed that in case of overall losses there will be no dividend payments or capital injections. In such cases, the surplus ratio will increase (the solvency ratio will decrease) and in more extreme cases, or following a series of poor results, the deterioration in surplus may lead to a ratings downgrade. This approach is used to test the resilience of the plans and ultimately the capital requirements of the company.

Classes, Premiums and losses

There were seven classes of business with annual premiums exceeding $150m and a number of much smaller classes with aggregate net premiums of less that $65m. We grouped these smaller classes together for the modelling ending with just seven classes, Home, PPA, CAL, WC, CMP, Commercial Other and Short-Tail. Premium, loss and loss payment pattern characteristics were then obtained from the financial data supplied.

For the purposes of the evaluation, we assumed that the company exposures are stable, with growth in premiums and claims costs arising purely as a result of economic variables, such as inflation and interest expectations affecting premiums and claims amounts. This implicitly assumes that future prices are being set to maintain the 'premium adequacy' at the base year (Year 2000) levels. Pricing cycles and price-volume changes could be included in the modelling but this was not considered necessary for this analysis.

Expenses And Allocations

Commission and expense figures were only available in aggregate and these were allocated to the above classes of business using broad assumptions, checked for overall reasonableness only. These allocations do not have a significant impact on the overall projections or results, except in that they limit what can be said with any degree of confidence in regard to the actual pricing adequacy of any of these classes. It is, however, still possible to make useful comments on the required risk-sensitive performance requirements for these classes.

For the modelling, loss related expenses were included in the loss projections. Commissions and other expenses by class were then modelled by two class specific percentages. These percentages were set for the whole of the projection period.

Assets And Allocations

Detailed asset information was available in aggregate form. The DFA model actually maintains invested funds for each class of business and the capital account (policyholder surplus) separately to facilitate better matching of assets and liabilities, if required. Choosing to mis-match assets and liabilities in this way may result in an increase in any systematic risk associated with the particular class of business and may well be a strategy that could be investigated in our framework.

The assets were grouped into seven main classes, cash, equities and bonds of durations of 1, 3, 5, 10 and 30 years. The initial invested funds for each class were estimated from the financial information and then allocated to the available asset classes broadly to reflect the term of the liabilities. Equity investments were assumed to be from shareholder funds (surplus). These initial allocations were deemed to reflect the company asset strategy and were maintained through the projection period. The actual amounts and allocations are given in Table 11, Appendix B

Large and peril losses and Reinsurances

The company buys a significant level of excess of loss and catastrophe reinsurance. We have modelled these reinsurances for all classes except for the short tail class. We estimated reinsurance premiums from the financial information and used the limits of reinsurance purchased to help us select a likely large loss frequency and severity distribution for each class that provided a reasonable match to both the cost of the reinsurances and the amount of cover purchased.

We made an assumption that the price of these reinsurances is around twice expected risk cost. This may be considered relatively expensive cover. Clearly, assuming that these reinsurances are priced at below risk or expected cost will result in a clear benefit emerging from the purchase of the reinsurance, particularly if it is assumed, as will often be the case with such modelling, that there is no resulting credit risk associated with such low reinsurance costs.

In the case of the catastrophe cover, we used the amount of cover and the indicated PML exposure information to identify an appropriate set of loss generating parameters. Here the cost is assumed to carry a heavier risk loading of 2.66 times expected risk cost. This value is equivalent to pricing the catastrophe reinsurance using the Wang proportional hazards transform with a risk aversion index of 1 6. See Christofides (1998) for more details of this approach and a justification for the choice of the loading factor.

In all cases, the Poisson distribution was used to generate the number of large claims and catastrophe occurrences. The loss amounts were generated by a new distribution, which we call the Parbull, and which is a Pareto with a Weibull tail. This distribution has three parameters, the usual two parameters of the Pareto, a scale and a shape, and the value at which the Weibull takes over.

Catastrophe exposures, losses and reinsurance costs, were assumed to fall 80% on the Home class with the other 20% on the CMP class. The average annual catastrophe retained losses are approximately 10% of premium for the Home class and 2.5% of premium for CMP. This is a key assumption as it has a significant impact on the class results. The company has significant exposures in the North East, where the coastal region has a high hurricane exposure. With our limited knowledge of the US market, we assumed that most of this exposure falls on the Home account. The choice of affected classes and allocation was selected to demonstrate the implications of such losses on the risk characteristics of the affected classes more easily, rather than reflect the actuality at class level. The overall company catastrophe risk impact is not affected by this allocation.

The main parameter assumptions for the DFAIC calibration are given in Appendix B.

3. MODELLING MARKET INTERACTIONS

EMPIRICAL NATURE OF DFA MODELS

Our discussion of the DFA modelling process so far has been largely empirical. There is no general theory to tell us whether large loss distributions should be Pareto, lognormal, gamma or some other family. The decision is a matter of historical data, and in the absence of data, experienced guesswork.

This empirical aspect identifies a number of possible problems with a model. Other authors analysing the same data are likely to built significantly different models. Given *different* data sets, but relating to the same company, two analysts' models would diverge still further. It is clear that our calibration is subject to significant model and parameter error.

In many cases, there is little that can be done about this error, other than to acknowledge its existence and exercise caution in interpreting model output. To model the parameter error itself requires the construction of meta-models in which the parameters themselves are treated stochastically. Vast arrays of meta-parameters proliferate further, rapidly exhausting the degrees of freedom in the data. This way madness lies.

SAMPLING ERROR AND OPTIMISATION

There is one situation where a purely empirical approach to model estimation can be more dangerous. This arises in situations where one or more players are competing – for example, in capital markets and premium rates (both direct and reinsurance). In this case, a reasonable prior view would be that competitive pressures cause convergence in profitability between alternative investments or lines of business.

If this prior view is not reflected in a DFA model, we risk overestimating the extent of any capital allocation opportunities. For example, let us suppose (naively) that competitive pressures forced 10 lines of business towards equal expected profit margins as a proportion of premium. An examination of historic data is unlikely to show equal actual profitability; sampling error causes variations in the estimated results by line.

If we ignored the prior convergence view, we would estimate one line as being more profitable than the others. This would be an example of a *non-competitive* model. We would allocate most, if not all, of the company's premium capacity to this most profitable line. We would overestimate aggregate projected profitability, and we would mistakenly forsake diversification in favour of hoped-for profits. To avoid such misleading results, it is important to consider the effect of competition in a more structured way.

MODELLING COMPETITIVE EFFECTS

An ambitious way to model competitive effects would be to construct explicit models of current and potential future competitors and their actions. In an insurance context, this many require a model of many dozens of firms; capital markets have millions of participants. Such models quickly become unmanageable.

Is there a practical alternative to simple models that assume no competition? The other extreme is to use economic models based on *perfect competition*. Economists have built these models to describe the effect of a many parties competing with each other. In this case, major structural simplifications apply which avoid the need separately to model each individual participant in a perfectly competitive market.

Competitive models contain many other useful pieces of information. For example, competitive market models imply predictive theories of how markets will price certain products. We use this pricing information to estimate the effect of strategic choices on the price of an insurer's share.

The use of competitive models creates biases in the opposite direction from non-competitive models We underestimate profit opportunities. A competitive market provides no profitable niches; every cash flow is fairly priced. There is nothing to be gained from smart resource allocation. The best strategy is to diversify as far as possible and to track market resource allocation decisions.

Whether we want to model a non-competitive, or a perfectly competitive, situation will depend on the characteristics of the markets we wish to model – that is, how competitive we think the market is. It also depends on the outputs we wish to examine; if we need to estimate future market prices, there is no practical alternative to the use of competitive models. On the other hand, if we model all markets as competitive, then the optimal strategy becomes a foregone conclusion – simply conform to a peer average. We must identify some competitive failures if DFA modelling is to be of any value.

MARKET PRICING AND DEFLATORS

Probably the best contenders for the competitive market approach, in an insurance context, are the capital markets. This does not imply we think capital markets are perfect. There are specialist securities firms who have competitive advantages in terms of information or execution, who can extract excess profits from capital markets by proprietary trading. However, most of these institutions are not insurers, and in particular, DFAIC is not one of them. It is prudent to assume DFAIC faces competitive capital markets.

Such an approach provides the added boon of a market pricing capability, which we have implemented using deflators. Our competitive asset model (TSM) explains traded asset prices in terms of their future cash flow distributions. We can also use the deflators to interpolate market pricing, thus valuing cash flows for which a market price is not directly observable. In this case, the competitive market framework provides a risk-sensitive equilibrium value for that cash flow stream.

This is vitally important for evaluating different corporate strategies. Our deflator approach provides market values for strategic alternatives. It is calibrated to replicate market prices of traded assets, so market consistency is guaranteed. The result of the modelling process is a clear ranking of attractiveness of different strategies, according to the value the market would put on DFAIC should it adopt that strategy

To create meaningful valuations, the cash flow model needs to be good enough. Deflators are widely used in the pricing of financial products such as options Unlike financial products, there is no contractual formula linking insurance profit streams to capital market inputs. The links are via actuarial formulas containing all sorts of estimated parameters and leaving out all sorts of remote contingencies. We investigate some of these contingencies in the final section of this chapter.

OLIGOPOLY PROFIT

The next step from a competitive market model is to model some forms of market imperfection. This intermediate step is an *oligopoly*. An oligopoly may provide economic profits, for example because of barriers to entry, economies of scale, regulatory capture, or niches of asymmetric information. The oligopoly profit is an explicit adjustment between a competitive market price (for example, for reinsurance) and a price used in a DFA model. Such a modelling structure ensures that modelled premium rates respond appropriately, for example, to a change in interest rates or in inflation expectations.

Simpler modelling approaches, for example based on loss ratios, do not respond in the right way to changes in the economic outlook. Instead, oligopoly profits in the loss ratio approach become implicit items. Even when inputs appear consistent, the implied oligopoly profit under the loss ratio approach is the difference between two large numbers in the calibration, and may behave erratically unless deliberate thought is given to the issue.

In capital markets we have modelled oligopoly profits to be zero. In insurance markets we recognise a number of specific imperfections, which impact prices. As optimal corporate strategies are driven by deviations from perfect markets, our DFA approach involves optimising the impact of these imperfections.

SYSTEMATIC AND NON-SYSTEMATIC RISK

Deflators provide competitive capital market pricing for any cash flows, including insurance cash flows. Consistent with capital market theory, this methodology implies a reward for investors who are exposed to systematic risk, that is, market risk that remains even in a diversified portfolio.

This mechanism provides no reward for non-systematic, that is diversifiable or specific risk. There may be investors who face high diversification costs. Will these investors bid up the price of specific risk? No, investors with high diversification costs will favour investments, such as pooled vehicles, that are already diversified. Such investors will see as unattractive any insurance shares carrying material specific risk. Insurance shares will instead be sold at higher prices to investors who face lower diversification costs.

Nevertheless, there is a widely held conviction in the insurance community that specific risks should carry some (non-zero) price. This is manifested in pricing practices such as standard deviation loads or proportional hazards transformations. It is also implicit in most approaches to capital allocation, which often look at percentiles, put option prices or other measures of total volatility, without distinguishing the systematic and non-systematic components.

We make the distinction between perfection in capital markets, in contrast to the impact of well documented distorting costs embedded in insurance pricing. An insurer may well enjoy some form of competitive advantage in its core markets, where it has bought its way through entry barriers, building customer relationships, branding, developing specific expertise and managing relations with third parties such as regulators, distributors and analysts. It is less plausible to believe that international capital markets, with far lower barriers to entry, offer any special terms to insurers. This point is commonly misunderstood; for example, we often encounter the misconception that risk loadings in insurance markets necessarily imply a mis-pricing of traded financial securities.

FRICTIONAL COSTS AND RISK LOADING

The fact that deflators do not associate a premium with non-systematic risks has some important consequences. For example, in their ground breaking 1958 paper, Modigliani & Miller demonstrated that the way that a firm was financed, either using debt or equity, made no fundamental difference to the value that a market would place on a firm. Their argument showed that swapping equity capital for bond capital just increased the gearing of the firm and hence the return required by equity holders. They concluded that the capital structure of the firm was irrelevant to the firm's valuation. A similar argument explains that, within the context of perfect capital markets, changes in investment strategy would similarly leave unchanged the market value of an insurance company.

Modigliani and Miller considered a simple model of a company, which ignored a number of items that are important in practice. They are sometimes called *frictional costs*, or *operational risks*

Examples of frictional costs include:

- Future business terms being sensitive to credit risk.
- Project disruption & wastage of unbudgeted flows.
- Optimistic plans survive longer in uncertain world
- Convex tax formulas – not able to use tax losses.
- Back office / processing expense which is convex in transaction flow.
- Capital raising, distribution, restructuring costs.
- Double taxation of income on retained surplus.
- Operational risk of cash misuse.
- Management time opportunity cost.

Frictional costs may in the past have been given little attention because they have been regarded as small, compared for example to claim payments. More dangerously, future frictional costs are also often ignored within the planning process and even within DFA models. The model projections are overly optimistic. Although most actuarial models do not allow explicitly for frictional costs, there may be implicit allowance inside a hurdle rate of shareholder return that seems puzzlingly high or in the use of total risk measures in an efficient frontier construction.

We prefer, instead, to build an explicit model of frictional costs. We allow management a constrained choice within a family of convex functions, each of which relates frictional costs to profit. This enables us not only to measure current frictional costs, but also to understand the impact of possible risk mitigation initiatives, such as asset-liability matching or reinsurance. In so far as they can minimize the frictional costs the management can then influence the market value of the firm.

MODELLING FRICTIONAL COST

Our model for frictional costs is an extended proportional hazards approach. It is based on 'ideal profit' as an independent variable. We define and relate true profit, ideal profit and frictional cost as follows:

true profit = ideal profit – frictional costs

The ideal profit is a measure coming out of a business plan or DFA model, which may contain optimism, either in parameter estimates or in cash flows omitted from the model

We model frictional costs as a function of the ideal profit. This function is determined by a combination of:

- management choices, relating to softer decisions on how they run their business
- market constraints on the minimised frictional costs for various aspects of the business.

We would usually expect frictional cost functions to be a convex function of ideal profit, so that businesses whose profits are more risky also attract higher frictional costs. We would expect them to be minimised for some finite value of profit, and to increase more steeply on the left than on the right. This is because unexpectedly low ideal profits typically generate more frictional costs than unexpectedly high ideal profits.

We would expect frictional cost families to give at least the following flexibilities to managers:

- managers should be able to translate the frictional cost function by a scalar, so that an addition of a constant (risk free) amount to the profit would not affect the frictional costs
- managers should be able to choose between risk tolerant cost functions that are more or less flat but high, compared to risk averse cost functions. A risk averse cost function would have a lower minimum but would increase faster if ideal profits moved away from that minimum.

There are many possible choices of frictional cost function families tnat satisfy these criteria. Our chosen functions are of the form:

$$\theta(x) = \lambda \int_{-\infty}^{x} G(y)^{1-\lambda}\, dy + \int_{x}^{\infty} \left[(1-\lambda) G(y)^{-\lambda} + \lambda G(y)^{1-\lambda} - 1 \right] dy$$

where:

- x is the ideal profit
- θ is the frictional cost
- λ is a risk loading parameter between 0 and 1, and is a determined by the overall level of costs in the market. $\lambda = 0$ corresponds to zero risk loading; $\lambda = 1$ implies that all risks are priced at their maximum value.
- y is a dummy integration variable
- $G(y)$ is a function which increases from 0 to 1 as y moves from $-\infty$ to ∞. The increase is not necessarily strict, nor continuous.

Management Decision Process

The softer management decisions are assumed to affect the choice of the function G. In our model, they can choose this function in the knowledge of the distribution of ideal profit. However, management cannot peek ahead to the actual outcome of ideal profit.

We assume that management will choose the optimal profit to minimise the market value of the frictional cost In other words, given a value of λ between 0 and 1, management are assumed to pick a function G to minimise the expectation:

$$\mathbf{E}[\, D\theta(X) \,]$$

where D is the state price deflator.

We can solve this optimisation problem as follows. Let $F(x)$ be the cumulative probability function of the ideal profit X. Let $D(x)$ be the expected deflator, conditional on the ideal profit taking the value x. Then the value of the frictional cost is given by

$$\int_{-\infty}^{\infty} D(x)\theta(x)dF'(x)$$

On simplification, we finally obtain the following frictional cost:

$$\mathbf{E}[D\theta(X)] = \int_{-\infty}^{\infty}\left\{\lambda G(x)^{1-\lambda}\left[\int_{-\infty}^{\infty} D(y)dF(y)\right] - \left[1-(1-\lambda)G(x)^{-\lambda}\right]\left(\int_{-\infty}^{x} D(y)dF(y)\right)\right\}dx$$

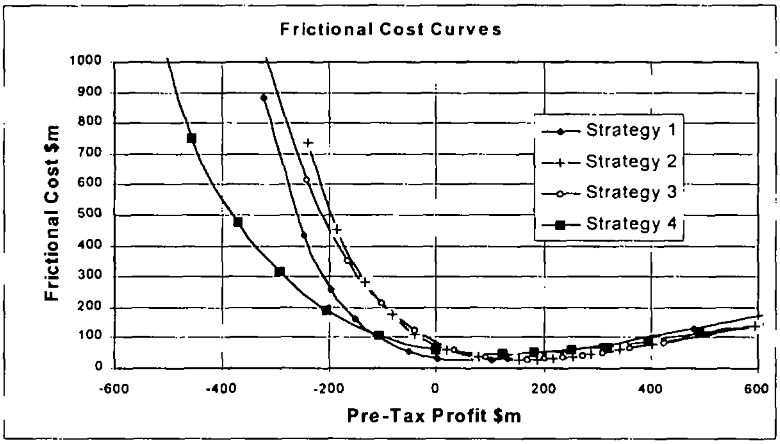
We seek to choose G to minimise this quantity. The optimum is achieved when

$$G(x) = \frac{\int_{-\infty}^{x} D(y)dF(y)}{\int_{-\infty}^{\infty} D(y)dF'(y)}$$

We recognise this as the cumulative distribution function of G under the risk neutral law.

The typical shape of the frictional cost functions selected for use in the analysis of DFAIC is shown in Figure 1.

Figure 1: Typical Frictional Cost Curves from the DFA model



The minimised frictional cost is then

$$\mathrm{E}[D\theta(X)] = \int_{-\infty}^{\infty} D(y)dF(y)\left[\int_{-\infty}^{\infty} xdG(x) - xd\left[G(x)^{1-\lambda}\right]\right]$$

and so

$$\mathrm{E}\left[DX - D\theta(X)\right] = \int_{-\infty}^{\infty} D(y)dF(y)\int_{-\infty}^{\infty} xd\left[G(x)^{1-\lambda}\right]$$

Thus, the mean value of the realistic profit is equal to the mean idealistic profit, but under an adjusted risk neutral probability law. The second adjustment involves raising the cumulative distribution function to a power of 1-λ. This always has the effect of increasing the cumulative distribution function, or, alternatively, of shifting it to the left. This is equivalent to the proportional hazards transform proposed by Wang (1995). Wang's version has some sign changes relative to ours, as he deals with insurance losses where we deal with overall profit. Our analysis has shown how Wang's method deals with the non-systematic component of risk, as represented by frictional costs Our analysis, in using a risk neutral law, generalises Wang's work to cover both systematic and non-systematic risks.

Allocation of Risk Cost by Line

We now move on to the allocation of frictional costs by line of business. We do not seek to allocate the costs on each simulation by line of business. Instead, it is the deflated value we allocate. Thus, we can either allocate total frictional costs by simulation, or by line of business, but not by both at once.

Our approach requires a decomposition of ideal profit into the sum of a number of components, one for each line of business, and one for the capital account. The ideal profit for each line should add up to the ideal profit for the total.

The idea then is to allocate the total frictional cost according to the marginal impact of each line of business.

Let $y(x)$ denote the conditional mean of the line 1 profit y conditional on the total profit being x. Our expression for the allocated frictional cost is given by the integral:

$$\int_{-\infty}^{\infty} D(y)dF(y)\left[\int_{-\infty}^{\infty} y(x)dG(x) - y(x)d\left[G(x)^{1-\lambda}\right]\right]$$

It is clear from this expression that the total of the allocated costs for each line gives the value of frictional costs for the business as a whole, as it should.

4. DFA ANALYSIS OF DFAIC

This section presents the main results of the DFA analysis of DFAIC. These results are based on a set of assumptions made from very limited data and with no access to management. The analysis could be improved with more information and with access to management. Such information may have a significant impact on absolute values, such as an estimate of the market value of DFAIC, but may have less of an impact on the risk costs calculated or their allocation.

RECENT PERFORMANCE AND FUTURE PLAN ASSUMPTIONS

The company experienced an operating ratio of around 105.5% and paid dividend to stockholders in excess of $300m, reducing policyholder surplus in the year by nearly $60m. Accident Year losses for 1999 looked somewhat higher than the more developed positions of the earlier accident years and this may indicate some initial redundancy in the most recent claims provisions

The 1999 accident year net loss and loss expense ratio was over 7% points higher than the revenue year figure. We have assumed that the opening balance sheet claims reserves as well as all future claims reserves are set on a best-estimate basis without any margins. Any surplus in the opening loss reserves will be 'lost' as it will be assumed paid as a loss or loss expense. This is an area that would receive much more attention in practice. In a real DFA analysis, a reserve review would often be a necessary first step of the DFA exercise.

The company has cash balances totalling 18% of assets, which seems a little high for a company with a relatively diversified portfolio and with relatively low exposure to catastrophe losses The bond portfolio also appears to be of longer duration than the insurance liabilities it is supporting Equity investments are almost insignificant, at 12% of overall assets. The impact of increasing the equity investments and moving the bond portfolio nearer to the duration of the liabilities will be considered in the analysis section.

The company buys a considerable amount of reinsurance, to provide protection to relatively low levels of retention at class of business level. A cursory review of gross and net accident year loss ratios did not indicate any significant smoothing or benefit from these reinsurances at company level. The impact of the main excess of loss reinsurance treaties will be investigated to see whether these reinsurances actually reduce the company results variability or simply protect class of business results at a real cost to shareholders.

THE BASE SCENARIO

The select 'Plan Scenario' has parameter selections that result, on average, in an operating ratio that is below that reported in 1999 before allowing for frictional costs. Once allowance is made for these costs, the average operating result is in line with the base figures.

For the purposes of the exercise, it is assumed that the company continues to write the same volume of business at the same fair premium levels, all protected with the same reinsurance arrangements at similar costs. Expense and commission rates remain the same throughout the five-year projection period and asset allocation as initially derived for each 'fund' are rebalanced at the end of every year to their initial percentages.

Finally, any surplus in excess of a premium to surplus ratio of 1.43 (70% solvency ratio) is paid as dividends to shareholders. This dividend strategy is selected to test the ability of the company to maintain its market rating and fund its inflationary premium growth without recourse to shareholders. Clearly, such a policy results in variable flows of dividends that could be zero, in cases where results are poor or surplus adequacy levels are recovering.

Some summary plan statistics of the base scenario are given in Appendix B

ALTERNATIVE SCENARIOS FOR DFA ANALYSIS

In order to test alternative strategies, a number of other sets of assumptions, or scenarios, were selected to demonstrate the use of the DFA model and explore some of the questions listed.

Eight different scenarios are used in the analysis These are as follows.
Scenario 1: Base or assumed Plan with the reinsurance and asset strategies as in 1999
Scenario 2: Base but with no Class Excess of Loss Reinsurance
Scenario 3. Base but with no Class or Catastrophe Reinsurance
Scenario 4: Base but with reinsurance at risk cost (cover at risk or expected loss costs)
Scenario 5. Base but with 100% of surplus in Equities rather than 35%
Scenario 6: Base but with all investments in bonds with mean terms matching the liabilities
Scenario 7: Base, no reinsurance, surplus in equities (Scenario 3+Scenario 5)
Scenario 8: Base, lower capital, no class reinsurance, investments in bonds

Looking at these alternatives, Scenario 4 will be better than Scenario 1, if cheaper reinsurance can be purchased without an increase in credit risk and should provide a benchmark for evaluating the other reinsurance alternatives. Scenarios 5 to 7 are intended to help evaluate the asset or investment strategy. Scenario 7 should prove to be a high risk one. Scenario 8 was developed after some initial evaluation of the results of the earlier scenarios.

EVALUATION OF ALTERNATIVE STRATEGIES

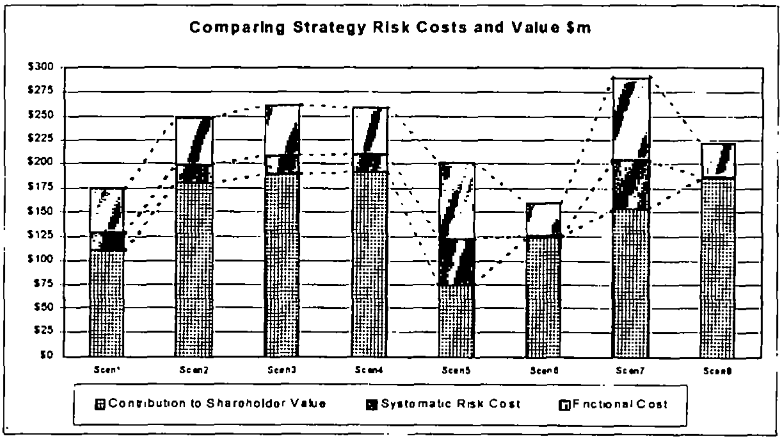
Results from the economic evaluation of the above scenarios, using 2,500 simulations over the five-year projection period, are shown in Table 1. The calculations are based on the pre-tax profit values, adjusted for systematic and non-systematic risk, using a frictional cost index of $\lambda = 0.33$. The impact of such a choice is considered later. The 'P-TP@rfr' row shows the average value of the 'plan period Pre-Tax Profits discounted at the risk free rates. The CSV (Contribution to Shareholder Value) line shows the result of adjusting the earlier discounted values for both systematic and operational risk, using the method outlined in Section 2. As these calculations are based on the pre-tax profit figures as the independent variable, the CSV is a gross of tax value. Post tax values are discussed later when we discuss the value of dividends and of the company. In the meantime these gross figures are sufficient for the first evaluation of alternative strategies and for risk cost allocations. The values shown are the averages over the five year period.

Table 1: Scenario Value and Risk Cost comparisons

Value	Scen1	Scen2	Scen3	Scen4	Scen5	Scen6	Scen7	Scen8
P-TP @rfr	175,123	248,122	261,594	257,945	202,009	159,400	290,307	221,456
Sys Cost	18,593	19,218	19,257	18,687	48,725	1,574	50,617	1,381
Frictional Cost	46,567	48,969	53,027	47,490	79,948	33,532	85,632	34,543
Total Risk$	65,160	68,187	72,284	66,176	128,673	35,106	136,249	35,924
CSV (Gross)	109,963	179,936	189,310	191,769	73,337	124,294	154,057	185,532

The following figure shows the results much more clearly.

Figure 2. Scenario Value and Risk Cost comparisons



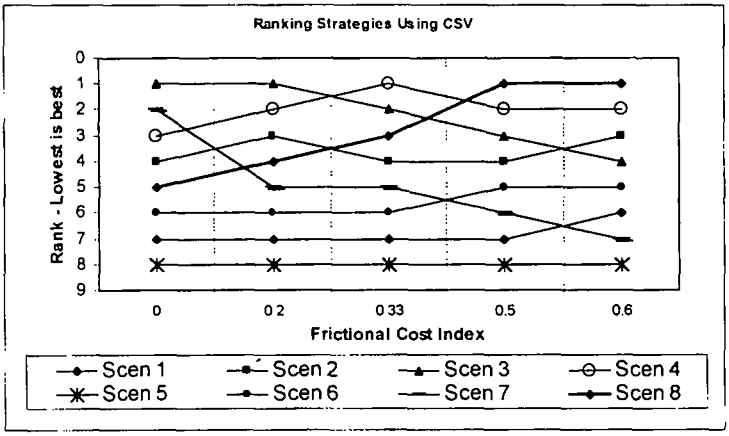
Both Table 1 and Figure 2 show results that are in line with expectations. It is expected, for example, that cheap, good quality, reinsurance (Scenario 4) should be more valuable than expensive reinsurance (Scenario 1). This is confirmed. The results also indicate that, in terms of shareholder value, reinsurance at much above risk cost (Scenario 1) is not of value to shareholders who may be much better off with less reinsurance (Scenarios 2) or no reinsurance at all (Scenario 3).

These observations are, however, based purely on the shareholders' perspective. Later analysis will consider what these strategies may do to the security of the policyholder and also the interest of the other key stakeholder, the manager, who may also lose if the company loses its rating as a result of strategies that are of benefit to well diversified shareholders.

Looking at the next four scenarios, it is clear that investing the surplus in higher risk assets, such as equities, increases the average return but may also significantly increase both systematic and non-systematic risk costs. This strategy (Scenario 5) is shown to be less valuable than any other of the tested strategies. Investing the surplus in bonds reduces average returns but increases value (Scenario 6). Scenario 7 is a high risk one. There in no reinsurance and the surplus is invested in equities. The average return is now maximised, but this strategy generates high risk costs. We will see later that it is also a high risk strategy for policyholders and managers (Table 7). Scenario 8 was developed by reviewing the earlier results and attempts to show a realistic practical strategy that management can adopt to maintain the financial strength, or rating, of the company whilst improving returns to shareholders. This strategy has lower initial capital, no class excess of loss reinsurances or any equity investments.

The calculations assume a frictional cost index of 0.33. Figure 3 and Table 2 show the sensitivity of these conclusions to the choice of index. At a value of 0, we only allow for systematic costs.

Figure 3: Impact of the Frictional Cost Index on Strategy value contribution



The actual dollar value impact of varying this key index is shown more clearly in Table 2.

Table 2: Impact of the Frictional Cost index on Shareholder Value Contributions

CSV $k	Impact of λ on Value Contribution							
λ	Scen1	Scen2	Scen3	Scen4	Scen5	Scen6	Scen7	Scen8
0.00	156,530	228,904	242,337	239,259	153,285	157,826	239,689	220,075
0.20	131,413	202,477	213,993	213,621	109,478	140,100	192,952	201,786
0.33	109,963	179,936	189,310	191,769	73,337	124,294	154,057	185,532
0.50	71,245	139,323	143,624	152,420	11,611	93,729	86,700	154,228
0.60	38,869	105,436	104,446	119,607	-36,382	65,904	33,375	125,841

Table 2 shows that the Scenario 8, which was developed after the initial evaluation of the earlier scenarios, does create more value once we allow for non-systematic risk. Scenario 4 starts better but begins to lose as we increase the frictional costs. Scenario 4 is the one that assumes that reinsurance can be bought at risk cost, which is clearly unrealistic. Scenario 8 has starting surplus that is $100m less than the other scenarios as explained earlier.

RISK COST AND CAPITAL ALLOCATIONS

The interpretation methodology can be extended to the allocation of risk costs to the underlying classes of business, including the capital account.

Table 3: Overall Risk Cost Allocations – Systematic and Non-Systematic Risk

Value	Scen1	Scen2	Scen3	Scen4	Scen5	Scen6	Scen7	Scen8
Capital	38,504	40,341	38,796	39,829	111,311	5,300	115,430	4,636
Home	8,082	7,751	13,641	7,952	4,355	10,411	8,519	10,173
PPA	5,276	5,711	5,097	5,228	3,603	5,430	2,940	6,010
CAT	1,298	1,581	1,418	1,283	966	1,219	960	1,512
WC	2,349	2,652	2,400	2,331	1,834	1,899	1,423	2,192
CMP	5,445	5,577	6,769	5,405	3,732	5,634	4,239	5,791
OC	785	1,304	1,100	770	581	709	680	1,246
S-Tail	3,420	3,270	3,063	3,378	2,292	4,503	2,058	4,365
Total Risk $	65,160	68,187	72,284	66,176	128,673	35,106	136,249	35,924

Table 3 shows allocation of the Frictional cost values shown in Table 1. The impact of the high equity investment of the capital account (surplus) can be easily seen in the results for Scenarios 5, and 7. The impact of removing the catastrophe reinsurance protection can be seen in the Home and CMP risk cost increases for Scenario 3. It is interesting to note that in the case of Scenario 7, the high equity investment increases overall risk charges with a greater proportion now falling on the capital account. In other words, variability from this source is more significant than variability from the liabilities. These costs can be subdivided further as shown in Table 4.

Table 4: Risk Cost Allocations – Frictional Costs

Value	Scen1	Scen2	Scen3	Scen4	Scen5	Scen6	Scen7	Scen8
Capital	21,550	22,630	20,986	22,453	64,225	4,965	65,840	4,342
Home	7,841	7,507	13,445	7,763	4,114	10,197	8,350	9,955
PPA	4,980	5,529	4,916	5,013	3,306	5,241	2,873	5,943
CAT	1,148	1,414	1,252	1,162	815	1,091	818	1,370
WC	2,223	2,521	2,269	2,269	1,707	1,880	1,405	2,173
CMP	4,784	4,897	6,101	4,794	3,071	5,082	3,682	5,222
OC	705	1,283	1,080	742	501	650	684	1,249
S-Tail	3,336	3,186	2,979	3,294	2,207	4,426	1,981	4,288
Total Risk $	46,567	48,969	53,027	47,490	79,948	33,532	85,632	34,543

The risk cost allocations can be used to derive the benchmarks needed for relative performance measurement. For example, they can be used to 'allocate capital'. It is instructive to tabulate the risk costs to premiums to see what they indicate Table 5 shows such an analysis.

Table 5: Total Risk Costs to Premiums

Value	Scen1	Scen2	Scen3	Scen4	Scen5	Scen6	Scen7	Scen8
Capital	1.54%	1.52%	1.45%	1.54%	4.46%	0.21%	4.33%	0.18%
Home	2.34%	2.18%	3.67%	2.20%	1.26%	3.02%	2.29%	2.86%
PPA	0.82%	0.84%	0.75%	0.79%	0.56%	0.84%	0.43%	0.88%
CAT	0.75%	0.84%	0.75%	0.71%	0.56%	0.71%	0.51%	0.80%
WC	1.10%	1.07%	0.97%	1.01%	0.86%	0.89%	0.58%	0.89%
CMP	1.53%	1.47%	1.77%	1.46%	1.05%	1.58%	1.11%	1.53%
OC	1.17%	1.40%	1.18%	0.96%	0.87%	1.06%	0.73%	1.34%
S-Tail	0.49%	0.47%	0.44%	0.48%	0.33%	0.64%	0.29%	0.62%
Total to Prm	2.61%	2.58%	2.71%	2.56%	5.15%	1.41%	5.11%	1.36%

The percentages shown for the capital, or surplus, account are the risk costs associated with the investment of the surplus as a percentage of the overall premium. An alternative approach would be to allocate the capital associated frictional costs to the classes of business. These risk cost allocations can be turned into risk-sensitive profit targets by class of business, which can in turn, be expressed as target loss or operating ratios. All the information necessary to do this, such as claim payment and premium receipt patterns, is available from the DFA calibration but these results would be highly dependent on the accuracy of the expense cost assumptions. As both commissions and expenses were allocated from overall figures, based on no more than inspired guesswork, this has not been done for this analysis

Whether and how overall risk costs should be allocated to classes of business is a matter of choice. For example, looking at Scenario 5, it is clear that the increase in investment risk, whilst increasing overall risk costs actually decreases the class of business allocations. Our class of business risk cost allocations for cost of capital or capital allocation purposes, would include the apportionment of the frictional costs associated with the capital (surplus) account. What is also happening here is that this higher risk strategy actually requires a higher level of capital in order to provide the same level of security to the policyholders. This is considered in the next section.

CAPITAL EVALUATION

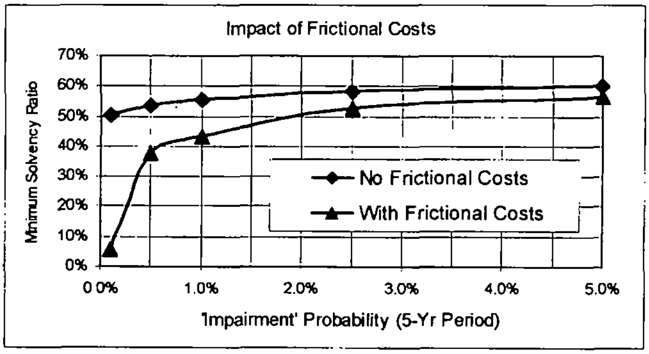
Insurance companies need to maintain a level of surplus which is considered sufficient by regulators and market security analysts and which provides a minimum necessary degree of protection to policyholders. Holding excess capital dilutes the returns to shareholders and may encourage managers to take on projects or business that they may otherwise consider unattractive.

Identifying the 'correct or optimal' amount of capital for an insurance company is a particularly demanding task as so much depends on future utilisation and management competence, as well as market conditions, competition and many other factors that are often outside the control of managers. This optimal amount of capital should also reflect the financial exposures the company faces, both from the type of business it underwrites, its investment strategy and also its degree of geographic and business diversification. In practice, financial impairment, or insolvency in extreme cases, of insurance companies is often associated with mis-management or operational risk.

Traditional capital evaluations using DFA or ALM or Solvency models, have tended to concentrate on financial impairment or probability of ruin using outputs from these models without any attempt to allow for such operational risks. The result is that increasingly remote probability levels have to be used in order to derive capital requirement values that look believable in the context of market experience and practice. The incorporation of frictional costs which include operational risk, changes impairment assessment and provides a basis for capital evaluation using DFA that was previously unavailable.

The process requires that the frictional costs be allocated to individual simulations. The family of cost functions described in Section 2 facilitate these associations using the pre-tax profit values as the 'ideal profit' variable. The results produce impairment probability estimates that appear much more realistic and useable. The following diagram illustrates the differences between the raw DFA output and the frictional cost adjusted results for the Base scenario.

Figure 4: Impairment assessment comparison



The two lines plot the probability that surplus, at some time during the 5-year projection period, as measured relative to annual premiums, drops to levels that may result in the company losing its security rating. In the absence of frictional costs, the surplus rarely drops below a 50% ratio (premium to surplus ratio of 2). The diagram shows the significant impact of frictional costs in these evaluations It shows, for example, that there is a 1% chance that solvency drops below 43% (premium to surplus ratio of 2.3).

This may well result in a downgrade of the company. It is a useful benchmark to adopt as defining the optimal level capital of DFAIC, consistent with its assumed plan strategy, to help investigate the capital implications implicit in the other strategies described above. The following table shows the results of such an impairment evaluation of all these strategies.

Table 6: Impairment impact of alternative strategies

Impairment	Minimum Solvency Ratio During Plan (5-yr) Period							
Probability	Scen1	Scen2	Scen3	Scen4	Scen5	Scen6	Scen7	Scen8
0.25%	28%	28%	21%	31%	18%	30%	16%	23%
0.5%	38%	41%	31%	42%	21%	38%	21%	37%
1.0%	43%	46%	38%	48%	25%	48%	29%	43%
2.5%	53%	54%	51%	56%	34%	58%	38%	52%
5.0%	57%	58%	56%	60%	42%	61%	45%	55%
10.0%	60%	61%	60%	63%	48%	64%	51%	58%
25.0%	65%	66%	65%	67%	56%	67%	59%	60%

The first thing to note is that Scenario 5 and Scenario 7 are the ones with a greater chance of impairment as measured by the minimum surplus to premium ratio at, say, the 1.0% level. For both of these scenarios, the surplus ratio is as low as 25% and 29% respectively, compared to the Plan 70% level that is assumed to be the level required to maintain the company A.M. Best, A rating. In practice, ratings may be impacted by changes of 20% in the 'surplus' amount backing the rating. In the example above, this will occur with a probability of 5% under the assumed Plan strategy (Scenario 1) and a disturbing probability of 25% in the case of Scenario 5, where 100% of the policyholder surplus is invested in equities.

Keeping to the 1% benchmark level, the results show that Scenario 2 (no class reinsurance) requires less capital as more premium and profit, are retained. This is not the case with the catastrophe reinsurance cover, as is shown by the results for Scenario 3. Clearly, this reinsurance increases the amount of capital required under the capital benchmark assumption in order to increase the minimum surplus ratio from the estimated 38% to the required 43% of the current scenario. By far the best option is Scenario 4, which assumes reinsurance protections at risk cost. This is, however, a somewhat artificial Scenario and unlikely to be available to DFAIC. It does help to demonstrate that with class reinsurances, much depends on the assumed price against the benefits implicit in the DFA large loss parameters.

Scenario 8 is the one developed by a preliminary analysis of the other scenarios. Its impairment shape, as indicated by the results in Table 6, is very close to the base scenario, Scenario 1. The select scenario has a lower surplus level, 60% of premium compared to the 70% of premium assumed for all the other scenarios and yet it is no more susceptible to impairment on this basis than Scenario 1.

The DFA model can be used to help identify the level of capital required to meet the impairment objective identified above for any of the other scenarios. The scenario in question is run with varying initial surplus levels and the 1% impairment level percentages are identified. Simple numerical approximation usually produces the exact capital level associated with the scenario. In such an exercise it is important to remember to adjust the dividend strategy to each new level of capital.

This is now demonstrated with Scenario 5 In this scenario the surplus is wholly invested in equities, compared to the current 35%, so the task is to identify the new surplus level that is required to support such a change in investment strategy. We define three new scenarios, identical to Scenario 5 but with initial surplus set at 85%, 100% and 125% of 1999 premiums, remembering also to amend the dividend strategy ratios in each case. The results are shown in the following table.

Table 7: Impairment table for capital evaluation

Impairment Probability	Min Solvency Ratio (5-yr)				
	Base	Scen 5 (70%)	Cap 85%	Cap 100%	Cap 125%
0.1%	6%	7%	15%	21%	26%
0.5%	38%	21%	27%	32%	43%
1.0%	43%	25%	31%	39%	49%
2.5%	53%	34%	43%	51%	63%
5.0%	57%	42%	52%	60%	76%
10.0%	60%	48%	59%	69%	86%
25.0%	65%	56%	68%	80%	99%

The results indicate that the required level of capital needed to support the particular change in investment strategy is somewhere between 100% and 125% of annual premium. The actual answer turns out to be 111% of premium. This is a near 60% increase in surplus, which would be very difficult to justify to shareholders.

The analysis does not stop at the identification of the surplus required to support a new strategy, whilst maintaining the previous or desired level of impairment criteria. Each new capital level will generate a different flow of dividends and these will need to be valued in order to see whether such a change in strategy is of benefit to shareholders. This is relatively easy to do, using deflators.

THE VALUE OF DFAIC

A publicly quoted company has an on-going market valuation in its capitalisation value. The methodology presented in this paper values variable cash flows in a manner that is consistent with the way the market values such flows.

One way to value a company, which is often implicit in multiplier approaches used in practice, is to project the stream of dividends and then risk discount the mean flows to present value. The DFA methodology described in this paper facilitates these calculations by using deflators and frictional risk cost adjustments to 'stochastically discount' the individual projected dividend streams.

DFA models are not intended to project over the longer term, with typical projection periods in practice ranging from three to seven years. These models are capable of valuing both the dividend streams and the retained end surplus at the end of the projection period. It is then possible to use these values, together with some simple assumptions, to estimate market values of the study company.

The following simple example illustrates how this can be done in practice.

Define a variable M as the ratio of market value to surplus, that is:

Market Value = M * Surplus

At the beginning of the projection period we have a value for the surplus. Now use the DFA model to project the dividends over the plan period and also the value of the ending surplus. Both the dividends and the ending surplus are already adjusted for frictional costs, so all that remains is to apply deflators to these values to obtain their market value at the beginning of the projection period.

We can now express the current market value as the value of the dividends and the market value of the ending surplus. This enables us to deduce the implicit multiple M, which we can then multiply by the current surplus to get an estimate of the company market value.

Using the Base assumptions, the DFA model estimates the present (deflated) value over the five-year projection period at $310m. The present (deflated) value of the end surplus is $1,431m. The assumed initial surplus was taken as $1,604m.

The multiplier M is then 1.79 [310/(1604-1431)], which estimates the market value of DFAIC at $2,874m. This estimate is dependent on the dividend stream projected and the particular choice of the frictional cost index.

The DFA model outputs that are used to estimate the likely market value of DFAIC rely on both the input business assumptions, particularly those relating to profitability levels as well as the choice of frictional cost index. DFA derived values that look abnormally high or low may simply indicate a poor calibration of the model business assumptions or a poor choice of the frictional cost index.

We prefer to view the availability of a market value as a very helpful element of the DFA calibration process. The approach described above is simply reversed to ensure that the level of 'ideal profits' being projected and the frictional cost index being used to adjust these 'ideal profits' to more realistic values are consistent with the company market capitalisation

The frictional cost index provides a link between the company plan, as defined by the 'ideal profit' to a market view of value. This helps to illustrate that the appropriate frictional costs index for a company, at a given time, will be highly dependent on the 'quality' of the underlying Plan and the markets' view on the likelihood that plan profits will be delivered In turn, this is influenced by the markets' assessment of the quality of the management team. We have demonstrated that in certain cases it is possible to identify the frictional cost index that provides the link between the company plan and the market's valuation of the company

Often, a company plan will be improved during the planning process until it meets an expected level of performance. This is sometimes achieved by reducing projected future losses or expenses. Unless such improvements are justified by changes in strategy the only real change may simply be the removal of some costs from these plans. Such plan changes are unlikely to be reflected in immediate increases in the market value of the company unless the managers convince the market of their viability.

In our formulation, what has happened is that the 'ideal profits' have been increased and we simply need to increase the frictional cost level to reflect any un-justified increase in these profits

5. CONCLUSION

In this paper we have seen that DFA can be a practical, powerful and flexible strategic management tool.

In particular we have described how DFA can help management with the:

- Evaluation of capital requirements
- Evaluation of capital utilisation and risk allocations
- Evaluation of asset and reinsurance strategies
- Identification of appropriate dividend strategies
- Identification of Shareholder Value Contributions

In order to achieve such functionality, the model has to have.

- A sound economic scenario generator
- Proper economic linkages between the liability and asset developments
- A methodology for turning the huge volume of outputs to summary information

Finally, we are aware that there is still a significant amount of scepticism as to whether DFA models, particularly complex ones, can be truly valuable or practical tools. Our experience has convinced us of the value that a focused DFA analysis can bring – we hope this paper will encourage many more to build and learn from DFA models.

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APPENDIX A: DESCRIPTION OF THE DFA MODEL

BASIC FEATURES

The model is a multi-line, multi-period, multi-currency, multi-strategy, multi-asset stochastic plan generator. It has small, large and peril losses, class excess, stop loss and whole account catastrophe excess of loss reinsurances, flexible investment and dividend strategies.

Economic files are pre-generated using a proprietary economic scenario generator (TSM – see next appendix) and include term structures and deflators. Returns for eight asset classes, including cash, stocks and bonds of various durations are available, split into income and gains.

DFA Scenarios are characterised by an economic file and a strategy file. The strategy file contains the user inputs that describe the company financials and plan characteristics. The model can run a number of scenarios at the same time These scenarios may use different strategy assumptions, for example in the amount of reinsurance to be bought, or may use different economic files to test the sensitivity of results from random economic assumptions. There is complete flexibility in the number of scenarios, simulations, seed numbers and the level of raw simulation data and summary statistical data that is saved. A Results Analyser facilitates the calibration and evaluation processes.

INPUTS

There are four main types of inputs required in order to run the model.

1: Company capital structure, accounting currency, assets, dividend and asset strategies, taxation rate and details of any whole account reinsurances.

2: Class of business details including patterns for receipt and earning of premiums, distributions to generate future losses and pay future and outstanding claims amounts, indices to generate future premiums and loss volumes, large loss frequency and severity distributions and reinsurance details, including the share of any whole account cover costs. Commission and expense information as well as the investment policy for the class policyholder funds is also required.

3: Peril losses (catastrophe events). Distributions for the frequency and severity of each such peril event, the loss amount main currency, the payment pattern and cost allocation to classes of business, for each of these peril events

4: Simulation control inputs, including scenarios to run, number of simulations, seed number (if required), name and format of the outputs database and the level of information to be output.

CAPITAL ACCOUNT, TAXES AND DIVIDENDS

Capital comprises issued, retained earnings and subordinated debt. A facility exists for consolidating an investment by year and simulation.

A single tax rate is input. Tax calculated is assumed paid at the end of the year. The user specifies the rate at which capital gains are to be realised for tax purposes. The model keeps track of unrealised gains and deferred taxes.

A number of alternative dividend strategies are accommodated, including no dividends, fixed dividends, fixed dividends in real terms, dividends calculated as % of post-tax profits and dividends calculated to reduce solvency or surplus margin to specified %.

ASSETS AND CURRENCIES

The model is a multi-currency one. Each class of business has a currency and this may or may not be the company accounting currency. This feature was not used in the DFAINC analysis as the company was assumed to have no exposures or investments in currency except the US$.

Assets classes for each economy include cash, equities, index linked bonds and government bonds of durations 1, 3, 5, 10 and 30 years. Asset mix for both shareholder funds as well as policyholder funds by class of business is specified for the projection period and can be in any of the available currencies and asset types. The model rebalances each 'fund' at each projection point.

PREMIUM AND LOSS CALCULATIONS

Premium income is determined as a product of a volume and pricing adequacy (cycle) with a fair premium and inflation adjustment, including any super-imposed inflation for the class of business·

The amount of variability of the first plan year premiums can be controlled by the user by class·

The theoretical 'fair' premium for a class for each period, is calculated taking account of the conditional expectations of future cash flows and the time value of money, allowing both for the claims payment and premium receipt patterns and inflation and interest term structures from the economic file. Premium receipt and earning patterns allow for multi-year policies

Losses for each class of business include normal or attritional losses, individual large losses and a share of any event or catastrophe losses.

Attritional losses are based on a user selected base year loss distribution for each class. Actual losses for a particular class/year/simulation are calculated from simulated values of the base year distribution, indexed by volume and inflation changes since the base year.

Large individual losses for each class are projected for each year using a loss frequency and loss severity distribution with the frequency adjusted for volume changes and the severity for inflation changes.

Event, peril or catastrophe losses for any number of perils (storms, flood, etc) are modelled at company level and allocated to classes according to an initial percentage and then adjusted for volume changes and inflation. Each peril event has its own payment pattern and a main loss currency.

Loss payments, before claims inflation (stochastic consumer price inflation plus a stochastic class specific super-inflation component), are determined by a payment pattern by class with random variability in the payments determined by an error distribution applied to the disposal rate by development year. Actual loss payments are subject to claims inflation including any class specific superimposed inflation, at the time of payment.

Loss and loss adjustment reserves are set at each evaluation point and for each accident year and class, taking account of pre-inflated amounts at the time and expected class specific inflation at the time, using the CPI term structure projected by the ESG, of expected payment. For each class, claims reserves may be discounted or undiscounted and may or may not contain margins defined by a percentage.

CORRELATIONS, REINSURANCE AND STATISTICAL DISTRIBUTIONS

Correlation between classes is generated implicitly as a result of a number of contributory influences, including premium market price indices, the impact of inflation on claims and the occurrence of catastrophe or peril losses that impact more than one class of business. The overall correlations resulting from these assumptions are then validated for reasonableness during the calibration process.

Reinsurance modelling is available at both class of business and overall company level, with excess of loss and stop loss in the case of classes and catastrophe excess of loss for the overall account. More complex reinsurances are modelled externally, before inflation, and net distributions so derived are used directly in the DFA model to allow for timing and inflation impacts to be evaluated.

Many statistical distributions are available through a distributions dynamic link library (dll) including the usual standard ones, such as normal, lognormal, poisson, pareto, weibull, extreme value as well as a number of user defined options and a new distribution, the Parbull, which is a pareto with a weibull tail and is described by three parameters, a (pareto) scale, a (pareto) shape and the point, or large value, at which the pareto tail becomes a weibull one. This distribution has been found particularly useful in modelling catastrophe event and large claim loss amounts.

THE ECONOMIC SCENARIO GENERATOR (TSM)

The Smith Model (TSM) is a proprietary macro-economic model calibrated to major world economies. It is a comprehensive, coherent, innovative and robust economic scenario generator.

It describes interest rates, inflation, exchange rates and equity returns (split between income and capital gains). Where inflation-linked bonds have been issued, these too are modelled.

The building block for The Smith Model is the numeraire, which is an economic cash flow quantity, which is modelled statistically. Examples of numeraires include currencies, inflation indices and equity dividend indices. Numeraires are treated within The Smith Model in an entirely symmetric manner. No single accounting unit holds a central role; any numeraire can be expressed in terms of any other numeraire.

Term Structures within financial markets consist of traded claims on future numeraires. For example, bonds denominated in various currencies can be considered to be future claims on that currency. Every different redemption date defines a different bond, which is modelled separately. This gives rise to a 'term structure' of interest rates, which describes how bond yields vary by term to redemption. Similarly, inflation linked bonds are considered as future claims on an inflation index. Even equities can be thought of as a special kind of bond whose cash flows are linked to a dividend numeraire - but this bond market is the least developed of all because investors only trade perpetuities

It is an efficient market, arbitrage free model. It generates asset prices by equating the supply of different investments to the demand of a representative investor. The equilibrium construction enables us to model risk and return consistently. The model can output the state price deflator, a weight which when applied to each simulation translates from the 'true' probability measure to the risk neutral version.

This enables market-consistent valuations to be assigned to awkwardly constructed cash flows; for example, insurance benefits or statutory profit. It is based on a Levy process, which in any time interval has both a large number of small jumps and also a small number of large jumps. These jumps apply to all asset classes, including interest rates, currencies and equities. However, the large jumps are more noticeable in some markets than others. It is these large jumps that capture the failure of traditional hedging techniques. It is implemented fully in continuous time

APPENDIX B: DFAIC DFA ASSUMPTIONS AND RESULTS

INPUT ASSUMPTIONS

Table 8: Starting balances, premium and expense ratios

	Loss and Loss Adjustment Reserves as at 12/99							
Acc Yr	All	Home	PPA	CAL	WC	CMP	COther	STail
1990 & Prior	196749	1494	11364	3108	96680	36031	48072	0
1991	34077	298	2349	856	22199	7500	875	0
1992	41579	195	3669	1339	25927	9217	1232	0
1993	49207	1858	4924	2154	28236	10615	1420	0
1994	74124	2042	10584	3977	29623	24854	2273	771
1995	114253	4348	22652	9735	36253	34205	5438	1622
1996	167455	5683	40134	15902	42806	49406	7418	6106
1997	278784	13638	86525	34418	57325	67729	16433	2716
1998	463891	23968	170166	65478	78934	100238	24468	639
1999	910056	85414	335722	98710	137297	166178	29751	56984
Total	2330175	138938	688089	235677	555280	505973	137380	68838

Other Balances 12/99	All	Home	PPA	CAL	WC	CMP	COther	STail
UPR	985422	181628	211134	77721	85323	164745	28658	236213
Agents Balances	445133	82045	95373	35108	38542	74418	12945	106702
RI due	49609	2958	14649	5018	11822	10772	2925	1466
Drafts	186209	11103	54987	18833	44374	40433	10978	5501
Funds for Inv	3007064	246666	844187	292106	634613	625961	161146	202385

Ratios	All	Home	PPA	CAL	WC	CMP	COther	STail
Earn Yr1		50.0%	68 5%	56.8%	67.8%	54.4%	64.5%	64 5%
Prem Receipt Yr1		74 7%	83.9%	78.6%	80.8%	78.0%	79.8%	84.0%
Com RatioNet		15 0%	14.0%	14.0%	9.0%	20.0%	20.0%	18.0%
U/W ExpenseNet		15.6%	12.0%	16 8%	19.2%	19.2%	13.2%	18.0%
Com Ratio Gr		13 9%	13.2%	12.8%	7.8%	18.6%	14.3%	18.0%
U/W ExpenseGr		14.5%	11 3%	15.4%	16 6%	17.8%	9.4%	18.0%

Table 9. Summary statistics from the economic (TSM) simulations.

Averages	Inflation	Cash	Equity	1Yr B	3Yr B	5Yr B	10Yr B	30Yr B
2000	1 6%	5 1%	8.5%	5.1%	5.1%	5 1%	5.2%	5.4%
2001	1.6%	5 1%	8.0%	5.1%	5.1%	5 1%	5.1%	5.2%
2002	1 6%	5.1%	8.4%	5.1%	5.1%	5 1%	5.2%	5.3%
2003	1 6%	5.1%	8.0%	5.1%	5.1%	5 1%	5.1%	5.1%
2004	1 6%	5 1%	8.5%	5.1%	5.1%	5 1%	5.2%	5.3%

Stats Yr2002	Inflation	Cash	Equity	1Yr B	3Yr B	5Yr B	10Yr B	30Yr B
St Dev	0 9%	0.6%	14.1%	0.6%	1.0%	1.4%	2 3%	4.7%
Skewness	5.0%	23.8%	38.8%	29 3%	16.7%	12.7%	12.6%	20.5%
1% Percentile	-0.6%	3.9%	-20 2%	3.9%	2.9%	2.0%	0 0%	-5.1%
5% Percentile	0.1%	4.2%	-12 8%	4.2%	3.6%	2.9%	1 6%	-2.1%
50% Percentile	1 6%	5 1%	7.1%	5.1%	5.1%	5 1%	5 1%	5.1%
95% Percentile	3 2%	6 1%	33.4%	6.1%	6.8%	7 4%	9 0%	13.3%
99% Percentile	3 8%	6 5%	45.4%	6.5%	7.5%	8.4%	10 7%	17.3%

Table 10: Class calibration assumptions

Loss Basis		Home	PPA	CAL	WC	CMP	COther	S-Tail
Base Premium		361086	645127	182675	239438	371117	90529	679254
Target N L/R		80.0%	80.0%	75.0%	72.5%	72.5%	67.5%	65.0%
Target RI Xol Pm		10000	39000	15000	34000	22000	25000	0
Parbull shape		1.6	1.5	1.5	1.5	1.4	1.5	0
Parbull scale		250	250	250	250	250	250	0
Parbull large		5000	15000	15000	15000	5000	15000	0
Parbull Mean		620	700	700	700	734	700	0
Number Large		38	60	23	51	46	38	0
Gross Large		23560	42000	16100	35700	33764	26600	0
Retained Large		18620	23400	8970	19890	23414	14820	0
Cats allocation		0.8	0	0	0	0.2	0	0
Retained Cats		33512	0	0	0	8378	0	0
Cat Premium		16000	0	0	0	4000	0	0
Attritional base		210981	441651	109748	123421	201414	24452	432459
Coeff Variation		7.5%	5.0%	5.0%	7.5%	7.5%	10.0%	5.0%
Log Par Attritional		12.257	12.997	11.605	11.721	12.210	10.099	12.976

Pre-Inflated Loss and Loss Adjustment Cumulative payment patterns								
Development Year		Home	PPA	CAL	WC	CMP	COther	S-Tail
1		70.2%	34.0%	27.3%	24.3%	39.1%	29.5%	87.6%
2		89.3%	66.0%	50.2%	49.5%	56.0%	41.2%	99.8%
3		94.6%	81.1%	71.5%	61.1%	68.1%	63.4%	99.3%
4		97.0%	90.2%	84.4%	68.4%	74.5%	77.1%	98.3%
5		97.9%	93.9%	90.7%	73.6%	83.4%	82.1%	99.5%
6		98.6%	96.8%	95.2%	79.0%	84.1%	83.1%	100.0%
7		98.7%	98.4%	96.7%	79.7%	91.4%	87.7%	100.0%
8		99.8%	98.6%	98.0%	82.2%	93.2%	91.8%	100.0%
9		99.7%	99.0%	98.8%	85.7%	94.7%	94.4%	100.0%
10		99.4%	98.3%	99.4%	86.8%	95.7%	95.5%	100.0%
Ultimate		100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%

Table 11: Base Asset Allocation

Asset Allocation	Cap Acc	Home	PPA	CAL	WC	CMP	COther	STail
Invested Amount $m	1,605	248	844	292	634	625	161	202
Adjusted Cash	0.0%	20.0%	20.0%	20.0%	15.0%	20.0%	20.0%	25.0%
Stocks	35.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
One Year	0.0%	5.0%	5.0%	10.0%	10.0%	5.0%	5.0%	30.0%
5 Year	5.0%	25.0%	25.0%	25.0%	25.0%	40.0%	25.0%	45.0%
10 Year	10.0%	50.0%	40.0%	35.0%	25.0%	15.0%	25.0%	0.0%
20 Year	45.0%	0.0%	10.0%	10.0%	10.0%	10.0%	20.0%	0.0%
30 Year	5.0%	0.0%	0.0%	0.0%	15.0%	10.0%	5.0%	0.0%

SAMPLES OF SUMMARY OUTPUTS

Table. 12: Base Scenario sample statistics

Year 2000	Average	StDev	Skew	1%	25%	50%	75%	99%
uwPremiumWritten	2400220	20904	-0.007	2351219	2385894	2400532	2414449	2450377
plPremiumEarned	2405048	12799	-0.006	2374785	2396187	2405293	2413764	2435062
PlClaimsIncd	1770595	75331	0.831	1619260	1721246	1765984	1813548	1967819
PcLossRatio	73.6%	3.0%	0.948	67.7%	71.7%	73.4%	75.3%	81.5%
plCommissionEarned	380745	1975	-0.007	376117	379393	380774	382088	385485
PlcfExpenses	328541	1670	-0.007	324675	327391	328570	329661	332503
plProfitOperating	-74833	71405	-0.957	-264724	-116222	-70055	-28135	67692
pcOperatingRatio	103.1%	3.0%	0.950	97.2%	101.2%	102.9%	104.8%	111.0%
PlProfitPre	130098	155833	-4.149	-353384	71478	149894	222726	375393
plFrictionalCost	-53707	65573	-17.110	-215091	-59451	-39883	-29959	-26545
plProfitPost	104079	124667	-4.149	-282707	57182	119915	178181	300314
diviDeclPaid	60094	63177	0.891	0	0	43410	104517	231547
bsRetainedProfit	87584	91153	-9.151	-239107	91087	110326	123081	147985
bsShareCapital	1560700	0	0.000	1560700	1560700	1560700	1560700	1560700
pcSolvencyRatio	68.7%	3.8%	-9.023	54.9%	69.1%	70.0%	70.0%	70.0%
pcReturnOnCapital	7.7%	13.4%	-16.480	-24.2%	4.4%	9.1%	13.5%	22.9%

Year 2002	Average	StDev	Skew	1%	25%	50%	75%	99%
uwPremiumWritten	2497660	52980	0.044	2379413	2482131	2497575	2532438	2621087
plPremiumEarned	2478598	47924	0.031	2369439	2446263	2478451	2510168	2592612
plClaimsIncd	1814255	74578	0.263	1651212	1762010	1812505	1861023	2008594
pcLossRatio	73.2%	2.9%	0.341	67.0%	71.3%	73.1%	75.1%	80.5%
plCommissionEarned	393187	7446	0.028	376502	388108	393113	398120	410725
plcfExpenses	338319	6324	0.026	323976	334029	338338	342530	353171
plProfitOperating	-67163	71445	-0.321	-248886	-113289	-65377	-19754	89037
pcOperatingRatio	102.7%	2.9%	0.342	96.4%	100.8%	102.6%	104.6%	110.0%
plProfitPre	157482	143995	-1.518	-289853	87820	173342	249467	433754
plFrictionalCost	-51561	43274	-7.154	-186881	-59385	-38444	-28628	-25449
plProfitPost	125985	115196	-1.518	-231883	70256	138673	199574	347003
diviDeclPaid	92435	84142	0.624	0	0	83161	153356	306995
bsRetainedProfit	153278	115614	-8.057	-293461	140048	175469	202649	267040
bsShareCapital	1560700	0	0.000	1560700	1560700	1560700	1560700	1560700
pcSolvencyRatio	68.6%	4.5%	-9.021	51.1%	69.9%	70.0%	70.0%	70.0%
pcReturnOnCapital	9.1%	8.9%	-2.141	-20.4%	5.2%	10.2%	14.5%	25.0%

Year 2004	Average	StDev	Skew	1%	25%	50%	75%	99%
uwPremiumWritten	2597184	73499	0.012	2425501	2546307	2597353	2646529	2772055
plPremiumEarned	2577289	67944	0.014	2419730	2530246	2577521	2621790	2743855
plClaimsIncd	1886714	88397	0.469	1698149	1827210	1885738	1941250	2102865
pcLossRatio	73.2%	3.0%	0.843	66.8%	71.2%	73.1%	74.9%	80.8%
plCommissionEarned	408557	10562	0.010	383809	401359	408637	415417	434390
plcfExpenses	351181	8971	0.006	330103	345147	351234	356997	373098
plProfitOperating	-69163	76667	-0.795	-265417	-114267	-67683	-18451	97298
pcOperatingRatio	102.7%	3.0%	0.840	96.3%	100.7%	102.6%	104.4%	110.2%
plProfitPre	168058	163430	-1.770	-332893	92188	184091	271421	475625
plFrictionalCost	-56408	51799	-8.982	-207115	-65350	-42888	-31175	-27766
plProfitPost	134446	130744	-1.770	-266314	73751	147273	217137	380500
diviDeclPaid	101272	93503	0.650	0	0	90898	170727	336902
bsRetainedProfit	218162	127586	-6.120	-227882	191991	241702	280795	364876
bsShareCapital	1560700	0	0.000	1560700	1560700	1560700	1560700	1560700
pcSolvencyRatio	68.5%	4.6%	-7.558	50.8%	69.8%	70.0%	70.0%	70.0%
pcReturnOnCapital	9.3%	10.5%	-4.139	-23.0%	5.1%	10.3%	15.2%	26.9%

Table 13: Class of Business Operating Results

Year 2000	Average values over 2,500 simulations							
Item	Total Ins	Home	PPA	CAL	WC	CMP	C Other	S Tail
uwPremiumWritten	2400220	334501	605209	167381	205069	344522	65411	678128
plPremiumEarned	2405048	335836	613519	166280	213455	340354	61998	673606
plClaimsIncd	1770595	269219	502894	124601	144720	246526	41620	441016
pcLossRatio	73.6%	80.2%	82.0%	74.9%	67.8%	72.4%	67.1%	65.5%
plCommissionEarned	380745	50293	86126	23203	19299	68138	12436	121249
plcfExpenses	328541	44142	61985	23565	34640	54950	6870	102388
plProfitOperating	-74833	-27819	-37486	-5089	14796	-29260	1072	8953
pcOperatingRatio	103.1%	108.3%	106.1%	103.1%	93.1%	108.6%	98.3%	98.7%

Year 2000	Standard deviations from 2,500 simulations							
Item	Total Ins	Home	PPA	CAL	WC	CMP	C Other	S Tail
uwPremiumWritten	20904	2821	8809	1379	1720	2844	584	5381
plPremiumEarned	12799	1391	5916	754	1109	1503	330	3470
plClaimsIncd	75331	37038	33073	9787	13956	18586	7123	30887
pcLossRatio	3.0%	11.0%	5.0%	5.8%	6.4%	5.4%	11.4%	4.6%
plCommissionEarned	1975	199	831	105	100	297	66	625
plcfExpenses	1670	174	598	107	180	240	37	528
plProfitOperating	71405	36938	31083	9621	13649	18286	7085	30675
pcOperatingRatio	3.0%	11.0%	5.0%	5.8%	6.4%	5.4%	11.4%	4.6%

Year 2000	1st percentile (1%) from 2,500 simulations							
Item	Total Ins	Home	PPA	CAL	WC	CMP	C Other	S Tail
uwPremiumWritten	2351219	327993	584652	164199	201101	337961	64063	665714
plPremiumEarned	2374785	332627	599714	164540	210896	336886	61238	665599
plClaimsIncd	1619260	210167	427766	103748	115792	206693	26134	373603
pcLossRatio	67.7%	62.7%	70.8%	62.9%	54.5%	60.9%	42.1%	55.4%
plCommissionEarned	376117	49835	84188	22960	19068	67452	12284	119808
plcfExpenses	324675	43740	60590	23319	34224	54397	6786	101171
plProfitOperating	-264724	-137489	-113528	-30801	-17686	-74689	-16466	-63513
pcOperatingRatio	97.2%	90.8%	95.0%	91.0%	79.8%	97.1%	73.3%	88.6%

Year 2000	99th percentile (99%) from 2,500 simulations							
Item	Total Ins	Home	PPA	CAL	WC	CMP	C Other	S Tail
uwPremiumWritten	2450377	341192	626250	170652	209149	351267	66797	690890
plPremiumEarned	2435062	339134	627650	168068	216086	343920	62781	681838
plClaimsIncd	1967819	379983	584073	150776	178570	292720	59164	514312
pcLossRatio	81.5%	112.7%	94.1%	90.3%	82.9%	85.8%	95.4%	76.2%
plCommissionEarned	385485	50765	88110	23452	19537	68844	12593	122731
plcfExpenses	332503	44556	63412	23819	35067	55519	6957	103639
plProfitOperating	67692	30794	30754	14972	42865	9826	16314	76243
pcOperatingRatio	111.0%	140.9%	118.3%	118.4%	108.2%	121.9%	126.6%	109.4%

Table 14: Plan period results – Base Scenario (Scen 1)

Base Scenario	Revenue Year				
Average values	2000	2001	2002	2003	2004
UwPremiumWritten	2400220	2449568	2497660	2546847	2597184
PlPremiumEarned	2405048	2430564	2478598	2527378	2577289
PlClaimsIncd	1770595	1780205	1814255	1850992	1886714
PcLossRatio	73.6%	73.2%	73.2%	73.2%	73.2%
PlCommissionEarned	380745	385699	393187	400787	408557
PlcfExpenses	328541	332046	338319	344682	351181
PlProfitOperating	-74833	-67386	-67163	-69083	-69163
PcOperatingRatio	103.1%	102.8%	102 7%	102 7%	102 7%
PlProfitPre	130098	145088	157482	156764	168058
PlFrictionalCost	-53707	-52119	-51561	-53257	-56408
PlProfitPost	104079	116071	125985	125411	134446
DiviDeclPaid	60094	83927	92435	93702	101272
BsRetainedProfit	87584	119728	153278	184988	218162
BsShareCapital	1560700	1560700	1560700	1560700	1560700
PcSolvencyRatio	68.7%	68.6%	68.6%	68.5%	68.5%
PcReturnOnCapital	7.7%	8.3%	9.1%	8.9%	9.3%

Base Scenario	Revenue Year				
Standard Deviations	2000	2001	2002	2003	2004
UwPremiumWritten	20904	44231	52980	62734	73499
PlPremiumEarned	12799	34868	47924	57474	67944
PlClaimsIncd	75331	74632	74578	81619	88397
PcLossRatio	3.0%	3.0%	2.9%	2.9%	3.0%
PlCommissionEarned	1975	5404	7446	8930	10562
PlcfExpenses	1670	4583	6324	7582	8971
PlProfitOperating	71405	72568	71445	73728	76667
PcOperatingRatio	3.0%	3 0%	2.9%	2.9%	3.0%
PlProfitPre	155833	152335	143995	153316	163430
PlFrictionalCost	65573	64686	43274	52453	51799
PlProfitPost	124667	121868	115196	122653	130744
DiviDeclPaid	63177	78372	84142	86758	93503
BsRetainedProfit	91153	109268	115614	121915	127586
BsShareCapital	0	0	0	0	0
PcSolvencyRatio	3.8%	4.4%	4.5%	4.6%	4.6%
PcReturnOnCapital	13.4%	18.9%	8.9%	10.1%	10.5%

Table 15: Scenario Results comparison – Year 2000

Year 2000 Averages	Scen 1	Scen 2	Scen 3	Scen 4	Scen 5	Scen 6	Scen 7	Scen 8
uwPremiumWritten	2,400,220	2.545,070	2,565,070	2,485,703	2,400,220	2,400,220	2.565,070	2,545,070
plClaimsIncd	1,770,595	1,845,638	1,852,388	1,770,595	1,770,595	1,770.595	1,852,388	1,845,638
pcLossRatio	73.6%	72.4%	72.1%	71.1%	73.6%	73.6%	72.1%	72.4%
plProfitOperating	-74,833	-5,026	8.224	10,650	-74,833	-74.833	8,224	-5.026
pcOperatingRatio	103.1%	100.2%	99.7%	99.6%	103.1%	103.1%	99.7%	100.2%
plProfitPre	130,098	201,921	210,401	217,710	122,672	126,853	206,833	193,731
plFrictionalCost	-53,707	-54,913	-60,108	-53.765	-94,100	-38,162	-96,078	-39.079
plProfitPost	104,079	161,537	168,320	174,168	98,138	101,482	165,466	154,985
dviDeclPaid	60,094	35,024	33,734	67,251	88,654	47,063	63,441	139,473
bsRetainedProfit	87,584	170,113	178,186	150,517	53,084	98,019	145,625	59,112
pcSolvencyRatio	68.7%	68.0%	67.8%	68.8%	67.2%	69.1%	66.5%	59.7%
pcReturnOnCapital	7.7%	11.7%	12.0%	12.8%	6.9%	7.0%	11.7%	11.8%

Year 2000 St Dev	Scen 1	Scen 2	Scen 3	Scen 4	Scen 5	Scen 6	Scen 7	Scen 8
uwPremiumWritten	20,904	21,688	21,688	21,296	20,904	20,904	21,688	21,688
PlClaimsIncd	75,331	79,434	90,228	75,331	75,331	75,331	90.228	79,434
PcLossRatio	3.0%	3.0%	3.4%	2.9%	3.0%	3.0%	3.4%	3.0%
plProfitOperating	71,405	75,334	86.602	71,258	71,405	71,405	86,602	75,334
pcOperatingRatio	3.0%	3.0%	3.4%	2.9%	3.0%	3.0%	3.4%	3.0%
PlProfitPre	155,833	158,839	180.960	155,963	255,462	130,360	261.730	132,329
plFrictionalCost	65,573	66,005	82,920	65,571	75,119	75,765	79,291	76,048
DviDeclPaid	63.177	49,504	48,885	66,360	110,013	44,526	93,588	65,998
bsRetainedProfit	91.153	102,935	123,222	89.038	135,787	86,692	155,692	74,764
pcSolvencyRatio	3.8%	4.1%	4.9%	3.6%	5.7%	3.6%	6.1%	2.9%
pcReturnOnCapital	13.4%	12.7%	18.4%	12.7%	17.8%	37.9%	17.8%	50.4%

Year 2000 Skew	Scen 1	Scen 2	Scen 3	Scen 4	Scen 5	Scen 6	Scen 7	Scen 8
PcLossRatio	0.948	0.825	1.495	0.948	0.948	0.948	1.495	0.825
plProfitOperating	-0.957	-0.839	-1.511	-0.962	-0.957	-0.957	-1.511	-0.839
PlProfitPre	-4.149	-3.984	-5.094	-4.138	-1.269	-11.656	-1.464	-11.106
plFrictionalCost	-17.110	-16.274	-15.826	-17.077	-5.979	-27.882	-6.291	-26.926
DviDeclPaid	0.891	1.507	1.571	0.778	1.307	0.716	1.721	-0.385
bsRetainedProfit	-9.151	-6.986	-7.778	-9.599	-3.819	-18.776	-3.408	-25.060
pcSolvencyRatio	-9.023	-6.791	-7.598	-9.510	-3.811	-18.456	-3.383	-25.732
pcReturnOnCapital	-16.480	-14.240	-22.009	-15.472	-2.871	-46.792	-3.294	-47.696

Year 2000 99th-tile	Scen 1	Scen 2	Scen 3	Scen 4	Scen 5	Scen 6	Scen 7	Scen 8
PcLossRatio	81.5%	80.4%	82.3%	78.7%	81.5%	81.5%	82.3%	80.4%
pcOperatingRatio	111.0%	108.2%	109.9%	107.2%	111.0%	111.0%	109.9%	108.2%
PlProfitPre	-353,384	-308,069	-360,059	-266,230	-674,462	-192,890	-626,045	-143,058
plFrictionalCost	-215,091	-230,304	-264,078	-215,268	-355,141	-142.564	-357,656	-153,000
PlProfitPost	-282,707	-246,456	-288,047	-212,984	-539,570	-154,312	-500,836	-114,446
pcSolvencyRatio	54.9%	53.2%	51.2%	55.8%	43.5%	59.7%	43.0%	53.9%
pcReturnOnCapital	-24.2%	-20.8%	-24.7%	-17.8%	-50.5%	-12.6%	-46.2%	-9.9%